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The Inverse Kinematics Solutions of Industrial Robot Manipulators

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Abstract

The inverse kinematics problem of robot manipulators is desired to be solved analytically in order to have complete and simple solutions to the problem. This approach is also called as a closed form solution of robot inverse kinematics problem. In this paper, the inverse kinematics of sixteen industrial robot manipulators classified by Huang and Milenkovic [1] were solved in closed form. Each robot manipulator has an Euler wrist (Figure 1) whose three axes intersect at a common point. Basically, five trigonometric equations were used to solve the inverse kinematics problems. Robot manipulators can be mainly divided into four different group based on the joint structure. In this work, the inverse kinematics solutions of SN (cylindrical robot with dome), CS (cylindrical robot), NR (articulated robot) and CC (selectively compliant assembly robot arm-SCARA, Type 2) robot manipulator belonging to each group mentioned above are given as an example. The number of the inverse kinematics solutions for the other robot manipulator was also summarized in a table.

1. Introduction

A robot manipulator is composed of a serial chain of rigid links connected to each other revolute or prismatic joints. A revolute joint rotates about a motion axis and a prismatic joint slide along a motion axis. Each robot joint location is usually defined relative to neighboring joint. The relation between successive joints is described by *4x4* homogeneous transformation matrices that have orientation and position data of robots. The number of those transformation matrices determines the degrees of freedom of robots. The product of these transformation matrices produces final orientation and position data of a n degrees of freedom robot manipulator. Given the sets of joint angles, calculate the position and orientation of the end-effector of the robot manipulator, is called forward kinematics [2]. Forward kinematics problem is straightforward and there is no complexity deriving the equations [5]. Hence, there is always a forward kinematics solution of a robot manipulator. Tasks to be performed by a robot manipulator are in the Cartesian space, whereas actuators work in joint space. Cartesian space includes orientation matrix and position vector. However, joint space is represented by joint angles. The conversion of the position and orientation of a robot manipulator end-effector from Cartesian space to joint space is called as inverse kinematics problem [3,4]. This relationship between joint space and Cartesian space is illustrated in Figure 2.

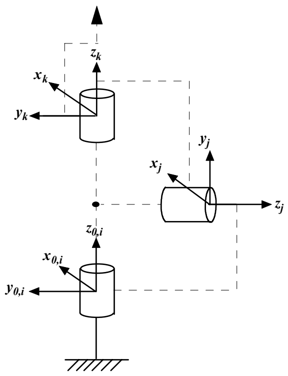

Figure 1. The configuration of Euler wrist.

The inverse kinematics problem of six joint robot manipulators has been studied for decades. The reason of this study comes from widesprade use of six jointed robot manipulators in industry. The complexity of the inverse kinematics problem of industrial robot manipulators arises from their geometry and including nonlinear equations. Some other difficulties in inverse kinematics: kinematics equations are coupled and multiple solutions and singularities may exist. Matematical solutions for inverse kinematics problem may not always correspond to physical solutions and method of its solution depends on the robot configuration [6-8].

There are three types of inverse kinematic solution: complete analytical solution (closed form solution), numerical solutions and semi-analitical solutions. In the first type, all of the joint variables are solved analytically according to given configuration data. Closed form solution is preferable because in many applications where the manipulator supports or is to be supported by a sensory system, the results from kinematics computations need to be supplied rapidly in order to have control actions. In the second type of solution, all of the joint variables are obtained iterative computational procedures. There are four disadvantages in these: a) incorrect initial estimations, b) before executing the inverse kinematics algorithms, convergence to the correct solution can not be guarantied, c) multiple solutions are not known, d) there is no solution, if Jacobian matrix is singular [9]. In the third type, some of the joint variables are determined analytically in terms of two or three joints variables and these joint variables computed numerically.

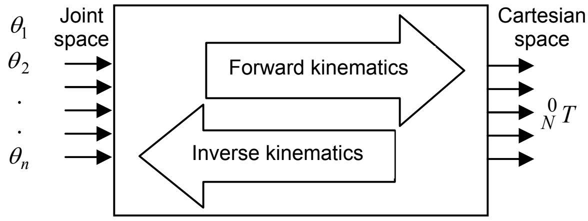

Figure 2. The schematic representation of forward and inverse kinematics.

This paper is written following manner. In Section II, the properties of homogenous transformation matrices were identified and inverse kinematics problem formulation was given. In Section III, the two-Letter code description of robot configurations were explained and the fundamental robot manipulators were classified according to their joint properties. In Section IV, the inverse kinematics solutions of SN, CS, NR and CC robot manipulator which were given in closed form and the numbers of inverse kinematics solution of sixteen robot manipulator were presented as table. Finally, the contribution of this work is insisted in Section V.

2. Problem Formulation

For a six jointed robot manipulator, the position and orientation of the end-effector with respect to the base is given by,

$$ {}^{base}_{end-effector}T = \begin{bmatrix} r_{11} & r_{21} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1) $$

where r_{ij}'s represent the rotational elements of transformation matrix (i and j=1, 2 and 3). p_x, p_y and p_z are the elements of position vector. Using complete analytical solution technique, it should be started by equating the known transformation with the product of the link transformations.

$$ {}^{0}_{6}T = {}^{0}_{1}T(q_1)\,{}^{1}_{2}T(q_2)\,{}^{2}_{3}T(q_3)\,{}^{3}_{4}T(q_4)\,{}^{4}_{5}T(q_5)\,{}^{5}_{6}T(q_6) \quad (2) $$

where q_i is the joint variable (revolute joint or prismatic joint) for joint i. To find the inverse kinematics solution, it should be solved for q_i as function of the known elements of ${}^{base}_{end-effector}T$. To find q_1 firstly as a function of the known elements, the link transformation inverses are premultiplied as follows.

$$ \left[{}^{0}_{1}T(q_1)\right]^{-1} {}^{0}_{6}T = \left[{}^{0}_{1}T(q_1)\right]^{-1} {}^{0}_{1}T(q_1) \\ {}^{1}_{2}T(q_2)\,{}^{2}_{3}T(q_3)\,{}^{3}_{4}T(q_4)\,{}^{4}_{5}T(q_5)\,{}^{5}_{6}T(q_6) \quad (3) $$

where $\left[{}^{0}_{1}T(q_1)\right]^{-1}\,{}^{0}_{1}T(q_1) = I$, I is identity matrix. Then equation 3 can be simplified as follows.

$$ \left[{}^{0}_{1}T(q_1)\right]^{-1} {}^{0}_{6}T = {}^{1}_{2}T(q_2)\,{}^{2}_{3}T(q_3)\,{}^{3}_{4}T(q_4)\,{}^{4}_{5}T(q_5)\,{}^{5}_{6}T(q_6) \quad (4) $$

To find the other joint variables, the following equations are obtained in a similar manner.

$$ \left[{}^{0}_{1}T(q_1)\,{}^{1}_{2}T(q_2)\right]^{-1} {}^{0}_{6}T = {}^{2}_{3}T(q_3)\,{}^{3}_{4}T(q_4)\,{}^{4}_{5}T(q_5)\,{}^{5}_{6}T(q_6) \quad (5) $$

$$ \left[{}^{0}_{1}T(q_1)\,{}^{1}_{2}T(q_2)\,{}^{2}_{3}T(q_3)\right]^{-1} {}^{0}_{6}T = {}^{3}_{4}T(q_4)\,{}^{4}_{5}T(q_5)\,{}^{5}_{6}T(q_6) \quad (6) $$

$$ \left[{}^{0}_{1}T(q_1)\,{}^{1}_{2}T(q_2)\,{}^{2}_{3}T(q_3)\,{}^{3}_{4}T(q_4)\right]^{-1} {}^{0}_{6}T = {}^{4}_{5}T(q_5)\,{}^{5}_{6}T(q_6) \quad (7) $$

$$ \left[{}^{0}_{1}T(q_1)\,{}^{1}_{2}T(q_2)\,{}^{2}_{3}T(q_3)\,{}^{3}_{4}T(q_4)\,{}^{4}_{5}T(q_5)\right]^{-1} {}^{0}_{6}T = {}^{5}_{6}T(q_6) \quad (8) $$

There are 12 simultaneous set of nonlinear equations to be solved. The only unknown on the left hand side of equation 4 is q_1. The 12 nonlinear matrix elements of right hand side are either zero, constant or functions of q_2 through q_6. If the elements of on the left hand side that are a function of q_1 are equated with elements on the right hand side then the joint variable q_1 can be solved as functions of r_{11}, r_{12} …..r_{33}, p_x, p_y and p_z and fixed link parameters. Once q_1 is found, then other joint variables are solved by same procedure as before. There is no necessity that first equation will produce q_1 and second q_2 etc. To find suitable equation to solve the inverse kinematics problem, any equation defined above (equations 4-8) can be used arbitrary. Some trigonometric equations used in the solution of inverse kinematics problem were given as Table in Appendix A.

3. Description of Robot Configurations

B.Huang and V. Milenkovic [1] used a two-letter code to classify a robot configuration. The first letter characterizes the first joint and the first joint's relationship to the second joint. The second letter identifies the third joint and third joint's association to the second joint. The code letters and their meanings are:

S : Slider
C : Rotary parallel to slider
N : Rotary perpendicular to rotary
R : Rotary perpendicular to rotary, or rotary parallel to rotary

The combination of these rotary and prismatic joints compose the sixteen robot configurations which are named as

SS, SC, SN, CS, CC, CR, NS, NN, NR, RC, RN, RR, RS, SR, CN and NC.

Table 1 gives the joint structure of sixteen robot manipulator (P, R, F, S, T and M represent prismatic, revolute, first, second and third joints and manipulator, respectively).

Table 1. The joint structure of sixteen robot manipulator.

M	SS	SC	SN	CS	CC	CR	NS	NN	NR	RC	RN	RR	RS	SR	CN	NC
F	P	P	P	R	R	R	R	R	R	R	R	R	R	P	P	R
S	P	P	R	P	P	P	R	R	R	P	R	P	R	R	R	R
T	P	R	R	P	R	R	P	R	R	R	R	R	P	R	R	P

The fundamental robot manipulators can be divided in to four types according to their joint properties as follows. In the first Type, there are five orthogonal robot manipulator (SS, SN, NS, NN and RR) which have three joints are perpendicular each other. In the second Type, there are four robot manipulators (CS, CR, RN and CN) whose the first two joints are parallel to each other. In the third Type, there are four robot manipulators (SC, NR, RC and NC) whose the last two joints are parallel to each other. In the fourth Type, there are three robot manipulators (CC, RS and SR) whose three joints are parallel to each other.

3. The Inverse Kinematics

In this section, the inverse kinematics of SN, CS, NR and CC robot manipulators illustrated in Figure 3 were driven. The forward transformation matrices of SN, CS, NR and CC robot manipulator used in inverse kinematics problem were given in Appendix B.

3.1. The Inverse Kinematics for SN Robot

Revolute joint variable θ_3 can be determined by equating (2,4) matrix elements each of sides (on the left hand side and right hand side) in equation 4.

$$\theta_3 = A\tan 2(\pm\sqrt{1-(\frac{d_2-p_y}{d_4})^2}\ ,\ \frac{d_2-p_y}{d_4}) \qquad (9)$$

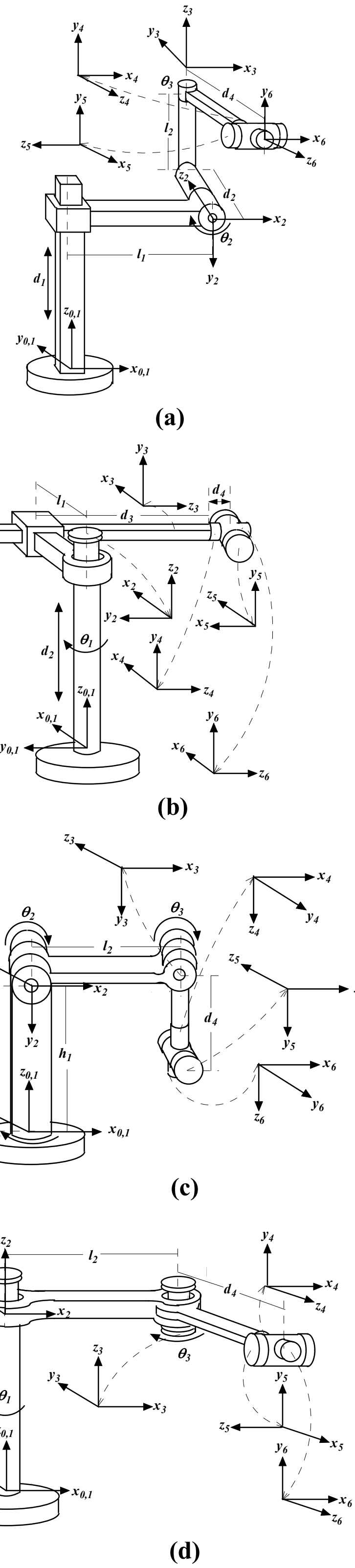

Figure 3. (a) SN robot manipulator. **(b)** CS robot manipulator. **(c)** NR robot manipulator. **(d)** CC robot manipulator.

Taking square of (1,4), (2,4) and (3,4) matrix elements of each sides and adding the results then, prismatic joint variable d_1 can be computed.

$$d_1 = p_z - \sqrt{d_4^2 + l_2^2 - 2d_2 d_4 c\theta_3 - p_x^2 - l_1^2 + 2p_x l_1 - p_y^2} \quad (10)$$

Revolute joint variable θ_2 can be found by equating (1,3) matrix elements of each sides in equation 4.

$$\theta_2 = A\tan 2(d_4 s\theta_3, l_2) \pm A\tan 2(\sqrt{l_2^2 + d_4^2 s^2\theta_3 - (p_z - d_1)^2}, p_z - d_1) \quad (11)$$

Revolute joint variable θ_5 can be determined by equating (2,3) matrix elements of each sides in equation 6

$$\theta_5 = A\tan 2(\pm\sqrt{1 - (c\theta_2 s\theta_3 r_{13} - c\theta_3 r_{23} + s\theta_2 s\theta_3 r_{33})^2}, c\theta_2 s\theta_3 r_{13} - c\theta_3 r_{23} + s\theta_2 s\theta_3 r_{33}) \quad (12)$$

Extracting $\cos\theta_4$ and $\sin\theta_4$ from (1,3) and (3,3), $\cos\theta_6$ and $\sin\theta_6$ from (2,1) and (2,2) matrix elements of each sides in equation 6, revolute joint variables θ_4 and θ_6 can be computed, respectively.

$$\theta_4 = A\tan 2([-s\theta_2 r_{13} + c\theta_2 r_{33}]/s\theta_5, [c\theta_2 c\theta_3 r_{13} + s\theta_3 r_{23} + s\theta_2 c\theta_3 r_{33}]/s\theta_5) \quad (13)$$

$$\theta_6 = A\tan 2([-c\theta_2 s\theta_3 r_{12} + c\theta_3 r_{22} - s\theta_2 s\theta_3 r_{32}]/s\theta_5, [c\theta_2 s\theta_3 r_{11} - c\theta_3 r_{21} + s\theta_2 s\theta_3 r_{31}]/s\theta_5) \quad (14)$$

3.2. The Inverse Kinematics for CS Robot

Revolute joint variable θ_1 can be determined by equating (1,4) matrix elements of each sides in equation 4.

$$\theta_1 = A\tan 2(p_y, p_x) \pm A\tan 2(\sqrt{p_x^2 + p_y^2 - l_1^2}, l_1) \quad (15)$$

Prismatic joint variables d_2 and d_3 can be attained by equating (3,4) and (2,4) matrix elements of each sides, respectively in equation 4.

$$d_2 = p_z \quad (16)$$

$$d_3 = s\theta_1 p_x - c\theta_1 p_y - d_4 \quad (17)$$

Revolute joint variable θ_4 can be determined by equating (2,3) matrix elements of each sides in equation 7.

$$\theta_4 = A\tan 2(-r_{33}, -c\theta_1 r_{13} - s\theta_1 r_{23}) \quad \text{or}$$

$$\theta_4 = A\tan 2(r_{33}, c\theta_1 r_{13} + s\theta_1 r_{23}) \quad (18)$$

Extracting $\cos\theta_5$ and $\sin\theta_5$ from (1,3) and (3,3), $\cos\theta_6$ and $\sin\theta_6$ from (2,2) and (2,1) matrix elements of each sides in equation 7, revolute joint variables θ_5 and θ_6 can be computed, respectively.

$$\theta_5 = A\tan 2(c\theta_1 c\theta_4 r_{13} + s\theta_2 c\theta_4 r_{23} + s\theta_4 r_{33}, s\theta_1 r_{13} - c\theta_1 r_{23}) \quad (19)$$

$$\theta_6 = A\tan 2(-c\theta_1 s\theta_4 r_{11} - s\theta_1 s\theta_4 r_{21} + c\theta_4 r_{31}, -c\theta_1 s\theta_4 r_{12} - s\theta_1 s\theta_4 r_{22} + c\theta_4 r_{32}) \quad (20)$$

3.3. The Inverse Kinematics for NR Robot

Revolute joint variable θ_1 can be determined by equating (2,4) matrix elements of each sides in equation 4.

$$\theta_1 = A\tan 2(-p_y, -p_x) \quad \text{or} \quad \theta_1 = A\tan 2(p_y, p_x) \quad (21)$$

Taking square of (1,4) and (3,4) matrix elements of each sides in equation 4 and adding the results then, revolute joint variable θ_3 can be computed.

$$\theta_3 = A\tan 2\left(\frac{(c\theta_1 p_x + s\theta_1 p_y)^2 + (p_z - h_1)^2 - d_4^2 - l_2^2}{2d_4 l_2}, \pm\sqrt{1 - \left(\frac{(c\theta_1 p_x + s\theta_1 p_y)^2 + (p_z - h_1)^2 - d_4^2 - l_2^2}{2d_4 l_2}\right)^2}\right) \quad (22)$$

Revolute joint variable θ_2 can be determined by equating (1,4) matrix elements of each sides in equation 4.

$$\theta_2 = A\tan 2(d_4 c\theta_3, d_4 s\theta_3 + l_2) \pm A\tan 2(\sqrt{d_4^2 + l_2^2 + 2l_2 d_4 s\theta_3 - (c\theta_1 p_x + s\theta_1 p_y)^2}, c\theta_1 p_x + s\theta_1 p_y) \quad (23)$$

Revolute joint variable θ_5 can be determined by equating (2,3) matrix elements of each sides in equation 6.

$$\theta_5 = \pm A\tan 2(\sqrt{1 - (c\theta_1 s\theta_{23} r_{13} + s\theta_1 s\theta_{23} r_{23} - c\theta_{23} r_{33})^2}, c\theta_1 s\theta_{23} r_{13} + s\theta_1 s\theta_{23} r_{23} - c\theta_{23} r_{33}) \quad (24)$$

Extracting $\cos\theta_4$ and $\sin\theta_4$ from (1,3) and (3,3), $\cos\theta_6$ and $\sin\theta_6$ from (2,1) and (2,2) matrix elements of each sides in equation 3, θ_4 and θ_6 revolute joint variables can be computed, respectively.

$$\theta_4 = A\tan 2([s\theta_1 r_{13} - c\theta_1 r_{23}]/s\theta_5\ ,\ [c\theta_1 s\theta_{23} r_{13} + s\theta_1 c\theta_{23} r_{23} + s\theta_{23} r_{33}]/s\theta_5) \quad (25)$$

$$\theta_6 = A\tan 2([c\theta_1 s\theta_{23} r_{12} + s\theta_1 s\theta_{23} r_{22} - c\theta_{23} r_{32}]/s\theta_5\ ,\ [-c\theta_1 s\theta_{23} r_{11} - s\theta_1 s\theta_{23} r_{21} + c\theta_{23} r_{31}]/s\theta_5) \quad (26)$$

3.4. The Inverse Kinematics for CC Robot

Prismatic joint variable d_2 can be determined by equating (3,4) matrix elements of each sides in equation 4.

$$d_2 = p_z \quad (27)$$

Taking square of (1,3) and (2,3) matrix elements of each sides in equation 4 and adding the results then, revolute joint variable θ_3 can be computed.

$$\theta_3 = A\tan 2\left(\frac{p_x^2 + p_y^2 - d_4^2 - l_2^2}{2l_2 d_4},\ \pm\sqrt{1 - \left(\frac{p_x^2 + p_y^2 - d_4^2 - l_2^2}{2l_2 d_4}\right)^2}\right) \quad (28)$$

Revolute joint variable θ_1 can be determined by equating (1,3) matrix elements of each sides in equation 4.

$$\theta_1 = A\tan 2(p_y, p_x) \pm A\tan 2\left(\sqrt{p_x^2 + p_y^2 - (d_4 s\theta_3 + l_2)^2}\ ,\ d_4 s\theta_3 + l_2\right) \quad (29)$$

Revolute joint variable θ_4 can be found by equating (2,3) matrix elements of each sides in equation 7.

$$\theta_4 = A\tan 2(-r_{33}\ , -c\theta_{13} r_{13} - s\theta_{13} r_{23}) \quad \text{or} \quad \theta_4 = A\tan 2(r_{33}\ , c\theta_{13} r_{13} + s\theta_{13} r_{23}) \quad (30)$$

Extracting $\cos\theta_5$ and $\sin\theta_5$ from (3,3) and (1,3), $\cos\theta_6$ and $\sin\theta_6$ from (2,2) and (2,1) matrix elements of each sides in equation 7, revolute joint variable θ_5 and θ_6 can be computed respectively.

$$\theta_5 = A\tan 2(-c\theta_4 c\theta_{13} r_{13} - s\theta_{13} c\theta_4 r_{23} - s\theta_4 r_{33}\ ,\ s\theta_{13} r_{13} - c\theta_{13} r_{23}) \quad (31)$$

$$\theta_6 = A\tan 2(-c\theta_{13} s\theta_4 r_{11} - s\theta_{13} s\theta_4 r_{21} + c\theta_4 r_{31},\ -c\theta_{13} s\theta_4 r_{12} - s\theta_{13} s\theta_4 r_{22} + c\theta_4 r_{32}) \quad (32)$$

Table 2 gives the numbers of inverse kinematics solution for sixteen robot manipulator.

Table 2. The numbers of inverse kinematics solution for sixteen robot manipulator.

M	SS	SC	SN	CS	CC	CR	NS	NN	NR	RC	RN	RR	RS	SR	CN	NC
N	2	4	8	4	8	8	8	16	16	8	16	8	4	4	8	8

N represents the number of solutions of sixteen robot manipulators.

4. Conclusion

The inverse kinematics solutions of sixteen industrial robot manipulators were found in closed form. These manipulator designs have efficient closed form solutions, because they allow for the decoupling of the position and orientation kinematics. The geometric feature that generates this decoupling is the intersection of joint axes. Having closed form solutions of robot manipulators avoids from computation cost and time consuming, while inverse kinematics solution data are evaluated in the microprocessor. Therefore, it is desirable to find robot structure that gives closed form solution. In literature, there are no criteria to test which robot structure gives closed form solution or to obtain closed form solution what kind of robot structure should be had. It can be said that six degree of freedom robot manipulators with Euler wrist always produce closed form solutions.

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Appendix A

Table 3. Some trigonometric equations and solutions used in inverse kinematics.

Equations	Solutions
$a\sin\theta + b\cos\theta = c$	$\theta = A\tan 2(a,b)$ $\mp A\tan 2(\sqrt{a^2+b^2-c^2}\,,\,c)$
$a\sin\theta + b\cos\theta = 0$	$\theta = A\tan 2(-b\ ,\ a)$ or $\theta = A\tan 2(b\ ,-a)$
$\cos\theta = a$ and $\sin\theta = b$	$\theta = A\tan 2\left(b,\ a\right)$
$\cos\theta = a$	$\theta = A\tan 2\left(\mp\sqrt{1-a^2}\ ,\ a\right)$
$\sin\theta = a$	$\theta = A\tan 2\left(a\ ,\mp\sqrt{1-a^2}\right)$

Appendix B

Table 4. The forward transformation matrices of (I) SN, (II) CS, (III) NR and (IV) CC robot manipulators.

T	I	II	III	IV
${}^0_1T$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
${}^1_2T$	$\begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & l_1 \\ 0 & 0 & -1 & d_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
${}^2_3T$	$\begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 0 \\ 0 & 0 & 1 & l_2 \\ -\sin\theta_3 & -\cos\theta_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & l_2 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & l_2 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
${}^3_4T$	$\begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 0 \\ 0 & 0 & -1 & -d_4 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 0 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 0 \\ 0 & 0 & -1 & -d_4 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 0 \\ 0 & 0 & -1 & -d_4 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
${}^4_5T$	$\begin{bmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_5 & \cos\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_5 & -\cos\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_5 & -\cos\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_5 & \cos\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
${}^5_6T$	$\begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_6 & -\cos\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_6 & -\cos\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

T represents the index of transformation matrices.

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