

# Deep Learning for Speech Recognition

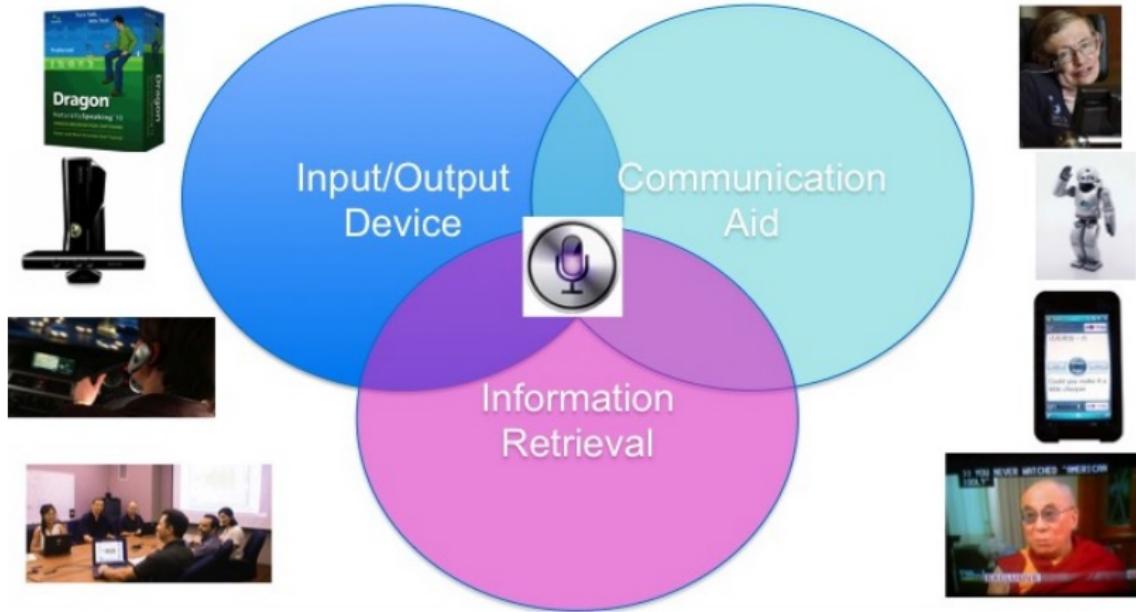
Mark Gales

July 2017

# Apple Siri (2011)

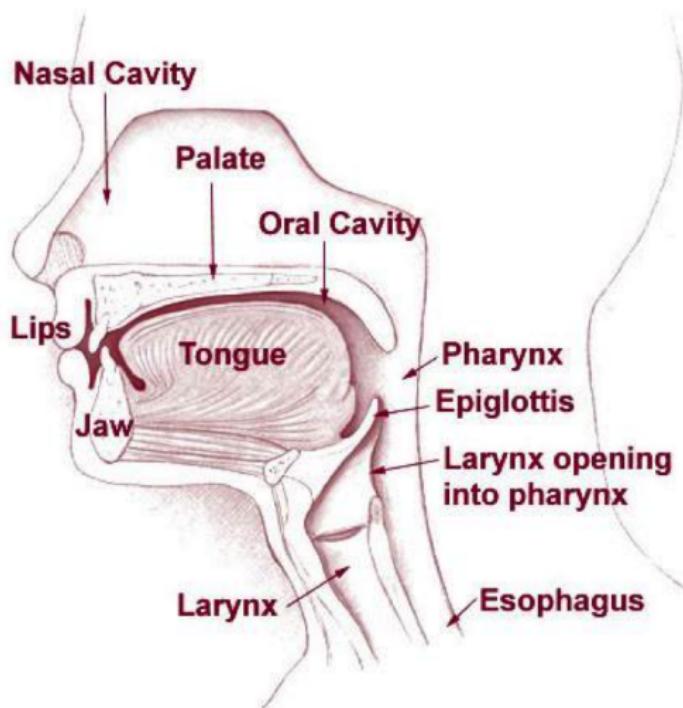


# Speech Application Areas

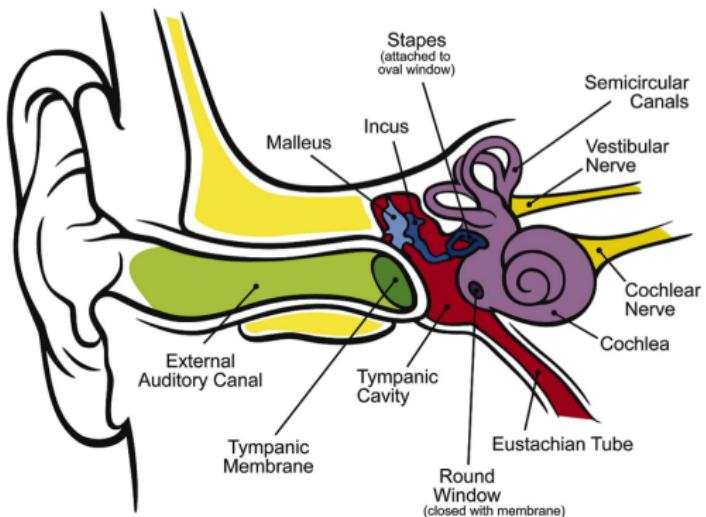


# Speech Processing: Proof of Concept

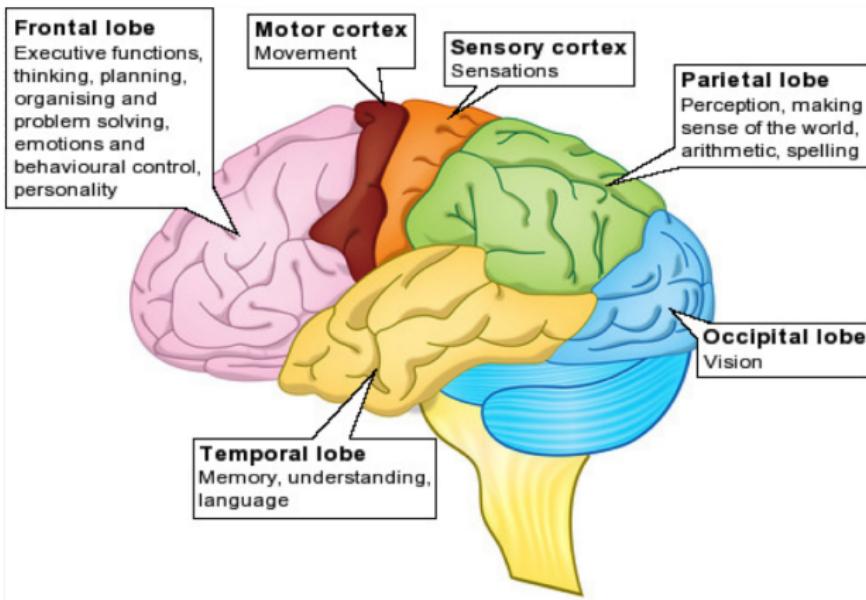
# Speech Production (Synthesis)



# Speech Perception (Recognition)

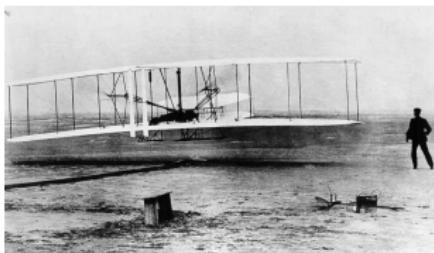


# Speech Understanding

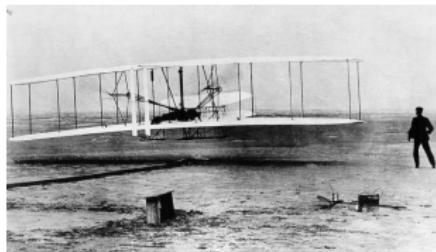


# Should Speech Recognisers have Ears?

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# Should Speech Recognisers have Ears?



No - I'm an Engineer!

# Speech Recognition

# Speech Recognition



Waveform



ya

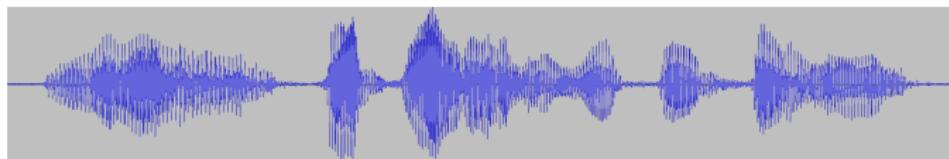
uphethiloli

wona

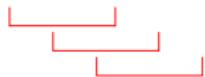
usuwuthengile

Words

# Speech Recognition (Traditional)



Waveform



Features

ya

uphethiloli

wona

usuwuthengile

Words

# Speech Recognition (Traditional)



Waveform



Features

/w/ /O/ /n/ /a/

Phones

ya

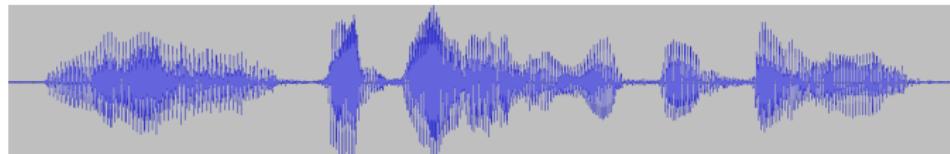
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Words

# Speech Recognition (Traditional)



Waveform



Features

/w/-/O/+/n/

Context-Dependent  
Phones

/w/ /O/ /n/ /a/

Phones

ya

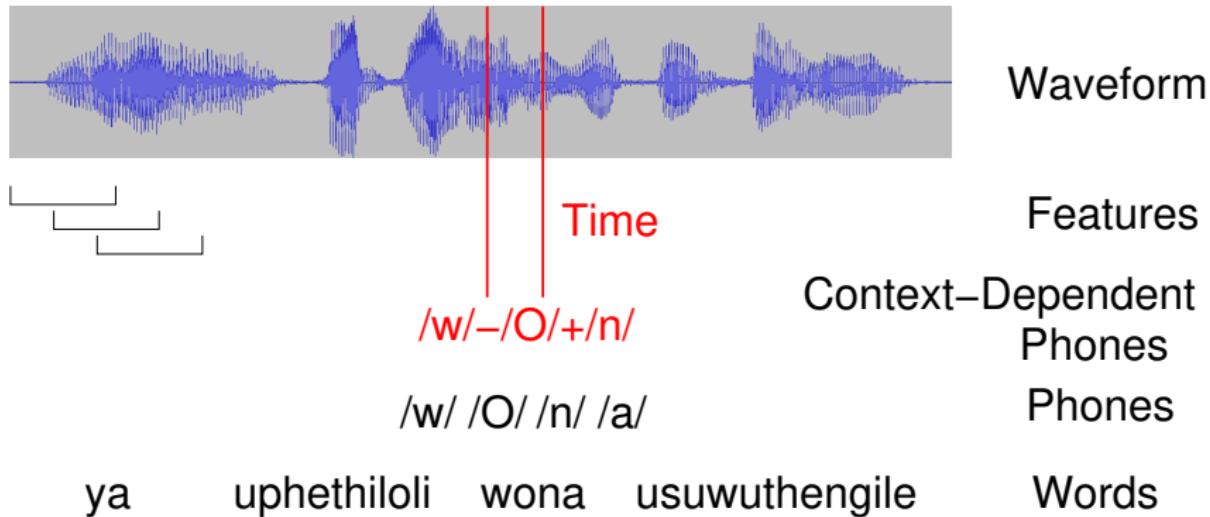
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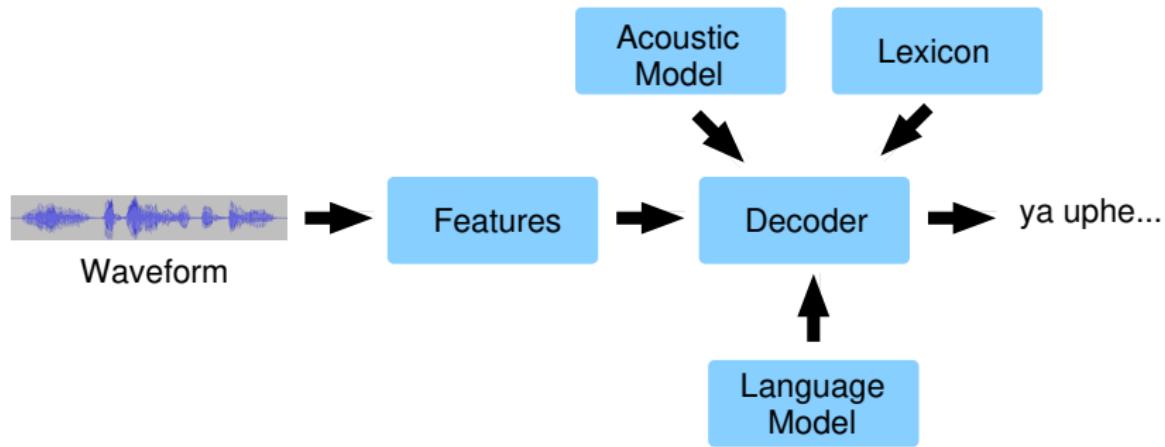
# Speech Recognition (Traditional)



# Sequence-to-Sequence Modelling

- Sequence-to-sequence modelling central to speech/language:
  - machine translation:  
word sequence (discrete) → word sequence (discrete)
  - speech synthesis:  
word sequence (discrete) → waveform (continuous)
  - speech recognition:  
waveform (continuous) → word sequence (discrete)
- The sequence lengths on either side can differ
  - waveform sampled at 10ms/5ms frame-rate -  $T$ -length  $x_{1:T}$
  - word/token sequences -  $L$ -length  $\omega_{1:L}$

# Speech Recognition Framework (Traditional)



- Acoustic model: likelihood model generating observed features
- Language model: probability of **any** word sequence
- Lexicon: maps words to sub-word units (phones)

- Consider two sequences (note  $L \leq T$ ):
  - features:  $\mathbf{x}_{1:T} = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_T\}$
  - words:  $\omega_{1:L} = \{\omega_1, \omega_2, \dots, \omega_L\}$
- Consider generative model

$$p(\omega_{1:L}, \mathbf{x}_{1:T}) = P(\omega_{1:L}) p(\mathbf{x}_{1:T} | \omega_{1:L})$$

- $P(\omega_{1:L})$ : language model
- $p(\mathbf{x}_{1:T} | \omega_{1:L})$ : acoustic model

## Language Model: N-grams [15]

< s >   the   cat   sat   on   the   mat   </ s >

---

< s >   the   cat   sat   on   the   mat   </ s >

< s >   the   cat   sat   on   the   mat   </ s >

< s >   the   cat   sat   on   the   mat   </ s >

< s >   the   cat   sat   on   the   mat   </ s >

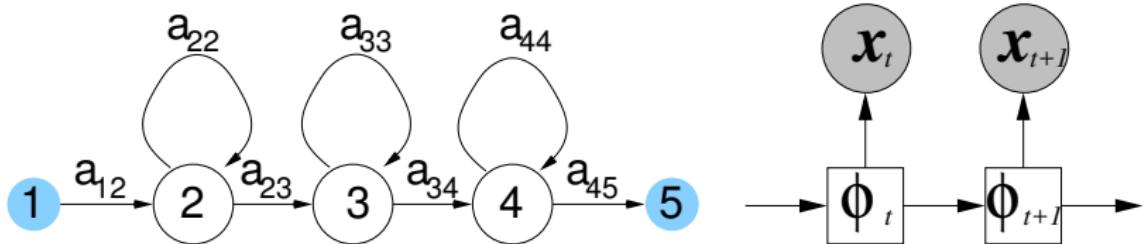
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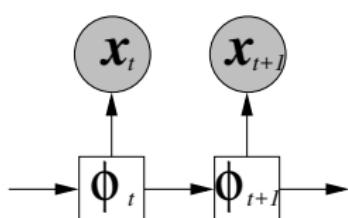
$$P(\omega_{1:L}) = \prod_{i=1}^L P(\omega_i | \omega_{1:i-1}) \approx \prod_{i=1}^L P(\omega_i | \omega_{i-N+1:i-1})$$

## Acoustic Model: Hidden Markov Models [1, 8, 23]



- HMMs standard model for many years (1970s-2010s)
  - each (context-dependent) phone modelled by an HMM
  - typically 3-emitting state topology, left-right
  - non-emitting (end) states used for “gluing” models together
- $\phi_{1:T}$  is the  $T$ -length state-sequence
  - $\phi_t$  indicates the HMM-state at time instance  $t$

- Important sequence model: hidden Markov model (HMM)
  - an example of a dynamic Bayesian network (DBN)



- discrete **latent variables**
  - $\phi_t$  describes discrete state-space
  - conditional independence assumptions

$$P(\phi_t | \phi_{1:t-1}) = P(\phi_t | \phi_{t-1})$$

$$p(x_t | x_{1:t-1}, \phi_{1:t}) = p(x_t | \phi_t)$$

- The likelihood of the data is

$$p(x_{1:T} | \omega_{1:L}) = \sum_{\phi_{1:T} \in \Phi_{\omega_{1:L}}} \left( \prod_{t=1}^T p(x_t | \phi_t) P(\phi_t | \phi_{t-1}) \right)$$

- Use Bayes' Decision Rule

$$\begin{aligned}\hat{\omega} &= \arg \max_{\omega} \{P(\omega | x_{1:T})\} \\ &= \arg \max_{\omega} \{P(\omega, x_{1:T})\} \\ &= \arg \max_{\omega} \{P(\omega) p(x_{1:T} | \omega)\}\end{aligned}$$

- need to efficiently search over all possible word sequences
- Viterbi decoding used for efficiency with HMMs & N-grams
  - leverages model conditional independence assumptions

# Deep Learning and Recurrent Neural Networks

# What is Deep Learning?

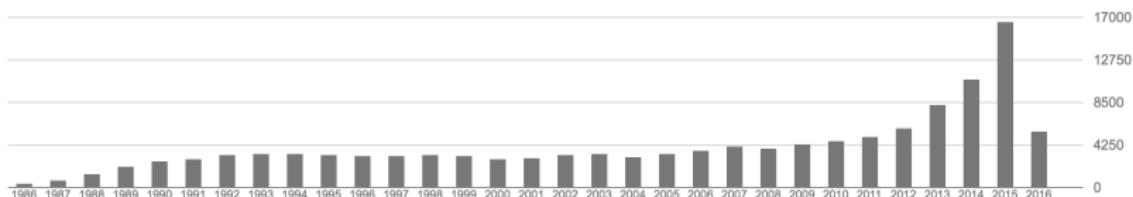
From Wikipedia:

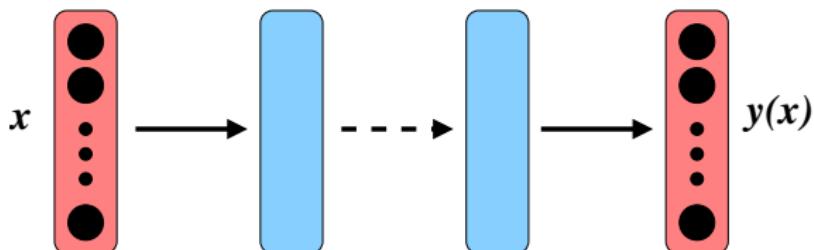
*Deep learning is a branch of machine learning based on a set of algorithms that attempt to model high-level abstractions in data by using multiple processing layers, with complex structures or otherwise, composed of multiple non-linear transformations.*

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From Wikipedia:

*Deep learning is a branch of machine learning based on a set of algorithms that attempt to model high-level abstractions in data by using multiple processing layers, with complex structures or otherwise, composed of multiple non-linear transformations.*



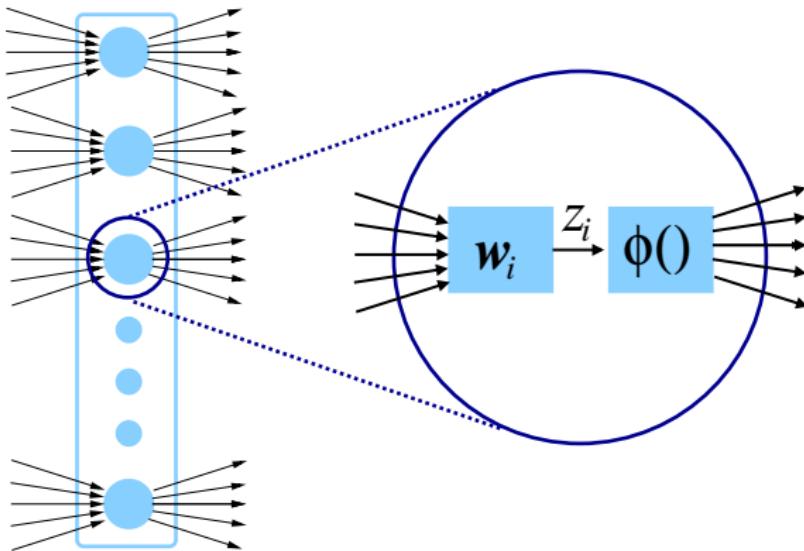


- General mapping process from input  $x$  to output  $y(x)$

$$y(x) = \mathcal{F}(x)$$

- deep refers to number of **hidden** layers
- Output from the previous layer connected to following layer:
  - $x^{(k)}$  is the input to layer  $k$
  - $x^{(k+1)} = y^{(k)}$  the output from layer  $k$

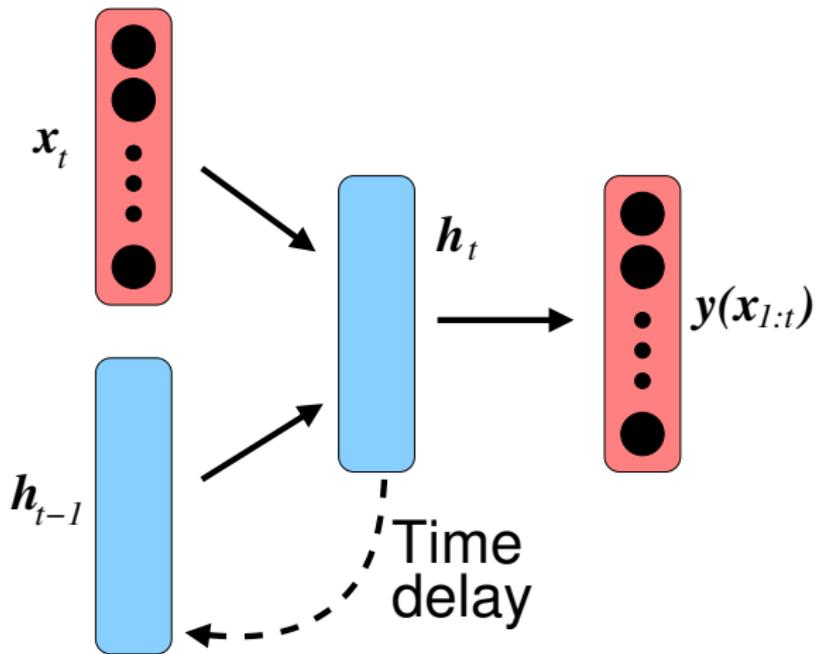
# Neural Network Layer/Node



- General form for layer  $k$ :

$$y_i^{(k)} = \phi(\mathbf{w}'_i \mathbf{x}^{(k)} + b_i) = \phi(z_i^{(k)})$$

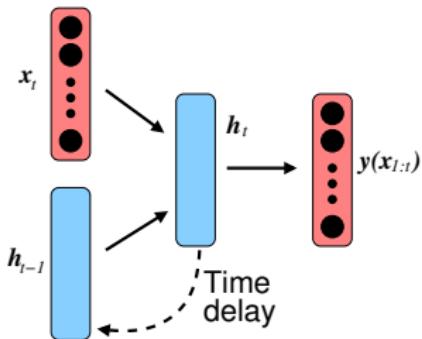
## Recurrent Neural Networks [19, 18]



# Recurrent Neural Networks

- Consider a **causal** sequence of observations  $\mathbf{x}_{1:t} = \{\mathbf{x}_1, \dots, \mathbf{x}_t\}$

- Introduce recurrent units



$$\mathbf{h}_t = \mathbf{f}^h (\mathbf{W}_h^f \mathbf{x}_t + \mathbf{W}_h^r \mathbf{h}_{t-1} + \mathbf{b}_h)$$
$$\mathbf{y}(\mathbf{x}_{1:t}) = \mathbf{f}^f (\mathbf{W}_y \mathbf{h}_t + \mathbf{b}_y)$$

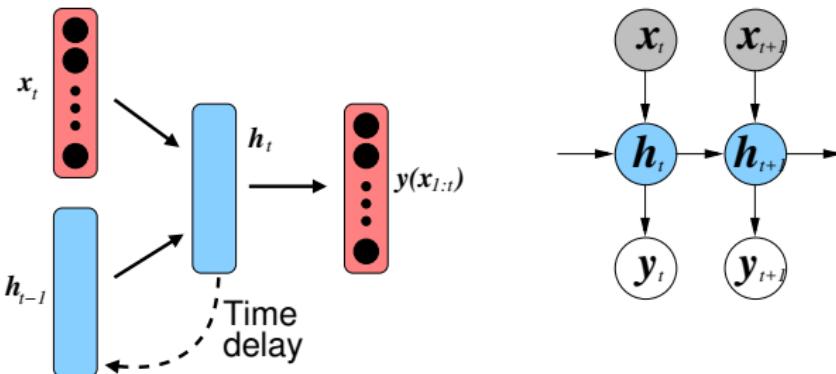
- $\mathbf{h}_t$  history vector at time  $t$

- Uses approximation to model **history of observations**

$$\mathcal{F}(\mathbf{x}_{1:t}) = \mathcal{F}(\mathbf{x}_t, \mathbf{x}_{1:t-1}) \approx \mathcal{F}(\mathbf{x}_t, \mathbf{h}_{t-1}) \approx \mathcal{F}(\mathbf{h}_t) = \mathbf{y}(\mathbf{x}_{1:t})$$

- network has (causal) memory encoded in **history vector** ( $\mathbf{h}_t$ )

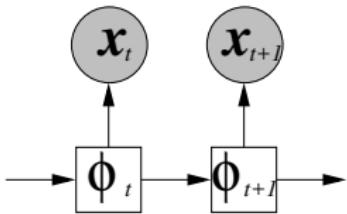
# RNN: Dynamic Bayesian Network



- Maps between two sequences  $\mathbf{x}_{1:T} \rightarrow \mathbf{y}_{1:T}$
- Figure on right is **unwrapped in time**
  - shows dependencies - shaded blue are **deterministic mappings**
- Seen similar models - HMMs, CRFs, SSVMs ..
  - doesn't handle sequence length mappings in ASR

- Extensions of standard RNN structure:
  - bi-directional RNN (depends on future and past)
  - latent-variable RNNs (continuous latent variables)
- Modification to the recurrent units (gating)
  - long-short term memory units (LSTMs)
  - gated recurrent units (GRUs)
  - highway connections (gating in time)

# Acoustic Modelling



- Discrete **latent variables**
  - $\phi_t$  describes discrete **state-space**
  - conditional independence assumptions

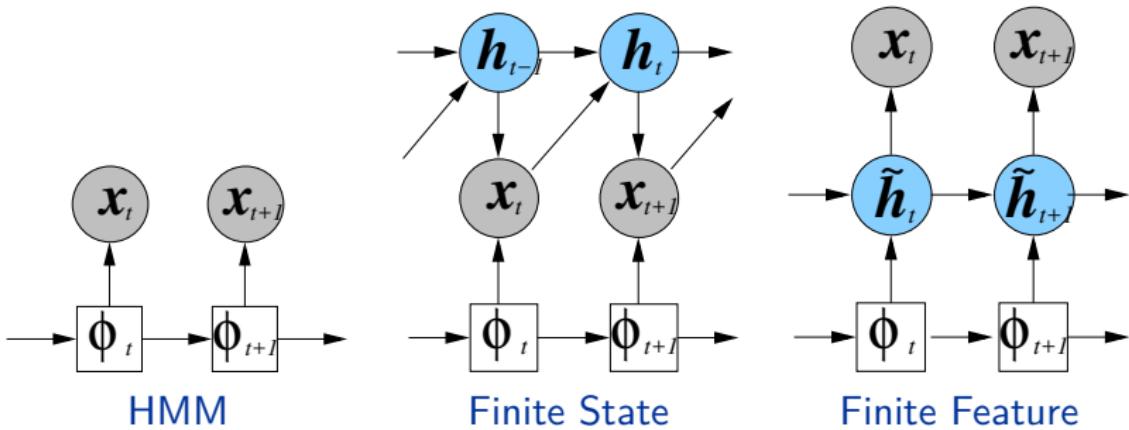
$$P(\phi_t | \phi_{1:t-1}) = P(\phi_t | \phi_{t-1})$$

$$p(x_t | x_{1:t-1}, \phi_{1:t}) = p(x_t | \phi_t)$$

- The likelihood of the data is

$$\begin{aligned} p(x_{1:T} | \omega_{1:L}) &= \sum_{\phi_{1:T} \in \Phi_{\omega_{1:L}}} p(x_{1:T} | \phi_{1:T}) P(\phi_{1:T}) \\ &= \sum_{\phi_{1:T} \in \Phi_{\omega_{1:L}}} \left( \prod_{t=1}^T p(x_t | \phi_t) P(\phi_t | \phi_{t-1}) \right) \end{aligned}$$

# History Approximations and Inference



$$\prod_{t=1}^T p(\boldsymbol{x}_t | \phi_t)$$

$$\prod_{t=1}^T p(\boldsymbol{x}_t | \phi_t, \boldsymbol{h}_{t-1})$$

$$\prod_{t=1}^T p(\boldsymbol{x}_t | \tilde{\boldsymbol{h}}_t)$$

- Inference costs significantly different:
  - finite state: all past history **observed** - deterministic
  - finite feature: past history **unobserved** - depends on path

- HMM: simplest form of approximation

$$p(\mathbf{x}_{1:T} | \phi_{1:T}) \approx \prod_{t=1}^T p(\mathbf{x}_t | \phi_t)$$

- Finite State:

$$p(\mathbf{x}_{1:T} | \phi_{1:T}) \approx \prod_{t=1}^T p(\mathbf{x}_t | \phi_t, \mathbf{x}_{1:t-1}) \approx \prod_{t=1}^T p(\mathbf{x}_t | \phi_t, \mathbf{h}_{t-1})$$

- Finite Feature:

$$p(\mathbf{x}_{1:T} | \phi_{1:T}) \approx \prod_{t=1}^T p(\mathbf{x}_t | \phi_{1:t}) \approx \prod_{t=1}^T p(\mathbf{x}_t | \tilde{\mathbf{h}}_t)$$

## “Likelihoods” [3]

- Deep learning can be used to estimate distributions
  - mixture density neural network (MDNN)
  - more often trained as a **discriminative** model
  - need to convert to a “likelihood”

## “Likelihoods” [3]

- Deep learning can be used to estimate distributions
  - mixture density neural network (MDNN)
  - more often trained as a **discriminative** model
  - need to convert to a “likelihood”
- Most common form (for RNN acoustic model):

$$\begin{aligned} p(\mathbf{x}_t | \phi_t, \mathbf{h}_{t-1}) &= \frac{P(\phi_t | \mathbf{x}_t, \mathbf{h}_{t-1}) p(\mathbf{x}_t | \mathbf{h}_{t-1})}{P(\phi_t | \mathbf{h}_{t-1})} \\ &\propto \frac{P(\phi_t | \mathbf{x}_t, \mathbf{h}_{t-1})}{P(\phi_t | \mathbf{h}_{t-1})} \\ &\approx \frac{P(\phi_t | \mathbf{x}_t, \mathbf{h}_{t-1})}{P(\phi_t)} \end{aligned}$$

- $P(\phi_t | \mathbf{x}_t, \mathbf{h}_{t-1})$ : modelled by a standard RNN
- $P(\phi_t)$ : state/phone prior probability

# “Baseline” Acoustic Training Criteria

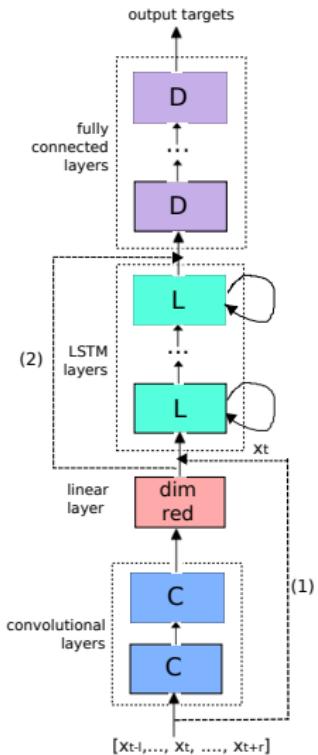
- Originally generative models (GMM-HMM systems) used ML

$$\begin{aligned}\mathcal{F}_{\text{ml}} &= \log(p(\mathbf{x}_{1:T} | \omega_{\text{ref}})) \\ &= \log \left( \sum_{\phi_{1:T} \in \Phi_{\omega_{\text{ref}}}} p(\mathbf{x}_{1:T} | \phi_{1:T}) P(\phi_{1:T}) \right)\end{aligned}$$

- Neural networks: Cross-Entropy with **fixed alignment**,

$$\begin{aligned}\mathcal{F}_{\text{ce}} &= - \sum_{t=1}^T \log(P(\hat{\phi}_t | \mathbf{x}_t, \mathbf{h}_{t-1})) \\ \hat{\phi}_{1:T} &= \arg \max_{\phi_{1:T} \in \Phi_{\omega_{\text{ref}}}} \{P(\phi_{1:T} | \mathbf{x}_{1:T})\}\end{aligned}$$

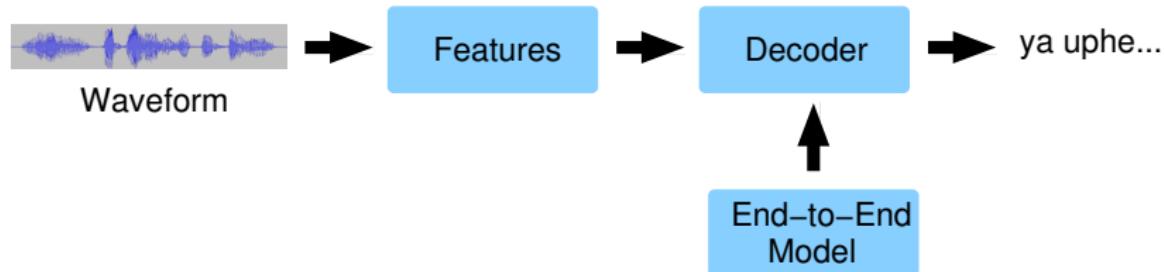
# Example “Generative” Acoustic Model [20]



- Example Architecture from Google (2015)
  - C: CNN layer (with pooling)
  - L: LSTM layer
  - D: fully connected layer
- Two multiple layer “skips”
  - (1) connects input to LSTM input
  - (2) connects CNN output to DNN input
- Additional linear projection layer
  - reduces dimensionality
  - and number of network parameters!

# Discriminative Models ("End-to-End" Models)

# Speech Recognition Framework



- Apply Bayes' Decision Rule

$$\hat{\omega} = \arg \max_{\omega} \{P(\omega | \mathbf{x}_{1:T})\}$$

- Directly train model to solve task (“speech-to-text”)
  - single model trained
  - no separate acoustic and language models
- More complicated to incorporate additional LM data

- Compute posterior of word sequence

$$P(\omega_{1:L} | \mathbf{x}_{1:T}) = \sum_{\phi_{1:T} \in \Phi_{\omega_{1:L}}} P(\omega_{1:L} | \phi_{1:T}) P(\phi_{1:T} | \mathbf{x}_{1:T})$$

# Discriminative Models

- Compute posterior of word sequence

$$P(\omega_{1:L} | \mathbf{x}_{1:T}) = \sum_{\phi_{1:T} \in \Phi_{\omega_{1:L}}} P(\overbrace{\omega_{1:L} | \phi_{1:T}}^1) P(\phi_{1:T} | \mathbf{x}_{1:T})$$

# Discriminative Models

- Compute posterior of word sequence

$$P(\omega_{1:L} | \mathbf{x}_{1:T}) = \sum_{\phi_{1:T} \in \Phi_{\omega_{1:L}}} P(\omega_{1:L} | \cancel{\phi_{1:T}}) \overset{1}{P}(\phi_{1:T} | \mathbf{x}_{1:T})$$

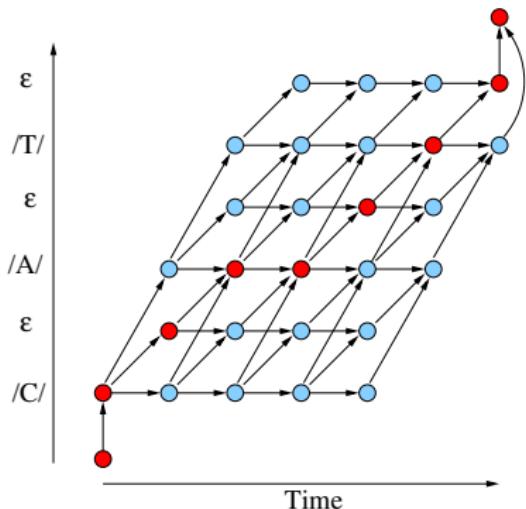
- finite state RNNs used to model history/alignment

$$\begin{aligned} P(\phi_{1:T} | \mathbf{x}_{1:T}) &\approx \prod_{t=1}^T P(\phi_t | \mathbf{x}_{1:t}) \\ &\approx \prod_{t=1}^T P(\phi_t | \mathbf{x}_t, \mathbf{h}_{t-1}) \approx \prod_{t=1}^T P(\phi_t | \mathbf{h}_t) \end{aligned}$$

- Expression does not have a language model

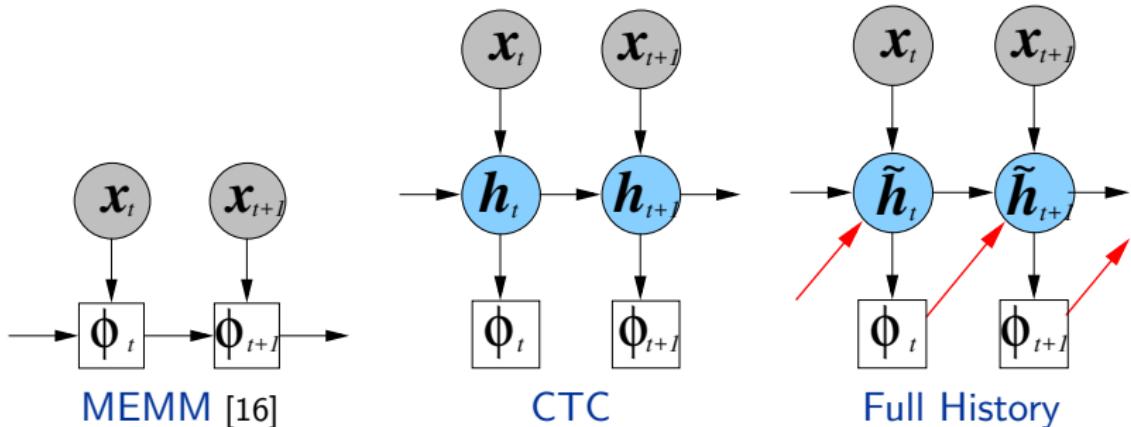
# Connectionist Temporal Classification [10]

- CTC: discriminative model, no explicit alignment model
  - introduces a **blank** output symbol ( $\epsilon$ )



- Consider word: CAT
  - Pronunciation: /C/ /A/ /T/
- Observe 7 frames
  - possible state transitions
  - example path:  
/C/  $\epsilon$  /A/ /A/  $\epsilon$  /T/  $\epsilon$

# Including State History?



- Interesting to consider state dependencies (right)

$$P(\phi_{1:T} | \mathbf{x}_{1:T}) \approx \prod_{t=1}^T P(\phi_t | \mathbf{x}_{1:t}, \phi_{1:t-1}) \approx \prod_{t=1}^T P(\phi_t | \tilde{\mathbf{h}}_t)$$

# Nature of Targets

- One trend for discriminative models:  
**Graphemes** (letters) rather than context-dependent phones
- Take the example of the lexicon entry cat: /k/ /a/ /t/

sil	k	a	t	sil
sil	sil-/k/+/a/	/k/-/a/+/t/	/a/-/t/+sil	sil
sil	sil-/c/+/a/	/c/-/a/+/t/	/a/-/t/+sil	sil
sil	c	a	t	sil

- Can be run at the character level
  - no need to have a lexicon (hence no OOVs)
  - language model implicit by history vector (of features)

- No language models in (this form of) discriminative model
  - in CTC the word history “captured” in frame history
  - no explicit dependence on state (word) history
- Treat as a **product of experts** (log-linear model): for CTC

$$P(\omega_{1:L} | \mathbf{x}_{1:T}) = \frac{1}{Z(\mathbf{x}_{1:T})} \exp \left( \alpha^T \left[ \begin{array}{c} \log \left( \sum_{\phi_{1:T} \in \Phi_{\omega_{1:L}}} P(\phi_{1:T} | \mathbf{x}_{1:T}) \right) \\ \log (\tilde{P}(\omega_{1:L})) \end{array} \right] \right)$$

- $\alpha$  trainable parameter (related to LM scale)
- $\tilde{P}(\omega_{1:L})$  standard “prior” (language) model
- Normalisation term not required in decoding
  - $\alpha$  often empirically tuned

# Encoder-Decoder Style Models

- Directly model relationship

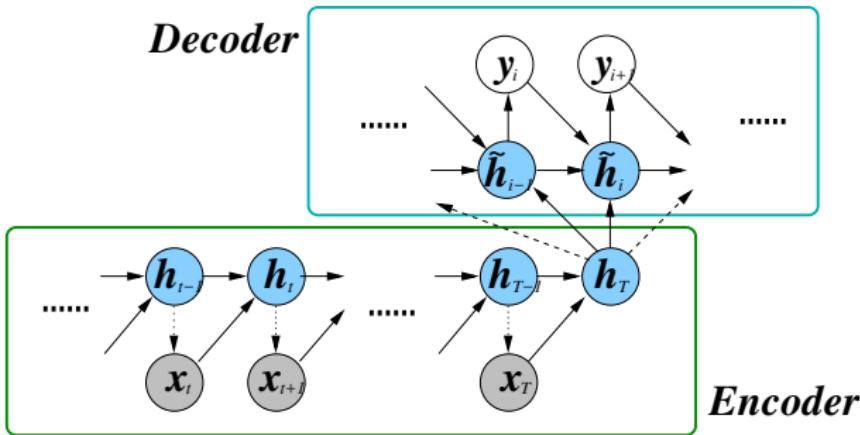
$$\begin{aligned} P(\omega_{1:L} | \mathbf{x}_{1:T}) &= \prod_{i=1}^L P(\omega_i | \omega_{1:i-1}, \mathbf{x}_{1:T}) \\ &\approx \prod_{i=1}^L P(\omega_i | \omega_{i-1}, \tilde{\mathbf{h}}_{i-2}, \mathbf{c}) \end{aligned}$$

- looks like an **RNN LM** with additional dependence on  $\mathbf{c}$

$$\mathbf{c} = \phi(\mathbf{x}_{1:T})$$

- $\mathbf{c}$  is a fixed length vector - like a **sequence kernel**

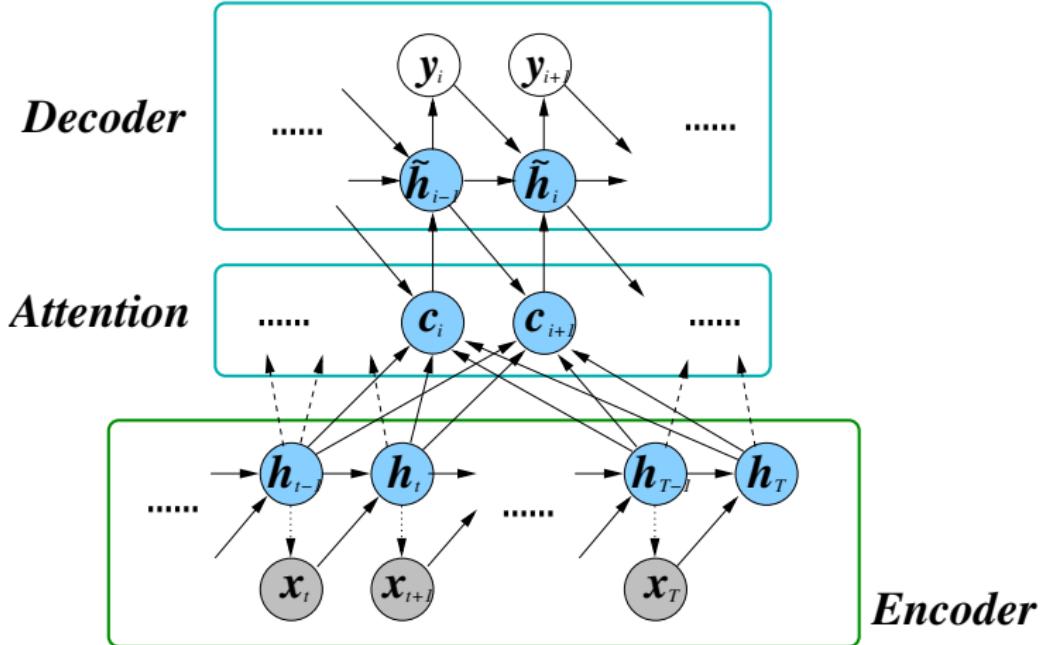
# RNN Encoder-Decoder Model [9, 17]



- Simplest form is to use **hidden unit** from acoustic RNN/LSTM

$$\mathbf{c} = \phi(\mathbf{x}_{1:T}) = \mathbf{h}_T$$

- dependence on context is global via  $\mathbf{c}$  - possibly limiting



# Attention-Based Models

- Introduce **attention** layer to system
  - introduce dependence on locality  $i$

$$P(\omega_{1:L} | \mathbf{x}_{1:T}) \approx \prod_{i=1}^L p(\omega_i | \boldsymbol{\omega}_{i-1}, \tilde{\mathbf{h}}_{i-2}, \mathbf{c}_i) \approx \prod_{i=1}^L p(\omega_i | \tilde{\mathbf{h}}_{i-1})$$

$$\mathbf{c}_i = \sum_{\tau=1}^T \alpha_{i\tau} \mathbf{h}_\tau; \quad \alpha_{i\tau} = \frac{\exp(e_{i\tau})}{\sum_{k=1}^T \exp(e_{ik})}, \quad e_{i\tau} = f^e(\tilde{\mathbf{h}}_{i-2}, \mathbf{h}_\tau)$$

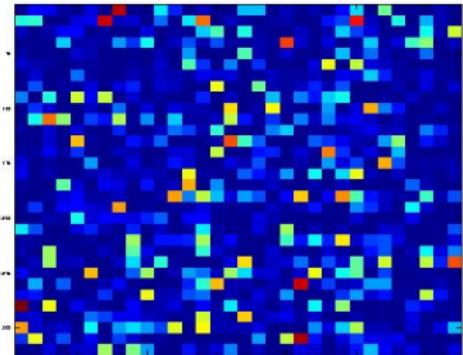
- $e_{i\tau}$  how well position  $i-1$  in input matches position  $\tau$  in output
- $\mathbf{h}_\tau$  is representation (RNN) for the input at position  $\tau$
- Attention can “wander” with large input size ( $T$ )
  - use a **pyramidal network** to reduce frame-rate for attention

# Conclusions

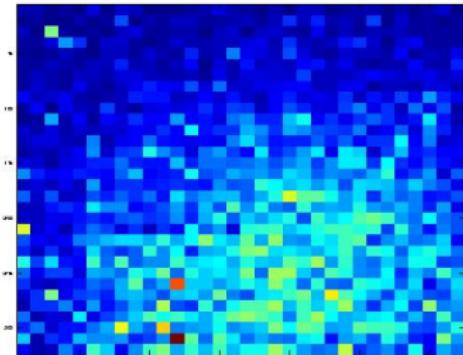
It's an interesting time!

- Deep learning integrated into standard speech toolkits
  - Kaldi, HTK etc
- Rich variety of models and topologies supported by:
  - large quantities of training data
  - GPU-based training (and parallel implementations)
  - array of software tools: TensorFlow, CNTK, Theano ...
- Most state-of-the-art still “generative”
  - but next conference in August ...

## Network Interpretation [24]



Standard /ay/



Stimulated /ay/

- Deep learning usually highly distributed - hard to interpret
  - awkward to adapt/understand/regularise
  - modify training - add **stimulation regularisation**
  - improves ASR performance ...

# Thank-you!

- [1] L. Baum and J. Eagon, "An Inequality with Applications to Statistical Estimation for Probabilistic Functions of Markov Processes and to a Model for Ecology," *Bull Amer Math Soc*, vol. 73, pp. 360–363, 1967.
- [2] C. M. Bishop, *Pattern Recognition and Machine Learning*. Springer Verlag, 2006.
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