closed monoidal category

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Defines	left closed
Defines	right closed
Defines	biclosed
Defines	symmetric monoidal closed

Let $\mathcal{C}$ be a monoidal category, with tensor product $\otimes$. Then we say that

- $\mathcal{C}$ is *closed*, or *left closed*, if the functor $A \otimes -$ on $\mathcal{C}$ has a right adjoint $[A, -]_l$
- $\mathcal{C}$ is *right closed* if the functor $- \otimes B$ on $\mathcal{C}$ has a right adjoint $[B, -]_r$
- $\mathcal{C}$ is *biclosed* if it is both left closed and right closed.

A biclosed symmetric monoidal category is also known as a *symmetric monoidal closed category*. In a symmetric monoidal closed category, $A \otimes B \cong B \otimes A$, so $[A, B]_l \cong [A, B]_r$. In this case, we denote the right adjoint by $[A, B]$.

Some examples:

- Any cartesian closed category is symmetric monoidal closed.
- In particular, as a category with finite products is symmetric monoidal, it is biclosed iff it is cartesian closed.
- An example of a biclosed monoidal category that is not symmetric monoidal is the category of bimodules over a non-commutative ring. The right adjoint of $A \times -$ is $[A, -]_l$, where $[A, B]_l$ is the collection of all left R-linear bimodule homomorphisms from A to B, while the right adjoint of $- \times A$ is $[A, -]_r$, where $[A, B]_r$ is the collection of all right R-linear bimodule homomorphisms from A to B. Unless R is commutative, $[A, B]_l \neq [A, B]_r$ in general.

more to come...