A Thesis Report on

MODELING, SIMULATION AND REAL-TIME CONTROL OF UNDERACTUATED SYSTEMS: A CART-PENDULUM EXAMPLE

Submitted in partial fulfilment of requirement for the award of degree of

MASTER OF ENGINEERING
IN
CAD/CAM ENGINEERING

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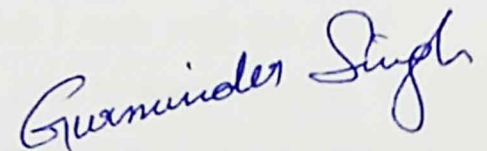
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DECLARATION

I hereby declare that work done in this thesis report entitled, "**Modeling, Simulation and Real-Time Control of Underactuated Systems: A Cart-Pendulum Example**" is an authentic record of my study carried out as requirement for the award of degree of **Master of Engineering in CAD/CAM Engineering** during 4th semester (Jan 2016 – June 2016) at Thapar University, Patiala, under the supervision of **Dr. Ashish Singla** (Assistant Professor, Mechanical Engineering Department).

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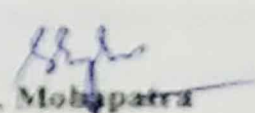
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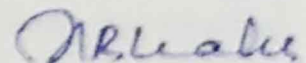Thapar University, Patiala (INDIA)

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After the first semester, department advised the whole class to choose our thesis guide on the basis of our research interest. As, I was interested in mathematics, I choose FEM field for my research career and started finding my guide. Also, I was very motivated to do work in the FEM field by Dr. I V Singh (IIT Roorkee) since their guest lecture about Advance FEM in our university. I started doing my literature survey work, which was our first work of thesis under the guidance of Dr. J S Saini (Thapar University) and Dr. I V Singh (IIT Roorkee). As, we are GATE qualified students, we have to do teaching assistant (TA) work. From the God's order, my TA load comes under the superb personality **Dr. Ashish Singla, Assistant Professor, Mechanical Engineering Department, TU, Patiala.**. Due to traveling and some my personal reasons, it became difficult to make co-ordination with my outsider guide. I was very upset, because there was nothing happening in my research subject. After lot of discussions with Dr. Ashish Singla, he gave me an opportunity to do work under him in the area of dynamics and control. However, Ashish sir knows that I had not studied any of these two subjects in my past, he gave me push and motivate to take this

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ABSTRACT

The present work deals with the modeling, simulation and real-time control of the cart-pendulum system, which is underactuated in nature. An effort is made, to control unstable nature of the cart-pendulum system. The underactuated cart-pendulum system is a typical benchmark problem in the control field, having one control input for two degrees-of-freedom (DOFs) system. It has nonlinear structure, which can be used to validate different nonlinear and linear controllers and has wide range of real-time (realistic) applications like rockets propeller, tank missile launcher, self-balancing robot, stabilization of ships, design of earthquake resistant buildings, etc. As there are broad range of applications, the study of the controllers on the cart-pendulum system should be done on the real-time basis, for significant results.

Generally, the cart-pendulum system deals with two type of problems−stabilization of pendulum at unstable equilibrium point and swing-up of the pendulum from hanging position to upward position by moving cart under the restricted track length. The literature study present in this work reveals several types of controllers developed by different researchers to address these problems. The implementation of the many controllers on the real-time basis have not been exploited well in the literature. The present thesis is an attempt to use modern control theory controllers to address the two control problems, swing-up and stabilization in the simulation tasks along with verification on the real-time experimental setup. The simulation tasks are performed in the MATLAB environment and experiments are performed on the Googoltech Linear Inverted Pendulum (GLIP) setup, which works in the SIMULINK environment. To obtain the accurate dynamic model of the system, actuator dynamics is considered. The mathematical model is developed by using Euler-Lagrange approach.

In this dissertation, modern control theory is used to address the stabilization problem of the cart-pendulum system. The main aim of this task is to investigate the performance of two different controllers in stabilizing the inverted pendulum at the unstable equilibrium point. This regulation problem is addressed by developing the Pole Placement Controller (PPC) and Linear Quadratic Regulator Controller (LQRC). The analytical results are found in close agreement with the experimental results.

Another significant contribution of this work is the swing-up control of the cart-pendulum system under the restricted track length. In this task, the performance of two

control strategies is investigated − first to swing-up the pendulum to near unstable equilibrium region and second to stabilize the pendulum at unstable equilibrium point. The swing-up problem is addressed by using energy controller in which cart is accelerated by providing a force to the cart with a AC servo motor with the help of timing pulley arrangement. The initial velocity of the cart is taken into account to confirm swing-up in the restricted track length. The cart displacement in the restricted track length is verified by simulation and experimental test-run. The second control strategy is addressed using Pole Placement Controller (PPC) and LQR Controller (LQRC), which was used in the first stage of this dissertation. The analytically and experimental results show that, the energy controller along with constrained on the linear and angular velocities is able to swing-up the pendulum from hanging position to unstable equilibrium position under the restricted track length. In order to demonstrate the effect of both the stabilizing controllers on the performance of the system, comparison of the experimental results is carried out. It is demonstrated experimentally that LQR controller outperforms the Pole Placement controller, in terms of reduction in the oscillations of the inverted pendulums, as well as the magnitude of maximum control input. Further, robustness of the closed-loop system is investigated by providing external disturbances.

Contents

List of Symbols

x = cart linear displacement,

$\dot{x}$ = cart linear velocity,

$\ddot{x}$ = cart linear acceleration,

ϕ = pendulum angular displacement,

$\dot{\phi}$ = pendulum angular velocity,

$\ddot{\phi}$ = pendulum angular acceleration,

M = cart mass,

m = pendulum mass,

l = pendulum length from pivot to center of gravity,

b = cart coefficient of friction,

f = horizontal force acting on the cart,

x_p = pendulum x-axis co-ordinate,

y_p = pendulum y-axis co-ordinate,

$\mathbf{z}(t)$ = state vector of the system,

$\dot{\mathbf{z}}(t)$ = derivative of the state vector,

$\mathbf{z_d}(t)$ = desired state vector,

K_{Cart} = kinetic energy of the cart,

K_{Pend} = kinetic energy of the pendulum,

K = total kinetic energy of the system,

P_{Cart} = cart potential energy,

P_{Pend} = pendulum potential energy,

P = system's total potential energy,

$\mathcal{L}$ = Lagrangian,

R = Rayleigh's dissipation function,

J_p = total inertia,

$\phi_m(t)$ = rotor angular displacement,

$I_m(t)$ = rotor inertia,

B_m = viscous-friction coefficient,

$T_m(t)$ = torque generated by a motor,

$T(t)$ = load torque used to move the cart through a timing pulley and a synchronous belt,

$i_a(t)$ = current winding,

K_m = torque constant,

K_b = back emf constant,

$V_c(t)$ = voltage applied to ac servo motor,

e_m = back emf,

R_m = motor resistance,

r = radius of the timing pulley,

$\boldsymbol{q}(t)$ = generalized co-ordinates vector,

$\boldsymbol{M}(\boldsymbol{q})$ = positive-definite inertia matrix,

$\boldsymbol{N}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ = Coriolis and centripetal force vector,

$\boldsymbol{g}(\boldsymbol{q})$ = gravitational force vector,

$\boldsymbol{y}(t)$ = output vector,

$\boldsymbol{u}(t)$ = general control input vector,

$\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ = generalized state weighting matrices,

$\boldsymbol{P}$ = generalized controllability test matrix,

$\boldsymbol{\alpha}$ = coefficient vector,

$\boldsymbol{K}$ = generalized feedback gain matrix,

$\boldsymbol{T}$ = transformation matrix,

$\boldsymbol{W}$ = controller companion matrix,

J_∞ = objective function of LQR controller,

$\boldsymbol{M_0}$ = solution matrix of riccati equation,

$\boldsymbol{R}$ = state weighting matrix pf LQR controller,

$\boldsymbol{Q}$ = control cost matrix of LQR controller,

E_p = total energy of the system,

$\boldsymbol{K_e}$ = control gain matrix of the energy controller,

$\boldsymbol{K_{PPC}}$ = pole placement controller gain matrix,

$\boldsymbol{K_{LQRC}}$ = LQR controller gain matrix,

u_{swing} = swing-up control input,

$u_{stabilize}$ = stabilization control input,

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[1]   Ashish Singla, Gurminder Singh, Gurvinder Singh Virk, **"Matlab/SimMechanics Based Control of Four-Bar Passive Lower-Body Mechanism for Rehabilitation,"** *Perspectives in Science*, 2016. http://dx.doi.org/10.1016/j.pisc.2016.04.072. **[In Press]**

[2]   Gurminder Singh, Ashish Singla, Gurvinder Singh Virk, **"Modeling and Simulation of a Passive Mechanism Based on Four-Bar Mechanism for Rehabilitation,"** *22nd National Conference on Mechanical Engineering and Technology (COMET-2016),* pp.124-129, IIT BHU, 2016.

[3]   Gurminder Singh, Ashish Singla, **"Modeling, Analysis and Control of a Single Stage Linear Inverted Pendulum,"** *IEEE International Conference on Robotics and Automation for Humanitarian Applications (RAHA-2016)*, Amrita University, Kerala, India, 2016. [**Paper Communicated**]

[4]   Gurminder Singh, Ashish Singla, "**Real-Time Stabilization of an Underactuated System: A Cart-Pendulum Example,"** *Transactions of the Institute of Measurement and Control*, 2016. [**Paper Communicated**]

[5]   Gurminder Singh, Ashish Singla, "**Real-Time Swing-up and Stabilization Control of a Cart-Pendulum System with Constrained Cart Movement,"** *Automatica*, 2016. [**Paper Communicated**]

[6]   Gurminder Singh, Ashish Singla, "**Effect of Actuator Dynamics on the Modeling of a Real-Time Cart-Pendulum System,"** *Simulation*, 2016. [**Paper Communicated**]

Chapter 1

Introduction

1.1 Underactuated Systems

Underactuated mechanical systems (UMS) typically possess lesser number of control inputs than their respective degrees of freedom (DOFs). In these systems, control inputs cannot accelerate the system in every direction. Underactuated systems are gaining popularity as they are found suitable in diverse applications like *marine engineering, flexible systems, aerospace systems, mobile robots, etc.* Several types of underactuated systems [1] are found in literature like cart-pendulum system [2], pendubot [3], acrobat [4], rotating pendulum [5], beam-and-ball system [6], vertical take-off and landing (VTOL) aircraft [7], etc. Underactuated systems can be classified by reasons of underactuation [8] (dynamic of the system by nature, actuator failure, etc.), by system constraints [9] (holonomic, non-holonomic), by control problems [10] (trajectory tracking and planning, set-point regulation), etc.

Development of underactuated mechanical systems is not limited to cost reduction, but possess other benefits like, lower power consumption, improved dexterity, fault-tolerant design and lower environmental impact. Following no-free-lunch policy, along with the above merits, the underactuated systems comes with some demerits also. One of the major demerits is that underactuated systems are difficult to control due to less control inputs and also they are difficult to model due to their complex structure. Due to the broad applications of the underactuated systems in different disciplines, this class of mechanical system becomes an important subject of scientific research.

1.2 A Cart-Pendulum System

In our childhood or at some point of our life, we have attempted to balance a broomstick on the palm of our hand. Inverted pendulum recalls us of broomstick, with the difference that the broomstick is free to move in the three dimensional space, while

inverted pendulum is pivoted on the cart and obliged to move in the linear plane. As like broomstick, the inverted pendulum pivoted on the cart is a highly unstable system.

In 1990, a set of real control problems is published by the International Federation of Automatic Control (IFAC) theory committee, that can be used to perform different type of control experiments to find new control methods, which are called as benchmark problems. The control of Single Inverted Pendulum (SIP) is one of them. There are generally two types of Single Inverted Pendulum (SIP) systems – rotary pendulum system and linear cart-pendulum system. The controllers for these two systems are similar in nature, but these systems have different actuator dynamics. This dissertation deals in the direction of cart-pendulum system.

Cart-pendulum system is highly nonlinear underactuated mechanical system and chosen as a typical experimental device in the control field. The cart-pendulum setup is well used for testing various types of control theories, thus tempted the recognition of many researchers and scholars. The cart-pendulum system is a two-DOF system with pivoted pendulum on the moving cart. The cart movement is constrained in the linear horizontal direction only with finite track length, whereas the pendulum can move in both horizontal and vertical planes. Due to its underactuated nature, there is only one control input to the system, which is the force acting on the cart. The AC or DC motor is used to control the linear motion of the cart with the help of timing pulley and synchronous belt, which helps to convert the rotary motion of the motor to the linear motion to accelerate the cart. There are two equilibrium points in the system, stable point in the downward direction and unstable point in the upward direction. The cart-pendulum system can be controlled at upward equilibrium indirectly by the cart actuation with the help of controllers.

Three types of control problems related to cart-pendulum system are found in literature, given as

- **Swing-up control problem** – It is a highly nonlinear problem, in which the pendulum is controlled to swing-up from hanging stable equilibrium position to unstable equilibrium point with the constrained track movement [11, 12].

- **Stabilization control problem** – It is a linear problem, in which the pendulum is lifted manually from hanging position to upward position, until it comes in the

linear stage of operation i.e. $\phi = \pm 20°$. As it is a linear problem, but it is difficult to design a controller for the stabilization of the pendulum at unstable equilibrium point (upward position) [13] .

- **Tracking control problem** – In this type of problem, cart is accelerated so that the pendulum or cart can track a desired trajectory [14]. It can be a linear or nonlinear control problem.

1.3 Applications of a Cart-Pendulum System

In practice, stabilization of the underactuated cart-pendulum system at upward position has wide range of real-time applications, as its shape and dynamics resembles many real word systems. There are also many robotics controls, which are based on the problems of the cart-pendulum system. Some of the potential applications of the cart-pendulum system are given below:

- **Mobile wheeled inverted pendulum** – Many mobile wheeled systems are designed and implemented based on the stabilization principle of inverted pendulum. Segways [15], hoverboard [16], unicycle [17] are some of the popular commercially available systems, which are used as transportation devices in the small areas.

Figure 1.1: (a) Segways, (b) Hoverboard, (c) Unicycle.

- **Stabilization of ships and rockets –** The pendulum stabilization problem upright position helps to design the control for the rocket propeller stabilization during arrival and for stabilization of the ships in the vertical position for travelling in the oceans [18].

Figure 1.2: (a) Rocket stabilization, (b) Stabilization of ships in the vertical upward direction.

- **Design of earthquake resistant building –** During earthquake, buildings experiences many disturbances. The dynamics of building during an earthquake resembles the dynamics of inverted pendulum. Hence, it is easy to design earthquake resistant buildings using robustness character of the cart-pendulum system [19].
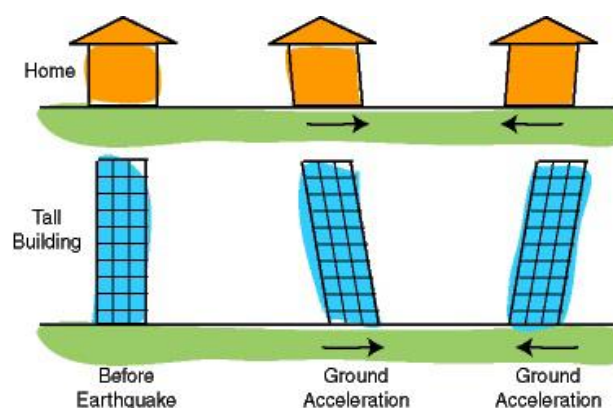

Figure 1.3: Design of earthquake resistant building.

- **Adequate model of human standing still –** For the daily activities of people, the ability to maintain stability while standing straight plays an important aspect. A human standing still is an excellent example of inverted pendulum. It's amazing

how our central nervous system (CNS) indexes the pose and alteration in the pose of the human body, and operate the muscles in order to sustain balance [20].

- **Dynamic walking** − Human walking can be considered as combination of simple pendulum and inverted pendulum. Hence to obtain dynamic walking it is important to study inverted pendulum [21].
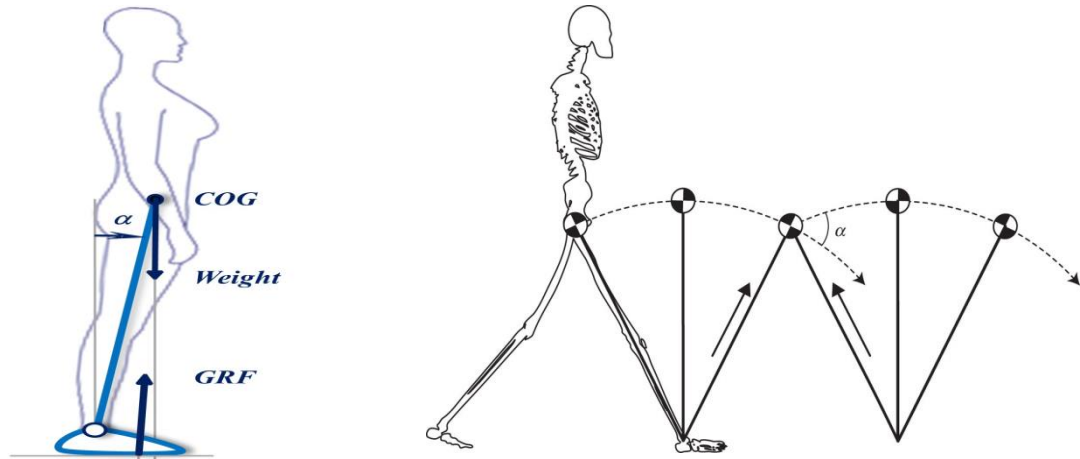
Figure 1.4: (a) Human standing still, (b) Dynamic walking.

1.4 Scope of the Thesis

Cart-pendulum system is underactuated in nature, due to one control input for the two degrees-of-freedom system. From the control aspects, cart-pendulum system plays a major role as the benchmark problem to check efficacy of new or updated controllers. Due to two equilibrium positions − unstable and stable, there are generally three types of problems for the cart-pendulum system. These are − swing-up of pendulum from hanging position to upward unstable position, stabilization of pendulum at unstable equilibrium position and tracking problem of cart and pendulum. It is important from the control and dynamics perspective to design an accurate mathematical model of the cart-pendulum system. There are different control theories available in literature to address the control problems of the cart-pendulum system like, classical control theory, modern control theory and intelligent control theory.

Stabilization of the cart-pendulum system has wide range of applications in the filled of marine engineering, flexible systems, aerospace systems, mobile robots, defense system, etc. such as segways, unicycles, stabilization of tank missile launchers,

design of earthquake resistant buildings, dynamic walking, etc. As there are many broad applications regarding stabilization problem of the cart-pendulum system, the designed controller must be checked by implementation on the real-time setups. The nonlinear nature of the swing-up problem creates a challenge for researchers to design a highly nonlinear control law with less control input magnitude for the underactuated cart-pendulum system. The robustness of the controller also plays a vital role for cart-pendulum system. As real-time example, the missile launcher of the tank must tolerate all type of disturbances and should stable at a desired direction. If the launcher is less robust, the direction will shift to other side with the movement of tank from one position to other. To study the real-time cart-pendulum system, the following challenges must be accomplished: –

- Accurate mathematical model of the underactuated system with all important aspects.
- Controllers for swing-up and stabilization problems with less control efforts on the real-time basis.

In the present thesis, an attempt has been made to address the above challenges. To derive the accurate differential equations for the modelling of the cart-pendulum system, effort has been made to study the effect of actuator dynamics on the modeling of the system. Full-order model augmented with actuator dynamics has been resulted as the accurate model for the system dynamics, which is verified by real-time experiments. Due to ability of dealing with multi-input and multi-outputs with less cumbersome mathematics, modern control theory has been preferred to address the control problems of the cart-pendulum system. The stabilization problem of the cart-pendulum system has been addressed by using two linear controllers, Pole Placement Controller (PPC) and Linear Quadratic Regulator Controller (LQRC) with the same design parameters for settling time of 2 seconds. The nonlinear swing-up problem is addressed with using energy controller having constrained on linear and angular velocities. The results and discussions has been made on the basis of the values of maximum overshoot, settling time and control input, analytically as well as real-time experimentally. As compared to the existing literature, the present work has following distinguished features:

- In order to restrict the cart movement within acceptable track length limits, the authors used the non-zero initial condition of the cart-velocity.

- To obtain more realistic results, the actuator dynamics (Panasonic AC servo motor) is considered in the mathematical modeling.
- The swing-up controller with non-zero initial cart velocity, is very easy to tune. As compared to other energy-based controller, the proposed controller uses only one parameter to be tuned.
- Algorithm is used along swing-up controller, in which constrained is used that $\dot{\phi} > 0$ if $\dot{x} < 0$ and vice-versa.
- The present work also reports the comparison of LQRC with PPC experimentally in real-time domain. It is found that LQRC outperforms the PPC in terms of reduction in control input and oscillations of cart and pendulum displacement.
- The robustness of the proposed controller is demonstrated experimentally to reject the external disturbances.

Switch condition is used to switch from non-linear to linear controller, whenever the pendulum reaches the linear area ($|\phi| < 20°$) at unstable equilibrium point. Analytical work is done by using MATLAB language by solving dynamics of the system with swing-up and stabilizing controller. Experiment work is performed on Googoltech Linear Inverted Pendulum (GLIP), which works by real-time toolbox in MATLAB and SIMULINK environment.

1.5 Organization of the Thesis

The present dissertation is organized as follows− Chapter 2 represents the detailed literature survey of the cart-pendulum system. The survey starts, with the modeling of the cart-pendulum system, where the different modeling techniques used by researchers are discussed. Further, the controls used to address the stabilization and swing-up problem of cart-pendulum system are mentioned. The controllers developed by different researchers for the cart-pendulum system are classified into three theories− classical, modern and artificial intelligence control theory. The conclusions from the literature survey is summarized in the end of this chapter.

Chapter 3 deals with the cart-pendulum system dynamics. The mathematical model is developed by using Euler-Lagrange method. The actuator dynamics is augmented with cart-pendulum model for better accuracy. Further, the real-time

apparatus Googoltech Linear Inverted Pendulum (GLIP) is discussed along with system parameters, which has been used for the experimental work in this thesis.

Chapter 4 presents development of the stabilization and swing-up controllers for the cart-pendulum system. The important aspects of the controller design are discussed in the first part of the chapter, which plays a vital role to develop controllers. The topics include linearization of the dynamic model, state-space representation of the linearized model and controllability of the system. First, the mathematical model is linearized to calculate state-space representation. Further, three different controllers are discussed, which has been used for the control scheme in this thesis. Pole Placement and LQR controllers are discussed for the stabilization problem and energy controller with a constraint on linear and angular velocities is mentioned for the swing-up problem.

Chapter 5 deals with the stabilization problem of the cart-pendulum system at the unstable equilibrium point. The chapter follows the control scheme, in which stabilization is considered as a regulation problem. The Pole Placement Controller (PPC) and Linear Quadratic Regulator (LQRC) are used for this control scheme. Both controllers are designed with the step response settling time of 2 seconds. Further, simulation and experimental results are carried out on the basis of cart displacement response, pendulum displacement response and control input response.

Chapter 6 presents the swing-up problem of the cart-pendulum system from hanging position to unstable equilibrium point. Two-step control scheme is used to address this problem. First, the swing-up is carried out with energy controller with some constraints on linear and angular velocities, then PPC and LQRC are used for stabilization of the pendulum at upward position. Switch method is used to switch between nonlinear and linear controllers. Further, result and discussions are drawn on the basis of simulations and real-time experiments.

Chapter 7 represents the closing remarks and future direction of this thesis.

Chapter 2

Literature Survey

2.1 Introduction

In this chapter, the literature related to modeling and control of cart-pendulum system is reviewed. The direction of the literature survey is towards the cart-pendulum system, which has wide range of real-time applications in the field of *marine engineering, flexible systems, aerospace systems, mobile robots, etc.* In spite, the structure of the cart-pendulum system is simple, it is considered as the fundamental benchmark example among the other underactuated examples, which are mentioned in the Chapter 1. In 1844, James Forbes used the concept of an inverted pendulum in the design of seismometer. Forbes used the fact that the upright equilibrium position is very sensitive to disturbances due to its unstable nature, which is the earliest notification in the literature of an inverted pendulum [22].

There are different versions exist in the family of cart-pendulum, which create interesting control challenges for research scholars and scientists. In defiance of a cart-pendulum system, which have single pendulum, there are also other versions exist in the literature like cart with rotational double link pendulum [23], the parallel type dual inverted pendulum [24], the quadruple inverted pendulum [25], the triple inverted pendulum [26] and 3D pendulum [27], which is generally known as a spherical pendulum. These versions of cart-pendulum systems are not widely common. Due to diverse applications of single cart-pendulum system such as rocket propeller, tank missile launcher, self-balancing robot, stabilization of ships, the design of earthquake resistant buildings, etc., it is most useful version of the cart-pendulum system family. The orientation of the dissertation is basically towards the single cart-pendulum system. In the present chapter, works are classified on the basis of modeling techniques, control of cart-pendulum system which is further classified in to three different control theories – classical control theory, modern control theory and intelligent control theory.

2.2 Modeling of a Cart-Pendulum System

The first step in the design and control of any physical system is to establish the accurate model of the system. There are different types of methods available in the literature to model the dynamic cart-pendulum system. Generally, mathematical techniques are preferred to model cart-pendulum system over the other methods like computer software based or graph based methods. Modeling of cart-pendulum system involves the mathematical model of two-degree freedom system. There is involvement of modeling of different types of forces and energies in the system, which is the challenging task in the mathematical modeling of this system. The mathematically modeling of the cart-pendulum system is classified into two types − Newton-Euler method and Euler-Lagrange method. The mathematical model of the cart-pendulum system can be represented in *"Transform function form"* or *"state-space form"*, it depends upon whether the system is a SISO (Single-Input-Single-Output) system or a MIMO (Multi-Input-Multi-Output) system. The two mathematical techniques are broadly discussed in the following sub-sections.

2.2.1 Newton-Euler Method

In Newton-Euler method, the cart-pendulum system is modeled on the basis of forces acting on it. This method is based on the famous Newton's second law, which gives the exact relationship between force, mass and acceleration of the dynamic system [28]. The law states that, *"The acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass of the object"*. It involves all the forces and moments acting on the individual system links. The equations obtained from the Newton-Euler method include the constraint forces i.e. external forces, gravitational force, Coriolis force, damping force, etc., acting between adjacent links. This method is widely used by many researchers [29, 30] to model the cart-pendulum system. Due to involvement of several types of forces, some authors [31, 32] neglect the effect of air friction and force acting on the cart due to pendulum's action. Kennedy [33] discuss the effect of different types of friction on the modeling of the cart-pendulum system to produce less oscillatory behavior during stabilization.

2.2.2 Euler-Lagrange Method

In Euler-Lagrange method, the system is described in the terms of work and energy using kinematics (generalized co-ordinates). The kinetic and potential energies of the system is used to formulate the equation of motion of the system [34]. In deriving the differential equation of the system, it needs to define generalized coordinates and the Lagrangian. The resultant equations provide a closed-form expression in the terms of torques and displacements.

The Lagrangian is defined as

$$\mathcal{L}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t) = T(\boldsymbol{q}, \dot{\boldsymbol{q}}, t) - V(\boldsymbol{q}, t) \tag{2.1}$$

where $\mathcal{L}$ is the Lagrange operator, $\boldsymbol{q}$ is vector of the generalized coordinates, T denotes the Kinetic energy and V denotes potential energy of the system.

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = f_i \quad \forall \quad i = 1,2,\dots,n \tag{2.2}$$

where f_i is the ith generalized force acting on the system and R is Rayleigh's dissipation function. Different author's [35, 36] has used this method to model the cart-pendulum system mathematically. This method provides easier derivative calculations to calculate differential equations of the system.

In spite of the above two methods, there are other methods available in the literature to develop the dynamic model like bond-graph [37] and modeling using commercial software like SimMechanics [38, 39], which are not so popular.

2.3 Control of a Cart-Pendulum System

Since 1950s, the cart-pendulum system is the benchmark example for teaching and studying different types of control theories [40]. Because of its wide range applications, there are many type of controls available in literature to address the cart-pendulum system unstable problem. Boubaker [41] have briefly discussed, the variety of controllers with their advantages and disadvantages for the inverted pendulum in a survey article, which was highly cited. However, many controllers available in published literature has been tested in the simulation environment only, not

demonstrated on real-time experimental setups. The simulation results are often found to be different, when these are compared with real-time experiment results. This is due to the use simple model of the cart-pendulum system in the simulations. Moreover, most of the simulations ignore the physical restrictions like constrained track length, etc., effects of friction and additional dynamics like of actuator, timing pulley, gears, etc. In this dissertation, the popular control methods for cart-pendulum system are classified into three categories, on the basis of their parent theories, which are discussed in next subsections.
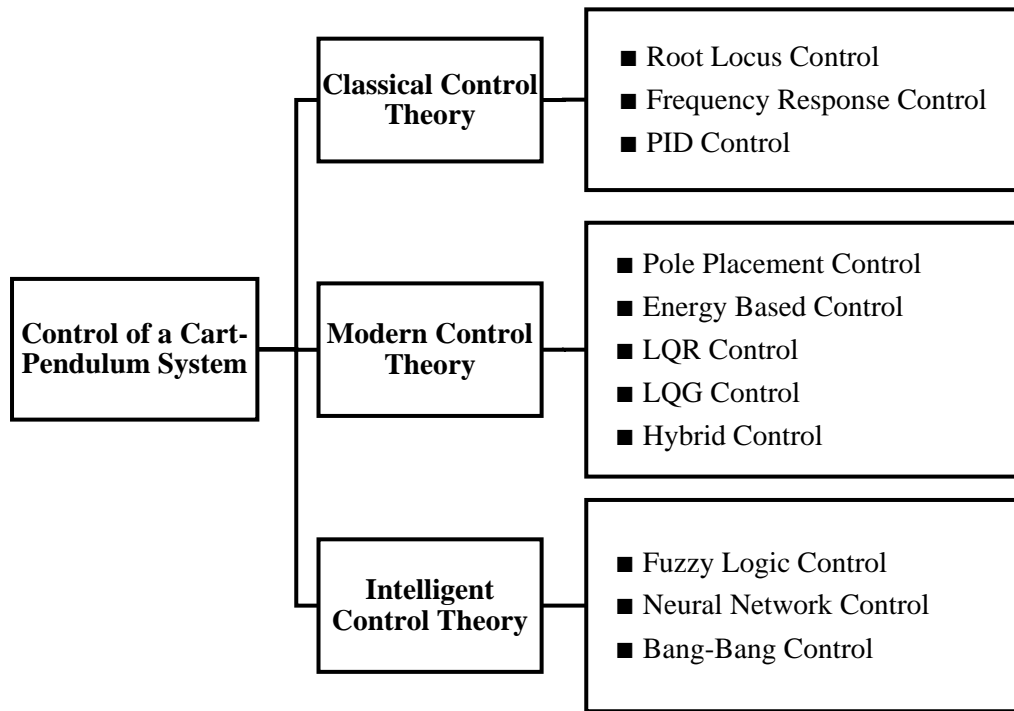

Figure 2.1: Different types of control theories.

2.3.1 Classical Control Theory

Classical control theory is the branch of control theory, that deals with the characteristics of dynamic systems in *frequency* domain. Generally, this type of theory is applied on SISO (Single-Input-Single-Output) systems [42]. The major advantage of using this theory on the cart-pendulum system is that, the controllers under this theory are easy to apply. Due to the use of transfer function in this theory, there is a pole-zero

cancellation problem, which causes difficulty in stabilizing the cart-pendulum system. Three different examples of classical control theory are discussed below.

A. Root Locus Control

Root locus control method is used for stability analysis of the closed loop system. It determines the stability of the system by investigating the return difference (property of feedback loop) [42]. System stability is checked by calculating roots of the characteristic equation of the return difference, whether the roots are positive (unstable system) or negative roots (stable system). System is controlled by designing locus around the unstable roots to stable the system. Root locus method has been well applied on the cart-pendulum system with help of transfer function concept by different authors [43, 44, 45].

B. Frequency Response Control

Nyquist and Bode plots are used in the frequency response control method to check the stability of the system. Generally, the system response to the sinusoidal signal is called as frequency response. In this control method, the system input signal frequency is varied to certain range to stable the cart-pendulum system. The experimental and simulation results have been well performed and mentioned in the Googoltech Linear Inverted Pendulum manual [43].

C. PID Control

Proportional Integral Derivative (PID) is the simplest closed-loop controller of the classical control theory, which has been extensively used on the cart-pendulum system for the stability. PID controller works as a feedback controller, which calculates the error signal between output and desired output reference. It works on three control gain values− proportional gain (K_p) for present value of error, integral gain (K_i) for the past value of error and derivative gain (K_d) for possible future value of the error. Many researchers have used PID controller to address the stabilization problem of the pendulum to unstable equilibrium position [46, 47]. This method is well used for cart-pendulum system stability, but it requires frequent tuning. Self-tuning PID controller is applied on the cart-pendulum system with the help of Lyapunov function by Chang et

al. [48], but the whole research was based on simulations only, not verified by applying control law on the experimental setup.

2.3.2 Modern Control Theory

Modern control theory is the branch of the control theory that deals with the MIMO type of systems. This type of theory is based on *time* domain, in which the whole system deals in state-space representation [28]. Modern controllers do what they have told or in other words, modern controllers work in a manner designed by designer. Some controllers which are widely used to stable cart-pendulum system are summarized below.

A. Pole Placement Control

Pole placement control technique has been extremely used for the regulation problems of the systems. In this technique, the eigenvalues of the system are placed at the desired place by designing a full-state feedback controller. Roshdy et al. [24] proposed the pole separation factor to stabilize the real-time cart-pendulum system [49]. The work is done experimentally to ensure robustness and stability. Song et al. [50] performed the comparison study of PID controller, pole placement controller and fuzzy logic controller on the cart-pendulum system, which concluded that real-time control using PID control results in small overshoots, whereas state-space pole placement method results in good dynamic and steady state performance.

B. Energy Based Control

Energy based control method is one of the popular method to address the swing-up problem of the cart-pendulum system, which is nonlinear in nature. The main study on the swing-up problem was conducted by Astrom and Furuta [51]. Energy controller as proposed by them to address the swing-up problem, in which the pendulum energy is made to converge to zero at upright unstable equilibrium position by providing a control law in the form of cart acceleration. However, the cart track length was not taken into account. Lozano et al. [52] proposed a Lyapunov function to address swing-up problem by considering the cart track length into account. The function is derived by sum of squares of mechanical energy, cart position and cart velocity. The swing-up control law has four parameters, which are difficult to tune. Chatterjee et al. [53] used a potential

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well concept to address the cart length limitation problem. The control law constructed in such a way that as the cart reaches the boundaries, it experiences a repulsive force. The simulation and experimental results taken by the proposed controller shows that their method is effective. Major limitation in their control law is to design five coefficients for the potential well by trial and error, which is highly cumbersome.

This problem is addressed by Yang et al. [54] by using control law to solve the infinite track length problem, which is given by sum of square of pendulum energy and weighted square of cart velocity. The complete proof by using LaSalle's theorem is reported in the paper. As compared to previous controllers, Yang et al. [54] used two parameters only, which makes the tuning process easy during simulation and experimentation. However, in their work, the stabilization was not discussed and the actuator dynamics was not taken into account, while designing the Lyapunov function. There are many other researchers [55, 56, 57] used Lyapunov based energy controller to address swing-up issue of the cart-pendulum system.

C. LQR Control

Linear Quadratic Regulator (LQR) is a feedback type of controller, which is simple and easy to apply to solve the stabilization problem of the cart-pendulum system at unstable equilibrium position [58]. It is an optimal control used to minimize the cost associated with generating control inputs. The cost function consists of two matrices, Q and R, the state weighting matrix and control cost matrix. For a linearized plant, the objective of LQR controller is to minimize the integration of sum of these two matrices by using a feedback regulator. The main limitation of the LQR controller is to design state weighting matrix and control cost matrix for the cart-pendulum system. Generally, the tuning of the design parameters is tuned manually, which is quite cumbersome. Trimpe et al. [59] proposed a stochastic optimization algorithm based self-tuned LQR approach to stable the cart-pendulum system at unstable equilibrium point. The study was done on the simulation based test runs, not on any real-time experiment setup. There are also other optimization techniques [60, 61], which are used by researcher to develop self-tuned LQR controllers.

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D. LQG Control

Linear Quadratic Gaussian (LQG) is a controller in which LQR technique is augmented with Kalman Filter technique. Two different techniques are combined to improve disturbance rejection. Eide et al. [62] used this method on mobile inverted pendulum robot, however, results show that LQR gives better response when superimposed to LQG approach.

E. Hybrid Control

Hybrid controls are based on energy method that accomplish both the stabilization and swing-up problems of the cart-pendulum system. This control approach does not need any switching technique to switch from nonlinear to linear controllers. Many researches [63, 64, 65] has used the hybrid control approach to solve swing-up and stabilization problem combined.

2.3.3 Intelligent Control Theory

Intelligent control theory is based on the various artificial intelligence computing approaches [66]. Artificial intelligence control techniques are used to solve problems of the dynamic system by observing the behavior of the system by feedback response. Intelligent controllers do what it wants to do or in other words, the intelligent controllers itself re-controlled to achieve the objective. This type of control theory usually avoids unnecessary and lengthy calculations.

A. Fuzzy Logic Control

Fuzzy logic control work on the principle of many-value logic, in which the truth values can take any real value between 0 and 1 [66]. This control law can be used along with the controllers based on modern control theory to solve high nonlinear problems. Muskinja and Tovornik [12] proposed a method for swing-up problem of cart-pendulum system under restricted track length by augmenting energy controller with fuzzy logic control. The proposed work was demonstrated on real-time cart-pendulum system. In the recent article, Elsayed et al. [67] demonstrated the fuzzy swing-up method augmented with energy controller for swing-up problem of cart-pendulum system experimentally and verified their results by mathematically technique. Sliding mode controller (SMC) and Linear Quadratic Controller (LQRC) were used to stabilize

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the pendulum at unstable equilibrium position, in which SMC outperforms the LQRC with less control input value.

B. Neural Network Control

Neural network control is based on the structure of nervous system of human being [68], in which signal is transfers from one neuron to other neuron. The control method acts as feedback controller with less mathematical equations. Neural network control can be used along with classical and modern control theories to control the unstable dynamic systems. Omatu et al. [69] used neural network control method integrated with PID control for better robustness of the stable cart-pendulum system. Many researchers [70, 71] have used optimization techniques along with neural network controller to stabilize the cart-pendulum system at the unstable equilibrium point.

C. Bang-Bang Control

Bang-Bang control system can be used along with the controllers based on classical and modern control theories to increase the robustness of the system. Bang-bang theory works between two bounds, upper and lower. Binary numbers are used to create saturation functions in the bang-bang controller. Muskinja et al. [12] has used bang-bang controller along with energy controller to increase the robustness of the cart-pendulum system to resist any disturbances during swing-up of the pendulum from hanging position to upward position. Furuta et al. [72] proposed a feedback control method using bang-bang theory for the swing-up problem and has used LQR controller to address stabilization problem of the cart-pendulum system.

There are many other type of controllers exists in literature, to address the problems of the cart-pendulum system, such as, sliding model control [73, 74], approximate linearization based controllers [75, 76, 77], State-Dependent Riccati Equation (SDRE) based controller [78, 79], Pulse Width Modulation (PWM) [80], PV controller [81], etc.

2.4 Conclusions from Literature Review

Through the literature review of the works reported by many authors on the modeling and control of the underactuated cart-pendulum system, following observations are drawn.

- Most of the authors have reported the system dynamics derivation of the cart-pendulum system using mathematical techniques – Newton-Euler technique and Euler-Lagrange technique. Due to nonlinear nature, the structure of the cart-pendulum system is complex. Many authors have not derived the actuator dynamics, which can play a vital role in the modeling of the cart-pendulum system.

- To check the efficacy of the proposed controllers, several authors in the past has used experimental way. To make an accurate and better controller, it has been reviewed that the validation of the proposed controllers is necessary, so that they can be used in the real-time world applications.

- It has been observed that more than 12 experiments [43] can be performed on the cart-pendulum system, out of which 77.8% can be performed using classical control theory, whereas the complete 100% can be performed using modern control theory. The classical control theory is difficult to implement on the complex systems whereas, the modern control theory is easy to apply, simple and less time consuming than intelligent control theory.

- In order to address the swing-up problem of the cart-pendulum system, it has been reviewed that energy controller is efficient than the other controllers. The major problem by using energy controller is to swing-up the pendulum in the restricted track length. Many authors have successfully proposed controllers to swing the pendulum to upward position within the track by using different algorithms or techniques augmented with energy technique. These methods are reported cumbersome to use or to implement on real-time systems.

In this thesis, author has studied the effect of actuator dynamics on the modeling of cart-pendulum system by using Euler-Lagrange mathematical technique for deriving differential equations. Further, author has addressed two control problems of the cart-pendulum system− swing-up problem, which is nonlinear in nature and stabilization problem, in which selection of controller is the major issue. The energy controller with constraint on linear and angular velocities of the system is used to control the swing-up problem. Pole Placement and LQR controller from modern control theory are used for the stabilization of the pendulum at unstable equilibrium point. These two controllers are compared to check the efficacy of the controllers.

Chapter 3

System Dynamics

3.1 Introduction

The cart-pendulum system is having two degrees of freedom – cart linear displacement and pendulum angular displacement. The cart is free to move in horizontal linear direction with pendulum pivoted on it, which can move in both vertical and horizontal planes. There is only one control input to the cart-pendulum system i.e. force acting on the cart in horizontal direction, which makes the system underactuated in nature. The stabilization problem of the cart-pendulum system has broad range of real-time applications in the field of robotics, marine engineering, defense, etc.

To control any system, the first task is to derive an accurate dynamic model of the system. The cart-pendulum system is highly nonlinear in nature having both linear and angular motions. Different methods are available in literature to model the cart-pendulum system, which are discussed in the literature review section of Chapter 2. The cart-pendulum system can be model by using computational software's such as SimMechanics. Simwise 4D, etc. There are also different methods like bond graph methods and mathematical methods to design the dynamic model of the cart-pendulum system. In this dissertation, the mathematical approach is used to model the dynamic model of the cart-pendulum system. As mentioned in Chapter 2, there are two popular mathematical approaches are available in literature – Newton-Euler method and Euler-Lagrange method. Newton-Euler approach is based on the Newton second law of motion and Euler-Lagrange approach is based on energy method of calculating Lagrangian.

This chapter represents the differential equations of the cart-pendulum system. The Euler-Lagrange method is used to derive the mathematical model. The actuator dynamics is augmented with the cart-pendulum system dynamics to design an accurate model. Further, Googoltech Linear Inverted Pendulum (GLIP) is discussed, which is used for the experimental verifications of the controllers in this dissertation. Summary of the system dynamics is drawn at the end of this chapter.

3.2 Mathematical Modeling of Cart-Pendulum System

The schematic diagram of the cart-pendulum system is shown in Fig. 3.1, which is a two DOF system with pivoted pendulum on the moving cart. Let, M and m represent the mass of cart and pendulum respectively, l represents the distance from the rod axis rotation center to the rod mass center, b represents the coefficient of viscous friction of the cart. Moreover, $x(t)$ represents the horizontal position of the cart from static equilibrium position, $\phi(t)$ represents the angular position of the inverted pendulum from the vertical axis, and $f(t)$ is the horizontal force acting on the cart. Further, $x_p(t), y_p(t)$ represents the position of the center of gravity of the pendulum.

The model is derived with assumption that there is only viscous friction. Air friction and force on cart due to pendulum's action are considered negligible. For brevity, time dependence of the relevant variables is shown at important steps only.
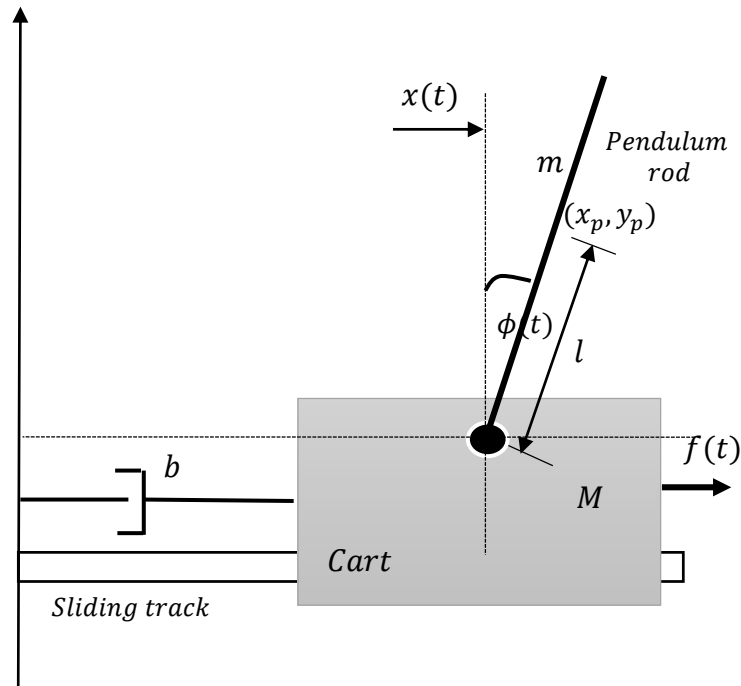
Figure 3.1: Schematic diagram of the cart-pendulum system.

The state vector of the system is taken as

$$\mathbf{z}(t) = \begin{bmatrix} x_p \\ y_p \end{bmatrix} = \begin{bmatrix} x + l \sin \phi \\ l \cos \phi \end{bmatrix}, \tag{3.1}$$

and its derivative is

$$\dot{z}(t) = \begin{bmatrix} \dot{x}_p \\ \dot{y}_p \end{bmatrix} = \begin{bmatrix} \dot{x} + l \cos \phi \, \dot{\phi} \\ -l \sin \phi \, \dot{\phi} \end{bmatrix}. \tag{3.2}$$

3.2.1 Kinetic Energy

Total kinetic energy of the system is given as the summation of translational kinetic energy of cart and rotational kinetic energy of pole, given as

$$
\begin{aligned}
K &= K_{Cart} + K_{Pend} \\
&= \frac{1}{2} M \dot{x}^2 + \frac{1}{2} m \left(\dot{x}_p{}^2 + \dot{y}_p{}^2 \right) + \frac{1}{2} I \dot{\phi}^2, \\
&= \frac{1}{2} (M + m) \dot{x}^2 + \frac{1}{2} m l^2 \dot{\phi}^2 + m l \cos \phi \, \dot{\phi} \dot{x} \\
&\quad + \frac{1}{2} I \dot{\phi}^2.
\end{aligned}
\tag{3.3}
$$

3.2.2 Potential Energy

The potential energy contains two components generally, i.e. the gravitational potential energy and elastic potential energy. The first one accounts for relative position of the rigid body in space and the second one accounts for the strain energy stored in the elastic member of the system due to its deformation. As the cart moves in horizontal direction only, thus the contribution to potential energy comes from the pendulum only.

The total potential energy of the system is given as

$$
\begin{aligned}
P &= P_{Cart} + P_{Pend} \\
&= 0 + (-mg(l - l \cos \phi)).
\end{aligned}
\tag{3.4}
$$

3.2.3 Lagrange Equations

Lagrange $\mathcal{L}$ of the system is given by subtraction of total kinetic energy and total potential energy of the system, given as

$$\mathcal{L} \quad = \quad K - P$$

$$= \quad \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}ml^2\dot{\phi}^2 \ + ml\cos\phi\,\dot{\phi}\dot{x} \qquad (3.5)$$

$$+ \ \frac{1}{2}I\dot{\phi}^2 \ + mg(l - l\cos\phi).$$

To drive damping force of the system due to frictional effects, Rayleigh's dissipation function is taken into account as

$$R = \frac{1}{2}b\dot{x}^2. \qquad (3.6)$$

Due to two generalized coordinates, the two equation of motion of the system are given as

$$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{x}}\right) - \frac{\partial\mathcal{L}}{\partial x} + \frac{\partial R}{\partial\dot{x}} = f, \qquad (3.7)$$

$$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{\phi}}\right) - \frac{\partial\mathcal{L}}{\partial\phi} + \frac{\partial R}{\partial\dot{\phi}} = 0. \qquad (3.8)$$

By substituting the values of Lagrangian and Rayleigh's dissipation function from (3.5) and (3.6) into in (3.7) and (3.8), the equations of motion are calculated as

$$(M+m)\ddot{x} + ml\cos\phi\,\ddot{\phi} - ml\sin\phi\,\dot{\phi}^2 + b\dot{x} = f, \qquad (3.9)$$

$$J_p\ddot{\phi} + ml\cos\phi\,\ddot{x} - mgl\sin\phi = 0, \qquad (3.10)$$

where,

$$J_p = \ I + ml^2.$$

The experimental setup used for experimental work in this thesis uses a Panasonic AC servo motor, which is controlled by motion controller card (SV-400) of Googoltech

company via motor drive amplifier. Therefore, the voltage input to a AC servo motor is more appropriate variable than the acceleration for better and accurate results. In order to develop a more realistic dynamic model, the dynamics of AC servo motor is also taken into account, which is given as

$$I_m \ddot{\phi}_m + B_m \dot{\phi}_m = T_m - T. \qquad (3.11)$$

where,

$$
\begin{aligned}
\phi_m(t) &= \text{rotor angular displacement} \\
I_m(t) &= \text{rotor inertia} \\
B_m &= \text{viscous-friction coefficient} \\
T_m(t) &= \text{torque generated by a motor} \\
T(t) &= \text{load torque used to move the cart through a} \\
&\quad \text{timing pulley and a synchronous belt}
\end{aligned}
$$
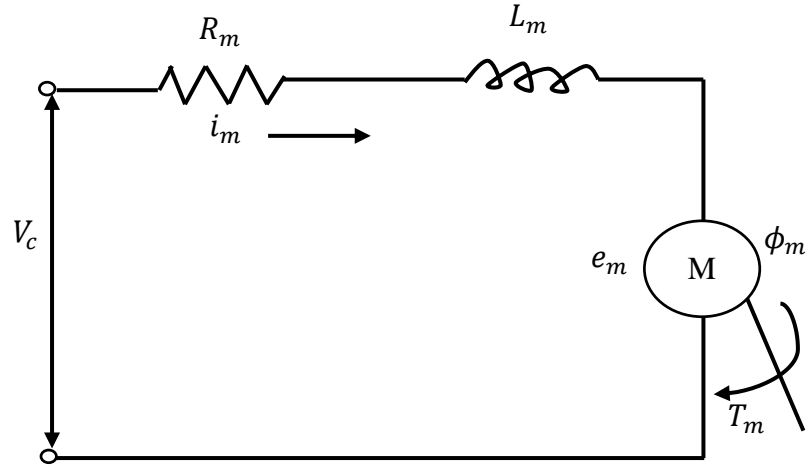
Figure 3.2: Electric circuit of a motor.

By Kirchhoff's law, the circuit given in Fig. 3.2 can be written as

$$\frac{di_m}{dt} = \frac{V_c}{L_m} - \frac{R_m}{L_m} i_m - \frac{e_m}{L_m}, \qquad (3.12)$$

For maximum current put derivative equal to zero. (3.12) can be rewritten as

$$V_c - R_m i_m - e_m = 0, \qquad (3.13)$$

Further, the back emf (e_m) is written as

$$e_m = K_b \dot{\phi}_m, \tag{3.13}$$

Therefore, (3.13) can be written as

$$i_m = -\frac{K_b}{R_m}\dot{\phi}_m + \frac{V_c}{R_m}, \tag{3.14}$$

The torque generated by motor can be represent in the form of voltage input, as

$$T_m = K_m i_a = K_m\left(-\frac{K_b}{R_m}\dot{\phi}_m + \frac{V_c}{R_m}\right), \tag{3.15}$$

where,

$$
\begin{aligned}
i_a(t) &= \text{winding current} \\
K_m &= \text{torque constant} \\
K_b &= \text{back emf constant} \\
V_c(t) &= \text{voltage applied to AC servo} \\
&\quad\ \text{motor}
\end{aligned}
$$

The Googoltech setup used for experimental work consist of timing pulley and synchronous belt arrangement, to convert rotational motion to translation motion. If r is the radius of timing pulley, $x(t)$ represent the translation motion variable and $\phi_m(t)$ represents the rotational motion variable, then the relation can be written as

$$\dot{x} = r\dot{\phi}_m, \tag{3.16}$$

$$\ddot{x} = r\ddot{\phi}_m. \tag{3.17}$$

Using (3.15), motor torque can be written as

$$T_m = -\frac{K_m K_b}{R_m r}\dot{x} + \frac{K_m}{R_m}V_c. \tag{3.18}$$

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From (3.16) and (3.17), load torque can be calculated as

$$T = -\frac{I_m}{r}\ddot{x} - \left(\frac{B_m}{r} + \frac{K_m K_b}{R_m r}\right)\dot{x} + \frac{K_m}{R_m}V_c, \tag{3.19}$$

And, the force acting on cart is given as

$$f = \frac{T}{r} = -\frac{I_m}{r^2}\ddot{x} - \left(\frac{B_m}{r^2} + \frac{K_m K_b}{R_m r^2}\right)\dot{x} + \frac{K_m}{R_m r}V_c. \tag{3.20}$$

Substituting f into (3.9), the dynamic equation of motion of the cart-pendulum is represented as

$$\left(M + m + \frac{I_m}{r^2}\right)\ddot{x} + ml\cos\phi\,\ddot{\phi} - ml\sin\phi\,\dot{\phi}^2 + \left(b + \frac{B_m}{r^2} + \frac{K_m K_b}{R_m r^2}\right)\dot{x} = \frac{K_m}{R_m r}V_c. \tag{3.21}$$

Equation (3.21) and (3.10) represent the nonlinear equations of motion of the cart-pendulum system. These equations can be written in standard form as

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{N}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{f}, \tag{3.22}$$

where $\boldsymbol{q} = [x\ \phi]^T$, is the vector of generalized coordinates, $\boldsymbol{M}(\boldsymbol{q})$ is the positive-definite inertia matrix, $\boldsymbol{N}(\boldsymbol{q},\dot{\boldsymbol{q}})$ is vector of the Coriolis and centripetal forces, $\boldsymbol{g}(\boldsymbol{q})$ is the vector of the gravitational force and $\boldsymbol{f}$ is vector of the generalized forces, given as

$$\boldsymbol{M}(\boldsymbol{q}) = \begin{bmatrix} \dfrac{MR_m r}{K_m} + \dfrac{mR_m r}{K_m} + \dfrac{I_m R_m}{rK_m} & \dfrac{R_m rml\cos\phi}{K_m} \\ ml\cos\phi & J_p \end{bmatrix},$$

$$\boldsymbol{N}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \begin{bmatrix} \dfrac{bR_m r}{K_m} + \dfrac{B_m R_m}{K_m r} + \dfrac{K_b}{r} & -\dfrac{R_m rml\sin\phi\,\dot{\phi}}{K_m} \\ 0 & 0 \end{bmatrix},$$

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$$g(q) = \begin{bmatrix} 0 \\ -mgl\sin\phi \end{bmatrix},$$

$$f = \begin{bmatrix} V_c \\ 0 \end{bmatrix}.$$

3.3 Experimental Setup

Figure 3.3: Googotech Linear Inverted Pendulum (GLIP) setup.

The cart-pendulum setup used for the experimental purpose is made by Googoltech, Hong Kong (HK), as shown in Fig. 3.3. Googoltech Linear Inverted Pendulum (GLIP) consists of a motor driven cart and a pole (pendulum) freely pivoted above it, with feedback sensors and electric circuit. It is a two DOF system with one control input, i.e. the sliding cart on the horizontal track. The cart is controlled by an AC servo motor with the help of a timing pulley and a synchronous belt. The timing pulley helps to convert the angular motion of the motor to linear motion of the sliding cart. The different modules of the setup are shown in Fig. 3.4. The system works in the MATLAB and SIMULINK environment. Googoltech Simulink toolbox is used for communication between setup and PC.

3.3.1 Design Specifications

The software commands the setup via motion controller card (Googoltech SV-400), which convey signals in the form of pulses to AC servo motor (Panasonic, Model-MSMJ022G1U). The motor has 200 W power and works on 10 V supply. The track length of the cart is ±0.35 m. The incremental encoder of the motor is used to get the

cart's position as feedback and rotary encoder (Nemicon Corporation, Model- OVW2-06-2MD resolution 600 P/R) is also used to measure the angular position of the pendulum.
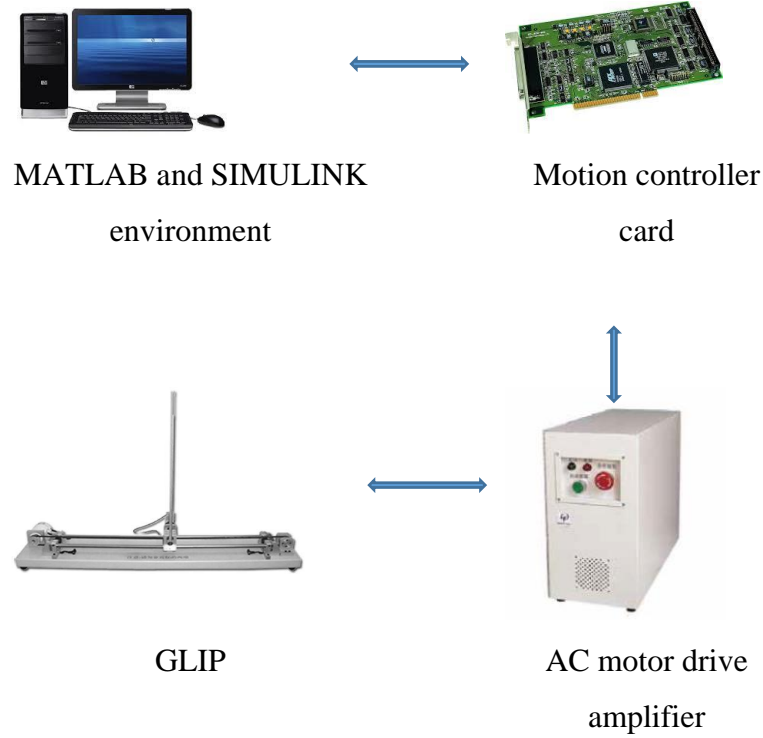
Figure 3.4: Modules of GLIP.

The whole system works in a closed-loop structure. The conversion factors are used at both the encoders to convert the pulses signals to angular and linear measurements, which are given below

From pulses to angular measurement: −

$$\phi = \frac{360°}{\mathcal{M}} N, \tag{3.23}$$

From pulses to linear measurement: −

$$x = \frac{3.14\, r}{\mathcal{M}} N, \tag{3.24}$$

where, ϕ is angle in degrees, x is displacement in meters, N no. of pulses, $\mathcal{M}$ is the resolution with quadrature in pulse/revolution and r is the radius of timing pulley.

The cart linear acceleration is used as an input to the motion controller card and motor drive amplifier, which further converts this input to voltage signal for the motor. The system parameters of the GLIP is listed in Table 3.1.

Table 3.1: System parameter of GLIP [43].

Parameter	Units	Symbol	Value
Cart mass	kg	M	1.096
Pendulum mass	kg	m	0.109
Coefficient of friction		b	0.1
Pendulum rod inertia	kgm^2	I	0.0034
Pendulum length from pivot to centre of mass	m	l	0.25
Acceleration due to gravity	m/s^2	g	9.8
Motor rotor inertia	kgm^2	I_m	1.4×10^{-5}
Motor viscous friction coefficient	Nms	B_m	0.03
Motor torque constant	kgm^2	K_m	2
Motor back emf constant	Ns/rad	K_b	0.1
Motor armature resistance	Ω	R_m	2.5
Radius of timing pulley	m	r	0.02

3.4 Summary

This chapter represents the dynamic model of the cart-pendulum system, which have been used in next chapters during the control phase. The Euler-Lagrange method has been used to derive the differential equations of the cart-pendulum system. The Googoltech Linear Pendulum (GLIP) has been discussed in this chapter, which is used for experimental demonstrations in this dissertation. The actuator dynamics along with timing pulley dynamics has been used to design the accurate mathematical model of the cart-pendulum system. The derived mathematical model has been used in Chapter 4 to check the controllability of the system.

Chapter 4

Control of the Cart-Pendulum System

4.1 Introduction

In this chapter, the controllability method for the cart-pendulum system is discussed by linearizing the dynamic model of the system, which has been derived in Chapter 3. At the outset, the state-space model is derived from the differential equations of the cart-pendulum system. To address the stabilization and swing-up problem of the cart-pendulum system, three controllers are discussed. In which, pole placement and LQR controllers have been used to address the stabilization problem of the cart-pendulum system at unstable equilibrium position. The energy controller is discussed in the subsection of controller design, which have been used to address the swing-up problem of the cart-pendulum system in this dissertation.

4.2 Linearization of the Dynamic Model

4.2.1 State-Space Model

For stabilization of the cart-pendulum system around upright equilibrium point, the nonlinear system can be linearized by neglecting the square term and taking $\sin \phi = \phi$, $\cos \phi = 1$. The design of controller requires the mathematical model of the system in state-space form. By linearizing (3.9) and (3.21),the state-space representation of the cart-pendulum system is given as

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{A}\boldsymbol{z}(t) + \boldsymbol{B}\boldsymbol{u}(t), \qquad (4.1)$$
$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{z}(t) + \boldsymbol{D}\boldsymbol{u}(t).$$

where $\boldsymbol{z}(t) = [x \; \dot{x} \; \phi \; \dot{\phi}]^{T}$, is called as the state-vector, $\boldsymbol{y}(t) = [x \; \phi]^{T}$ as the output vector, $\boldsymbol{u}(t)$ is known as the control input vector and $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ are called as state-weighing coefficient matrices.

And,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \dfrac{m^2 l^2 R_m r \Lambda_2}{\Lambda_1{}^2 \Lambda_3 k_m} - \dfrac{\Lambda_2}{\Lambda_1} & -\dfrac{m^2 g l^2 R_m r}{\Lambda_1 \Lambda_3 k_m} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{ml\Lambda_2}{\Lambda_1 \Lambda_3} & \dfrac{mgl}{\Lambda_3} & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ \dfrac{ml}{\Lambda_1} + \dfrac{1}{\Lambda_1} \\ 0 \\ \dfrac{-ml}{\Lambda_1 \Lambda_3} \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$$\Lambda_1 = \left(M + m + \frac{I_m}{r^2} \right) \frac{R_m r}{K_m},$$

$$\Lambda_2 = \left(b + \frac{B_m}{r^2} + \frac{K_m K_b}{R_m r^2} \right) \frac{R_m r}{K_m},$$

$$\Lambda_3 = I + ml^2 + \frac{m^2 l^2 R_m r}{\Lambda_1 k_m}.$$

4.2.2 Controllability

Before designing the controller, the first and foremost requirement is to ensure the controllability of the plant. Controllability is defined as the property by which it is possible to take the system from any initial state to any final state in a finite time by means of the input vector. Controllability of any system can be checked easily by calculating the controllability test matrix P, given as

$$P = [B \ AB \ A^2 B \ \ldots\ldots\ldots A^{n-1} B]. \tag{4.2}$$

If rank $(P) == n$, the system is controllable [42].

4.3 Controller Design

The cart-pendulum system is highly nonlinear in nature. As discussed in Chapter 1, there are three control problems for the cart-pendulum system – *stabilization problem, swing-up problem* and *tracking problem.* The stabilization and swing-up problems are addressed in this thesis by designing three different controllers. Two linear controllers – Pole Placement and LQR from the modern control theory family are used to address the regulation problem. The nonlinear energy controller is used to overcome the swing-up problem of the cart-pendulum system. These controllers are briefly discussed in the next subsections.

4.3.1 Pole Placement Controller (PPC)

Pole placement method places the eigenvalues of the close-loop system at the desired place by designing a full-state feedback controller in order to satisfy the transient and steady-state performance requirements of the system. Pole placement design possesses the following steps.

1) Verify the controllability of the plant.
2) Calculate the coefficient vector $\boldsymbol{a} = [a_1, a_2 \dots \dots a_n]$ of the characteristic polynomial of state dynamic matrix $\boldsymbol{A}$

$$|s\boldsymbol{I} - \boldsymbol{A}| = s^n + a_1 s^{n-1} + \dots\dots\dots a_{n-1}s + a_{n.} \qquad (4.3)$$

3) Determine the matrix T that transform the state-space representation to controller companion form, as

$$\boldsymbol{T} = \boldsymbol{PW}, \qquad (4.4)$$

where, $\boldsymbol{P}$ is the controllability test matrix, and

$$\boldsymbol{W} = \begin{bmatrix} a_{n-1} & a_{n-2} & \dots & a_1 & 1 \\ a_{n-2} & a_{n-3} & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_1 & 1 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \end{bmatrix}. \qquad (4.5)$$

4) Formulate the desired polynomial by using the desired eigenvalues

$$(s - \mu_1)(s - \mu_2) \dots \dots \dots (s - \mu_n) = s^n + \alpha_1 s^{n-1} + \dots + \alpha_{n-1} s + \alpha_{n.} = 0. \quad (4.6)$$

And determine the *coefficient* vector $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \dots \dots \alpha_n]$.

5) Calculate the state feedback gain matrix $\boldsymbol{K_{PPC}}$ as

$$\boldsymbol{K_{PPC}} = (\boldsymbol{\alpha} - \boldsymbol{a})\boldsymbol{T}. \quad (4.7)$$

4.3.2 LQR Controller (LQRC)

Linear Quadratic Regulator is the optimal control used to minimize the cost associated with generating control inputs. The objective cost function consists of two matrices, $\boldsymbol{Q}$ and $\boldsymbol{R}$, the state weighting matrix and control cost matrix. For a linearized plant, the objective of LQR controller is to minimize the integration of sum of these two matrices by using a feedback regulator. The objective function is given as:

$$\boldsymbol{J_\infty}(t) = \int_0^\infty (\boldsymbol{z}^T(\tau)\boldsymbol{Q}\boldsymbol{z}(\tau) + \boldsymbol{u}^T(\tau)\boldsymbol{R}\boldsymbol{u}(\tau)d\tau, \quad (4.8)$$

and $\boldsymbol{K_{LQRC}}$ is the optimal state feedback control matrix given as

$$\boldsymbol{K_{LQRC}} = \boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{M_0}, \quad (4.9)$$

where, $\boldsymbol{M_0}$ is the solution of the following algebraic Riccati equation

$$\boldsymbol{M_0}\boldsymbol{A} + \boldsymbol{A}^T\boldsymbol{M_0} - \boldsymbol{M_0}\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{M_0} + \boldsymbol{Q} = 0. \quad (4.10)$$

From the above equation, it can be observed that the optimal gain matrix $\boldsymbol{K_{LQRC}}$, depends upon $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{Q}$ and $\boldsymbol{R}$. Out of these four, $\boldsymbol{A}$ and $\boldsymbol{B}$ are mainly involved in the structure and parameters of the plant. Thus, the optimal gain matrix $\boldsymbol{K_{LQRC}}$ depends mainly on $\boldsymbol{Q}$ and $\boldsymbol{R}$. These matrices can be found by manual tuning, however for optimal weight matrices, special types of algorithms are available. In the current chapter, the

control gain matrix K_{LQRC} is derived using two different approaches, which will be used in the next chapter for comparing the results.

4.3.3 Energy Controller

The energy based approach is used to swing-up the pendulum form downward to upward position. The total energy of the pendulum is the sum of its rotational kinetic energy and its potential energy, which is given by

$$E_p = \frac{1}{2}J_p\dot{\phi}^2 + mgl(\cos\phi - 1). \tag{4.11}$$

The aim of the controller is to increase the pendulum energy, until the upright position is reach. By differentiating (4.16), the equation become

$$\frac{dE_p}{dt} = J_p\dot{\phi}\ddot{\phi} - mgl\sin\phi\,\dot{\phi}, \tag{4.12}$$

Thus using Lagrange's equation (3.9), it can be rewrite as

$$\frac{dE_p}{dt} = -ml\ddot{x}\cos\phi\,\dot{\phi}, \tag{4.13}$$

The energy of the pendulum should be converging to zero at upright position. The following control law is used to address energy criteria.

$$u_{swing} = k_e E_p sign(\cos\phi\,\dot{\phi}). \tag{4.14}$$

where

$$sign(\theta) = \begin{cases} 1 & if\ \theta \geq 0 \\ -1 & else \end{cases} \tag{4.15}$$

The working of the control law is given by Lyapunov approach.

Consider the Lyapunov function

$$L = \frac{1}{2}(E_p)^2,$$ (4.16)

taking derivative of (4.21)

$$\frac{dL}{dt} = E_p \frac{dE_p}{dt},$$ (4.17)

substituting (4.16) in (4.22)

$$\frac{dL}{dt} = -E_p m l \ddot{x} \cos \phi \, \dot{\phi},$$ (4.18)

Here, acceleration to the cart is the control input to the system. So we can say, $u_{swing} = \ddot{x}$ and (4.23) can be written as

$$\frac{dL}{dt} = -ml \cos \phi \, \dot{\phi} k_e E_p{}^2 sign(\cos \phi \, \dot{\phi}),$$ (4.19)

From (4.24), it can be noted that the derivative is negative semi negative and hence E_p will tend to zero as long as $\dot{\phi} \neq 0$ and $\cos \phi \neq 0$. If $\phi = \pi$ and $\dot{\phi} = 0$, then $u_{swing} = -2k_e mgl$, which entails that the cart will experience an initial acceleration from rest and be forces out of the downward position. Since, $\dot{\phi}$ cannot remain identical and ϕ can't remain in horizontal direction ($\phi = \pm 90°$), which is impossible. So the system energy will converge to zero by using (4.10). In order to restrict the cart with track limits, velocity of cart is taken into account. The algorithm is used with a condition that when $\dot{\phi} > 0, \dot{x} < 0$ and vice-versa. The algorithm works well with non-zero initial velocity of the cart.

4.4 Summary

This chapter presents the controllability method, which has been used to check the controllability of three linearized model later in the thesis. To address the stability issues of the cart-pendulum system, three different controllers has been discussed in

this chapter. Pole placement and LQR controllers has been discussed for the stabilization problem of the cart-pendulum system at unstable equilibrium point. The energy controller has been discussed with some constrained on the cart and pendulum velocity, to resist the cart position in the finite track length during swing-up of the pendulum from downward hanging position to upward unstable position.

Chapter 5

Stabilization Control

5.1 Introduction

There are two equilibrium position in the cart-pendulum system – stable equilibrium point at hanging downward position and unstable equilibrium point at upward position. In practice, stabilization of the cart-pendulum system at upward position has wide range of real-time applications such as rockets propeller, tank missile launcher, self-balancing robot, biped walking, etc. Stabilization of such a system is a linear problem, which is easier than the respective nonlinear problem, but still needs a well-designed controller. As discussed in Chapter 2, there are several controllers available to address the stabilization problem of the cart-pendulum system. The main aim of this chapter is to investigate the performance of two different controllers in stabilizing the inverted pendulum at the unstable equilibrium point. This regulation problem is addressed by developing the controller using Pole Placement approach and Linear Quadratic Regulator (LQR) approach, which has been discussed in Chapter 4. Both the controllers are compared analytically and experimentally using the Googoltech Linear Inverted Pendulum (GLIP) setup and analytical results are simulated in MATLAB and SIMULINK environment. In order to demonstrate the effect of both the controllers on the performance of the system, both analytical and experimental comparison of the results is reported in this chapter.

5.2 Control Scheme

In this chapter, controller design with two different approaches is discussed for the cart-pendulum system. As the objective of both the controllers, is to stabilize the plant at the unstable equilibrium point, the problem can be treated as a regulation problem as

$$\boldsymbol{u}(t) = \boldsymbol{K}\big(\boldsymbol{z_d}(t) - \boldsymbol{z}(t)\big), \qquad\qquad (5.1)$$

where $\boldsymbol{u}(t), \boldsymbol{z}(t), \boldsymbol{z}_d(t)$ and $\boldsymbol{K}$ are control input matrix, state vector, desired state vector and control gain vector. For the regulation problem, $\boldsymbol{z}_d(t) = 0$, which leads to

$$\boldsymbol{u}(t) = -\boldsymbol{K}\boldsymbol{z}(t). \tag{5.2}$$

In the next sub-section, the control gain matrix $\boldsymbol{K}$ is derived using two different controllers namely pole placement and LQR controller

5.3 Results and Discussions

The efficacy of the both pole placement and LQR controller is checked on analytical and experiment tasks. First, the controllability of the cart-pendulum system is checked. The control gain matrices are calculated for both controllers by designing with same parameters. Both controllers are designed, so that they can stable the system within 2 seconds of settling time. The efficiency of the controllers is checked by taking step response. Both the controllers are demonstrated experimentally and analytically to examine the performance on cart-pendulum system. Simulink based closed loop diagram for real-time experimentation is shown in Fig. 5.1. The details of controller block and Real-Control are shown in Figs. A1 and A3 of Annexure A.

Substitute parameters of the plant and motor, as given in Table 2.1 into (4.6), following state-space representation is obtained.

$$\begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -250.85 & 0.75 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -811.96 & 34.15 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 33.94 \\ 0 \\ 115.12 \end{bmatrix} u, \tag{5.3}$$

$$\begin{bmatrix} x \\ \phi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u.$$

The eigenvalues of the state dynamic matrix $\boldsymbol{A}$ are $[0, -250.85, -5.62, 5.64]$. As one of the eigenvalue of the open-loop system lies in the right half plane, system is unstable to start with. Thus, in order to stabilize the system at the inverted position, a controller needs to be designed in order to shift the eigenvalues in the left half plane.
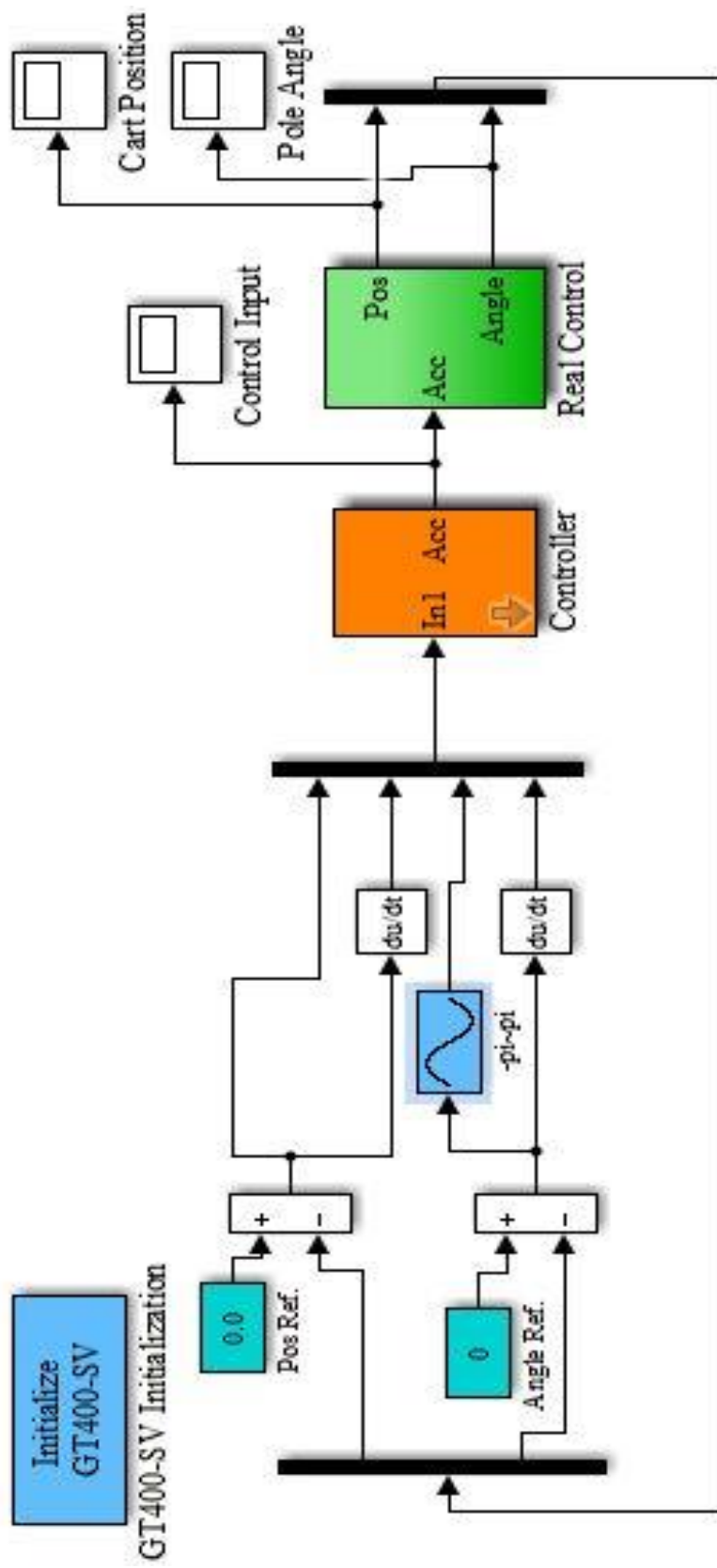
Figure 5.1: Main Simulink closed loop diagram to run stabilization control problem.

The cart-pendulum system has four state variables, i.e. $\mathbf{z} = [x\ \dot{x}\ \phi\ \dot{\phi}]^T$, where x is the linear displacement, $\dot{x}$ is the linear velocity of the cart, ϕ is the angular displacement and $\dot{\phi}$ is the angular velocity of the pendulum. Thus, $n = 4$ for controllability matrix, which is given as

$$\mathbf{P} = [\mathbf{B}\ \ \mathbf{AB}\ \ \mathbf{A^2B}\ \ \mathbf{A^3B}],$$

$$\mathbf{P} = \begin{bmatrix} 0 & 33.94 & -85.13 \times 10^2 & 21.35 \times 10^5 \\ 33.94 & -85.13 \times 10^2 & 21.35 \times 10^5 & -53.57 \times 10^7 \\ 0 & 115.12 & -27.55 \times 10^3 & 69.16 \times 10^5 \\ 115.12 & -27.55 \times 10^3 & 69.16 \times 10^5 & -17.3 \times 10^8 \end{bmatrix}. \tag{5.4}$$

The rank of the controllability matrix is found as four. Hence, the cart-pendulum system is controllable.

5.3.1 Analytical and Experimental Results

A. Pole Placement Controller

The desired closed-loop poles for pole placement controller are selected as [-10, -10, -2+$j\sqrt{3}$, -2-$j\sqrt{3}$]. These poles are selected to achieve the settling time within 2 seconds. Gains for the closed-loop system are calculated as $\mathbf{K_{PPC}} = [-54.4218,\ -24.4898,\ 93.2739,\ 16.1633]$. The step response for pole-placement gains is shown in Fig. 5.2.

The choice of the desired poles is justified, as it can be observed in Fig. 5.2, that both the cart as well as pendulum reach the desired state within 2 seconds. Both analytical and experimental results using pole placement controller are obtained for the validation.

In the first test run, the control objective is to stabilize the pole at upright position i.e. $\phi = 0$, while the cart maintains the reference position on the horizontal track i.e. $x = 0$. First, the experimental run needs to be executed, as it will decide the initial condition of ϕ, where the linear controller starts operating.
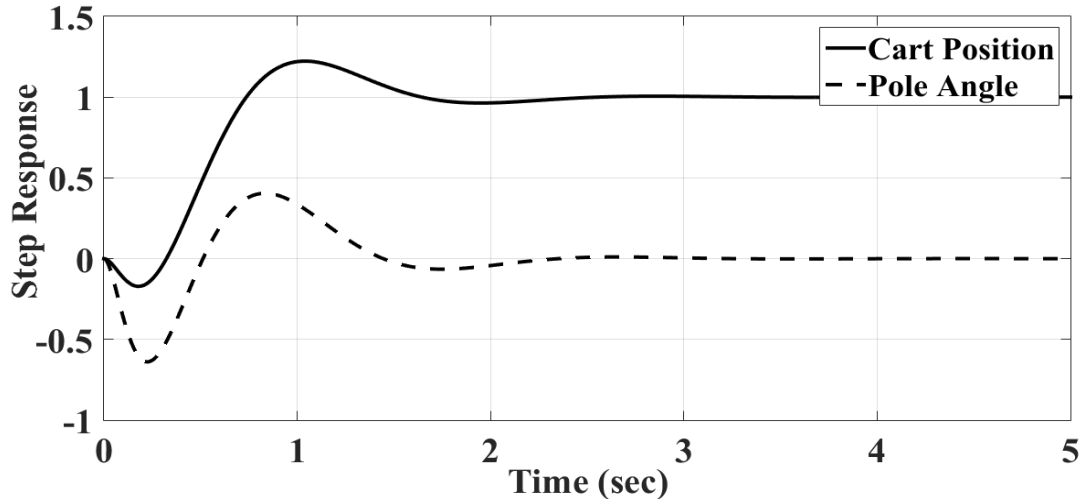
Figure 5.2: Step response using pole placement controller.

At that start of any test run, the GLIP setup is in its home position (stable equilibrium points of the pendulum), i.e. $x = 0$ and $\phi = 0$. Now, the pendulum is lifted manually from hanging position to upward position, until it comes in the linear stage of operation i.e. $\phi = \pm 20°$. As the pendulum is lifted, at $\phi = -19.5°$, the linear pole placement controller becomes active and stabilize the pendulum at its unstable equilibrium point with an overshoot of 5°, shown in Fig. 5.3.
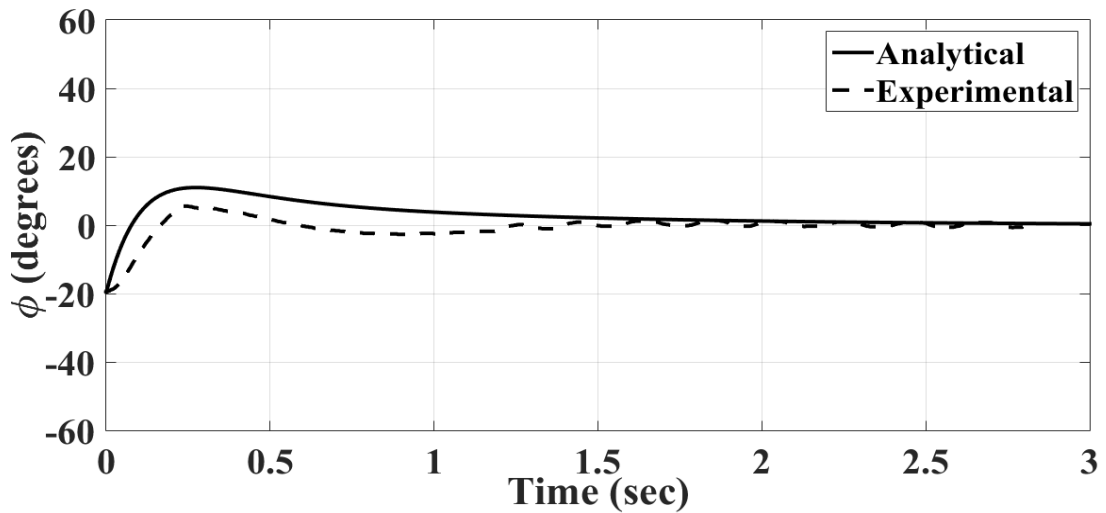
Figure 5.3: Angular displacement of pendulum using pole placement controller.

It can be observed from Fig. 5.4 that in order to stabilize the pendulum, the cart executes an overshoot of 0.15 m from its reference position and the closed-loop system settles within 2 seconds, as expected due to the choice of eigenvalues.

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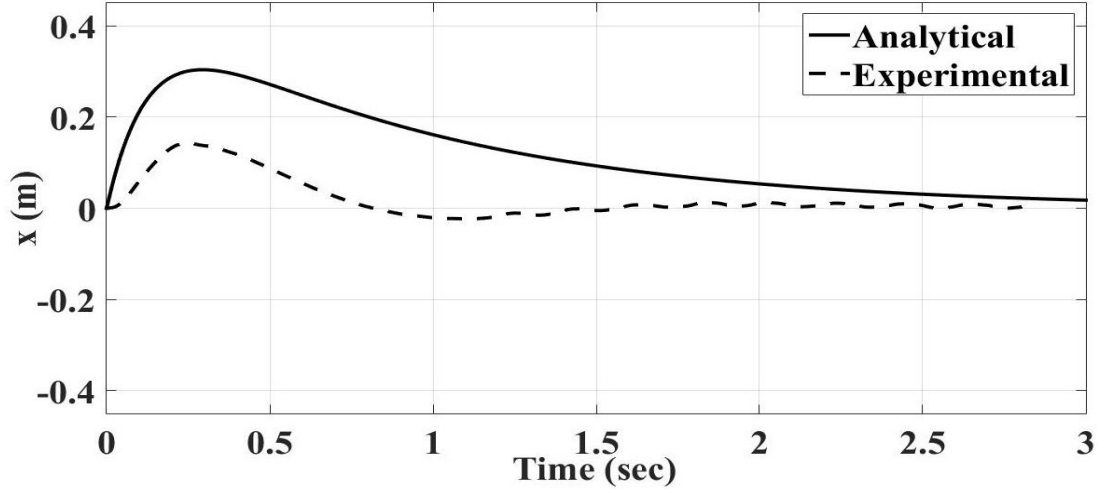
Figure 5.4: Linear displacement of cart using pole placement controller.

After getting the initial condition of ϕ from the experimental run, we switch to the simulation run for validation the response of the closed-loop system. Similar trend is observed in the analytical results, performed in the MATLAB environment, as both the cart and pendulum position settle in 2 seconds. It can be seen in Fig. 5.3 and 5.4, that the overshoot for the simulation run in the case of $\phi(t)$ and $x(t)$ is found as 11°and 0.3 m, as compared to 5° and 0.15 m respectively in the experimental results. This discrepancy in the overshoot results due to the activation of the linear controller in real time.
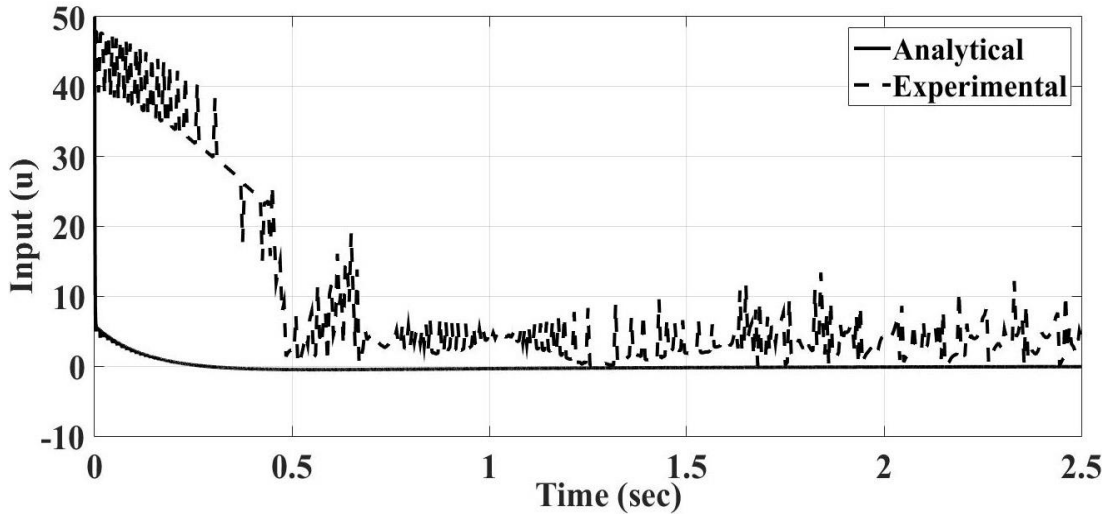
Figure 5.5: Comparison of control input using pole placement controller.

The comparison of control input is shown in Fig. 5.5, where the magnitude of maximum control input is found as 50 units for the simulation run, whereas it is observed as 48 units for the experimental run. Thus, it is observed that the maximum control input is

of the same order in both the cases, which validates the efficacy of the mathematical model and actuator dynamics.

B. LQR Controller

In this section, the control gain matrix is calculated using optimal control theory. The initial state vector i.e. $z = [0, 1.5, -19.5, 0]$. Selection of Q and R matrices for LQR controller is done by manual tuning, with the objective of achieving the settling time within 2 seconds. For this Q and R taken as

$$Q = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 200 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad R = [1].$$

Using (4.8)-(4.10), the optimal control gain matrix for LQR controller is calculated as $K_{LQRC} = [-31.628, -20.1507, 72.718, 13.152]$. The selection of Q and R is justified by taking step response of the plant with resulted gains. The cart and pole reaches the desired state within 2 seconds, as shown in Fig. 5.6.
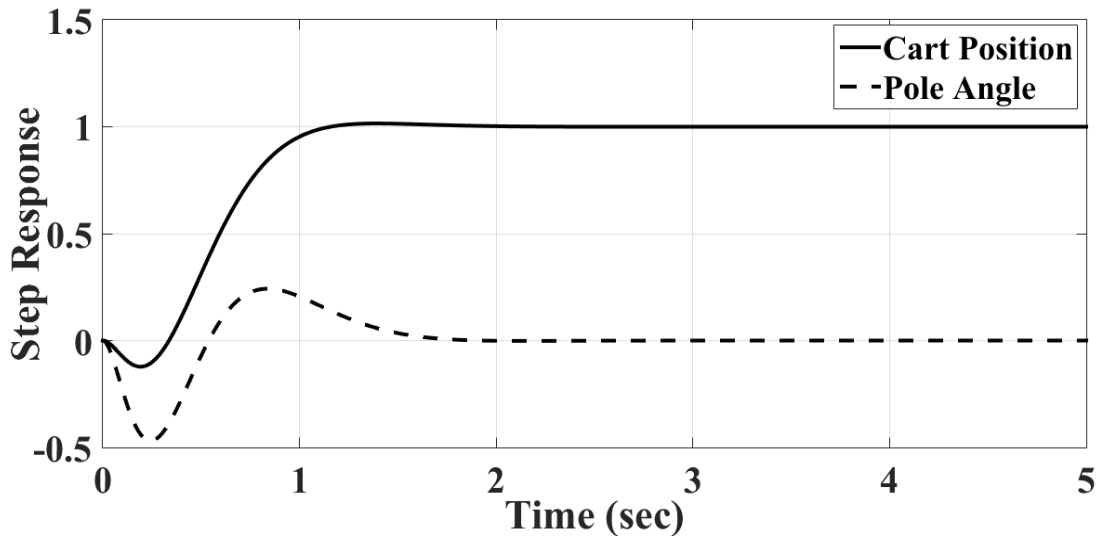
Figure 5.6: Step response using LQR controller

In order to stabilize the pendulum at upright position, the next test run is executed with LQR controller, in the same manner, as explained earlier with pole placement controller. The pendulum stabilizes at its inverted position with a settling time of 2 seconds and a marginal overshoot difference of 6° between experimental and analytical

results, as shown in Fig. 5.7. It can be observed from Fig. 5.8 that the maximum overshoot value for cart displacement is 0.17 m for analytical results and 0.12 m for experimental results. LQR controller is able to stabilize the cart to its desired position within 2 seconds. The comparison of control input is shown in Fig. 5.9, where the maximum control input magnitude is found as 25 for the experimental run and 20 for the simulation run.
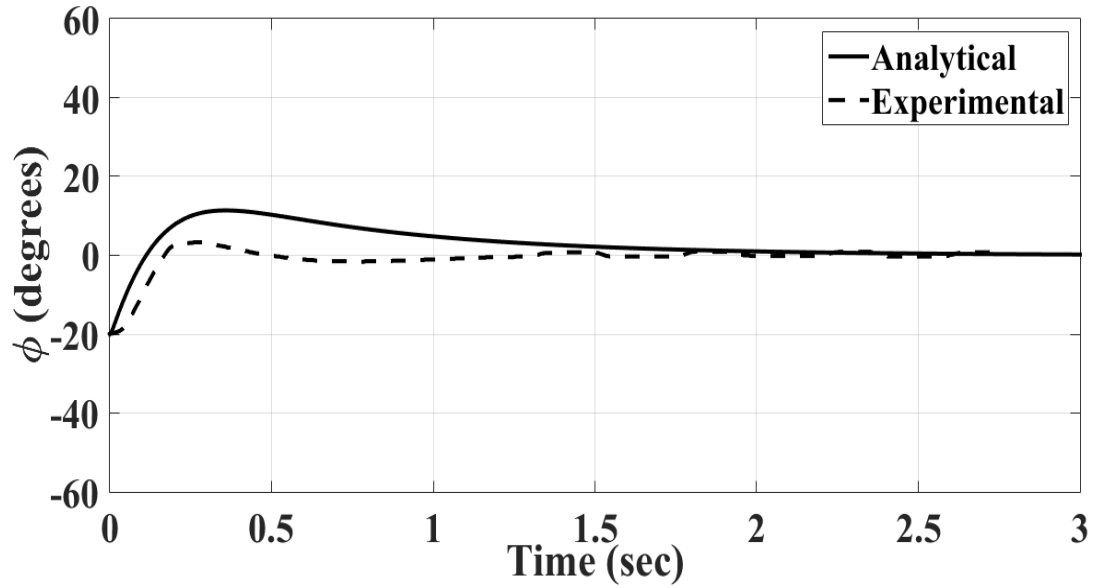
Figure 5.7: Linear displacement of the cart by LQR controller.
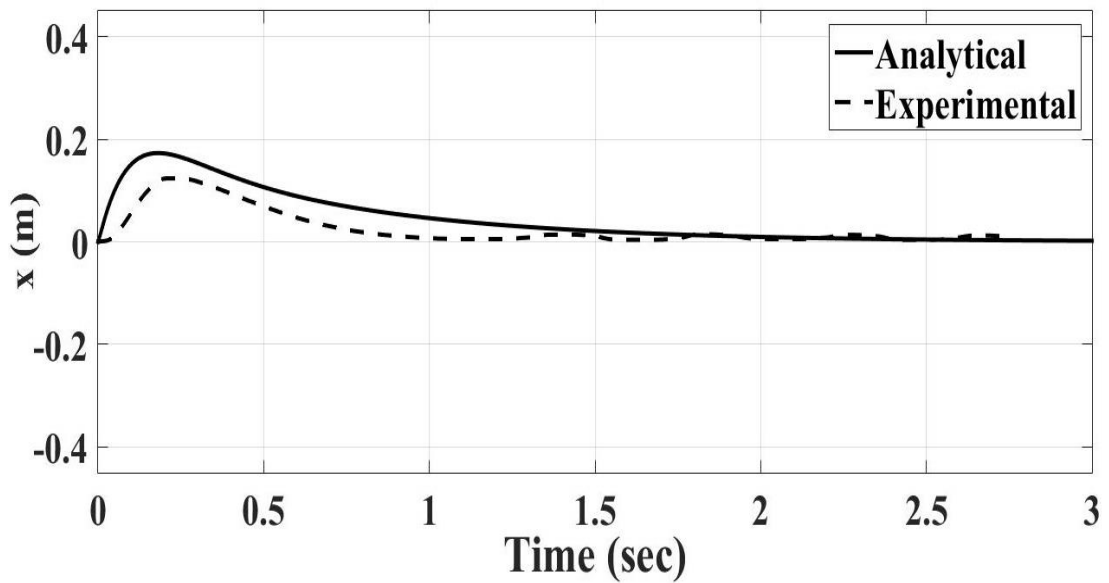
Figure 5.8: Angular displacement of the pole by LQR controller.
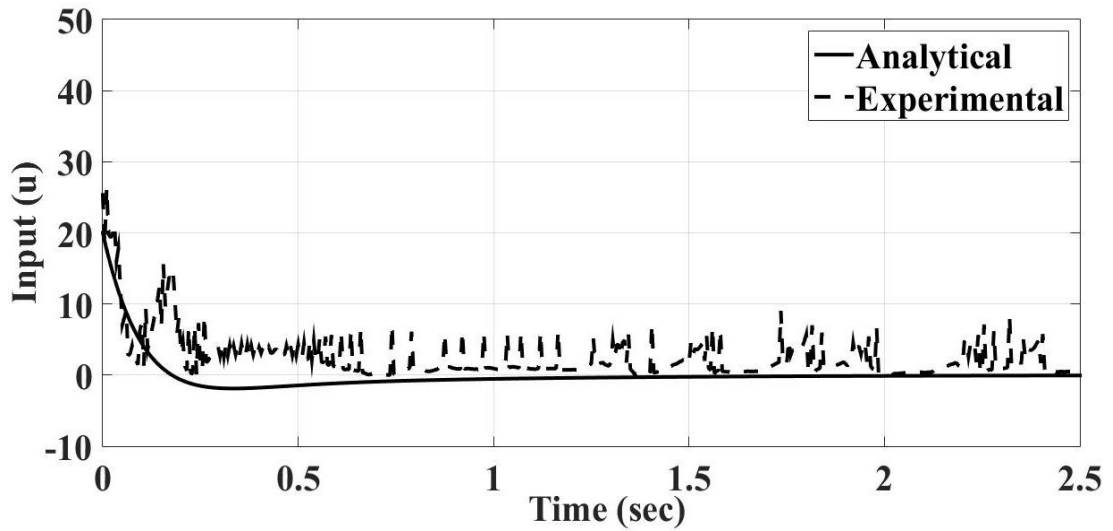
Figure 5.9: Comparison of control input using LQR controller.

5.3.2 Comparison Between PPC and LQRC

In this subsection, the comparison between two controllers is discussed. Both the controllers are able to stabilize the cart-pendulum system. From the experimental and analytical results, it is observed that the settling time for both controllers is near 2 seconds for stabilization. The control input magnitude by both the controllers from experimental results is compared in Fig. 5.10. From the results, it is found that the maximum control input magnitude for stabilization of inverted pendulum using pole placement controller is 48 units, whereas, it is found as 25 units with the LQR controller. Hence, LQR controller performs better than the pole placement controller, as it has resulted in 47.9% reduction in of maximum control input.
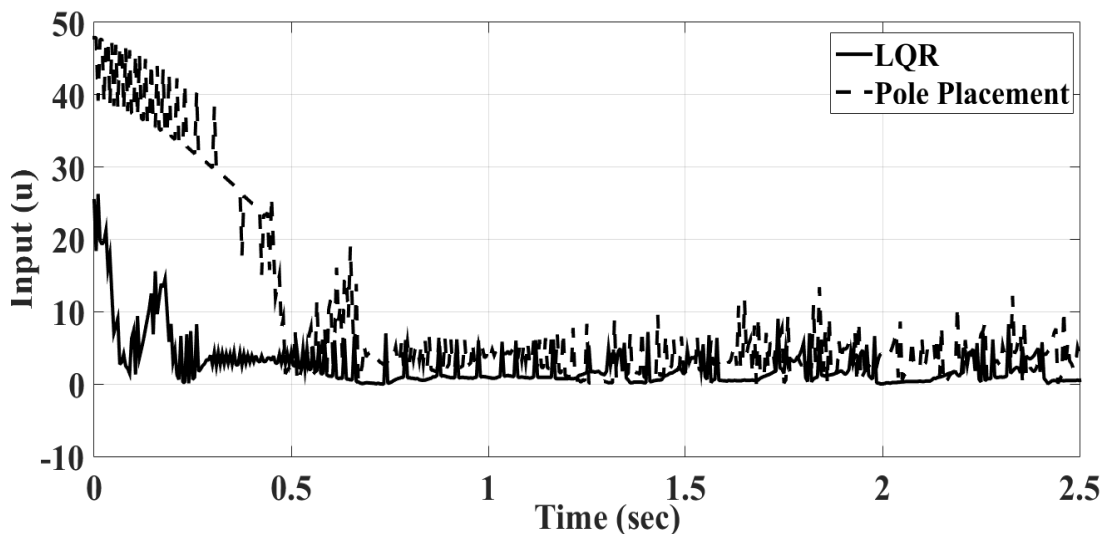
Figure 5.10: Comparison between control inputs of pole placement and LQR controller.

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5.4 Summary

This chapter has investigated the real-time control of cart-pendulum system, which is underactuated in nature. With one control input, it is difficult to stabilize the pendulum to its unstable equilibrium point (upward position). Two linear controllers (pole placement and LQR) has been used to solve this problem. Both the controllers have been designed to stabilize the system within 2 seconds. The actuator dynamics has been incorporated in the mathematical model to ensure the accuracy of the analytical model, which is validated through the close agreement of the simulation results with the experimental results. Control input magnitude by both controllers to the system is compared, which results that LQR controller outperforms the pole placement controller as it results in 47.9% reduction in maximum control-input magnitude.

Chapter 6

Swing-up Control

6.1 Introduction

The main aim of this work is to investigate the performance of two different control strategies- first to swing-up the pendulum to near unstable equilibrium region and second to stabilize the pendulum at unstable equilibrium point. The swing-up problem is addressed by using energy controller in which cart is accelerated by providing a force to the cart with a AC servo motor with the help of timing pulley arrangement. The initial velocity of the cart is taken into account to confirm swing-up in the restricted track length. The cart displacement in the restricted track length is verified by simulation and experimental test-run. The regulation problem of stabilization of pendulum is addressed by developing the controller using pole placement controller (PPC) and LQR controller (LQRC).

6.2 Controller Design

In this section, controller is designed for the cart-pendulum system. Controller design is broken up into two tasks. First task is to swing-up the pendulum to unstable equilibrium point (upright position) from the stable equilibrium point (downward position). This problem is addressed by using energy based approach, which is discussed in first subsection.

Second task is to stabilize the pendulum at upright position, which is carried out by two different approaches− LQR controller (LQRC) and pole placement controller (PPC), which are described in second subsection. Further, the switch algorithm is discussed to switch from nonlinear to linear controller when $|\phi| < 20°$.The control scheme is shown as a block diagram in Fig. 6.1.
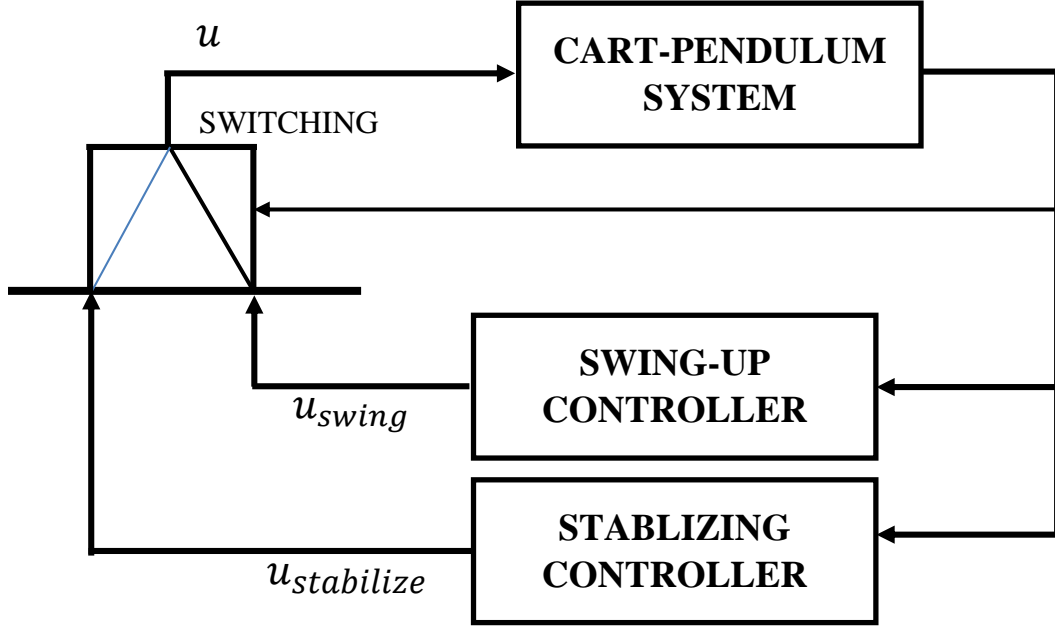
Figure 6.1: Control scheme for the cart-pendulum system.

One move switch is developed to switch between swing-up and stabilizing controllers. This switch ensures that the energy controller is activated only once. When pendulum reaches the upward point, the stabilization controller is activated permanently. Switch output (u) is represented as follows

$$u = \begin{cases} u_{swing}, & \text{if } 20° < |\phi| < 180° \\ u_{stabilize}, & \text{if } |\phi| > 20° \end{cases} \tag{6.1}$$

6.3 Results and Discussions

6.3.1 Analytical and Experiment Results

Simulations are performed in MATLAB environment. The mathematical model of the cart-pendulum system is simulated along with swing-up and stabilizing control laws. Switch is used between linear and nonlinear controller as discussed in previous section. As discussed, energy controller is used for swing-up problem with gain value $k_e = 2.3$, and initial conditions $[0, 0.341, -178.2°, 0]$ for state vector $[x, \dot{x}, \phi, \dot{\phi}]$. Cart velocity's initial condition is taken into account for restricting the cart track length. Pole placement controller gains are calculated as K_{PPC} = [-54.4218, -24.4898, 93.2739,

16.1633] and for LQR controller gains are K_{LQRC} = [-31.628, -20.1507, 72.718, 13.152]. Both controllers are designed for same configuration results, which are elaborated in Chapter 5.

In the first simulation run, the swing-up controller with PPC is used. As shown in Fig. 6.2, the pendulum is at the downward position ($\phi = 180°$) to start with and the nonlinear swing-up controller is able to swing the pendulum near the upward position within 8.5 sec, i.e. angle ϕ approaches from $180°$ to $0°$. As soon as, the pendulum enters the linear band about the unstable equilibrium point ($|\phi| < 20°$), the one-way switch gets activates, and the linear PPC comes into action. Figure 6.1 shows that pole placement controller is able to stabilize the pendulum at unstable equilibrium point, i.e. $\phi = 0°$. Cart displacement and control input response is shown in Figs. 6.3 and 6.4. Both the curves decay to zero to achieve stability. The cart displacement takes near about 6.5 swings in -0.105 to 0.09 m track range to achieve stability in 12 seconds. Control input takes 11.5 sec to decay to zero with the bandwidth of 0.55 from -2 to 2 range. Phase portrait is shown in Fig. 6.5, which demonstrate that the pendulum energy decays to zero as the time evolves.
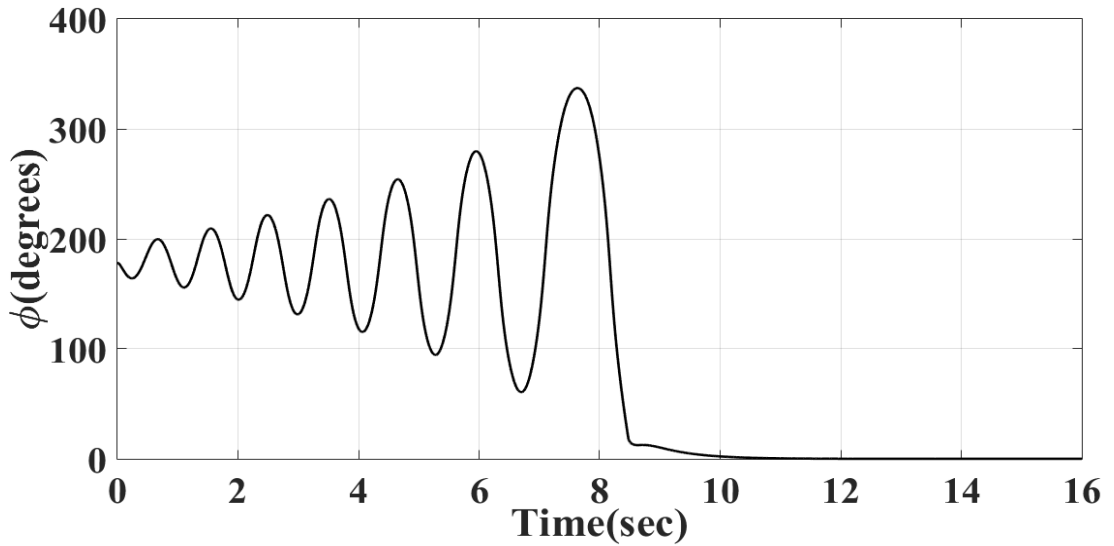
Figure 6.2: Simulation pendulum angular position response for energy swing-up with PPC.
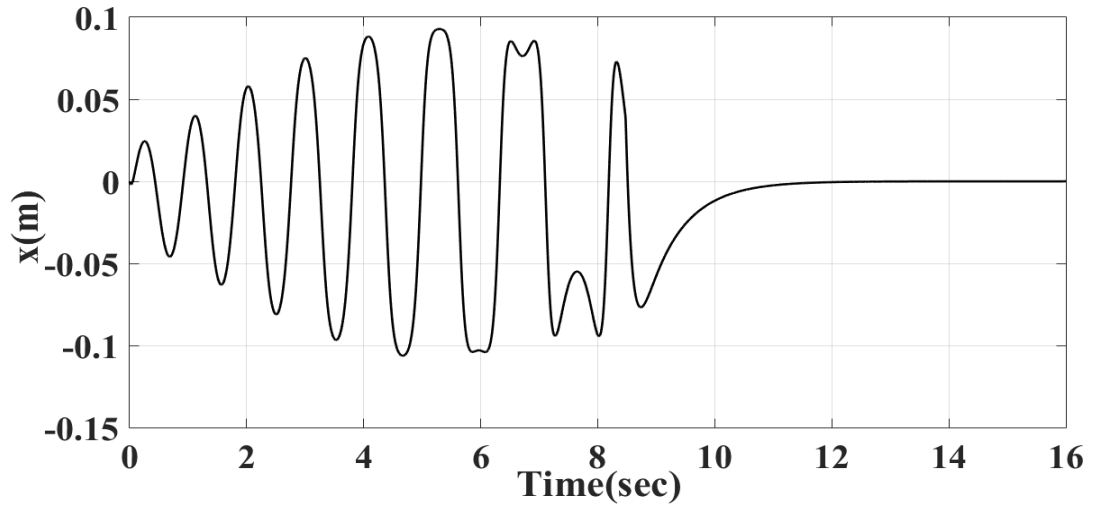
Figure 6.3: Simulation cart position response for energy swing-up with PPC.
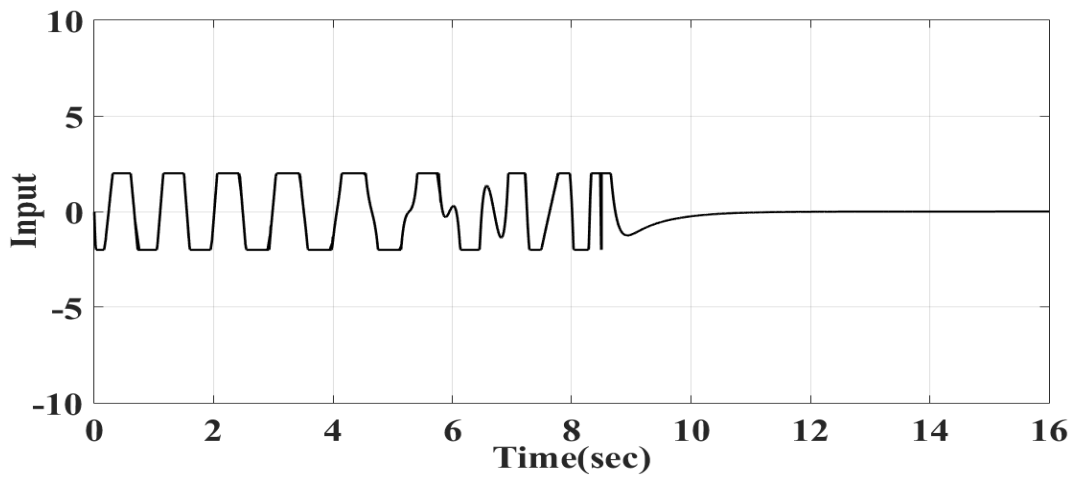
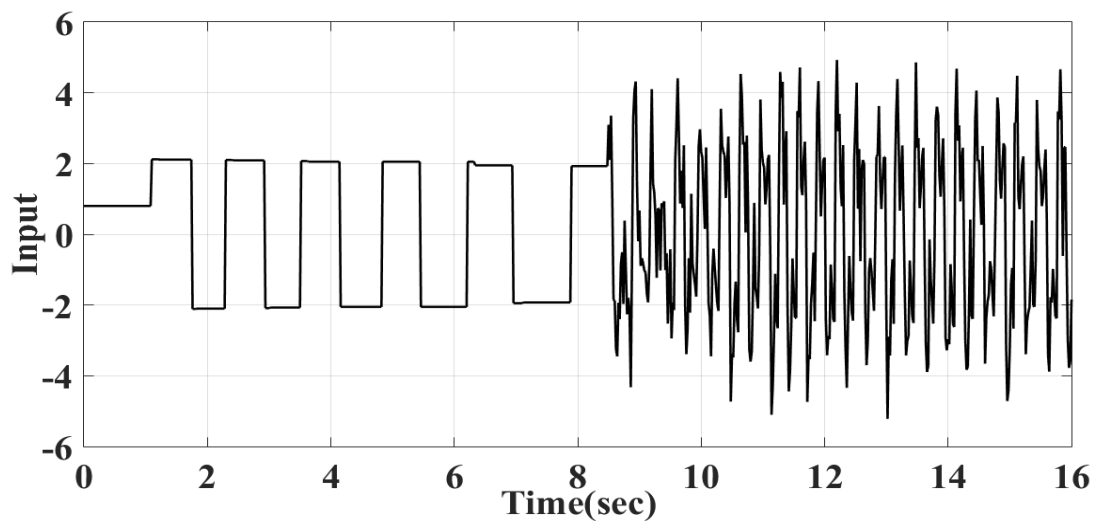**Figure 6.4:** Simulation control input response for energy swing-up with PPC.
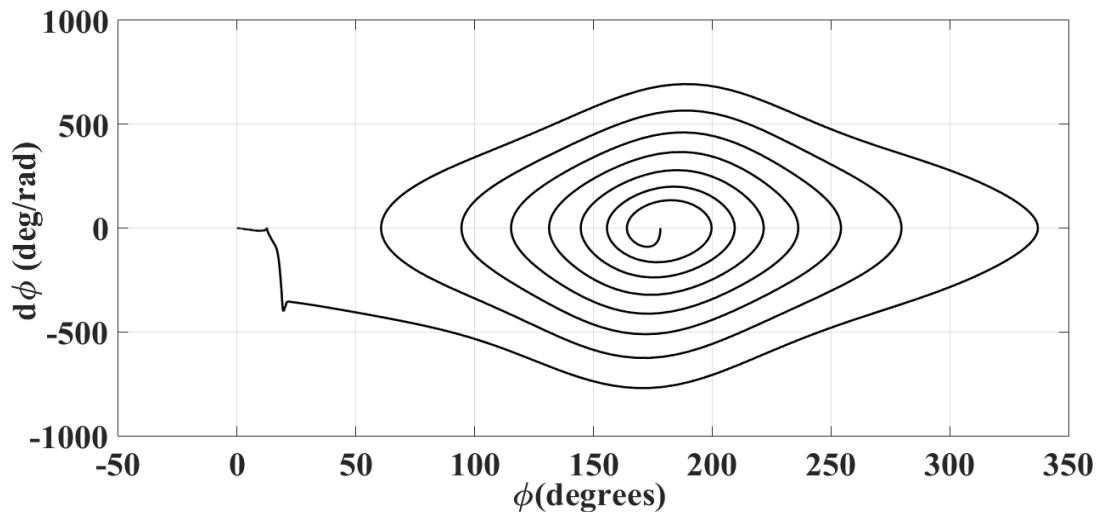
Figure 6.5: Simulation phase portrait for energy swing-up with PPC.

The results of energy swing-up controller with LQR stabilizing controller are shown in Figs. 6.6 to 6.9. All the figures (pendulum angular response, cart position response, control input response and phase portrait) reveals same trend. Both PPC and LQRC are able to stabilize the cart-pendulum system to unstable equilibrium point due to their same design configuration. It can be expected that both controllers should give same trend in experimental results also.
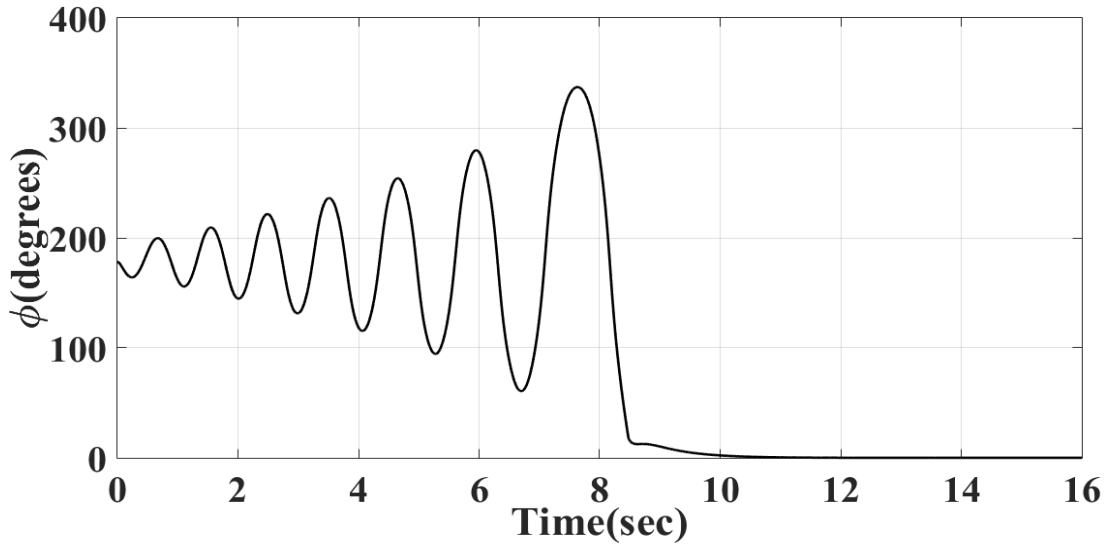
Figure 6.6: Simulation pendulum angular position response for energy swing-up with LQRC.
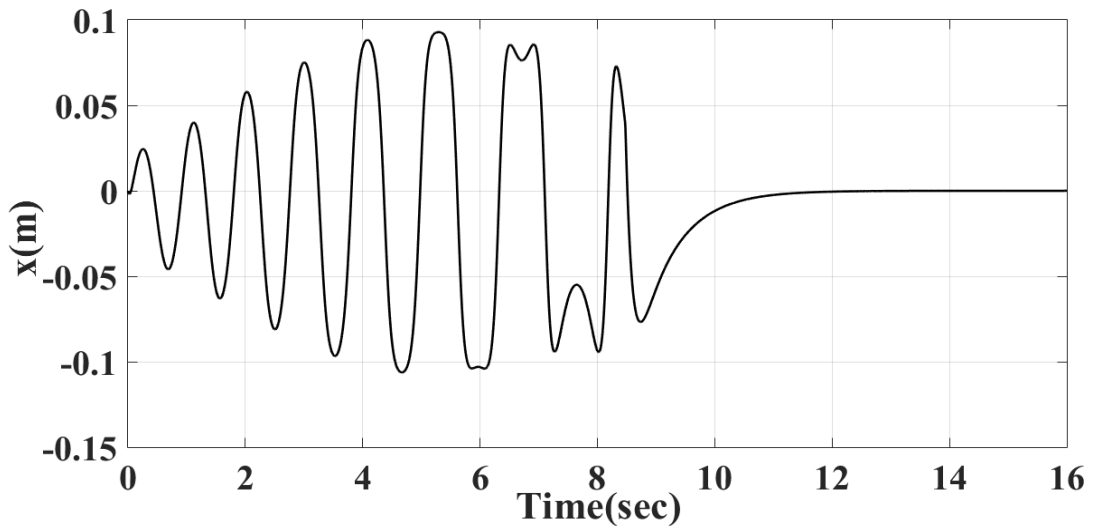
Figure 6.7: Simulation cart position response for energy swing-up with LQRC.
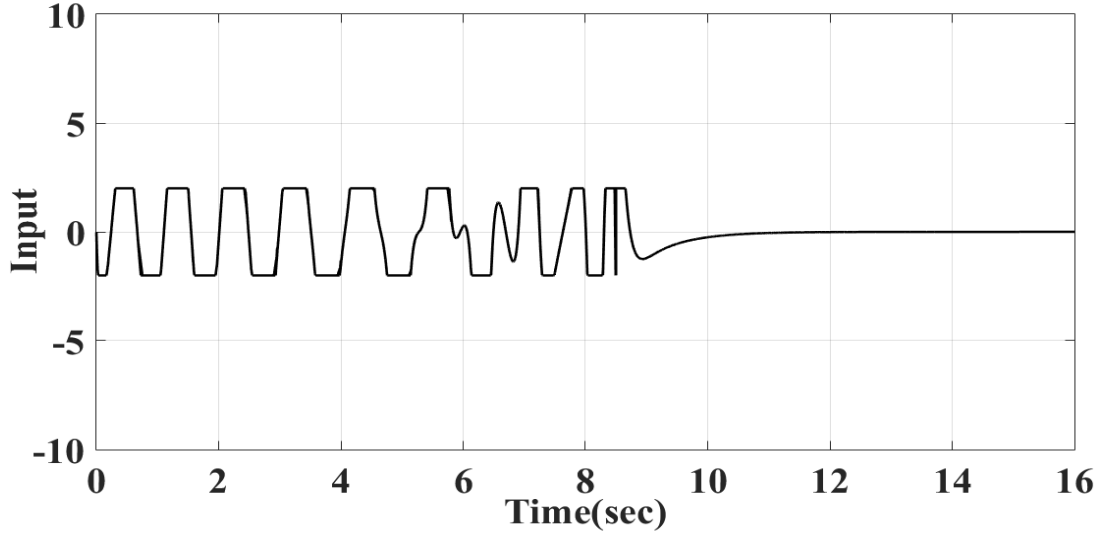
Figure 6.8: Simulation control input response for energy swing-up with LQRC.
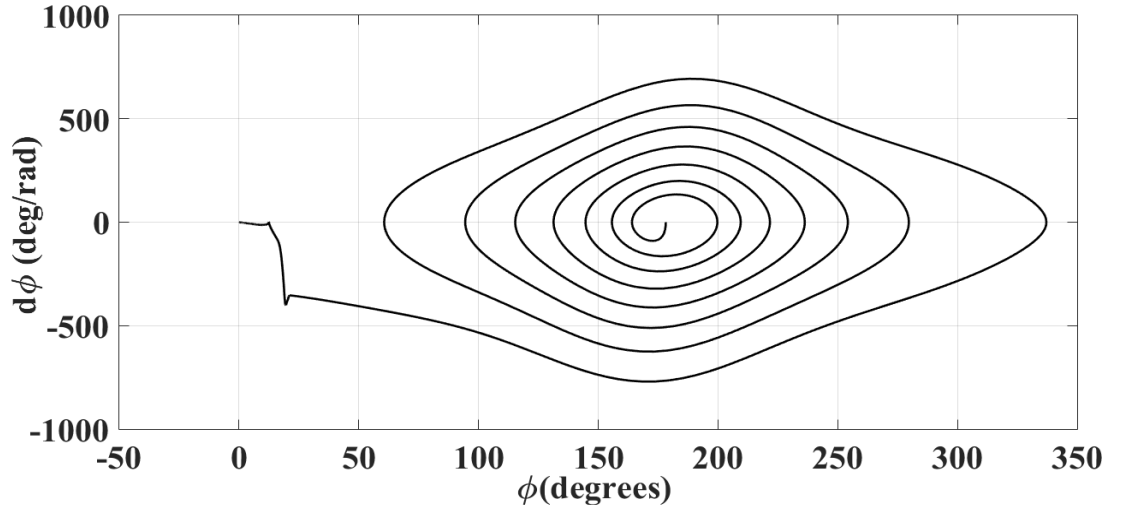
Figure 6.9: Simulation phase portrait for energy swing-up with LQRC.

The experiments are carried out on Googoltech setup, which is well described in Section 2. The control algorithm is established by SIMULINK and MATLAB integration with real-time implementation. The closed loop Simulink control scheme is shown in Fig. 6.10. The details for Stabilization, Swing-up and Real-Time control blocks are shown in Figs. A1, A2 and A3 of Annexure A. S-function is used to execute swing-up algorithm, in which cart is initially accelerated to 0.05 m distance in right direction to move pendulum from its initial rest position. Thereafter the swing-up controller is executed, which is earlier designed in simulation results and discussed in previous section. The swing-up controller is tested experimentally along with PPC and LQRC for stabilization of cart-pendulum system at unstable equilibrium point. The

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controller parameters for both the controllers are kept same as used for simulations purpose.
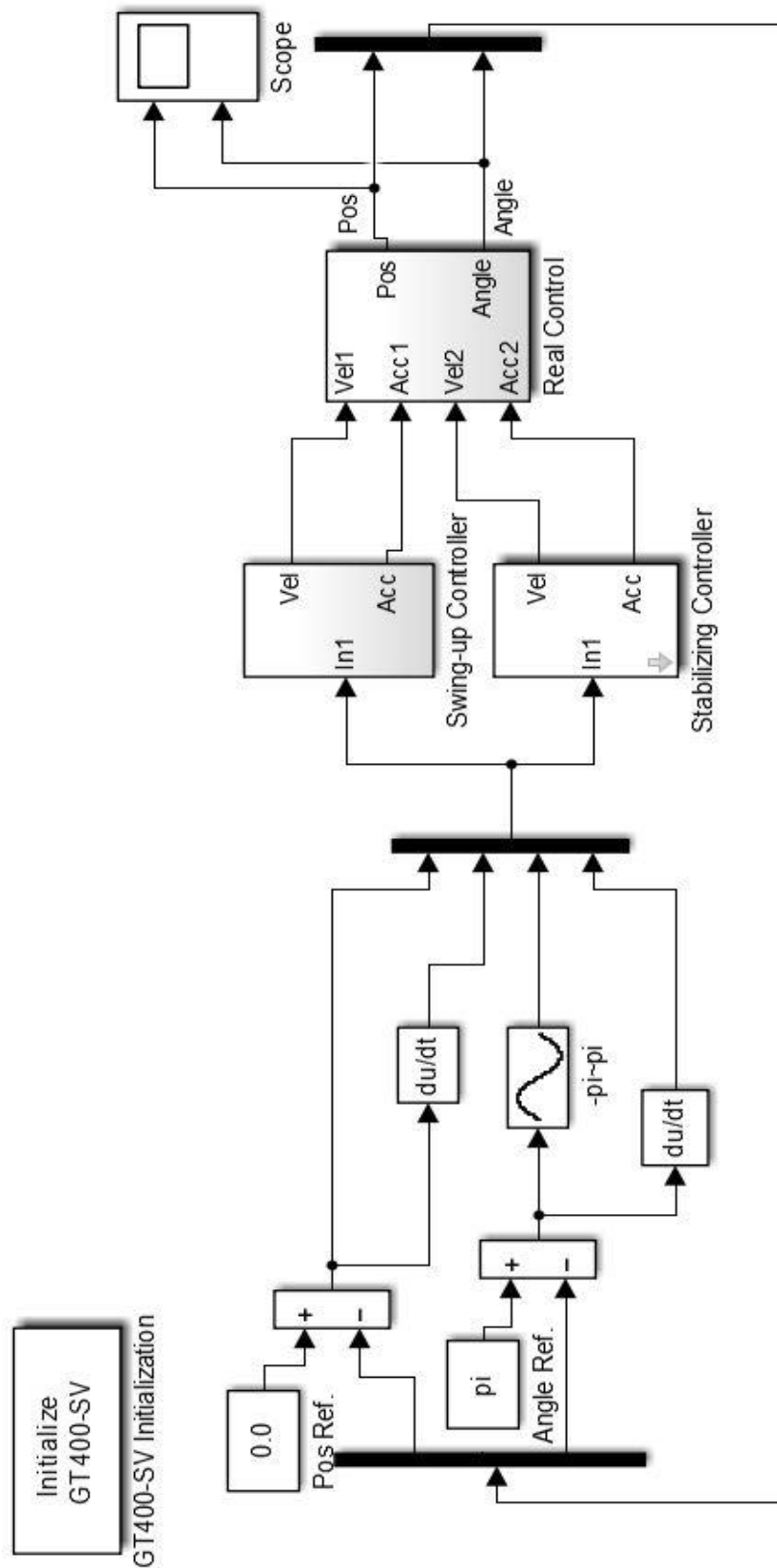
Figure 6.10: Main Simulink diagram for swing-up control problem.

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Figures 6.11 to 6.14 present the results of the real-time implementation of the energy swing-up controller with PPC. The pendulum swings up in 9 sec with 6 swings in before PPC is applied. As shown in Fig. 6.11, the pendulum reaches near the upward position in 9 seconds, where the switching takes place and the PPC takes change and able to stabilize the pendulum at upward position. The cart and pendulum displacement response are plotted near the upward position with some oscillations. As shown in Fig. 6.12, the cart moves in range of -0.125 to 0.075. The control input plot is shown in Fig. 6.13, where it can be clearly seen that during the swing-up stage, the control input lies in the range of [-2,2], whereas during the stabilizing stage, the control input lies in range of [-15,15]. The phase portrait using PPC is shown in Fig. 6.14, which resembles with simulation phase portrait result.
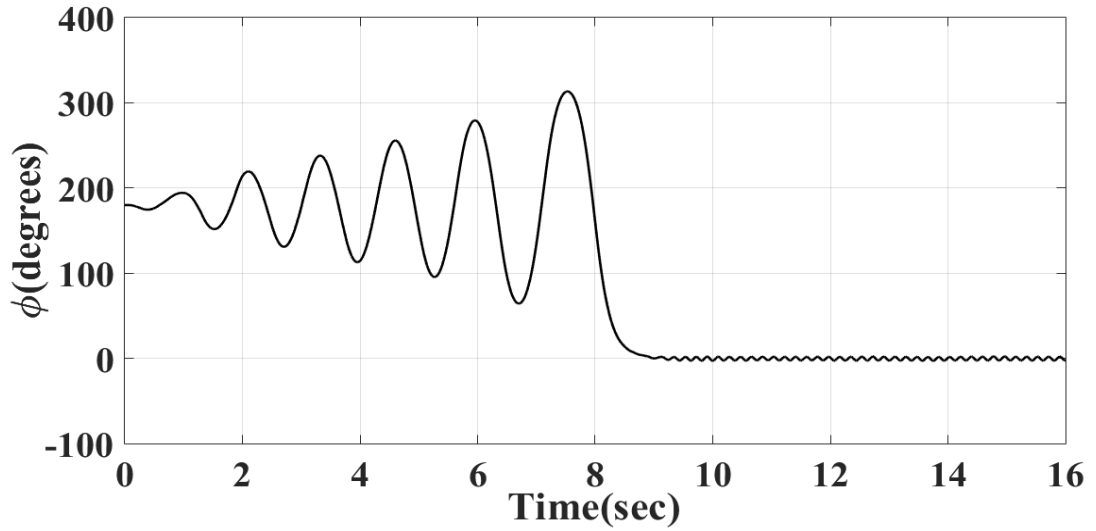
Figure 6.11: Experimental pendulum angular position response for energy swing-up with PPC.
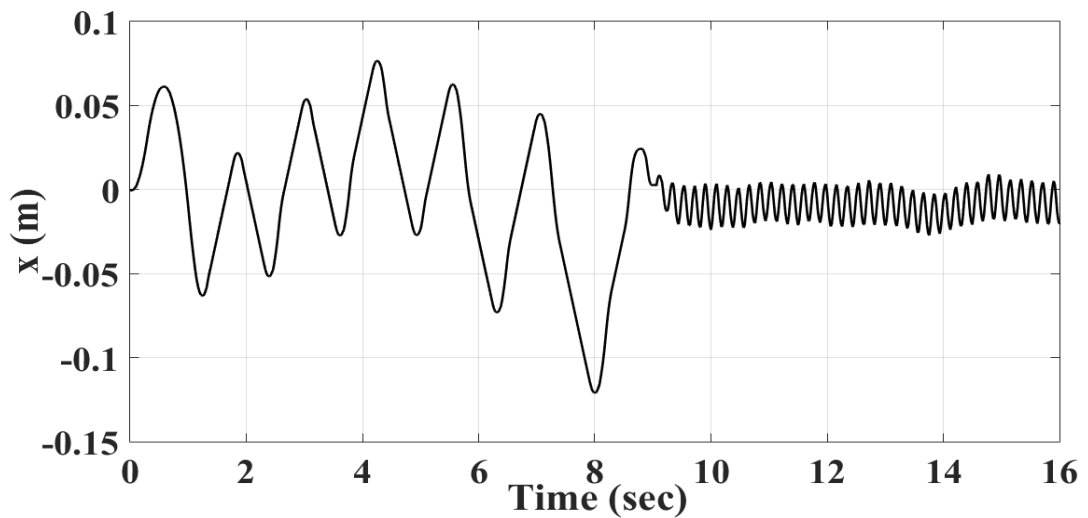
Figure 6.12: Experimental cart position response for energy swing-up with PPC.

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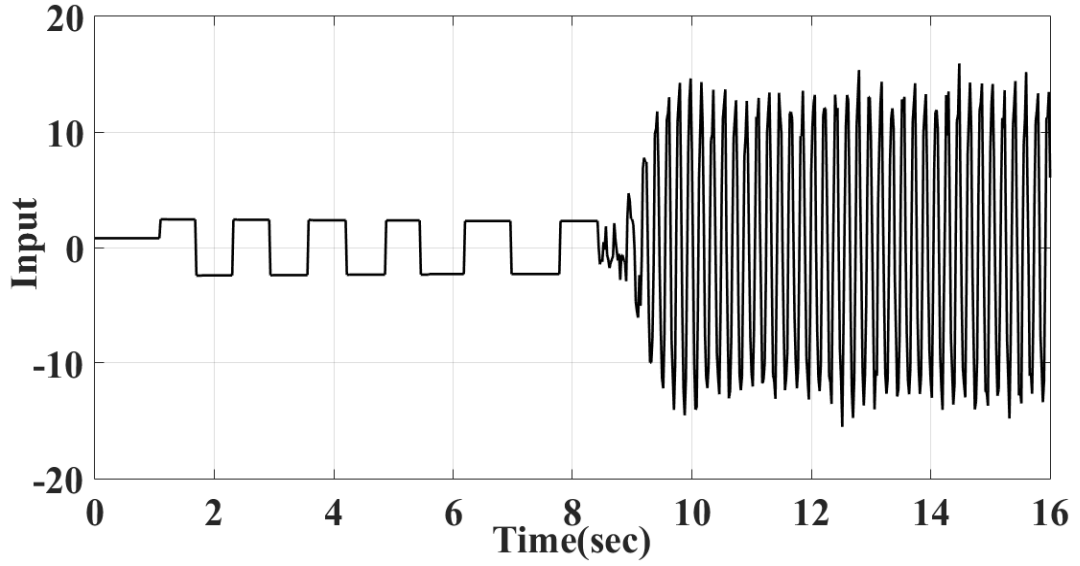
Figure 6.13: Experimental control input response for energy swing-up with PPC.
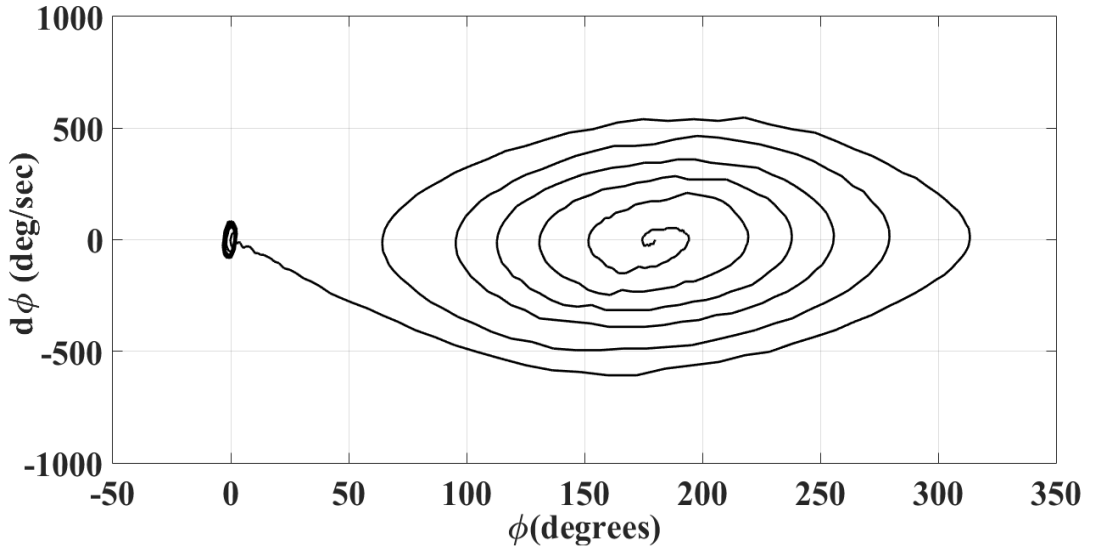
Figure 6.14: Experimental phase portrait for energy swing-up with PPC.

The energy controller is also tested with LQRC and the results are shown in Figs. 6.15 to 6.18. Swing-up controller behaves in a similar manner as PPC uses swings the pendulum up in 9 sec. LQR controller also stables the pendulum. Control input plots follows similar trend, as observed with PPC. In this case, the control input varies in the range of [-2,2] during swing-up stage and [-5,5] during stabilizing stage.

The oscillations are also observed in case of LQRC in all three cases (pendulum displacement, cart displacement and control input) which are less then PPC case. The comparison between both the controllers are discussed in next subsection.

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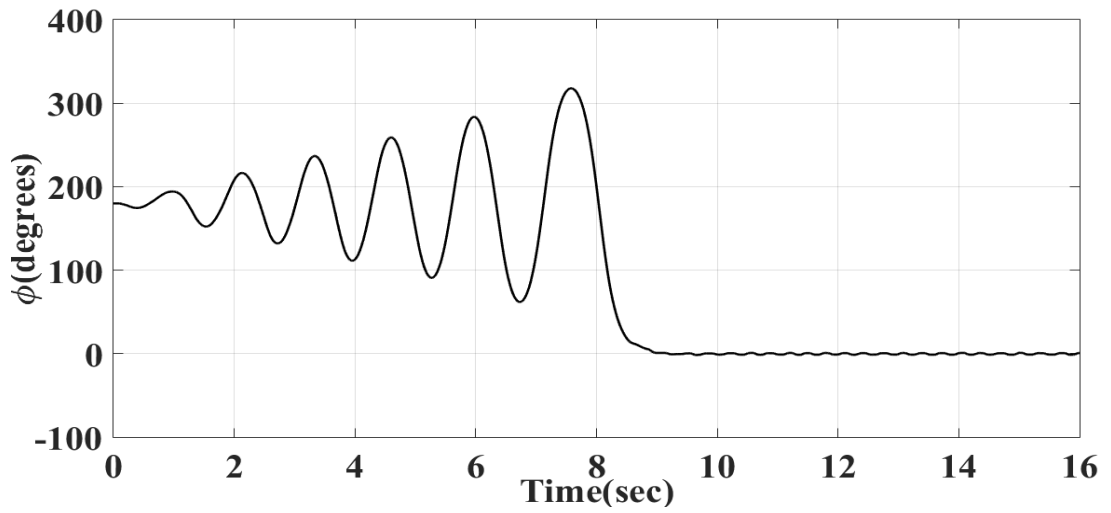
Figure 6.15: Experimental pendulum angular position response for energy swing-up with LQRC.
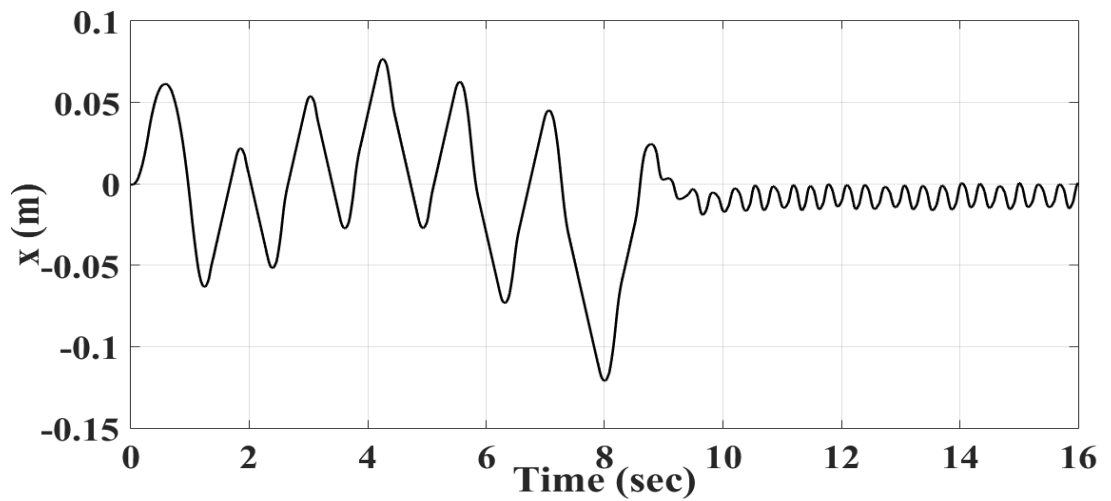
Figure 6.16: Experimental cart linear position response for energy swing-up with LQRC.

Figure 6.17: Experimental control input response for energy swing-up with LQRC.

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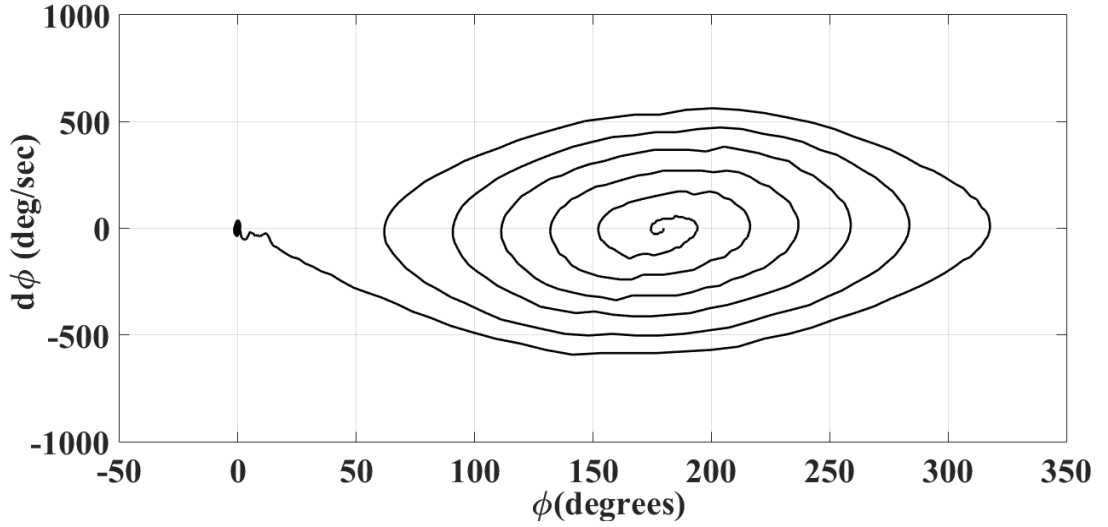
Figure 6.18: Experimental phase portrait for energy swing-up with LQRC.

It has been observed that both LQRC and PPC are able to stabilize the cart-pendulum system along with swing-up controller, both analytically and experimentally. It can be observed from analytical as well as experimental plots that pendulum displacement response is similar in both cases except there is one less swing in experimental case which is due to some initial acceleration applied to the cart to move the pendulum from its rest position. As far as the cart position is concerned, similar trend is observed in simulation and experimental runs, except their behavior in the swing-up stage. It has been observed in Fig. 6.6, that the cart position is centered about its mean position, which is not the case experimentally, as shown in Fig. 6.16. This slight discrepancy in cart behavior might occur due to several reasons like un-modeled dynamics of friction and backlash, sensitivity of the setup due to temperature variation, signal noise, digital quantization and possibility of relative movement between motor shaft and timing pulley. Further, during the stabilizing stage, slight different in behavior of PPC and LQRC is observed, which is reported in the next subsection.

6.3.2 Comparison between PPC and LQRC

In this subsection, the comparison between two controllers is discussed during stabilizing stage. Both the controllers are able to stabilize the cart-pendulum system, with settling time near 2 seconds. The aim of the comparison is to show the validity and effectiveness of LQRC with PPC, especially in terms of control input.

Figure 6.19 clearly shows that pendulum angle steady state oscillations are significantly decreased with LQRC as compared to PPC. In LQRC case, the pendulum

angle oscillation range is between -1.38° to 1.04°, which is very less as compared to -3° to 2.5° in PPC case. The cart displacement comparison is shown in Fig. 6.20. Marginal steady state error is observed with both the controllers, which is of the same order i.e. 0.02 m. However, higher frequencies are observed in PPC as compared to LQRC, which signifies that there is lot of jerks observed in PPC.
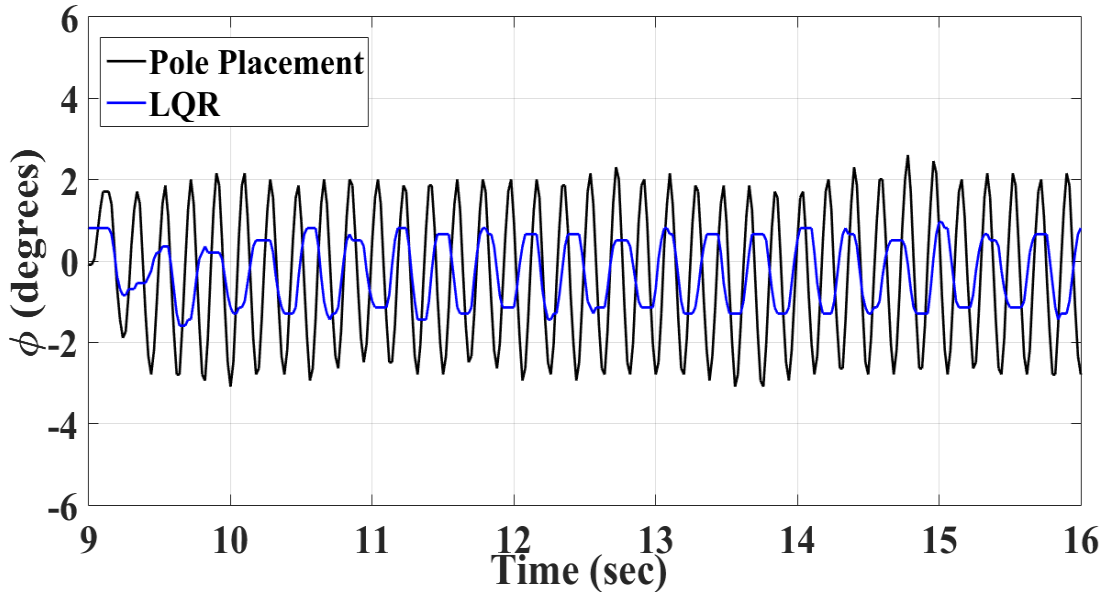
Figure 6.19: Pendulum angle experimental results.
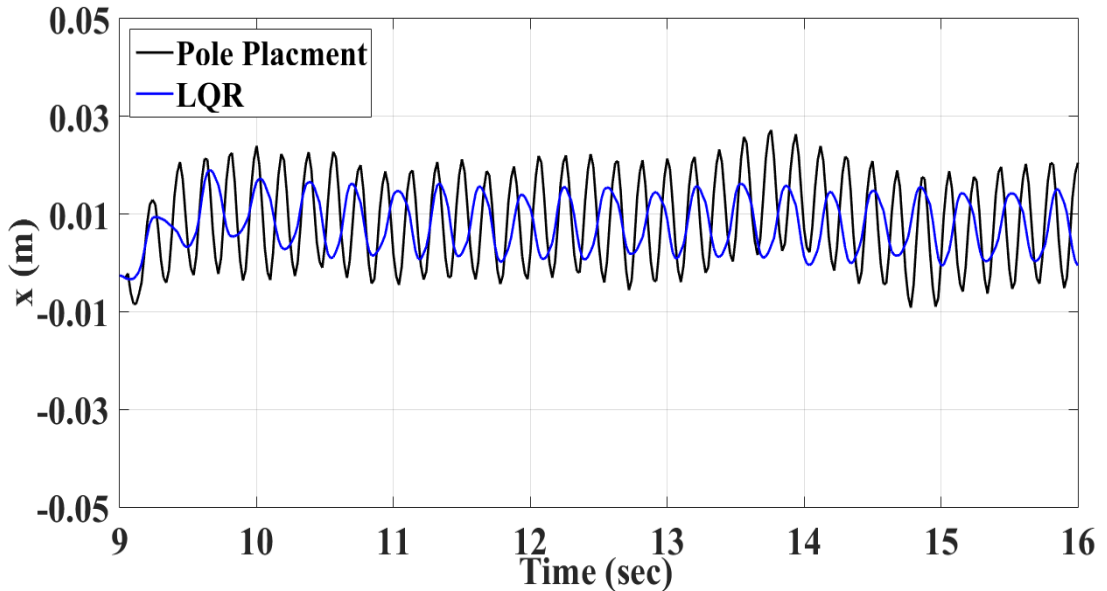
Figure 6.20: Cart displacement experimental results.

Control input magnitudes from both PPC and LQRC are also compared, which is shown in Fig. 6.21. It can be clearly observed that the control input magnitude in PPC is three

time more than the corresponding magnitude in LQRC.

The control input magnitude range in LQRC is -5 to 5, which is very less as compared to -15 to 15 in case of PPC. Hence, LQRC is capable of stabilizing the system with less control signal noise and lower consumption power as well.
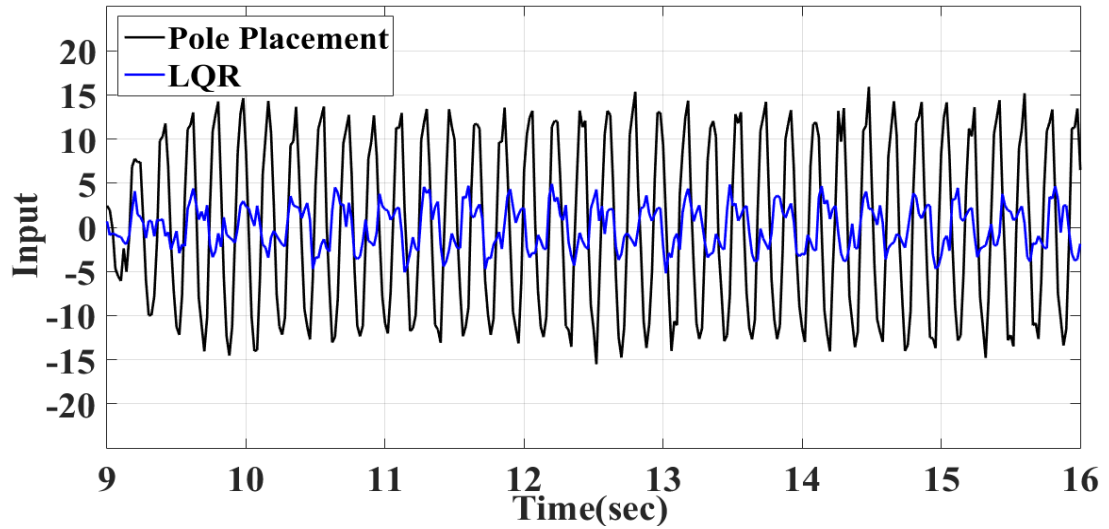
Figure 6.21: Control input experimental results.

Hence, the pendulum observed less oscillatory motion using LQR controller as compared to pole placement controller.

6.3.3 Robustness

Moreover, in order to investigate the robustness of the closed-loop control system, LQR controller is preferred as it gives less oscillatory response, as observed in Fig. 6.19. For this purpose, external disturbance is applied to the pendulum at its upright position, at time 1.1 second and 2.4 second with the help of a stick. It is observed in Fig. 6.22, that the pendulum returns to its stable position within 0.4 seconds.

Comparison plots shows the effectiveness of LQRC over the PPC however, more enhanced results could be achieved by using higher resolution sensors. In order to increase the robustness of the controllers, acceleration sensors can be utilized, but generally not preferred due to high noises during real-time implementation.

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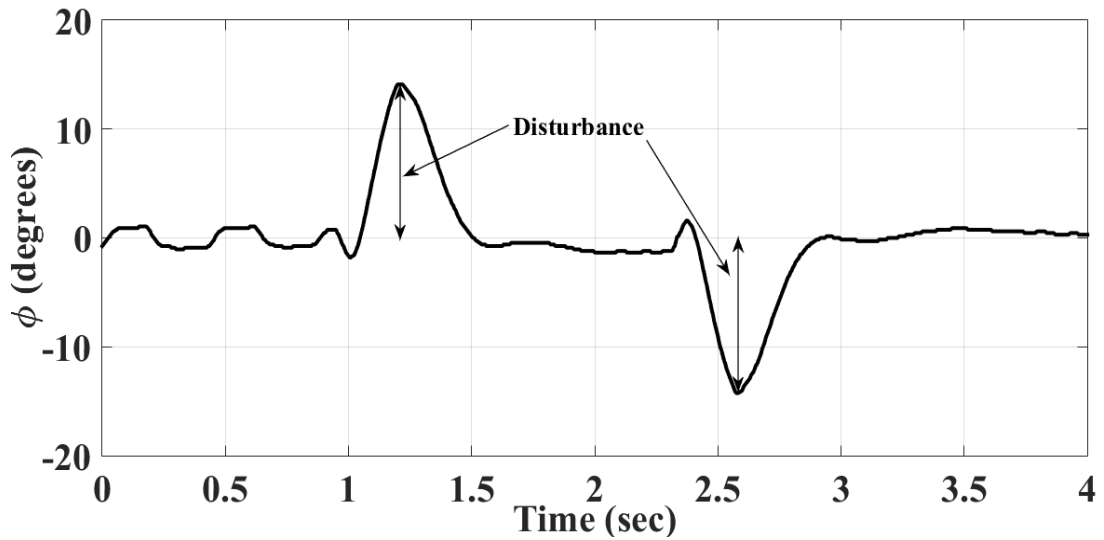
Figure 6.22: External disturbance applied to stable pendulum.

6.4 Summary

This chapter has reported the swing-up control problem of the cart-pendulum system by using two control schemes. The first control scheme is based on the energy method, which is nonlinear in nature and is mainly responsible to swing-up the pendulum from stable equilibrium point to the unstable equilibrium point. The second control scheme is based on full-state feedback control, which is linear in nature and responsible for the stabilization of the pendulum at its upright position. The cart movement has been constrained to move within the acceptable track length limits by choosing the non-zero initial condition of the cart velocity. During the stabilization stage, both PPC and LQRC has been used for comparing the results analytically as well as experimentally. During real-time experimentation runs, significant reductions has been observed in terms of oscillations of the pendulum (56%), as well as maximum control input (66.7%). Finally, the robustness of the LQR controller has been demonstrated experimentally by providing external disturbance to the pendulum at different times.

Chapter 7

Closing Remarks

7.1 Conclusions

Based on the dynamic modeling, controller design and implementation on the GLIP apparatus, following observations are made.

- It has been found that, the accurate mathematical modeling of the underactuated cart-pendulum system plays a vital role in the controlling part. The differential equations of the systems are derived by using Euler-Lagrange approach by taking actuator dynamics into account. The simulations and experimental results has been discussed for this system in this work. The real-time experiment results have been approximately resembled with the simulations results, which has been simulated by using mathematical modeling in MATLAB environment.

- It is found in literature that the benefits of using modern control theory on the real-time setups have not been exploited thoroughly. In the second part of the dissertation, PPC and LQRC are compared for the stabilization control problem. Both controllers are designed with having settling time of 2 seconds. Control input magnitude by both controllers to the system is compared, which results that LQR controller outperforms the pole placement controller as it results in 47.9% reduction in maximum control-input magnitude.

- Several energy control strategies have been listed in the literature to address the swing-up control problem of the cart-pendulum system. In the past, different authors has been used energy controller with five gains, later on, which has been simplified with two gains. In this dissertation, an attempt is made by developing simple energy controller along with constrained on linear and angular velocities of the systems, with one control gain only. It has been demonstrated that the developed energy controller is able to swing-up pendulum from the hanging

positon to upright equilibrium positon under the restricted track length. PPC and LQRC are compared for catching the pendulum at upright position. It has been that LQRC outperforms the PPC in the swing-up control problem of the cart-pendulum system in terms of oscillations of the pendulum (56%), as well as maximum control input (66.7%).

- To check the robust behavior of LQR controller, external disturbance has been applied to the pendulum at its upright position, at time different time intervals with the help of a stick. It has been observed, that the pendulum returns to its stable position within 0.4 seconds.

7.2 Future Work

- The work has been successfully implanted on the underactuated cart-pendulum system, which can be extended to other classes of pendulum family, such as rotary pendulum, planar pendulum, etc.
- The proposed work can be implemented on the real-time applications, due to its real-time verification.
- The dynamic model can be derived more accurately by considering static friction, stick slip friction, working table friction, etc. So that, the analytical and experimental results can be resembled more closely.
- Modern control theory linear controller can be used along power series based controllers for less oscillation and more robustness response.
- The developed energy controller strategy can be make more robust by integration with the intelligent control theory controllers, (neural network control, fuzzy-logic control, etc.) and other nonlinear controllers (sliding mode control, feedback linearization, computed torque control, etc.).
- Optimization algorithms such as genetic algorithm, ant-colony algorithm, etc. can be used to calculate the optimized gains for the stabilization and swing-up controllers.
- The sensory noise can be addressed by using Kalman filters.
- As the real-time setup can accommodate more input and output channels, the current setup can be extended for double-link inverted pendulum case.

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Annexure

A: Simulink Diagrams of Subsystems
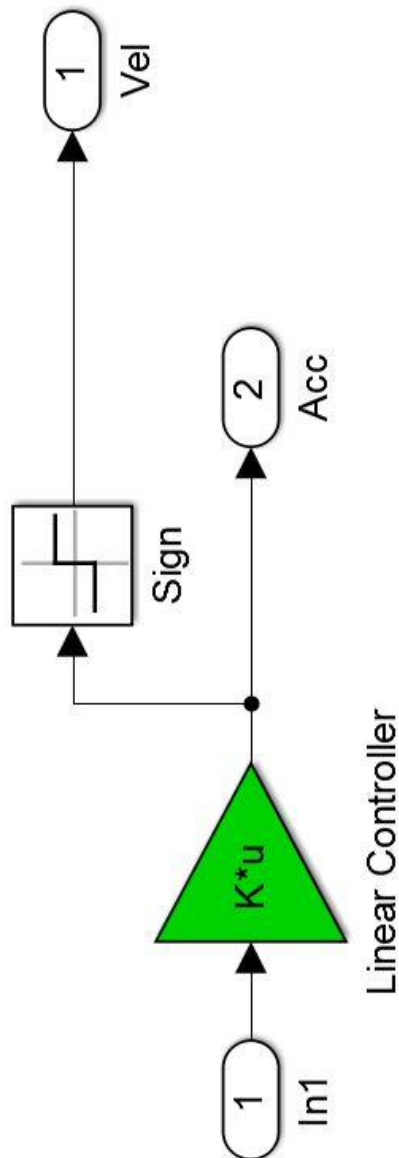
Figure A.1: Simulink subsystem used to compute stabilization controller.
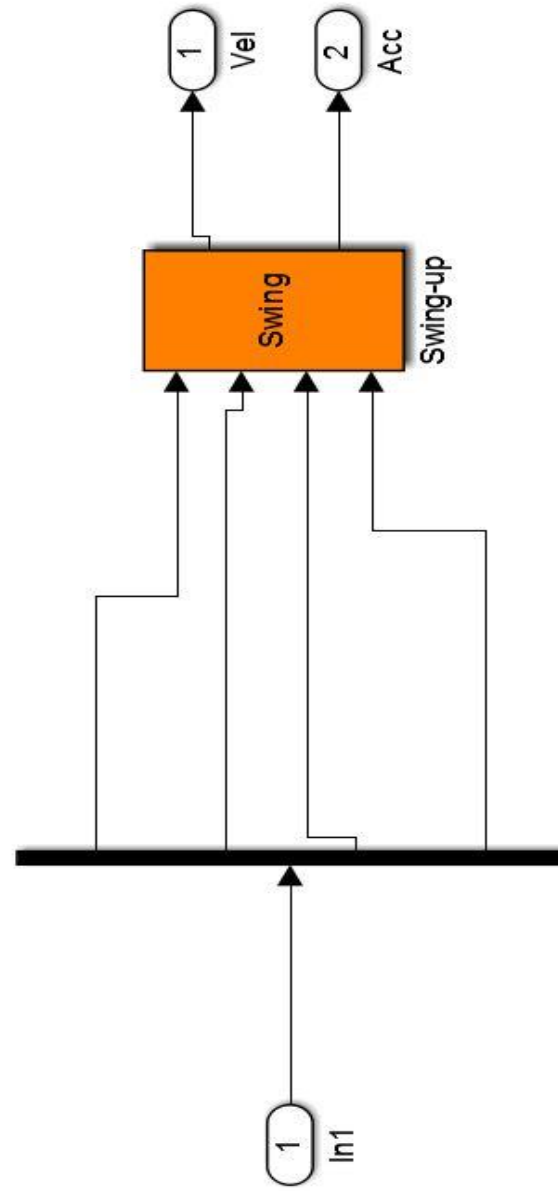
Figure A.2: Simulink subsystem to compute swing-up controller.
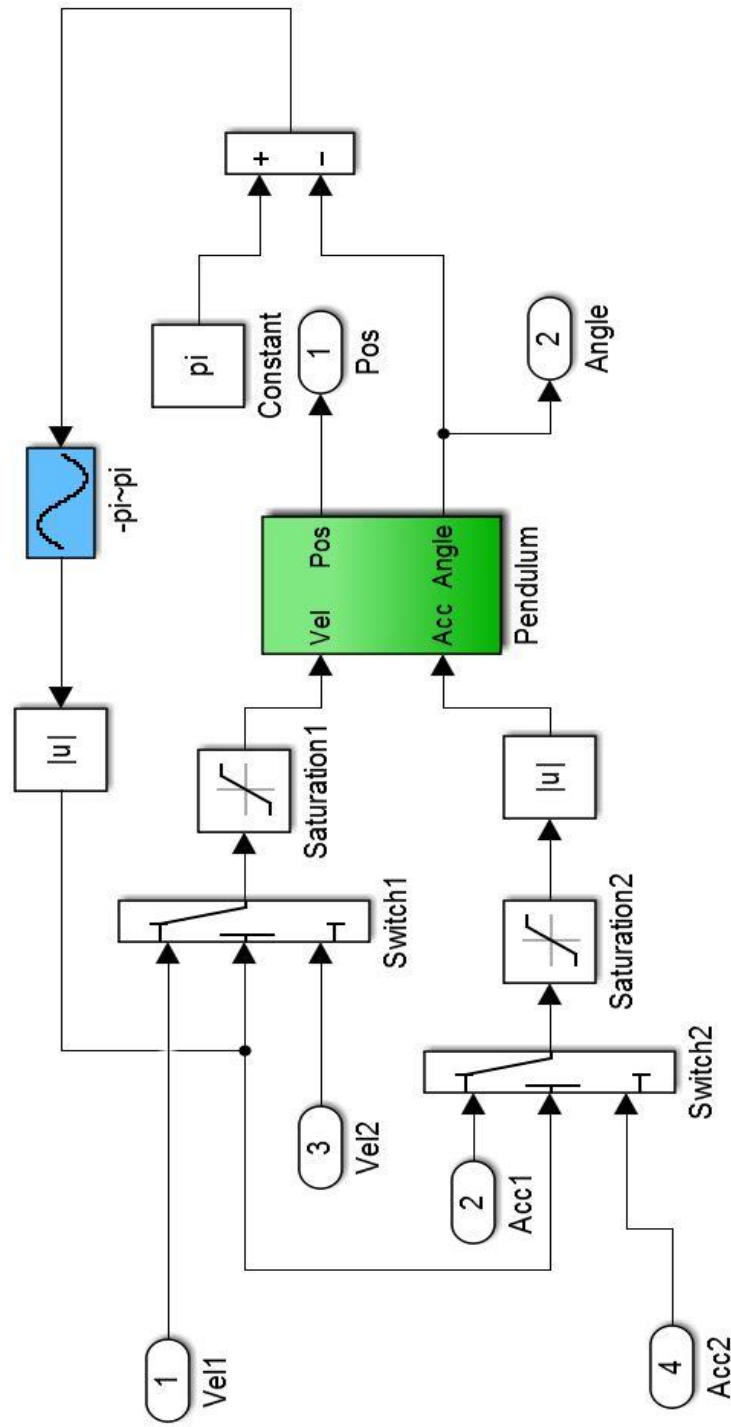
Figure A.3: Simulink subsystem to compute real-time control. Switch blocks are used for signal switching between swing-up and stabilization controllers. The details of the Pendulum block is given in Fig. A.4.
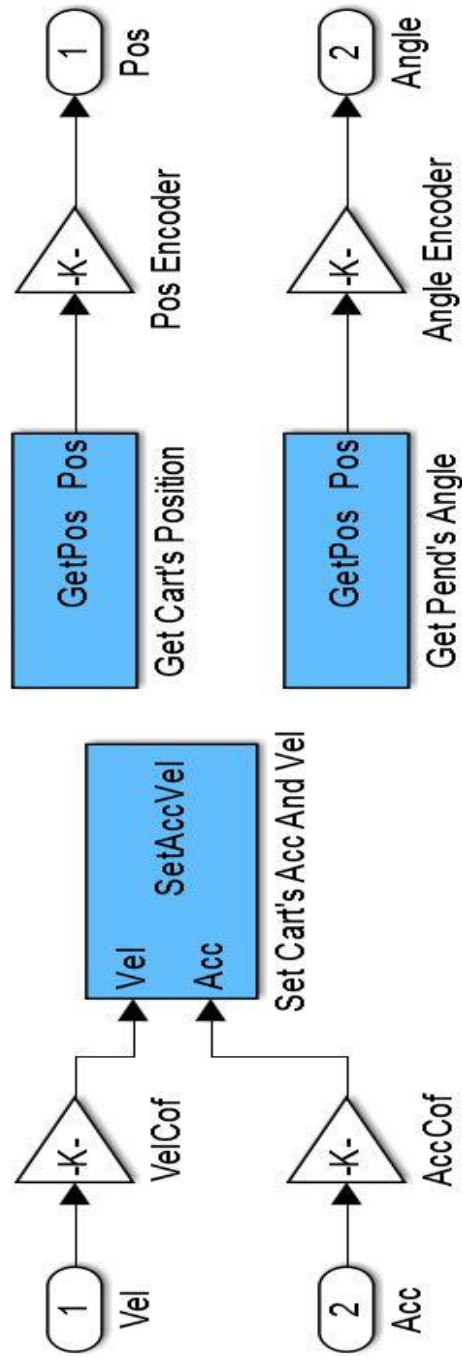
Figure A.4: Simulink subsystem to compute real-time pendulum control.

B: Plagiarism Report

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