# 02562 Rendering - Introduction

## Ray Tracing

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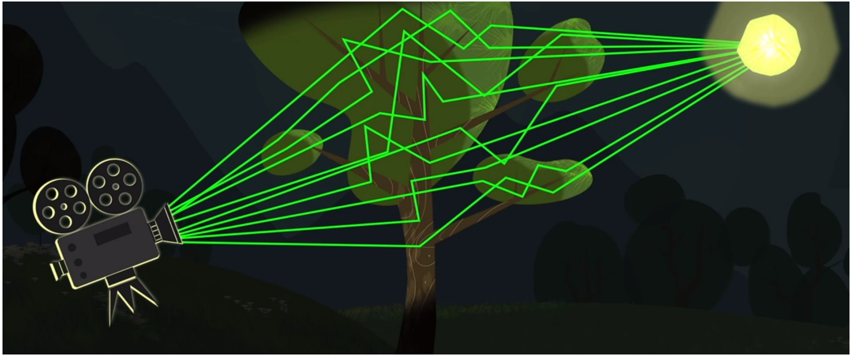
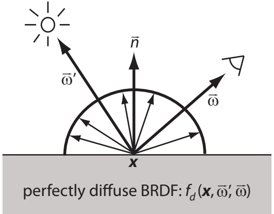
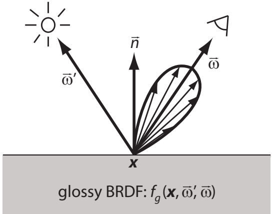
September 2023

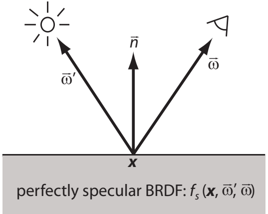
# Interaction of eye rays with specular and glossy materials







# Geometrical Optics

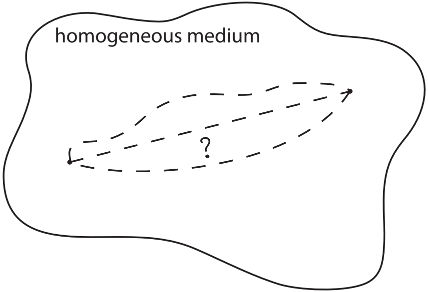
If we neglect wavelength ($\lambda \to 0$), we can derive the following properties of light from wave optics:

1. Light travels in the form of rays.
   - Rays are emitted from light sources.
   - Rays are observed when they reach an optical detector.
2. An optical medium is characterized by an *index of refraction* (or *refractive index*)

$$n = c_0/c$$

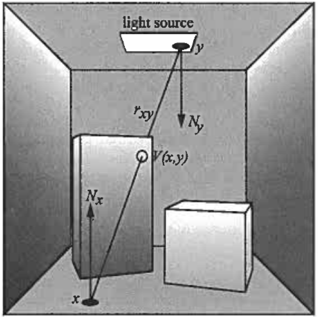
   - $c$ is the speed of light in a medium,
   - $c_0$ is the speed of light in a vacuum.
3. In an inhomogeneous medium, the refractive index $n(\boldsymbol{x})$ is a function of the position $\boldsymbol{x}$ in the medium.
4. **Fermat's Principle** [1662].
   Rays traveling between two points follow the path of least time.

# Hero's Principle




- What is the fastest path between two points in a homogeneous medium?
- In a homogeneous medium, the refractive index $n$ is constant.
- The speed of light in the medium is then constant $c = c_0/n$.
- The path of least time is then also the path of minimum distance.
- **Hero's Principle** [$\sim$ 50 A.D.].
  Light rays travel in straight lines in a homogenous medium.

# Shadow Rays

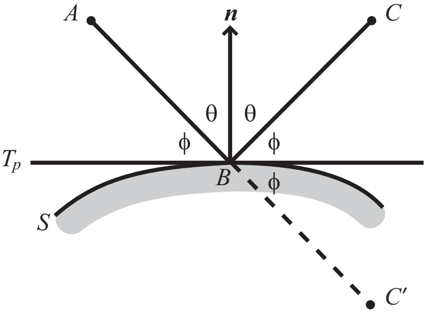



- Rays that start on a surface need an offset distance $t_{\text{min}} = \epsilon$ $(= 10^{-4}$, e.g.).
- Shadow rays need a cutoff distance $t_{\text{max}} = \|\boldsymbol{p} - \boldsymbol{x}\| - \epsilon$, where
  - $\boldsymbol{p}$ is the position of the light,
  - $\boldsymbol{x}$ is the surface position to be shaded

  (unless the source is infinitely far away, like a directional light).
- Shadow rays are only used for checking occlusion:
  Create a `Ray` and a `HitInfo` and use the `intersect_scene` function.

# Reflected rays

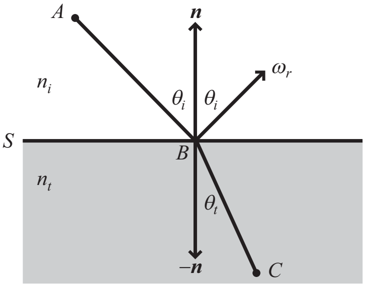
plane of incidence: containing $B$, spanned by $\boldsymbol{n}$ and $A - B$




$$
\begin{aligned}
\vec{\omega}_i &= (A-B)/\|A-B\| \\
\vec{n} &= \boldsymbol{n}/\|\boldsymbol{n}\| \\
\cos\theta_i &= \vec{\omega}_i \cdot \vec{n} \\
\vec{\omega}_r &= (C-B)/\|C-B\| \\
&= 2\cos\theta_i \vec{n} - \vec{\omega}_i \\
&= 2(\vec{\omega}_i \cdot \vec{n})\vec{n} - \vec{\omega}_i \\
&= \texttt{reflect}(-\vec{\omega}_i, \vec{n})
\end{aligned}
$$

- Light path from $A$ to $C$ by reflection off a mirror surface $S$.
- Applying Hero's principle:
  Choose a point $B$ on $S$ such that the distance $\overline{AB} + \overline{BC}$ is minimal.
- Let $C'$ be the mirror image of $C$ in the tangent plane $T_p$ at $B$.
  Then $\overline{AB} + \overline{BC} = \overline{AB} + \overline{BC'}$ is minimal if $\overline{AC'}$ is straight.
- This requires that the vector $C' - B$ is in the plane of incidence.
- **Law of reflection**. The reflected ray lies in the plane of incidence; the angle of reflection equals the angle of incidence.

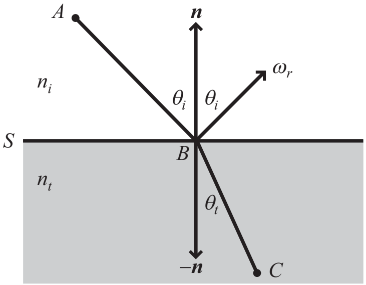
# Refracted rays




$$
\begin{aligned}
A &= (u_1, v_1) \\
C &= (u_2, v_2) \\
\overline{AB} &= \sqrt{(u-u_1)^2 + v_1^2} \\
\overline{BC} &= \sqrt{(u_2-u)^2 + v_2^2} \\
\frac{d(nd)}{du} &= n_i \frac{u-u_1}{\overline{AB}} - n_t \frac{u_2-u}{\overline{BC}} \\
0 &= n_i \sin\theta_i - n_t \sin\theta_t
\end{aligned}
$$

- Light path from a point $A$ in a medium of refractive index $n_i$ to a point $C$ in a medium of refractive index $n_t$.
- Time it takes for light to travel a distance $d$ is $d/c = nd/c_0$.
- Applying Fermat's principle: Choose a point $B$ on $S$ such that the optical distance $nd = n_i\overline{AB} + n_t\overline{BC}$ is minimal.
- Let the surface tangent $\boldsymbol{t}$ and the normal $\boldsymbol{n}$ define the $u$-axis and the $v$-axis in the plane of incidence, then $B = (u, 0)$.
- **Law of refraction**. The refracted ray lies in the plane of incidence; the angle of refraction $\theta_t$ is related to the angle of incidence $\theta_i$ by $n_i \sin\theta_i = n_t \sin\theta_t$.

# Refraction and total internal reflection




$$
\begin{aligned}
\cos\theta_i &= \vec{\omega}_i \cdot \vec{n} \\
\sin^2\theta_i &= 1 - (\vec{\omega}_i \cdot \vec{n})^2 \\
\sin\theta_t &= \frac{n_i}{n_t}\sin\theta_i \\
\cos^2\theta_t &= 1 - \left(\frac{n_i}{n_t}\right)^2 (1 - (\vec{\omega}_i \cdot \vec{n})^2) \\
\vec{t} &= \frac{\cos\theta_i \vec{n} - \vec{\omega}_i}{\sin\theta_i} \\
\vec{t}\sin\theta_t &= \frac{n_i}{n_t}((\vec{\omega}_i \cdot \vec{n})\vec{n} - \vec{\omega}_i)
\end{aligned}
$$

- In the plane of incidence:
  - $\vec{\omega}_i = (A - B)/\|A - B\|$ is the direction of incidence,
  - $\vec{t} = \boldsymbol{t}/\|\boldsymbol{t}\|$ is the unit length tangent of $S$ at $B$,
  - $\vec{n} = \boldsymbol{n}/\|\boldsymbol{n}\|$ is the unit length normal of $S$ at $B$,
  - $\vec{\omega}_t = \vec{t}\sin\theta_t - \vec{n}\cos\theta_t$ is the direction of the refracted ray.
- We have total internal reflection if $\cos^2\theta_t < 0$ (all is reflected, no refracted ray, $n_i > n_t$).
- Otherwise: $\vec{\omega}_t = \frac{n_i}{n_t}((\vec{\omega}_i \cdot \vec{n})\vec{n} - \vec{\omega}_i) - \vec{n}\sqrt{1 - \left(\frac{n_i}{n_t}\right)^2 (1 - (\vec{\omega}_i \cdot \vec{n})^2)}$

# Glossy materials

- Point lights are not reflected as they have no physical extent.
- The Phong illumination model provides reflection highlights for point sources.
- The Phong model is however not energy conserving (reflectance goes to infinity at grazing incidence).
- The modified Phong model only has an energy loss [Lewis 1994, Lafortune and Willems 1994]:

$$\begin{aligned} L_r &= (k_d + k_s \cos^s \alpha) L_i \cos\theta_i \\ &= \left( \frac{\rho_d}{\pi} + \rho_s \frac{s+2}{2\pi} (\vec{\omega}_o \cdot \vec{\omega}_r)^s \right) L_i \, (\vec{\omega}_i \cdot \vec{n}) \ , \end{aligned}$$

- where
  - $\rho_d$ and $\rho_s$ are the diffuse and specular reflectances ($\rho_d + \rho_s \leq 1$),
  - $s$ is the shininess (or Phong exponent),
  - $\vec{\omega}_o$ is the unit length direction vector toward the observer,
  - $\vec{\omega}_i$, $\vec{\omega}_r$, and $\vec{n}$ are as in the previous slides.

# Exercises

- Trace shadow rays.
- Trace reflected rays.
- Trace refracted rays.
- Compute shading of glossy surfaces.

- New info that you need to record in the `HitInfo` struct:
  - Shader index, `shader: u32`.
  - Reciprocal relative index of refraction $\frac{n_1}{n_2}$, `ior1_over_ior2: f32`.
  - Specular reflectance $\rho_s$, `specular: f32`.
  - Phong exponent $s$, `shininess: f32`.
- To continue a path, let the shader
  - overwrite the `Ray` data,
  - set `(*hit).has_hit` in `HitInfo` to `false`, and
  - increment `(*hit).depth` in `HitInfo`.

# Related references (chronologically in groups)

## History

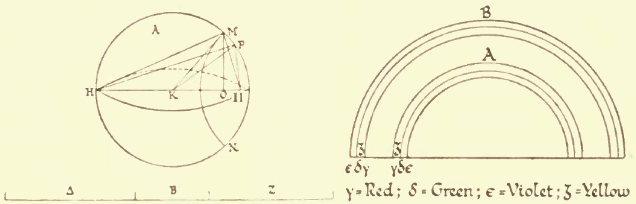

- Euclid, ∼300 B.C., 1895. Catoptrics. In J. L. Heiberg, ed., *Euclidis opera omnia*, Vol. 7, pp. 285–343, Teubner, 1895. English excerps by Smith [1999].
- Hero of Alexandria, ∼50 A.D., 1900. Catoptrics. In W. Schmidt, ed. and trans., *Heronis Alexandrini opera quae supersunt omnia*, Vol. 2:1, pp. 301–373, Teubner, 1900. English excerps by Smith [1999].
- Ibn Sahl, ∼984, 1993. On the burning instruments. In R. Rashed *Géométrie et dioptrique au X^e siècle: Ibn Sahl, al-Qūhī et Ibn al-Haytham*, Les Belles Lettres, 1993. English excerpts by Rashed [1990].
- Fermat, P. d. 1891–1912. *Œuvres de Fermat: Publiées par les soins de MM. Paul Tannery et Charles Henry sous les auspices du Ministère de l'instruction publique*, vol. 2. Gauthier-Villars et fils. Correspondance.
- Sommerfeld, A., and Runge, J. 1911. Anwendung der Vektorrechnung auf die Grundlagen der Geometrischen Optik. *Annalen der Physik 340*, pp. 277–298.
- Rashed, R. 1990. A pioneer in anaclastics: Ibn Sahl on burning mirrors and lenses. *Isis 81*, pp. 464–491.
- Smith, A. M. 1999. *Ptolemy and the Foundations of Ancient Mathematical Optics: A Source Based Guided Study*, Vol. 89.3 of *Transactions of the American Philosophical Society*. American Philosophical Society, Philadelphia.


## Graphics


- Phong, B. T. 1975. Illumination for computer-generated pictures. *Communications of the ACM 18*(6), pp. 311–317.
- Kay, D. S., and Greenberg, D. 1979. Transparency for computer synthesized images. In *Proceedings of SIGGRAPH 1979*, pp. 158–164, ACM.
- Whitted, T. 1980. An improved illumination model for shaded display. *Communications of the ACM 23*(6), pp. 343–349.
- Lewis, R. R. 1994. Making shaders more physically plausible. *Computer Graphics Forum 13*(2), pp. 109–120.
- Lafortune, E. P., and Willems, Y. D. 1994. Using the modified Phong reflectance model for physically based rendering. Technical Report CW 197, Department of Computer Science, KU Leuven.
- Dutré, P., Bekaert, P., and Bala, K. 2003. *Advanced Global Illumination*. A K Peters, Natick, Massachusetts.
- Eisenacher, C., Nichols, G., Selle, A. and Burley, B. 2013. Sorted deferred shading for production path tracing. *Computer Graphics Forum 32*(4), pp. 125–132.

# The Aristotelian rainbow

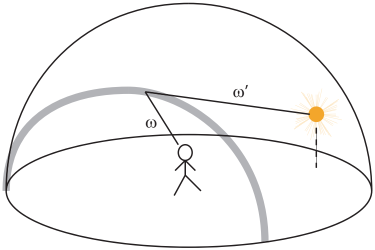
- Today, we have the computational power to compute the visual results predicted by the theories of scientists and philosophers of old.
- The rainbow as described by Aristotle (Meteorology) around 350 B.C.:




- Aristotle's rainbow theory is endearingly simple:
  - The rainbow appears where there is a special angle between observer, cloud, and sun.
  - The extension of the solar disk determines the size of the bow and the change in colours.


- Frisvad, J. R., Christensen, N. J., and Falster, P. 2007. The Aristotelian rainbow: from philosophy to computer graphics. In *Proceedings of GRAPHITE 2007*, pp. 119-128+311, ACM.

# Rendering the Aristotelian rainbow




- The rainbow is where $\omega \cdot \omega' = \cos 42^\circ$ .
- The sun is not a point, we have

$$a = \omega \cdot \omega'_{\text{high}} \qquad , \qquad b = \omega \cdot \omega'_{\text{low}} \ .$$

- The colour spectrum is a 1D array/texture. 
- When $a < \cos 42^\circ < b$, look-up using:

$$\text{smoothstep}(a, b, \cos 42^\circ) = 3\left(\frac{0.7431 - a}{b - a}\right)^2 - 2\left(\frac{0.7431 - a}{b - a}\right)^3 ,$$

a smooth Hermite interpolation between 0 and 1 (built-in function on GPUs).