approximation theorem for an arbitrary space

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Defines	unique colimit of a sequence of cellular inclusions of CW-complexes

Theorem 0.1. *(*Approximation theorem for an arbitrary topological space in terms of the colimit of a sequence of cellular inclusions of CW-complexes*):*

> *"There is a functor* $\Gamma : \boldsymbol{hU} \longrightarrow \boldsymbol{hU}$ *where* $\boldsymbol{hU}$ *is the homotopy category for unbased spaces , and a natural transformation* $\gamma : \Gamma \longrightarrow Id$ *that asssigns a* CW*-complex* ΓX *and a weak equivalence* $\gamma_e : \Gamma X \longrightarrow X$ *to an arbitrary space* X*, such that the following diagram commutes:*
>
> $$\begin{tikzcd} \Gamma X \arrow[r, "\Gamma f"] \arrow[d, "\gamma(X)"'] & \Gamma Y \arrow[d, "\gamma(Y)"] \\ X \arrow[r, "f"] & Y \end{tikzcd}$$
>
> *and* $\Gamma f : \Gamma X \to \Gamma Y$ *is unique up to homotopy equivalence."*

*(viz. p. 75 in ref. [**?**]).*

Remark 0.1. The CW-complex specified in the `http://planetmath.org/ApproximationTheoremForAnAr` theorem for an arbitrary space is constructed as the colimit ΓX of a sequence of cellular inclusions of CW-complexes $X_1, ..., X_n$, so that one obtains $X \equiv colim[X_i]$. As a consequence of J.H.C. Whitehead's Theorem, one also has that:

$\gamma* : [\Gamma X, \Gamma Y] \longrightarrow [\Gamma X, Y]$ is an isomorphism.

Furthermore, the homotopy groups of the CW-complex ΓX are the colimits of the homotopy groups of X_n and $\gamma_{n+1} : \pi_q(X_{n+1}) \longmapsto \pi_q(X)$ is a group epimorphism.

References

[1] May, J.P. 1999, *A Concise Course in Algebraic Topology.*, The University of Chicago Press: Chicago