# Foundations of Audio Signal Processing
# Assignment 7

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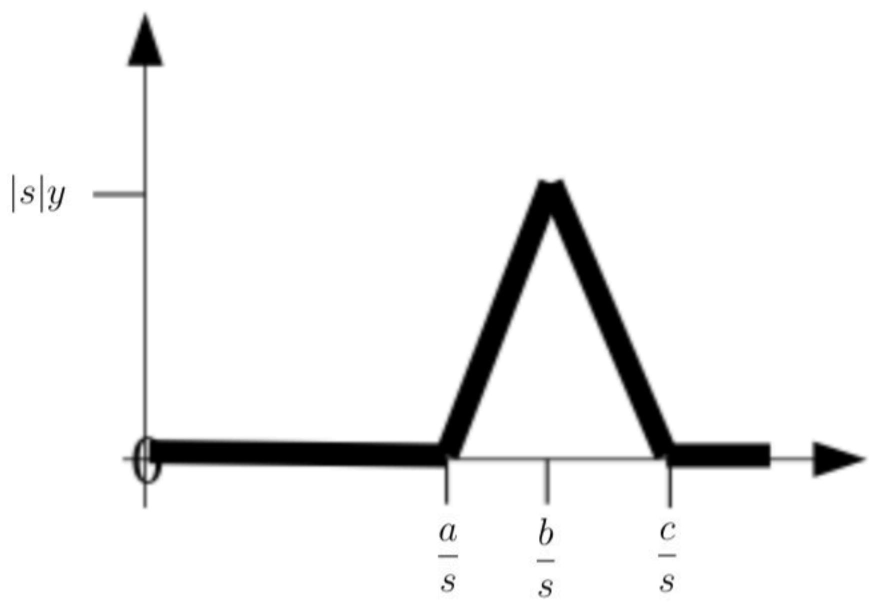
## Exercise 7.1

Knowing that one of the properties of the Fourier Transform is:
For $s \in \mathbb{R}$ the s-scaled version $t \to f(t/s)$ is also in $L^2(\mathbb{R})$ and

$$\widehat{f(./s)}(\omega) = |s| \cdot \hat{f}(\omega s)$$

We can conclude that the Fourier transform of the scaled function $t \to f(t/s)$ for $s \in \mathbb{R}_{>0}$ should be:



# Exercise 7.2

**a.** Linearity: $x(\widehat{\omega) + y}(\omega) = \hat{x}(\omega) + \hat{y}(\omega)$

$$
\begin{aligned}
x(\widehat{\omega) + y}(\omega) &= \sum_{n\in\mathbb{Z}} (x(n) + y(n)) \cdot e^{-2\pi i\omega n} \\
&= \sum_{n\in\mathbb{Z}} x(n) \cdot e^{-2\pi i\omega n} + y(n) \cdot e^{-2\pi i\omega n} \\
&= \sum_{n\in\mathbb{Z}} x(n) \cdot e^{-2\pi i\omega n} + \sum_{n\in\mathbb{Z}} y(n) \cdot e^{-2\pi i\omega n} = \hat{x}(\omega) + \hat{y}(\omega)
\end{aligned}
$$

Linearity: $\widehat{\lambda x(\omega)} = \lambda \hat{x}(\omega)$

$$
\begin{aligned}
\widehat{\lambda x(\omega)} &= \sum_{n\in\mathbb{Z}} \lambda x(n) \cdot e^{-2\pi i\omega n} \\
&= \lambda \sum_{n\in\mathbb{Z}} x(n) \cdot e^{-2\pi i\omega n} = \lambda \hat{x}(\omega)
\end{aligned}
$$

**b.** Time Shift: $\hat{x_k}(\omega) = e^{-2\pi i\omega k}\hat{x}(\omega)$, $n' = n - k, n = n' + k$

$$
\begin{aligned}
\hat{x_k}(\omega) &= \sum_{n\in\mathbb{Z}} x_k(n) \cdot e^{-2\pi i\omega n} \\
&= \sum_{n\in\mathbb{Z}} x(n-k) \cdot e^{-2\pi i\omega n} \\
&= \sum_{n'\in\mathbb{Z}} x(n') \cdot e^{-2\pi i\omega (n'+k)} \\
&= \sum_{n'\in\mathbb{Z}} x(n') \cdot e^{-2\pi i\omega n'} \cdot e^{-2\pi i\omega k} \\
&= e^{-2\pi i\omega k} \sum_{n'\in\mathbb{Z}} x(n') \cdot e^{-2\pi i\omega n'} \quad = e^{-2\pi i\omega k}\hat{x}(\omega)
\end{aligned}
$$

**c.** Frequency Shift: $\widehat{x^{\omega_0}}(\omega) = \hat{x}(\omega + \omega_0)$

$$
\begin{aligned}
\widehat{x^{\omega_0}}(\omega) &= \sum_{n\in\mathbb{Z}} x^{\omega_0}(n) \cdot e^{-2\pi i\omega n} \\
&= \sum_{n\in\mathbb{Z}} e^{-2\pi i\omega_0 n} x(n) \cdot e^{-2\pi i\omega n} \\
&= \sum_{n\in\mathbb{Z}} x(n) \cdot e^{-2\pi i\omega_0 n - 2\pi i\omega n} \\
&= \sum_{n\in\mathbb{Z}} x(n) \cdot e^{-2\pi i n(\omega + \omega_0)} \quad = \hat{x}(\omega + \omega_0)
\end{aligned}
$$

**d.** Frequency Reversal: $y(n) = \overline{x(n)} \implies \hat{y}(\omega) = \overline{\hat{x}(-\omega)}$

$$\begin{aligned}
\hat{y}(\omega) &= \sum_{n\in\mathbb{Z}} y(n) \cdot e^{-2\pi i\omega n} \\
&= \sum_{n\in\mathbb{Z}} \overline{x(n)} \cdot e^{-2\pi i\omega n} \\
&= \overline{\sum_{n\in\mathbb{Z}} x(n) \cdot e^{2\pi i\omega n}} \\
&= \overline{\sum_{n\in\mathbb{Z}} x(n) \cdot e^{-2\pi i(-\omega)n}} = \overline{\hat{x}(-\omega)}
\end{aligned}$$

**e.** Time Reversal: $\forall n,$ if $y(n) = x(-n)$ then $\hat{y}(\omega) = \hat{x}(-\omega), m = -n$

$$\begin{aligned}
\hat{y}(\omega) &= \sum_{n\in\mathbb{Z}} y(n) \cdot e^{-2\pi i\omega n} \\
&= \sum_{n\in\mathbb{Z}} x(-n) \cdot e^{-2\pi i\omega n} \\
&= \sum_{n\in\mathbb{Z}} x(m) \cdot e^{2\pi i\omega m} \\
&= \sum_{n\in\mathbb{Z}} x(m) \cdot e^{-2\pi i(-\omega)m} = \hat{x}(-\omega)
\end{aligned}$$

# Exercise 7.3

**a-b.** The solutions can be found inside the `code` folder.