LINEAR ALGEBRA

STRANG, SECTION 4.2

1. The assignment

- Read section 4.2 of Strang (pages 206-214).
- Read the following and complete the exercises below.

2. Orthogonal Projection

One good use of the geometry in $\mathbb{R}^n$ is the concept of orthogonal projection. The basic idea is to mimic the behavior of shadows under sunlight. Our everyday experience leads us to thinking about the projection of a vector onto a plane (the ground–its roughly a plane), but if you imagine holding out a pencil you can summon up the visual of projection onto a line, too.

The key concept is to use the basic condition of orthogonality ($u \cdot v = 0$) to figure things out.

Note that everything in this section is done by projecting onto *subspaces*! This is a bit of a restriction. In practice, this restriction can be removed by translating your whole problem to have a new origin.

3. Sage instructions

I have made a Sage worksheet file with some basic commands that you might find useful in investigating matrices. The file is called `section4_2.sagews`.

4. Questions for Section 4.2

Exercise 123. Find the projection matrix which computes projections of vectors in $\mathbb{R}^2$ onto the line $3x + 2y = 0$. (Since it goes through zero, it is a subspace.)

Find the orthogonal projection of the vector $(17, 3)$ onto this line.

Exercise 124. Find the projection matrix which computes projections of vectors in $\mathbb{R}^3$ onto the line which is the intersection of the planes $x - 2y + 3z = 0$ and $y + 2z = 0$. (Again, that is a subspace.)

Find the orthogonal projection of the vector $(1, 1, 1)$ onto this line.

Exercise 125. Find the projection matrix which computes projections of vectors in $\mathbb{R}^3$ onto the plane $-2x + y + 3z = 0$.

Find the orthogonal projection of the vector $(9, 7, -5)$ onto this plane.

Exercise 126. Find the projection matrix which computes projections of vectors in $\mathbb{R}^4$ onto the plane which is the intersectoin of $5x + y + w = 0$ and $z + y + z + w = 0$. (This subspace is the 2 dimensional plane where these two 3-dimensional hyperplanes meet.)

Find the orthogonal projection of the vector $(-3, 1, -3, 1)$ on this plane.