Euler's equation for rigid bodies

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Let 1 be an inertial frame body (a rigid body) and 2 a rigid body in motion respect to an observer located at 1. Let Q be an arbitrary point (fixed or in motion) and C the center of mass of 2. Then,

$$\mathbf{M}_Q = \mathbb{I}^Q \boldsymbol{\alpha}_{21} + \boldsymbol{\omega}_{21} \times (\mathbb{I}^Q \boldsymbol{\omega}_{21}) + m\overline{\mathbf{QC}} \times \mathbf{a}_1^{Q2}, \tag{1}$$

where m is the mass of the rigid body, $\overline{\mathbf{QC}}$ the position vector of C respect to Q, $\mathbf{M}_Q$ is the moment of forces system respect to Q, $\mathbb{I}^Q$ the tensor of inertia respect to orthogonal axes embedded in 2 and origin at $Q2$ [1], and $\mathbf{a}_1^{Q2}$, $\boldsymbol{\omega}_{21}$, $\boldsymbol{\alpha}_{21}$, are the acceleration of $Q2$, the angular velocity and acceleration vectors respectively, all of them measured by an observer located at 1.
This equation was got by Euler by using a fixed system of principal axes with origin at $C2$. In that case we have $Q = C$, and therefore

$$\mathbf{M}_C = \mathbb{I}^C \boldsymbol{\alpha}_{21} + \boldsymbol{\omega}_{21} \times (\mathbb{I}^C \boldsymbol{\omega}_{21}). \tag{2}$$

Euler used three independent scalar equations to represent (2). It is well known that the number of degrees of freedom associate to a rigid body in free motion in $\mathbb{R}^3$ are six, just equal the number of independent scalar equations necessary to solve such a motion. (Newton's law contributing with three)
Its is clear if 2 is at rest or in uniform and rectilinear translation, then $\mathbf{M}_Q = \mathbf{0}$, one of the necessary and sufficient conditions for the equilibrium of the system of forces applied to a rigid body. (The other one is the force resultant $\mathbf{F} = \mathbf{0}$)

[1] That is possible because the kinematical concept of frame extension.