Implementation of a confinement dependent LITS model for concrete

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Implementation of a confinement dependent LITS model for concrete

Contents

1. Industrial context

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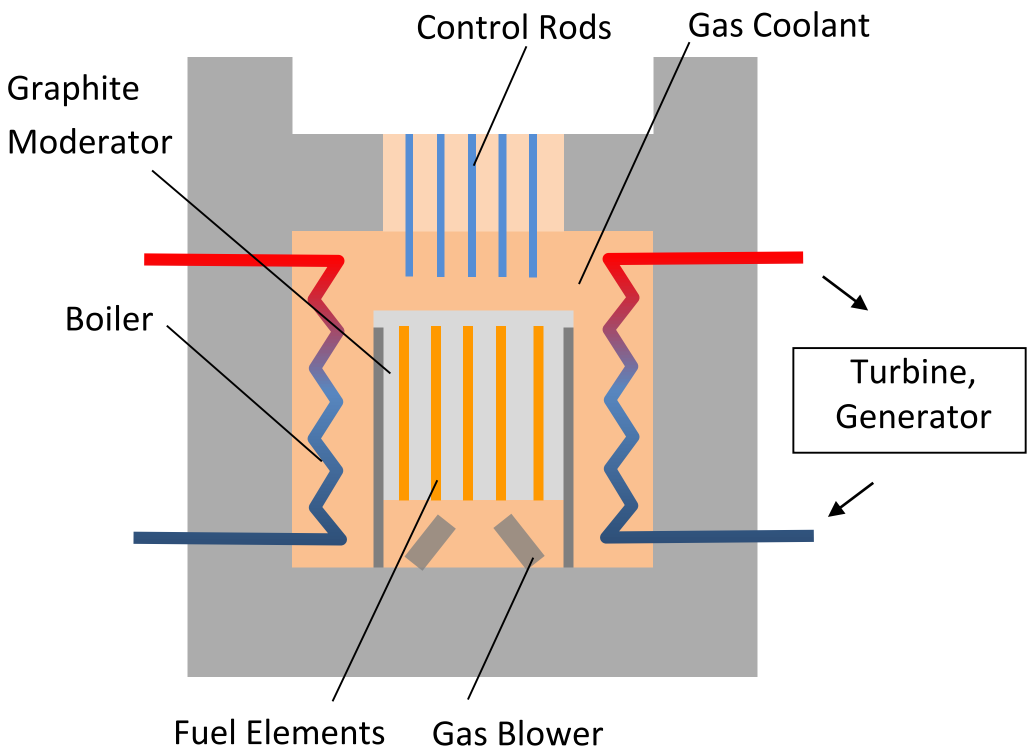
Industrial context

Assessment of the AGRs' Pre-stressed Concrete Pressure Vessels (PCPVs) under severe fault conditions



Service Conditions:

$T_{GAS} \approx 600$ °C

$T_{CONCRETE} \approx 50$ °C

⇩

Accidental Conditions:

$T_{GAS} \approx 600$ °C

$T_{CONCRETE} > 50$ °C

(Ex: fault of the cooling pipes system)

1. Industrial context

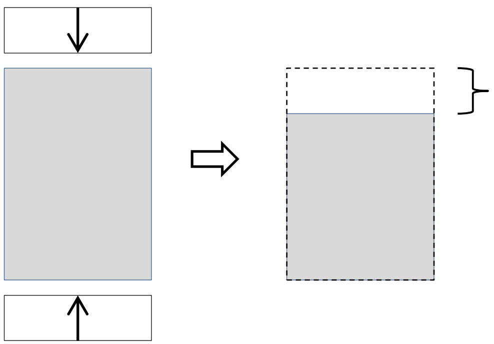
Objectives

- Implement, verify and validate a LITS model able to capture:
  - Triaxial behaviour
  - Irrecoverability
- Apply the implemented model to a hypothetical industrial case.

2. Definition of LITS

2. Definition of LITS

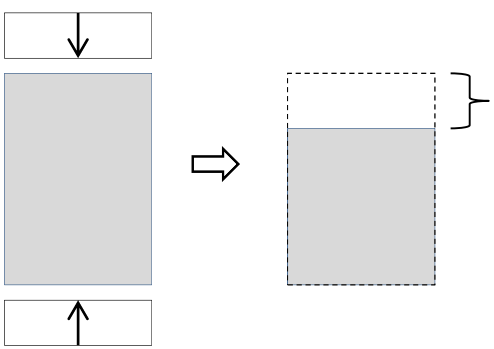
Strain decomposition for ambient temperatures



$$\varepsilon_{tot} = \overbrace{\varepsilon_{ela} + \varepsilon_{cr} + \varepsilon_{dc}}^{\varepsilon_\sigma \text{ MECHANICAL STRAIN}} + \overbrace{\varepsilon_{sh} + \varepsilon_{th}}^{\varepsilon^0 \text{ STRESS-INDEPENDENT STRAIN}}$$

ELASTIC
BASIC CREEP
DRYING CREEP
SHRINKAGE
THERMAL

Strain decomposition for high temperatures: introduction of LITS



$$\varepsilon_{tot} = \overbrace{\varepsilon_{ela} + \varepsilon_{lits}}^{\varepsilon_\sigma \text{ MECHANICAL STRAIN}} \overbrace{+ \varepsilon_{sh} + \varepsilon_{th}}^{\varepsilon^0 \text{ STRESS-INDEPENDENT STRAIN}}$$

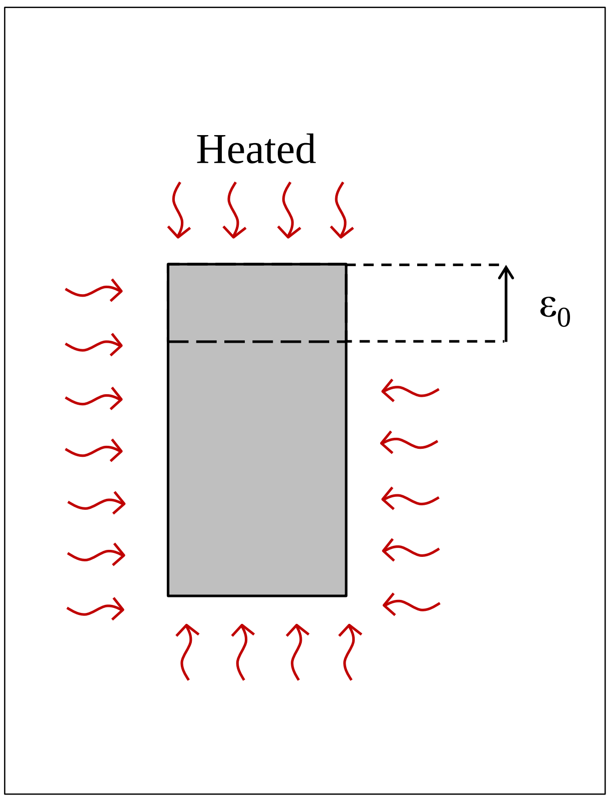
ELASTIC
LITS
(Comprehends an elastic component)
SHRINKAGE
THERMAL

Note
Some authors consider the ε_{ela} independent on the temperature and take into account the stiffness variation in the ε_{lits} term.

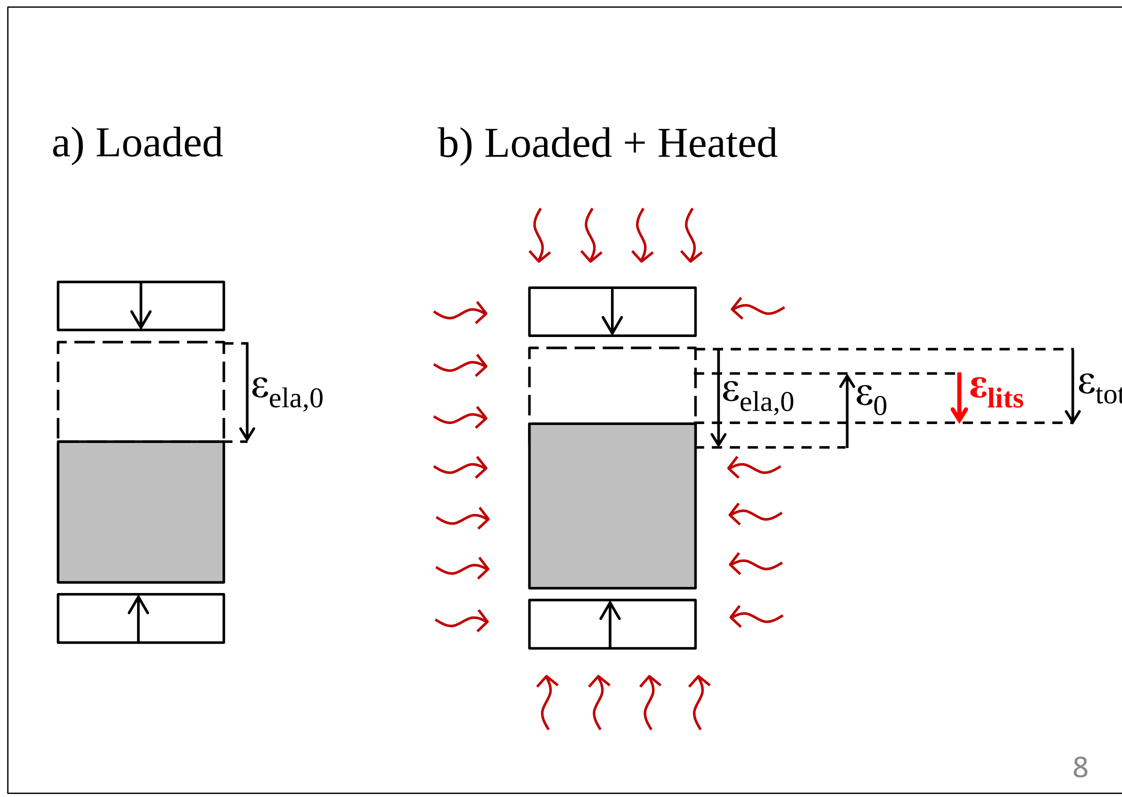
2. Definition of LITS

Definition of LITS

- Thermal load
- Stress-free specimen



- Thermal load
- Constant compressive stress

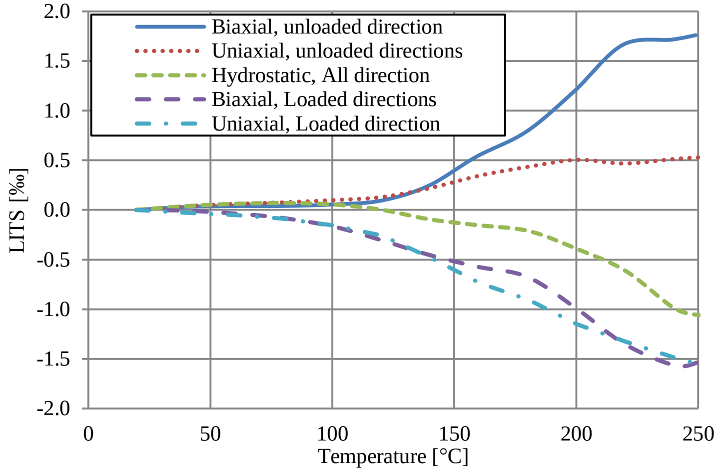


2. Definition of LITS

Gap in the existing LITS models

- LITS is commonly modelled as a confinement independent phenomenon (actually, it significantly depends on the confinement[1])

LITS during uniaxial and biaxial and hydrostatic LTH test [1].



[1] Petkovski M, Crouch RS (2008).
Strains under transient hygro-thermal states in concrete loaded in multiaxial compression and heated to 250 °C. Cement and Concrete Research 2008; 38:586–96.

3. 3D confinement-dependent bilinear LITS model

3. 3D Confinement-dependent bilinear LITS model

General strain decomposition

$$\dot{\varepsilon}_{ij}^{tot} = \underbrace{\frac{1+\upsilon}{E}\dot{\sigma}_{ij} - \frac{\upsilon}{E}\dot{\sigma}_{kk}\delta_{ij}}_{\dot{\varepsilon}_{ij}^{el}} + \underbrace{\eta\frac{\beta(T)}{\sigma_{u0}}\left(-\upsilon_{lits}\sigma_{kk}^{-}\delta_{ij} + (1+\upsilon_{lits})\sigma_{ij}^{-}\right)\dot{T}}_{\dot{\varepsilon}_{ij}^{lits}} + \underbrace{\alpha(T)\delta_{ij}\dot{T}}_{\dot{\varepsilon}_{ij}^{th}}$$

ELASTIC STRAIN

3D LITS

NON-LINEAR THERMAL STRAIN

3. 3D Confinement-dependent bilinear LITS model

Common approach for extending uniaxial LITS models to 3D [1-2]

CLASSIC APPROACH

$$\dot{\varepsilon}_{ij}^{lits} = \frac{\beta(T)}{\sigma_{u0}}\left(-\upsilon_{lits}\sigma_{kk}^{-}\delta_{ij} + (1+\upsilon_{lits})\sigma_{ij}^{-}\right)\dot{T}$$

Where:

- υ_{lits} is a material property analogous to the elastic Poisson's modulus υ– LITS strain in the unloaded direction
- $\beta(T)$ is the LITS functions, i.e. a generic function of the temperature aimed at fitting the uniaxial temperature-LITS curve
- σ_{u0} the compressive strength of the material
- σ_{ij}^{-} is the i-th j-th component of the negative part of the stress tensor
- T is the temperature of the point

[1] Pearce CJ, Nielsen C V., Bićanić N. 1987
Gradient enhanced thermo-mechanical damage model for concrete at high temperatures including transient thermal creep. Int J Numer Anal Methods Geomech 2004

[2] Thelandersson S. 1987
Modeling of combined thermal and mechanical action in concrete. J Eng Mech

3. 3D Confinement-dependent bilinear LITS model

Adopted approach for extending uniaxial LITS models to 3D

CLASSIC APPROACH

$$\boxed{\dot{\varepsilon}_{ij}^{lits} = \frac{\beta(T)}{\sigma_{u0}}\left(-\upsilon_{lits}\sigma_{kk}^{-}\delta_{ij} + (1+\upsilon_{lits})\sigma_{ij}^{-}\right)\dot{T}}$$

$\Downarrow$

ADOPTED APPROACH

$$\boxed{\dot{\varepsilon}_{ij}^{lits} = \eta\frac{\beta(T)}{\sigma_{u0}}\left(-\upsilon_{lits}\sigma_{kk}^{-}\delta_{ij} + (1+\upsilon_{lits})\sigma_{ij}^{-}\right)\dot{T}}$$

Where:

- η confinement coefficient – captures the triaxiality of the stress state

$$\eta = 1 + (C_m - 1)\gamma$$

$C_m = -\frac{\sigma_1^- + \sigma_2^- + \sigma_3^-}{\sqrt{(\sigma_1^-)^2 + (\sigma_2^-)^2 + (\sigma_3^-)^2}}$ Triaxiality index

γ Triaxiality scaling factor (recommended γ= 1.5)

Note:
$\uparrow C_m \uparrow \eta$
$\uparrow \gamma \quad \uparrow \eta$

3. 3D Confinement-dependent bilinear LITS model

Adopted approach for extending uniaxial LITS models to 3D

CLASSIC APPROACH

$$\boxed{\dot{\varepsilon}_{ij}^{lits} = \frac{\beta(T)}{\sigma_{u0}}\left(-\upsilon_{lits}\sigma_{kk}^{-}\delta_{ij} + (1+\upsilon_{lits})\sigma_{ij}^{-}\right)\dot{T}}$$

$$\Downarrow$$

ADOPTED APPROACH

$$\boxed{\dot{\varepsilon}_{ij}^{lits} = \eta\frac{\beta(T)}{\sigma_{u0}}\left(-\upsilon_{lits}\sigma_{kk}^{-}\delta_{ij} + (1+\upsilon_{lits})\sigma_{ij}^{-}\right)\dot{T}}$$

Where:

- η confinement coefficient – captures the triaxiality of the stress state

According to the adopted modelling approach, LITS:

1. Develops only in compression ($\boldsymbol{\sigma_{kk}^{-}}$)
2. Develops just during the first heating under compressive load – does not recover on cooling or unloading (***T***$\boldsymbol{T_{MAX}}$)
3. Depends on the confinement of the stress state ($\boldsymbol{\eta}$)

3. 3D Confinement-dependent bilinear LITS model

Dependency on the confinement - Confinement factor η

CLASSIC APPROACH

$$\boxed{\dot{\varepsilon}_{ij}^{lits} = \frac{\beta(T)}{\sigma_{u0}}\left(-\upsilon_{lits}\sigma_{kk}^{-}\delta_{ij} + (1+\upsilon_{lits})\sigma_{ij}^{-}\right)\dot{T}}$$

⇩

ADOPTED APPROACH

$$\boxed{\dot{\varepsilon}_{ij}^{lits} = \eta\frac{\beta(T)}{\sigma_{u0}}\left(-\upsilon_{lits}\sigma_{kk}^{-}\delta_{ij} + (1+\upsilon_{lits})\sigma_{ij}^{-}\right)\dot{T}}$$

Different uniaxial models, and consequently different $\beta(T)$, have been extended to 3D:

1. Anderberg & Thelandersson model [1]
2. Terro's model [2]
3. Bilinear model (T<250°C)

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[1] Anderberg Y, Thelandersson S.
A Constitutive Law for Concrete at Transient High Temperature Conditions. J Am Concr Institute Spec Publ 1978
[2] Terro MJ.
Numerical modeling of the behavior of concrete structures in fire 1998

3. 3D Confinement-dependent bilinear LITS model

Dependency on the confinement - Confinement factor η

CLASSIC APPROACH

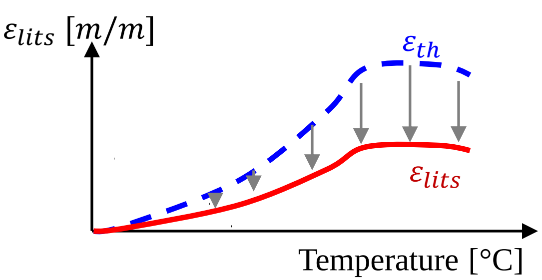
$$\boxed{\dot{\varepsilon}_{ij}^{lits} = \frac{\beta(T)}{\sigma_{u0}}\left(-\upsilon_{lits}\sigma_{kk}^{-}\delta_{ij} + (1+\upsilon_{lits})\sigma_{ij}^{-}\right)\dot{T}}$$

⇩

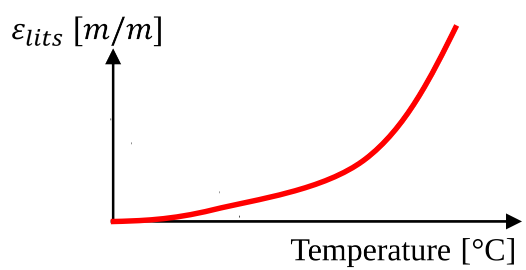
ADOPTED APPROACH

$$\boxed{\dot{\varepsilon}_{ij}^{lits} = \eta\frac{\beta(T)}{\sigma_{u0}}\left(-\upsilon_{lits}\sigma_{kk}^{-}\delta_{ij} + (1+\upsilon_{lits})\sigma_{ij}^{-}\right)\dot{T}}$$

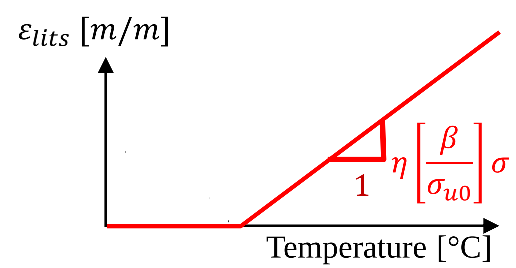
Anderberg & Thelandersson model [1]



Terro's model [2]



Bilinear model



[1] Anderberg Y, Thelandersson S.
A Constitutive Law for Concrete at Transient High Temperature Conditions. J Am Concr Institute Spec Publ 1978
[2] Terro MJ.
Numerical modeling of the behavior of concrete structures in fire 1998

4. Implementation with MFront

4. Implementation with MFront

Key passages of the .mfront script

```
@Parser Implicit;
[…]
@Algorithm NewtonRaphson_NumericalJacobian
[…]
@MaterialProperty stress young;
young.setGlossaryName("YoungModulus");
@MaterialProperty real nu;
nu.setGlossaryName("PoissonRatio");

@MaterialProperty real alpha;
@MaterialProperty real beta;
@MaterialProperty real gamma;
@MaterialProperty real sigmultimate;
@MaterialProperty real nulits;
@MaterialProperty real tcrit;
```

Integration method

Iterative method

Material properties

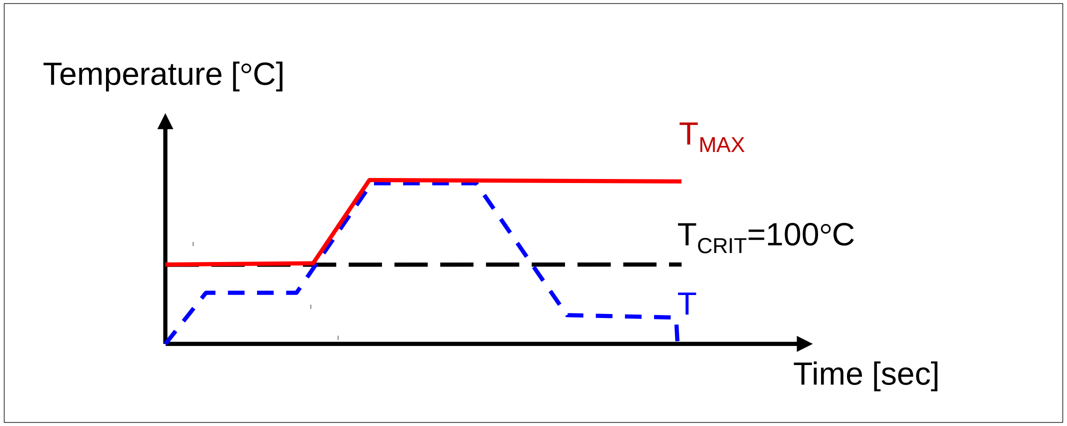
4. Implementation with MFront

Key passages of the .mfront script

```
@StateVariable Stensor    ETTC;
@StateVariable Stensor    ETH;
@AuxiliaryStateVariable real tmax;
```

State variables: LITS and FTS tensors

Auxiliary state variable: T_{MAX}



Schematic evolution of the internal variable T_{MAX}, for a given temperature history and a critical temperature T_{CRIT}.

4. Implementation with MFront

Key passages of the .mfront script

```
@Integrator{
[…]
real cm=1.;
real eta=1.;
real s_den;

s_den=sqrt(pow(s1n,2)+pow(s2n,2)+pow(s3n,2));

if (s_den>1.e-6*young)
        {cm=-(s1n+s2n+s3n)/(s_den);}
         else {cm=1;}
eta=1+(cm-1)*gamma;
```

Definition of the confinement coefficient:

$\eta = 1 + (C_m - 1)\gamma$

4. Implementation with MFront

Key passages of the .mfront script

```
[…]
// A)---- LITS increment ----
  if((T+dT>tmax)&&(s1+s2+s3<0)){

      incr_ttc=(beta/sigmultimate)*eta*((1.+nulits)
      *(sigma_n)-
      nulits*trace(sigma_n)*Stensor::Id())
        *(T+dT-tmax);
  } else {
        incr_ttc=0.*Stensor::Id();}
[…]
// C)----TOTAL STRAIN DECOMPOSITION---
        fETTC=dETTC-(incr_ttc);
        fETH=dETH-(incr_eth);
        feel=deel-(deto-dETTC-dETH);
```

LITS increment in the time step.

LITS develops:

- just on first heating
- under compressive load.

General strain decomposition: $\varepsilon^{tot} = \varepsilon^{el} + \varepsilon^{th} + \varepsilon^{lits}$

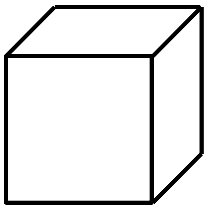
5. Verification studies

5. Verification studies

Verification test cases

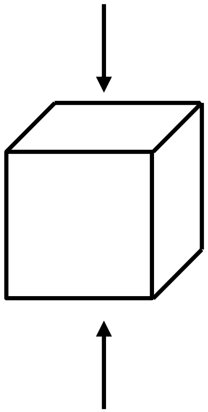
Verify that:
- Convergence in reached in a reasonable time
- Numerical results are consistent with implemented equations

1) FREE THERMAL STRAIN



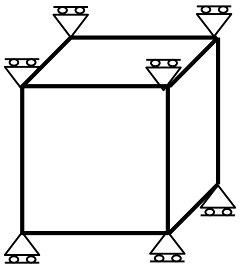
Stress-free specimen

2) LOAD-THEN-HEAT



Uniaxially loaded specimen

3) RESTRAINED THERMAL STRAIN



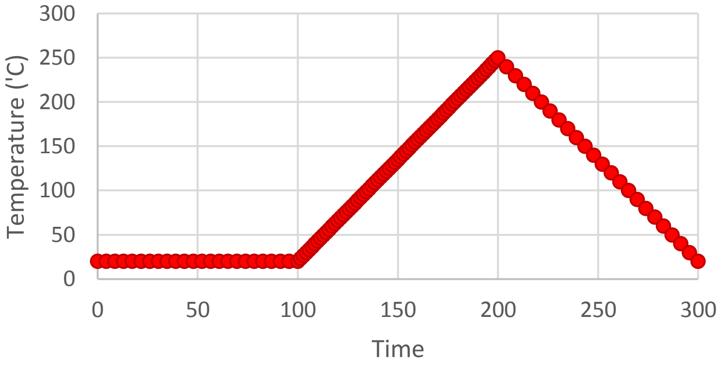
Uniaxially restrained specimen

5. Verification studies

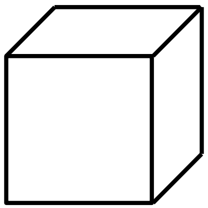
Verification example n.1
Specimen subjected to free thermal strain
(linear thermal expansion)

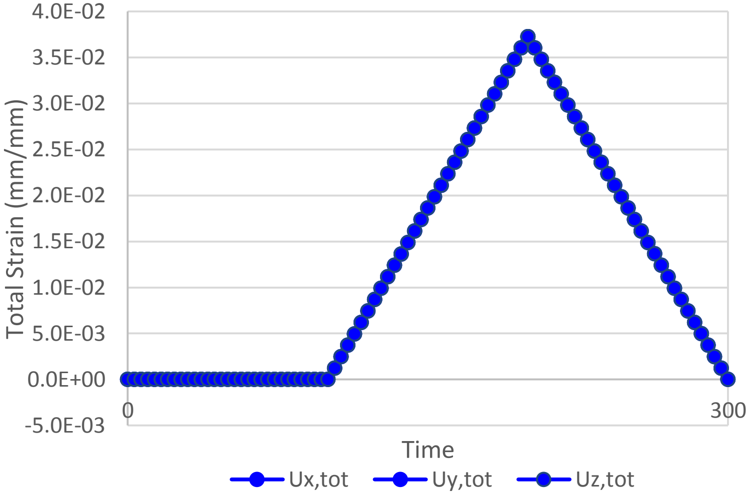
Constitutive equation (Anderberg and Thelandersson model)

$$\underbrace{\dot{\varepsilon}_{ij}^{el} = \frac{1+\upsilon}{E}\dot{\sigma}_{ij} - \frac{\upsilon}{E}\dot{\sigma}_{kk}\delta_{ij}}_{\dot{\varepsilon}_{ij}^{el}} + \underbrace{\frac{k_{tr}\alpha}{\sigma_{u0}}\left(-\upsilon_{lits}\sigma_{kk}\delta_{ij} + (1+\upsilon_{lits})\sigma_{ij}\right)\dot{T}}_{\dot{\varepsilon}_{ij}^{lits}} + \underbrace{\alpha\delta_{ij}\dot{T}}_{\dot{\varepsilon}_{ij}^{th}}$$



Stress-free specimen



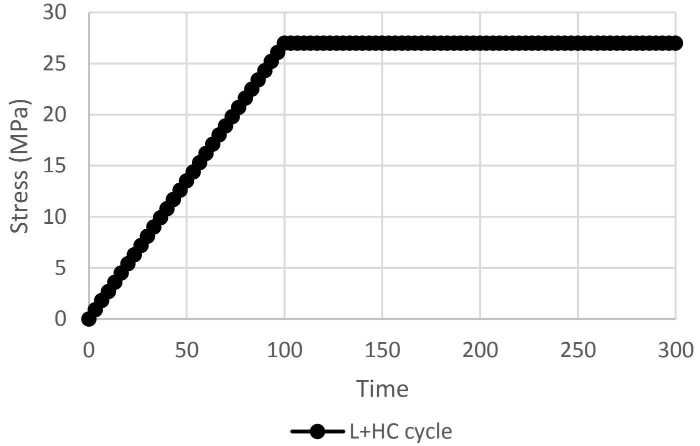


5. Verification studies

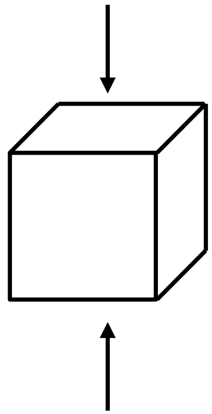
Verification example n.2
Specimen subjected to Load-Then-Heat regime (bilinear LITS model)

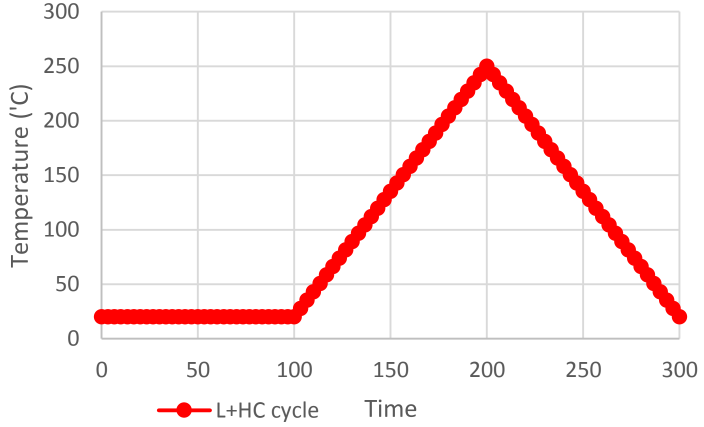
Constitutive equation (Anderberg and Thelandersson model)

$$\dot{\varepsilon}_{ij}^{el} = \underbrace{\frac{1+\upsilon}{E}\dot{\sigma}_{ij} - \frac{\upsilon}{E}\dot{\sigma}_{kk}\delta_{ij}}_{\dot{\varepsilon}_{ij}^{el}} + \underbrace{\frac{k_{tr}\alpha}{\sigma_{u0}}\left(-\upsilon_{lits}\sigma_{kk}\delta_{ij} + (1+\upsilon_{lits})\sigma_{ij}\right)\dot{T}}_{\dot{\varepsilon}_{ij}^{lits}} + \underbrace{\alpha\delta_{ij}\dot{T}}_{\dot{\varepsilon}_{ij}^{th}}$$



Uniaxially loaded specimen





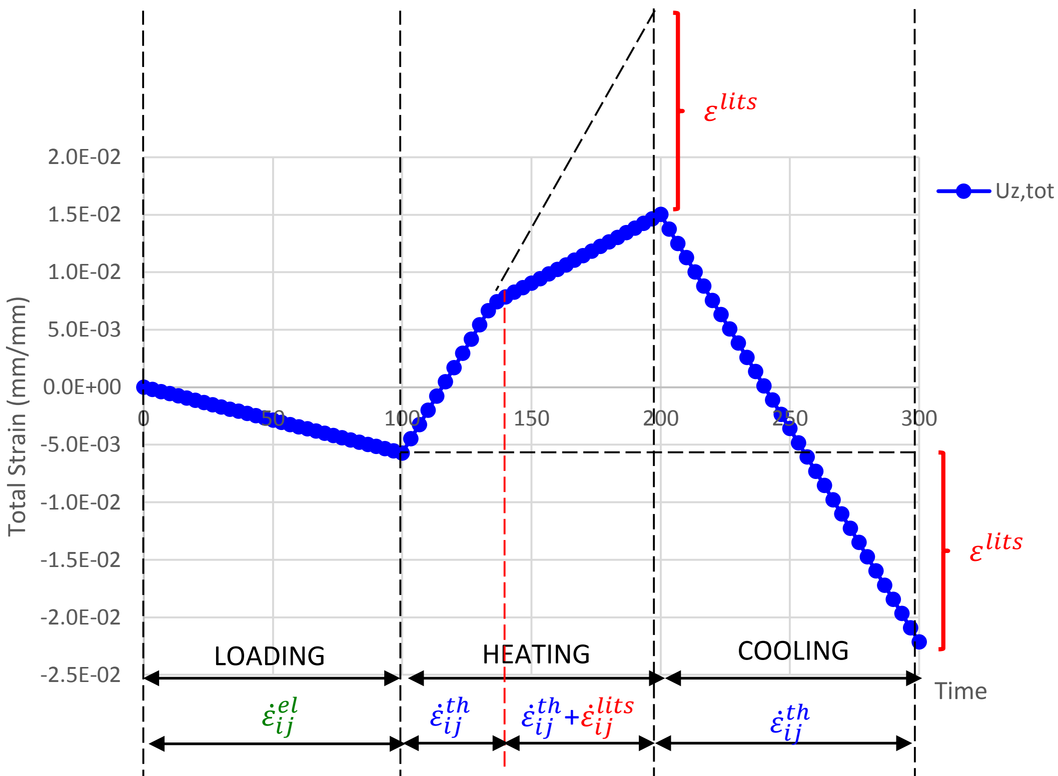
5. Verification studies

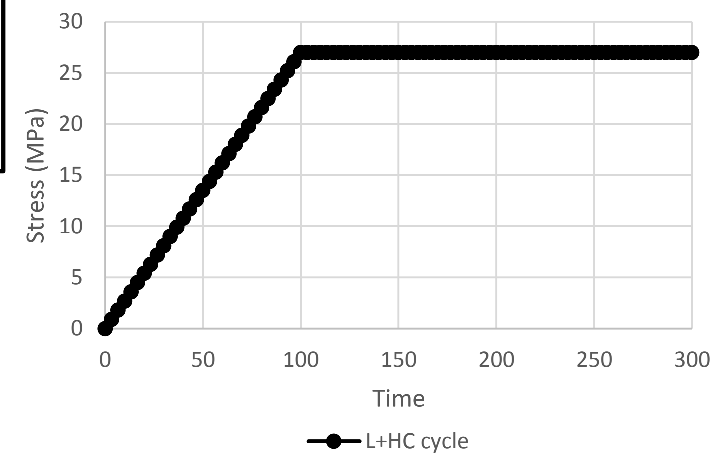
Verification example n.2

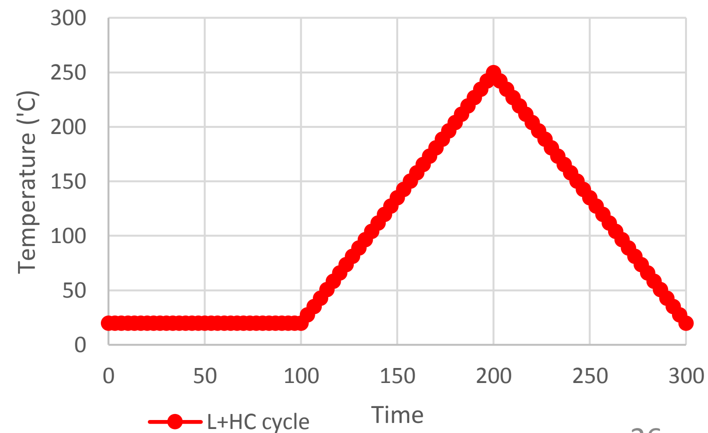
Specimen subjected to Load-Then-Heat regime (bilinear LITS model)

Constitutive equation (Anderberg and Thelandersson model)

$$\dot{\varepsilon}_{ij}^{el} = \underbrace{\frac{1+\upsilon}{E}\dot{\sigma}_{ij} - \frac{\upsilon}{E}\dot{\sigma}_{kk}\delta_{ij}}_{\dot{\varepsilon}_{ij}^{el}} + \underbrace{\frac{k_{tr}\alpha}{\sigma_{u0}}\left(-\upsilon_{lits}\sigma_{kk}\delta_{ij} + (1+\upsilon_{lits})\sigma_{ij}\right)\dot{T}}_{\dot{\varepsilon}_{ij}^{lits}} + \underbrace{\alpha\delta_{ij}\dot{T}}_{\dot{\varepsilon}_{ij}^{th}}$$







5. Verification studies

Verification example n.3

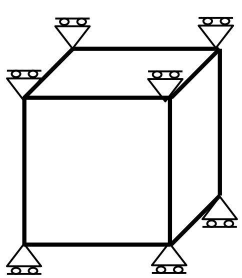
Specimen subjected to Restrained Thermal Strain (bilinear LITS model)

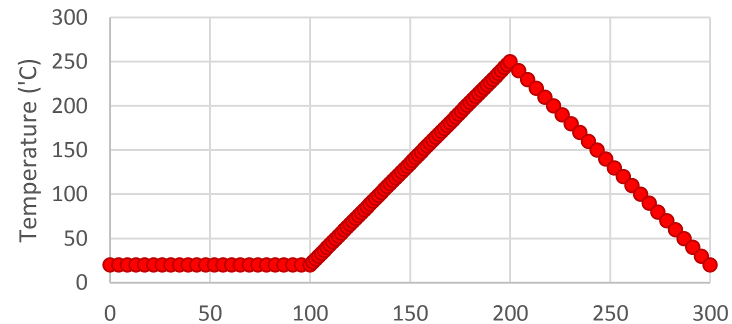
Constitutive equation (Anderberg and Thelandersson model)

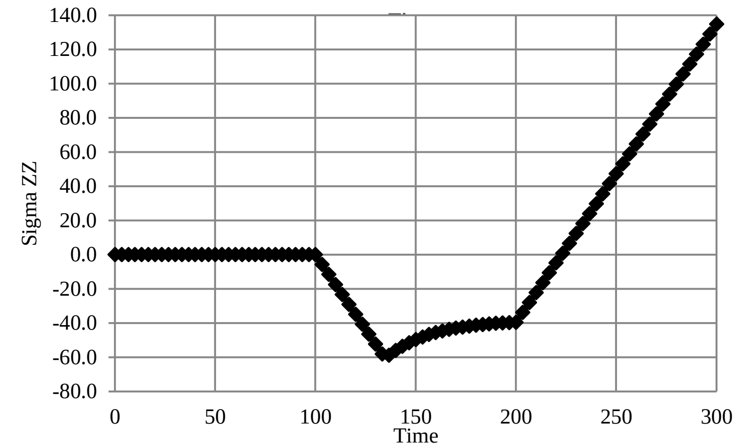
$$\dot{\varepsilon}_{ij}^{el} = \underbrace{\frac{1+\upsilon}{E}\dot{\sigma}_{ij} - \frac{\upsilon}{E}\dot{\sigma}_{kk}\delta_{ij}}_{\dot{\varepsilon}_{ij}^{el}} + \underbrace{\frac{k_{tr}\alpha}{\sigma_{u0}}\left(-\upsilon_{lits}\sigma_{kk}\delta_{ij} + (1+\upsilon_{lits})\sigma_{ij}\right)\dot{T}}_{\dot{\varepsilon}_{ij}^{lits}} + \underbrace{\alpha\delta_{ij}\dot{T}}_{\dot{\varepsilon}_{ij}^{th}}$$

Note: T_{boil}=100°C

Uniaxially restrained specimen







ε^{th} ($\uparrow \sigma_{zz}$)
ε^{lits} ($\downarrow \sigma_{zz}$)

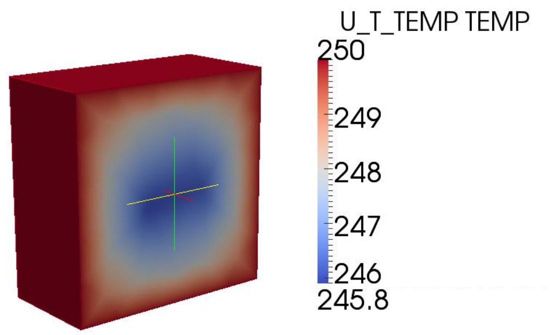
ε^{th}($\downarrow \sigma_{zz}$)

6. Validation studies

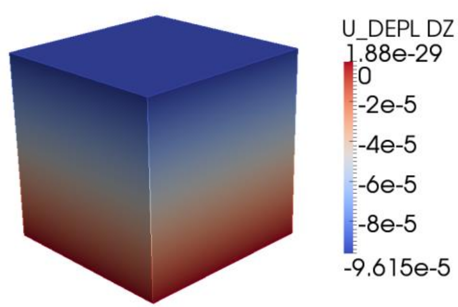
6. Validation studies

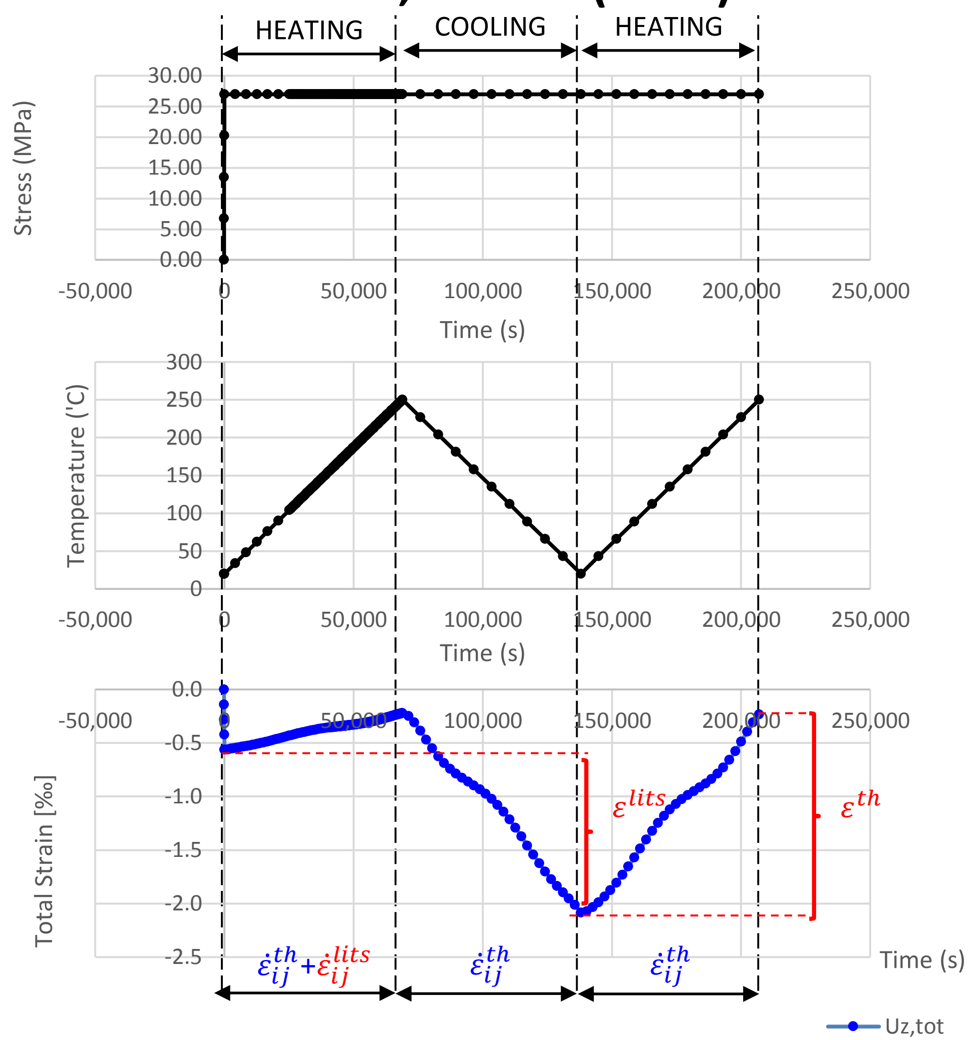
Multiaxial loading conditions
Modelling of experiments Petkovski, Crouch (2008)

1. THERMAL ANALISYS



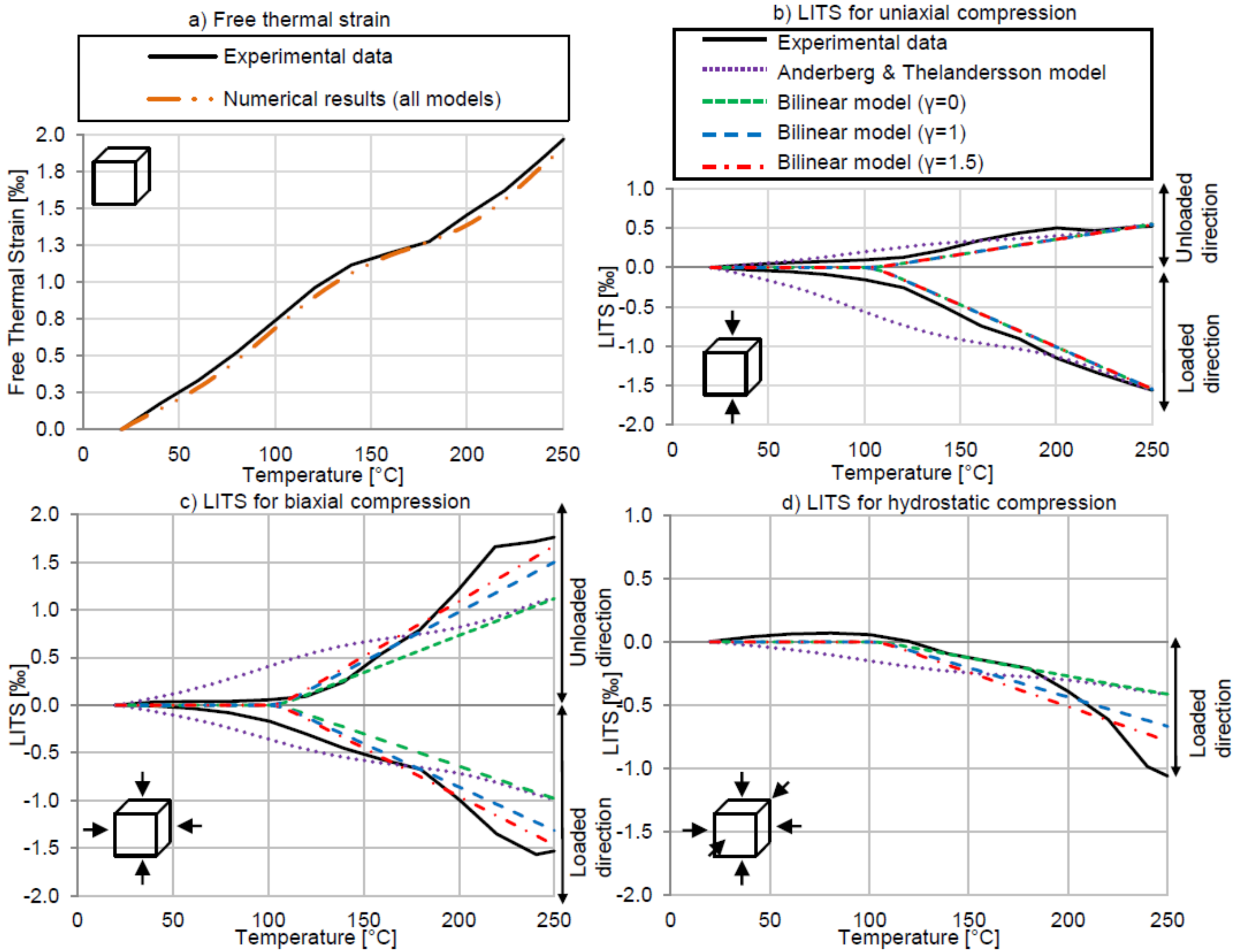
2. MECHANICAL ANALYSIS





6. Validation studies

Multiaxial loading conditions
Modelling of experiments Petkovski, Crouch (2008)



7. Application to a test case

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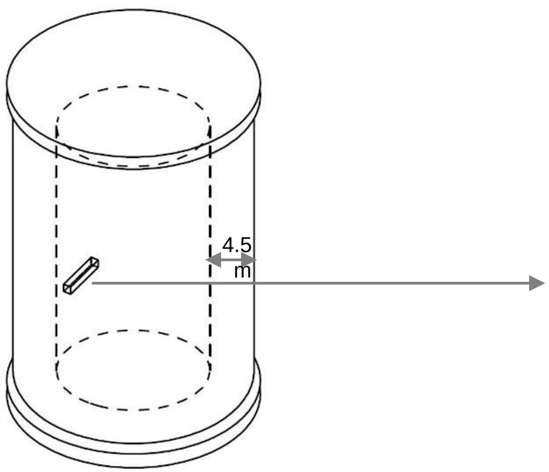
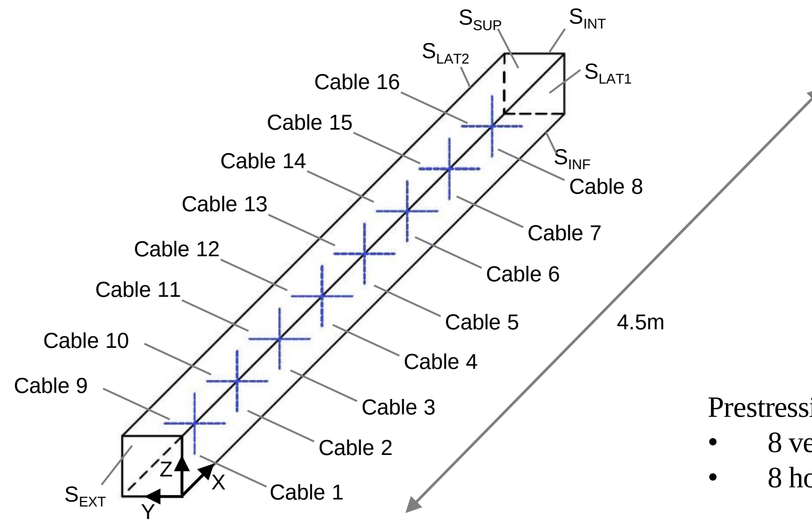
Loss of prestress in Pressure Vessel subjected to HC cycle

Typical PCPV geometry

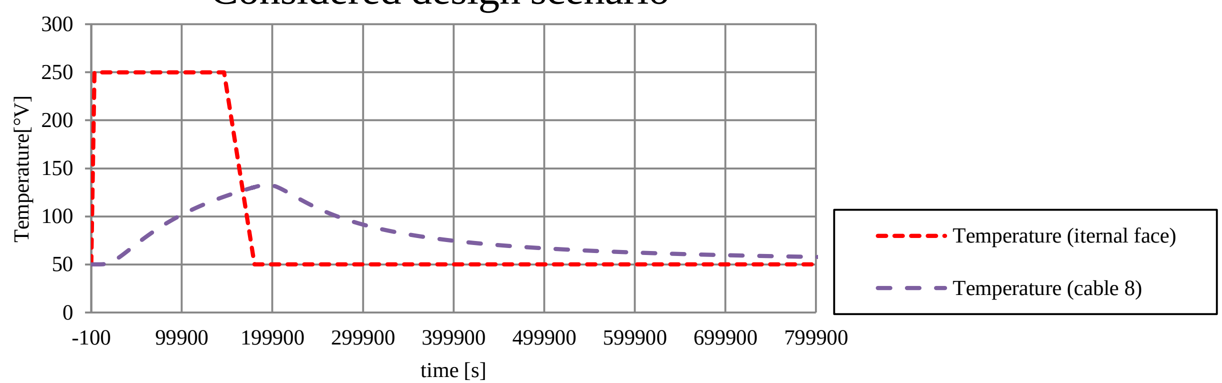


Studied representative portion



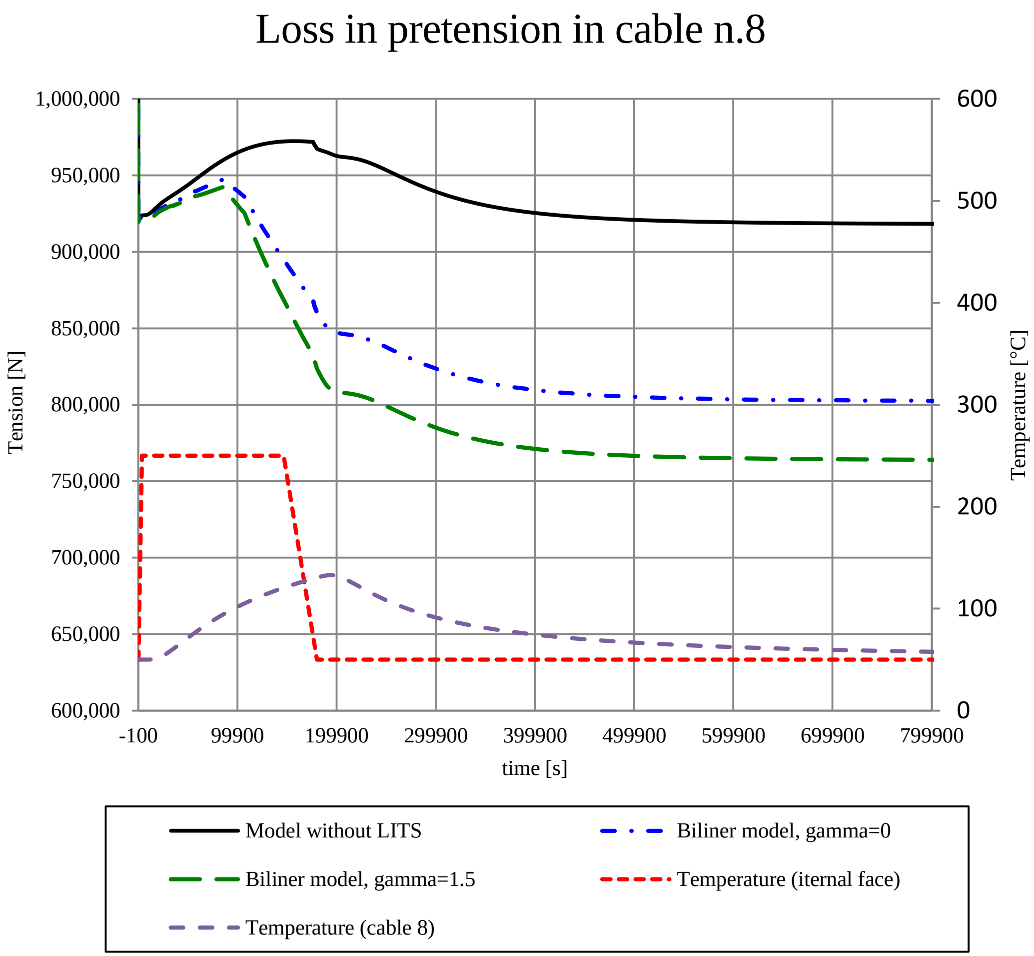
Prestressing system:
- 8 vertical tendons
- 8 horizontal tendons

Considered design scenario



7. Application to a test case

Loss of prestress in Pressure Vessel subjected to HC cycle



LOSS IN PRETENSION
16.80%

7. Application to a test case

Conclusions

- The introduced **triaxiality coefficient** is crucial to capture the behaviour of structures working in biaxial compression

- In case of prestressed structures subjected to HC cycles, LITS can produce a loss in prestress of about 15-20%. Therefore**, it is essential to include a LITS model** for assessing the stress redistribution in case of fault condition of nuclear PCPV.

8. Questions

Implementation of a confinement dependent LITS model for concrete

Giacomo Torelli[1], Partha Mandal[1], Martin Gillie[1], Van Xuan Tran[2]

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