in which the "upset" molecules were not completely removed by light reversed rubbing the orientation of the remaining material was found to have been unaltered by the second rubbing. It seems quite possible that the material which was not removed had never been actually touched by the lens paper.[5]

It may be interesting to describe a mechanical model which might be expected to exhibit similar different responses to rubbing. If very soft bristles were supported in an inclined position in small holes in a solid surface, rubbing in the direction of their inclination would remove many bristles but rubbing in the opposite direction would only wedge them in more tightly.

It may be possible to carry the analysis of diffraction patterns like that of Fig. 5 considerably farther than has been done here. Also diffraction patterns from lightly rubbed films obtained with the primary beam parallel to the rubbing direction would yield precise values of the "b" spacing and probably other data. Intriguing as these matters are, they seem to us to be unrelated to the purpose of this simple study of rubbed films.

[5] Finch has reported very briefly [G. I. Finch and S. Fordham, Chem. and Ind. **56**, 637 (1937); G. I. Finch, J. Chem. Soc. 1144 (1938); Nature **141**, 547 (1938)] similar orientation of stearic acid molecules produced by rubbing. Our observations differ from his in some ways. From his very brief published statements we understand that his rubbed films are fairly resistant to rubbing in the opposite direction; this we do not find.

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On the Origin of Great Nebulae

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The formation of condensation due to gravitational instability is discussed in a uniformly expanding space. It is shown that such condensations cannot be formed at the present stage of the development of the universe but could have been formed in the past when all linear dimensions were 600 times smaller. This corresponds to the stage at which, according to astronomical observations, nebulae have been separated from each other. To get the correct dimensions of nebulae it is necessary to accept that the velocities of particles at the moment of separation were about 140 km/sec. which strongly suggests that these particles were stars and not atoms. The type of expansion necessary for the formation of nebulae indicates that space is infinite and unlimitedly expanding.

I. General Properties of Great Nebulae

During recent years, considerable progress has been made in the study of great (or extragalactic) nebulae, due mainly to the investigations of Hubble.[1] We know that these objects represent immensely large accumulations of matter scattered in general uniformly through space, the average distance between neighboring nebulae being about 1.7×10^6 light years or 1.6×10^{24} cm.

The nebulae themselves possess radii varying from one thousand light years (10^{21} cm) for spherical forms up to four or five thousand light years for very elongated nebulae which are usually accompanied by more or less opened spiral arms. (These arms often extend for more than forty thousand light years from the center.) It is usually accepted that different shapes of nebulae are due to rotation, and the spiral arms represent the material thrown out by centrifugal force from the equatorial plane. The study of nebular rotation permits estimation of their masses which come out of the order of 10^9 sun masses.[1] The observed total luminosities average 10^9 suns and the spectral class varies between $F9$ and $G4$ indicating an effective temperature slightly lower than that of the sun. From these data it can be definitely concluded that we must consider great nebulae as consisting of separate stars analogous to those of our own galaxy. In fact, if we would suppose that nebulae are continuous masses of gas emitting a continuous spectrum, then the luminosity of nebulae as

[1] E. Hubble, *The Realm of the Nebulae* (Yale University Press, 1936).

compared to that of the sun would be roughly proportional to the ratio of the surfaces. However, although the radii of nebulae are 10^{10} times larger than the sun's radius, their luminosities are not 10^{20} but only 10^9 times larger, indicating that only 10^{-11} part of their surface is actually luminous. If, on the other hand, we suppose that nebulae consist of separate stars with average luminosity somewhat smaller than that of the sun, their masses and luminosities can be easily understood.

One of the most important discoveries concerning the nebulae is that they show a definite recession-effect, which is usually interpreted in terms of the expanding universe. The velocity of recession (v) increases proportionally to the distance and we can write

$$v=\alpha R, \tag{1}$$

where α has (at present) the numerical value 1.8×10^{-17} sec.$^{-1}$.

A random motion of nebulae is superimposed on this recession but is comparatively so small that it overbalances the recession of neighboring nebulae in only a few cases.

From the values of recession velocities it can be concluded that nebulae separated from each other about 1.8×10^9 years ago, if we suppose that the relative velocity of two neighboring nebulae did not change since the time of separation (uniform expansion). Before that time, space must have been uniformly populated by stars, or by gas molecules, if we suppose that the formation of stars took place after the separation of nebulae.

II. Formation of Nebulae in an Expanding Space

We want to consider now the conditions under which the continuous distribution of matter will show the tendency to form separate nebulae. Let us consider space uniformly populated by particles (stars or molecules) moving with certain random velocities (V). The condition for the formation of condensation due to gravitational instability[2] can be written in the form

$$\frac{G\rho(4\pi R^3/3)}{R}\geqslant\frac{V^2}{2}, \tag{2}$$

where G is Newton's constant of gravitation, ρ is the density at the time in question, and R is the radius inside of which the condensation will take place. It is clear that condition (2) can always be satisfied if we choose R sufficiently large. Thus, if space was originally filled with a gas at any given temperature, the condensation of sufficiently large masses into stars was always taking place. The size of stars formed by such a process must be limited by the fact that the nuclear energy, liberated in central regions of too massive condensations, makes them unstable and leads to splitting up into smaller stars.

The situation, however, will be rather different if we accept that the distribution of matter in space is in the state of a permanent expansion as given by formula (1). In fact, under such conditions the effective velocity of particles at large distances will be mainly given by the expansion velocity, $v=\alpha R$. Substituting this velocity into Eq. (2) we obtain

$$\frac{G\rho(4\pi R^3/3)}{R}\geqslant\frac{\alpha^2R^2}{2} \tag{3}$$

or

$$\rho\geqslant 3\alpha^2/8\pi G=\rho_0. \tag{4}$$

This formula[3] no longer contains the radius of the condensing region. It shows that in an expanding space the formation of gravitational condensations can take place only when the average density is above a certain critical density ρ_0. Substituting $\alpha=1.8\times 10^{-17}$ sec.$^{-1}$ and $G=6.7\times 10^{-8}$ cm^3 sec.$^{-2}$ g^{-1} we obtain $\rho_0=0.6\times 10^{-27}$ g cm^{-3}. This density is much smaller than the present mean density in space. In fact according to astronomical estimates, the mean density of matter in space is of the order of magnitude 10^{-30} g/cm^{-3};[4] and we must conclude that, *under present conditions the formation of condensations in the universe on a great scale is impossible whatever the masses or velocities of particles may be.* Thus the great nebulae must have been formed before the average density of the universe reached a certain critical value ρ_0'.

[2] J. H. Jeans, *Astronomy and Cosmogony* (Cambridge University Press, 1928).

[3] A similar formula has been derived by G. Lemaître (National Academy of Science **20**, 12 (1934)) but his interpretation of its role in the problems pertaining to the great nebulae is rather different from ours. In his work particular attention is paid to nebular clusters.

[4] The density may be much larger if a considerable amount of dark matter is present in the intranebular space. However, there is no direct experimental evidence of such matter.

It can also be shown that existing nebulae could not have been formed a long time before the critical value of density was attained. In fact, for accumulations formed due to gravitational instability at such early stage of expansion the velocity between two neighbors would not be large enough according to (3) to separate them entirely. Therefore separation becomes possible only if the random velocities of the newly formed condensations are larger than the expansion velocities. A system of nebulae receding from each other without a strong random motion can thus be formed only when the density is close to the critical value corresponding to the given stage of expansion.

In order to find when in the past development of the universe the condition for the formation of nebulae was satisfied, we must make a definite hypothesis concerning the rate of expansion during this time. We shall accept here that *the expansion is uniform in time;* i.e., *that the relative velocity of two nebulae always remains constant.* According to (1) this means that the value of α is inversely proportional to the "dimensions" of the expanding universe. If we denote by l the length, the change of which characterizes the expansion, and denote by primed symbols the quantities corresponding to a certain passed time, we may write

$$\alpha' = \alpha l/l'; \tag{5}$$

similarly

$$\rho' = \rho(l/l')^3. \tag{6}$$

Now we can write our condition for the formation of nebulae in the form

$$\left(\frac{l}{l'}\right)^3 \rho = \frac{3}{8\pi}\frac{\alpha^2}{G}\left(\frac{l}{l'}\right)^2 \tag{7}$$

or

$$\frac{l}{l'} = \frac{3}{8\pi}\frac{\alpha^2}{G\rho} = \frac{\rho_0}{\rho}. \tag{7'}$$

Substituting numerical values for ρ and ρ_0 we find $l/l' \sim 600$, i.e., formation of nebulae took place when all linear dimensions were six hundred times smaller than at present. Since after separation the nebulae should have kept roughly their original dimensions,[5] the size of nebulae should be about six hundred times smaller than present internebular distances. This is in agreement with observation.

We proceed now to calculate the velocity of particles at the moment of formation of nebulae necessary to secure the observed dimensions of nebulae. This can be estimated from the relation:

$$V = \alpha' R = \alpha(l/l')R. \tag{8}$$

If R (the radius of a nebula) is assumed to be

$$\frac{1}{2}\times\frac{1}{600}\times 1.7\times 10^6 ly = 1.4\times 10^3 ly,$$

$$V = 1.8\times 10^{-17}\times 600\times 1.3\times 10^{21} = 100 \text{ km/sec}. \tag{8'}$$

This velocity is common in the world of stars.[6] On the other hand if we assumed that the particles participating in the formation of nebulae were gas molecules, the temperature of the gas must have been around one million degrees. At such temperatures and and at densities corresponding to the present mean densities in nebulae the mass of the radiation would be more than 10^9 times that of matter. Therefore we conclude that *most of the stars have been formed before the separation of nebulae* though, of course, further formation might continue in separate nebulae even at present.

III. Cosmological Consequences

As we have seen in the previous section, our theory of nebular formation requires that the velocity of expansion remain nearly constant while the distances between nebulae increase by a factor 600. According to the general relativistic theory of the expanding universe,[7] the factor representing the time dependence of linear dimension, usually denoted by $e^{\frac{1}{2}g(t)}$, is given by the equation:

$$\frac{d}{dt}e^{\frac{1}{2}g(t)} = \pm\left(\frac{8\pi G}{3}\rho e^{g(t)} - \frac{c^2}{\mathfrak{R}^2}\right)^{\frac{1}{2}}, \tag{9}$$

[5] Because at the time of separation they were in dynamical equilibrium and the loss of energy through various processes is relatively small.

[6] Actually observed stellar velocities in our galaxy are grouped around 30 km/sec. However, we should not forget that stars near our sun are located rather eccentrically relative to the center of galaxy and might therefore possess smaller velocities. The second reason for the present velocities of stars to be smaller than at the time of the formation of nebulae is that during its life a nebula might lose rapidly moving stars. The average velocity is thus reduced.

[7] R. C. Tolman, *Relativity, Thermodynamics and Cosmology* (Oxford, Clarendon Press, 1934).

where the cosmological constant Λ is taken to be zero.

If we use for the present mean density $\rho=10^{-30}$ g cm^{-3} and for the present expansion velocity $\alpha=(d/dt)\log e^{\frac{1}{2}g(t)}=1.8\times10^{-17}$ sec.$^{-1}$, Eq. (9) is inconsistent with the real value of $\Re$ (closed space) and also with $\Re=\infty$. The only remaining possibility is to assume an imaginary value of $\Re$. This gives for sufficiently late stages of expansion a constant expansion velocity, i.e., an infinite infinitely expanding space. For sufficiently early stages of expansion the Eq. (9) approaches the condition (4), and small density fluctuations in these early stages could have given rise to the condensations discussed above. Thus *in order to understand the formation of great nebulae and to satisfy the condition of continuity at the moment of their separation, it is necessary to accept the hypothesis that space is infinite and ever expanding.*

The only way to estimate the type of the curvature of our universe from direct observations is, at present, the method of Hubble and Tolman[8] based on the changes of observed density distribution of nebulae at great distances. Their analysis indicates that the apparent density of nebular distribution increases with increasing distance, which brings them to the conclusion that our universe has positive curvature and is closed in itself. However, we must notice that these conclusions are based on the hypothesis that the absolute luminosities of nebulae, which they use for the estimate of distances, do not change with the age of the nebulae. *If, however, we suppose that the absolute luminosities of very distant nebulae are slightly higher, because we see them at an earlier stage of evolution, the observational material no longer contradicts the assumption of an opened hyperbolic space.* At the present stage of our knowledge it is, of course, difficult to predict the expected changes of nebulae with their age, but since we deal with a period comparable to the total age of nebulae small changes are quite plausible. For example a decrease of luminosity might be accounted for by the above-mentioned possibility that the nebulae are permanently losing stars possessing too large velocities to be kept back by the gravitational attraction of the nebula.

We consider it our pleasant duty to express thanks to Mr. C. G. H. Tompkins for having suggested the topic of this paper and to H. A. Bethe for valuable discussions.

[8] E. Hubble and R. C. Tolman, Astrophys. J. **82**, 302 (1935).

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Perpendicular Vibrations of the Ammonia Molecule

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The vibration frequency ν_2 of NH_3 is identified by means of the combination bands $\nu_2\pm\nu_3$ at 2.2μ and 4.0μ, in each of which the spacing between component bands is about 16 cm^{-1}. From the difference band the numerical value of ν_2 is fixed at 3415 cm^{-1}. The fundamental band may be recognized in the weak and complex background of the absorption at 3μ. The coefficient ζ_2 of the vibrational angular momentum is approximately +0.6.

The second perpendicular fundamental band at 6μ is partially resolved, and is in good agreement with the expected pattern if ζ_4 equals -0.3. The indicated value of ν_4 is 1628 cm^{-1}. The numerical values of ζ_2 and ζ_4 do not agree well with those predicted theoretically.

The parallel component of $2\nu_4$ is found at 3220 cm^{-1}, and two pairs of parallel combination bands, $\nu_1+\nu_3$ at 4270 and 4303 cm^{-1} and $2\nu_4+\nu_3$ at 4177 and 4217 cm^{-1}.

The four atoms in the ammonia molecule, as is well known, form a pyramid with axial symmetry, and must have four different fundamental vibration frequencies. All four should be active both in infra-red and in Raman spectra. Two of these, represented by the bands at 3μ and at 10μ, involve oscillations parallel to the axis of symmetry and are readily identified. Each gives rise to a double band with P, Q and R branches, the levels occur in pairs because of the two potential minima defining alternative equilibrium positions for the nitrogen atom. A third