

Foundations of Audio Signal Processing

Assignment 3

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Exercise 3.1

a.

$$4 + i4\sqrt{3} = 1 + i\sqrt{3}$$

$$a = 1, b = \sqrt{3}$$

$$r = \sqrt{1 + 3} = 2$$

$$\cos \phi = \frac{1}{2}$$

$$\sin \phi = \frac{\sqrt{3}}{2}$$

$$\phi = \frac{\pi}{3}$$

b.

$$\begin{aligned} (-1 + i\sqrt{3})^4 &= (1 - i2\sqrt{3} - 3)^2 \\ &= (-2 - i2\sqrt{3})^2 \\ &= 4(1 + i2\sqrt{3} - 3) \\ &= 4(-2 + i2\sqrt{3}) = -8 + i8\sqrt{3} \end{aligned}$$

$$a = -8, b = 8\sqrt{3}$$

$$r = \sqrt{64 + 192} = 16$$

$$\cos \phi = -\frac{8}{16} = -\frac{1}{2}$$

$$\sin \phi = \frac{8\sqrt{3}}{16} = \frac{\sqrt{3}}{2}$$

$$\phi = \frac{2\pi}{3}$$

c. Here we use the solution from exercise b to solve the numerator.

$$\begin{aligned}
 \frac{(-1 + i\sqrt{3})^4}{4 + i4\sqrt{3}} &= \frac{-8 + i8\sqrt{3}}{4 + i4\sqrt{3}} \\
 &= \frac{-2 + i2\sqrt{3}}{1 + i\sqrt{3}} \\
 &= \frac{(-2 + i2\sqrt{3})(1 - i\sqrt{3})}{(1 + i\sqrt{3})(1 - i\sqrt{3})} \\
 &= \frac{-2 + i2\sqrt{3} + i2\sqrt{3} + 6}{(1 + 3)} \\
 &= \frac{4 + i4\sqrt{3}}{4} = 1 + i\sqrt{3}
 \end{aligned}$$

$a = 1$, $b = \sqrt{3}$, which are the same as in exercise a, and thus lead to the same solution.

d.

$$\begin{aligned}
 2e^{\frac{\pi}{2}i}(1 + i) &= 2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)(1 + i) \\
 &= 2(0 + i)(1 + i) = -2 + 2i
 \end{aligned}$$

$$a = -2, b = 2$$

$$\begin{aligned}
 r &= \sqrt{4 + 4} = 2\sqrt{2} \\
 \cos \phi &= -\frac{2}{2\sqrt{2}} = -\frac{\sqrt{2}}{2} \\
 \sin \phi &= \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2} \\
 \phi &= \frac{3\pi}{4}
 \end{aligned}$$

Exercise 3.2

a.

Exercise 3.3

a.