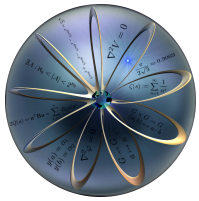

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rigged Hilbert space

Canonical name	RiggedHilbertSpace
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Entry type	Definition
Classification	msc 81Q20
Synonym	Gelfand triple
Defines	dual Hilbert space
Defines	adjoint map
Defines	eigen spectrum

In extensions of Quantum Mechanics [**?**, **?**], the concept of rigged Hilbert spaces allows one "to put together" the discrete spectrum of eigenvalues corresponding to the bound states (eigenvectors) with the continuous spectrum (as , for example, in the case of the ionization of an atom or the photoelectric effect).

Definition 0.1. A *rigged Hilbert space* is a pair $(\mathcal{H}, \phi)$ with $\mathcal{H}$ a Hilbert space and ϕ is a dense subspace with a topological vector space structure for which the inclusion map i is continuous. Between $\mathcal{H}$ and its dual space $\mathcal{H}^*$ there is defined the *adjoint map* $i^* : \mathcal{H}^* \to \phi^*$ of the continuous inclusion map i. The duality pairing between ϕ and ϕ^* also needs to be compatible with the inner product on $\mathcal{H}$:

$$\langle u, v \rangle_{\phi \times \phi^*} = (u, v)_{\mathcal{H}}$$

whenever $u \in \phi \subset \mathcal{H}$ and $v \in \mathcal{H} = \mathcal{H}^* \subset \phi^*$.

References

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[2] J-P. Antoine, "Quantum Mechanics Beyond Hilbert Space" (1996), appearing in *Irreversibility and Causality, Semigroups and Rigged Hilbert Spaces*, Arno Bohm, Heinz-Dietrich Doebner, Piotr Kielanowski, eds., Springer-Verlag, $ISBN 3 - 540 - 64305 - 2$.