transfer function

Canonical name	TransferFunction
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Entry type	Definition
Classification	msc 93A10
Defines	frequency domain
Defines	stable
Defines	unstable

The *transfer function* of a linear dynamical system is the ratio of the Laplace transform of its output to the Laplace transform of its input. In systems theory, the Laplace transform is called the "frequency domain" representation of the system.

Consider a canonical dynamical system

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t)\end{aligned}$$

with input $u : R \mapsto R^n$, output $y : R \mapsto R^m$ and state $x : R \mapsto R^p$, and (A, B, C, D) are constant matrices of conformable sizes.

The frequency domain representation is

$$y(s) = (D + C(sI - A)^{-1}B)u(s),$$

and thus the transfer function matrix is $D + C(sI - A)^{-1}B$.

In the case of single-input-single-output systems ($m = n = 1$), the transfer function is commonly expressed as a rational function of s:

$$H(s) = \frac{\prod_{i=0}^{Z}(s - z_i)}{\prod_{i=0}^{P}(s - p_i)}.$$

The values z_i are called the zeros of $H(s)$, and the values p_i are called the poles. If any of the poles has positive real part, then the transfer function is termed *unstable*; if all of the poles have strictly negative real part, it is *stable*.