Foundations of Audio Signal Processing
Assignment 3

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Exercise 4.1

$$\begin{aligned}
\|\sum_{j=1}^{n} x_j\|^2 &= \langle\sum_{j=1}^{n} x_j, \sum_{j=1}^{n} x_j\rangle && \text{Scalar product norm definition} \\
&= \sum_{j=1}^{n}\langle x_j, x_j\rangle && \text{Linearity of scalar product} \\
&= \sum_{j=1}^{n} \|x_j\|^2 && \text{Scalar product norm definition}
\end{aligned}$$

Exercise 4.2

a. $d(x, y) = |x - y|$
Let $x = a_1 + b_1 i, y = a_2 + b_2 i, z = a_3 + b_3 i, \forall x, y, z \in \mathcal{C}$ then

$$|x - y| = \sqrt{(a_1 + b_1)^2 - (a_2 + b_2)^2} = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$$

1. $\sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2} \geq 0$ holds.
2. $\sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2} = 0$ only if $a_1 = a_2$ and $b_1 = b_2$ so when $x = y$.
3. $\sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2} = \sqrt{(a_2 - a_1)^2 + (b_2 - b_1)^2}$
4. $\sqrt{(a_1 - a_3)^2 + (b_1 - b_3)^2} \leq \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2} + \sqrt{(a_2 - a_3)^2 + (b_2 - b_3)^2}$

And according to Minkownsky Inequality

$$\left(\sum_{k=1}^{n} |x_k + y_k|^p\right)^{\frac{1}{p}} \leq \left(\sum_{k=1}^{n} |x_k|^p\right)^{\frac{1}{p}} + \left(\sum_{k=1}^{n} |y_k|^p\right)^{\frac{1}{p}}$$

If we consider the case with $p = 2$, then our inequality holds.

b. $d(x, y) = |x| \cdot |y|$
In this case the second property ($d(x, y) = 0$ iff $x = y$) does not hold because $d(x, y) = 0$ with $x = 0$ and $y = 2 - 5i$.

c. $d(x, y) = \begin{cases} 1 & x \neq y \\ 0 & \text{else} \end{cases}$

$\forall x, y, z \in \mathcal{C}$:
1. $d(x, y) \geq 0$ holds, because the possible values are $0, 1$.
2. $d(x, y) = 0$ iff $x = y$ holds for the definition of $d(x, y)$.
3. $d(x, y) = d(y, x)$ holds, because $x \neq y$ and $y \neq x$ are the same.
4. $d(x, z) \leq d(x, y) + d(y, z)$ holds because if $x \neq y \neq z$ then $1 \leq 1 + 1$. If $x = y \neq z$ then $1 \leq 0 + 1$. If $x \neq y = z$ then $1 \leq 1 + 0$. If $x = z \neq y$ then $0 \leq 1 + 1$. If $x = y = z$ then $0 \leq 0 + 0$.

Exercise 4.3

a-b. The solutions can be found inside the `code` folder.

Exercise 4.4

a-b. The solutions can be found inside the `code` folder.