```
In [1]: import matplotlib.pyplot as plt
    from itertools import product
    import numpy as np
    from collections import Counter
    from sklearn.base import BaseEstimator, RegressorMixin, ClassifierMixi
    from sklearn.tree import DecisionTreeClassifier, DecisionTreeRegressor
    from sklearn.ensemble import GradientBoostingClassifier, GradientBoost
    import graphviz
    from IPython.display import Image
    %matplotlib inline
```

Load Data

```
In [2]: data_train = np.loadtxt('svm-train.txt')
    data_test = np.loadtxt('svm-test.txt')
    x_train, y_train = data_train[:, 0: 2], data_train[:, 2].reshape(-1, 1
    x_test, y_test = data_test[:, 0: 2], data_test[:, 2].reshape(-1, 1)
In [3]: # Change target to 0-1 label
    y_train_label = np.array(list(map(lambda x: 1 if x > 0 else 0, y_train))
```

Decision Tree Class

Problem 1

```
In [4]: | def compute_entropy(label_array):
            Calulate the entropy of given label list
            :param label_array: a numpy array of binary labels shape = (n, 1)
            :return entropy: entropy value
            # Your code goes here
            #Only considering binary classification
            entropy = 0 #Initialize entropy
            prob_one = label_array.sum() / len(label_array) #Calculate the pro
            prob_array = [1-prob_one,prob_one]
                                                             #Likewise for clas
            #Make sure we don't bug out trying to take log 2(0)
            if prob one == 0 or prob one == 1:
                return 0
            #Calculate entropy sum(-log_2(prob)*prob)
            for i in [0,1]:
                entropy == np.log2(prob_array[i]) * prob_array[i]
            #Return entropy
            return entropy
        def compute_gini(label_array):
            Calulate the gini index of label list
            :param label array: a numpy array of labels shape = (n, 1)
            :return gini: gini index value
            #Only considering binary classification
            qini = 0
                                                             #Initialize gini i
            prob_one = label_array.sum() / len(label_array) #Calculate the pro
            prob_array = [1-prob_one,prob_one]
                                                             #Likewise for clas
            #Calculate gini index -> sum(prob_class(1-prob_class))
            for i in [0,1]:
                gini += prob_array[i]*(1-prob_array[i])
            return gini #Return gini index
```

. . .

```
Initialize the decision tree classifier
    :param split_loss_function: method with args (X, y) returning
    :param leaf_value_estimator: method for estimating leaf value
    :param depth: depth indicator, default value is 0, representing
    :param min_sample: an internal node can be splitted only if it
    :param max depth: restriction of tree depth.
    self.split loss function = split loss function
    self.leaf value estimator = leaf value estimator
    self.depth = depth
    self.min_sample = min_sample
    self.max_depth = max_depth
    self.is_leaf = False
    #Add these variables to the constructor
    self.right = None
                             #Left child node
    self.left = None
                             #Right child node
    self.split id = None
                            #Best column to split on
    self.split_value = None #Best value to split on within best
    self.value = None
                             #Value to return if
def fit(self, x, y):
    This should fit the tree classifier by setting the values self
    self.split_id (the index of the feature we want ot split on, i
    self.split_value (the corresponding value of that feature wher
    and self.value, which is the prediction value if the tree is a
    splitting the node, we should also init self.left and self.rig
    objects corresponding to the left and right subtrees. These su
    the data that fall to the left and right, respectively, of self
    This is a recurisive tree building procedure.
    :param X: a numpy array of training data, shape = (n, m)
    :param y: a numpy array of labels, shape = (n, 1)
    :return self
    # Your code goes here
    #Check break condition, if we've exceeded max depth or are led
    if self.depth >= self.max depth or len(y) <= self.min sample:</pre>
        self.is leaf = True
        self.value = self.leaf_value_estimator(y)
          if y.sum() / len(y) <= .5:
              self.value = 0
          else:
              self_{\bullet}value = 1
```

#

#

```
else:
        #Calculate best splitting point
        self.find best feature split(x,y)
        #Split data in two depending on criteria
        all_data = np.append(x,y,axis=1) #Create one big matrix (e
        #Filter data by split column / split point
        left_data = all_data[all_data[:,self.split_id]<=self.split</pre>
        #Again but look for greater for right tree
        right_data = all_data[all_data[:,self.split_id]>self.split
        left_x_node = left_data[:,0:-1]
        left_y_node = left_data[:,-1].reshape(-1,1)
        right_x_node = right_data[:,0:-1]
        right_y_node = right_data[:,-1].reshape(-1,1)
        #Create left and right nodes
        self.left = Decision_Tree(self.split_loss_function, #Pass
                                 self.leaf value estimator, #Pass
                                 depth=self.depth+1,
                                 min sample = self.min sample, #Pa
                                 max depth = self.max depth
        self.right = Decision_Tree(self.split_loss_function, #Pass
                                 self.leaf_value_estimator, #Pass
                                 depth=self.depth+1,
                                                             #Pass
                                 min sample = self.min_sample, #Pa
                                 max depth = self.max depth
        #Fit the left/right nodes
        self.left.fit(left_x_node,left_y_node)
        self.right.fit(right_x_node,right_y_node)
    return self
def find_best_split(self, x_node, y_node, feature_id):
    For feature number feature id, returns the optimal splitting p
    for data X_node, y_node, and corresponding loss
    :param X: a numpy array of training data, shape = (n_node)
    :param y: a numpy array of labels, shape = (n_node, 1)
    # Your code
   \#x \ copy = x \ node.copy()
   y_copy = y_node.copy()
    feature_vals = x_node[:,feature_id].copy() #Grab the feature v
    sorting = feature_vals.argsort() #Prepare index for arg sor
    y_{copy} = y_{copy}[sorting]
                                        #Sort the y_node by x inde
    feature_vals.sort()
                                        #Sort the feature grabbed
    #Initialize entropy variable
    best_loss = 100
```

```
split value = -1
    #Iterate over the feature vals
    for i in range(1,len(feature vals)):
        #Seperate sorted (single) feature vals into two halves
        top_half = y_copy[:i]
        bottom_half = y_copy[i:]
        #Calculate weighted entropy for each half
        top ratio = (len(top half)/len(y copy))
        bottom_ratio = (len(bottom_half)/len(y_copy))
        top_half_entropy = top_ratio * self.split_loss_function(t
        bottom_half_entropy = bottom_ratio * self.split_loss_funct
        #Calculate Loss = Total Weighted Entropy
        loss = top_half_entropy + bottom_half_entropy
        #Check if we've reached a smaller loss
        if loss <= best_loss:</pre>
            best loss = loss #Update smaller loss
            if i == 1:
                split_value = (feature_vals[i]+feature_vals[i+1])/
            #Update the best split value via midpoint value
            #Take midpoint of point before after if best split poi
            else:
                split_value = (feature_vals[i]+feature_vals[i-1])/
    return split value, best loss
def find best feature split(self, x node, y node):
    Returns the optimal feature to split and best splitting point
    for data X_node, y_node.
    :param X: a numpy array of training data, shape = (n_node, 1)
    :param y: a numpy array of labels, shape = (n_node, 1)
    best_feature_loss = 100
    #Iterate over all of the columns
    for i in range(x_node.shape[1]):
        #Use self.find_best_split to
        split_value, best_loss = self.find_best_split(x_node,y_nod
        #Check if we've found a column to split better on
        if best_loss <= best_feature_loss:</pre>
                                               #Update Loss Accord
            best_feature_loss = best_loss
            self.split_id = i
                                                #Update the column
            self.split_value = split_value
                                                #Update the column
dof prodict instance (solf instance).
```

```
Predict label by decision tree

:param instance: a numpy array with new data, shape (1, m)

:return whatever is returned by leaf_value_estimator for leaf

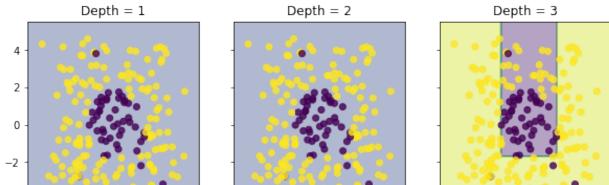
if self.is_leaf:
    return self.value
if instance[self.split_id] <= self.split_value:
    return self.left.predict_instance(instance)
else:
    return self.right.predict_instance(instance)</pre>
```

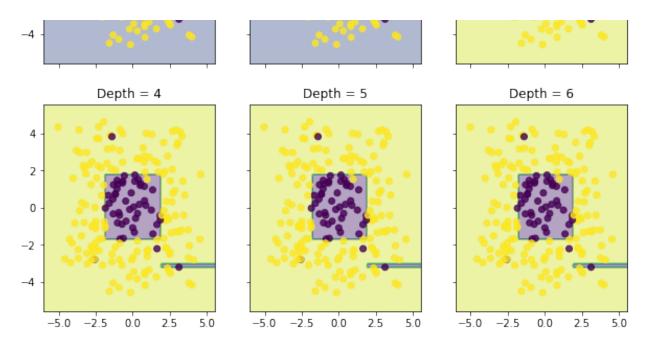
Decision Tree Classifier

Decision Tree Boundary

```
In [8]:
```

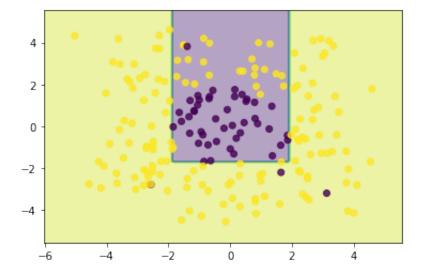
```
# Training classifiers with different depth
clf1 = Classification_Tree(max_depth=1, min_sample=2)
clf1.fit(x_train, y_train_label)
clf2 = Classification_Tree(max_depth=2, min_sample=2)
clf2.fit(x_train, y_train_label)
clf3 = Classification_Tree(max_depth=3, min_sample=2)
clf3.fit(x_train, y_train_label)
clf4 = Classification_Tree(max_depth=4, min_sample=2)
clf4.fit(x train, y train label)
clf5 = Classification_Tree(max_depth=5, min_sample=2)
clf5.fit(x_train, y_train_label)
clf6 = Classification_Tree(max_depth=6, min_sample=2)
clf6.fit(x_train, y_train_label)
# Plotting decision regions
x_{min}, x_{max} = x_{train}[:, 0].min() - 1, <math>x_{train}[:, 0].max() + 1
y_{min}, y_{max} = x_{train}[:, 1].min() - 1, <math>x_{train}[:, 1].max() + 1
xx, yy = np.meshgrid(np.arange(x_min, x_max, 0.1),
                     np.arange(y_min, y_max, 0.1))
f, axarr = plt.subplots(2, 3, sharex='col', sharey='row', figsize=(10,
for idx, clf, tt in zip(product([0, 1], [0, 1, 2]),
                         [clf1, clf2, clf3, clf4, clf5, clf6],
                         ['Depth = \{\}'.format(n) for n in range(1, 7)])
    Z = np.array([clf.predict_instance(x) for x in np.c_[xx.ravel(), y
    Z = Z.reshape(xx.shape)
    axarr[idx[0], idx[1]].contourf(xx, yy, Z, alpha=0.4)
    axarr[idx[0], idx[1]].scatter(x_train[:, 0], x_train[:, 1], c=y_tr
    axarr[idx[0], idx[1]].set_title(tt)
plt.show()
```

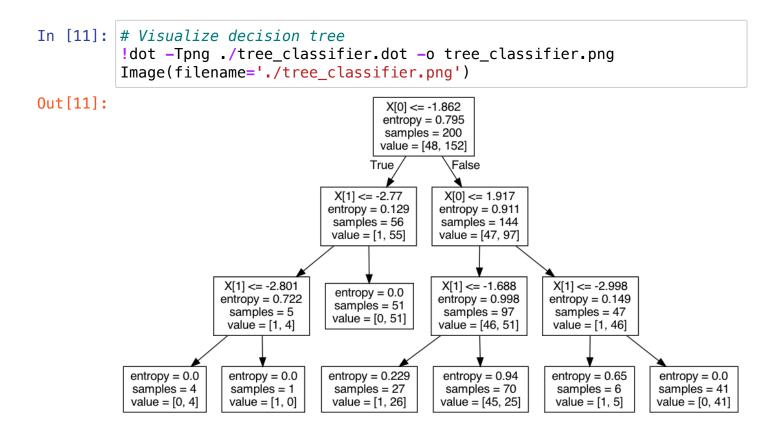




Compare decision tree with tree model in sklearn

Out[10]: <matplotlib.collections.PathCollection at 0x7ff5505a2670>





Decision Tree Regressor

In [12]:

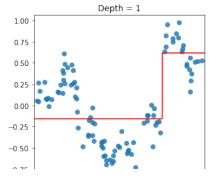
```
# Regression Tree Specific Code
def mean_absolute_deviation_around_median(y):
    Calulate the mean absolute deviation around the median of a given
    :param y: a numpy array of targets shape = (n, 1)
    :return mae
    1.1.1
    # Initialize mae / median
    mae = 0
    median = np.median(y)
    #Iterate over y's and calculate absolute deviation from median
    for y_hat in y:
        mae += abs(y_hat-median)
    #Take average
    mae = mae / len(y)
    #Return mae
    return mae
```

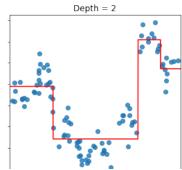
```
In [13]:
         class Regression_Tree():
              :attribute loss_function_dict: dictionary containing the loss fund
              :attribute estimator_dict: dictionary containing the estimation fu
             loss_function_dict = {
                  'mse': np.var,
                  'mae': mean_absolute_deviation_around_median
             estimator_dict = {
                  'mean': np.mean,
                  'median': np.median
             }
             def __init__(self, loss_function='mse', estimator='mean', min_samp
                 Initialize Regression_Tree
                  :param loss_function(str): loss function used for splitting in
                  :param estimator(str): value estimator of internal node
                 self.tree = Decision_Tree(self.loss_function_dict[loss_function_dict[])
                                             self.estimator_dict[estimator],
                                            0, min_sample, max_depth)
             def fit(self, X, y=None):
                  self.tree.fit(X.v)
                  return self
             def predict_instance(self, instance):
                 value = self.tree.predict_instance(instance)
                  return value
```

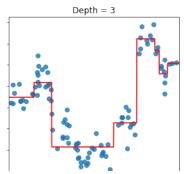
Fit regression tree to one-dimensional regression data

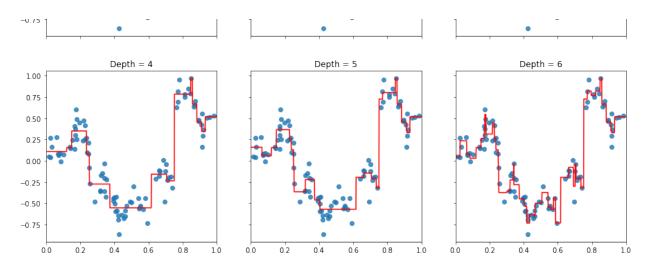
```
In [14]:
```

```
data_krr_train = np.loadtxt('krr-train.txt')
data_krr_test = np.loadtxt('krr-test.txt')
x_krr_train, y_krr_train = data_krr_train[:,0].reshape(-1,1),data_krr_
x_krr_test, y_krr_test = data_krr_test[:,0].reshape(-1,1),data_krr_test
# Training regression trees with different depth
clf1 = Regression_Tree(max_depth=1, min_sample=3, loss_function='mae'
clf1.fit(x_krr_train, y_krr_train)
clf2 = Regression Tree(max depth=2,
                                     min sample=3, loss function='mae'
clf2.fit(x_krr_train, y_krr_train)
clf3 = Regression_Tree(max_depth=3,
                                     min sample=3, loss function='mae'
clf3.fit(x_krr_train, y_krr_train)
clf4 = Regression_Tree(max_depth=4,
                                     min_sample=3, loss_function='mae'
clf4.fit(x_krr_train, y_krr_train)
clf5 = Regression Tree(max depth=5,
                                     min_sample=3, loss_function='mae'
clf5.fit(x_krr_train, y_krr_train)
clf6 = Regression_Tree(max_depth=10,
                                      min_sample=3, loss_function='mae
clf6.fit(x_krr_train, y_krr_train)
plot size = 0.001
x_range = np.arange(0., 1., plot_size).reshape(-1, 1)
f2, axarr2 = plt.subplots(2, 3, sharex='col', sharey='row', figsize=(1
for idx, clf, tt in zip(product([0, 1], [0, 1, 2]),
                        [clf1, clf2, clf3, clf4, clf5, clf6],
                        ['Depth = \{\}'.format(n) for n in range(1, 7)])
   y range predict = np.array([clf.predict instance(x) for x in x ran
   axarr2[idx[0], idx[1]].plot(x_range, y_range_predict, color='r')
   axarr2[idx[0], idx[1]].scatter(x_krr_train, y_krr_train, alpha=0.8
   axarr2[idx[0], idx[1]].set_title(tt)
    axarr2[idx[0], idx[1]].set_xlim(0, 1)
plt.show()
```

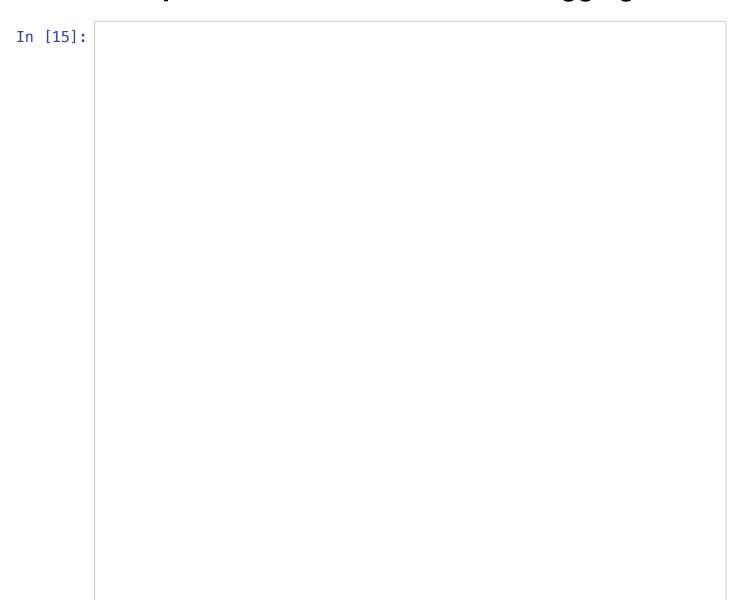




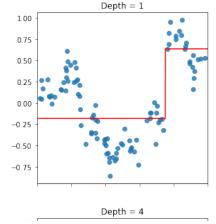


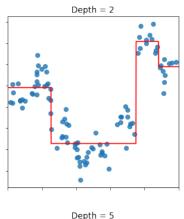


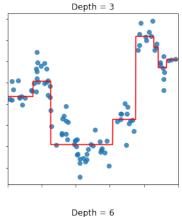
Compare with scikit-learn for debugging

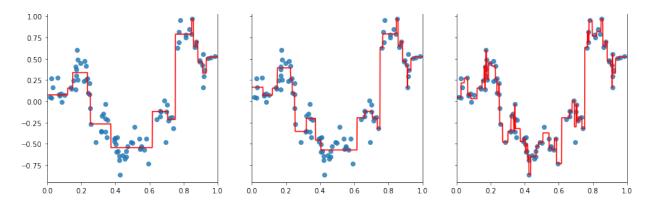


```
# Training regression trees with different depth
clf1 = DecisionTreeRegressor(criterion='absolute_error', max_depth=1,
Regression_Tree(max_depth=1, min_sample=3, loss_function='mae', estim
clf1.fit(x_krr_train, y_krr_train)
clf2 = DecisionTreeRegressor(criterion='absolute_error', max_depth=2,
clf2.fit(x_krr_train, y_krr_train)
clf3 = DecisionTreeRegressor(criterion='absolute_error', max_depth=3,
clf3.fit(x_krr_train, y_krr_train)
clf4 = DecisionTreeRegressor(criterion='absolute error', max depth=4,
clf4.fit(x_krr_train, y_krr_train)
clf5 = DecisionTreeRegressor(criterion='absolute_error', max_depth=5,
clf5.fit(x_krr_train, y_krr_train)
clf6 = DecisionTreeRegressor(criterion='absolute_error', max_depth=10,
clf6.fit(x_krr_train, y_krr_train)
#Compare Plots
plot_size = 0.001
x_range = np.arange(0., 1., plot_size).reshape(-1, 1)
f2, axarr2 = plt.subplots(2, 3, sharex='col', sharey='row', figsize=(1
for idx, clf, tt in zip(product([0, 1], [0, 1, 2]),
                        [clf1, clf2, clf3, clf4, clf5, clf6],
                        ['Depth = \{\}'.format(n) for n in range(1, 7)])
   y_range_predict = clf.predict(np.array([x for x in x_range]).resha
   axarr2[idx[0], idx[1]].plot(x_range, y_range_predict, color='r')
   axarr2[idx[0], idx[1]].scatter(x_krr_train, y_krr_train, alpha=0.8
   axarr2[idx[0], idx[1]].set_title(tt)
    axarr2[idx[0], idx[1]].set_xlim(0, 1)
plt.show()
```









Gradient Boosting Method

Problem 5

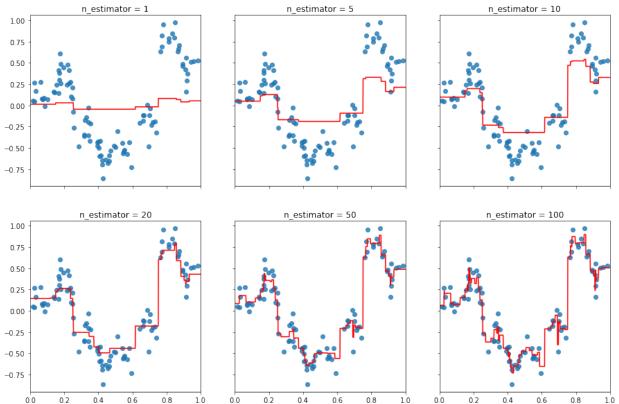
```
In [27]: class gradient_boosting():
             Gradient Boosting regressor class
             :method fit: fitting model
             def __init__(self, n_estimator, pseudo_residual_func, learning_rat
                          min_sample=5, max_depth=5):
                 Initialize gradient boosting class
                 :param n estimator: number of estimators (i.e. number of round
                 :pseudo_residual_func: function used for computing pseudo-resi
                 :param learning_rate: step size of gradient descent
                 self.n_estimator = n_estimator
                 self.pseudo_residual_func = pseudo_residual_func
                 self.learning rate = learning rate
                 self.min_sample = min_sample
                 self.max depth = max depth
                 self.estimators = [] #will collect the n estimator models
             def fit(self train data train tarmet).
```

```
wer ittooch, chath_data, chath_target/.
   Fit gradient boosting model
    :train_data array of inputs of size (n_samples, m_features)
    :train_target array of outputs of size (n_samples,)
    #Initialize array of 0's of size = (len(train_target))
    base_grad = np.zeros(len(train_target))
    self.estimators.append(base_grad) #Append base case
   #Set up our base case - Sk Learns Regression Tree
    base_case = DecisionTreeRegressor(criterion='squared_error',
                                      min samples split=self.min s
                                      max_depth=self.max_depth)
    #Fit regression model
    base_case.fit(train_data, train_target.flatten())
    #Append Estimators
    self.estimators.append(base_case)
   #Iterate for however many rounds of gradient boosting we're us
    for i in range(1,self.n_estimator):
        #Compute Predictions
        predictions = self.predict(train_data)
        #Compute Residuals
        residuals = train target.flatten() - predictions
        #Fit regression model to -q
        base case = DecisionTreeRegressor(criterion='squared_error
                                      min_samples_split=self.min_s
                                      max_depth=self.max_depth)
        #Fit new function
        base case.fit(train data, residuals)
        #Append estimators
        self.estimators.append(base case)
    return self
def predict(self, test_data):
    Predict value
    :train_data array of inputs of size (n_samples, m_features)
    #Initialize prediction
    test predict = np.zeros(len(test data))
    #Iterate over the estimators we have saved in our .fit method
    for i in range(1,len(self.estimators)):
        #Add estimator_i prediction to test_predict, but scale by
        test predict += self.estimators[i].predict(test data) * se
```

return test_predict

1-D GBM visualization - KRR data

Question 6



Sklearn implementation for Classification of images

Question 9

Gradient Boosting Classifier

In [29]: from sklearn.datasets import fetch openml

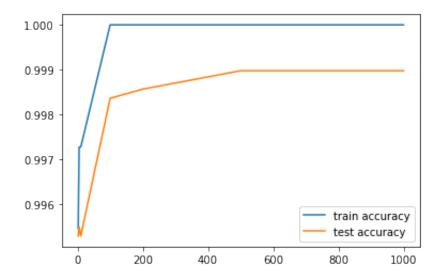
```
from sklearn.linear_model import LogisticRegression
         from sklearn.model selection import train test split
         from sklearn.preprocessing import StandardScaler
         from sklearn.utils import check_random_state
In [30]: | def pre_process_mnist_01():
             Load the mnist datasets, selects the classes 0 and 1
             and normalize the data.
             Args: none
             Outputs:
                 X_train: np.array of size (n_training_samples, n_features)
                 X_test: np.array of size (n_test_samples, n_features)
                 y_train: np.array of size (n_training_samples)
                 y_test: np.array of size (n_test_samples)
             X_mnist, y_mnist = fetch_openml('mnist_784', version=1,
                                              return X y=True, as frame=False)
             indicator_01 = (y_mnist == '0') + (y_mnist == '1')
             X mnist 01 = X mnist[indicator 01]
             y_mnist_01 = y_mnist[indicator_01]
             X train, X test, y train, y test = train test split(X mnist 01, y
                                                                  test size=0.33
                                                                  shuffle=False)
             scaler = StandardScaler()
             X train = scaler.fit transform(X train)
             X_test = scaler.transform(X_test)
             y_{test} = 2 * np_array([int(y) for y in y_test]) - 1
             y_train = 2 * np.array([int(y) for y in y_train]) - 1
             return X_train, X_test, y_train, y_test
```

```
In [31]: X_train, X_test, y_train, y_test = pre_process_mnist_01()
```

```
In [32]: #Initalize helper variables
         loss_dict = {'train':[],'test':[]}
         estimators = [2,5,10,100,200,500,1000]
         #Iterate over the estimators
         for n in estimators:
             #Helper Print Statement to let me know it isn't dead
             print(f'Now fiitting GBC - {n} Estimators used')
             #Initliaze GBC Estimator
             gbc = GradientBoostingClassifier(n_estimators=n, loss='deviance',
             #Fit the GBC estimator
             gbc.fit(X_train, y_train)
             #Append results
             loss_dict['train'].append(gbc.score(X_train, y_train)) #Append Tra
             loss_dict['test'].append(gbc.score(X_test,y_test)) #Append Tes
         #Plot our results
         plt.plot(estimators, loss_dict['train'], label = 'train accuracy') #Pl
         plt.plot(estimators, loss_dict['test'], label = 'test accuracy')
         plt.legend()
         plt.plot()
```

fitting gbc with 2 estimators fitting gbc with 5 estimators fitting gbc with 10 estimators fitting gbc with 100 estimators fitting gbc with 200 estimators fitting gbc with 500 estimators fitting gbc with 1000 estimators

Out[32]: []



Random Forest Classifier

```
In [36]: #Initalize helper variables
         loss_dict = {'train':[],'test':[]}
         estimators = [2,5,10,100,200,500,1000]
         #Iterate over our estimators
         for n in estimators:
             #Initalize and fit a Random Forest Classifier from sklearn
             gbrf = RandomForestClassifier(n_estimators=n, criterion = 'entropy
             gbrf.fit(X_train, y_train)
             #Append our train / test loss
             loss_dict['train'].append(gbrf.score(X_train, y_train))
             loss dict['test'].append(gbrf.score(X test,y test))
         #Plot our results
         plt.plot(estimators, loss_dict['train'], label = 'train accuracy') #Pl
         plt.plot(estimators, loss dict['test'], label = 'test accuracy')
         plt.legend()
         plt.plot()
```

Out[36]: []

