Printed Pages : 4

Roll No.

322351(14)

BE (3rd Semester)
Examination, Nov.-Dec., 2018

(New Scheme)

Mathematics—III

Time Allowed : 3 hours *Maximum Marks* : 80
Minimum Pass Marks : 28

Note : (*i*) Answer **all** questions. Part *(a)* is compulsory from each question. Attempt any **two** from *(b)*, *(c)* and *(d)* of each question.

(*ii*) The figures in the right-hand margin indicate marks.

1. *(a)* Check which function is even or odd : [2]

(i) $f(x) = x \sin x$

(ii) $f(x) = x(1 - \cos x)$

(b) Prove that

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{n^2}, \quad -\pi < x < \pi$$

Hence show that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$. [7]

(c) Find the half-range sine series for the function $f(x) = x - x^2$ in the interval (0, 1). [7]

(d) Obtain the first three coefficients in the Fourier series for y, where y is given in the following table : [7]

x	0	1	2	3	4	5
y	4	8	15	7	6	2

2. *(a)* Find the Laplace transform of $f(t) = t \cos 2t$. [2]

(b) Evaluate the integrals by Laplace transform

$$\int_0^{\infty} te^{-2t} \cos t \, dt$$ [7]

(c) Use convolution theorem to find the inverse Laplace transform of the function

$$L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}$$ [7]

(d) Solve $ty'' + 2y' + ty = \cos t$, given that $y(0) = 1$. [7]

3. *(a)* Find the value of $\int_{|z|=3} \left(\frac{z}{z-1}\right) dz$. [2]

(b) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though C-R equations are satisfied thereof. [7]

(c) If $f(z) = u + iv$ is an analytic function of z, find $f(z)$ if $u - v = (x - y)(x^2 + 4xy + y^2)$. [7]

(d) Find the Laurent series expansion of

$$\frac{z^2 - 1}{z^2 + 5z + 6}$$

about $z = 0$ in the region $2 < |z| < 3$. [7]

4. (a) Form the partial differential equation by eliminating the arbitrary function $z = f\left(\frac{xy}{z}\right)$. [2]

(b) Solve : [7]

$$4\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 16 \log (x + 2y)$$

(c) Solve : [7]

$$x^2(y - z)p + y^2(z - x)q = z^2(x - y)$$

(d) Using the method of separation of variables, solve

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u, \text{ where } u(x, 0) = 6e^{-3x}$$ [7]

(Turn Over)

5. (a) Define random variable. [2]

(b) A die is tossed thrice. A success is 'getting 1 or 6' on a toss. Find the mean and variance of the number of success. [7]

(c) The probability of a bad reaction from a certain injection is 0·001. Determine the chance that out of 2000 individuals more than two will get a bad reaction. [7]

(d) Fit a normal curve to the following distribution : [7]

x :	2	4	6	8	10
f :	1	4	6	4	1