LINEAR ALGEBRA

STRANG, SECTION 2.1

1. The assignment

- Read section 2.1 of Strang (pages 31-40).
- Read the following and complete the exercises below.

2. Some Notes on the Three Perspectives

Now we have built a little experience with vectors and related things, it is time to be aware of what we have done so we can use it as a foundation for future work.

Does the following picture make sense to you?

Row Picture		**Column Picture**
Algebra: system of linear equations	$\longleftrightarrow$	Algebra: equation on column vectors
Geometry: (hyper)planes intersecting		Geometry: linear combination of vectors

$\searrow\nwarrow$ $\swarrow\nearrow$

Transformational Picture
Algebra: matrix equation
Geometry: ????

A deep understanding of linear algebra will involve a level of comfort with each of the three views of the subject in the diagram, and also the ability to pass back and forth between them.

2.1. The Transformational View.

We have seen that matrices can be made to "act upon" vectors by a kind of multiplication. In particular, if A is an $m \times n$ matrix, then A can be multiplied (on the left) with a column vector of size m, and the result is a column vector of size n.

This makes A into a kind of *function*. (We will use the synonyms *mapping* or *transformation*, too.) For every vector v of size m, the matrix A allows us to compute a new vector $T_A(v) = Av$ of size n. This is the basic example of what we will eventually call a *linear transformation*.

$$\mathbb{R}^m \xrightarrow{T_A} \mathbb{R}^n$$

$$v \longmapsto Av$$

One of our long term goals is to find a way to think about the geometry of linear algebra from this viewpoint.

3. Sage instructions

I have made a Sage worksheet file with some basic commands that you might find useful in investigating linear systems. The file is called `section2_1.sagews`.

4. Questions for Section 1.1

Exercise 1. Find an example of a vector b so that the equation

$$\begin{pmatrix} -1 & 2 \\ 5 & -9 \end{pmatrix} v = b$$

has no solution v, or explain why it is impossible to find such an example.

Exercise 2. Consider the matrix equation

$$\begin{pmatrix} 1 & 2 & 4 \\ 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}.$$

a) Draw a diagram representing the row picture of this equation.
b) Draw a diagram representing the column picture of this equation.
c) Draw a diagram representing the transformational picture of this equation.

Exercise 3. Find an example of a matrix B which has the following effect:

a) $B\left(\begin{smallmatrix} x \\ y \end{smallmatrix}\right) = \left(\begin{smallmatrix} y \\ x \end{smallmatrix}\right)$
b) Rotates vectors through $45°$ counter-clockwise.
c) Reflects vectors across the y-axis.
d) $B\left(\begin{smallmatrix} x \\ y \end{smallmatrix}\right) = \left(\begin{smallmatrix} X+y \\ y \end{smallmatrix}\right)$