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Conference Paper · January 2015

DOI: 10.20906/CPS/COB-2015-1292

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Abstract. *The cart-pole system is an underactuated system, with two possible configurations: (i) Gantry, it is an stable system, but under perturbation the system shows a strong swing; (ii) Inverted pendulum, it is an inherently unstable system. The system consists on a single pole mounted on a linear cart whose axis of rotation is perpendicular to the direction of motion of the cart, it is the Linear Motion Servo Plant of the Quanser. The backstepping is a recursive design procedure which finds a Lyapunov function and a control law, this is called "backstepping", such that the controlled system is asymptotically stable. The backstepping procedure is applied to both configurations and for linear and nonlinear models. The purpose of this paper is to show that both configurations, under linear and nonlinear backstepping controllers track a defined trajectory (here there is a square wave signal) for the cart with minimum swing of the pole, as well as these controllers are appropriated to control underactuated cart-pole system of Quanser.*

Keywords: *Backstepping Controller, Cart-Pole System, Trajectory Tracking, Real-Time Application.*

1. INTRODUCTION

Underactuated systems is a very interesting field of recent research as it is well presented and classified by Olfati-Saber (2001). These systems have less actuators than the number of degree of freedom which become a challenge to the control design, but it is also an interesting way to downsize costs and/or plan an emergency strategy in case of malfunction, leading the system to an underactuated behavior. The other aspect of this work is the backstepping methodology, which is discussed in Krstic *et al.* (1995); Khalil (2002) and it was applied to control (a) chaotic dynamics Lü and Zhang (2001); Mascolo and Grassi (1999); (b) wheeled inverted pendulum Do and Seet (2010); Chiu *et al.* (2011a,b); and (c) ship courses Fossen and Strand (1999). The backstepping approach provides a recursive method to construct the feedback control law and the associated Lyapunov function in a systematic manner, such that the controlled system is asymptotically stable.

In Altinoz (2007) is designed an adaptive integral backstepping for an inverted pendulum, and the controlled system is simulated in Matlab/Simulink. In Man and Lin (2010) is applied the backstepping design scheme for the balancing control of some pendulum-like plant, which is a class of underactuated systems but do not fit the exactly structure needed for a backstepping project and those system were simulated subjected to backstepping. In Rudra (2010) is studied the stabilization problem of inverted pendulum and tracking control problem of cart-pole system, applying the backstepping control design methodology. An example of the gantry configuration can be seen in Boschetti *et al.* (2012) as two dimensional pendulum. On the other hand, there is Ebrahim and Murphy (2005) where is studied the inverted configuration controlling the angle of the pendulum.

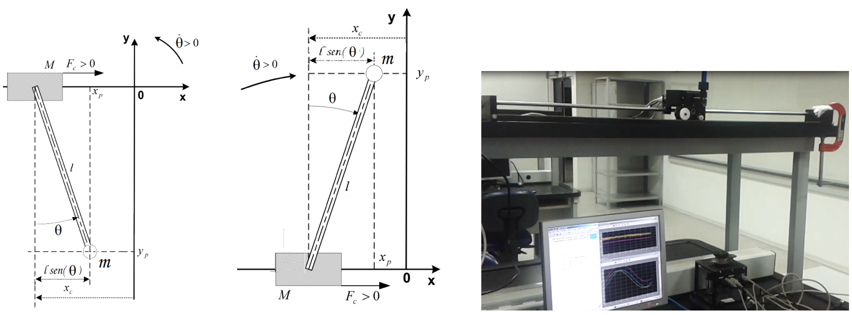
The paper is organized as follows: in Section 2 is shown the nonlinear and linear system models; in Section 3 is determined the backstepping controllers; in Section 4 is shown the computational simulations of systems under the backstepping controllers; in Section 5 is shown the controller implementations; and finally in Section 6 is shown the conclusion of the paper.

2. SYSTEM MODELS

The two configurations used in this work have the coordinates given by:

$$x_p = x_c + l \cdot \sin(\theta) \quad ; \quad y = \mp l \cdot \cos(\theta) \tag{1}$$

and are described by Fig. 1(a) named *Gantry*, where y is negative, and by Fig. 1(b) named *Inverted*, where y is positive. Figure 1(c) is a photo from the real plant. The physical parameters are: M is the mass of the cart, m is the pendulum mass, l is the length of the pole and its moment of inertia is I; gravity is present in the opposite direction on y^+-axis. The system also has a modelled friction B_{eq} for the linear coordinate and B_p for angle. The dynamic system model was



(a) *Gantry.* (b) *Inverted.* (c) Real Plant.

Figure 1. Coordinates from both configuration of Cart-Pole system.

obtained with the Lagrange-Euler formalisms which is described as:

$$\frac{d}{dt}\left(\frac{dL(q_i,\dot{q}_i)}{d\dot{q}_i}\right)-\frac{dL(q_i,\dot{q}_i)}{dq_i}=\tau_i-d_i\ ;\ \forall i \tag{2}$$

which i is the index for all generalized coordinate q_i adopted. τ_i: generalized forces; d_i: dissipative force; L: Lagrangian function defined as the difference between kinetic (T) and potential (V) energies; the L functions here are:

$$L=\frac{1}{2}\left(M+m\right)\dot{x}_c^2+ml\cos\theta\cdot\dot{x}_c\cdot\dot{\theta}+\frac{1}{2}(ml^2+I)\cdot(\dot{\theta})^2\pm mgl\cos(\theta) \tag{3}$$

where "+" describes the *Gantry* Lagrangian and "−" describes the *Inverted* one.

2.1 Nonlinear model

Both systems are described, *Gantry* and *Inverted*, with the notation from:

$$M_M(q)\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix}+\begin{bmatrix}C_{12}(q,\dot{q})\dot{q}_2\\ \beta\sin q_2\end{bmatrix}=\begin{bmatrix}u-B_{eq}\dot{q}_1\\ -B_p\dot{q}_2\end{bmatrix};\ \text{where:}M_M(q)=\begin{bmatrix}M_{11} & M_{12}(q_2)\\ M_{12}(q_2) & M_{22}\end{bmatrix}=\begin{bmatrix}(M+m) & ml\cos(q_2)\\ ml\cos(q_2) & \left(I+ml^2\right)\end{bmatrix} \tag{4}$$

as derived from the matrix notation for second order differential system, which allow us to develop a single algorithm for both configurations: $M_M(q)\cdot\ddot{q}+C(q,\dot{q})\cdot\dot{q}+G(q)=\underline{\tau}-\underline{d}$, where $M_M(q)\in\mathbb{R}^{2\times2}$ is the mass matrix, $C(q,\dot{q})\in\mathbb{R}^{2\times2}$ is the Centripetal and Coriolis matrix and $G(q)\in\mathbb{R}^{2\times1}$ is the vector of gravity term, $\underline{\tau}\in\mathbb{R}^{2\times1}$ are the conservative forces and our control u (it is the Force F_c at Fig. 1 and it is converted to voltage signal input to the motor of the cart), $\underline{d}\in\mathbb{R}^{2\times1}$ is the vector of dissipative forces like friction, which is modeled as linear dependent of the velocities, the state variable vector is defined as: $[q_1,q_2,\dot{q}_1,\dot{q}_2]^T=[x_c,\theta,\dot{x}_c,\dot{\theta}]^T$, as x_c describes the cart position and $\dot{x_c}$ its velocity, θ describes the angle between the $y-axis$ and the pole; the positive direction depends on the configuration, it is anticlockwise positive for *Gantry*, and clockwise positive for *Inverted* and $\dot{\theta}$ is the angular velocity. Finally it is necessary to define the variables $C_{12}(q,\dot{q})$ and β as follows:

$$C_{12}(q,\dot{q})=-ml\sin(q_2)\cdot\dot{q}_2\ ;\quad \beta=\pm mgl \tag{5}$$

where, "+" describes the *Gantry* and "−" describes the *Inverted* one.

2.2 Linear model

The linearized models calculated at their respective origins are described by:

$$\begin{bmatrix}m_{11} & a_{12}\\ a_{12} & m_{22}\end{bmatrix}\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix}+\begin{bmatrix}0\\ \beta\cdot q_2\end{bmatrix}=\begin{bmatrix}u-B_{eq}\dot{q}_1\\ -B_p\dot{q}_2\end{bmatrix} \tag{6}$$

for both systems, A refers to the linearized mass matrix from M_M, where $a_{12}=ml$ and m_{11} and m_{22} do not change relative to M_M, but the use of small letters means that they are from the linear model.

3. Controllers design

3.1 Backstepping linear design

Isolating $\ddot{q}_1$ and $\ddot{q}_2$ from Eq. (6), the first line $\ddot{q}_1$ is defined equal to the new control v, and the result is used to rewrite the second line denoted initially with "?". The determinant $\Delta(A)=m_{11}m_{22}-{a_{12}}^2$ comes from the linearized mass matrix A:

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \frac{1}{\Delta(A)} \begin{bmatrix} m_{22}\left(u - B_{eq}\dot{q}_1\right) - a_{12}\left(-B_p\dot{q}_2 - \beta \cdot q_2\right) \\ -a_{12}\left(u - B_{eq}\dot{q}_1\right) + m_{11}\left(-B_p\dot{q}_2 - \beta \cdot q_2\right) \end{bmatrix} = \begin{bmatrix} v \\ ? \end{bmatrix} \tag{7}$$

the control $u = u(v)$ is defined as:

$$u = \frac{(m_{11}m_{22} - a_{12}^2)v + a_{12}\left(-B_p\dot{q}_2 - \beta \cdot q_2\right)}{m_{22}} + B_{eq}\dot{q}_1 \tag{8}$$

and the new dynamical system from Eq. (7) can be written as follows:

$$\therefore \begin{cases} \ddot{q}_1 = v \\ \ddot{q}_2 = -\dfrac{(B_p\dot{q}_2 + \beta \cdot q_2)}{m_{22}} - \dfrac{a_{12}}{m_{22}}v = -\zeta_1 - \zeta_2 v \end{cases} \tag{9}$$

Then, we begin the *Backstepping* controller defining the new variable z_1:

$$z_1 = q_2 + k_1 q_1 + k_2\left(a_{12}\dot{q}_1 + m_{22}\dot{q}_2\right) \tag{10}$$

and also the first temporal derivative $\dot{z}_1$:

$$\dot{z}_1 = \underbrace{\dot{q}_2 + \underbrace{k_1\dot{q}_1 - k_2\left(B_p\dot{q}_2 + \beta \cdot q_2\right) + c_1 z_1}_{-\alpha_1}}_{z_2} - c_1 z_1 \tag{11}$$

which let us define the pseudo control α_1 and the second variable of our new state space z_2:

$$z_2 = \dot{q}_2 - \alpha_1 \quad ; \quad \alpha_1 = -k_1\dot{q}_1 + k_2\left(B_p\dot{q}_2 + \beta \cdot q_2\right) - c_1 z_1 \tag{12}$$

Now $\dot{z}_2$ is calculated:

$$\dot{z}_2 = -\frac{(B_p\dot{q}_2 + \beta \cdot q_2)}{m_{22}} - \frac{a_{12}}{m_{22}}v + k_1 v - k_2\left(B_p\ddot{q}_2 + \beta \cdot \dot{q}_2\right) + c_1\dot{z}_1 \tag{13}$$

Then, it is defined the Lyapunov function $V_1(z_1, z_2)$ and its time derivative $\dot{V}_1(z_1, z_2)$:

$$\begin{aligned} &V_1(z_1, z_2) = (z_1^2 + z_2^2)/2 \\ &\dot{V}_1(z_1, z_2) = z_1\dot{z}_1 + z_2\dot{z}_2 = z_1\left(-c_1 z_1 + z_2\right) + z_2\dot{z}_2 = -c_1 z_1^2 - c_2 z_2^2 \end{aligned} \tag{14}$$

Then, we must have $\dot{z}_2 = -c_2 z_2 - z_1$ such that $\dot{V}_1$ will be a negative definite function, the control v is written to achieve it:

$$\underbrace{\left(\frac{m_{22}k_1 - a_{12}}{m_{22}}\right)}_{\sigma_1} v = \underbrace{\frac{(B_p\dot{q}_2 + \beta \cdot q_2)}{m_{22}}}_{\sigma_2} + k_2\left(B_p\ddot{q}_2 + \beta \cdot \dot{q}_2\right) + \underbrace{-c_1\left(\dot{q}_2 + k_1\dot{q}_1 - k_2\left(B_p\dot{q}_2 + \beta \cdot q_2\right)\right) - c_2 z_2 - z_1}_{+\sigma_3} \tag{15}$$

Using the ζ_i notation from Eq. (9), it is possible to identify the control v as:

$$v = \frac{\sigma_2 + k_2\left(-B_p\zeta_1 + \beta \cdot \dot{q}_2\right) + \sigma_3}{(\sigma_1 + k_2 B_p\zeta_2)} \tag{16}$$

which is used to calculate the original control u from Eq. (8) and it is implemented using the state feedback covetor K written as $u = -K \cdot q$, $K \in \mathbb{R}^{1x4}$.

3.2 Backstepping nonlinear design

The nonlinear controller has the same idea and flow of the linear controller, so it is possible to understand the steps just comparing with the linear one. First, let us isolate $\ddot{q}_1$ and $\ddot{q}_2$. The determinant $\Delta(M_M) = M_{11}M_{22} - M_{12}^2(q_2)$ is almost the same as the previous one, because now $M_{12}(q_2)$ is no longer a simple constant.

Following the steps used for the linear controller, the variables are defined as:

$$\bar{M}(q_2) = M_{22}^{-1}(M_{11}M_{22} - M_{12}^2(q_2)) \quad ; \quad H(q,\dot{q}) = M_{22}^{-1}M_{12}(q_2)\left(-B_p\dot{q}_2 - \beta\sin q_2\right) \tag{17}$$

it is now possible to define $u = u(v)$ as:

$$u = \bar{M}(q_2)\cdot v + H(q,\dot{q}) + C_{12}(q,\dot{q})\dot{q}_2 + B_{eq}\dot{q}_1 \tag{18}$$

The new dynamical system coordinates from Eq. (4) are given:

$$\begin{cases} \ddot{q}_1 = v \\ \ddot{q}_2 = -\left(+B_p\dot{q}_2 + \beta\sin q_2\right)\cdot M_{22}^{-1} - M_{12}(q_2)\cdot M_{22}^{-1}\cdot v = -\zeta_3 - \zeta_4\cdot v \end{cases} \tag{19}$$

Then, the new variables z_3 and z_4 are defined as follows:

$$z_3 = q_2 + k_3 q_1 + k_4\left(M_{12}(q_2)\dot{q}_1 + M_{22}\dot{q}_2\right) \tag{20}$$

Before continuing, some auxiliary functions should be defined:

$$\dot{M}_{12}(q_2) = -ml\sin(q_2)\dot{q}_2 = f(q_2)\cdot\dot{q}_2 \quad ; \quad \dot{M}_{22} = \frac{d}{dt}\left(ml^2 + I\right) = 0\ ;\ L(q,\dot{q}) = \left(1 + k_4 f(q_2)\cdot\dot{q}_1\right) \tag{21}$$

Then, we calculate $\dot{z}_3$ and define z_4 and α_2 at the process:

$$\dot{z}_3 = L(q,\dot{q})\cdot\dot{q}_2 + k_3\dot{q}_1 - k_4\cdot\left(B_p\dot{q}_2 + \beta\sin q_2\right) + c_3 z_3 - c_3 z_3 \tag{22}$$

which let us define:

$$z_4 = L(q,\dot{q})\cdot\dot{q}_2 - \alpha_2 \quad ; \quad \alpha_2 = -k_3\dot{q}_1 + k_4\cdot\left(B_p\dot{q}_2 + \beta\sin q_2\right) - c_3 z_3 \quad ; \quad \dot{z}_3 = z_4 - c_3 z_3 \tag{23}$$

Again, some derivatives of auxiliary functions are computed:

$$\begin{aligned} &\dot{\alpha}_2 = -k_3\ddot{q}_1 + k_4\cdot\left(B_p\ddot{q}_2 + \beta\cos q_2\cdot\dot{q}_2\right) - c_3\dot{z}_3 \qquad ; \quad \dot{L}(q,\dot{q}) = k_4\left(f'(q_2)\cdot\dot{q}_2\cdot\dot{q}_1 + f(q_2)\cdot\ddot{q}_1\right) \\ &f'(q_2) = \frac{df(q_2)}{dq_2} = \frac{d\left(-ml\sin(q_2)\right)}{dq_2} = -ml\cos(q_2) \end{aligned} \tag{24}$$

And they are applied to calculate $\dot{z}_4$ which is written with σs functions to simplify comprehension:

$$\begin{aligned} &\dot{z}_4 = \underbrace{k_4 f'(q_2)\cdot\dot{q}_1\cdot\dot{q}_2^2}_{\sigma_4} + \underbrace{\left(k_4 f(q_2)\cdot\dot{q}_2 + k_3\right)}_{\sigma_5}\cdot v + \underbrace{\left(1 + k_4 f(q_2)\cdot\dot{q}_1 - k_4 B_p\right)}_{\sigma_6}\cdot\ddot{q}_2\,\underbrace{-k_4\beta\cos q_2\cdot\dot{q}_2 + c_3\dot{z}_3}_{\sigma_7} \\ &\dot{z}_4 = \sigma_4 + \sigma_5\cdot v + \sigma_6\cdot\ddot{q}_2 + \sigma_7 \end{aligned} \tag{25}$$

The Lyapunov function $V_2(z_3, z_4)$ for the nonlinear controller is defined like the one for the linear controller:

$$\begin{aligned} &V_2(z_3,z_4) = (z_3^2 + z_4^2)/2 \\ &\dot{V}_2(z_3,z_4) = z_3\dot{z}_3 + z_4\dot{z}_2 = z_3\left(-c_3 z_3 + z_4\right) + z_4\dot{z}_4 = -c_3 z_3^2 - c_4 z_4^2 \end{aligned} \tag{26}$$

Then, we must have $\dot{z}_4 = -c_4 z_4 - z_3$ such that $\dot{V}_1$ will be a negative definite function. Applying Equations (25), (26) and the ζs functions from Eq. (19) the new control v have been found for the nonlinear system:

$$v = \frac{\eta_1(q,\dot{q})}{\eta_2(q,\dot{q})} \quad ; \quad \eta_1(q,\dot{q}) = \sigma_6\zeta_3 - \sigma_4 - \sigma_7 - c_4 z_4 - z_3 \quad ; \quad \eta_2(q,\dot{q}) = (\sigma_5 - \sigma_6\zeta_4) \tag{27}$$

and the original control u is obtained from Eq. (18) with v from Eq. (27).

4. COMPUTATIONAL SIMULATIONS

All combinations of the two system configurations with the backstepping controllers were tested to track a square-wave signal function and a range of parameter were tested in order to find the regions with better response with a low effort of the controller, *i.e.*, low value of voltage was applied to the motor while controlling the position of the cart tracking the desired trajectory. Table 1 presents the values of the physical parameters of the pendulum system for both configurations.

Table 1. Numerical values of physical parameters.

$M\ (Kg)$	$m\ (Kg)$	$l\ (m)$	$I\ (kg\cdot m^2)$	$B_{eq}\ (N\cdot s/m)$	$B_p\ (N\cdot m\cdot s/rad)$
0,7031	0,127	0,1778	0,0012	4,3	0,0024

4.1 Backstepping linear

The linear backstepping was developed to show a response like a first order ordinary differential equation in response to the square wave reference signal. From many possible combinations, it was selected the one with settling time smaller than 3 seconds, which means that the system should move itself almost at 20 mm/s and no overshoot when achieving the desired new position, although there is always a overshoot (if we may say so) at the beginning of the movement which emerges naturally from the objective of keeping the angle oscillation as small as possible, for all tests it is always smaller than 5 degrees. Fig. 2 illustrates the performance for linear *Gantry*, and Fig. 3 for linear *Inverted*.

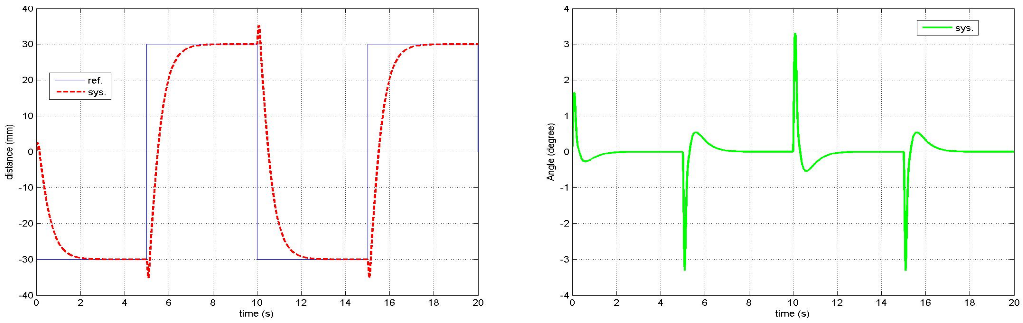


Figure 2. Simulation results for linearised *Gantry*. Parameter values: $k_1 = 4$, $k_2 = 4$, $c_1 = 4$, $c_2 = 3$.

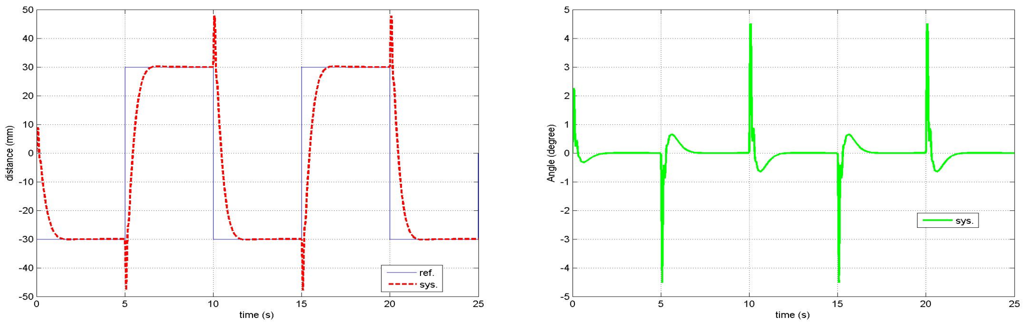


Figure 3. Simulation results for linearised *Inverted*. Parameter values: $k_1 = 4,5$, $k_2 = 3$, $c_1 = 3$, $c_2 = 3$.

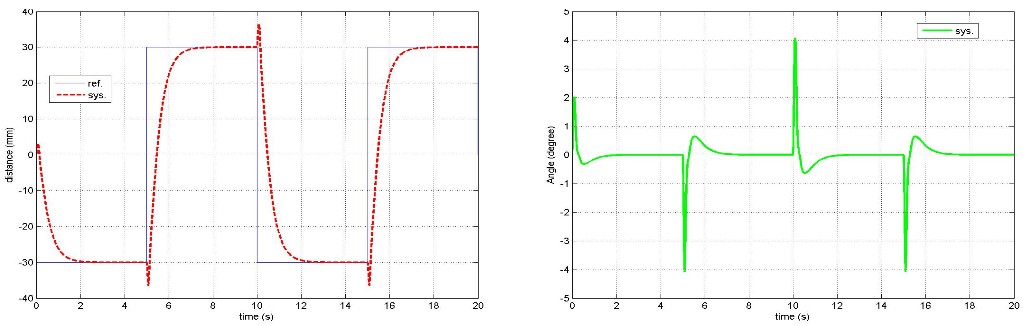


Figure 4. Simulation results for nonlinear *Gantry*. Parameter values: $k_3 = 4$, $k_4 = 2$, $c_3 = 3$, $c_4 = 5$.

4.2 Backstepping nonlinear

For *Gantry*, we chose parameters for the nonlinear controller such as we could achieve a similar response as we had for the linear controller, as it is showed in Fig. 4. For *Inverted* there were no satisfactory combination of parameters with a settling time smaller than 3 seconds, so we chose a second-order-like response under this constrain as Fig. 5 shows.

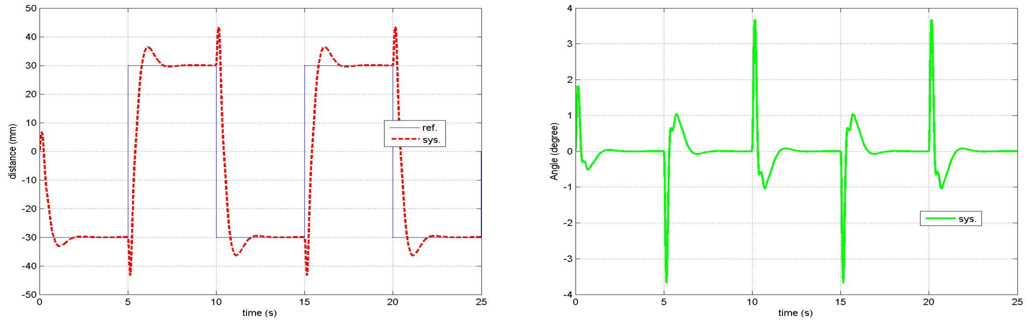


Figure 5. Simulation results for nonlinear *Inverted*. Parameter values: $k_3 = 5$, $k_4 = 1$, $c_3 = 3$, $c_4 = 2$.

5. CONTROLLER IMPLEMENTATION

Controllers were applied to the real plant and also plotted the previous simulated response to be possible to compare the desired result and the real one. Unfortunately it was possible to save only the last 10 seconds of the real test[1]. Therefore period of the reference signal was changed from the simulation, so it was possible to record at least two changes at the trajectory.

5.1 Backstepping linear

Figure 6 shows the performance of the linear *Gantry* without overshoot and 3% of steady-state error for position and for the angle it show 5 degrees of maximum variation and no measurable steady-state error. For the *Inverted*, Fig. 7 shows that the controller had in real plant the same 3% steady state error for the cart position and also a variation at the angle smaller than 5 degrees.

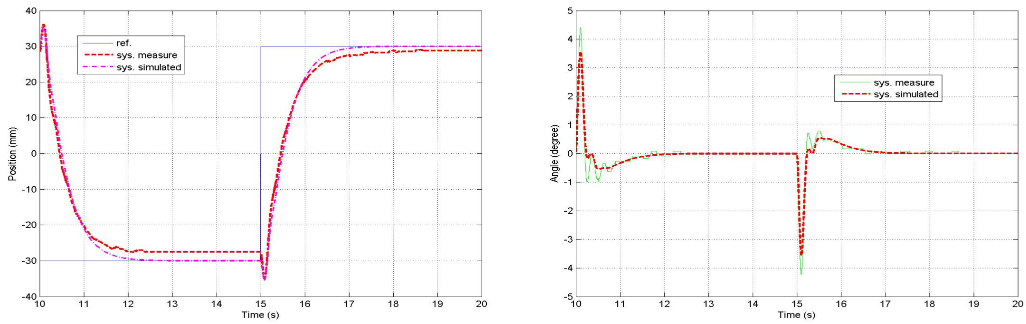


Figure 6. Real plant results for linearised *Gantry*. Parameter values: $k_1 = 4$, $k_2 = 4$, $c_1 = 4$, $c_2 = 3$. Source Fig. 2.

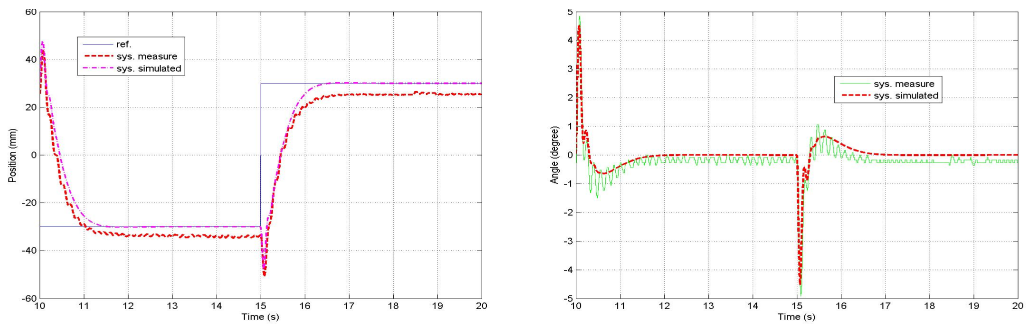


Figure 7. Real plant results for linearised *Inverted*. Parameter values: $k_1 = 4, 5$, $k_2 = 3$, $c_1 = 3$, $c_2 = 3$. Source Fig. 3.

[1]This is a feature from our plant which was designed for didactic proposes and don't let us save any data out of the gap of "last 10 seconds".

5.2 Backstepping nonlinear

Figure 8 shows the performance of the nonlinear *Gantry*, we may see the angle oscillating a little bit more than the expected from simulation with ± 1 degree at the transitions

Figure 9 shows the performance of the nonlinear *Inverted*, here we see how the real plant can behave quite different from the simulations. From Fig. 5 we expected a second order response and no great oscillations at the steady-state phase, but what happened instead was a $\pm 10mm$ position oscillation of the cart around the trajectory set point keeping the angle oscillation inside the ± 0.5 degree gap.

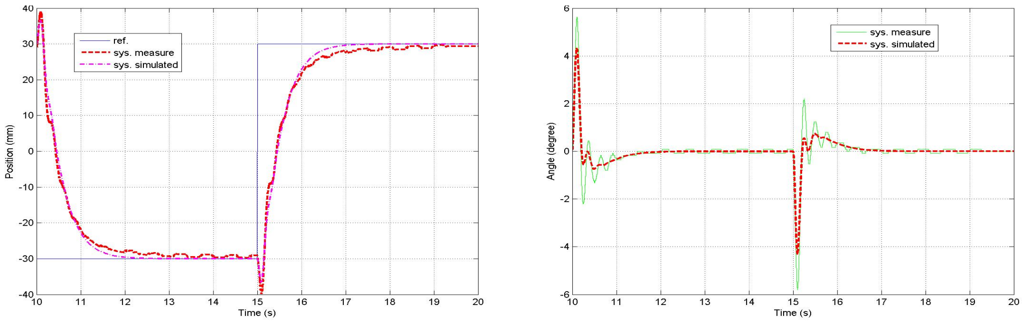


Figure 8. Real plant results for nonlinear *Gantry*. Parameter values: $k_3 = 4$, $k_4 = 2$, $c_3 = 3$, $c_4 = 5$. Source Fig. 4.

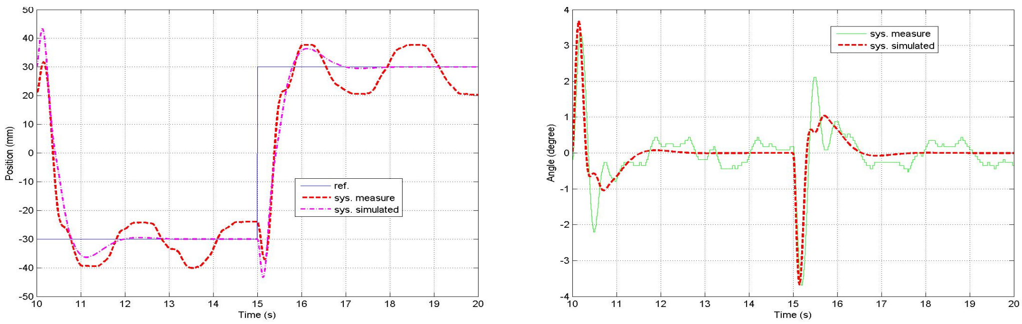
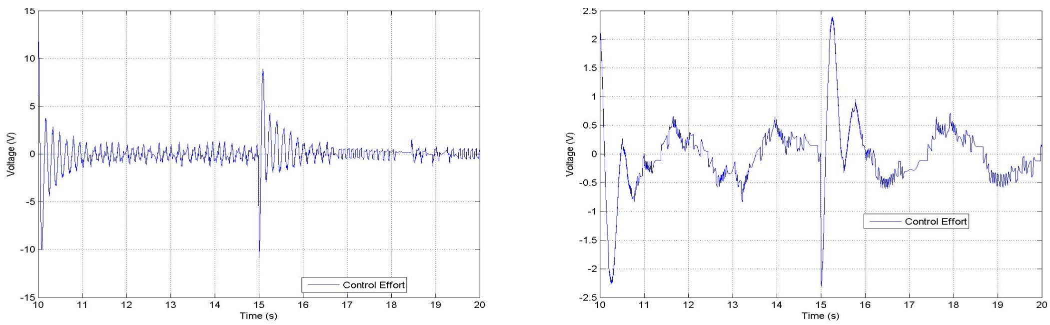


Figure 9. Real plant results for nonlinear *Inverted*. Parameter values: $k_3 = 5$, $k_4 = 1$, $c_3 = 3$, $c_4 = 2$. Source Fig. 5.

To understand this misbehaviour, we analyse the control input signal u, as measured voltage applied to the cart motor. Fig. 10 (a) presents the control input applied by the linear controller to *Inverted* and it applies an absolute value of $10V$ at the transitions of the set point, which is around four to five times the absolute value seen at Fig. 10 (b) with absolute values between 2 and $2.5V$. This difference was significant when applying the results from simulations to the real plant.



(a) Control input from Fig. 7. (b) Control input from Fig. 9.

Figure 10. Comparison between linear and nonlinear control inputs u for *Inverted*.

6. CONCLUSIONS

In this paper, the underactuated pendulum system was controlled for both configurations, *Gantry* and *Inverted*, using the linear and nonlinear backstepping design, keep the cart-pole with minimum swing ± 5 degrees while oscilation the cart position ± 60 mm in 3 seconds and also tracking a desired trajectory.

The linear controller was useful to understand the possibles behaviours, according with the parameters set. The backstepping design procedure proved to be suitable for the control of the Quanser equipment for both models and both configurations. The design of the nonlinear control law for the nonlinear control is the most complex, but it is the most robust. The research project is to extend and improve the control law, wherefore this paper shows the first results. A way to improve the control is considering an adaptive backstepping control with optimal parameters

Another future research will determine the controller parameters using an optimization algorithm, as it has been done in Sahab and Ziabari (2012), where was used different algorithm types to compute the optimal parameters for the generalized backstepping controller of DC motor system.

7. ACKNOWLEDGEMENTS

Authors would like to thanks technicians, Gustavo and Marcos, from the Control and Servomechanisms Laboratory at the Federal University of ABC.

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