# present value

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| Defines | net present value |
| Defines | future value |

Suppose you are going to receive $10,000, to be paid in two payments at the end of the next two years. You have the following two options

| options | year 1 | year 2 |
|---|---|---|
| option 1 | $6,000 | $4,000 |
| option 2 | $4,000 | $6,000 |

Which option would you select in order to have the maximum gain? Of course, if there is no interest, both options are equal. If any non-zero interest rates are involved, one option may be preferable than the other.

By calculating the *present values* of these options, one may be able to compare the "present" values of these payments and figure out which is the preferable option. So what is a *present value*?

**Definition**. Let $P$ be the amount of a payment at sometime $t > 0$ in the future. then the *present value* $\mathrm{PV}(P)$ of $P$ is simply the value of this payment at time $t = 0$. Specifically, if the interest rate from 0 to $t$ is $r$, then

$$\mathrm{PV}(P) = \frac{P}{1+r}.$$

In other words, if we invest $\mathrm{PV}(P)$ today, earning an interest at a rate of $r$ between times 0 and $t$, then at time $t$, we would have made $P$.

Now, suppose in the example above, both options have an `http://planetmath.org/InterestRa` annual interest rate of 5% `http://planetmath.org/CompoundInterest`compounded annually, then the present value of option 1 is

$$\frac{\$6,000}{1.05} + \frac{\$4,000}{(1.05)^2} \approx \$9,342.40$$

whereas the second option has present value

$$\frac{\$4,000}{1.05} + \frac{\$6,000}{(1.05)^2} \approx \$9,251.70$$

Clearly, the first option is superior than the second one.

**Remarks**.

- Of course, the result will be the same if one instead computes the *future values* of these options, which are the values of the payments at a specific future time $t > 0$: if payment is valued at $P$ at time 0, its value at some future time $t > 0$, or its *future value* is

$$\mathrm{FV}(P) = P(1+r),$$

if $r$ is the interest rate from 0 to $t$.

- An accompanying concept is that of the *net present value* NPV. It is the present value of all the future payments minus the initial investment: suppose an investment $I$ is made where an initial amount of $A$ is made at time 0, and payments $P_1, \ldots, P_n$ are returns as a result of this investment. Then

$$\mathrm{NPV}(I) = \Big(\mathrm{PV}(P_1) + \mathrm{PV}(P_2) + \cdots + \mathrm{PV}(P_n)\Big) - A.$$

If we treat the initial invsetment $A$ as a "negative" return, $A = -P_0 = -\mathrm{PV}(P_0)$, then the net present value of the investment can be written

$$\mathrm{NPV}(I) = \mathrm{PV}(P_0) + \mathrm{PV}(P_1) + \cdots + \mathrm{PV}(P_n) = \sum_{i=0}^{n} \mathrm{PV}(P_i).$$

One would usually want to invest in something with a positive net present value. Net present values are commonly used when one is interested in comparing car loans or home mortgages.