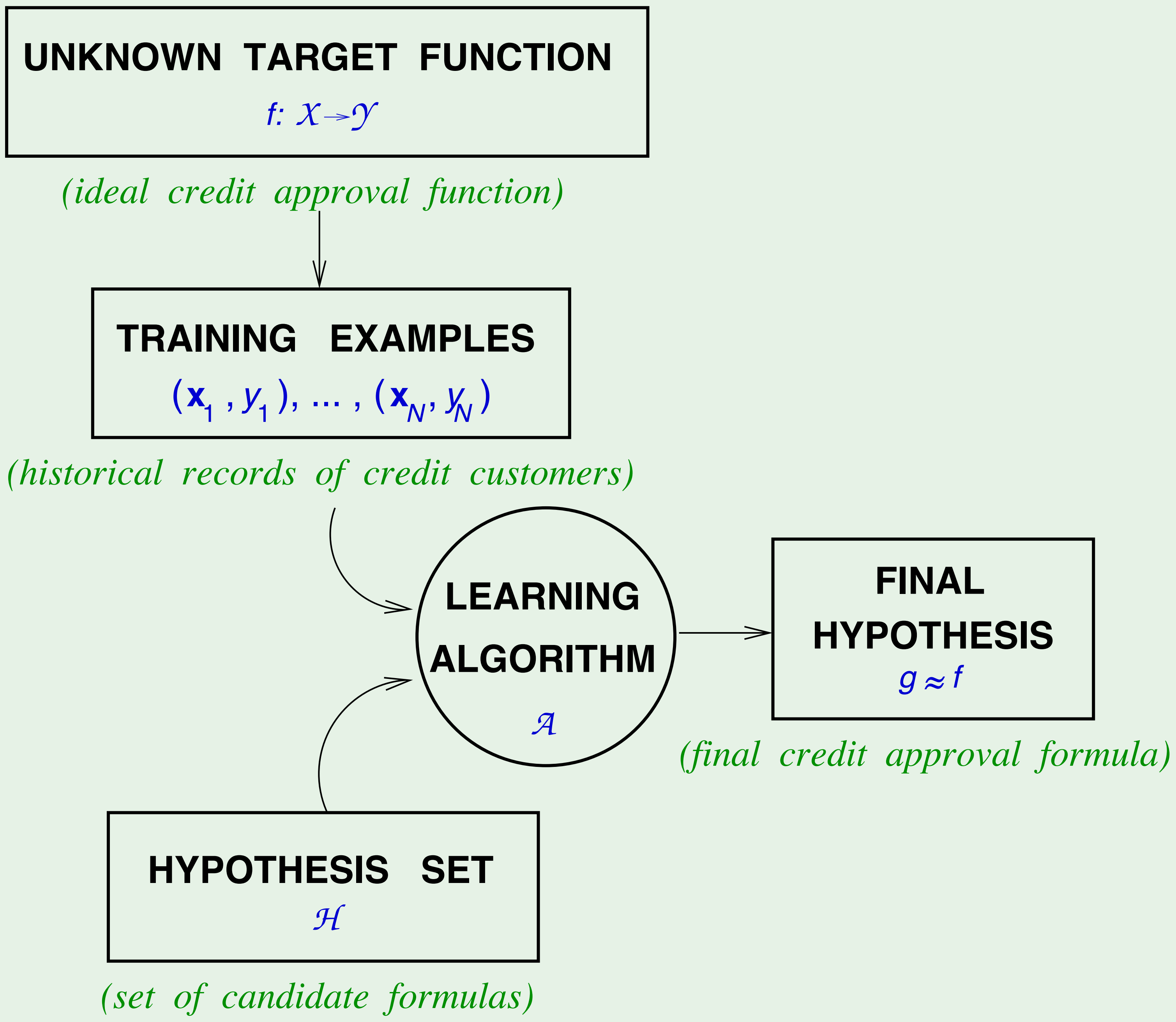
Components of learning

Formalization:

- Input: $\mathbf{x}$ *(customer application)*
- Output: y *(good/bad customer?)*
- Target function: $f : \mathcal{X} \to \mathcal{Y}$ *(ideal credit approval formula)*
- Data: $(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \cdots, (\mathbf{x}_N, y_N)$ *(historical records)*

$$\downarrow \quad \downarrow \quad \downarrow$$

- Hypothesis: $g : \mathcal{X} \to \mathcal{Y}$ *(formula to be used)*



UNKNOWN TARGET FUNCTION
$f: \mathcal{X} \to \mathcal{Y}$
(ideal credit approval function)
TRAINING EXAMPLES
$(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_N, y_N)$
(historical records of credit customers)
LEARNING ALGORITHM
$\mathcal{A}$
FINAL HYPOTHESIS
$g \approx f$
(final credit approval formula)
HYPOTHESIS SET
$\mathcal{H}$
(set of candidate formulas)

Solution components

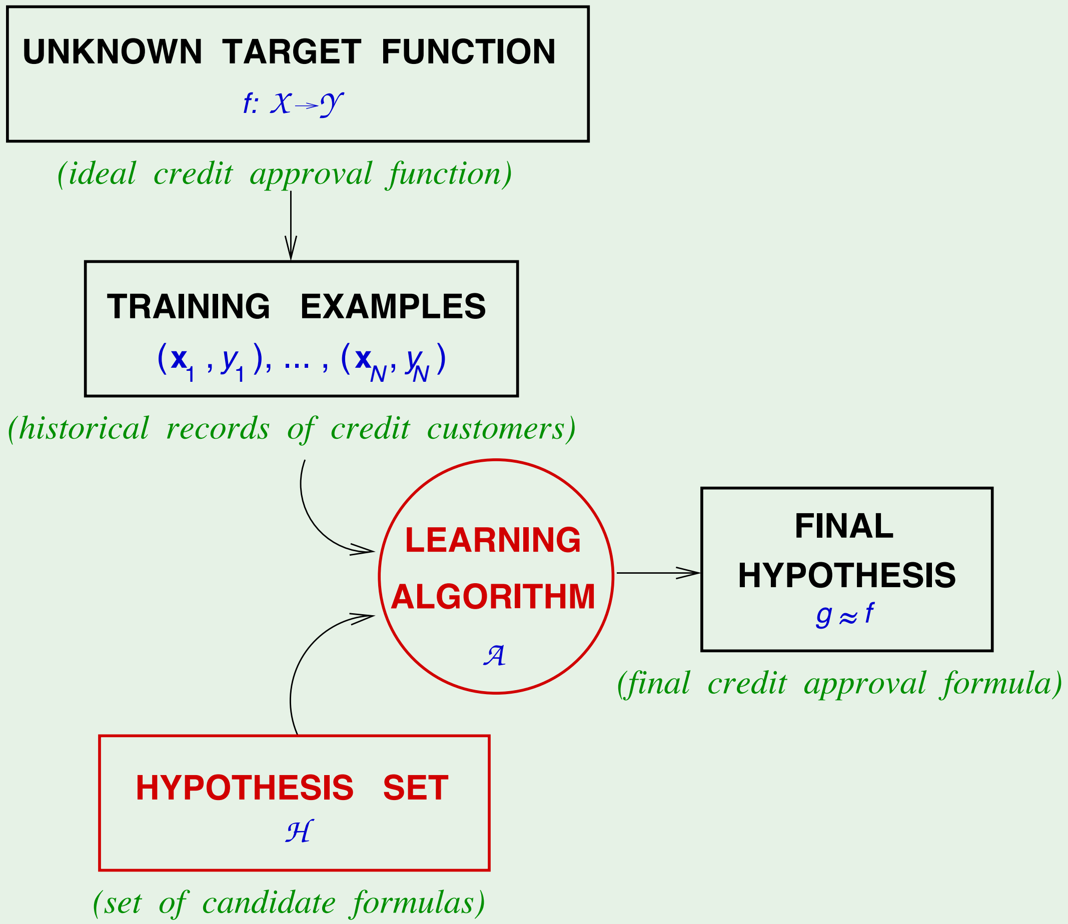
The 2 solution components of the learning problem:

- The Hypothesis Set

  $\mathcal{H} = \{h\} \qquad g \in \mathcal{H}$

- The Learning Algorithm

Together, they are referred to as the *learning model*.



Error measures

What does "$h \approx f$" mean?

Error measure: $E(h, f)$

Almost always *pointwise definition*: $\mathrm{e}\left(h(\mathbf{x}), f(\mathbf{x})\right)$

Examples:

Squared error: $\mathrm{e}\left(h(\mathbf{x}), f(\mathbf{x})\right) = \left(h(\mathbf{x}) - f(\mathbf{x})\right)^2$

Binary error: $\mathrm{e}\left(h(\mathbf{x}), f(\mathbf{x})\right) = [\![h(\mathbf{x}) \neq f(\mathbf{x})]\!]$

From pointwise to overall

Overall error $E(h, f)$ = average of pointwise errors $\mathrm{e}\left(h(\mathbf{x}), f(\mathbf{x})\right)$.

In-sample error:

$$E_{\mathrm{in}}(h) = \frac{1}{N}\sum_{n=1}^{N} \mathrm{e}\left(h(\mathbf{x}_n), f(\mathbf{x}_n)\right)$$

Out-of-sample error:

$$E_{\mathrm{out}}(h) = \mathbb{E}_{\mathbf{x}}\left[\mathrm{e}\left(h(\mathbf{x}), f(\mathbf{x})\right)\right]$$

A simple hypothesis set - the 'perceptron'

For input $\mathbf{x} = (x_1, \cdots, x_d)$ 'attributes of a customer'

$$\text{Approve credit if} \quad \sum_{i=1}^{d} w_i x_i > \text{threshold},$$

$$\text{Deny credit if} \quad \sum_{i=1}^{d} w_i x_i < \text{threshold}.$$

This linear formula $h \in \mathcal{H}$ can be written as

$$h(\mathbf{x}) = \text{sign}\left(\left(\sum_{i=1}^{d} w_i x_i\right) - \text{threshold}\right)$$

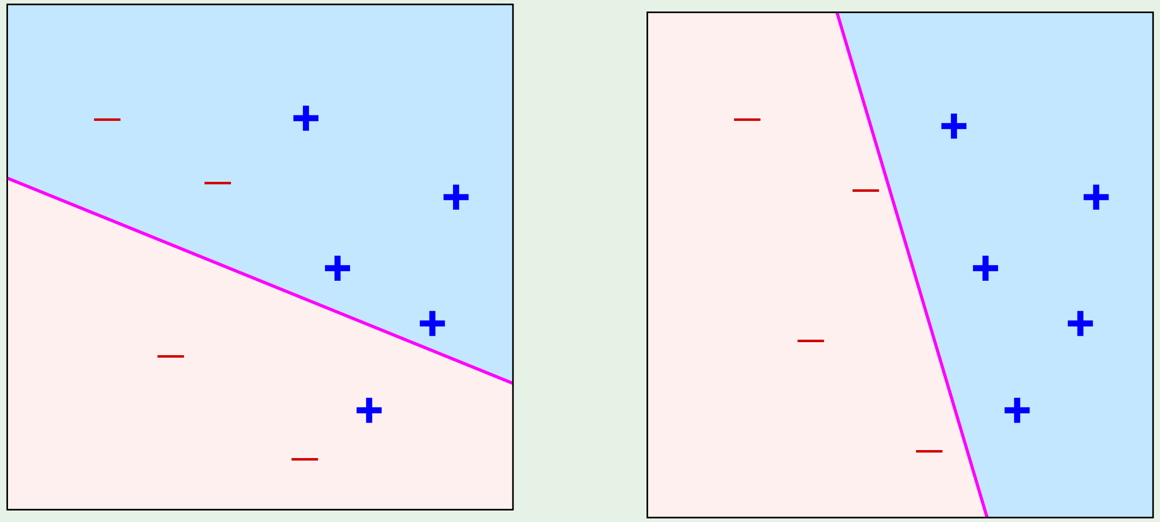
$$h(\mathbf{x}) = \text{sign}\left(\left(\sum_{i=1}^{d} w_i\ x_i\right) + \quad w_0 \quad\right)$$

Introduce an artificial coordinate $x_0 = 1$:

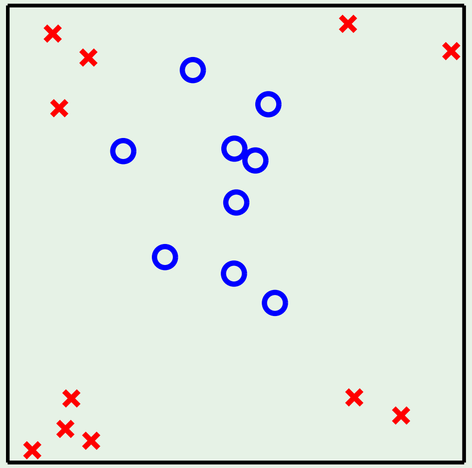
$$h(\mathbf{x}) = \text{sign}\left(\sum_{i=0}^{d} w_i\ x_i\right)$$

In vector form, the perceptron implements

$$h(\mathbf{x}) = \text{sign}(\mathbf{w}^{\mathsf{T}}\mathbf{x})$$

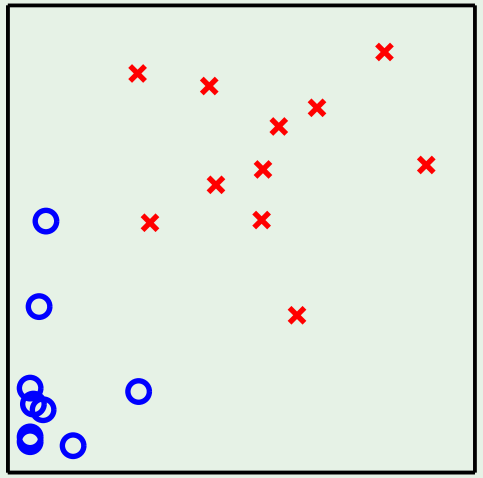


'linearly separable' data



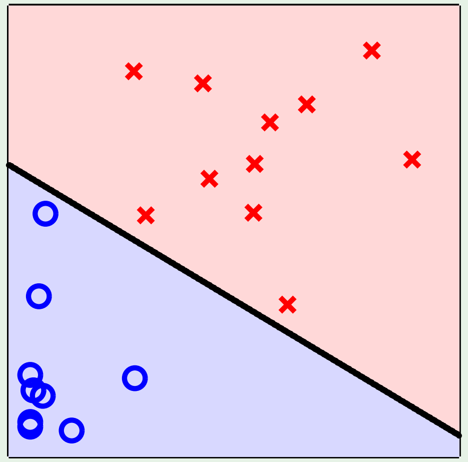
1. Original data
$\mathbf{x}_n \in \mathcal{X}$

$\xrightarrow{\Phi}$



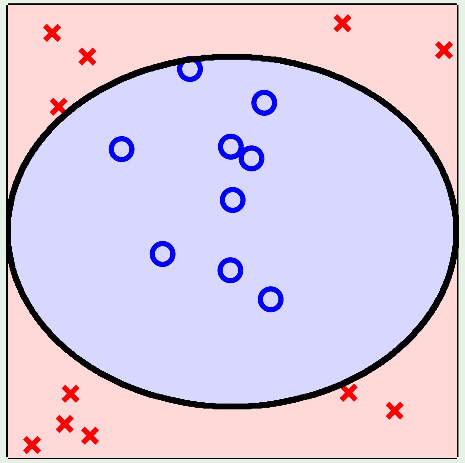
2. Transform the data
$\mathbf{z}_n = \Phi(\mathbf{x}_n) \in \mathcal{Z}$

$\downarrow$



3. Separate data in $\mathcal{Z}$-space
$\tilde{g}(\mathbf{z}) = \text{sign}(\tilde{\mathbf{w}}^{\mathsf{T}}\mathbf{z})$

$\xleftarrow{`\Phi^{-1}'}$



4. Classify in $\mathcal{X}$-space
$g(\mathbf{x}) = \tilde{g}(\Phi(\mathbf{x})) = \text{sign}(\tilde{\mathbf{w}}^{\mathsf{T}}\Phi(\mathbf{x}))$

What transforms to what

$$\mathbf{x} = (x_0, x_1, \cdots, x_d) \quad \xrightarrow{\Phi} \quad \mathbf{z} = (z_0, z_1, \cdots\cdots\cdots, z_{\tilde{d}})$$

$$\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_N \quad \xrightarrow{\Phi} \quad \mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_N$$

$$y_1, y_2, \cdots, y_N \quad \xrightarrow{\Phi} \quad y_1, y_2, \cdots, y_N$$

No weights in $\mathcal{X}$ $\qquad$ $\tilde{\mathbf{w}} = (w_0, w_1, \cdots\cdots\cdots, w_{\tilde{d}})$

$$g(\mathbf{x}) \quad = \quad \mathrm{sign}(\tilde{\mathbf{w}}^{\mathsf{T}} \Phi(\mathbf{x}))$$

Basic premise of learning

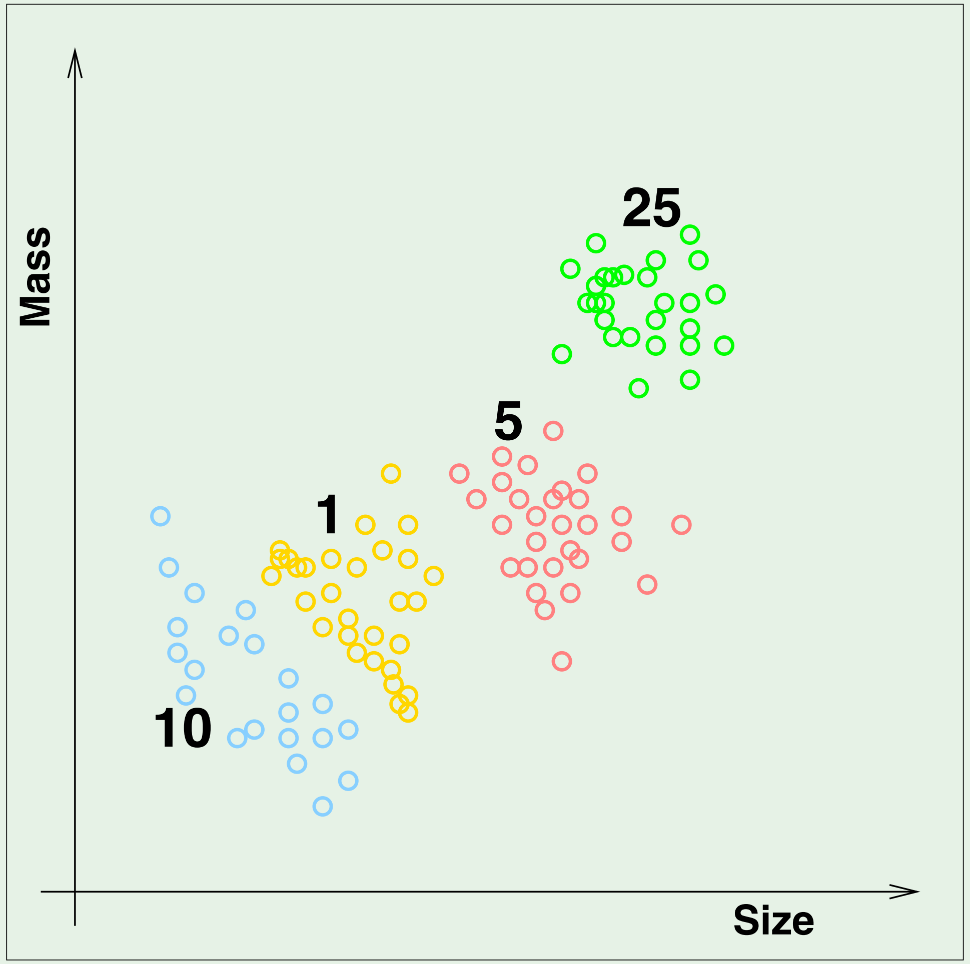
"using a set of observations to uncover an underlying process"

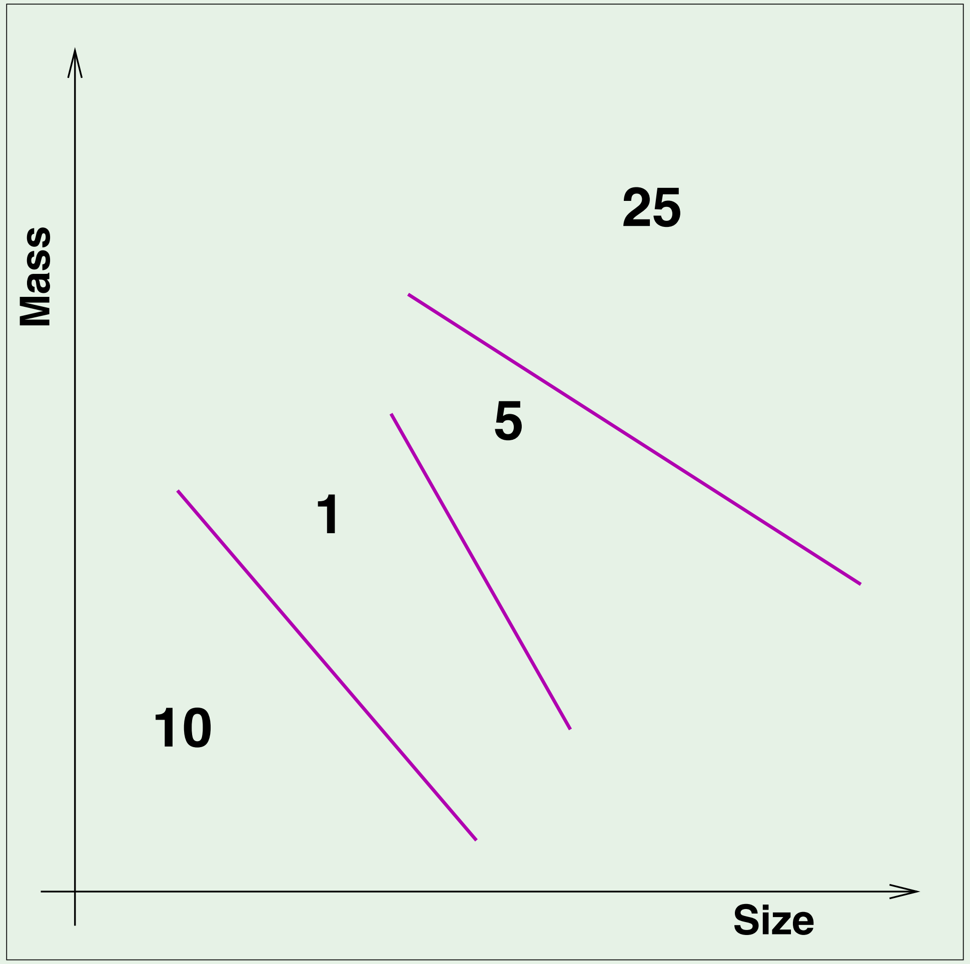
broad premise $\Longrightarrow$ many variations

- Supervised Learning
- Unsupervised Learning
- Reinforcement Learning

Supervised learning

Example from vending machines – **coin recognition**





Unsupervised learning

Instead of (input, correct output), we get (input, ?)