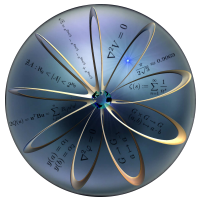

planetmath.org

Math for the people, by the people.

examples of normal form games

Canonical name	ExamplesOfNormalFormGames
Date of creation	2013-03-22 12:51:27
Last modified on	2013-03-22 12:51:27
Owner	Henry (455)
Last modified by	Henry (455)
Numerical id	9
Author	Henry (455)
Entry type	Example
Classification	msc 91A10
Classification	msc 91A06
Classification	msc 91A05
Defines	prisoners dilemma
Defines	battle of the sexes

A few example normal form games:

Prisoner's dilemma

Probably the most famous game theory example, the prisoner's dilemma is a two player game where $S_1 = S_2 = C, D$ and:

$$u_1(s_1, s_2) = \begin{cases} 5 & \text{if} \quad s_1 = C \quad \text{and} \quad s_2 = C \\ 10 & \text{if} \quad s_1 = D \quad \text{and} \quad s_2 = C \\ -5 & \text{if} \quad s_1 = C \quad \text{and} \quad s_2 = D \\ 0 & \text{if} \quad s_1 = D \quad \text{and} \quad s_2 = D \end{cases}$$

$$u_2(s_1, s_2) = \begin{cases} 5 & \text{if} \quad s_1 = C \quad \text{and} \quad s_2 = C \\ 10 & \text{if} \quad s_1 = C \quad \text{and} \quad s_2 = D \\ -5 & \text{if} \quad s_1 = D \quad \text{and} \quad s_2 = C \\ 0 & \text{if} \quad s_1 = D \quad \text{and} \quad s_2 = D \end{cases}$$

Traditionally this is interpreted as the case of two criminal partners separately being interrogated and asked to give up the other partner. C stands for cooperating (with their partners) by refusing to give up information, and D stands for defecting and agreeing to testify against the partner. The different payoffs reflect different jail sentences, ranging from nothing (+10) to a long jail sentence (-5), with amounts in between depending on the evidence against them.

The (much more convenient) normal form is:

	C	D
C	5,5	-5,10
D	10,-5	0,0

Notice that (C, C) Pareto dominates (D, D), however (D, D) is the only Nash equilibrium.

Battle of the Sexes

Another traditional two player game. The normal form is:

	O	F
O	2,1	0,0
F	0,0	1,2

A Degenerate Example

One more, rather pointless, example which illustrates a game where one player has no choice:

	X	Y	Z
A	2,100	1,7	14,-5

Undercut

A game which illustrates an infinite (indeed, uncountable) strategy space. There are two players and $S_1 = S_2 = \mathbb{R}^+$.

$$u_1(s_1, s_2) = \begin{cases} 1 & \text{if} \quad s_1 < s_2 \\ 0 & \text{if} \quad s_1 \geq s_2 \end{cases}$$

$$u_2(s_1, s_2) = \begin{cases} 1 & \text{if} \quad s_2 < s_1 \\ 0 & \text{if} \quad s_2 \geq s_1 \end{cases}$$