Theoretical Results

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1 Intermediate points of fixed points compatible words lie in A

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2 Coordinates of intermediate points

Claim 2.1. *Given a fixed word $\omega \in \mathbb{F}_2 \langle H, V \rangle$ and a compatible homotopy class α, the coordinates of fixed points of $\Phi_\lambda(\omega)$ are determined up to $O(1/\lambda)$*

Proof. Assume ω starts with H^N. Due to our assumption that intermediate points lie in A, we know that

$$y + N\lambda(1 - |x|) = [ML - 1, ML + 1]$$

therefore

$$1 - |x| \in \left[\frac{ML - 1 - y}{N\lambda}, \frac{ML + 1 - y}{N\lambda}\right]$$

or

$$1 - |x| = \frac{ML}{N\lambda} + O(\frac{1}{\lambda})$$

Same calculation shows this for words starting with V. □

3 Actions

Claim 3.1. *Given a fixed word $\omega \in \mathbb{F}_2 \langle H, V\rangle$ and a compatible homotopy class α, all the actions relative to the loop having (alternatingly) $x = 0, y = 0$ are determined up to $O(1)$.*

Proof. For each part of type H^N or V^N the action is

$$\mathsf{A} = NF(x) - N\lambda x(1 - |x|)$$

Now, $F(x) = \lambda x(1 - \frac{|x|}{2})$ and therefore

$$\mathsf{A} = \lambda N x(1 - \frac{|x|}{2}) - \lambda N x + N\lambda |x| = N\lambda x\frac{|x|}{2}$$

For positive x we get:

$$\mathsf{A} = N\lambda\frac{x^2}{2} = \frac{N\lambda}{2}\left(1 - \frac{ML}{N\lambda} - O(\frac{1}{\lambda})\right)^2 = \frac{N\lambda}{2} - ML = -\frac{N\lambda}{2} + O(1)$$

For a general word, we have a sum of such terms, up to double counting the area at the corners. However, there is a fixed amount of corners depending on the word, and the area of each corner is bounded by 4. □