Auction in Multi-Path Multi-Hop Routing

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***Abstract*—We model the multi-path multi-hop routing in networks with selfish nodes as an auction and provide a novel solution from the game-theoretical perspective. We design a mechanism that results in Nash equilibria rather than the traditional strategyproofness, which alleviates the over-payment problem of the widely used VCG mechanism. Through theoretical analysis, the proposed protocol is shown to be effective.**

***Index Terms*—Mechanism design, game theory, Nash equilibrium.**

I. Introduction

With advantages such as load balancing and robustness to failures, multi-path routing has long been studied as an important routing technique in various networks. In Internet, traffic may be routed over multiple paths due to both technical and administrative reasons. In wireless ad hoc networks, in order to avoid excessive interference and draining the device battery on a specific path, traffic may also be allocated to multiple paths. When all the participating nodes are self-organized, multi-path routing is a very interesting problem. Helping forward packets incurs cost and thus a selfish participant is not likely to offer the service free of charge. Therefore how to stimulate cooperation among these rational nodes is a challenge.

In this letter, we analyze the rational behaviors in the multi-path routing scenario and model the multi-path routing game as an auction. In previous works, the Vickrey-Clark-Groves (VCG) payment mechanism is widely used in both single-path routing [1][2] and multi-path routing [3] scenarios. However, VCG suffers from the inevitable over-payment problem [5][6]. Efforts to alleviating this problem have been made in single-path routing scenario [9]. In order to alleviate the over-payment in the multi-path routing scenario, we propose a mechanism that uses generalized second price (GSP) auction originating from Internet advertising [7][8] and tailors it to the multi-path routing game. Our mechanism provably results in Nash equilibria for all player nodes to behave honestly. Through theoretical analysis, our mechanism is shown to effectively reduce the over-payment. To the best of our knowledge, this is the first work that adopts and analyzes GSP mechanism in the network routing context.

Manuscript received December 1, 2008. The associate editor coordinating the review of this letter and approving it for publication was A. Banchs.

This work was supported by a grant from the Research Grants Council of the Hong Kong SAR, China [Project No. CityU 111208].

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Digital Object Identifier 10.1109/LCOMM.2009.082021

II. System Model

A. Network Abstract

A network is formed by a finite number of nodes, which are denoted by $\mathcal{V} = \{1, 2, \ldots, n\}$. The edge $(i, j) \in \mathcal{E}$ between node i and node j represents the communication link between them. In this way, the network could be represented by a weighted graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{W}\}$, where $\mathcal{W}$ is the set of weights representing the cost on each edge (i, j).

The player set of this multi-path routing game are the intermediate nodes. We assume each intermediate node incurs a per-packet cost for forwarding traffic, and this cost is private to herself. For the sake of simplicity, we assume in this paper that there is no collusion among the nodes. In the route discovery process, each node bids with the reported cost of the outgoing links. After obtaining all the link information and constructing the network graph, the routing protocol orders the node-disjoint paths as the Least Cost Path (LCP) candidates denoted as $\{LCP_i\}$, with $i < j$ if $\mathcal{C}_i < \mathcal{C}_j$, where $\mathcal{C}_i$ is the cost of LCP_i. Assume that the first m LCP candidates are selected for packet forwarding. A fraction of data traffic f_i will be forwarded through LCP_i. The per-packet payment is calculated according to the routing decision and the bids placed by intermediate nodes.

The bid of each intermediate node is kept confidential by encryption and can only be exposed to the destination and source nodes. Once the route discovery process is finished, nodes cannot change their bids before the transmission is complete or rerouting is triggered. Therefore, we model this auction as a simultaneous-move, one-shot strategic game.

Following the definition in game theory [10], node i's per-packet payoff or utility u_i is given by

$$u_i = p_i - c_i, \tag{1}$$

where c_i is node i's cost and p_i is the payment made by the source to node i.

While it is obvious each intermediate node i's objective is to maximize her utility by giving a proper bid, the goal of the entire system is to minimize the total transmission cost by allocating proper traffic among the m paths, subject to certain constraints $\{\mathcal{P}^{(n)}\}$, which represent a set of policies we will discuss in detail later. Therefore, we have

$$\begin{aligned} \text{node } i: &\quad \max\{u_i f_i\} \\ \text{system}: &\quad \min\left\{\sum_{j=1}^{m}(\mathcal{C}_j f_j)\right\} \\ &\quad s.t. \quad \{\mathcal{P}^{(n)}\} \end{aligned} \tag{2}$$

B. Mechanism Design

We need to design a mechanism to calculate the payment. The objective of such a mechanism is to stimulate the rational players being honest without hurting their utility.

VCG (sometimes is also called *second price auction*) payment is a widely used strategyproof mechanism for network routing [1][2][4]. In classical auction context, VCG requires each player i in the auction pays the opportunity cost that her presence introduces to all the other players [11]. For the multi-path routing game, if m LCP candidates are used, the VCG per-packet payment to node i for serving LCP_k is given by

$$\begin{aligned} p_{i,k}^{VCG} &= \frac{1}{f_k}\left[\sum_{j=k}^{m}\mathcal{C}_{j+1}f_j - \left(\sum_{j=k+1}^{m}\mathcal{C}_j f_j + [\mathcal{C}_k - c_{i,k}]f_k\right)\right] \\ &= \frac{\sum_{j=k}^{m}[\mathcal{C}_{j+1}-\mathcal{C}_j]f_j}{f_k} + c_{i,k}, \end{aligned} \tag{3}$$

where $c_{i,k}$ denotes the cost of node i serving LCP_k.

VCG is strategyproof, however, it suffers from the over-payment problem [5][6]. In order to reduce the over-payment, we propose to adopt another mechanism called generalized second price (GSP) auction. GSP is currently used by Yahoo! and Google in Internet advertising auction [7][8], where advertisers bid for multiple advertisement positions for each keyword appearing on the search engine. Instead of calculating the opportunity cost of all the other players, GSP only considers the player who obtains the next position. Therefore in the multi-path routing game, if node i is on LCP_k, then GSP per-packet payment only considers the cost of other nodes on LCP_k (excluding herself) and LCP_{k+1}, and is given by

$$p_{i,k}^{GSP} = \frac{\mathcal{C}_{k+1}f_k - [\mathcal{C}_k - c_{i,k}]f_k}{f_k} = \mathcal{C}_{k+1} - \mathcal{C}_k + c_{i,k} \tag{4}$$

Although GSP achieves lower over-payment than VCG, the existing GSP mechanism used in Internet advertising auction has a unpleasant flaw that generally it does not have any truth-telling equilibrium [7][8], where each node truthfully reveals her private type. According to our analysis, this flaw arises from the fact that Yahoo! and Google do not have the ability to control or adjust the number of clicks by users per unit time for each advertisement position. In our mechanism design, we eliminate this flaw by adding a policy which controls the traffic allocation among the selected LCP candidates.

III. Policy Discussion

In the network abstract, the constraints represent a set of policies $\{\mathcal{P}^{(n)}\}$. Such policies are an important part of our mechanism design. The basic policies are as follows:

$\mathcal{P}^{(1)}$: *The number of selected LCP candidates m is always less than the total number of LCP candidates between the source and destination.* This policy is natural and easy to understand. Note that the case with $m = 1$ is trivial since it reduces the problem to single-path routing, where our proposed mechanism behaves exactly the same as VCG does. Therefore, we only consider $m \geq 2$ in the following discussion.

$\mathcal{P}^{(2)}$: *The fraction of data traffic forwarded through each selected LCP follows that $f_1 > f_2 > \cdots > f_m > 0$ and $\sum_{i=1}^{m} f_i = 1$.* We emphasize that $\forall i, f_i > 0$ since any $f_i = 0$ will reduce m. Also, the fact that traffic is allocated to the m LCP candidates in descending order is compatible to the entire system's goal since LCP_i with larger i introduces higher cost.

$\mathcal{P}^{(3)}$: *For any $p < q$, we have $[\mathcal{C}_{p+1} - \mathcal{C}_p]f_p > [\mathcal{C}_{q+1} - \mathcal{C}_p]f_q > [\mathcal{C}_{q+1} - \mathcal{C}_q]f_q$.* We refer policy $\mathcal{P}^{(3)}$ as the *traffic allocation condition*. We use this condition to guarantee a truth-telling node's utility if some other nodes are trying to game the routing protocol.

$\mathcal{P}^{(4)}$: *Any other practical policies that do not conflict with $\mathcal{P}^{(1)} \sim \mathcal{P}^{(3)}$ but are essential to the network under consideration.* We introduce $\mathcal{P}^{(4)}$ to reflect those technical and/or administrative polices existing in practical networks and routing protocols. For example, in wireless networks, a certain link usually has a limit for traffic carried over unit time due to device battery life or transmission interference. It is also common to observe in the Internet routing that sometimes one AS does not want her traffic through a certain intermediate AS exceeding a threshold due to administrative reasons. A simple example of $\mathcal{P}^{(4)}$ could be the constraints that $f_i \leq \mathcal{L}_S^i, (1 \leq i \leq m)$, where $\mathcal{L}_S^i$ is the limit of source's traffic fraction sent over LCP_i. The detailed discussion of $\mathcal{P}^{(4)}$ is beyond the scope of this paper. We only present it here to provide a means of combining our incentive polices discussed in this paper with the existing network architecture and routing protocols in the literature. Once $\{\mathcal{P}^{(n)}\}$ have a feasible region, the selection of $\mathcal{P}^{(4)}$ does not affect the analysis of our proposed mechanism. Alternatively, as in [3], one can take into account bandwidth constraints explicitly when formulating the traffic allocation policy.

IV. Theoretical Analysis

In this section, we give theoretical analysis of the proposed mechanism.

Theorem 1: In the multi-path routing game, under the traffic allocation condition $\mathcal{P}^{(3)}$, there are Nash equilibria for all player nodes to honestly bid their true cost.

Proof: Let $a_k^{(r)}$ be the action of node k bidding her true cost of link k' in route discovery. Let $\overline{a}_k^{(r)} \neq a_k^{(r)}$ be a different action. Let $a_{-k}^{(r)}$ be the action profile of all the other nodes behave honestly in this stage. For the prospective utility, we will show that $u_k(\overline{a}_k^{(r)}, a_{-k}^{(r)}) \leq u_k(a_k^{(r)}, a_{-k}^{(r)})$. There are several cases.

- Case (1): With $a_k^{(r)}$, Link k' was on LCP_j, $j > m$, and node k exaggerates the cost of k'.
- Case (2): With $a_k^{(r)}$, Link k' was on LCP_j, $j > m$, and node k understates the cost of k'.
- Case (3): With $a_k^{(r)}$, Link k' was on LCP_i, $1 < i \leq m$, and node k exaggerates the cost of k'.
- Case (4): With $a_k^{(r)}$, Link k' was on LCP_i, $1 < i \leq m$, and node k understates the cost of k'.

In case (1), exaggerating the cost could not increase the chance of LCP_j being selected, i.e., $u_k(\overline{a}_k^{(r)}, a_{-k}^{(r)}) = u_k(a_k^{(r)}, a_{-k}^{(r)}) = 0$.

In case (2), understating the cost could increase the chance of LCP_j being selected. Recall that $\mathcal{C}_j$ as the cost of LCP_j if node k bids the true cost. After understating the cost of link k', if LCP_j is still not selected, then node k will not make a profit and $u_k(a_k^{(r)}, a_{-k}^{(r)}) = u_k(\overline{a}_k^{(r)}, a_{-k}^{(r)}) = 0$. If LCP_j is selected after understating the cost of k', it becomes one of the selected LCP candidates $LCP_i, 1 \leq i \leq m$. Note for true cost $\mathcal{C}_j > \mathcal{C}_{i+1}$. Therefore $u_k(\overline{a}_k^{(r)}, a_{-k}^{(r)}) = [\mathcal{C}_{i+1} - \mathcal{C}_j] \cdot f_i < 0 = u_k(a_k^{(r)}, a_{-k}^{(r)})$.

In case (3), there are three possibilities. First, with $\overline{a}_k^{(r)}$, if link k' becomes not selected, then $u_k(\overline{a}_k^{(r)}, a_{-k}^{(r)}) = 0 < u_k(a_k^{(r)}, a_{-k}^{(r)})$. Second, with $\overline{a}_k^{(r)}$, if link k' is moved from LCP_i to $LCP_j, i < j \leq m$, according to policy $\mathcal{P}^{(3)}$, we have $u_k(\overline{a}_k^{(r)}, a_{-k}^{(r)}) = [\mathcal{C}_{j+1} - \mathcal{C}_i]f_j < [\mathcal{C}_{i+1} - \mathcal{C}_i]f_i = u_k(a_k^{(r)}, a_{-k}^{(r)})$. Third, with $\overline{a}_k^{(r)}$, if link k' is still on LCP_i, the nodes on LCP_{i-1} do not have incentive (or fear) to underbid because their utilities are guaranteed by policy $\mathcal{P}^{(3)}$, which coincides with the locally envy-free property [7]. Hence $u_k(\overline{a}_k^{(r)}, a_{-k}^{(r)}) = u_k(a_k^{(r)}, a_{-k}^{(r)})$.

In case (4), there are two possibilities. First, with $\overline{a}_k^{(r)}$, if link k' is still on LCP_i, then $u_k(\overline{a}_k^{(r)}, a_{-k}^{(r)}) = u_k(a_k^{(r)}, a_{-k}^{(r)})$. Second, with $\overline{a}_k^{(r)}$, link k' is moved from LCP_i to LCP_j, $1 \leq j < i$. Note for true cost $\mathcal{C}_i > \mathcal{C}_{j+1}$. Therefore $u_k(\overline{a}_k^{(r)}, a_{-k}^{(r)}) = [\mathcal{C}_{j+1} - \mathcal{C}_i]f_j < 0 < u_k(a_k^{(r)}, a_{-k}^{(r)})$.

Therefore node k can only maximize her utility by bidding the true cost of link k', i.e., revealing the private type. Thus the proof is completed. ∎

Theorem 1 is important in the sense that it demonstrates that by policy $\mathcal{P}^{(3)}$, our mechanism results in a set of Nash equilibria where each player node reveals the true cost. Any additional police $\mathcal{P}^{(4)}$ will lead to a certain Nash equilibrium among them. Based on Theorem 1, we now analyze the over-payment at any of these equilibria.

Theorem 2: In any of these Nash equilibria, the over-payment of GSP is always less than VCG.

Proof: It is easy to show that the amount of over-payment to each intermediate node actually equals her utility, and it is the same for all the intermediate nodes on the same path. Let n_i, $1 \leq i \leq m$, denote the number of intermediate nodes on LCP_i. Consider the ratio r_i of the over-payment introduced by GSP to VCG on LCP_i:

$$r_i = \frac{[\mathcal{C}_{i+1} - \mathcal{C}_i] \cdot f_i \cdot n_i}{\sum_{j=i}^{m} [\mathcal{C}_{j+1} - \mathcal{C}_j] \cdot f_j \cdot n_i} \tag{5}$$

Remember $\mathcal{P}^{(3)}$ guarantees that for any $p < q$, we have $[\mathcal{C}_{p+1} - \mathcal{C}_p] \cdot f_p > [\mathcal{C}_{q+1} - \mathcal{C}_q] \cdot f_q$, therefore we can derive the lower and upper bounds for this ratio as:

$$\begin{aligned} r_i^{lower} &> \frac{[\mathcal{C}_{i+1} - \mathcal{C}_i] \cdot f_i}{[\mathcal{C}_{i+1} - \mathcal{C}_i] \cdot f_i \cdot (m - i + 1)} \\ &= \frac{1}{m - i + 1} \end{aligned} \tag{6}$$

$$r_i^{upper} < \frac{[\mathcal{C}_{i+1} - \mathcal{C}_i] \cdot f_i}{[\mathcal{C}_{m+1} - \mathcal{C}_m] \cdot f_m \cdot (m - i + 1)} \tag{7}$$

Now we consider the ratio r between the total over-payment introduced by GSP to VCG, and derive its lower and upper bounds as:

$$r^{lower} > \min_i(r_i^{lower}) = r_1^{lower} = \frac{1}{m} \tag{8}$$

$$r^{upper} < \max_i(r_i^{upper}) = r_m^{upper} = 1 \tag{9}$$

Therefore, we conclude that at any of these Nash equilibria, the ratio of over-payment by GSP to VCG is bounded by $\frac{1}{m} < r < 1$. Thus the proof is completed. ∎

V. APPLICATION

The proposed mechanism can be easily applied to the existing routing architecture with minor modifications. For example, considering our mechanism is used in ad hoc networks with Dynamic Source Routing (DSR) protocol. The basic route discovery and route maintenance are handled by DSR as before. The only modification is that together with the hop-count metric, the encrypted bid placed by each node is also included in the route request (RREQ) message. Accordingly, the route reply (RREP) message sent from destination to source includes the network graph constructed from the information gathered in RREQ messages. Then the routing protocol determines $f_1, f_2, \ldots, f_m$ by solving the optimization problem specified by (2). Due to limited space, we refrain from discussing the implementation and performance evaluation in detail here.

VI. CONCLUSION

We propose a novel game-theoretical solution to the multi-path multi-hop routing problem in selfish networks. By incorporating different policies, the proposed mechanism is highly compatible to existing routing protocols, which results in Nash equilibria where each player honestly reveals the true cost. By using Nash equilibrium solution as opposed to the traditional strategyproof solution, our mechanism effectively alleviates the over-payment of the widely-used VCG mechanism.

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