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discretization of continuous systems

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Author	mathcam (2727)
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Consider a continuous-time system with the following state space representation

$$P: \quad \left\{ \begin{array}{rcl} \dot{x}(t) & = & Ax(t) + Bu(t), \\ y(t) & = & Cx(t) + Du(t), \end{array} \right. \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^r$ and $y(t) \in \mathbb{R}^m$ are the state vector, input vector and output vector of the system, respectively; $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times r}$, $C \in \mathbb{R}^{m\times n}$ and $D \in \mathbb{R}^{m\times r}$ are the constant real or complex matrices.

Suppose that the sampling interval is τ. By using the step invariance transform or the zero-order hold (ZOH), i.e., $u(t) = u(k\tau),\ k\tau \le t < (k+1)\tau$, discretizing the system in (**??**) gives a discrete-time model,

$$P_\tau: \quad \left\{ \begin{array}{rcl} x(k\tau + \tau) & = & G_\tau x(k\tau) + F_\tau u(k\tau), \\ y(k\tau) & = & Cx(k\tau) + Du(k\tau),\ \ k = 0, 1, 2, \cdots \end{array} \right. \tag{2}$$

where $x(k\tau) = x(t)\,|_{t=k\tau}$, $y(k\tau) = y(t)\,|_{t=k\tau}$, and

$$G_\tau := \mathrm{e}^{A\tau},\ \ F_\tau := \int_0^\tau \mathrm{e}^{At}\mathrm{d}t\ B. \tag{3}$$