

322351(14)**B. E. (Third Semester) Examination,****April-May, 2019****(New Scheme)****(CSE Engg. Branch)****MATHEMATICS-III****Time Allowed : Three hours****Maximum Marks : 80****Minimum Pass Marks : 28**

Note : Part (a) of each question compulsory and carrying 2 marks each and attempt any two parts (b), (c) and (d) and carrying 7 marks. each.

1. (a) Write the Dirichlet conditionss for Fourier series.

(b) If

$$f(x) = \begin{cases} 0, & -\pi \leq x \leq 0 \\ \sin x, & 0 \leq x \leq \pi \end{cases}$$

Prove that $f(x) = \frac{1}{\pi} + \frac{\sin x}{2} - \frac{2}{\pi} \sum \frac{\cos 2nx}{4n^2 - 1}$

(c) Obtain the Fourier expansion of $x \sin x$ as cosine series in $(0, \pi)$.

(d) The displacement y of a part of mechanism is tabulated with corresponding angular movement x° of the crank. Expression y as a Fourier series neglecting the harmonic above the third.

x°	y
0	1.80
30	1.10
60	0.30
90	0.16
120	1.50
150	1.30
180	2.16
210	1.25
240	1.30
270	1.52
300	1.76
330	2.00

2. (a) Write the change of scale property for Laplace transform.

(b) Write the properties of Laplace transform :

(i) Multiplication by t^n

(ii) Division by t

(iii) Find the Laplace transform of :

$$\frac{\cos at - \cos bt}{t} + t \sin at$$

(c) Find the inverse Laplace transform of :

$$\frac{5s+3}{(s-1)(s^2+2s+5)}$$

(d) Solve by method of transforms, the equation

$$y''' + 2y'' - y' - 2y = 0$$

given $y(0) = y'(0) = 0$ and $y''(0) = 6$.

3. (a) Write Cauchy's theorem for complex integration.

(b) Determine the analytic function $f(z) = u + iv$, if

$$u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$$

and $f(\pi/2) = 0$.

(c) Obtain Laurent's expansion for the function

$$f(z) = 1/z^2 \sinh z \text{ and evaluate}$$

$$\oint_C \frac{z}{z^2 \sinh z} dz,$$

where C is circle $|z-1|=2$.

(d) Evaluate :

$$\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$$

4. (a) Form the partial differential equation

$$Z = a \log \left\{ \frac{b(y-1)}{1-x} \right\}$$

(b) Solve

$$x^2(y-z)p + y^2(z-x)q = z^2(x-y)$$

(c) Solve

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = y \cos x$$

(d) Using the method of separation of variables, solve

$$\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u,$$

$$\text{where } u(x, 0) = 4e^{-x}.$$

5. (a) Find the mean to the set of observations :

x	:	0	1	2	3	4
f	:	122	60	15	2	1

(b) The diameter of an electric cable is assumed to be a continuous variate with p.d.f. $f(x) = 6x(1-x)$; $0 \leq x \leq 1$. Verify that the above is a p.d.f. Also find the mean and variance.

(c) The probability that a bomb dropped from a plane will strike the target is $1/5$. If six bombs are dropped, find the probability that :

- (i) exactly two will strike the target,
- (ii) at least two will strike the target.

(d) X is a normal variate with mean 30 and S.D.5, find the probabilities that :

- (i) $26 \leq x \leq 40$
- (ii) $x \geq 45$
- (iii) $|x - 30| > 5$