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symmetric monoidal category

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A monoidal category $\mathcal{C}$ with tensor product $\otimes$ is said to be *symmetric* if for every pair A, B of objects in $\mathcal{C}$, there is an isomorphism

$$s_{AB} : A \otimes B \cong B \otimes A$$

that is natural in both A and B such that the following diagrams are commutative

1. (*unit coherence for s*):

$$\begin{tikzcd} A \otimes I \arrow[rr, "s_{AI}"] \arrow[dr, "r_A"'] & & I \otimes A \arrow[dl, "l_A"] \\ & A & \end{tikzcd}$$

2. (*associativity coherence for s*):

$$\begin{tikzcd} (A \otimes B) \otimes C \arrow[r, "s_{AB} \otimes 1_C"] \arrow[d, "a_{ABC}"'] & (B \otimes A) \otimes C \arrow[d, "a_{BAC}"] \\ A \otimes (B \otimes C) \arrow[d, "s_{A, B \otimes C}"'] & B \otimes (A \otimes C) \arrow[d, "1_B \otimes s_{AC}"] \\ (B \otimes C) \otimes A \arrow[r, "a_{BCA}"'] & B \otimes (C \otimes A) \end{tikzcd}$$

3. (inverse law):

$$\begin{tikzcd} & B \otimes A \arrow[dr, "s_{BA}"] & \\ A \otimes B \arrow[ur, "s_{AB}"] \arrow[rr, equal, "1_{A \otimes B}"'] & & A \otimes B \end{tikzcd}$$

In the diagrams above, a, l, r are the associativity isomorphism, the left unit isomorphism, and the right unit isomorphism respectively.

Some examples and non-examples of symmetric monoidal categories:

- The category of sets. The tensor product is the set theoretic cartesian product, and any singleton can be fixed as the unit object.
- The category of groups. Like before, the tensor product is just the cartesian product of groups, and the trivial group is the unit object.
- More generally, a category with finite products is symmetric monoidal. The tensor product is the direct product of objects, and any terminal object (empty product) is the unit object.
- The category of bimodules over a ring R is monoidal. However, this category is only symmetric monoidal if R is commutative.

Remark. A symmetric monoidal category is a braided monoidal category such that the inverse law: $s_{BA} \circ s_{AB} = 1_{A \otimes B}$ holds.