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MATH-IV MINOR TEST

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Q1. i) solution
For Continuous Random Variable

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

Since, $x \geq 0$, $\therefore \int_{0}^{\infty} f(x)\,dx = 1$

Here, $f(x) = kxe^{-\lambda x} \Rightarrow \therefore \int_{0}^{\infty} kxe^{-\lambda x}\,dx = 1$

let $\lambda x = u$

$\Rightarrow \lambda\,dx = du$

$$\therefore k\int_{0}^{\infty} \frac{u}{\lambda} e^{-u} \frac{du}{\lambda} = 1$$

$$\Rightarrow \frac{k}{\lambda^2}\int_{0}^{\infty} ue^{-u}\,du = 1$$

Comparing above equation with $\int_{0}^{\infty} e^{-x}x^n\,dx = n!$

we get $n = 1$ and $\boxed{\int_{0}^{\infty} e^{-x}x\,dx = 1!}$

$$\therefore \Rightarrow \frac{k}{\lambda^2}\int_{0}^{\infty} ue^{-u}\,du = 1$$

Replace u by x

$$\Rightarrow \frac{k}{\lambda^2}\int_{0}^{\infty} xe^{-x}\,dx = 1$$

$$\therefore \frac{k}{\lambda^2}\{(1!)\} = 1$$

$$\therefore k = \lambda^2$$

Ans

ii) Now, Mean And Variance,

$$E(x) = \int_0^\infty x f(x)\,dx$$

$$= \int_0^\infty x\,(kxe^{-\lambda x})\,dx$$

$$= k\int_0^\infty x^2 e^{-\lambda x}\,dx$$

$$= k\left[\frac{x^2e^{-\lambda x}}{-\lambda} - \frac{2xe^{-\lambda x}}{(-\lambda)^2} + \frac{2e^{-\lambda x}}{(-\lambda)^3}\right]_0^\infty$$

$$= k\left[-\frac{x^2e^{-\lambda x}\lambda^2 + 2xe^{-\lambda x}\lambda + 2e^{-\lambda x}}{-\lambda^3}\right]_0^\infty$$

$$= k\left[\frac{e^{-\lambda x}(\lambda^2x^2 + 2x\lambda + 2)}{-\lambda^3}\right]_0^\infty$$

$$= k\left[0 - \frac{2}{-\lambda^3}\right] = \lambda^2 \times \frac{2}{\lambda^3} = \frac{2}{\lambda}$$

Now, Mean $= \frac{2}{\lambda}$

Now, $E(x^2) = \int_0^\infty x^2 f(x)\,dx = k\int_0^\infty x^3 e^{-\lambda x}\,dx$

$$= k\left[\frac{x^3e^{-\lambda x}}{-\lambda} - \frac{3x^2e^{-\lambda x}}{(-\lambda)^2} + \frac{6x\cdot e^{-\lambda x}}{(-\lambda)^3} - \frac{6e^{-\lambda x}}{(-\lambda)^4}\right]_0^\infty$$

$$= \lambda^2 \times \frac{6e^0}{(-\lambda)^4} = \frac{6}{\lambda^2}$$

$\therefore$ Var $= E(x^2) - (E(x))^2 = \frac{6}{\lambda^2} - \frac{4}{\lambda^2} = \frac{2}{\lambda^2}$ Ans

Q2. Given $f(x) = 4xy$; joint density function

Now, $0 \le x \le 1$ and $0 \le y \le 1$ and 0 otherwise

Marginal Probability (of x) distribution $= \int_0^1 4xy \, dy$

$= \left.\frac{4xy^2}{2}\right]_0^1$

$= 2x$

Again,

Marginal Probability (of y) distribution $= \int_0^1 4xy \, dx$

$= 2y$.

And for independent events,

We know,

$F(x,y) =$ Marginal prob(x) $\times$ Marginal prob(y)

$= 2x \times 2y$

$= 4xy$,

which is equal to $F(x,y)$

$\therefore$ x and y are independent events.

Now, $E(x,y) = \int_0^1 \int_0^1 xy \, F(x,y) \, dy \, dx$

$= \int_0^1 \int_0^1 xy \, 4xy \, dy \, dx$

$= \int_0^1 \left[\frac{4xy^3}{3}\right]_0^1 dx = \int_0^1 \frac{4}{3} x \, dx$

$= \left.\frac{4x^2}{3 \times 2}\right]_0^1 = 2/3$

$\therefore$ Hence, $E(x,y) = 2/3$. Ans

Q3. solution

$x: 65, 66, 67, 67, 68, 69, 70, 72$

$y: 67, 68, 65, 68, 72, 72, 69, 71$

Now, $\bar{X} = \frac{65+66+67+67+68+69+70+72}{8}$

$= 68$

$\bar{Y} = \frac{\Sigma Y}{n} = \frac{67+68+65+68+72+72+69+71}{8}$

$= 69$

Now, $\sigma x^2 = \frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+ \cdots \cdots (x_8-\bar{x})^2}{n}$

$= \frac{(65-68)^2+(66-68)^2+ \cdots \cdots +(72-68)^2}{8}$

$= \frac{3^2+2^2+2\times1^2+0+1^2+2^2+4^2}{8}$

$= \frac{\cancel{36}\,9}{\cancel{8}\,2} = 9/2$

So, $\sigma x^2 = 9/2$

$\therefore \sigma x = \sqrt{9/2} = 3/\sqrt{2} = 2.12$

Similarly,

$\sigma y^2 = \frac{(y_1-\bar{y})+(y_2-\bar{y})+ \cdots \cdots +(y_8-\bar{y})^2}{n}$

$= \frac{2^2+1^2+4^2+1^2+3^2+3^2+0^2+2^2}{8}$

$\sigma y^2 = \frac{\cancel{44}\,11}{\cancel{8}\,2} = 11/2$

$\therefore \sigma y = \sqrt{\frac{11}{2}} = 2.345$. Ans

$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})$	$(y_i - \bar{y})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
-3	9	-2	4	6
-2	4	-1	1	2
-1	1	-4	16	4
-1	1	-1	1	1
0	0	3	9	0
1	1	3	9	3
2	4	0	0	0
4	16	2	4	8
	$\Sigma = 36$		$\Sigma = 44$	$\Sigma = 24$

$$r_{\text{(correlation coef.)}} = \frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma (x_i - \bar{x})^2 \, \Sigma (y_i - \bar{y})^2}}$$

$$= \frac{24}{\sqrt{36 \times 44}}$$

$$= 0.603$$

Q4.

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i) solution

If a coin is tossed 3 times, then sample space for this experiment is;

$\{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

Since X denotes the number of tails,

$\therefore X$ can take values as,

X = Number of tails in three tosses $= \{0, 1, 2, 3\}$

Now, $P(X=0) = P(0) = 1/8 = 0.125$

$P(X=1) = P(1) = 3/8 = 0.375$

$P(X=2) = P(2) = 3/8 = 0.375$

$P(X=3) = P(3) = 1/8 = 0.375$

$\therefore$ the probability distribution of X is given by

X	$P(x)$	$xP(x)$	$x^2P(x)$
0	$1/8$	0	0
1	$3/8$	$3/8$	$3/8$
2	$3/8$	$3/4$	$3/2$
3	$1/8$	$3/8$	$9/8$

$\therefore \sum x(P(x)) = \frac{12}{8} = 3/2$

$\sum x^2(P(x)) = \frac{24}{8} = 3$

$\therefore E(X) = \sum xP(x) = 3/2 = 1.5$ Ans.

Q.4

ii) solution

$$F(x,y) = 1/y \;;\; 0 < x < y,\; 0 < y < 1$$

$$\therefore E(x|y) = \int_{-\infty}^{\infty} x F(x,y)\,dx$$

$$= \int_{-\infty}^{\infty} \frac{x}{y}\,dx$$

$$= \frac{x^2}{2y}$$ Ans.

Q4.

iii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\Rightarrow \int_0^1 3x^2 dx = [x^3]_0^1 = 1$

$\therefore f(x) = \begin{cases} 3x^2 & ; 0 \le x \le 1 \\ 0 & ; \text{otherwise} \end{cases}$

(a) $P(x \le a) = P(x > a)$

$\because$ X is continuous Random Variable

$P(x \le a) = P(x \ge a)$

$\Rightarrow 2P(x \le a) = 1$

$\therefore P(x \le a) = 1/2$

$\therefore \int_0^a 3x^2 dx = \frac{1}{2} \Rightarrow a^3 = \frac{1}{2}$

$\Rightarrow a = \sqrt[3]{1/2}$

(b) $P(x > b) = 0.05$

$\because$ X is a continuous Random Variable

$P(x \ge b) = 0.05$

$\int_b^1 3x^2 dx = \frac{5}{100} = \frac{1}{20}$

or, $1 - b^3 = \frac{1}{20}$

$\Rightarrow b^3 = 1 - \frac{1}{20} = \frac{19}{20}$

$\therefore b = \sqrt[3]{\frac{19}{20}} = \sqrt[3]{\frac{19}{20}}$ Ans