02562 Rendering - Introduction

Measuring light (radiometry and photometry)

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Physics of light

- The three basic actions of quantum electrodynamics:
  1. A photon goes from place to place.
  2. An electron goes from place to place.
  3. An electron emits or absorbs a photon.
- The probability for a photon to be at a position $\boldsymbol{x}$ follows a time-varying wave function.
- The energy E of the photon is related to the angular frequency ω of this wave:

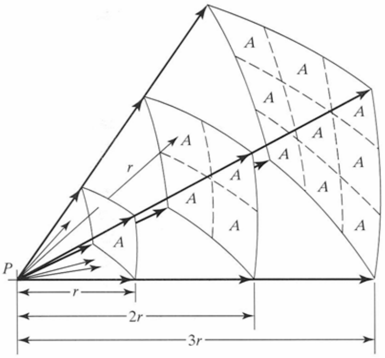
$$E = \hbar\omega = hf = \frac{hc}{\lambda}\,, \quad \hbar = \frac{h}{2\pi}\,, \quad f = 2\pi\omega\,, \quad h = 6.626 \cdot 10^{-34}\ \mathrm{J\,s}\,,$$

  where λ is the wavelength and $c = 2.9979 \cdot 10^{8}$ m/s is the speed of light.
- Light can be a vector field if we average over a large number of photons.
- The number of photons arriving per second determines the magnitude of the energy flux (J/s or W) in an electromagnetic field.
- We can measure such radiant energy.

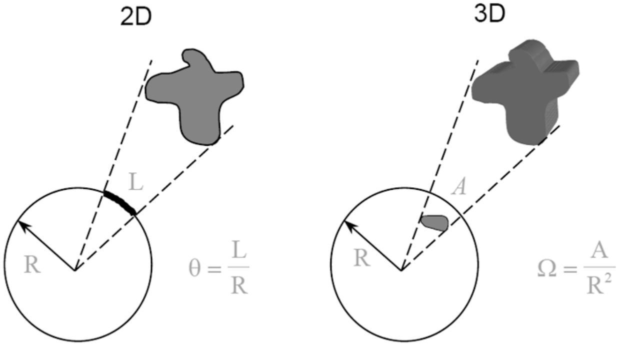
Physics	Radiometry			Photometry	
Energy	Radiant Energy	Q	joule J ($\mathrm{kg\,m^2/s^2}$)	Luminous Energy	talbot T (lm s)
Flux (Power)	Radiant Flux	$\Phi = \frac{\mathrm{d}Q}{\mathrm{d}t}$	watt W (J/s)	Luminous Flux	lumen lm (T/s)
Flux Density (incoming)	Irradiance	$E = \frac{\mathrm{d}\Phi}{\mathrm{d}A}$	$\mathrm{W/m^2}$	Illuminance	lux ($\mathrm{lm/m^2}$)
Flux Density (outgoing)	Radiant Exitance	$M = \frac{\mathrm{d}\Phi}{\mathrm{d}A}$	$\mathrm{W/m^2}$	Luminous Exitance	lux ($\mathrm{lm/m^2}$)
Flux Density (outgoing)	Radiosity (diffuse)	$B = \frac{\mathrm{d}\Phi}{\mathrm{d}A}$	$\mathrm{W/m^2}$	Luminosity	lux ($\mathrm{lm/m^2}$)
Angular Flux Density	Radiance	$L = \frac{\mathrm{d}\Phi}{\mathrm{d}A^{\perp}\mathrm{d}\omega}$	$\mathrm{W/(m^2\,sr)}$	Luminance	nit ($\mathrm{cd/m^2}$)
Intensity	Radiant Intensity	$I = \frac{d\Phi}{d\omega}$	W/sr	Luminous Intensity	candela cd (lm/sr)

Light emission

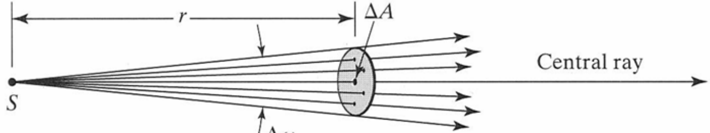
- An electric light emits light according to its power P and emissivity/efficiency ε:
$$\Phi = \varepsilon P = \varepsilon VC\,,$$
where V is voltage (volts) and C is current (amps).
- If the source emits light isotropically in the solid angle Ω, its intensity is
$$I = \frac{\mathrm{d}\Phi}{\mathrm{d}\omega} = \frac{\Phi}{\Omega}\,.$$
- If the source emits light homogeneously across a surface, the radiant exitance is
$$M = \frac{\mathrm{d}\Phi}{\mathrm{d}A} = \frac{\Phi}{A}\,.$$
- The radiance from an isotropic point light reaching an observer at the distance r is
$$L_i = \frac{I}{r^2}\,.$$



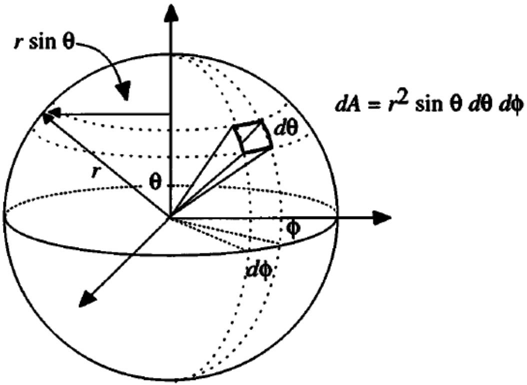
Solid angles (radians vs. steradians)



$$d\omega = \frac{dA^{\perp}}{r^2}$$



Calculating solid angles using spherical coordinates



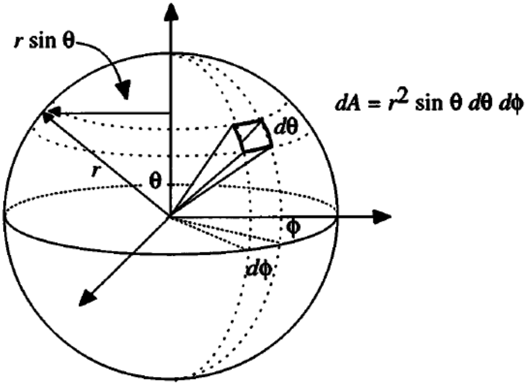
$$d\omega = \frac{dA}{r^2} = \sin\theta \, d\theta \, d\phi$$

- Full sphere:

$$\begin{aligned}
\Omega &= \int_0^{2\pi} \int_0^{\pi} \sin\theta \, d\theta \, d\phi \\
&= \int_0^{2\pi} d\phi \int_{\cos\pi}^{\cos 0} d(\cos\theta) \\
&= 2\pi(1 - (-1)) \\
&= 4\pi
\end{aligned}$$

$$\frac{d(\cos\theta)}{d\theta} = -\sin\theta$$

Integrating over a cosine-weighted hemisphere



$$d\omega = \frac{dA}{r^2} = \sin\theta \, d\theta \, d\phi$$

$$\begin{aligned}
\int_{2\pi} \cos\theta \, d\omega &= \int_0^{2\pi}\int_0^{\frac{\pi}{2}} \cos\theta \sin\theta \, d\theta \, d\phi \\
&= 2\pi \int_0^{\frac{\pi}{2}} (-\cos\theta) \, d(\cos\theta) \\
&= 2\pi \left[-\frac{1}{2}x^2 \right]_{\cos 0}^{\cos\frac{\pi}{2}} \\
&= 2\pi \left(-\frac{1}{2}(0-1) \right) \\
&= \pi
\end{aligned}$$

$$\frac{d(\cos\theta)}{d\theta} = -\sin\theta$$

A diffuse emitter

- Total emitted radiant flux:

$$
\begin{aligned}
\Phi &= \int_A \int_{2\pi} L \cos\theta \, \mathrm{d}\omega \, \mathrm{d}A_o \\
&= L \int_A \mathrm{d}A_o \int_0^{2\pi} \mathrm{d}\phi \int_0^{\frac{\pi}{2}} \cos\theta \sin\theta \, \mathrm{d}\theta \\
&= LA \, 2\pi \frac{1}{2} = LA\pi \, .
\end{aligned}
$$

- Radiosity:

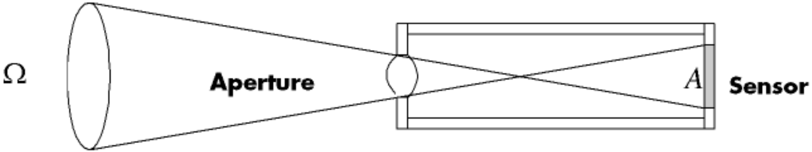
$$
\begin{aligned}
B &= \int_{2\pi} L \cos\theta \, \mathrm{d}\omega \\
&= L \, 2\pi \int_0^{\frac{\pi}{2}} \cos\theta \sin\theta \, \mathrm{d}\theta = L\pi \, .
\end{aligned}
$$

The measurement equation

- Radiance is flux per projected area per solid angle: $L = \frac{d\Phi}{dA^{\perp}\,d\omega} = \frac{d\Phi}{\cos\theta\, dA\, d\omega}$
- Suppose we let R denote the relative responsivity of the sensor.
- The energy received per second by a sensor of area A and solid angle Ω is then

$$\Phi_i(A,\Omega) = \int_A \int_\Omega R(\boldsymbol{x}_i,\vec{\omega}_i) L_i(\boldsymbol{x}_i,\vec{\omega}_i)(\vec{\omega}_i \cdot \vec{n}\,)\, d\omega_i\, dA_i$$

where $\vec{n}$ is the surface normal of the sensor area $(\cos\theta = \vec{\omega}_i \cdot \vec{n}\,)$.



Irradiance due to a source of a given intensity

- Element of solid angle subtended by an element of surface area at distance r:

$$\mathrm{d}\omega = \frac{\mathrm{d}A^{\perp}}{r^2} = \frac{\mathrm{d}A\cos\theta}{r^2} \quad \Leftrightarrow \quad \frac{\mathrm{d}\omega}{\mathrm{d}A} = \frac{\cos\theta}{r^2}\,,$$

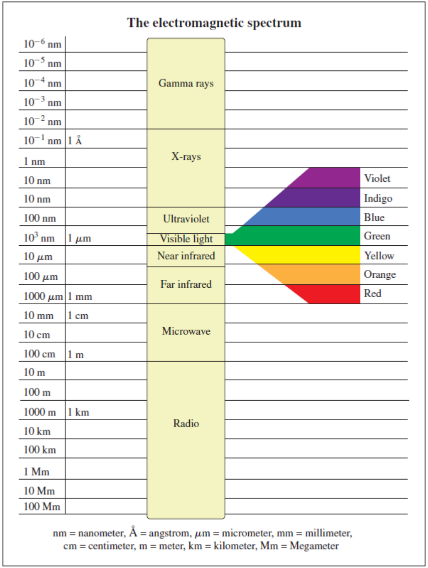
  where θ is the angle between the surface normal and the central ray of the solid angle.

- Relation between irradiance and source intensity:

$$E = \frac{\mathrm{d}\Phi}{\mathrm{d}A} = \frac{\mathrm{d}\Phi}{\mathrm{d}\omega}\frac{\mathrm{d}\omega}{\mathrm{d}A} = I\frac{\mathrm{d}\omega}{\mathrm{d}A} = I\frac{\cos\theta}{r^2}\,.$$

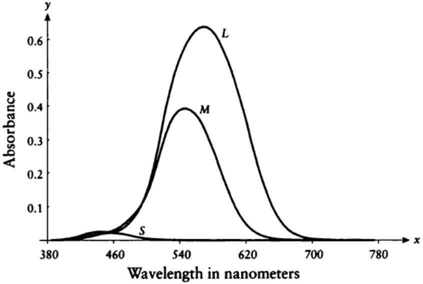
- Intensity of an isotropic point light: $I = \dfrac{\mathrm{d}\Phi}{\mathrm{d}\omega} = \dfrac{\Phi}{\Omega} = \dfrac{\Phi}{4\pi}\,.$
- Irradiance due to an isotropic point light: $E = I\dfrac{\cos\theta}{r^2} = \dfrac{\Phi}{4\pi}\dfrac{\cos\theta}{r^2}\,.$

Colour perception

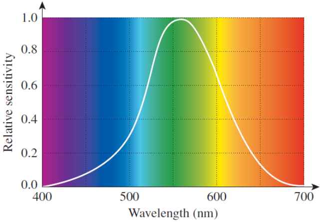


Photopigment absorption

(short-, medium-, and long-wavelength cone response)



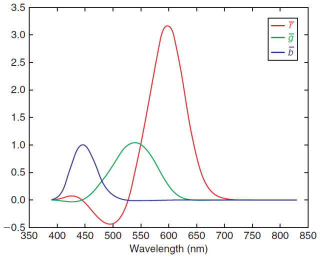
Luminous efficiency



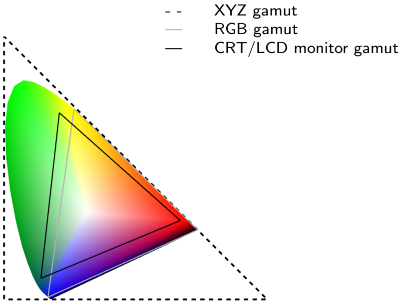
Photometry

- For every spectral radiometric quantity $f_r(\lambda)$, there is a related photometric quantity f_p measuring the "usefulness" of this quantity to a human observer.
- Conversion formula:
$$f_p = 683 \frac{\mathrm{lm}}{\mathrm{W}} \int_{380\,\mathrm{nm}}^{800\,\mathrm{nm}} \overline{y}(\lambda) f_r(\lambda)\,\mathrm{d}\lambda\,,$$
where $\overline{y}$ is the luminous efficiency function (sometimes called V).
- Example: radiance to luminance
$$Y = 683 \frac{\mathrm{lm}}{\mathrm{W}} \int_{380\,\mathrm{nm}}^{800\,\mathrm{nm}} \overline{y}(\lambda) L(\lambda)\,\mathrm{d}\lambda\,,$$
- Luminance Y is also one of three components in the XYZ color space, and $\overline{y}$ is one of three color matching function: $\overline{x}$, $\overline{y}$, $\overline{z}$.
- Similarly, there are color matching functions for the RGB color space: $\overline{r}$, $\overline{g}$, $\overline{b}$.

Colorimetry



CIE color matching functions



The chromaticity diagram

$$
\begin{aligned}
R &= \int_{\mathscr{V}} C(\lambda)\bar{r}(\lambda)\,\mathrm{d}\lambda \\
G &= \int_{\mathscr{V}} C(\lambda)\bar{g}(\lambda)\,\mathrm{d}\lambda \\
B &= \int_{\mathscr{V}} C(\lambda)\bar{b}(\lambda)\,\mathrm{d}\lambda \ ,
\end{aligned}
$$

where $\mathscr{V}$ is the interval of visible wavelengths and $C(\lambda)$ is the spectrum that we want to transform to RGB.