

# Calculation of Losses in Ferro- and Ferrimagnetic Materials Based on the Modified Steinmetz Equation

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**Abstract** - This paper discusses the influence of non-sinusoidal flux-waveforms on the remagnetization losses in ferro- and ferrimagnetic materials of inductors, transformers and electrical machines used in power electronic applications. The non-sinusoidal changes of flux originate from driving these devices by non-sinusoidal voltages and currents at different switching frequencies. A detailed examination of a dynamic hysteresis model shows that the physical origin of losses in magnetic material is the average remagnetization velocity rather than the remagnetization frequency. This principle leads to a modification of the most common calculation rule for magnetic core losses, i.e., to the "Modified Steinmetz Equation" (MSE). In the MSE the remagnetization frequency is replaced by an equivalent frequency which is calculated from the average remagnetization velocity. This approach allows, for the first time, to calculate the losses in the time domain for arbitrary waveforms of flux while using the available set of parameters of the classical Steinmetz equation. DC-premagnetization of the material, having a substantial influence on the losses, can also be included. Extensive measurements verify the Modified Steinmetz Equation presented in this paper.

## I. INTRODUCTION

An exact prediction of the remagnetization losses of ferro- or ferrimagnetic material is critical for the design of inductors, transformers and electrical machines. In power electronic applications this task is difficult because in most applications the magnetic material is exposed to non-sinusoidal flux waveforms. As an example, Fig. 1 shows the primary and secondary transformer currents of a flyback converter as a function of time.

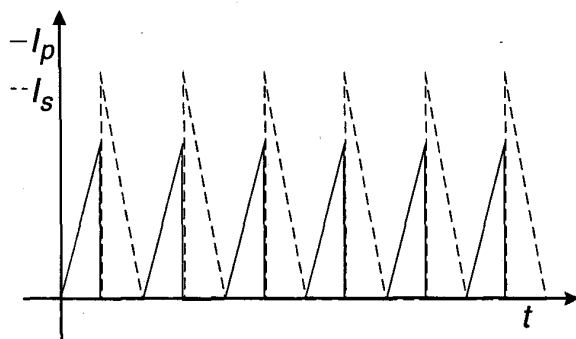


Fig. 1: Idealized transformer currents in the windings of a flyback converter

In another example, Fig. 2 illustrates the phase current and the corresponding flux linkage in the stator pole of a switched reluctance (SR) machine. Clearly, both examples show that the remagnetization processes are non-sinusoidal.

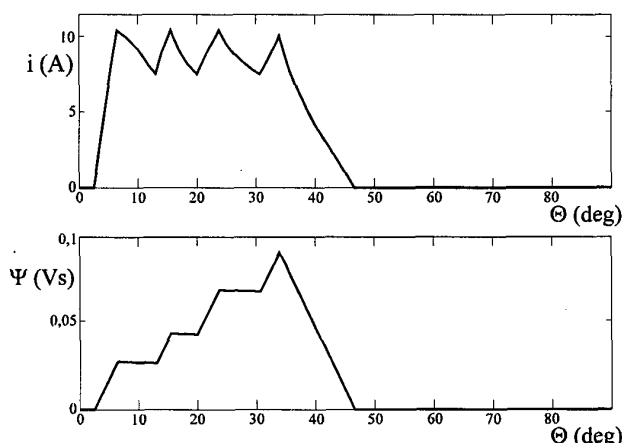


Fig. 2: Phase current and flux linkage of a SR machine in chopping mode

Although magnetic materials have been subjected to research for over a century, the exceptional conditions and requirements in power electronic circuits have not been taken into detailed consideration. The reason for this might be that most design rules and loss formulas for magnetic materials were formulated a long time ago for sinusoidal processes. When applied to inductors, transformers and machines that are exposed to high induction levels and switching frequencies, they loose their validity

Therefore, this paper discusses the physical justification for representing the properties of magnetic components in the frequency domain.

## II. PHYSICAL ORIGIN OF LOSSES

The most common concept for calculating remagnetization losses is the addition of two separate terms, i.e., the so-called hysteresis losses and the so-called eddy current losses. Hence, it was assumed that two separate physical effects are contributing to the remagnetization losses. Although many specialists from material sciences and physics have contradicted this hypothesis, most engineers and technicians still believe this loss separation is correct.

To shed some light on this topic, a brief overview over the theory of magnetization and magnetization losses that has been discussed in literature is given below.

A very commendable introduction to magnetic materials has been given by Fish [15]. Serious discussions of magnetization theory and the reasons for remagnetization losses can also be found in the work of Bertotti [16],[17], Graham [1] and Becker [18].

It is known that the magnetization in ferro- and ferrimagnetic materials is not uniform. As shown in Fig. 3, the internal structure of the material can be subdivided into saturated domains which differ from each other by the orientation of their magnetization vectors. The magnetic domains are separated from each other by domain walls. A change in the global magnetization of the material can only be achieved by movement of these domain walls.

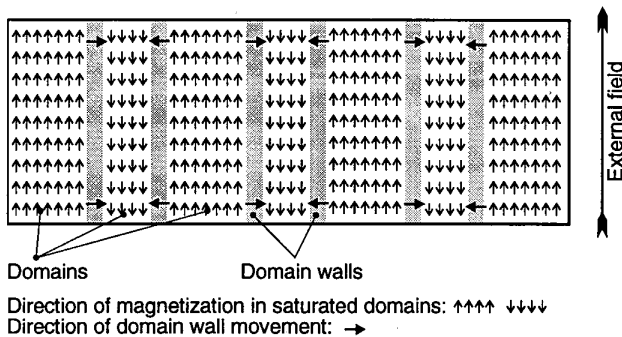


Fig. 3: Change in a domain structure

This means that the magnetization changes in a highly localized way, rather than uniformly through the material. The magnetization change is discrete in terms of space.

Impurities and imperfections inside the material hinder the domain wall motion and cause rapid movements of the domain walls, the so-called Barkhausen-Jumps.

Therefore, the movement of the domain walls is not regular. The local velocity of the walls is not equal to the change of rate of the external field. **This means that the magnetization change is discrete in terms of time.**

If the change of magnetization is discrete in terms of space and in terms of time there have to be rapid local changes of magnetization, even if the external field changes with an infinitesimal low rate, i.e., the quasi-static case. Associated with magnetization changes are local energy losses caused by eddy currents and by spin-relaxation. These losses are determined by the local- and time-distribution of the changes.

Consequently, there is no physical difference between “hysteresis” losses and “eddy current” losses. As Graham pointed out there is no physical distinction to be made between the static losses and the dynamic losses [1]. There is only one physical origin of remagnetization losses, namely, the damping of domain wall movement by eddy currents and spin-relaxation.

### III. CONVENTIONAL CALCULATION

The detailed knowledge about the origin of remagnetization losses does not provide a practical means of calculating losses. In general, the rather chaotic time- and space- distribution of the magnetization changes is unknown and cannot be described exactly.

To work around this lack of a microscopic physical remagnetization model, several macroscopic and empirical approaches have been formulated in the past. They can be subdivided into hysteresis models, empirical equations and the loss separation approach.

#### A. Hysteresis Models

The vast number of hysteresis models can be separated into two branches. One part is based on the Jiles-Atherton model and the other part traces back to Preisach’s work.

The Jiles-Atherton model [19] is based on a macroscopic energy calculation. It consists of a differential equation that describes the static behavior of ferro- and ferrimagnetic behavior. An iterative procedure has to be used to estimate the parameters of the model. The model can be extended to dynamic calculations [20] which increase the number of required parameters to seven. Additional parameters have to be used to describe the temperature behavior and to calculate minor hysteresis loops. Although this model leads to a better understanding of the remagnetization process, it is of limited practical use.

Preisach’s model as described e.g. by Hui [23] introduces a statistical approach for the description of the time- and space distribution of domain-wall movement. A weight function represents the material characteristics. The classical model exhibits two major drawbacks - the limited congruency of minor loops and the static character. The model can be extended to dynamic effects but the identification problem connected with the weight functions results in a tremendous experimental effort that is not justified by the incremental increase of accuracy.

#### B. Empirical Equations

One well-known empirical equation to calculate remagnetization losses traces back to the original work of Steinmetz more than a century ago and is formulated by means of an empirical equation [4]:

$$p_v = C_m f^\alpha \hat{B}^\beta \quad (1)$$

It states that the power losses  $p_v$  per volume are dependent on exponential functions of the remagnetization frequency  $f$  and the peak induction  $\hat{B}$ , using three empirical parameters  $C_m$ ,  $\alpha$ ,  $\beta$ . Both exponents are non-integer numbers, i.e.,  $1 < \alpha < 3$  and  $2 < \beta < 3$ . The appearance of the remagnetization frequency  $f$  in this equation has to be explained by the empirical character of the studies made by Steinmetz a century ago. The equation and the corresponding set of parameters is only valid for sinusoidal remagnetization, which is a major

drawback for the implementation in power electronic applications.

Gradzki [21] and Severns [22] try to overcome this problem by using a Fourier expansion of the arbitrary non-sinusoidal waveforms. Equation (1) is then applied to each single Fourier component. Finally, the individual losses of the fundamental and all harmonics are superimposed and summarized to calculate the total losses. The fact that the induction exponent  $\beta$  of the equation has a typical value of  $\beta = 2.5$  indicates that there is an extremely non-linear relation between losses and peak-induction. The method of superposition is mathematically only applicable for linear systems. In case of non-linear magnetic materials its application is not valid and the results of this procedure are invalid [7],[8],[9].

For ferromagnetic materials, that are normally used in form of sheets, laminations or tapes with a fixed thickness, the losses are specified by the manufacturers as a function of material, quality and sheet thickness. Equation (1) is used to extract the parameters from these specifications. This finally leads to a different set of parameters for each individual material and sheet thickness. Consequently, (1) gives the total remagnetization losses including static and dynamic eddy-current losses.

In case of ferrites, losses are specified depending on the material grade. The dependence on geometry is usually neglected in these specifications. Therefore, it is necessary to introduce an additional term into the loss equation that accounts for geometric effects:

$$p_v = C_m f^\alpha \hat{B}^\beta + C_2 f^2 \hat{B}^2 \quad (2)$$

The parameter  $C_2$  of this equation is dependent on the cross-section and the conductivity of the core. For medium frequencies below 100 kHz, the conductivity of ferrites is typically very low, which means that the geometrical influence on the total losses can be neglected. However, above 100 kHz ferrite may be subjected to dispersion that leads to a significant increase in conductivity.

### C. Loss-Separation Approach

The third loss calculation method traces back to Jordan [3] and separates the total losses  $P_t$  in two parts, i.e., the static hysteresis loss  $P_h$  and the dynamic eddy-current loss  $P_e$ .

$$P_t = P_h + P_e$$

It has already been shown that this approach lacks theoretical justification. But even the practical use is limited because of the fact that the results are inaccurate. Many papers report that calculation errors between 200% and 2000% can occur. Therefore a third artificial loss component is introduced, the so called "eddy-current anomaly loss"  $P_a$ .

Only the eddy current loss  $P_e$  of the three components can be calculated. The hysteresis loss and the anomalous losses have to be determined experimentally.

Non-sinusoidal remagnetization can be taken into account on the assumption that hysteresis and anomalous losses are

not influenced by the remagnetization waveform. In this case Maxwell's theory can be applied and the classic eddy current calculation finally leads to several form-factors for typical non-sinusoidal waveforms. These form-factors have the same poor accuracy as the loss-separation approach.

## IV. THE NOVEL MSE APPROACH

The empirical Steinmetz equation (1) has proven to be the most useful tool for the calculation of remagnetization losses. It requires only three parameters which are usually published by the manufacturer. For sinusoidal flux-waveforms it provides a high accuracy and is quite simple to use.

Therefore, it is desirable to extend this equation to non-sinusoidal problems. This can be done with the help of the physical understanding taken from the development of dynamic hysteresis models. It has been shown that the macroscopic remagnetization velocity  $dM/dt$  is directly related to the core losses [20]. Therefore, the task is quite simple: the empirical loss parameter frequency  $f$  of (1) has to be replaced by the physical loss parameter  $dM/dt$ , which is proportional to the rate-of-change of the induction  $dB/dt$ .

As a first step, the induction change-rate  $dB/dt$  is averaged over a complete remagnetization cycle, thus from maximum induction  $B_{max}$  down to its minimum  $B_{min}$  and back:

$$\dot{B} = \frac{1}{\Delta B} \oint \frac{dB}{dt} dB, \quad \Delta B = B_{max} - B_{min} \quad (3)$$

This integral can be transformed:

$$\dot{B} = \frac{1}{\Delta B} \int_0^T \left( \frac{dB}{dt} \right)^2 dt \quad (4)$$

The second step consists of finding a relationship between the remagnetization frequency  $f$  and the averaged remagnetization velocity  $\dot{B}$ . It has been shown by Dürbaum [10] that (4) can be normalized with respect to a sinusoidal case. From the averaged remagnetization velocity an equivalent frequency  $f_{eq}$  can be calculated using the normalization constant  $2 / \Delta B \pi^2$ :

$$f_{eq} = \frac{2}{\Delta B^2 \pi^2} \int_0^T \left( \frac{dB}{dt} \right)^2 dt \quad (5)$$

Similar to the empirical formula of Steinmetz the specific energy loss  $w_v$  of every remagnetization cycle can now be determined using this equivalent frequency:

$$w_v = C_m f_{eq}^{\alpha-1} \hat{B}^\beta \quad (6)$$

If the remagnetization is repeated with the period  $T_r = 1 / f_r$  the power losses are:

$$p_v = \left( C_m f_{eq}^{\alpha-1} \hat{B}^\beta \right) f_r \quad (7)$$

This Modified Steinmetz Equation (MSE) describes the physical origin of the losses and gives the opportunity to

calculate the core losses in the time domain for arbitrary shapes of induction. Compared to the original Steinmetz equation, no additional parameters are needed.

## V. EXPERIMENTAL VERIFICATION

To validate the MSE, an experimental setup according to the European Standard CECC 25 300 and CECC 25 000, as shown in Fig. 4, is used. This setup is chosen because of its high accuracy even for non-linear materials and because it provides more information about the core material than just the core losses. According to Carsten [11] it is the only core loss measurement technique without technical disadvantages.

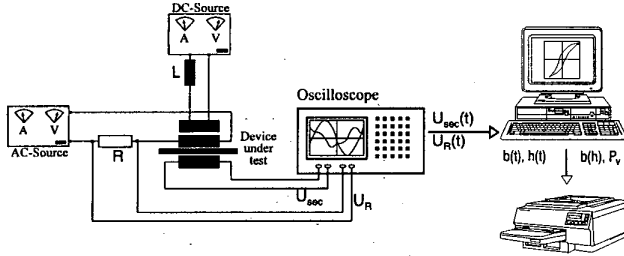


Fig. 4: Measurement setup

The device under test (DUT) carries three windings, nl. a primary AC-winding, a secondary sense winding and a third winding to introduce a DC-premagnetization via a DC source. Via a special low-inductive shunt  $R$  the primary winding is connected to an AC-power amplifier, producing arbitrary remagnetization cycles. The induced voltage at the DUT is measured by the secondary sense winding. This induced voltage and the voltage drop over the shunt resistor are sampled by a LeCroy digital storage oscilloscope with 2,5 GS/s and a bandwidth of 300 MHz. The sampled waveforms are transferred to a PC, used to calculate the magnetic field and the flux density in the core. The core losses can then directly be determined by the surface of the hysteresis loop.

The phase angle between primary current and secondary voltage of the DUT is related directly to the core losses. For low-loss components it differs only slightly from  $90^\circ$ . Hence, the error is typically introduced by the current measurement device. Therefore, it is crucial important to use high-precision low-inductive shunts.

### A. Measurement Results - Ferrimagnetic

The first experiment is performed with an E42/42/15 Philips 3C85 ferrite core, which is exposed to constant triangular remagnetization cycles, as is shown in Fig. 5. In Fig. 6, the measured core losses are compared to different calculations. Firstly, the remagnetization losses are calculated from the original Steinmetz equation (1). Secondly, the modified equation (7) is used. In this case it simplifies into:

$$P = C_m \frac{1}{T_r} \left( \frac{2}{\pi^2} \frac{4}{T} \right)^{\alpha-1} B_0^\beta \quad (8)$$

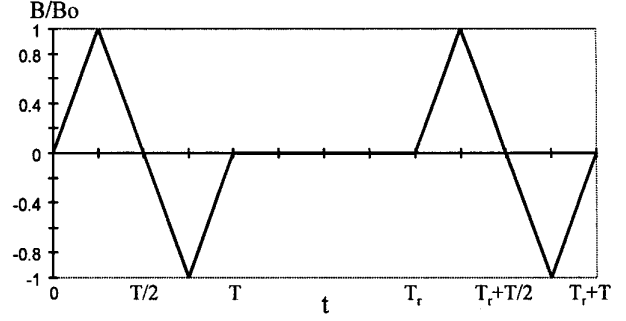


Fig. 5: Triangular remagnetization with varying delay time,  $B_0 = 200 \text{ mT}$ ,  $T^{-1} = 20 \text{ kHz}$ ,  $\nu = 100^\circ\text{C}$

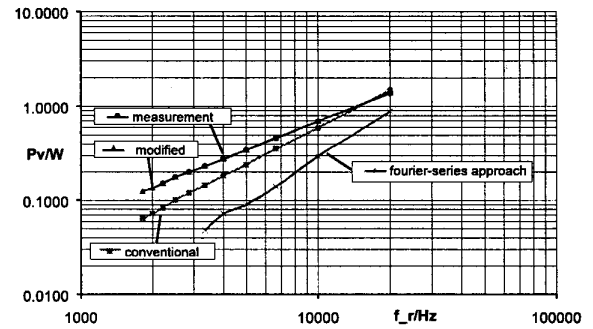


Fig. 6: Comparison between calculation and measurement for triangular remagnetization

Thirdly, the calculation of the core losses is performed by the Fourier series approach. Fig. 6 shows that the results of the MSE are in very good agreement with the experimental results. It also shows that the use of the original Steinmetz equation, which is only valid for sinusoidal remagnetization, is actually more accurate than the calculation by the Fourier series approach.

For the next experiment, the duty cycle of the flux waveform is varied, as shown in Fig. 7.

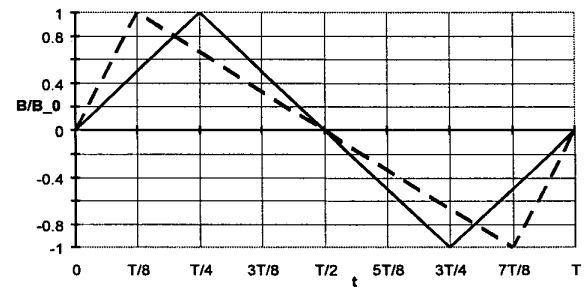


Fig. 7: Remagnetization with different duty cycles  $B_0 = 220 \text{ mT}$ ,  $1/T = 20 \text{ kHz}$ ,  $\nu = 100^\circ\text{C}$

Fig. 8 shows a comparison of the core losses calculated from (1) and from (7) with the experimental results. It indicates that the measured core losses increase significantly with increasing duty cycle. This behavior is also represented by



results derived from the modified core loss equation, but can not be predicted with the conventional equation.

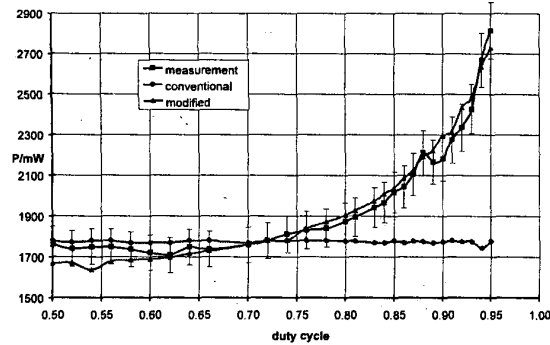


Fig. 8: Comparison between measurement and calculation as a function of duty cycle.

### B. Measurement Results - Ferromagnetic

The first experiment with ferromagnetic material is performed with a Surahmams Bruk CK27 material with a lamination thickness of 0.35 mm, which is used in a 4-phase, 30 kW switched reluctance (SR) machine [12]. To be able to test the influence of different remagnetizations, independently of the lamination geometry, a toroidal core of the same lamination material was used as the DUT in the setup of Fig. 4 for the first tests. Subjecting the toroidal core to an alternating block voltage (duty cycle=50%,  $T_r=T$ ) and measuring the losses, gives the results shown in Fig. 9. Again, the experimental results are compared to the calculated losses from (1) and (7).

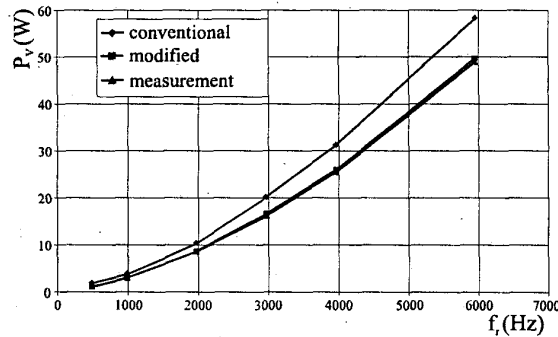


Fig. 9: Comparison between calculation and measurement for triangular remagnetization ( $T_r = T$ )

Increasing the period of the cycle, i.e.  $T_r > T$  (see Fig. 5), leads to an even larger error of the calculation with (1).

In single pulse operation, a SR-drive is operated with a block voltage scheme, leading to triangular remagnetization in the poles. From this, the waveforms in the different yoke sections can be obtained. As an example the flux waveforms for the 4-phase machine in one specific working point are shown in Fig. 10.

To calculate the entire iron losses of a SR-machine for each specific set of control parameters, a simulation program

can be used to find the flux waveforms in the different parts of the machine. For the piecewise linear waveforms (see Fig. 10), equation (7) is used to calculate the losses in the poles and the yoke sections [13]. Due to the non-uniform flux distribution and the saturation effects always present in SR machines, a precise calculation of the iron losses is extremely difficult. Experiments with the 4-phase machine have shown, that the error of the iron-loss calculation is smaller than 10% for all operating conditions [12]. This accuracy can not be achieved with any other previously derived method.

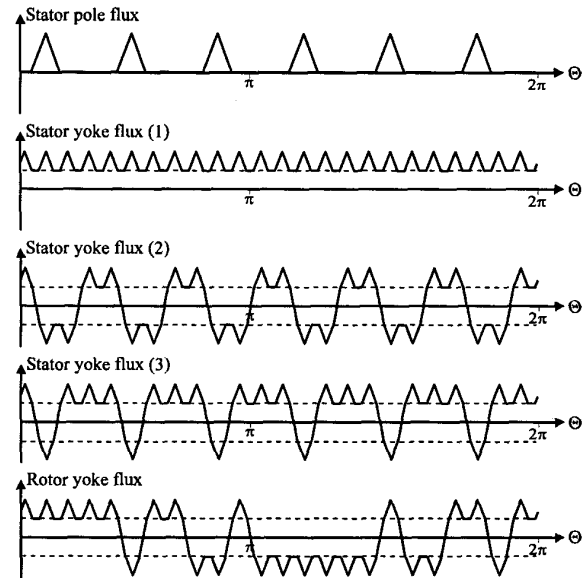


Fig. 10: Flux waveforms in different core parts of a 4-phase SR machine in single pulse operation

### C. Influence of DC-premagnetization

From Fig. 10 it can readily be seen, that during the operation of SR-machines smaller hysteresis sub-loops are encountered under the influence of premagnetization. From Fig. 1 it is evident, that premagnetization is also commonly experienced in ferrimagnetic materials used in power electronic applications. Although manufacturers of magnetic materials never supply data of the influence of premagnetization, it has been shown that it has a major influence on the losses in both ferromagnetic [14] and ferrimagnetic materials [7],[24].

This influence has been proven by measurement for the ferromagnetic toroidal core described in section B, as shown in Fig. 11. It can be seen that the losses at a constant AC-induction and frequency increase continuously with the DC part of the flux density. Similar to the original Steinmetz equation and any other calculation method, the MSE (7) cannot incorporate the influence of a premagnetization. In an empirical approach the loss parameter  $C_m$  in the MSE can be used to adapt to the influence of premagnetization, as shown in (9) [7],[12]:

$$C_{m,new} = C_{m,old} (1 + K_1 B_{DC} e^{\frac{-B_{AC}}{K_2}}) \quad (9)$$

$B_{DC}$  and  $B_{AC}$  relate to the constant and the alternating part of the flux density. The constants  $K_1$  and  $K_2$ , found by measurements at different frequency and magnetization, describe the material-dependent influence of premagnetization.

For example, to calculate the iron losses in SR machines, the magnetization waveforms in the different sections are divided into their main and sub-loops. Once having determined  $K_1$  and  $K_2$  the additional losses of the hysteresis loops under the influence of premagnetization are readily available for any operating condition.

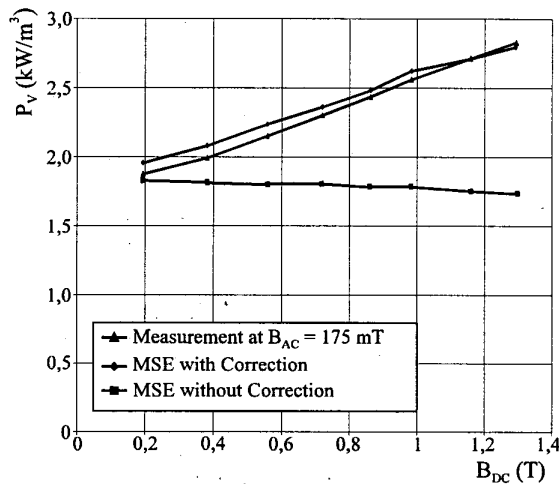


Fig. 11: Influence of DC-premagnetization on losses

## VI. CONCLUSIONS

In this paper the physical justification and the experimental verification of a novel method for the calculation of core losses for non-sinusoidal induction are presented. This calculation method, called the Modified Steinmetz Equation (MSE), is particularly useful for the design of magnetic components for power electronic applications and electric machine theory due to its simplicity and because all necessary parameters are readily available.

The tests undertaken for the ferro- and ferrimagnetic material show that the modified equation enables accurate calculation of remagnetization losses for arbitrary flux waveforms.

## REFERENCES

- [1] C.D. Graham Jr., "Physical origin of losses in conducting ferromagnetic materials (invited)", *Journal of Applied Physics*, 53:8276-8280, 11, 1982
- [2] A. Brockmeyer and L. Schülting, "Modeling of dynamic losses in magnetic material", in *EPE'93 Proc.* Vol. 3, pp 112-117, Brighton (UK), 1993
- [3] H. Jordan, "Die ferromagnetischen Konstanten für schwache Wechselfelder", *Elektr. Nach. Techn.*, 1924
- [4] Ch. P. Steinmetz, "On the law of hysteresis", reprint, *Proc IEEE*, 72(2), pp. 196-221, 2 1984.
- [5] S. A. Mulder, "Fit formulae for power loss in ferrites and their use in transformer design", in *PCIM'93 Proc.*, pp.345-359, ZM Communications, Nürnberg, Germany, 6 1993
- [6] D. Grätzer, "Ummagnetisierungsverluste weichmagnetischer Werkstoffe bei nichtsinusförmiger Aussteuerung", *Zeitschrift für angewandte Physik*, 32(3), pp. 241-246, 1971
- [7] A. Brockmeyer, "Dimensionierungswerkzeug für magnetische Bauelemente in Stromrichteranwendungen" (in German), PhD. Thesis, Verlag Augustinus Buchhandlung Aachen, ISBN 3-86073-239-0
- [8] A. Brockmeyer, M. Albach, T. Dürbaum, "Remagnetization losses of ferrite materials used in power electronic applications" in *PCIM'96 Proc.*, ZM Communications, Nürnberg, Germany, 1996
- [9] M. Albach, T. Dürbaum, A. Brockmeyer, "Calculating core losses in transformers for arbitrary magnetization currents- a comparison of different approaches" in *PESC'96 Proc.*, 1996
- [10] T. Dürbaum and M. Albach, "Core losses in transformers with an arbitrary shape of the magnetization current", in *EPE Proc.*, Vol.1, pp 1.171-1.176, Sevilla (Spain), 1995
- [11] B. Carsten, "Why a magnetics designer should measure core loss; with a survey of loss measurement techniques and a low cost, high accuracy alternative", in *PCIM'95 Proc.*, pp. 163-179, ZM Communications, Nürnberg, Germany, 1995
- [12] J. Reinert, "Optimierung der Betriebseigenschaften von Antrieben mit geschalteter Reluktanzmaschine" (in German), PhD. Thesis, Verlag Augustinus Buchhandlung Aachen, ISBN 3-86073-682-5
- [13] J. Reinert, R. Inderka, R.W. De Doncker, "A Novel Method for the Prediction of Losses in Switched Reluctance Machines", in *EPE'97 Proc.* Vol. 3, pp 3.608-3.612, Trondheim, 1997
- [14] R. M. Bozorth, "Ferromagnetism", IEEE Press Reprint, Piscataway NJ, USA, 1993
- [15] G.E. Fish, "Soft magnetic materials", *Proc. IEEE*, Vol. 78, No. 6, June 1990
- [16] G. Bertotti, "General properties of power losses in soft ferromagnetic materials", *IEEE Trans. Mag.*, Vol. 24, No. 1, Jan. 1988
- [17] G. Bertotti, F. Fiorillo, P. Mazetti, "Basic principles of magnetization processes and origin of losses in soft magnetic materials", *Journal of Magnetism and Magnetic Materials* 112 (1992) 146-149, North Holland
- [18] J. J. Becker, "Magnetization changes and losses in conducting ferromagnetic materials", *Jour. Appl. Phys.*, Vol. 34, No. 4, April 1963
- [19] D. C. Jiles, D. L. Atherton, "Theory of ferromagnetic hysteresis", 1986, *Journal of Magnetism and Magnetic Materials* 61, 48-60, North Holland
- [20] A. Brockmeyer, L. Schülting, "Modelling of dynamic losses in magnetic material", *EPE Proc.* 1993, Brighton, U.K.
- [21] P. M. Gradzki, M. M. Jovanovic, F. C. Lee, "Computer aided design for high-frequency power transformers", in *IEEE APEC Proc.* 1990, 336-343
- [22] R. Severns, "HF-core losses for non-sinusoidal waveforms", in *HFPC Proc.* 1991, 140-148
- [23] S. Y. R. Hui, J. Zhu, "Magnetic hysteresis modelling and simulation using the preisach theory and TLM technique", in *IEEE PESC Proc.* 1994, 837-842
- [24] A. Brockmeyer, "Experimental evaluation of the influence of DC-premagnetization on the properties of power electronic ferrites", in *IEEE APEC Proc.*, 1996