Game Theory Applications in Network Reliability

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Abstract— **Recently, game theoretic perspectives have been used for formulating and solving network reliability problems. These include enforcement of cooperation in ad hoc networks using bribery and punishment, and modeling network reliability as a game between a router and an imaginary tester. In this paper, we explore the formulation of these problems as optimization problems, and we numerically solve the latter problem, extending the available formulation to treat randomness in the model.**

I. Introduction

While traditionally networks have been centrally designed and managed, protocols for new wireless networks and technologies, which are far more unpredictable and dynamic than their predecessors, often become too complex for centralized management. In recent years, game theory has been applied to various areas of wireless networks to decentralize network management, with the hope of creating more efficient and robust protocols. Game theory has been applied in both physical layer problems such as power control and interference avoidance, as well as network routing problems such as cooperation and reliability.

There are two areas where a network reliability can be compromised; network performance can be compromised by internal factors such as non-cooperating users within the network, or external conditions such as link failures. In this paper, we discuss these two particular problems from a game theoretic perspective. First, we discuss the modeling and enforcing of node cooperation in ad hoc networks with possibly selfish nodes. We survey bribery and punishment as game theoretic formulations to ensure cooperation and, in turn, reliability. We then discuss an application of game theory to characterize network reliability and vulnerability in terms of link failure costs, as presented by Bell in [1]. We extend the results by adapting the formulation to better model random link costs in wireless networks. Our results demonstrate the effect of link cost on the optimal probabilities of path selection.

II. Node Cooperation

In a cooperative ad hoc network, nodes are expected to cooperate by relaying messages from the source to the destination. Although this is a valid assumption in military and disaster relief applications, it is not generally applicable in commercial applications where nodes are usually rational (or selfish)[2]. Relaying messages requires the node to use up its limited energy resources without getting any immediate benefit. Relaying may also delay the transmission of the node's own data while it is busy forwarding messages. Hence, the node-centric choice is to never cooperate, which may be detrimental to network reliability.

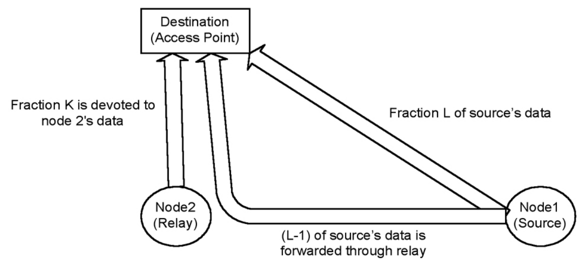

Fig. 1. Simple forwarding network settings

Cooperation can be induced by offering the nodes incentives for relaying. Nodes will then try to maximize their benefit in the network by reaching a compromise between self interest and cooperation. The nature of this problem suggests the use of a microeconomic framework based on game theory to reach an optimal solution. In fact, the literature contains several different formulations of this problem. For instance, Srinivasan et al. consider rational nodes with a lifetime constraint and determine the optimal throughput that each node should receive using a distributed acceptance algorithm that leads to a Nash equilibrium [3]. Conversely, Ileri et al. introduce a mechanism that fosters cooperation through bribery [4]. Bandyopadhyay and Bandyopadhyay in [5] show analytically that, given a suitable punishment mechanism, individual nodes can be deterred from their selfish behavior. We will examine and expand on the last two approaches since they present clever game theoretic formulations of this problem.

A. Cooperation By Bribery

Ileri et al. in [4] propose a scheme that fosters cooperation through bribery. This scheme was further explored in [2]. This simple model considers a network composed of an access point (AP), one source node and one relay node. At the start of the game the AP sets the prices for channel utilization λ (per unit throughput) as well as the forwarding fees μ (per unit throughput) that the network awards the forwarding node for its cooperation. In response, the nodes adjust their transmit power (p_i) and their forwarding preferences (L and K) to maximize their return as illustrated in Figure 1. This interaction between the nodes and the network continues until the network's revenue is maximized. The nodes are assigned the utility function

$$u_i(p_i) = \frac{T_i(p_i)}{p_i}, \tag{1}$$

which is the same as the one used in [6], where T_i is the total throughput of the node. T_{ij}, the throughput between nodes

i and j, is related to power (p), signal bandwidth (W), and signal-to-noise-ratio (γ) by

$$T_{ij}(p_i, W_{ij}) = W_{ij} * f(\gamma) \tag{2}$$

$$f(\gamma) = (1 - 2 * BER(\gamma))^M; \quad \gamma = \frac{hp_i}{N_0 W_{ij}} \tag{3}$$

where h, N_0, M, and W_{ij} are the channel path gain, the noise density, frame length, and the signal bandwidth respectively. The nodes pay the network λ for channel use and are paid μ for forwarding messages. From this, we can formulate the optimization problem for Node 1 as given in [4]

$$\max_{p_1, W_{1A}, W_{12}} \left(\frac{1}{p_1} - \lambda\right)(T_{1A} + T_{12}) \tag{4}$$

$$\text{subject to} \left\{ \begin{array}{c} 0 \le p_1 \le p_{max} \\ W_{1A} + W_{12} = B \\ T_{12} \le KT_{2A} \end{array} \right\}$$

where W_{12} is the signal bandwidth from Node 1 to 2, W_{1A} is the signal bandwidth between Node 1 and the AP, and B is the total available bandwidth. Here, T_{2A} is fixed, T_{1A} depends on W_{1A} and p_1, and T_{12} depends on W_{12} and p_1. The last inequality constraint is due to the fact that Node 1 cannot use more bandwidth than permitted by Node 2 from its forwarding allocation K.

We can derive a similar formulation for the forwarding Node 2 taking into account the payoff term $\mu \min\{KT_{2A}(p_2, W2A), T_{12}\}$ that it receives from AP for relaying. Using the same reasoning for maximum bandwidth utilization as in the previous problem, we obtain the following formulation for Node 2

$$\max_{p_2, k} \left\{ \frac{(1-K)T_{2A}}{p_2} - \lambda T_{2A} + \mu K T_{2A} \right\} \tag{5}$$

$$\text{subject to} \left\{ \begin{array}{c} 0 \le p_2 \le p_{max} \\ 0 \le K \le 0.5 \\ KT_{2A} \le T_{12} \end{array} \right\}$$

where T_{2A} uses the entire bandwidth B and depends on p_2 only, and T_{12} is fixed. The inequality constraint on K is explained numerically in [4]. The AP (network) tries to maximize its revenue given by

$$\lambda (T_{1A} + T_{12} + T_{2A}) - \mu K T_{2A}. \tag{6}$$

This formulation models the external game involving the AP similar to a Stackelberg game [8] where the leader (the AP) moves first by picking a quantity (pricing in our case) then the followers (the nodes) observe the leader's choice and decide on their own quantities (power levels and forwarding preferences). Next, using the current λ and μ the nodes engage in a non-cooperative game that can achieve a Nash equilibrium. It is shown in [4] that this equilibrium is highly dependent on the network geometry. The external game stops when the AP maximizes its revenue. This two-node model is easily extended to a multinode one with multi-hop transmission [4]. The main drawback of this formulation is that it requires the AP to act as a facilitator, which centralizes network management. We next explore a truly decentralized approach to enforce node cooperation.

B. Cooperation Through Punishment

Bandyopadhyay and Bandyopadhyay in [5] use a different game theoretic approach to solve the node cooperation problem. They consider an ad hoc network with n selfish nodes with at least one packet each to transmit to another node at a fixed energy cost c_t. Reception energy cost is also fixed at c_r. The benefit of successfully sending a packet is b, and that of receiving a packet at the destination node is zero. Each node i in the network has a pure strategy space $S_i = \{C_i, D_i\}$. C is the strategy of cooperating and D is defecting. We assume that a fraction α out of the n nodes is cooperating and that on average, a packet goes through h hops to get to its destination. The benefit b is incurred only if the packet is successfully transmitted through all h hops. The network traffic is assumed to be uniformly distributed, meaning that each cooperating node transmits and receives h/α packets in each stage of the game. A non-cooperating node transmits just its own packet. Every node can have one of four payoffs in the single-stage game:

$$\begin{array}{ll} \pi_C^S = b - c_r\left(\frac{h}{\alpha}\right) - c_t\left(\frac{h}{\alpha}\right); & \pi_D^S = b - c_t \\ \pi_C^F = c_r\left(\frac{h}{\alpha}\right) - c_t\left(\frac{h}{\alpha}\right); & \pi_D^F = -c_t \end{array} \tag{7}$$

The superscripts S and F denote successful or failed transmission and the subscripts C and D denote cooperation or defection strategies, respectively. The above equations are not valid for $\alpha = 0$ (no cooperation) or $\alpha = 1$ (full cooperation). It is clear that the payoffs are monotonic in α. We can analyze this problem as a one-shot strategic game [8]. Without loss of generality, we consider a network of two nodes, where transmission is always successful. The payoff matrix is

$$\begin{array}{ccc} & & \textbf{Node 2} \\ & & \begin{array}{cc} C & D \end{array} \\ \textbf{Node 1} & \begin{array}{c} C \\ D \end{array} & \left[\begin{array}{cc} (\pi_C^S, \pi_C^S) & (\pi_C^S, \pi_D^S) \\ (\pi_D^S, \pi_C^S) & (\pi_D^S, \pi_D^S) \end{array} \right] \end{array} \tag{8}$$

The value in each cell is Node 1 and Node 2's respective payoffs. This strategy game can then be solved using iterated strict dominance [8] to eliminate the less attractive options for each player. After one round of eliminations we conclude that the nodes will choose to defect since this strategy consistently yields a better payoff. This solution is a strictly dominating pure strategy, therefore it constitutes the unique Nash equilibrium of the game [8]. Since this game is similar to the prisoners' dilemma [8], the Nash equilibrium reached in the single-stage game holds for the iterated dominance game. The same result is obtained when we n-node network with possibly failing transmissions.

This simplistic problem formulation proves that the nodes will not cooperate, resulting in poor network reliability. The main shortcoming of this setup is the fixed energy cost assumption regardless of distance between the communicating nodes. Although this might be valid for some network geometries, it is not true for most cases where the need for cooperation rises from the fact that the source's message cannot reach the destination node because of signal power limitations. Moreover, unlike the one presented in [4], this model fails to provide strong incentives for the nodes to cooperate. In order to induce cooperation, the authors in [5] propose an alternate model that uses a punishment mechanism to deters nodes from selfish behavior.

III. LINK FAILURE COST

In addition to internal network challenges such as node cooperation, network reliability must also take into account external forces that affect link conditions, these may include intelligent adversaries or oblivious environmental conditions. Traditionally, one of two measures is used to characterize network reliability: connectedness and capacity [1]. In each measure, the change in connectedness (or capacity) is derived from the probabilities of node or link failure. However, a game theoretic interpretation can be used to develop another way of characterizing reliability. Consider the game between a router and an imaginary tester presented by Bell [1]. The cost of routing is assumed to be routing delay. The objective of the router, given a set of paths to a destination, is to choose a path that minimizes the cost of routing a packet. The objective of the tester is to fail a link such that the expected cost to the router is maximized. We consider mixed strategies, where the strategy of each player is characterized by a set of probabilities (probability of choosing a path for the router, and probability of failing a link for the tester). The game is played as follows, for n even:

Move n: The tester assigns probabilities of failing each link such that the expected cost of the path selected by the router, based on the probabilities the router chose on the last move, will increase by the largest amount.

Move $n+1$: Based on the new probabilities of failure, the router assigns new probabilities of choosing each path such that the expected cost is minimized.

Generally, if played in this manner, the players' choices will not converge. However, as we will discuss below, there exists a Nash Equilibrium such that if we carefully choose the initial probabilities, each turn will result in the same probabilities being chosen. That is, given the strategy of the other player, neither player has a better strategy. We can determine which links are critical to network performance by finding the mixed strategy Nash Equilibrium of the game.

A. Mathematical Model

Let us assume that link failure is independent of traffic and the state of other links, and that only a single link can fail at a time. Without loss of generality, we denote scenarios $j = 1, ..., M$, where M is the total number of links in the network, and link j fails in scenario j. Let q_j be the probability that the tester chooses scenario j (i.e. chooses to fail link j). $c_{i,j}$ denotes the cost of link i in scenario j where $c_{i,j} = c_i^N$ if $i \neq j$ (normal link cost) and $c_{i,j} = c_i^F$ if $i = j$ (failed link cost). Now, suppose that K paths are available between a source and a destination, and let the router choose each path $k = 1, ..., K$ with probability h_k. The expected cost of each path is $\sum_{i=1}^{M} a_{i,k} c_{i,j}$, where $a_{i,k} = 1$ if link i is on path k, and $a_{i,k} = 0$ otherwise. So, the overall expected cost is

$$COST = \sum_{j=1}^{M} \sum_{i=1}^{M} \sum_{k=1}^{K} h_k a_{i,k} c_{i,j} q_j. \tag{9}$$

If we let $\mathbf{P} = [p_{k,j}], p_{k,j} = \sum_{i=1}^{M} a_{i,k} c_{i,j}$, the maximin problem formulation for the tester is

$$\max_{\mathbf{q}} \min_{\mathbf{h}} COST = \max_{\mathbf{q}} \min_{\mathbf{h}} \mathbf{h}^{\mathbf{T}} \mathbf{P} \mathbf{q}, \tag{10}$$

and the minimax problem formulation for the router is

$$\min_{\mathbf{h}} \max_{\mathbf{q}} COST = \min_{\mathbf{h}} \max_{\mathbf{q}} \mathbf{h}^{\mathbf{T}} \mathbf{P} \mathbf{q}, \tag{11}$$

subject to $\mathbf{q} \geq 0$, $\sum q_j = 1$, $\mathbf{h} \geq 0$, and $\sum h_k = 1$. Now, since the problem is linear it is easy to show that strong duality holds, and we can rewrite Eqn. (10) in terms of the minimization problem's dual with dual variable C as

$$\max_{\mathbf{q}} \left\{ \max_{C} C, \text{ s.t. } \mathbf{P}\mathbf{q} - \mathbf{1}C \geq 0 \right\}. \tag{12}$$

Likewise, we can manipulate Equation (11) to obtain the two optimization problems:

$$\max_{\mathbf{q}, C} C, \text{ s.t. } \mathbf{P}\mathbf{q} - \mathbf{1}C \geq 0, \mathbf{q} \geq 0, \sum_{j=1}^{M} q_j = 1 \tag{13}$$

$$\min_{\mathbf{h}, D} D, \text{ s.t. } \mathbf{P}\mathbf{h} - \mathbf{1}D \leq 0, \mathbf{h} \geq 0, \sum_{k=1}^{K} h_k = 1 \tag{14}$$

It turns out that these two problems are the duals of each other and have the same optimal value $C = D$, which is the expected cost. This shows that a Nash Equilibrium exists for the game, where given the strategy of the other player, neither player has a better strategy.

B. Interpretation of the Nash Equilibrium

At the Nash Equilibrium, we have one solution for the router $\mathbf{h}$ and one solution for the tester $\mathbf{q}$. From the perspective of the router, suppose that at equilibrium $\{\mathbf{P}\mathbf{q}\}_k > C$, where $\{...\}_k$ denotes the k^{th} row. Since h_k is the dual variable of that particular constraint, $h_k = 0$. This means that if the expected cost of path k is greater than the minimum expected worst-case cost, then the path should have zero probability of being used. On the other hand, if path k is selected with positive probability ($h_k > 0$), then equality is necessary for the constraint ($\{\mathbf{P}\mathbf{q}\}_k = C$), meaning that the expected cost of that path is equal to the minimum expected worst-case cost. Thus, $\mathbf{h}$ yields the optimal probabilities for selecting paths such that the expected worst-case cost is minimized. With similar arguments, we see that, from the perspective of the tester, if at equilibrium $\{\mathbf{P}\mathbf{h}\}_j < C$, then $q_j = 0$. This means that the tester would never fail link j, because the link is not critical to network performance (after the router has been allowed to adapt to network conditions). Note that $\mathbf{q}$ does not reflect the actual probability of link failure, but rather, how critical the link is to network performance. Thus, in designing a network, critical links should be made more reliable.

C. Random Cost Problem Formulation

The model used in [1] assumes fixed failed link cost, or failed link cost that depends on traffic. However, the cost of a failed link may neither be fixed, nor dependent on traffic. We extend the problem formulation from [1] by assuming that costs for failed links are not fixed, but independent and identically distributed random variables. Consider a mobile ad hoc network where packets are forwarded over multiple hops to other nodes in the network. Suppose that the delay for transmission over a normal link is fixed, and in the case where the link to the next hop has failed, the intermediate node stores the packet until the link is restored. We model the duration that the link is down using an exponential random variable. We fix the normal link cost at $c_i^N = \Delta = L/R$, and let the failed link incur an additional exponentially distributed cost of X_i (i.e. $c_i^F = \Delta + X_i$) with finite mean λ_i^{-1}. We set the required probability that C must be correct (i.e. C is in fact the maximum delay over all scenarios) to be η. The inequality constraint (13) from the fixed cost problem formulation thus becomes

$$\mathbf{Pr}\left\{\sum_{k=1}^{K}\sum_{i=1}^{M} a_{ik}c_{ij}h_k - C \leq 0\right\} \geq \eta,\ \forall j = 1...N \quad (15)$$

where $c_{ij} = \Delta$ if $i \neq j$ and $c_{jj} = \Delta + X_j$. Let

$$\alpha_j^T = \left[\begin{array}{ccc} \sum_{i=1}^{M} a_{i1}\Delta & ... & \sum_{i=1}^{M} a_{iK}\Delta \end{array}\right]$$

$$\beta_j^T = \left[\begin{array}{ccc} a_{j1} & ... & a_{jK} \end{array}\right]$$

We can rewrite (15) as

$$\mathbf{Pr}\left\{X_j \leq \frac{-\alpha_j^T\mathbf{h} + C}{\beta_j^T\mathbf{h}}\right\} \geq \eta,\ \forall j = 1...M \quad (16)$$

Now, since X_j is exponentially distributed with mean λ_j^{-1},

$$\mathbf{Pr}\left\{X_j \leq x\right\} = 1 - e^{-\lambda_j x} \quad (17)$$

and we have

$$-\lambda_j\left(\frac{-\alpha_j^T\mathbf{h} + C}{\beta_j^T\mathbf{h}}\right) \leq \log(1-\eta). \quad (18)$$

Finally, we have the problem formulation

$$\min_{\mathbf{h},C} C \quad \text{subject to} \quad (19)$$

$$\left(\alpha_j^T - \frac{\log(1-\eta)}{\lambda_j}\beta_j^T\right)\mathbf{h} - C \leq 0, \forall j = 1...M$$

$$h_k \geq 0,\ \forall k = 1...K,\ \sum_{k=1}^{K} h_k = 1,$$

1) Scenario: Random paths with random failure cost: Using this new model, we conduct an experiment to look at the probabilities of path selection when paths are randomly selected. In this case, paths are not assumed to be disjoint, and may share one or multiple links. For this experiment, we generate random graphs with 20 nodes, where a link exists between any two nodes with equal and independent probability. The graphs are such that every node has on average 5 links. We generate random paths by performing random walks on the graph starting from node 1, and stopping when we reach node 20. Cycles are removed from the walks to yield paths.

For 100 trials, each with a different random graph, random walks were obtained, and simplified to paths. Figure 2 shows

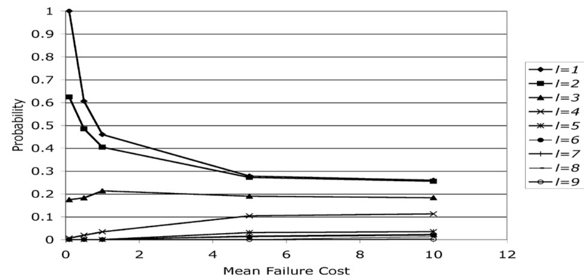

Fig. 2. Probability of selecting route of length l (given that a path of such length exists)

the average probability of selecting a path of length l, given that a path of length l was generated. (Note that the sample space is not the set of path lengths l, so the probabilities do not sum to 1.) The mean failure cost is λ^{-1}. The results suggest that the selection of short paths is favored for small mean failure costs, and the selection probabilities across all paths even out as the mean failure cost increases. The important point to note from the data is that while the probabilities tend towards each other, they do not appear to converge as in the case with disjoint paths. The probabilities are non-trivially affected by path length and edge overlap. Given a set of paths, this game theoretic approach is able to find path selection probabilities that balance path length and edge overlap to maximize reliability.

IV. Conclusions

In this paper, we discussed two interesting areas where game theory has provided a deeper understanding of network reliability. We surveyed how selfish nodes can be modeled and controlled to manage cooperation in ad hoc networks, and discussed in detail two problem formulations that use bribery and punishment. We also looked at a use of game theory to characterize network reliability by casting the problem as a game between a router and an imaginary tester. Simulation results, that used an extended problem formulation to accommodate random link failure costs, showed the relationship between mean link failure cost and optimal path selection probabilities.

References

[1] M. Bell, "The use of game theory to measure the vulnerability of stochastic networks" *IEEE Trans. Reliability*, vol. 52, no. 1, pp. 63-68, 2003.

[2] N. Shastry and R.S. Adve, "Stimulating Cooperative Diversity in Wireless Ad Hoc Networks through Pricing" *To appear in IEEE, ICC 2006*.

[3] V. Srinivasan, P. Nuggehalli, C.F. Chiasserini and R.R Rao, "Cooperation in wireless ad hoc networks" *INFOCOM 2003*. vol. 2, pp. 808-817, 2003.

[4] O. Ileri, Siun-Chuon Mau and N.B. Mandayam, "Pricing for enabling forwarding in self-configuring ad hoc networks," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 1, pp. 151-162, 2005.

[5] S. Bandyopadhyay and S. Bandyopadhyay, "A game-theoretic analysis on the conditions of cooperation in a wireless ad hoc network," *Third International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks*, pp. 54-58, 2005.

[6] D. J. Goodman and N. B. Mandayam, "Power control for wireless data," *IEEE Personal Communications*, vol. 7, no. 2, pp. 48-54, 2000.

[7] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge, UK: Cambridge University Press, 2004.

[8] D. Fudenberg and J. Tirole, *Game Theory*, Cambridge, MA: MIT Press, 1991.