example of Nash equilibrium

Canonical name	ExampleOfNashEquilibrium
Date of creation	2013-03-22 12:52:48
Last modified on	2013-03-22 12:52:48
Owner	Henry (455)
Last modified by	Henry (455)
Numerical id	6
Author	Henry (455)
Entry type	Example
Classification	msc 91A99

Consider the first two games given as examples of normal form games.

In Prisoner's Dilemma the only Nash equilibrium is for both players to play D: it's apparent that, no matter what player 1 plays, player 2 does better playing D, and vice-versa for 1.

Battle of the Sexes has three Nash equilibria. Both (O, O) and (F, F) are Nash equilibria, since it should be clear that if player 2 expects player 1 to play O, player 2 does best by playing O, and vice-versa, while the same situation holds if player 2 expects player 1 to play F. The third is a mixed equilibrium; player 1 plays O with $\frac{2}{3}$ probability and player 2 plays O with $\frac{1}{3}$ probability. We confirm that these are equilibria by testing the first derivatives (if 0 then the strategy is either maximal or minimal). Technically we also need to check the second derivative to make sure that it is a maximum, but with simple games this is not really necessary.

Let player 1 play O with probability p and player 2 plays O with probability q.

$$u_1(p, q) = 2pq + (1 - p)(1 - q) = 2pq - p - q + pq = 3pq - p - q$$

$$u_2(p, q) = pq + 2(1 - p)(1 - q) = 3pq - 2p - 2q$$

$$\frac{\partial u_1(p, q)}{\partial p} = 3q - 1$$

$$\frac{\partial u_2(p, q)}{\partial q} = 3p - 2$$

And indeed the derivatives are 0 at $p = \frac{2}{3}$ and $q = \frac{1}{3}$.