linear time invariant system

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A *linear time invariant system* (LTI) is a linear dynamical system $T(p)$,

$$y(k) = T(p)\, u(k),$$

with parameter p that is time independent. $y(k)$ denotes the system output and $u(k)$ denotes the input. The independent variable k can be denoted as time, index for a discrete sequences or differential operaters (e.g. such as s in Laplace domain or ω in frequency domain).

For example, for a simple mass-spring-dashpot system, the system parameter p can be selected as the mass m, spring constant k and damping coefficient d. The input u to the said system can be chosen as the force applied to the mass and the output y can be chosen as the mass's displacement.

LTI system has the following properties.

Linearity: If $y_1 = Tx_1$ and $y_2 = Tx_2$, then

$$T\{\alpha x_1 + \beta x_2\} = \alpha y_1 + \beta y_2$$

Time Invariance: If $y(k) = Tx(k)$, then

$$y(k + \delta_k) = Tx(k + \delta_k)$$

Associative:

$$T_1 \cdot (T_2 \cdot T_3) = (T_1 \cdot T_2) \cdot T_3$$

Commutative:

$$T_1 \cdot T_2 = T_2 \cdot T_1$$

A LTI system can be represented with the following:

- Transfer function of Laplace transform variable s, which is commonly used in control systems design.
- Transfer function of Fourier transform variable ω, which is commonly used in communication theory and signal processing.
- Transfer function of z-transform variable z^{-1}, which is commonly used in digital signal processing (DSP).
- State-space equations, which is commonly used in modern control theory and mechanical systems.

Note that all transfer functions are LTI systems, but not all state-space equations are LTI systems.