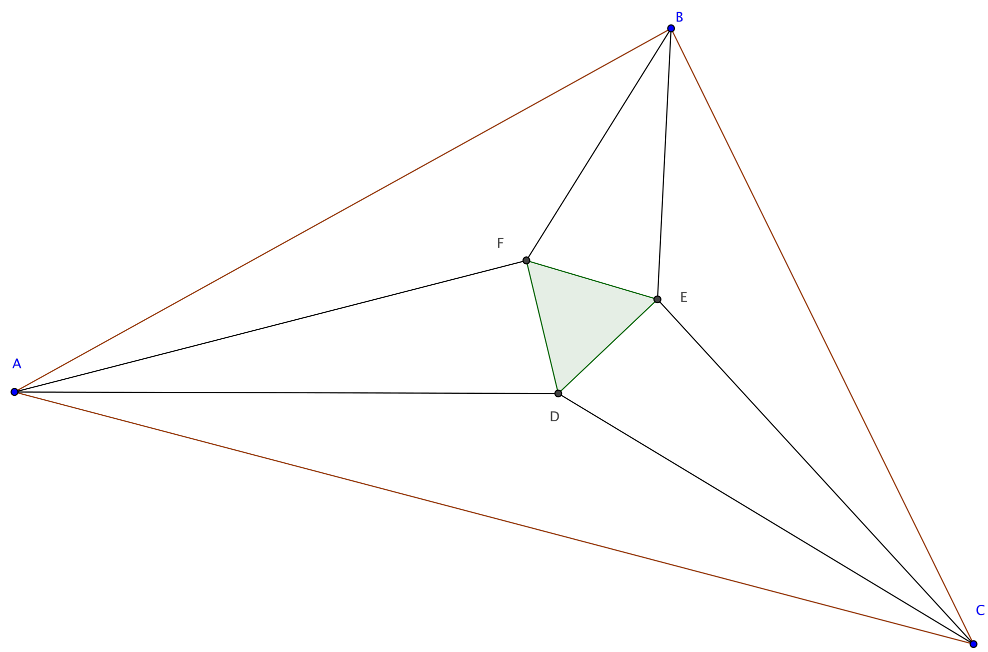
Transactions in **Euclidean Geometry**



Issue # 1

Opposite Angles in a Rhombus are Congruent

Thomas Bieber, Joshua Hawkins

September 3, 2014

Theorem 1.1. Let ABCD be a rhombus. Then angle ABC is congruent to angle ADC.

Proof. First, we are given a rhombus. By the definition of a rhombus, sides AB, BC, CD, and DA are congruent. Next, we draw a line from A to C and call it line AC by Postulate 1. We notice that triangles ABC and CDA share the side AC, which is congruent to itself. Since line AB is congruent with line AD, line BC is congruent with line DC, and line AC is congruent with line AC, then triangle ABC is congruent with triangle ADC by Euclid I.8. Since the triangles are congruent, we can conclude that all their angles are also congruent. So, angle ABC is congruent with angle ADC □

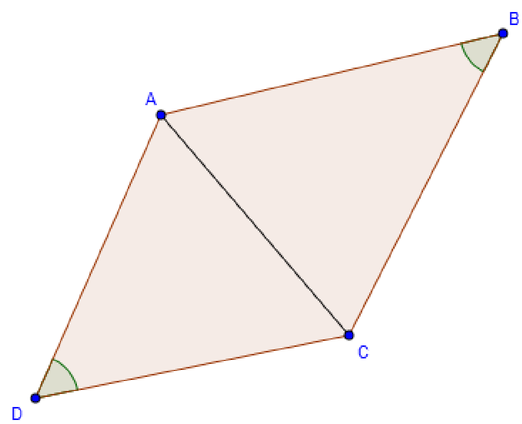


Figure 1: Rhombus

Angle Congruency in a Rhombus

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September 11, 2014

Conjecture 1.1. Let ABCD be a rhombus. If line AC is greather than line BD, then angles BDC is greater than angle BAC. If line BD is greater than line AC, then angle BAC is greater than angle BDC.

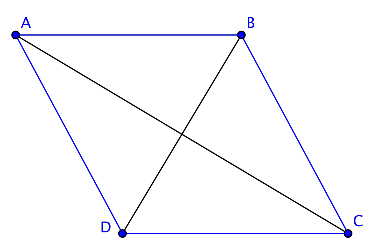
To disprove Conjecture 1.1, we will prove Theorem 1.1.

Theorem 1.1. Let ABCD be a rhombus. If line AC is not congruent to line BD, then angle BDC is not congruent to angle BAC.

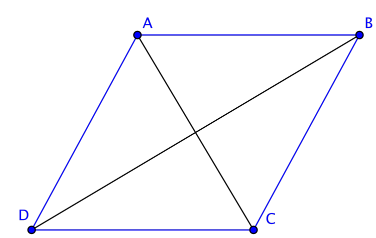
Proof. Let ABCD be a rhombus. Then line AB is congruent to line BC, and line BC is congruent to line CD.

Case 1: Let line AC be greater than line BD. Then by Euclid Proposition I.25, angle ABC is greater than angle BCD. By the Bieber-Hawkins Theorem, angle ABC is congruent to angle ADC and angle BCD is congruent to angle BAD. Since angle ABC is greater than angle BCD, angle ADC is greater than angle BAD. By the Bieber-Hawkins Theorem, angle BDA is congruent to angle BDC. Thus angle BDC is half of angle ADC. Also by the Bieber-Hawkins Theorem, angle DAC is congruent to angle BAC. Thus angle BAC is half of angle BAD. Since whenever line AC is greater than line BD angle ADC is greater than angle BAD, angle BDC is greater than angle BAC.

Case 2: Let line BD be greater than line AC. Then by Euclid Proposition I.25, angle BCD is greater than angle ABC. By the Bieber-Hawkins Theorem, angle BCD is congruent to angle BAD, and angle ABC is congruent to angle ADC. Since angle BCD is greater than angle ABC, angle BAD is greater than angle ADC. By the Bieber-Hawkins Theorem, angle BDA is congruent to angle BDC. Thus angle BDC is half of angle ADC. Similarly, angle BAC is half of angle BAD. Since whenever line AC is greater than line BD angle BAD is greater than angle ADC, angle BAC is greater than angle BDC. □



(a) Case 1



(b) Case 2

Congruent Diagonals in a Rhombus

John Fisher

September 8, 2014

Theorem 1.1b. Let ABCD be a rhombus. If AC is congruent to BD, then angle BAC is congruent to BDC.

Proof. Let ABCD be a rhombus. Then AB is congruent to BC, BC is congruent to CD, and CD is congruent to DA, by the definition of a rhombus.

Let AC be congruent to BD. Since we know AB is congruent to AD, CD is congruent to AD, and the diagonals (AC and BD) are congruent, we know that triangle BAD is congruent to ADC. By proposition 8, the corresponding angles of the triangles are also congruent. Thus angle BAD is congruent to angle ADC. Since triangle ABC is congruent to triangle ADC by the Bieber-Hawkins Theorem, we know angle BAC is congruent to angle DAC, which is half of angle BAD. Similarly, angle ADB is congruent to BDC, which is half of angle ADC. Since angle ADC is congruent to angle BAD, angle BAC is half of BAD, and angle BDC is half of angle ADC, then angle BAC must be congruent to angle BDC. □

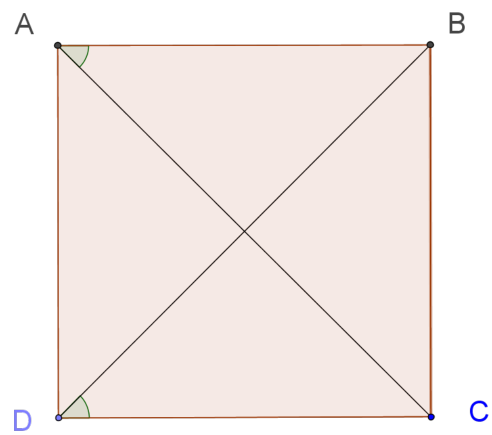


Figure 1: Rhombus ABCD with congruent diagonals

The Crossing of Rhombus Diagonals

Emily Herbst

September 11, 2014

Introduction: For the theorem below, we will need the following conjecture:

Conjecture D. Let AC be a segment and x a point on this segment. Suppose that segment BX meets AC at right angles, and segment DX meets AC at right angles. Then the points B, X, and D are collinear.

Assume that Conjecture D is in fact true.

Theorem 1.2. Let ABCD be a rhombus. The diagonals, AC and BD, must cross.

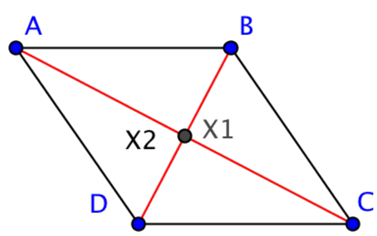


Figure 1: Rhombus ABCD

Proof. Let ABCD be a rhombus. Let there be the line AC. By Euclid Proposition I.9, choose a point X1 on the line AC where angle ABX1 is congruent to angle CBX1. Since angle ABX1 is congruent to angle CBX1, line AB is congruent to line BC, and line BX1 is congruent to line BX1, then by Euclid Proposition I.4, triangle ABX1 is congruent to triangle CBX1. Thus line AX1 is congruent to line CX1, so X1 is considered the midpoint of line AC. Since the straight line BX1 was created on the straight line AC with X1 being the midpoint, then by Euclid Definition I.1 angle BX1C and angle BX1A are right angles.

By Euclid Propositon I.9, choose a point X2 on the line AC where angle ADX2 is congruent to angle CDX2. Since angle ADX2 is congruent to angle CDX2, line AD is congruent to line CD, and line DX2 is congruent to line DX2, then by Euclid Proposition I.4, triangle ADX2 and triangle CDX2 are congruent. Thus line AX2 is congruent to line CX2, so X2 is the midpoint of line AC. Since the straight line BX2 was created on the straight line AC with X2 being the midpoint, then by Euclid Definition I.1 angle DX2C and angle DX2A are right angles.

Since X1 and X2 are both midpoints on the line AC, X1 is equal to X2. For clarification purposes, because X1 is equal to X2, let X1 and X2 be renamed X. By assuming that Conjecture D is true, points B, X, and D are collinear and thus causing the creation of the line BD and X is the midpoint of line AC. Thus line AC must cross line BD.

□

Challenge 1.4: Rhombus Construction

Emily Bachmeier

September 16, 2014

Theorem 1.4. Given a segment AB, it is possible to find a compass and straightedge construction of a rhombus $ABCD$ having AB as one of its sides.

Proof. 1. Construct a circle with center A through B.
2. Construct a circle with center B through A.
3. Construct the point of intersection of these two circles, C.
4. Construct a circle with center C through A.
5. Construct the point of intersection of circle C and circle A, D.
6. Connect all four points A, B, C, D, to form rhombus $ABCD$. (Figure 1) $\square$

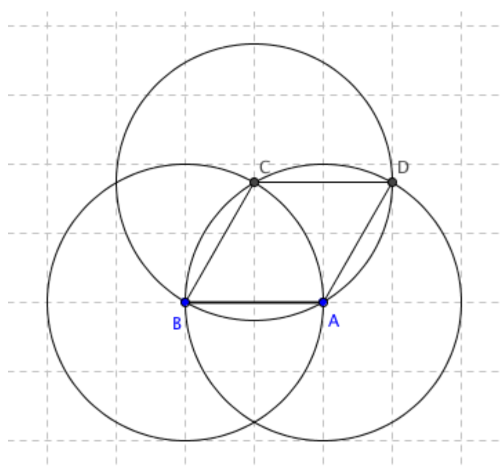


Figure 1: Step 7

Proof. By Euclid I.1, it can be shown that segment BA is congruent to segment BC because both segments stem from the point B in the center of the circle to a point (A and C) on the edge of the same circle. Thus, these segments BA and BC are both inscribed in the same circle and congruent to one another.

Likewise, segments AB and AD are congruent because, by Euclid I.1, these two segments are inscribed in circle AB and both go through the middle point A. Therefore, segments AB and AD are congruent. Similarly, segments CD and CB are congruent by the same arguement.

Therefore, it can be concluded that each of the segments are mutually congruent to each other because, by Euclid I.1, things whic are congruent to the same thing are also congruent to each other. Thus, since a rhombus consists of four mutually congruent sides, $ABCD$ is a rhombus. (Figure 1)

$\square$

How to Construct Non-congruent Rhombi

Eric Scheidecker

September 12, 2014

Ms. Brockmeier met Challenge 1.4 by constructing a rhombus in part using Proposition 1 from Euclid's Elements; a construction of an equilateral triangle on a straight line. Unfortunately, her process can only form rhombi with one of its diagonals creating two equilateral triangles with the edges of the rhombus.

For any points A,B, and C the notation $\overline{AB}$ should be read as the line segment between A and B and the notation $\odot AB$ should be read as the circle centered on A passing through B.

Theorem 1.5. Let segment AB be given. It is possible to construct with a straightedge and compass a large family of rhombi having AB as a side.

Here are the steps for such a construction.

We are given $\overline{AB}$.

1. Make $\odot AB$ and choose some point C on $\odot AB$ not on the line through A and B.
2. Make $\odot CA$.
3. Make $\odot BA$ and choose a point D on the intersection of $\odot BA$ and $\odot CA$.
3. Make $\overline{AC}$.
4. Make $\overline{CD}$.
5. Make $\overline{BD}$.

Proof. By definitions 15 and 16, any two line segments that extend from the center of the same circle to the circle's edge will be congruent to each other.

In our enumerated construction, points points A and D lie on a circle centered on C, by our definitions, $\overline{AC}$ and $\overline{CD}$ are congruent. Similarly, since A and D lie on the circle centered on B, $\overline{AB}$ and $\overline{BD}$ are congruent. Since B and C lie on the circle centered on A, then $\overline{AB}$ and $\overline{AC}$ are congruent. Since $\overline{AB}$ is congruent to $\overline{BD}$, $\overline{AC}$ is congruent to $\overline{BD}$. Similarly, $\overline{AB}$ is congruent to $\overline{CD}$. Therefore, all edges are congruent to each other and they must form a rhombus.

Note that the one step changed was how to choose C. In the original construction, C was chosen at the intersection of $\odot AB$ and $\odot BA$ whereas we just choose it anywhere on $\odot AB$ that doesn't intersect a line passing through A and B. $\square$

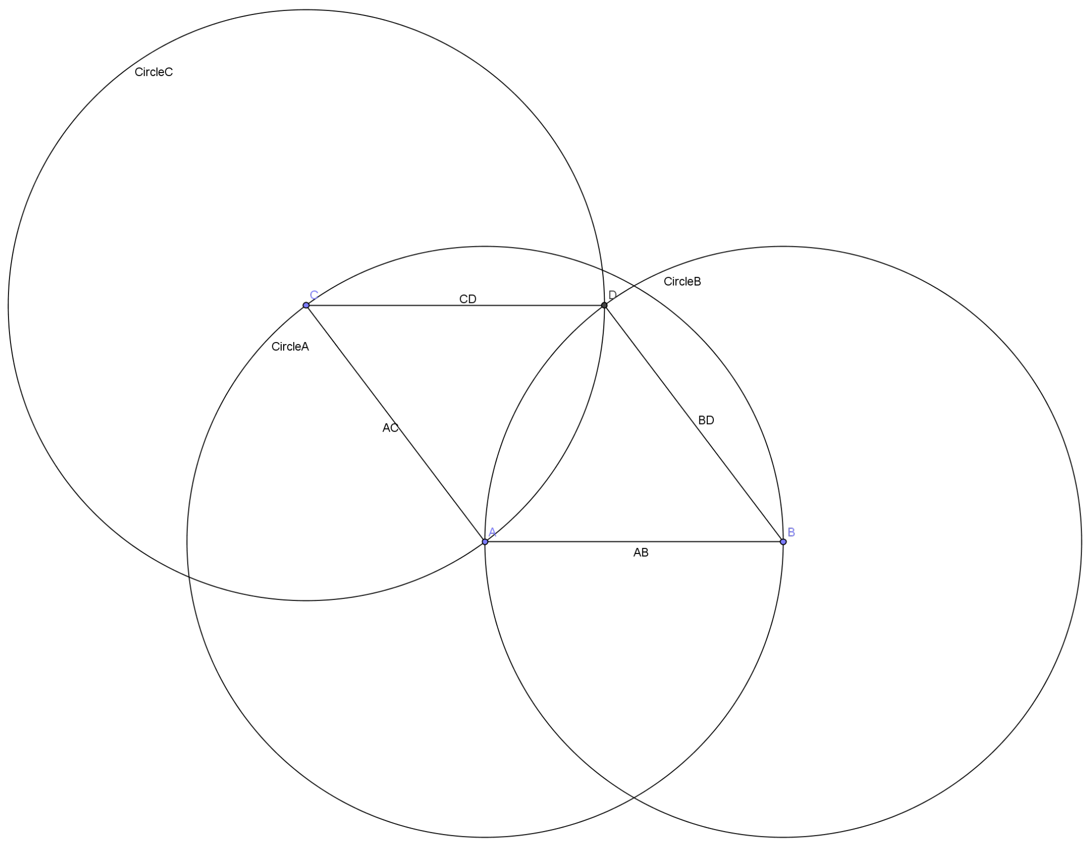


Figure 1: The construction of a rhombus

From Rhombus to Parrallelogram

Katy Goodmundson

September 13, 2014

Theorem 1.6. Let ABCD be a quadrilateral. If ABCD is a rhombus, then ABCD is a parallelogram.

Proof. Let ABCD be a rhombus. A rhombus is a quadrilateral with all four sides mutually congruent, by definition. By Postulate 1, let line segment AC be joined. Line AB is congruent to DC and line AD is congruent to BC, by definition of a rhombus. Also, line AC is congruent to itself. By Proposition 8, angle BAC is congruent to angle DCA. So, by Proposition 8, the triangles ABC and CDA are congruent. By Postulate 2, extend lines AB and line DC. Then, by Proposition 27, since AC cuts through AB and DC and angle BAC is congruent to DCA, AB is parallel to DC. Similarly, AC cuts through AD and BC. By CPCT, the angle BCA is congruent to angle DAC. Then, by Proposition 27, since AC cuts through AD and BC, AD is also parrallel to BC. Thus, the rhombus is a parallelogram. □

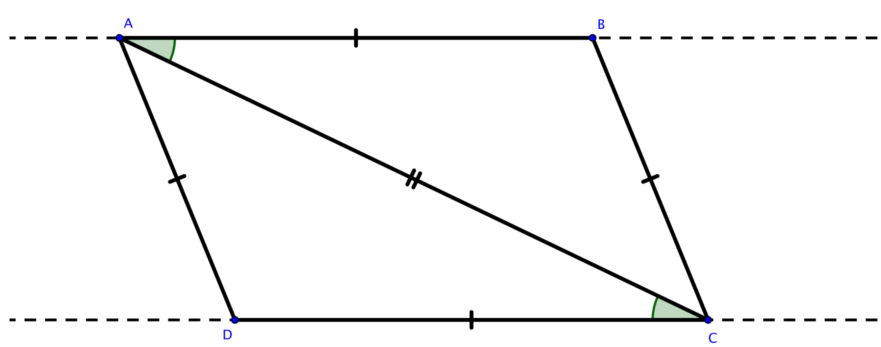


Figure 1: Rhombus ABCD

The Diagonals of a Rhombus are Perpendicular

Jalen Raymond

September 11, 2014

Theorem 1.7. Let ABCD be a rhombus. Suppose that the diagonals AC and BD meet at a point X. The angle AXB is a right angle.

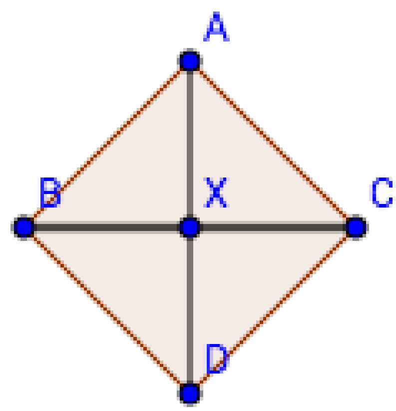


Figure 1:

Theorem 1.7A. Triangle ABX is congruent to Triangle ACX and Triangle CXD and Triangle DCB.

Proof. By Definition of a Rhombus, Line AB is congruent to line CD and line AC is congruent to line BD. According to the definition of a diagonal, the line must connect two non-adjacent points. Thus, Let line AD be a diagonal that goes through point X. Also, Let Line BC be a diagonal that goes through point X.

Because line AB is congruent to line AC, line BD is congruent to line CD, and line AD is congruent to line AD, triangle ABD is congruent to triangle ACD based on proposition 8. Triangle ABD and triangle ACD are both isosceles triangles because they both have two equal sides. Because line AB is congruent to line BD, line AC is congruent to line CD and line BC is congruent to line BC, trangle BAC is congruent to triangle BDC based on proposition 8. Triangle BAC and triangle BDC are both isosceles triangles because they both have two equal sides.

According to proposition 5, the base angles of an isoceles triangle are congruent. Therefore, angle BAX is congruent to angle CAX, angle BCX is congruent to angle CDX, angle ABX is congruent to angle DBX, and angle ACX is congruent to angle DCX.

Because AB is congruent to line AC, AX is congruent to line AX, and angle BAX is congruent to CAX, it is true that triangle BAX is congruent to line CAX based on proposition 4. Because line CA is congruent to line CD, CX is congruent to line CX, and angle ACX is congruent to angle DCX, it is true that triangle ACX is congruent to triangle DCX based on proposition 4. Because line CD is congruent to line BD, line DX is congruent to line DX, and angle BDX is congruent to CDX, it is true that triangle CDX is congruent to triangle BDX based on proposition 4.

Based on this, triangle ABX is congruent to triangle ACX, triangle DCX, and triangle DBX.

□

Theorem 1.7B. The diagonals of a rhombus are perpendicular

Proof. Based on the fact that angle BAX is congruent to angle CAX, then by definition 10, line AD creates right angles on line BC at point X. Similarly, this works for angle BDX and angle CDX. Based on the fact that angle ACX is congruent to angle DCX, then by definition 10, line CB creates right angles on line AD at point X. Similarly, this works for angle ABX and angle DBX

Based on this, the diagonals of a rhombus are perpendicular.

□