ETAS interpretation

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0.1 Introduction

ETAS is the acronym for the *"Elementary Theory of Abstract Supercategories"* as defined by the *axioms of metacategories and supercategories.*

The following are simple examples of supercategories that are essentially interpretations of the eight *ETAC* axioms reported by W. F. Lawvere (1968), with one or several *ETAS* axioms added as indicated in the examples listed. A family, or class, of a specific level (or 'order') $(n + 1)$ of a supercategory $\mathbb{S}_{n+1}$ (with n being an integer) is defined by the specific ETAS axioms added to the eight ETAC axioms; thus, for $n = 0$, there are no additional ETAS axioms and the supercategory $\mathbb{S}_1$ is the limiting, lower type, currently defined as a category with only one composition law and any standard interpretation of the eight ETAC axioms. Thus, the first level of 'proper' supercategory $\mathbb{S}_2$ is defined as an interpretation of ETAS axioms **S1** and **S2**; for $n = 3$, the supercategory $\mathbb{S}_4$ is defined as an interpretation of the eight ETAC axioms plus the additional three ETAS axioms: **S2**, **S3** and **S4**. Any (proper) recursive formula or 'function' can be utilized to generate supercategories at levels n higher than $\mathbb{S}_4$ by adding composition or consistency laws to the ETAS axioms **S1** to **S4**, thus allowing a digital computer algorithm to generate any finite level supercategory $\mathbb{S}_n$ syntax, to which one needs then to add semantic interpretations (which are complementary to the computer generated syntax).

0.2 Simple examples of ETAS interpretation in supercategories

1. *Functor categories* subject only to the eight *ETAC* axioms;

2. *Functor supercategories*, $\mathcal{F}_{\mathsf{S}} : \mathcal{A} \to \mathcal{B}$, with both $\mathcal{A}$ and $\mathcal{B}$ being 'large' categories (i.e., $\mathcal{A}$ does not need to be small as in the case of *functor categories*);

3. A *topological groupoid category* is an example of a particular supercategory with all invertible morphisms endowed with both a topological and an agebraic structure, still subject to all ETAC axioms;

4. *Supergroupoids* (also definable as crossed complexes of groupoids), and *supergroups* –also definable as crossed modules of groups– seem to be

of great interest to mathematicians currently involved in 'categorified' mathematical physics or physical mathematics.)

5. A *double groupoid category* is a 'simple' example of a higher dimensional supercategory which is useful in higher dimensional homotopy theory, especially in non-Abelian algebraic topology; this concept is subject to all eight ETAC axioms, plus additional axioms related to the definition of the double groupoid (generally non-Abelian) structures;

6. An example of 'standard' supercategories was recently introduced in mathematical (or more specifically 'categorified') physics, on the web's `http://golem.ph.utexas.edu/category/2007/07/supercategories.html`n-Category café's web site under *"Supercategories"*. This is a rather 'simple' example of supercategories, albeit in a much more restricted sense as it still involves only the standard categorical homo-morphisms, homo-functors, and so on; it begins with a somewhat standard definiton of super-categories, or 'super categories' from category theory, but then it becomes more interesting as it is being tailored to supersymmetry and extensions of 'Lie' superalgebras, or superalgebroids, which are sometimes called graded 'Lie' algebras that are thought to be relevant to quantum gravity ([**?**] and references cited therein). The following is an almost exact quote from the above n-Category cafe' s website posted mainly by Dr. Urs Schreiber: A *supercategory* is a *diagram* of the form:

   $$\diamond \diamond Id_C \diamond \mathbf{C} \diamond \diamond s$$

   in **Cat**–the category of categories and (homo-) functors between categories– such that:

   $$\diamond \diamond Id \diamond \diamond Id_C \diamond \mathbf{C} \diamond \mathbf{C} \diamond \diamond s \diamond \diamond s = \diamond \diamond Id_C \diamond Id_C \diamond \diamond Id,$$

   (where the 'diamond' symbol should be replaced by the symbol 'square', as in the original Dr. Urs Schreiber's postings.)

   This specific instance is that of a supercategory which has only **one object**– the above quoted superdiagram of diamonds, an arbitrary abstract category **C** (subject to all ETAC axioms), and the standard category identity (homo-) functor; it can be further specialized to the previously introduced concepts of *supergroupoids* (also definable as crossed complexes of groupoids), and *supergroups* (also definable as crossed modules of groups), which seem to be of great interest to mathematicians involved in 'Categorified' mathematical physics or physical mathematics.) This was then continued with the following interesting example. "What, in this sense, is a *braided monoidal supercategory ?*". Dr. Urs Schreiber, suggested the following answer: "like an ordinary braided monoidal catgeory is a 3-category which in lowest degrees looks like the trivial 2-group, a braided monoidal supercategory is a 3-category which in lowest degree looks like the strict 2-group that comes from the crossed module $G(2) = (\diamond 2 \diamond Id \diamond 2)$". Urs called this generalization of stabilization of n-categories, $G(2)$-*stabilization*. Therefore, the claim would be that 'braided monoidal supercategories come from $G(2)$-stabilized 3-categories, with $G(2)$ the above strict 2-group';

7. An *organismic set* of order n can be regarded either as a category of algebraic theories representing organismic sets of different orders $o \leq n$ or as a *discrete topology* organismic supercategory of algebraic theories (or supercategory only with discrete topology, e.g. , a *class* of objects);

8. Any 'standard' topos with a (commutative) Heyting logic algebra as a subobject classifier is an example of a commutative (and distributive) supercategory with the additional axioms to ETAC being those that define the Heyting logic algebra;

9. The generalized LM_n (Łukasiewicz–Moisil) toposes are supercatgeories of *non-commutative*, algebraic n-valued logic diagrams that are subject to the axioms of LM_n *algebras of n-valued logics*;

10. n-categories are supercategories restricted to interpretations of the ETAC axioms;

11. An *organismic supercategory* is defined as a supercategory subject to the ETAC axioms and also subject to the ETAS axiom of complete self–reproduction involving π–entities (*viz.* Löfgren, 1968; [**?**]); its objects are classes representing organisms in terms of morphism (super) diagrams or equivalently as heterofunctors of organismic classes with variable topological structure;

Definition 0.1. *Organismic Supercategories ([**?**])* An example of a class of supercategories interpreting such ETAS axioms as those stated above was previously defined for organismic structures with different levels of complexity ([**?**]); *organismic supercategories* were thus defined as *superstructure interpretations of ETAS* (including ETAC, as appropriate) in terms of triplets $\mathbf{K} = (C, \Pi, N)$, where C is an arbitrary category (interpretation of ETAC axioms, formulas, etc.), Π is a category of complete self–reproducing entities, π, ([**?**]) subject to the negation of the axiom of restriction (for elements of sets): $\exists S : (S \neq \oslash)$ *and* $\forall u : [u \in S) \Rightarrow \exists v : (v \in u)$ *and* $(v \in S)]$, (which is known to be independent from the ordinary logico-mathematical and biological reasoning), and N is a category of non-atomic expressions, defined as follows.

Definition 0.2. An *atomically self–reproducing entity* is a unit class relation u such that $\pi\pi\langle\pi\rangle$, which means "π stands in the relation π to π", $\pi\pi\langle\pi, \pi\rangle$, etc.

An expression that does not contain any such atomically self–reproducing entity is called a *non-atomic expression.*

References

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