322351(14)

BE (3rd Semester)

Examination, Nov.-Dec., 2017

(New Scheme)

Mathematics-III

Time Allowed : 3 hours | *Maximum Marks* : 80

Minimum Pass Marks : 28

Note : (*i*) Part (*a*) of each question is compulsory. Attempt any **two** parts from (*b*), (*c*) and (*d*) of each question.

(*ii*) The figures in the right-hand margin indicate marks.

1. (*a*) Explain Dirichlet's conditions for a Fourier expansion of function. [2]

(*b*) An alternating current after passing through a rectifier has the form

$$i = I_0 \sin x \quad \text{for } 0 \le x \le \pi$$
$$= 0 \quad \text{for } \pi \le x \le 2\pi$$

where I_0 is the maximum current and the period is 2π. Express i as a Fourier series and evaluate

$$\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\cdots\cdots\infty$$ [7]

(*c*) Obtain Fourier for the function $f(x)$ given by

$$f(x) = 1+\frac{2x}{\pi} \quad -\pi \le x \le 0$$
$$= 1-\frac{2x}{\pi} \quad 0 \le x \le \pi$$

Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots = \pi^2/8$ [7]

(*d*) The following table gives the variations of periodic current over a period :

t sec	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
A amp	1.98	1.30	1.05	1.30	−0.88	−0.25	1.98

Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first harmonic. [7]

(*a*) State convolution theorem to find inverse Laplace transform. [2]

(*b*) Find the Laplace transform of

(*i*) $\frac{e^{-at}-e^{-bt}}{t}$ (*ii*) $t^2 e^{-3t}.\sin 2t$ [7]

(c) Find the inverse Laplace transform of

(i) $\dfrac{s}{\left(s^2+a^2\right)^2}$ by convolution theorem

(ii) $\cot^{-1}(s+1)$ [7]

(d) Solve $\dfrac{d^2x}{dt^2}+9x=\cos 2t$, if $x(0)=1$, $x(\pi/2)=-1$. [7]

3. (a) Write Cauchy's integral formula. [2]

(b) If $f(z)$ is a regular function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$$ [7]

(c) Obtain Laurent's expansion for the function

$f(z)=\dfrac{1}{z^2\sinh z}$ and evaluate

$\displaystyle\oint_C \frac{dz}{z^2\sinh z}$, where C is the circle $|z-1|=2$ [7]

(d) Apply the calculus of residues, to prove that

$$\int_0^{2\pi}\frac{d\theta}{1-2p\sin\theta+p^2}=\frac{2\pi}{1-p^2}\quad(0<p<1)$$ [7]

4. (a) Form the partial differential equation from

$$z=y^2+2f\left(\frac{1}{x}+\log y\right)$$ [2]

(b) Solve the following equation : [7]

$(z^2-2yz-y^2)p+(xy+zx)q=xy-zx$

(c) Solve the following equation : [7]

$(D^3+D^2D'-DD'^2-D'^3)z=e^x\cos 2y$.

(d) Solve the following equation by the method of separation of variables :

$$4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u,\ \text{given } u=3e^{-y}-e^{-5y}$$

when $x=0$ [7]

5. (a) Define moment generating function of discrete and continuous probability distribution. [2]

(b) If on an average 1 vessel in every 10 is wrecked, find the probability that out of 5 vessels expected to arrive, at least 4 will arrive safely. [7]

(c) The incidence of occupational disease in an industry is such that the workmen have a 10% chance of suffering from it. What is the probability that in a group of 7, 5 or more will suffer from it? [7]

(d) Assuming that the diameters of 1000 brass plugs that consecutively from a machine form a normal distribution with mean 0.7515 cm and standard deviation 0.0020 cm, how many of the plugs are likely to be rejected if the approved diameter is 0.752 ± 0.004 cm? [7]