

How to calculate  $T_{I/O}$  for an HDD given the number of blocks

$$T_{I/O} = [(1 - DL) * (T_{seek} + T_{latency}) + T_{transfer} + T_{CTRL}] * N$$

$$T_{latency} = \frac{T_{rotation}}{2} \rightarrow T_{rotation} = \frac{60 * 1000}{RPM}$$

If I don't know RPM (o  $T_{seek}$ )  $\rightarrow N * (1 - DL) * T_{service\ time} + N * DL * (T_{transfer} + T_{CTRL})$

### RAID Disks: Reliability Calculation

MTTF = Mean Time To Failure

- RAID 0
  - $MTTF_{RAID\ 0} = MTTF_{single\ disk} / N$
- RAID 1 (per N=2)
  - $MTTF_{RAID\ 1} = (MTTF_{single\ disk} / N) * (1 / (MTTR / MTTF_{single\ disk}))$
- RAID 1+0
  - $MTTF_{RAID\ 1+0} = (MTTF_{single\ disk}^2) / (N * MTTR)$
- RAID 0+1
  - $MTTF_{RAID\ 0+1} = (MTTF_{single\ disk}^2) / (N * G * MTTR)$ 
    - $G = (N/2)$  is the number of disks in a stripe group
- RAID 4
  - $MTTF_{RAID\ 4} = (MTTF_{single\ disk}^2) / (N * N - 1 * MTTR)$
- RAID 5
  - $MTTF_{RAID\ 5} = (MTTF_{single\ disk}^2) / (N * N - 1 * MTTR)$
- RAID 6
  - $MTTF_{RAID\ 6} = 2 * (MTTF_{single\ disk}^3) / (N * N - 1 * N - 2 * MTTR)$

|   | RAID 0 | RAID 1       | RAID 4      | RAID 5      | RAID 6 |
|---|--------|--------------|-------------|-------------|--------|
| Capacity                                      | N      | N / 2        | N - 1       | N - 1       | N - 2  |
| Seq. Read                                     | N * S  | (N / 2) * S  | (N - 1) * S | (N - 1) * S |        |
| Seq. Write                                    | N * S  | (N / 2) * S  | (N - 1) * S | (N - 1) * S |        |
| Random Read                                   | N * R  | N * R        | (N - 1) * R | N * R       |        |
| Random Write                                  | N * R  | (N / 2) * R  | R / 2       | (N / 4) * R |        |
| Reliability<br>(how many<br>disk(s) can fail) | NO     | 1 (or N / 2) | 1           | 1           | 2      |

Best performance and most capacity?  $\rightarrow$  RAID 0

Greatest error recovery?  $\rightarrow$  RAID 1 (1+0 or 0+1) or RAID 6

Balance between space, performance, and recoverability?  $\rightarrow$  RAID 5

## Performance

Operational laws are based on observable variables:

- $T$ , the length of time we observe the system
- $A_k$ , the number of request arrivals we observe for resource k
- $C_k$ , the number of request completions we observe at resource k
- $B_k$ , the total amount of time during which the system is busy
- $N_k$ , the average number of jobs in the resource k

From these observed values, we can derive the following 4 quantities:

1.  $\lambda_k = A_k/T$ , the arrival rate
2.  $X_k = C_k/T$ , the throughput or completion rate
3.  $U_k = B_k/T$ , the utilisation
4.  $S_k = B_k/C_k$ , the mean service time per completed job

(The upper formulas are also valid if we have total data and not partial ones like  $X = C/T$ )

**Utilization Law:**  $U_k = X_k * S_k$  and  $X_k = U_k/S_k$

**Little's Law:**  $N_{insystem} = X * R$

**Response Time Law:**  $R = N/X - Z$  (where N number of users, X system throughput and Z think time)

**Forced Law:**  $X_k = v_k * X$

Other laws:

|                           |  |
|---------------------------|--|
| $V_k = C_k/C$             | $R_k(\text{residence time}) = V_k * R_k(\text{response time})$ |
| $D_k = V_k * S_k = B_k/C$ | $R = \sum R_k$   |
| $U_k = X * D_k$           | $N = \sum N_k$   |
| $X_{sys} = U_k/D_k$       |  |

$$R = \sum_{i=1}^k \frac{D_k}{1 - \lambda D_k}$$

$$R(\lambda) = \frac{D_{CPU}}{1 - \lambda D_{CPU}} + \frac{D_{disk}}{1 - \frac{\lambda D_{disk}}{n}}$$

Fundamentals laws:

$$D = \sum_{k=1}^K D_k \text{ and } D_{max} = \max_k D_k \quad N^* = \frac{D+Z}{D_{max}}$$

|             |   |
|-------------|---|
| batch       | $\frac{1}{D} \leq X(N) \leq \min\left(\frac{N}{D}, \frac{1}{D_{max}}\right)$      |
| terminal    | $\frac{N}{ND+Z} \leq X(N) \leq \min\left(\frac{N}{D+Z}, \frac{1}{D_{max}}\right)$ |
| transaction | $X(\lambda) \leq \frac{1}{D_{max}}$   |
| batch       | $\max(D, ND_{max}) \leq R(N) \leq ND$   |
| terminal    | $\max(D, ND_{max} - Z) \leq R(N) \leq ND$   |
| transaction | $D \leq R(\lambda)$   |

Transaction = Open Models  
Batch & Terminal = Closed Models

## Dependability

MTTF: mean time to (first) failure, the up time before the first failure

MTBF: mean time between failures =  $\frac{\text{total operating time}}{\text{number of failures}}$  or  $\frac{1}{\lambda}$  where  $\lambda = \frac{\text{number of failures}}{\text{total operating time}}$

Block diagram:

- Component in series**

a. 2 blocks, C1 and C2

$$R_S = R_{C1} * R_{C2}$$

In general, if system S is composed by components with a reliability having an exponential distribution (very common case)

$$R_S(t) = e^{-\lambda_s t} \text{ where } \lambda_s = \sum_{i=1}^n \lambda_i \rightarrow MTTF_S = \frac{1}{\sum 1/MTTF_i}$$

A special case: when all components are identical

$$R_S(t) = e^{-\lambda_s t}$$



$$R_S(t) = e^{-n\lambda t} = e^{-\frac{nt}{MTTF_1}} \quad MTTF_S = \frac{MTTF_1}{n}$$

Availability:

$$A_S = \prod_{i=1}^n \frac{MTTF_i}{MTTF_i + MTTR_i}$$

When all components are the same:

$$A_S(t) = A_1(t)^n \quad A = \left( \frac{MTTF_1}{MTTF_1 + MTTR_1} \right)^n$$

- Component in parallel**

a. 2 blocks, C1 and C2

$$R_S = R_{C1} + R_{C2} - R_{C1} * R_{C2}$$

$$MTTF_{parallel} = MTTF * \sum_{n=1}^N \frac{1}{n} \quad (\text{if all MTTF are equal})$$

$$MTTF_{parallel} = \sum_{i=1}^N MTTF_i - \frac{1}{\sum_{i=1}^N \frac{1}{MTTF_i}} \quad (\text{if all MTTF are different})$$

System P composed by  $n$  components

$$R_p(t) = 1 - \prod_{i=1}^n (1 - R_i(t))$$

Availability

$$A_p(t) = 1 - \prod_{i=1}^n (1 - A_i(t))$$

$$A_p = 1 - \prod_{i=1}^n (1 - A_i) = 1 - \prod_{i=1}^n \frac{MTTR_i}{MTTF_i + MTTR_i}$$