Klein-Gordon equation

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The Klein-Gordon equation is an equation of mathematical physics that describes spin-0 particles. It is given by:

$$\Box\psi = \left(\frac{mc}{\hbar}\right)^2 \psi$$

Here the $\Box$ symbol refers to the wave operator, or D'Alembertian, ($\Box = \nabla^2 - \frac{1}{c^2}\partial_t^2$) and ψ is the wave function of a particle. It is a Lorentz invariant expression.

0.1 Derivation

Like the Dirac equation, the Klein-Gordon equation is derived from the relativistic expression for total energy:

$$E^2 = m^2c^4 + p^2c^2$$

Instead of taking the square root (as Dirac did), we keep the equation in squared form and replace the momentum and energy with their operator equivalents, $E = i\hbar\partial_t$, $p = -i\hbar\nabla$. This gives (in disembodied operator form)

$$-\hbar^2\frac{\partial^2}{\partial t^2} = m^2c^4 - \hbar^2c^2\nabla^2$$

Rearranging:

$$\hbar^2\left(c^2\nabla^2 - \frac{\partial^2}{\partial t^2}\right) = m^2c^4$$

Dividing both sides by $\hbar^2c^2$:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) = \frac{m^2c^2}{\hbar^2}$$

Identifying the expression in brackets as the D'Alembertian and right-multiplying the whole expression by ψ , we obtain the Klein-Gordon equation:

$$\Box\psi = \left(\frac{mc}{\hbar}\right)^2 \psi$$