



POLITECNICO
MILANO 1863

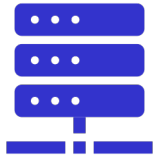
Computing Infrastructures

-

System Dependability

Reliability Block Diagrams

The topics of the course: what are we going to see today?



HW Infrastructures:

System-level: Computing Infrastructures and Data Center Architectures, Rack/Structure;

Node-level: Server (computation, HW accelerators), Storage (Type, technology), Networking (architecture and technology);

Building-level: Cooling systems, power supply, failure recovery



SW Infrastructures:

Virtualization: Process/System VM, Virtualization Mechanisms (Hypervisor, Para/Full virtualization)

Computing Architectures: Cloud Computing (types, characteristics), Edge/Fog Computing, X-as-a service



Methods:

Reliability and availability of datacenters (definition, fundamental laws, **RBDs**)

Disk performance (Type, Performance, RAID)

Scalability and performance of datacenters (definitions, fundamental laws, queuing network theory)



Reliability Block Diagrams

An inductive model where a system is divided into blocks that represent distinct elements such as components or subsystems.

Every element in the RBD has its own reliability (previously calculated or modelled)

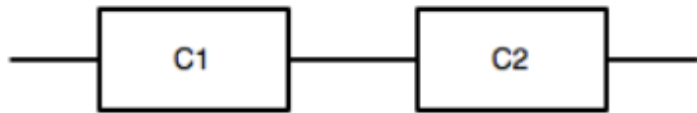
Blocks are then combined together to model all the possible *success paths*

- **Model Topology can be different from the actual system topology**



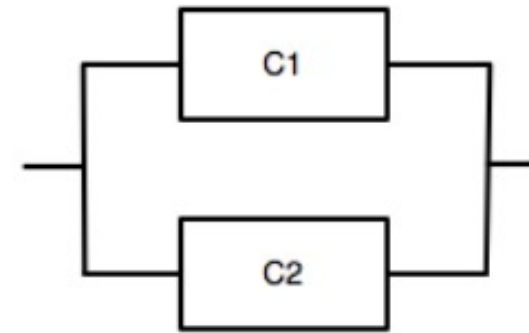
Reliability Block Diagrams

RBDs are an approach to compute the reliability of a system starting from the reliability of its components



components in series

All components must be healthy for the system to work properly



components in parallel

If one component is healthy the system works properly

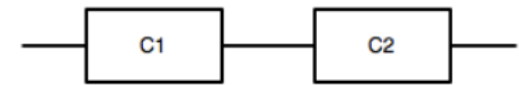


Reliability Block Diagrams

- **Series:** System failure is determined by the failure of the *first* component

$$R_S(t) = \prod_{i=1}^n R_i(t)$$

$$R_S(t) = R_{C1}(t) * R_{C2}(t)$$

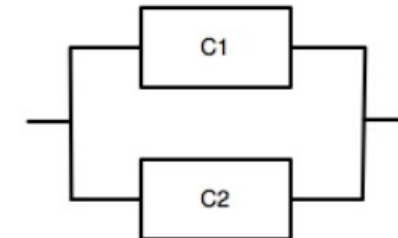


- **Parallel:** System fails when the *last* component fails

$$R_S(t) = 1 - \prod (1 - R_i(t))$$

$$R_S(t) = 1 - [(1 - R_{C1}(t)) * (1 - R_{C2}(t))]$$

$$R_S(t) = R_{C1}(t) + R_{C2}(t) - R_{C1}(t) * R_{C2}(t)$$



Reliability Block Diagrams

series

In general, if system S is composed by components with a reliability having an exponential distribution (very common case)

$$R_s(t) = e^{-\lambda_s t}$$

where

Failure in time

$$\lambda_s = \sum_{i=1}^n \lambda_i$$



Reliability Block Diagrams

series

In general, if system S is composed by components with a reliability having an exponential distribution (very common case)

$$R_s(t) = e^{-\lambda_s t}$$

where

Failure in time

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$$MTTF_s = \frac{1}{\lambda_s} = \frac{1}{\sum_{i=1}^n \lambda_i} = \frac{1}{\sum_{i=1}^n \frac{1}{MTTF_i}}$$



Reliability Block Diagrams

series

A special case: when all components are identical

$$R_s(t) = e^{-\lambda_s t}$$



$$R_s(t) = e^{-n\lambda t} = e^{-\frac{nt}{MTTF_1}} \quad MTTF_s = \frac{MTTF_1}{n}$$



Reliability Block Diagrams

series

Availability:

$$A_S = \prod_{i=1}^n \frac{MTTF_i}{MTTF_i + MTTR_i}$$

When all components are the same:

$$A_S(t) = A_1(t)^n \quad A = \left(\frac{MTTF_1}{MTTF_1 + MTTR_1} \right)^n$$



Reliability Block Diagrams

parallel

System P composed by n components

$$R_P(t) = 1 - \prod_{i=1}^n (1 - R_i(t))$$

Availability

$$A_P(t) = 1 - \prod_{i=1}^n (1 - A_i(t))$$

$$A_P = 1 - \prod_{i=1}^n (1 - A_i) = 1 - \prod_{i=1}^n \frac{MTTR_i}{MTTF_i + MTTR_i}$$




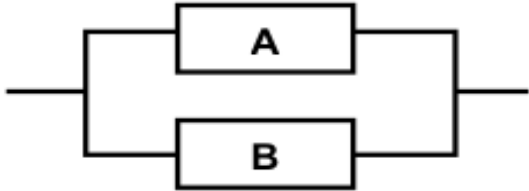
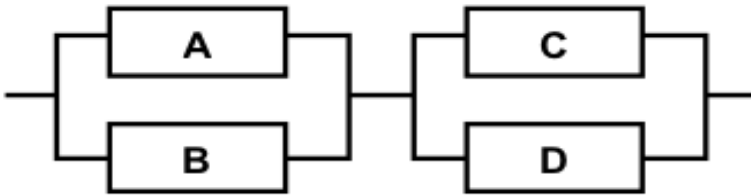
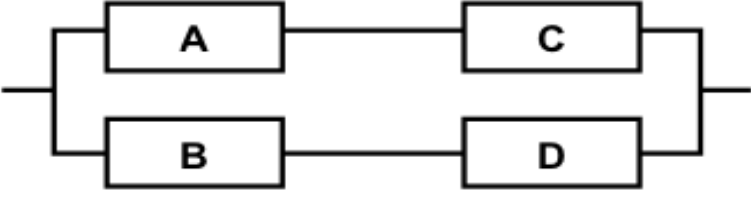
Reliability Block Diagrams (recap)

$$R_s = \prod_i^n R_i$$

$$R_s = 1 - \prod_i^n (1 - R_i)$$

Component redundancy

System redundancy

Type	Block Diagram Representation	System Reliability (R_S)
Series		$R_S = R_A R_B$ R_A = reliability, component A R_B = reliability, component B
Parallel		$R_S = 1 - (1 - R_A)(1 - R_B)$
Series-Parallel		$R_S = [1 - (1 - R_A)(1 - R_B)]^* [1 - (1 - R_C)(1 - R_D)]$ R_C = reliability, component C R_D = reliability, component D
Parallel-Series		$R_S = 1 - (1 - R_A R_C)^* (1 - R_B R_D)$



Example 1

RBDs

What is the Reliability of the entire system knowing the Reliability of each component?

$$R_A = 0.95$$

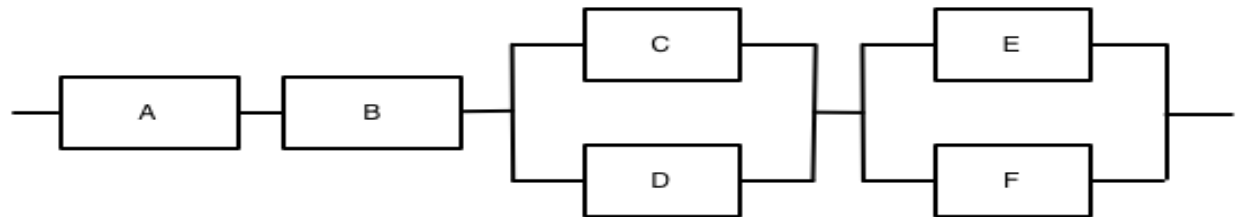
$$R_B = 0.97$$

$$R_C = 0.99$$

$$R_D = 0.99$$

$$R_E = 0.92$$

$$R_F = 0.92$$



Example 1

RBDs

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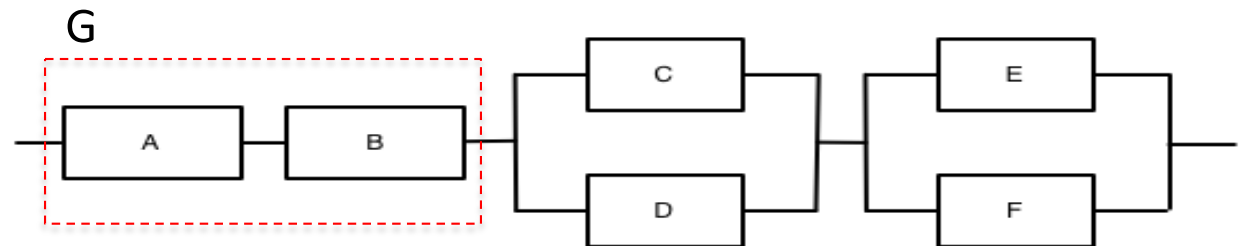
$$R_B = 0.97$$

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$$R_F = 0.92$$



$$R_G = R_A * R_B$$

$$R_G = 0.9215$$



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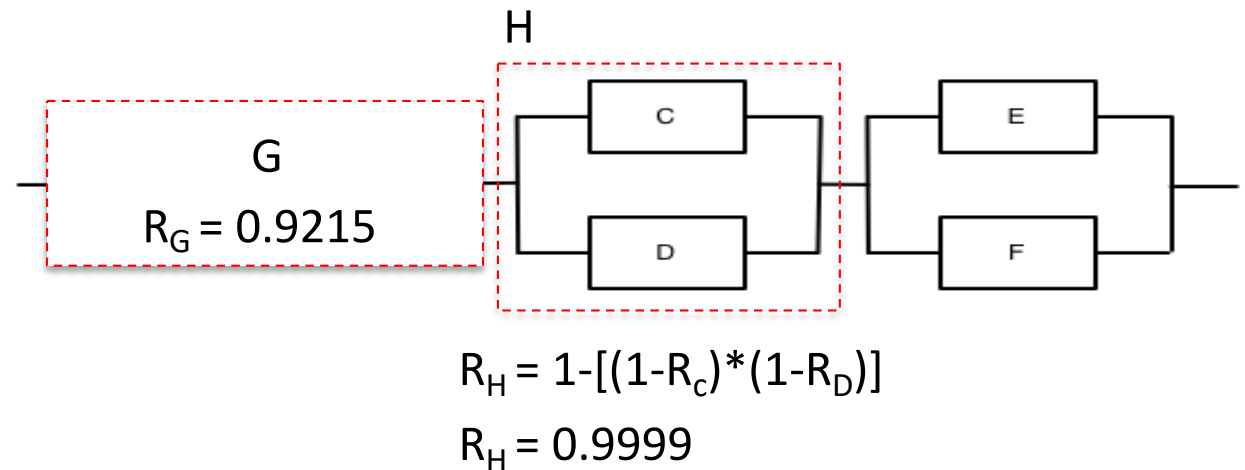
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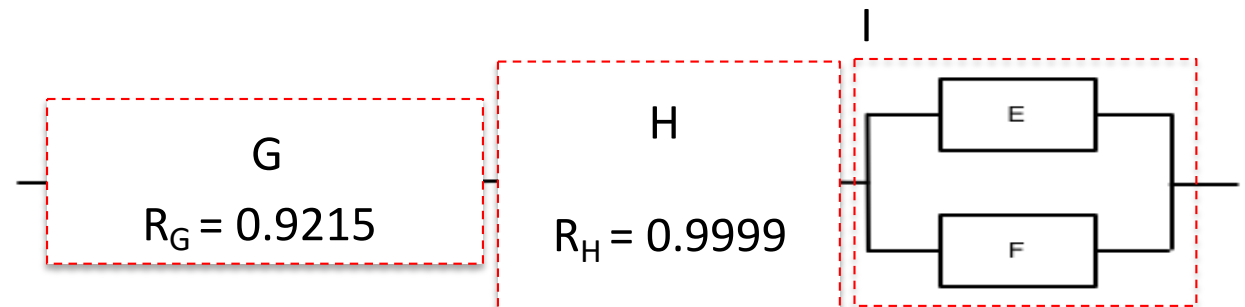
$$R_B = 0.97$$

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$$R_E = 0.92$$

$$R_F = 0.92$$



$$R_I = 1 - [(1 - R_E) * (1 - R_F)]$$

$$R_I = 0.9936$$



Example 1

RBDs

What is the Reliability of the entire system knowing the Reliability of each component?

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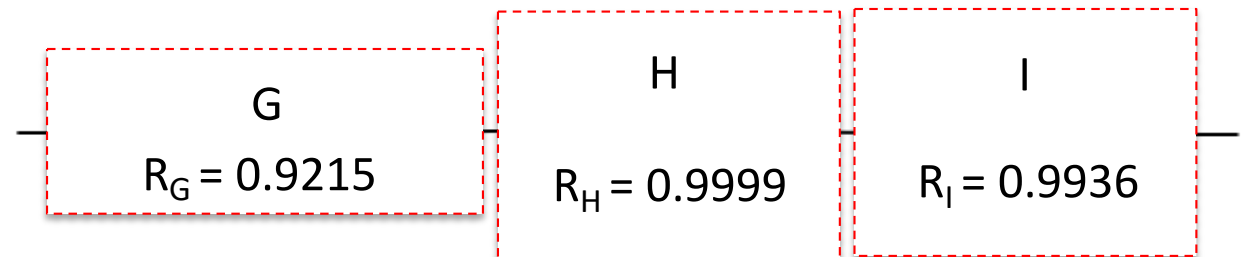
$$R_B = 0.97$$

$$R_C = 0.99$$

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$$R_S = R_G * R_H * R_I = 0.9155$$



Example 1

What is the Reliability of the entire system knowing the Reliability of each component?

$$R_A = 0.95$$

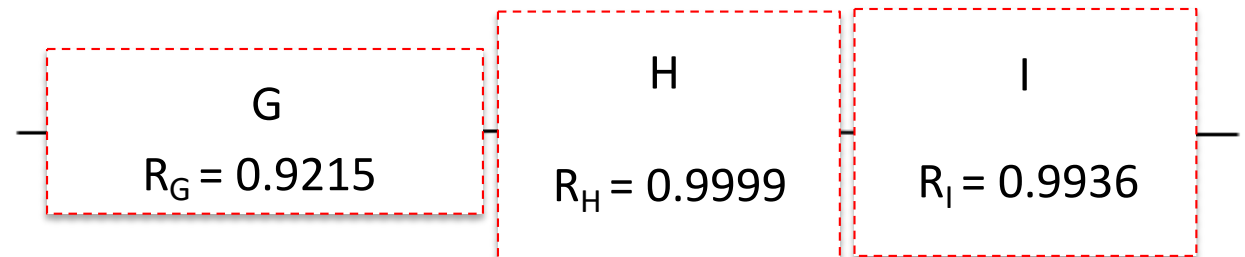
$$R_B = 0.97$$

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$$R_F = 0.92$$



$$R_S = R_G * R_H * R_I = 0.9155$$

$$R_S = R_A R_B [1-(1-R_C)(1-R_D)] [1-(1-R_E)(1-R_F)]$$

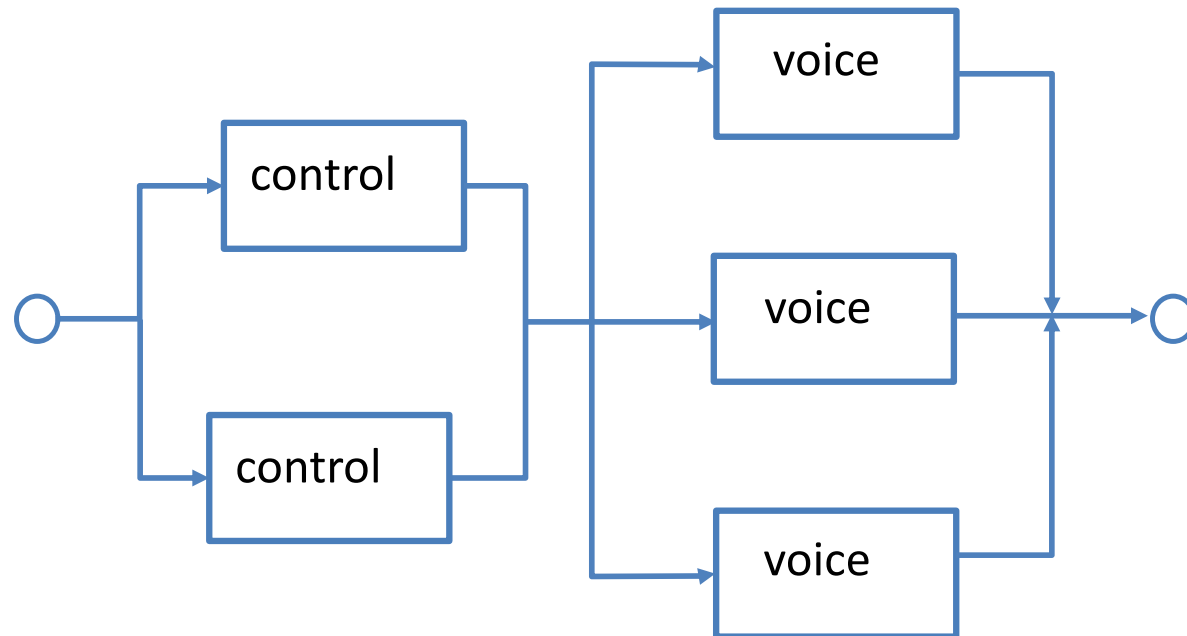
$$R_S = (0.95)(0.97)[1-(1-0.99)(1-0.99)] [1-(1-0.92)(1-0.92)]$$



Example 2

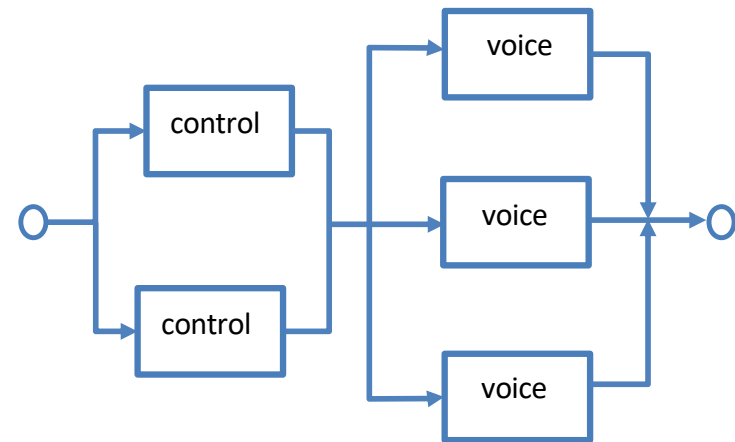
2 control blocks and 3 voice channels:

- system is up if at least 1 control channel and at least 1 voice channel are up



Example 2 - cont'd

- Each control channel has reliability R_c
- Each voice channel has reliability R_v
- Reliability:

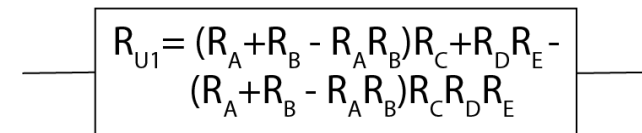
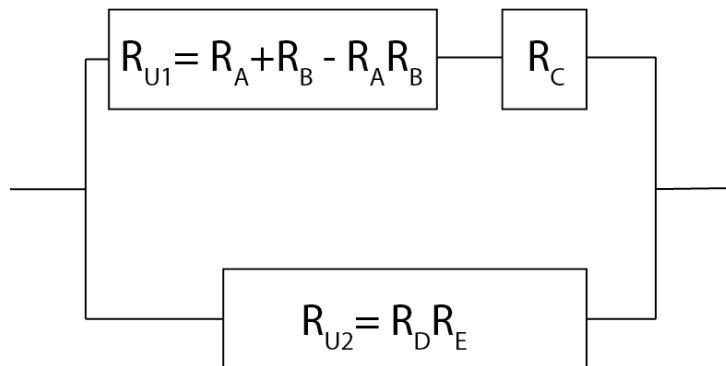
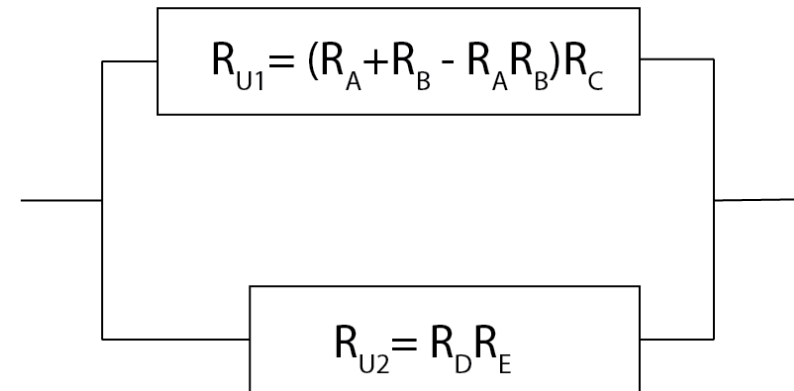
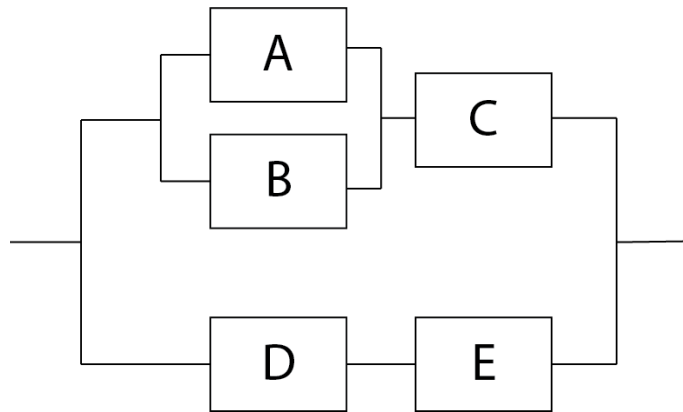


$$R = [1 - (1 - R_c)^2][1 - (1 - R_v)^3]$$



Example 3

RBDs

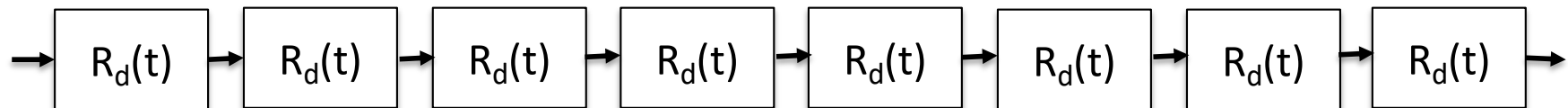


RBD: used to model a system and calculate its reliability

RBDs

We have a RAID-0 storage system composed of 8 parallel disks; each disk of the system may fail independently of the others;
If the reliability of each disk is $R_d(t)$, what is the overall reliability of the storage system?

How would you model the entire storage system using an RBD?



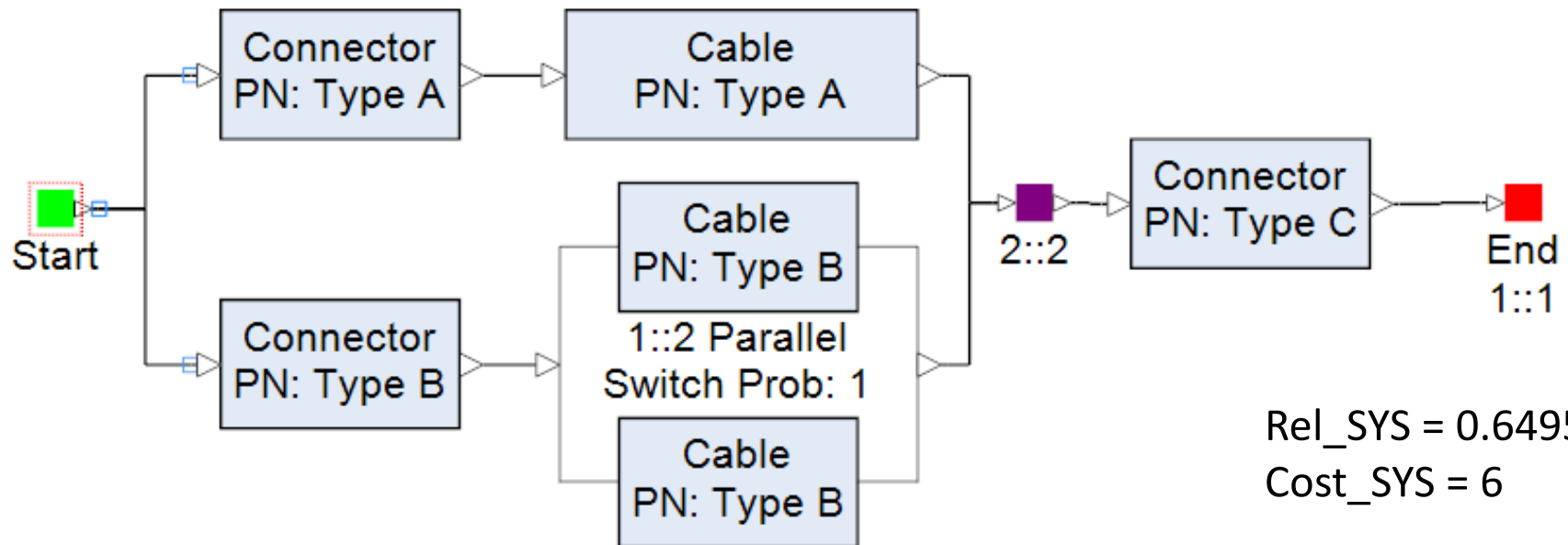
RBD: used to compare different alternatives

RBDs

Cable Bundle

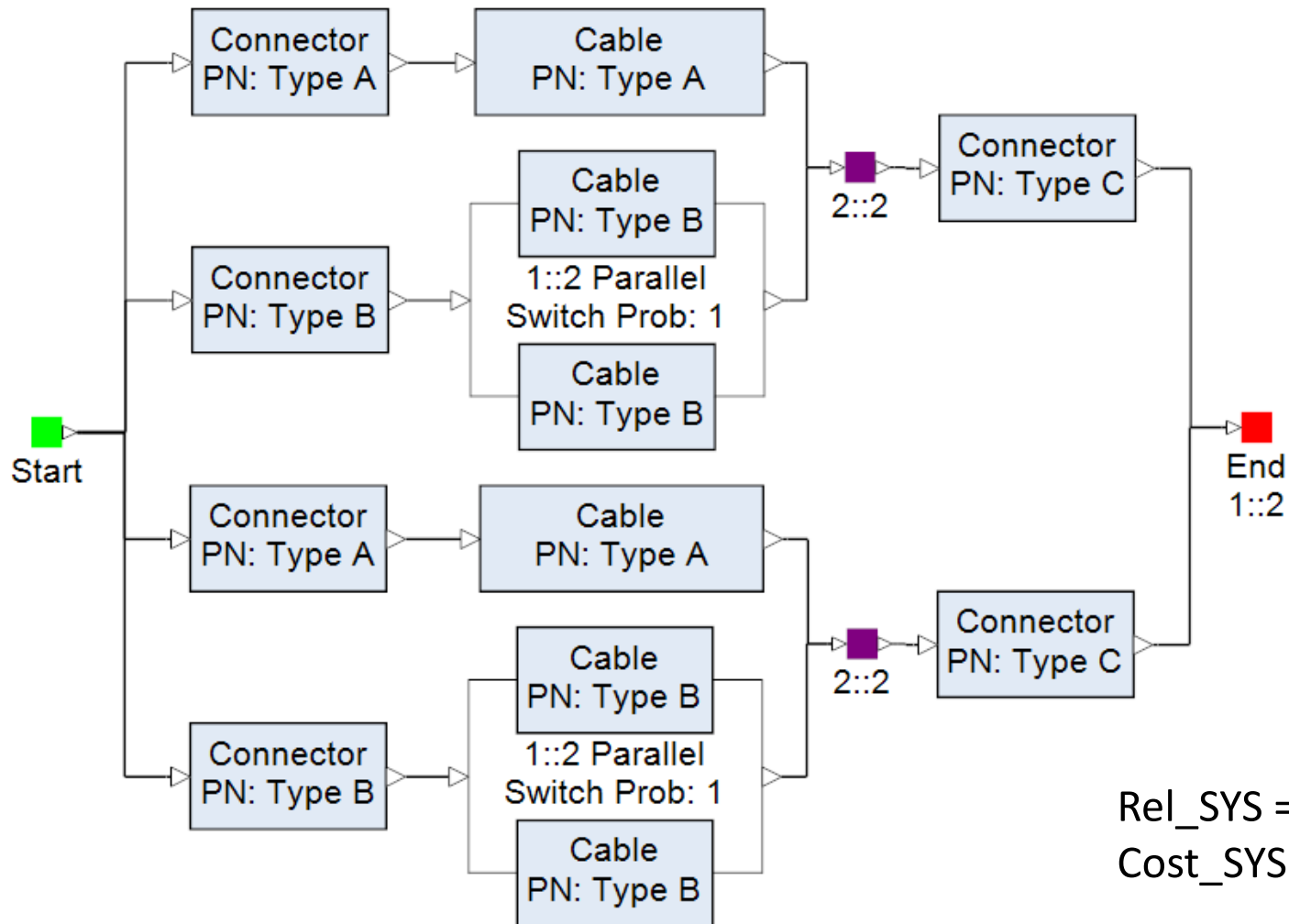
Each block has $R = 0.9$

Each block costs 1



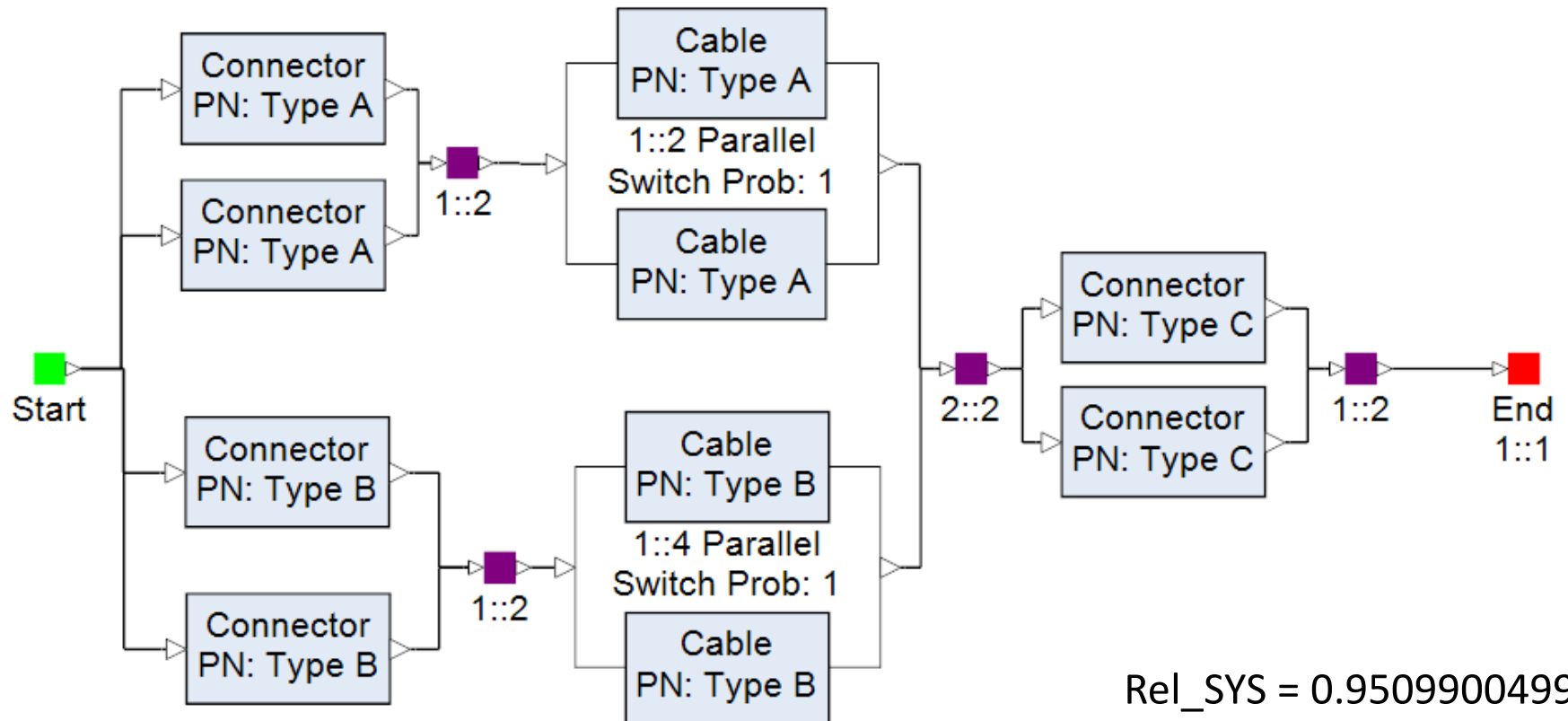
Alternative 1

RBDs



Alternative 2

RBDs



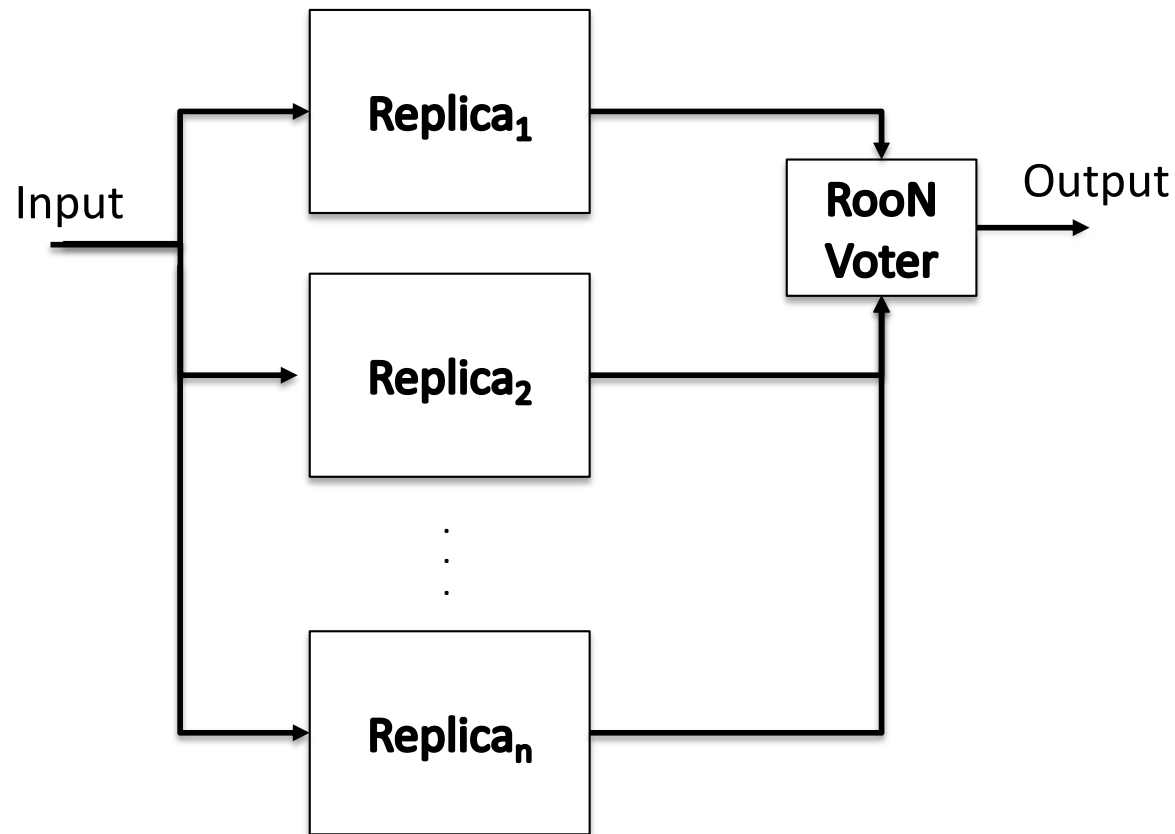
Rel_SYS = 0.9509900499

Cost_SYS = 10



r out of n redundancy (RooN)

A system composed of n identical replicas where at least r replicas have to work fine for the entire system to work fine



r out of n redundancy (RooN)

RBDs

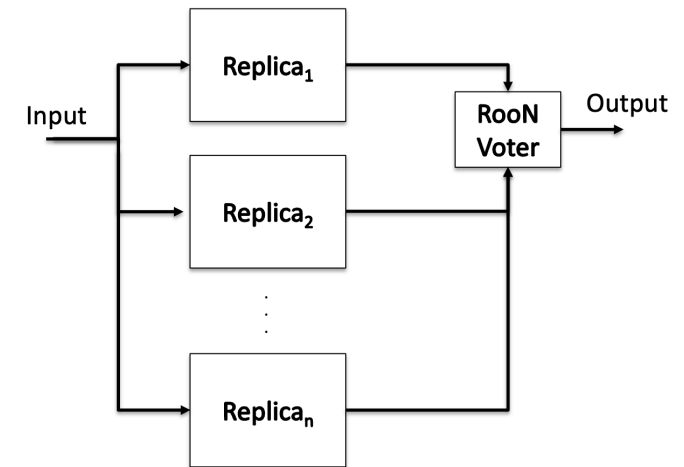
R_s = System reliability

R_c = Component reliability

R_v = Voter Reliability

n = Number of components

r = Minimum number of components which must survive



$$R_s(t) = R_v \sum_{i=r}^n R_c^i (1 - R_c)^{n-i} \underbrace{\frac{n!}{i! (n-i)!}}_{\text{Binomial coefficient}}$$

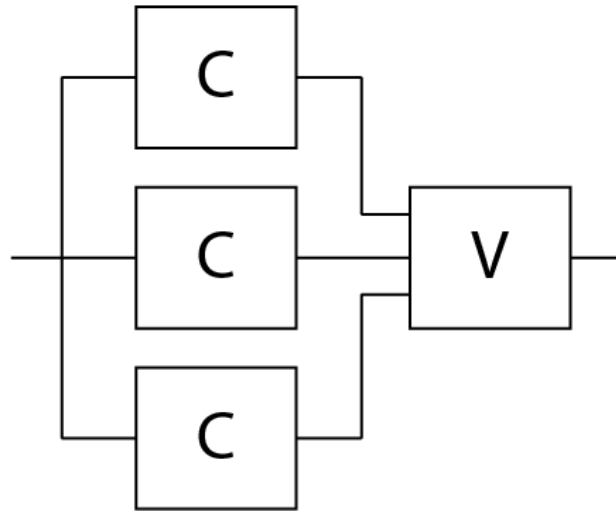
Binomial coefficient

$$\binom{n}{i}$$



Triple Modular Redundancy – TMR

RBDs



System works properly if

- 2 out of 3 components work properly
AND the voter works properly

$$R_{TMR} = R_v \left[\sum_{i=2}^3 \binom{3}{i} R_m^i (1 - R_m)^{3-i} \right] = R_v [R_m^3 + 3R_m^2(1 - R_m)] = R_v (3R_m^2 - 2R_m^3)$$

$$\begin{aligned} MTTF_{TMR} &= \int_0^{\infty} R_{TMR} dt = \int_0^{\infty} R_v (3R_m^2 - 2R_m^3) dt = \int_0^{\infty} e^{-\lambda_v t} (3e^{-2\lambda_m t} - 2e^{-3\lambda_m t}) dt \\ &= \frac{3}{2\lambda_m + \lambda_v} - \frac{2}{3\lambda_m + \lambda_v} \cong \frac{3}{2\lambda_m} - \frac{2}{3\lambda_m} = \left(\frac{5}{6}\right) \left(\frac{1}{\lambda_m}\right) = \frac{5}{6} MTTF_{simplex} \end{aligned}$$



Triple Modular Redundancy – TMR – GOOD or BAD?

RBDs

- $MTTF_{TMR}$ is shorter than $MTTF_{symplex}$
- Can tolerate transient faults and permanent faults
- Higher reliability (for shorter missions)

When do we have the same reliability?

- $R_{TMR}(t) = R_C(t)$

$$3e^{-2\lambda_m t} - 2e^{-3\lambda_m t} = e^{-\lambda_m t}$$

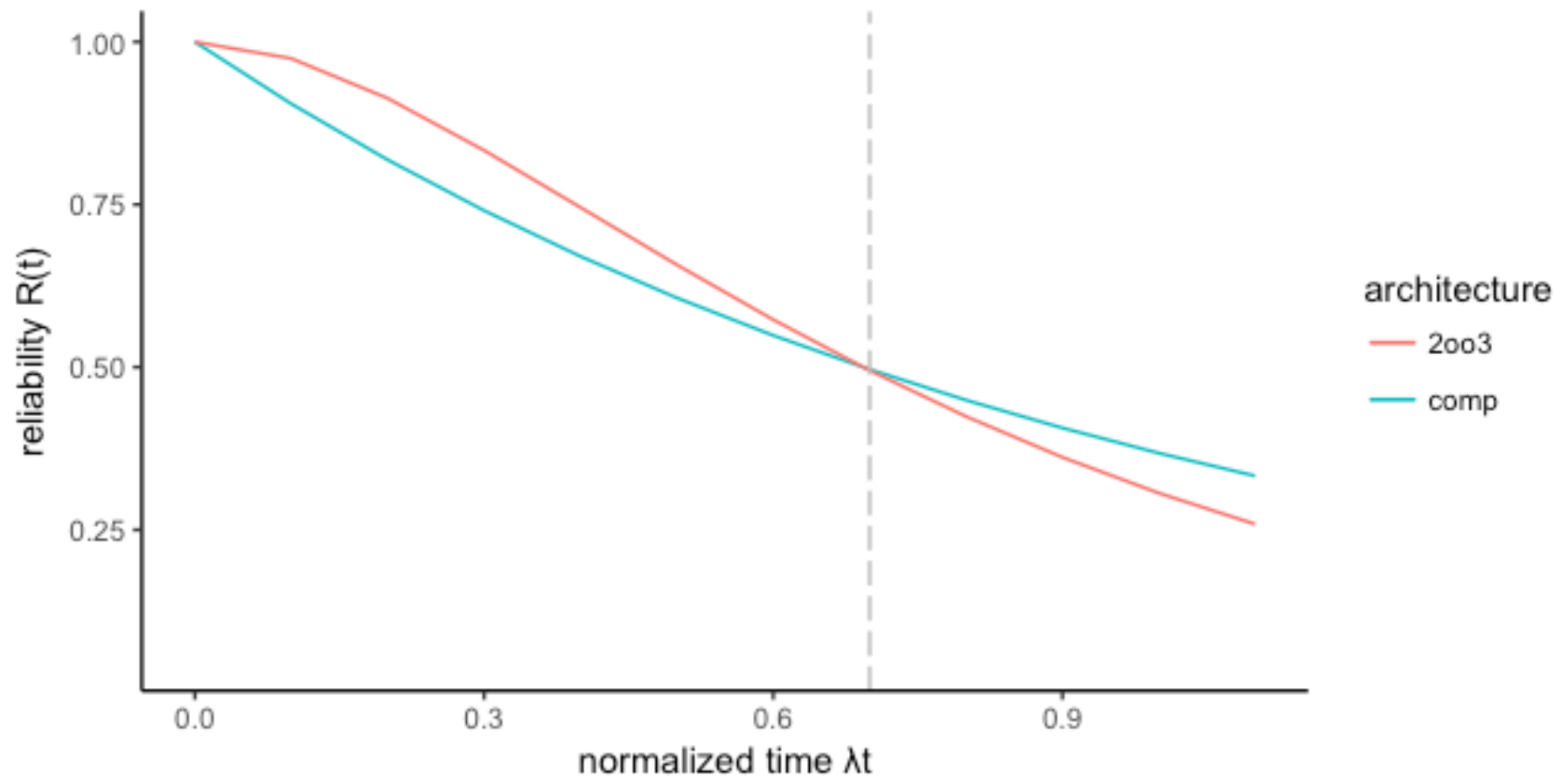
$$t = \frac{\ln 2}{\lambda_m} \cong 0.7 MTTF_C$$



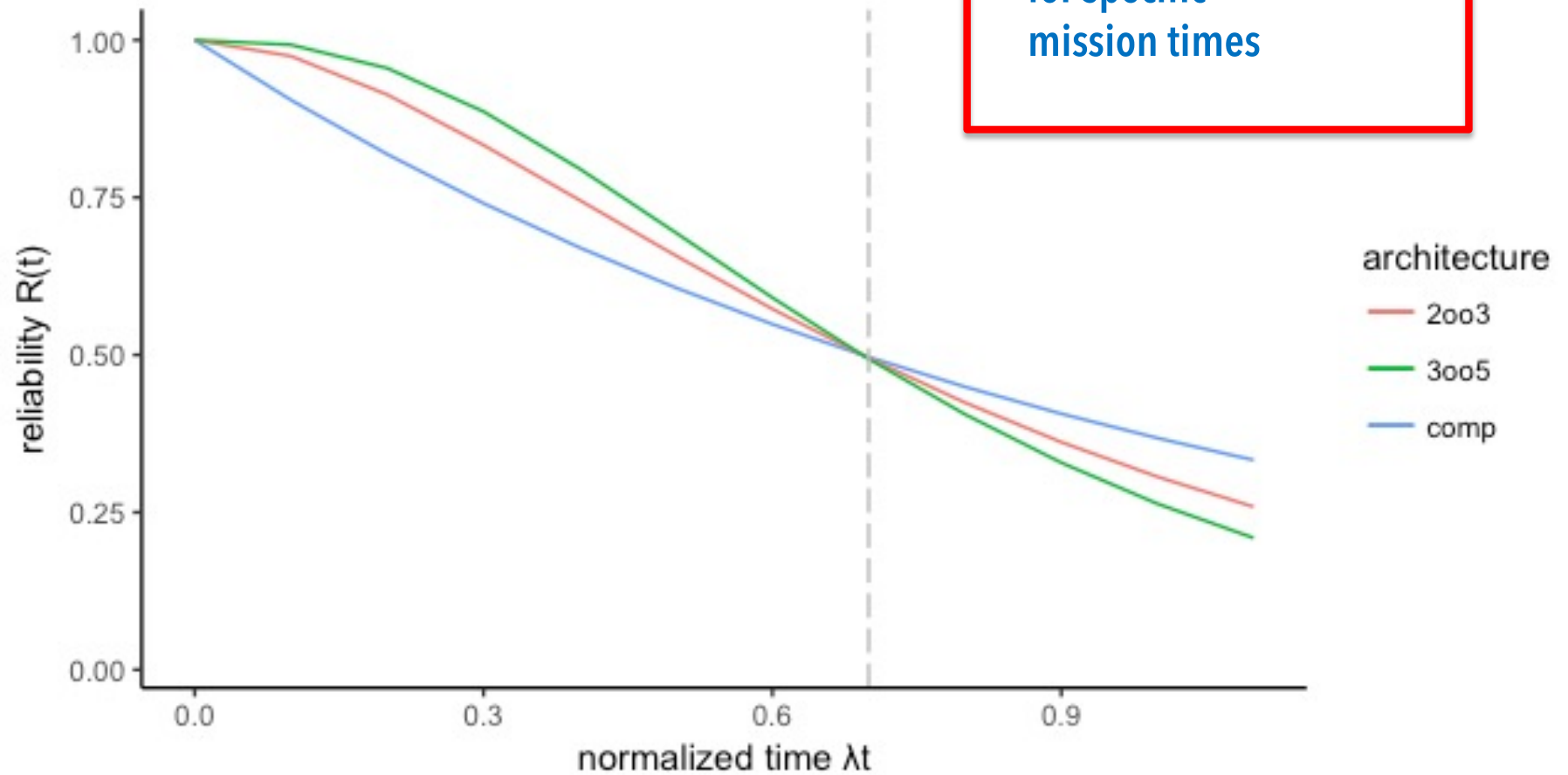
$R_{TMR}(t) > R_C(t)$ when the mission time is shorter than 70% of $MTTF_C$



TMR: 2 out of 3 components (voter is a 'perfect' element)



TMR: 2oo3 and nMR: 3oo5



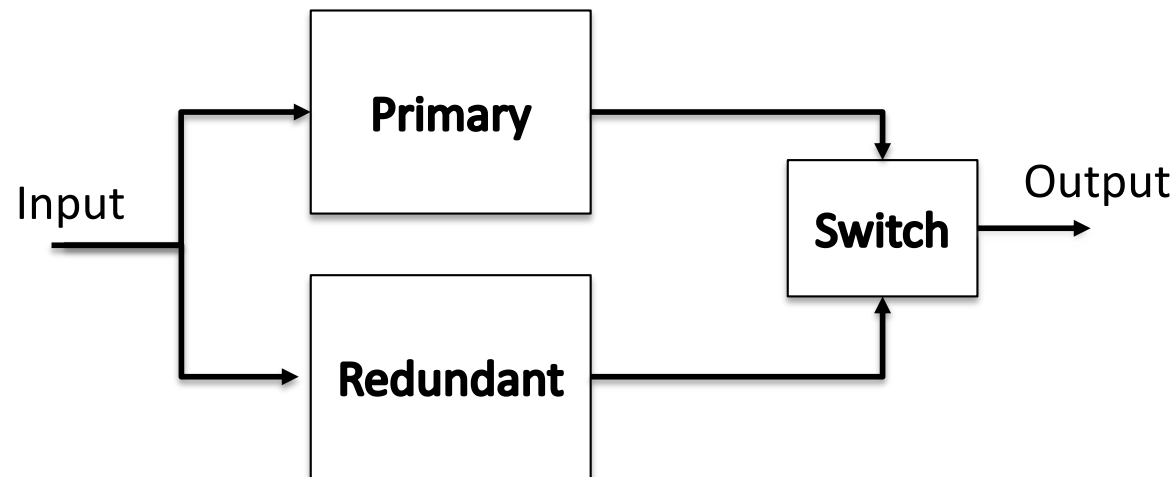
Redundancy is useful
for specific
mission times



Standby redundancy

A system may be composed of two parallel replicas:

- The primary replica working all time, and
- The redundant replica (generally disable) that is activated when the primary replica fails



Standby redundancy

RBDs

A system may be composed of two parallel replicas:

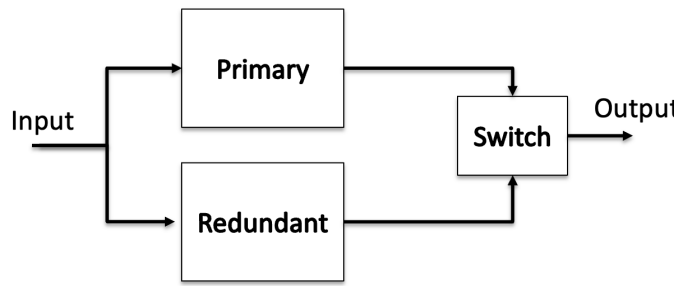
- The primary replica working all time, and
 - The redundant replica (generally disable) that is activated when the primary replica fails
-
- **What do we need for such a redundancy to be operational?**

Obviously, we need:

- A mechanism to determine whether the primary replica is working properly or not (on-line self check)
- A dynamic switching mechanism to disable the primary replica and activate the redundant one



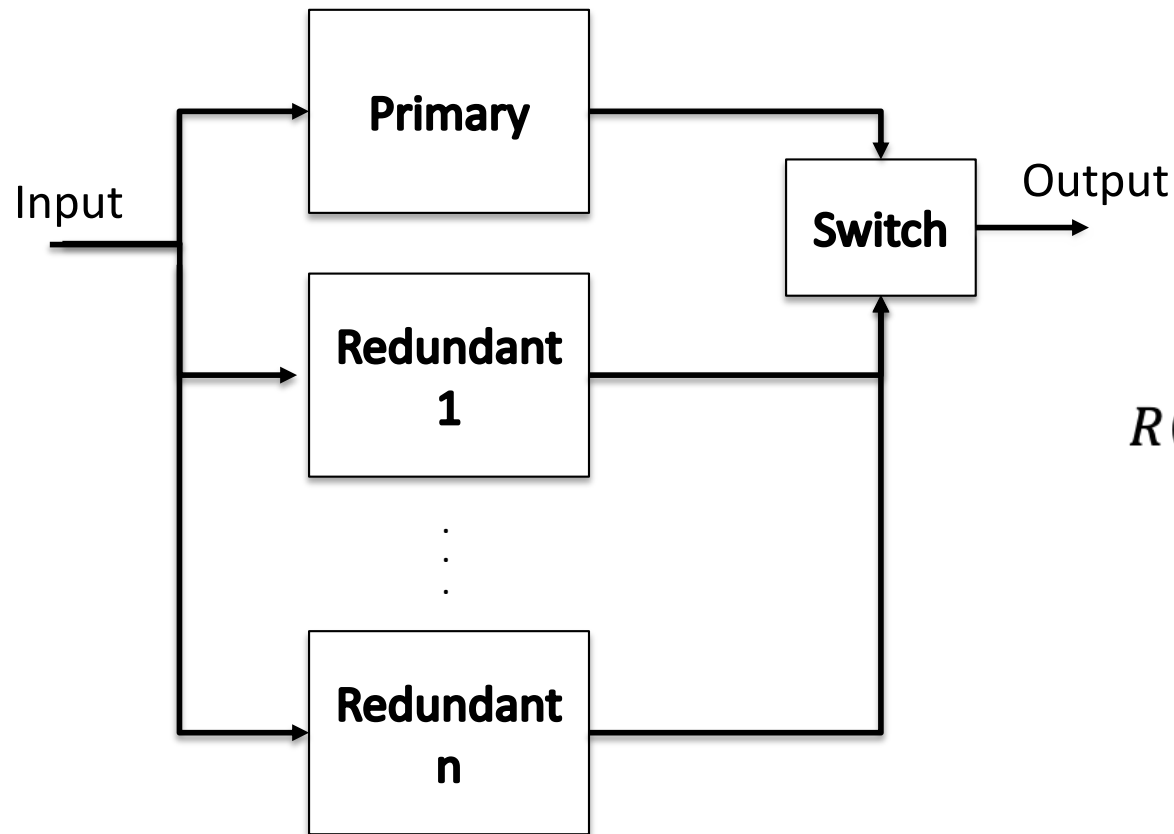
Standby redundancy – Quick Formulas

Standby Parallel Model	System Reliability
Equal failure rates, perfect switching	$R_s = e^{-\lambda t} (1 + \lambda t)$
Unequal failure rates, perfect switching	$R_s = e^{-\lambda_1 t} + \lambda_1 (e^{-\lambda_1 t} - e^{-\lambda_2 t}) / (\lambda_2 - \lambda_1)$
Equal failure rates, imperfect switching	$R_s = e^{-\lambda t} (1 + R_{\text{switch}} \lambda t)$
Unequal failure rates, imperfect switching	$R_s = e^{-\lambda_1 t} + R_{\text{switch}} \lambda_1 (e^{-\lambda_1 t} - e^{-\lambda_2 t}) / (\lambda_2 - \lambda_1)$
<p>where</p> <p>R_s = System reliability λ = Failure rate t = Operating time R_{switch} = Switching reliability</p> 	



Standby redundancy – Quick Formulas

More in general, a system having one primary replica and n redundant replicas (with identical replicas and perfect switching)



$$R(t) = e^{-\lambda t} \sum_{i=0}^{n-1} \frac{(\lambda t)^i}{i!}$$

