

# Foundations of Audio Signal Processing

## Assignment 2

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### Exercise 2.1

a.

$$\begin{aligned}2e^{\frac{\pi}{2}i}(1+i) &= 2(\cos(\frac{\pi}{2}) + i\sin(\frac{\pi}{2}))(1+i) \\&= 2i(1+i) \\&= -2 + 2i \\a &= -2 \\b &= 2 \\r &= \sqrt{(-2)^2 + (2)^2} \\&= 2\sqrt{2}\end{aligned}$$

Knowing that  $a = \cos(\phi)$  and  $b = \sin(\phi)$ :

$$\begin{aligned}\cos(\phi) &= \frac{-2}{2\sqrt{2}} = -\frac{\sqrt{2}}{2} \\ \sin(\phi) &= \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \phi &= \frac{3}{4}\pi \\ z &= 2\sqrt{2}e^{\frac{3}{4}\pi i}\end{aligned}$$

b. Considering that  $z = re^{\phi i}$ ,  $\bar{z} = re^{-\phi i}$  and  $|z| = r$

$$\begin{aligned}z\bar{z} &= re^{\phi i}re^{-\phi i} \\&= r^2e^0 = |z|^2\end{aligned}$$

c.

$$\begin{aligned}\frac{1}{2i}(e^{ia} - e^{-ia}) &= \frac{1}{2i}(\cos(a) + i\sin(a) - (\cos(-a) + i\sin(-a))) \\&= \frac{1}{2i}(\cos(a) + i\sin(a) - \cos(a) + i\sin(a)) \\&= \frac{1}{2i}(2i\sin(a)) = \sin(a)\end{aligned}$$

## Exercise 2.2

a. For  $n = 4$ :

$$\Omega_4^1 = \cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right) = i$$

$$\Omega_4^2 = \cos(\pi) + i \sin(\pi) = -1$$

$$\Omega_4^3 = \cos\left(3\frac{\pi}{2}\right) + i \sin\left(3\frac{\pi}{2}\right) = -i$$

$$\Omega_4^4 = 1$$

So the fourth roots of unity are  $\{1, -1, i, -i\}$  and, since 4 is the smallest integer for which  $\Omega = \{i, -i\}$ , then these are its primitives.

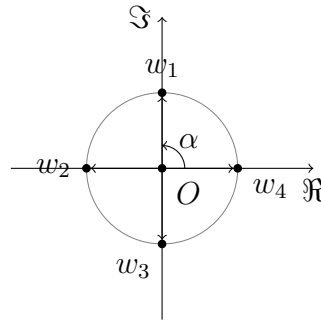


Figure 1: Illustration of all fourth roots of unity.

For  $n = 6$ :

$$\Omega_6^1 = \cos\left(\frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3}\right) = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$\Omega_6^2 = \cos\left(2\frac{\pi}{3}\right) + i \sin\left(2\frac{\pi}{3}\right) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$\Omega_6^3 = \cos(\pi) + i \sin(\pi) = -1$$

$$\Omega_6^4 = \cos\left(4\frac{\pi}{3}\right) + i \sin\left(4\frac{\pi}{3}\right) = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$\Omega_6^5 = \cos\left(5\frac{\pi}{3}\right) + i \sin\left(5\frac{\pi}{3}\right) = \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$\Omega_6^6 = 1$$

So the sixth roots of unity are  $\{1, -1, \frac{1}{2} + \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i\}$  and, since 4 is the smallest integer for which  $\Omega = \{-\frac{1}{2} + \frac{\sqrt{3}}{2}i, \frac{1}{2} + \frac{\sqrt{3}}{2}i\}$ , then these are its primitives.

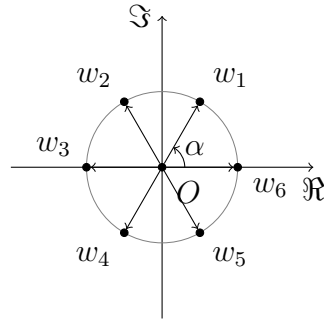


Figure 2: Illustration of all sixth roots of unity.

b. By definition of roots of unity

$$\sum_{k=0}^{n-1} \Omega_k = \omega^0 + \omega^1 + \dots + \omega^{n-1}$$

where  $\omega^k = e^{\frac{2k\pi i}{n}}$ . Since  $\omega^0 = 1$ , we can write the definition as a finite geometric series

$$\sum_{k=0}^{n-1} e^{\frac{2k\pi i}{n}} = \frac{1 - e^{\frac{2n\pi i}{n}}}{1 - e^{\frac{2\pi i}{n}}}$$

since  $e^{\frac{2n\pi i}{n}} = 1$ , then

$$\sum_{k=0}^{n-1} \Omega_k = \frac{0}{1 - e^{\frac{2\pi i}{n}}} = 0$$

## Exercise 2.3

a-c. The solutions can be found inside the `code` folder.

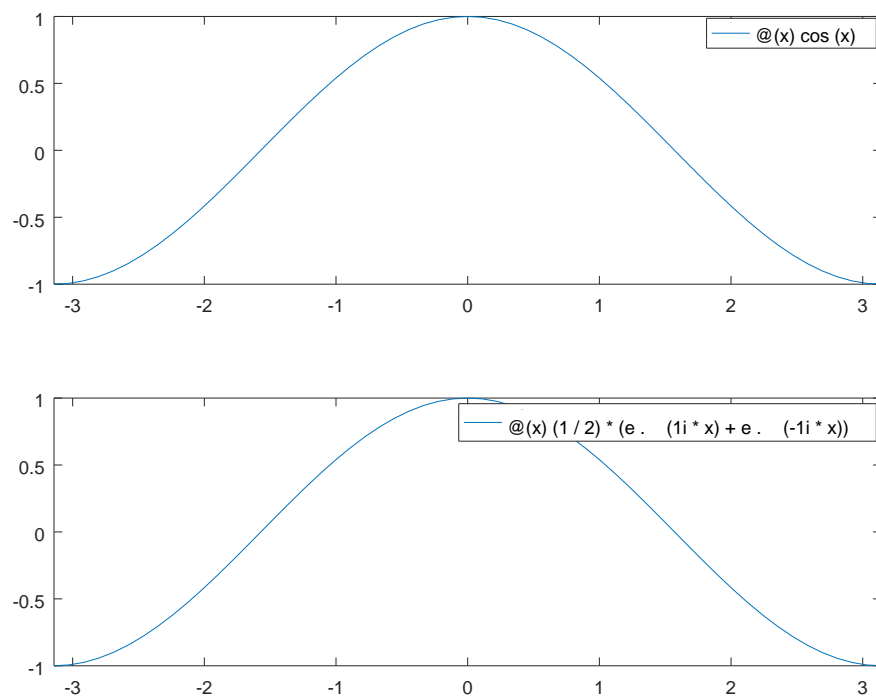


Figure 3: Visualization of the statement  $\cos(\alpha) = \frac{1}{2} \cdot (e^{i\alpha} + e^{-i\alpha})$