Foundations of Audio Signal Processing
Assignment 5

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Exercise 5.1

a. The $L^1(\mathbb{R})$ norm of f:

$$\begin{aligned}\|f\|_1 &= \int_0^1 |f(t)|dt \\ &= \int_0^1 \frac{1}{\sqrt{t}}dt \\ &= \left|2\sqrt{t}\right|_0^1 \\ &= 2 - 0 \qquad = 2\end{aligned}$$

The $L^2(\mathbb{R})$ norm of f:

$$\begin{aligned}\|f\|_2 &= \left(\int_0^1 |f(t)|^2 dt\right)^{\frac{1}{2}} \\ &= \left(\int_0^1 \frac{1}{t}dt\right)^{\frac{1}{2}} \\ &= \left(|\log(t)|_0^1\right)^{\frac{1}{2}} \\ &= (0+\infty)^{\frac{1}{2}} \qquad = \infty\end{aligned}$$

b. The $L^1(\mathbb{R})$ norm of g:

$$\begin{aligned}\|g\|_1 &= \int_1^\infty |g(t)|dt \\ &= \int_1^\infty \frac{1}{t}dt \\ &= |\log(t)|_1^\infty \\ &= \infty - 1 \qquad = \infty\end{aligned}$$

The $L^2(\mathbb{R})$ norm of g:

$$\begin{aligned}\|g\|_2 &= \left(\int_1^\infty |g(t)|^2 dt\right)^{\frac{1}{2}} \\ &= \left(\int_1^\infty \frac{1}{t^2} dt\right)^{\frac{1}{2}} \\ &= \left(\left|-\frac{1}{t}\right|_1^\infty\right)^{\frac{1}{2}} \\ &= \left(-\frac{1}{\infty}+1\right)^{\frac{1}{2}} \qquad = 1\end{aligned}$$

Exercise 5.2

A signal is said to belong to the vector space $\ell^p(\mathbb{Z})$ if $\|x(n)\|_p < \infty$.
a. $x(n) = e^n$
The ℓ^1 norm:

$$\begin{aligned}\|x(n)\|_1 &= \sum_{n\in\mathbb{Z}}^{\infty} |x(n)| \\ &= \sum_{n\in\mathbb{Z}}^{\infty} e^n\end{aligned}$$

The ℓ^2 norm:

$$\begin{aligned}\|x(n)\|_2 &= \sum_{n\in\mathbb{Z}}^{\infty} |x(n)|^2 \\ &= \sum_{n\in\mathbb{Z}}^{\infty} e^n\end{aligned}$$

The ℓ^∞ norm:

$$\begin{aligned}\|x(n)\|_\infty &= \sup_{n\in\mathbb{Z}} |x(n)| \\ &= \sup_{n\in\mathbb{Z}} e^n\end{aligned}$$

b. $x(n) = e^{2\pi i n}$
The ℓ^1 norm:

$$\begin{aligned}\|x(n)\|_1 &= \sum_{n\in\mathbb{Z}}^{\infty} |x(n)| \\ &= \sum_{n\in\mathbb{Z}}^{\infty} e^{2\pi i n}\end{aligned}$$

The ℓ^2 norm:

$$\begin{aligned}\|x(n)\|_2 &= \sum_{n\in\mathbb{Z}}^{\infty} |x(n)|^2 \\ &= \sum_{n\in\mathbb{Z}}^{\infty} e^{2\pi i n}\end{aligned}$$

The ℓ^∞ norm:

$$\begin{aligned}\|x(n)\|_\infty &= \sup_{n\in\mathbb{Z}} |x(n)| \\ &= \sup_{n\in\mathbb{Z}} e^{2\pi i n}\end{aligned}$$

c. $x(n) = \frac{1}{\sqrt{n}}, n > 0$
The ℓ^1 norm:

$$\begin{aligned}\|x(n)\|_1 &= \sum_{n\in\mathbb{Z}}^{\infty} |x(n)| \\ &= \sum_{n\in\mathbb{Z}(0,\infty)}^{\infty} \frac{1}{\sqrt{n}}\end{aligned}$$

The ℓ^2 norm:

$$\begin{aligned}\|x(n)\|_2 &= \sum_{n\in\mathbb{Z}}^{\infty} |x(n)|^2 \\ &= \sum_{n\in\mathbb{Z}(0,\infty)}^{\infty} \frac{1}{\sqrt{n}}\end{aligned}$$

The ℓ^∞ norm:

$$\begin{aligned}\|x(n)\|_\infty &= \sup_{n\in\mathbb{Z}} |x(n)| \\ &= \sup_{n\in\mathbb{Z}(0,\infty)} \frac{1}{\sqrt{n}}\end{aligned}$$

Exercise 5.3

a-b. The solutions can be found inside the `code` folder.