MID-SEMESTER EXAMS

2012174
Tushar Rathi

Name:- Tushar Rathi
Scholar ID:- 2012174
Subject:- Maths-IV

Q1 i) solution

Total number of people = 3 boys + 2 girls = 5

Total number of comitte of 3 members than can be made from 5 people = ${}^5C_3$

For atleast one boy,

Now, Probability of 1 boy = $P(1) = \dfrac{{}^3C_1 \times {}^2C_2}{{}^5C_3}$

$= \dfrac{3}{{}^5C_3}$

Now, Probability of 2 boys = $P(2) = \dfrac{{}^3C_2 * {}^2C_1}{{}^5C_3}$

$= \dfrac{3 \times 2}{{}^5C_3} = \dfrac{6}{{}^5C_3}$

Also, Probability of 3 boys = $P(3) = \dfrac{{}^3C_3 \times {}^2C_0}{{}^5C_3}$

$= \dfrac{1}{{}^5C_3}$

$\therefore$ Total probability of atleast one boy $= \dfrac{3+6+1}{{}^5C_3} = \dfrac{10}{10} = 1.$

2012174

Q 1.ii) solution

Total number of balls = 28

There are 4 options,

(2R, 1W), (2W, 1R), (0R, 3W), (3R, 0W)

∴ Total number of ways of selecting 3 balls = ${}^{56}C_3$

Now, P(containing a color of matching pairs)

$$= \frac{P(2\text{ red}) + P(3\text{ red}) + P(2\text{ white}) + P(3\text{ white})}{{}^{56}C_3}$$

$$= \frac{{}^{34}C_2 \cdot {}^{22}C_1 + {}^{22}C_2 \times {}^{34}C_1 + {}^{34}C_3 \times {}^{22}C_0 + {}^{22}C_3 \times {}^{34}C_0}{{}^{56}C_3}$$

$$= \frac{17 \times 33 \times 22 + 11 \times 21 \times 34 + \frac{34 \times 33 \times 32}{6} + \frac{22 \times 21 \times 20}{6}}{\frac{56 \times 55 \times 54}{6}}$$

$$= \frac{12342 + 7854 + 5984 + 1540}{27720}$$

$$= \frac{27720}{27720} = 1$$ Ans.

∴ Answer is 1.

Q2. solution 2012174.

Number of shoes = 20 (10 pairs)

For one box, Number of possibilities of shoes from different pairs : 4, 3, 2

For 4 different pairs,

No. of possibilities = $^{10}C_4 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1$

$= 1680$

For 3 different pairs,

Possibilities = $^{10}C_3 \times {}^3C_2 \times {}^2C_1 \times {}^2C_1$

$= 120 \times 3 \times 4$

$= 1440$

For 2 different pairs,

possibilities = $^{10}C_2 = 45$.

Total number of ways to store shoes = $^{20}C_4$

$= 4845$

∴ No. of possible students shoes packed in same box

$$= \left(4 \times \frac{1680}{4845} + 3 \times \frac{1440}{4845} + \frac{45 \times 2}{4845}\right) \Big/ 3$$

$$= \frac{6720 + 4320 + 90}{4845 \times 3} = \frac{11130}{14535} = 0.765$$

∴ Above probability for 5-boxes = 0.765×5

$= 3.825$ Ans.

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93. solution

Given, let Temp = X, No. of coff drinks = Y

Temp	75	81	85	105	93	113	121	125
No. of cold drinks	35	45	59	75	43	79	87	95

X	Y	$X.Y$	X^2	Y^2
75	35	2625	5625	1225
81	45	3645	6561	2025
85	59	5015	7225	3481
105	75	7875	11025	5625
93	43	3999	8649	1849
113	79	8927	12769	6241
121	87	10527	14641	7569
125	95	11875	15625	9025
$\Sigma X = 798$	$\Sigma Y = 518$			

$\Sigma X.Y = 54488$

$\Sigma X^2 = 82120$

$\Sigma Y^2 = 37040$

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i = \frac{798}{8} = 99.75$$

$$\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i = \frac{518}{8} = 64.75$$

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$$\sigma_x^2 = \sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2$$

$$= 82120 - \frac{(798)^2}{8} = 2519.5$$

$$\sigma_y^2 = \sum_{i=1}^{n} y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2$$

$$= 37040 - \frac{(518)^2}{8} = 3499.5$$

$$\text{cov}(x, y) = \sum_{i=1}^{n} x_i y_i - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)$$

$$= 54488 - \frac{798 \times 518}{8}$$

$$= 2817.5$$

Now, $r = \dfrac{\text{cov}(x, y)}{\sigma_x \sigma_y} = \dfrac{2817.5}{\sqrt{2519.5 \times 3499.5}}$

$$= \frac{2817.5}{2969.34}$$

$$= 0.9488$$

This is a ~~st~~ strong positive co-relation which means that high X variable score (Temp) go with high Y variable (cold drinks Number) scores and vice versa.

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Q4. solution

$$f_{xy} = \begin{cases} 2x - xy, & 0<x<1,\ 0<y<2 \\ 0, & \text{otherwise} \end{cases}$$

$$f_y(y) = \int_{x=0}^{1} (2x - xy)\,dx$$

$$= x^2 - \frac{x^2 y}{2}\Big|_0^1 = 1 - \frac{y}{2}$$

$$\therefore P(x < 1/2 \mid y < 1) = \frac{P(x < 1/2,\ y < 1)}{P(y < 1)}$$

$$= \frac{\int_{y=0}^{1}\int_{x=0}^{1/2} (2x - x y)\,dx\,dy}{\int_0^1 (1 - \frac{y}{2})\,dy}$$

$$= \frac{\int_{y=0}^{1} x^2 - \frac{x^2 y}{2}\Big|_0^{1/2} \cdot dy}{\int_0^1 (1 - \frac{y}{2})\,dy}$$

$$= \frac{\int_0^1 (1/4 - \frac{y}{8})\,dy}{1 - 1/4}$$

$$= \frac{\frac{y}{4} - \frac{y^2}{16}\Big|_0^1}{1 - 1/4} = \frac{\frac{1}{4} - \frac{1}{16}}{\frac{3}{4}}$$

$$= \frac{3 \times 4}{16 \times 3} = 1/4$$

ans

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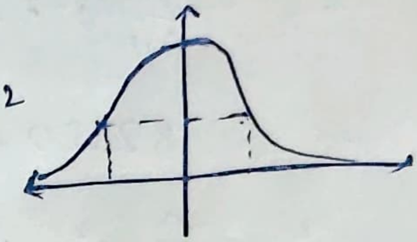
Q5. solution

$\mu = 65, \sigma = 9$

$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$



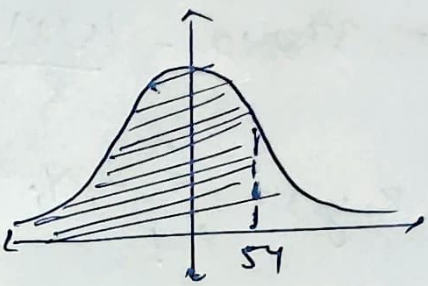
i) $P(x<54)$

let $Z = \frac{x-\mu}{\sigma}$

$Z < \frac{54-65}{9}$

or, $Z < -\frac{11}{9}$

or $Z < -1.22$



$\therefore P(Z<-1.22) = 0.112$

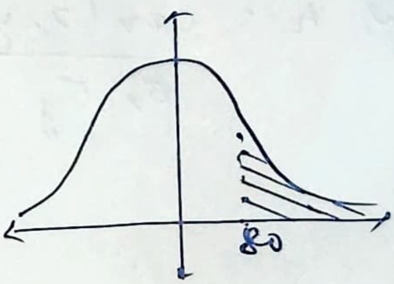
$\therefore$ Percentage score for $x<54$ = $0.112 \times 100 = 11.2\%$

ii) $x > 80$

$Z = \frac{x-\mu}{\sigma}$

or, $Z > \frac{80-65}{9}$

or $Z > 1.667$



$P(Z>1.667) = 1 - P(Z<1.667)$

$= 1 - 0.9525$

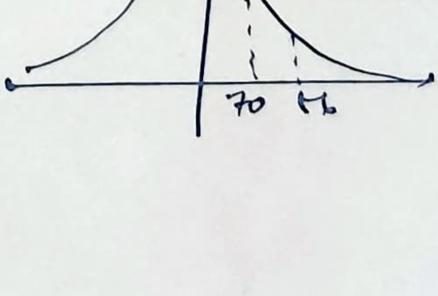
$= 0.0475$

$\therefore$ Percentage score for $(x>80) = 0.0475 \times 100\% = 4.75\%$

iii) $70 < x < 86$.

or, $Z < \frac{86-65}{9}$

or $Z < 2.33$ — (i)



And $Z > \frac{70-65}{9}$

$Z > 0.55$ — (ii)

Q5 (ii) cont.... 2012174

From (i) and (ii)

$$0.55 < Z < 2.33$$

$\therefore P(0.55 < Z < 2.33) = P(Z < 2.33) - P(Z < 0.55)$

$= 0.9901 - 0.7123$

$= 0.2778.$

Percentage scores for $(70 < x < 86) = 0.2778 \times 100\%$

$= 27.78\%.$

Thank You