Prob: 2

$y(t) = ?$

$h(t) = =$

$\downarrow LT$

$H(s) = \frac{Y(s)}{X(s)} \Rightarrow Y(s) = H(s) \cdot X(s)$

$\downarrow ILT$

$y(t) = h(t) * x(t)$

Convolution.

Convolution operation

A convolution is an integral that expreces the amount of overlap of one function when it is shifted over another function.

Different operation

① Shifting ② Scaling ③ Differenciation

④ Integration ⑤ Convolution.

(Calculate the o/p of LTI System)

$$\boxed{\text{Convolution operator } (*) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) \cdot d\tau.}$$

$x(t)$

$\downarrow$

$x(\tau)$ ① Task

$h(t)$

$\downarrow$

$h(\tau)$

② Task

$\downarrow$ Time Reversal (Scaling)

$h(-\tau)$ ③ Task

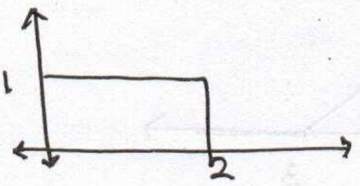
$\downarrow$ Time Shifting.

$h[-(\tau - t)] = h(t-\tau)$

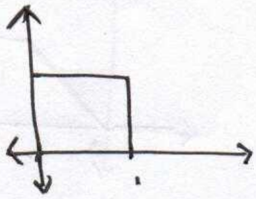
④ $x(\tau) \cdot h(t-\tau)$

⑤ $\int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) \cdot d\tau$

Example

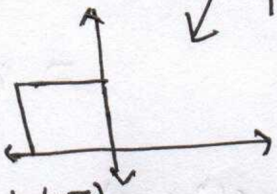


$x(\tau)$



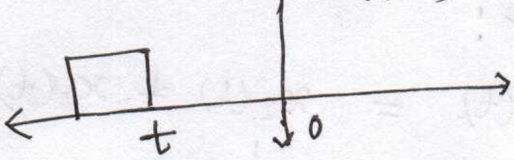
$h(\tau)$

Time Reversal.

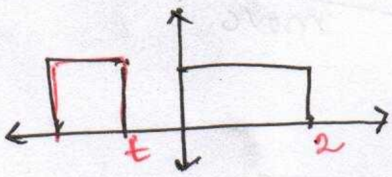


$h(-\tau)$

$h(t-\tau)$

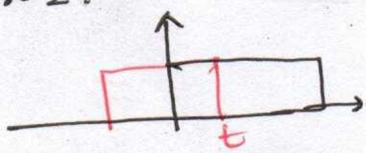


Case 1: $t<0$



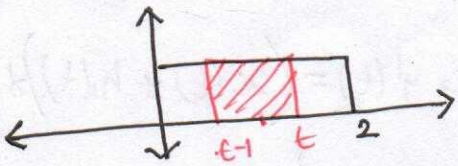
$$y(t)=0$$

Case 2: $0<t<1$



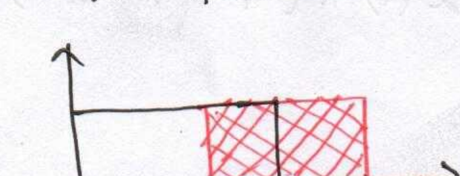
$$y(t)=\int_0^t 1 \cdot d\tau = t$$

Case 3: $1<t<2$



$$y(t)=\int_{t-1}^{t} 1 \cdot d\tau = 1$$

Case 4: $2<t<3$



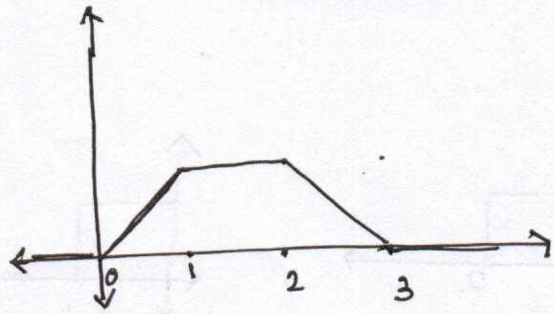
$$y(t)=\int_{t-1}^{2} 1 \cdot d\tau = 3-t$$

Case 5: $t>3$

$$y(t)=0$$

$$y(t) = \begin{cases} 0, & t<0 \\ t & 0<t<1 \\ 1, & 1<t<2 \\ 3-t, & 2<t<3 \\ 0, & t>3 \end{cases}$$

Ans



$$y(t) = 0 + 1\cdot r(t-0) - 1\cdot r(t-1) - 1\cdot r(t-2) + 1\cdot r(t-3)$$

$$= r(t) - r(t-1) - r(t-2) + r(t-3)$$

Ans

Properties of Convolution (10 Diff^n prop)

1. Commutative Property:

$$x_1(t) * x_2(t) = y(t) = x_2(t) * x_1(t)$$

$$\underset{\text{fixed}}{x_1(\tau)} \quad \underbrace{x_2(t-\tau)}_{\text{move}} \qquad \underbrace{x_2(\tau)}_{\text{fix}} \quad \underbrace{x_1(t-\tau)}_{\text{move}}$$

$$x(t) \rightarrow \underset{\text{LTI}}{\boxed{h(t)}} \rightarrow y(t) \qquad h(t) \rightarrow \boxed{x(t)} \rightarrow y(t)$$

2. Associative Property:

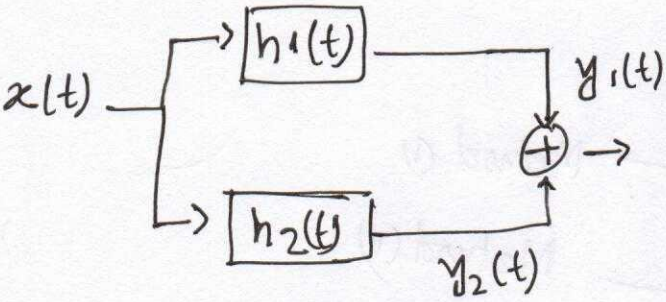
$$\big(x_1(t) * x_2(t)\big) * x_3(t) = y(t) = x_1(t) * \big(x_2(t) * x_3(t)\big)$$

$$x(t) \longrightarrow \overset{\text{1st System}}{\boxed{h_1(t)}} \longrightarrow y(t) \quad \overset{\text{2nd Syst.}}{\boxed{h_2(t)}} \rightarrow y'(t) = \big(x(t) * h_1(t)\big) * h_2(t)$$

$$x(t) \longrightarrow \boxed{h_1(t) * h_2(t)} \rightarrow y'(t) = x(t) * \big(h_1(t) * h_2(t)\big)$$

3. Distributive Property:

$$x_1(t) * (x_2(t) + x_3(t)) = y(t) = x_1(t) * x_2(t) + x_1(t) * x_3(t).$$



$$\rightarrow y(t) = x_1(t) * h_1(t) + x_1(t) * h_2(t)$$

$$x(t) \longrightarrow \boxed{h_1(t) + h_2(t)} \rightarrow x(t) * (h_1(t) + h_2(t)).$$

4. Property of Delta function: → Impulse Signal.

$$\boxed{x(t) * \delta(t-t_1) = x(t-t_1)}$$

if $t_1 = 0$

$$\boxed{x(t) * \delta(t) = x(t).}$$

$$\boxed{x(t) * A\,\delta(t-t_1) = A \cdot x(t-t_1).}$$

$$\boxed{x(t) * A\delta(t) = A \cdot x(t)}$$

Ex: $x(t) = r(t)$

$$r(t) * \delta(t-2) = r(t-2)$$

Ans

Ex: $x(t) = u(t)$.

$$u(t+3) * \delta(t-1) = u(t-1+3) = u(t+2)$$

5. Derivatives.

$$y(t) = x(t) * h(t).$$

$$\frac{dy(t)}{dt} = \frac{d}{dt}\left[x(t) * h(t)\right]$$

$$= x(t) * \frac{d}{dt} h(t) \text{ — Method-(I)}$$

$$= \frac{d}{dt} x(t) * h(t). \text{ — Method (II)}$$

Ex: $y(t) = r(t) * u(t)$

M-I $\quad \frac{dy(t)}{dt} = r(t) * \frac{d}{dt} u(t) = r(t) * \delta(t)$ [using Prop. 4]

$$= r(t). \text{ Ans}$$

M-II

$$\frac{d}{dt} y(t) = \frac{d}{dt} r(t) * u(t)$$

$$\boxed{= u(t) * u(t) = r(t)}$$

6. Integration

$I(D[y(t)]) = y(t)$

$$\boxed{y(t) = x(t) * u(t) = \int_{-\infty}^{t} x(\tau) d\tau}$$

Proof: $\quad y(t) = x(t) * u(t)$

$$\Rightarrow \frac{d}{dt} y(t) = x(t) * \frac{d}{dt} u(t)$$

$$\Rightarrow \frac{d}{dt} y(t) = x(t) * \delta(t)$$

$$\Rightarrow \int_{-\infty}^{t} \frac{d}{d\tau} y(\tau) d\tau = \int_{-\infty}^{t} x(\tau) * \delta(\tau) d$$

$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau.$$

Suppose, Eg.

$$y(t) = r(t) * u(t)$$

$$= \int_{-\infty}^{t} r(\tau)\,d\tau = p(t). = \frac{t^2}{2} \cdot u(t).$$

7. Time delay:

$$x_1(t) * x_2(t) = y(t)$$

delay t_1

$$x(t-t_1) * x(t-t_2) = y[t-(t_1+t_2)]$$

Example: $u(t-1) * u(t-2) = ?$

$$u(t) * u(t) = \int_{-\infty}^{t} u(\tau)\cdot d\tau . = r(t).$$

$$u(t-1) * u(t-2) = r(t-(1+2)) = r(t-3). \text{ Ans.}$$

$\boxed{\text{H.W}}$ $r(t+2) * u(t-2) = ?$

8. Time Scaling:

$$x_1(t) * x_2(t) = y(t)$$

Time Scaling by a | Time Scaling by a

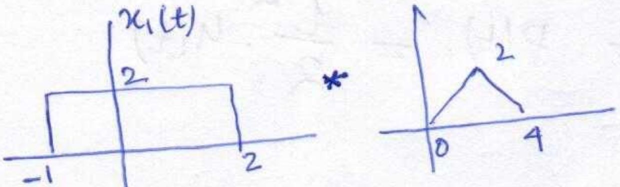
$$x_1(at) \quad x_2(at) = \frac{1}{|a|}\, y(at). \quad, a \neq 0$$

Same Amount Scaling

9. Property of area:-

$x_1(t) * x_2(t) = y(t)$

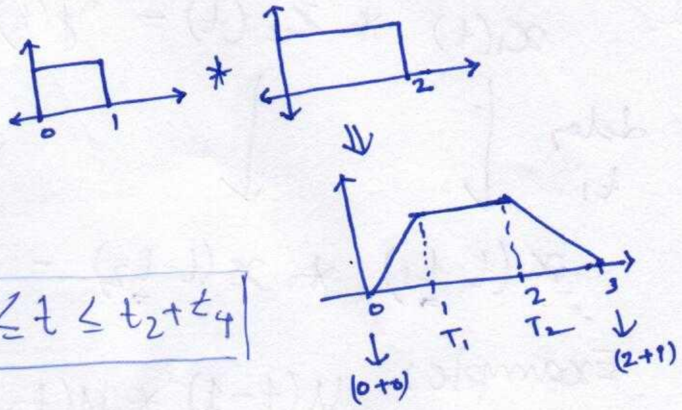
$A_1 \times A_2 = A$



$= y(t) = 6 \times 4 = 24.$

$A_1 = 6$ $\qquad$ $A_2 = 4.$

10. Duration / Extension :-



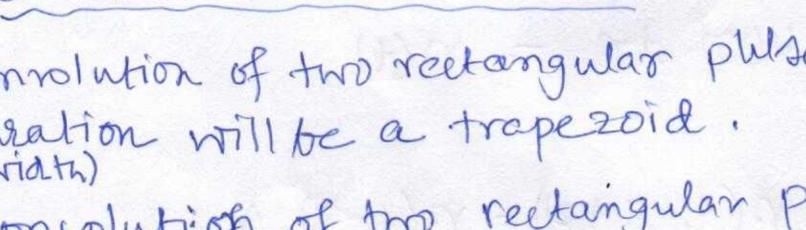
$x_1(t)$ $\qquad t_1 \leq t \leq t_2$

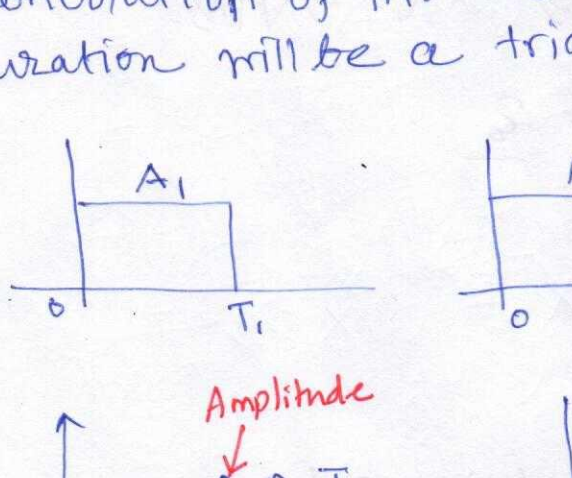
$x_2(t)$ $\qquad t_3 \leq t \leq t_4.$

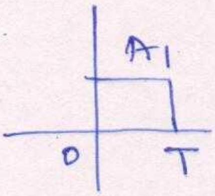
$\boxed{\text{Let, convolution } y(t),\ t_1+t_3 \leq t \leq t_2+t_4}$

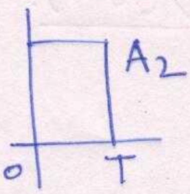
Convolution Method Shortcut

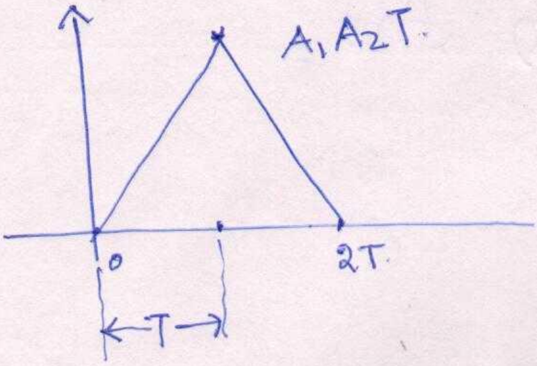
- Convolution of two rectangular pulses of unequal duration (width) will be a trapezoid.
- Convolution of two rectangular pulses of equal duration will be a triangle.



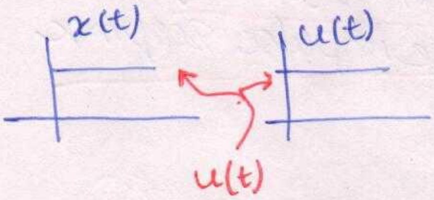




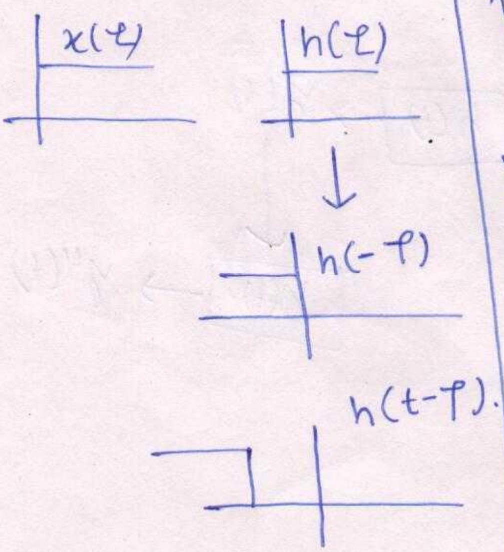




Q. Find out the O/P of the LTI System



Method - 1.



$-\infty < t \leq 0$

$y(t) = 0$

$t > 0$

$y(t) = \int_0^t 1 \cdot d\tau$

$= t$

M-II

$y(t) = x(t) * h(t)$

$= u(t) * u(t)$

$\downarrow$ LT

$Y(s) = \frac{1}{s} \cdot \frac{1}{s}$

$= \frac{1}{s^2}$

$\downarrow$ ILT

$y(t) = r(t).$

M-III

$x(t) * u(t) = \int_{-\infty}^{t} x(\tau) \cdot d\tau$

$= \int_{-\infty}^{t} u(\tau) \cdot d\tau$

$= r(t)$

Q. convolution of $x(t+5)$ with $\delta(t-7)$.

Solution.

$\cancel{x(t) * \delta(t-7) = x(}$

$$x(t+5) * \delta(t-7) = x(t-7+5)$$
$$= x(t-2) \quad \text{Ans}$$

Q. Solve, $x(-t) * \delta(t-t_0) = ?$

(a) $x(-t+t_0)$ (b) $x(t-t_0)$

(c) $x(t+t_0)$ (d) $x(-t-t_0)$

Q. The impulse response of a system $h(t) = \delta(t-0.5)$ if such two systems are cascaded, the impulse response of the over all system is

(a) $0.5\,\delta(t-0.25)$ (b) $\delta(t-0.25)$

(c) $\delta(t-1)$ (d) $0.5\,\delta(t-1)$

$$y(t) = x(t) * h(t)$$
$$= x(t) * \delta(t-0.5)$$
$$= x(t-0.5)$$

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) \rightarrow \boxed{h(t)} \rightarrow y''(t)$

$$y''(t) = y(t) * h(t)$$
$$= x(t-0.5) * \delta(t-0.5)$$
$$= x(t-0.5-0.5)$$
$$= x(t-1).$$

$x(t) \rightarrow \boxed{h_0(t)} \rightarrow y''(t)$

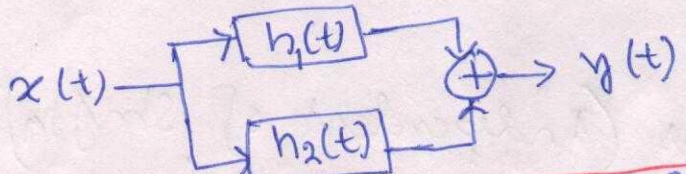
$$y''(t) = x(t) * h_0(t)$$
$$\Rightarrow x(t-1) = x(t) * h_0(t)$$
$$\downarrow LT$$
$$e^{-s} X(s) = X(s) \cdot H_0(s)$$

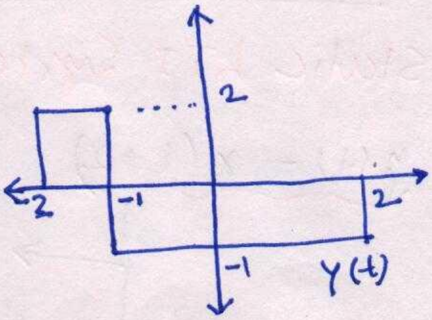
$$H_0(S) = e^{-S} \xrightarrow[\text{ILT}]{} \delta(t-1).$$

Q. A LTI with impulse response $h(t)$ produces $y(t)$ when i/p $x(t)$ is applied. When the I/P $x(t-\tau)$ is applied to a system with impulse response $h(t-\tau)$, the o/p will be

(a) $y(t)$ (b) $y(2(t-\tau))$
(c) $y(t-\tau)$ (d) $y(t-2\tau)$.

Q.

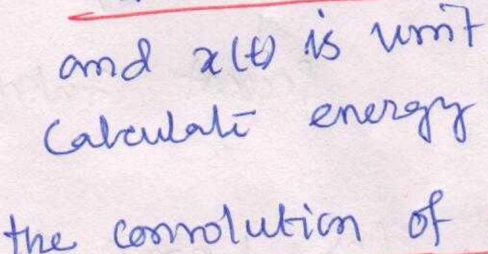




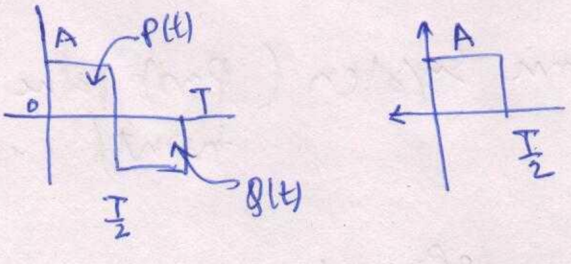
Given $h_1(t) = 2\delta(t+2) - 3\delta(t+1)$
$h_2(t) = \delta(t-2)$
and $x(t)$ is unit step signal, then
calculate energy of $y(t) = ?$

Q. Draw the convolution of $u(t) - u(t-1) * u(t) - u(t-2)$.

Ans:



Q.



$x(t) = p(t) + g(t)$ $y(t) = p(t)$