Quantum Information and Computing 2021-2022

8^{th} week assignment

Real State Renormalization Group

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- the transverse-field Ising model of N $\frac{1}{2}-$spins is described by the Hamiltonian

$$\hat{H}_N = -\sum_{i=1}^{N} \sigma_i^x \sigma_{i+1} + \lambda \sum_{i=1}^{N} \sigma_i^z, \quad \hat{H}_N : \mathbb{C}^{d^N} \to \mathbb{C}^{d^N}, \quad d = 2$$

  where λ is the magnitude of the magnetic field applied along $\hat{z}$
- the notation used implies that

$$\sigma_i^x \sigma_{i+1} = \mathbb{1}_1 \otimes \cdots \otimes \mathbb{1}_{i-1} \sigma_i^x \otimes \sigma_{i+1} \otimes \mathbb{1}_{i+2} \cdots \otimes \mathbb{1}_N$$

$$\sigma_i^z = \mathbb{1}_1 \otimes \cdots \otimes \mathbb{1}_{i-1} \otimes \sigma_i^z \otimes \mathbb{1}_{i+1} \cdots \otimes \mathbb{1}_N$$

- to retrieve the Ground State energy density of this system (in the thermodynamic limit) the Real State Renormalization Group (RSRG) algorithm is exploited
- the RSRG algorithm is based on the hypothesis that the ground state of the global system is composed by the lowest energy eigenstates of its sub-partitions. This is trivial in the case in which the sub-systems of which it is composed are not interacting.

Given the Hamiltonian $\hat{H}_N$ of an $N-$units system, the RSRG algorithm iterations follows this steps:

- build the Hamiltonian $\hat{H}_{2N}$ of the doubled system as

$$\hat{H}_{2N} = \hat{H}_N \otimes \mathbb{1}_N + \mathbb{1}_N \otimes \hat{H}_N + \hat{H}_{int}, \quad \hat{H}_{2N} : \mathbb{C}^{d^{2N}} \to \mathbb{C}^{d^{2N}}$$

  with

$$H_{int} = \hat{H}_L + \hat{H}_R$$

  that for the Transverse Field Ising model reads as

$$\hat{H}_{int} = \left[\bigotimes_{1}^{N-1} \mathbb{1} \otimes \sigma^x \right] \otimes \left[\sigma^x \otimes \bigotimes_{1}^{N-1} \mathbb{1} \right]$$

- diagonalize $\hat{H}_{2N}$ and construct the projector $\hat{P} : \mathbb{C}^{d^{2N}} \to \mathbb{C}^{d^N}$ along the d^N lowest eigenvectors
- retrieve the truncated Hamiltonian of the doubled system

$$\tilde{H}_{2N} = \hat{P}^\dagger \hat{H}_{2N} \hat{P}, \quad \mathbb{C}^{d^N} \to \mathbb{C}^{d^N}$$

  and the truncated interaction terms

$$\tilde{H_L} = P^\dagger \left(\bigotimes_{1}^{N} \mathbb{1} \otimes H_L \right) \hat{P}, \quad \tilde{H_R} = P^\dagger \left(H_R \otimes \bigotimes_{1}^{N} \mathbb{1} \right) \hat{P}$$

- iterate until the desired number of iterations/convergence parameter is reached

Code

```
subroutine RSRG_iteration(H_N, H_L, H_R, N)

    double complex :: H_N(:,:), H_L(:,:), H_R(:,:)
    integer        :: N
    double complex :: H_2N(2**(2*N), 2**(2*N))
    double complex :: PJ(2**(2*N), 2**N)
    double complex :: PJ_ADJ(2**N, 2**(2*N))
    double complex :: H_L_int(2**(2*N), 2**(2*N)), H_R_int(2**(2*N), 2**(2*N))
    double complex :: eigvctrs(2**(2*N), 2**(2*N))
    real(8)        :: eigvls(2**(2*N))


    H_2N =  kronecker_product_cmplx( H_N, ising_identity(N) ) +&
            kronecker_product_cmplx( ising_identity(N), H_N ) +&
            kronecker_product_cmplx( H_L, H_R )


    call ising_diagonalize_hamiltonian(H_2N, eigvctrs, eigvls)

    PJ = eigvctrs(:, :2**N)
    PJ_ADJ = ADJOINT( PJ )

    H_N = MATMUL ( MATMUL( PJ_ADJ, H_2N ), PJ)

    H_L_int = kronecker_product_cmplx( ising_identity(N), H_L )
    H_R_int = kronecker_product_cmplx( H_R , ising_identity(N) )

    H_L = MATMUL( MATMUL( PJ_ADJ , H_L_int), PJ)
    H_R = MATMUL( MATMUL( PJ_ADJ , H_R_int), PJ)

end subroutine
```

Figure: RSRG algorithm implementing subroutine

- The results reported are evaluated for a starting number of spins $N = 2, 3, 4$ and $n_{iters} = 100$
- the displayed quantity is the ground-state energy density $\rho_{GS} = \frac{E_{GS}}{1+d^{n_{iters}}}$, where E_{GS} is the Ground State energy obtained diagonalizing the truncated Hamiltonian obtained after the last iteration
- the comparison with the mean-field result is performed

$$\rho_{GS} = \begin{cases} -1 - \frac{\lambda^2}{4}, & \lambda \in [-2, 2] \\ |\lambda|, & \lambda \notin [-2, 2] \end{cases}$$