Foundations of Audio Signal Processing
Assignment 9

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Exercise 9.2

a

Upsampling operators are not time-invariant.
Consider

$$\uparrow M[x](n) = \begin{cases} x(\frac{n}{M}), & \text{if } M|n \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

then

$$((\uparrow M) \circ T^k)[x](n) = \begin{cases} x(\frac{n}{M} - k), & \text{if } M|n \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

but

$$(T^k \circ (\uparrow M))[x](n) = \begin{cases} x(\frac{n-k}{M}), & \text{if } M|n \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

and since they are not equal, these operators are not time-invariant.

b

Frequency-shift operators are time-invariant only in the $\omega = 0$ case.
Consider

$$E_\omega[x](n) = e^{-2\pi i \omega n} \cdot x(n), \omega \in [0, 1] \tag{4}$$

then

$$\begin{aligned} (T^k \circ E_\omega)[x](n) &= e^{-2\pi i \omega n \cdot (-k)} \cdot x(n-k) && (5) \\ &= e^{2\pi i \omega n k} \cdot x(n-k) && (6) \end{aligned}$$

but

$$\begin{aligned} (E_\omega \circ T^k)[x](n) &= e^{-2\pi i \omega (n-k)} \cdot x(n-k) && (7) \\ &= e^{-2\pi i \omega n + 2\pi i \omega k} \cdot x(n-k) && (8) \end{aligned}$$

and since they are not equal (except when $\omega = 0$, where then they both become $1 \cdot x(n-k)$), these operators are not time invariant.