Unit-3 Laplace transform

Q. 1. Write the Laplace transform of $e^{at}\sinh t$.

Q. 2. Write the Laplace transform of $\cos t \sin t$.

Q. 3. Write the condition for existence of Laplace transform.

Q. 4. Define unit impulse function.

Q. 5. Find the Laplace transform of

i. $(e^{-t}\sin t)t$

ii. $\left(\sqrt{t}-\frac{1}{\sqrt{t}}\right)^3$

iii. $\frac{\cos at-\cos bt}{t}$.

Q. 6. If $L\{f(t)\}=\bar{f}(s)$. Then prove that

i. $L\{t^n\}=\frac{n!}{s^{n+1}}, n=0,1,2,\ldots$

ii. $L\{e^{at}f(t)\}=\bar{f}(s-a)$.

Q. 7. Find the Laplace transform of

$$\frac{1-\cos t}{t^2}.$$

Q. 8. Find the inverse Laplace transform of

i. $\frac{s^2+6}{(s^2+1)(s^2+4)}$

ii. $\frac{s}{(s^2+1)(s^2+4)}$ using Convolution theorem.

Q. 9. Find the inverse Laplace transform of

i. $\frac{s+2}{s^2-4s+3}$

ii. $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ using Convolution theorem.

Q. 10. Find the inverse Laplace transform of

i. $\frac{s^2+s-2}{s(s+3)(s-2)}$

ii. $\tan^{-1}\frac{2}{s^2}$.

Q. 11. Apply convolution theorem evaluate $L^{-1}\left\{\frac{8}{(s^2+1)^3}\right\}$.

Q. 12. Apply convolution theorem to prove that

$$\beta(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}dx, m>n>0=\frac{\Gamma(m)\Gamma(\mathrm{n})}{\Gamma(\mathrm{m}+\mathrm{n})}$$

Q. 13. Use the method of partial fraction to find the inverse transform of $\frac{s}{s^4+s^2+1}$

Q. 14. (a)Find the Laplace transform of $\frac{1-\cos}{t}$

(b) Evaluate the following: $\int_0^\infty te^{-3t}\sin t\,dt$.

Q. 15. (a)Find the Laplace transform of $\sin 2t\sin 3t+\cos^2 t$.

(b) Show that $\int_0^\infty te^{-2t}\cos t\,dt=\frac{3}{25}$

Q. 16. Solve the differential equation by transform method $\frac{d^2x}{dt^2} + 9x = \cos 2t\,, when\ x(0) = 1,$ $x\left(\frac{\pi}{2}\right) = -1.$

Q. 17. Solve the differential equation by transform method $ty'' + 2y' + ty = \sin t\,, when\ y(0) = 1.$

Q. 18. Solve the differential equation by transform method $ty'' + (1 - 2t)y' - 2y = 0, when\ y(0) = 1\ and\ y'(0) = 2.$

Q. 19. Solve the differential equation by transform method $y'' - 3y' + 2y = 4t - e^{3t}, when\ y(0) = 1\ and\ y'(0) = -1.$

Q. 20. Solve $(D^2 + m^2)x = a\cos nt\,, t > 0, when\ x = x_0\ and\ Dx = x_{1,}\ when\ t = 0,\ \ m \neq n.$

Q. 21. Solve $(D^3 - 3D^2 + 3D - 1)y = t^2e^t,\ when\ y(0) = 1, Dy(0) = 0\ and\ D^2y(0) = -2.$

UNIT-3 (RANDOM VARIABLES)

2 marks Questions

1. Define probability density function.
2. Define moment generating function of discrete and continuous probability distribution.
3. Define expectation and variance.
4. Define random variable and random experiment.
5. Write applications of binomial distribution.

4 marks Questions

1. If a random variable has a Poisson distribution such that P(1) = P(2), find mean of the distribution and P(4).
2. A variate X has a probability distribution

x	:	-3	6	9
P(X = x)	:	1/6	½	1/3

Find E(X) and $E(X^2)$. Hence evaluate $E(2x+1)^2$?

3. Verify that the area under the curve $f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}$ is unity. Find the mean value.

4. The mean and variance of binomial distribution are 4 and 4/3 respectively. Find $P(X \geq 1)$.

5. In 256 sets of 12 tosses of a coin, in how many cases one can expect 8 heads and 4 tails.

8 marks Questions

1. The probability density function of a variate X is

X	: 0	1	2	3	4	5	6
P(X)	: k	3k	5k	7k	9k	11k	13k

(i) Find $P(X < 4)$, $P(X \geq 5)$, $P(3 < X \leq 6)$.
(ii) What will be the minimum value of k so that $P(X \leq 2) > 0.3$.

2. The probability density p(x) of a continuous random variable is given by $p(x) = y_0 e^{-|x|}$, $-\infty < x < \infty$ Prove that $y_0 = ½$. Find the mean and variance of the distribution.
3. If x is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} kx & (0 \leq x \leq 2) \\ 2k & (2 \leq x < 4) \\ -kx + 6k & (4 \leq x < 6) \end{cases}$$

Find k and mean value of x.

4. Find the moment generating function of the exponential distribution
$f(x) = \frac{1}{c} e^{-x/c}, 0 \leq x \leq \infty, c > 0$
Hence find its mean and S.D.

5. A bag contains 5 black, 6 white and 7 red balls. Four balls are drawn at random from it. If x denotes the number of white balls, then find E(x).
6. The probability that a pen manufactured by a company will be defective is 1/10. If 12 such pens are manufactured find the probability that
   (a) Exactly 2 will be defective.
   (b) Atleast two will be defective.
   (c) None will be defective.
7. Fit a binomial distribution for the following data and compare the theoretical frequencies with the actual ones.

x	0	1	2	3	4	5
f	2	14	20	34	22	8

8. Out of 800 families with 5 children each, how many would you expect to have
   (a) 3 boys (b) 5 girls (c) Either 2 or 3 boy?
   Assume equal probabilities for boys and girls.
9. In a certain factory turning out razor blades there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 10,000 packets.
10. Fit a Poisson distribution to the set of observation:

X	0	1	2	3	4
F	122	60	15	2	1

11. A car hire firm has 2 cars which it hires out day by day. The number of demands for a car on each day is distribution as a Poisson distribution with mean 1.5. Calculate the probability of day
    (i) on which there is no demand
    (ii) on which demand is refused. ($e^{-1.5} = 0.2231$).
12. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D of the distribution.
13. Fit a normal curve to the following distribution.

X	2	4	6	8	10
F	1	4	6	4	1

14. In a precision bombing attack there is a 50% chance that any bomb will strike the target. Two direct hits are needed to destroy the target completely. How many bombs must be dropped to give a 99% chance or better of completely destroying the target?
15. In a test on 2000 electric bulbs, it was found that the life of particular make, was normally distributed with an average life of 2040 hours and S.D of 60 hours. Estimate the number of bulbs likely to burn for
    (a) More than 2150 hours.
    (b) Less than 1950 hours and
    (c) More than 1920 hours and less than 2160 hours.

Unit-5(Complex functions)

Q. 1. State Cauchy's Residue theorem.

Q. 2. Define Analytic function.

Q. 3. Write the Taylor series.

Q. 4. Find the regular function whose imaginary part is: $\frac{x-y}{x^2+y^2}$.

Q. 5. Show that $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though C-R equation are satisfied thereat.

Q. 6. An electrostatic field in the xy-plane is given by the potential function $\varphi = 3x^2y - y^3$, find the stream function.

Q. 7. Determine the analytic function $f(z) = u + iv$ if $u - v = e^x(\cos y - \sin y)$.

Q. 8. Determine the analytic function $f(z) = u + iv$ if $u - v = \frac{\cos x + \sin x - \ \ ^{-y}}{2(\cos x - \cosh y)}$ and $f(\pi/2) = 0$.

Q. 9. If $f(z)$ is a Holomorphic function of z, show that

$$\left\{\frac{\partial}{\partial x}|f(z)|\right\}^2 + \left\{\frac{\partial}{\partial y}|f(z)|\right\}^2 = |f'(z)|^2.$$

Q. 10. Evaluate $\int_C \frac{3z^2+2}{(z-1)(z^2+9)}dz$, where C is the circle: |z-2|=2.

Q. 11. Evaluate $\int_C \frac{z-3}{z^2+2z+5}dz$, where C is the circle

(a) $|z| = 1$,

(b) $|z + 1 - i| = 2$.

Q. 12. State Cauchy's integral formula and use this formula to evaluate $\int_C \frac{z^2-z+1}{z-1}dz$, where C is the circle

(c) $|z| = 1$,

(d) $|z| = \frac{1}{2}$.

Q. 13. Prove that :

$$\log z = (z-1) - \frac{(z-1)^2}{2} + \frac{(z-1)^3}{3} - \frac{(z-1)^4}{4} + \cdots + (-1)^{n-1}\frac{(z-1)^n}{n},$$
$$|z-1| < 1.$$

Q. 14. Obtain Laurent expansion for the function $f(z) = \frac{1}{z^2 \sinh}$, where C is the circle $|z - 1| = 2$.

Q. 15. Obtain Laurent expansion for the function $f(z) = \frac{z^2-1}{z^2+5z+6}$, about z=0 in the region $2 < |z| < 3$.

Q. 16. Obtain Laurent expansion for the function $f(z) = \frac{7z-2}{(z+1)z(z-2)}$, in the region $1 < |z + 1| < 3$.

Q. 17. Determine the poles of the function: $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and the residue at each pole. Hence evaluate $\int_C f(z)dz$, where C is the circle |z|=2.5.

Q. 18. Apply calculus of residue to prove that

$$\int_0^{2\pi} \frac{\cos 2\theta \, d\theta}{1-2\,a\cos\theta+a^2} = \frac{2\pi a^2}{1-a^2}, (a^2 < 1).$$

Q. 19. Apply calculus of residue to prove that

$$\int_0^{\pi} \frac{d\theta}{17-8\cos\theta} = \frac{\pi}{15}.$$

Q. 20. By integrating around a unit circle, evaluate:

$$\int_0^{2\pi} \frac{\cos 3\theta \, d\theta}{5-4\cos\theta}.$$