Stat4DS / Homework 01

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Due Tuesday, November 07, 2017, 23:00 PM on Moodle

General Instructions

I expect you to upload your solutions on Moodle as a **single running** `R Markdown` file (`.rmd`), named with your surnames.

You will give the commands to answer each question in its own code block, which will also produce plots that will be automatically embedded in the output file. Your responses must be supported by both textual explanations and the code you generate to produce your results. *Just examining your various objects in the "Environment" section of RStudio is insufficient – you must use scripted commands and functions.*

R Markdown Test

To be sure that everything is working fine, start `RStudio` and create an empty project called `HW1`. Now open a new `R Markdown` file (`File > New File > R Markdown...`); set the output to `HTML mode`, press `OK` and then click on `Knit HTML`. This should produce a web page with the knitting procedure executing the default code blocks. You can now start editing this file to produce your homework submission.

Please Notice

- For more info on `R Markdown`, check the support webpage that explains the main steps and ingredients: R Markdown from RStudio.
- For more info on how to write math formulas in LaTex: Wikibooks.
- Remember our **policy on collaboration**: *Collaboration on homework assignments with fellow students is **encouraged**. However, such collaboration should be clearly acknowledged, by listing the names of the students with whom you have had discussions concerning your solution. You may **not**, however, share written work or code after discussing a problem with others. The solutions should be written by **you**.*

Exercise 01: Educated guesses...

You're new in Rome and you just moved into a new house. Surprise surprise, the phone is connected as in the old days! Now, you're pretty sure the phone number is `067405111`, but not as sure as you would like to be. As an experiment, you pick up the phone and dial `067405111` – don't try it at home please! – you obtain a "busy" signal.

Are you now more sure of your phone number? If so, how much?

Please notice: to get a realistic answer, you will have to make a series of reasonable, educated guesses on the unknowns involved in this exercise. Think carefully and do your best.

Exercise 02: Spice up T^3!

Tic–tac–toe is a boring game that always ends up in a tie if players play optimally. Instead, we may consider random variations of tic–tac–toe.

- **First variation**: Each of the 9 squares is labeled either $\times$ or $\bigcirc$ according to an independent and balanced coin flip. In other words, we fill the entire (3×3) board in one random shot. If only one of the players has one (or more) winning tic–tac–toe combinations, that player wins. Otherwise, the game is a tie. Determine the probability that $\times$ wins. Write a `R` program to help run through the configurations. (Hint: the function `sample()` may be useful).

- **Second variation**: $\times$ and $\bigcirc$ take turns, with the $\times$ player going first. On the $\times$ player's turn, an $\times$ is placed on a square chosen independently and uniformly at random from the squares that are still vacant; $\bigcirc$ plays similarly. The first player to have a winning tic–tac–toe combination wins the game, and a tie occurs if neither player achieves a winning combination. Find the probability that each player wins. Again, write an `R` program to help you.

Exercise 03: Shaking Monty

We talked about Monty's three doors game a bit ago. Now imagine that during a show, the contestant had initially chosen *door 1* and, just as Monty is about to open one of the other doors, a very violent earthquake rattles the building and one of the three doors flies open.

It happens to be *door 3*, and it happens **not** to have the prize behind it. Well, since none of the rules was violated by the shaking, Monty decided to keep calm and carry on... the show must go on!

The question is obvious: should the contestant stick with *door 1*, or switch to *door 2*, or does it make any difference? (You may assume that the prize was placed randomly, the Monty does **not** know where it is, and the door flew open just because of the earthquake).

To complete the exercise, setup a simulation in `R` to validate your math and comment the results.

Exercise 04: To be or not to be... random?

Consider the so–called "tent" transform:

$$D(x) = \begin{cases} 2x & x \leqslant 1/2 \\ 2(1-x) & x > 1/2. \end{cases}$$

The sequence $x_{n+1} = D(x_n)$ should have the same behaviour as a sequence of random numbers distributed according to the uniform distribution $\mathrm{Unif}(0,1)$ – start with $x_0 \in (0,1)$.

1. Show that if $U \sim \mathrm{Unif}(0,1)$, then $X = D(U)$ is still distributed as a $\mathrm{Unif}(0,1)$; in other words, show that the uniform is invariant under the "tent" transform.
2. Implement the tent generator $X_{n+1} = D(X_n)$ in `R` to generate 100 uniform random variables. Check the properties of the resulting sequence with appropriate plots.
3. Does the resulting sequence have a desirable behaviour? Why? (Hint: Computing issues related to the finite representation of numbers. Examine what happens when the sequence starts at a value of the form $1/2^k$ for some k)
