Feynman path integral

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A generalisation of multi-dimensional integral, written

$$\int \mathcal{D}\phi \exp\left(\mathcal{F}[\phi]\right)$$

where ϕ ranges over some restricted set of functions from a measure space X to some space with reasonably nice algebraic structure. The simplest example is the case where

$$\phi \in L^2[X, \mathbb{R}]$$

and

$$F[\phi] = -\pi \int_X \phi^2(x) d\mu(x)$$

in which case it can be argued that the result is 1. The argument is by analogy to the Gaussian integral $\int_{\mathbb{R}^n} dx_1 \cdots dx_n e^{-\pi \sum x_j^2} \equiv 1$. Alas, one can absorb the π into the measure on X. Alternatively, following Pierre Cartier and others, one can use this analogy to *define* a measure on L^2 and proceed axiomatically.

One can bravely trudge onward and hope to come up with something, say à la Riemann integral, by partitioning X, picking some representative of each partition, approximating the functional F based on these and calculating a multi-dimensional integral as usual over the sample values of ϕ. This leads to some integral

$$\int \cdots d\phi(x_1) \cdots d\phi(x_n) e^{f(\phi(x_1),\ldots,\phi(x_n))}.$$

One hopes that taking successively finer partitions of X will give a sequence of integrals which converge on some nice limit. I believe Pierre Cartier has shown that this doesn't usually happen, except for the trivial kind of example given above.

The Feynman path integral was constructed as part of a re-formulation of by Richard Feynman, based on the sum-over-histories postulate of quantum mechanics, and can be thought of as an adaptation of Green's function methods for solving initial/boundary value problems. No appropriate measure has been found for this integral and attempts at pseudomeasures have given mixed results.

Remark: Note however that in solving quantum field theory problems one attacks the problem in the Feynman approach by 'dividing' it *via* Feynman diagrams that are directly related to specific quantum interactions;

adding the contributions from such Feynman diagrams leads to high precision approximations to the final physical solution which is finite and physically meaningful, or observable.

References

[1] Hui-Hsiung Kuo, *Introduction to Stochastic Integration.* New York: Springer (2006): 250 - 253

[2] J. B. Keller & D. W. McLaughlin, "The Feynman Integral" *Amer. Math. Monthly* **82** 5 (1975): 451 - 465