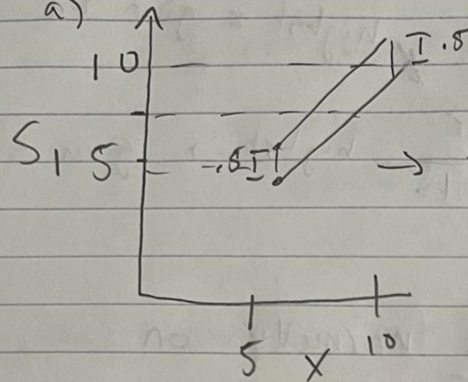
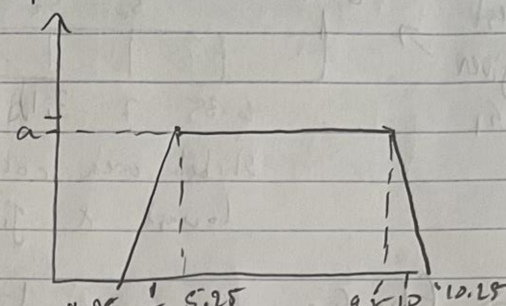
Problem 3

a)



$\rightarrow f_{S_1}(s_1)$

pdf sketch of S_1



$\int_{4.75}^{10.25} f_1(s_1) = 1$ lets use geometry, we have 2 triangles and 1 ~~tri~~ rectangle.

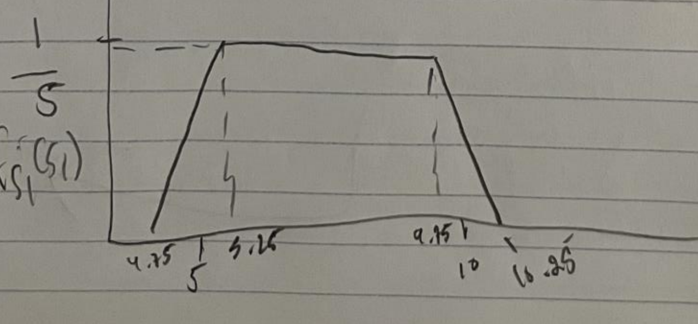
↳ $.5a + 4.5a = 1$

$5a = 1$

$a = 1/5$

therefore

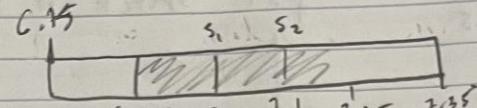
$$f_{S_1}(s_1) = \begin{cases} \frac{2}{5}s - \frac{19}{10} & 4.75 < s_1 < 5.25 \\ 1/5 & 5.25 \le s_1 \le 9.75 \\ -\frac{2}{5}s_1 + \frac{41}{10} & 9.75 \le s_1 \le 10.25 \end{cases}$$



Part B

Problem 3 b) $6.85 \le x \le 7.25$

lowest x given s_1



highest x given s_2

highest x given s_1

shaded area corresponds to lowest x given s_2

$\Rightarrow$ Therefore x is distributed normally on the range $6.85 \le x \le 7.25$

$$f_x(x \mid 6.85 \le x \le 7.25)$$

$$\int_{6.85}^{7.25} f_{\hat{x}}(x \mid 6.85 \le x \le 7.25) = 1$$

$$\hookrightarrow f(x \mid 6.85 \le x \le 7.25) = 2.5$$

Part

c) $\Rightarrow$ Part c) $f_{s_1, s_2}(\hat{s}_1, \hat{s}_2) = \int_5^{10} f(\hat{s}_1, \hat{s}_2, x) = \int_5^{10} \underbrace{f(\hat{x})}_{1/5} f(\hat{s}_1 \mid \hat{x}) + f(\hat{s}_2 \mid x)\,dx$

$$\int_{\max(5,\, s_1 - .25,\, s_2 - .25)}^{\min(10,\, s_1 + .25,\, s_2 + .25)} \frac{4}{5}\,dx$$

$$\int_{x-0.25}^{x+0.25} f(s_1 \mid x) = 1 \Rightarrow f(\hat{s}_1 \mid x) = 2$$

$$\int_{x-.25}^{x+.25} f(\hat{s}_2 \mid x) = 1 \Rightarrow f(s_2 \mid x) = 2$$

$$\Rightarrow f(x) f(\hat{s}_1 \mid \hat{x}) f(\hat{s}_2 \mid x) = 4/5$$

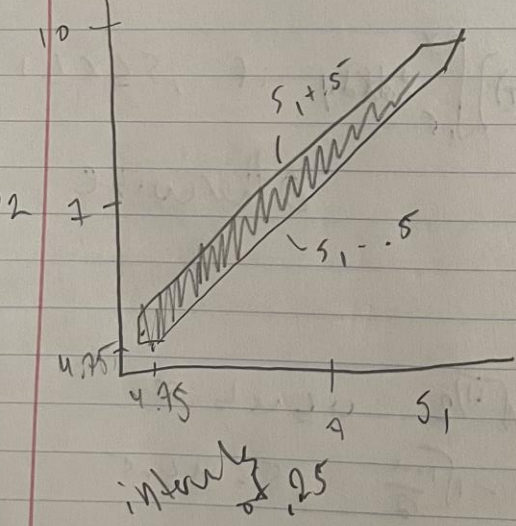
c continued

$$= \begin{cases} \frac{4}{5}\left[\min(10, s_1+.25, s_2+.25) - \max(5, s_1-.25, s_2-.25)\right] \\ \text{if } \left(\begin{array}{l} 4.75 \le s_1 \le 10.25 \\ \text{and } 4.75 \le s_1 \le 10.25 \end{array}\right) \text{ and } (|s_1 - s_2| = .5) \\ 0 \text{ otherwise} \end{cases}$$

check that $f_{s_1,s_2}(6,6) = f_{s_1}(6)\, f_{s_2}(6)$

$$\frac{4}{5}(6.25 - 5.75) = \left(\frac{1}{5}\right)\left(\frac{1}{5}\right)$$

$$\frac{2}{5} \neq \frac{1}{25} \quad \text{Not Independent!}$$



It makes sense that they are not independent, as one observation influences the range of possibilities for the other.

Problem 4

Firstly we need to obtain a pdf of the tringle.

$$\left(\frac{1}{2} b h\right) c = 1 \Rightarrow \frac{1}{2} c = 1$$
$$c = 2$$

$$f_{X,Y}(x,y) = \begin{cases} 2 & \text{for } X,Y \text{ in triangle} \\ 0 & \text{otherwise} \end{cases}$$

Now that we have the pdf we can integrate to get the cdf

$$F_{X,Y}(x,y) = \begin{cases} \frac{1}{2}\int_0^{2x}\int_0^{2y} 2\,dy\,dx & 0 \le x \le .5 \\ .5 + \frac{1}{2}\left[1 + (2-2x)\right]\int_{.5}^{x} 2\,dx\,dy & \text{if } .5 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Now, compute inverse cdf

$$\text{inverse cdf} = \begin{cases} \sqrt{u/2} & 0 \le u \le .5 \\ 1 - \sqrt{\frac{1-u}{2}} & .5 < u \le .5 \end{cases}$$

Now we can plot uniformly distributed samples