



Math for the people, by the people.

control system

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| Canonical name   | ControlSystem       |
| Date of creation | 2013-03-22 14:22:59 |
| Last modified on | 2013-03-22 14:22:59 |
| Owner            | ppirrip (5555)      |
| Last modified by | ppirrip (5555)      |
| Numerical id     | 9                   |
| Author           | ppirrip (5555)      |
| Entry type       | Definition          |
| Classification   | msc 93A10           |
| Related topic    | SystemDefinitions   |

The sole objective of control system is to generate feasible inputs to the plant (e.g. dynamic systems) such that it will operate as it intended to under a wide range of operating conditions.

Examples of control systems: Cruise control, auto pilot, rice cooker.

For a general finite dimensional dynamic system in its ODE form,

$$\begin{aligned}\dot{x} &= f(x(t), t) + g(x(t), u(t), t), \\ y &= h(x(t), t),\end{aligned}\tag{1}$$

where  $x \in \mathbb{R}^n$  is the state,  $y \in \mathbb{R}^l$  is the output and  $u \in \mathbb{R}^m$  is the control input of the system. In the control literature, equation ?? is general referred as the plant, where the function  $f : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^n$  governs the system dynamics,  $g : \mathbb{R}^m \times \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^n$  determines how the input (control signals) influence the state  $x$  via *actuators* (e.g. gas turbine) and  $h : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^l$  determines how the state generates the output signal. If  $m$  is equal to  $n$ , the plant is *fully actuated*. If  $m < n$  then the plant is *under actuated* and otherwise the plant is *over actuated*. For a plant that is not explicitly dependent in time  $t$ , such system is called a *Autonomous* system. The main difference between a control system and a general dynamic system is the additional signal  $u(t)$ .

For example, to control an airplane, the control system has to control the *thrust*, *flaps*, *aileron* and *rudder*, which they are the control signals of the system  $u$ . Those control input influence the system state  $x$  such as *speed* (with thrust), *attitude* and *orientation* (with flaps, aileron and rudder). To physically alter the state of the airplane, actuators such as gas turbines are needed, which are controlled by the control signals  $u$ .

The control signal  $u$  can be generated in a *closed-loop* fashion or *open-loop* fashion. An open-loop control system generates  $u$  with the user, or operator supplied reference state  $x_d$  or output  $y_d$  only; meanwhile closed-loop control system uses both reference and *feedback* signals that are usually measured from *sensors*. In the airplane attitude control example, the desired attitude is usually represented in roll-pitch-yaw angle representation, and these signals are measurable by attaching sensors to the flaps, aileron and rudder. In engineering practice, only closed-loop control systems should be used, since open-loop systems are not *robust* against uncertainties, modeling errors and measurement errors.

If a closed-loop control system is based on state feedback, such control

system is called a *state-feedback* control system. By the same token, a *output-feedback* control system is based on output feedback only. Notice that output signals are available for feedback by definition, however in reality not all the states are measurable. If a state-feedback control system with all the states available for feedback, it is called a *full-state feedback* system and otherwise is called *partial-state feedback* system, which usually requires a *state observer* (e.g. Kalman filter) to estimate the unavailable states.

To illustrate the simple concept of control systems, we will use a simple example. A truck driver is required to travel 1000 Km in 10 hours. To relieve the stress on the driver's heel, he has placed a stick to the gas paddle so the car travels at 100Km/h. Under perfect conditions, the driver will reach the destination in the allocated time. However, a certain section of the road is up-hill, so the truck slowed down by a considerable amount and will not arrive its destination in time. To remedy this problem, the driver 'implemented' a simple solution using the speed-o-meter such that the gas paddle position  $p_{set}$  of the truck is now depends on the current speed  $v_{current}$  of the truck,  $p_{set} = -K(v_{current} - 100\text{Km/h})$ , where  $K$  is just an adjustable parameter. So if the truck is running too slow (e.g. up-hill),  $p_{set}$  will be positive (more gas to the engine) hence speed will increase to maintain the desired speed, so vice-versa for the down-hill case.

In this example, we have outlined all the major components of a typical control system:

- Actuator: engine,
- Sensor: speed-o-meter,
- Plant: truck,
- Control input: gas paddle,
- Control objective: 1000 Km in 10 hrs,
- Control law:  $p_{set} = -K(v_{current} - 100\text{Km/h})$ ,

The science aspect of control systems is the study of design, synthesis and analysis of control systems using mathematical concepts, and the engineering aspect of controls systems is to implement, construct and adjust the control system according to real-life situation and limitations.