Transform Calculus (MA20101)

Assignment – 2

1) Let $f(t)$ be defined by

$$f(t) = \begin{cases} t , & 0 \le t \le \pi/2 \\ \pi/2 , & \pi/2 \le t \le \pi \\ \pi - \frac{t}{2} , & \pi \le t \le 2\pi . \end{cases}$$

Then sketch the graph of $f(t)$ and determine a Fourier series of $f(t)$ by assuming $f(t) = f(t+2\pi)$.

2) Determine the Fourier series for $f(x) = H(x)$, the Heaviside Unit Step function, in the range $[-\pi, \pi]$, $f(x) = f(x+2\pi)$.

Hence find the value of the series

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$$

3) Find the Fourier series of the functions –

$$f(x) = \begin{cases} \sin(x/2) & 0 \le x \le \pi \\ -\sin(x/2) & \pi < x \le 2\pi \end{cases}$$

with $f(x) = f(x+2\pi)$.

4) Determine the Fourier series for the function $f(x) = 1 - x^2$, $f(x) = f(x+2\pi)$. Suggest the possible value of $f(x)$ at $x = \pi$

5) Find the Fourier series expansion of the function $f(t)$ where

$$f(t) = \begin{cases} \pi^2 & -\pi < t < 0 \\ (t-\pi)^2 & 0 \leq t < \pi, \end{cases}$$

with $f(t) = f(t+2\pi)$.

Hence determine the values of the series —

(i) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (ii) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$.

Determine the two Fourier half-range series for the above function $f(t)$ and sketch the graphs of the function in both cases over the range $[-2\pi \leq t \leq 2\pi]$.

6) Given the half range sine series

$$f(\pi - t) = \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)t}{(2n-1)^3} \quad 0 \leq t \leq \pi,$$ use Parseval's theorem to deduce the value of the series $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^6}$. Hence, deduce the value of the series $\sum_{n=1}^{\infty} \frac{1}{n^6}$.

7) Expand $f(x) = \sin x$, $0 < x < \pi$ in Fourier cosine series

8) Expand the following functions in Fourier sine series:

a) $f(x) = \cos x$ $(0 < x < \pi)$

b) $f(x) = x^3$ $(0 \leq x < \pi)$

9) Graph each of the following functions & find its corresponding Fourier series, using properties of even and odd functions ~~whenever~~ wherever applicable –

a) $$f(x) = \begin{cases} 8 & 0 < x < 2 \\ -8 & 2 < x < 4 \end{cases}$$

Periodicity of f is 4.

b) $$f(x) = \begin{cases} 2x & 0 \le x \le 3 \\ 0 & -3 < x < 0. \end{cases}$$

Periodicity of f is 6.

c) $f(x) = 4x \qquad 0 < x < 10$, Periodicity of f is 10

9A) Find the Fourier Series of the function –

$$f(x) = \begin{cases} \sin \frac{x}{2} & \text{for } 0 \le x \le \pi \\ -\sin x/2 & \text{for } 0 \le x \le 2\pi \end{cases}$$

$$f(x) = f(x + 2\pi)$$

9B) Derive Fourier Series for e^{-ax}, $-\pi < x < \pi$ and deduce series for $\frac{\pi}{\sinh \pi}$.

10) Expand $f(x) = x^2, -\pi < x < \pi$ in Fourier series and show that

a) $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

b) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$

c) $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$

11) Find the Fourier series for the function f defined by $f(x) = x - x^2, -\pi < x < \pi$. Deduce that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}.$$

12) If a is a real number, find the Fourier series of the function f defined by –

$$f(x) = e^{ax}, \quad -\pi < x < \pi$$

$$f(x+2\pi) = f(x), \quad x \in \mathbb{R}$$

Deduce the value of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{a^2+n^2}$

13) Expand $f(t) = (1-t^2), -1 \leq t \leq 1$ in Fourier series.

Q.13 A) Determine Fourier series expansion of the function –

$$f(x) = \begin{cases} 2, & 0 < x < \frac{2\pi}{3} \\ 1, & \frac{2\pi}{3} < x < \frac{4\pi}{3} \\ 0, & \frac{4\pi}{3} < x < 2\pi \end{cases}$$

14) Determine the Fourier Series of the square wave function f defined by

$$f(x) = \begin{cases} -k & \text{for } -\pi < x < 0 \\ k & \text{for } 0 < x < \pi \end{cases}$$

$$f(x) = f(x + 2\pi)$$

Deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} = \frac{\pi}{4}$.

15) Expand $f(x) = x^2$, $0 < x < 2\pi$ in a Fourier Series assuming that the function is of period 2π.

16) Develop Fourier series for $f(x) = x \sin x$, $0 < x < 2\pi$

17) Determine the Fourier series of the half-wave rectified sinusoidal defined by

$$f(t) = \begin{cases} \sin t & \text{for } 0 < t < \pi \\ 0 & \text{for } \pi < t < 2\pi \end{cases}$$

$$f(t) = f(t + 2\pi)$$

Deduce that

a) $\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \cdots = \frac{1}{2}$

b) $\frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - = \frac{\pi - 2}{4}$

18) Determine half-rang sine series for the function f defined by $f(t) = t^2 + t$, $0 \le t \le \pi$

19) Find the Fourier series of the following function:

$$f(x) = \begin{cases} x^2 & \text{for } 0 \le x \le \pi \\ -x^2 & \text{for } -\pi \le x \le 0 \end{cases}$$