The Best Score in the Worst Case of a Memory Game is 2N - 1

Canonical name	TheBestScoreInTheWorstCaseOfAMemoryGameIs2N1
Date of creation	2013-03-22 19:04:15
Last modified on	2013-03-22 19:04:15
Owner	ubershmekel (18723)
Last modified by	ubershmekel (18723)
Numerical id	5
Author	ubershmekel (18723)
Entry type	Theorem
Classification	msc 91A99

For a memory game with N pairs (2*N cards on the table) the "best case" (the best result for one session) is to have N lucky turns where every second card picked is the matching one.

Theorem: For a perfect player, it takes $2N-1$ turns to solve the worst case of a Memory Game.

Proof: With out loss of generality lets assume that the player gets no lucky shots where two matching cards are flipped over by chance.

Definition: a "hint" is the first time the player sees a card type. Seeing the matching card is not considered a "hint".

Every turn the player gets either 0, 1 or 2 new hints: 0 hints - The player's first pick was a card he'd already seen and knows how to match. 1 hint - The player's first pick is a card not yet seen and the second pick was a card seen in a previous turn. 2 hints - Both cards picked did not appear in a previous turn.

Finding a card that the player had already seen isn't considered a "hint" because it's dead giveaway for a perfect player. Notice the maximum number of "hints" is N.

Using this definition for "hints" a perfect player needs N turns to finish the game after collecting N hints. This is because the player can pick any card that he hasn't flipped over yet and he'll know where it's matching card is 100

If the player flips over only unique cards (2 hints per turn) until he reaches N hints, the player will finish the game in a total of $(3/2)N$ turns. $N/2$ turns were used for gathering the hints and another N turns for finishing the game.

The player always gets 2 hints the first turn (if he's unlucky as is our assumption) then his next turn can possibly give him only 1 hint, too bad for the player. The next turns can also give the player only 1 hint until the hints are exhausted. See the following example.

The turns the perfect player in his worst case will be similar to: 1. Flipped over cards A and B - 2 hints for A and B 2. Flipped over cards C and A - 1 hint for C and a dead giveaway for A (there were 2 options for a dead giveaway - A or B) 3. Flipped over cards D and C - 1 hint for D and a dead giveaway for C (there were 2 options for a dead giveaway - B or C) 4. Flipped over cards E and B - 1 hint for E and a dead giveaway for B (there were 2 options for a dead giveaway - D or B) ... N - 1. Flipped over 2 cards - the last hint is given, a dead giveaway is given as well. Only 2 cards left unseen on the board, but the player already has hints for both cards.

Thus, the game can continue until N hints are collected, one at a time,

for $N-1$ turns, it's (N - 1) because the worst case for the first turn is always 2 hints.

Thus the worst case for playing the Memory Game is $2 * N - 1$. The amount of turns needed for gathering hints is (N - 1) and another N turns are used for finishing the game.

QED

Closing Note: A simple algorithm for solving the game always wins in $2N$ turns, simply flipping over everything and then matching. Seeing as how the perfect player with the worst of luck is only 1 turn better than a player with a constant algorithm, it might make one think that luck is more important than brains.