GROUP 22 SII FINAL PROJECT

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Contents

1 Original text of the project

Consider a simplified insurance company whose assets and liabilities sides are characterized as follows:

ASSETS

- there is a single fund all made of equity, $F_t = S_t$
- at the beginning (t=0) the value of the fund is equal to the invested premium $F_0 = C_0 = 100,000$
- equity features
  - listed in the regulated markets in the EEA
  - No dividend yield
  - to be simulated with a Risk Neutral GBM ($sigma = 25\%$) and a time varying instantaneous risk free rate r

LIABILITIES

- contract terms
  - benefits
    * in case of lapse, the beneficiary gets the value of the fund at the time of lapse, without penalties applied
    * in case of death, the beneficiary gets the maximum between the 110% of the invested premium and the value of the fund
    * when a benefit is paid to the beneficiary, a fixed cost of 20 euro is applied reducing the paid benefit
  - others
    * Regular Deduction, RD of 2.00%
    * Commissions to the distribution channels, COMM (or trailing) of 1.40%
    * No External Fees
- model points
  - just 1 model point
  - male with insured aged $x = 60$ at the beginning of the contract
- operating assumptions

- mortality: rates derived from the life table SI2021 ($https : //demo.istat.it/index_e.php$)
- lapse: flat annual rates $l_t = 15\%$
- expenses: constant unitary (i.e. per policy) cost of 50 euros per year, that grows following the inflation pattern

- economic assumption
  - risk free: rate r derived from the yield curve (EIOPA IT without VA 31.03.22), supposing linear interpolation of the zero rates and using the formula $DF_{t+dt} = DF_t * exp[-r_t * dt]$
  - o inflation: flat annual rate of 2%

Other specifications:

- time horizon for the projection: 50 years. In case there still was an outstanding portfolio in $T = 50$, let all the people leave the contract with a massive surrender
- the interest rates dynamic is deterministic, while the equity one is stochastic

QUESTIONS

1. code a Matlab script to compute the Basic Solvency Capital Requirement via Standard Formula and provide comments on the results obtained. The risks to be considered are:
   - Market Interest
   - Market equity
   - Market spread
   - Life mortality
   - Life lapse
   - Life cat
   - Expense
2. Calculate the Macaulay BEL duration in all the cases and provide comments on the results obtained
3. Split the BEL value into its main PV components: premiums (=0), death benefits, lapse benefits, expenses, and commissions
4. Replicate the same calculations in an Excel spread sheet using a deterministic projection. Do the results differ from 1? If so, what is the reason behind?

5. Open questions:

    - what happens to the asset and liabilities when the risk free rate increases/decreases with a parallel shift of, say, 100bps? Describe the effects for all the BEL components
    - what happens to the liabilities if the insured age increases? What if there were two model points, one male and one female?

2 Summary tables

In this section we show the results obtained with Matlab via Standard Formula.

	Assets	Liabilities	BoF	DBoF	Dur L
BASE	100 000	98 320.89	1679.11	0	5.7074
IR up	100 000	97 553.05	2446.95	0	5.6436
IR down	100 000	98 524.45	1475.55	203.55	5.7324
Equity	61 000	63 374.47	-2374.47	4053.57	5.8836
Mortality	100 000	98 506.04	1493.96	185.151	5.6666
Lapse down	100 000	99 082.47	917.53	761.5798	9.3624
Lapse up	100 000	98 485.71	1514.29	164.821	4.0482
Lapse mass	100 000	98 896.41	1103.59	575.515	3.4777
Expenses	100 000	98 371.41	1628.59	50.5171	5.7086
CAT	100 000	98 340.99	1659.01	20.0963	5.6993

SCR IR	SCR Equity	SCR Mortality	SCR Lapse	SCR Expenses	SCR CAT
203.5540	4053.57	185.1505	761.5798	50.5171	20.0963

$$\text{BSCR} = 4435.2144 \tag{1}$$

3 Formulas adopted for the calculations

3.1 Assets

First of all, for the computation of the Basic Solvency Capital Requirement, is necessary to calculate the value of the fund for each year $F_t = S_t$. To do so we start with the portfolio of assets, which in our case is composed by a stock. The stock has initial value $S_0 = 100K$ evolves like a Geometric Brownian Motion with $sigma = 0.25$. The risk free rates are taken from the yield curve of EIOPA (31.03.22) without Volatility Adjustment. From the rates r(t) we can obtain the discounting factor as $exp(-r(t) \cdot t)$.
The stock dynamic is stochastic and evaluated with classical $Black\&Scholes$ formula with the simulation of a Brownian Motion $B'_t(w)$, taking into account the regulatory reduction RD:

$$S_t = (1-RD)\cdot S_{t-1}\cdot exp[fwd(t_{i-1}, t_i)\cdot(t_i - t_{i-1}) - \frac{\sigma^2}{2}\cdot(t_i - t_{i-1}) + \sigma\cdot(B'_{t_i}(w) - B'_{t_{i-1}}(w))] \quad (2)$$

3.2 Liabilities

To complete the evaluation of the fund we need to compute the liabilities. In our case the liabilities are for a contract term with term $T = 50$ years, for a male aged 60. The contract consists on a whole life policy in which the single premium has been already cashed in. The benefits given in case of lapse is the value of the fund at time of lapse (without any penalties) and in case of death is the maximum value between the 110% of the invested premium and the value of the fund at time of death. Every time that a benefits is paid a fixed cost of 20 euro is applied reducing the paid benefit
Moreover we have to consider No External Fees, an RD of 2.00% and a COMM of 1.40%. An important step is the computation of the probability of survival until the year i=1,2... that is

$$p_{surv_i} = (1 - qx_i) \cdot (1 - lx_i) \quad (3)$$

where
q_i is the mortality rate derived from the life table SI2021 (ISTAT website)
l_x is the lapse rate defined in the text

For the computation of liabilities we need then to estimate the cash flows during the years. In case of lapse we assume that one can lapse only at the end of the year. In this we don't repeat to pay the benefit to one which lapse and then die in the same year:

- $Lapse_{bel}(i) = ((F_{i+1} - comm_{if\_benefit}) \cdot lx_i \cdot (1 - qx_i)) \cdot P_{surv_i}$
- $Death_{bel}(i) = ((max(1.1 * S0, F_{i+1}) - comm_{if\_benefit}) \cdot qx_i) \cdot P_{surv_i}$
- $Expen_{bel}(i) = expenses_i) \cdot P_{surv_i}$
- $Commissions_{bel}(i) = (F_{i+1}/factor_{RD} \cdot COMM) \cdot P_{surv_i}$

So the cash flows can be written as:

$$CF_t = Lapse_{bel}(i) + Death_{bel}(i) + Expen_{bel}(i) + Commissions_{bel}(i) \tag{4}$$

Then we discount it to have the present value of them:

$$Cashflow = CF_t \cdot exp(-r(t) \cdot t) \tag{5}$$

Where 20€is the commission that the insurance company takes when the benefits are paid (both in the case of lapse and death) Finally, we can evaluate liabilities as the sum of all the cash flows of the year.
For the computation of the duration of the liabilities we use the Macaulay Duration, that is given by the formula below

$$Duration = \frac{\sum_{i=1}^{n} t_i \cdot CF_i}{\sum_{i=1}^{n} CF - i} = \frac{\sum_{i=1}^{n} t_i \cdot CF_i}{Liabilities} (6)$$

3.3 Basic Own Funds

Basic Own Funds consist of the excess of assets over liabilities:

$$BOF = Assets - Liabilities \tag{7}$$

In our project we consider different types of risks and then we evaluate the change of BOF due to the stressed scenario as

$$\Delta BOF = BOF_{based} - BOF_{stressed} \tag{8}$$

As we can see from the formula ΔBOF is defined to be positive if we have a loss in BOF in the stressed scenario. However, if we have an increase in the BOF of the stressed scenario which leads to a negative ΔBOF, it means that there are no risks for the undertaking so we floored it to 0.

4 Basic Solvency Capital Requirement Evaluation

The Basic Solvency Capital Requirement evaluates a risk-based capital to face risks coming from different areas. In our stressed scenarios we take into account these risks:

- Market Risks: originate from the level and the volatility of the financial instruments the company has
  - Interest rate risk
  - Equity risk
- Life Underwriting Risks: come from uncontrollable factors
  - Mortality risk
  - Lapse risk
  - Catastrophe (CAT) risk
  - Expenses risk

4.1 Interest Rate Risk

We have to consider Interest Rate Risk because all the instruments inside the Portfolio are sensitive to variations of the term structure of Risk Free Rates. We examine two possible scenarios:

- Decrease in the term structure each year. We apply the new rates only in case they are positive, according to EIOPA:
  $IR^{dw} = IR^{base} - stress \times IR^{base}$
- Increase in the term structure each year:
  $IR^{up} = IR^{base} + MIN(1\%, stress \times |IR^{base}|)$

In the first scenario, the one with the interest down, EIOPA does not apply the downward shift for all the rates. It begins to decrease the rates only when they become positive. So, this shift provides in our cases a result which is not as evident as will be in the second scenario. As in the usual financial theory, in our case a decrease in rates causes an initial increase in the value of assets due to the presence of bond. At the beginning the stock starts decreasing for same years and then starts increasing but the general trend stays below the base scenario. As a consequence, the value of the portfolio decreases with respect to the base scenario. Since the value of the stock is now lower, we expect that the liability increases, and our results confirm that. A totally inverted situation is in the case of the second

scenario where the upward shift shows evident effects. The stock value increases each year. This effect for the company is evident, we expect that the liability decreases, and our results confirm that.
Then, the SCR_{IR} is evaluated as:

$$SCR_{IR} = MAX(SCR_{IR^{up}}, SCR_{IR^{down}}) \tag{9}$$

4.2 Equity Risk

We consider the Equity Risk because the equities are subject to changes of prices due to volatility in the market. All the equities come from countries members of the European Economic Area (EEA) and so we have to apply the shock scenario Type 1. This consists of a decrease of 39% of the total amount: $E^{shock} = (1 - 0.39) \cdot E$
The shock scenario results in a determinant decrease of value of the assets which is a critical event for the company especially when the capital is guaranteed, as fro the case of death where it is guaranted the 110% of S_0 (100K€). Indeed, we can notice that, if company has to guarantee the nominal value, assets are not enough to cover losses because liabilities remain similar to base scenario. In conclusion dBOF becomes positive, and this risk is our principal component of the value of the BSCR, as we can see from the final results.

4.3 Mortality Risk

We consider Life Mortality Risk to cover possible changes of mortality tables and consequently in the mortality rates. Actually, extraordinary events can cause permanent increase in these rates and cause problems to insurances. To study this possibility is applied a permanent increase of 15% on the mortality rates $q_{mor} = min(q \cdot 1.15, 1)$.
This increase causes a lasting effect on the cash flows and consequently on the liabilities, these are reduced because the endowments are more expensive to payback. This change of rates increase a little the liabilities while assets stays constant.

4.4 Lapse Risk

We take into account in our computation the lapse risk. This risk concerns the loss of the adverse change in liabilities that occurs when either one party fails to act on its obligations, or it is allowed by the contract.
To evaluate this risk, we have three stressed scenarios:

- **Lapse up shock**: all the computations are done taking into consideration the $Lapse_{up} = min(1.5 \times Lapse, 1)$
- **Lapse down shock**: all the computations are done taking into consideration the $Lapse_{down} = max(0.5 \times Lapse, Lapse - 0.2)$
- **Lapse mass shock**: all the computations are done taking into consideration the Lapse with a discontinuance of 40% in the first year

Then the SCR_{lapse} in computed as

$$SCR_{lapse} = MAX(SCR_{lapseup}, SCR_{lapsedown}, SCR_{lapsemass}) \tag{10}$$

The effect of these shocks is applicable to liabilities since the probability of entering in the new year changes because of lapse rates. We begin to analyze the simplest cases which are Lapse UP and Lapse MASS. If in Lapse UP rates increase, liabilities increase as well. This happens because cash flows are higher at the beginning because of money to give back, but then at the end of the contract, at year 50, very few people are still in it. It also the increase in lapse diminishes the probability of entering in the next year. The same happens with Lapse MASS and again dBOF is positive. Also with Lapse DOWN, even if a lot of people stay in the contract until the end, liabilities are higher. This fact is explainable as before, the endowments are very expensive to pay back and these augment the liabilities making a dBOF positive.

An important factor to analyze in this evaluated lapse risk is the duration of the liabilities. Indeed, while in the other evaluated risks the duration of liabilities does not change significantly from the base scenario, in this case it does. As we can see from the tables the duration of liabilities is much higher in the case of lapse down, lower in the lapse up case and lower in the lapse mass. This effect can be easily explained by the change in the cash flows mentioned above. As a trivial example, if we look at the lapse down case, as the probability of lapse is lower than the base scenario, it is clear that policies will last longer.

4.5 CAT Risk

The last risk to consider is the one of a catastrophe during the year. This risk consists in an augment of the mortality rates of 1.5 per mille for just the first year $q_{CAT}(0) = q(0) + 0,0015$. As in the case of mortality risk this augment in rates increase little the liabilities so there are little changes in dBOF. Futhermore, the variation is so imperceptible that the overall values are identical to the base scenario.

4.6 Expenses Risk

The expenses risk impact very little on the computaation of the BSCR. This fact is due to the very little influence of the expenses on the total value of the liabilities. In fact the fund values stay around 100K€, but the expenses are in the order of 50€increasing with inflation. Even after the shock of the stress test, the maximum value of the expenses is below 300 €, almost nothing compared to the stock values. In fact both the Liabilities and the durantion does not change very much.

4.7 BSCR evaluation

In the evaluation of the BSCR all the SCR computed above are aggregated through linear correlations to take into account the diversification and the mitigation effect.

For the **Market Risk** we aggregate the risks with the formula:

$$SCR_{market} = \sqrt{\sum_{i,j} Corr_{i,j} \times SCR_i \times SCR_j} \tag{11}$$

where the correlation matrix, as we consider only the interest risk and the equity risk, is given by

	Interest	**Equity**
Interest	1.00	A
Equity	A	1.00

where $A = 0.00$ if exposed to IR_{up} and $A = 0.50$ if exposed to IR_{down}

For the **Life Underwriting Risk** we aggregate the risks with the formula:

$$SCR_{life} = \sqrt{\sum_{i,j} Corr_{i,j} \times SCR_i \times SCR_j} \tag{12}$$

where the correlation matrix, as we consider only the mortality risk,the lapse risk,the expenses risk and the CAT risk, is given by

Finally, for the computation of the **BSCR** we aggregate the modules above with the formula:

$$BSCR = \sqrt{\sum_{i,j} Corr_{i,j} \times SCR_i \times SCR_j} \tag{13}$$

where the correlation matrix is given by

	Mortality	**Lapse**	**Expenses**	**CAT**
Mortality	1.00	0.00	0.25	0.25
Lapse	0.00	1.00	0.50	0.25
Expenses	0.25	0.50	1.00	0.25
CAT	0.25	0.25	0.25	1.00

	Market	**Life**
Market	1.00	0.25
Life	0.25	1.00

5 Deterministic calculations and comments on the results

In this section we illustrate the same calculations above with a deterministic projection using an Excel spread sheet.

	Assets	Liabilities	BoF	D BoF	Dur L
BASE	100 000	98 297.0199	1702.980	0	5.7066
IR up	100 000	97 581.6669	2418.333	0	5.6453
IR down	100 000	98 581.366	1418.633	284.347	5.733
Equity	61 000	63 379.83	-2 379.83	4 082.81	5.88
Mortality	100 000	98 483.34	1516.66	186.32	5.66
Lapse down	100 000	99 065.423	934.58	768.40	9.36
Lapse up	100 000	98 467.02	1 532.98	170.00	4.0476
Lapse mass	100 000	98 871.01	1 128.99	573.987	3.4773
Expenses	100 000	98 347.56	1652.44	50.54	5.71
CAT	100 000	98 317.15	1 682.85	20.13	5.6985

SCR IR	SCR Equity	SCR Mortality	SCR Lapse	SCR Expenses	SCR CAT
284.34	4 082.81	186.32	768.40	50.54	20.13

$$\textbf{BSCR} = 4372.8353$$

(14)

As we can see from the tables, all the values seem pretty similar to the ones computed with the stochastic evaluation. However, if we have a closer look at the data we can see that there is a variation in the liabilities value. This difference can be explained by the volatlity of the GBM applied to the stock value. In fact, the only value that changes significantly is the SCR Equity.

6 Open questions

What happens to the asset and liabilities when the risk free rate increases/decreases with a parallel shift of, say, 100bps? Describe all the effects

When we consider the shift UP of rates, the price of assets change. The stock initially has more or less the same value. Proceeding in time, the stock price becomes higher than the one in base case. Hence, the value of assets starts to be bigger than standard one. For the liabilities the situation is very similar to interest rate UP risk where a higher stock, especially during last years of contract, makes liabilities decrease.

With a shift DOWN in interest rates, the results are not different from our expectations. This shift leads to a very big decrease of the stock values.

As we expected, for liabilities we obtain the opposite result of the previous case, since a decrease in interest rates makes liabilities increase, even if all the rates get negative.

What happens to the liabilities if the insured age increases? What if there were two model points, one male and one female?
If the insured age increases, we expect that the mortality rates increase as well. As a consequence of that, the probability of entering the year decreases. In fact, we remember that the probability of entering the year i is given by $p_{surv\_i} = (1 - q_i) \cdot (1 - l_x) \cdot p_{i-1}$.
Hence, since in the computation of cash flows we take into account the probability p_i, we expect that the cash flows increase and so do the liabilities. In fact we have an higher probability to pay the death benefit.
We made as prove of this thesis an example in which we have a male insured aged $x = 70$.

	Liabilities
x=60	$0.9832 \cdot 10^5$
x=70	$1.0011 \cdot 10^5$

As we can see from the table liabilities increase.

If we consider two model points, one male and one female, we expect that the mortality rates for female decrease due to the fact that the probability of death is lower than the one for males. Hence, the probability of entering the year increases as well as the liabilities.

	Liabilities
male	$0.9832 \cdot 10^5$
female	$0.9836 \cdot 10^5$

As expected, even if by a little, liabilities increase.

7 Matlab code

```
%GROUP 22 SII project

close all;
clearvars;
clc
rng(1)


%% Data

% Asset data
C0 = 100000;              % Insured capital
F0 = 100000;              % The value of the fund at t_0
S0 = F0;                  % Equity price at t_0 (all equity)
sigma = 0.25;             % Volatility
delta_BOF = 0;            % Dividend yield
T = 50;                   % Maturity (in years)

% extra data
lx = 0.15;                % Fixed lapse rate
inflation = 0.02;         % Inflation rate
expenses_t0 = 50;         % Expenses per year (growing with inflation rate)
RD = 0.02;                % Regular deduction
COMM = 0.014;             % Commission to the distribution channel
comm_if_benefit = 20;     % Commission if benefit is paid

% Maturitues
times = (1:1:T)';


%% Rates from EIOPA IT with no VA 31.03.22

% Spot rates
rates = xlsread('EIOPA_RFR_20220331_Term_Structures', 'RFR_spot_no_VA', 'S11:S60');

```

```
% Rates shock UP
r_up = xlsread('EIOPA_RFR_20220331_Term_Structures', 'Spot_NO_VA_shock_UP', 'S11:S60');

% Rates shock DOWN
r_down = xlsread('EIOPA_RFR_20220331_Term_Structures', 'Spot_NO_VA_shock_DOWN', 'S11:S60');


%% Computing The Forward Rates

rates = rates + 0.01;

% Discount
B = exp(-rates.*times);
B_up = exp(-r_up.*times);
B_down = exp(-r_down.*times);

% computing forward discounts
fwd_B = B(2:end)./B(1:end-1);
fwd_up_B = B_up(2:end)./B_up(1:end-1);
fwd_down_B = B_down(2:end)./B_down(1:end-1);

% computing forward rates
fwd_rates = [rates(1); -log(fwd_B)];
fwd_rates_up = [r_up(1); -log(fwd_up_B)];
fwd_rates_down = [r_down(1); -log(fwd_down_B)];


%% Computing The Probabilities

% Probability of survival (per thousand) ISTAT 2021
life_table = readmatrix('Life_table_male_2021');
px = life_table(64:114,2);

% computing the probability of death
qx = 1-px(2:end)./px(1:end-1);

% mortality shock
qx_mor = min((1 + 15/100)*qx(1:end),1);
```

```
% mortality cat
qx_cat = [(qx(1) + 1.5/1000); qx(2:end)];

% Expenses
expenses = [expenses_t0; expenses_t0 * (1+inflation).^times
    (1:end-1)];

% Expenses shock
expenses_t0_bumped = expenses_t0*1.1;
expenses_bumped = [expenses_t0_bumped; expenses_t0_bumped *
    (1+(inflation+0.01)).^times(1:end-1)];

% equity shock
e_shock=0.61*S0;

% lapse up
lx_up = min(1.5*lx,1) .* ones(T,1);

% lapse down
lx_down = max(0.5*lx, lx-0.2) .* ones(T,1);

% lapse mass
lx_mass = [lx + 0.4; lx .* ones(T-1,1)];

% lapse spot
lx = lx .* ones(T,1);


%% BASE SCENARIO

% simulation
S = EquitySimulation(S0,fwd_rates,sigma,T,RD);

% Liabilities, Durantion and cash-flows computations
[Liabilities_1, Duration, Cf, Bel_Lapse, Bel_Death,
    Bel_Expen, Bel_Commissions] = Liabilities(S0, S, rates,
    times, lx, qx, comm_if_benefit, expenses,RD, COMM);

% Sum of all the BEL
```

```
sum_bel = Bel_Lapse + Bel_Death + Bel_Expen +
    Bel_Commissions;

% Basic Own Fund
Basic_Fund = S0 - Liabilities_1;

% print the output
disp('BEL:')
disp(sum_bel)
disp('Liabilities:')
disp(Liabilities_1)
disp('Duration:')
disp(Duration)


%% Interest Rate Up Risk

S_IR_up = EquitySimulation(S0, fwd_rates_up, sigma, T,RD);

[Liabilities_IR_up, Duration_IR_up, Cf_IR_up] = Liabilities
    (S0, S_IR_up, r_up, times, lx, qx, comm_if_benefit,
    expenses, RD, COMM);

Basic_Fund_IR_Up = S0 - Liabilities_IR_up;
delta_BOF_IR_Up = max(Basic_Fund - Basic_Fund_IR_Up,0);


%% Interest Rate Down Risk

S_IR_down = EquitySimulation(S0, fwd_rates_down, sigma, T,
    RD);

[Liabilities_IR_down, Duration_IR_down, Cf_IR_down] =
    Liabilities(S0, S_IR_down, r_down, times, lx, qx,
    comm_if_benefit, expenses,RD, COMM);

Basic_Fund_IR_Down = S0 - Liabilities_IR_down;
delta_BOF_IR_Down = max(Basic_Fund - Basic_Fund_IR_Down,0);


```

```
%% Equity Risk

S_Eq_R = EquitySimulation(e_shock,fwd_rates,sigma,T,RD);

[Liabilities_Eq_R, Duration_Eq_R, Cf_Eq_R] = Liabilities(S0
    , S_Eq_R, rates, times, lx, qx, comm_if_benefit,
    expenses,RD, COMM);

Basic_Fund_Eq_R = e_shock - Liabilities_Eq_R;
delta_BOF_Eq_R = max(Basic_Fund - Basic_Fund_Eq_R,0);


%% Mortality Risk

S_Mor_R = S;

[Liabilities_Mor_R, Duration_Mor_R, Cf_Mor_R] = Liabilities
    (S0, S_Mor_R, rates, times, lx, qx_mor, comm_if_benefit,
     expenses, RD, COMM);

Basic_Fund_Mor_R = S0 - Liabilities_Mor_R;
delta_BOF_Mor_R = max(Basic_Fund - Basic_Fund_Mor_R,0);


%% Lapse Down Risk

S_L_down = S_Mor_R;

[Liabilities_L_down, Duration_L_down, Cf_L_down] =
    Liabilities(S0, S_L_down, rates, times, lx_down, qx,
    comm_if_benefit, expenses,RD, COMM);

Basic_Fund_L_down = S0 - Liabilities_L_down;
delta_BOF_L_down = max(Basic_Fund - Basic_Fund_L_down,0);


%% Lapse Up Risk

S_L_up = S_Mor_R;

```

```
[Liabilities_L_up, Duration_L_up, Cf_L_up] = Liabilities(S0
    , S_L_up, rates, times, lx_up, qx, comm_if_benefit,
    expenses,RD, COMM);

Basic_Fund_L_up = S0 - Liabilities_L_up;
delta_BOF_L_up = max(Basic_Fund - Basic_Fund_L_up,0);


%% Lapse Mass Risk

S_L_mass = S_Mor_R;

[Liabilities_L_mass, Duration_L_mass, Cf_L_mass] =
    Liabilities(S0, S_L_mass, rates, times, lx_mass, qx,
    comm_if_benefit, expenses,RD, COMM);

Basic_Fund_L_mass = S0 - Liabilities_L_mass;
delta_BOF_L_mass = max(Basic_Fund - Basic_Fund_L_mass,0);


%% Catastrophe Risk

S_Cat = S_Mor_R;

[Liabilities_Cat, Duration_Cat, Cf_Cat] = Liabilities(S0,
    S_Cat, rates, times, lx, qx_cat, comm_if_benefit,
    expenses,RD, COMM);

Basic_Fund_Cat = S0 - Liabilities_Cat;
delta_BOF_Cat = max(Basic_Fund - Basic_Fund_Cat,0);


%% Expenses Risk

S_Exp = S_Mor_R;

[Liabilities_Exp, Duration_Exp, Cf_Exp] = Liabilities(S0,
    S_Exp, rates, times, lx, qx, comm_if_benefit,
    expenses_bumped, RD, COMM);

```

```
Basic_Fund_Exp = S0 - Liabilities_Exp;
delta_BOF_Exp = max(Basic_Fund - Basic_Fund_Exp,0);


%% Computation of BSCR

% Computation of SCR: Market Risk

SCR_IR_up = max(delta_BOF_IR_Up, 0);
SCR_IR_down= max(delta_BOF_IR_Down, 0);
SCR_IR = max(SCR_IR_up, SCR_IR_down);

SCR_Eq_R = max(delta_BOF_Eq_R, 0);

MRKT = [SCR_IR SCR_Eq_R]';


 if SCR_IR == SCR_IR_up
      CORR = [1 0 ; 0 1 ];
 else
      CORR = [1 0.5 ; 0.5 1 ];
end

%
SCR_MRKT = sqrt(MRKT'*CORR*MRKT);

% Computation of SCR: Life Risk
SCR_L_down = max(delta_BOF_L_down, 0);
SCR_L_mass = max(delta_BOF_L_mass, 0);
SCR_L_up = max(delta_BOF_L_up, 0);
SCR_Cat = max(delta_BOF_Cat, 0);
SCR_Mor_R = max(delta_BOF_Mor_R, 0);
SCR_Exp = max(delta_BOF_Exp, 0);

SCR_L_A = max([SCR_L_down SCR_L_mass SCR_L_up]); %max
    change lapse

LIFE = [SCR_Mor_R SCR_L_A SCR_Exp SCR_Cat]';
COR_L = [1 0 0.25 0.25; 0 1 0.5 0.25; 0.25 0.5 1 0.25 ;
    0.25 0.25 0.25 1];
```

```
SCR_LIFE = sqrt(LIFE'*COR_L*LIFE);

% Computation of BSCR
SCR = [SCR_MRKT SCR_LIFE];
COR = [1 0.25; 0.25 1];
BSCR = sqrt(SCR*COR*SCR')
```

```
function S = EquitySimulation(S0,forward_rates,sigma,T,RD)
% function which simulate the equity using the GBM model
%
% INPUTS:
% S0:                Equity at time t=0
% sigma:             Volatility
% forward_rates:     Forward rates from the EOPIA
% T:                 Maturity
%
% OUTPUT:
% SimEquity:         vector of the simulated equity per year


%% Generate the B.M

N = 1e6;
dt = 1;
g = randn(N,T);


%% Inizialitation

S = zeros(N,T+1);
S(:,1) = S0;
factor_RD = 1-RD;


%% Compute Equity using GBM via a monte carlo simulation

for t=2:T
    S(:,t) = S(:,t-1).*exp( (forward_rates(t-1)-0.5*sigma
        ^2)*dt+sigma*sqrt(dt)*g(:,t-1) );
    S(:,t) = factor_RD*S(:,t);
end
```

```

S = mean(S,1)';


end
```

```
function [Liabilities, Duration, Cf, Bel_Lapse, Bel_Death, Bel_Expen, Bel_Commissions] = Liabilities(S0, S, rates, times, lx, qx, comm_if_benefit, expenses, RD, COMM)
% function which computes the value of the liabilities, the duration and
% cash flows
%
% INPUTS:
% S0:                 initial value of the stock
% S:                  value of the stock simulated
% rates:              risk-free rates
% times:              vector of times
% lx:                 vector of probability of lapse for each year
% qx:                 vector of probability of death for each year
% comm_if_benefit:    commission of the benefit is paid
% expenses:           expenses each year
% RD:                 Regulatory capital
% COMM:               commission to the distribution channel
%
% OUTPUTS:
% Liabilities:        value of the Liabilities
% Duration:           maculay duration
% Cf:                 vector of future cash flows
% Bel_Lapse:          liabilities of lapse
% Bel_Death:          liabilities of death
% Bel_Expen:          liabilities of expenses
% Bel_Commissions:    liabilities of commissions
%

% Initializations
T = times(end);
F = S;

```

```
% factor of the regulatory capital
factor_RD = 1-RD;

%Survival Probability
P_surv = cumprod([1;(1 - qx(1:end-1)).*(1 - lx(1:end-1));
    0]); %prob di rimanere nel contratto fino a anno i

%BEL components
Lapse_bel = zeros(T,1);
Death_bel = zeros(T,1);
Expen_bel = zeros(T,1);
Commissions_bel = zeros(T,1);

% initialization
C = zeros(T,1);
for i = 1:T

    % Bel computation
    Lapse_bel(i) = ((F(i+1) - comm_if_benefit) * lx(i) *
        (1-qx(i))) * P_surv(i);
    Death_bel(i) = ((max(1.1*S0,F(i+1)) - comm_if_benefit)
        * qx(i)) * P_surv(i);
    Expen_bel(i) = expenses(i) * P_surv(i);
    Commissions_bel(i) = (F(i+1)/factor_RD*COMM) * P_surv(i
        );

    % computation of the costs each year
    C(i) = Lapse_bel(i) + Death_bel(i) + Expen_bel(i) +
        Commissions_bel(i);

end

% Discounts computation
discounts = exp(-rates.*times);

% Computation of Liabilities
Cf = C.*discounts;
Liabilities = sum(Cf)';

% Duration computation
```

```
Num = sum(Cf.*times)';
Duration = Num/Liabilities;

%BEL components
Bel_Lapse = sum(Lapse_bel.*discounts);
Bel_Death = sum(Death_bel.*discounts);
Bel_Expen = sum(Expen_bel.*discounts);
Bel_Commissions = sum(Commissions_bel.*discounts);


end
```