Hamiltonian operator

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Defines	QM
Defines	energy conservation in finite quantum systems
Defines	Heisenberg formulation of QM

Definition 0.1 The Hamiltonian operator **H** introduced in quantum mechanics (QM) by Schroedinger (and thus sometimes also called the *Schroedinger operator*) on the Hilbert space $L^2(\mathbb{R}^n)$ is given by the action:

$$\psi \mapsto [-\nabla^2 + V(x)]\psi, \quad \psi \in L^2(\mathbb{R}^n),$$

The operator defined above $[-\nabla^2 + V(x)]$, for a potential function $V(x)$ specified as the real-valued function $V\colon \mathbb{R}^n \to \mathbb{R}$ is called the *Hamiltonian operator*, $\mathbb{H}$, and only very rarely the *Schrödinger operator*.

0.1 Schroedinger formulation of QM

The energy conservation (quantum) law written with the operator $\mathbb{H}$ as the Schrödinger equation is fundamental in quantum mechanics and is perhaps the most utilized, mathematical computation device in quantum mechanics of systems with a finite number of degrees of freedom. There is also, however, the alternative approach in the Heisenberg picture, or formulation, in which the observable and other operators are time-dependent whereas the state vectors ψ are time-independent, which reverses the time dependences betwen operators and state vectors from the more popular Schrödinger formulation. Other formulations of quantum theories occur in quantum field theories (QFT), such as QED (quantum electrodynamics) and QCD (quantum chromodynamics).

0.2 Heisenberg formulation of QM

Although the two formulations, or pictures, are unitarily (or mathematically) equivalent, however, sometimes the claim is made that the Heisenberg picture is "more natural and fundamental than the Schrödinger" formulation because the Lorentz invariance from General Relativity is also encountered in the Heisenberg picture, and also because there is a 'correspondence' between the commutator of an observable operator with the Hamiltonian operator, and the Poisson bracket formulation of classical mechanics. If the state vector ψ, or $|\psi\rangle$ does not change with time as in the Heisenberg picture, then the 'equation of motion' of a (quantum) observable operator is :

$$\frac{d}{dt} A_{quantum} = (i\hbar)^{-1}[A, H] + \left(\frac{\partial A}{\partial t}\right)_{classical}$$