LINEAR ALGEBRA

STRANG, SECTION 1.3

1. The assignment

- Read section 1.1 of Strang (pages 22-27).
- Read the following and complete the exercises below.

2. Matrices

3. Sage instructions

I have made a Sage worksheet file with some basic commands that you might find useful in investigating with matrices. The file is called *section1-3.sagews*. It also has some interactive demonstrations about how to deal with vectors.

4. Questions for Section 1.3

Exercise 25. Make an example of a matrix $\left(\begin{smallmatrix} 1 & \bullet \\ -1 & \bullet \end{smallmatrix}\right)$ so that the equation

$$\begin{pmatrix} 1 & \bullet \\ -1 & \bullet \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

has exactly one solution, or explain why this is not possible.

Interpret this as a statement about 2-vectors and draw the picture which corresponds.

Exercise 26. Make an example of a matrix $\left(\begin{smallmatrix} 4 & 8 & \bullet \\ 3 & 6 & \bullet \\ 1 & 2 & \bullet \end{smallmatrix}\right)$ so that the equation

$$\begin{pmatrix} 4 & 8 & \bullet \\ 3 & 6 & \bullet \\ 1 & 2 & \bullet \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \\ 2 \end{pmatrix}$$

has exactly one solution, or explain why this is not possible.

Interpret this as a statement about 3-vectors and draw the picture which corresponds.

Exercise 27. Make an example of a matrix $\left(\begin{smallmatrix} 2 & -1 \\ \bullet & \bullet \end{smallmatrix}\right)$ so that the equation

$$\begin{pmatrix} 2 & -1 \\ \bullet & \bullet \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$$

has exactly one solution, or explain why this is not possible.

Interpret this as a statement about a pair of lines in the plane and draw the picture which corresponds.

Exercise 28. Make an example of a matrix $\left(\begin{smallmatrix} 1 & 0 & 1 \\ 1 & 1 & 3 \\ \bullet & \bullet & \bullet \end{smallmatrix}\right)$ so that the equation

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 3 \\ \bullet & \bullet & \bullet \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

has no solutions, or explain why this is not possible.

Interpret this as a statement about a planes in space and draw the picture which corresponds.

Exercise 29. Find a triple of numbers x, y, and z so that the linear combination

$$x \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + y \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} + z \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$$

yields the zero vector, or explain why this is not possible.

Rewrite the above as an equation which involves a matrix.

Plot the three vectors and describe the geometry of the situation.

Exercise 30. The vectors

$$r_1 = \begin{pmatrix} 1 \\ 4 \\ 7 \end{pmatrix}, \qquad r_2 = \begin{pmatrix} 2 \\ 5 \\ 8 \end{pmatrix}, \qquad \text{and} \qquad r_3 = \begin{pmatrix} 3 \\ 6 \\ 9 \end{pmatrix}$$

are linearly dependent because they lie in a common plane (through the origin). Find a normal vector to this plane.

Since the vectors are linearly dependent, there must be (infinitely) many choices of scalars x, y, and z so that $xr_1 + yr_2 + zr_3 = 0$. Find two sets of such numbers.

Exercise 31. Consider the equation

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}.$$

We are interested in being able to solve this for x and y for any given choice of the numbers b_1 and b_2. Figure out a way to do this by writing x and y in terms of b_1 and b_2.

Rewrite your solution in the form

$$\begin{pmatrix} x \\ y \end{pmatrix} = b_1 \begin{pmatrix} \bullet \\ \bullet \end{pmatrix} + b_2 \begin{pmatrix} \bullet \\ \bullet \end{pmatrix}.$$

How is this related to the inverse of the matrix $A = \left(\begin{smallmatrix} 2 & 1 \\ 1 & 1 \end{smallmatrix}\right)$?

Exercise 32. Find an example of a number c and a vector $\left(\begin{smallmatrix} b_1 \\ b_2 \end{smallmatrix}\right)$ so that the equation

$$\begin{pmatrix} 3 & 51 \\ c & 17 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

does not have a solution, or explain why no such example exists.

Explain your solution in terms of

- lines in the plane,
- 2-vectors and linear combinations, and
- invertibility of a matrix.