Conjugacy tables with fixed margins

1 Background

Let $\lambda = (\lambda_1, \ldots, \lambda_m)$ and $\mu = (\mu_1, \ldots, \mu_n)$ be compositions of a natural number d. Let M be an $m \times n$ matrix with entries in $\mathbb{N}$. Say that the pair (λ, μ) is the *margin* of M if the sum of entries in the i-th row of M is λ_i, and the sum of entries in the j-th column of M is μ_j. Write A^λ_μ for the set of such matrices.

For example:

$$A^{(3,3)}_{(2,2,2)} = \{ \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 2 & 1 \\ 2 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 2 \\ 2 & 1 & 0 \end{pmatrix} \}.$$

A summary of results and applications of such matrices can be found in [DG95]. In statistical applications these matrices arise as conjugacy tables with fixed margins (alternatively called fixed-margin matrices). Such matrices also play a role in group theory and representation theory. For instance A^λ_μ is in bijection with double cosets of the symmetric group $\mathfrak{S}_d$ by Young subgroups $\mathfrak{S}_\lambda$ and $\mathfrak{S}_\mu$. These results and more are summarised in [DG95].

2 Fixed-Margin matrix calculator (how to use)

This github repo contains a single python script to calculate the matrices with margin (λ, μ) for a given λ, μ. After running the script you will be prompted to enter the row sequence λ, then the column sequence μ. Enter these as a list of integers separated by commas. If the sum of entries in λ and μ are equal, the console will return the list of all matrices with margin (λ, μ) and tell you how many such matrices there are. Otherwise an error will be raised and you will be prompted to enter the input sequences again. Type q to quit.

References

[DG95] Persi Diaconis, Anil Gangolli. Rectangular arrays with fixed margins, in: *Discrete Probability and Algorithms*, Springer-Verlag, Berlin/New York, 15-41, 1995