## Dark matter and dark energy from Bose-Einstein condensate

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We show that Dark Matter consisting of bosons of mass of about  $1\ eV$  or less has critical temperature exceeding the temperature of the universe at all times, and hence would have formed a Bose-Einstein condensate at very early epochs. We also show that the wavefunction of this condensate, via the quantum potential it produces, gives rise to a cosmological constant which may account for the correct dark energy content of our universe. We argue that massive gravitons or axions are viable candidates for these constituents. In the far future this condensate is all that remains of our universe.

The basic contents of our universe in terms of Dark Matter (DM), Dark Energy (DE), visible matter and radiation, and also its accelerated expansion in recent epochs has been firmly established by a number of observations now [1–4]. However although DM constitutes about 25% and DE about 70% of all matter-energy content, the constituents of DM and the origin of a tiny cosmological constant, or DE, of the order of  $10^{-123}$  in Planck units, which drives this acceleration, remains to be understood. In this paper we show that if DM is assumed to consist of a gas of bosons of mass m, then for  $m \leq 1eV$ , the critical temperature below which they will form a Bose-Einstein condensate (BEC) exceeds the temperature of the universe at all times. Therefore they would form such a condensate at very early epochs, in which a macroscopic fraction of the bosons fall to the ground state with little or no momentum and zero pressure, and therefore may be considered as viable candidates for cold DM (CDM). Further, via the quantum potential that it produces, the macroscopic wavefunction of the condensate gives rise to a positive cosmological constant in the Friedmann equation, whose magnitude depends on m, and for  $m \simeq 10^{-32} \ eV$ , one obtains the observed value of the cosmological constant. Therefore bosons with this tiny mass can account for both DM and DE in our universe. We argue that massive gravitons or axions are viable candidates for these bosons. Finally we speculate on the ultimate fate of our universe, and end with some open problems.

To compute the critical temperature of an ideal gas of bosons constituting DM, we first note that these must have a mass, however small, and with average interparticle distances  $(N/V)^{-1/3}$  (where N = total number of bosons in volume V) comparable or smaller than the

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thermal de Broglie wavelength  $hc/k_BT$ , such that quantum effects start to dominate. Identifying this temperature of a bosonic gas to the critical temperature  $T_c$  (below which the condensate forms) we get  $k_BT_c \simeq hc(N/V)^{1/3}$ . A more careful calculation for ultra-relativistic noninteracting bosons with a tiny mass gives [5–7]  $^{1-2}$ 

$$T_c = \frac{\hbar c}{k_B} \left( \frac{N\pi^2}{V\eta\zeta(3)} \right)^{1/3} . \tag{1}$$

In the above  $N=N_B+N_R$ ,  $N_B$  being the number of bosons in the BEC, and  $N_R$  outside it, both consisting of bosons of small mass as discussed earlier,  $\eta$  is the polarization factor and  $\zeta(3)\approx 1.2$ . Also a is the cosmological scale factor,  $L=L_0a$  is the Hubble radius,  $V=L^3=L_0^3a^3=V_0a^3$  (subscript 0 here and in subsequent expressions denote current epoch, when we also assume a=1). Note that for boson temperature  $T< T_c$ , a BEC will necessarily form, even when there are interactions [8]. As stated before, identifying the BEC of bosons all in their ground states with zero momenta and only rest energies with DM in any epoch, we estimate  $N_B/V=\rho_{DM}/m=0.25\rho_{crit}/ma^3$  i.e.  $N\simeq N_B$ , and obtain from Eq.(1)

$$T_c = \frac{6 \times 10^{-12}}{m^{1/3} a} K , \qquad (2)$$

(m in kg in the above). Therefore if m < 1 eV,  $T_c > 2.7/a$ , the universe background temperature at all times,

<sup>&</sup>lt;sup>1</sup> One can also consider a shallow three dimensional harmonic oscillator trapping potential with angular frequency  $\omega$ , for which the Gaussian wavefunction is a coherent ground state, one has  $T_c = (N/\eta)^{1/3}\hbar\omega/k_B$  [5], which on using  $L_0 = \sqrt{\hbar/m\omega}$ ,  $m = h/cL_0$  and the fact that N/V is constant, gives  $T_c = (\hbar c/k_B) (N/\eta V)^{1/3}$ , virtually identical to Eq.(1) up to a factor of order unity, lending it further credence.

<sup>&</sup>lt;sup>2</sup> For previous studies of superfluids and BEC in cosmology see [9–24]. See note at the end for more information.

and a BEC of the constituent bosons will form at very early epochs. As mentioned earlier, when the bosons are in the BEC state, they have little or no momentum, and behave as CDM.

Furthermore, as is well-known, a BEC is a macroscopic quantum state, described by a wavefunction  $\phi$ , which we decompose in the form  $\phi = \mathcal{R}e^{iS}$  ( $\mathcal{R}(x^{\alpha}), S(x^{a}) =$  real functions, and  $|\phi|^{2} = \mathcal{R}^{2}$  represents the spatial density of the bosons in the BEC). Next we replace classical geodesics by quantal (Bohmian) trajectories defined by the velocity field  $u_{a} = \hbar \ \partial_{a}S/m$  (as one should, in a quantum mechanical description) [25, 26], and also define the induced metric  $h_{ab} = g_{ab} - u_{a}u_{b}$ , using which it is found that the quantum corrected second order Friedmann equation (Raychaudhuri equation) for the scale factor a(t) is [27–29] <sup>3</sup>

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + 3p\right) + \frac{\hbar^2}{3m^2} h^{ab} \left(\frac{\Box \mathcal{R}}{\mathcal{R}}\right)_{:a:b} , \qquad (3)$$

where  $\rho = \rho_{vis} + \rho_{DM}$  is the sum of densities from visible matter and DM (similarly for the pressure). We interpret the last  $\mathcal{O}(\hbar^2)$  term as the quantum mechanical contribution to the cosmological constant

$$\Lambda_Q = \frac{\hbar^2}{m^2 c^2} h^{ab} \left(\frac{\Box \mathcal{R}}{\mathcal{R}}\right)_{a:b} . \tag{4}$$

In the above,  $V_Q \equiv (\hbar^2/m^2c^2)\Box \mathcal{R}/\mathcal{R}$  is known as the relativistic quantum potential [27]. Note that  $V_Q$  or  $\Lambda_Q$ are not ad-hoc, but in fact always present in a quantum description of the contents of our universe, and vanishes in the  $\hbar \to 0$  limit. From Eq.(4) it follows that  $\Lambda_O$  depends on the amplitude  $\mathcal{R}$  of the wavefunction  $\phi$ , which we take to be the ground state of a condensate. Note that with this choice,  $\mathcal{R}$  is also time-independent. Its exact form is not important to our argument however, except that it is non-zero and spread out uniformly over the range  $L_0$  of the observable universe, or the Hubble radius, with minute non-uniformities present at much smaller scales. This follows from the cosmological principle (homogeneity and isotropy of our universe) as well as from causality, the latter requiring that anything outside the Hubble radius would not influence the accessible wavefunction. Also as shown in [32] modes with wavelengths greater than this radius decay rapidly. Thus either using a straightforward dimensional argument, or a generic wavefunction such as a Gaussian with a large spread  $\mathcal{R} = \mathcal{R}_0 \exp(-r^2/L_0^2)$ , which is also the ground state for a shallow three dimensional harmonic oscillator potential [5], or one which results when an interaction of strength g is included such that  $\mathcal{R} = \mathcal{R}_0 \tanh(r/L_0\sqrt{2})$  (g > 0)and  $\mathcal{R} = \sqrt{2} \mathcal{R}_0 \operatorname{sech}(r/L_0)$  (g < 0) [33], one obtains

 $(\Box \mathcal{R}/\mathcal{R})_{;a;b} \simeq 1/L_0^4$ . Furthermore from quantum mechanics,  $L_0$  also determines the characteristic range of the wavefunction  $\phi$ , and as such may be identified with the Compton wavelength of the bosons of mass m under consideration, i.e.  $L_0 = h/mc$  [34], from which it follows

$$\Lambda_Q = \frac{1}{L_0^2} = \left(\frac{mc}{h}\right)^2 , \qquad (5)$$

Thus for the current Hubble radius  $L_0 = 1.4 \times 10^{26} \ metre$ , one obtains  $m \simeq 10^{-68} \ kg$  or  $10^{-32} \ eV$ , Finally, inserting the above value of  $L_0$  or m in Eq.(5), one obtains

$$\Lambda_Q = 10^{-52} \ (metre)^{-2}$$
 (6)

$$=10^{-123} \ell_{Pl}^{-2}$$
 (Planck units), (7)

where  $\ell_{Pl} = 1.6 \times 10^{-35} \ metre$  is the Planck length. This entirely accounts for the observed value of the cosmological constant, without the need to put it in the Friedmann equation by hand. We therefore see that bosons of such tiny mass in a BEC can account for both the DM (via its density) and DE (via quantum potential potential of its macroscopic wavefunction). Also in this case, from Eq.(2), the critical temperature is  $T_c = 10^{11} \ a^{-1} \ K$  much higher than the universe temperature at all times as stated earlier, confirming that the BEC will indeed form at the very early stages of our universe.

But what could be these bosons constituting DM and giving rise to DE? There are at least two viable candidates. First, we consider massive gravitons. Although gravitons derived from general relativity are massless, there has been considerable progress recently, both in the theoretical and experimental fronts, in having a consistent picture of massive gravitons in extensions of general relativity. For example, on the theoretical side, as early as in the 1930s gravitons of mass  $\mathcal{O}(10^{-32})~eV$  had been proposed [35]. More recently it was shown that massive gravitons can appear due to spontaneous symmetry breaking [36, 37], in a completely covariant non-linear completion of the Fierz-Pauli type massive gravity action [38], and in ghost free theories [39], solving an age old problem of having a covariant theory of massive gravitons. These theories also clearly admit graviton mass  $\mathcal{O}(10^{-32}) \ eV$ . Also as was recently shown, study of cosmology within these theories gives rise to additional densities in the Friedmann equation, which can be included in our definition of  $\rho$  in Eq.(3) [40–42]. Other theoretical approaches also point to graviton mass in this range, [43– 45]. In the Newtonian limit, for gravitons of mass m, the corresponding gravitational field follows a Yukawa type of force law  $F \propto \frac{k}{r^2} \ e^{-r/L_0}$ . Since gravity has not been tested beyond this length scale, such an interpretation is natural and cannot be ruled out. Also if one invokes periodic boundary conditions, this is also the mass of the lowest Kaluza-Klein modes. All bounds on graviton masses obtained from observations too suggest graviton mass  $\mathcal{O}(10^{-32}) \ eV$  [46]. Finally, gravitons as DM during the inflationary stage may also have observational consequences at present [47].

 $<sup>^3</sup>$  see also [30] and [31] for application of Bohmian mechanics in cosmology.

The second possibility is axions. Although axions were originally proposed to solve the strong CP problem in quantum chromodynamics, they also arise in the context of string theory, and have long been advocated as DM candidates [48], and BEC of axionic DM have also been explored [14]. Axion mass depends on the form of the action considered and couplings therein, but masses  $\mathcal{O}(10^{-32})$  eV are certaintly not ruled out [49, 50]. Experiments to detect axionic DM are also in progress [51]. It must be kept in mind however that until detected, axions remain as hypothetical particles, requiring extension of the otherwise well-tested standard model of particle physics, with their masses and couplings put in by hand.

As for the dynamics of DM, the BEC wavefunction and its fluctuations should be governed by the Gross-Pitaevskii equation (GPE) or its relativistic generalization [52, 53]

$$\left[\Box + m^2 + g|\phi|^2\right]\phi = 0 ,$$
 (8)

which in a Hubble background, and for negligible self-interactions ( $g \ll 1$ ) is consistent with the dynamical equation (13) of [49] and perturbation equations derived from it. It may be noted that the above equation holds for a BEC of gravitons as well. It has also been shown that at galaxy length scales, a BEC of light particles naturally gives rise to DM density profiles which match well with observed galaxy rotation curve velocities (see e.g. [12, 13, 20]).

At late times with universe temperature  $T \ll T_c$ , one has [5]

$$\frac{N_B}{N} = 1 - \left(\frac{T}{T_c}\right)^3 \to 1 , \qquad (9)$$

i.e.  $N_B \to N, N_R \to 0$ , and most available bosons would be subsumed within the condensate, accounting for the DM in our universe. The density of the latter of course falls off as  $1/a^3$ , while that of DE (via the  $\Lambda_Q$  term) remains constant. It follows that in the end the latter is all that remains, just as predicted by the DE hypothesis, with the universe being described by a giant quantum state of the condensate. This situation is depicted in Figure 1, where one can see that the BEC dominates at later times and lower background temperatures, when as we argued, the condensate provides a viable source of DE and DM. This may also be one of the best evidences for the graviton [54].

In summary, we have shown here that bosons of tiny mass should form a BEC at early times and may account for the DM content of the universe, while its macroscopic wavefunction can account for the DE. Both massive gravitons and axions are viable candidates of bosons of this mass, and hence that of DM and DE. While the

former requires a modification of general relativity, the latter requires extension of the standard model. While this picture predicts a high degree of homogeneity and isotropy at large scales (as observed), it still allows for relatively small variations of densities, temperatures etc.

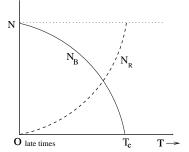


FIG. 1:  $N_B$  and  $N_R$  vs. T.

at smaller scales. It will be interesting to investigate other testable predictions of this BEC, such as its heat capacity, the distribution of DM, response to galaxy rotations etc. We hope to report on these elsewhere.

## Note added

In this note we list features of previous works in BEC in the context of cosmology. In [9], the authors use the same formula for  $T_c$  as our Eq.(1), and conjectured that the infinite heat conductivity of the BEC may account for the uniform microwave background temperature. In [10], it was proposed that the BEC manifests itself as DE and DM at different epochs. In [11], DM candidate of a BEC formed out of the 'phion' field in modified gravity was considered. In [12], a cold star composed of a dilute BEC was studied. In [13], [16], [17], [18], [20] and [21], properties of various forms of BEC DM were studied. In [14] and [24], the possibility of axions as a BEC was examined. Background geometries and black holes made up of BEC was considered in [15]. DM composed of a BEC of particles obeying infinite statistics was studied in [19]. In [22] BEC in loop quantum cosmology was studied. In [23] it was shown that DM can be well approximated by BEC at large scales, although their estimate of boson mass was higher than that proposed in this paper.

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