APPLIED MATHEMATICS - III

ASSIGNMENT - 1

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ROLL NO - 16

SECTION - B

SEMESTER - 3RD

BRANCH - CSE

Unit - 1 (Partial differential Equation)

Q1) Solve $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x+3y)$

Soln Given,

$$\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x+3y) \quad —\ ①$$

Integrating eqⁿ ① wrt x

$$\int \frac{\partial^3 z}{\partial x^2 \partial y} \partial x = \int \cos(2x+3y)\, \partial x$$

$$\Rightarrow \frac{\partial^2 z}{\partial x \partial y} = \frac{\sin(2x+3y)}{2} + f(y)$$

$\Rightarrow$ Integrating above eqⁿ wrt y.

$$\int \frac{\partial^2 z}{\partial x \partial y} \partial y = \frac{1}{2}\int \sin(2x+3y)\, \partial y + \int f(y)\, \partial y$$

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{1}{2}\left(-\frac{\cos(2x+3y)}{3}\right) + g(y) + c(x)$$

Integrating above eqⁿ wrt x

$$\int \frac{\partial z}{\partial x} \partial x = \frac{-1}{6}\int \cos(2x+3y)\, \partial x + \int g(y)\, \partial x + \int c(x)\, \partial x$$

$$z = \frac{-1}{6}\left(\frac{\sin(2x+3y)}{2}\right) + x\, g(y) + p(x) + q(y)$$

$$\boxed{z = \frac{-1}{12}\sin(2x+3y) + x\, g(y) + q(y) + p(x)}$$

Q2) Form the partial differential equation from

$$z = f(x^2+y^2,\ z-xy)$$

sol^n → We have to form a partial differential eq^n from

$$z = f(x^2+y^2,\ z-xy)$$

let $x^2+y^2 = U$

$z - xy = V$

so that $F(U,V) = 0$

$$\frac{\partial F}{\partial x} = \frac{\partial F}{\partial U} \times \frac{\partial U}{\partial x} + \frac{\partial F}{\partial V} \times \frac{\partial V}{\partial x}$$

$$\frac{\partial F}{\partial x} = \frac{\partial F}{\partial U} \cdot \frac{\partial (x^2+y^2)}{\partial x} + \frac{\partial F}{\partial V} \times \frac{\partial (z-xy)}{\partial x} = 0$$

$$\Rightarrow \frac{\partial F}{\partial U}(2x) + \frac{\partial F}{\partial V}(p-y) = 0 \qquad \left[p = \frac{\partial z}{\partial x}\right] \quad \text{— ①}$$

$$\frac{\partial F}{\partial y} = \frac{\partial F}{\partial U} \times \frac{\partial U}{\partial y} + \frac{\partial F}{\partial V} \times \frac{\partial V}{\partial y}$$

$$\frac{\partial F}{\partial y} = \frac{\partial F}{\partial U} \cdot \frac{\partial (x^2+y^2)}{\partial y} + \frac{\partial F}{\partial V} \cdot \frac{\partial (z-xy)}{\partial y} = 0$$

$$\frac{\partial F}{\partial U}(2y) + \frac{\partial F}{\partial V}(q-x) = 0 \quad \text{— ②} \quad \left[q = \frac{\partial z}{\partial y}\right]$$

Eliminating $\frac{\partial F}{\partial U}$ and $\frac{\partial F}{\partial V}$ from ① and ② we get

$$\begin{vmatrix} 2x & p-y \\ 2y & q-x \end{vmatrix} = 0$$

$$\boxed{xq - yp = x^2 - y^2}$$

⇒

Q3) Solve $p - q = \log(x+y)$

Sol^n Given,

$$p - q = \log(x+y) \quad — (1)$$

Comparing eq^n (1) with standard lagrange's linear equation we get

$$Pp + Qq = R$$

$$P = 1 \;,\; Q = -1 \;,\; R = \log(x+y)$$

substituting value of P, Q, R in subsidiary eq^n we get

$$\frac{dx}{1} = \frac{dy}{-1} = \frac{dz}{\log(x+y)}$$

Taking $\dfrac{dx}{1} = \dfrac{dy}{-1}$

$$\int dx = -\int dy$$

$$x + y = c_1 \quad — (2)$$

Taking $\dfrac{dy}{-1} = \dfrac{dz}{\log(x+y)}$

sub value of $x+y$ from (2)

$$\frac{dy}{-1} = \frac{dz}{\log c_1}$$

$$\int -\log c_1 dy = \int dz$$

$$-\log c_1 \cdot y = z + c_2$$

$$-z - y\log c_1 = c_2$$

$$z + y\log c_1 = c_2 \quad — a$$

$$z + y\log(x+y) = c_2 \quad — (3)$$

Sol^n of eq^n (1) is

$$\phi(c_1, c_2) = 0$$

$$\boxed{\phi(x+y,\ z+y\log(x+y)) = 0} \quad (\text{from (2) and (3)})$$

Q4) Solve $(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x+2y)$.

<u>Sol^n</u> Given,

$$(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x+2y)$$

$$\phi(D, D') = D^2 + 2DD' + D'^2 - 2D - 2D'$$

$$\phi(D, D') = (D+D')(D+D'-2)$$

Since the sol^n corresponding to the factor $D - mD' - c$ is known to be

$$z = e^{cx} f(y+mx)$$

we get

$$C.F = f_1(y-x) + e^{2x} f_2(y-x)$$

Now

$$P.I = \frac{1}{(D^2 + 2DD' + D'^2 - 2D - 2D')} \sin(x+2y)$$

Here $a = 1 \quad b = 2$

$$D^2 = -a^2 = -1, \quad DD' = -ab = -2$$

$$D'^2 = -b^2 = -4$$

$$P.I = \frac{1}{-1+2(-2)+(-4)-2D-2D'} \cdot \sin(x+2y)$$

$$P.I = \frac{1}{-2(D+D')-9} \sin(x+2y)$$

$$P.I = -\frac{1}{2(D+D')+9} \sin(x+2y)$$

$$P.I = -\frac{2(D+D')-9}{(2(D+D'))^2-(9)^2} \sin(x+2y)$$

$$P.I = -\frac{2(D+D')-9}{4(D^2+2DD'+D'^2)-81} \sin(x+2y)$$

$$P.I = -\frac{2(D+D')-9}{4(-1+2(-2)+(-4))-81} \sin(x+2y)$$

$$P.I = -\frac{2(D+D')-9 \, . \sin(x+2y)}{(-117)}$$

$$P.I = \frac{2(D+D')(\sin(x+2y))}{117} - \frac{9}{117} \sin(x+2y)$$

$$P.I = \frac{2}{117}(3.\cos(x+2y)) - \frac{9}{117} \sin(x+2y)$$

$$P.I = \frac{1}{39}[2\cos(x+2y) - 3\sin(x+2y)]$$

Hence complete solution is

$$z = C.F + P.I$$

$$z = f_1(y-x) + e^{2x} f_2(y-x) + \frac{1}{39}[2\cos(x+2y)-3\sin(x+2y)]$$

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Q5) Solve $(D^2+3DD'+2D'^2)z = 24xy$

Soln Given,

$$(D^2+3DD'+2D'^2)z = 24xy$$

$$D^2+3DD'+2D'^2 = 0$$

A.E is given by ~~D=m~~

$$m^2+3m+2=0$$

$$m^2+2m+m+2=0$$

$$(m+2)(m+1)=0$$

$$m=-1,-2$$

$$C.F = f_1(y-x)+f_2(y-2x)$$

$$P.I = \frac{1}{(D^2+3DD'+2D'^2)} 24xy$$

$$P.I = 24 \cdot (D^2+3DD'+2D'^2)^{-1} xy$$

$$P.I = 24 \cdot \frac{1}{D^2}\left(1+\left(\frac{3D'}{D}+\frac{2D'^2}{D^2}\right)\right)^{-1} xy$$

$$P.I = 24 \cdot \frac{1}{D^2}(1-t+t^2-t^3)xy \quad \left(t = \frac{3D'}{D}+\frac{2D'^2}{D^2}\right)$$

$$P.I = 24 \cdot \frac{1}{D^2}\left(1-\left(\frac{3D'}{D}+\frac{2D'^2}{D^2}\right)\right)xy$$

$$P.I = 24 \cdot \frac{1}{D^2}\left(1-\frac{3D'}{D}\right)xy$$

$$P.I = 24 \cdot \left(\frac{1}{D^2}-\frac{3D'}{D^3}\right)xy$$

$$P.I = 24\left[\frac{1}{D^2}(xy) - \frac{3D'}{D^3}(xy)\right]$$

$P.I = 24 \cdot \left[\iint xy\, dx - 3 \iiint x\, dx \right]$

$P.I = 24 \left(\frac{x^3 y}{6} - \frac{x^4}{8} \right)$

$P.I = 4x^3 y - 3x^4$

Hence complete solution is given by

$Z = C.F + P.I$

$$\boxed{Z = f_1(y-x) + f_2(y-2x) + 4x^3 y - 3x^4}$$

Q6) Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2\sin y$ when $x=0$ and $z=0$ when y is an odd multiple of $\frac{\pi}{2}$

Sol^n Given equation,

$$\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y \quad -(1)$$

Integrating eq^n (1) wrt x

$$\int \frac{\partial^2 z}{\partial x \partial y} \partial x = \int \sin x \sin y\, \partial x$$

$$\Rightarrow \quad \frac{\partial z}{\partial y} = -\cos x \sin y + f(y) \quad -(2)$$

when $x=0$ $\quad \frac{\partial z}{\partial y} = -2\sin y$

$$\therefore \quad -2\sin y = -\sin y + f(y)$$

$$\text{or } f(y) = -\sin y$$

Sub $f(y)$ in ③ we get

$$\frac{\partial z}{\partial y} = -\cos x \sin y - \sin y$$

Now integrating wrt y

$$\int \frac{\partial z}{\partial y} dy = \int -\cos x \sin y \, dy - \int \sin y \, dy$$

$$z = \cos x \cos y + \cos y + g(x)$$

when y is odd multiple of $\pi/2$ we get $z=0$

$$0 = 0 + 0 + g(x) \quad \text{or} \quad g(x) = 0 \quad \left[\because \cos(2n+1)\frac{\pi}{2} = 0\right]$$

$$z = \cos x \cos y + \cos y$$

or

$$\boxed{z = \cos y\,(1 + \cos x)}$$

Q7) Solve by method of seperation of variable

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u \quad \text{where} \quad u(x,0) = 6e^{-3x}$$

sol^n Assume the solution

$$u(x,t) = f(x) \cdot g(t) \quad — ①$$

substituting in eq^n we get

$$\frac{\partial u}{\partial x} = f'g = \frac{\partial f}{\partial x} g$$

$$\frac{\partial u}{\partial t} = fg' = f\frac{\partial g}{\partial t}$$

$$f'g = 2fg' + fg$$

$$(f' - f)g = 2fg'$$

$$\frac{(f'-f)g}{2f} = \frac{g'}{g} = k$$

$\therefore$ $f'-f = 2fk$

$f'-f-2fk = 0$

$\frac{f'}{f} = 1+2k$

$\log f = (1+2k)x + \log c$

$f = ce^{(1+2k)x}$

$\frac{g'}{g} = k$

$\log g = kt + \log c'$

$g = c'e^{kt}$

$u(x,t) = cc'e^{(1+2k)x}e^{kt}$ — (2)

$u(x,0) = 6e^{-3x}$

$6e^{-3x} = cc'^{(1+2k)x}$

Comparing $cc' = 6$ $\quad 1+2k = -3$

$k = -2$

substituting values in (2)

$u = 6e^{-3x}e^{-2t}$

i.e $\boxed{u = 6e^{-(3x+2t)}}$

Unit - 2

FOURIER SERIES

Q1) Find fourier series for function

$$f(x) = x\sin x \quad . \quad 0 \le x \le 2\pi$$

Sol^n Fourier series for function $f(x)$ in the interval (α to $\alpha + 2\pi$) is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad —(1)$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} x\sin x\,dx$$

$$a_0 = \frac{1}{\pi}\left[x\int \sin x\,dx - \int \frac{d(x)}{dx}\int \sin x\,dx\right]_0^{2\pi}$$

$$a_0 = \frac{1}{\pi}\left[-x\cos x + \sin x\right]_0^{2\pi}$$

$$a_0 = \frac{1}{\pi}\left[-2\pi\right]$$

$$\boxed{a_0 = -2}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x(\sin x\cos nx)\,dx$$

$$a_n = \cancel{\frac{1}{\pi}\left[x\right.}\ \frac{1}{2\pi}\int_0^{2\pi} x(2\sin x\cos nx)\,dx$$

$$a_n = \frac{1}{2\pi}\int_0^{2\pi} x\left[\sin(n+1)x - \sin(n-1)x\right]dx$$

$$a_n = \frac{1}{2\pi}\left[\int_0^{2\pi} x\sin(n+1)x\,dx - \int_0^{2\pi} x\sin(n-1)x\,dx\right]$$

$$a_n = \frac{1}{2\pi}\left[x\left(-\frac{\cos(n+1)x}{n+1}\right) + \frac{\sin(n+1)x}{(n+1)^2} - \left(x\left(-\frac{\cos(n-1)x}{n-1}\right) + \frac{\sin(n-1)x}{(n-1)^2}\right)\right]_0^{2\pi}$$

$$a_n = \frac{1}{2\pi}\left[x\left(-\frac{\cos(n+1)x}{n+1} + \frac{\cos(n-1)x}{n-1}\right) + \frac{\sin(n+1)x}{(n+1)^2} - \frac{\sin(n-1)x}{(n-1)^2}\right]_0^{2\pi}$$

$$a_n = \frac{1}{2\pi}\left[2\pi\left(-\frac{\cos(n+1)2\pi}{n+1} + \frac{\cos(n-1)2\pi}{n-1}\right)\right]$$

$$a_n = \frac{2}{n^2-1} \quad (n \neq 1)$$

when $n=1$ $\quad a_1 = \frac{1}{\pi}\int_0^{2\pi} x\sin x\cos x\,dx$

$$a_1 = \frac{1}{2\pi}\int_0^{2\pi} x\sin 2x\,dx$$

$$a_1 = \frac{1}{2\pi}\left[-\frac{x\cos 2x}{2} + \frac{\sin 2x}{4}\right]_0^{2\pi}$$

$$a_1 = \frac{1}{2\pi}\left[-\frac{2\pi\cos 4\pi}{2} +\right] = -\frac{1}{2}$$

$$a_1 = -\frac{1}{2}$$

Now

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x\sin x\sin nx\,dx$$

$$b_n = \frac{1}{2\pi}\int_0^{2\pi} x(2\sin x\sin nx)\,dx$$

$$b_n = \frac{1}{2\pi}\int_0^{2\pi} x\left[\cos(n-1)x\,\cancel{dx} - \cos(n+1)x\right]dx$$

$$b_n = \frac{1}{2\pi}\left[\int_0^{2\pi} x\cos(n-1)x\,dx - \int_0^{2\pi}\cos(n+1)x\,dx\right]$$

$$b_n = \frac{1}{2\pi}\left[x\left(\frac{\sin(n-1)x}{n-1} - \frac{\sin(n+1)}{n+1}\right) - 1\left(-\frac{\cos(n-1)x}{(n-1)^2} + \frac{\cos(n+1)x}{(n+1)^2}\right)\right]_0^{2\pi}$$

$$b_n = \frac{1}{2\pi}\left[\frac{\cos\,2(n-1)\pi}{(n-1)^2} - \frac{\cos(2n+1)\pi}{(n+1)^2} - \frac{1}{(n-1)^2} + \frac{1}{(n+1)^2}\right]$$

$$b_n = 0 \quad (n \neq 1)$$

when $n=1$ $\quad b_1 = \frac{1}{\pi}\int_0^{2\pi} x\sin x\sin x\,dx = \frac{1}{2\pi}\int_0^{2\pi} x(1-\cos 2x)\,dx$

$$b_1 = \frac{1}{2\pi}\left[\frac{x^2}{2} - \left(\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right)\right]_0^{2\pi}$$

$$b_1 = \frac{1}{2\pi}\left[\frac{4\pi^2}{2} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}}\right]$$

$$b_1 = \pi$$

substituting values of a_0, b_n a_n, b_n in (1)

$$x\sin x = \frac{-\cancel{2}}{\cancel{2}} + \sum_{n=2}^{\infty}\left(\frac{2}{n^2-1}\right)\cos nx + \sum_{n=2}^{\infty}(0)\sin nx + \left(\frac{-1}{2}\right)^{\cos x} + \pi\sin x$$

$$x\sin x = -1 + \pi\sin x - \frac{1}{2}\cos x + \frac{2}{2^2-1}\cos 2x + \frac{3}{3^2-1}\cos 3x + \cdots$$

Q2) Find Fourier series for periodic function

$f(x) = x + x^2$, $-\pi < x < \pi$ and $f(x) = \pi^2$, $x = \pm\pi$

Deduce the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$

Sol^n Given,

$$f(x) = x + x^2 \quad , \quad -\pi < x < \pi$$

$$f(x) = \pi^2 \quad , \quad x = \pm\pi$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad \text{—(1)}$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} (x + x^2)\, dx$$

$$a_0 = \frac{1}{\pi}\left[\frac{x^2}{2} + \frac{x^3}{3}\right]_{-\pi}^{\pi} = \frac{1}{\pi}\left(\cancel{\frac{\pi^2}{2}} + \frac{\pi^3}{3} - \cancel{\frac{\pi^2}{2}} + \frac{\pi^3}{3}\right)$$

$$a_0 = \frac{1}{\pi}\left(\frac{2\pi^3}{3}\right) = \frac{2\pi^2}{3}$$

$$\boxed{a_0 = \frac{2\pi^2}{3}}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (x + x^2) \cos nx\, dx$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} x \cos nx\, dx + \int_{-\pi}^{\pi} x^2 \cos nx\, dx\right]$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} x^2 \cos nx\, dx\right]$$

$$a_n = \frac{1}{\pi}\left[x^2\frac{\sin nx}{n} - (2x)\left(-\frac{\cos nx}{n^2}\right) + 2\left(-\frac{\sin nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[-2\pi\left(-\frac{\cos n\pi}{n^2}\right) + 2(-\pi)\left(-\frac{\cos n(-\pi)}{n^2}\right)\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{2\pi\cos n\pi}{n^2} + \frac{2\pi\cos n\pi}{n^2}\right]$$

$$a_n = \frac{4(-1)^n}{n^2} \quad [\cos n\pi = (-1)^n]$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(x+x^2)\sin nx\,dx$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi}x\sin nx\,dx + \int_{-\pi}^{\pi}x^2\sin nx\,dx\right]$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi}x\sin nx\,dx\right]$$

$$b_n = \frac{1}{\pi}\left[-x\frac{\cos nx}{n} - (\cancel{1})\left(\frac{-\sin nx}{n^2}\right)\right]_{-\pi}^{\pi}$$

$$b_n = \frac{1}{\pi}\left(-\frac{\pi\cos n\pi}{n} + \frac{(-\pi)\cos n(-\pi)}{n}\right)$$

$$b_n = \frac{1}{\cancel{\pi}}\left(-\frac{2\cancel{\pi}\cos n\pi}{n}\right) = \frac{-2(-1)^n}{n}$$

subs a_0, a_n, b_n in (1)

$$f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty}\frac{4(-1)^n}{n^2}\cos nx + \sum_{n=1}^{\infty}\frac{-2(-1)^n}{n}\sin nx$$

$$f(x) = \frac{\pi^2}{3} + 4\left(\frac{-\cos x}{1^2} + \frac{\cos 2x}{2^2} - \frac{\cos 3x}{3^2}+\right) - 2\left(-\frac{\sin x}{1^2} + \frac{\sin 2x}{2^2} - \frac{\sin 3x}{3^2} + \cdots\right)$$

when $x = \pi$ $\quad f(x) = \pi^2$

$$\pi^2 = \frac{\pi^2}{3} + 4\left(\frac{-\cos\pi}{1^2} + \frac{\cos 2\pi}{2^2} - \frac{\cos 3\pi}{3^2}\right) - 2(0) \quad (\sin n\pi = 0)$$

$$\pi^2 = \frac{\pi^2}{3} + 4\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2}\right)$$

$$\frac{2\pi^2}{3} = 4\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2}\right)$$

$$\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}\cdots\right) = \frac{\pi^2}{6} \quad — (2)$$

when $x=0$ $f(x)=0$

$$0 = \frac{\pi^2}{3} + 4\left(-\frac{1}{1^2}+\frac{1}{2^2}-\frac{1}{3^2}\cdots\right)$$

$$\cancel{-}\frac{\pi^2}{3} = \cancel{-}4\left(\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}+\cdots\right)$$

$$\frac{\pi^2}{12} = \left(\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}+\cdots\right) - (3)$$

Adding (2) and (3) we get

$$\frac{\pi^2}{8} = \left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\right)$$

Q3) Find half range sine series for

$$f(x) = x - x^2 \ , \ 0<x<1$$

sol^n Half range sine series for $f(x) = \cancel{x^2}\, x - x^2$ in $(0,1)$ is

$$f(x) = \sum_{n=1}^{\infty} b_n \sin(n\pi x) \quad — (1)$$

$\cancel{b_n = \frac{1}{\pi}\int}$ $b_n = 2\int_0^1 (x-x^2)\sin n\pi x \, dx$

$$b_n = 2\left[(x-x^2)\left(-\frac{\cos n\pi x}{n\pi}\right) - (1-2x)\left(-\frac{\sin n\pi x}{n^2\pi^2}\right) + (-2)\left(\frac{\cos n\pi x}{n^3\pi^3}\right)\right]_0^1$$

$$b_n = 2\left[-2\frac{\cos n\pi}{n^3\pi^3} + \frac{2\,\cancel{\cos}\,\pi}{n^3\pi^3}\right]$$

$$b_n = \frac{4}{n^3\pi^3}\left(-(-1)^n+1\right)$$

$$f(x) = \sum_{n=1}^{\infty} \frac{4}{n^3\pi^3} \left(\overset{1-(-1)^n}{\cancel{(-1)^n-1}}\right) \sin(n\pi x)$$

$$f(x) = \frac{4}{\pi^3} \sum_{n=1}^{1} \left(\frac{\overset{1-(-1)^n}{\cancel{(-1)^n-1}}}{n^3}\right) \sin(n\pi x)$$

$$f(x) = \frac{4}{\pi^3}\left[\frac{2\sin(\pi x)}{1^3} + \frac{2}{3^3}\sin(3\pi x) + \frac{2}{5^3}\sin(5\pi x) + \cdots\right]$$

Half range sine series is given by

$$x - x^2 = \frac{8}{\pi^3}\left[\frac{\sin(\pi x)}{1^3} + \frac{\sin(3\pi x)}{3^3} + \frac{\sin(5\pi x)}{5^3} + \cdots\right]$$

(Q4) Find half range cosine series for

$$f(x) = \begin{cases} kx & \text{if } 0 \le x \le l/2 \\ k(l-x) & \text{if } l/2 \le x \le l \end{cases}$$

Hence deduce the sum of series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$

Sol^n: Half range cosine series is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

Then ~~a_0 =~~ Here $f(x) = \begin{cases} kx & \text{if } 0 \le x \le l/2 \\ k(l-x) & \text{if } l/2 \le x \le l \end{cases}$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx = \frac{2}{l}\left[\int_0^{l/2} kx\,dx + \int_{l/2}^{l} k(l-x)\,dx\right]$$

$$a_0 = \frac{2}{l}\left[\left[\frac{Kx^2}{2}\right]_0^{l/2} + \left[Klx - \frac{Kx^2}{2}\right]_{l/2}^{l}\right]$$

$$a_0 = \frac{2}{l}\left[\frac{K}{2}\cdot\frac{l^2}{4} + Kl^2 - \frac{Kl^2}{2} - \frac{Kl^2}{2} + \frac{K}{2}\cdot\frac{l^2}{4}\right]$$

$$a_0 = \frac{2}{l}\left(\frac{Kl^2}{4}\right) = \frac{Kl}{2}$$

$$a_0 = \frac{Kl}{2}$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$a_n = \frac{2}{l}\left[\int_0^{l/2} kx\cos\left(\frac{n\pi x}{l}\right)dx + \int_{l/2}^{l} k(l-x)\cos\left(\frac{n\pi x}{l}\right)dx\right]$$

$$a_n = \frac{2}{l}\left[\left[\frac{Kx\sin\left(\frac{n\pi x}{l}\right)}{\frac{n\pi}{l}} - \frac{K\left(-\cos\left(\frac{n\pi x}{l}\right)\right)}{\frac{n^2\pi^2}{l^2}}\right]_0^{l/2}\right.$$

$$\left.+\left[\frac{K(l-x)\sin\left(\frac{n\pi x}{l}\right)}{\frac{n\pi}{l}} - \frac{K(-1)\left(-\cos\left(\frac{n\pi x}{l}\right)\right)}{\frac{n^2\pi^2}{l^2}}\right]_{l/2}^{l}\right]$$

$$a_n = \frac{2}{l}\left[\cancel{\frac{Kl}{2}\times\frac{l}{n\pi}\sin\left(\frac{n\pi}{2}\right)} + K\cos\left(\frac{n\pi}{2}\right)\times\frac{l^2}{n^2\pi^2} - \frac{Kl^2}{n^2\pi^2}\right.$$

$$\left. - K\cos n\pi\times\frac{l^2}{n^2\pi^2} \cancel{- \frac{Kl}{2}\sin\left(\frac{n\pi}{2}\right)\times\frac{l}{n\pi}} + K\cos\left(\frac{n\pi}{2}\right)\times\frac{l^2}{n^2\pi^2}\right.$$

$$a_n = \frac{2}{l}\left[2K\cos\left(\frac{n\pi}{2}\right)\times\frac{l^2}{n^2\pi^2} - \frac{Kl^2}{n^2\pi^2} - K\cos n\pi\times\frac{l^2}{n^2\pi^2}\right]$$

$$a_n = \frac{2}{l}\times\frac{K\times l^2}{n^2\pi^2}\left(2\cos\frac{n\pi}{2} - 1 - (-1)^n\right)$$

$$a_n = \frac{2Kl}{n^2\pi^2}\left(2\cos\frac{n\pi}{2} - 1 - (-1)^n\right)$$

$$f(x) = \frac{Kl}{4} + \sum_{n=1}^{\infty} \cdot \frac{2Kl}{n^2\pi^2}\left(2\cos\frac{n\pi}{2} - 1 - (-1)^n\right)\cos\left(\frac{n\pi x}{l}\right)$$

$$f(x) = \frac{Kl}{4} + \cancel{\sum_{n=1}^{\infty}} \frac{2Kl}{\pi^2}\sum_{n=1}^{\infty}\left(\frac{2\cos\frac{n\pi}{2} - 1 - (-1)^n}{n^2}\right)\cos\left(\frac{n\pi x}{l}\right)$$

$$f(x) = \frac{Kl}{4} + \left(0 - \frac{8Kl}{2^2\pi^2} + 0 - \frac{8Kl}{6^2\pi^2} + 0 - \frac{8Kl}{10^2\pi^2} + \cdots\right)\cos\left(\frac{n\pi x}{l}\right)$$

$$f(x) = \frac{Kl}{4} - \frac{8Kl}{\pi^2}\left(\frac{\cos\left(\frac{2\pi x}{l}\right)}{2^2} + \frac{\cos\left(\frac{6\pi x}{l}\right)}{6^2} + \frac{\cos\left(\frac{10\pi x}{l}\right)}{10^2} + \cdots\right)$$

putting $x = l$ we get $f(x) = 0$

$$0 = \frac{Kl}{4} - \frac{8Kl}{\pi^2}\left(\frac{1}{2^2} + \frac{1}{6^2} + \frac{1}{10^2} + \cdots\infty\right)$$

$$0 = \frac{Kl}{4} - \frac{8Kl}{4\pi^2}\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\infty\right)$$

$$\frac{\cancel{8Kl}}{\cancel{4}\pi^2}\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\infty\right) = \frac{Kl}{4}$$

$$\boxed{\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\infty = \frac{\pi^2}{8}}$$

(Q5). Obtain the first three coefficients ~~of~~ in the fourier cosine series expansion of y as given in the following table

x	0	1	2	3	4	5
y	4	8	15	7	6	2

Sol^n Taking interval as 60° we have

$\theta = 0^\circ \quad 60^\circ \quad 120^\circ \quad 180^\circ \quad 240^\circ \quad 300^\circ$

$x = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$

$y = 4 \quad 8 \quad 15 \quad 7 \quad 6 \quad 2$

Fourier series in the interval $(0, 2\pi)$ is

$$y = \frac{a_0}{2} + a_1\cos\theta + a_2\cos 2\theta + a_3 \cos 3\theta + \cdots$$

θ°	$\cos\theta$	$\cos 2\theta$	$\cos 3\theta$	y	$y\cos\theta$	$y\cos 2\theta$	$y\cos 3\theta$
0°	1	1	1	4	4	4	4
60°	1/2	-1/2	-1	8	4	-4	-8
120°	-1/2	-1/2	1	15	-7.5	-7.5	15
180°	-1	1	-1	7	-7	7	-7
240°	-1/2	-1/2	1	6	-3	-3	6
300°	1/2	-1/2	-1	2	1	-1	-2
			$\Sigma =$	42	-8.5	-4.5	8

Hence $a_0 = 2\cdot\left(\frac{42}{6}\right) = 14$

$a_1 = 2\left(\frac{-8.5}{6}\right) = -2.8$

$a_2 = 2\left(\frac{-4.5}{6}\right) = -1.5$

$a_3 = 2\left(\frac{8}{6}\right) = 27$

(Q6) The following table gives the variations of periodic current over a period

t (sec)	0	T/6	T/3	T/2	2T/3	5T/6	T
A (amp)	1.98	1.30	1.05	1.30	−0.88	−0.25	1.98

Show that there is a direct current of 0.75 amp in the variable current and obtain the amplitude of the first harmonic

Sol^n Here Length of Interval is T, i.e $C = T/2$

$$A = \frac{a_0}{2} + a_1\cos\left(\frac{2\pi t}{T}\right) + b_1\sin\left(\frac{2\pi t}{T}\right) + a_2\cos\left(\frac{4\pi t}{T}\right) + b_2\sin\left(\frac{4\pi t}{T}\right) \cdots$$

The desired values are tabulated as follows

t	$2\pi t/T$	$\cos(2\pi t/T)$	$\sin(2\pi t/T)$	A	$A\cos(2\pi t/T)$	$A\sin(2\pi t/T)$
0	0	1.0	0.000	1.98	1.980	0.000
T/6	π/3	0.5	0.866	1.30	0.650	1.126
T/3	2π/3	−0.5	0.866	1.05	−0.525	0.909
T/2	π	−1.0	0.000	1.30	−1.300	0.000
2T/3	4π/3	−0.5	−0.866	−0.88	0.440	0.762
5T/6	5π/3	0.5	−0.866	−0.25	−0.125	0.217
			Σ =	4.5	1.12	3.014

$$a_0 = 2\cdot\frac{1}{6}\Sigma A = 2\cdot\frac{1}{6}(4.5) = 1.5$$

$$a_1 = 2 \cdot \frac{1}{6} \sum A \cos \frac{2\pi t}{T}$$

$$a_1 = \frac{1}{3} \cdot (1.12) = 0.373$$

$$\cancel{a_0} \cdot b_1 = 2 \cdot \frac{1}{6} \sum A \sin\left(\frac{2\pi t}{T}\right)$$

$$b_1 = \frac{1}{3}(3.014) = 1.005$$

Thus direct current part of variable current $= a_0/2 = 0.75$

and amplitude of first harmonic $= \sqrt{(a_1^2 + b_1^2)}$

$$= \sqrt{(0.373)^2 + (1.005)^2}$$

$$= 1.072$$