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Computational numerical models for seismic response of structures isolated by sliding systems

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SUMMARY

The problem of sliding structures is discontinuous one as different sets of equations of motion with varying forcing functions are required for the sliding and non-sliding phases. This is inconvenient for the numerical integration of the governing equations for the response of sliding structures. To overcome such difficulties continuous hysteretic models of the sliding systems have been presented in the past. In the present study, a comparison of the response of structures (i.e. multi-storey buildings and bridges) isolated by sliding systems with conventional and hysteretic models of the frictional force of the sliding system is carried out to investigate the comparative performance and computational efficiency of the two models. The seismic response of isolated structures is obtained by solving the governing equations of motion using a step-by-step method under single and two horizontal components of real earthquake motions. For comparative study, the seismic response of a multi-storey building obtained by the conventional model is compared with the corresponding response by the hysteretic model under different sliding isolation systems, numbers of storeys and values of the fundamental time period of the superstructure. It is found that the conventional and hysteretic models of sliding systems predict a similar seismic response for isolated structures. However, the difference in the response between the two models is relatively more for the pure friction system as compared with the sliding systems with restoring force. Further, the conventional model is relatively more computationally efficient as compared with the hysteretic model.

KEY WORDS: base isolation; earthquake; frictional force; hysteretic model; computational efficiency; sliding system

1. INTRODUCTION

A significant amount of past research in the area of base isolation has focused on the use of frictional elements to concentrate the flexibility of the structural system and to add damping to the isolated structure. The most attractive feature of frictional base isolation systems is their effectiveness over a wide range of frequency input. The other advantage of a frictional system is that it ensures the maximum acceleration transmissibility equal to the maximum limiting frictional force. The simplest sliding system device is pure friction (PF) system without any

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restoring force [1–2]. Sliding isolation systems with restoring force include: the resilient-friction base isolator (RFBI) system [3], the Electricité de France (EDF) system [4], the friction pendulum system (FPS) [5], the sliding resilient friction (SRF) base isolator [6] and elliptical rolling rods [7]. The sliding systems perform very well under a range of severe earthquake loading and are quite effective in reducing the large levels of the superstructure's acceleration without inducing large base displacements [3]. Comparative study of different base isolation systems has shown that the response of a sliding system does not vary with the frequency content of earthquake ground motion [6,8,9]. In addition, sliding systems are less sensitive to the effects of torsional coupling in asymmetric base-isolated buildings [10].

The problem of a structure isolated by sliding systems is a discontinuous one in which different sets of equations of motion with varying forcing functions are required for the sliding and non-sliding phases. This becomes relatively inconvenient for numerical solution of the governing equations of motion for the response to specified time-history of earthquake ground motion. To overcome such difficulties a continuous hysteretic model of a sliding system is presented by Constantinou *et al.* [11] in which the rigid-plastic behaviour of the frictional force of the sliding systems is modelled by a nonlinear differential equations. Such a continuous model ensures that the same numbers of equations of motion are required for both sliding and non-sliding phases. However, this method may not be computationally efficient as a very high initial stiffness is required to represent the non-sliding phase, as a result, a very small time interval (or more computational time) must be used for stable response of the system. In the past, the response of structures with sliding systems had been studied with both types of model. However, the comparison of the seismic response obtained by these models and their computational efficiency is not reported.

In the present study, the response of multi-storey shear-type buildings and bridges isolated by the sliding systems is presented. The frictional force of the sliding systems is represented by two models, namely the conventional and hysteretic models. The response of isolated structures with the conventional model is compared with the corresponding response by the hysteretic model to distinguish the difference. Further, the computational efficiency of the two models of sliding mechanism and their relative advantages and disadvantages are also presented.

2. MODELLING OF MULTI-STOREY BUILDING WITH SLIDING SYSTEMS

Figure 1 shows the structural system under consideration which is an idealized base-isolated N-storey shear-type building with rigid floors. The force–deformation behaviour of the superstructure is considered to be linear with viscous damping. The sliding isolation system is installed between the base mass and foundation of the structure. The restoring force provided by the sliding systems is considered as linear (i.e. proportional to relative displacement). In addition, the damping provided other than friction is modelled in the form of viscous damping (i.e. damping in the rubber core of RFBI system). Thus, the sliding system is characterized by the three parameters: the lateral stiffness k_b, the damping constant c_b and the coefficient of friction μ (Figure 1b). The isolated structural system is subjected to two horizontal components of the earthquake motion applied in two horizontal directions of the building (referred as x- and y-directions). At each floor and base mass two lateral dynamic degrees-of-freedom are considered. Therefore, for the N-storey superstructure the dynamic degrees-of-freedom are $2 \times (N+1)$. The equations governing the motion of an isolated N-storey flexible shear-type

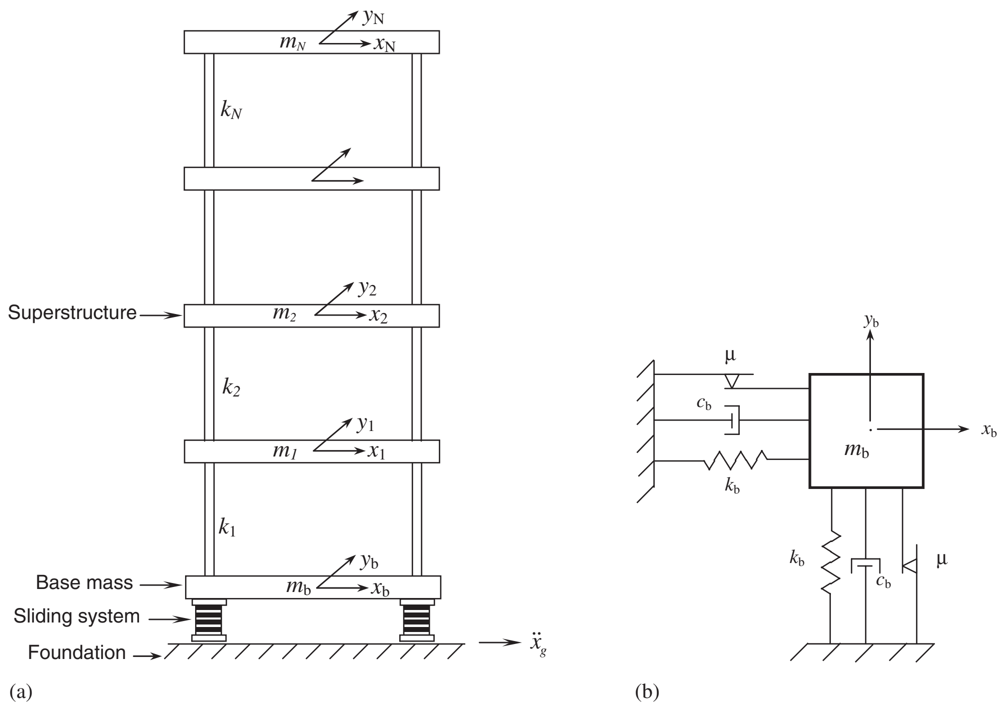


Figure 1. Model of N-storey building isolated by the sliding system. (a) Building model; (b) Schematic view of sliding system.

structure with sliding systems under the two horizontal components of earthquake excitation are expressed as

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = -[M][r]\{\ddot{z}_g\} \tag{1}$$

$$m_b\ddot{x}_b + c_b\dot{x}_b + k_b x_b + F_x - c_1\dot{x}_1 - k_1 x_1 = -m_b\ddot{x}_g \tag{2}$$

$$m_b\ddot{y}_b + c_b\dot{y}_b + k_b y_b + F_y - c_1\dot{y}_1 - k_1 y_1 = -m_b\ddot{y}_g \tag{3}$$

where $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices of the superstructure, respectively of size $2N \times 2N$; $\{x\} = \{x_1, x_2, \ldots, x_N, y_1, y_2, \ldots, y_N\}^T$ is the displacement vector of the superstructure relative to the base mass; x_i and y_i are the lateral displacement of ith floor relative to the base mass in the x- and y-directions, respectively; m_b is the mass of base raft; x_b and y_b are the displacement of base mass relative to the ground in the x- and y-directions, respectively; c_b and k_b are the viscous damping and the stiffness of the sliding system, respectively; F_x and F_y are the frictional forces of the sliding system in the x- and y-directions, respectively; c_1 and k_1 are the damping and stiffness of the first storey, respectively; $[r]$ is the influence coefficient matrix; $\{\ddot{z}_g\} = \{\ddot{x}_g + \ddot{x}_b, \ddot{y}_g + \ddot{y}_b\}^T$ is the vector of base acceleration; $\ddot{x}_g$ and $\ddot{y}_g$ are the earthquake ground acceleration in the x- and y-directions, respectively; and T denotes the transpose.

3. MODELING OF SLIDING SYSTEMS

The stiffness and viscous damping constant of the sliding system is designed to provide the specific values of the two parameters, namely the period of isolation T_b and the damping ratio ξ_b expressed by

$$T_{\mathrm{b}} = 2\pi\sqrt{\frac{(m_{\mathrm{b}} + \sum_{i=1}^{N} m_i)}{k_{\mathrm{b}}}} \tag{4}$$

$$\xi_{\mathrm{b}} = \frac{c_{\mathrm{b}}}{2(m_{\mathrm{b}} + \sum_{i=1}^{N} m_i)\omega_{\mathrm{b}}} \tag{5}$$

where m_i is the mass of ith floor of the superstructure; and $\omega_b = 2\pi/T_b$ is the base isolation frequency.

The friction coefficient of sliding system is assumed to be independent on the relative velocity at the sliding interface. This is based on the findings that such effects do not have noticeable effects on the peak response of the isolated structural system [8,12]. The limiting value of the frictional force F_S, to which the sliding system can be subjected (before sliding), is expressed as

$$F_{\mathrm{S}} = \mu\left(m_{\mathrm{b}} + \sum_{i=1}^{N} m_i\right)g \tag{6}$$

where g is the acceleration due to gravity.

Two models referred as conventional and hysteretic model are used for the frictional forces of the sliding system. The conventional model is a discontinuous one and number of stick–slide conditions requires different sets of equations to be solved and checked at every stage of solution of equations of motion. On the other hand, the hysteretic model is continuous and the required continuity is automatically maintained by the hysteretic displacement component specified in the model equation.

3.1. Conventional model

In the conventional model, the frictional force in the isolation system is evaluated by considering the equilibrium of the base mass (during the non-sliding phase) and the limiting value of the frictional force (during the sliding phase). This model had been extensively used in the past by many researchers for evaluation of the dynamic response of structures with sliding systems [1,3,7–10,13,14]. The system remains in the non-sliding phase ($\dot{x}_b = \ddot{x}_b = 0$ and $\dot{y}_b = \ddot{y}_b = 0$) if the frictional forces mobilized at the interface of sliding system are less than the limiting frictional force (i.e. $|F_x| < F_S$ and $|F_y| < F_S$). During the non-sliding phase, the equations of motion of the superstructure are only integrated and the corresponding frictional force is evaluated by

$$F_x = c_1\dot{x}_1 + k_1x_1 - k_{\mathrm{b}}x_{\mathrm{b}} - m_{\mathrm{b}}\ddot{x}_{\mathrm{g}} \tag{7}$$

$$F_y = c_1\dot{y}_1 + k_1y_1 - k_{\mathrm{b}}y_{\mathrm{b}} - m_{\mathrm{b}}\ddot{y}_{\mathrm{g}} \tag{8}$$

The system starts sliding ($\dot{x}_b \neq \ddot{x}_b \neq 0$ or $\dot{y}_b \neq \ddot{y}_b \neq 0$) as soon as the frictional force attains the limiting frictional force (i.e. $|F_x| = F_S$ or $|F_y| = F_S$). The governing equations of motion of the base mass are also included for the solution during the sliding phase of motion. Whenever the relative velocity of the base mass becomes zero (i.e. $\dot{x}_b = 0$ or $\dot{y}_b = 0$), the phase of the motion has to be checked in order to determine whether the system remains in the sliding phase or sticks to the foundation. Thus, the conventional model is a discontinuous model in which different set of equations of motion are to be solved for evaluating the seismic response of the system, depending upon the phase of motion.

3.2. Hysteretic model

The hysteretic model is a continuous model of the frictional force proposed by Constantinou *et al*. [11], using the Wen equation [15]. The frictional forces mobilized in the sliding system is expressed by

$$F_x = F_S Z_x \tag{9}$$

$$F_y = F_S Z_y \tag{10}$$

where F_S is the limiting frictional force expressed by Equation (6); and Z_x and Z_y are the dimensionless hysteretic components satisfying the following nonlinear first-order differential equations

$$q\frac{dZ_x}{dt} = A\dot{x}_b + \beta|\dot{x}_b|Z_x|Z_x|^{n-1} - \tau\dot{x}_b|Z_x|^n \tag{11}$$

$$q\frac{dZ_y}{dt} = A\dot{y}_b + \beta|\dot{y}_b|Z_y|Z_y|^{n-1} - \tau\dot{y}_b|Z_y|^n \tag{12}$$

where q is the displacement quantity representing the yield displacement of frictional force loop; β, τ, n and A are dimensionless parameters of the hysteresis loop. The parameters β, τ, n and A control the shape of the loop and are selected such that to provide a rigid-plastic behaviour (typical Coulomb-friction behaviour). The recommended values are: $q = 0.25$ mm, $A = 1$, $\beta = 0.9$, $\tau = 0.1$ and $n = 2$. The hysteretic displacement component, Z_x and Z_y are bounded by peak values of ± 1 to account for the conditions of sliding and non-sliding phases. Thus, the hysteretic model is a continuous model in which the system is always analyzed for same degrees-of-freedom, irrespective of the phases of motion.

3.3. Bidirectional interaction of frictional forces

The frictional forces of the sliding systems expressed in the Sections 3.1 and 3.2 assume that there is no interaction exists in two orthogonal horizontal directions. However, the frictional forces in the sliding phase of the motion are coupled as the system starts sliding when the resultant of the frictional forces exceeds the limiting value. The existence of these interaction effects have been reported experimentally [16] and analytically [17–18], in which the displacement of sliding systems are under-estimated if these effects are ignored.

In the conventional method, the frictional forces are considered to be coupled by the equation expressed as [17]

$$F_x^2 + F_y^2 = F_S^2 \tag{13}$$

Equation (13) implies that the system starts sliding as soon as the resultant of frictional forces attains the limiting frictional force. Under such conditions, the governing equations of motion of the isolated building during the sliding phases are coupled in two orthogonal directions and these effects are ignored when the building is idealized independently in two directions.

In the hysteretic model, the interaction between the frictional forces is expressed by coupling of the hysteretic displacements components Z_x and Z_y in two directions [19]. The coupled Z_x and Z_y are obtained by the following expression as

$$q\begin{Bmatrix} \dot{Z}_x \\ \dot{Z}_y \end{Bmatrix} = \begin{bmatrix} A - \beta\mathrm{sign}(\dot{x}_\mathrm{b})|Z_x|Z_x - \tau Z_x^2 & -\beta\mathrm{sign}(\dot{y}_\mathrm{b})|Z_y|Z_x - \tau Z_x Z_y \\ -\beta\mathrm{sign}(\dot{x}_\mathrm{b})|Z_x|Z_y - \tau Z_x Z_y & A - \beta\mathrm{sign}(\dot{y}_\mathrm{b})|Z_y|Z_y - \tau Z_y^2 \end{bmatrix} \begin{Bmatrix} \dot{x}_\mathrm{b} \\ \dot{y}_\mathrm{b} \end{Bmatrix} \tag{14}$$

The interaction between the frictional forces of the sliding system in two horizontal directions is due to the coupling through the off-diagonal terms of the matrix in Equation (14). Further details can be obtained on the bidirectional model used for the sliding system [20].

4. SOLUTION OF EQUATIONS OF MOTION

The frictional forces mobilized in the sliding system are nonlinear functions of the displacement and velocity of the system; as a result, the governing equations of motion are solved in incremental form, using Newmark's step-by-step method, assuming linear variation of acceleration over small time interval Δt. For the hysteretic model, an iterative procedure is used for evaluating the incremental hysteretic displacement components. The iterations are necessary because of the dependence of Z_x and Z_y on the response of the system at the end of each time step. The incremental hysteretic displacement components for each time step are obtained by solving the first-order differential equations (Equations 11,12,14) by a fourth-order Runge–Kutta method.

5. NUMERICAL STUDY FOR A MULTI-STOREY BUILDING

For the present study, the mass matrix of the superstructure $[M]$ is diagonal and characterized by the mass of each floor which is kept constant (i.e. $m_i = m$ for $i = 1, 2, \ldots, N$). Also, for simplicity the stiffness of all the floors in two horizontal directions is taken as constant expressed by the parameter k. The value of k is selected to provide the required fundamental time period of superstructure as a fixed base. The damping matrix of the superstructure $[C]$ is not known explicitly. It is constructed by assuming the modal damping ratio, which is kept constant in each mode of vibration. Thus, the model of the isolated structural system under consideration can be completely characterized by the parameters fundamental time period of the superstructure T_S, damping ratio of the superstructure ξ_S, number of storeys in the superstructure N, ratio of base mass to superstructure floor mass m_b/m, period of base isolation T_b, damping ratio of the sliding system ξ_b and coefficient of friction of the sliding system μ. In the present study, the parameters ξ_S and m_b/m are held constant with $\xi_\mathrm{S} = 0.05$ and $m_\mathrm{b}/m = 1$.

Three types of commonly proposed sliding base isolation systems, i.e. the pure friction (PF) system, the friction pendulum system (FPS) and the resilient friction base isolator (RFBI) are considered for comparison of the seismic response by two models of frictional forces of the sliding mechanism. The parameters of the PF, FPS and RFBI systems considered are $\mu = 0.1$, $T_\mathrm{b} = 2.5$ s and $\mu = 0.05$, and $T_\mathrm{b} = 4$ s, $\xi_\mathrm{b} = 0.1$ and $\mu = 0.04$, respectively.

5.1. Unidirectional response

First, the seismic response of isolated multi-storey building with two models of frictional forces of sliding system is investigated under a single horizontal component of earthquake ground motion applied in the x-direction only. The north–south (NS) component of the three earthquake motions namely El Centro 1940, Northridge 1994 (recorded at Sylmar station) and Kobe 1995 (recorded at JMA) are selected to study the seismic response isolated building. The response quantities of interest are the top floor absolute acceleration (i.e. $\ddot{x}_a = \ddot{x}_N + \ddot{x}_b + \ddot{x}_g$) and the relative base displacement x_b. The absolute acceleration is directly proportional to the forces exerted in the superstructure due to earthquake ground motion. On the other hand, the relative base displacement is crucial from the design point of view of the sliding system.

The time variation of top floor absolute acceleration and relative base displacement of the five-storey structure isolated by the PF, FPS and RFBI systems is shown in Figure 2 under El Centro 1940 earthquake ground motion. The fundamental time period of the superstructure, T_S is taken as 0.5 s and the response is shown for both conventional and hysteretic models of the frictional forces of the sliding system. It is observed from the figure that the seismic responses obtained by the conventional and hysteretic models are the same, indicating that the both models predict the same trend and peak response of the isolated structure. Further, comparison of superstructure acceleration between isolated and non-isolated conditions indicates that all the sliding systems are effective in reducing the earthquake response of the structure. The percentage reduction in the peak top floor acceleration due to isolation by conventional model is 55.6, 66.7 and 72.1 in PF, FPS and RFBI systems, respectively, while, for the hysteretic model, the corresponding reduction is 57.1, 66.9 and 71.5. The peak base displacement obtained by conventional model is 5.64, 7.36 and 6.07 cm in PF, FPS and RFBI systems, respectively, while the corresponding base displacement predicted by the hysteretic model is 5.87, 7.52 and 6.34 cm. This analysis shows that both models predict almost the same peak response of the system, but the difference can be relatively more for the sliding displacement of the isolator. Among the three isolation systems the FPS and RFBI systems perform better than the PF system which reduces lesser superstructure acceleration and undergoes large residual sliding displacement.

In Figure 3, variation of the frictional force of the sliding systems is plotted against the base displacement for both models under El Centro 1940 earthquake ground motion. The figure indicates the same variation of the frictional force in the two models. However, there is a smooth transition of the frictional force in the hysteretic model, which is expected since the hysteretic model is a continuous whereas the conventional model is a discontinuous model.

It has been observed in the past that the response of structure with sliding systems is quite sensitive to the time step Δt, used for the solution of the equations of motion. In order to study this fact, the peak values of top floor absolute acceleration and base displacement is plotted against the time step division N_t in Figure 4 under different earthquake ground motions for both conventional and hysteretic models. The actual time step considered for solution of equations of motion is $\Delta t = 0.02/N_t$ s (i.e. 0.02 s is the digitized time interval of all the earthquake ground motions). It is observed from the figure that the peak seismic response of a multi-storey building is sensitive to the value of time step division in the range 1–10 (i.e. Δt in the range 0.002–0.02) for the conventional model. The effects of the time interval are more pronounced on the base displacement in comparison with the top floor acceleration of the isolated structure. Further, the response of structure with the PF system is more sensitive to the time interval as compared to the FPS and RFBI systems. This is expected, owing to the fact that the sliding base displacement

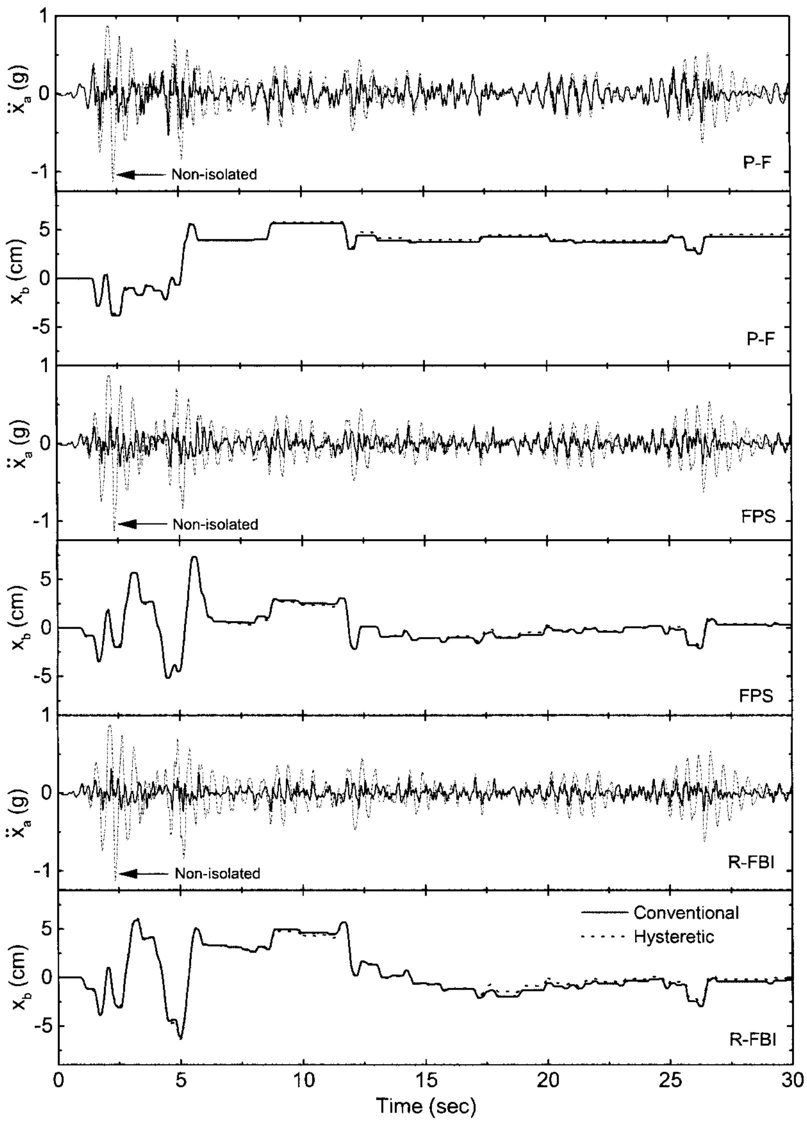


Figure 2. Time variation of the response of five-storey building isolated by the PF, FPS and RFBI systems to El Centro 1940 earthquake motion ($T_S = 0.5$ s).

is quite sensitive in the PF system to the initial conditions at the sliding and non-sliding phases [21]. Further, the number of divisions required to obtain a stable solution for the hysteretic model are more in comparison with the conventional model. This is expected since the hysteretic model is a continuous model and the time interval must be very small to predict the exact rigid-plastic behaviour. The minimum value of N_t required for the hysteretic model for a stable response is 20 for the El Centro 1940 earthquake. This number is found to be order of 100 for

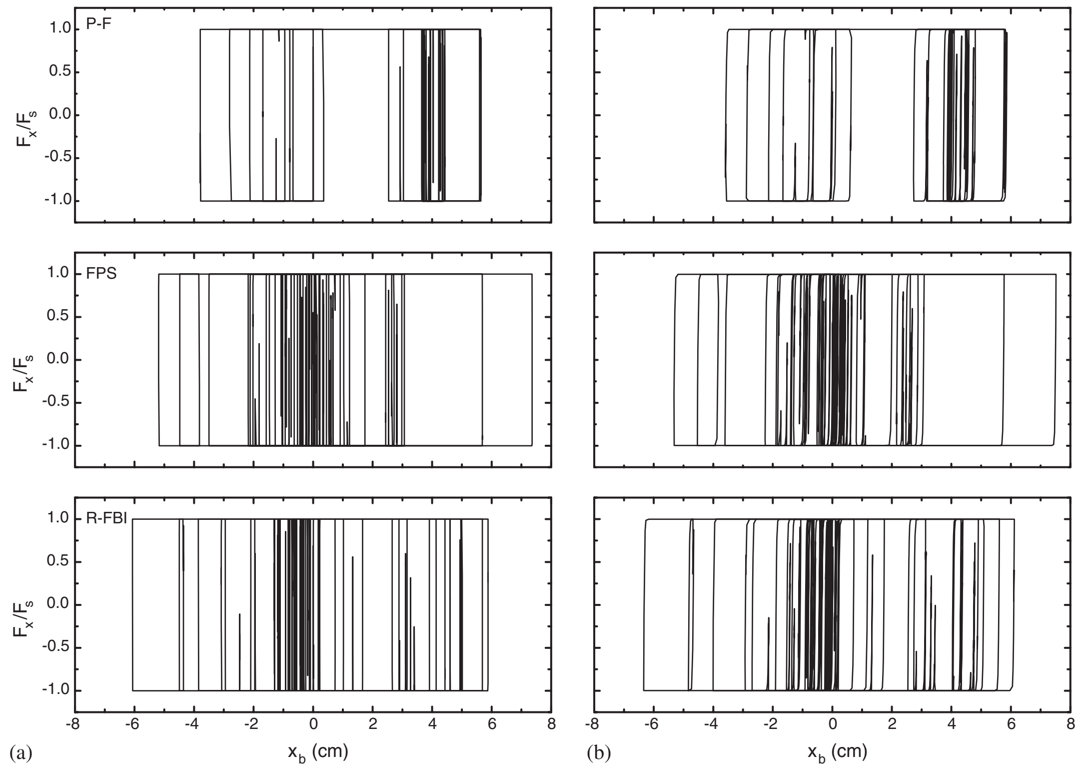


Figure 3. Variation of frictional force in different systems isolating five-storey building against the base displacement under El Centro 1940 earthquake motion ($T_S = 0.5$ s). (a) Conventional model; (b) Hysteretic model.

the Northridge 1994 and Kobe 1995 earthquake ground motions because these earthquake records are more severe than the El Centro 1940. The ratio of computation time for hysteretic model to the conventional model was about 40 for the El Centro 1940 and 200 for the Northridge 1994 and Kobe 1995 earthquakes. Thus, the computation time required for hysteretic model is significantly higher as compared with the conventional model, owing to the small time interval required to follow the rigid-plastic behaviour and the additional iterations required in each time step for the convergence of hysteretic displacement component.

The effects of superstructure time period, T_S on the peak seismic response of isolated and non-isolated structure is shown in Figures 5 and 6 for one- and five-storey structure, respectively. The response for both models of frictional forces is compared under different earthquake 'motions and isolation systems. It is observed that the superstructure acceleration due to isolation is considerably reduced and remains constant for all values T_S in the range of 0–1 s. The superstructure acceleration and base displacement of isolated building predicted by the conventional model closely matches the corresponding response by the hysteretic model. However, the difference in the response between the two models is relatively more for the PF system as compared with the FPS and RFBI systems for all earthquake ground motions.

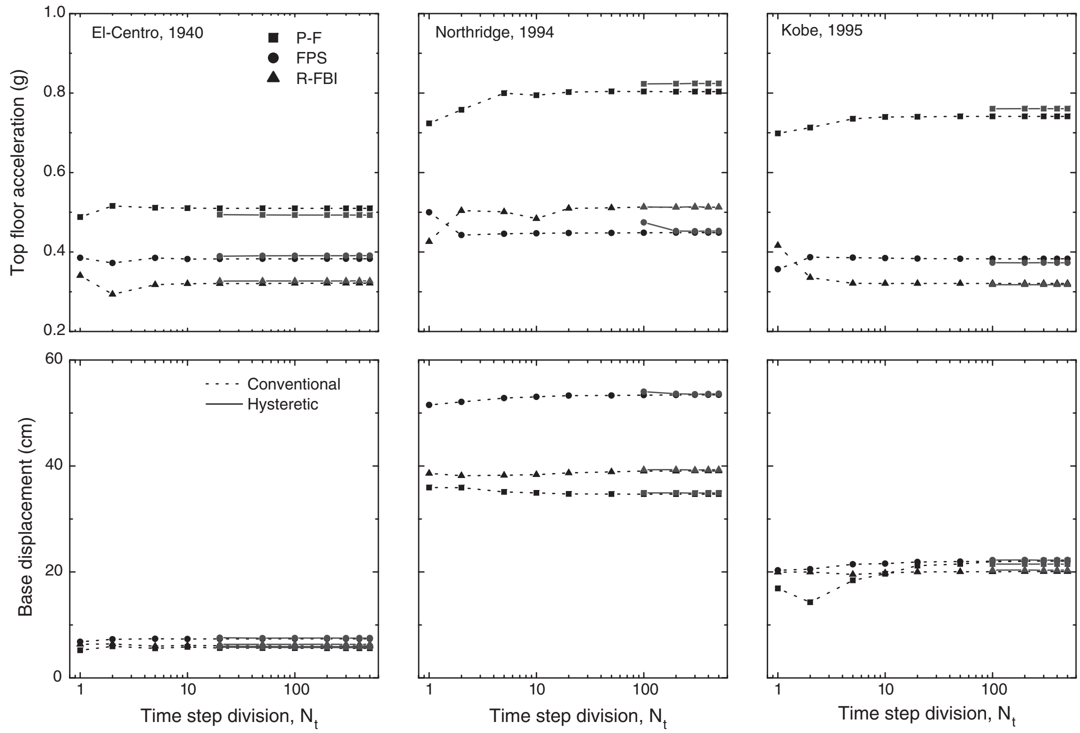


Figure 4. Effects of time interval on the peak seismic response of five-storey building with different systems ($T_S = 0.5$ s).

5.2. Bidirectional response

In this section, the response of a multi-storey building with two models of sliding systems is compared under two horizontal components of the earthquake motions. The bidirectional interaction between the frictional forces of the sliding systems in two horizontal directions is duly considered. The NS and east–west (EW) component of the earlier selected ground motions are applied in the x- and y-directions, respectively. The response quantities of interest are the top floor absolute accelerations and the relative base displacement.

The time variation of the top floor absolute accelerations (i.e. $\ddot{x}_a$ and $\ddot{y}_a$), relative base displacements (x_b and y_b) and frictional forces (F_x and F_y) of the five-storey structure isolated by the FPS system is shown in Figure 7 under 1995 Kobe earthquake motion. The fundamental time period of the superstructure T_S is taken as 0.5 s and the response is shown for both conventional and hysteretic models of the frictional force of the sliding mechanism. It is observed from the figure that the seismic response of the building in both directions obtained by the conventional and hysteretic models are the same, indicating that both models predict the same trend and peak response of the isolated structure. The peak base displacement obtained by the conventional model in the x- and y-directions is 25.03 and 16.73 cm, respectively. The corresponding peak base displacement predicted by the hysteretic model is 25.17 and 16.9 cm in the x- and y-directions, respectively. This analysis

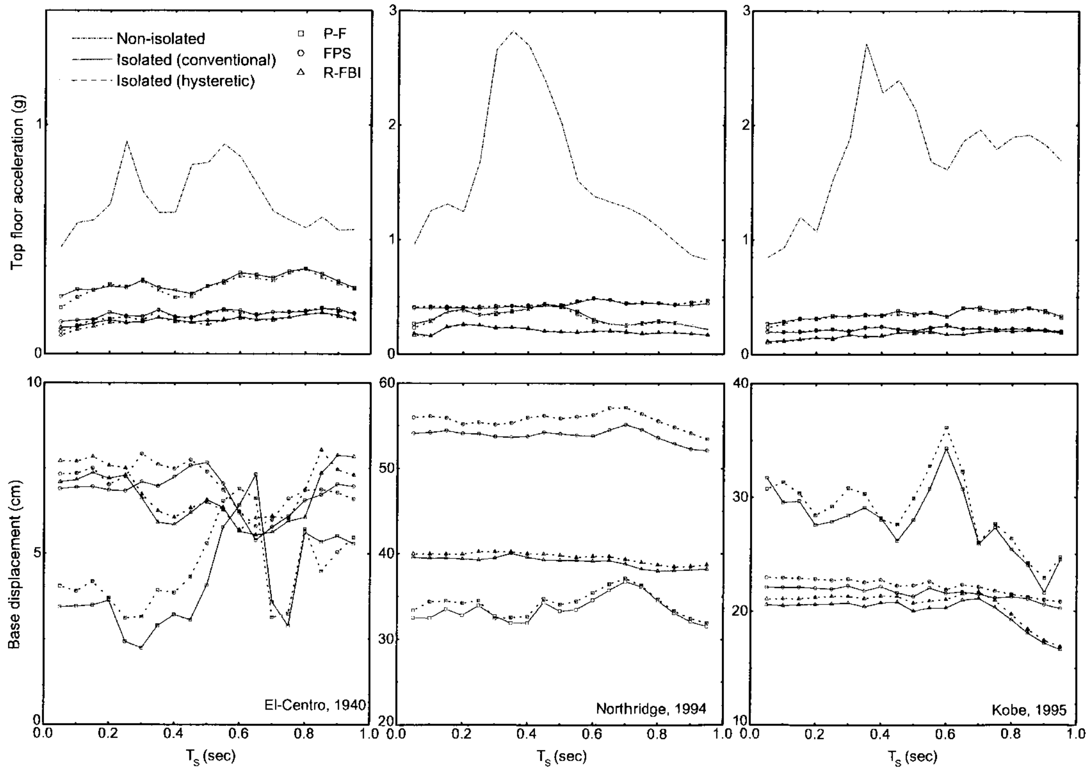


Figure 5. Comparison of peak response of one-storey isolated structure by conventional and hysteretic models against fundamental time period of superstructure under different isolation systems and earthquake motions.

shows that both models predict almost the same peak response of the system. Comparison of superstructure acceleration between isolated and non-isolated conditions indicates that the FPS is quite effective in reducing the earthquake forces of the structure. Thus, the conventional and hysteretic models of frictional forces of the sliding system also predict the identical response under bidirectional earthquake excitation.

Figure 8 shows the variation of the peak resultant top floor acceleration and base displacement of a one-storey building against the time period of the superstructure T_S. The resultant response by conventional and hysteretic models is compared for different isolation systems under three earthquake motions. In general, the superstructure acceleration and base displacement of an isolated building predicted by the conventional model matches with the corresponding response by hysteretic model for all earthquake motions under bidirectional excitation. However, the difference in the base displacement between the two models is relatively more for the PF system which is the similar observation made for the building under unidirectional excitation. Similar effects of comparison between the two models of the sliding mechanism are depicted in Figure 9 for the five-storey structure.

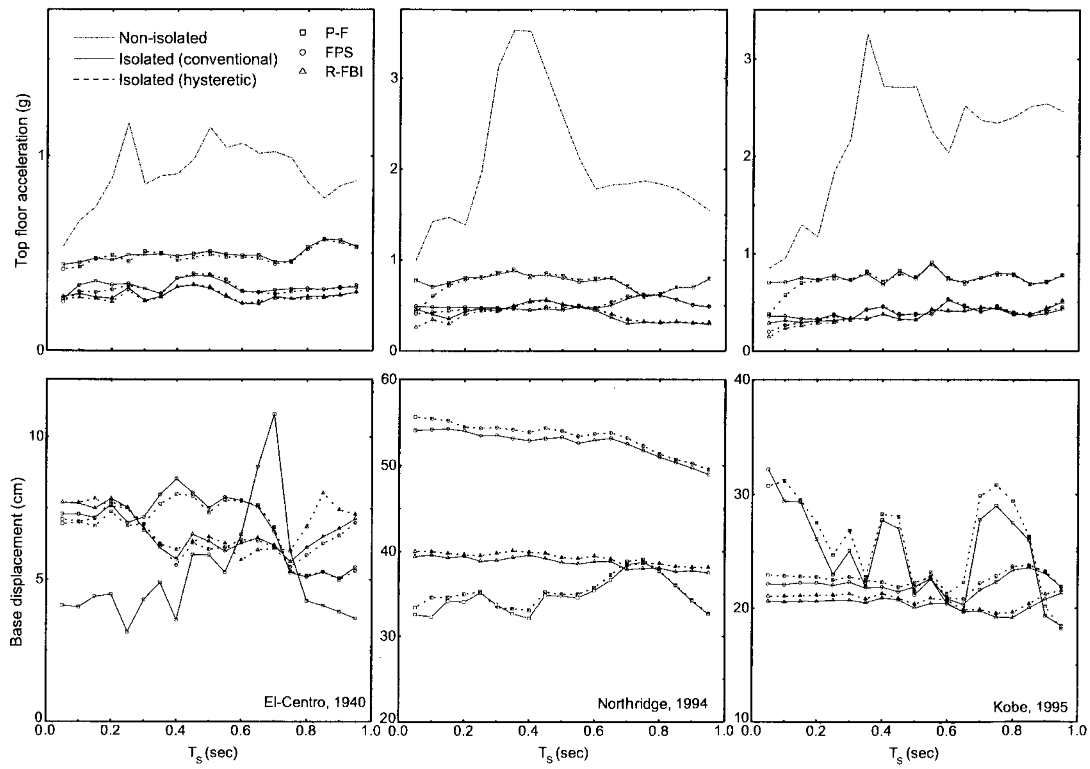


Figure 6. Comparison of peak response of five-storey isolated structure by conventional and hysteretic model against fundamental time period of superstructure under different isolation systems and earthquake motions.

6. RESPONSE OF A BRIDGE WITH SLIDING SYSTEMS

The comparison of the two models of the frictional forces of the sliding system studied previously was for a symmetric multi-storey building. In such a sliding structure the application of the conventional approach is simpler, as there is a difference of only one degree-of-freedom at a specified base mass location during the sliding and non-sliding phases. On the other hand, the use of the conventional method may not be much attractive for structures such as multi-span continuous deck bridges isolated by the sliding systems (owing to sliding supports at several locations). In such cases the use of hysteretic models can be preferred, but the computational time required may become enormous. In view of this, it is interesting to compare the response of an isolated bridge with two models of the frictional forces of sliding systems. Figure 10(a) shows the general elevation of a bridge consisting of a multi-span continuous deck supported by a sliding system at piers and abutment levels. The effectiveness of sliding systems for isolation of bridges has been demonstrated in the past by many researchers [12,22,23]. For the present study, the FPS sliding system is selected for isolation of the bridge, as successfully used in an actual implementation [24]. The substructure of bridge consists of rigid abutments and reinforced concrete piers. Bridge superstructure and piers are assumed

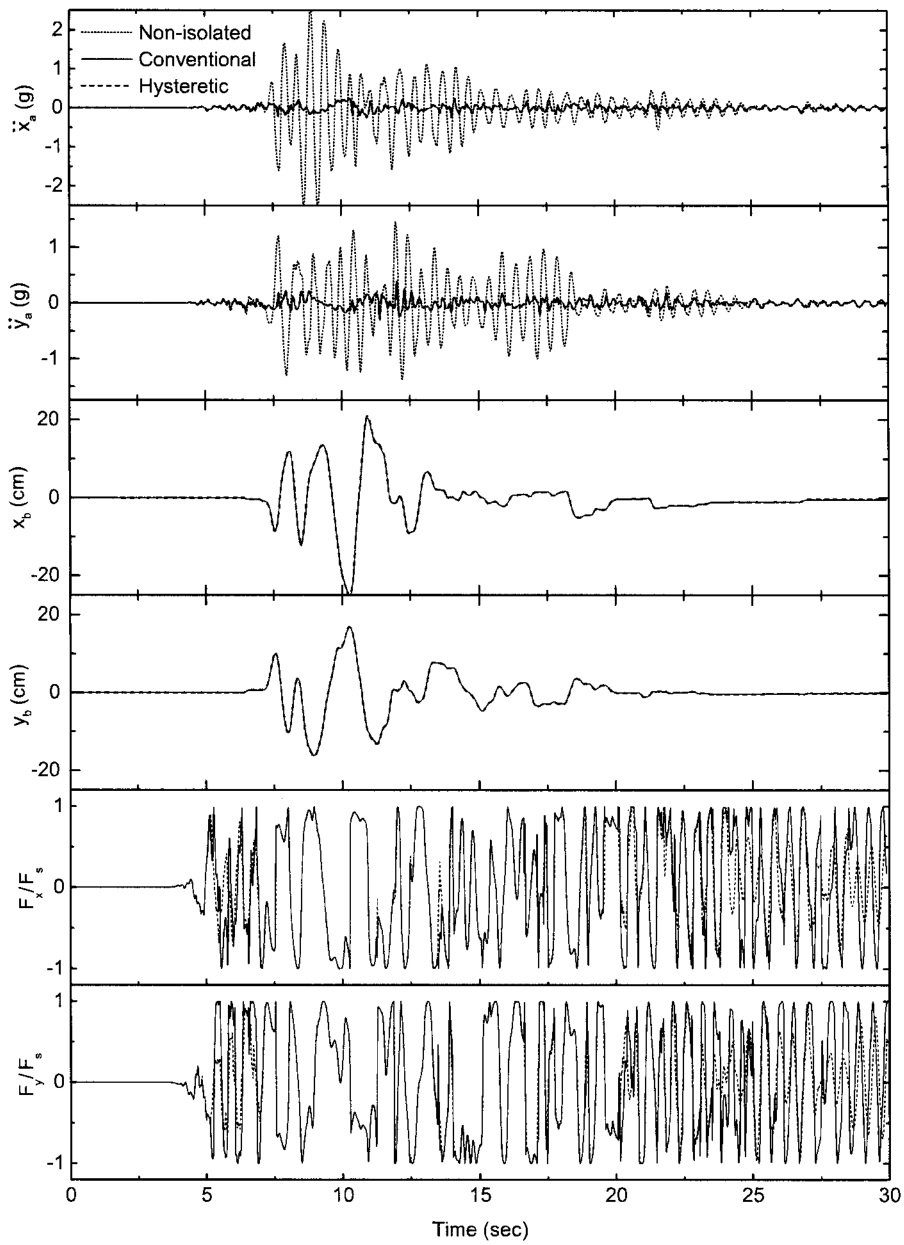


Figure 7. Time variation of the response of five-storey building isolated by the FPS system under Kobe 1995 earthquake motion ($T_S = 0.5$ s).

to remain in the elastic state during the earthquake excitation. This is a reasonable assumption as the isolation attempts to reduce the earthquake response in such a way that the structure remains within the elastic range. The superstructure and substructure of the bridge are

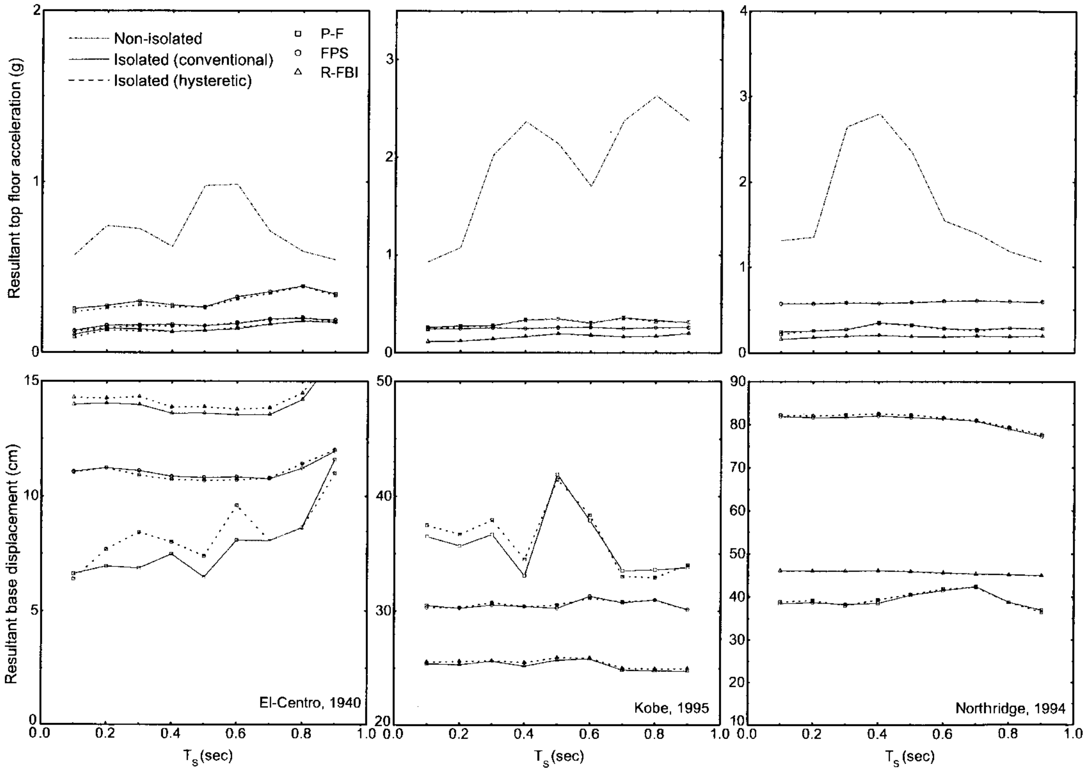


Figure 8. Comparison of peak resultant response of one-storey isolated structure by conventional and hysteretic models against fundamental time period of superstructure under different isolation systems and earthquake motions.

modelled as lumped mass system divided into number of small discrete segments. Each adjacent segment is connected by a node and at each node two degrees-of-freedom are considered. The masses of each segment are assumed to be distributed between the two adjacent nodes in the form of point masses. These assumptions lead to the mathematical model of the isolated bridge system, as shown in Figure 10(b).

The FPS isolators installed at the piers and abutments have the same dynamic characteristics. The FPS is characterized with system parameters, namely the friction coefficient and the isolation period based on the rigid superstructure and pier condition by the following expression

$$T_{\mathrm{b}} = 2\pi\sqrt{\frac{m_{\mathrm{d}}}{\sum k_{\mathrm{b}}}} \tag{15}$$

where m_{d} is the deck mass and $\sum k_{\mathrm{b}}$ is the total stiffness of the FPS provided by its curved surface.

The governing equations of motion of the isolated bridge are derived for the two models of the sliding mechanism. The seismic response of the bridge is evaluated under two horizontal

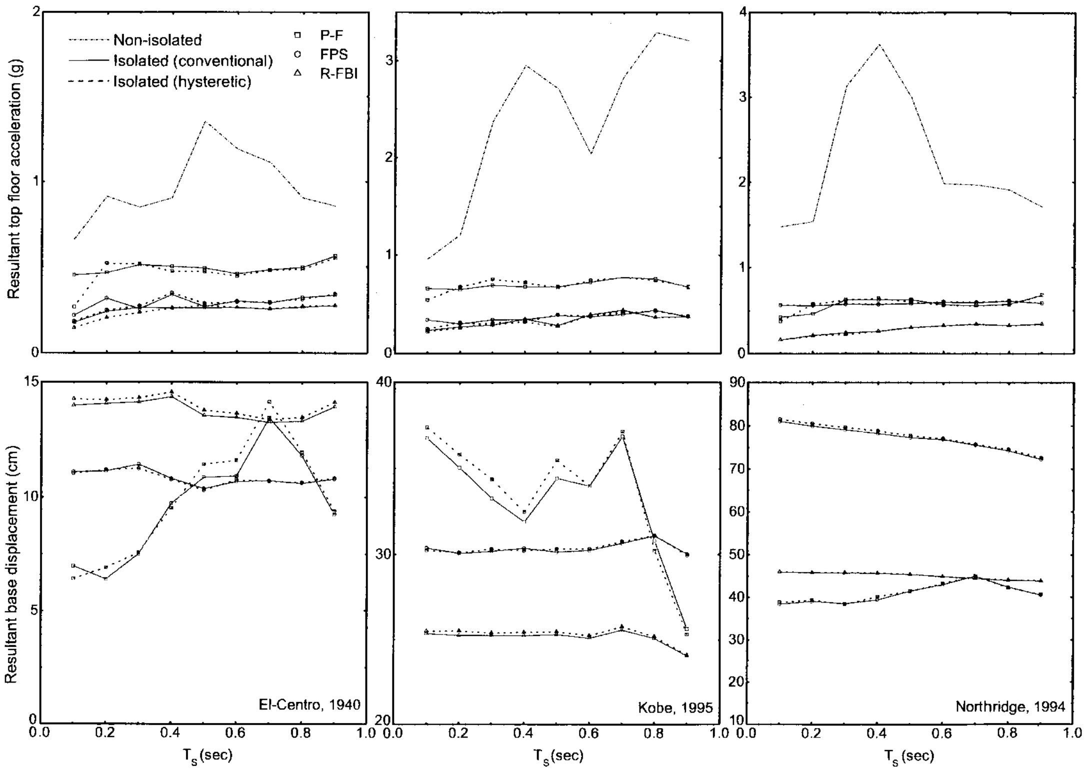
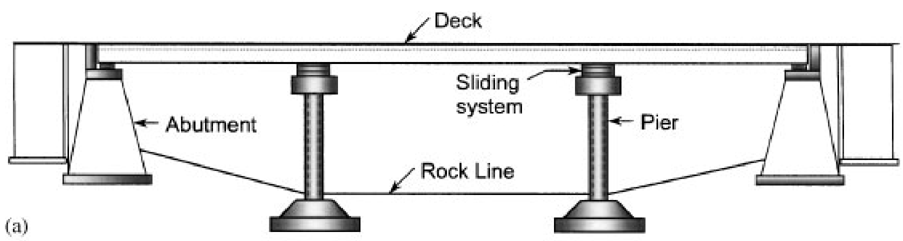


Figure 9. Comparison of peak resultant response of five-storey isolated structure by conventional and hysteretic model against fundamental time period of superstructure under different isolation systems and earthquake motions.

components of earthquake ground motions by considering and ignoring the bidirectional interaction effects. The NS and ES component of the three selected earthquake motions is applied in the longitudinal and transverse directions, respectively.

The three-span continuous deck with reinforced concrete piers and prestressed concrete box girders is considered for the present study. The dynamic properties of this bridge are given in Table I, taken from Wang *et al.* [12] where the seismic response of the same bridge isolated by sliding systems was investigated. The fundamental time period of this bridge without isolation system is 0.45 s in both directions. The values of the parameters of the FPS considered are $T_b = 2.5$ s and $\mu = 0.05$. The response quantities of interest for the bridge system under consideration (in both longitudinal and transverse directions) are the absolute acceleration at the centre of bridge deck ($\ddot{x}_d$ and $\ddot{y}_d$) and the relative displacement of the FPS (x_a or y_a) at the abutments. The x and y in the response quantities refers to the response in the longitudinal and transverse directions of the bridge, respectively. The absolute acceleration of the deck is directly proportional to the forces exerted in the bridge system due to earthquake ground motion. On the other hand, the relative displacement of the FPS is crucial from the design point of view of isolation system and expansion joints.



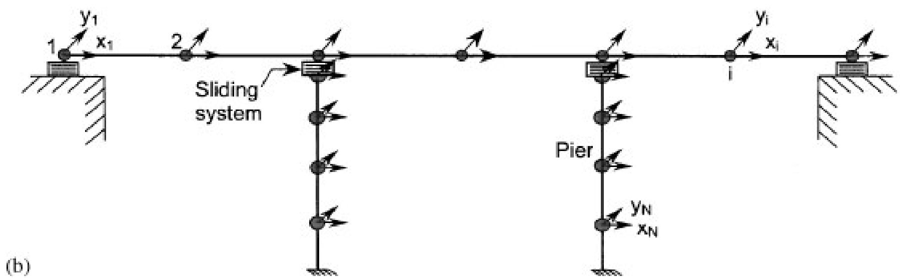


Figure 10. General elevation of a bridge seismically isolated by the FPS system and its mathematical model.

Table I. Properties of the bridge deck and piers.

Properties	Deck	Piers
Cross-sectional area (m^2)	3.57	4.09
Moment of inertia (m^4)	2.08	0.64
Young's modules of elasticity (N/m^2)	20.67×10^9	20.67×10^9
Mass density (kg/m^3)	2.4×10^3	2.4×10^3
Length/height (m)	$3 \times 30 = 90$	8

The time variation of the deck acceleration, relative sliding displacement of the isolated bridge and frictional forces in both the longitudinal and transverse directions are shown in Figure 11 under Kobe 1995 earthquake motion. The response is compared between hysteretic and conventional models of the sliding system without considering the interaction of the frictional forces. It is observed from the figure that both models of the sliding system predict an identical response. The peak sliding displacement at abutment location in the longitudinal direction of the bridge predicted by conventional and hysteretic models are 28.56 and 28.76 cm, respectively. The corresponding sliding displacements in the transverse direction are 24.18 and 24.57 cm by conventional and hysteretic models, respectively. Similar comparison of the response between conventional and hysteretic models is depicted in Figure 12 where the corresponding response is plotted by considering the bidirectional interaction of frictional forces.

In Table II, the peak responses of bridge to different earthquake motions is compared between conventional and hysteretic model of sliding systems, both ignoring and considering the bidirectional interaction. The table confirms that the conventional and hysteretic models provide identical responses for all earthquake motions. The bidirectional interaction of

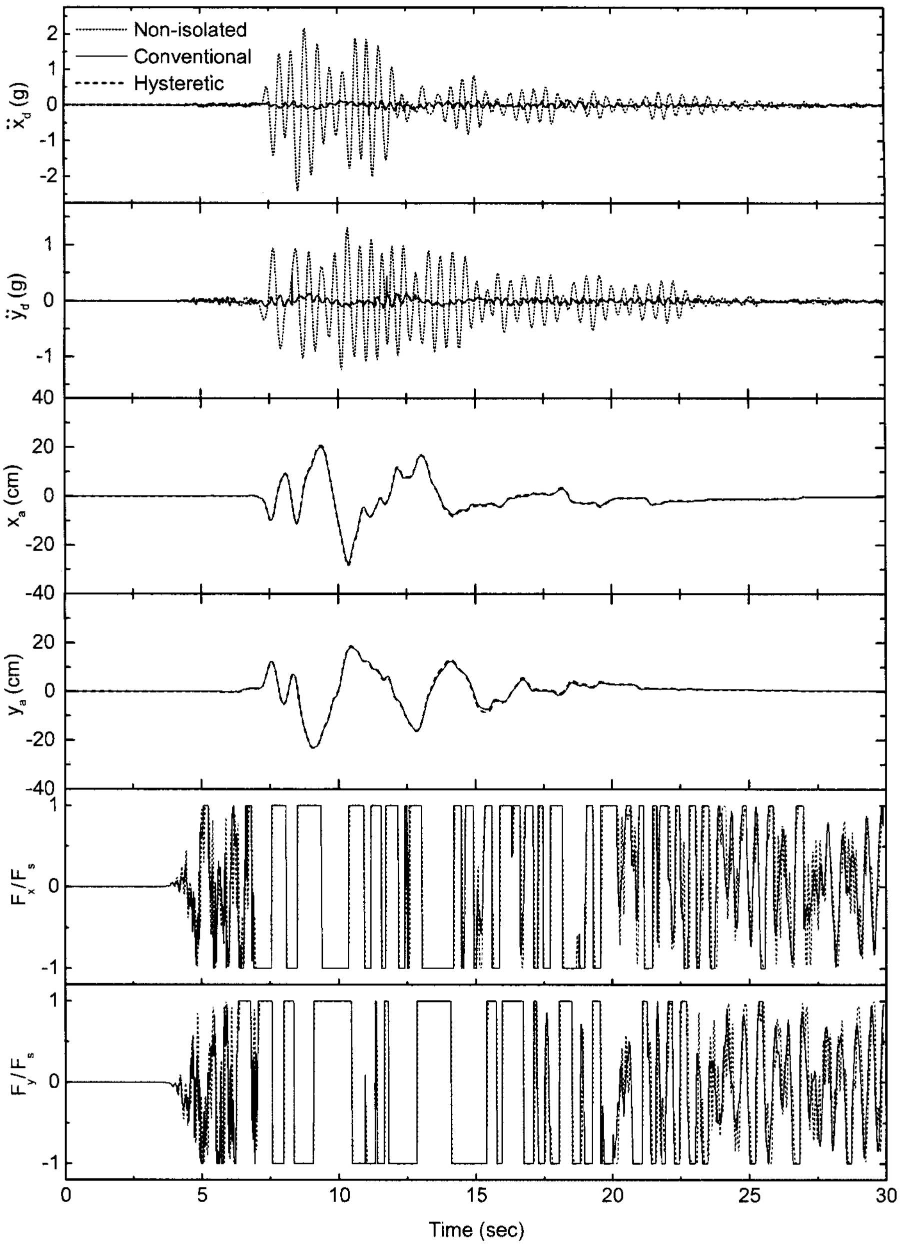


Figure 11. Time variation of deck acceleration and bearing displacement by conventional and hysteretic models without interaction under Kobe 1995 earthquake motion.

frictional forces increases the displacement in the isolation system which is crucial from the design point of view. Table II also indicates that there is significant reduction in the absolute deck acceleration, confirming the effectiveness of the FPS for aseismic design of bridges.

Table II. Peak response quantities of bridge for isolated and non-isolated conditions under different earthquake motions.

		El Centro 1940		Kobe 1995		Northridge 1994	
Response quantity	Model	Longitudinal	Transverse	Longitudinal	Transverse	Longitudinal	Transverse
Non-isolated deck acceleration (g)		00.826	00.570	02.396	01.314	02.388	01.211
Unidirectional response without interaction							
Isolated deck acceleration (g)	Conventional	00.122	00.123	00.163	00.147	00.226	00.169
Isolated deck acceleration (g)	Hysteretic	00.125	00.122	00.170	00.155	00.235	00.153
Bearing displacement (cm)	Conventional	10.69	18.05	27.53	23.01	56.29	42.49
Bearing displacement (cm)	Hysteretic	11.24	18.50	28.32	23.23	57.33	43.48
Bidirectional response with interaction							
Isolated deck acceleration (g)	Conventional	00.092	00.121	00.128	00.137	00.198	00.165
Isolated deck acceleration (g)	Hysteretic	00.080	00.118	00.134	00.143	00.207	00.166
Bearing displacement (cm)	Conventional	15.72	22.43	28.56	24.18	57.57	50.06
Bearing displacement (cm)	Hysteretic	16.31	22.90	28.76	24.57	57.96	50.26

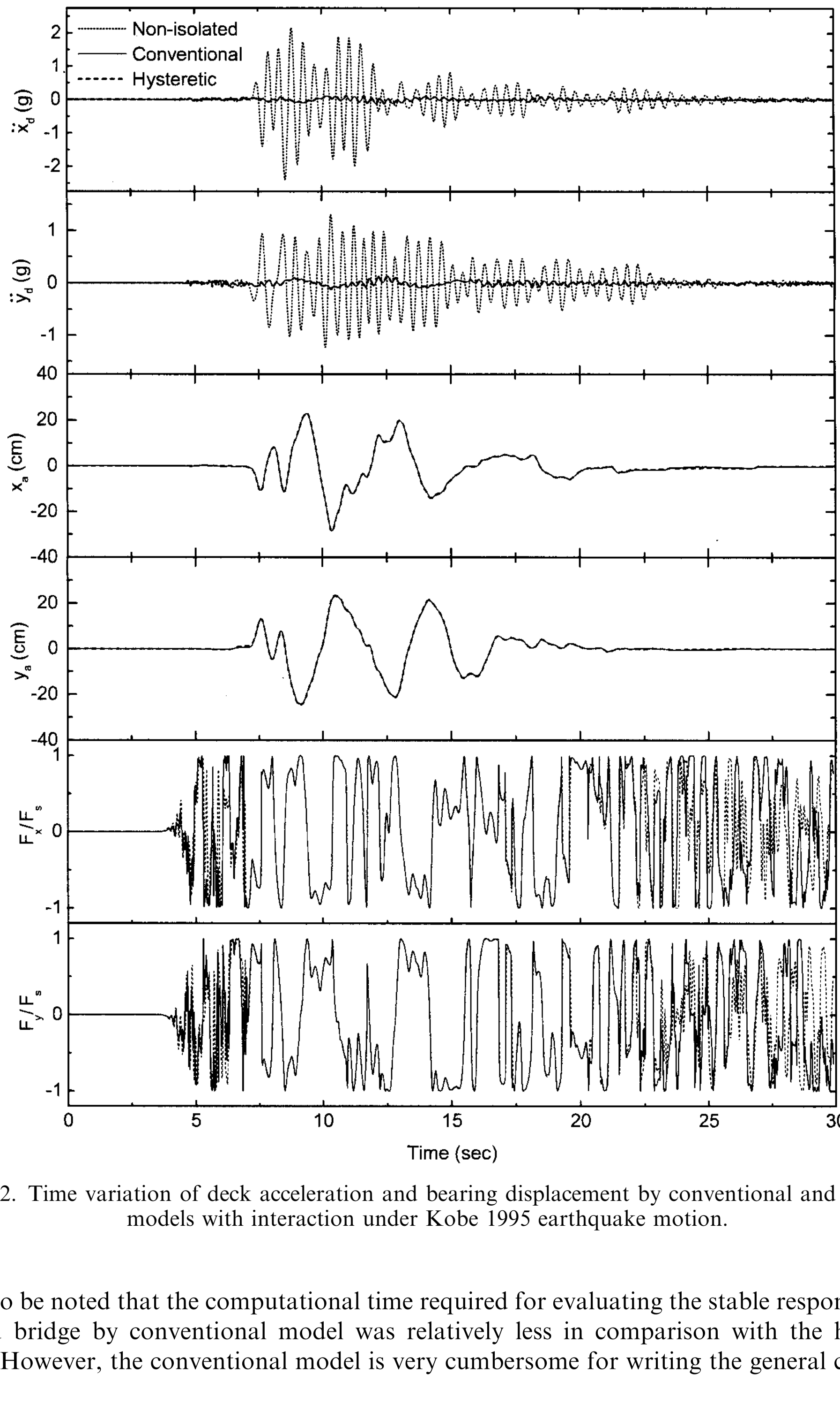


Figure 12. Time variation of deck acceleration and bearing displacement by conventional and hysteretic models with interaction under Kobe 1995 earthquake motion.

It is to be noted that the computational time required for evaluating the stable response of the isolated bridge by conventional model was relatively less in comparison with the hysteretic model. However, the conventional model is very cumbersome for writing the general computer code as different sets of equations of motion are to be constructed for each time step, depending upon the phase of motion at various locations of sliding interface.

7. CONCLUSIONS

The seismic response of a multi-storey shear-type structures and bridges isolated by sliding systems is investigated. Two models, referred to as conventional and hysteretic models, are used for the frictional force of the sliding system. A comparison of the seismic response of an isolated structure obtained from the two models is made to distinguish the difference in the peak response. In addition, the computational efficiencies of the two models and their relative advantages and disadvantages are also discussed. From the trends of the results of the present study, the following conclusions can be drawn:

1. The peak seismic response, such as superstructure acceleration and bearing displacement, predicted by the conventional and hysteretic models of frictional forces of sliding systems matches closely.
2. The difference in the response of an isolated structure between conventional and hysteretic models is relatively more for the PF system as compared with the FPS and RFBI systems.
3. The response of the sliding structure is found to be sensitive to the time interval used for solution of the equations of motion. These effects are more pronounced for the PF system as compared with the FPS and RFBI systems.
4. The computational time required for the conventional model is significantly less in comparison with the hysteretic model.
5. The conventional model is found to be very cumbersome for writing the general computer code in comparison with the hysteretic model, especially for multi-span continuous deck bridges supported on a sliding system.

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