Foundations of Audio Signal Processing
Assignment 3

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Exercise 4.1

$$
\begin{aligned}
\|\sum_{j=1}^{n} x_j\|^2 &= \langle\sum_{j=1}^{n} x_j, \sum_{i=1}^{n} x_i\rangle && \text{Scalar product norm definition} \\
&= \sum_{i,j=1}^{n} \langle x_j, x_i\rangle && \text{Linearity of scalar product} \\
&= \sum_{j=1}^{n} \langle x_j, x_j\rangle && \text{Ortogonality of } x_j \\
&= \sum_{j=1}^{n} \|x_j\|^2 && \text{Scalar product norm definition}
\end{aligned}
$$

Exercise 4.2

a. $d(x, y) = |x - y|$
b. $d(x, y) = |x| \cdot |y|$
In this case the second property ($d(x, y) = 0$ iff $x = y$) does not hold because $d(x, y) = 0$ with $x = 0$ and $y = 2 - 5i$.

c. $d(x, y) = \begin{cases} 1 & x \neq y \\ 0 & \text{else} \end{cases}$
$\forall x, y, z \in \mathcal{C}$:
1. $d(x, y) \geq 0$ holds, because the possible values are $0, 1$.
2. $d(x, y) = 0$ iff $x = y$ holds for the definition of $d(x, y)$.
3. $d(x, y) = d(y, x)$ holds, because $x \neq y$ and $y \neq x$ are the same.
4. $d(x, z) \leq d(x, y) + d(y, z)$ holds because if $x \neq y \neq z$ then $1 \leq 1 + 1$. If $x = y \neq z$ then $1 \leq 0 + 1$. If $x \neq y = z$ then $1 \leq 1 + 0$. If $x = z \neq y$ then $0 \leq 1 + 1$. If $x = y = z$ then $0 \leq 0 + 0$.

Exercise 4.3

a-b. The solutions can be found inside the `code` folder.

Exercise 4.4

a-b. The solutions can be found inside the `code` folder.