MFront in Biomechanics: Abdominal muscle simulation.

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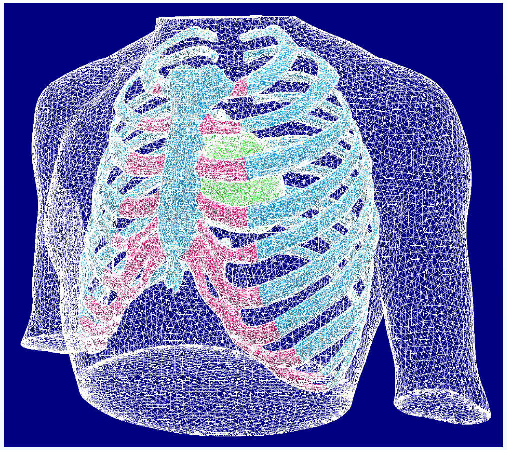
MedSim Main Objective:

To work in biomechanics problems arising directly from medical professional experience

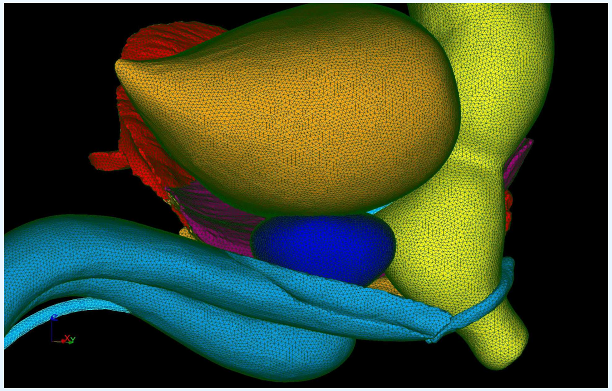
using exclusively Open Source Software (OSS)

Current Projects

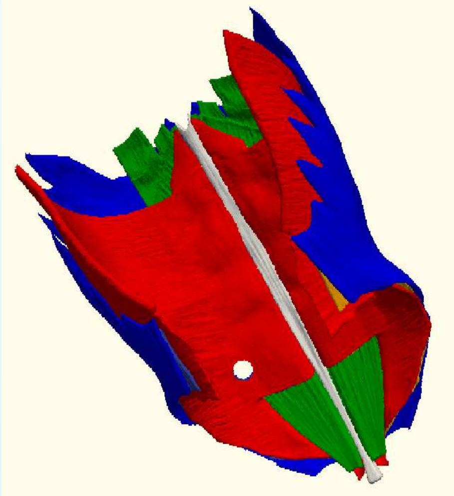
Inferior Vena Cava Filters



Cardiopulmonary Resuscitation



Dynamics of Prostatic Region



Dynamics of Abdominal Wall

Abdominal muscle simulation

- We propose a new transversely isotropic hyperelastic model (TIHM) for the human abdominal wall tissues.
- The novelty of our formulation is that both the isotropic and the fiber contributions to the strain energy function are characterized exclusively by polynomial convex functions.
- We studied the following abdominal wall tissues: linea alba, rectus sheath, external oblique muscle, internal oblique muscle, transversus abdominis muscle and rectus abdominis muscle.
- Our formulation, closely reproduces tensile test data for each tissue in the corresponding FE numerical simulation.

Formulation

Soft tissues are usually modelled as hyperelastic materials for which a strain energy function (SEF), also known as Helmholtz free-energy function, is used. In the case that tissue is assumed to be slightly compressible, the SEF is decoupled into a volumetric and an isochoric part:

$$\Psi(J, \mathbf{C}) = U(J) + \Psi_{ich}(J, \mathbf{C})$$

and the second Piola–Kirchoff stress tensor, $\mathbf{S}$, is:

$$\mathbf{S} = \mathbf{S}_{vol}(J) + \mathbf{S}_{ich}(J, \mathbf{C}) = 2\left(\frac{\partial U(J)}{\partial \mathbf{C}}\right) + 2\left(\frac{\partial \Psi_{ich}(J, \mathbf{C})}{\partial \mathbf{C}}\right)$$

$\mathbf{C}$ denotes the right Cauchy–Green symmetric tensor, defined as $\mathbf{C} = \mathbf{F}^T\mathbf{F}$, where $\mathbf{F}$ is the deformation gradient and J is the determinant of $\mathbf{F}$.

Model

In the TIHM, the soft tissue is assumed to be a composite formed by a ground isotropic material and one family of fibers which have a preferred direction, $\mathbf{a}_0$. In terms of the modified SEF, $\overline{\Psi} = \overline{\Psi}(\overline{\mathbf{C}}, \mathbf{a}_0)$, where $\overline{\mathbf{C}} = J^{-2/3}\mathbf{C}$ is the modified right Cauchy–Green tensor

In the TIHM formulation, a fibrous tissue is modeled by decomposing the modified SEF, $\overline{\Psi}$, into a ground isotropic contribution, $\overline{\Psi}_{iso}$, plus a fiber contribution, $\overline{\Psi}_{fib}$:

$$\overline{\Psi}(\overline{\mathbf{C}}, \mathbf{a_0}) = \overline{\Psi}_{iso}(\overline{\mathbf{C}}) + \overline{\Psi}_{fib}(\overline{\mathbf{C}}, \mathbf{a_0}) =$$

$$\overline{\Psi}_{iso}\left(\overline{I}_1(\overline{\mathbf{C}}), \overline{I}_2(\overline{\mathbf{C}})\right) + \overline{\Psi}_{fib}\left(\overline{I}_4(\overline{\mathbf{C}}, \mathbf{a}_0), \overline{I}_5(\overline{\mathbf{C}}, \mathbf{a}_0)\right)$$

Polynomial

where:

$$
\begin{aligned}
\overline{I}_1(\overline{\mathbf{C}}) &= \text{tr}(\overline{\mathbf{C}}) \\
\overline{I}_2(\overline{\mathbf{C}}) &= \frac{1}{2}\left[\left(\text{tr}(\overline{\mathbf{C}})\right)^2 - \text{tr}\left(\overline{\mathbf{C}}^2\right)\right] \\
\overline{I}_4(\overline{\mathbf{C}}, \mathbf{a}_0) &= \overline{\mathbf{C}} : (\mathbf{a}_0 \otimes \mathbf{a}_0) = \mathbf{a}_0^T \cdot \overline{\mathbf{C}} \cdot \mathbf{a}_0 = \lambda^2 \\
\overline{I}_5(\overline{\mathbf{C}}, \mathbf{a}_0) &= \overline{\mathbf{C}}^2 : (\mathbf{a}_0 \otimes \mathbf{a}_0) = \mathbf{a}_0^T \cdot \overline{\mathbf{C}}^2 \cdot \mathbf{a}_0
\end{aligned}
$$

We set the target on polynomial functions of the form:

$$
\overline{\Psi}(\overline{\mathbf{C}}, \mathbf{a_0}) = \overline{\Psi}_{iso}(\overline{\mathbf{C}}) + \overline{\Psi}_{fib}(\overline{\mathbf{C}}, \mathbf{a_0})
$$

$$
\begin{aligned}
\overline{\Psi}_{iso} &= C_1\left(\overline{I}_1 - 3\right) + C_2\left(\overline{I}_1 - 3\right)^2 \\
\overline{\Psi}_{fib} &= C_3\left(\overline{I}_4 - 1\right)^2 + C_4\left(\overline{I}_4 - 1\right)^4
\end{aligned}
$$

Jacobian

The global Jacobian has to be provided in the form of the fourth order tangent tensor, $\mathbb{E}$:

$$\mathbb{E} = \mathbb{E}_{vol} + \mathbb{E}_{ich}$$

$$\mathbb{E}_{vol} = K\left(J^2\mathbf{C}^{-1}\otimes\mathbf{C}^{-1} - \left(J^2 - 1\right)\mathbf{C}^{-1}\odot\mathbf{C}^{-1}\right)$$

$$\begin{aligned}\mathbb{E}_{ich} \quad = \quad & \mathbb{P}^T : \overline{\mathbb{E}} : \mathbb{P} - \frac{1}{3}J^{-2/3}\left(\overline{\mathbf{S}}\otimes\mathbf{C}^{-1} + \mathbf{C}^{-1}\otimes\overline{\mathbf{S}}\right) + \\ & + \frac{1}{3}\left(\overline{\mathbf{S}} : \overline{\mathbf{C}}\right)\left(\mathbf{C}^{-1}\odot\mathbf{C}^{-1} + \frac{1}{3}\mathbf{C}^{-1}\otimes\mathbf{C}^{-1}\right)\end{aligned}$$

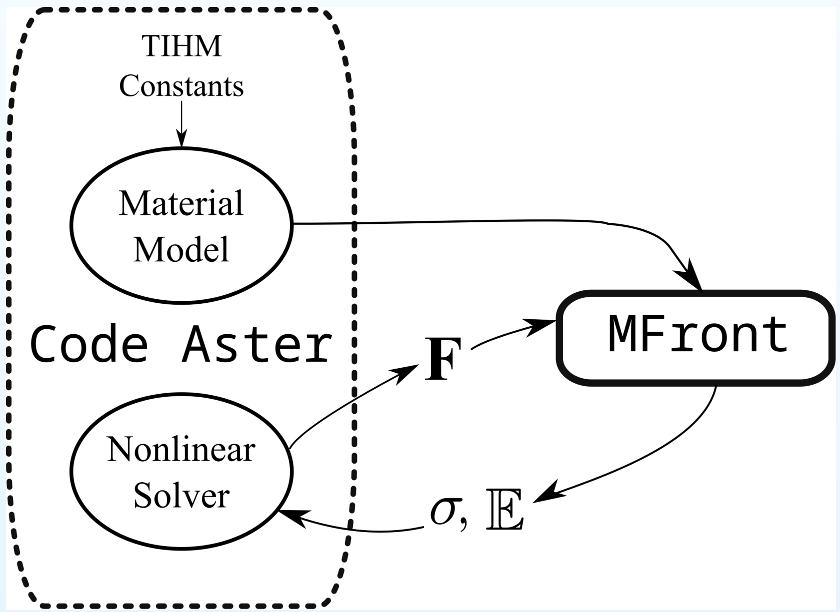
MFront: example

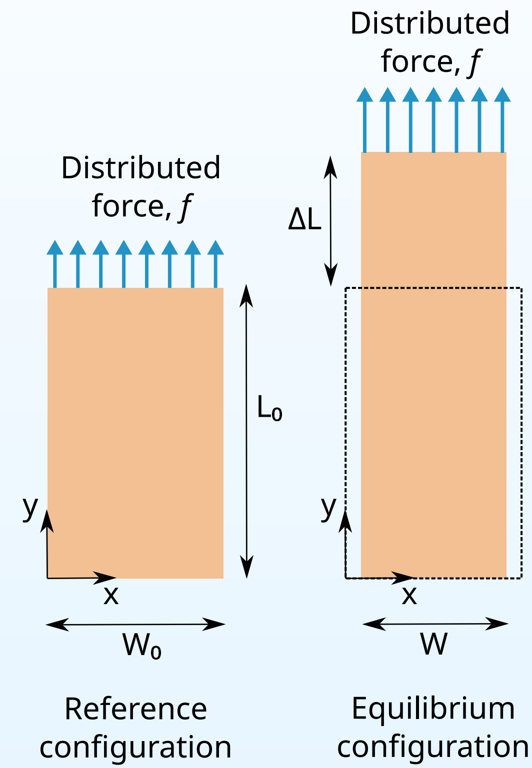
$$\mathbb{P}^T : \overline{\mathbb{E}} : \mathbb{P} = \frac{1}{9}\left(\overline{\delta}_1 \overline{I}_1^2 + \overline{\delta}_4 \overline{I}_4^2\right) \mathbb{C}_{\otimes}^{-1} + \overline{\delta}_1 J^{-4/3} \left(\mathbf{I} \otimes \mathbf{I}\right)$$
$$+\overline{\delta}_4 J^{-4/3} \left(\mathbf{a}_0 \otimes \mathbf{a}_0 \otimes \mathbf{a}_0 \otimes \mathbf{a}_0\right)$$
$$-\frac{1}{3}\overline{\delta}_1 \overline{I}_1 J^{-2/3} \left(\mathbf{I} \otimes \mathbf{C}^{-1} + \mathbf{C}^{-1} \otimes \mathbf{I}\right)$$
$$-\frac{1}{3}\overline{\delta}_4 \overline{I}_4 J^{-2/3} \left(\left(\mathbf{a}_0 \otimes \mathbf{a}_0\right) \otimes \mathbf{C}^{-1} + \mathbf{C}^{-1} \otimes \left(\mathbf{a}_0 \otimes \mathbf{a}_0\right)\right)$$

$$\mathtt{(1./9.)}*(\overline{\delta}_1*\overline{I}_1*\overline{I}_1+\overline{\delta}_4*\overline{I}_4*\overline{I}_4)*\mathbb{C}_{\otimes}^{-1}+$$
$$(\overline{\delta}_1*J^{-2/3}*J^{-2/3})*(\mathtt{Stensor::Id()}\wedge\mathtt{Stensor::Id()})$$
$$+(\overline{\delta}_4*J^{-2/3}*J^{-2/3})*(\mathbf{A}_0\wedge\mathbf{A}_0)$$
$$\mathtt{-(1./3.)}*(\overline{\delta}_1*\overline{I}_1\ *J^{-2/3})*(\mathtt{Stensor::Id()}\wedge\mathbf{C}^{-1}$$
$$+\mathbf{C}^{-1}\wedge\mathtt{Stensor::Id()})$$
$$\mathtt{-(1./3.)}*(\overline{\delta}_4*\overline{I}_4*J^{-2/3})*(\mathbf{A}_0\wedge\mathbf{C}^{-1}+\mathbf{C}^{-1}\wedge\mathbf{A}_0)$$

Simulations

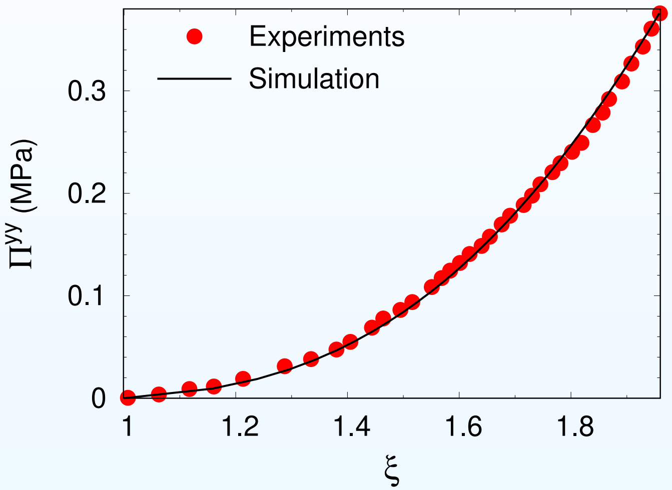
First simulations are done for a brick–shaped sample:



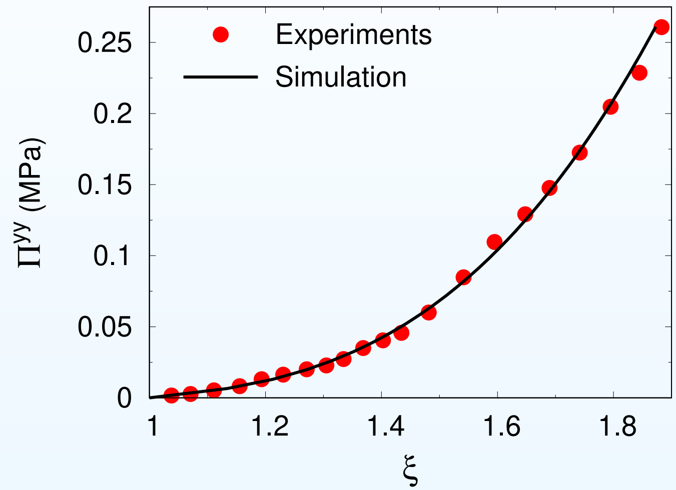


Stretch parallel to the fibers

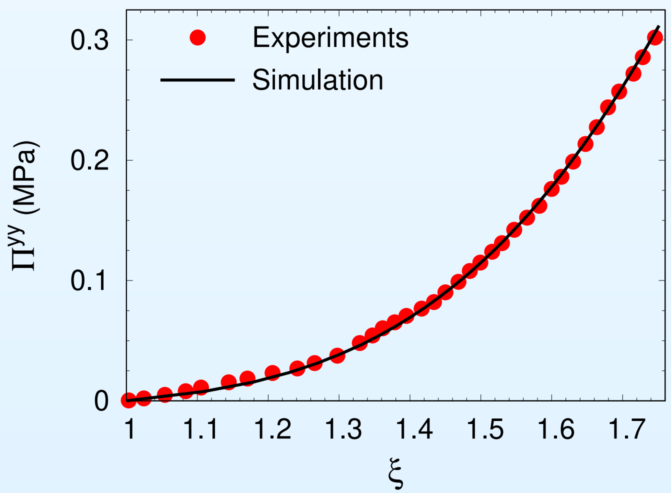
(a) Transverse muscle



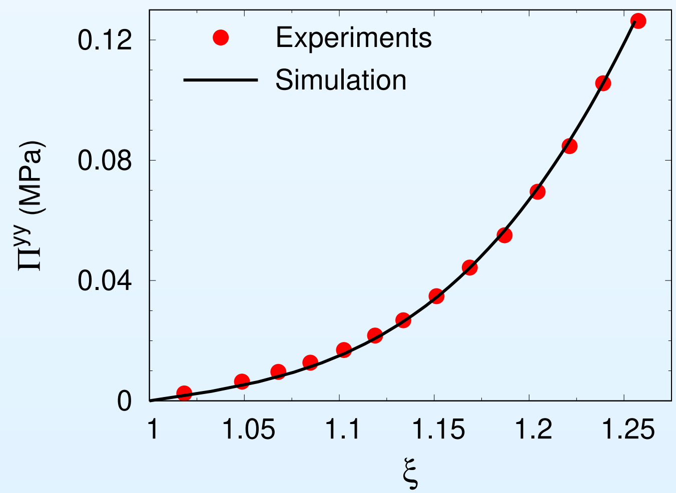
(b) External oblique muscle



(c) Internal oblique muscle

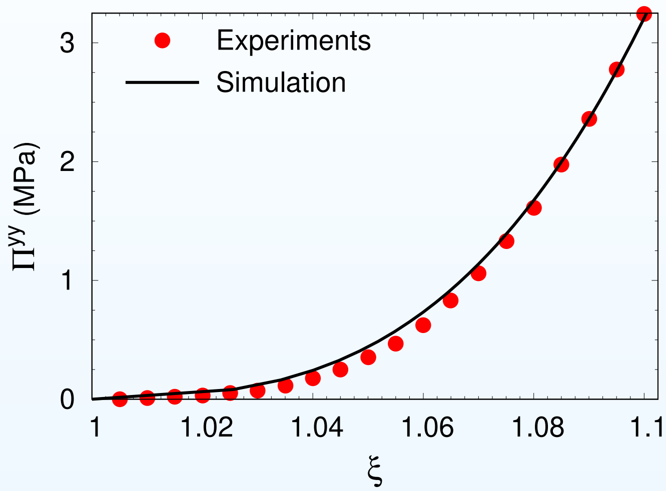


(d) Rectus abdominalis muscle

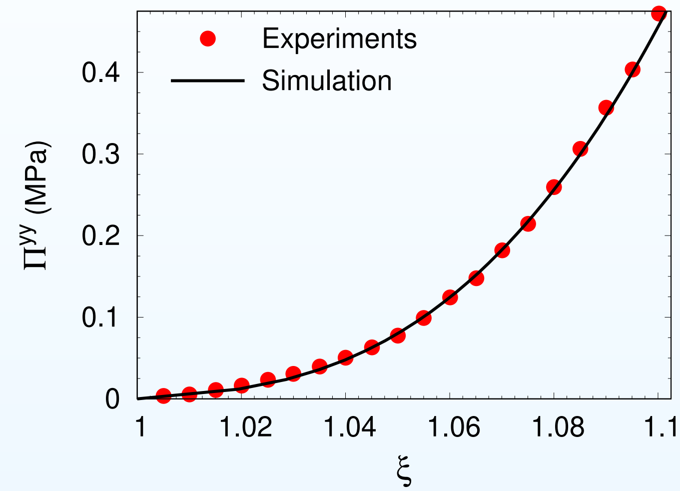


Stretch parallel and transverse to the fibers

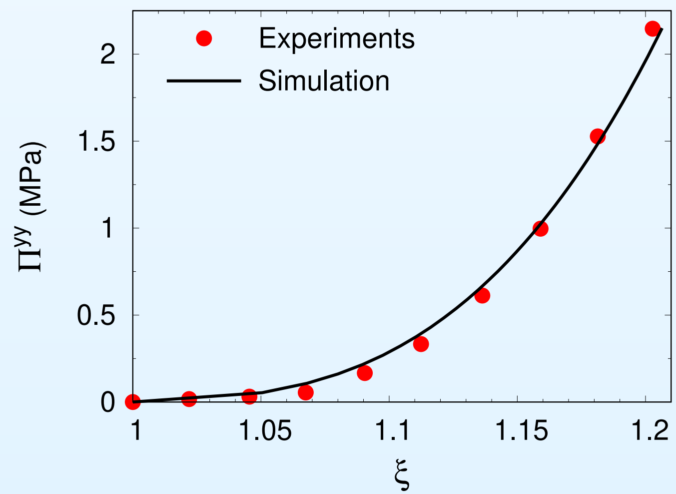
(a) Linea Alba parallel



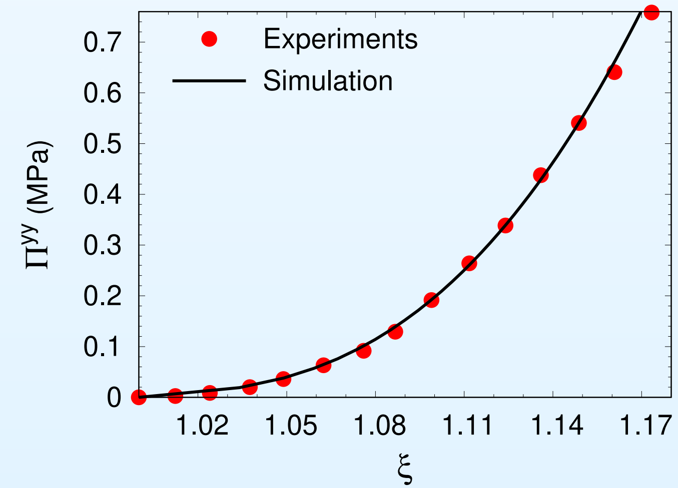
(b) Linea Alba transverse



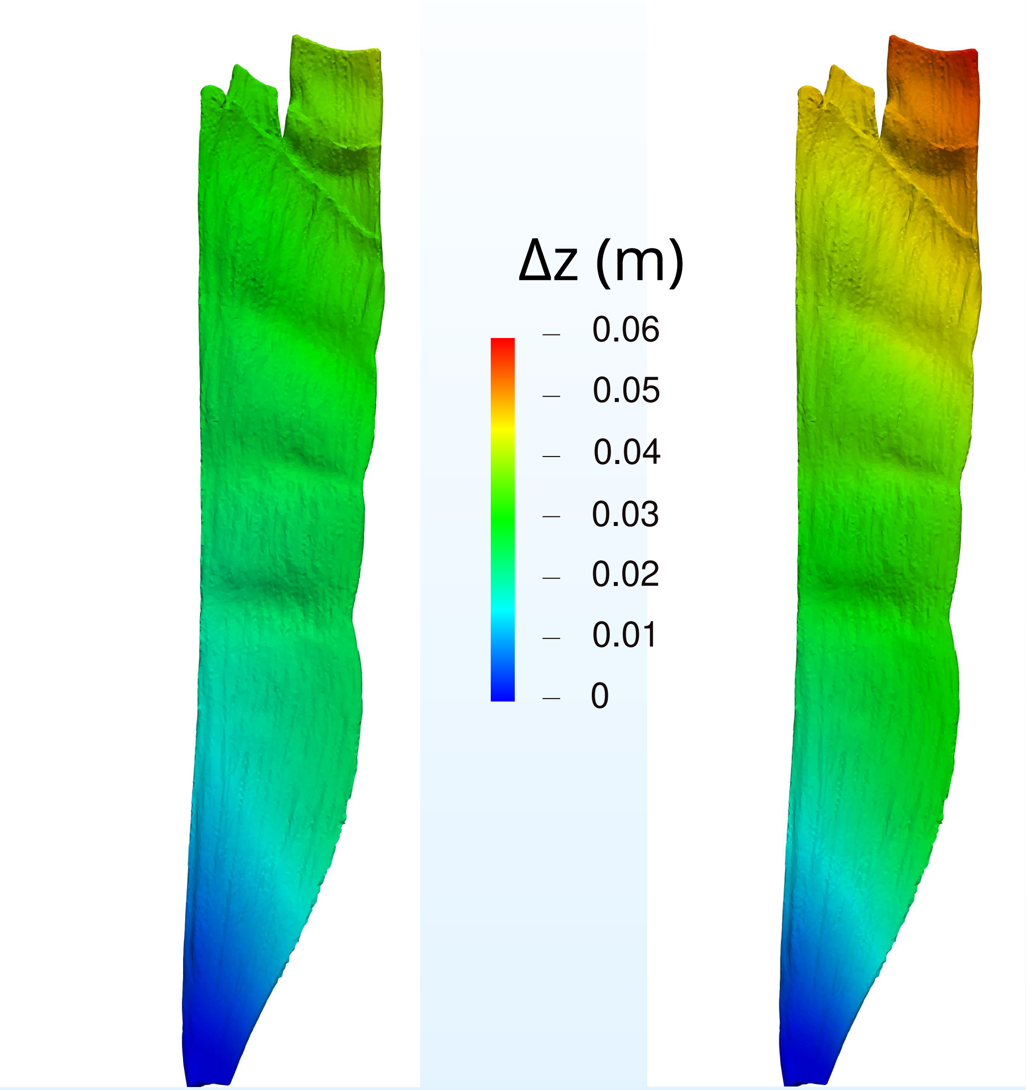
(a) Rectus sheath parallel



(d) Rectus sheath transverse



Results. Simulation of one full muscle.



$5\,\mathrm{kPa}$ $10\,\mathrm{kPa}$

Conclusions

- The methodology implemented in the present study can be easily extended in the future to develop and implement a TIHM for active muscles and/or a different type of constitutive model which might be suitable to characterize other tissues of biomedical interest.
- The new TIHM formulation is suitable for a future numerical investigation of the abdominal wall, which will in turn help us to assess the best zone to practice a colostomy.
- MFront is a well suited tool to provide answers to very diverse kind of problems that arise in biomechanics.

See: *Implementation of a new constitutive model for abdominal muscles. Ll. Tuset, G. Fortuny, J. Herrero, D. Puigjaner, J. M. López. Computer Methods and Programs in Biomedicine. 179, October 2019*

Thanks!

We want to emphasize especially that this work has been possible thanks to the versatility of MFront.

and ... thank you very much for your attention.

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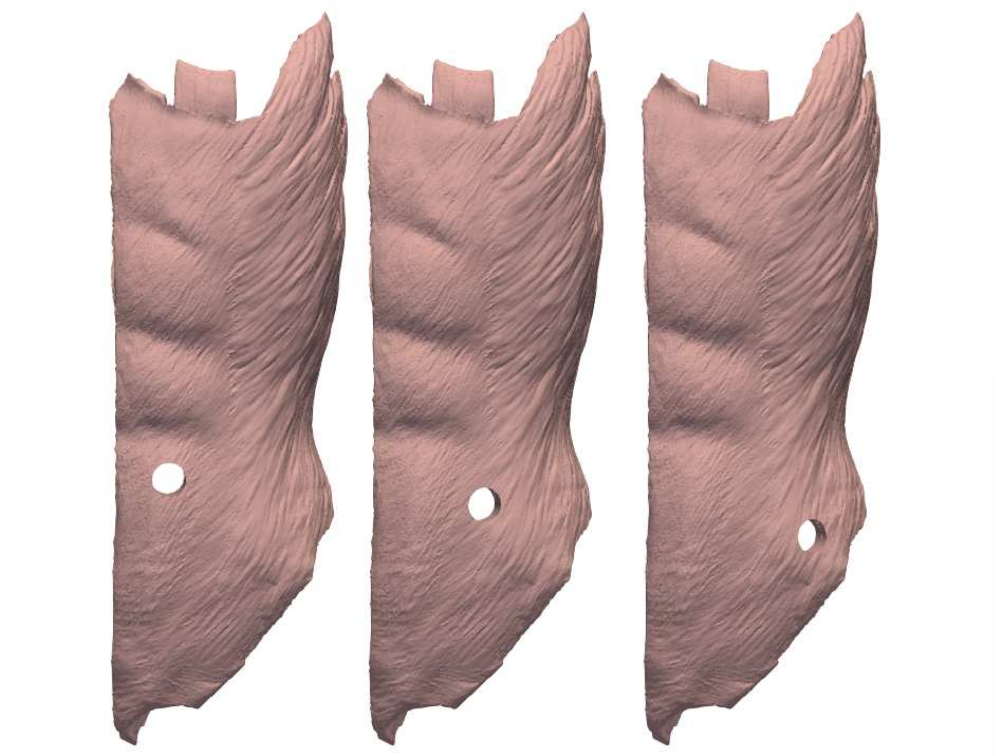
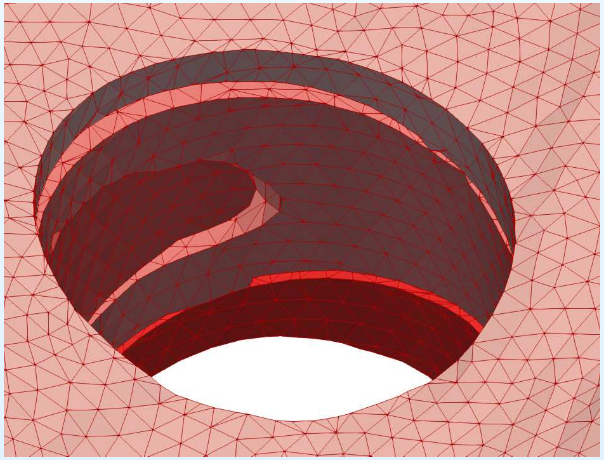
Stomas.



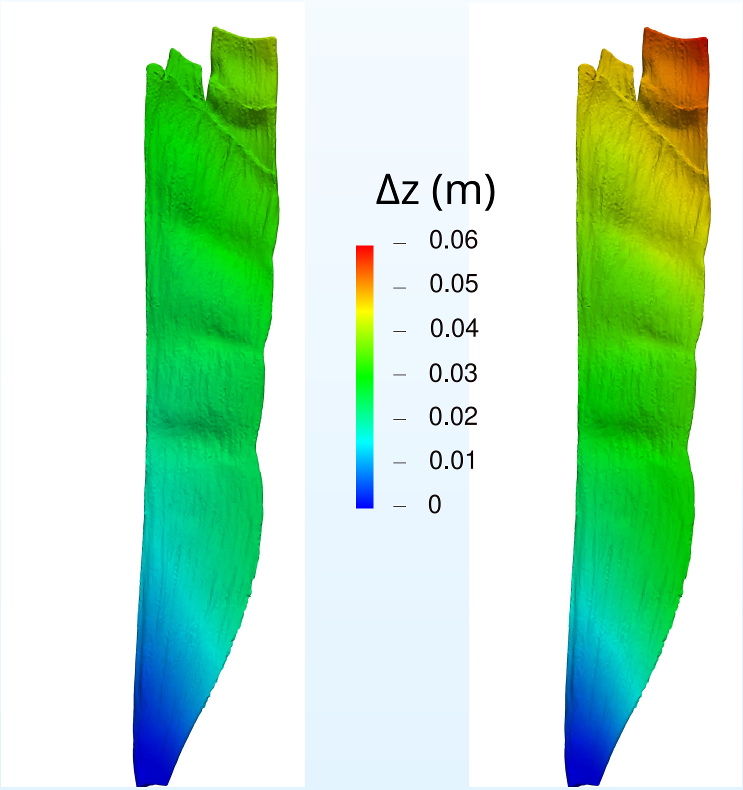


Pseudocode

#	Variables	Equations	MFront expression
1	Real J	$\det(\mathbf{F})$	det(F1)
2	Stensor $\mathbf{C}$	$\mathbf{F}^T\mathbf{F}$	computeRightCauchyGreenTensor(F1)
3	Stensor $\mathbf{C}^{-1}$		invert($\mathbf{C}$)
4	Real $J^{-2/3}$		pow(J,−2./3.)
5	Stensor $\bar{\mathbf{C}}$	$J^{-2/3}\mathbf{C}$	$(J^{-2/3})^*\mathbf{C}$
6	Stensor $\mathbf{A}_0$	$\mathbf{a}_0\otimes\mathbf{a}_0$	buildFromVectorDiadicProduct($\mathbf{a}_0$)
7	Real $\bar{I}_1$	(12)	trace($\bar{\mathbf{C}}$)
8	Real $\bar{I}_4$	(14)	$\bar{\mathbf{C}} \vert \mathbf{A}_0$
9	Real $\bar{\gamma}_1$	(22)	$2^*C_1+4^*C_2{}^*(\bar{I}_1-3)$
10	Real $\bar{\gamma}_4$	(23)	$4^*C_3{}^*(\bar{I}_4-1)+8^*C_4{}^*$pow($\bar{I}_4-1$,3)
11	Real $\bar{\delta}_1$	(33)	4^*C_2
12	Real $\bar{\delta}_4$	(34)	$4^*C_3+24^*C_4{}^*(\bar{I}_4-1)^*(\bar{I}_4-1)$
13	Stensor $\bar{\mathbf{S}}$	(21)	$\bar{\gamma}_1{}^*$Stensor::Id()$+\bar{\gamma}_4{}^*\mathbf{A}_0$
14	Stensor $\mathbf{S}_{ich}$	(5)	$(J^{-2/3})^*(\bar{\mathbf{S}}-(\bar{\mathbf{S}} \vert \mathbf{C})/3^*\mathbf{C}^{-1})$
15	Stensor $\mathbf{S}_{vol}$	(19)	$K^*(J^*J-1)^*\mathbf{C}^{-1}$
16	Stensor $\boldsymbol{\sigma}$	(2), (37)	convertSecondPiolaKirchoffStress toCauchyStress($\mathbf{S}_{vol}+\mathbf{S}_{ich}$,F1)
17	Stensor4 $\mathbb{C}_{\otimes}^{-1}$	$\mathbf{C}^{-1}\otimes\mathbf{C}^{-1}$	$\mathbf{C}^{-1}\wedge\mathbf{C}^{-1}$
18	Stensor4 $\mathbb{C}_{\odot}^{-1}$	(35)	circledot($\mathbf{C}^{-1}$)
19	Stensor4 $\mathbb{E}_{vol}$	(31)	$K^*J^*J^*\mathbb{C}_{\otimes}^{-1}-K^*(J^*J-1)^*\mathbb{C}_{\odot}^{-1}$
20	Stensor4 $\bar{\mathbb{E}}$	(32)	$\bar{\delta}_1{}^*$(Stensor::Id()$\wedge$Stensor::Id())$+\bar{\delta}_4{}^*(\mathbf{A}_0\wedge\mathbf{A}_0)$
21	Stensor4 $\mathbb{P}$	(7)	$(J^{-2/3})^*$(Stensor4::Id()$-(1./3.)^*(\mathbf{C}\wedge\mathbf{C}^{-1})$)
22	Stensor4 $\mathbb{P}^T$		$(J^{-2/3})^*$(Stensor4::Id()$-(1./3.)^*(\mathbf{C}^{-1}\wedge\mathbf{C})$)
23	Stensor4 $\mathbb{E}_{ich}$	(29)	$\mathbb{P}^{T*}\bar{\mathbb{E}}^*\mathbb{P}-(1./3.)^*(J^{-2/3})^*(\bar{\mathbf{S}}\wedge\mathbf{C}^{-1}+\mathbf{C}^{-1}\wedge\bar{\mathbf{S}})+$ $(1./3.)^*(\bar{\mathbf{S}} \vert \bar{\mathbf{C}})^*(\mathbb{C}_{\odot}^{-1}+(1./3.)^*\mathbb{C}_{\otimes}^{-1})$
24	Stensor4 $\mathbb{E}$	(27)	$\mathbb{E}_{vol}+\mathbb{E}_{ich}$

Results. Simulation of one full muscle.

Vertical (z) component of the predicted deformation on the surface of the right RA muscle. The 3D mesh used in this simulation, consisting of $310\,497$ tetrahedra, was generated from a surface mesh consisting of $65\,188$ triangles. The dimensions of the bounding box surrounding the right RA model are 0.0762 m $\times$ 0.0727 m in the $x - y$ (horizontal) plane and 0.394 m in the z (vertical) direction. The boundary conditions in these simulations were roughly equivalent to the ones prescribed earlier for the rectangular tissue sample; that is, the right RA model was fixed at the bottom edge ($\Delta z = 0$) and a uniform stress load of either (a) 5 kPa (left) or (b) 10 kPa (right) was applied to the upper edge boundaries. The muscle fibers were assumed to be initially aligned with the $z-$axis.



$5\,\mathrm{kPa}$ $10\,\mathrm{kPa}$