linear programming

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A *linear programming problem*, or *LP*, is a problem of optimizing a given linear objective function over some polyhedron. The standard maximization LP, sometimes called the primal problem, is

$$\begin{aligned} \text{maximize } & c^T x \\ \text{s.t. } & Ax \le b \\ & x \ge 0 \end{aligned} \tag{P}$$

Here $c^T x$ is the objective function and the remaining conditions define the polyhedron which is the feasible region over which the objective function is to be optimized. The dual of (**??**) is the LP

$$\begin{aligned} \text{minimize } & y^T b \\ \text{s.t. } & y^T A \ge c^T \\ & y \ge 0 \end{aligned} \tag{D}$$

The linear constraints for a linear programming problems define a convex polyhedron, called the *feasible region* for the problem. The weak duality theorem states that if $\hat{x}$ is feasible (i.e. lies in the feasible region) for (**??**) and $\hat{y}$ is feasible for (**??**), then $c^T \hat{x} \le \hat{y}^T b$. This follows readily from the above:

$$c^T \hat{x} \le (\hat{y}^T A)\hat{x} = \hat{y}^T (A\hat{x}) \le y^T b.$$

The strong duality theorem states that if both LPs are feasible, then the two objective functions have the same optimal value. As a consequence, if either LP has unbounded objective function value, the other must be infeasible. It is also possible for both LP to be infeasible.

The `http://planetmath.org/SimplexAlgorithm`simplex method of G. B. Dantzig is the algorithm most commonly used to solve LPs; in practice it runs in polynomial time, but the worst-case running time is exponential. Two polynomial-time algorithms for solving LPs are the ellipsoid method of L. G. Khachian and the interior-point method of N. Karmarkar.

References

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[3] Korte, B. and J. Vygen, *Combinatorial optimization: theory and algorithms*, Springer-Verlag, 2002.