

symmetric monoidal category

Canonical name SymmetricMonoidalCategory

Date of creation 2013-03-22 18:30:22 Last modified on 2013-03-22 18:30:22

Owner CWoo (3771) Last modified by CWoo (3771)

Numerical id 6

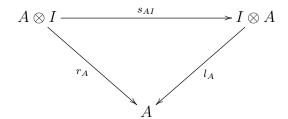
Author CWoo (3771)
Entry type Definition
Classification msc 81-00
Classification msc 18-00
Classification msc 18D10

A monoidal category \mathcal{C} with tensor product \otimes is said to be *symmetric* if for every pair A, B of objects in \mathcal{C} , there is an isomorphism

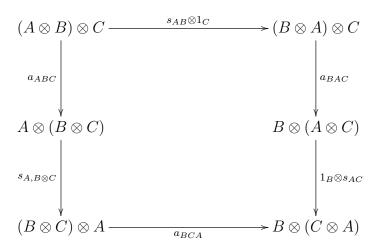
$$s_{AB}: A \otimes B \cong B \otimes A$$

that is natural in both A and B such that the following diagrams are commutative

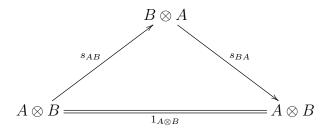
1. $(unit\ coherence\ for\ s)$:



2. (associativity coherence for s):



3. (inverse law):



In the diagrams above, a, l, r are the associativity isomorphism, the left unit isomorphism, and the right unit isomorphism respectively.

Some examples and non-examples of symmetric monoidal categories:

- The category of sets. The tensor product is the set theoretic cartesian product, and any singleton can be fixed as the unit object.
- The category of groups. Like before, the tensor product is just the cartesian product of groups, and the trivial group is the unit object.
- More generally, a category with finite products is symmetric monoidal. The tensor product is the direct product of objects, and any terminal object (empty product) is the unit object.
- \bullet The category of bimodules over a ring R is monoidal. However, this category is only symmetric monoidal if R is commutative.

Remark. A symmetric monoidal category is a braided monoidal category such that the inverse law: $s_{BA} \circ s_{AB} = 1_{A \otimes B}$ holds.