# MEETING 04, GROUP WORK

LINEAR ALGEBRA SECTION 01, SPRING 2014

The focus of today's seminar work is to get comfortable with possible matrix representations for the algebra involved in studying (hyper)planes and vectors. A lot of this will build on the activities from meeting 02.

## Part One: 2D Warm Up

Consider the line in the Cartesian plane described in parametric vector form as

$$\ell = \left\{ X = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \end{pmatrix} \mid t \text{ is a real number} \right\}.$$

**Task 1.** Rewrite the vector parametric equation here in the form $X = (1-t)P + tQ$ for some vectors $P$ and $Q$. (*Hint: What is the value of $X$ when $t = 0$? What is the value of $X$ when $t = 1$?*)

**Task 2.** Using this new form, find equations for the coordinates $x$ and $y$ of a generic point $X = \begin{pmatrix} x \\ y \end{pmatrix}$ on $\ell$ as functions of the parameter $t$.

**Task 3.** Rewrite the linear combination equation for this situation as a matrix-vector equation involving the vectors $X$ and $\begin{pmatrix} 1 \\ t \end{pmatrix}$.

**Task 4.** Eliminate the parameter $t$ from the equations in task 1 to write the standard form of the equation of $\ell$.

**Task 5.** Rewrite the equation you just found as a matrix-vector equation involving the vector $\begin{pmatrix} x \\ y \end{pmatrix}$.

## Part Two: working in 3D

For the rest of this investigation, we shall consider the two points $P$ and $Q$ in $\mathbb{R}^3$ given by the vectors

$$P = \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} \qquad \text{and} \qquad Q = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

**Task 6.** Write the vector parametric form of the line $m$ in $\mathbb{R}^3$ which passes through $P$ and $Q$.

**Task 7.** Rewrite that vector parametric description as a matrix-vector equation which involves the vector $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

**Task 8.** Find some equations in the coordinates $x, y$ and $z$ of a generic point $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ which describe exactly when the point $X$ lies on the line $m$. (The parameter $t$ should no longer appear.)

**Task 9.** Rewrite that system of equations as a matrix-vector equation involving the vector $X$.

## Part III: A linear combination equation

Now consider these four vectors in $\mathbb{R}^3$.

$$u = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}, \quad v = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}, \quad w = \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

**Task 10** (*Strogatz*, p 10 #31)**.** Write down three equations for $c$, $d$, and $e$ so that $cu + dv + ew = b$.

**Task 11.** Rewrite the system of equations as a matrix-vector equation involving the vector $\begin{pmatrix} c \\ d \\ e \end{pmatrix}$.