CW-complex approximation of quantum state spaces in QAT

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Defines	CW-complex approximation of quantum state spaces in QAT

Theorem 1.

Let $[QF_j]_{j=1,\ldots,n}$ be a complete sequence of commuting quantum spin 'foams' (QSFs) in an arbitrary `http://planetmath.org/QuantumSpaceTimes`quantum state space (QSS), and let (QF_j, QSS_j) be the corresponding sequence of pair subspaces of QST. If Z_j is a sequence of CW-complexes such that for any j , $QF_j \subset Z_j$, then there exists a sequence of n-connected models (QF_j, Z_j) of (QF_j, QSS_j) and a sequence of induced isomorphisms ${f_*}^j : \pi_i(Z_j) \to \pi_i(QSS_j)$ for $i > n$, together with a sequence of induced monomorphisms for $i = n$.

Remark 0.1. There exist *weak* homotopy equivalences between each Z_j and QSS_j spaces in such a sequence. Therefore, there exists a CW–complex approximation of QSS defined by the sequence $[Z_j]_{j=1,\ldots,n}$ of CW-complexes with dimension $n \geq 2$. This CW–approximation is unique up to *regular* homotopy equivalence.

Corollary 2.

The n-connected models (QF_j, Z_j) of (QF_j, QSS_j) form the *Model Category* of `http://planetmath.org/Sp`
Spin Foams (QF_j), *whose morphisms are maps* $h_{jk} : Z_j \to Z_k$ *such that* $h_{jk} \mid QF_j = g : (QSS_j, QF_j) \to (QSS_k, QF_k)$, *and also such that the following diagram is commutative:*

$$\begin{array}{ccc} Z_j & \xrightarrow{f_j} & QSS_j \\ {\scriptstyle h_{jk}}\downarrow & & \downarrow{\scriptstyle g} \\ Z_k & \xrightarrow{f_k} & QSS_k \end{array}$$

Furthermore, the maps h_{jk} are unique up to the homotopy rel QF_j , and also rel QF_k.

Remark 0.2. Theorem 1 complements other data presented in the `http://planetmath.org/QuantumAlgebra`
entry on QAT.