MEETING 04, GROUP WORK

LINEAR ALGEBRA SECTION 01, SPRING 2014

Basic Advice: Successful students in linear algebra learn to speak with abstract terminology, compute with algebra, and think with geometry. If you feel stuck, try drawing a picture.

The focus of today's seminar work is to get comfortable with possible matrix representations for the algebra involved in studying (hyper)planes and vectors. A lot of this will build on the activities from meeting 02.

Part One: 2D Warm Up

Task 1. Consider the line in the Cartesian plane described in parametric vector form as

$$\ell = \left\{ X = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \end{pmatrix} \mid t \text{ is a real number} \right\}.$$

Write the pair of parametric equations for the coordinates x and y of a generic point $X = \begin{pmatrix} x \\ y \end{pmatrix}$ on ℓ as functions of the parameter t.

Task 2. Rewrite the linear combination equation for this situation as a matrix-vector equation involving the vectors X and $\begin{pmatrix} 1 \\ t \end{pmatrix}$.

Task 3. Eliminate the parameter t from the equations in task 1 to write the standard form of the equation of ℓ.

Task 4. Rewrite the equation you just found as a matrix-vector equation involving the vector $\begin{pmatrix} x \\ y \end{pmatrix}$.

Part Two: working in 3D

For the rest of this investigation, we shall consider the two points P and Q in $\mathbb{R}^3$ given by the vectors

$$P = \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} \qquad \text{and} \qquad Q = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Task 5. Write the vector parametric form of the line m in $\mathbb{R}^3$ which passes through P and Q.

Task 6. Rewrite that vector parametric description as a matrix-vector equation which involves the vector $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

Task 7. Find some equations in the coordinates x, y and z of a generic point $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ which describe exactly when the point X lies on the line m. (The parameter t should no longer appear.)

Task 8. Rewrite that system of equations as a matrix-vector equation involving the vector X.

Part III: A linear combination equation

Now consider these four vectors in $\mathbb{R}^3$.

$$u = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}, \quad v = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}, \quad w = \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Task 9 (*Strogatz*, p 10 #31)**.** Write down three equations for c, d, and e so that $cu + dv + ew = b$.

Task 10. Rewrite the system of equations as a matrix-vector equation involving the vector $\begin{pmatrix} c \\ d \\ e \end{pmatrix}$.