Printed Pages : 4

Roll No.

322452(14)

BE (4th Semester)
Examination, Nov.-Dec., 2017

(New Scheme)

Discrete Structures

Time Allowed : 3 hours *Maximum Marks* : 80
Minimum Pass Marks : 28

Note : (*i*) Part (*a*) of each question is compulsory. Attempt any **two** parts from (*b*), (*c*) and (*d*).

(*ii*) The figures in the right-hand margin indicate marks.

1. *(a)* Define Tautology and Contradiction. [2]

(b) Test the validity of the argument : If 8 is even then 2 does not divide 9. Either 7 is prime or 2 divides 9. But 7 is prime, therefore 8 is odd. [7]

(c) Prove that–

(*i*) $(a.b)' = a' + b'$

(*ii*) $(a + b)' = a' \cdot b'$

for every elements a and b of a Boolean algebra B. [7]

(d) Construct the switching circuit for the function $F(x, y, z) = (x + y) \cdot (y + z) \cdot (z' + x')$. Simplify it and draw the equivalent circuit. [7]

2. *(a)* Define Hasse diagram with example. [2]

(b) Prove that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$ [7]

(c) If R and S be an equivalence relation in the set X, then prove that $R \cap S$ is an equivalence relation in X. [7]

(d) Define bijective function. If Q is the set of rational numbers and $f : Q \to Q$ is defined by $f(x) = 3x + 5$, $x \in Q$, then prove that f is bijective. Find also f^{-1}. [7]

3. *(a)* State Lagrange's theorem. [2]

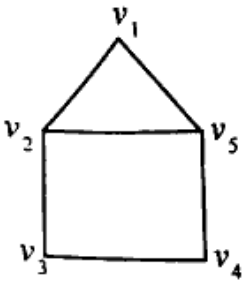
(b) Show that the set of all integers I forms a group with respect to the binary operation '$*$' defined by the rule

$$a * b = a + b + 1 \quad \forall a, b \in I$$ [7]

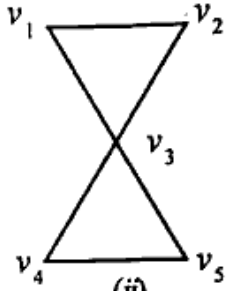
(c) Prove that the intersection of two subring is a subring. [7]

(d) Consider the two groups G and G' where $G = (I, +)$, i.e., additive group of integers and $G' = (\{2^m : m \in I\}.)$, i.e., multiplicative group of all positive and negative powers of an integer 2. Then show that G is isomorphic to G'. [7]

4. *(a)* Which of the following graphs have a Hamiltonian circuit? [2]



(i)



(ii)

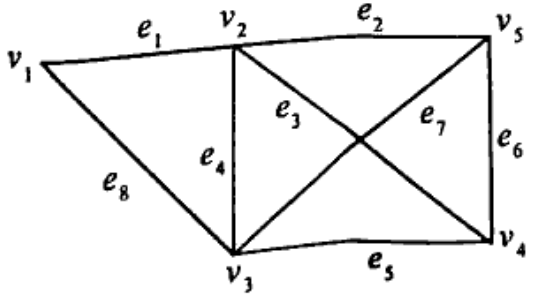
(b) Show that the maximum number of edges in a simple graph with n vertices is $n(n-1)/2$. [7]

(c) Explain the following :

(*i*) Cut-Set

(*ii*) Rank and nullity of a graph

Also find the rank and nullity of the following graph : [7]



(Turn Over)

(d) Draw the multigraph whose adjacency matrix M is given by

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 4 & 0 & 0 \\ 4 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 2 & 2 \end{bmatrix} \end{array}$$

[7]

5. *(a)* A farmer buys 4 cows, 3 goats and 2 hens from a seller who has 6 cows, 5 goats and 7 hens. How many choices does the farmer have? [2]

(b) Write the principle of mathematical induction. Also show that

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}, n \geq 1$$

by mathematical induction. [7]

(c) Find the generating function for the infinite sequence $1, \alpha, \alpha^2, \alpha^3, \cdots$ where α is a fixed constant. [7]

(d) Solve the recurrence relation

$$a_r - 4a_{r-1} + 4a_{r-2} = 2^r$$

[7]