

(c) If  $x$  and  $y$  denote any pair of real numbers for which  $0 < x < y$ , prove by mathematical induction  $0 < x^n < y^n$  for all natural number  $n$ . [7]

(d) A person invests Rs. 10,000 @ 12% interest compounded annually. How much will be there at the end of 15 years using recurrence relation? [7]

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BE (4<sup>th</sup> Semester)

Examination, April - May, 2017

[ New Scheme ]

Discrete Structures

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Time Allowed : 3 hours

Maximum Marks : 80

Minimum Pass Marks : 28

**Note :** (i) Part (a) of each question is compulsory. Attempt any two parts from (b), (c) and (d) of each question.

(ii) The figures in the right-hand margin indicate marks.

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I. (a) Write the following statement in symbolic form :

If either Jerry takes calculus or Ken takes Sociology, then Larry will take English. [2]

(b) Write an equivalent expression for  $(p \Rightarrow q \wedge r) \vee (r \Leftrightarrow s)$  which contains neither the conditional nor the biconditional. [7]

(c) Express the Boolean function  $f(a, b, c) = ab + a'c$  as a product of maxterms. [7]

(d) Prove that if  $H_1, H_2, \dots, H_m$  and  $P$  imply  $Q$ , then  $H_1, H_2, \dots, H_m$  imply  $P \rightarrow Q$ . [7]

2. (a) Prove that the null set  $\phi$  is a subset of every set.

[2]

(b) How many reflexive relation and transitive relations are there on a set with  $n$  elements?

[7]

(c) Let  $X = \{2, 3, 6, 12, 24, 36\}$  and the relation  $\leq$  be such that  $x \leq y$  if  $x$  divides  $y$ . Draw the Hass diagram of  $(x, \leq)$ .

[7]

(d) If  $f(x) = x+2$ ,  $g(x) = x-2$ ,  $h(x) = 3x$  for  $x \in R$  where  $R$  is the set of real numbers. Find  $g \circ f$ ,  $f \circ g$ ,  $f \circ h$ ,  $h \circ f$ ,  $h \circ g$  and  $g \circ h$ .

[7]

3. (a) Define semigroup.

[2]

(b) Prove that the set  $\{-1, 1\}$  forms an abelian group with respect to multiplication but with respect to addition it does not form a group.

[7]

(c) Prove that the intersection of any two subgroups of a group  $G$  is also a subgroup of  $G$  but the union of any two subgroups of a group  $G$  is not necessarily a subgroup of  $G$ .

[7]

(d) The parity check matrix for a 6-bit linear code is

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

The words 111001 and 101011 are received. Use the matrix to decide whether or not the words are likely to have been correctly transmitted.

[7]

4. (a) Prove that a group which contains a triangle cannot be bipartite.

[2]

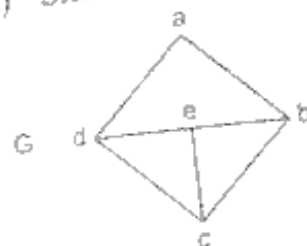
(b) Prove that a group with  $n$  vertices and  $k$  components cannot have more than

$$\frac{(n-k)(n-k+1)}{2}$$

[7]

edges.

(c) Show that the graph  $G$  and  $G'$  are isomorphic. [7]



(d) Show that the minimum height of a binary tree on  $n$  vertices is  $\log_2(n+1)-1$  and maximum possible height is  $\frac{n-1}{2}$ .

[7]

5. (a) Prove that  ${}^nC_r + {}^nC_{r-1} = {}^{n-1}C_r$ .

[2]

(b) Prove that in any room of people who have been doing handshaking, there will always be at least two people who have shaken hands the same number of times.

[7]