Printed Pages : 4

Roll No.

322452(14)

BE (4^{th} Semester)

Examination, April-May, 2018

(New Scheme)

Discrete Structures

Time Allowed : 3 hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note : (*i*) Part (*a*) of each question is compulsory. Attempt any **two** parts from (*b*), (*c*) and (*d*) of each question.

(*ii*) The figures in the right-hand margin indicate marks.

Unit-I

1. (*a*) Define disjunctive normal form and conjunctive normal form. [2]

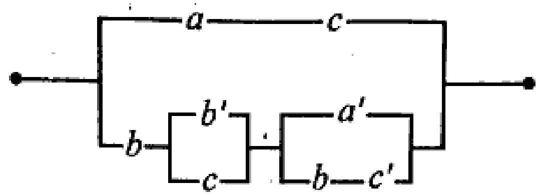
(*b*) State and prove De Morgan's law for Boolean algebra. [7]

(*c*) Prove that

(*i*) $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology

(*ii*) $p \to (q \vee r)$ and $(p \to q) \vee (p \to r)$ are. equivalent [7]

(*d*) Replace the following switching circuit by a simpler circuit : [7]



Unit-II

2. (*a*) Define sets and power sets. [2]

(*b*) Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$. [7]

(*c*) If R is an equivalence relation in the set A, then prove that R^{-1} is an equivalence relation in the set A. [7]

(*d*) If $f : X \to Y$ and $g : Y \to Z$ be one-one and onto mappings, then prove that the mapping $g \circ f : X \to Z$ is also one-one and onto. Also prove that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. [7]

Unit-III

3. (*a*) Define Algebraic structure with examples. [2]

(*b*) Let Q_+ be the set of all positive rational numbers and '$*$' is a binary operation on Q_+ defined as

$$a * b = \frac{ab}{3}, \quad a, b \in Q_+$$

Show that $(Q_+, *)$ is a group. [7]

(*c*) State and prove Lagrange's theorem. [7]

(*d*) Define group code. Show that the encoding function $E : B^2 \rightarrow B^5$ defined by

$$E(00) = 00000 \ ; \ E(01) = 01110;$$

$$E(10) = 10101 \ ; \ E(11) = 11011$$

is a group code. [7]

Unit-IV

4. (*a*) Define Directed and Undirected graphs. [2]

(*b*) Let G be a simple graph with n vertices. If G has k components, then the maximum numbers of edges that G can have are $\frac{(n-k)(n-k+1)}{2}$. Prove [7]

(*c*) Define the following : [7]

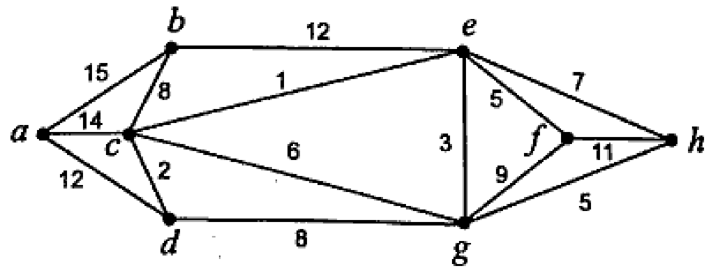
(*i*) Complete graph

(*ii*) Isomorphic graph

(*iii*) Paths and circuits

(*iv*) Cutsets

(*d*) Find the minimum spanning tree of the following graph : [7]



Unit-V

5. (*a*) How many ways are there to arrange the nine letters in the word 'ALLAHABAD'? [2]

(*b*) Show that

$$1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}, n \geq 1$$

by mathematical induction. [7]

(*c*) How many positive integers not exceeding 1000 are divisible by 5, 7 or 10? [7]

(*d*) Solve the following recurrence relation using generating function method : [7]

$$a_{r+2} - 3a_{r+1} + 2a_r = 0, \ r \geq 0$$

given that $a_0 = 2, \ a_1 = 3$
