Foundations of Audio Signal Processing
Assignment 6

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Exercise 6.1

a. $\hat{f'}(\omega) = 2\pi i\omega \hat{f}(\omega)$
Using the integration by parts:

$$\begin{aligned} u &= e^{-2\pi i\omega t} \\ du &= -2\pi i\omega e^{-2\pi i\omega t} dt \\ v &= f(t) \\ dv &= f'(t)dt \end{aligned}$$

$$\begin{aligned} \hat{f'}(\omega) &= \int_{-\infty}^{\infty} f'(t) \cdot e^{-2\pi i\omega t} dt \\ &= \left[f(t) \cdot e^{-2\pi i\omega t}\right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} -2\pi i\omega \cdot f(t) e^{-2\pi i\omega t} dt \\ &= \left[f(t) \cdot e^{-2\pi i\omega t}\right]_{-\infty}^{\infty} + 2\pi i\omega \int_{-\infty}^{\infty} f(t) e^{-2\pi i\omega t} dt \end{aligned}$$

Since $f(t) \in L^2$ then:

$$= 2\pi i\omega \int_{-\infty}^{\infty} f(t) e^{-2\pi i\omega t} dt = 2\pi i\omega \hat{f}(\omega)$$

b. $\hat{f}'(\omega) = -2\pi i\hat{g}(\omega)$

$$\begin{aligned} \hat{f}'(\omega) &= \frac{d}{d\omega}\hat{f}(\omega) \\ &= \frac{d}{d\omega}\int_{-\infty}^{\infty} f(t) e^{-2\pi i\omega t} dt \\ &= \int_{-\infty}^{\infty} \frac{d}{d\omega}(f(t) e^{-2\pi i\omega t}) dt \\ &= \int_{-\infty}^{\infty} (-2\pi i t) f(t) e^{-2\pi i\omega t} dt \\ &= -2\pi i \int_{-\infty}^{\infty} t f(t) e^{-2\pi i\omega t} dt \\ &= -2\pi i \int_{-\infty}^{\infty} g(t) e^{-2\pi i\omega t} dt \quad = -2\pi i\hat{g}(\omega) \end{aligned}$$

c.

$$\begin{aligned}\hat{f}(\omega) &= \int_{-\infty}^{\infty} f(t)e^{-2\pi i\omega t}dt \\ &= \int_{-\infty}^{\infty} f(t)(\cos(2\pi\omega t) + i\sin(2\pi\omega t))dt \\ &= \int_{-\infty}^{\infty} f(t)(\cos(2\pi\omega t)) + \int_{-\infty}^{\infty} f(t)(i\sin(2\pi\omega t))dt\end{aligned}$$

Since $f(t)$ is real, then the first integral is real too (because there are not any imaginary components) and the second integral is imaginary.

d. Assuming that $f(\omega)$ is real and even, it holds that $f(\omega) = f(-\omega)$, and for $\hat{f}$ to be even we need to prove that $\hat{f}(\omega) = \hat{f}(-\omega)$.

$$\begin{aligned}\hat{f}(\omega) &= \int_{-\infty}^{\infty} f(t)e^{-2\pi i\omega t}dt \\ &= \int_{-\infty}^{\infty} f(-t)e^{2\pi i\omega(-t)}dt\end{aligned}$$

We now substitute $-t$ with u:

$$\begin{aligned}&= \int_{-\infty}^{\infty} f(u)e^{2\pi i\omega(u)}du \\ &= \int_{-\infty}^{\infty} f(u)e^{2\pi(-i)(-\omega)(u)}du \\ &= \int_{-\infty}^{\infty} f(u)e^{-2\pi i(-\omega)(u)}du \ = \hat{f}(-\omega)\end{aligned}$$

We now have to prove that if $f(\omega)$ is real and even, $\hat{f}$ is real.

$$\begin{aligned}\hat{f}(\omega) &= \int_{-\infty}^{\infty} f(t)e^{-2\pi i\omega t}dt \\ &= \int_{-\infty}^{\infty} f(t)(\cos(2\pi\omega t) + i\sin(2\pi\omega t))dt \\ &= \int_{-\infty}^{\infty} f(t)(\cos(2\pi\omega t))\end{aligned}$$

which is real.

Exercise 6.3

a-b. The solutions can be found inside the `code` folder.