R-algebroid

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Defines	R-module
Defines	convolution product
Defines	R-algebroid

Definition 0.1. If G is a groupoid (for example, regarded as a category with all morphisms invertible) then we can construct an R-algebroid, $R\mathsf{G}$ as follows. Let us consider first a module over a ring R, also called a R-*module*, that is, a `http://planetmath.org/Module`module M_R that takes its coefficients in a ring R. Then, the object set of $R\mathsf{G}$ is the same as that of G and $R\mathsf{G}(b,c)$ is the free R-module on the set $\mathsf{G}(b,c)$, with composition given by the usual bilinear rule, extending the composition of G.

Definition 0.2. Alternatively, one can define $\bar{R}\mathsf{G}(b,c)$ to be the set of functions $\mathsf{G}(b,c) \longrightarrow R$ with finite support, and then one defines the *convolution product* as follows:

$$(f * g)(z) = \sum \{(fx)(gy) \mid z = x \circ y\} \ . \tag{0.1}$$

Remark 0.1. As it is very well known, only the second construction is natural for the topological case, when one needs to replace the general concept of 'function' by the topological-analytical concept of 'continuous function with compact support' (or alternatively, with 'locally compact support') for all quantum field theory (QFT) extended symmetry sectors; in this case, one has that $R \cong \mathbb{C}$. The point made here is that to carry out the usual construction and end up with only an algebra rather than an algebroid, is a procedure analogous to replacing a groupoid G by a semigroup $G' = G \cup \{0\}$ in which the compositions not defined in G are defined to be 0 in G'. We argue that this construction removes the main advantage of groupoids, namely the presence of the *spatial component* given by the set of objects of the groupoid.

More generally, a `http://planetmath.org/RCategory`R-category is similarly defined as an extension to this R-algebroid concept.

References

[1] R. Brown and G. H. Mosa: Double algebroids and crossed modules of algebroids, University of Wales–Bangor, Maths Preprint, 1986.

[2] G. H. Mosa: *Higher dimensional algebroids and Crossed complexes*, PhD thesis, University of Wales, Bangor, (1986). (supervised by R. Brown).