# Game Theory Final Project

A Game Theoretical Application on Internet Protocols: Gateway Selection Game in Cyber-Physical Systems

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# Summary

- Analysis of the Gateway Selection Problem
- Analogies between Gateway Selection Problem and Game Theory
- Modeling of the problem
- Gateway Selection Algorithm
- Implementation of a simulator for dynamic analysis

# Scenario

- Working area with devices and gateways
- Devices with two different wireless communication technologies
- Gateways with a fixed and constant bandwidth

# Devices’ and Gateways’ Dynamics

- Devices are divided into 2 groups according to their wireless communication technologies
- The devices of the same group can exchange information with each other
- Each device has to be connected with one and only one gateway which is reachable from its position
- The bandwidth of a gateway is equally distributed among all the devices connected
- Each device chooses its gateway in order to maximize the obtained bandwidth

# Assumptions

- Each gateway has the same bandwidth $W$
- Each device must be connected to one and only one gateway
- Devices do not have a minimum bandwidth’s demand
- Each device has to be in the range of at least one gateway

# Model Adopted

Network Model:

- Gateways' set $G = \{g_1, g_2, \ldots, g_k\}$
- Devices' sets differentiated by different wireless communication technologies:

$$A = \{a_1, a_2, \ldots, a_n\} \qquad B = \{b_1, b_2, \ldots, b_m\}$$

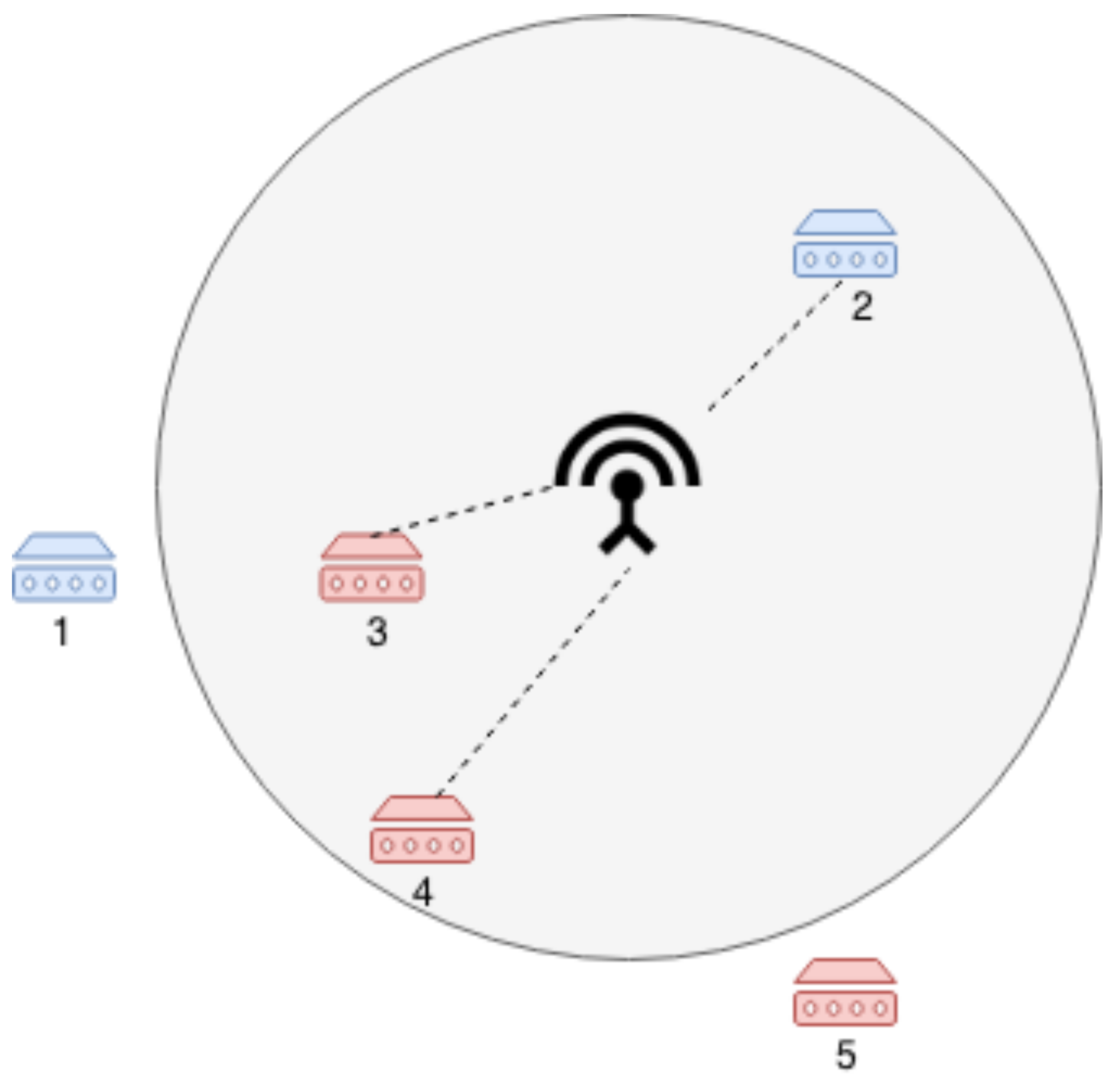
# Model Adopted

Bandwidth Allocation Model:

- Each gateway $g_i$ has a total bandwidth $W$
- The gateway's bandwidth assigned to each client connected is

$$w_{a,i} = w_{b,i} = \frac{W}{n_i + m_i}$$

# Practical Example




- Devices 2, 3 and 4 are connected to the reachable gateway *i*
- $n_i = 2 \quad m_i = 1$
- The bandwidth obtained by each connected device is:

$$w_{a,i} = w_{b,i} = \frac{W}{n_i + m_i} = \frac{W}{3}$$

# Model Adopted

Gateway Selection Game:

- It is based on clients that want to maximize their benefits
- If a device increases its bandwidth, then all the devices of the same kind will have a benefit in it
- A device changes the associated gateway if and only if this migration will increase the total bandwidth of its client

# Game Theory Interpretation

The Gateway Selection Game is:

- Sequential: the devices sequentially choose their associated gateways
- Iterated: each device continues to change gateway until an equilibrium is reached
- Non-zero sum: the sum of the clients' utilities is the sum of the related gateways' bandwidths (> 0)
- Perfect Information: each client is aware of all the devices connected with each gateway
- Non-cooperative: each clients aims to maximize its own total bandwidth, without considering the other client's utility

# Mathematical Formulation

*Sets*

$A = \{a_1, a_2, \ldots, a_n\}$ *is the set of the devices type A*

$B = \{b_1, b_2, \ldots, b_m\}$ *is the set of the devices type B*

$P = \{A, B\}$ is the set of the players, where $A$ and $B$ are respectively *client A* and *client B*

$G = \{g_1, g_2, \ldots g_k\}$ is the set of the *k-available gateways*

*Strategies*

$S_A = \{s_{a_1}, s_{a_2}, \ldots, s_{a_n}\}$, where $s_i$ is the strategy of the device $a_i \in A$, i.e. $s_i \subseteq G$ and contains all the possible gateways reachable by $a_i$

$S_B = \{s_{b_1}, s_{b_2}, \ldots, s_{b_m}\}$, *where* $s_i$ *is the strategy of the device* $b_i \in B$, *i.e.* $s_i \subseteq G$ *and contains all the possible gateways reachable by* $b_i$

*Decision Variables*

$$x_{i,g} = \begin{cases} 1 & \text{if the device i selects the gateway g and g} \in s_i \\ 0 & \text{otherwise} \end{cases} \qquad i \in A \cup B, g \in G$$

$n_g$ = number of devices type A connected to gateway g, i.e. $n_g = \sum_{i=1}^{n} x_{a_i,g} \quad a_i \in A, g \in G$

$m_g$ = number of devices type B connected to gateway g, i.e. $m_g = \sum_{i=1}^{m} x_{b_i,g} \quad b_i \in B, g \in G$

$w_{i,j}$ = bandwidth obtained by player i connected to gateway g,

i.e. $w_{i,g} = \begin{cases} \frac{W}{n_g + m_g} & \text{if } n_g + m_g > 0 \\ 0 & \text{otherwise} \end{cases} \qquad i \in P, g \in G$

$W_A$ = total bandwidth obtained by player $A$, i.e. $W_A = \sum_{g=1}^{k} w_{A,g}$

$W_B$ = total bandwidth obtained by player $B$, i.e. $W_B = \sum_{g=1}^{k} w_{B,g}$

*Payoffs*

$W_A$ and $W_B$ are respectively the payoffs of player $A$ and player $B$.

*Nash Equilibrium*

The game reaches the Nash Equilibrium when each player selects the strategy which maximizes its payoff:

$$\max \quad W_A + W_B$$

*s.t.*

$$\sum_{g=1}^{k} x_{i,g} = 1 \quad \forall i \in A \cup B$$ (each device must be connected to one and only one gateway)

$$W_A = \sum_{g=1}^{k} w_{A,g}$$ (the total bandwidth of player A is the sum of the bandwidth given by each gateway)

$$W_B = \sum_{g=1}^{k} w_{B,g}$$ (the total bandwidth of player B is the sum of the bandwidth given by each gateway)

where $w_{i,j}$ has been previously defined.

# Gateway Selection Algorithm

| | |
|---|---|
| Input | • Device's accessible gateways set<br>• Device's current gateway<br>• Parameter of each gateway  in  (bandwidth)<br>• Number of devices of type  and devices type  connected to |
| Output | • Decision to migration<br>• Eventually, the new gateway selected for the migration |
| Algorithm | 1. **for each** gateway $g_i$ **in** $G_i$<br>2. calculate the increment $\Delta w_i$ of the total payoff of the population after migration<br>3. **if** $\Delta w_i > 0$<br>4. device changes its gateway to $g_i$<br>5. **else**<br>6. device keeps its current gateway |

To calculate $\Delta w_i$ , we adopt the following formula:

$$\Delta w_i = \left[(n_0 - 1)\frac{W}{n_0 + m_0 - 1} + (n_i + 1)\frac{W}{n_i + m_i + 1}\right] - \left(n_0\frac{W}{n_0 + m_0} + n_i\frac{W}{n_i + m_i}\right)$$

Where:

- n and m are used to differentiate the two kinds of devices
- the elements subscripted by 0 are connected with the starting gateway
- the element subscripted by $i$ are connected to the destination gateway

There are some values of the attributes that make this formula unfeasible. For each of them, an adapted formula is proposed (next slides).

- $n_0 + m_0 = 1$ **and** $n_i + m_i = 0$

$$\Delta w_i = \left[(n_i + 1)\frac{W}{n_i + m_i + 1}\right] - \left(n_i\frac{W}{n_i + m_i}\right) = 0$$

So the client will **not** change the gateway

- $n_0 + m_0 > 1$ **and** $n_i + m_i = 0$

$$\Delta w_i = \left[ (n_0 - 1)\frac{W}{n_0 + m_0 - 1} + (n_i + 1)\frac{W}{n_i + m_i + 1} \right] - \left( n_0 \frac{W}{n_0 + m_0} \right) > 0$$

So the client will change the gateway

- $n_0 + m_0 = 1$ **and** $n_i + m_i > 0$

$$\Delta w_i = \left[(n_i + 1)\frac{W}{n_i + m_i + 1}\right] - \left(n_0\frac{W}{n_0 + m_0} + n_i\frac{W}{n_i + m_i}\right) < 0$$

So the client will **not** change the gateway

# Theoretical Results

***Theorem 1.*** *If the number of clients A and clients B of the original gateway equals the number of clients A and clients B of candidate gateway, respectively, none of these clients will change its strategy.*

***Theorem 2.*** *If the number of one kind of clients in the original gateway is equal to that in the candidate gateway, the other kind of clients in original gateway will change its strategy iff there are at least two clients more in original gateway than in candidate gateway.*

***Theorem 3.*** *Denote the number of clients B in $g_1$ and $g_2$ by $n_1$ and $n_2$, and denote the number of clients B in $g_1$ and $g_2$ by $m_1$ and $m_2$. When $n_1 = n_2 = n$ and $m_1 \neq m_2$, we have the following.*

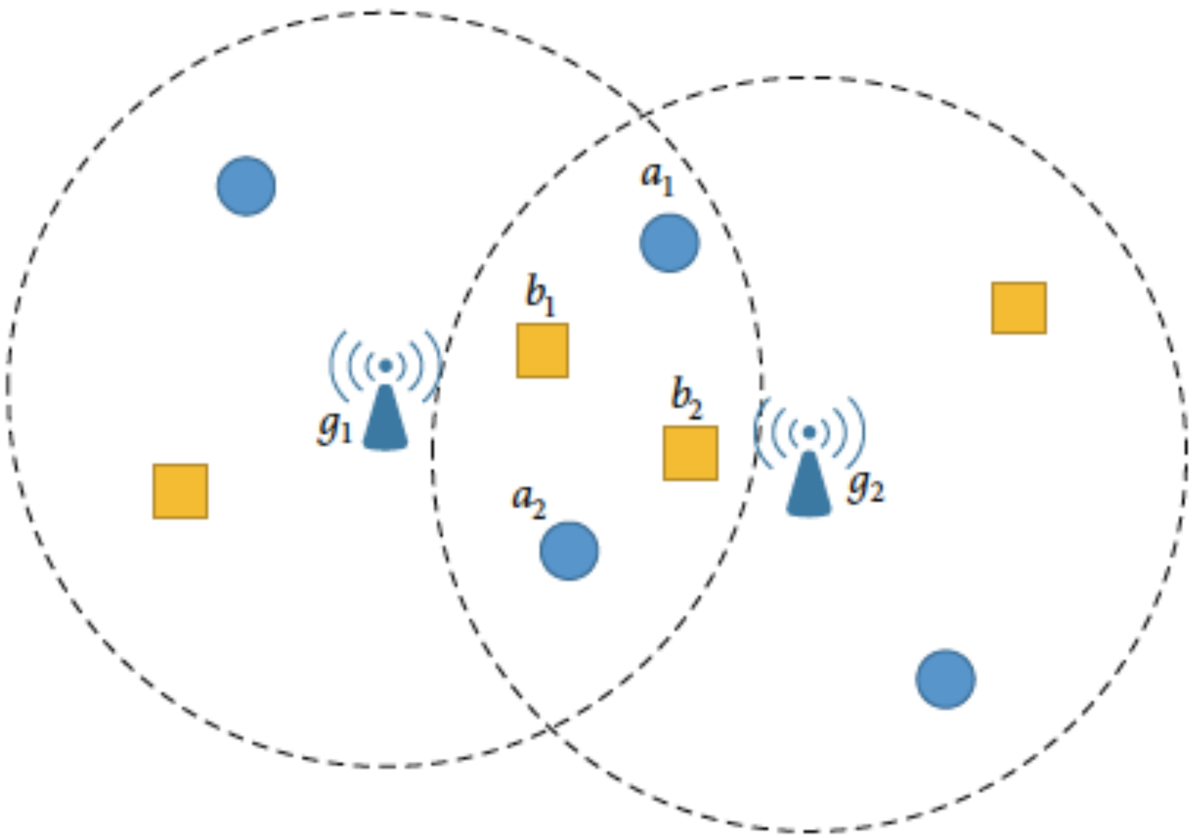
1. *If $m_1 = m_2 + 1$, none of players of clients A in $g_1$ will change its strategy to $g_2$.*
2. *If $m_1 < m_2$ and $n^2 < m_1 \cdot m_2$ or $m_1 < m_2$ and $n^2 > m_1 \cdot m_2$, none of players of clients A in $g_1$ will change its strategy to $g_2$.*

***Theorem 4.*** *If only one kind of clients can change their strategies, this gateway selection game will always come to a Nash Equilibrium.*

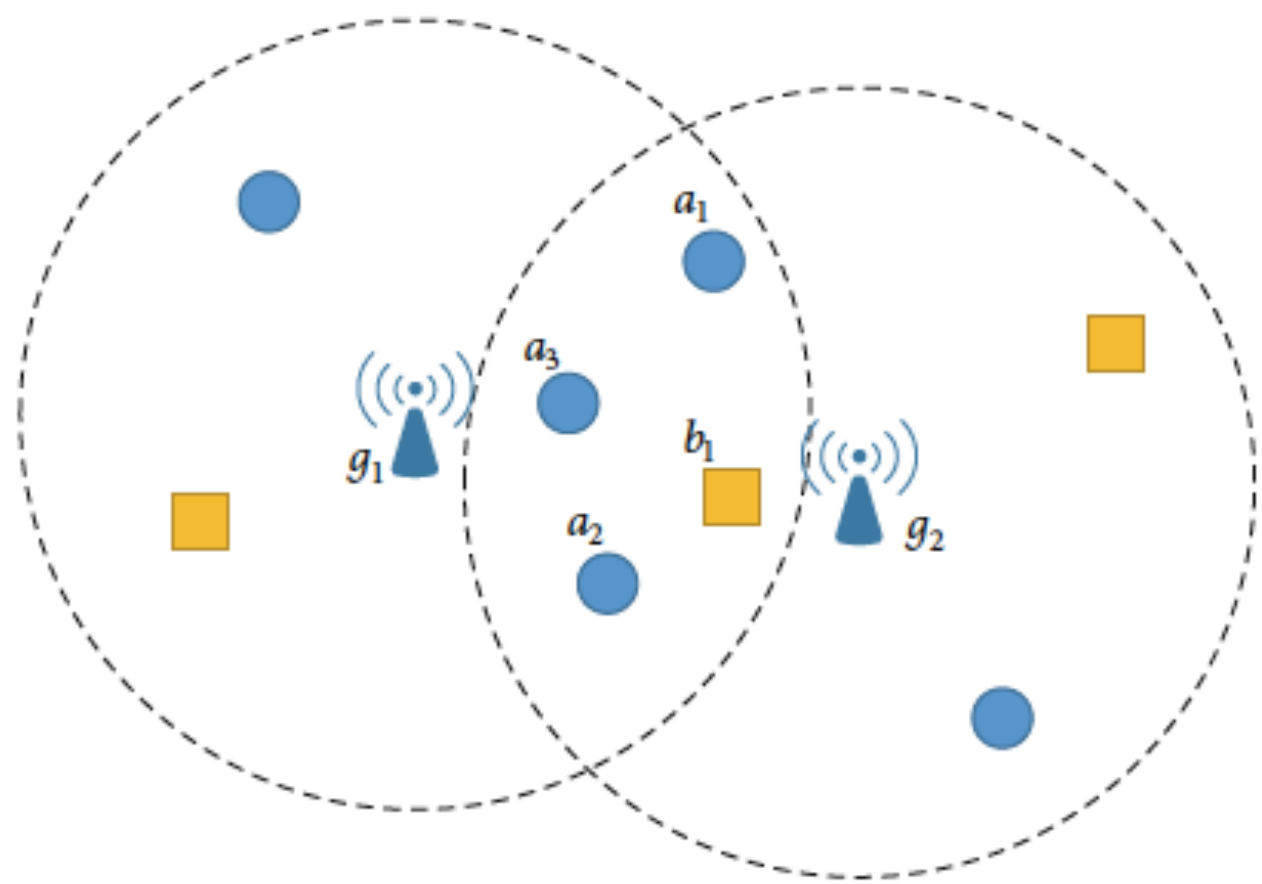
# The Termination Problem

- The termination of the algorithm is not guaranteed because loops can occur if the Devices continue to change their associated Gateway
- The previously seen ***Theorem 4*** guarantees the termination if just one kind of devices is running the algorithm
- It is possible to detect loops by checking if the total score (i.e. the payoffs) does not change in sequential steps of the algorithm
- Our simulator terminates after a fixed number of consecutive loops (threshold) in which the total score does not change

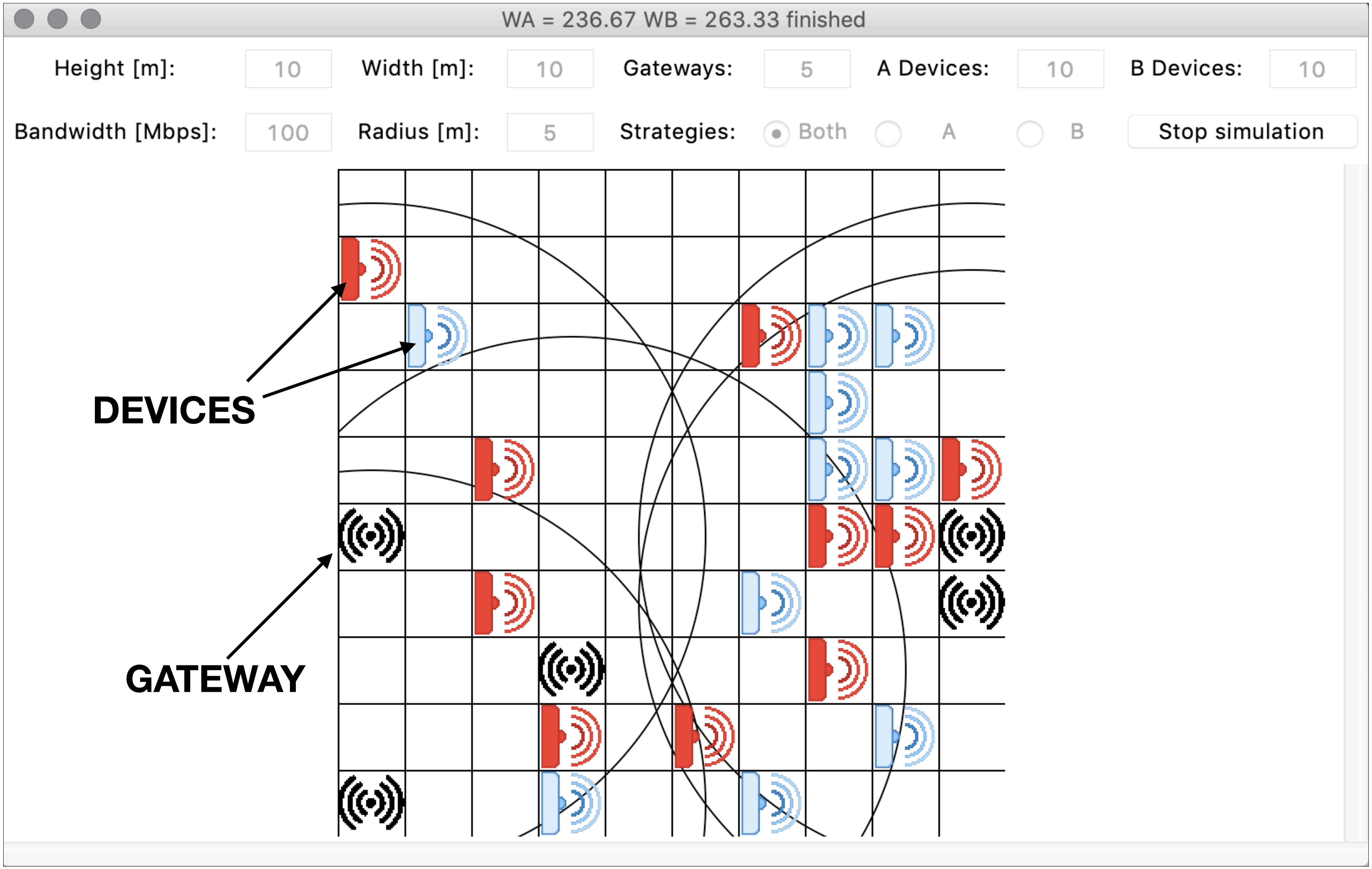
**An example which has Nash Equilibrium**




**An example which has no Nash Equilibrium**



# Simulation


WA = 236.67 WB = 263.33 finished
Height [m]:
10
Width [m]:
10
Gateways:
5
A Devices:
10
B Devices:
10
Bandwidth [Mbps]:
100
Radius [m]:
5
Strategies:
Both
A
B
Stop simulation
DEVICES
GATEWAY

# References

[1] Hao Wang, Jianzhong Li, and Hong Gao, “Gateway Selection Game in Cyber-Physical System”, International Journal of Distributed Sensor Network, Vol. 2016, Atricle ID 7190767

# Thank you!