



Addressing the Energy Efficiency of Deep Learning Algorithms in Finance

by

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Declaration

I, Tom Maxwell Potter, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the dissertation.

Abstract

This thesis investigates the efficiency of deep learning training algorithms, exploring how deep neural networks can be trained in a way that minimises energy consumption and training data requirements. These energy and data-efficient training algorithms are applied to financial modelling, demonstrating how the computational cost of deep learning in finance can be reduced. The financial sector has long been associated with negative environmental, social, and governance (ESG) impacts, including being a major contributor to global carbon emissions. Despite the attempts by some to prioritise *sustainable finance*, the recent expansion of financial technology—incorporating new, expensive methods such as *deep learning* (DL)—has only worsened the energy consumption attributed to this industry, accelerating its carbon emissions. To address these negative impacts, this project develops energy-efficient DL training algorithms for financial models. This research explores *Green AI*—a research domain aiming to mitigate the energy budget of DL algorithms—and applies these methods for the first time to the models and algorithms used in finance. To exemplify the benefits of these methods, a performant financial volatility model is developed that not only produces accurate results but prioritises generating this performance in an efficient manner, minimising the energy and data resources required during training. This research aims to demonstrate how the principles of Green AI are applicable within the financial sector, furthering the scope of sustainable finance by improving the sustainability of deep learning for finance and, hence, minimising the ESG impacts of the financial sector.

Outline of research:

- Chapter 3: *Methodology*. This research commences by implementing a baseline model, dataset, and training algorithm characteristic of those used for DL in finance. A particular focus will be given to the domain of financial volatility modelling, as this is a major application of deep learning in finance, and the *long short-term memory* networks typically exploited for such tasks. This analysis acts as an exemplar of the computational cost of typical deep learning algorithms used in finance.
- Chapter 4: *Energy-Efficient Training Algorithms*. Alternative training algorithms are then implemented that prioritise reducing the energy consumed by DL whilst maintaining an accurately performing model. Specifically, *progressive training*, and *mixed-precision* will be explored, proving the feasibility of efficient training algorithms within the chosen domain.
- Chapter 5: *Data-Efficient Training Algorithms*. Training algorithms that prioritise data efficiency are then implemented, further lowering resource requirements and mitigating the computational costs of data storage and processing at data centres.
- Chapter 6: *Results & Discussion*. An analysis is made between the baseline and extended

models, comparing their performance and efficiency, and discussing the overall success of the application of Green AI to finance.

Contributions to science:

1. *Expanding the applications of Green AI.* The first application of Green AI to the finance sector, demonstrating the utility and importance of Green AI in lowering the environmental cost of deep learning outside of the limited existing research applications.
2. *Reducing the environmental impact of financial technology.* Mitigating the costs of DL in finance and advancing sustainable finance to include the new research domain of *sustainable deep learning for sustainable finance*.
3. *Improving the inclusivity of finance.* Lowering the bar-to-entry to engage in deep learning for finance, allowing more individuals to leverage financial technology and analytics.

Keywords: Green AI, Energy Efficiency, Data Efficiency, Sustainable Finance, Financial Technology, Financial Volatility Modelling, Long Short-Term Memory

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Chapter 1

Introduction

1.1 Topic & Background

Escalating concern about how industrialisation has contributed to climate change has led to many industries coming under increased pressure to monitor and rectify their environmental impact. This pressure is typically directed towards high carbon industries that constitute the major pollutant sectors of the economy, such as transport, energy supply, and agriculture. However, both the finance and technology industries are also responsible for an alarming amount of carbon emissions.

1.1.1 Sustainable Finance

The finance sector has long been closely associated with sustainability concerns, being a major contributor to global carbon emissions both directly and indirectly. The most visible environmental impact of the financial industry is its direct emissions from business practices such as the distribution of cash through the economy, card payment processing centres, and everyday operational costs (Hanegraaf et al., 2018). However, indirect emissions attributable to services such as investing and lending (termed *financed emissions* by Power et al. (2020)) have been estimated to contribute over 700 times more to the carbon footprint of the financial industry than all direct emissions (Power et al., 2020). In their survey of 700 global financial institutions, Power et al. (2020) estimated that the production of over 1.04 billion tons of carbon dioxide was attributable to financed emissions in 2020 (approximately 3.1% of global emissions). Furthermore, a recent report by *Greenpeace* and the *WWF* concluded that the combined carbon emissions of the largest banks and investors in the UK totalled 805 million tons in 2019, which (if consolidated into its own country) would rank 9th in the global list of total emissions per country (Greenpeace, 2021). This figure is 1.8 times higher than the total emissions of the UK (455 million tons), and almost 90% of the global emissions from commercial aviation (918 million tons) in the same year (Graver et al., 2020).

Increasing concern surrounding these negative *environmental, social, and governance* (ESG) impacts of the finance industry has led to the exploration of financial practices that prioritise the *sustainable development goals* (SDGs). These initiatives, known as *sustainable finance*, aim to incorporate ESG impacts and SDGs into financial decisions to improve sustainability. In particular, a focus is given to those practices that produce a measurable improvement to the social and environmental impact of the finance industry, global economy, and wider society.

Recent research into sustainable finance has attempted to put into practice the sustainability goals set out by the *Paris Agreement* and SDGs, mitigating the negative ESG impacts of finance.

This work commonly investigates sustainable investment fields such as *impact investing* (Agrawal and Hockerts, 2021) and *ESG investing* (Alessandrini and Jondeau, 2020), and has significantly grown in prominence over recent years. In their review of 936 research papers, Kumar et al. (2022) found that almost 70% of sustainable finance research had been published between 2015–2020, and an exponential trend was exhibited in the increase in papers being published each year; additionally, they found that in 2020, \$400 billion of new sustainability funds were raised on capital markets. Thus, it is clear that the scope and impact of sustainable finance are currently on the rise, predominantly driven by a renewed focus on the ESG impacts of financial practices, resources, and investments.

1.1.2 Financial Technology and the Issues with Sustainable Finance

Despite these achievements, these sustainability funds only amount to 0.98% of the value of the US equity market, and recent investigations have uncovered the prevalence of *greenwashing* (Popescu et al., 2021), where institutions misleadingly classify investments as sustainable without supporting evidence. Such issues are common within sustainable finance, as Cunha et al. (2021) asserted the field is currently “excessively fragmented”. This indicates that whilst attention around the sustainability of financial practices is growing, this domain is still not widely recognised, and further work is necessary to increase the adoption of sustainable practices and methods within finance.

An additional concern is that the tools used to conduct financial practices are becoming increasingly resource-hungry. A clear example of this is the increased adoption of technology throughout the finance industry. Recently, a surge of developments in financial technology (*Fintech*) has revolutionised financial methods and practices, from those used within large financial institutions and Fintech companies, to groups of academic researchers. This Fintech revolution has transformed many aspects of finance, enhancing existing financial services and delivering new, innovative financial products. Palmié et al. (2020) assert that a significant reason behind the accelerating adoption of Fintech in recent years is a new reliance upon *artificial intelligence* (AI). Driven by the promise of increased automation and computing power, the utilisation of AI within the financial sector has been rapidly expanding over recent years, becoming a core component of many of the financial products and services used today: from accurate real-time financial fraud detection (Sadgali et al., 2019), to automated analysis of financial statements (Amel-Zadeh et al., 2020).

1.1.3 Deep Learning for Finance

AI methods have been extensively used in recent research to produce state-of-the-art results over a plethora of applications. This research typically uses *machine learning* (ML), where large collections of data are used to train computational models to perform certain tasks independently (Samuel, 1959). Recent innovations in ML have increasingly taken advantage of *deep learning* (DL) methods, which use large, complex models to produce state-of-the-art performance (Witten et al., 2017).

The highly publicised successes of complex DL algorithms has driven increased adoption of DL further afield, such as within the finance industry. It is estimated that 70% of financial institutions rely upon ML (Gokhale et al., 2019) and that global spending on AI is predicted to double in value by 2024, from \$50 billion in 2020 to approximately \$110 billion (Nassr et al., 2021). In a recent survey of financial professionals, Chartis Research Staff (2019) found that 44% of respondents cited “greater accuracy of process and analysis” as a key motivation behind their adoption of AI methods. Hence,

the superior accuracy provided by ML provides a compelling alternative to traditional statistical models. Hence, the promise of more accurate performance is a major driving factor behind recent ML adoption within Fintech.

1.1.4 The Issues with Deep Learning

Whilst cutting-edge DL models push the boundaries of computational accuracy, few of these systems prioritise the efficient use of energy and data. This has led to DL inflicting a great cost upon the environment, as the energy-intensive algorithms, long training processes, and the power-hungry data centres they utilise inflict a high carbon footprint (Lacoste et al., 2019). Schwartz et al. (2019) label these accurate but energy-intensive DL algorithms as *Red AI*, defining such work as “research that seeks to improve accuracy through the use of massive computational power while disregarding the cost”. Schwartz et al. assert that the performance of these systems is often achieved by employing extensive computational resources, such as complex models with many parameters, large datasets, and power-hungry computer hardware. Bender et al. (2021) illustrate this trend through the progression of language models: whilst the *BERT* (Devlin et al., 2018) produced state-of-the-art performance in 2019 using an architecture of 340 million parameters in total and 16GB of training data, the cutting-edge models of 2020 (*GPT-3* by Brown et al. (2020)) and 2021 (*Switch-C* by Fedus et al. (2021)) utilised 175 billion and 1.57 trillion parameters respectively, and 570GB and 745GB datasets. In fact, between 2012 and 2018 the computational resources used in cutting-edge training algorithms increased by a factor of 300,000, outpacing *Moore’s Law* (Amodei and Hernandez, 2018).

The intense computational load of these DL algorithms does not come for free; the large parameter and data sets mean training, storing, and computing with DL models draws a significant amount of energy (Bietti and Vatanparast, 2019). Partly due to this inefficiency, the data centres upon which DL algorithms rely for storage and cloud computing have become a significant hidden contributor to carbon emissions (Al-Jarrah et al., 2015). Studies such as Masanet et al. (2020) and Malmmodin and Lundén (2018) have estimated that processing at these data centres consumes around 200–300TWh of electricity a year, with the cost of data transmission exceeding this at up to 340TWh per year (IEA, 2022). These estimates suggest that global data centres use more electricity than most countries in the world, ranking above both Australia and Spain (EIA, 2019), and account for around 1–1.5% of global electricity consumption (rising to 2.4% when including transmission costs). Furthermore, this energy is likely not entirely carbon-neutral; Cook et al. (2017) showed that of their total electricity demand, *Amazon Web Services* only powered 12% through renewable sources, and *Google Cloud* 56% (with the latter figure ranging between 4% and 94% depending on location). Thus, these large energy budgets generate considerable carbon emissions, resulting in the use of data centres coming alongside significant environmental detriment. Specifically, Hockstad and Hanel (2018) assert that in 2018 cloud computing at data centres generated the equivalent of 31 million tons of carbon dioxide in the US alone, equalling the total emissions attributable to electricity generation in California (IEA, 2022).

Beyond these general figures, Strubell et al. (2019) demonstrated the carbon emissions directly produced by DL training algorithms. They found that training the base language model that underlies BERT, which utilised 110M parameters and trained for 96 hours over 16 TPUs, produced the equivalent of 1438lbs of CO_2 —the same as a trans-American flight. Strubell et al. also found that whilst the *Evolved Transformer* of So et al. (2019) improves state-of-the-art translation accuracy by 0.1 *BLEU* (in English-German translation), this model cost \$3.2 million to train (on GPU

hardware) and produced 626,155lbs of CO_2 (five times the lifetime emissions of an average car).

The large financial cost associated with intensive DL systems also inflicts a great social cost. As DL models get larger and more complex, the price of storing the model and its training data, as well as the cost of running its training algorithm, becomes prohibitively high (Schwartz et al., 2019). This financial barrier restricts who can engage in cutting-edge research to only those with the backing of a large institution. Thus, this lack of accessibility drives a “rich get richer” funding cycle (Strubell et al., 2019) where only research within the interest of these institutions receives adequate funding. This stifles creativity and leaves the allocation of who benefits from these systems (and who bears the negative side effects) to a handful of large corporations. Bender et al. (2021) highlight this disconnect between the benefits of energy-intensive DL research and the environmental consequences it inflicts: “is it fair or just, for example, that the residents of the Maldives [...] pay the environmental price of training and deploying ever larger English [language models]”.

Thus, whilst DL provides state-of-the-art computational accuracy across a range of fields, its ESG impacts cannot be ignored. This poses sustainability concerns for the accelerating reliance on ML in Fintech, contributing further to the negative ESG impacts of the financial sector; furthermore, this generates a clear conflict between the growing use of DL in Fintech and the growing need for sustainable finance. Both of these fields provide great utility to the finance sector: Fintech (including DL) provides innovative services to consumers and accurate tools for institutions, and sustainable finance ensures the industry inflicts minimal ESG costs. Moreover, DL has been shown to have utility to sustainable finance itself, for example using ML to analyse the ESG factors of potential investments (Mehra et al., 2022). Hence, a compromise between the use of DL Fintech and the prioritisation of sustainable finance must be reached.

1.1.5 Green AI

The gap between the utility of DL and its environmental consequences has recently started to gather attention from ML researchers. In what has become known as *Green AI* (Schwartz et al., 2019), new research has begun to consider how to mitigate the negative ESG impacts of ML by improving the efficiency of DL models. These efficient models reduce the energy and resources required for training and deployment, minimising the carbon emissions they contribute towards.

Alongside *mobile computing*—which prioritises energy efficiency due to the hardware constraints of mobile devices—the research domains of *Natural Language Processing* (NLP) and *Computer Vision* (CV) are currently the predominant fields focussing on Green AI. NLP focuses on the processing and understanding of language, typically using complex, intensive DL algorithms to match and exceed human accuracy in language modelling (e.g. translation between languages). CV uses complex models with expensive methods—such as the *convolution operation*—to analyse, classify, and map visual environments. Within these fields, Green AI researchers have been working to promote the utilisation of efficient methods that reduce the energy, time, and data requirements of model training by using parameters, data, and operations more intelligently. However, these methods have yet to garner significant attention outside of the specific research domains of NLP, CV, and mobile computing. Therefore, the utility of these methods to improving the sustainability of ML has largely not been demonstrated further afield, meaning their potential benefits to reduce the energy consumption and carbon emissions of ML have yet to be seen on a wide scale.

1.2 Research Motivations

As demonstrated, the energy-intensive DL systems cause several environmental and social issues, including their carbon footprint and financial cost. Thus, the accelerating adoption of ML and DL within Fintech raises concerns about increasing the negative ESG impacts of the finance industry. Namely, the side effects of these complex DL algorithms produced by exploiting large parameter and data sets and long training processes (and hence high energy budgets) are in direct conflict with the goal of sustainable finance to reduce the negative ESG impacts of finance. However, the performance benefits of DL in Fintech have been shown, making their continued adoption inevitable.

Therefore, to minimise the negative ESG impacts of Fintech, the adoption of Green AI is paramount. In the first study of its kind, this thesis investigates how energy-efficient Green AI algorithms can be adapted for DL in finance, aiming to demonstrate how Fintech does not have to inflict such environmental and social costs. To exemplify the promise of introducing Green AI to Fintech and sustainable finance, energy-efficient methods will be applied to one of the most popular applications of DL in finance: *financial volatility forecasting*. Namely, a DL-based model that accurately predicts the future volatility of the *S&P 500* market will be developed using training algorithms that require minimal energy consumption and training data. Volatility forecasting is commonly used to give insight into the risk of a financial market or asset (French et al., 1987); it has also been extensively explored by researchers to showcase the prediction capabilities of DL (Zhang et al., 2022). Hence, this domain is often cited as a core application of DL in finance (Thakkar and Chaudhari, 2021), and so has been chosen as the specific application within Fintech to demonstrate the promise of using Green AI within the finance industry.

By showing the utility of Green AI within Fintech, this thesis aims to advance the field of sustainable finance, creating the new field *sustainable deep learning for sustainable finance*. This work is the first study to address the conflict of interest between sustainable finance and the growing reliance of Fintech on DL algorithms with high energy consumption and concerning ESG impacts. Hence, this research aims to demonstrate how energy and data-efficient algorithms can be exploited by the DL systems used in finance, increasing the scope and impact of sustainable finance by allowing the use of DL without compromising the SDGs, and reducing the ESG impacts of Fintech.

1.3 Contributions to Science

The contributions of this research to scientific literature and the finance industry are as follows:

1. *Expanding the applications of Green AI*. This thesis is the first application of Green AI to the field of finance. This will further demonstrate the utility of Green AI by proving that efficient algorithms can provide compelling performance with a lower environmental cost in new domains outside of the existing research focus on NLP, CV, and mobile computing.
2. *Reducing the environmental impact of financial technology*. The following work combines the research fields of Fintech, deep learning for finance, sustainable finance, and Green AI, to create the new research domain of *sustainable deep learning for sustainable finance*. This further improves the ESG impact of the financial sector beyond previous work in sustainable finance by mitigating the conflict between the utility of DL algorithms for sustainability modelling and the intrinsic carbon footprint of these energy-intensive programs.

3. *Improving the inclusivity of finance.* The reduction in the financial, environmental, and social cost of DL for finance also increases the inclusivity of this field. Improving the energy and data efficiency of these algorithms reduces resource requirements and lowers the bar-to-entry to using DL in finance, allowing more industry players, developers, and individual traders to utilise the advantages brought by Fintech and DL.

1.4 Research Objectives

The initial research objective is to analyse the efficiency of traditional training algorithms used for DL in finance. In particular, the resource requirements in terms of training time and training dataset size are inspected. A particular focus is given to the use of *long short-term memory* (LSTM) networks, as these are the typically used DL model for sequence forecasting tasks in general, and recent research into volatility forecasting (Xiong et al., 2015a). The second objective is to then implement an energy-efficient training algorithm for the baseline model, adapting the traditional training algorithm originally utilised such that the total training time is reduced. The third objective is to build a data-efficient training algorithm for the implemented DNN, which facilitates the same performance within the chosen financial domain but requires less training data. Finally, the efficiency and performance of each training algorithm will be discussed and their utility evaluated.

1.5 Research Structure

The research discussed within this thesis is organised as follows:

- Chapter 2: *Background & Literature Review*. The relevant literature to this research is first explored to give an overview of the key concepts and methodologies used. This begins with an exploration of ML, DL in finance, and the specific algorithms implemented in this research, giving a clear outline of the domain of financial volatility forecasting. Following this, Green AI is explored, focusing on the efficient algorithms at the core of this study.
- Chapter 3: *Methodology*. Initially, an outline of the implemented modelling task, dataset, and network architecture is presented. This DNN is then optimised through a traditional training algorithm, producing a baseline model that exemplifies the computational cost and resource requirements of typical DL models and methods used within financial volatility forecasting.
- Chapter 4: *Energy-Efficient Training Algorithms*. Alternative energy-efficient training algorithms are then implemented and evaluated. This section explores these methods, explaining how they work and their utility in decreasing the energy consumption of training.
- Chapter 5: *Data-Efficient Training Algorithms*. A focus is then given to data-efficient training algorithms, discussing their origins within Green AI and explaining how they aim to reduce the data requirements and computational cost of training DNNs.
- Chapter 6: *Results & Discussion*. An extensive analysis of the benefit of Green AI algorithms to this domain is then made, quantifying the facilitated reduction in time and energy. This compares both the accuracy of each optimised model and the efficiency of each training algorithm to the baseline approach, evaluating any accuracy-efficiency compromises made. A general conclusion is then given as to the utility and viability of Green AI within Fintech.

Chapter 2

Background & Literature Review

To understand how to improve the efficiency of DL algorithms in finance, a detailed understanding of DL must be conveyed. This chapter explores these algorithms, beginning with a general summary of ML and DL, before focussing on the specific models and algorithms used within volatility forecasting. The second part of the chapter outlines the field of Green AI, its motivations, applications, and efficiency improvements, acting as the basis for the studies that follow.

2.1 Machine Learning

Machine learning, first discussed by Arthur Samuel in his 1953 exploration of “mechanical brains” (Samuel, 1959), is a subset of artificial intelligence that uses data to allow computer programs to independently learn how to solve a given problem or task without explicit information about the task’s governing rules. This learning typically involves a *training* procedure, where the program is supplied with instances of experience describing the problem from the training dataset D . The program learns from D how to perform the task by taking input of a data instance and outputting the result it believes is the correct response. This result is then evaluated through some performance metric (typically computed through a *loss function*). For example, in a *prediction* task, the program takes input of a collection of values and outputs the value it predicts should come next: e.g. Xiong et al. (2015a) use a dataset of S&P 500 values to predict future stock movements. Alternatively, in a *classification* task, a data point is input into the program to be assigned to a distinct class: for example, Sadgali et al. (2019) use financial transaction data to classify whether a particular transaction is fraudulent or not. Such tasks are typically trained through *supervised learning*, where the correct result (known as the *label*) is provided to the program after it has produced its output; the correctness of the program’s solution is then evaluated, and its behaviour adapted to improve the correctness of its outputs.

2.1.1 Neural Networks

Since the conception of ML, a core goal has been the development of algorithms that can accurately mimic how the brain processes information. These algorithms, known as *artificial neural networks* (ANNs), model the complex communication system between neurons in the brain. The first implementation of such a model was Frank Rosenblatt’s *Mark I Perceptron*, which attempted to perform binary classification of images (Rosenblatt, 1958). Each pixel in the input image was represented as a single value in a 2D matrix of input neurons known as the *input layer*. These values were then

passed to the single internal neuron through *weighted channels* to compute the weighted sum of all input values, the result of which was passed through an *activation function* to normalise the result to zero or one (representing an output classification of ‘class zero’ or ‘class one’).

2.1.2 Deep Learning

Whilst Rosenblatt’s simple model produced underwhelming results, its design became a fundamental building block of the higher complexity ANNs used today. Modern networks consist of wider and deeper *network architectures* containing more internal neurons arranged into layers. Recent work has explored *deep learning*, which pushes the boundaries of neural network performance by designing deeper and deeper architectures, known as *deep neural networks* (DNNs). These networks process data by passing information between the input layer $l^{(0)}$ and output layer $l^{(L-1)}$ along weighted channels between each neuron; the *activation value* $a_n^{(i)}$ assigned to each neuron n in layer i is computed as the dot product between the weights $W^{(i-1,i)}$ of channels entering n and the values $x^{(i-1)}$ of neurons in the preceding layer $i - 1$ connected to n (Witten et al., 2017). This product is then summed with a *bias* term $b^{(i)}$ and passed through the activation function α (Equation 2.1).

$$a_n^{(i)} = \alpha\left(W^{(i-1,i)} \cdot x^{(i-1)} + b^{(i)}\right) \quad (2.1)$$

This computation is performed over all layers of the network, starting with the input layer—whose neurons take the values of the input vector—and propagating through to the output layer—the values of which are the final network output. During training, the values of the network’s parameters (i.e. the elements of weight matrix W and bias vector b) are adjusted as the model learns, manipulating its outputs to more closely align with the true labels.

The full training process is typically implemented over many iterations over the entire training dataset. The network’s parameter set is updated based upon data sampled from D using *backpropagation*, where the gradient of the loss function with respect to the parameters is determined by propagating the error of the network on the current input backwards through all its layers (Zaras et al., 2022). These parameters are then adjusted in the direction of the negative gradient through *gradient descent* towards the minimum of the loss function (representing optimal performance).

2.1.3 Sequence Learning Problems

In ML, a *sequence* is an ordered set of data elements, each of uniform type and dimension: for example, a stream of text can be modelled as a sequence of strings. Mathematically, temporal sequences (known as *time series*) can be denoted by their data elements x and time indices t :

$$seq = x_0, x_1, \dots, x_t, \dots, x_T. \quad (2.2)$$

Sequence learning uses ANNs to model, analyse, and generate these sequences; the problem is common in fields such as NLP. One task typically explored within sequence learning is the *sequence-to-sequence* problem, where a model both takes input of, and outputs, a data sequence. This problem involves either *translation*—where the $M + 1$ elements in the input sequence are converted into a new sequence of length N —or *prediction*, where an input sequence x_0, \dots, x_M is used to predict the subsequent sequence x_{M+1}, \dots, x_{M+N} . One such prediction task is *time series forecasting*, where the values of a time series at future timesteps are predicted: for example, forecasting the progression of stock prices (Darapaneni et al., 2022). In this case, the output sequence $y = x_{t+1}, \dots, x_{t+n}$ is

computed based upon the preceding time series x_0, \dots, x_t , building a model M_θ (parameterised by θ) that approximates the mapping $y = f(x_0, \dots, x_t | \theta)$ between past and future time series.

2.2 Recurrent Neural Networks

2.2.1 Neural Networks for Sequence Learning

Forecasting sequences poses a challenge for traditional ANNs, whose architectures have a fixed input structure that requires all elements of an input sequence to be fed into the network at the same time, with one element per neuron in the input layer. Hence, an input sequence would contain no intrinsic sense of order, severely limiting the model as the order of a sequence is crucial to understanding and forecasting it (Tsantekidis et al., 2022). Additionally, modelling a sequence requires the persistence of information independent of its location within that sequence. Namely, the network must use previously observed information to contextually analyse the current input element, wherever they appeared in the preceding sequence. This is not possible with traditional ANNs as the parameters of channels connected to different input neurons are not shared. Therefore, when the parameters of a specific neuron and its channels are optimised to learn an important subsequence pattern, this information will only prove useful if that pattern appears again in the exact same location over the input neurons, as if it occurs at a different position it will be input through different neurons that have not previously learned to correctly handle this subsequence.

2.2.2 Recurrent Networks

To overcome these issues, *recurrent neural networks* (RNNs) were proposed that use recurrent loops to allow varying input lengths and the persistence of learned sequential information (Sharma et al., 2022). These networks take input of a single sequence element x_t at each timestep t , using this and previously observed elements to inform the internal state h_t , which acts as internal memory. This state is updated sequentially to incorporate information learned from the current input x_t through the function g , parameterised by $\theta^{(g)} = \{W^{(g)}, U^{(g)}, b^{(g)}\}$, where $W^{(g)}$ and $U^{(g)}$ are weight matrices, and $b^{(g)}$ a bias vector (Sharma et al., 2022):

$$h_t = g(h_{t-1}, x_t | \theta^{(g)}) \quad (2.3)$$

$$= W^{(g)} \cdot h_{t-1} + U^{(g)} \cdot x_t + b^{(g)}. \quad (2.4)$$

The state h_t is output along a recurrent loop that connects the output of the network at time $t - 1$ to the input of the network at the next timestep t . This allows information to be passed between successive versions of the same ANN at different points in time. At the final timestep T , the state h_T is output as the final network output. The concept can be demonstrated by unrolling the network in time, shown in Figure 2.1 depicting the same network at different timesteps, with the connecting arrows between each signifying that the output at time $t - 1$ is being passed on to the same network at the subsequent timestep t .

To capture more information from the input sequences, RNNs can consist of multiple internal layers (Figure 2.2), allowing them to learn higher-dimensional internal representations (Bengio, 2009). In this case, each layer i retains its own internal state $h_t^{(i)}$, which is passed on to both the

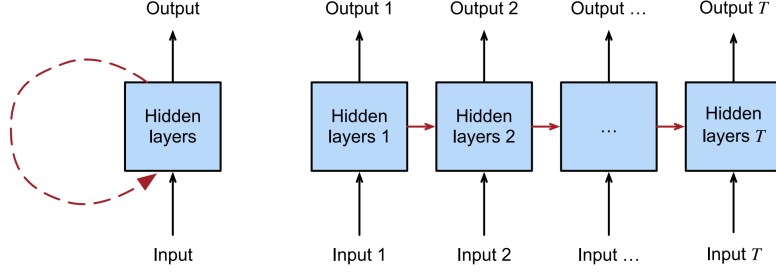


Figure 2.1: Diagram of an RNN at a single timestep t (left) and unrolled in time between timesteps $t = 1$ and $t = T$ (right) (Zhang et al., 2021).

next layer—to compute its state $h_t^{(i+1)}$ at the same input timestep—and recurred back into itself at the subsequent timestep to compute the state $h_{t+1}^{(i)}$ (Zhang et al., 2021).

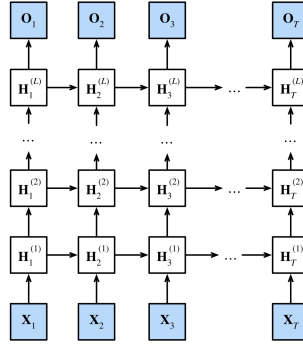


Figure 2.2: Diagram of a multi-layer RNN (showing layers 1 to L) with each layer unrolled in time between timesteps 1 and T (Zhang et al., 2021).

RNNs are trained through a similar backpropagation process to that used by traditional DNNs. This is done by computing the final output of the network (given an input sequence) and evaluating the loss function between this and the true label to compute the network’s error, which is then propagated backwards through both the network’s layers and through time to the network’s state at each previous timestep (Zhang et al., 2021). This allows the training algorithm to adjust both the network parameters (weights W and biases b) and the parameters $\theta^{(g)}$ of the state function g .

2.2.3 Applications

Recurrent networks have been employed in most recent research into sequence learning. Lipton et al. (2015) explain how NLP tasks and time-series forecasting are the most common avenues explored by researchers utilising RNNs. These two areas have been extensively surveyed in literature; for example, Hewamalage et al. (2021) survey current and future applications of RNNs for time series forecasting, highlighting the recent success of this model architecture at forecasting competitions, such as an RNN-based model by Smyl (2020) winning the *M4 Competition* (in 2019) with an accuracy nearly 10% greater than the utilised baseline (Makridakis et al., 2020).

2.3 Long Short-Term Memory

RNNs have been shown to accurately model short-term dependencies within an input sequence. In many cases, however, to understand the full context of a sequence long-term dependencies need

to be taken into account. Unfortunately, Hochreiter (1991) and Bengio et al. (1994) have shown that RNNs cannot accurately capture or exploit these long-term dependencies, as contextually relevant elements observed many timesteps ago can be forgotten prematurely. This issue is known as the *short-term memory problem* of RNNs. Whilst several solutions to this problem have been proposed—such as *skip connections* and *leaky recurrent units*—the most widespread approach is the use of *long short-term memory* (LSTM), presented by Hochreiter and Schmidhuber (1997).

2.3.1 Long Short-Term Memory Networks

An LSTM is an RNN that uses multiple internal states and parameter matrices to model both short-term and long-term dependencies within sequences. Similarly to an RNN, this architecture contains a core network of one or more layers (each of which is an *LSTM cell*) and information is passed both between these cells (from the input to the output layer) and between timesteps (recurring the output at time $t - 1$ to be input at time t). However, instead of a single recurrent state h_t , the LSTM contains two: a short-term memory state h_t and long-term memory state c_t used to retain information from far back in the input sequence.

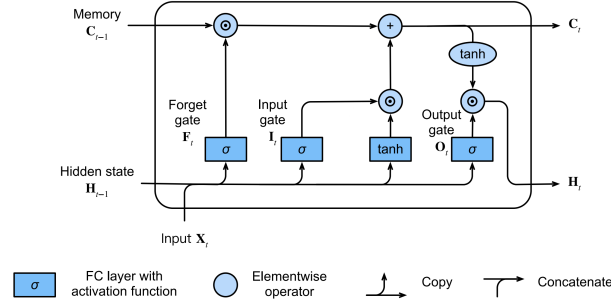


Figure 2.3: Diagram of the logical gates within an LSTM cell (Zhang et al., 2021).

At each timestep, the preceding long-term state c_{t-1} is input into the LSTM cell (alongside the current sequence element x_t) to compute the new states h_t and c_t . These updates are achieved through several logical *gates* within the network (Figure 2.3), each of which is implemented as a single fully-connected ANN layer. Firstly, the *forget gate* is used to decide what information to discard from the previous state c_{t-1} . This is computed by multiplying the previous short-term state h_{t-1} and input element x_t with the forget gate’s weight matrices $W^{(F)}$ and $U^{(F)}$ and adding the bias $b^{(F)}$, the result of which is passed through the *sigmoid* activation function to produce the intermediate state value $\tilde{c}_t^{(F)}$, shown in Equation 2.5 (Zhang et al., 2021).

$$\tilde{c}_t^{(F)} = \sigma\left(W^{(F)} \cdot h_{t-1} + U^{(F)} \cdot x_t + b^{(F)}\right) \quad (2.5)$$

Secondly, when new relevant information is seen in the input sequence, the *input gate* is used to add it to the long-term memory c_t . This process similarly uses the short-term state h_{t-1} and current input x_t , but evaluates both a sigmoid and *hyperbolic tangent* activation function in parallel on these inputs to decide both which parts of c_{t-1} must be updated and by how much. The result of these two operations is multiplied together to give the second intermediate state value $\tilde{c}_t^{(I)}$, shown in Equation 2.8 (Zhang et al., 2021). Both of these intermediate state values $\tilde{c}_t^{(F)}$ and $\tilde{c}_t^{(I)}$ are then combined to give the updated long-term state c_t (Equation 2.9).

$$\left(\tilde{c}_t^{(I)}\right)_\sigma = \sigma\left(W^{(I,0)} \cdot h_{t-1} + U^{(I,0)} \cdot x_t + b^{(I,0)}\right) \quad (2.6)$$

$$\left(\tilde{c}_t^{(I)}\right)_{\tanh} = \tanh\left(W^{(I,1)} \cdot h_{t-1} + U^{(I,1)} \cdot x_t + b^{(I,1)}\right) \quad (2.7)$$

$$\tilde{c}_t^{(I)} = \left(\tilde{c}_t^{(I)}\right)_\sigma \cdot \left(\tilde{c}_t^{(I)}\right)_{\tanh} \quad (2.8)$$

$$c_t = \tilde{c}_t^{(F)} \cdot c_{t-1} + \tilde{c}_t^{(I)} \quad (2.9)$$

The third and final gate is the *output gate*, which computes the short-term state h_t . The output gate uses the newly computed long-term state c_t , input x_t , and previous short-term state h_{t-1} . A biased weighted sum of h_{t-1} and x_t is computed and fed into the sigmoid function, which is then multiplied by the hyperbolic tangent function evaluated on c_t (Equation 2.10). Similarly to within the input gate, the sigmoid evaluation determines the values of h_{t-1} to be updated, and the tangent evaluation determines by how much (Zhang et al., 2021).

$$h_t = \sigma\left(W^{(O)} \cdot h_{t-1} + U^{(O)} \cdot x_t + b^{(O)}\right) \cdot \tanh(c_t) \quad (2.10)$$

These gates occur in every LSTM cell, passing state information between the network’s input and output layers at all timesteps. When the network reaches the final timestep, the short-term state is output as the final network output y ; note, if the sequence learning problem involves outputting a generated sequence of length N (e.g. forecasting multiple future values), this is produced one element at a time over N additional timesteps that take no input and output the value $y_i = h_{T+i}$.

2.3.2 Applications

Both Lipton et al. (2015) and Yu et al. (2019) assert that most state-of-the-art applications within the field of sequence learning use LSTM networks, largely due to their ability to model both long and short-term dependencies, facilitating more accurate predictions; hence, LSTMs have been widely used in both NLP and time series forecasting. For example, Shi et al. (2022) compared the performance of several network architectures for predicting Beijing air quality over time, concluding that LSTMs generated more accurate predictions than simpler RNNs, and demonstrated their improved long-term memory by showing LSTMs outperform other networks even when the context window available to the model is small. This architecture has also been shown to permit the accurate forecasting of financial markets, with research highlighting its ability to accurately capture the complex dependencies within highly variable financial variables (Li and Tam, 2017); this has facilitated models with impressive performance across financial applications, such as forecasting stock market indices like the S&P 500 (Fjellström, 2022), currency pairs on the Foreign Exchange (Qi et al., 2021), and the fluctuations of financial market risk (Du et al., 2019).

2.3.3 The Efficiency of Recurrent Networks

Unfortunately, these performance gains inflict a significant computational cost, as research into recurrent architectures has shown that the sequential processing methods vital to their performance also make these models highly inefficient. This tradeoff between accuracy and efficiency has inspired research addressing the energy consumption of these architectures. In their study of compression techniques for LSTMs, Wang et al. (2018) identify the efficiency of such architectures as a core

limiting factor restricting their use in resource-constrained domains. This is similarly identified by Zarzycki and Ławryńczuk (2021) in their modelling of chemical reactors; they found that whilst the number of parameters utilised by an LSTM was directly proportional to its modelling performance, higher complexity models inflicted a significant computational cost.

Numerous sources, such as Cao et al. (2017) and Feliz (2021), have deduced that this inefficiency is directly caused by the sequential nature of RNNs. They assert that the many dependencies within LSTM cells (between cells and to the input sequence) restrict how these models can be trained and used. Specifically, cells in an LSTM network take input from both the input sequence and preceding cells; each cell at every network layer and timestep receives both an input vector and two memory states, introducing both temporal dependencies (between cell states at each timestep) and layerwise dependencies (between the output and input of each network layer). Both Cao et al. (2017) and Feliz (2021) identify that this means LSTMs cannot effectively exploit parallelisation, as variables with sequential dependencies must be computed serially. Thus, these networks cannot fully take advantage of the computational speedups provided by specialised hardware such as *Graphics Processing Units* (GPUs) and *Tensor Processing Units* (TPUs) that have been effectively used to parallelise other DL algorithms, making training long and energy intensive.

2.4 Financial Volatility Modelling

Predicting stock price movements is a common example of sequence-to-sequence modelling that has received great attention within the field of DL for finance, with numerous reviews extensively exploring the area (such as Berat Sezer et al. (2019) and Jiang (2021)) and innovative approaches pushing the boundaries of prediction accuracy (Darapaneni et al., 2022). Despite this plethora of research, it is still a challenging task to accurately model price movements in real financial markets due to the complexity of financial systems, non-linear relationships between financial variables, and high-frequency variations in values (Timmermann and Granger, 2004).

2.4.1 Financial Risk & Volatility

The challenges associated with market price prediction mean that within the finance industry a more helpful use case of sequence-to-sequence modelling is forecasting financial risk. In their white paper explaining the applications and benefits of ML for financial modelling, LaPlante and Rubtsov (2019) exemplify this sentiment, asserting that ANNs provide the most utility for modelling potential risks. In fact, a recent survey of financial professionals by Chartis Research Staff (2019) found that 70% of respondents use ML tools in the financial risk sector, majoritively for analysing “market risk for the trading book” (51% of respondents) and “market risk for the banking book” (44%). Peng and Yan (2021) similarly assert that modelling risk is currently in high demand due to the large fluctuations, uncertainty, and high volatility exhibited in financial markets, making risk an important consideration when making financial decisions.

Hence, modelling financial market risk is a popular application of DL; commonly this involves forecasting the *volatility* of an asset or market (Peng and Yan, 2021). Cavalcante et al. (2016) define volatility as the scale of fluctuations in the pricing of an asset within a financial market over time and is intrinsically related to the risk associated with that asset. Cavalcante et al. assert that volatility is a vital measure to financial analysis, as it is a key indicator of the state of many economic and external factors that affect financial markets; for instance, in times of economic crisis

or political instability, volatility tends to rise, meaning large price variations of assets are likely, and the market is high risk (Berat Sezer et al., 2019). Because of this relationship between volatility and risk, Tino et al. (2001) highlight that volatility changes are typically used as buy and sell signals to investors; Ge et al. (2022) similarly assert market volatility dictates many decisions of market players. Hence, forecasting market volatility is a popular modelling domain.

2.4.2 Measures of Volatility

The review of Ge et al. (2022) and volatility model implementation of Tino et al. (2001) both provide comprehensive overviews of the most commonly used measures quantifying volatility: *historical volatility*, *realised volatility*, and *implied volatility*. Implied volatility (IV) is computed given a specific option price, computing the expected volatility associated with that price. Alternatively, realised volatility (RV) is a measure computed directly from the price movements of a market and hence is the volatility actually realised in the underlying financial market. Historical volatility (HV) is a type of realised volatility, which computes the volatility over a preceding time period based upon market closing prices.

Due to the differences in what these measures represent, each is computed in a slightly different way. Since IV represents an expectation of volatility, it cannot be computed directly from financial market data; instead, IV is estimated through an option pricing model such as the *Black-Scholes model* (Black and Scholes, 1973). On the contrary, RV is computed directly from market price movements over a time window $\tau \rightarrow t$ (Ge et al., 2022). This measure is typically calculated as the variation in logarithmic returns r_t of an asset (Equation 2.11) over the specified time period. Typically, this time window covers returns observed in the past, which is the HV of that period. When surveying various volatility measures, Ge et al. (2022) found HV to be the most popular, primarily because it provides an intuitive metric that is simple to compute. The most common method of computing HV is to approximate the volatility v_t over the N values of the interval $\tau \rightarrow t$ as the standard deviation of logarithmic returns; this is shown in Equation 2.12, where $\mu_{\tau,t}$ is the mean return over the time interval (Ge et al., 2022). This method of computing HV is used across a wide range of applications within volatility modelling, such as the ANN of Lahmiri (2017) which forecasts the volatility of exchange rates between currency pairs.

$$r_t = \log \left(\frac{c_t}{c_{t-1}} \right) \quad (2.11)$$

$$v_t = \sqrt{\frac{1}{N} \sum_{t'=\tau}^t (r_{t'} - \mu_{\tau,t})^2} \quad (2.12)$$

In their review of 35 volatility forecasting ANNs, Ge et al. (2022) found HV was the most popular metric, constituting 71% of research papers; this was followed by other RV measures (17%), and finally IV (9%). Additionally, Ge et al. (2022) explored the most common domains for forecasting volatility, finding the S&P 500 market to be the most popular (accounting for 12 out of the 35 research papers), primarily due to its accessibility. However, a vast spectrum of models have been developed outside of these domains; for instance, Ge et al. (2022) also explored the use of IV, RV, and HV within the context of stocks, bonds, and other financial indices. Hence, the selection of a volatility measure is typically a domain-specific choice, based upon the different benefits and drawbacks associated with each metric. For instance, due to its reliance upon an options pricing model

IV requires options data, and hence cannot be used in other domains. There are also challenges associated with RV and HV; whilst RV has been shown to tend towards an estimate indistinguishable from the latent volatility (Andersen et al., 2001), this accuracy requires data with a high sampling frequency. Additionally, several issues unanimously encompass all volatility estimates, including price irregularities at the tail-ends of return distributions (Ozbayoglu et al., 2020) and the complex dependencies between financial variables (Timmermann and Granger, 2004).

2.4.3 Forecasting Financial Volatility

Due to the utility of modelling risk, forecasting market volatility has become a popular domain within both Fintech research (Ozbayoglu et al., 2020) and the finance industry (Chartis Research Staff, 2019). Traditionally, financial institutions and researchers have exploited *generalised autoregressive conditional heteroscedasticity* (GARCH) models to forecast volatility; these models have been shown to accurately and reliably capture the characteristics of financial time series (Lahmiri, 2017). GARCH models calculate the volatility of a market from a sequence of logarithmic returns r_0, \dots, r_t , approximating the volatility v_t at time t as the conditional variance σ_t^2 of returns over the preceding time period. Early work in this field by Akgiray (1989) showed that GARCH models fit training data and forecast volatility more accurately than other models in its class. Similar studies, such as that of Hansen and Lunde (2005), have also demonstrated the impressive forecasting accuracy of GARCH models within the foreign exchange market and shown GARCH’s ability to effectively determine clusters of high and low volatility. Largely because of this accuracy, GARCH has become one of the most frequently utilised methods of estimating the volatility of financial time series. Even despite modern methods, this model is still a popular choice, often being used as a baseline to judge the performance of newly proposed volatility forecasting models; for example, Rodikov and Antulov-Fantulin (2022) used GARCH to contextualise the results of their DNN.

2.4.4 Deep Learning for Volatility Forecasting

Whilst traditional forecasting methods have been shown to produce accurate estimations of financial market risk, a significant amount of recent work has applied DL to this domain; for example, Chartis Research Staff (2019) found that 70% of financial institutions use ML for “market risk” forecasting. Through analysing the publication rate of research papers in 2018, Berat Sezer et al. (2019) found that volatility forecasting was in the top five uses of DL within financial research. Additionally, in their review of volatility models, Ge et al. (2022) asserted that almost all recent work on forecasting financial volatility relied on ML, largely due to it facilitating higher accuracy modelling of complex time series. DNNs are used to forecast volatility through sequence-to-sequence modelling; a time series of logarithmic returns r_0, \dots, r_t and of previously calculated volatilities v_0, \dots, v_t are used to predict the following volatility sequence v_{t+1}, \dots, v_{t+n} starting at timestep $t + 1$ and continuing to forecast a specified number of steps into the future. Namely, the volatility model M_θ is developed that computes the output sequence $y = v_{t+1}, \dots, v_{t+n}$ by approximating the mapping $y = f(r_0, \dots, r_t, v_0, \dots, v_t | \theta)$ between past and future time series.

The performance improvement provided by DL has been exemplified through several studies; for example, Zhang et al. (2022) demonstrated the superiority of DNNs at handling complex interactions and dependencies between financial variables, as their high-dimensional parameter sets allow them to act as better function approximators. Through an evaluation of volatility forecasting models

over 23 different stocks, Rahimikia and Poon (2020) found that DL gives stronger forecasting power than autoregressive methods like GARCH. Rodikov and Antulov-Fantulin (2022) drew a similar conclusion, finding that LSTMs could generate a higher test accuracy than other popular models. They also found that RNN architectures were effective at estimating financial market variables like volatility as they do not need to know the parameters of the underlying distribution of the variable being forecast. This is different from models such as GARCH, which uses *maximum likelihood estimation* to approximate the parameters of the return and variance functions of the market.

Whilst some researchers opt for alternative architectures (such as the CNN-based model of Chen et al. (2018)), the improved accuracy offered by RNNs and LSTMs has established them as a core focus of DL models for volatility forecasting. In their survey of 35 research papers, Ge et al. (2022) found that 9 out of the 21 pure models utilised an RNN, as these networks are a natural fit for time series data. This research typically explores the use of RNNs for forecasting realised volatility within the S&P 500 market, as Bucci (2020) showed that RNNs outperform all other traditional methods in this domain. Their experimentation demonstrated the ability of an LSTM to capture long-term dependencies within the financial time series, which enabled this architecture to produce accurate forecasts during highly volatile periods.

Thus, the use of DNNs and LSTMs is an expanding field within financial volatility forecasting, primarily due to the increased prediction accuracy facilitated by these models and algorithms.

2.5 Green AI

Whilst complex DNNs are seeing increased adoption across financial modelling, many of these implementations ignore the efficiency concerns associated with DL, such as the memory and energy inefficiency of recurrent networks. The ignorant use of these resource-hungry Red AI models inflicts large ESG costs, as their extreme computational load generates thousands of pounds of carbon emissions over training (Strubell et al., 2019), and limits who can research and employ within high-performance ML systems (Bender et al., 2021). These impacts have spurred recent attention around Green AI, where energy and data-efficient algorithms are developed to reduce resource requirements, mitigate environmental costs, and promote inclusivity (Schwartz et al., 2019).

2.5.1 Quantifying the Energy Efficiency of Deep Learning

As a simple initial approach to mitigating the ESG cost of DL, many proponents of Green AI suggest that new research papers presenting cutting-edge DNNs should report their training time and resource requirements. In their analysis of 60 DL research papers, Schwartz et al. (2019) found that 90% of papers in the ACL Anthology, 80% of those in the NeurIPS conference, and 75% in the CVPR conference cited accuracy improvements as the main contribution of their work, with only 10% of ACL papers and 20% of CVPR papers contributing a new efficiency result. Schwartz et al. (2019) argue that this demonstrates the lack of reporting surrounding the energy and data efficiency of DL algorithms, highlighting that describing performance contextually with respect to training budgets improves both the sustainability and inclusivity of this work by allowing future work to be compared despite fewer training resources.

However, a ubiquitous hardware-independent measure of the computational cost of DL algorithms has yet to be agreed upon. Schwartz et al. (2019) describe how the expense $Cost(R)$ of processing the result R using a DNN is proportional to the cost of processing a single instance I ,

the size of the training dataset D , and the number of hyperparameters H to be optimised (Equation 2.13). They survey several cost metrics, including runtime, parameter count, electricity usage, and carbon emissions; however, they highlight that each of these poses its own challenges, such as the hardware-dependent nature of electricity usage and difficulty in directly measuring carbon emissions. Schwartz et al. (2019) conclude that reporting the total number of *floating-point operations* (FLOPs) to generate the result is the most reliable way to quantify the computational cost, and hence energy efficiency, of an algorithm.

$$Cost(R) = I \cdot D \cdot H \quad (2.13)$$

Schwartz et al. (2019) use the competing models *ResNet* (he-2015) and *ResNeXt* (xie-2017) to exemplify how reporting FLOPs can contextualise performance advancements with model efficiency: whilst ResNeXt exhibited a 0.5% accuracy improvement over its predecessor (over the *ImageNet* dataset), it required 35% more FLOPs. Amodei and Hernandez (2018) echo these benefits; however, they highlight that computing the FLOP count of a program is not always trivial. Instead, they assert that the computational cost $Cost(T)$ of a training algorithm T in *petaFLOP/s-days* (pfs-days) is given by the product of the number of GPUs used (N_{gpu}), the processing power per GPU in FLOP/s (P_{GPU}), the training time (T_{days}), and the estimated average utilisation of these GPUs (U), typically taken as 33% (Equation 2.14). To exemplify this approximation, Amodei and Hernandez calculated that whilst 2012’s leading image classifier AlexNet (Krizhevsky et al., 2012) had an approximate computational cost of 0.0058pfs-days, five years later the innovative DNN *Xception* (Chollet, 2016) had a cost of 5.0pfs-days (an increase by a factor of 862).

$$Cost(T) = N_{gpu} \cdot P_{gpu} \cdot T_{days} \cdot U \quad (2.14)$$

Alternatively, further work has attempted to directly quantify the carbon emissions generated by DL algorithms. For instance, Lacoste et al. (2019) proposed the *Machine Learning Emissions Calculator* that computes the CO_2 -equivalents of an ML algorithm, measuring the environmental detriment caused by energy-intensive Red AI, allowing a direct insight into its environmental cost. However, this approach has yet to receive significant attention.

This work demonstrates that quantifying the computational, energy, and carbon cost of DNNs is an expanding field within DL research, solidifying quantification methods as a vital first step to raising awareness about the ESG impacts of DL algorithms.

2.5.2 Efficient Neural Network Architectures

Once the ESG impact of DNNs has been quantified, new Green AI systems can be developed that minimise their computational cost. As Schwartz et al. (2019) identify, the cost associated with generating a result from a DNN is proportional to that of processing a single instance I . Hence, much research into Green AI has focussed on how the architecture of DNNs can be redesigned to minimise computational complexity and the energy expended to process data instances.

Quantisation

The record-setting performance of complex DNNs establishes these models are alluring tools for industry players looking to upgrade their workflows; however, in many cases, the upper echelons of performance are neither necessary nor beneficial. For example, Kumar et al. (2020) highlight that

in resource-constrained applications (e.g. mobile computing), state-of-the-art DNNs are not feasible as they would quickly engulf the limited power and memory resources. Therefore, researchers have begun to explore how accuracy-efficiency tradeoffs can be made that produce significant decreases in the energetic cost of DNNs whilst minimising the resultant accuracy drop.

Much of this research—explored by Xu et al. (2021) and Cai et al. (2022)—has focussed on decreasing computational costs by using lower precision representations of variables and parameters. Typically this is implemented through *quantisation*, where the value of each variable is mapped to discrete quantisation levels that require fewer bits to store. In practice, *deterministic quantisation* is commonly used, with approaches ranging from *uniform quantisation*, where floating-point numbers are mapped to their closest low-bit fixed-point representations, to *clustering quantisation*, where parameters are clustered by value and replaced by the mean of their cluster (Xu et al., 2021). A spectrum of accuracy-efficiency compromises can be implemented through quantisation: for example, *BinaryConnect* (Courbariaux et al., 2015) implements an extreme quantisation approach, using only single-bit representations, but inflicts an accuracy drop of 19%, alternatively, *Stripes* (Judd et al., 2016) reduce precision from 32 to 8-bits whilst inflicting an accuracy drop of only 1%.

Quantisation-Aware Training

These methods implement post-training quantisation, which improves the efficiency of models after they have been developed. Further research has explored quantisation-aware training, which aims to reduce the performance drop inflicted by quantisation by utilising it throughout training. For example, Fan et al. (2020) quantised parameters during training to produce a model that achieved an ImageNet top-1 accuracy score of 80.0% (equivalent to 2017’s highest accuracy model ResNeXt) using only 3.3MB of memory (only 3% of the memory required by ResNeXt). Furthermore, Cai et al. (2022) assert that this training process can be adapted even further to conduct *low-bit training*, where parameters, activation values, and error gradients are all quantised. *DoReFa-Net* (Zhou et al., 2016) implemented such low-bit representations, using 1-bit parameters, 2-bit activations, and 6-bit gradients to boost training speed whilst generating an accuracy comparable to AlexNet using 32-bit representations. This demonstrates that quantisation and low-bit representations can be effectively used to produce models with significantly lower memory constraints and energy consumption. Research has also been conducted into quantisation for RNNs and LSTMs: Hubara et al. (2016) explored the effectiveness of quantising weights and activations within recurrent networks, whilst He et al. (2016) proposed quantising LSTM gates. However, whilst investigating quantisation for RNNs, Ott et al. (2016) found that low-bit training was not ubiquitously effective in this context; thus, they concluded *mixed-precision training* should be used for RNNs, where weight matrices are quantised, but activation values retain higher precision representations.

2.5.3 Efficient Training Algorithms

Whilst the efficiency of a DNN’s architecture is essential to minimising the energy expended on each data instance, it is also important to minimise the length of DNN training. Hence, a significant amount of research within Green AI focuses on energy-efficient training algorithms that attempt to reduce the number of iterations necessary to train an accurate model.

Transfer Learning

Possibly the most common method of reducing the length of DNN training is using a pre-trained model developed for an analogous task. This method—discussed by Strubell et al. (2019), Walsh et al. (2021), and Schwartz et al. (2019)—is known as *transfer learning*, which uses an existing pre-trained *base model*, and tunes its parameters over a new training dataset that covers a new domain. Since the base model has already learned a general ability to complete a similar task, the new target model requires only a short, inexpensive retraining process, where the model is fine-tuned to accurately capture information from the new domain. Both Strubell et al. (2019) and Walsh et al. (2021) identify that transfer learning significantly reduces the resources required to train a DNN and allows their application to fields with limited amounts of data. For example, Wang et al. (2020) used transfer learning to apply ResNet to the *Stanford Dogs 120* dataset, a small collection of only 20,580 images (only 1.7% of the size of the ImageNet dataset used to train ResNet). The use of transfer learning allowed Wang et al. to develop a model that had an accuracy 11.07% greater than any other DNN developed for this domain and reduced the required training iterations by a factor of 10. This demonstrates that transfer learning is a simple but effective method for reducing the training cost of DNNs, which allows models to be developed for niche domains and minimises the energy required to train accurate ML models.

Initialisation

Instead of directly copying a base model, its parameter values can be used as a starting point to optimise the parameter set of a new DNN. This is known as *initialisation*, which is the process through which the initial values of a parameter set are chosen. Initialisation is incredibly important, as Xu et al. (2021) found that the rate of convergence of a network’s parameters towards their optimal values heavily depends upon their initial values. Furthermore, Hanin and Rolnick (2018) found that primitive initialisation methods (such as random initialisation) can lead to slow training, or even result in parameter values never converging.

Both Hanin and Rolnick (2018) and Xu et al. (2021) evaluate alternative methods for initialising a DNN. *Feature-based initialisation* assigns borrowed parameter values to a subset of the new DNN’s parameters and keeps these fixed during training (improving generalisability), whereas *fine-tuning-based initialisation* trains all new and borrowed parameters, providing greater specialisation. Alternatively, *supervised initialisation* pre-trains the new DNN over similar datasets or for analogous tasks, then reuses these representations as a starting point for training over the target task and dataset. For example, Lin et al. (2020) initially pre-trained their language model as a general multilingual translator, before specifically applying the network to translate between language pairs. *Self-supervised initialisation* takes this concept further, allowing pre-training to be conducted unsupervised with an unlabelled dataset (Peters et al., 2018).

Hence, this established research demonstrates the importance of selecting a suitable initialisation method when developing a new DNN based upon the chosen application and architecture, as the starting point of training can drastically reduce the energy expended to generate an accurate model.

Progressive Training

Progressive training (Xu et al., 2021), also known as *greedy layer-wise pre-training* (Xu et al., 2018), builds upon this concept of pre-training. This training approach constructs a model layer-by-layer

by iteratively adding a new layer and tuning its parameters. This builds the DNN in a bottom-up approach, whereby the network first trains lower layers to accurately represent low-level features of the input, then trains higher layers based upon the learned parameters of the preceding layers. After adding the desired number of layers, all parameters are optimised in a concluding tuning phase. In theory, this reduces the computational cost of training a DNN, as training shallow, single-layer networks is significantly simpler and less resource intensive than training full, deep networks (Xu et al., 2021). Furthermore, Xu et al. (2021) assert that the parameters of later layers are optimised faster, as values are updated based on information already learnt about the features of the input during preceding pre-training rounds. This approach has been successfully used by Yang et al. (2020) who reduced the total training time required by the language model BERT by over 110% (from 85 to 40 hours) without degrading accuracy.

Progressive training can be implemented through either *supervised layer-wise pre-training* or *unsupervised layer-wise pre-training*. Supervised approaches, such as that of Ienco et al. (2019), train individual layers through supervised learning; for example, if building a forecasting model, each layer is independently trained to predict the future elements of a sequence (Xu et al., 2018). Starting by training a shallow network consisting of only an input layer, one hidden layer, and an output layer, the model iteratively gets deeper; this is achieved by first removing (and saving) the output layer, then fixing the values of all previously trained parameters, adding a new hidden layer, appending the saved output layer, and finally training the parameters of this new layer. Once a network of the desired depth has been constructed, the parameters of all layers are unfixed, and a short training round is conducted over the full network, where the parameters found during pre-training are used as a starting point for deducing the optimal global parameter set.

Unsupervised approaches—such as the implementations of Xu et al. (2018) and Sagheer and Kotb (2019)—train each layer as an *auto-encoder*. Autoencoders are a type of ANN that learns to output an encoded representation of its input; they are trained through unsupervised learning, minimising the difference between the network’s input and output vectors (Lopez Pinaya et al., 2020). In unsupervised layer-wise pre-training, network layers are also built up iteratively as unsupervised autoencoders, learning to output a representation of the input vector. This unsupervised learning is repeated over the entirety of pre-training, producing an autoencoder network of the desired depth (Sagheer and Kotb, 2019). Subsequently, the full network is tuned, repurposing the model to the desired task by discarding the autoencoder output layer used during pre-training, and appending a new output layer for the desired task. With all parameters unfixed, the network then undergoes training, starting from the parameter set found during pre-training, and optimising the entire network’s performance in the chosen domain (Sagheer and Kotb, 2019).

Research into layer-wise pre-training typically centres around improving the efficiency of training recurrent networks, as RNNs have been identified as one of the most sensitive architectures to parameter initialisation. Ienco et al. (2019) showcase how supervised layer-wise pre-training can be used to produce an RNN-based sequence classifier that exceeded the accuracy of comparable models in a variety of applications. Both Xu et al. (2018) and Sagheer and Kotb (2019) demonstrate that unsupervised layer-wise pre-training is effective at improving training efficiency while preserving accuracy. By comparing LSTM models initialised with layer-wise pre-training to a simple randomised initialisation, Xu et al. (2018) exemplified the performance and efficiency benefits of this technique across several domains, including image recognition and sequence-to-sequence modelling. Xu et al. (2018) found that unsupervised layer-wise pre-training of LSTMs induced faster convergence toward

the optimal parameter set, as their initialised values were located close to local minima of the loss function. They also found that the resultant model exhibited a smaller test error, suggesting this training method improves the generalisability of LSTMs. Similarly, Sagheer and Kotb (2019) compared unsupervised pre-training and random parameter initialisation, focussing on deep LSTMs for time series forecasting. They discovered that the layer-wise method produced better and faster convergence, especially when forecasting collections of correlated variables; this suggests unsupervised layer-wise pre-training is a good fit for forecasting complex financial time series.

Hence, progressive training has been shown to decrease the time and energy required to train DNNs and RNNs, and develop networks that capture complex dependencies between variables, indicating that this algorithm will be effective for modelling complex financial time series.

2.5.4 Data Efficiency

A common trend in state-of-the-art DL algorithms is a reliance upon vast datasets; Bender et al. (2021) summarise this work as following a philosophy of “there’s no data like more data”. For example, the image classification model of Mahajan et al. (2018) produced record-breaking accuracy but achieved this by training over a dataset of 3.5 billion images, three orders of magnitude larger than the commonly used *Open Images Dataset*. Schwartz et al. (2019) highlighted that these expansive datasets are a significant contributor to the computational and energetic cost of Red AI; thus, Green AI often attempts to minimise these datasets and their associated costs.

Both Bender et al. (2021) and Walsh et al. (2021) highlight that much of this computational cost is due to the inefficient use of data, as a large proportion of the instances within these vast training datasets are not beneficial to learning. Bender et al. suggest that more time should be put into carefully curating specialised datasets for each model and domain, minimising the time and energy wasted on unhelpful data instances. Al-Jarrah et al. (2015) further note that current ML algorithms are not intelligent enough to efficiently deal with significantly increased data loads, as commonly used optimisation methods (e.g. gradient descent) require substantial work to locate global optima; thus, when applied to large datasets, these algorithms accumulate an extreme computational cost.

Furthermore, there can often be a conflict between the architecture size and training dataset size, as small ANNs typically require more data to achieve comparable accuracy. Bender et al. (2021) found that the model *ALBERT* (Lan et al., 2019), which attempts to recreate the performance of the complex network BERT using a smaller parameter set, required large amounts of data to force advances in accuracy beyond that typically achieved by compact networks. This demonstrates that when developing efficient DL algorithms, a balance must be drawn between model and dataset sizes.

Reducing Data Requirements

Several avenues have been proposed within Green AI for reducing data requirements whilst preserving accurate performance. Xu et al. (2021) discuss a variety of dataset reduction techniques; these methods typically rely on pre-trained models analogous to those described in Section 2.5.3. Namely, pre-trained models can exploit self-supervised learning to initialise near-optimal parameters without needing labelled data. This reduces data requirements as energy does not need to be expended labelling the data required for pre-training, and the initialised model converges faster downstream during the full training stage (Xu et al., 2021). *Contrastive learning* (Chen et al., 2020) is raised by Xu et al. (2021) as a commonly used pre-training method for reducing data requirements in CV. This approach focuses on learning pairwise relationships between data instances, representing

similar pairs close together in the data space and pushing divergent pairs far apart, thus allowing the dataset to be constricted. Xu et al. (2021) further highlight the effectiveness of *prompt learning* (Liu et al., 2021), where data instances are labelled with a task-specific template known as a *prompt*, shrinking the dataset as a single prompt can represent up to 100 individual data instances.

Active Learning

Possibly the most widely explored method for reducing the data requirements of training DNNs is *active learning*. This method, whose utility to DL has been extensively explored by surveys such as that of Ren et al. (2020), aims to reduce training costs by selecting a subset of data instances from the full data space that are believed to provide the most utility to the model’s learning process.

Active learning adapts the traditional algorithm used to train DNNs over a training dataset D (of N instances) by splitting D into smaller chunks, utilising it as either a *stream* or a *pool*. *Stream-based active learning* individually picks data instances from D and evaluates whether to train over this single instance, feeding useful instances into the DNN (Ren et al., 2020). More commonly, *pool-based active learning* is implemented; at each round of training, this algorithm selects a pool of the n_{sample} most useful instances from the full dataset and trains the network over these (Ren et al., 2020). Specifically, pool-based approaches divide D into the pool set P and validation set V ; at the start of training, the pool set $P^{(0)}$ is empty, and the validation set $V^{(0)}$ contains all elements of the full dataset D . To begin training, an initial pool $P^{(0)}$ of n_{sample} data instances (known as the *seed*) must be sampled from the full dataset; most simplistically, these are drawn at random. These sampled values are then moved from the initial validation set $V^{(0)}$ to the new pool; hence we have the new datasets $P^{(1)}$ (where $|P^{(1)}| = n_{sample}$) and $V^{(1)}$ (where $|V^{(1)}| = N - n_{sample}$). After the seed has been selected, the first round of training can be conducted, where the DNN is trained over pool $P^{(1)}$. This partially trained DNN is then used to predict the labels of the remaining $N - n_{sample}$ data points in the validation set; these predictions are fed into an *importance function* scoring each instance in $V^{(1)}$ according to its utility to the learning process of the DNN. The n_{sample} instances with the highest importance score are then selected from the validation set and moved into the pool; thus, we have the new datasets $P^{(2)}$ and $V^{(2)}$ where $|P^{(2)}| = 2n_{sample}$ and $|V^{(2)}| = N - 2n_{sample}$. This process is repeated over multiple training iterations, evaluating the importance function and expanding the pool set, until either a dataset size, iteration count, or accuracy threshold has been reached (Ren et al., 2020).

Both Ren et al. (2020) and Xu et al. (2021) assert that active learning drastically increases the data efficiency of training DNNs. Ren et al. (2020) found that this algorithm can theoretically achieve an exponential improvement in the time required to label training data; they also found it could successfully deal with high-dimensional data and train efficient but accurate models for sequential data. Xu et al. (2021) similarly assert that active learning significantly reduces the time and energy wasted on redundant data instances that do not benefit the DNN’s learning. Furthermore, Ren et al. (2020) note that the resources required to evaluate the importance function do not undermine the efficiency gains provided by the iterative learning process, as this function can be efficiently implemented to inflict a negligible computational cost.

Hence, active learning has been shown to reduce the data requirements of DL training algorithms. Furthermore, whilst training over time series data introduces further complexity due to the temporal nature of variables, Peng et al. (2017) and Zimmer et al. (2018) have demonstrated the feasibility of active learning in this field, suggesting this approach may improve the efficiency of volatility models.

Chapter 3

Methodology

3.1 Introduction to Experiments

This study investigates how energy and data-efficient Green AI algorithms can be applied to finance. Specifically, this research will explore how DNNs for financial volatility forecasting over the S&P 500 market can be trained efficiently whilst preserving accuracy, to demonstrate how the resource requirements, energy expenditure, and environmental impacts of Fintech can be reduced. Hence, this experimentation highlights the importance of considering the ESG impacts of DL, and demonstrates that high-performance DNNs can be utilised in this domain without sacrificing the SDGs of sustainable finance. Additionally, this study intends to illustrate how efficient training algorithms can be applied outside of the existing research applications of NLP, CV, and mobile computing, expanding the scope and impact of Green AI.

To illustrate the utility of Green AI to finance, two core research hypotheses are tested. Firstly, this study explores the question: *can using Green AI algorithms reduce the training time required to develop an accurate volatility forecasting model?* Experimentation into the hypothesis that energy-efficient algorithms can produce accurate volatility models aims to fill the research gap between the goals of sustainable finance and the expanding use of DL in Fintech by minimising the computational cost of training DNNs. Secondly, this research examines the further question: *can Green AI algorithms reduce the data requirements of training an accurate volatility forecasting model?* This extended study explores the hypothesis that Green AI not only facilitates a direct reduction in training time and energy consumption, but additionally provides a way to reduce the data requirements of DL. By minimising data requirements, the cost of training DNNs can further be reduced, as less energy is expended labelling training data and learning over redundant instances that do not benefit performance (Schwartz et al., 2019). Furthermore, the use of smaller datasets improves the inclusivity of this field, as researchers and market players do not need to invest in expansive computer memory or cloud storage to train accurate volatility models (Strubell et al., 2019). Hence, the exploration of both hypotheses contributes to the field in several ways: expanding the applications of Green AI, reducing the environmental cost of Fintech, and improving the inclusivity of finance.

To investigate these hypotheses, this research is divided into three core studies. Initially, the dataset over which volatility forecasting is conducted is presented in Section 3.2, and the metrics used to analyse the accuracy and efficiency of the developed DL models and algorithms are outlined in Section 3.3. Following this, then model architecture implemented in this research is presented in Section 3.4; a traditional training algorithm is presented through which a baseline model is

developed. Subsequently, Chapter 4 explores the Green AI algorithms implemented that attempt to improve the energy efficiency of training the LSTM network. A comprehensive explanation of these methods and their utility is given, exploring their potential to reduce the computational cost of DNN training. Finally, Chapter 5 explores the minimisation of data requirements through Green AI methods; an adapted training algorithm is presented that aims to decrease the training dataset size, hence facilitating further computational cost, energy consumption, and carbon emission reductions.

3.2 Dataset

To evaluate the use of Green AI in finance, the specific domain of financial market volatility forecasting was chosen, as it is currently one of the most popular applications of DL within finance; in fact, volatility forecasting is within the top five most popular uses of DL within financial research (Berat Sezer et al., 2019). Furthermore, Chartis Research Staff (2019) demonstrated that this domain is a popular application of DL within the financial industry, finding that 70% of financial institutions use ML for risk analysis, most commonly for forecasting “market risk”. This extensive use is primarily due to the superior accuracy provided by DL: Chartis Research Staff (2019) found that 44% of financial firms are motivated to use DL by the promise of “greater accuracy”, and DNNs have been shown to outperform traditional volatility forecasting methods (Rodikov and Antulov-Fantulin, 2022). For this reason, it is clear that volatility forecasting is a characteristic example of the use of DL within finance; hence, showing the applicability and benefit of Green AI within this domain acts as an exemplar of how the energy consumption of DL for finance in general can be reduced, and the ESG impacts of Fintech mitigated.

The *Standard & Poor’s 500 Index* (S&P 500) is a stock market index capturing the price movements of 500 leading US companies. It is often regarded as one of the best indicators of the performance of the global stock market; thus, S&P 500 data has become a popular target for time series modelling (Thakkar and Chaudhari, 2021). Ge et al. (2022) found that DNNs were most commonly used for forecasting the volatility of the S&P 500 market (accounting for 12 of the 35 works surveyed), primarily due to the accessibility of data. Therefore, to ensure universal results characteristic of this domain, S&P 500 data was chosen as the specific modelling domain explored within this research.

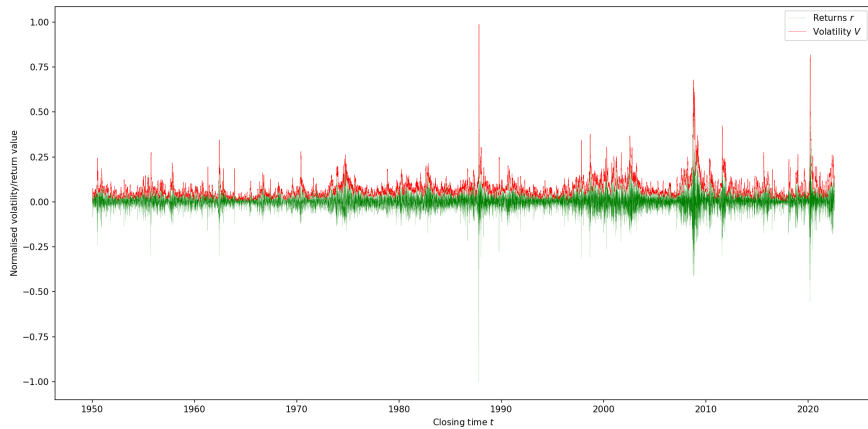


Figure 3.1: Time series of daily returns of prices over the S&P 500 market, overlaid with the computed historical volatility time series for the same period.

To enable sequence learning, the S&P 500 data is represented as a multivariate time series

consisting of 18261 daily sampled timesteps between the fifth of January 1950 and the first of August 2022. This period and sampling frequency were specifically chosen as daily data strikes an effective compromise between maintaining a reasonable dataset size and having a high enough data frequency to permit detailed analysis (Rodikov and Antulov-Fantulin, 2022). Each timestep maintains the value of seven variables of the S&P 500 market: the daily *open*, *close*, *high*, and *low* prices, trading *volume*, associated *return*, and market *volatility*. The return r_t is computed as the logarithmic change in closing price from the previous day (Equation 2.11), and used to calculate the HV as the standard deviation σ_t^2 of returns over the past 10 days; namely, σ_t^2 is computed from the time series r_τ, \dots, r_t over the time interval $\tau \rightarrow t$ (where τ is 9 timesteps behind the current timestep t). The HV is calculated through Equation 2.12, iteratively constructing the volatility time series v_0, \dots, v_t ; this echoes the approach of Lahmiri (2017) who also developed an ANN for volatility forecasting. The time series of returns and volatilities over the full period is showcased in Figure 3.1. Historical volatility was chosen as the metric to be forecast as it was identified by Ge et al. (2022) as the most commonly explored way of measuring market volatility. Furthermore, HV has been utilised in many innovative explorations of this field, such as Rahimikia and Poon (2020) and Rodikov and Antulov-Fantulin (2022) who both showcased the exceptional forecasting accuracy of DNNs for this metric.

The implemented DNNs use the volatility time series v_0, \dots, v_t to forecast the daily HV of the S&P 500 at a single future timestep $t + 1$ (outputting the prediction \hat{v}_{t+1}). To train and test the model, the full time series is broken down into individual sequences of 10 timesteps, where each data instance is a 10-step subsequence of the entire time series, enumerating the progression of the seven intrinsic variables. Hence, each data instance in the full dataset is in essence 7 small time series; one for the closing prices, returns, volatilities, and so on. Each of these variables was normalised through min-max scaling, restricting their values to the range 0–1 (except the returns, which were allowed to fluctuate between -1 and 1 to permit negative values); this scaling approach was similarly used by Rodikov and Antulov-Fantulin (2022), who highlighted that normalisation is essential to a stable and efficient training process.

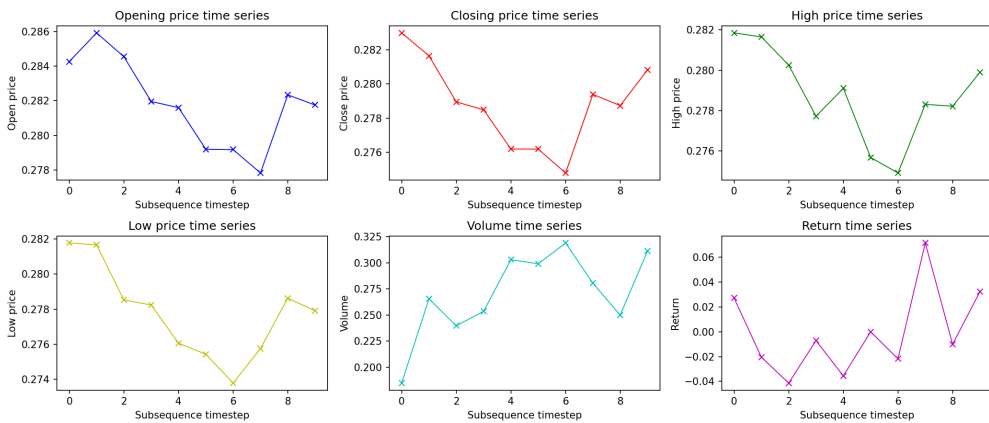


Figure 3.2: Subsequences comprising the multivariate time windowed time series for an example instance from the training dataset.

The modelling task was set up as a sequence-to-sequence problem, where each multivariate 10-step time series is input into the DNN to predict the subsequent value $\hat{y}_t = \hat{v}_{t+1}$. Thus, each pass through the model predicts the volatility of the S&P 500 market at timestep $t + 1$ through analysing the preceding multivariate time series over steps $(t - 9) \rightarrow t$. This is shown through Figure 3.2,

which (for a randomly selected data instance) gives an example of subsequences that make up each multivariate 10-step time series, and Figure 3.3 demonstrating a volatility subsequence with the true label v_{t+1} to be predicted by the model.

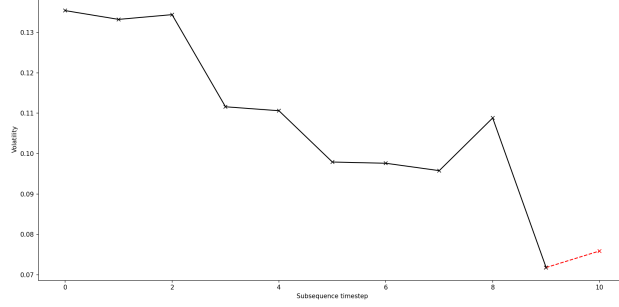


Figure 3.3: Volatility subsequence for an example data instance, including the true label.

This multivariate, time windowed approach was chosen as it has been effectively used by a host of popular studies on the use of DNNs for time series prediction, and in particular volatility forecasting. For example, Xiong et al. (2015b) used 25-dimensional multivariate time series (including daily open, close, high, and low prices, and returns) to demonstrate the prediction accuracy and robustness of their RNN for the S&P 500. Furthermore, Xiong et al. (2015b), Bucci (2020), and Rodikov and Antulov-Fantulin (2022) all utilised a dataset consisting of time windowed subsequences; Xiong et al. (2015b) specifically made predictions using a 10-step rolling time window, as they found this to be consistent with the majority of comparable models in this field. Hence, a dataset has been developed in keeping with popular modelling implementations within this field.

3.3 Analysis Approach

To demonstrate the performance and efficiency of each training algorithm, after each DNN had finished training it was tested over the testing dataset D_{test} , constructed by implementing a train-test split of the full dataset D , partitioning it into two distinct subsets D_{train} and D_{test} . In this research, an 80: 20 split is implemented, using 80% of data instances for training and the remaining 20% for testing (determined experimentally to produce the best results).

3.3.1 Accuracy Metrics

After training, the DNNs predictions over D_{test} are analysed through statistical metrics that quantify the modelling error ε and prediction accuracy. These accuracy metrics measure the divergence between the predicted volatility time series value $\hat{y}_t = \hat{v}_{t+1}$ and the true label $y_t = v_{t+1}$. These metrics are important as they demonstrate the success of all training algorithms, contextualising the forecasting performance generated by each approach.

One metric commonly used to quantify forecasting performance is the *coefficient of determination* R^2 (Zhang et al., 2022), which computes the ratio between the estimated variance of the prediction error ε and the variance of the variable y_t being predicted, shown in Equation 3.1 for the predictions \hat{y}_t (and sample mean of labels μ_y). The value of R^2 for a given set of predictions represents the proportion of the variability in the label set that is correctly captured by the model’s predictions; it ranges between $-\infty$ and 1, with a score of 1 meaning the true labels have been perfectly represented. This is an effective metric for demonstrating how well a model represents

data, and has been used to showcase the performance of several volatility models such as the ANN implementation of Zhang et al. (2022). However, it also has several limitations. Most notably, this metric doesn't explicitly quantify whether a model is good or bad, it only shows whether the testing performance is noticeably better than that of a comparison *constant model* that outputs predictions simply as the sample mean of the observed inputs; hence, its results can sometimes be misleading.

$$R^2 = 1 - \frac{\sum_{t=0}^n (y_t - \hat{y}_t)^2}{\sum_{t=0}^n (y_t - \mu_y)^2} \quad (3.1)$$

Because of this limitation, several other statistical metrics are commonly used to quantify the prediction accuracy of DNNs. Two such measures are the *mean absolute error* (MAE) and *root mean squared error* (RMSE). Given the set of predictions \hat{y}_t and labels y_t , MAE measures the average magnitude of the prediction error ε , taken as the mean absolute difference between \hat{y}_t and y_t over the entire test set (Equation 3.2). RMSE also measures the magnitude of deviations, computing this as the standard deviation of prediction errors (Equation 3.3). Low MAE and RMSE (close to 0) both indicate that the model is accurate; however, the squared component of RMSE gives increasing weight to large errors and hence is useful for penalising highly anomalous predictions more than slight deviations, whereas MAE gives a uniform representation of ε . One limitation of both MAE and RMSE is that they are scale-dependent; when predicting small values, the magnitude of the calculated error will always be smaller than that generated predicting large values, irrespective of the model's accuracy. Hence, these metrics are typically utilised to compare various models over the same domain, such as by Rodikov and Antulov-Fantulin (2022) who used both MAE and RMSE to compare the performance of LSTMs and GARCH. The MAE and RMSE accuracy metrics will be used similarly in the research of this thesis, comparing the performance of the traditionally trained DNN to the DNNs using energy and data-efficient training algorithms.

$$MAE = \frac{1}{n} \sum_{t=0}^n |y_t - \hat{y}_t| \quad (3.2)$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{t=0}^n (y_t - \hat{y}_t)^2} \quad (3.3)$$

To further quantify the accuracy of the implemented models, and address the inherent issues of preceding measurements, a scale-independent measure is required. One such metric is the *mean absolute percentage error* (MAPE) of predictions, which computes the absolute difference between \hat{y}_t and y_t over the test set as a percentage average (Equation 3.4). This provides an objective and easily interpretable picture of the accuracy of a model and has been used to analyse numerous volatility forecasting DNNs, including by Xiong et al. (2015b) and Zhang et al. (2022). Hence, this research additionally computes the MAPE of each implementation to independently demonstrate the performance of each trained model.

$$MAPE = \frac{1}{n} \sum_{t=0}^n \left| \frac{y_t - \hat{y}_t}{y_t} \right| * 100\% \quad (3.4)$$

3.3.2 Efficiency Metrics

Beyond generating accurate performance, the efficiency of each training algorithm is of vital importance to this research. Several efficiency metrics have been presented by Schwartz et al. (2019), who assert that the computational cost of a DNN is proportional to the cost of passing a single data instance through the network, the size of the utilised datasets, and the number of hyperparameters being tuned (Equation 2.13). Since the architecture used in this research is fixed, and the hyperparameters predetermined, these variables are not covered by the efficiency calculations analysed; however, dataset size is inspected for data-efficient training algorithms.

Amodei and Hernandez (2018) also explore the efficiency of DNN training, asserting that the cost of a training algorithm is dependent on the utilised hardware and training time. As the implemented studies all use the same hardware, metrics that target hardware efficiency are redundant to this research. However, training time is an important aspect of computational cost, and hence to quantify energy efficiency the total time required to train a DNN with accurate performance is recorded. Training time is a simple but effective efficiency metric, as more time typically means a higher computational cost. Although it is important to note that training time is not a completely representative metric, as it is reliant upon the underlying hardware and software dependencies of the system (Schwartz et al., 2019). However, since this efficiency metric is used for comparison of the relative differences in efficiency between implementations (all of which use the same underlying hardware and software), the calculation of training time is both valid and beneficial in this context.

The efficiency of a training algorithm can additionally be determined by inspecting how fast and smoothly the minimum of the loss function is converged upon. As outlined in Section 2.1.2, a DNN is trained by adjusting the network’s parameter set in the direction of the negative gradient of the loss function, determined through backpropagation of the DNN’s prediction error. Hence, the purpose of DNN training is to find the optimal parameter set that corresponds to the minimum of that loss function. The efficiency of a DL training process can therefore be determined by the speed at which this minimum point is approached; this is known as the *convergence rate* of training, which calculates the size of the step taken towards the loss function’s minimum during each epoch. The convergence rate is an important factor when considering the efficiency of a DL training algorithm, as the faster a DNN’s parameters converge towards their optimal values, the more effective the early stages of training (as the DNN learns more over each epoch), and thus the less time, fewer calculations, and lower energy consumption is required to develop an accurate model. Hence, the convergence rate is also evaluated to demonstrate the efficiency of each training algorithm.

3.4 Baseline Model Implementation

Before analysing the efficiency of each training algorithm, a model architecture characteristic of those used in Fintech must be implemented. This should ensure that the findings of this research are universal, demonstrating on a wide scale how the ESG impacts of DL for finance can be mitigated.

3.4.1 Baseline Volatility Forecasting Model

To conduct volatility forecasting, an LSTM architecture was chosen, as RNNs are proven to produce superior forecasting accuracy over time series, accurately modelling both long and short-term dependencies in sequences, and capturing complex dependencies between highly variable features.

Furthermore, they have been successfully applied to forecasting financial market volatility, with Bucci (2020) showing that LSTMs outperform traditional volatility forecasting models.

The hyperparameters regulating the number of hidden layers and neurons per layer within the DNN were then determined, constructing a hyperparameter set representative of typical models in this field. Existing models in the literature were found to typically exploit between 1 (Bucci, 2020) and 5 (Kim and Won, 2018) layers; therefore, to strike a balance between these implementations the implemented model relies on 3 LSTM layers, each containing 24 neurons (Table 3.1). Since the output layer is used to return a single prediction \hat{v}_{t+1} , this layer contains a single neuron fully-connected to its preceding hidden layer, from which the network’s single output value is read.

Layer	Type	Output Shape	Parameter Count
Input layer	Vector	(32, 10, 7)	-
Hidden layer 1	LSTM	(32, 10, 24)	3072
Hidden layer 2	LSTM	(32, 10, 24)	4704
Hidden layer 2	LSTM	(32, 10, 24)	4704
Output layer	Fully-connected	(32, 1)	25

Table 3.1: Architecture of the LSTM model implementation.

3.4.2 Baseline Training Algorithm

To train the DNN within this initial study, a baseline training algorithm was implemented using backpropagation and gradient descent. The model (using the aforementioned architecture) was trained over a specified number of iterations over the full dataset (each iteration is known as an *epoch*), within which batches of data are sampled and input into the model. Before training could commence, the algorithm’s *learning rate*, *epoch count*, and *batch size* hyperparameters had to be determined. The surveyed models typically used a learning rate of 0.001 (Zhang et al., 2022), trained over between 50 (Rahimikia and Poon, 2020) and 600 (Xiong et al., 2015b) epochs, and sampled data in batches of size 32 (Xiong et al., 2015b). Hence, to implement a model that was both characteristic of these statistics and experimentally determined to produce the best performance, a learning rate of 0.001, epoch count of 100, and batch size of 32 were chosen (Table 3.2).

Window Size	Time Series Variables	Prediction Length	Layers	Neurons	Learning Rate	Epochs	Batch Size
10	7	1	3	24	0.001	100	32

Table 3.2: Hyperparameters of baseline model implementation and training algorithm.

3.5 Summary

Hence, a financial volatility forecasting model was developed over the S&P 500 market using a 3-layer LSTM architecture. This model is trained over 100 epochs, recording the efficiency of training to give an insight into the computational cost of traditional DL algorithms, and contextualise the performance and efficiency of the efficient algorithms implemented later in this research.

Chapter 4

Energy-Efficient Training Algorithms

4.1 Aims of Study

Building upon the baseline approach, energy-efficient training algorithms are explored that minimise the energy expended training the implemented model. This research focuses on three energy-efficient training algorithms: mixed-precision training, supervised layer-wise pre-training, and unsupervised layer-wise pre-training. These implementations all use the same dataset and model architecture presented in Chapter 3. Initially, mixed-precision training is implemented, where network computations are quantised to utilise low-bit representations. This exploration aims to demonstrate how using lower precision variables can boost training speed whilst still producing an accurate model, thus showing that mixed-precision training is a viable mechanism for reducing the energetic cost of developing DNNs in finance. The experimentation then proceeds to explore progressive training methods; this begins by implementing a supervised pre-training process, that additionally aims to reduce training time and energy consumption. Unsupervised pre-training is then explored, where a more complex approach is taken to pre-train the proposed DNN to produce further computational cost reductions.

4.2 Mixed-Precision Training Algorithm

The first method presented by this research for reducing the energy consumption of DL is mixed-precision training. As explored in Section 2.5.2, quantisation is often used within Green AI to reduce resource requirements. These implementations reduce energy and memory costs by lowering the precision at which variables are stored and operations are evaluated (Fan et al., 2020). In real-world applications, quantisation is often realised through mixed-precision training, where a combination of high-precision and quantised representations are used to balance efficiency and accuracy (Ott et al., 2016). This research aims to demonstrate how mixed-precision training can be used to reduce the computational and memory cost of training DNNs, mitigating the energy consumption and ESG impacts of DL in finance.

To implement mixed-precision training, the *Keras mixed-precision API* (Abadi et al., 2016) was exploited. This API enables ML developers to adapt the precision of variables and operations within an implemented DNN, changing the default 32-bit precision. In this research, the mixed precision API is used to specify a ‘*mixed_float16*’ precision policy (Abadi et al., 2016), where a combination of 16-bit and 32-bit floating-point data types are used during training. Specifically, this exploits 16-bit floating-point variables for evaluating network computations, whilst maintaining a 32-bit

floating-point representation for storage. Using lower precision reduces the amount of memory used by network operations, increasing the speed of calculations and reading from memory by minimising their computational load and allowing increased hardware acceleration. Specifically, 16-bit representations are utilised as modern GPUs and TPUs have been shown to run operations significantly faster when using 16-bit precision, as batches of data consume half the amount of memory when being processed. High precision is still used for storing variables to maintain numerical stability, ensuring that the optimised model is as accurate as possible. Hence, this implementation of mixed-precision training strikes a compromise between reducing the energetic cost of network operations during training and preserving accurate performance.

The efficiency of mixed-precision training was evaluated using the same DNN architecture as proposed in Section 3.4. First, the use of a ‘mixed_float16’ precision policy is specified; the 3-layer LSTM network is then implemented using variables of the specified precision. This DNN is trained using the same training algorithm as in the baseline case (but evaluated using lower precision), for the same number of epochs (and the same hyperparameters).

4.3 Supervised Layer-wise Pre-Training Algorithm

Pre-training is also a popular approach within Green AI, which aims to allow accurate models to be developed more intelligently and efficiently. Progressive training is one such pre-training method, highlighted as being capable of reducing the length of DNN training through a computationally inexpensive pre-training phase (Xu et al., 2021). As discussed in Section 2.5.3, progressive training constructs a DNN layer-by-layer through an iterative process of adding new layers and training each individually. Research into this approach—such as the exploration of Ienco et al. (2019)—has shown its effectiveness at reducing the computational cost of training a DNN, as the inexpensive pre-training phase (which is computationally cheap since only single network layers are trained) allows a significant reduction in the number of epochs necessary to optimise the parameters of the full network. Both the low-cost pre-training and shortened full network training phases that progressive training is an effective way to reduce training time and energy consumption, providing a promising avenue for improving the ESG impact of DL in finance.

In this study, supervised layer-wise pre-training will be explored to demonstrate the benefits of progressive training. This method individually trains network layers through supervised learning over a labelled dataset outlining the modelling task (Ienco et al., 2019). In our case, the time series dataset outlined in Section 3.2 is used, with each round of pre-training optimising a new layer’s parameters to predict the HV \hat{v}_{t+1} at timestep $t + 1$ within an input sequence. In this research, the base model at the start of pre-training consists of a single LSTM layer combined with a fully-connected single-neuron output layer. To execute the training algorithm, each pre-training round (where a single layer is added and optimised) was conducted over 20 epochs. A further 30 epochs were then used to conclude training, where the parameters of the full network are tuned. This proportion of pre-training and tuning epochs was chosen to provide the best accuracy-efficiency tradeoff, minimising the total length of training but maintaining the model’s forecasting performance.

The first pre-training round focuses on training the base network. This shallow network is trained over the dataset D_{train} , optimising the parameters of the single LSTM layer and the DNN’s output layer. After 20 epochs, the parameters of the first layer are fixed, a new LSTM layer is

added between the initial hidden layer and the output layer, and another 20 pre-training epochs are executed. This iterative process was repeated until a DNN with 3 LSTM layers was constructed, meaning this experimentation utilised the same architecture as in the baseline case. After the full network is pre-trained, final tuning begins, where all network parameters are unfixed and optimised over 20 additional epochs, starting from the initialisation values deduced during pre-training.

4.4 Unsupervised Layer-wise Pre-Training Algorithm

Although supervised layer-wise pre-training is a simple method through which progressive training can be realised, greater performance and efficiency can be achieved through unsupervised layer-wise pre-training (as discussed in Section 2.5.3). Both Xu et al. (2018) and Sagheer and Kotb (2019) found this approach induced faster convergence toward the optimal parameter set and smaller prediction error than traditional training algorithms. Hence, to further improve the efficiency of DNN training in the chosen domain, an unsupervised layer-wise pre-training algorithm is also implemented.

Unsupervised layer-wise pre-training similarly begins by initialising a single-layer base network. The base architecture was identical to that used in the supervised case, except the output layer which used a different structure to enable unsupervised learning. This output layer consisted of the same dimensions as the input layer such that the network could be trained as an autoencoder. Since the same training dataset of multivariate time series subsequences was utilised, the unsupervised pre-training approach trained each layer to reconstruct this subsequence, meaning the network learned to output a representation of the input time series (tuning its parameters to minimise the difference between each layer’s inputs and outputs). Layer-wise pre-training was then implemented, optimising each LSTM layer independently as an autoencoder over 7 epochs to result in a 3-layer DNN. The concluding full network tuning stage (over 50 epochs) then converts this autoencoder into a forecasting model that predicts the volatility \hat{v}_{t+1} based upon each input time series. To achieve this, the autoencoder output layer is discarded and replaced by a single fully-connected output neuron, which is optimised (alongside the three hidden layers) over the tuning stage.

4.5 Summary

Hence, to demonstrate how Green AI can improve the energy efficiency of training DL models in finance, three algorithms were implemented: mixed-precision training, supervised layer-wise pre-training, and unsupervised layer-wise pre-training. To evaluate the success of each approach, a DNN with the same architecture and training dataset as presented in Chapter 3 was implemented and trained, recording the efficiency of the training algorithm and the accuracy of the resulting model. This involved the calculation of the R^2 , MAE, RMSE, and MAPE of test predictions, to deduce the accuracy of each developed model, and quantify the performance difference from the baseline model. The total training time and convergence rate were also recorded over the execution of each algorithm, which are put into context against those computed for the baseline training algorithm, demonstrating any efficiency improvements, and exemplifying the benefit of the adapted training algorithms for reducing the energy consumption, carbon emissions, and ESG impacts of financial volatility forecasting models.

Chapter 5

Data-Efficient Training Algorithms

5.1 Aims of Study

A core component of the computational cost of cutting-edge DL algorithms is the use of extremely large datasets, which force performance gains through training a model over expanses of data, most of which is not beneficial to learning (Bender et al., 2021). Whilst this has been shown to facilitate impressive accuracy, traditional DL training algorithms typically do not deal with data intelligently or efficiently, wasting a significant amount of time and energy learning over redundant data instances that do not benefit performance (Al-Jarrah et al., 2015). Therefore, this research explores data-efficient training algorithms that minimise training data requirements, further improving the efficiency of training and minimising memory and energy costs. Specifically, active learning is employed, which adapts the training process to intelligently select data instances that are the most beneficial to learning, discarding redundant instances. Thus, the training dataset size is reduced, lowering the time and energy expended during training.

Furthermore, Bender et al. (2021) showed that compact networks with small parameter sets typically require increased amounts of training data to achieve comparable performance to larger models. Thus, data-efficient training algorithms additionally allow compact networks to be used without inflicting the further computational costs associated with large datasets.

5.2 Active Learning Algorithm

As described in Section 2.5.4, active learning has been shown to drastically improve the data efficiency of training DNNs by reducing the time and energy wasted on redundant data instances (Xu et al., 2021), as well as achieving an exponential improvement in the time required to label training data, and successfully dealing with high-dimensional sequential data (Ren et al., 2020). The active learning training process implemented in this research aims to reduce computational costs by training over a subset of the data space containing instances believed to provide the most utility to the model’s learning process. In this experimentation, pool-based active learning is implemented, which uses two datasets $P \subset D_{train}$ and $V \subseteq D_{train}$ to split useful and unuseful data according to an importance function. Beginning with an initial seed $P^{(1)}$ of n_{sample} instances, an iterative process is exploited where the DNN is trained over pool P and used to predict the outputs of all data instances in the validation set V ; these predictions are then evaluated through the importance function, and the n_{sample} most important data instances moved into the pool before the next training round is conducted.

In this research, data instances are selected through two algorithms: *greedy sampling on the inputs* (GSx) and *greedy sampling on the outputs* (GSy). These methods, conceived by Wu et al. (2019), implement a performant greedy sampling approach that aims to increase the diversity of the input and output spaces of a model, improving training efficiency. In our implementation, GSx is used to specify the initial seed pool $P^{(1)}$, and GSy is used in the following training iterations to select data instances to be moved from the validation set $V^{(i)}$ to the pool set $P^{(i+1)}$.

GSx (Wu et al., 2019) is an algorithm for constructing the seed pool $P^{(1)}$ by selecting n_{sample} data instances from the full dataset $V^{(0)} = D_{train}$ of size N (Wu et al., 2019). Firstly, GSx selects the single data instance from the full validation set $V^{(0)}$ that is the shortest distance from the dataset’s mean instance (in this case, the *Euclidean distance* is used). Once the data instance closest to the dataset’s mean is found, it moved from $V^{(0)}$ to $P^{(1)}$. This instance is then used to iteratively find the remaining $n_{sample} - 1$ elements of the seed; to find instance x_i , we first compute the Euclidean distance between each data instance x_m already in the seed pool and each instance x_n remaining in $V^{(0)}$, constructing a 2D distance matrix (Equation 5.1). Then, for each instance x_n remaining in $V^{(0)}$ we select the corresponding seed value v_m that minimises the distance $x_n \rightarrow x_m$, building a 1D distance vector enumerating the closest seed instance to each remaining validation set element (Equation 5.2). Finally, the data instance x_n furthest from all existing seeds (i.e. the maximum of the distance vector) is selected and moved from $V^{(0)}$ into $P^{(1)}$. This process is repeated until we have a full seed $P^{(1)}$ of n_{sample} data instances, and a validation set $V^{(1)}$ of $N - n_{sample}$ instances. By adding seeds that are furthest from any others in the pool, GSx ensures that the seed pool used to initially train the DNN is as diverse in the data space as possible, giving the model a complete picture of the breadth of the full training dataset whilst only using a small subset of instances.

$$d_{n,m}^{(x)} = \|x_n - x_m\| \quad (5.1)$$

$$d_n^{(x)} = \min_m d_{n,m}^{(x)} \quad (5.2)$$

GSy (Wu et al., 2019) aims to capture diverse instances from the output space of the model M_θ that realises the function $y = f(x|\theta)$ (where x is the model’s input, in this case multivariate time series instances). The GSy algorithm implements an iterative approach where data instances are selected from the validation set $V^{(i)}$ to add to the pool $P^{(i+1)}$ such that the prediction error of the function $f(x|\theta)$ over the total dataset is minimised. Namely, when considering an input instance x_n , if the model produces an output $y_n = f(x_n|\theta)$ that is noticeably distinct from the outputs generated from other data instances, the algorithm selects x_n to add to the training pool as it helps refine the parameters defining the model’s predictions. GSy then works by the assertion that the more diverse the pool instances x_m collected, the more accurately the optimal parameters of the model can be determined. However, this algorithm requires a functioning model $f(x|\theta)$ to exist (such that model outputs can be generated and analysed), so it can only be used to populate a pool that has already been initialised with seed instances. Therefore, Wu et al. (2019) assert that to implement active learning, GSx should be used for determining the seed, followed by GSy to further populate the pool set at later training iterations.

Given a pool $P^{(1)}$ of n_{sample} data instances, and validation set $V^{(1)}$ of $N - n_{sample}$ instances, the GSy algorithm selects n_{sample} instances to move from $V^{(1)}$ to $P^{(2)}$ believed to provide the most utility to training. This is done by first evaluating the model $f(x_n|\theta)$ over all data instances

$x_n \in V^{(1)}$, to create the set of predicted labels $\hat{Y}^{(1)} = \{\hat{y}_n: \hat{y}_n = f(x_n | \theta), \forall x_n \in V^{(1)}\}$. Once all $N - n_{sample}$ predicted labels have been computed over the validation set $V^{(1)}$, a 2D distance matrix is constructed enumerating the distance (as the absolute difference) between each prediction \hat{y}_n and all true labels y_m of the data instances x_m that already exist within the pool set $P^{(1)}$ (Equation 5.3). From the distance matrix a 1D distance vector is constructed over the $N - n_{sample}$ outputs of the validation set $V^{(1)}$, enumerating the shortest distance of each prediction \hat{y}_n to a true label y_m of an instance x_m in the pool set (Equation 5.4). The data instances in the validation set corresponding to the n_{sample} largest distances in the distance vector are then selected and moved from the validation set to the pool, creating the new datasets $P^{(2)}$ and $V^{(2)}$. This process is repeated at each training round, allowing the pool to be iteratively populated with data instances that stimulate diverse outputs from the model being trained. Thus, a DNN is developed that can produce a wide range of appropriate outputs to accurately represent a spectrum of different predictions (allowing the DNN to model a plethora of different possible scenarios), despite only being trained over a limited number of data instances.

$$d_{n,m}^{(y)} = |\hat{y}_n - y_m| = |f(x_n | \theta) - y_m| \quad (5.3)$$

$$d_n^{(y)} = \min_m d_{n,m}^{(y)} \quad (5.4)$$

In their research, Wu et al. (2019) found that GSx and GSy improved the data efficiency of training and produced a model that exhibits impressive generalisability when compared to approaches that select data instances through random sampling. This demonstrates the effectiveness and robustness of active learning training that uses GSx and GSy. Hence, the experimentation into data-efficient training for DNNs presented in this thesis utilises GSx for picking the seed pool of $n_{sample} = 100$ data instances and GSy at each following training iteration i to select a further 100 data instances to move into the pool set. The full 3-layer DNN proposed in Section 3.4 is utilised over this iterative process, training over the expanding pool set for 100 iterations.

5.3 Summary

Hence, to prove that data-efficient training algorithms are effective within financial domains, an active learning algorithm was implemented, initially selecting a seed pool using the GSx algorithm and iteratively building up the training dataset through the GSy algorithm (Wu et al., 2019). To evaluate the performance and efficiency of this training algorithm, a comparative analysis is conducted between the baseline algorithm and the active learning implementation. A close inspection of the amount of data utilised over training is made, demonstrating how active learning can reduce the data requirements of training DNNs. Whilst proposing the GSx and GSy algorithms, Wu et al. (2019) highlight the limitations of this approach; they found that when the pool size during the early stages of training is too small the model being built exhibits very high variance, meaning its performance is not reliable. Therefore, inspecting the convergence rate of the DNN's parameters over training (to show the stability of training) is essential to exploring the benefits and drawbacks of active learning for improving the efficiency of DNN training.

Chapter 6

Results & Discussion

The following chapter explores the results of the presented studies. Initially, the data collected from each experiment is presented, summarising what is shown by the research findings, highlighting their significance, and examining their implications for the research hypotheses. Once all results have been outlined, a detailed discussion surrounding the success of each experiment is undertaken, evaluating how effective Green AI is at reducing the computational cost of DNN training within the chosen application of financial volatility forecasting.

Each result was derived by tuning the hyperparameters of each training algorithm to maximise model accuracy whilst maintaining an efficiency greater than that of the baseline training approach. This prioritised developing models with low prediction error, then evaluated the efficiency of the training process undertaken to develop that model. To evaluate each training algorithm in a robust and reproducible manner, each was employed five times, optimising a model from scratch over a fixed number of epochs; the results of the most successful training round were then collected as the outcome of this research. This was to ensure that a representative result of the true ability of each training algorithm was obtained, and that the outcome of this research was not biased or degraded by the stochastic nature of DL training algorithms, which can cause results to vary over different executions. To ensure reproducible and generalisable results, the model and training process are implemented in *Python* with the popular DL framework *TensorFlow* (Abadi et al., 2016). The implementations were run on *Google Colaboratory Pro* to achieve higher training performance by exploiting high-end ML hardware, executing each algorithm using a *Tesla T4* GPU.

6.1 General Predictive Performance

To demonstrate the success of each algorithm, each trained model was employed to forecast volatility over the test dataset. Figure 6.1 shows the forecasts of all models, showcasing the predicted volatilities \hat{v}_{t+1} over all timesteps of the testing period, contextualised against the true values v_{t+1} . Figure 6.1 demonstrates that all models generate predictions that accurately follow the general trend of the true volatility sequence. However, close inspection of how the different models handle rapid volatility jumps highlights the differences in forecasting characteristics. If one inspects the significant rise in volatility in late 2008 (corresponding to the 2008 financial crisis), the baseline model (optimised through a traditional training algorithm) is shown to underpredict the true volatility, undercutting the true peak by a fairly significant margin ($\hat{v}_{t+1} \approx 0.6$ when the true v_{t+1} is closer to 0.7). A similar characteristic can be seen in early 2020 (corresponding to the COVID-19 pandemic), where the significant fluctuations in volatility are consistently underestimated by the DNN's predictions.

This suggests that whilst the baseline DNN appears to model the smaller volatility changes very accurately, large, rapid changes in the true volatility result in less accurate predictions.

Predictions of All Models

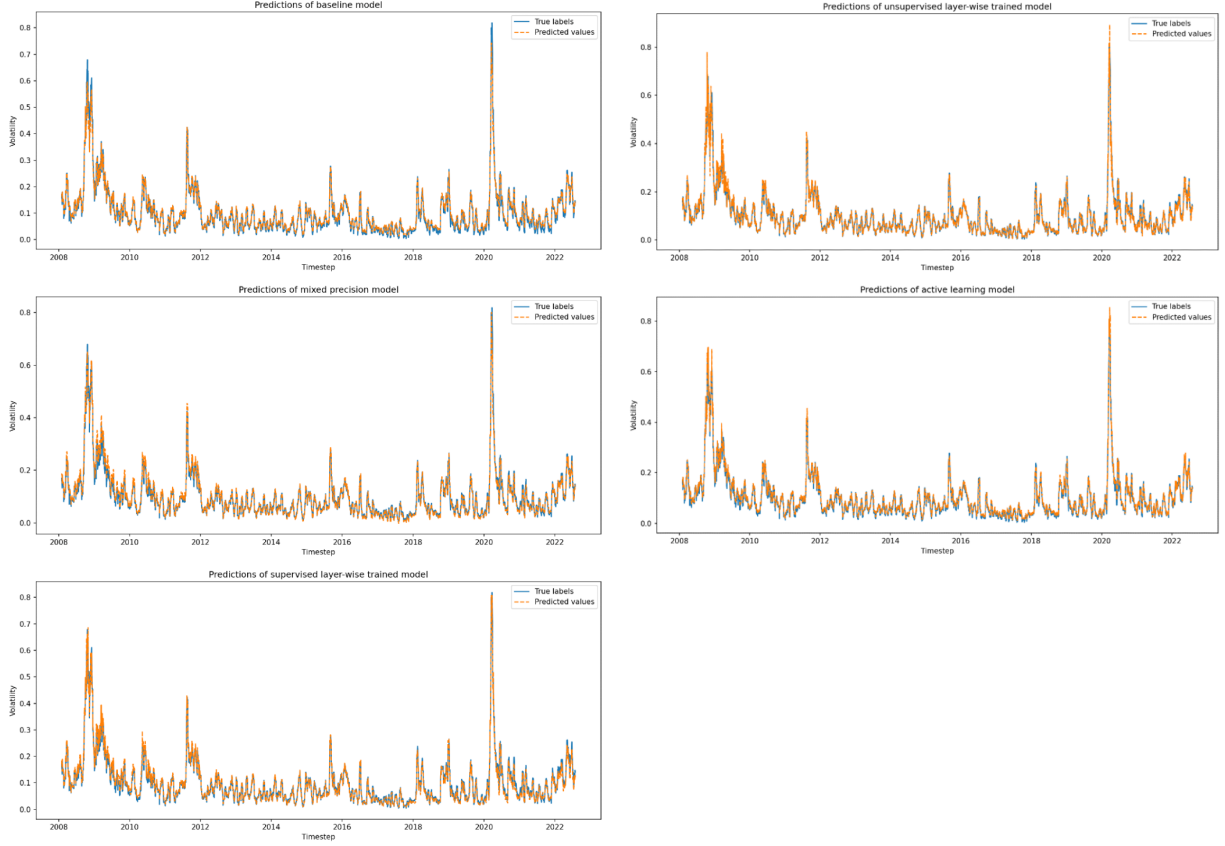


Figure 6.1: Predicted volatility time series of each tested model (orange dashed line) against the true volatility time series (blue) over the testing dataset.

When the predicted time series of the other models are compared to that of the baseline, the differences in forecasting ability can be observed. Figure 6.1 shows that the models using supervised and unsupervised layer-wise pre-training most accurately represent both small and large fluctuations in volatility. These plots show the predictions \hat{v}_{t+1} are significantly closer to the labels v_{t+1} at times of high volatility (e.g. in 2008 and 2020). The forecast of the model trained through unsupervised layer-wise pre-training appears slightly less accurate than that of its supervised counterpart, as this model overestimates the volatility jumps in 2008 and 2020, however less extreme movements are modelled very accurately. The model developed through mixed-precision training also appears to generate predictions that lie closer to the true volatility than the baseline model, although with an accuracy slightly below that of the layer-wise pre-trained models.

When considering the model trained through active learning, the forecast time series is also shown to accurately follow the path of the true volatility, with most fluctuations accurately captured. However, these predictions seem to slightly miss the true movements, as the scale of peaks and troughs seems to often miss the exact movements of the true volatility, suggesting this model is slightly less performant than other training algorithms. However, the general trend of the predictions on a wider scale seems (at least) commensurate with those of the baseline model, indicating that the implemented active learning training process does not degrade forecasting performance.

6.2 Accuracy Results

To demonstrate the performance of each trained model more precisely, Table 6.1 presents the results generated by evaluating each optimised model’s accuracy (through the metrics presented in Section 3.3.1). This began with the baseline model; it was found to exhibit an impressive test accuracy, demonstrating the great ability of the LSTM architecture to model sequences such as the utilised financial volatility time series. The coefficient of determination of the baseline model was found to be 0.977414, showing that the sum of squared prediction errors between \hat{v}_{t+1} and v_{t+1} is close to zero (Equation 3.1). This means that the true labels have been near-perfectly represented by the model, demonstrating the DNN trained by a traditional training process can generate excellent one-step-ahead forecasts of the market volatility of the S&P 500. Similarly, the baseline model produced impressive MAE and RMSE scores of 0.008407 and 0.013830, demonstrating that the average magnitude of the prediction error is near zero. Importantly, both the MAE and RMSE are similar, indicating that the model is averse to uniformly distributed prediction errors (highlighted by the MAE) and highly deviant predictions (identified by the RMSE). The model also generated an impressive MAPE of 14.662217%, indicating that the DNN has fit the training data well.

Method	R^2	MAE	RMSE	MAPE (%)
Baseline	0.9774	0.008407	0.01383	14.66
Mixed-precision	0.9774	0.009121	0.01384	12.13
Supervised layer-wise pre-training	0.9856	0.007814	0.01103	9.932
Unsupervised layer-wise pre-training	0.9841	0.007080	0.01162	8.805
Active Learning	0.9711	0.01045	0.01564	11.09

Table 6.1: Accuracy data for all methods, evaluated over the testing dataset.

After certifying the performance of the baseline model, the accuracy achieved by alternative training algorithms could be contextualised. This exploration began with an analysis of mixed-precision training; impressively, the mixed-precision model had higher prediction accuracy than the baseline across almost all metrics, generating a 15.10% lower MAE, 10.50% lower MAPE, 8.53% lower RMSE (Table 6.1). These scores indicate that this training algorithm was capable of optimising a model that can forecast volatility more accurately than the baseline approach.

Following this result, the two models using progressive training were analysed. The first model, trained through supervised layer-wise pre-training, exhibited higher forecasting accuracy than both the baseline and mixed-precision models that used traditional training algorithms. The optimised model produced an outstanding R^2 score of 0.985642, indicating that using supervised layer-wise pre-training facilitated an almost perfect correspondence between \hat{v}_{t+1} and v_{t+1} ; additionally, this model reduced the MAPE of the baseline model by 32.26% and the mixed-precision model by 18.09%. The accuracy improvement was observed across the board, generating better R^2 , MAE, RMSE, and MAPE scores than both the baseline and mixed-precision methods. The model trained through unsupervised layer-wise pre-training also exhibited impressive performance, generating predictions of similar accuracy to its supervised counterpart, and significantly outperforming the baseline with 0.6800% higher R^2 , 15.78% lower MAE, 15.98% lower RMSE, and an exceptional decrease in MAPE

of 39.95%. When directly comparing the models trained through supervised and unsupervised layer-wise pre-training, however, the differences are not so explicit. The supervised approach produced marginally better performance across the R^2 and RMSE metrics: the resultant model exhibited a 0.1607% higher R^2 and 5.103% lower RMSE. On the contrary, the unsupervised approach produced a model with a 9.393% smaller MAE and 11.35% smaller MAPE. This suggests that both supervised and unsupervised layer-wise pre-training produced a more accurate model than the baseline training algorithm, but each reduced prediction error differently: for example, the lower RMSE of the supervised pre-trained DNN indicates this model less frequently generates predictions that are highly incompatible with the true value. Thus, the accuracy results in Table 6.1 show that both supervised and unsupervised layer-wise pre-training are the most effective training algorithms for developing high-performance DNNs for volatility forecasting.

The active learning algorithm produced a model impressive MAPE, exhibiting a 24.38% lower test MAPE than the baseline model. However, when considering other accuracy metrics, the model was not shown to exhibit such an accuracy benefit, having a slightly worse coefficient of determination (0.6440% lower) and noticeably increased test MAE (24.33% higher) and RMSE (13.08% higher). Thus, the performance of the active learning-based model was roughly equivalent to the baseline, outperforming it on some metrics but underperforming on others. When different parameterisations of the active learning training algorithm were explored, a few implementations did produce higher accuracy than the baseline; for example, using 200 training iterations and a sample size of 70 facilitated a 26.83% lower RMSE and 34.72% lower MAPE (Table 6.4). Hence, these results indicate that active learning can be used to develop a model with similar performance to a traditionally trained baseline, supporting the assertion that this method can be tuned to only inflict a marginal accuracy detriment.

6.3 Efficiency Results

6.3.1 Training Time Analysis

As outlined in Section 3.3.2, total training time is one of the most applicable metrics for quantifying the energy efficiency of DL training algorithms; hence, to examine the success of each training algorithm the time (in seconds) taken to optimise the DNN’s parameters was quantified by profiling each implementation, the results of which are exhibited in Table 6.2. This analysis found that the baseline training approach took a total of 198.602 seconds to optimise the DNN; this result was then used to contextualise the efficiency of each subsequent training method. Firstly, mixed-precision training was examined, which produced a fairly disappointing result: the algorithm took 18.98% longer to complete than in the baseline, despite using the same number of epochs. This suggests that utilising 16-bit floating-point representations for evaluating operations during training did not directly improve the energy efficiency of training the DNN.

On the contrary, both progressive training algorithms were shown to have significantly smaller training times, with unsupervised layer-wise pre-training being the fastest to complete. The unsupervised method facilitated a 34.40% decrease in training time, whilst the supervised method improved on the baseline by 22.43%, but took 18.24% longer than its unsupervised counterpart. These results indicate that unsupervised layer-wise pre-training is the most efficient way to train DNNs, taking the least amount of time to produce an accurate model, and hence likely being the most energy-efficient algorithm. Furthermore, it demonstrates that supervised unsupervised layer-

Method	Total Training Time (s)
Baseline	198.6
Mixed-precision	236.3
Supervised layer-wise pre-training	155.0
Unsupervised layer-wise pre-training	130.3
Active learning	212.5

Table 6.2: Total training time of each training algorithm (to produce an accurate model).

wise pre-training are both viable ways to improve the energy efficiency of DNN training, as they both took significantly less time to train an accurate forecasting model than the baseline algorithm.

When the training time of active learning is analysed, the results do not indicate that this is an effective method for reducing the overall length of training. Namely, the active learning algorithm using hyperparameters detailed in Section 5.2 marginally exceeded the total time spent training by the baseline approach by 6.974%. Although, a small proportion of the active learning parameterisations tested did result in a reduction in training time over the baseline algorithm, majoritively achieved by drastically limiting the number of training iterations and training dataset size, and hence in general also inflicted an accuracy drop (Table 6.4). Despite this result, it is not concerning that total training time was not reduced, as active learning majoritively focuses on improving the data efficiency of training, not explicitly the energy efficiency. This process instead intends to improve the ESG impacts of DL by reducing resource requirements, data storage and labelling costs, and the energy consumption of large data centres.

6.3.2 Convergence over Training

To further quantify the efficiency of each training algorithm, the convergence of the training error towards the minimum of the loss function was evaluated (Figure 6.2). The plots show the prediction error output by the loss function on input of the training dataset and on input of a smaller validation set (a collection of data instances held back from the training process) during each epoch.

When the convergence of mixed-precision training is compared to the baseline algorithm, a very similar trend can be seen; both exhibit closely aligned training and validation error curves, implying the models fit well to the training data, and both exhibit a significant decrease in MAE at the start of training with this tapering off in later epochs. The close similarities between both plots suggest that whilst introducing mixed-precision representations has not sped up the training process in terms of convergence per epoch, using these lower bit representations has not degraded the convergence rate of the prediction error towards the minimum of the loss function. Although, upon close inspection of the progression of the validation error over the mixed-precision training process, the curve appears to be slightly rougher than in the baseline case, exhibiting marginally larger fluctuations around the general convergence trend. This suggests that the mixed-precision training algorithm may be slightly less stable than when using high-precision representations.

Comparison of the convergence of the two progressive training algorithms demonstrates the

Convergence Rate of All Tested Models

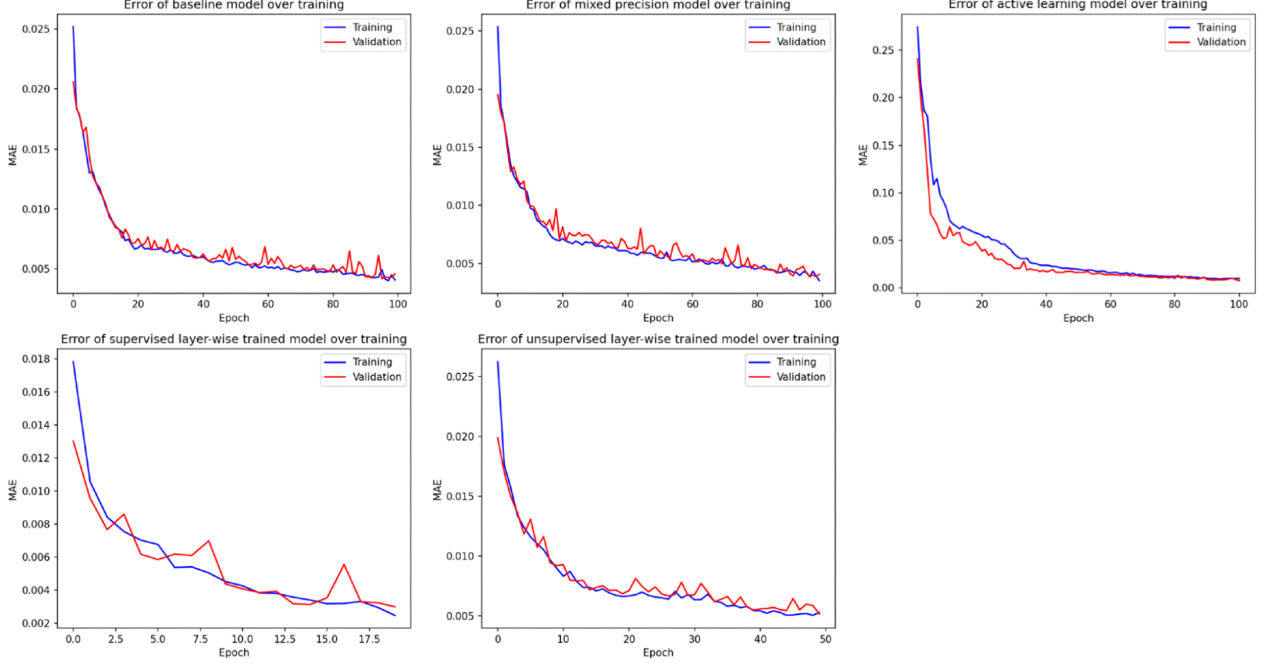


Figure 6.2: Convergence over each training algorithm, shown as the progression of the output of the loss function (MAE) at each epoch to determine the prediction error of the network (a steeper gradient indicates a higher convergence rate).

differences between these algorithms. Firstly, one can see that the pre-trained DNNs require significantly fewer epochs to optimise the full parameter set than the other training algorithms: the supervised and unsupervised methods only required 20 and 50 epochs respectively over the full network. Additionally, the initial epochs of both algorithms are shown to exhibit a much faster convergence, reducing both the training and validation error more rapidly. Furthermore, the majority of their minimisation of prediction error occurs within the first 10 epochs, whereas the same reduction is spread over the first 20 epochs in the baseline case. For example, of the total reduction of validation error from ~ 0.013 to ~ 0.003 produced by the supervised approach, 90% occurs within the first 10 epochs (decreasing the MAE to ~ 0.004). Similarly, 80% of the total reduction in validation MAE ($\sim 0.02 \rightarrow \sim 0.005$) in the unsupervised case happens over the first 11 epochs of full network training ($\sim 0.02 \rightarrow \sim 0.008$). This indicates that progressive training is a significantly more efficient training algorithm as it induces faster learning within the DNN.

Analysis of the convergence over active learning similarly highlights its ability to induce faster learning within a DNN than the baseline algorithm. This is indicated through the incredibly sharp decline in both training and validation error over the early stages of training, generating approximately 83% of the total reduction in validation MAE over the first 8 epochs. Furthermore, whilst in the baseline cas, the convergence rate slows considerably after the first few epochs (significantly plateauing after 20 epochs), during active learning the convergence rate stays higher for longer. Namely, a significant reduction in MAE is still observed up to approximately the 32nd epoch. This suggests that the learning process is more efficient as the network’s parameters are optimised faster.

6.3.3 Efficiency Breakdown of Progressive Training

To further analyse the benefit provided by progressive training, a detailed study was conducted into the time taken by the supervised and unsupervised layer-wise pre-training algorithms, breaking down how these approaches improve training efficiency (Table 6.3). These results demonstrate the different approaches taken by the progressive training algorithms, laying out how much time (and how many epochs) was spent on pre-training and tuning.

Pre-Training Method	Pre-Training Time (s)	Pre-Training Epochs	Layer 1 (%)	Layer 2 (%)	Layer 3 (%)	Tuning Time (s)	Tuning Epochs	Total Time (s)
None	-	-	-	-	-	198.6	100	198.6
Supervised	107.9	20	32.6	29.0	38.4	47.07	20	155.0
Unsupervised	43.77	7	27.4	47.8	24.7	86.52	50	130.3

Table 6.3: The time taken by pre-training and full network tuning for both supervised and unsupervised layer-wise pre-training (including a breakdown of the time spent pre-training each hidden layer), compared to the total time taken by the baseline method.

This study found that supervised layer-wise pre-training was most effective when 20 epochs were used to pre-train each layer, and another 20 epochs optimising the full network; in comparison, the unsupervised approach generated the best performance when employing only 7 epochs per pre-training round, then a final tuning round of 50 epochs. Thus, in total the supervised method used 60 low-cost epochs (training only a single layer) and 20 high-cost epochs, whereas the supervised method only employed 21 low-cost epochs but required 50 high-cost epochs. Table 6.3 illustrates this divergence; the supervised approach allocated 69.63% of the total training time to pre-training, compared to only 33.59% during the unsupervised algorithm. Therefore, whilst unsupervised layer-wise pre-training had a shorter total training time, supervised pre-training facilitated a greater reduction in the required number of full DNN tuning epochs (exhibiting an 80% decrease in tuning epochs, compared to a 50% in the unsupervised case).

Table 6.3 also breaks down how each pre-training round contributed to the overall computational cost of training. Of the total pre-training time, the supervised method spent 32.6% on the first layer of the network, 29.0% on the second, and 38.4% on the final layer. Hence, the second hidden layer was faster to pre-train than the initial base model (suggesting the transfer of knowledge between these rounds), but the final layer was the most computationally intensive of all (possibly due to deep layers optimising higher-dimensional representations). A different trend was seen over unsupervised pre-training, where the second hidden layer was the most computationally intensive (using 32.6% of the pre-training time), and the final layer was the fastest (24.7% of pre-training); this is likely due to the 2nd layer building up the majority of the encoding (of the input vector) within the autoencoder, meaning the 3rd hidden layer was not required to add as much to the representation learning process (simply building upon the knowledge of its preceding autoencoder layer).

6.3.4 Data Efficiency of Active Learning

Once the energy efficiency of the baseline, mixed-precision, and progressive training algorithms had been thoroughly explored, the data efficiency of active learning was examined. Since this approach focuses on simultaneously minimising both the training dataset size and the accuracy drop inflicted,

active learning can implement a spectrum of accuracy-efficiency compromises. To explore these varying compromises, different parameterisations of active learning algorithms were experimented with, manipulating the hyperparameters controlling the number of training iterations and sample size n_{sample} .

Method	Number of Iterations	Sample Size	Final Pool Size	Utilisation of Full Training Dataset (%)	RMSE	MAPE	Training Time (s)
Baseline	-	-	14600	100	0.01383	14.66	198.6
Active Learning	200	70	14070	96.36	0.01012	9.571	480.6
		45	9045	61.95	0.01492	10.48	436.7
		20	<i>4020</i>	<i>27.53</i>	<i>0.01863</i>	<i>14.04</i>	<i>222.6</i>
	150	90	13590	93.08	0.01404	12.25	363.5
		55	8305	56.88	0.01595	13.57	326.7
		20	3020	20.68	0.02579	25.93	274.5
	100	140	14140	96.85	0.01740	15.03	239.6
		80	<i>8080</i>	<i>55.34</i>	<i>0.01564</i>	<i>11.09</i>	<i>212.5</i>
		20	2020	13.84	0.03527	29.83	175.6
	50	250	12750	87.33	0.01735	13.69	176.1
		135	6885	47.16	0.02580	22.40	124.0
		20	1020	6.990	0.03883	41.61	89.70

Table 6.4: Results of the experimentation into how varying the number of training iterations and sample size effect the data efficiency, model accuracy, and training time of an active learning based training algorithm (using GSx and GSy). N.B. notable results are emphasised in italics.

Table 6.4 outlines the results of this study, enumerating the different hyperparameters used and the resulting data efficiency, accuracy, and training time. The results demonstrate that general the greater number of active learning iterations, and the more data added at each iteration, the higher the achieved model accuracy but the lower the data and time efficiency of training. For example, the hyperparameters $(N_{iters}, n_{sample}) = (200, 70)$ facilitated a highly performant DNN with a test MAPE of 9.57100 (a 34.72% decrease from the baseline model), but used most of the training data (96.36% of the full dataset) and a considerably longer training time (142.0% increase over the baseline).

However, by tuning these hyperparameters the ability of active learning to facilitate accurate performance whilst considerably reducing training data requirements was shown. Under the hyperparameters $(N_{iters}, n_{sample}) = (200, 20)$, this approach surpassed the performance of the baseline model (reducing the test MAPE by 4.259%) using only 27.53% of the amount of training data. Furthermore, whilst this did require a longer training time than the baseline, an increase of only 12.07% was observed, suggesting the direct computational cost was not significantly inflated. Another notable parameterisation $(N_{iters}, n_{sample}) = (100, 80)$ was able to produce a model with significantly improved MAPE (24.38% lower than the baseline) using only 55.34% of the training data, and only requiring a 6.974% longer training time. The impressive result solidified this parameterisation as the implementation used to compare the performance of active learning to the other implemented training algorithms in the preceding section.

6.3.5 Summary

Hence, mixed-precision training and both progressive training algorithms were shown to provide an accuracy benefit over the baseline model, with unsupervised layer-wise pre-training facilitating the greatest performance improvement. Additionally, active learning was shown to produce performance at least commensurate to that of the baseline. When considering the efficiency of each model, the mixed-precision algorithm generated disappointing results, exhibiting similar convergence but a longer training time. However, both progressive training methods were able to significantly reduce training time, whilst active learning was able to considerably restrict the training dataset without impacting performance; additionally, both progressive training and active learning achieved greater convergence rates, showing these algorithms learn more intelligently than the baseline approach.

6.4 Extended Discussion

Given the presented results outlining the accuracy and efficiency of each training algorithm, an evaluation can be made surrounding how well each reduces the computational cost and resource requirements of training, demonstrating their utility for lowering the ESG costs of Fintech.

6.4.1 Mixed-Precision Training

The implemented mixed-precision training algorithm attempts to reduce the computational cost of developing DNNs by lowering the precision at which operations within a DNN are evaluated. The experimentation found that whilst mixed-precision training did result in a higher accuracy model than the baseline, the training process took 18.98% longer, suggesting this approach did not directly improve energy efficiency. However, a nearly identical convergence rate towards the minimum of the loss function over this training process was demonstrated, suggesting that reducing the memory requirements of DNN training by using smaller 16-bit representations does not negatively impact the stability of training and performance of the resulting model.

Since the mixed-precision implementation was not shown to facilitate a reduction in training time, there is no evidence found in this research to suggest that mixed-precision representations are an effective method for improving the energy efficiency of training DNNs for financial volatility forecasting. This is a disappointing conclusion, as it does not echo the results of reduced precision implementations in other fields that have boosted training speed, such as DoReFa-Net (Zhou et al., 2016) which showed the benefit of low-bit training to CNN-based image classifiers. However, this is not that surprising, as DoReFa-Net inflicted a significant reduction in precision (using 1-bit parameters, 2-bit activations, and 6-bit gradients) which was not undertaken in this research; instead, 16-bit representations were used, which cannot be expected to facilitate the same efficiency improvement as that of Zhou et al. (2016). Furthermore, the presented findings more closely resemble those of Ott et al. (2016), who found that low-bit training was not ubiquitously effective for improving the efficiency of training RNNs; hence, the results presented in this thesis may suggest that LSTM-based volatility forecasting may be one such application.

The lack of efficiency benefit may also be explained by the model architecture and hardware utilised. Namely, the implemented DNN (and training dataset) is relatively small compared to state-of-the-art implementations typically used by large institutions. Due to the overhead costs within training algorithms, smaller models do not typically benefit from mixed-precision to the

same extent as those with more complex architectures, as these overheads constitute a large portion of the training time. Furthermore, hardware accelerators (such as the GPU relied upon in this study) do not provide as much benefit to smaller models, as the matrix multiplications are not as computationally intensive and so do not benefit as much from increased parallelisation. Therefore, whilst no training time reduction was observed in this research, this method may still hold promise in improving the energy efficiency of training higher complexity systems used by financial institutions. Moreover, one limitation of this research is that the memory consumption over DNN training was not explored; thus, this research does not quantify any reductions to memory usage that may have been facilitated by mixed-precision representations, which could have revealed other efficiency improvements.

6.4.2 Progressive Training

On the contrary, both progressive training approaches were able to significantly reduce total training time whilst producing a DNN with higher prediction accuracy. Supervised layer-wise pre-training improved the MAPE of the baseline model by 32.26%, whereas the unsupervised approach generated a $\sim 15\%$ decrease in MAE and RMSE and an impressive 39.95% reduction in MAPE. These results indicate that the true volatility time series has been accurately represented by the predictions of the trained model, enabling accurate predictions. Furthermore, a high convergence rate is exhibited over both progressive training approaches, demonstrating the stability and robustness of the training algorithms. The significant reduction in training time facilitated by both supervised and unsupervised approaches (22.43% and 34.40% faster respectively) also suggests that these algorithms are successful at improving the energy efficiency of training DNNs in this context, as shorter training times likely correspond to lower energy consumption.

These results align with other research into progressive training completed further afield, similarly highlighting the efficiency of progressive training algorithms. For instance, Ienco et al. (2019) found supervised layer-wise pre-training improved the generalisability of sequence classifiers, which is similarly shown in the high testing accuracy of the presented models. Additionally, Xu et al. (2018) and Sagheer and Kotb (2019) found unsupervised layer-wise pre-training could induce faster convergence whilst improving accuracy, both of which are demonstrated in the results of this research. Hence, this study supports the findings of other research into progressive training, extending the known utility of these Green AI algorithms to the field of financial market volatility forecasting.

Another core success shown by both algorithms was their ability to significantly reduce the number of epochs spent optimising the parameter set of the full network. The supervised approach reduced the required tuning rounds by 80% to 20 epochs and the supervised approach lowered it by 50% to 50 epochs. Since optimising the full 3-layer LSTM network is significantly more computationally intensive than optimising only a single LSTM layer (due to the higher number of parameters, calculations, and updates), this reduction in tuning likely facilitated a significant decrease in the computational cost of training. This was likely facilitated by the layer-wise pre-training stage inducing faster optimisation of the parameters, as the number of epochs in total (over both the pre-training and tuning stages) was still lower than when using the baseline training algorithm. Thus, the low-cost pre-training epochs were shown to be effective at teaching the DNN how to make accurate predictions, allowing it to learn faster than possible in the traditional baseline algorithm. Furthermore, this efficiency benefit was likely facilitated by the transfer of useful information learned by each layer over pre-training to each subsequently added layer, allowing

parameter updates to incorporate information already learned about the features of the input (as Xu et al. (2021) posited). This is shown through layers added later during pre-training (in general) making up a lower proportion of the total pre-training time. During supervised pre-training, 29.0% of the total time was allocated to training the second LSTM layer, compared to 32.6% spent on the first. Similarly, the unsupervised approach expended 27.4% of pre-training on the first hidden layer and only 24.7% on the last. Thus, these results demonstrate that supervised and unsupervised layer-wise pre-training facilitate a more intelligent and efficient learning process.

One limitation of layer-wise pre-training is the extra computational cost of fixing the parameters of previously pre-trained layers such that they aren't optimised in subsequent pre-training rounds. When profiling the progressive training algorithms, this cost was included in the pre-training time recorded per layer; thus, the total time calculated is not solely attributed to model training. This means there is a slight divergence between what is included in the total training time statistic between the progressive training algorithms and the baseline. Additionally, more time is spent on auxiliary tasks during each subsequent pre-training round than when training the initial base network, as the first round does not fix any parameters. This may contribute to why some hidden layers added during pre-training do not exhibit an increased training speed from the layer preceding it. However, these must be included in the total times attributed to progressive training to ensure a fair comparison to the baseline method which does not require these auxiliary tasks.

An interesting difference between the supervised and unsupervised algorithms is that the supervised method allocated 69.63% of the total training time to pre-training, compared to only 33.59% during the unsupervised approach. This distinction is likely due to the supervised algorithm training the DNN to forecast volatility from the start, whereas the unsupervised approach pre-trained the DNN as an autoencoder. Time series forecasting is known to be a challenging task; however, training an autoencoder is not so costly as simply minimising the reconstruction error between inputs and outputs is significantly easier than forecasting future values. Thus, more epochs were required to produce an accurate DNN over supervised pre-training than in the unsupervised case. However, later stages of unsupervised layer-wise pre-training required the developed autoencoder to be converted into a forecasting model, hence requiring more full network tuning epochs.

The observed lower training times, fewer epochs, and computational cost savings indicate that both supervised and unsupervised layer-wise pre-training algorithms improve the energy efficiency of training a DNN for financial volatility forecasting. Whilst the observed reductions in training time have been on the scale of seconds in this research, when the model architecture and training length are scaled up to be comparable to the large, energy-intensive DL systems utilised in the upper echelons of computational finance research and by large financial institutions, these savings will likely also scale, allowing more significant reductions in training time (in the scale of hours and days) and carbon emissions. This adequately fills the research gap by proving that progressive training is an effective Green AI algorithm for improving the energy efficiency of DL in finance, thus providing a possible route through which the carbon emissions of Fintech can be reduced, aligning Fintech more closely with the SDGs of sustainable finance. Moreover, this research has additionally improved the inclusivity of DL for finance, as shorter training times and smaller energetic costs lower the resource requirements of developing DNNs, reducing the expense of implementing such a system, and hence allowing more financial players and researchers to reap the benefits of DL.

6.4.3 Active Learning

As discussed by Bender et al. (2021) and Walsh et al. (2021), a significant portion of the computational cost of high-performance DNNs can be attributed to their inefficient use of data. Thus, to further mitigate the ESG impacts of Fintech this research explores a data-efficient training algorithm known as active learning; this approach selects data instances more intelligently, reducing the training dataset size and minimising memory and energy costs.

The completed study demonstrated the spectrum of efficiency-accuracy compromises achievable by active learning, ranging from the parameterisation $(N_{iters}, n_{sample}) = (200, 20)$ which maintained a comparable test MAPE to the baseline model using only 27.53% of the data, to the parameterisation $(100, 80)$ which facilitated a 24.38% improvement to MAPE using only 55.34% of the training data (Table 6.4). This shows that active learning is an effective approach for significantly reducing the amount of training data required to develop accurate DNNs in the chosen context; this lines up with the research of Ren et al. (2020), who showcase the ability of active learning to deal with high-dimensional data in CV, and the results of Peng et al. (2017) and Zimmer et al. (2018) who both demonstrated benefits of active learning to reducing dataset sizes in other time series modelling domains. Hence, the presented research has built upon this preceding work to demonstrate that active learning is effective at improving the data efficiency of DNNs used in finance.

By reducing the training data used within each epoch, it is likely active learning also reduced the total computational cost of training. One method of approximating the computational cost of each epoch is by estimating the number of parameter optimisation iterations (i.e. applications of gradient descent over the full parameter set) per epoch. If we have a training dataset of N instances and utilise a batch size b , there will be a total of $\frac{N}{b}$ full parameter optimisation iterations at each epoch, as the DNN's parameter set is iteratively updated over the full training dataset one batch at a time. In the baseline case, there are $\frac{14600}{32} \approx 456$ parameter optimisation iterations per epoch. If this cost is to be computed for an active learning algorithm, the equivalent figure for training dataset size must be approximated as the average pool set size over the entirety of training. For the parameterisation $(N_{iters}, n_{sample}) = (100, 80)$ we begin with a seed pool of 80 instances, and finish training with a pool set of size 8080, meaning on average a pool of $N_{pool} = 4080$ instances was employed. Therefore, the active learning algorithm on average employs $\frac{4080}{32} \approx 127$ parameter optimisation iterations per epoch (a 72.15% decrease from the baseline algorithm). Thus, this efficient algorithm required significantly fewer prediction errors and parameter updates, reducing the computational cost of each epoch and in total over the full training process.

Ren et al. (2020) assert that this reduction in training data is facilitated through more intelligent data selection. The presented findings support this assertion, as the active learning algorithm exhibits a higher convergence rate, maintained over a greater number of epochs; hence, this approach more rapidly optimises the parameter set, approaching the minimum of the loss function and optimal DNN parameterisation at a faster rate over the initial stages of training. The improved convergence is likely due to selected data allowing the DNN to learn more about its modelling task during each epoch, facilitating a faster and more efficient learning process. This indicates that the utilised GSx and GSy algorithms have successfully constructed a training pool that effectively represents the input and output spaces of the model, suggesting this algorithm more intelligently constructs a training dataset even over the complex, multivariate time series data used in finance.

This study achieved its aim to demonstrate how active learning can be used to produce equivalent (and even superior) performance to a traditional training approach whilst significantly reducing the

training dataset size. By reducing data requirements, another possible avenue for reducing the ESG impacts of DL for finance has been highlighted. Whilst not attempting to directly reduce the energy consumption of training by minimising training time (as is the case with progressive training), active learning does facilitate a reduction in the energetic cost of DL indirectly, mitigating environmental concerns and improving the inclusivity of this field. This is because reducing dataset size lowers the computational cost of training (as evaluating a DNN over a smaller dataset is cheaper) and decreases data storage and labelling costs. Furthermore, the datasets employed by state-of-the-art DL algorithms and industrial applications are often multiple orders of magnitude larger than that used in this research; hence, the energy and carbon savings generated if the successes of this research are applied to higher complexity models will likely be even more impressive. Additionally, this approach has the potential to reduce the energy consumption of large data centres by removing the requirement to store expansive datasets, lowering the cost of transferring these datasets between servers, and reducing the cost of executing DL algorithms there. This also lowers the bar-to-entry to using DL algorithms as developers do not need to gather or store vast datasets to produce accurate models.

Thus, the contribution of this study to Fintech research and the finance industry is twofold: firstly, an approach for decreasing the environmental cost of DL for finance has been presented, and secondly, the financial cost of high-performance DL algorithms has been reduced, improving the inclusivity of Fintech. Furthermore, the successful application of active learning to the new domain of financial volatility forecasting has expanded the space of known useful applications of Green AI, further demonstrating the utility of these algorithms, and providing a new avenue that research into Green AI can take focusing on improving the sustainability of DL in industry.

Chapter 7

Conclusions & Future Work

7.1 Summary & Contributions

In summary, this research improves the efficiency of DNN training algorithms used within Fintech. Due to the significant energy consumption of DL algorithms, their expanding use within finance raises concerns about inflating ESG costs. These concerns have been addressed within the research fields of CV, NLP, and mobile computing, through new Green AI algorithms that prioritise the efficient use of energy and data. Hence, to mitigate the environmental (and social) concerns associated with using DL in finance and ensure DNNs can be used without compromising the SDGs of sustainable finance, this research has explored how Green AI can be applied within this field.

To apply Green AI algorithms to finance, a volatility forecasting model was implemented using a DNN with an LSTM architecture; this model was optimised through efficient training algorithms that inflict a low computational cost (in terms of energy and data) whilst facilitating accurate performance. Mixed-precision training, progressive training, and active learning were explored, determining the efficiency of each algorithm. The experimentation found that progressive training was the most energy-efficient DL training algorithm, producing a model with higher performance over a shorter training period. Active learning was also shown to be incredibly beneficial to improving the data efficiency of training, further minimising the resource requirements of DL algorithms.

Based on these results, the contribution of this research is threefold. Firstly, applying Green AI to this new domain has expanded the known applications of efficient training algorithms, demonstrating the potential of progressive training and active learning to reduce the energy and carbon cost of DL in industrial applications (like finance). This proves that the benefits of Green AI are not only possible in state-of-the-art research domains like CV and NLP, or for low-resource applications such as mobile computing, as these algorithms are also effective at improving the sustainability of DL further afield. Hence, a new, impactful avenue of Green AI research has been created, exploring how these algorithms can be used to mitigate the ESG cost of DL in industry. Secondly, addressing the energy and data efficiency of DL algorithms in finance proves that these systems can be used to conduct highly accurate modelling at a low computational cost. This aligns Fintech closer to the SDGs of sustainable finance, providing an avenue for mitigating the carbon emissions attributed to this field; this introduces a new approach towards sustainable finance, namely *sustainable deep learning for sustainable finance*. Thirdly, lowering the resource requirements and computational cost of Fintech improves the social inclusivity of this field, lowering the bar-to-entry to engaging in computational finance research and to using DL algorithms to inform (and improve) the behaviour

of individual financial market players.

7.2 Limitations & Further Work

The research presented within this thesis does exhibit several limitations; most notably, the majority of the efficiency analyses heavily rely on evaluating the total training time of each training algorithm. Whilst this is an effective metric for enumerating the general computational cost of an algorithm, it is not without its flaws (Schwartz et al., 2019). Hence, to give a more holistic analysis of the utility of Green AI within this domain, a wider host of efficiency metrics could have been used. Another limitation is the use of a smaller model architecture, dataset, and training length (and inferior computer hardware) in this research compared to state-of-the-art systems used within DL research and Fintech (which are responsible for the bulk of the ESG costs attributed to DL). Hence, it is unknown how exactly the energy savings demonstrated in this research will scale when Green AI is employed in higher complexity systems.

However, these limitations do not restrict the presented research findings, as it is likely the improvements to energy and data efficiency observed are scalable to higher complexity DL systems. Namely, progressive training is likely to improve the energy efficiency of training complex DNNs employed by large financial institutions. Additionally, the benefits to data efficiency provided by active learning could be used to reduce the storage, communication, and computing costs of the large servers employed by data centres.

Based on these limitations, future work in this field could improve the scope and impact of the presented research by further analysing the implemented Green AI algorithms using a wider spectrum of efficiency metrics, facilitating a more comprehensive overview of their computational cost. For example, each algorithm’s memory consumption should be inspected, highlighting the reduction in memory usage facilitated by methods such as mixed-precision training. Explicitly quantifying the carbon emissions attributable to each algorithm would also be beneficial, possibly using the machine learning emissions calculator of Lacoste et al. (2019).

Furthermore, this research solely focused on improving the efficiency of DL training algorithms; however, both Bender et al. (2021) and Xu et al. (2021) assert that a significant proportion of the ESG impacts of DL algorithms are inflicted after these models have been deployed, not during training. These *inference* costs have been explored within Green AI, such as the research of Lacoste et al. (2019) and Cai et al. (2022). For example, *low-rank factorisation* (Xu et al., 2021) has been shown to effectively reduce the size of DNN architectures, reducing the computational cost of each computation made by the network. Hence, further work could exploit such methods to explore how the ESG impact of DL for finance could be reduced over the entirety of a model’s lifetime.

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