Foundations of Audio Signal Processing
Assignment 4

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November 23, 2018

Exercise 4.1

$$\begin{aligned}
\|\sum_{j=1}^{n} x_j\|^2 &= \langle\sum_{j=1}^{n} x_j, \sum_{i=1}^{n} x_i\rangle && \text{Scalar product norm definition} \\
&= \sum_{i,j=1}^{n} \langle x_j, x_i\rangle && \text{Linearity of scalar product} \\
&= \sum_{j=1}^{n} \langle x_j, x_j\rangle && \text{Ortogonality of } x_j \\
&= \sum_{j=1}^{n} \|x_j\|^2 && \text{Scalar product norm definition}
\end{aligned}$$

Exercise 4.2

a. $d(x,y) = |x-y|$
Let $x = a_1 + b_1 i, y = a_2 + b_2 i, z = a_3 + b_3 i, \forall x, y, z \in \mathcal{C}$ then

$$|x-y| = \sqrt{(a_1+b_1)^2 - (a_2+b_2)^2} = \sqrt{(a_1-a_2)^2 + (b_1-b_2)^2}$$

1. $\sqrt{(a_1-a_2)^2 + (b_1-b_2)^2} \geq 0$ holds, as the the square root of a positive number is always a positive number (resp. $\sqrt{0} = 0$)
2. $\sqrt{(a_1-a_2)^2 + (b_1-b_2)^2} = 0$ only if $a_1 = a_2$ and $b_1 = b_2$, which is the case when $x = y$.
3. $\sqrt{(a_1-a_2)^2 + (b_1-b_2)^2} = \sqrt{(a_2-a_1)^2 + (b_2-b_1)^2}$
4. $d(x,y) + d(y,z) = \sqrt{(a_1-a_2)^2 + (b_1-b_2)^2} + \sqrt{(a_2-a_3)^2 + (b_2-b_3)^2}$.
According to Minkowsky Inequality:

$$\left(\sum_{k=1}^{n} |x_k + y_k|^p\right)^{\frac{1}{p}} \leq \left(\sum_{k=1}^{n} |x_k|^p\right)^{\frac{1}{p}} + \left(\sum_{k=1}^{n} |y_k|^p\right)^{\frac{1}{p}}$$

If we consider the case with $p = 2$ and $n = 2$ (in our case $x_1 = (a_1 - a_2)$ and $x_2 = (b_1 - b_2)$ and similarly are y_1 and y_2 defined), then:

$$d(x,y) + d(y,z) \geq \sqrt{|x_1+y_1|^2 + |x_2+y_2|^2}$$

Since in our case $x_k, y_k \in \mathbb{R}$:

$$d(x,y) + d(y,z) \geq \sqrt{(a_1 - a_2 + a_2 - a_3)^2 + (b_1 - b_2 + b_2 - b_3)^2}$$

$$d(x,y) + d(y,z) \geq \sqrt{(a_1 - a_3)^2 + (b_1 - b_3)^2}$$

$$d(x,y) + d(y,z) \geq d(x,z)$$

b. $d(x,y) = |x| \cdot |y|$
In this case the second property ($d(x,y) = 0$ iff $x = y$) does not hold because $d(x,y) = 0$ with $x = 0$ and $y = 2 - 5i$.

c. $d(x,y) = \begin{cases} 1 & x \neq y \\ 0 & \text{else} \end{cases}$
$\forall x, y, z \in \mathcal{C}$:
1. $d(x,y) \geq 0$ holds, because the possible values are $0, 1$.
2. $d(x,y) = 0$ iff $x = y$ holds for the definition of $d(x,y)$.
3. $d(x,y) = d(y,x)$ holds, because $x \neq y$ and $y \neq x$ are the same.
4. $d(x,z) \leq d(x,y) + d(y,z)$ holds because if $x \neq y \neq z$ then $1 \leq 1+1$. If $x = y \neq z$ then $1 \leq 0 + 1$. If $x \neq y = z$ then $1 \leq 1 + 0$. If $x = z \neq y$ then $0 \leq 1 + 1$. If $x = y = z$ then $0 \leq 0 + 0$.

Exercise 4.3

a-b. The solutions can be found inside the `code` folder.

Exercise 4.4

a-b. The solutions can be found inside the `code` folder.