Managment and Analysis of Physics Dataset – 1 A.A. 2019-20

Digital Circuits - 5/6

Overview

G.Collazuol

- Digital Circuits properties
 - Asynchronous vs Synchronous circuits
 - Combinational vs Sequential
- Combinational circuits → synthesis and examples
- Sequential ciurcuits → latches and flip-flops

Asynchronous vs Synchronous Operation

Asynchronous operation serves for diverse specific processing that do not require using a clock (mismatches between delays can impair the results).

More reliable is when timing is controlled by a clock (synchronous operation). The period must be such as to accommodate the most time consuming processing section.

Often, the circuit includes sections where processing is performed in multiple clock periods.

Sequential Operation

A sequential system relies on a sequence of actions, required to occur in the right order.

Sequential architectures can be synchronous and asynchronous.

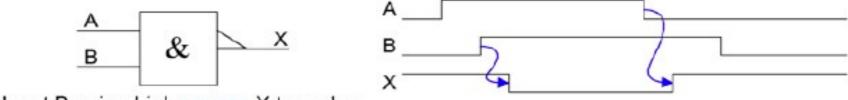
Use of a clock to establish the update times of the feedback signals or no time control the circuit sends back outputs continuously as soon they change value.

Sequential circuits, both synchronous and asynchronous architectures enable higher level processing than combinational schemes.

With given inputs possible instability. It persists until a different input configuration takes the output out of unstable conditions.

Combinational vs Sequential Operation

The task of combinational logics is just to relate signals at input and generate defined logic signals at output.



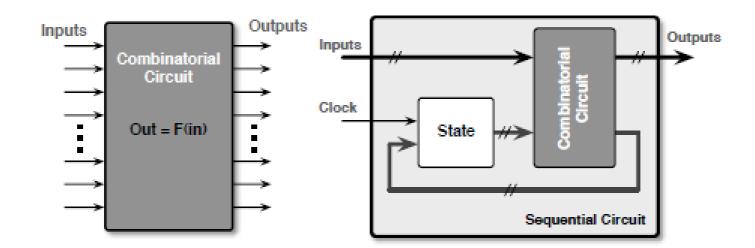
Input B going high causes X to go low

Input A going low causes X to go high

The output of combinational circuits depends only on input.

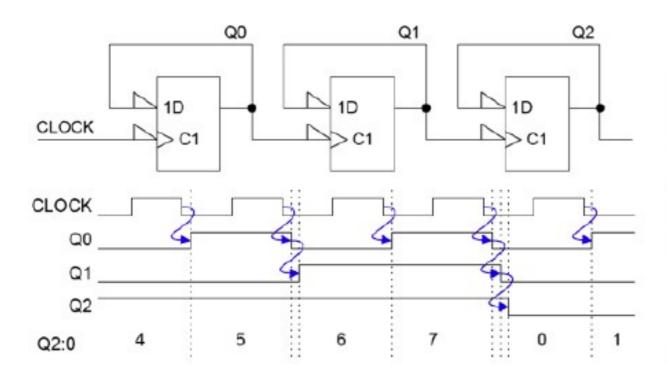
Sequential circuits produce output on the basis of both input and output.

For example, a memory circuit is essentially sequential, because its output depends on the input that occurred in the past. Instead, the addition of two inputs is combinational because the result just depends on changing inputs.



Ripple Counter

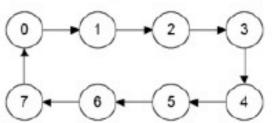
example of a D-FF used in a ripple counter.



- Notice inverters on the CLOCK and DATA inputs
- Least significant bit of a number is always labelled 0

State Diagram

(not including transient states):

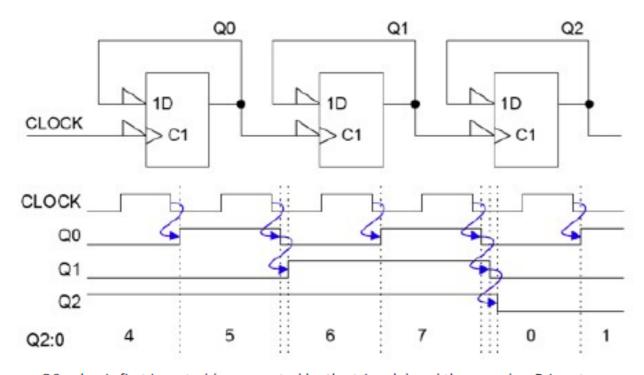


Propagation Delay: CLOCK Ψ to Q2 = 3 x 1ns = 3ns

This counter is also known as an **asynchronous sequential circuit**. It is "**asynchronous**" because the output signals are NOT synchronised to a single clock signal (since there are many clock signals), and "**sequential**" because its current output value (or state) depends on previous output values in the sequence.

Ripple Counter

example of a D-FF used in a ripple counter.



Q0 value is first inverted (represented by the triangle) and then used as D input on the next clock cycle. The flipflop is triggered on the FALLING edge of CLOCK. Therefore the Q output "TOGGLES" on each active edge of the clock (i.e. falling edge). Q0 is therefore changing at half the rate of CLOCK, hence this flipflop acts as a divide-by-2 circuit.

The Q0 signal is now used as clock input to the next D-FF. Hence Q1 is toggling at half the frequency of Q0. The circuit is effectively a binary counter.

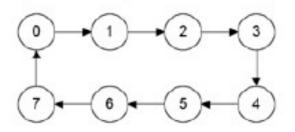
This is a simple finite state machine (FSM) because it has 8 states which cycles through in a sequence. FSM will be covered in some later lectures in details and it is a very important topic in digital designs.

We then use the Q0 output as the clock input the next stage etc. Note that because the 2nd stage only starts to work once the first stage is completed, the propagation of effects "ripples" through the circuit – hence we call this a "ripple counter".

- Notice inverters on the CLOCK and DATA inputs
- Least significant bit of a number is always labelled 0

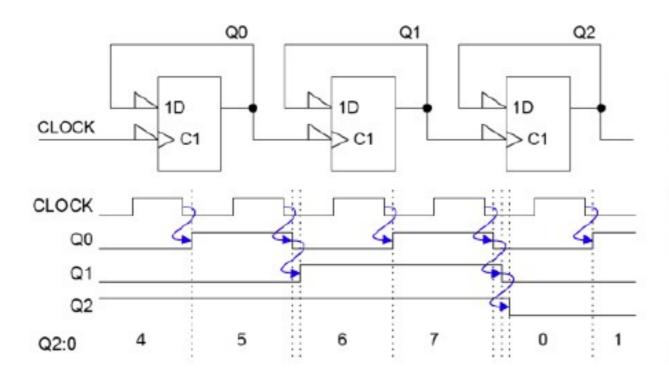
State Diagram

(not including transient states):



Ripple Counter

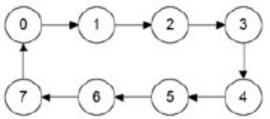
example of a D-FF used in a ripple counter.



- Notice inverters on the CLOCK and DATA inputs
- Least significant bit of a number is always labelled 0

State Diagram

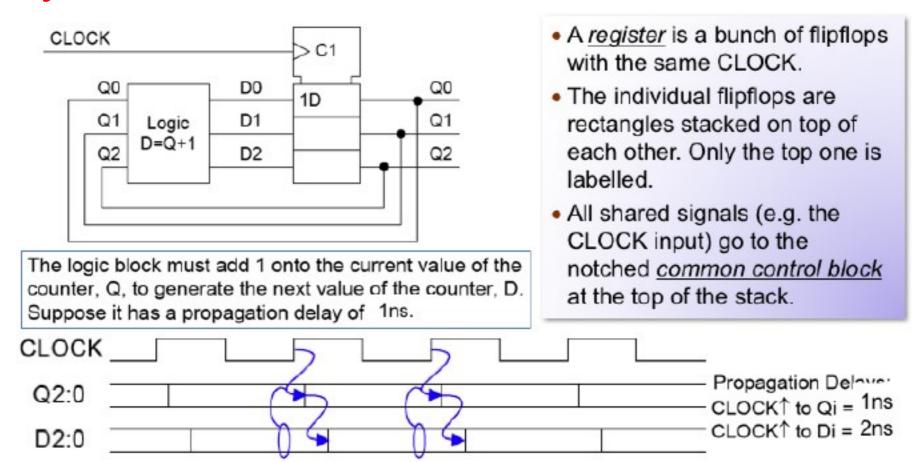
(not including transient states):



Propagation Delay: CLOCK Ψ to Q2 = 3 x 1ns = 3ns

The ripple counter is potentially slow. The delay between the active edge of the clock and the counter output giving the correct value is dependent on the number of flipflops in the circuit and therefore the size of the counter (i.e. how many stages).

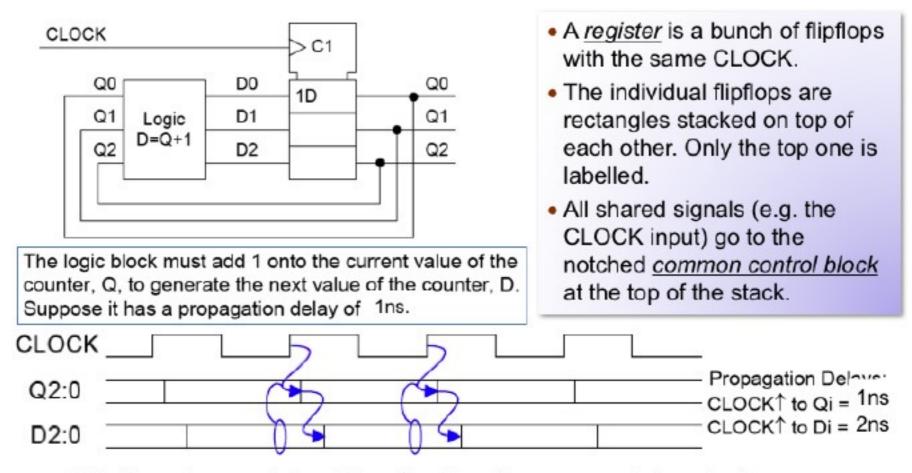
Synchronous Counter



All flipflops change state within a fraction of nanosecond of each other.

A far better approach is to use the flipflops TOGETHER as a group, and clock them using THE SAME CLOCK signal as shown here. The Logic Block is a combinatorial circuit which computes the next D value D2:0 from the current Q value Q2:0. (D has three bits D0, D1 and D2. We use the notation D2:0 to represent this.) The relationship between D and Q is simple: D2:0 = Q2:0 + 1.

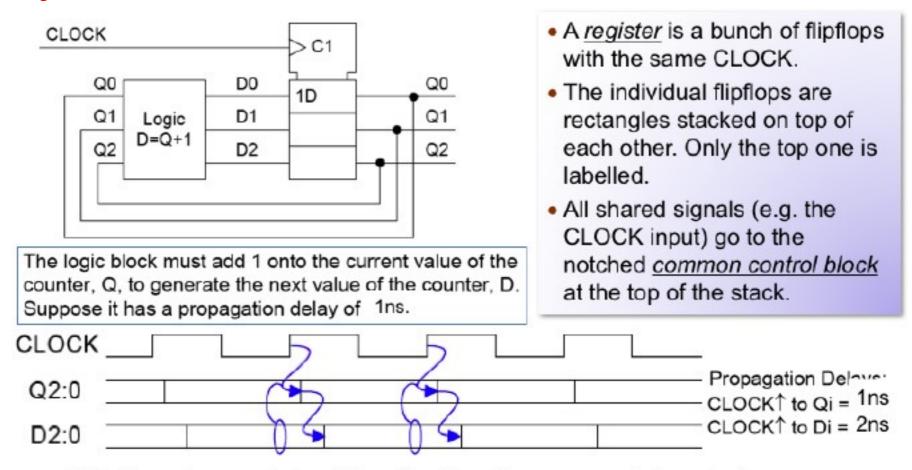
Synchronous Counter



All flipflops change state within a fraction of nanosecond of each other.

Since the three output bits Q2:0 change within a fraction of a nanosecond of each other, this circuit is: 1) faster than the ripple counter; 2) the "delay" is constant instead of dependent on the size of the counter.

Synchronous Counter



All flipflops change state within a fraction of nanosecond of each other.

This circuit is known as a **synchronous sequential circuit** because its function is synchronous to a single clock signal. If you regard the Q2:0 output value as a state value, it follows a finite number of states in a defined sequence. Therefore it is also a form of **Finite State Machine**.

Combinational Logic vs Sequential Logic

Combinational circuits

the steady-state outputs are logical functions of the steady-state inputs only,

- → outputs do not depend on the internal condition of the circuit
- → combinational circuits can always be constructed with gates

Examples: - adding or comparing integers

- deciding whether all the conditions for a decision are satisfied

Sequential circuits

when memory of past actions is required → combinational circuits do not suffice

Example: if we want to detect the second occurrence of a given action, we must register the fact that at some point there was a first occurrence of this action

Combinational Logic Circuits

- Synthesis of logical functions
- Examples of combinatorial circuits

Synthesis of combinatorial logical functions

Logical functions can be written in more than one way

Some versions are more economical than others (in terms of number of gates) eg: the function F=A+AB, for example, can be written more succintly as F=A+B

Criteria for synthesising

Minimizing the number of gates required to execute a logical function is not always a priority: speed or logical clarity, for example, almost always have higher priority

Programmable Logic Devices (eg FPGA) can generate arbitrary logical functions of a certain maximum number of variables when the interconnections of their internal gates are suitably programmed. In PLDs standardization is an additional criterium that has priority over economy: logical functions are therefore implemented in the well-defined canonical forms

Miniterms and Canonical form

Let us consider an arbitrary logical function of n variables F(A1, ..., An), and let us further consider n-term products of the form $(\overline{A}_1) \cdots (\overline{A}_n)$ in which each variable is represented once, either negated or not Such products of this form are called a miniterms

Example: in the case of two variables (A,B) the 4 miniterms are: AB,AB,AB,AB

Note: for each array of values of the variables $A_1...A_n$ there is one and only one minterm whose value is 1:

- → it is the minterm in which A_i is not negated if A_i is equal to 1,
- → and is negated if A_i is equal to 0

The number of miniterms is 2ⁿ (ie the number of possible arrays of values)

The function F is defined on the set of its minterms

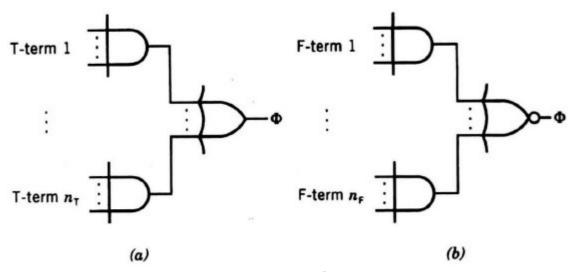
Note:

the number of the possible boolean functions of n boolean variables is 2^(2^n) (indeed two possible values can be assigned to each miniterm)

Miniterms and Canonical form

Let us divide the minterms of F into two type of terms:

- T-terms (corresponding to arrays of values for which F = 1 or TRUE)
- F-terms (corresponding to arrays of values for which F=0 or FALSE) If the number of T-terms is n_{τ} and the number of F-terms is n_{τ} then $n_{\tau}+n_{\epsilon}=2^{n}$
- → F is the sum of its T-terms and F is also the negated sum of its F-terms



These are canonical representations of F as **sum of products** of the n input variables

Note: similarly maxiterms are defined as the n-term sum of the n variables, where each variable is represented once, either negate or not. For each array of values of the variables $A_1...A_n$ there is one and only one maxterm = 0

Example: in the case of two variables (A,B) the 4 maxiterms are: $(\underline{A}+\underline{B}), (\underline{A}+B), (A+B)$

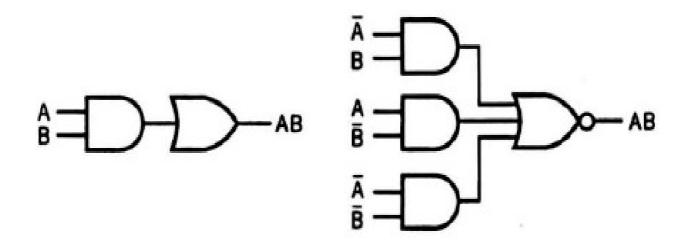
It can be seen that F can be represented as product of sums (namely T-maxterms)

Example: canonical form of F(A,B) = AB

The only T-term of F is AB
$$\rightarrow n_T = 1$$

The F-terms are AB, AB, AB $\rightarrow n_F = 3$

The canonical representations of AB in miniterms are thus:



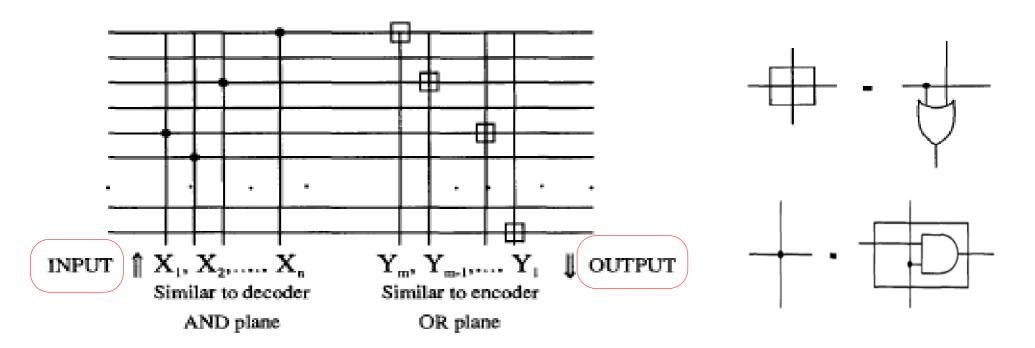
As a check: $\underline{A}B + \underline{A}\underline{B} + \underline{A}\underline{B} = \underline{A}(B + \underline{B}) + \underline{A}\underline{B} = \underline{A} + \underline{B}$ or $\underline{A}B + \underline{A}\underline{B} + \underline{A}\underline{B} = \underline{A}B + \underline{A}B + \underline{A}B = \underline{A}B + \underline{A}B + \underline{A}B = \underline{A}B + \underline{A}B + \underline{A}B + \underline{A}B + \underline{A}B + \underline{A}B = \underline{A}B + \underline{A}B +$

Note: Complete sets of operators

Given a basic set of Operations (eg {AND,OR,FANOUT,EXCHANGE}) then for any set of m output functions $\{F_i, i=1,...,m\}$ on n inputs $\{X_i, j=1,...,n\}$

$$Y_i = F_i(\{X\}), i=1,...,m$$

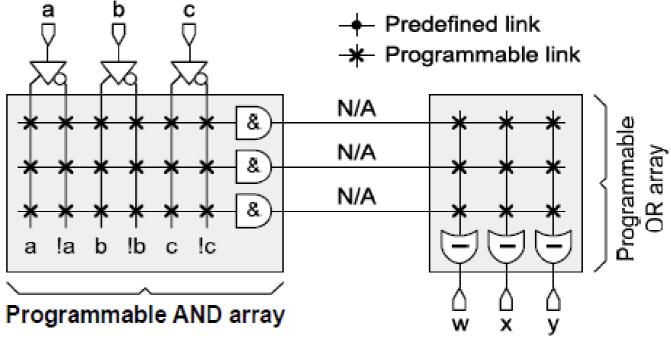
it can be shown that a circuit that can perform that functions can be built



Procedure quite simple (each output is a single value function) ... just many wires!

Programmable Logic Arrays

Una PLA (programmable logic array) è simile a una PROM ma ha sia il piano AND che il piano OR programmabili. A differenza delle PROM, la dimensione del piano AND è indipendente dal numero di ingressi. Lo svantaggio principale è una minore velocità perché il segnale in una PLA deve attraversare due livelli di collegamenti programmabili (che sono più lenti di un collegamento fisso).



Reminder!

Practical examples of combinatorial logic circuits

- Binary adder
- Arithmetic Logic Units
- Magnitude Comparator

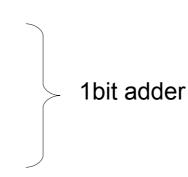
Binary addition with gates

In adding two binary words, at each position we need

- (1) to know whether there is a carry from the next-lowest position (carry-in)
- (2) to generate the carry for the next-highest position (carry-out)

At any given position, let

- A and B be the bits of the words to be added
- C the carry-in
- Σ the sum
- K the carry-out



 Σ is the low-order bit of the sum A+B+C, and it is 1 in only two cases:

- only one of the addends is 1, or
- all three are 1

If we interpret A, B, C, and Σ as logical variables \rightarrow

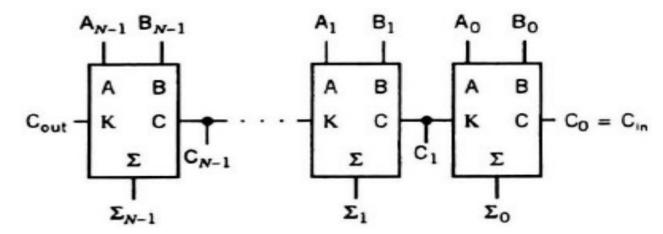
$$\Sigma = A\overline{B}\overline{C} + \overline{A}B\overline{C} + \overline{A}\overline{B}C + ABC$$

Similarly, K is 1 if any pair of addends is 1, and we obtain → Note: the term ABC is already included in AB+BC+AC indeed ABC=1 implies AB=BC=AC=1

$$K = AB + BC + AC$$

Binary addition with gates

If we want to add two N-bit words we can chain N 1-bit adders together



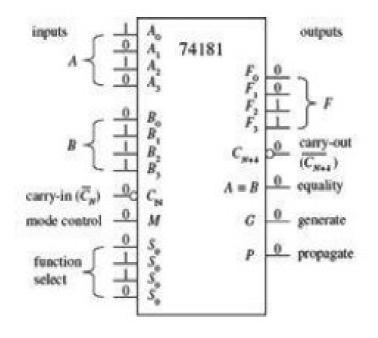
The adder for the lowest-order bit accepts a carry-in C_{in} from a possible lower-order word, and the adder for the highest-order bit generates a carry-out C_{out} that can feed a possible higher-order word

Note: the decision in a given bit has to wait for the decisions in all lower-order bits \rightarrow since each decision takes a finite time, the resulting delay in Cout might be considered excessive. This is the reason for the existence of look-ahead carry generators, circuits that consider all possible combinations of the inputs and decide with minimal delay whether C_{out} will be generated

Aritmetic Logic Units

An ALU is a multipurpose integrated circuit capable of performing various arithmetic and logic operations. To choose a specific operation to be performed, a binary code is applied to the IC's mode select inputs.

The 74181 fon instance is a 4-bit ALU that provides 16 arithmetic and 16 logic operations

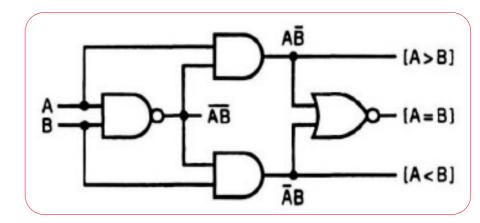


Mode select				Logic functions	Arithmetic operations			
S,	S2	S,	So	(M = 1)	$(M=0, \overline{C}_s=1)$			
0	0	0	0	$F = \overline{A}$	F = A			
0	0	0	1	$F = \overline{A + B}$	F = A + B			
0	0	1	0	$F = \overline{A}B$	$F = A + \overline{B}$			
0	0	1	1	F = 0	F = minus 1 (2's comp.)			
0	1	0	0	$F = \overline{AB}$	$F = A$ plus $A\overline{B}$			
0	1	0	1	$F = \overline{B}$	$F = (A + B)$ plus $A\overline{B}$			
0	1	1	0	$F = A \oplus B$	F = A minus B minus 1			
0	1 1 1 0	1	1	$F = A\overline{B}$	$F = A\overline{B}$ minus 1			
1	0	0	0	$F = \overline{A} + B$	F = A plus AB			
1	0	0	1	$F = \overline{A \oplus B}$	F = A plus B			
1	0	1	0	F = B	$F = (A + \overline{B}) \text{ plus } AB$			
1	0	1	1	F = AB	F = AB minus 1			
1	1	0	0	F = 1	F = A plus A			
1	1	0	1 0 1 0 1 0 1 0 1	F = A + B	F = (A + B) plus A			
1	1	1	0	F = A + B	$F = (A + \overline{B})$ plus A			
1	1	1	1	F = A	F = A minus 1			

Magnitude comparator for unsigned integers

First construct 1-bit magnitude comparator, which is a useful step because magnitude comparators usually have all 3 outputs for = > <

The conditions [A > B], [A = B], and [A < B] can be determined with the logical operations AB, (AB+AB) (ie XNOR), AB and correctly generated by the comparator in figure



A 2-bit comparator is then constructed as follows. As an example, let us build a circuit that decides whether the 2-bit word A_1A_0 is > (greater than) the 2-bit word B_1B_0 Note: the procedure for < (less than) is similar. Both can be used on widerwords

We observe first that the condition [A1 > B1] = 1 is sufficient

If it is not satisfied, we must require that [A1 = B1] and [A0 > B0] simultaneously be 1

We thus obtain \rightarrow

$$[A_1A_0>B_1B_0]=[A_1>B_1]+[A_1=B_1][A_0>B_0]$$

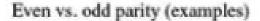
More combinatorial blocks

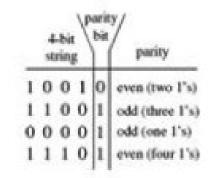
- Parity Generator / Checker
- Multiplexers / Demultiplexers
- Encoders / Decoders

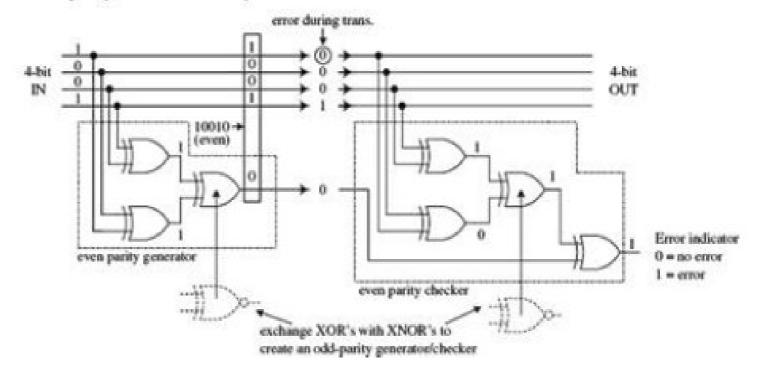
Parity generator / Checker

Often, external noise can corrupt binary information (cause a bit to flip from one logic state to the other) as it travels along a conductor from one device to the next

4-bit even-parity error-detection system





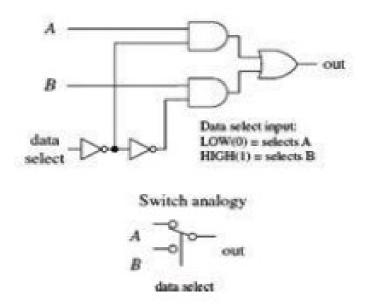


Multiplexers (Data Selectors)

Multiplexing is the technique of selecting and transmitting signals from multiple input lines to a single output line

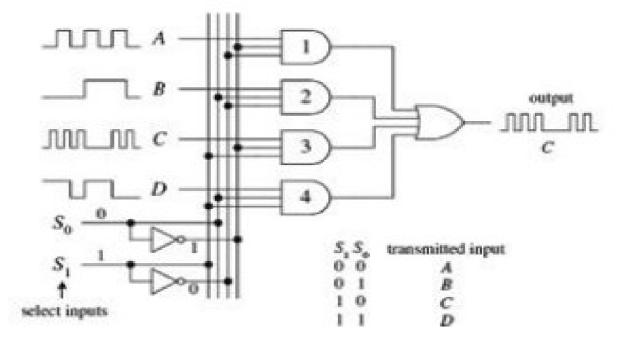
Multiplexers works as digitally controlled switches

Simple 2-to-1 data selector



AND gates are enable gates

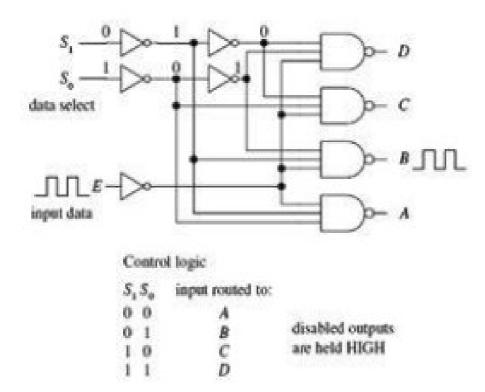
4-line to 1-line multiplexer



Demultiplexer (Data Distributor)

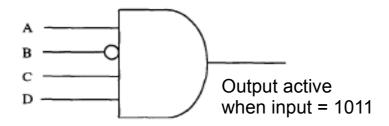
A demultiplexer (or data distributor) is the opposite of a multiplexer: it takes a single data input and routes it to one of several possible outputs

A simple 1-to-4 four-line demultiplexer



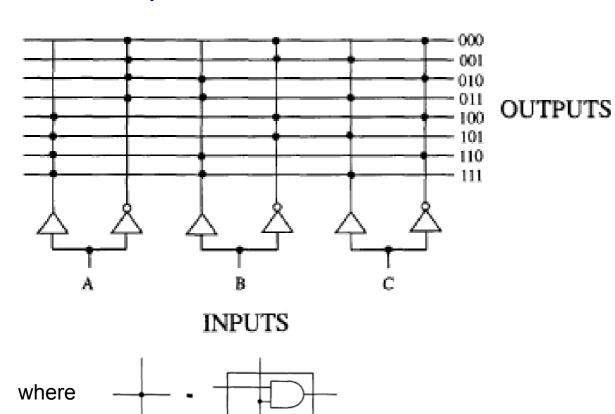
Binary Decoder

Simple binary decoder

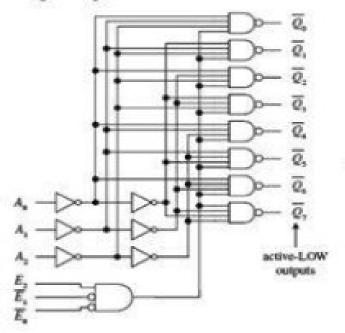


Data select inputs decide which outputs are active

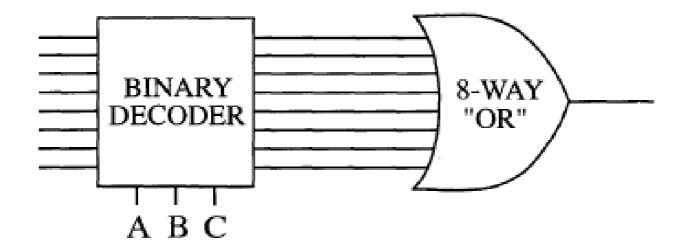
3-to-8 binary decoder



Logic diagram 74LS138 1-of-8 decoder



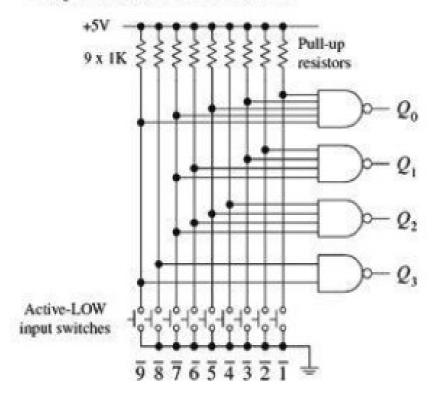
Note: Binary Decoder + OR = Multiplexer



Encoder

Encoders are the opposite of decoders. They are used to generate a coded output from a single active numeric input

Simple decimal-to-BCD encoder



Truth table

ī	2	3	4	3	<u>6</u>	7	$\overline{8}$	9	Q.	Q_1	Q_2	Q ₃	BCD (pos. logic)
Н	Н	Н	Н	Н	Н	Н	Н	Н	L	L	L	L	0000 (0,,)
L	H	H	H	H	H	H	H	H	L	L	L	H	0001 (1 10)
Н	L	H	H	H	H	H	H	H	L	L	H	L	0010 (2,0)
Н	H	L	H	H	H	Н	H	H	L	L	Н	H	0011 (3,0)
H	H	H	L	H	H	H	H	H	L	H	L	L	0100 (410)
H	H	н	H	L	H	H	Н	H	L	H	L	H	0101 (5,0)
Н	H	Н	H	H	L	H	H	H	L	H	H	L	0110 (610)
H	H	H	H	H	H	L	Н	H	L	H	H	H	0111 (7,0)
Н	H	Н	Н	Н	Н	H	L	H	Н	L	L	L	1000 (810)
H	H	H	Н	H	H	Н	Н	L	Н	L	L	Н	1001 (910)

H = High voltage level, L = Low voltage level

Note on obsolescence and digital electronics trend

The implementation of the logic block just illustrated can be made

- either in specific logic devices
- or in programmable logic devices

While the PLD are more and more used the specific logic devices (like for instance binary adders and code converters) are becoming obsolete

As an alternative, often easier to use than PLD's, the trend today is to use software-controlled devices such as microprocessors and microcontrollers to carry out arithmetic operations and code conversions

Microcontrollers can be used to collect data, store data, and perform logical operations using the input data. They also can generate output signals that can be used to control displays, audio devices, stepper motors, servos, and so on. The specific functions a microcontroller is designed to perform depend on the program you store in its internal ROM-type memory

Programming the microcontroller typically involves simply using a special programming unit provided by the manufacturer. The programming unit usually consists of a prototyping platform that is linked to a host computer (via a USB port) that is running a development environment. In the development environment, you typically write out a program in a high-level language such as C, or some other specialized language designed for a certain microcontroller, and then, with the press of a key, the program is converted into machine language (1s and 0s) and downloaded into the microcontroller's memory. In many applications, a single microcontroller can replace entire logic circuits composed of numerous discrete components

Sequential Logic Circuits

- Async / Syncronous systems
- Flip-Flops / Latches
- Counters and Registers
- State Machines
- Syncronizers
- Memories

Synchronous vs Asynchronous

- Synchronous sequential logic:
 - the time at which transitions between circuit states occurs is controlled by a common clock signal
 - changes in all variables occur simultaneously
- Asynchronous sequential logic:
 - state transitions occur independently of any clock, and normally depend on the time at which input variables change
 - outputs do not necessarily change simultaneously

Clock

- a clock signal is a square wave of a fixed frequency
- it is used to trigger state transitions at fixed times in synchronous circuits

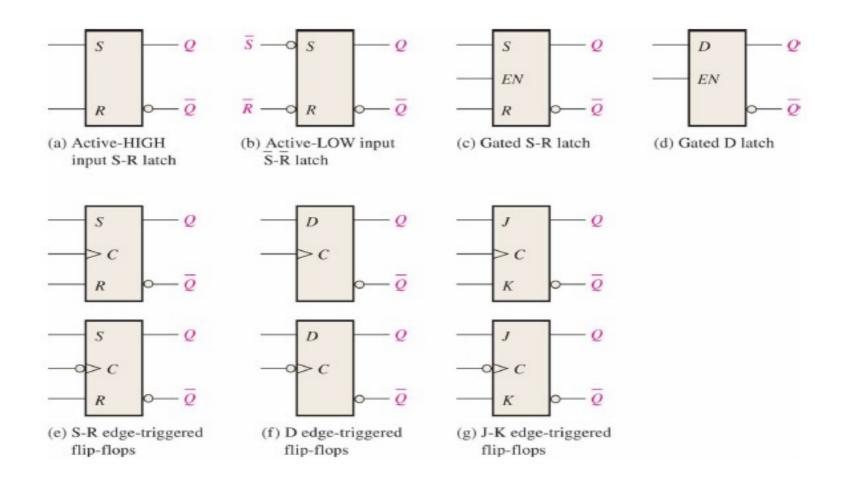
Flip-flops and Latches

- Flip-flops and latches are the fundamental elements of sequential circuits
 - bistable (two stable states)
- Flip-flops and latches are essentially 1-bit storage devices
 - outputs can be set to store either 1 or 0 depending on the inputs
 - even when the inputs are deasserted, the outputs retain their prescribed values
- Flip-flops and latches (normally) have 2 complementary outputs
 - usually denoted Q and \overline{Q}
- Three main types:
 - R-S J-K D-type

Name conventions

Standard distiction between:

- 1) Latches (serratura) → Flip-Flops which are sensitive to levels (high or low)
- 2) Flip-flops → Flip-Flops that are sensitive to transitions (signal edges)
- ... sometimes both of them are called Flip-Flop in any case ...



FLIP-FLOPs and Latches

A flip-flop is a memory device with two stable states:

→ by an appropriate choice of inputs, either state can be obtained

Flip-flops are needed to execute sequential logic, which requires memory of past actions

Flip-flops come in many forms:

- we focus on those obtained by cross-coupling logic gates
- we will assume that all gates have the same gate delay t

Asynchronous Flip-Flops

- Set-Reset Flip-Flop
- Arbitrator and metastability
- D Latch

Set-Reset FLIP-FLOPs - asyncronous

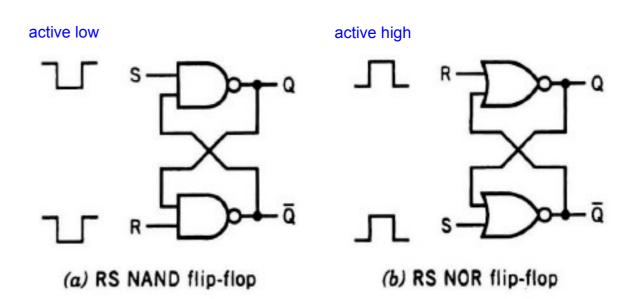
Consider the asynchronous RS NAND flip-flop

If the S(set) and R(reset)

inputs are inactive (R = S = 1) \rightarrow the outputs Q and Q can be in one of two self-consistent states:

either Q = 0 and Q = 1

or Q = 1 and Q = 0



- If S is made active (ie set to 0) → Q will become 1 after a time t (if it was not already 1) and Q will become 0 after a time 2t After this time, Q will remain at 1 even if S becomes inactive
- If R is made active (ie set to 0) for a time longer than 2t → Q can be set to 0

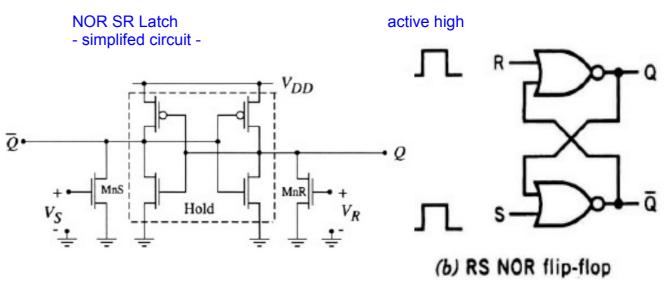
Note:

- 1) an analogous description is valid for the NOR flip-flop, where the only difference is that S and R are active when they are equal to 1
- 2) NAND FF is active low, NOR FF is active high

Set-Reset FLIP-FLOPs - asyncronous

Consider the asynchronous RS NAND flip-flop

If the S(set) and R(reset)
inputs are inactive (R = S = 1) \rightarrow the outputs Q and Q can be
one of two self-consistent state
either Q = 0 and Q = 1
or Q = 1 and Q = 0



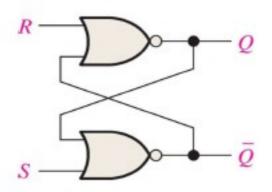
- If S is made active (ie set to 0) → Q will become 1 after a time t (if it was not already 1) and Q will become 0 after a time 2t After this time, Q will remain at 1 even if S becomes inactive
- If R is made active (ie set to 0) for a time longer than 2t → Q can be set to 0

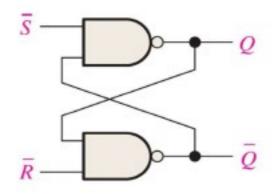
Note:

- 1) an analogous description is valid for the NOR flip-flop, where the only difference is that S and R are active when they are equal to 1
- 2) NAND FF is active low, NOR FF is active high

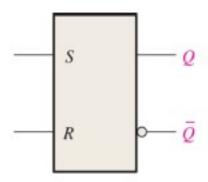
SR FLIP-FLOPs (two versions)

There are two versions of RESET-SET (R-S) Latch

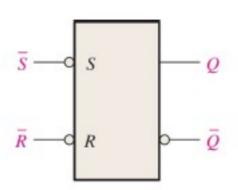




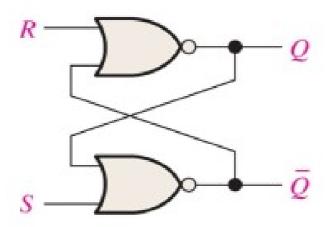
Active HIGH



Active LOW

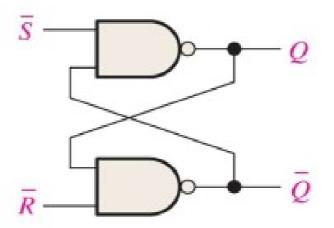


"Old Q" / "New Q" Analysis





Ing	out	Output
S	R	Q _{new}
0	0	Qold
0	1	0
1	0	1
1	1	0

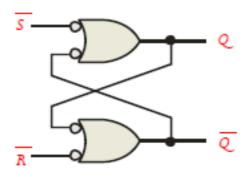


(b) Active-LOW input S-R latch

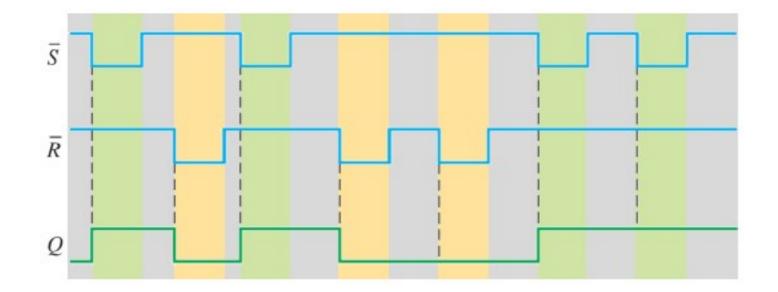
Inp	out	Output
s	R	Q _{new}
0	0	1
0	1	1
1	0	0
1	1	Qold

Example

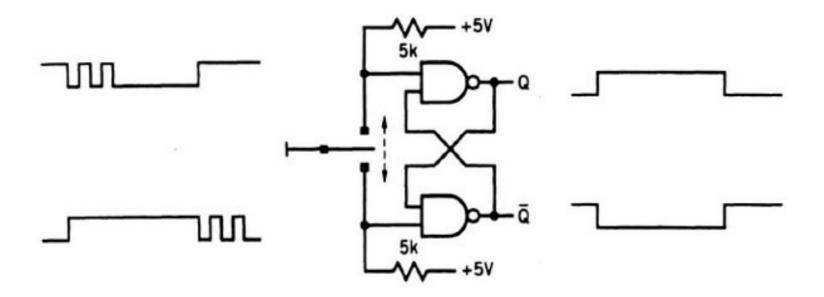
NAND type \rightarrow active low



Inp	ut s	Mode	
Ī	R	Mode	
0	1	SET	
1	0	RESET	
1	1	HOLD	



Example: RS latch debouncer

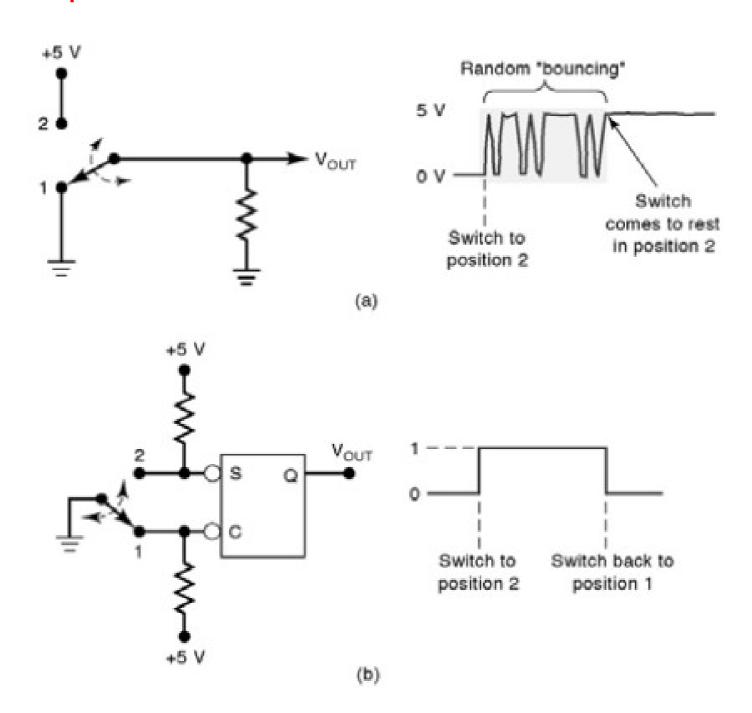


TTL RS flip-flop can be used in a debouncer to generate a single transition for instance when a mechanical single-pole double-throw switch changes state

The contacts bounce many times when the switch is thrown toward one pole or the other

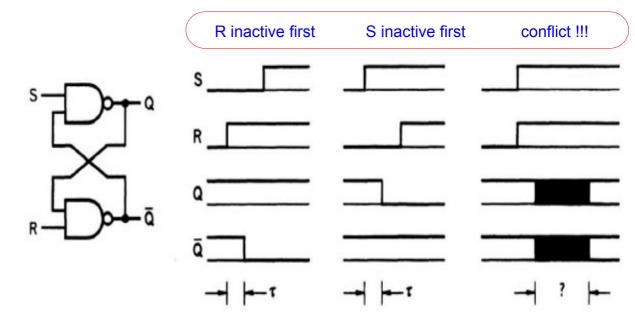
The duration of the first closure of the contacts, however, is far larger than any possible gate delay, so that the flip-flop is sure to change state. Furthermore, the bounce is never so severe that contact is made with the opposite pole

Example: RS latch debouncer



Set-Reset FLIP-FLOPs – Arbitrator

Consider the asynchronous RS NAND flip-flop



• If both inputs of are held active (R = S = 0) → both outputs are equal to 1 The flip-flop registers which input first becomes inactive and leaves it in a proper state ("arbitrator" function)

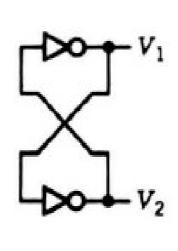
Example: if we consider S as a sampling signal when it goes to 1, and we then inquire about the state of the flip-flop, we can establish the value of R at the time of sampling (see clocked flip-flops, later)

... BUT there is a problem: the arbitrator works well if R and S become inactive at times that are separated by at least one gate delay: indeed the output that goes to 0 arrives in time to confirm the 0 at the input of the gate that remains at 1. For shorter intervals we must be careful... let us take a closer look at the structure of RS flip-flops then ...

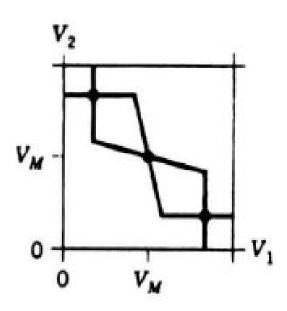
Set-Reset FLIP-FLOPs - Metastability

When both inputs of an RS flip-flop are inactive, the flip-flop reduces to a pair of cross-coupled inverters

!!! there is an additional
third non-digital equilibrium state
where both the output voltages
values between LOW and HIGH
If the inverters are identical their
output voltages are equal (V1=V2=VM)



Cross-coupled inverters

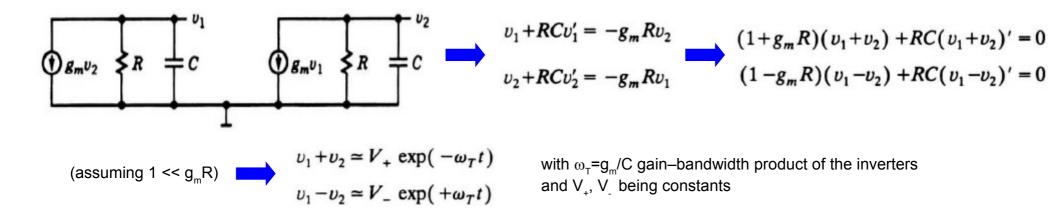


Static transfer characteristics
The equilibrium point at
V1=V2=VM is metastable

This state is metastable: any departure from equilibrium drives the flip-flop toward one of the two stable states: this happens because the inverters are on the steep part of their voltage transfer characteristic when they are near the metastable state and thus have a large incremental gain; since the feedback is positive, their small-signal response is unstable

Set-Reset FLIP-FLOPs – Metastability in detail

Assume that the inverters have a first-order response → small signal circuit



It is clear that v_1 and v_2 will grow exponentially if they differ initially in the least degree, and that a large gain—bandwidth product will result in a quick exit from the metastable state. These conclusions hold for higher-order gains

Set-Reset FLIP-FLOPs – Arbitrator in detail

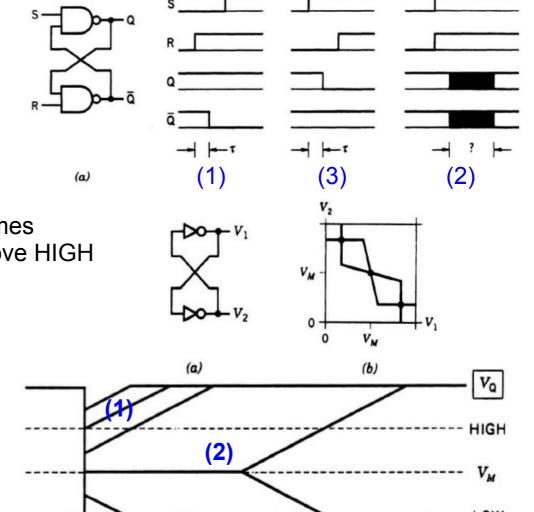
Back to the arbitrator: let us imagine to slowly shorten the interval between the times when R and S become inactive

(1) If we now make R precedes S by

less than one gate delay, nothing critical
happens for some time because even
though Q drops slightly, it remains
above HIGH. Beyond a certain point,
however, Q will drop below HIGH for longer times
although it will eventually return to a value above HIGH

(2) When the interval is reduced to zero, Q and Q are left in the metastable state.

(3) If we now make S precede R by an interval shorter than one gate delay, Q will eventually drop below LOW, but it will take longer and longer to do so as the interval is shortened



Waveforms of the voltage at the Q output

Set-Reset FLIP-FLOPs – Arbitrator in detail

 \rightarrow there exists a metastability window $\tau_w^-O(\tau)$ such that if S and R are separated by less than τ_w the flip-flop will become metastable (ie Q will not settle above HIGH or below LOW within one gate delay τ after S becomes inactive)

Now, if we assume that R goes from LOW to HIGH and back with an average frequency f, the probability that the flip-flop will become metastable when S becomes inactive is f_{τ_w}

Let us define $\Pi(T)$ as the probability that the flip-flop is still metastable a time T after becoming metastable. The probability P(T) that the flip-flop is still metastable a time T+ τ after S becomes inactive is the "probability of synchronization failure" or of "arbitration conflict" and is given by $P(T) = f\tau_{W}\Pi(T)$

P(T) cannot be reduced to zero in an RS flip-flop, but it can be made arbitrarily small

by waiting long enough before

examining Q because in practice,

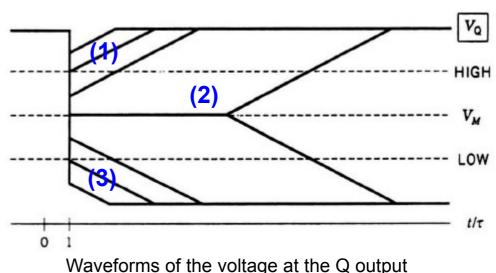
 $\Pi(T)$ decreases exponentially for large T

$$\Pi(T\gg\tau)\sim \exp(-T/\tau_M)$$

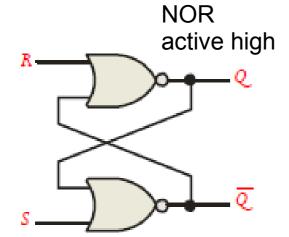
Note:

1) choice of T not easy ... failure to find good solution can result in catastrophes

2) synchronization failure can be avoided internally in clocked (or gated) systems

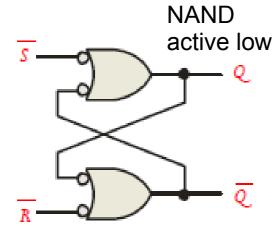


SR Flip-Flop – Short summary



Inp	uts	Output s		Mode	Comment
S	R	Q	$ar{oldsymbol{Q}}$	of Operation	Confinent
0	0	NC	NC	Hold	No change.
0	1	0	1	Reset	For RESET ting Q to 0
1	0	1	0	Set	For SETting Q to 1
1	1	0	0	Prohibited	Invalid Condition

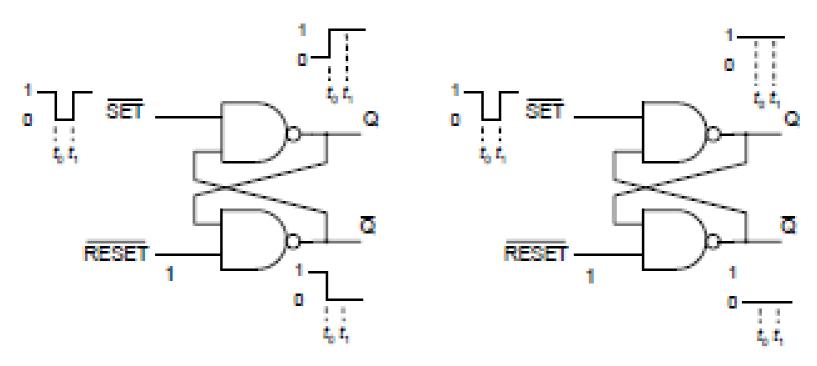
both active



Inp	uts	Out	puts	Mode	Commont
5	\overline{R}	Q	Q	of Operation	Comment
0	0	1	1	Prohibited	Invalid Condition
0	1	1	0	Set	For SETting Q to 1
1	0	0	1	Reset	For RESETting Q to 0
1	1	NC	NC	Hold	No change.

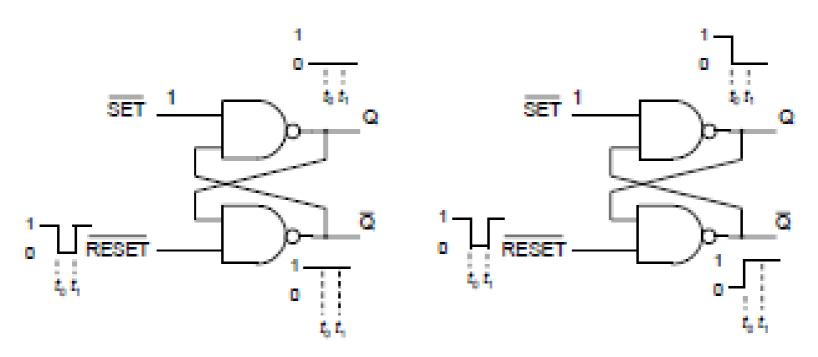
both active

SR Latch – Set operation



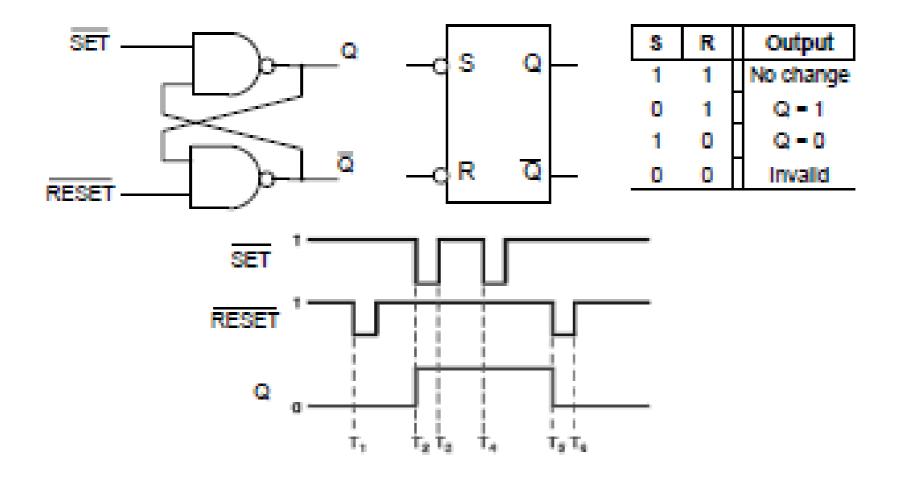
A negative pulse on SET puts the latch in HIGH (SET) state

SR Latch – ReSet operation



A negative pulse on RESET puts the latch in LOW (RESET) state

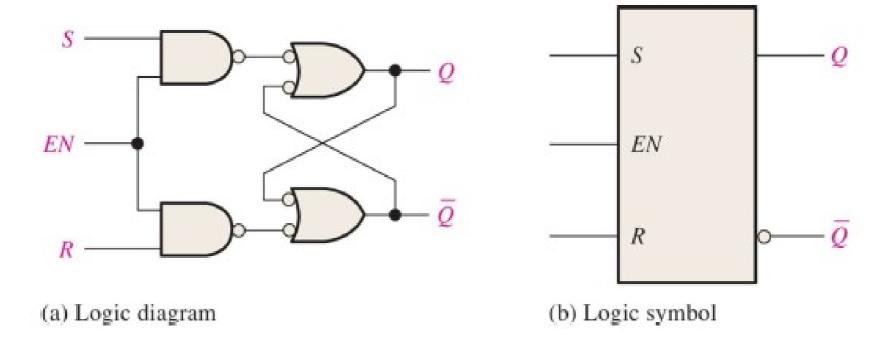
SR Latch – Set-Reset operation & Truth table



Gated Latch

A gated latch is a variation on the basic flip-flop

The gated latch has an additional input, called enable (EN) that must be HIGH in order for the latch to respond to the S and R inputs (active LOW)

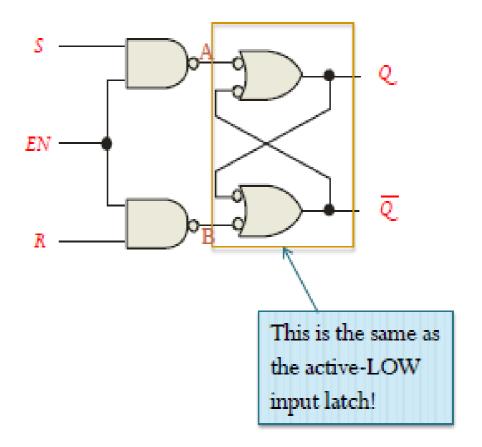


Gated Latch

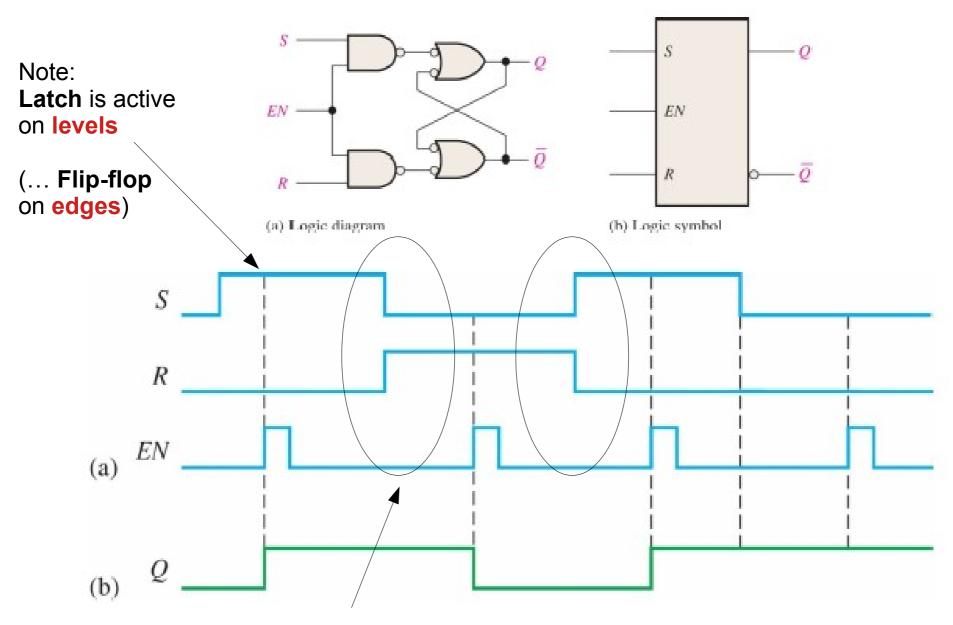
Observe that:

$$A = \overline{S \cdot EN} = \overline{S} + \overline{EN}$$
$$B = \overline{R \cdot EN} = \overline{R} + \overline{EN}$$

EN	Α	В
0 ⇔	1	1
1 ⇒	S	\overline{R}



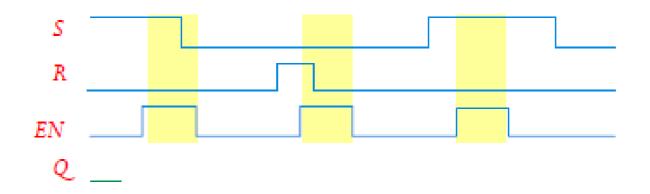
Example - Gated S-R Latch

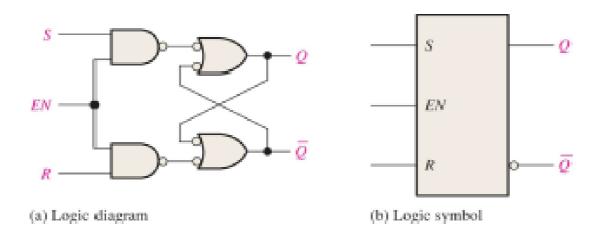


Note: Arbitration conflicts can be avoided

Exercise - Gated S-R Latch

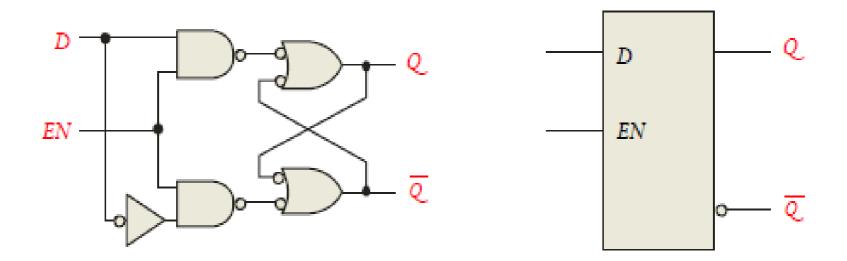
Show the Q output with relation to the input signals. Assume Q starts LOW.



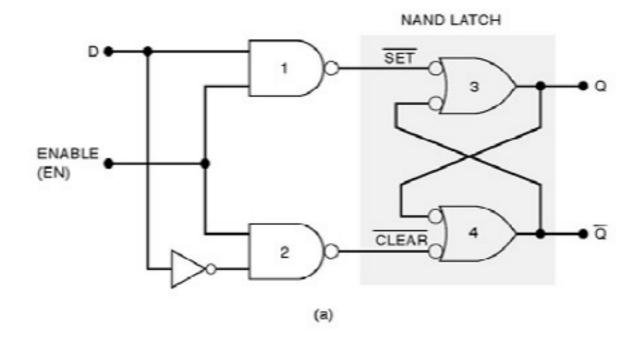


(Gated) D-Latch – Transparent Latch

- The D latch is a variation of the S-R latch.
- Has only one input in addition to EN.
 - This input is called the D (data) input.
- Combine the S and R inputs into a single D input.



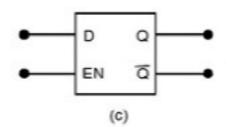
Gated D Latch



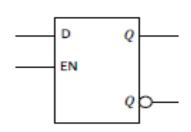
Inp	uts	Output	
EN	D	Q	
0	X	Q ₀ (no change)	
1	0	0	
1	1	1	

"X" indicates 'don't care'
Q₀ is state Q just
prior to EN going LOW

(b)



Gated D Latch



- A simple rule for the D latch is:
 - Q follows D when the Enable is active/asserted.
 - In this situation, the latch is said to be "open" and the path from D input to Q output is "transparent".
 - The circuit is often called a transparent latch for this reason.
- When EN is LOW, the state of the latch is not affected by the D input.
 - In this situation, the latch is said to be "close"
 - The Q output retains its last value and no longer changes in response to D, as long as EN remains negated.

Output is "latched" at the last value when the enable signal

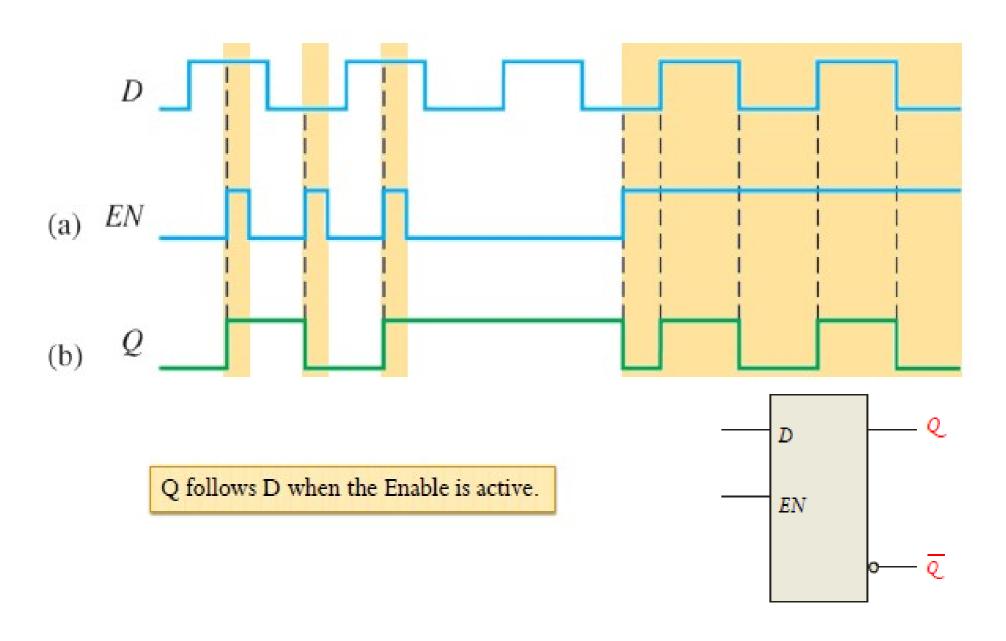
becomes not asserted.

Truth Table: —————

Q₀ is the prior output level before the indicated input conditions were established.

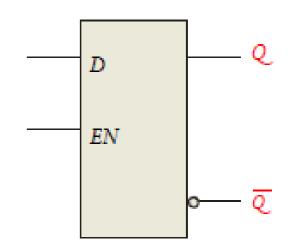
Inputs		Outputs		
D	EN	Q	Q	Comments
0	1	0	1	RESET
1	1	1	0	SET
Χ	0	Q ₀	\overline{Q}_{0}	No change

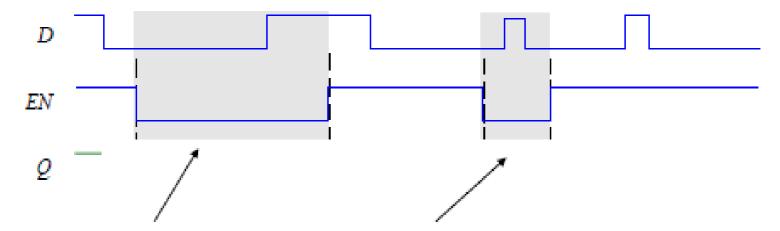
Example - Gated D Latch



Exercise - Gated D Latch

Determine the Q output for the D latch, given the inputs shown.

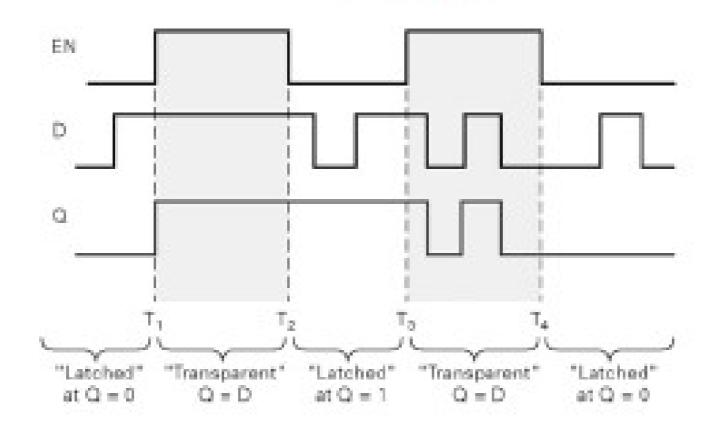




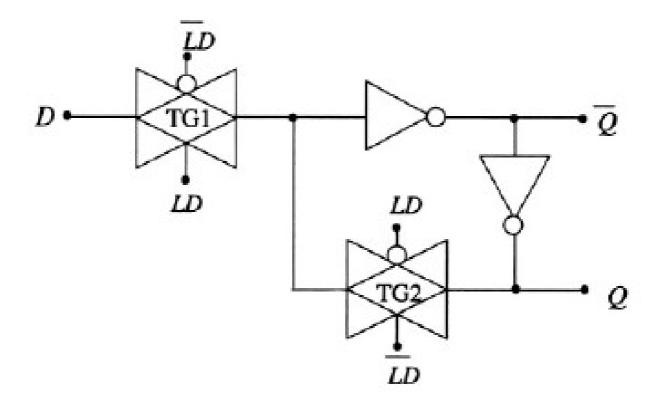
Notice that the Enable is not active during these times, so the output is latched.

Gated D Latch timing

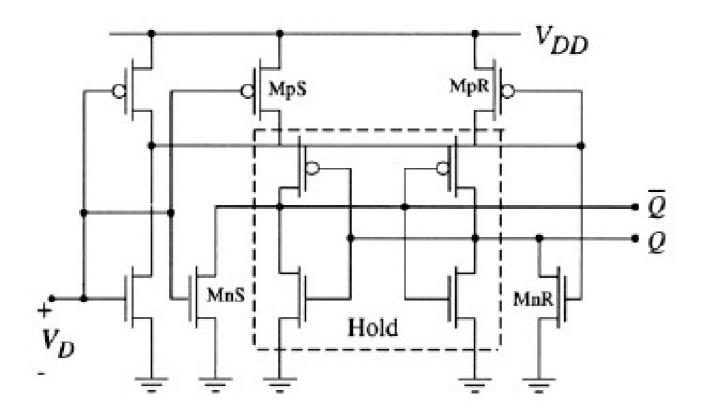
D latch timing



Gated D Latch circuit – Pass-Transistor Logic



Gated D Latch circuit - CMOS

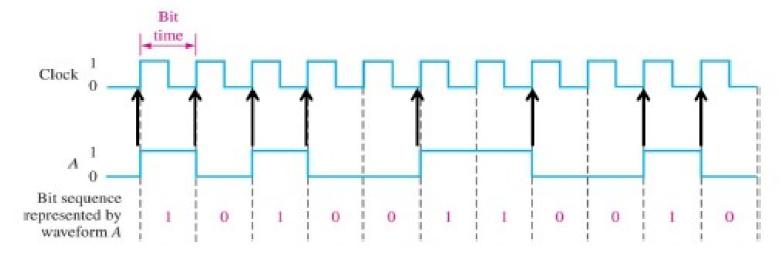


Synchronous Flip-Flops

- Clocked Flip-Flops and Synchronous systems
- Edge-Triggered Flip-Flops
- D Flip-Flops
- Shift Registers
- JK Flip-Flop
- Counters

Clocks (CLK)

- In digital synchronous systems, all waveforms are synchronized with a clock.
 - The clock waveform itself does not carry information.
- The clock is a periodic waveform in which each interval between pulses (the period) equals the time for one bit.



 Notice that change in level of waveform A occurs at the rising edge of the clock waveform.

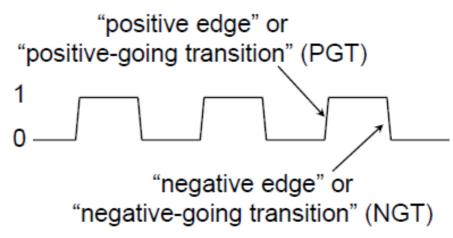
Note on "async" clocks

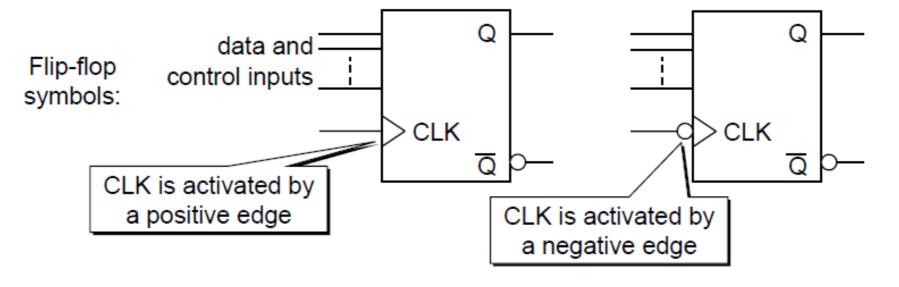
In some cases simple square pulses (either in regular trains or even in irregular sequences) can be used for synchronization ...

Clock signals and Flip-Flops

Synchronous digital systems:

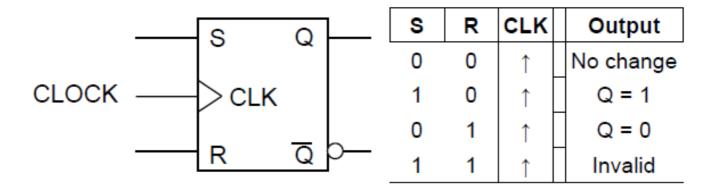
- the state bits of the circuit all change simultaneously
- the changes occur at fixed points in time
- the control signal which indicates it is time to change is called the clock



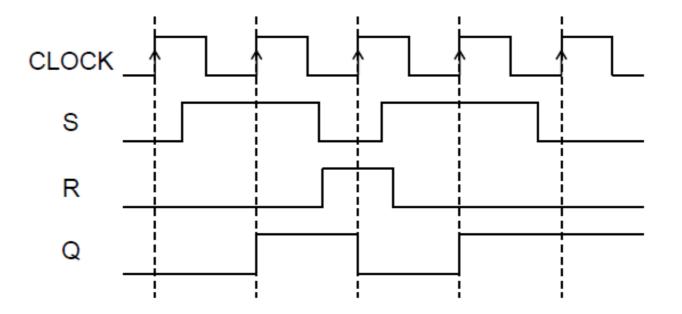


Set-Reset Flip-Flop

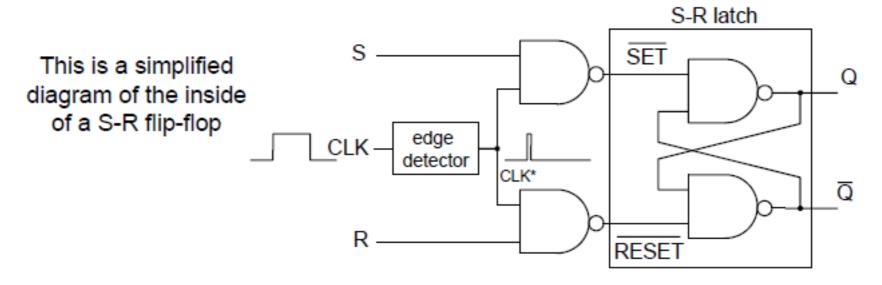
Clocked flip-flop: the output only changes at the positive edges of the clock



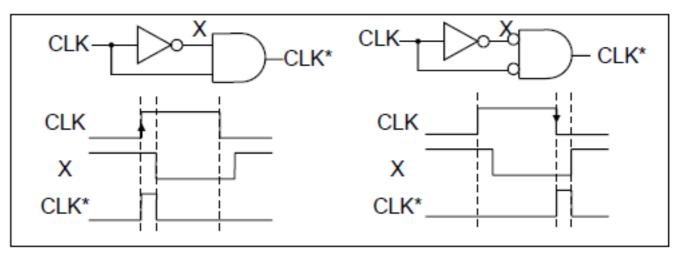
Flip-flops are sometimes called edge-triggered



Set-Reset Flip-Flop – internal circuitry



Edge detector circuits for both positive-edge and negative-edge flip-flops

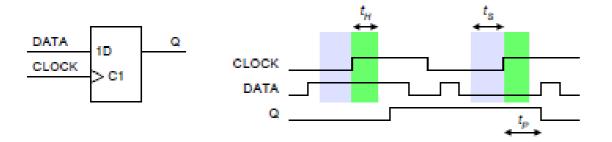


Clocked Flip-Flops – time constraints

Synchronization failure is avoided in clocked digital systems (synchronous) by

- gating all flip-flops with an (ideally) square-wave clock signal C and
- demanding that inputs other than the clock be kept steady for
 a setup time t_s before a clock transition that might cause a metastable condition
- demanding that inputs remain steady for a hold time t_H after the clock transition

The DATA input to a flipflop or register must not change at the same time as the CLOCK.



Typical values for a register: $t_S = 5$ ns, $t_H = 3$ ns

The setup and hold time define a window around each CLOCK ↑ edge within which the DATA must not change.

If these requirements are not met, the Q output may oscillate for many nanoseconds before settling to a stable value.

Setup Time: DATA must reach its new value at least

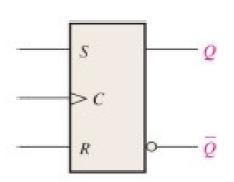
 t_S before the CLOCK \uparrow edge.

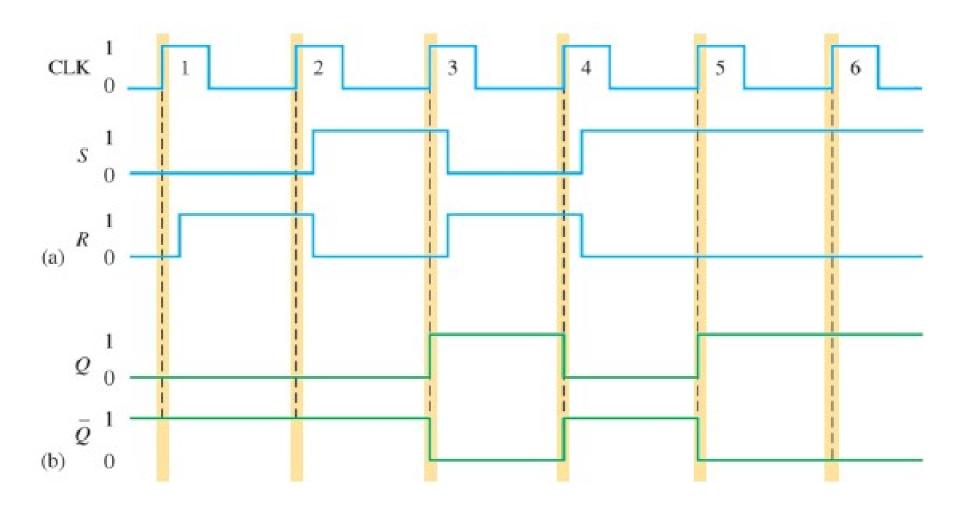
Hold Time: DATA must be held constant for at least

 t_H after the CLOCK \uparrow edge.

S-R type Flip-Flop

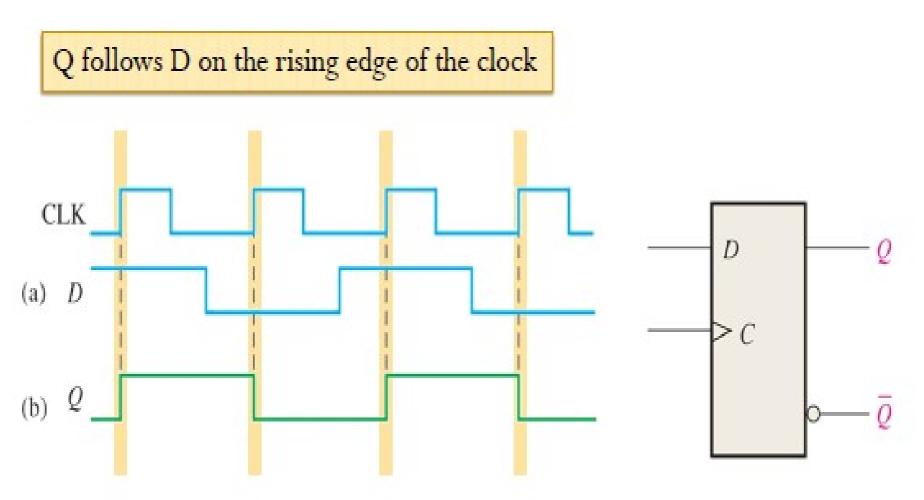
SR-Latch gated with a clock and including edge detector circuit → gives a SR-Flip-Flop





D type Flip-Flop

D-Latch gated with a clock and including edge detector circuit → gives a D-Flip-Flop



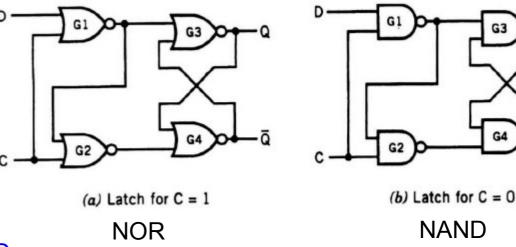
Let's discuss in more detail about timing issues ...

Example of clocked flip-flop: NOR / NAND D-Latch

NOR D-Latch:

When the clock (C) is 0, G1 and G2 act like inverters → the output Q of RS flip-flop G3|G4 follows the data input D

When the clock goes to 1 (C=1) the outputs of G1 and G2 \rightarrow 0 and Q registers the current state of D (latch for C=1)



- The setup time is t_s =2t when D goes 1 \rightarrow 0 because the output of G1 must be kept at 1 for a time 2t to ensure that G3|G4 is internally locked at Q = 0 In contrast, t_s =3t when D goes 0 \rightarrow 1 because of the extra delay in G2
- \bullet The hold time is t_H =0 because D loses control over G1 and G2 as soon as C=1

NAND D-Latch:

Note: the NAND latch registers the current state of D when C=0 (latch for C=0)

- t_s =3t when D goes from 1 \rightarrow 0, and t_s =2t when D goes 0 \rightarrow 1
- $t_H = 0$

Note: behaves similar to a Gated D-latch (but different timing diagrams)

Example of clocked flip-flop: edge-triggered FF

Edge-triggered flip-flops change state only at one of the clock signal transitions (We will normally take the transition $0 \rightarrow 1$ as the active one – rising edge)

The state of the output after the clock depends on the state of the output and the inputs just prior to the clock

- Time contraints
- (1) Clock must remain active for a time t_w (minimum clock-pulse width defined by the edge detection circuitry) Outside the interval defined by the setup and hold times, the inputs can change arbitrarily
- (2) Changes of state if any, occur a propagation time t_p after the clock Must be $t_w < t_p$ otherwise it would be impossible to build systems in which the response of a given flip-flop depends on the state of the flip-flop itself or other flip-flops at the time of the clock transition
- An edge-triggered flip-flop is described by an equation (or a truth table) that gives the output during clock period n + 1 as a function of the values of the inputs and the output during clock period n

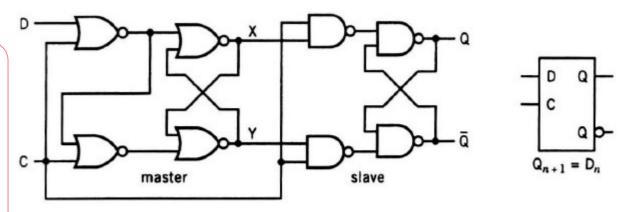
$$Q_{n+1} = F(inputs_n)$$

Example: the ubiquitous "D" type flip-flop, is governed only by its single data input D: $\mathbf{Q}_{n+1} = \mathbf{D}_n$

Example of clocked flip-flop: edge-triggered FF the "master-slave D FF"

The master–slave edge-triggered D flip-flop consists of a master NOR latch followed by a slave gated NAND latch

- When C=0 the master follows D
 → and we have X = D and Y = D
- When C flips 0→1, these values are held by the master and transmitted to the slave
 O thus takes on the value that
- → Q thus takes on the value thatD had just prior to the clock



Master-slave edge-triggered D flip-flop, its symbol and equation

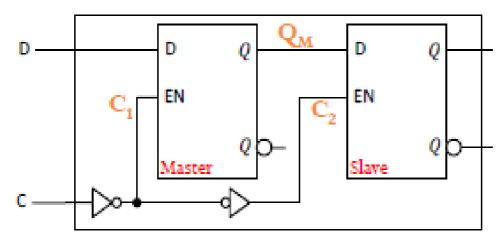
When C flips back 1→0, the slave continues to hold the current value of Q while the master accepts new data

The setup and hold times are the same as in the NOR latch The propagation time is 2t for the output when $0\rightarrow 1$ and 3t for the output when $1\rightarrow 0$

Note: two (gated) D-latches make a Master-Slave D-type Flip-Flop

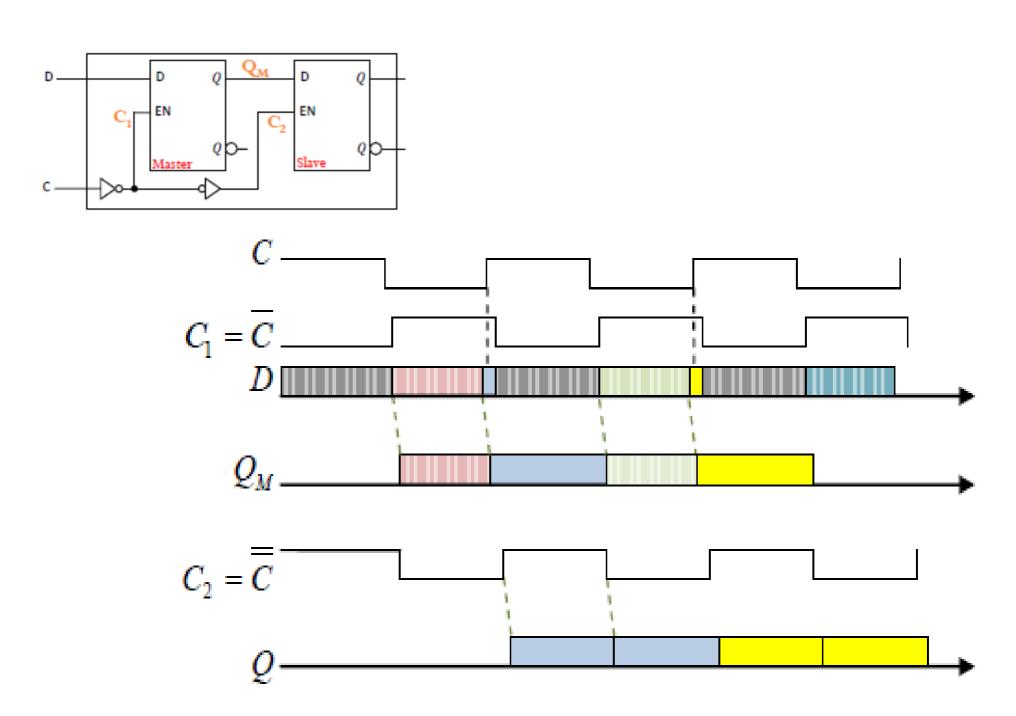
Master-Slave D Flip-Flop

Tie two D-latches together to make a D flip-flop



- When C is 0 (C₁ = 1), the master latch is open and follows the D input.
- When C is 1 (C₁ = 0, C₂ = 1), the master latch is closed and its output is transferred to the slave latch.
 - The slave latch is open all the while that C is 1, but changes only at the beginning of this interval, because the master is closed and unchanging during the rest of the interval.

Master-Slave D Flip-Flop



Example of clocked flip-flop: edge-triggered FF the "toggle flip-flop"

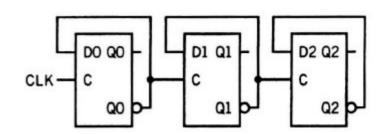
• If we connect the <u>Q</u> output of a D flip-flop to the D input, we obtain a toggle flip-flop that changes state at every (active) clock transition and therefore divides the frequency of the clock signal by 2.

Its equation is
$$Q_{n+1} = \overline{Q}_n$$

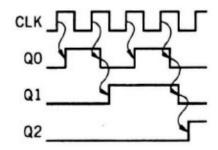
Note: that a toggle flip-flop is the ultimate example of why propagation times must be longer than hold times (t_w clock) – otherwise metastable state for D

• If we now chain together N toggle flip-flops by tying the images output of one flip-flop to the C input of the next, we obtain the divide-by-2N ripple counter (see figure for N = 3)

Although not strictly synchronous, the ripple counter is nonetheless widely used because it is simple and, having no inputs, has no problems with setup or hold times



The drawback it does have, however, is that it can take up to N propagation times to change from one state to the next

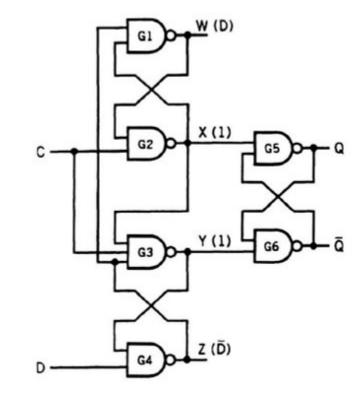


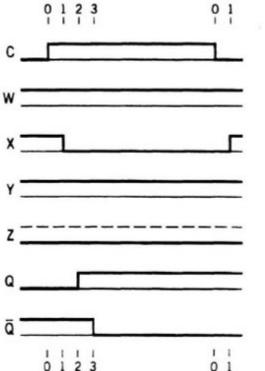
Example of clocked flip-flop: edge-triggered FF the "D flip-flop" LS-TTL 74LS74

Flip-flops G1|G2 and G3|G4 are both arbitrators which register the value of D when $C\rightarrow 1$

- The values in parentheses indicate the state of the internal nodes when C=0
- The waveforms (below) correspond to the case D = 1 and Q = 0

Note: drawing waveforms is often the easiest way to understand how digital circuits work, particularly when feedback is involved





If **D** = 1, we have W = 1 and Z = 0 when C = 0; then X \rightarrow 0 one gate delay τ after C 0 \rightarrow 1 and confirms W = 1 and Y = 1. After this time, Z can change without affecting X and Y (\rightarrow t_H=0) X must remain =0 for at least two gate delays in order to lock output Q of flip-flop G5|G6 at 1

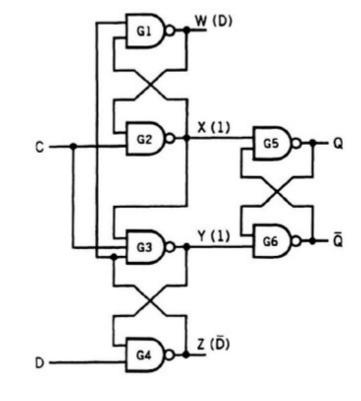
 \rightarrow The minimum clock-pulse width t_{w} is thus 2t

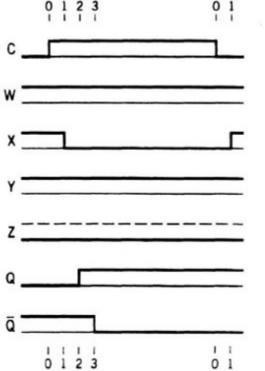
Example of clocked flip-flop: edge-triggered FF the "D flip-flop" LS-TTL 74LS74

Flip-flops G1|G2 and G3|G4 are both arbitrators which register the value of D when $C\rightarrow 1$

- The values in parentheses indicate the state of the internal nodes when C=0
- The waveforms (below) correspond to the case D = 1 and Q = 0

Note: drawing waveforms is often the easiest way to understand how digital circuits work, particularly when feedback is involved

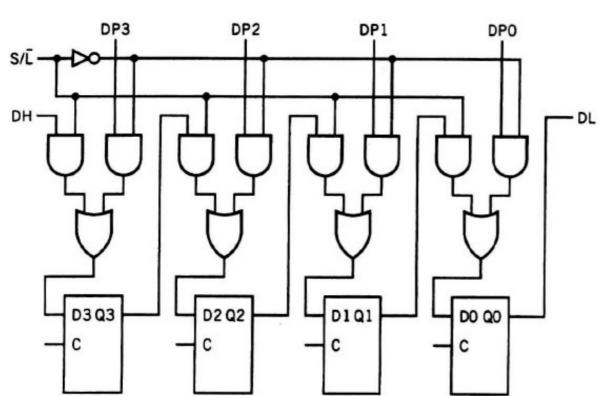




If **D** = **0**, we have W = 0 and Z = 1 when C = 0; in this case, $Y \rightarrow 0$ one gate delay τ after the clock transition and confirms Z = 1, W = 0 and X = 1

The hold time t_H is τ because D must remain steady until Y becomes 0. The setup time t_s is 2t, and the propagation time t_p to the Q output is 2t if Q goes from $0 \rightarrow 1$ and 3t if Q goes from $1 \rightarrow 0$

Example use of D flip-flop in synchronous system a 4-bit shift-register



The AND–OR gates are simple examples of multiplexers, that is, of circuits that route one of several inputs to their output according to the setting of their address or control inputs.

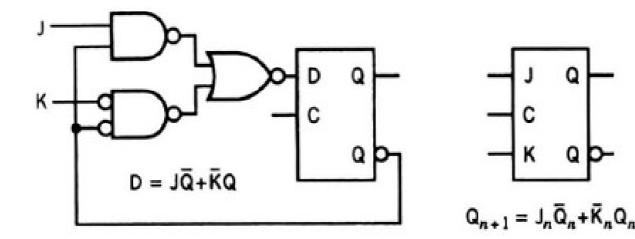
In this case, the only control input is S/L (SHIFT/LOAD): it determines whether the D input of a given flip-flop is the Q output of the flip-flop of the next-highest order (DH in the case of D3), or whether it is the corresponding bit of the 4-bit word DP3 ... DP0

- If S/L=1, the 4-bit word in the shift register is shifted one position to the right at the clock transition; DH is copied into Q3, and DL can be copied into the DH input of a lower-order word. Q3 is copied into itself if it is tied to DH; we can thus obtain an arithmetic shift-right or divide-by-2 circuit.
- If S/L=0, new parallel data are copied into the shift register (register load)

JK flip-flop – yet another clocked flip-flop

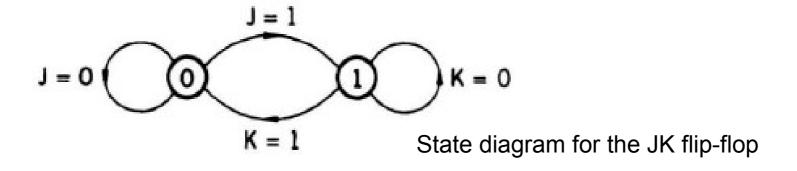
Less common than D flip-flops, but highly useful because of their versatility, are JK flip-flops with inputs J and K; their equation is $Q_{n+1} = J_n \overline{Q}_n + \overline{K}_n Q_n$

A JK flip-flop can be obtained by making the D input of a D flip-flop = JQ+KQ



J _n	K _n	Q_{n+1}
0	0	Q,
0	1	0
1	0	1
1	1	\bar{Q}_n

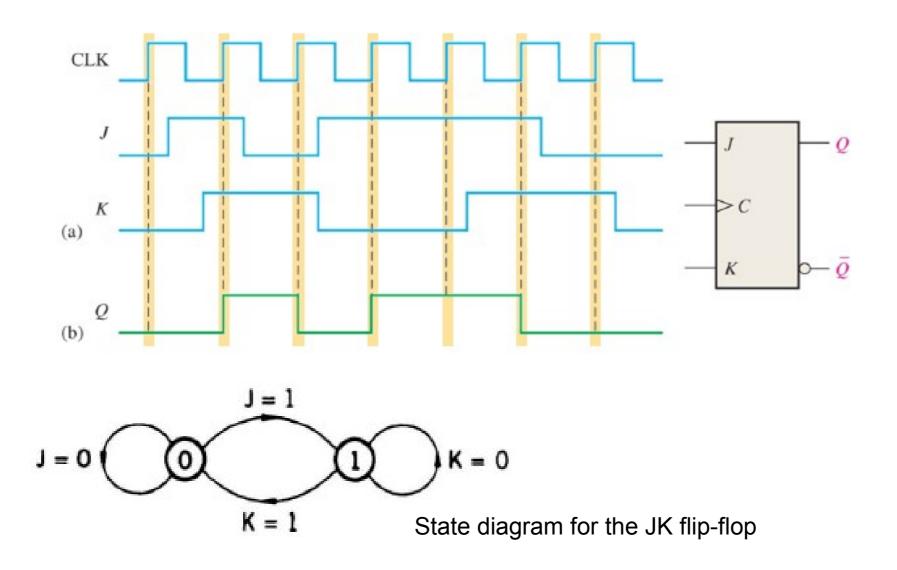
A JK flip-flop changes state if J and K are both equal to 1 and remains in the same state if J and K are both equal to 0



JK flip-flop – yet another clocked flip-flop

Less common than D flip-flops, but highly useful because of their versatility, are JK flip-flops with inputs J and K; their equation is $Q_{n+1} = J_n \overline{Q}_n + \overline{K}_n Q_n$

A JK flip-flop can be obtained by making the D input of a D flip-flop = JQ+KQ

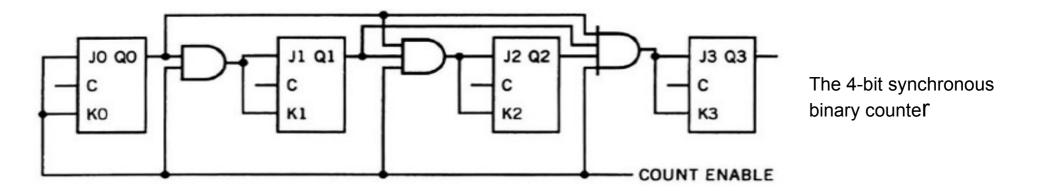


Example of use of JK flip-flop: 4-bit counter

If COUNT ENABLE = 0, the J and K inputs to all flip-flops are 0, and the counter remains wherever it is

If COUNT ENABLE = 1, the lowest-order bit Q0 toggles at every clock, and in each succeeding position the AND gates allow the bit to toggle only if all preceding bits are equal to 1

→ the counter thus cycles through the sequence 0, 1,..., 15, 0, and so on

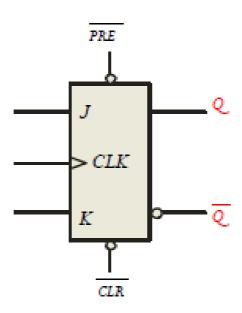


By exploiting a more complicated gating scheme, the counter can also count down, and (like the 4-bit shift register see previously) and can be set to an arbitrary value

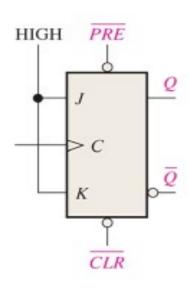
Note: loading a parallel word and then counting down to zero is useful when generating pulses of variable length because only one gate is needed to detect when the counter arrives at zero... in contrast, counting up to a given value requires a complete magnitude comparator (much more gates needed!)

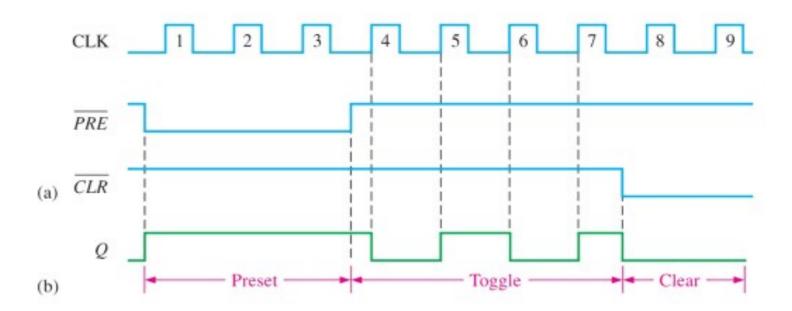
JK flip-flop with synchronous inputs

- Most flip-flops have other inputs that are asynchronous, meaning they affect the output independent of the clock.
- Two such inputs are normally labeled preset (PRE) and clear (CLR).
- These inputs are usually active-LOW.
- A J-K flip flop with active-LOW preset and CLR is shown.

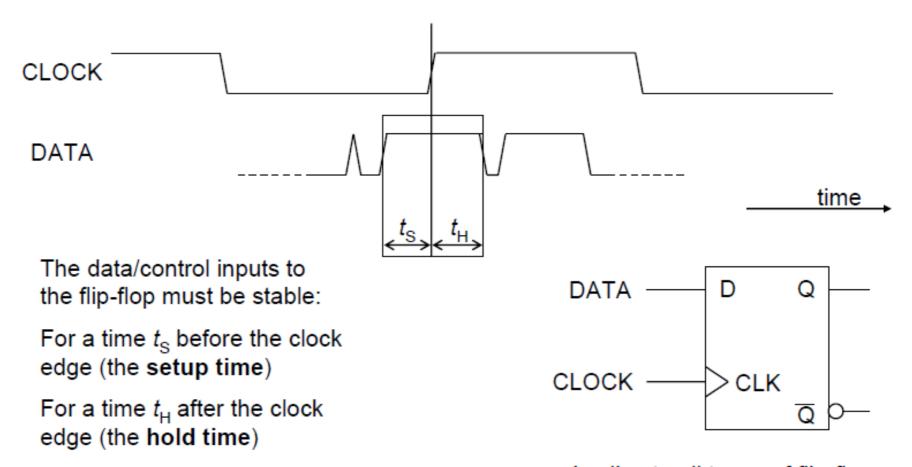


JK flip-flop with synchronous inputs: example





Setup and Hold times



Applies to all types of flip-flop

Note: Clock Skew

We learned that input must not change values at the moment that the clock is rising as this is the time that the flip-flops **read** the input values. Ok, but ...

... we have so far made the implicit assumption that there is no clock skew, that is, that the clock transition is fast, that it arrives simultaneously at all flip-flops and all of them have the same threshold voltage

Clock skew is never zero, but it must nonetheless be small enough to prevent one flip-flop from firing so far ahead of another that it changes state before the other has made a decision; in the worst of cases, a late clock can make a flip-flop become metastable

A slow clock rise time can result in a large clock skew if the scatter in the threshold voltages of different flip-flops is large; this problem particularly affects CMOS circuits, whose threshold voltages can lie anywhere between one-third and two-thirds of the power-supply voltage.

Clock skew also occurs if there are delays in the clock distribution circuit due either to gates or to transmission lines; this problem is of particular concern in ECL circuits