

# Foundations of Audio Signal Processing

## Assignment 7

Giulia Baldini, Luis Fernandes, Agustin Vargas Toro

December 14, 2018

### Exercise 7.1

### Exercise 7.2

a. Linearity:  $\widehat{x(\omega) + y(\omega)} = \hat{x}(\omega) + \hat{y}(\omega)$

$$\begin{aligned}\widehat{x(\omega) + y(\omega)} &= \sum_{n \in \mathbb{Z}} (x(n) + y(n)) \cdot e^{-2\pi i \omega n} \\ &= \sum_{n \in \mathbb{Z}} x(n) \cdot e^{-2\pi i \omega n} + \sum_{n \in \mathbb{Z}} y(n) \cdot e^{-2\pi i \omega n} \\ &= \sum_{n \in \mathbb{Z}} x(n) \cdot e^{-2\pi i \omega n} + \sum_{n \in \mathbb{Z}} y(n) \cdot e^{-2\pi i \omega n} = \hat{x}(\omega) + \hat{y}(\omega)\end{aligned}$$

Linearity:  $\widehat{\lambda x(\omega)} = \lambda \hat{x}(\omega)$

$$\begin{aligned}\widehat{\lambda x(\omega)} &= \sum_{n \in \mathbb{Z}} \lambda x(n) \cdot e^{-2\pi i \omega n} \\ &= \lambda \sum_{n \in \mathbb{Z}} x(n) \cdot e^{-2\pi i \omega n} = \lambda \hat{x}(\omega)\end{aligned}$$

b. Time Shift:  $\hat{x}_k(\omega) = e^{-2\pi i \omega k} \hat{x}(\omega)$ ,  $n' = n - k, n = n' + k$

$$\begin{aligned}\hat{x}_k(\omega) &= \sum_{n \in \mathbb{Z}} x_k(n) \cdot e^{-2\pi i \omega n} \\ &= \sum_{n \in \mathbb{Z}} x(n - k) \cdot e^{-2\pi i \omega n} \\ &= \sum_{n' \in \mathbb{Z}} x(n') \cdot e^{-2\pi i \omega (n' + k)} \\ &= \sum_{n' \in \mathbb{Z}} x(n') \cdot e^{-2\pi i \omega n'} \cdot e^{-2\pi i \omega k} \\ &= e^{-2\pi i \omega k} \sum_{n' \in \mathbb{Z}} x(n') \cdot e^{-2\pi i \omega n'} = e^{-2\pi i \omega k} \hat{x}(\omega)\end{aligned}$$

c. Frequency Shift:  $\widehat{x^{\omega_0}}(\omega) = \hat{x}(\omega + \omega_0)$

$$\begin{aligned}
 \widehat{x^{\omega_0}}(\omega) &= \sum_{n \in \mathbb{Z}} x^{\omega_0}(n) \cdot e^{-2\pi i \omega n} \\
 &= \sum_{n \in \mathbb{Z}} e^{-2\pi i \omega_0 n} x(n) \cdot e^{-2\pi i \omega n} \\
 &= \sum_{n \in \mathbb{Z}} x(n) \cdot e^{-2\pi i \omega_0 n - 2\pi i \omega n} \\
 &= \sum_{n \in \mathbb{Z}} x(n) \cdot e^{-2\pi i n(\omega + \omega_0)} = \hat{x}(\omega + \omega_0)
 \end{aligned}$$

d. Frequency Reversal:  $y(n) = \overline{x(n)} \implies \hat{y}(\omega) = \overline{\hat{x}(-\omega)}$

$$\begin{aligned}
 \hat{y}(\omega) &= \sum_{n \in \mathbb{Z}} y(n) \cdot e^{-2\pi i \omega n} \\
 &= \sum_{n \in \mathbb{Z}} \overline{x(n)} \cdot e^{-2\pi i \omega n} \\
 &= \overline{\sum_{n \in \mathbb{Z}} x(n) \cdot e^{2\pi i \omega n}} \\
 &= \overline{\sum_{n \in \mathbb{Z}} x(n) \cdot e^{-2\pi i (-\omega)n}} = \overline{\hat{x}(-\omega)}
 \end{aligned}$$

because  $\overline{\hat{x}(\omega)} = \sum_{n \in \mathbb{Z}} \overline{x(n)} \cdot e^{2\pi i \omega n}$ .

e. Time Reversal:  $\forall n$ , if  $y(n) = x(-n)$  then  $\hat{y}(\omega) = \hat{x}(-\omega)$ ,  $m = -n$

$$\begin{aligned}
 \hat{y}(\omega) &= \sum_{n \in \mathbb{Z}} y(n) \cdot e^{-2\pi i \omega n} \\
 &= \sum_{n \in \mathbb{Z}} x(-n) \cdot e^{-2\pi i \omega n} \\
 &= \sum_{n \in \mathbb{Z}} x(m) \cdot e^{2\pi i \omega m} \\
 &= \sum_{n \in \mathbb{Z}} x(m) \cdot e^{-2\pi i (-\omega)m} = \hat{x}(-\omega)
 \end{aligned}$$

## Exercise 7.3

a-c. The solutions can be found inside the `code` folder.