In [1]:
```
#Import necessary Libraries
import numpy as np
import scipy
from scipy.optimize import minimize
from functools import partial
import pandas as pd
import matplotlib.pyplot as plt
```

Question 5

In [2]:
```
def f_objective(theta, X, y, l2_param):
    '''
    Args:
        theta: 1D numpy array of size num_features
        X: 2D numpy array of size (num_instances, num_features)
        y: 1D numpy array of size num_instances
        l2_param: regularization parameter

    Returns:
        objective: scalar value of objective function
    '''
    #Helper variable for ERM Summation
    objective = 0
    #Iterate over data points
    for i in range(len(y)):
        #Calculate 1+e^margin
        m = -y[i]*theta@X[i,:]
        #Add the margin exponentiation to the ERM summation
        objective += np.logaddexp(0, m)
    #Calculate the regularization penalty
    l2_penalty = l2_param* (theta@theta)
    return (objective / len(y))+ l2_penalty
```

Question 6

In [3]:
```
def fit_logistic_reg(X, y, objective_function, l2_param):
    '''
    Args:
        X: 2D numpy array of size (num_instances, num_features)
        y: 1D numpy array of size num_instances
        objective_function: function returning the value of the object
        l2_param: regularization parameter

    Returns:
        optimal_theta: 1D numpy array of size num_features
    '''
    starting_theta = np.ones(X.shape[1])
    passing_func = partial(objective_function,X=X,y=y,l2_param=l2_para
    optimal_theta = minimize(passing_func,starting_theta).x
    return optimal_theta
```

In [4]:
```
#Load Data, then Clean it
X_train, X_val = pd.read_csv('X_train.txt',header=None), pd.read_csv('
y_train, y_val = pd.read_csv('y_train.txt',header=None), pd.read_csv('
#Fix y_train / val labels to -1 rather than 0
y_train[y_train==0], y_val[y_val==0] = -1, -1
#Standardize via Z-Scores, Add Bias Column
X_val = (X_val - X_train.mean() )/ X_train.std()
X_train = (X_train - X_train.mean() )/ X_train.std()
#X_val = (X_val - X_val.mean())/ X_val.std()
X_train['Bias'] = 1
X_val['Bias'] = 1

#Transform DataFrames to numpy ararys
X_train = X_train.to_numpy()
X_val = X_val.to_numpy()
y_train = y_train.to_numpy()
y_val = y_val.to_numpy()
```

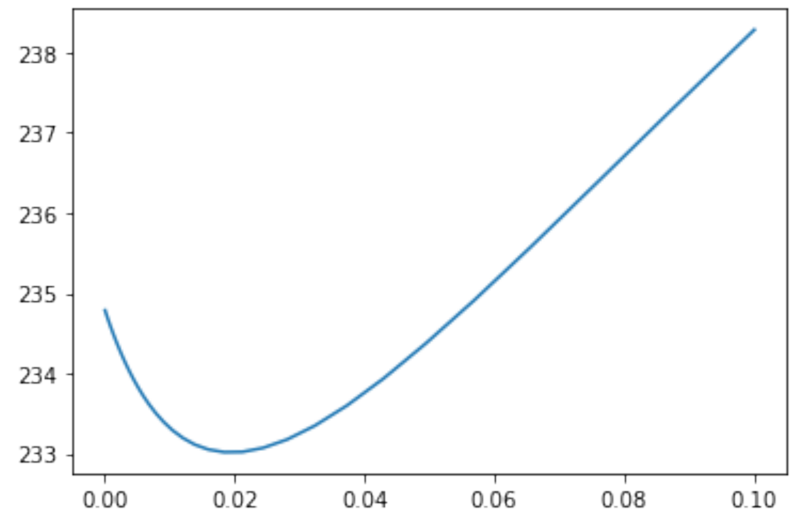
Problem 7

In [5]:
```
def NLL(X,y,theta):
    nll = 0
    for i in range(len(y)):
        m = -y[i]*theta@X[i,:]
        nll += np.logaddexp(0,m)
    return(nll)
```

In [6]:
```
#lambda_list = np.linspace(0,2,10)
lambda_list = np.logspace(-4,-1,num=50)

nll_list = []
for lamda_reg in lambda_list:
    optimal_theta = fit_logistic_reg(X_train, y_train,f_objective, l2_
    nll_list.append(NLL(X=X_val,y=y_val,theta=optimal_theta))
```

In [7]:
```
plt.plot(lambda_list,nll_list)
```

Out[7]: [<matplotlib.lines.Line2D at 0x7fd0119da700>]



In [8]:
```
min_nll = min(nll_list)
min_index = nll_list.index(min_nll)
min_lambda = lambda_list[min_index]
print("Min NLL:",min_nll,"Lambda:",min_lambda)
```

```
Min NLL: 233.0183069467581 Lambda: 0.018420699693267165
```

Minimum NLL achieved at $\lambda = 0.01842$, with $NLL = 233$

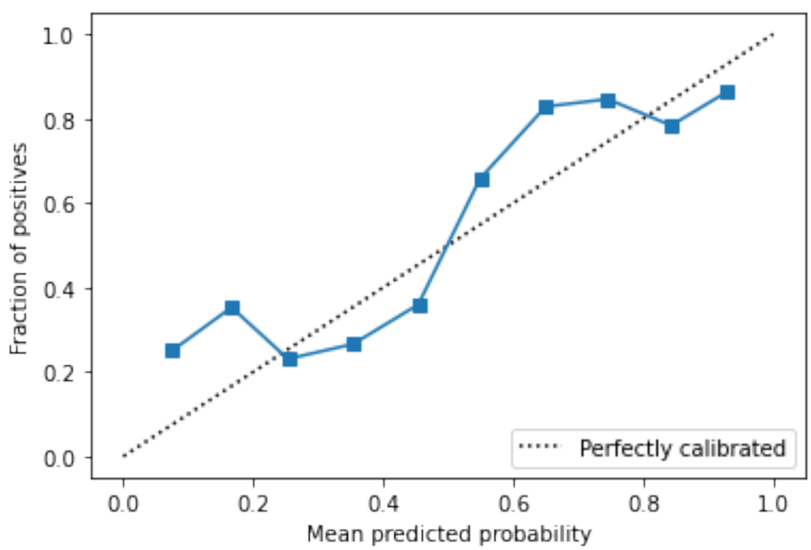
Problem 8

In [9]:
```
#Use the Optimal Theta Calculated in the Last Problem
optimal_theta = fit_logistic_reg(X_train,y_train,f_objective,l2_param=
```

In [10]:
```
#Initialize helper variable
scores = []
#Calculate a score for each observation in X_val
for i in range(len(X_val)):
    score = 1 / (1 + np.exp(-optimal_theta@X_val[i,:]))
    scores.append(score)
```

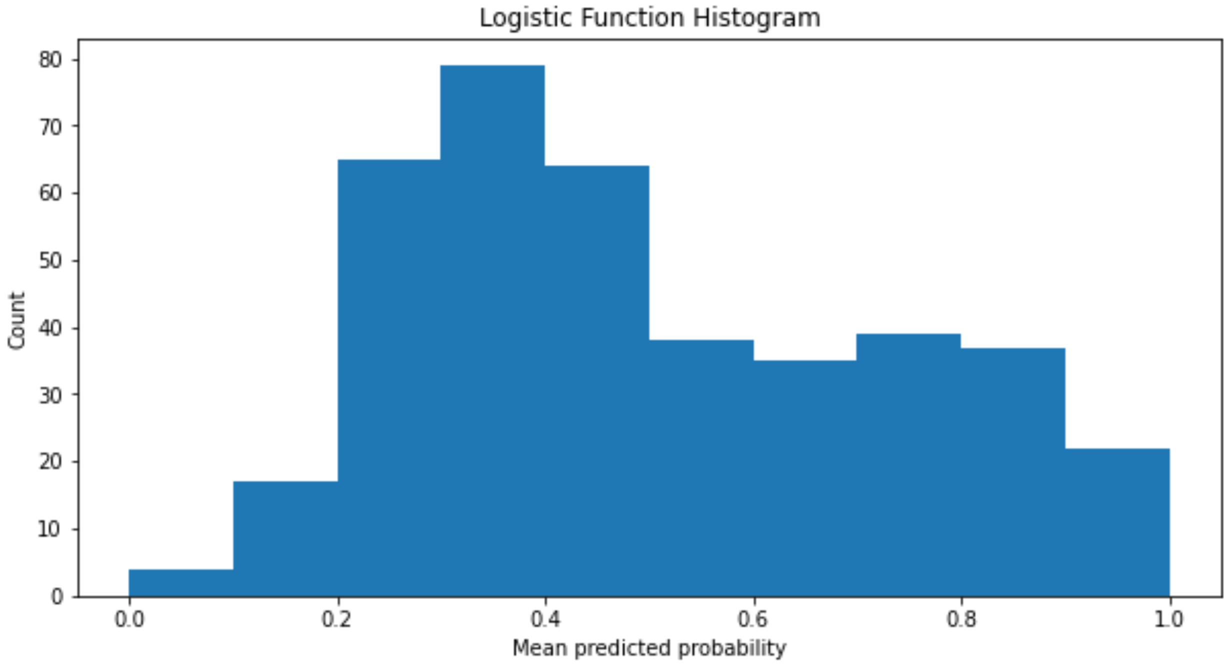
In [11]:
```
#Use Sci-Kit Learn Template to Show how Calibrated Our Model is
from sklearn.calibration import calibration_curve, CalibrationDisplay
y_true, y_prob = y_val, scores

prob_true, prob_pred = calibration_curve(y_true, y_prob, n_bins=10)
disp = CalibrationDisplay(prob_true, prob_pred,y_prob)
disp.plot()
```

Out[11]:
```
<sklearn.calibration.CalibrationDisplay at 0x7fd002044850>
```



In [12]:
```
plt.figure(figsize=(10,5))
plt.hist(disp.y_prob, range=(0,1), bins=10, histtype='bar')
plt.title('Logistic Function Histogram')
plt.xlabel("Mean predicted probability")
plt.ylabel("Count")
```

Out[12]: Text(0, 0.5, 'Count')



Commentary:

It appears that our model is not very well calibrated: using a bin size of 10, we can clearly see that the actual outcome percentage of positive results does not match the predicted probability. Visually, we can see this occuring as if our model was calibrated perfectly, we would have the points on our plot line up with the linear line $y = x$.

Interpolating further, our model is not calibrated well, as if we have a set of feature vectors $x_i \in X$, where $x_i^T w = .7$, we would expect about 70% of the labels to be positive, that is, $y = 1$. However, using our plot, we can see that the actual mix of positive and negative ground truth labels does not match our predicted 70%, and is actually 80%.

Furthermore, by analyzing the distribution of scores $x^T w$, we see many predictions that do not display the most confidence, or margin. This is apparant in the histrogram with bin size = 10, as we can see many of our predicted scores fall between the range $0.2 \leq x^T w \leq 0.6$, where as if our model was calibrated well, the majority of scores would fall in the $0.0 - 0.1$ bucket, and many others in the $0.9 - 1.0$ bucket. Since we used the lambda function that would minimize our NLL, I don't think the issue is caused by our model selection, but perhaps that the data we are analyzing is not well predicted by a logistic regression model.