Schrödinger operator

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Synonym	Hamiltonian operator
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Defines	quantum system dynamics and eigenvalues

Let $V\colon \mathbb{R}^n \to \mathbb{R}$ be a real-valued function. The *Schroedinger operator* $\mathbf{H}$ on the Hilbert space $L^2(\mathbb{R}^n)$ is given by the action

$$\psi \mapsto -\nabla^2 \psi + V(x)\psi, \quad \psi \in L^2(\mathbb{R}^n).$$

This can be obviously re-written as:

$$\psi \mapsto [-\nabla^2 + V(x)]\psi, \quad \psi \in L^2(\mathbb{R}^n),$$

where $[-\nabla^2 + V(x)]$ is the *Schrödinger* operator, which is now called the `http://planetmath.org/HamiltonianOperatorOfAQuantumSystem`Hamiltonian operator, $\mathbf{H}$.

For stationary quantum systems such as electrons in 'stable' atoms the *Schrödinger equation* takes the very simple form :

$$\mathbf{H}\psi = E\psi$$

, where E stands for energy eigenvalues of the stationary quantum states. Thus, in quantum mechanics of systems with finite degrees of freedom that are 'stationary', the Schrödinger operator is used to calculate the (time-independent) energy states of a quantum system with potential energy $V(x)$. Schrödinger called this operator the `http://planetmath.org/HamiltonianOperatorOfAQuantumS` operator, or the `http://planetmath.org/HamiltonianOperatorOfAQuantumSystem`Hamiltonian, and the latter name is currently used in almost all of quantum physics publications, etc. The eigenvalues give the energy levels, and the wavefunctions are given by the eigenfunctions. In the more general, non-stationary, or 'dynamic' case, the Schrödinger equation takes the general form:

$$\mathbf{H}\psi = (-i)\partial\psi/\partial t$$

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