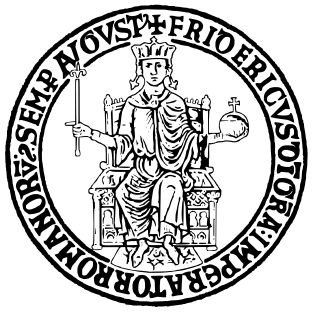
Università degli Studi di Napoli Federico II

Dipartimento di Ingegneria Elettrica e delle Tecnologie
dell'Informazione



Neuro BackPropagation Lab

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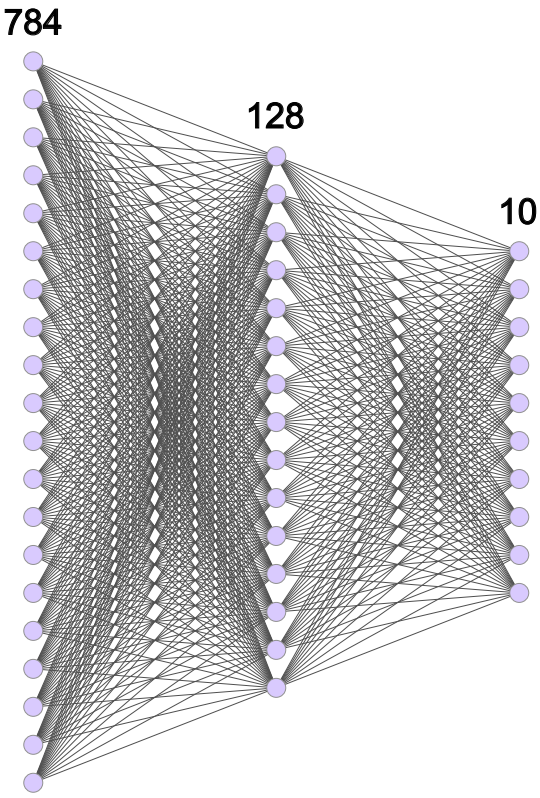
2025

Contents


784
128
10

Chapter 1

Prolusion

1.1 Goal

This report provides a comprehensive overview of a Python project whose goal is to develop and compare different adaptive backpropagation techniques involved in a machine learning process, as Rprop (Resilient BackPropagation). MNIST is the target of the learning model.

The project follows the "Empirical evaluation of the improved Rprop learning algorithms" article by Christian Igel and Michel Hüsken (2001).

1.2 Software Stack

- Python 3.9.6
- PyTorch 2.6.0

The project is equipped with a `requirements.txt` file which allows for seamless installation of dependencies, by executing `pip install -r requirements.txt`.

1.3 Project Structure

```
neuro-backprop-lab/
├── model/
├── tester/
│   ├── tester.py
│   └── <trained_model>.pt
├── trainer/
│   ├── irpropplus/
│   ├── rpropminus/
│   ├── rpropplus/
│   └── trainer.py
├── utils/
│   └── loader_dataset.py
├── test_model.py
└── train_model.py
```

- `model` includes the neural network model architecture.
- `tester` handles the testing flow of the ready-to-use `<trained_model>.pt`.
- `trainer` handles the examined backpropagation techniques and the training flow of the model, saving it as `<trained_model>.pt`.
- `utils` offers utility functions designed to support the root project scripts.

Chapter 2

Module Overview

The part of the project which is shared across all the examined Rprop techniques is presented as follows.

2.1 Train Model

This script runs the training flow of the model and saves the model for future testing phase.

Algorithm 1: `train_model.py`

$model \leftarrow newModel()$
$criterion, optimizer, epochs, train\_set, eval\_set \leftarrow get\_configuration()$

$Trainer.traineval(model, criterion, optimizer, train\_set, eval\_set, epochs)$

2.2 Test Model

This script runs the test flow of the model.

Algorithm 2: `test_model.py`

$model, optimizer \leftarrow load\_model()$
$criterion, test\_set \leftarrow get\_configuration()$

$Tester.test(model, criterion, test\_set)$

2.3 Model

This class, `model.py`, represents the artificial neural network model architecture to be trained and tested. What follows are empirical choices that are the result of various experiments.

The model is a shallow MultiLayer Perceptron based on `torch.nn.Module`[1] class. Its three layers are fully connected using `torch.nn.Linear(.)` and they feed forward as follows:

1. the first layer flattens the input MNIST image, by transforming it from a multidimensional vector to a 784-sized (since a 28×28-sized image is manipulated) one-dimensional vector;
2. the hidden layer receives the transformed vector and processes it into a 128-sized vector with a ReLU activation function to introduce non-linearity;
3. the output layer extracts the final predictions by transforming the 128-sized vector into a 10-sized vector, which corresponds to the number of possible classes for classification.

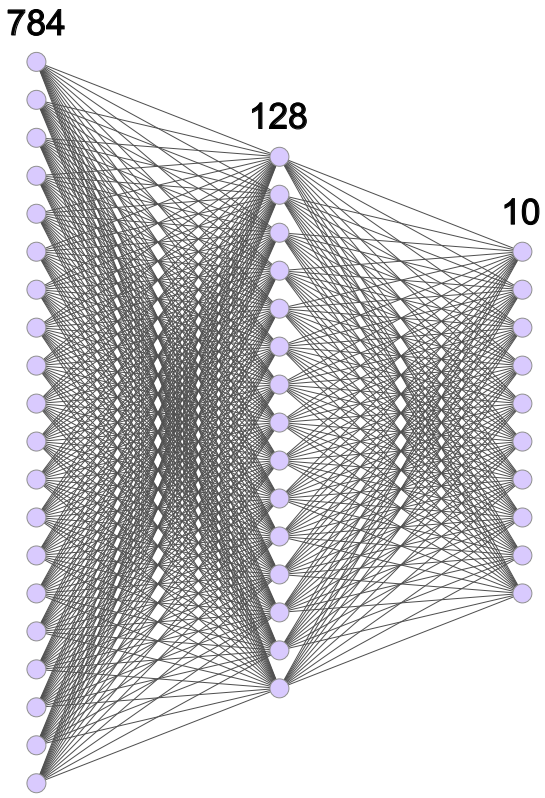


Figure 2.1: *A schematic representation of the model*[2]

[1] *https://pytorch.org/docs/stable/generated/torch.nn.Module.html* (accessed 2025)

[2] Representation generated using NN-SVG, a public tool available on GitHub (*https://github.com/alexlenail/NN-SVG*)

2.4 Trainer

This class, `trainer.py`, is responsible for training the model and saving it for future testing phase. The saved model corresponds to the epoch with the lowest evaluation error.

Data retrieval is addressed in subsection 2.6.1.

Algorithm 3: `trainer.py`

```
Function traineval(model, criterion, optimizer, train_set, eval_set, epochs):
    foreach epoch ∈ epochs do
        train_loss_avg, train_accuracy ←
          train(model, criterion, optimizer, train_set, train_loss_avgs, train_accuracies)
        eval_loss_avg, eval_accuracy ← eval(model, criterion, eval_set, eval_loss_avgs, eval_accuracies)
        if eval_loss_avg < eval_loss_avg_prev then
            savemodel(model)
        end
    end
    return train_loss_avgs, train_accuracies, eval_loss_avgs, eval_accuracies
return

Function train(model, criterion, optimizer, train_set, loss_averages, accuracies):
    foreach batch ∈ train_set do
        labels, loss, outputs ← trainstep(model, criterion, optimizer, batch)
        total_correct, total_loss, total_samples ←
          gather_metrics(labels, loss, outputs, total_correct, total_loss, total_samples)
    end
    loss_average, accuracy ←
      compute_metrics(total_correct, total_loss, total_samples, loss_averages, loss_accuracies)
    return loss_average, accuracy
return
Function trainstep(model, criterion, optimizer, batch):
    inputs, labels ← batch
    outputs ← model(inputs)
    loss ← criterion(outputs, labels)
    loss.compute_gradients()
    optimizer.step()
    return labels, loss, outputs
return
Function eval(model, criterion, eval_set, loss_averages, accuracies):
    foreach batch ∈ eval_set do
        labels, loss, outputs ← evalstep(model, criterion, batch)
        total_correct, total_loss, total_samples ←
          gather_metrics(labels, loss, outputs, total_correct, total_loss, total_samples)
    end
    loss_average, accuracy ←
      compute_metrics(total_correct, total_loss, total_samples, loss_averages, accuracies)
    return loss_average, accuracy
return
Function evalstep(model, criterion, batch):
    inputs, labels ← batch
    outputs ← model(inputs)
    loss ← criterion(outputs, labels)
    return labels, loss, outputs
return
```

2.5 Tester

This class, `tester.py`, is responsible for loading and testing a pre-trained model on unseen data.

Algorithm 4: `tester.py`

Input : model, criterion, test_set

Function *test*(*model, criterion, test_set*)**:**
  foreach $batch \in test\_set$ **do**
    $labels, loss, outputs \leftarrow eval(model, criterion, batch)$
    $total\_correct, total\_loss, total\_samples \leftarrow gather\_metrics(labels, loss, outputs)$
  end
  $loss\_average, accuracy \leftarrow compute\_metrics(total\_correct, total\_loss, total\_samples)$
return
Function *eval*(*model, criterion, batch*)**:**
  $inputs, labels \leftarrow batch$
  $outputs \leftarrow model(inputs)$
  $loss \leftarrow criterion(outputs, labels)$
  return *labels*, *loss*, *outputs*
return

2.6 Utils

2.6.1 Loader Dataset

This class, `loader-dataset.py`, is responsible for the methodology employed for data retrieval. It is the module that takes the most importance of the `utils` package, so it was worth describing it.

The method used to get the data is holdout, there is more than one function involved in this.

Algorithm 5: `loader_dataset.py`

Function *getdatasettraineval*(*dataset_size_train, batch_size_train, dataset_size_eval, batch_size_eval*):
    $learning\_set \leftarrow get\_MNIST\_dataset(learn\_mode = True)$
    $dataset\_train, dataset\_eval \leftarrow$
      $split(learning\_set, dataset\_size\_train, batch\_size\_train, dataset\_size\_eval, batch\_size\_eval)$
    return *dataset_train, dataset_eval*
return

Function *getdatasettest*(*dataset_size, batch_size*):
    $dataset\_test \leftarrow get\_MNIST\_dataset(learn\_mode = False)$
    return $dataset\_test(batch\_size)$
return

As planned, the learning data is completely separated from the testing data with the `learn_mode` parameter.

The learning set is, in turn, split into the train set for the training phase and in the eval set for the evaluation phase. Finally, the train set undergoes a data shuffle, this should make the train phase more robust. The eval set comes in handy to avoid overfitting.

It is also important to note that the MNIST allocates $60,000$ samples for training and $10,000$ for testing.

Chapter 3

Resilient Backpropagation Techniques

Rprop algorithms differ from the classical back-propagation algorithms by the fact that they are independent of the magnitude of the gradient, but depend on its sign only.

3.1 Implementations

Recall that the main concern of the documentation is readability. Hence, pseudocode and actual code implementations may slightly differ, as the Python scripting language allows for significant performance improvements through the use of native structures. These differences clearly don't affect the functionality of the implementations.

Each Rprop algorithm that will be described corresponds to a specialized `torch.optim.Optimizer.step()`[1] class method.

An Rprop algorithm is intended to perform the following steps:

1. Compute the gradient of the error function with respect to the model weights.
2. Update the step size based on a conditional logic of the current and previous gradient sign:

$$\Delta_{ij}^{curr} = \begin{cases} \min(\eta^+ \cdot \Delta_{ij}^{prev}, \Delta_{\max}) & \text{if} \frac{\partial E}{\partial w_{ij}}^{curr} \cdot \frac{\partial E}{\partial w_{ij}}^{prev} > 0 \\ \max(\eta^- \cdot \Delta_{ij}^{prev}, \Delta_{\min}) & \text{if} \frac{\partial E}{\partial w_{ij}}^{curr} \cdot \frac{\partial E}{\partial w_{ij}}^{prev} < 0 \\ \Delta_{ij}^{prev} & \text{otherwise} \end{cases}$$

3. Update the step size direction using either weight-backtracking or the gradient sign.
4. Update weights with the step size direction.

Subsequently, each variant of the algorithm implements its own adaptation of this general process.

[1] *https://pytorch.org/docs/main/optim.html* (accessed 2025)

3.1.1 Rprop without Weight-Backtracking

This Rprop version, also said Rprop$^-$, updates the step size direction with the gradient sign only.

Even though the gradient-product case resulting in zero is a no-op, it is kept for readability and consistency with other algorithms.

Algorithm 6: `RpropMinus.step()`

for $w_{ij} \in parameters$ **do**
  if $\frac{\partial E}{\partial w_{ij}}^{curr} \cdot \frac{\partial E}{\partial w_{ij}}^{prev} > 0$ **then**
    $\Delta_{ij}^{curr} = \min(\eta^+ \cdot \Delta_{ij}^{prev}, \Delta_{\max})$
  end
  if $\frac{\partial E}{\partial w_{ij}}^{curr} \cdot \frac{\partial E}{\partial w_{ij}}^{prev} < 0$ **then**
    $\Delta_{ij}^{curr} = \max(\eta^- \cdot \Delta_{ij}^{prev}, \Delta_{\min})$
  end
  if $\frac{\partial E}{\partial w_{ij}}^{curr} \cdot \frac{\partial E}{\partial w_{ij}}^{prev} = 0$ **then**
    $\Delta_{ij}^{curr} = \Delta_{ij}^{prev}$
  end
  $\Delta w_{ij}^{curr} = -\operatorname{sign}(\frac{\partial E}{\partial w_{ij}}^{curr}) \cdot \Delta_{ij}^{curr}$
  $w_{ij}^{curr} = w_{ij}^{prev} + \Delta w_{ij}^{curr}$
end

3.1.2 Rprop with Weight-Backtracking

This Rprop version, also said Rprop$^+$, updates the step size direction with the gradient sign when the gradient product is greater than or equal to zero. In the other case the algorithm performs a weight-backtracking, formally $\Delta w_{ij}^{curr} = -\Delta w_{ij}^{prev}$, then the current gradient is set to zero in order to activate the gradient-product case resulting in zero in the next iteration.

Algorithm 7: `RpropPlus.step()`

for $w_{ij} \in parameters$ **do**
  if $\frac{\partial E}{\partial w_{ij}}^{curr} \cdot \frac{\partial E}{\partial w_{ij}}^{prev} > 0$ **then**
    $\Delta_{ij}^{curr} = \min(\eta^+ \cdot \Delta_{ij}^{prev}, \Delta_{\max})$
    $\Delta w_{ij}^{curr} = -\operatorname{sign}(\frac{\partial E}{\partial w_{ij}}^{curr}) \cdot \Delta_{ij}^{curr}$
  end
  if $\frac{\partial E}{\partial w_{ij}}^{curr} \cdot \frac{\partial E}{\partial w_{ij}}^{prev} < 0$ **then**
    $\Delta_{ij}^{curr} = \max(\eta^- \cdot \Delta_{ij}^{prev}, \Delta_{\min})$
    $\Delta w_{ij}^{curr} = -\Delta w_{ij}^{prev}$
    $\frac{\partial E}{\partial w_{ij}}^{curr} = 0$
  end
  if $\frac{\partial E}{\partial w_{ij}}^{curr} \cdot \frac{\partial E}{\partial w_{ij}}^{prev} = 0$ **then**
    $\Delta_{ij}^{curr} = \Delta_{ij}^{prev}$
    $\Delta w_{ij}^{curr} = -\operatorname{sign}(\frac{\partial E}{\partial w_{ij}}^{curr}) \cdot \Delta_{ij}^{curr}$
  end
  $w_{ij}^{curr} = w_{ij}^{prev} + \Delta w_{ij}^{curr}$
end

3.1.3 Improved Rprop with Weight-Backtracking

This Rprop version, also said IRprop$^+$, extends the one described in subsection 3.1.2.

The only modification concerns the gradient-product case resulting in less than zero: weight-backtracking is adopted only if the current error is greater than the previous error (this is what is meant by 'improved'), otherwise the step size direction is set to zero.

Algorithm 8: `IRpropPlus.step()`

for $w_{ij} \in parameters$ **do**
  if $\frac{\partial E}{\partial w_{ij}}^{curr} \cdot \frac{\partial E}{\partial w_{ij}}^{prev} > 0$ **then**
    $\Delta_{ij}^{curr} = \min(\eta^+ \cdot \Delta_{ij}^{prev}, \Delta_{\max})$
    $\Delta w_{ij}^{curr} = -\operatorname{sign}(\frac{\partial E}{\partial w_{ij}}^{curr}) \cdot \Delta_{ij}^{curr}$
  end
  if $\frac{\partial E}{\partial w_{ij}}^{curr} \cdot \frac{\partial E}{\partial w_{ij}}^{prev} < 0$ **then**
    $\Delta_{ij}^{curr} = \max(\eta^- \cdot \Delta_{ij}^{prev}, \Delta_{\min})$
    if $E^{curr} > E^{prev}$ **then**
      $\Delta w_{ij}^{curr} = -\Delta w_{ij}^{prev}$
    else
      $\Delta w_{ij}^{curr} = 0$
    end
    $\frac{\partial E}{\partial w_{ij}}^{curr} = 0$
  end
  if $\frac{\partial E}{\partial w_{ij}}^{curr} \cdot \frac{\partial E}{\partial w_{ij}}^{prev} = 0$ **then**
    $\Delta_{ij}^{curr} = \Delta_{ij}^{prev}$
    $\Delta w_{ij}^{curr} = -\operatorname{sign}(\frac{\partial E}{\partial w_{ij}}^{curr}) \cdot \Delta_{ij}^{curr}$
  end
  $w_{ij}^{curr} = w_{ij}^{prev} + \Delta w_{ij}^{curr}$
end

3.1.4 Rprop with Weight-Backtracking by PyTorch

Additionally, the PyTorch version of Rprop$^+$ is provided as an extra implementation[2].

Thus, as said in sec. 3.1, this is the uncustomized `Optimizer.step()` method.

[2] *https://pytorch.org/docs/stable/generated/torch.optim.Rprop.html* (accessed 2025)

3.2 Comparisons

The four distinct techniques proposed for Rprop are compared through a structured experimental workflow. Each technique is measured in terms of loss and accuracy.

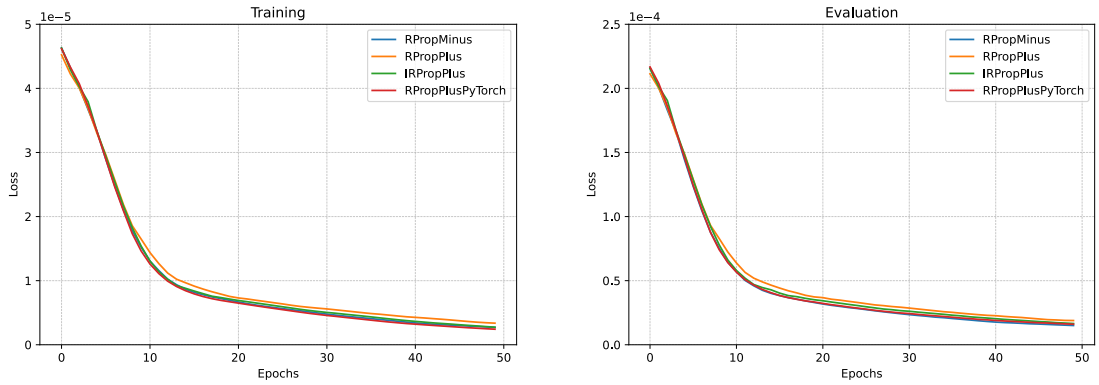
The loss function adopted is the cross-entropy loss:

$$\sum_{n=1}^{N} -\sum_{k=1}^{c} t_k^n \ln y_k^n$$

A total of $60,000$ elements was instantiated for the dataset, with $50,000$ allocated for the training set and $10,000$ for the evaluation set, using batch sizes of $50,000$ and $10,000$, respectively.
The learning rate η is set as 1e$-$3, η^- and η^+ are 0.5 and 1.2, $\Delta_{\min}$ and $\Delta_{\max}$ are 1e$-$6 and 50, the procedure was performed over 50 epochs. A coherent configuration file varying on a specific Rprop version is:

```
"criterion": "cross_entropy",
"optimizer": <rproptechnique>,
"learning_rate": 0.001,
"epochs": 50,
"train_set_size": 50000,
"train_batch_size": 50000,
"eval_set_size": 10000,
"eval_batch_size": 10000
```

3.2.1 Loss



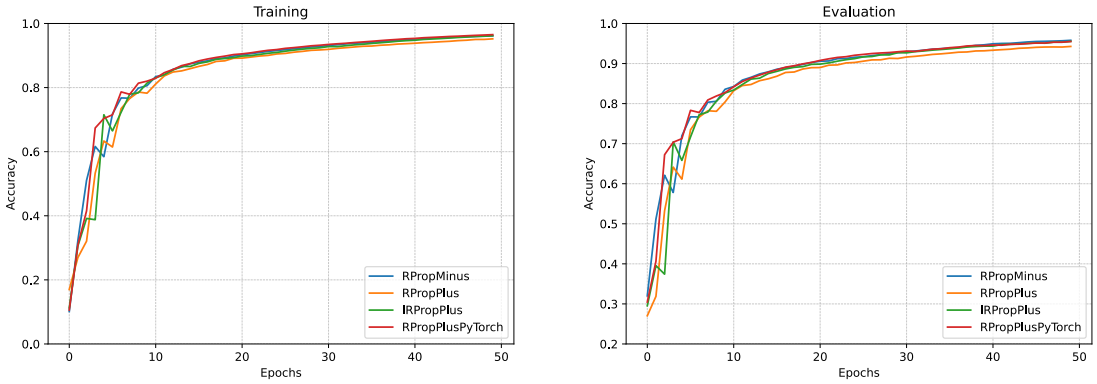
It can be instantly observed that the different versions of Rprop exhibit the same loss trend in the training phase: during the initial epochs, a steep decrease in loss is followed by a gradual decrease that starts from about the tenth epoch and persists until the last epoch.

Across these experiments, there is not a single technique that clearly stands out, they almost seem to overlap, although it can be seen that Rprop$^+$ slightly exhibits a worse loss.

Almost the same applies during the evaluation phase. However, a slight but greater discrepancy can be observed between each of the Rprop versions from the point at which the gradual decrease begins.
The Rprop$^-$ implementation seems to achieve the lowest error among all. Whereas, again, the Rprop$^+$ version exhibits the greatest error from the start of the gradual decrease onwards.
The evidence suggests that an early stopping criterion would not have been beneficial, as the evaluation loss does not exhibit any increasing curve from a certain point onwards. Even though it seems that the evaluation loss would have stopped improving from the 50^{th} epoch onwards.

An important remark is made on the difference in loss between training and evaluation phases: they differ by approximately an order of magnitude, this could be a warning of overfitting that could be worth monitoring.

3.2.2 Accuracy



In the training phase, the first 10 epochs were very bumpy, then the curves start to smooth out till the end of the training, reaching stability.

In the evaluation phase, oscillations are more visible for each of the Rprop versions, compared to the training phase. The performance of each Rprop version mirrors the trend observed during training: Rprop$^+$ shows the worst performance, while the Rprop$^-$ implementation reaches the peak.

Rprop Technique	**Evaluation Accuracy**	**Test Accuracy**
Rprop-	0.95800	0.95600
Rprop+	0.94270	0.94780
IRprop+	0.95540	0.95570
Rprop+ PyTorch	0.95500	0.95550

Table 3.1: *Test and evaluation performance metrics for Rprop implementations*

As highlighted in the table 3.1, the testing phase remained true to the evaluation phase, this emphasizes a good generalization by the model.

Acronyms

IRprop Improved Resilient BackPropagation 18

MLP MultiLayer Perceptron 10

MNIST Modified National Institute of Standards and Technology database 3, 10, 14

ReLU Rectified Linear Unit 10

Rprop Resilient BackPropagation 3, 7, 15–22