Foundations of Audio Signal Processing
Assignment 7

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Exercise 7.1

Exercise 7.2

a. Linearity: $\widehat{x(\omega) + y}(\omega) = \hat{x}(\omega) + \hat{y}(\omega)$

$$\begin{aligned}
\widehat{x(\omega) + y}(\omega) &= \sum_{n \in \mathbb{Z}} (x(n) + y(n)) \cdot e^{-2\pi i \omega n} \\
&= \sum_{n \in \mathbb{Z}} x(n) \cdot e^{-2\pi i \omega n} + y(n) \cdot e^{-2\pi i \omega n} \\
&= \sum_{n \in \mathbb{Z}} x(n) \cdot e^{-2\pi i \omega n} + \sum_{n \in \mathbb{Z}} y(n) \cdot e^{-2\pi i \omega n} = \hat{x}(\omega) + \hat{y}(\omega)
\end{aligned}$$

Linearity: $\widehat{\lambda x(\omega)} = \lambda \hat{x}(\omega)$

$$\begin{aligned}
\widehat{\lambda x(\omega)} &= \sum_{n \in \mathbb{Z}} \lambda x(n) \cdot e^{-2\pi i \omega n} \\
&= \lambda \sum_{n \in \mathbb{Z}} x(n) \cdot e^{-2\pi i \omega n} = \lambda \hat{x}(\omega)
\end{aligned}$$

b. Time Shift: $\hat{x_k}(\omega) = e^{-2\pi i \omega k} \hat{x}(\omega)$, $n' = n - k, n = n' + k$

$$\begin{aligned}
\hat{x_k}(\omega) &= \sum_{n \in \mathbb{Z}} x_k(n) \cdot e^{-2\pi i \omega n} \\
&= \sum_{n \in \mathbb{Z}} x(n-k) \cdot e^{-2\pi i \omega n} \\
&= \sum_{n' \in \mathbb{Z}} x(n') \cdot e^{-2\pi i \omega (n'+k)} \\
&= \sum_{n' \in \mathbb{Z}} x(n') \cdot e^{-2\pi i \omega n'} \cdot e^{-2\pi i \omega k} \\
&= e^{-2\pi i \omega k} \sum_{n' \in \mathbb{Z}} x(n') \cdot e^{-2\pi i \omega n'} = e^{-2\pi i \omega k} \hat{x}(\omega)
\end{aligned}$$

c. Frequency Shift: $\widehat{x^{\omega_0}}(\omega) = \hat{x}(\omega + \omega_0)$

$$\begin{aligned}
\widehat{x^{\omega_0}}(\omega) &= \sum_{n\in\mathbb{Z}} x^{\omega_0}(n) \cdot e^{-2\pi i\omega n} \\
&= \sum_{n\in\mathbb{Z}} e^{-2\pi i\omega_0 n} x(n) \cdot e^{-2\pi i\omega n} \\
&= \sum_{n\in\mathbb{Z}} x(n) \cdot e^{-2\pi i\omega_0 n - 2\pi i\omega n} \\
&= \sum_{n\in\mathbb{Z}} x(n) \cdot e^{-2\pi i n(\omega+\omega_0)} \qquad = \hat{x}(\omega + \omega_0)
\end{aligned}$$

d. Frequency Reversal: $y(n) = \overline{x(n)} \implies \hat{y}(\omega) = \overline{\hat{x}(-\omega)}$

$$\begin{aligned}
\hat{y}(\omega) &= \sum_{n\in\mathbb{Z}} y(n) \cdot e^{-2\pi i\omega n} \\
&= \sum_{n\in\mathbb{Z}} \overline{x(n)} \cdot e^{-2\pi i\omega n} \\
&= \sum_{n\in\mathbb{Z}} \overline{x(n)} \cdot e^{2\pi i(-\omega)n} \\
&= \overline{\hat{x}(-\omega)}
\end{aligned}$$

because $\overline{\hat{x}(\omega)} = \sum_{n\in\mathbb{Z}} \overline{x(n)} \cdot e^{2\pi i\omega n}$.

e. Time Reversal: $\forall n,$ if $y(n) = x(-n)$ then $\hat{y}(\omega) = \hat{x}(-\omega), m = -n$

$$\begin{aligned}
\hat{y}(\omega) &= \sum_{n\in\mathbb{Z}} y(n) \cdot e^{-2\pi i\omega n} \\
&= \sum_{n\in\mathbb{Z}} x(-n) \cdot e^{-2\pi i\omega n} \\
&= \sum_{n\in\mathbb{Z}} x(m) \cdot e^{2\pi i\omega m} \\
&= \sum_{n\in\mathbb{Z}} x(m) \cdot e^{-2\pi i(-\omega)m} = \hat{x}(-\omega)
\end{aligned}$$

Exercise 7.3

a-c. The solutions can be found inside the `code` folder.