Roll No. ......................

# 322452(14)

**B. E. (Fourth Semester) Examination, Nov.-Dec. 2019**

**(New Scheme)**

**(CSE Branch)**

**DISCRETE STRUCTURES**

***Time Allowed : Three hours***

***Maximum Marks : 80***

***Minimum Pass Marks : 28***

***Note :*** *Part (a) of each quesiton is compulsory and carries 2 marks. Attempt any* ***two*** *the remaining questions and carries 7 marks each.*

## Unit - I

**1.** (a) Define tautology.

(b) Establish the equivalance :

$$(P \vee q) \Rightarrow r \equiv (P \Rightarrow r) \wedge (q \Rightarrow r)$$

(c) Obtain the disjunctive normal form of the following function :

$$F(x, y, z) = (x + y) \cdot (x + z') + (y + z')$$

(d) Prove that for each $a \in B$, $a'$ is unique.

## Unit - II

2. (a) If $f : R \to R$ and $g : R \to R$ defined by the formula

$$f(x) = x + 2 \quad \forall x \in R$$

$$g(x) = x^2 \quad \forall x \in R \text{, then find } gof.$$

(b) Prove that the $R$ is an equivalance relation where $R$ be a relation in the set of integer $Z$ defined by

$$R = \{ (x, y) : x \in z, \ y \in z, \ x - y \text{ is divisible by } 6 \}.$$

(c) Prove that in a distributive lattice $(L, \leq)$ if $a \wedge b = a \wedge c$ and $a \vee b = a \vee c$ then $b = c$.





(d) Show that the mapping $f : R \to R$ be defined by $f(x) = ax + b$ where $a\ b \in R$ is invertible. Define its inverse.

## Unit - III

3. (a) Define semi-group.

(b) Prove that fourth root of unity $1, -1, i, -i$ from an abelian multiplicative group.

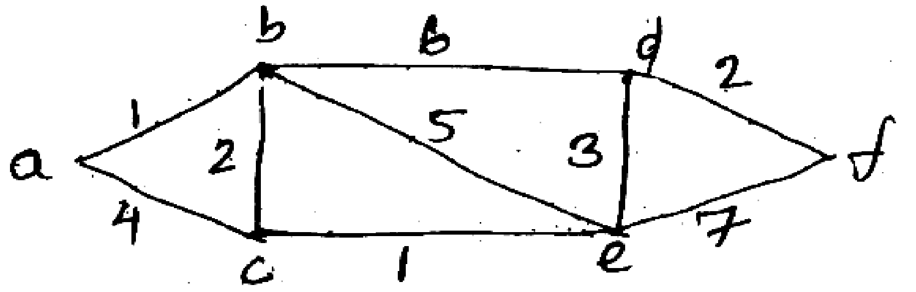
(c) State and prove Lagrange's theorem.

(d) Define Boolean ring. Prove that it is commutative.

## Unit - IV

4. (a) Define Eulerian and Hamiltonian graph.

(b) Apply Dijkstra's algorithm to the graph given below and find the shortest path from $a$ to $f$.



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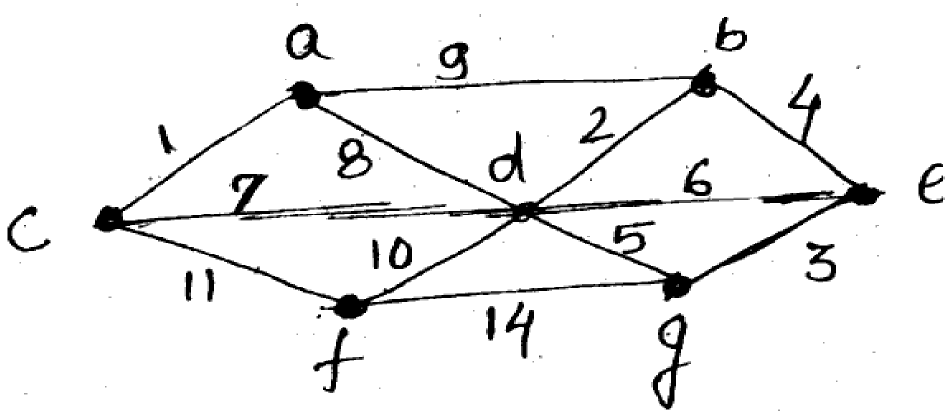
(c) Define spanning tree, find the minimum spanning tree of the following spanning tree of the following graph by using Kruskal's algorithm.




(d) Draw the graph represented by the following matrix A :

$$A = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{array} \begin{array}{c} \begin{array}{ccccc} V_1 & V_2 & V_3 & V_4 & V_5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$





## Unit - V

5. (a) Write the pigenhole principle.

(b) Show that $n^3 + 2n$ is divisible by 3 for all $n \geq 1$ by mathematical induction.

(c) How many positive integers not exceeding 500 are divisible by 7 or 11?

(d) Solve the recurrence relation by the method of characteristics roots $a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3}$ with initial condition $a_0 = 0, \quad a_3 = 3, a_5 = 10.$