Mercator projection

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In a Mercator Projection the point on the sphere (of radius R) with longitude L (positive East) and latitude λ (positive North) is mapped to the point in the plane with coordinates x, y:

$$x = RL$$
$$y = R\ln(\tan(\frac{\pi}{4} + \frac{\lambda}{2}))$$

The Mercator projection satisfies two important properties: it is conformal, that is it preserves angles, and it maps the sphere's parallels into straight line segments of length $2\pi R$. (A parallel of latitude means a small circle comprised of points at a specified latitude).

Starting from these two properties we can derive the Mercator Projection. First note that a parallel of latitude λ has length $2\pi R\cos(\lambda)$. To make the projections of the parallels all the same length a stretching factor in longitude of $\frac{1}{\cos(\lambda)}$ will have to be applied. For the mapping to be conformal, the same stretching factor must be applied in latitude also. Note that the stretching factor varies with λ so to map a specified latitude λ_0 to an ordinate y we must evaluate an integral.

$$y = \int_0^{\lambda_0} (1/\cos(\lambda))d\lambda$$

Early mapmakers such as Mercator evaluated this integral numerically to produce what is called a Table of Meridional Parts that can be used to map λ_0 into y. Later it was noticed that the integral of one over cosine actually has a closed form, leading to the expression for y shown above.