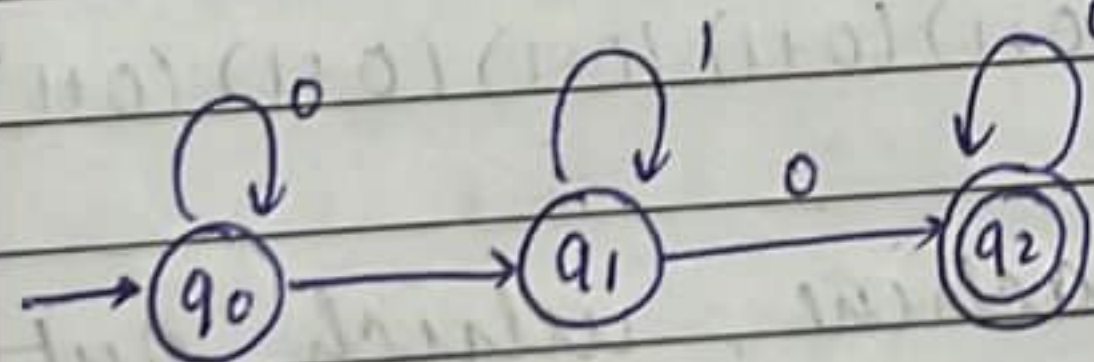
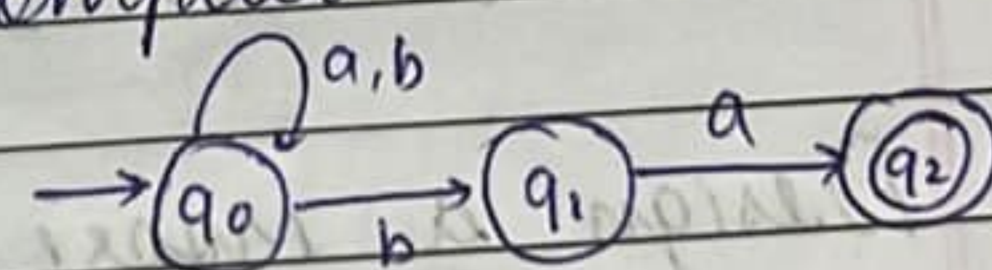
ASSIGNMENT 01

Q1. Differentiate between NFA and DFA with an example. Why DFA is a fast recognizer than NFA.

DFA	NFA
• Each transition leads to excatly one state called as deterministic	• A transition leads to a subset of states i.e some transition can be non-deterministic
• Accepts input if the last state is in final.	• Accepts input if one of the last states is in final
• Requires more space	• Requires less space
• Empty strings ~~are A~~ transition are not seen in DFA	• Primits ~~empty~~ empty string transition.
• $\delta : Q \times \Sigma \rightarrow Q$	• $\delta : Q \times \Sigma \longrightarrow 2^Q$
• DFA is more diffucult to construct	• NFA is easier to construct.
• DFA is a subset of NFA	• Need to convert NFA to DFA in the desgin of a compiler
• 	• 

DFA is a fast recognizer than NFA

Q2. Construct regular expressions for the following language

i. A language containing all the words over [a, b, c] ending ~~e~~ in a.

$$(a+b+c)^* a$$

ii A language containing all the words over [0, 1] not having "00" as a substring

$$(a+1)^* 00 (0+1)^*$$

iii Set of all strings over [a, b] containing exactly 2 a's

$$b^* a b^* a b^*$$

iv Write the regular expression for the set of strings of 0's and 1's whose tenth symbol from the right end is 1.

$$(0+1)^* 1 (0+1)^9$$ OR ~~$(0+1)^* 1 (0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)$~~

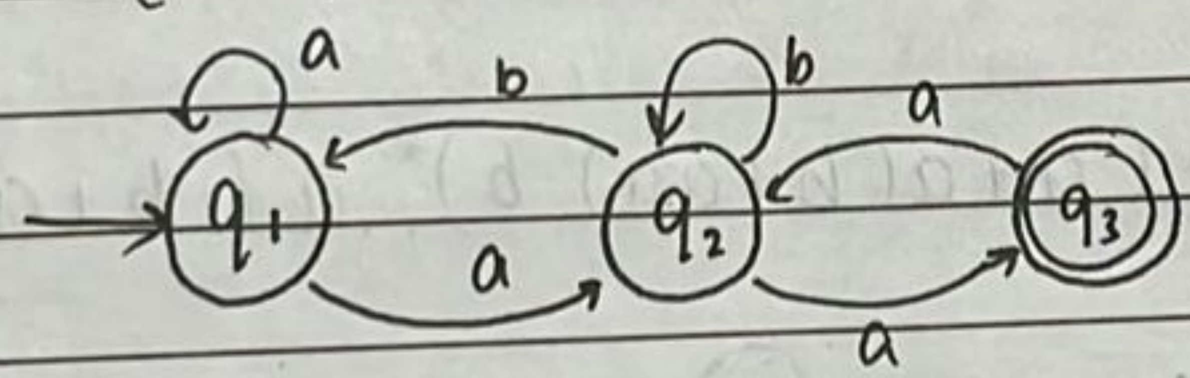
$$(0+1)^* 1 (0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)$$

Q3. Design a Moore machine, which outputs residue mod 3 for each binary input string treated as a binary integer.

$$\Sigma = \{0, 1\}$$
$$\Delta = \{0, 1, 2\}$$

PRESENT STATE	NEXT STATE		OUTPUT
	a=0	a=1	
q_0	q_0	q_1	0
q_1	q_2	q_0	1
q_2	q_1	q_2	2

Q4. Consider the ~~th~~ transition system and prove that the string recognized by it is $(a+a(b+aa)^*b)^*a(b+aa)^*a$



$q_1 = q_1 a + q_2 b + \epsilon$ — (1)

$q_2 = q_1 a + q_2 b + q_3 a$ — (2)

$q_3 = q_2 a$ — (3)

From (2)

$q_2 = q_1 a + q_2 b + q_3 a$

From (3)

$q_2 = q_1 a + q_2 b + q_2 a a$

$$q_2 = q_1 a + q_2(b + aa)$$
$$R = Q + RP$$
$$R = QP^*$$
$$q_2 = q_1 a (b+aa)^* \quad (4)$$

[Arden's Theorem
$R = q_2$
$Q = q_1 a$
$P = (b + aa)$

FROM (1)

$$q_1 = \epsilon + q_1 a + q_2 b$$
$$q_1 = \epsilon + q_1 a + [q_1 a(b+aa)^*] b$$
$\cancel{R = Q + RP}$
$$q_1 = \epsilon + q_1(a + a(b+aa)^* b$$
$$R = Q + RP$$
$$R = QP^*$$
$$q_1 = \epsilon (a + a(b+aa)^* b)^*$$
$$q_1 = (a + a(b+aa)^* b)^* \quad (5)$$

FROM q_5 & q_4

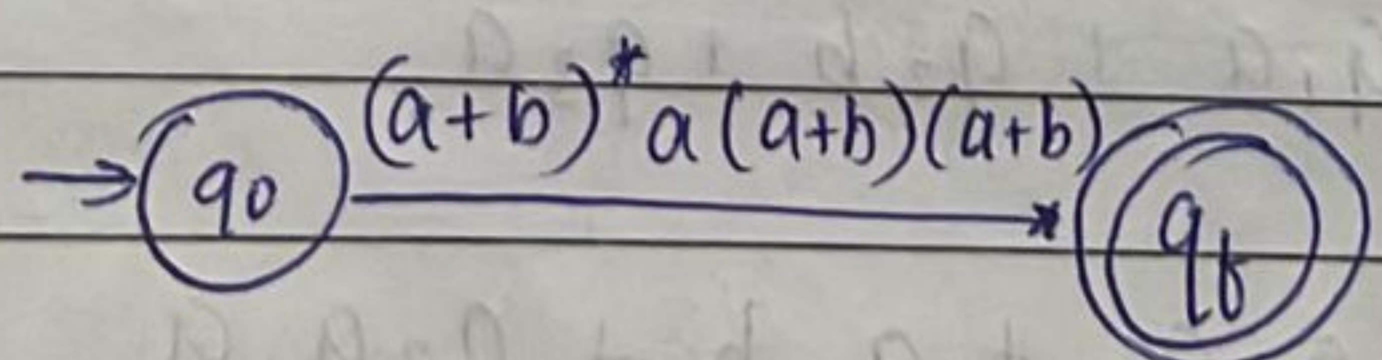
$$q_2 = (a + a(b+aa)^* b)^* a (b+aa)^* \quad (6)$$

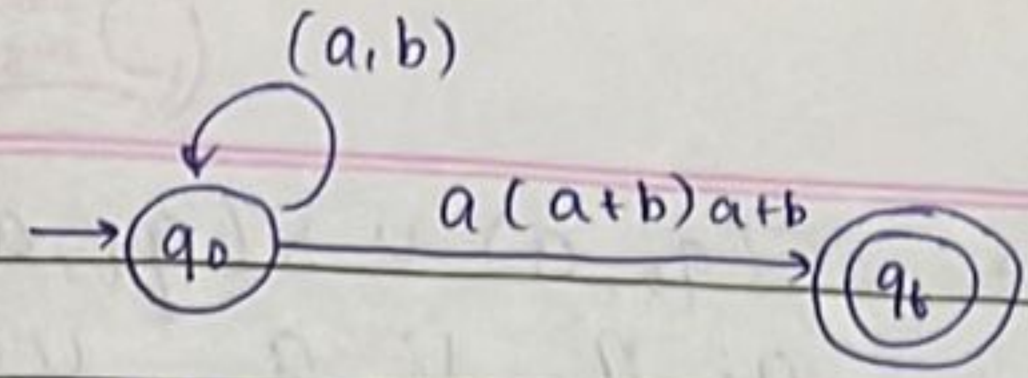
Putting value q_2 in (3)

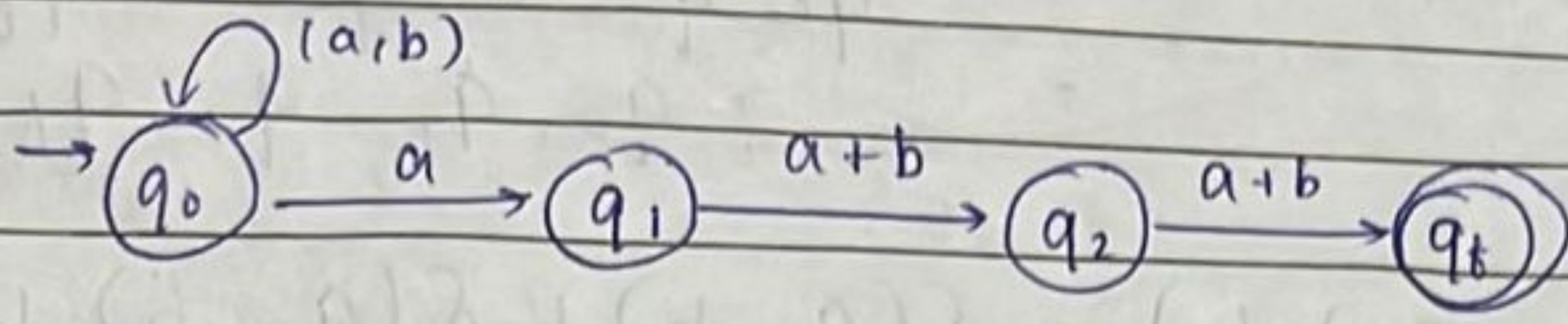
$$q_3 = (a + a(b+aa)^* b)^* a (b+aa)^* a$$

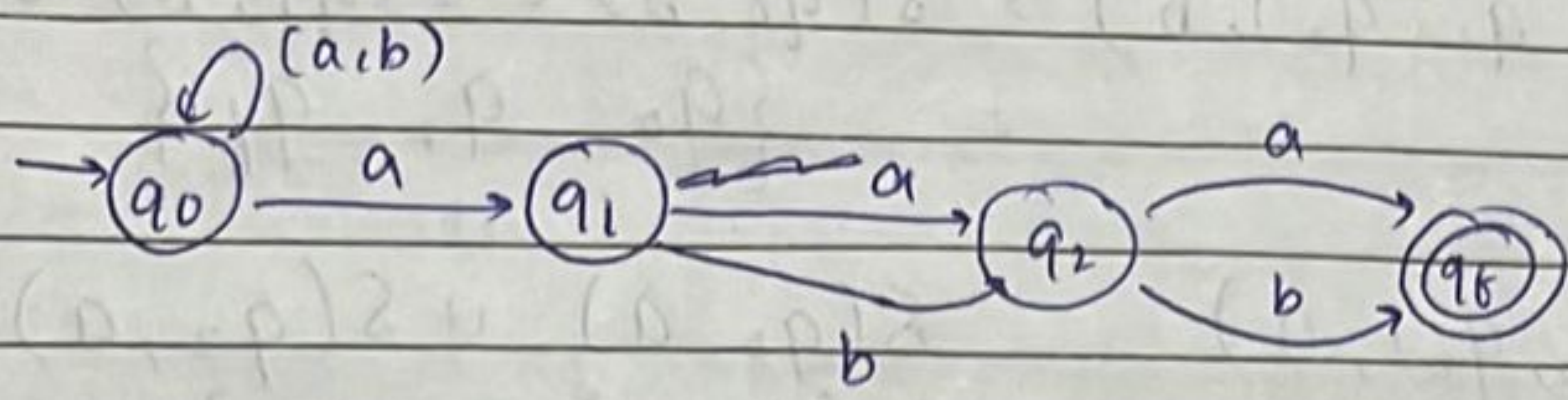
5. construct a minimum state DFA for the following regular expressions

i $(a+b)^* a (a+b)(a+b)$









State	NEXT	
	a	b
$\rightarrow q_0$	q_0, q_1	q_0
q_1	q_2	q_2
q_2	q_f	q_f
q_f	ϕ	ϕ

$$\delta'(q_0, a) = \delta(q_0, a)$$
$$= \{q_0, q_1\}$$ new

$$\delta'(q_0, b) = \delta(q_0, b)$$
$$= \{q_0\}$$

$$\delta'((q_0, q_1), a) = \delta(q_0, a) \cup (q_1, a)$$
$$= \{q_0, q_1\} \cup \{q_2\}$$
$$= \{q_0, q_1, q_2\}$$ new

$$\delta'((q_0, q_1), b) = \delta(q_0, b) \cup \delta(q_1, b)$$
$$= q_0 \cup q_2$$
$$= \{q_0, q_2\}$$ new.

$$\delta'((q_0, q_1, q_2), a) = \delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_2, a)$$
$$= q_0, q_1 \cup q_2 \cup q_f$$
$$= \{q_0, q_1, q_2, q_f\}$$

$$\delta'((q_0, q_1, q_2), b) = \delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_2, b)$$
$$= \{q_0, q_2, q_f\}$$

$$\delta'((q_0, q_2), a) = \delta(q_0, a) \cup \delta(q_2, a)$$
$$= \{q_0, q_1, q_f\}$$

$$\delta'((q_0, q_2), b) = \delta(q_0, b) \cup \delta(q_2, b)$$
$$= \{q_0, q_f\}$$

$$\delta'((q_0, q_1, q_2, q_f), a) = \delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_2, a) \cup \delta(q_f, a)$$
$$= \{q_0, q_1, q_2, q_f\}$$

$$\delta'((q_0, q_1, q_2, q_f), b) = \delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_2, b) \cup \delta(q_f, b)$$
$$= \{q_0, q_2, q_f\}$$

$$\delta'((q_0, q_2, q_f), a) = \delta(q_0, a) \cup \delta(q_2, a) \cup \delta(q_f, a)$$
$$= \{q_0, q_1, q_f\}$$

$$\delta'((q_0, q_2, q_f), b) = \delta(q_0, b) \cup \delta(q_2, b) \cup \delta(q_f, b)$$
$$= \{q_0, q_f\}$$

$$\delta'((q_0, q_1, q_f), a) = \delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_f, a)$$
$$= \{q_0, q_1, q_2\}$$

$$\delta'((q_0, q_1, q_f), b) = \delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_f, b)$$
$$= \{q_0, q_2\}$$

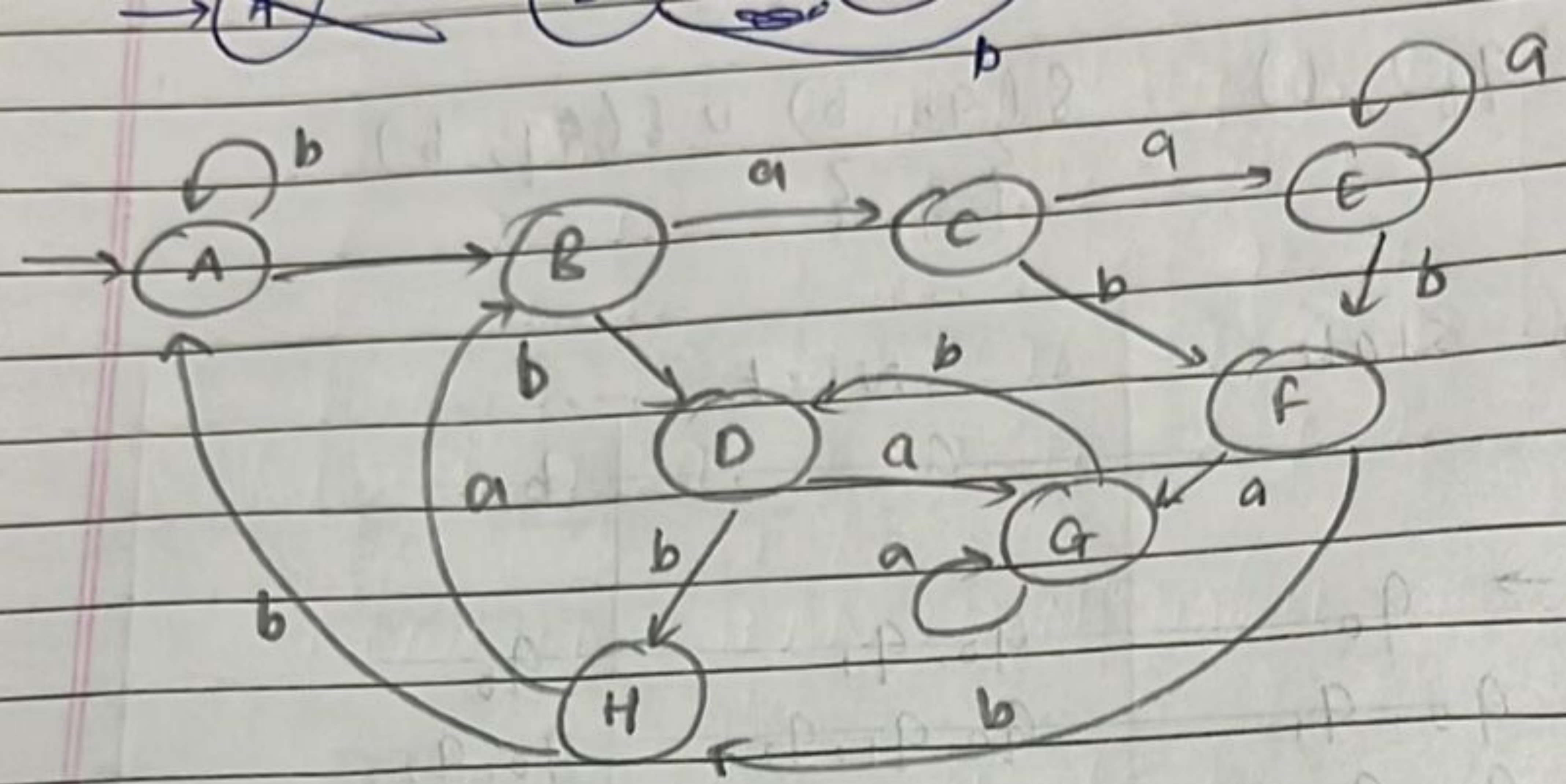
$$\delta'((q_0, q_f), a) = \delta(q_0, a) \cup \delta(q_f, a) = \{q_0, q_1\}$$

$$\delta'((q_0, q_f), b) = \delta(q_0, b) \cup \delta(q_f, b) = \{q_0\}$$

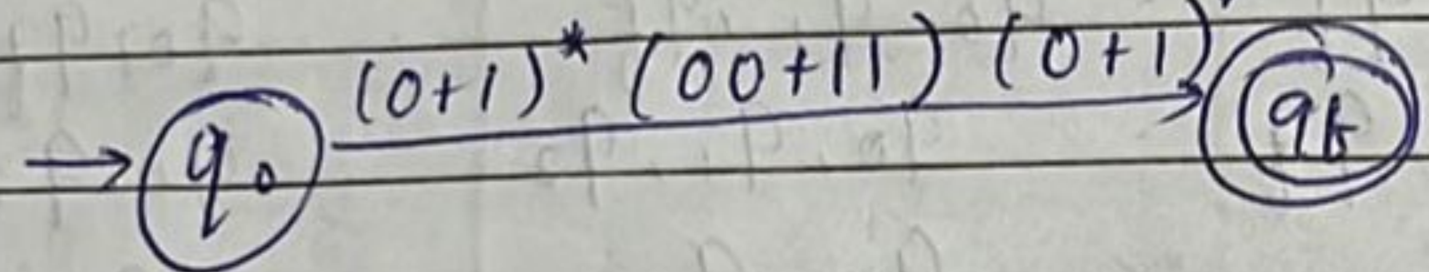
	State	Next a	Next b
A	$\rightarrow q_0$	q_0, q_1	q_0
B	q_0, q_1	q_0, q_1, q_2	q_0, q_2
C	q_0, q_1, q_2	q_0, q_1, q_2, q_f	q_0, q_2, q_f
D	q_0, q_2	q_0, q_1, q_f	q_0, q_f
E	q_0, q_1, q_2, q_f	q_0, q_1, q_2, q_f	q_0, q_2, q_f
F	q_0, q_2, q_f	q_0, q_1, q_f	q_0, q_f
G	q_0, q_1, q_f	q_0, q_1, q_2	q_0, q_2
H	q_0, q_f	q_0, q_1	q_0

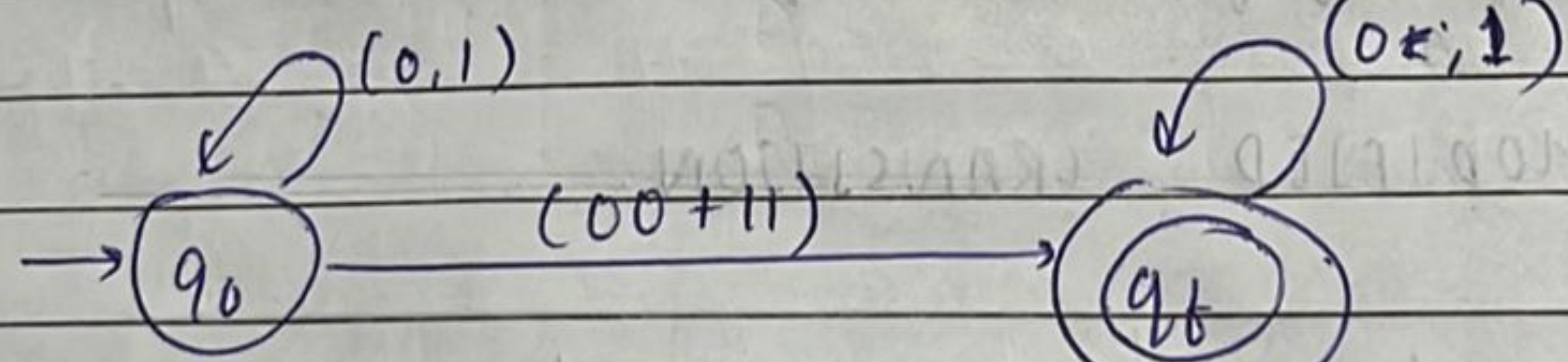
MODIFIED TRANSITION.

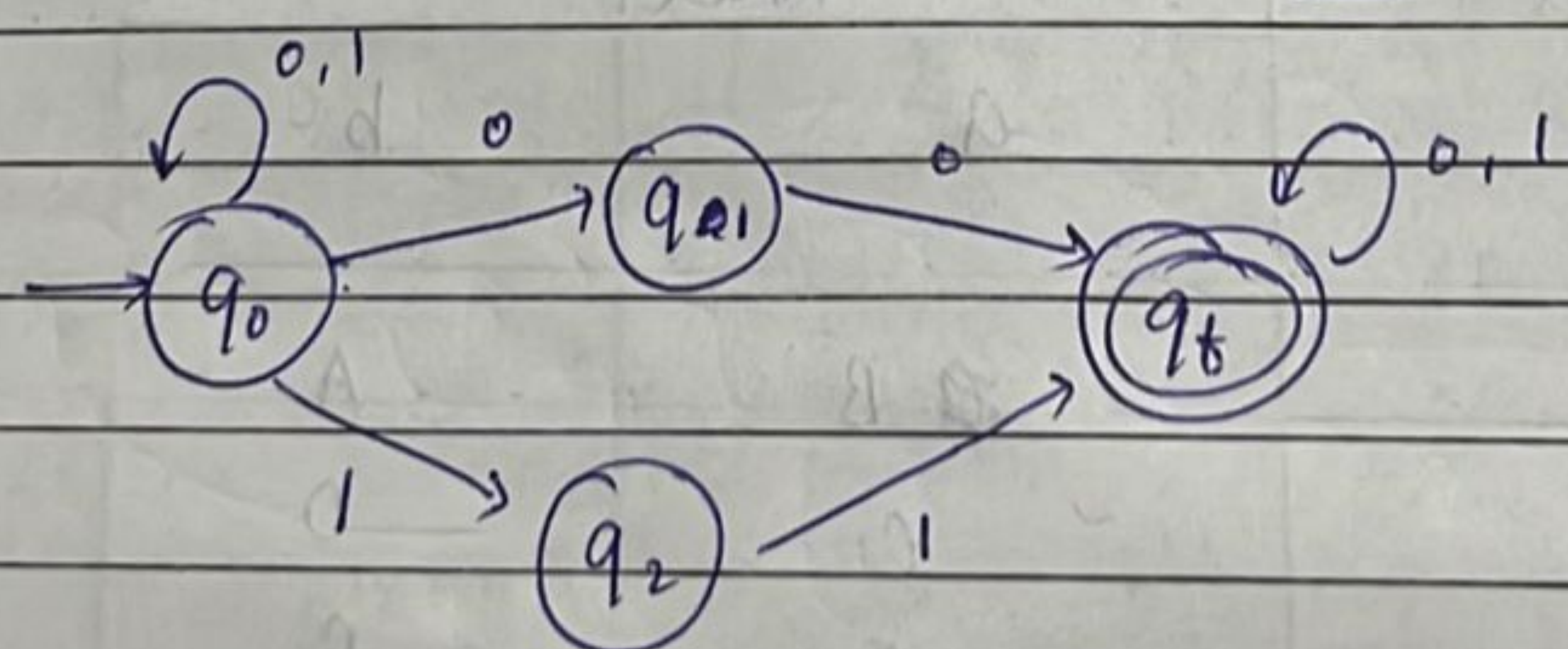
State	Next a	Next b
$\rightarrow$ A	B	A
B	C	D
C	E	F
D	G	H
E	E	F
F	G	H
G	G	D
H	B	A



ii) $(0+1)^* (00+11) (0+1)^*$







State	Next 0	1
q_0	q_0, q_1	q_0, q_2
q_1	q_f	ϕ
q_2	ϕ	q_f
q_f	q_f	q_f

$$\delta'(q_0, 0) = \delta(q_0, 0)$$
$$= \{q_0, q_1\}$$

$$\delta'(q_0, 1) = \delta(q_0, 1)$$
$$= \{q_0, q_2\}$$

$$\delta'((q_0, q_1), 0) = \delta(q_0, 0) \cup \delta(q_1, 0)$$
$$= \{q_0, q_1, q_f\}$$

$$\delta'((q_0, q_1)\}, 1) = \delta(q_0, 1) \cup \delta(q_1, 1)$$
$$= \{q_0, q_2\}.$$

$$\delta'((q_0, q_2), 0) = \delta(q_0, 0) \cup \delta(q_2, 0)$$
$$= \{q_0, q_1\}$$

$$\delta'((q_0, q_2), 1) = \delta(q_0, 1) \cup \delta(q_2, 1)$$
$$= \{q_0, q_2\}$$

$$\delta'(q_0, q_1, q_f), 0) = \delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_f, 0)$$
$$= \{q_0, q_1, q_f\}$$

$$\delta'(q_0, q_1, q_f), 1) = \delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_f, 1)$$
$$= \{q_0, q_2, q_f\}$$

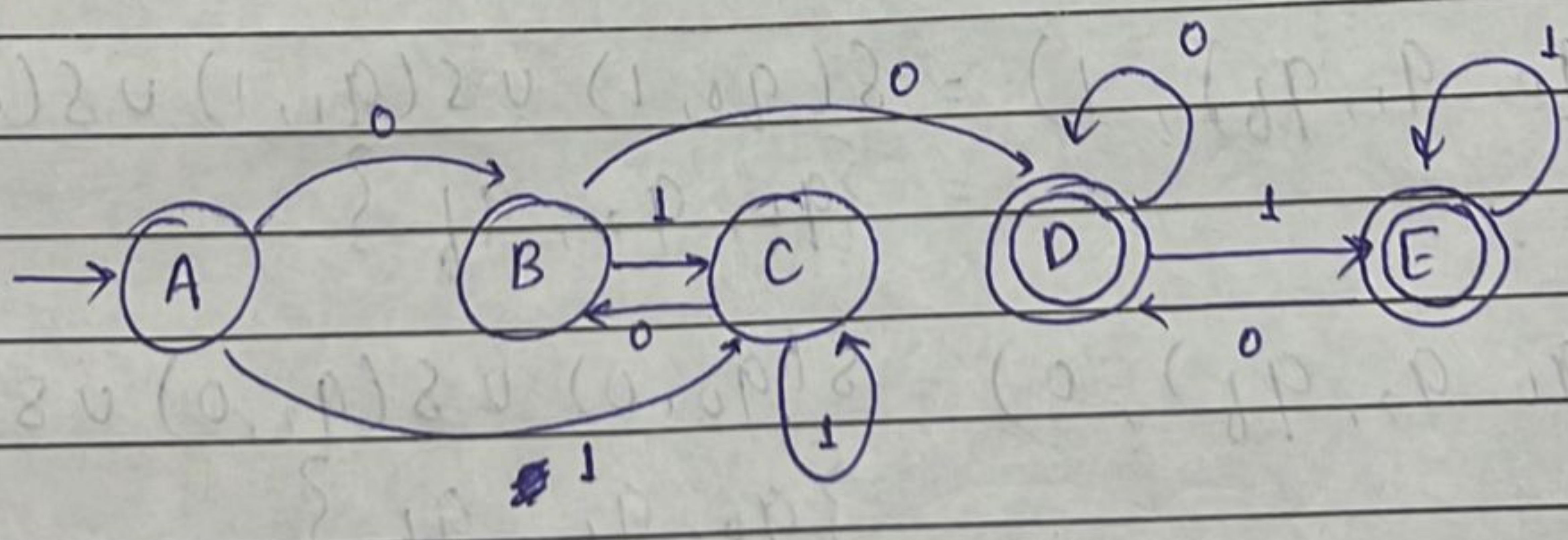
$$\delta'((q_0, q_2, q_f), 0) = \delta(q_0, 0) \cup \delta(q_2, 0) \cup \delta(q_f, 0)$$
$$\{q_0, q_1, q_f\}$$

$$\delta'((q_0, q_2, q_f), 1) = \delta(q_0, 1) \cup \delta(q_2, 1) \cup \delta(q_f, 1)$$
$$\{q_0, q_2, q_f\}$$

	State	Next	
		0	1
A	→ q_0	q_0, q_1	q_0, q_2
B	q_0, q_1	q_0, q_1, q_f	q_0, q_2
C	q_0, q_2	q_0, q_1	q_0, q_2
D	q_0, q_1, q_f	q_0, q_1, q_f	q_0, q_2, q_f
E	q_0, q_2, q_f	q_0, q_1, q_f	q_0, q_2, q_f

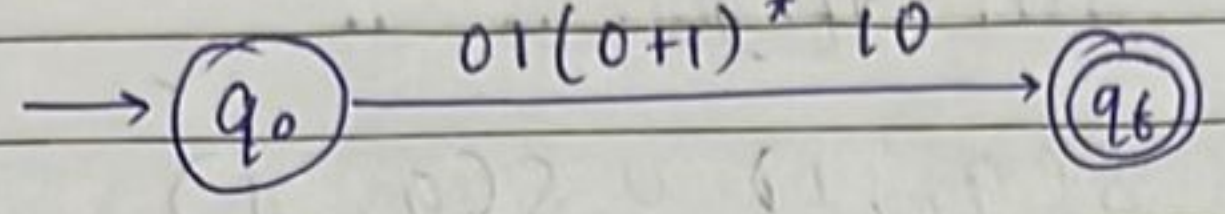
MODIFIED TRANSITION TABLE

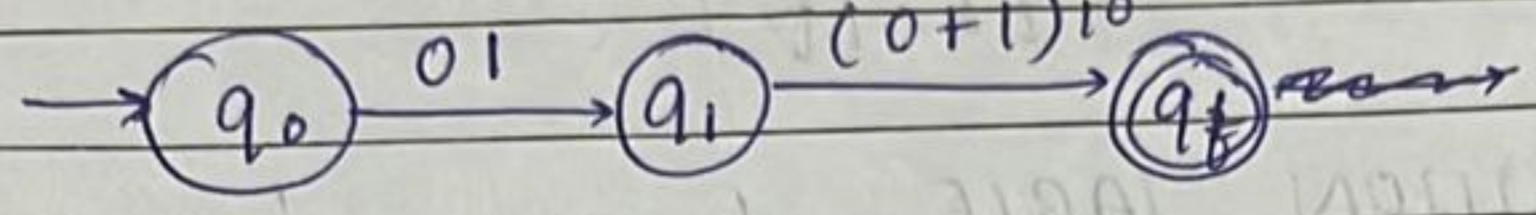
State	Next	
	0	1
→ A	B	C
B	D	C
C	B	C
D	D	E
E	D	E

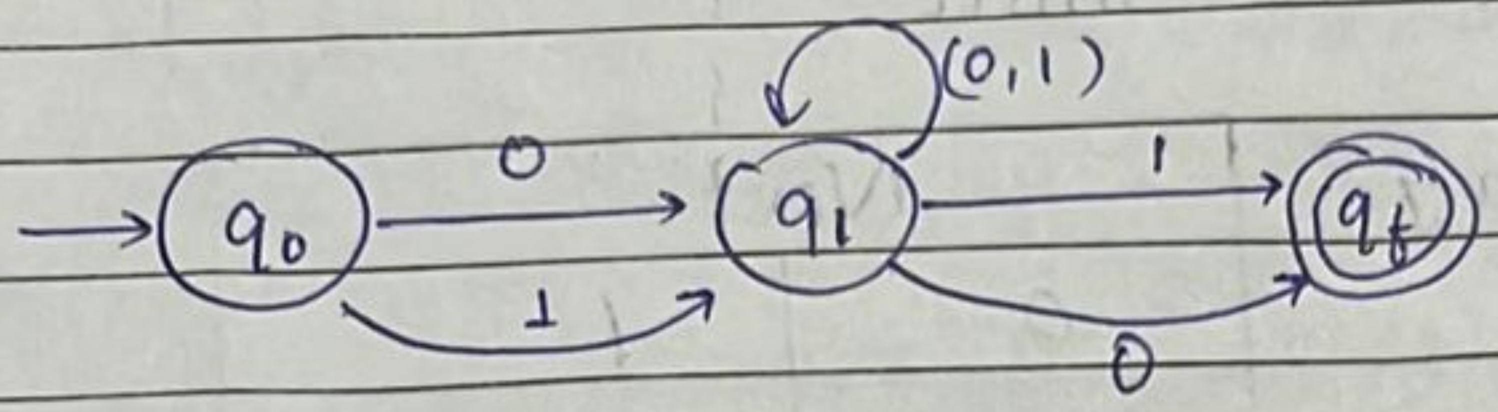


TRANSITION DIAGRAM

iii) $01(0+1)^*10$







State	Next 0	Next 1
q_0	q_1	q_1
q_1	q_1, q_f	q_1, q_f
q_f	ϕ	ϕ

$$\delta'(q_0, 0) = \delta(q_0, 0) = \{q_1\}$$

$$\delta'(q_0, 1) = \delta(q_0, 1) = \{q_1\}$$

$$\delta'(q_1, \cancel{q_f}, 0) = \delta(q_1, 0) = \{q_1, q_f\}$$

$$\delta'(q_1, 1) = \delta(q_1, 1) = \{q_1, q_f\}$$

$$\delta((q_1, q_f), 0) = \delta(q_1, 0) \cup \delta(q_f, 0)$$
$$= \{q_1, q_f\}$$

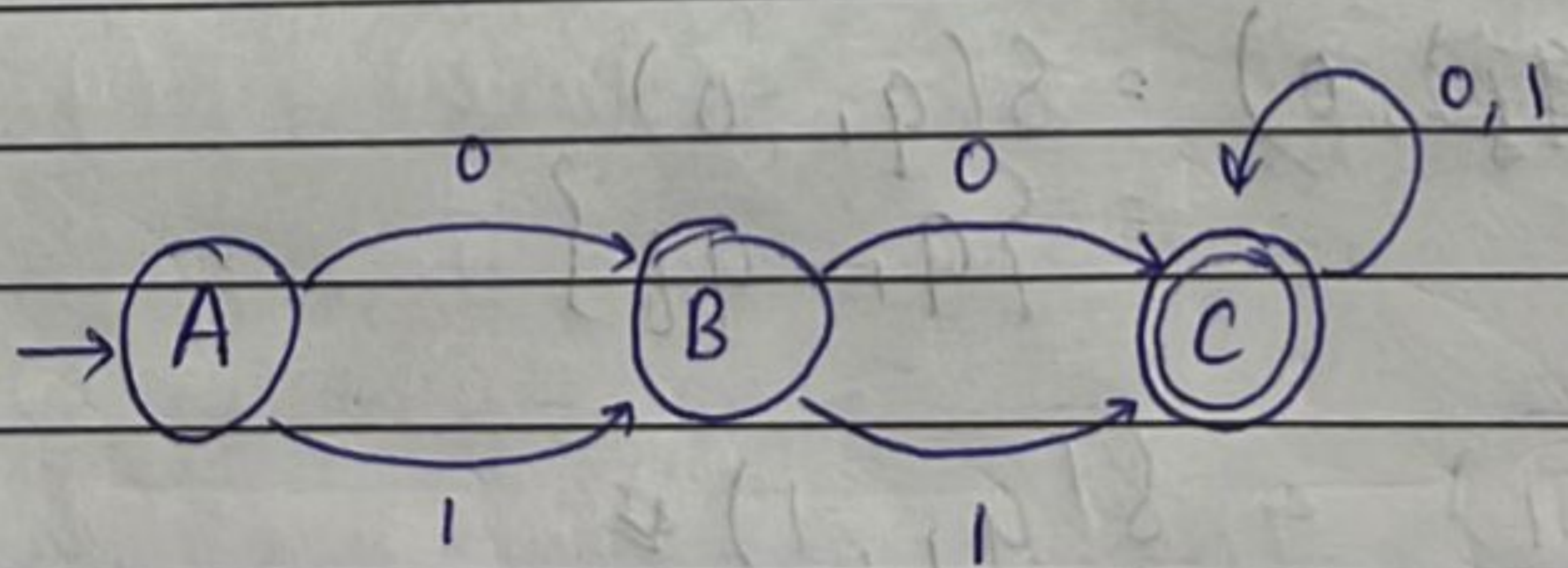
$$\delta((q_1, q_f), 1) = \delta(q_1, 1) \cup \delta(q_f, 1)$$
$$= \{q_1, q_f\}$$

TRANSITION TABLE

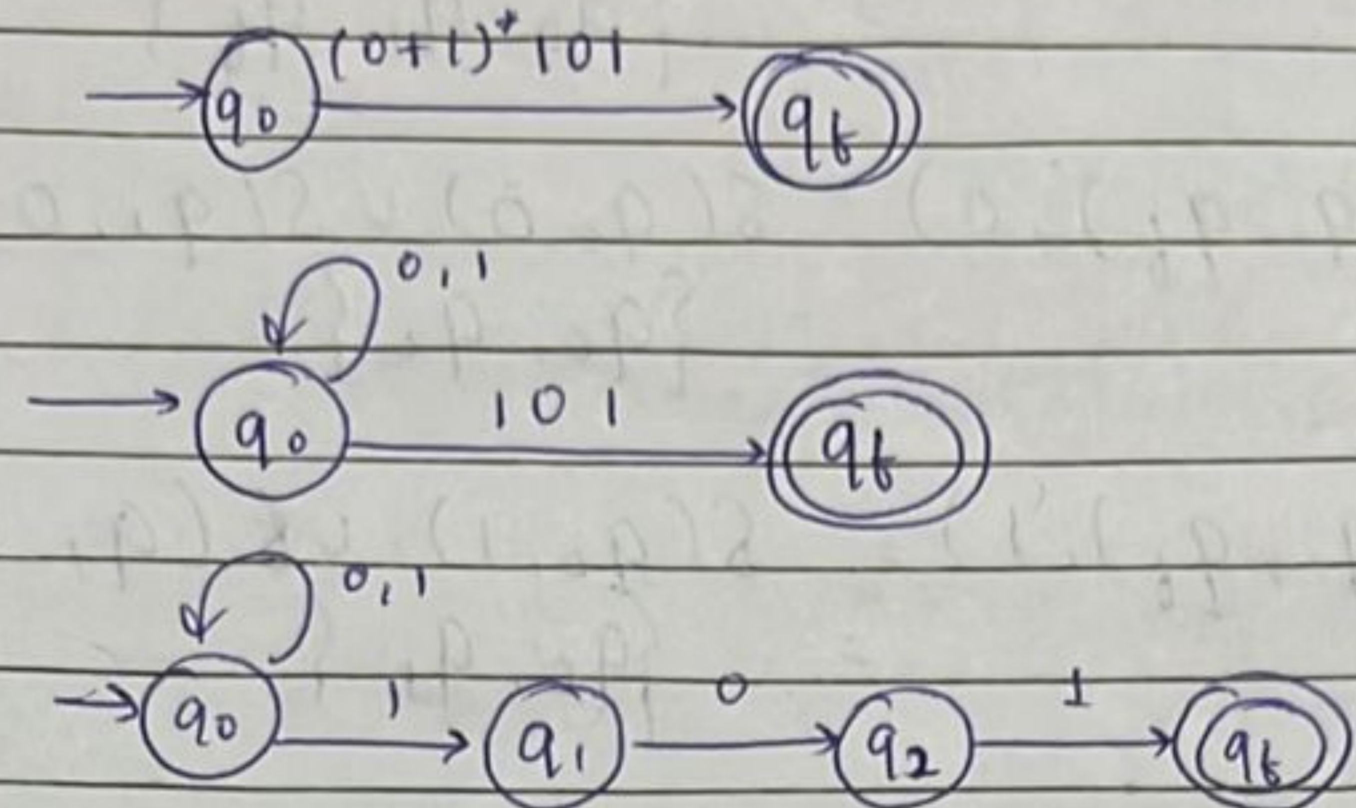
State	Next	
	0	1
$\rightarrow q_0$	q_1	q_1
q_1	q_1, q_f	q_1, q_f
q_1, q_f	q_1, q_f	q_1, q_f

~~II~~ MODIFING TRANSITION TABLE

State	Next	
	0	1
A	B	B
B	C	C
C	C	C



iv) $(0+1)^*101$



State	Next	
	0	1
$\rightarrow q_0$	q_0	q_0, q_1
q_1	q_2	ϕ
q_2	ϕ	q_f
q_f	ϕ	ϕ

$$\delta'(q_0, 0) = \delta(q_0, 0)$$
$$= \{q_0\}$$

$$\delta'(q_0, 1) = \delta(q_0, 1)$$
$$= \{q_0, q_1\}$$

$$\delta'((q_0, q_1), 0) = \delta(q_0, 0) \cup \delta(q_1, 0)$$
$$= \{q_0, q_2\}$$

$$\delta'((q_0, q_1), 1) = \delta(q_0, 1) \cup \delta(q_1, 1)$$
$$= \{q_0, q_1\}$$

$$\delta'((q_0, q_2), 0) = \delta(q_0, 0) \cup \delta(q_2, 0)$$
$$= \{q_0\}$$

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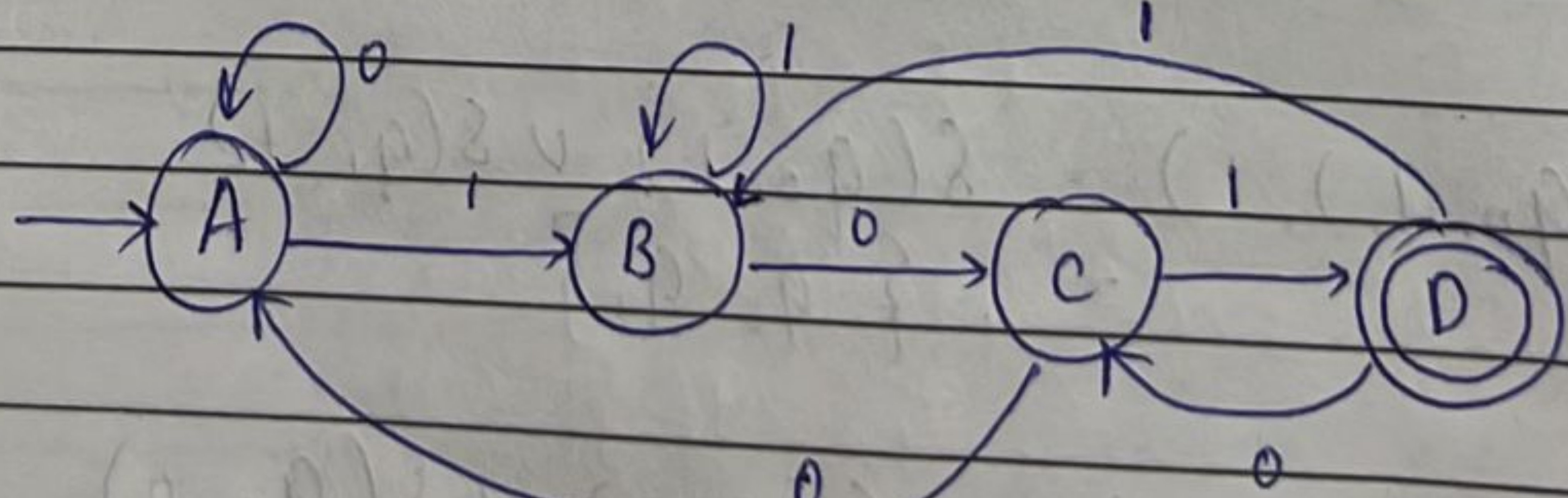
$$\delta'((q_0, q_2), 1) = \delta(q_0, 1) \cup \delta(q_2, 1)$$
$$= \{q_0, q_1, q_f\}$$

$$\delta'((q_0, q_1, q_f), 0) = \delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_f, 0)$$
$$= \{q_0, q_2\}$$

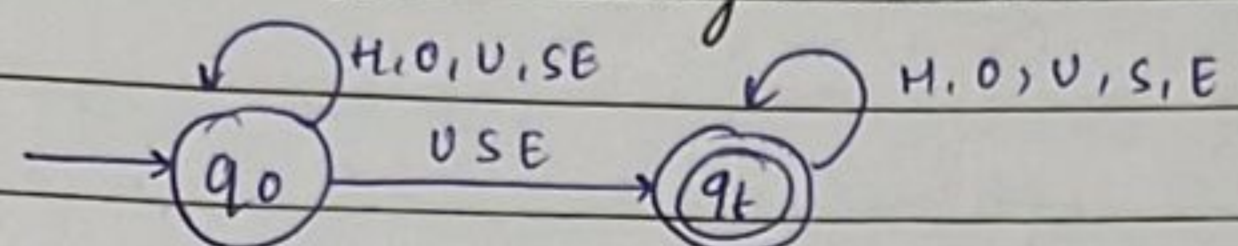
$$\delta'((q_0, q_1, q_f), 1) = \delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_f, 1)$$
$$= \{q_0, q_1\}$$

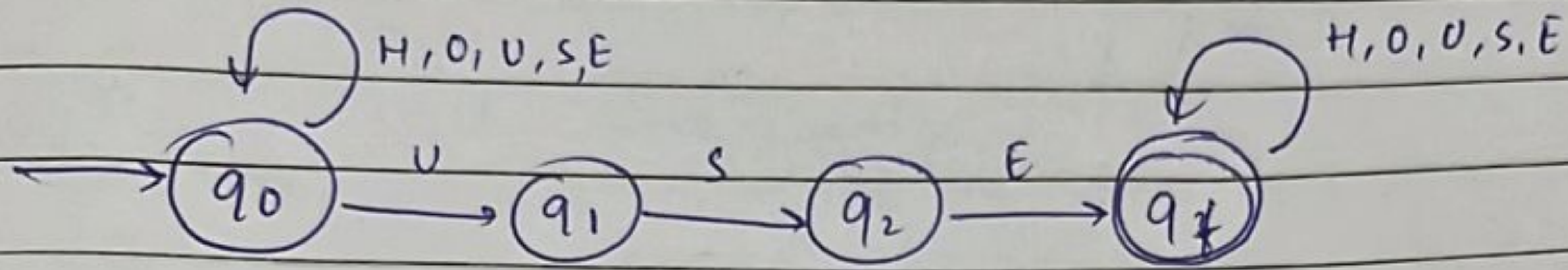
State	Next 0	Next 1
$\rightarrow q_0$	q_0	q_0, q_1
q_0, q_1	q_0, q_2	q_0, q_1
q_0, q_2	q_0	q_0, q_1, q_f
q_0, q_1, q_f	q_0, q_2	q_0, q_1

State	Next 0	Next 1
$\rightarrow$A	A	B
B	C	B
C	A	D
D	C	B



Q6. Design a finite automata that reads strings made up of letter in the word "HOUSE" & Recognizes those strings that contains the word "USE" at anywhere





Present State	Next H	O	U	S	E
$\rightarrow q_0$	q_0	q_0	q_0, q_1	q_0	q_0
q_1	ϕ	ϕ	ϕ	q_2	ϕ
q_2	ϕ	ϕ	ϕ	ϕ	q_f
q_f	q_f	q_f	q_f	q_f	q_f

$$\delta'(q_0, H) = \delta(q_0, H) = \{q_0\}$$

$$\delta'(q_0, O) = \delta(q_0, O) = \{q_0\}$$

$$\delta'(q_0, U) = \delta(q_0, U) = \{q_0, q_1\}$$

$$\delta'(q_0, S) = \delta(q_0, S) = \{q_0\}$$

$$\delta'(q_0, E) = \delta(q_0, E)$$
$$= q_0$$

$$\delta'((q_0, q_1), H) = \delta(q_0, H) \cup \delta(q_1, H)$$
$$= q_0 \cup \phi = q_0$$

$$\delta'((q_0, q_1), O) = \delta(q_0, O) \cup \delta(q_1, O)$$
$$= \{q_0\}$$

$$\delta'((q_0, q_1), U) = \delta(q_0, U) \cup \delta(q_1, U)$$
$$= \{q_0, q_1\}$$

$$\delta'((q_0, q_1), S) = \delta(q_0, S) \cup \delta(q_1, S)$$
$$= \{q_0, q_2\}$$

$$\delta'((q_0, q_1), E) = \delta(q_0, E) \cup \delta(q_1, E)$$
$$= \{q_0\}$$

$$\delta'((q_0, q_2), H) = \delta(q_0, H) \cup \delta(q_2, H) = \{q_0\}$$

$$\delta'((q_0, q_2), O) = \delta(q_0, O) \cup \delta(q_2, O) = \{q_0\}$$

$$\delta'((q_0, q_2), U) = \delta(q_0, U) \cup \delta(q_2, U) = \{q_0, q_1\}$$

$$\delta'((q_0, q_2), S) = \delta(q_0, S) \cup \delta(q_2, S) = \{q_0\}$$

$$\delta'(q_0, q_1), E) = \delta(q_0, E) \cup \delta(q_1, E) = \{q_0, q_f\}$$

$$\delta'((q_0, q_f), H) = \delta(q_0, H) \cup \delta(q_f, H)$$
$$= \{q_0, q_f\}$$

$$\delta'((q_0, q_f), O) = \delta(q_0, O) \cup \delta(q_f, O)$$
$$= \{q_0, q_f\}$$

$$\delta'((q_0, q_f), U) = \delta(q_0, U) \cup \delta(q_f, U)$$
$$= \{q_0, q_1, q_f\}$$

$$\delta'((q_0, q_f), S) = \delta(q_0, S) \cup \delta(q_f, S)$$
$$= \{q_0, q_f\}$$

$$\delta'((q_0, q_f), E) = \delta(q_0, E) \cup \delta(q_f, E)$$
$$= \{q_0, q_f\}$$

$$\delta'((q_0, q_1, q_f), H) = \{q_0, q_f\}$$

$$\delta'((q_0, q_1, q_f), O) = \{q_0, q_f\}$$

$$\delta'((q_0, q_1, q_f), U) = \{q_0, q_1, q_f\}$$

$$\delta'((q_0, q_1, q_f), S) = \{q_0, q_2, q_f\}$$

$$\delta'((q_0, q_1, q_f), E) = \{q_0, q_f\}$$

$$\delta'((q_0, q_2, q_f), H) = \{q_0, q_f\}$$

$$\delta'((q_0, q_2, q_f), O) = \{q_0, q_f\}$$

$$\delta'((q_0, q_2, q_f), U) = \{q_0, q_1, q_f\}$$

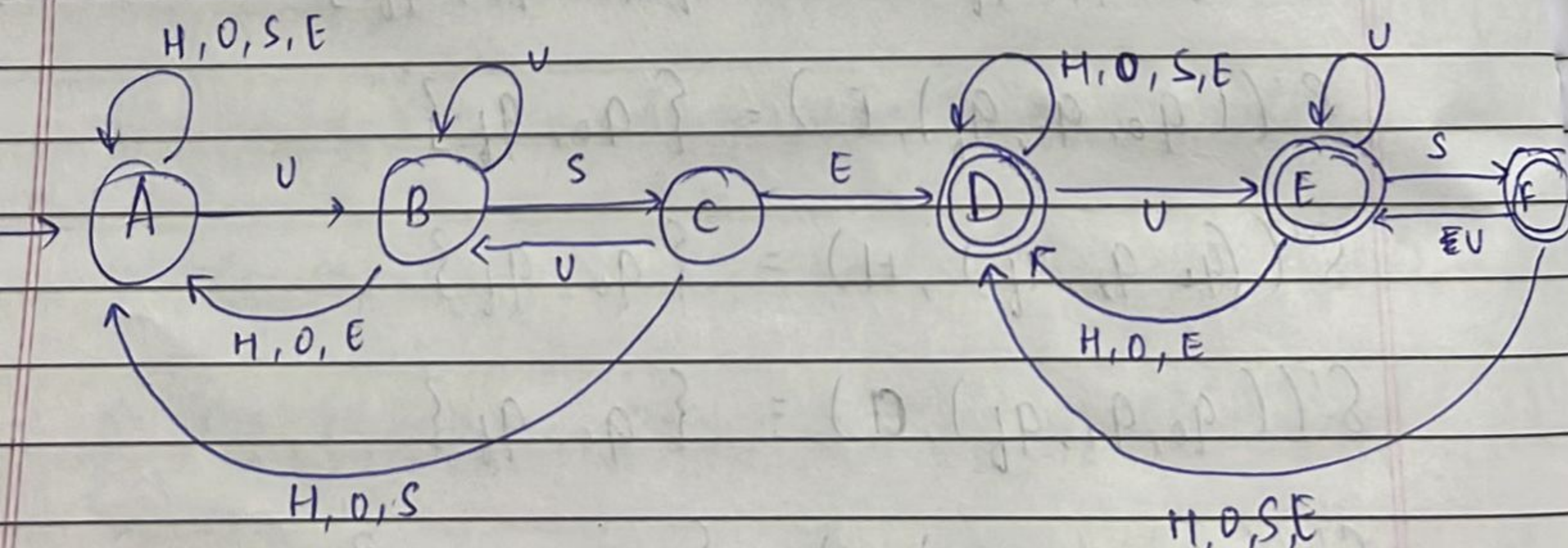
$$\delta'((q_0, q_2, q_f), S) = \{q_0, q_f\}$$

$$\delta'((q_0, q_2, q_f), E) = \{q_0, q_f\}$$

State	Next H	O	U	S	E
q_0	q_0	q_0	q_0, q_1	q_0	q_0
q_0, q_1	q_0	q_0	q_0, q_1	q_0, q_2	q_0
q_0, q_2	q_0	q_0	q_0, q_1	q_0	q_0, q_f
q_0, q_f	q_0, q_f	q_0, q_f	q_0, q_1, q_f	q_0, q_f	q_0, q_f
q_0, q_1, q_f	q_0, q_f	q_0, q_f	q_0, q_1, q_f	q_0, q_2, q_f	q_0, q_f
q_0, q_2, q_f	q_0, q_f	q_0, q_f	q_0, q_1, q_f	q_0, q_f	q_0, q_f

MODIFIED

State	Next H	O	U	S	E
→ A	A	A	B	A	A
B	A	A	B	C	A
C	A	A	B	A	D
D	D	D	E	D	D
E	D	D	E	F	D
F	D	D	E	D	D



Q7. What is acceptability of string by finite automata? Discuss the acceptability of DFA & NFA

ACCEPTABILITY OF STRING BY FINITE AUTOMATA

A string 'x' is accepted by the finite automata after reading x if it reaches the final state

$$L(M) = \{ x \mid \delta(q_0, x) \text{ is in } F \}$$

$$L(M) = \{ x \mid \delta(q_0, x) \in F \}$$

Consider the finite state machine whose transition function δ is given in the form of transition table, here

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0, 1\}$

$F = \{q_0\}$

given the entire sequence of state for the input string 110101 & ~~sta~~ transition table

State	Next 0	Next 1
$\rightarrow q_0$	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

$$L(M) = \{ x \mid \delta(q_0, x) \in F \}$$

$$\delta(q_0, \cancel{1}10101) = \delta(q_1, \cancel{1}0101)$$

$$= \delta(q_0, \cancel{0}101)$$

$$= \delta(q_2, \cancel{1}01)$$

$$= \delta(q_3, \cancel{0}1)$$

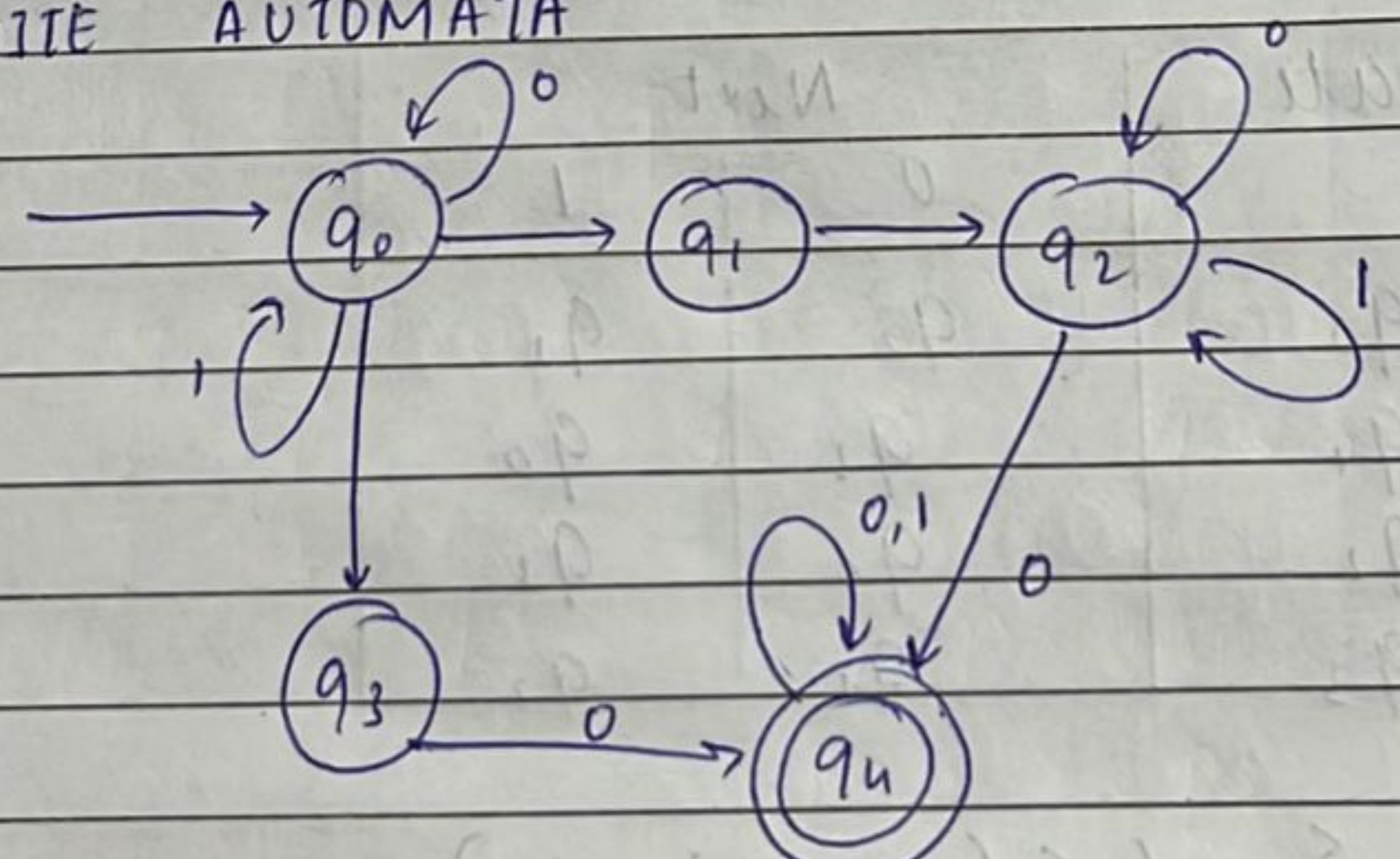
$$= \delta(q_1, \cancel{1})$$

$$= \delta(q_0, \epsilon) = q_0 \in F$$

After reading $x = 110101$ it reaches to the ~~final~~ state q_0 and this q_0 belongs to the final state so we can say that input string $x = 110101$ is excepted by finite automata

$$q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_0 \xrightarrow{0} q_2 \xrightarrow{1} q_3 \xrightarrow{0} q_1 \xrightarrow{1} q_0$$

ACCEPTIBILITY OF STRING BY A NON-DETERMINISTIC FINITE AUTOMATA



$x = 0100$

$q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_0 \xrightarrow{0} q_0 \xrightarrow{0} q_0$

$q_0 \xrightarrow{0} q_3$, $q_0 \xrightarrow{1} q_1$, $q_0 \xrightarrow{0} q_3 \rightarrow \boxed{q_4}$, $q_0 \xrightarrow{0} q_3$

States ~~&~~ reached while processing 0100

Atleast one path ~~&~~ reaches the final state then the string is accepted.

Q8. Construct DFA corresponding to the following NFA.

	0	1
$\rightarrow p$	q, s	q
q	r	q, r
r	s	p
s	-	p

$Q = \{p, q, r, s\}$

$\Sigma = \{0, 1\}$

q_0 = Initial state

$F = \{q_2\}$

$Q' = 2^{|Q|} = 2^{|4|} = 16$

$Q' = \{\phi,$ ~~q~~ $\{p\}, \{q\}, \{r\}, \{s\}, \{p, q\}, \{p, r\}, \{p, s\}, \{q, r\}, \{q, r, s\},$
$\{q, s\}, \{r, s\}, \{p, q, r\}, \{p, q, s\},$
$\{p, r, s\}, \{p, q, r, s\}\}$

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Page

$F = \{ \{q\}, \{s\}, \{p, q\}, \{p, s\}, \{q, r\}, \{q, s\}$
$\{r, s\}, \{p, q, r\}, \{p, q, s\}, \{q, r, s\}$
$\{p, r, s\}, \{p, q, r, s\} \}.$

$\delta'(p, 0) = \delta(p, 0) = \{q, s\}$ ✓

$\delta'(p, 1) = \delta(p, 1) = \{q\}$ ✓

$\delta'((q, s), 0) = \delta(q, 0) \cup \delta(s, 0)$
$= \{r\}$ ✓

$\delta'((q, s), 1) = \delta(q, 1) \cup \delta(s, 1)$
$= \{q, r, p\}$ ✓

$\delta'(q, 0) = \delta(q, 0) = \{r\}$ ✓

$\delta'(q, 1) = \delta(q, 1) = \{q, r\}$ ✓

$\delta'(r, 0) = \{s\}$ ✓

$\delta'(r, 1) = \{p\}$ ✓

$\delta'((q, r, p), 0) = \delta(q, 0) \cup \delta(r, 0) \cup \delta(p, 0)$
$= \{r, s, q\}$ ✓

$\delta'((q, r, p), 1) = \delta(q, 1) \cup \delta(r, 1) \cup \delta(p, 1)$
$= \{q, r, p\}$ ✓

$\delta'((q, r), 0) = \delta(q, 0) \cup \delta(r, 0)$
$= \{r, s\}$ ✓

$\delta'((q,r),1) = \delta(q,1) \cup \delta(r,1)$
$= \{q,r,p\}$ ✓

$\delta'(s,0) = \{\phi\}$ ✓

$\delta'(s,1) = \{p\}$ ✓

$\delta'((r,s,q),0) = \delta(r,0) \cup \delta(s,0) \cup \delta(q,0)$
$= \{s,r\}$ ✓

$\delta'((r,s,q),1) = \delta(r,1) \cup \delta(s,1) \cup \delta(q,1)$
$= \{p,q,r\}$ ✓

$\delta'((q,r,p),0) = \delta(q,0) \cup \delta(r,0) \cup \delta(p,0)$
$= \{r,s,q\}$

$\delta'((q,r,p),1) = \delta(q,1) \cup \delta(r,1) \cup \delta(p,1)$
$= \{q,r,p$

$\delta'((r,s),0) = \delta(r,0) \cup \delta(s,0)$
$= \{s\}$

$\delta'((r,s),1) = \delta(r,1) \cup \delta(s,1)$
$= \{p\}$

TRANSITION TABLE

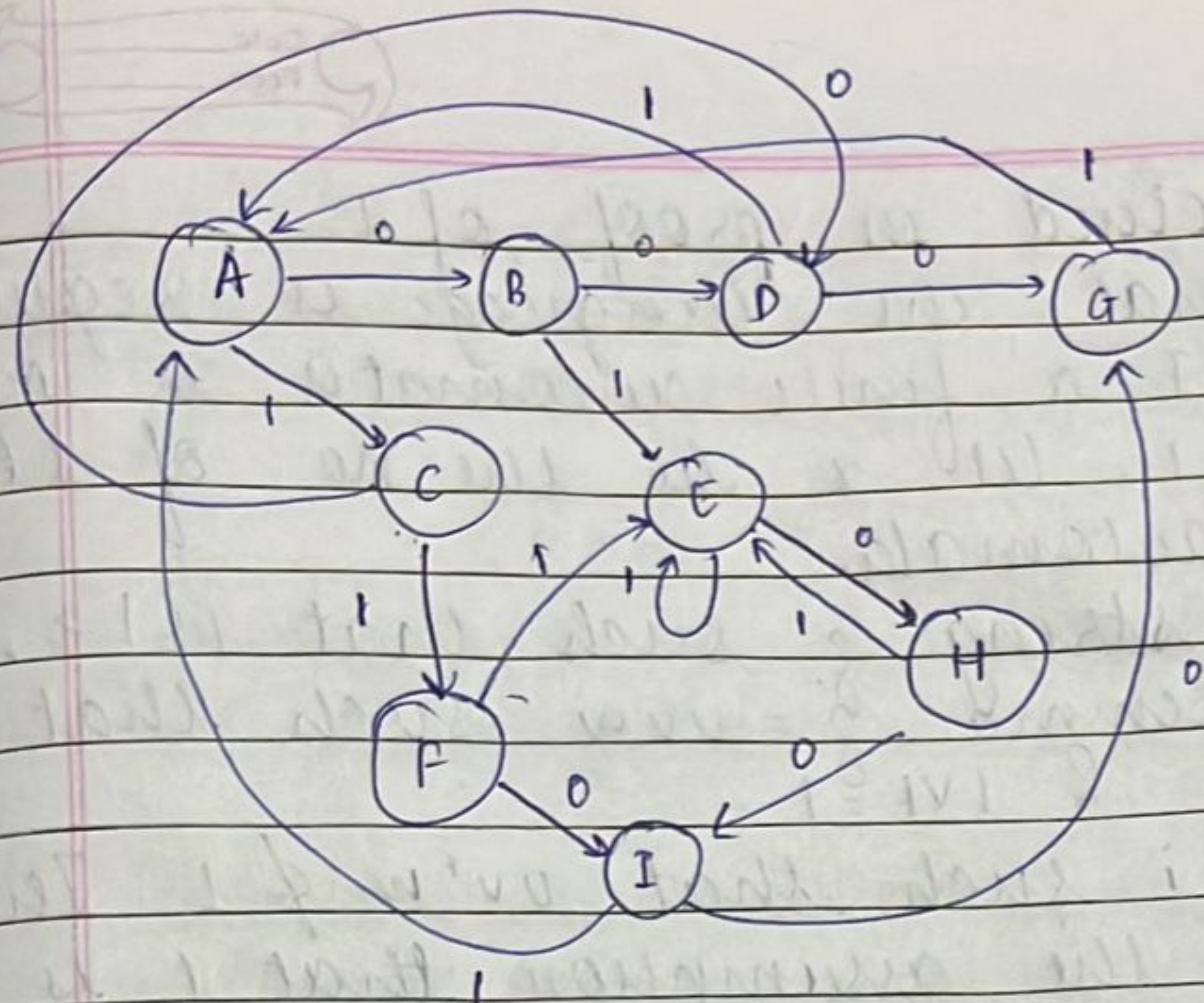
][a(b+aa)]a (6)

Date ______
Page ______

	State	Next	
		~~p~~0	1
A	p	q,s	q
B	q,s	r	q,p,r
C	q,	r	q,r
D	r	s	p
E	q,r,p	r,s,q	q,r,p
F	q,r	r,s	q,r,p
G	s	ϕ	p
H	r,s,q	s,r	p,q,r
I	r,s	s	p

MODIFIED TRANSITION TABLE

State	Next	
	0	1
A	B	C
B	D	E
C	D	F
D	G	A
E	H	E
F	I	E
G	ϕ	A
H	I	E
I	G	A



Q9. state and prove pumping lemma for regular sets. Prove that $L = \{a^{n^2} \mid n >= 1\}$ is not regular.

let L be a regular ~~expression~~ set, the ∃ n such that if z is any word in L & $|z| \geq n$ then we may write $z = uvw$ such that $|uv| \leq n$ and $|v| \geq 1$ for every $i \geq 0$, $uv^iw \in L$.
where n cannot be greater than the no. of state in the minimum state finite automata accepting L.

APPLICATION
→ To prove that language is not regular
→ To check whether the language accepted by the finite automata is finite or infinite

Steps involved in proof of L.

STEP 1 Assume that the language is regular then let ∃ a finite automata is M accepting L. Let x be the no of states in finite automata.

STEP 2 Choose a string z such that $|S| \geq n$ by pumping lemma $z = uvw$ such that $|uv| \leq n$ & $|v| = 1$

STEP 3 Find any i such that $uv^iw \notin L$. This contradicts the assumption that L is regular.

$L = \{a^{n^2} \mid n \geq 1\}$

1. assume L is regular then ∃ a finite automata accepting L let the no. of set in finite automata is L

2. consider $z = a^{n^2}$

$|z| \geq n$

$n^2 \geq n$

Such that by pumping lemma.

$z = uvw$

$|uv| \leq n$ &

$|v| \geq 1$ & $|v| \leq n$

3. Choose $n \leq z$

4. So,

$|uv^2w| = |uv^iw| + |v^i|$

$= n^2 + |v^i| \quad (|uv^iw| = |z| = n^2)$

$\leq n^2 + n$

$n^2 < |uv^2w| < (n+1)^2$

because no perfect sq. can exist between n^2 & $(n+1)^2$ so uv^2w does not belong to the language L

$$uv^2w \notin L$$

Therefore L is not regular as it contradicts the assumption.

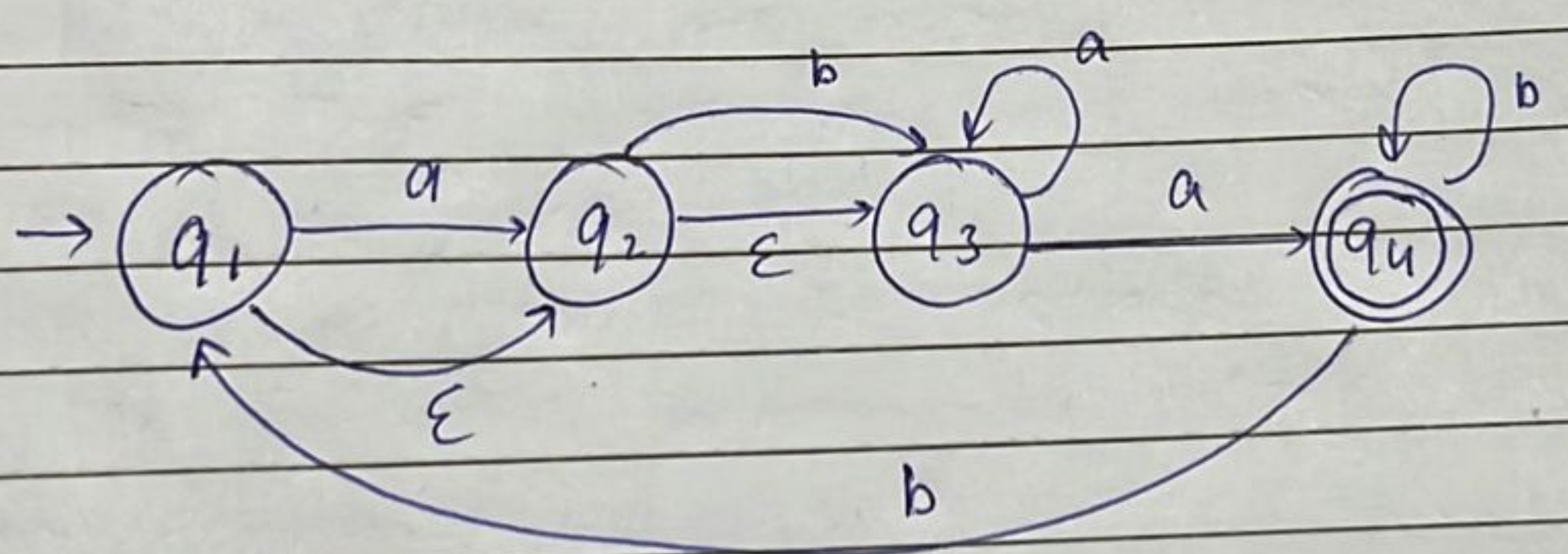
Q.10. Consider the machine M which is a NDFA with ε (Null) transition:

$M = \{q_1, q_2, q_3, q_4\}, \{a, b\}, \delta, q_1, \{q_4\}$

where

$\delta(q_1, a) \rightarrow q_2$	$\delta(q_3, a) \rightarrow q_3$
$\delta(q_1, \varepsilon) \rightarrow q_3$	$\delta(q_3, a) \rightarrow q_4$
$\delta(q_1, \varepsilon) \rightarrow q_2$	$\delta(q_4, b) \rightarrow q_1$
$\delta(q_2, b) \rightarrow q_3$	$\delta(q_4, b) \rightarrow q_4$

Convert this NDFA with ε ~~to~~ transition to one without ε-transition.



ϵ closure $q_1 = \{q_1, q_2, q_3\}$

ϵ closure $q_2 = \{q_2, q_3\}$

ϵ closure $q_3 = \{q_3\}$

ϵ closure $q_4 = \{q_4\}$

$\delta'(q_1, a) = \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_1), a))$

$= \epsilon\text{ closure}(\delta(q_1, q_2, q_3), a))$

$= \epsilon\text{ closure}(\delta(q_1, a) \cup \delta(q_2, a) \cup \delta(q_3, a))$

$= \epsilon\text{ closure}(q_2 \cup \phi \cup q_4)$

$= \epsilon\text{ closure}(q_2 \cup q_4)$

$= \{q_2, q_3, q_4\}$

$\delta'(q_1, b) = \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_1), b)$

$= \epsilon\text{ closure}(\delta(q_1, q_2, q_3), b)$

$= \epsilon\text{ closure}(\delta(q_1, b) \cup \delta(q_2, b) \cup \delta(q_3, b))$

$= \epsilon\text{ closure}(\phi \cup q_3 \cup \phi)$

$= \{q_3\}$

$\delta'(q_2, a) = \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_2), a)$

$= \epsilon\text{ closure}(\delta(q_2, q_3), a)$

$= \epsilon\text{ closure}(\delta(q_2, a) \cup \delta(q_3, a))$

$= \epsilon\text{ closure}(\phi \cup q_3, q_4)$

$= \epsilon\text{ closure}(q_3, q_4)$

$= \{q_3, q_4\}$

$\delta'(q_2, b) = \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_2), b)$

$= \epsilon\text{ closure}(\delta(q_2, q_3), b)$

$= \epsilon\text{ closure}(\delta(q_2, b) \cup \delta(q_3, b))$

$= \epsilon\text{ closure}(q_3, \phi)$

$= \{q_3\}$

$\delta'(q_3, a) = \epsilon\text{ closure}(\delta(\epsilon\text{ closure}(q_3), a)$

$= \epsilon\text{ closure}(\delta(q_3, a))$

$= \epsilon\text{ closure}(\cancel{\phi}\{q_3\}, q_4)$

$= \cancel{\phi}\{q_3, q_4\}$

$\delta'(q_3, b) = \epsilon\ closure\ (\delta(\epsilon\ closure\ (q_3), b))$
$= \epsilon\ closure\ (\delta(q_3, b))$
$= \epsilon\ closure\ (\phi)$
$= \{\phi\}$

$\delta'(q_4, a) = \epsilon\ closure\ (\delta(\epsilon\ closure\ (q_4), a))$
$= \epsilon\ closure\ (\delta(q_4, a))$
$= \epsilon\ closure\ (\phi)$
$= \{\phi\}$

$\delta'(q_4, b) = \epsilon\ closure\ (\delta(\epsilon\ closure\ (q_4), b))$
$= \epsilon\ closure\ (\delta(q_4), b))$
$= \epsilon\ closure\ (q_4, q_1)$
$= (q_4, q_1, q_2, q_3)$