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## quantum logic toposes

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Related topic	Topos
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Defines	quantum logics toposes
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This is a topic entry on extensions of standard and elementary toposes to *quantum topoi* founded upon many-valued logics.

**Definition 0.1.** A *quantum logic topos* (*QLT*) is defined as an *extension of the concept of a topos* in which the Heyting algebra or subobject classifier of the standard elementary topos is replaced by a *quantum logic* that is axiomatically defined by a *non-commutative* lattice structure such as that of a many valued,  $LM_n$ -logic algebra, modified to a non-distributive lattice structure corresponding to that of the quantum physics events.

**Remark 0.1.** Quantum logics topoi are thus generalizations of the Birkhoff and von Neumann definition of quantum state spaces based on their definition of a quantum logic (lattice), as well as a *non-Abelian*, higher dimensional extension of the recently proposed concept of a ‘quantum’ topos which employs the (*commutative*) Heyting logic algebra as a subobject classifier.

Some specific examples are considered in the following two recent references.

## References

- [1] Butterfield, J. and C. J. Isham: 2001, space-time and the philosophical challenges of quantum gravity., in C. Callender and N. Hugget (eds. ) *Physics Meets Philosophy at the Planck scale.*, Cambridge University Press, pp.33–89.
- [2] Butterfield, J. and C. J. Isham: 1998, 1999, 2000–2002, A topos perspective on the Kochen–Specker theorem I - IV, *Int. J. Theor. Phys.*, **37** No 11., 2669–2733 **38** No 3., 827–859, **39** No 6., 1413–1436, **41** No 4., 613–639.