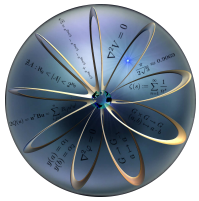

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dot product

Canonical name	DotProduct
Date of creation	2013-03-22 11:46:33
Last modified on	2013-03-22 11:46:33
Owner	drini (3)
Last modified by	drini (3)
Numerical id	13
Author	drini (3)
Entry type	Definition
Classification	msc 83C05
Classification	msc 15A63
Classification	msc 14-02
Classification	msc 14-01
Synonym	scalar product
Related topic	CauchySchwarzInequality
Related topic	CrossProduct
Related topic	Vector
Related topic	DyadProduct
Related topic	InvariantScalarProduct
Related topic	AngleBetweenLineAndPlane
Related topic	TripleScalarProduct
Related topic	ProvingThalesTheoremWithVectors
Defines	scalar square

Let $u = (u_1, u_2, \ldots, u_n)$ and $v = (v_1, v_2, \ldots, v_n)$ two vectors on k^n where k is a field (like $\mathbb{R}$ or $\mathbb{C}$). Then we define the *dot product* of the two vectors as:

$$u \cdot v = u_1 v_1 + u_2 v_2 + \cdots + u_n v_n.$$

Notice that $u \cdot v$ is NOT a vector but a scalar (an element from the field k).

If u, v are vectors in $\mathbb{R}^n$ and ϑ is the angle between them, then we also have

$$u \cdot v = \|u\| \|v\| \cos \vartheta.$$

Thus, in this case, $u \perp v$ if and only if $u \cdot v = 0$.

The special case $u \cdot u$ of scalar product is the *scalar square* of the vector u. In $\mathbb{R}^n$ it equals to the square of the length of u:

$$u \cdot u = \|u\|^2$$