Game Theory Final Project

A Game Theoretical Application on Internet Protocols: Gateway Selection Game in Cyber-Physical Systems

Alessandro Toschi - 118033990016
Edoardo Cappello - 118036990002
Giulia Serafini - 118036990003

Summary

- Analysis of the Gateway Selection Problem

- Analogies between Gateway Selection Problem and Game Theory

- Modeling of the problem

- Gateway Selection Algorithm

- Implementation of a simulator for dynamic analysis

Scenario

- Working area with devices and gateways

- Devices with two different wireless communication technologies

- Gateways with a fixed and constant bandwidth

Devices' and Gateways' Dynamics

- Devices are divided into 2 groups according to their wireless communication technologies

- The devices of the same group can exchange information with each other

- Each device has to be connected with one and only one gateway which is reachable from its position

- The bandwidth of a gateway is equally distributed among all the devices connected

- Each device chooses its gateway in order to maximize the obtained bandwidth

Assumptions

- Each gateway has the same bandwidth $\mathbb{W}$

- Each device must be connected to one and only one gateway

- Devices do not have a minimum bandwidth's demand

- Each device has to be in the range of at least one gateway

Model Adopted

Network Model:

- Gateways' set $\quad G = \{g_1, g_2, \ldots, g_k\}$

- Devices' sets differentiated by different wireless communication technologies:

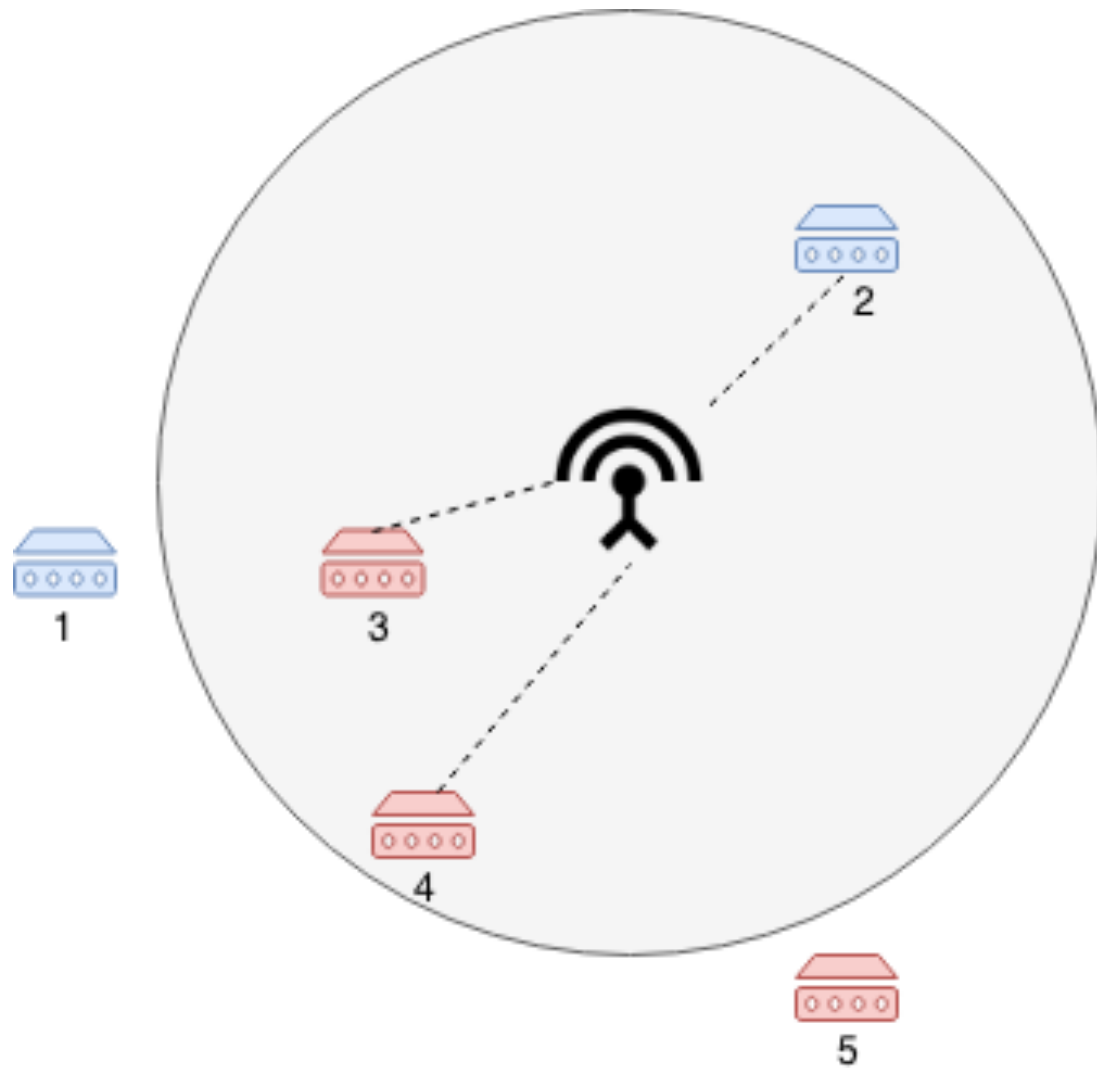$$A = \{a_1, a_2, \ldots, a_n\} \qquad B = \{b_1, b_2, \ldots, b_m\}$$

Model Adopted

Bandwidth Allocation Model:

- Each gateway g_i has a total bandwidth W

- The gateway's bandwidth assigned to each client connected is

$$w_{a,i} = w_{b,i} = \frac{W}{n_i + m_i}$$

Practical Example

- Devices 2, 3 and 4 are connected to the reachable gateway i

- $n_i = 2 \quad m_i = 1$

- The bandwidth obtained by each connected device is:

$$w_{a,i} = w_{b,i} = \frac{W}{n_i + m_i} = \frac{W}{3}$$

Model Adopted

Gateway Selection Game:

- It is based on clients that want to maximize their benefits

- If a device increases its bandwidth, then all the devices of the same kind will have a benefit in it

- A device changes the associated gateway if and only if this migration will increase the total bandwidth of its client

Game Theory Interpretation

The Gateway Selection Game is:

- Sequential: the devices sequentially choose their associated gateways

- Iterated: each device continues to change gateway until an equilibrium is reached

- Non-zero sum: the sum of the clients' utilities is the sum of the related gateways' bandwidths (> 0)

- Perfect Information: each client is aware of all the devices connected with each gateway

- Non-cooperative: each clients aims to maximize its own total bandwidth, without considering the other client's utility

Mathematical Formulation

Sets

$A = \{a_1, a_2, \ldots, a_n\}$ is the set of the devices type A

$B = \{b_1, b_2, \ldots, b_m\}$ is the set of the devices type B

$P = \{A, B\}$ is the set of the players, where A and B are respectively *client A* and *client B*

$G = \{g_1, g_2, \ldots g_k\}$ is the set of the *k-available gateways*

Strategies

$S_A = \{s_{a_1}, s_{a_2}, \ldots, s_{a_n}\}$, where s_i is the strategy of the device $a_i \in A$, i.e. $s_i \subseteq G$ and contains all the possible gateways reachable by a_i

$S_B = \{s_{b_1}, s_{b_2}, \ldots, s_{b_m}\}$, where s_i is the strategy of the device $b_i \in B$, i.e. $s_i \subseteq G$ and contains all the possible gateways reachable by b_i

Decision Variables

$$x_{i,g} = \begin{cases} 1 & \text{if the device i selects the gateway g and g} \in s_i \\ 0 & \text{otherwise} \end{cases} \quad i \in P, g \in G$$

n_g = number of devices type A connected to gateway g, i.e. $n_g = \sum\limits_{i=1}^{n} x_{a_i,g} \quad a_i \in A, g \in G$

m_g = number of devices type B connected to gateway g, i.e. $m_g = \sum\limits_{i=1}^{m} x_{b_i,g} \quad b_i \in B, g \in G$

$w_{i,j}$ = bandwidth obtained by device i connected to gateway g,

$$\text{i.e. } w_{i,g} = \begin{cases} \dfrac{W}{n_g + m_g} & \text{if } n_g + m_g > 0 \\ 0 & \text{otherwise} \end{cases} \quad i \in P, g \in G$$

W_A = total bandwidth obtained by player A, i.e. $W_A = \sum\limits_{g=1,i=1}^{k,n} x_{a_i,g} \cdot w_{a_i,g}$

W_B = total bandwidth obtained by player B, i.e. $W_B = \sum\limits_{g=1,i=1}^{k,m} x_{b_i,g} \cdot w_{b_i,g}$

Payoffs

W_A and W_B are respectively the payoffs of player A and player B.

Nash Equilibrium

The game reaches the Nash Equilibrium when each player selects the strategy which maximizes its payoff:

max $\quad W_A + W_B$

$\quad$ *s.t.*

$$\sum_{g=1}^{k} x_{i,g} = 1 \quad \forall i \in A \cup B \quad \text{(each device must be connected to one and only one gateway)}$$

$$W_A = \sum_{g=1,i=1}^{k,n} x_{a_{i,g}} \cdot w_{b_i,g} \qquad \text{(the total bandwidth of player A is the sum of the bandwidth given by each gateway)}$$

$$W_B = \sum_{g=1,i=1}^{k,m} x_{bi,g} \cdot w_{b_i,g} \qquad \text{(the total bandwidth of player B is the sum of the bandwidth given by each gateway)}$$

where $w_{i,j}$ has been previously defined.

Gateway Selection Algorithm

Input	• Device's accessible gateways set • Device's current gateway • Parameter of each gateway in (bandwidth) • Number of devices of type and devices type connected to
Output	• Decision to migration • Eventually, the new gateway selected for the migration
Algorithm	1. **for each** gateway g_i **in** G_i 2. calculate the increment Δw_i of the total payoff of the population after migration 3. **if** $\Delta w_i > 0$ 4. device changes its gateway to g_i 5. **else** 6. device keeps its current gateway

To calculate Δw_i , we adopt the following formula:

$$\Delta w_i = \left[(n_0 - 1)\frac{W}{n_0 + m_0 - 1} + (n_i + 1)\frac{W}{n_i + m_i + 1} \right] - \left(n_0\frac{W}{n_0 + m_0} + n_i\frac{W}{n_i + m_i} \right)$$

Where:

- n and m are used to differentiate the two kinds of devices

- the elements subscripted by 0 are connected with the starting gateway

- the element subscripted by i are connected to the destination gateway

There are some values of the attributes that make this formula unfeasible. For each of them, an adapted formula is proposed (next slides).

- $n_0 + m_0 = 1$ **and** $n_i + m_i = 0$

$$\Delta w_i = \left[(n_i + 1)\frac{W}{n_i + m_i + 1} \right] - \left(n_i \frac{W}{n_i + m_i} \right) = 0$$

So the client will **not** change the gateway

- $n_0 + m_0 > 1$ **and** $n_i + m_i = 0$

$$\Delta w_i = \left[(n_0 - 1)\frac{W}{n_0 + m_0 - 1} \ + \ (n_i + 1)\frac{W}{n_i + m_i + 1} \right] \ - \ \left(n_0\frac{W}{n_0 + m_0} \right) > 0$$

So the client will change the gateway

- $n_0 + m_0 = 1$ **and** $n_i + m_i > 0$

$$\Delta w_i = \left[(n_i + 1) \frac{W}{n_i + m_i + 1} \right] - \left(n_0 \frac{W}{n_0 + m_0} + n_i \frac{W}{n_i + m_i} \right) < 0$$

So the client will **not** change the gateway

Theoretical Results

Theorem 1. *If the number of clients A and clients B of the original gateway equals the number of clients A and clients B of candidate gateway, respectively, none of these clients will change its strategy.*

Theorem 2. *If the number of one kind of clients in the original gateway is equal to that in the candidate gateway, the other kind of clients in original gateway will change its strategy iff there are at least two clients more in original gateway than in candidate gateway.*

Theorem 3. *Denote the number of clients B in g_1 and g_2 by n_1 and n_2, and denote the number of clients B in g_1 and g_2 by m_1 and m_2. When $n_1 = n_2 = n$ and $m_1 \neq m_2$, we have the following.*
   1.   *If $m_1 = m_2 + 1$, none of players of clients A in g_1 will change its strategy to g_2.*
   2.   *If $m_1 < m_2$ and $n^2 < m_1 \cdot m_2$ or $m_1 < m_2$ and $n^2 > m_1 \cdot m_2$, none of players of clients A in g_1 will change its strategy to g_2.*

Theorem 4. *If only one kind of clients can change their strategies, this gateway selection game will always come to a Nash Equilibrium.*

The Termination Problem

- The termination of the algorithm is not guaranteed because loops can occur if the Devices continue to change their associated Gateway

- The previously seen *Theorem 4* guarantees the termination if just one kind of devices is running the algorithm

- It is possible to detect loops by checking if the total score (i.e. the payoffs) does not change in sequential steps of the algorithm

- Our simulator terminates after a fixed number of consecutive loops (threshold) in which the total score does not change
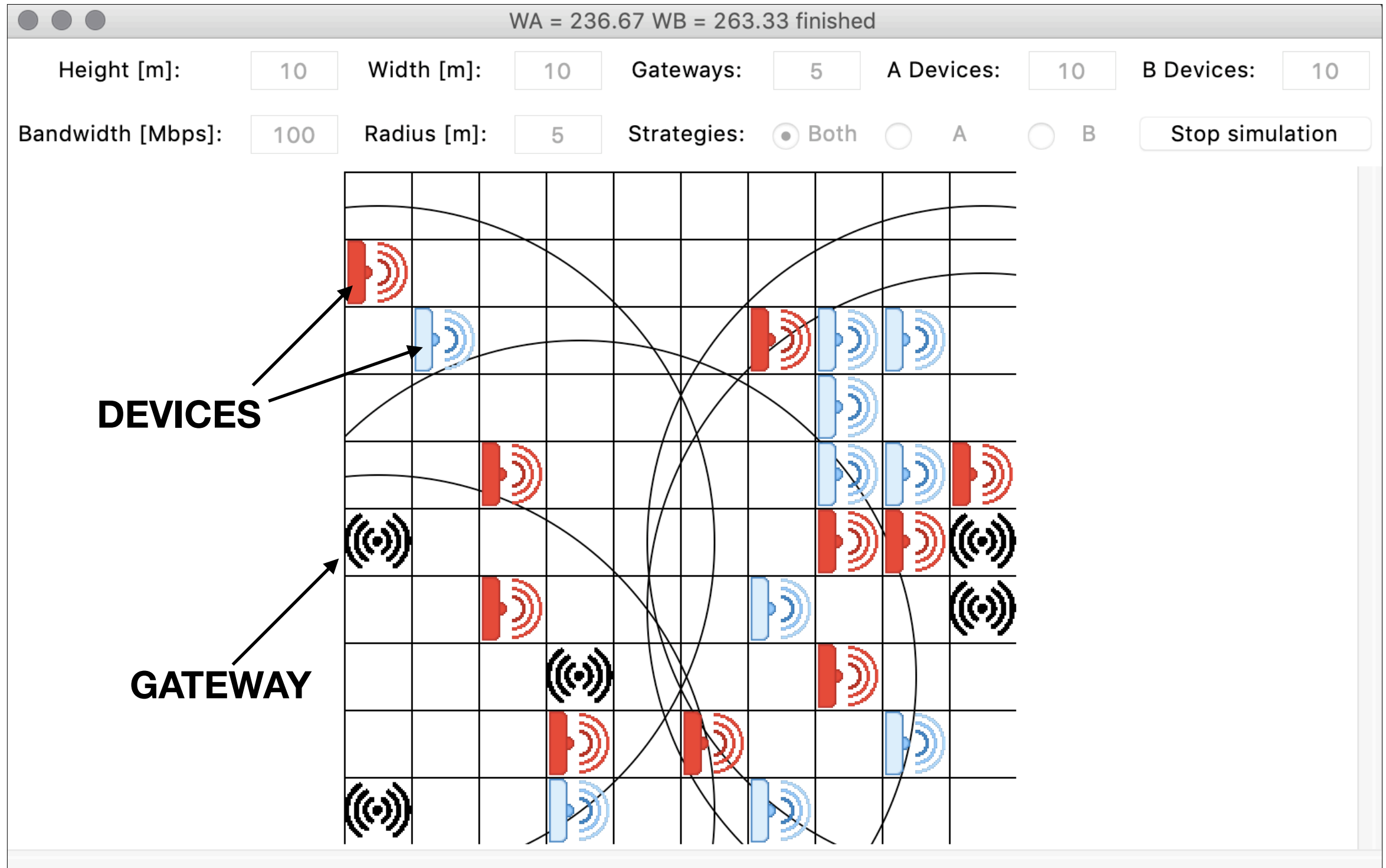
An example which has Nash Equilibrium
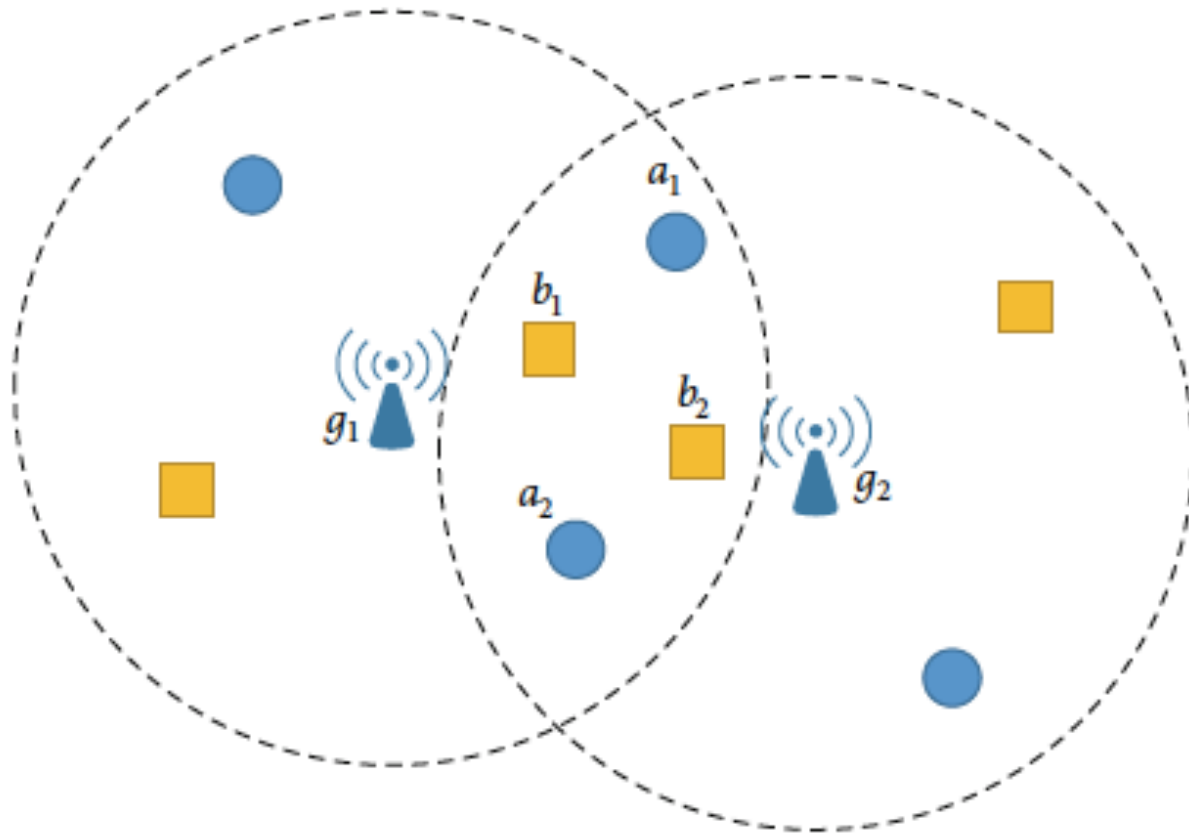
An example which has no Nash Equilibrium
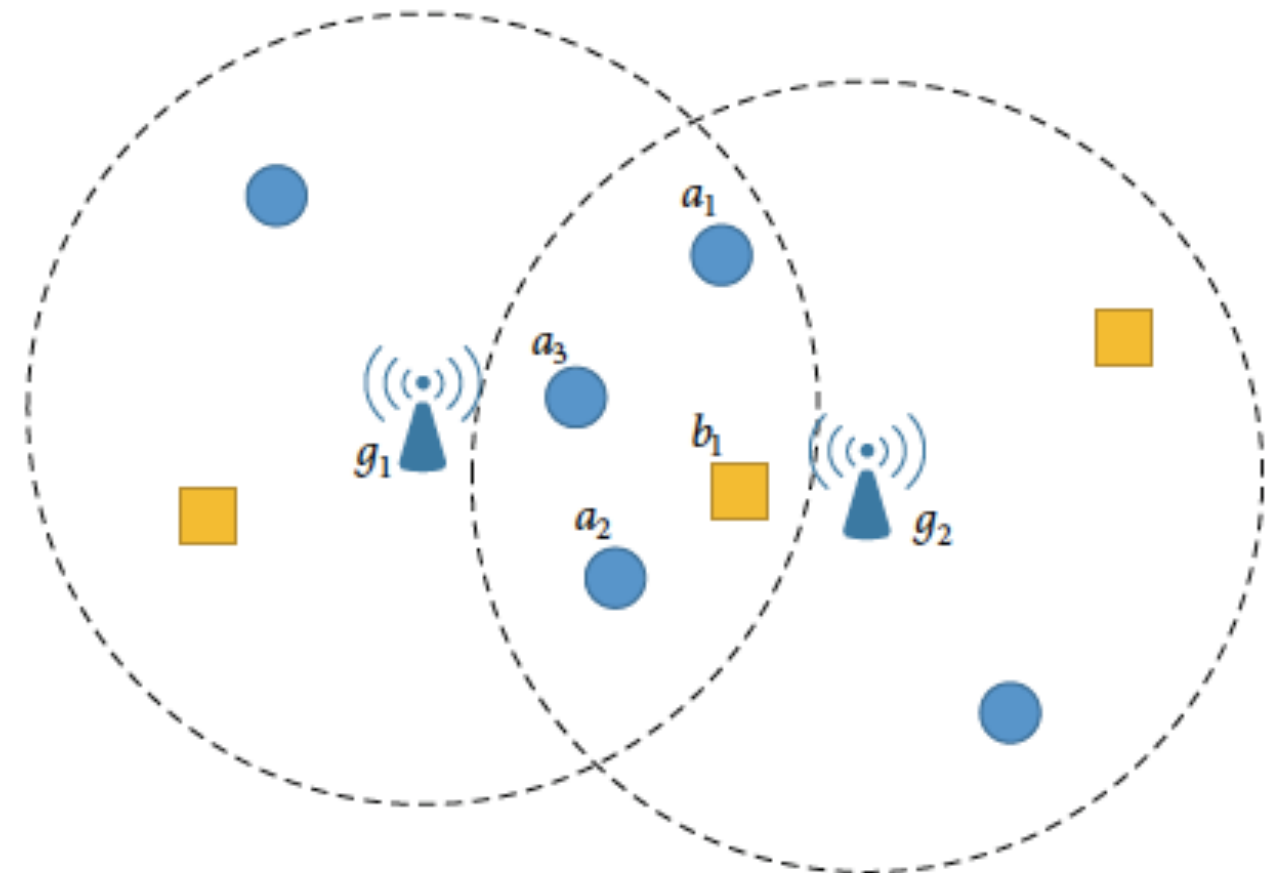
Simulation

References

[1] Hao Wang, Jianzhong Li, and Hong Gao, "Gateway Selection Game in Cyber-Physical System", International Journal of Distributed Sensor Network, Vol. 2016, Atricle ID 7190767

Thank you!