

Foundations of Audio Signal Processing

Assignment 4

Giulia Baldini, Luis Fernandes, Agustin Vargas Toro

November 17, 2018

Exercise 4.1

$$\begin{aligned}\left\|\sum_{j=1}^n x_j\right\|^2 &= \left\langle \sum_{j=1}^n x_j, \sum_{i=1}^n x_i \right\rangle && \text{Scalar product norm definition} \\ &= \sum_{i,j=1}^n \langle x_j, x_i \rangle && \text{Linearity of scalar product} \\ &= \sum_{j=1}^n \langle x_j, x_j \rangle && \text{Orthogonality of } x_j \\ &= \sum_{j=1}^n \|x_j\|^2 && \text{Scalar product norm definition}\end{aligned}$$

Exercise 4.2

a. $d(x, y) = |x - y|$

Let $x = a_1 + b_1i, y = a_2 + b_2i, z = a_3 + b_3i, \forall x, y, z \in \mathcal{C}$ then

$$|x - y| = \sqrt{(a_1 + b_1)^2 - (a_2 + b_2)^2} = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$$

1. $\sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2} \geq 0$ holds, as the the square root of a positive number is always a positive number (resp. $\sqrt{0} = 0$)
2. $\sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2} = 0$ only if $a_1 = a_2$ and $b_1 = b_2$, which is the case when $x = y$.
3. $\sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2} = \sqrt{(a_2 - a_1)^2 + (b_2 - b_1)^2}$
4. $d(x, y) + d(y, z) = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2} + \sqrt{(a_2 - a_3)^2 + (b_2 - b_3)^2}$.

According to Minkowsky Inequality:

$$\left(\sum_{k=1}^n |x_k + y_k|^p\right)^{\frac{1}{p}} \leq \left(\sum_{k=1}^n |x_k|^p\right)^{\frac{1}{p}} + \left(\sum_{k=1}^n |y_k|^p\right)^{\frac{1}{p}}$$

If we consider the case with $p = 2$ and $n = 2$ (in our case $x_1 = (a_1 - a_2)$ and $x_2 = (b_1 - b_2)$ and similarly are y_1 and y_2 defined), then:

$$d(x, y) + d(y, z) \geq \sqrt{|x_1 + y_1|^2 + |x_2 + y_2|^2}$$

Since in our case $x_k, y_k \in \mathbb{R}$:

$$d(x, y) + d(y, z) \geq \sqrt{(a_1 - a_2 + a_2 - a_3)^2 + (b_1 - b_2 + b_2 - b_3)^2}$$

$$d(x, y) + d(y, z) \geq \sqrt{(a_1 - a_3)^2 + (b_1 - b_3)^2}$$

$$d(x, y) + d(y, z) \geq d(x, z)$$

b. $d(x, y) = |x| \cdot |y|$

In this case the second property ($d(x, y) = 0$ iff $x = y$) does not hold because $d(x, y) = 0$ with $x = 0$ and $y = 2 - 5i$.

c.
$$d(x, y) = \begin{cases} 1 & x \neq y \\ 0 & \text{else} \end{cases}$$

$\forall x, y, z \in \mathcal{C} :$

1. $d(x, y) \geq 0$ holds, because the possible values are 0, 1.
2. $d(x, y) = 0$ iff $x = y$ holds for the definition of $d(x, y)$.
3. $d(x, y) = d(y, x)$ holds, because $x \neq y$ and $y \neq x$ are the same.
4. $d(x, z) \leq d(x, y) + d(y, z)$ holds because if $x \neq y \neq z$ then $1 \leq 1 + 1$. If $x = y \neq z$ then $1 \leq 0 + 1$. If $x \neq y = z$ then $1 \leq 1 + 0$. If $x = z \neq y$ then $0 \leq 1 + 1$. If $x = y = z$ then $0 \leq 0 + 0$.

Exercise 4.3

a-b. The solutions can be found inside the `code` folder.

Exercise 4.4

a-b. The solutions can be found inside the `code` folder.