```
import matplotlib.pyplot as plt
from itertools import product
import numpy as np
from collections import Counter
from sklearn.base import BaseEstimator, RegressorMixin, ClassifierMixin
from sklearn.tree import DecisionTreeClassifier, DecisionTreeRegressor, expor
from sklearn.ensemble import GradientBoostingClassifier, GradientBoostingRegr
import graphviz

from IPython.display import Image

%matplotlib inline
```

Load Data

```
In [2]:
    data_train = np.loadtxt('svm-train.txt')
    data_test = np.loadtxt('svm-test.txt')
    x_train, y_train = data_train[:, 0: 2], data_train[:, 2].reshape(-1, 1)
    x_test, y_test = data_test[:, 0: 2], data_test[:, 2].reshape(-1, 1)

In [3]:
# Change target to 0-1 label
    y_train_label = np.array(list(map(lambda x: 1 if x > 0 else 0, y_train))).residue.
```

Decision Tree Class

Problem 1

```
In [4]:
         def compute_entropy(label_array):
             Calulate the entropy of given label list
             :param label_array: a numpy array of binary labels shape = (n, 1)
             :return entropy: entropy value
             # Your code goes here
             #Only considering binary classification
             entropy = 0 #Initialize entropy
             prob one = label array.sum() / len(label array) #Calculate the probabilit
                                                              #Likewise for class = 0
             prob array = [1-prob one,prob one]
             #Make sure we don't bug out trying to take log 2(0)
             if prob one == 0 or prob one == 1:
                 return 0
             #Calculate entropy sum(-log 2(prob)*prob)
             for i in [0,1]:
                 entropy -= np.log2(prob_array[i]) * prob_array[i]
             #Return entropy
             return entropy
         def compute gini(label array):
             Calulate the gini index of label list
             :param label_array: a numpy array of labels shape = (n, 1)
             :return gini: gini index value
             #Only considering binary classification
             gini = 0
                                                              #Initialize gini index
             prob_one = label_array.sum() / len(label_array) #Calculate the probabilit
             prob_array = [1-prob_one,prob_one]
                                                             #Likewise for class = 0
             #Calculate gini index -> sum(prob class(1-prob class))
             for i in [0,1]:
                 gini += prob array[i]*(1-prob array[i])
             return gini #Return gini index
```

Problem 2

```
:param split loss function: method with args (X, y) returning loss
    :param leaf value estimator: method for estimating leaf value from ar
    :param depth: depth indicator, default value is 0, representing root
    :param min sample: an internal node can be splitted only if it contain
    :param max_depth: restriction of tree depth.
   self.split loss function = split loss function
   self.leaf_value_estimator = leaf_value_estimator
   self.depth = depth
   self.min sample = min sample
   self.max depth = max depth
   self.is leaf = False
   #Add these variables to the constructor
   self.right = None
                            #Left child node
   self.left = None
                           #Right child node
   self.split id = None #Best column to split on
   self.split_value = None #Best value to split on within best column
   self.value = None
                            #Value to return if
def fit(self, x, y):
   This should fit the tree classifier by setting the values self.is_lea
   self.split_id (the index of the feature we want ot split on, if we're
   self.split value (the corresponding value of that feature where the s
   and self.value, which is the prediction value if the tree is a leaf n
   splitting the node, we should also init self.left and self.right to b
   objects corresponding to the left and right subtrees. These subtrees
   the data that fall to the left and right, respectively, of self.split
   This is a recurisive tree building procedure.
   :param X: a numpy array of training data, shape = (n, m)
   :param y: a numpy array of labels, shape = (n, 1)
   :return self
   # Your code goes here
   #Check break condition, if we've exceeded max depth or are leg min sa
   if self.depth >= self.max depth or len(y) <= self.min sample:</pre>
        self.is leaf = True
       self.value = self.leaf value estimator(y)
          if y.sum() / len(y) <= .5:
             self.value = 0
         else:
             self.value = 1
   else:
       #Calculate best splitting point
       self.find_best_feature_split(x,y)
        #Split data in two depending on criteria
```

#

```
all data = np.append(x,y,axis=1) #Create one big matrix (easier t
        #Filter data by split column / split point
        left data = all data[all data[:,self.split id]<=self.split value]</pre>
        #Again but look for greater for right tree
        right_data = all_data[all_data[:,self.split_id]>self.split_value]
        left_x_node = left_data[:,0:-1]
        left_y_node = left_data[:,-1].reshape(-1,1)
        right x node = right data[:,0:-1]
        right_y_node = right_data[:,-1].reshape(-1,1)
        #Create left and right nodes
        self.left = Decision Tree(self.split loss function, #Pass split f
                                 self.leaf value estimator, #Pass leaf va
                                 depth=self.depth+1,
                                                            #Pass self.de
                                 min sample = self.min sample, #Pass min
                                 max depth = self.max depth
        self.right = Decision_Tree(self.split_loss_function, #Pass split
                                 self.leaf_value_estimator, #Pass leaf_va
                                 depth=self.depth+1,
                                                           #Pass self.de
                                 min sample = self.min sample, #Pass min
                                 max_depth = self.max_depth
        #Fit the left/right nodes
        self.left.fit(left_x_node,left_y_node)
        self.right.fit(right x node, right y node)
   return self
def find best split(self, x node, y node, feature id):
   For feature number feature id, returns the optimal splitting point
    for data X_node, y_node, and corresponding loss
    :param X: a numpy array of training data, shape = (n node)
    :param y: a numpy array of labels, shape = (n_node, 1)
    # Your code
   \#x \ copy = x \ node.copy()
   y_copy = y_node.copy()
   feature_vals = x_node[:,feature_id].copy() #Grab the feature vals
   sorting = feature vals.argsort() #Prepare index for arg sorting fe
                                       #Sort the y node by x index
   y copy = y copy[sorting]
   feature_vals.sort()
                                       #Sort the feature grabbed from x
    #Initialize entropy variable
   best loss = 100
   split value = -1
   #Iterate over the feature vals
    for i in range(1,len(feature_vals)):
        #Seperate sorted (single) feature vals into two halves
        top_half = y_copy[:i]
        bottom_half = y_copy[i:]
```

```
#Calculate weighted entropy for each half
        top ratio = (len(top half)/len(y copy))
        bottom ratio = (len(bottom half)/len(y copy))
        top half entropy = top ratio * self.split_loss_function(top_half
        bottom_half_entropy = bottom_ratio * self.split_loss_function(bot
        #Calculate Loss = Total Weighted Entropy
        loss = top half entropy + bottom half entropy
        #Check if we've reached a smaller loss
        if loss <= best loss:</pre>
            best loss = loss #Update smaller loss
            if i == 1:
                split value = (feature vals[i]+feature vals[i+1])/2 #Take
            #Update the best split value via midpoint value
            #Take midpoint of point before after if best split point is f
            else:
                split_value = (feature_vals[i]+feature_vals[i-1])/2 #Take
   return split value, best loss
def find_best_feature_split(self, x_node, y_node):
   Returns the optimal feature to split and best splitting point
    for data X node, y node.
    :param X: a numpy array of training data, shape = (n node, 1)
    :param y: a numpy array of labels, shape = (n node, 1)
   best feature loss = 100
    #Iterate over all of the columns
    for i in range(x node.shape[1]):
        #Use self.find best split to
        split_value, best_loss = self.find_best_split(x_node,y_node,i)
        #Check if we've found a column to split better on
        if best loss <= best feature loss:</pre>
            best feature loss = best loss
                                               #Update Loss Accordingly
            self.split id = i
                                               #Update the column split i
            self.split value = split value
                                             #Update the column split v
def predict instance(self, instance):
   Predict label by decision tree
    :param instance: a numpy array with new data, shape (1, m)
   :return whatever is returned by leaf_value_estimator for leaf contain
    if self.is_leaf:
        return self.value
```

```
if instance[self.split_id] <= self.split_value:
    return self.left.predict_instance(instance)
else:
    return self.right.predict_instance(instance)</pre>
```

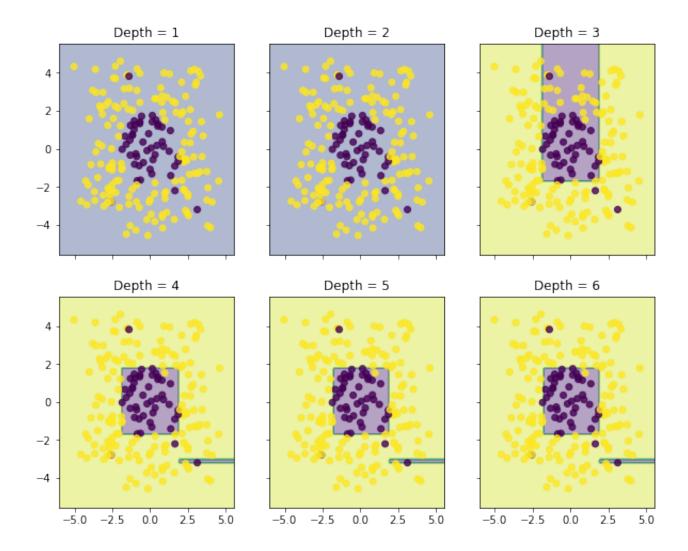
Decision Tree Classifier

```
In [6]:
         def most common label(y):
             Find most common label
             label cnt = Counter(y.reshape(len(y)))
             label = label_cnt.most_common(1)[0][0]
             return label
In [7]:
         class Classification Tree(BaseEstimator, ClassifierMixin):
             loss function dict = {
                 'entropy': compute entropy,
                 'gini': compute gini
             }
             def __init__(self, loss_function='entropy', min_sample=5, max_depth=10):
                 :param loss_function(str): loss function for splitting internal node
                 self.tree = Decision_Tree(self.loss_function_dict[loss_function],
                                          most common label,
                                          0, min_sample, max_depth)
             def fit(self, X, y=None):
                 self.tree.fit(X,y)
                 return self
             def predict instance(self, instance):
                 value = self.tree.predict_instance(instance)
                 return value
```

Problem 3

Decision Tree Boundary

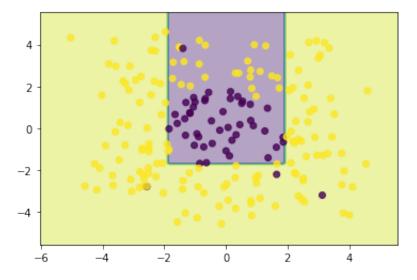
In [8]: # Training classifiers with different depth clf1 = Classification_Tree(max_depth=1, min_sample=2) clf1.fit(x_train, y_train_label) clf2 = Classification_Tree(max_depth=2, min sample=2) clf2.fit(x train, y train label) clf3 = Classification Tree(max depth=3, min sample=2) clf3.fit(x train, y train label) clf4 = Classification Tree(max depth=4, min sample=2) clf4.fit(x train, y train label) clf5 = Classification_Tree(max_depth=5, min_sample=2) clf5.fit(x_train, y_train_label) clf6 = Classification_Tree(max_depth=6, min_sample=2) clf6.fit(x_train, y_train_label) # Plotting decision regions $x \min, x \max = x \operatorname{train}[:, 0] \cdot \min() - 1, x \operatorname{train}[:, 0] \cdot \max() + 1$ y_min, y_max = x_train[:, 1].min() - 1, x_train[:, 1].max() + 1 xx, yy = np.meshgrid(np.arange(x_min, x_max, 0.1), np.arange(y min, y max, 0.1)) f, axarr = plt.subplots(2, 3, sharex='col', sharey='row', figsize=(10, 8)) for idx, clf, tt in zip(product([0, 1], [0, 1, 2]), [clf1, clf2, clf3, clf4, clf5, clf6], ['Depth = $\{\}$ '.format(n) for n in range(1, 7)]): Z = np.array([clf.predict_instance(x) for x in np.c_[xx.ravel(), yy.ravel Z = Z.reshape(xx.shape)axarr[idx[0], idx[1]].contourf(xx, yy, Z, alpha=0.4) axarr[idx[0], idx[1]].scatter(x_train[:, 0], x_train[:, 1], c=y_train_lab axarr[idx[0], idx[1]].set title(tt) plt.show()



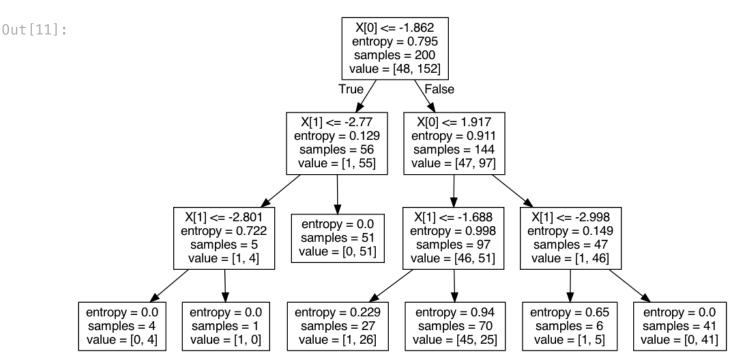
Compare decision tree with tree model in sklearn

```
In [9]:
    clf = DecisionTreeClassifier(criterion='entropy', max_depth=3, min_samples_sp
    clf.fit(x_train, y_train_label)
    export_graphviz(clf, out_file='tree_classifier.dot')
```

Out[10]: <matplotlib.collections.PathCollection at 0x7ff5505a2670>



```
In [11]:  # Visualize decision tree
!dot -Tpng ./tree_classifier.dot -o tree_classifier.png
Image(filename='./tree_classifier.png')
```



Problem 4

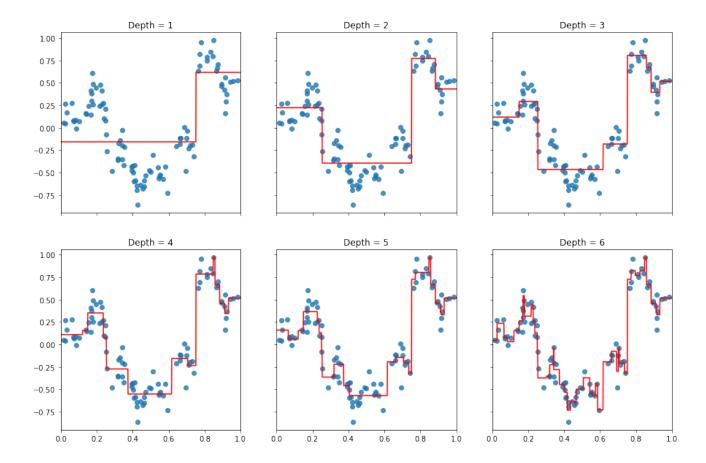
Decision Tree Regressor

```
In [12]:
          # Regression Tree Specific Code
          def mean absolute deviation around median(y):
              Calulate the mean absolute deviation around the median of a given target
              :param y: a numpy array of targets shape = (n, 1)
              :return mae
              # Initialize mae / median
              mae = 0
              median = np.median(y)
              #Iterate over y's and calculate absolute deviation from median
              for y hat in y:
                  mae += abs(y hat-median)
              #Take average
              mae = mae / len(y)
              #Return mae
              return mae
```

In [13]: class Regression_Tree(): :attribute loss function dict: dictionary containing the loss functions u :attribute estimator_dict: dictionary containing the estimation functions loss function dict = { 'mse': np.var, 'mae': mean absolute deviation around median estimator dict = { 'mean': np.mean, 'median': np.median def __init__(self, loss_function='mse', estimator='mean', min_sample=5, m Initialize Regression Tree :param loss_function(str): loss function used for splitting internal :param estimator(str): value estimator of internal node self.tree = Decision Tree(self.loss function dict[loss function], self.estimator dict[estimator], 0, min sample, max depth) def fit(self, X, y=None): self.tree.fit(X,y) return self def predict_instance(self, instance): value = self.tree.predict instance(instance) return value

Fit regression tree to one-dimensional regression data

In [14]: data_krr_train = np.loadtxt('krr-train.txt') data_krr_test = np.loadtxt('krr-test.txt') x krr_train, y krr_train = data krr_train[:,0].reshape(-1,1),data krr_train[: x krr_test, y krr_test = data_krr_test[:,0].reshape(-1,1),data_krr_test[:,1]. # Training regression trees with different depth clf1 = Regression Tree(max depth=1, min sample=3, loss function='mae', estim clf1.fit(x krr train, y krr train) clf2 = Regression Tree(max depth=2, min sample=3, loss function='mae', estim clf2.fit(x krr train, y krr train) clf3 = Regression Tree(max depth=3, min sample=3, loss function='mae', estim clf3.fit(x_krr_train, y_krr_train) clf4 = Regression_Tree(max_depth=4, min_sample=3, loss_function='mae', estim clf4.fit(x_krr_train, y_krr_train) clf5 = Regression Tree(max depth=5, min sample=3, loss function='mae', estim clf5.fit(x_krr_train, y_krr_train) clf6 = Regression Tree(max depth=10, min sample=3, loss function='mae', estil clf6.fit(x_krr_train, y_krr_train) plot size = 0.001x range = np.arange(0., 1., plot size).reshape(-1, 1)f2, axarr2 = plt.subplots(2, 3, sharex='col', sharey='row', figsize=(15, 10)) for idx, clf, tt in zip(product([0, 1], [0, 1, 2]), [clf1, clf2, clf3, clf4, clf5, clf6], ['Depth = $\{\}$ '.format(n) for n in range(1, 7)]): y range predict = np.array([clf.predict instance(x) for x in x range]).re axarr2[idx[0], idx[1]].plot(x_range, y_range_predict, color='r') axarr2[idx[0], idx[1]].scatter(x krr train, y krr train, alpha=0.8) axarr2[idx[0], idx[1]].set_title(tt) axarr2[idx[0], idx[1]].set xlim(0, 1)plt.show()

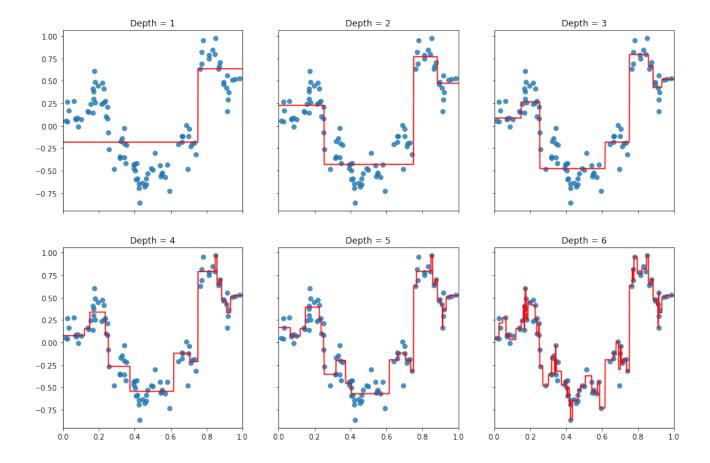


Compare with scikit-learn for debugging

In [15]: # Training regression trees with different depth clf1 = DecisionTreeRegressor(criterion='absolute_error', max_depth=1, min_sam) Regression_Tree(max_depth=1, min_sample=3, loss_function='mae', estimator='m clf1.fit(x_krr_train, y_krr_train) clf2 = DecisionTreeRegressor(criterion='absolute error', max depth=2, min sam clf2.fit(x krr train, y krr train) clf3 = DecisionTreeRegressor(criterion='absolute error', max depth=3, min sam clf3.fit(x krr train, y krr train) clf4 = DecisionTreeRegressor(criterion='absolute error', max depth=4, min sam clf4.fit(x krr train, y krr train) clf5 = DecisionTreeRegressor(criterion='absolute error', max depth=5, min sam clf5.fit(x_krr_train, y_krr_train) clf6 = DecisionTreeRegressor(criterion='absolute error', max depth=10, min sal clf6.fit(x_krr_train, y_krr_train) #Compare Plots plot size = 0.001x_range = np.arange(0., 1., plot_size).reshape(-1, 1) f2, axarr2 = plt.subplots(2, 3, sharex='col', sharey='row', figsize=(15, 10)) for idx, clf, tt in zip(product([0, 1], [0, 1, 2]), [clf1, clf2, clf3, clf4, clf5, clf6], ['Depth = {}'.format(n) **for** n **in** range(1, 7)]): y range predict = clf.predict(np.array([x for x in x range]).reshape(-1, axarr2[idx[0], idx[1]].plot(x range, y range predict, color='r') axarr2[idx[0], idx[1]].scatter(x_krr_train, y_krr_train, alpha=0.8)

axarr2[idx[0], idx[1]].set_title(tt)
axarr2[idx[0], idx[1]].set_xlim(0, 1)

plt.show()



Gradient Boosting Method

Problem 5

```
:param n estimator: number of estimators (i.e. number of rounds of gr
    :pseudo residual func: function used for computing pseudo-residual be
    :param learning rate: step size of gradient descent
   self.n estimator = n estimator
    self.pseudo_residual_func = pseudo_residual_func
   self.learning rate = learning rate
   self.min sample = min sample
   self.max_depth = max_depth
   self.estimators = [] #will collect the n estimator models
def fit(self, train data, train target):
   Fit gradient boosting model
    :train data array of inputs of size (n samples, m features)
    :train_target array of outputs of size (n_samples,)
   #Initialize array of 0's of size = (len(train target))
   base grad = np.zeros(len(train target))
   self.estimators.append(base_grad) #Append base case
   #Set up our base case - Sk Learns Regression Tree
   base_case = DecisionTreeRegressor(criterion='squared_error',
                                      min samples split=self.min sample,
                                      max depth=self.max depth)
   #Fit regression model
   base case.fit(train data, train target.flatten())
   #Append Estimators
   self.estimators.append(base case)
    #Iterate for however many rounds of gradient boosting we're using
    for i in range(1,self.n estimator):
        #Compute Predictions
        predictions = self.predict(train_data)
        #Compute Residuals
        residuals = train target.flatten() - predictions
        #Fit regression model to -g
        base case = DecisionTreeRegressor(criterion='squared error',
                                      min samples split=self.min sample,
                                      max depth=self.max depth)
        #Fit new function
        base case.fit(train data, residuals)
        #Append estimators
        self.estimators.append(base case)
   return self
def predict(self, test data):
```

```
Predict value
:train_data array of inputs of size (n_samples, m_features)
'''
#Initialize prediction
test_predict = np.zeros(len(test_data))

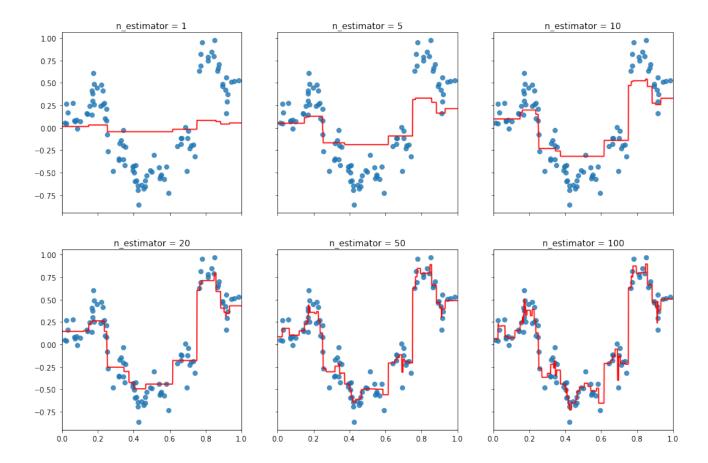
#Iterate over the estimators we have saved in our .fit method
for i in range(1,len(self.estimators)):

    #Add estimator_i prediction to test_predict, but scale by step si
    test_predict += self.estimators[i].predict(test_data) * self.lear

return test_predict
```

1-D GBM visualization - KRR data

Question 6



Sklearn implementation for Classification of images

Question 9

Gradient Boosting Classifier

```
In [29]:
```

```
from sklearn.datasets import fetch_openml
from sklearn.linear_model import LogisticRegression
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
from sklearn.utils import check_random_state
```

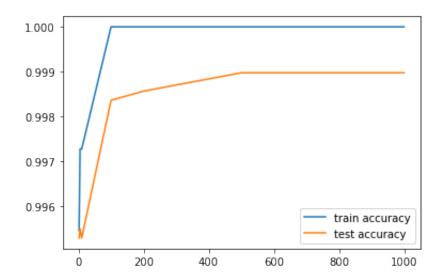
```
In [30]:
          def pre_process_mnist_01():
              Load the mnist datasets, selects the classes 0 and 1
              and normalize the data.
              Args: none
              Outputs:
                  X train: np.array of size (n training samples, n features)
                  X_test: np.array of size (n_test_samples, n features)
                  y train: np.array of size (n training samples)
                  y_test: np.array of size (n_test_samples)
              X mnist, y mnist = fetch openml('mnist 784', version=1,
                                               return X y=True, as frame=False)
              indicator_01 = (y_mnist == '0') + (y_mnist == '1')
              X_mnist_01 = X_mnist[indicator_01]
              y_mnist_01 = y_mnist[indicator_01]
              X_train, X_test, y train, y test = train_test_split(X_mnist_01, y_mnist_0
                                                                   test size=0.33,
                                                                   shuffle=False)
              scaler = StandardScaler()
              X train = scaler.fit transform(X train)
              X_test = scaler.transform(X_test)
              y test = 2 * np.array([int(y) for y in y test]) - 1
              y_train = 2 * np.array([int(y) for y in y_train]) - 1
              return X train, X test, y train, y test
```

```
In [31]: X_train, X_test, y_train, y_test = pre_process_mnist_01()
```

```
In [32]:
          #Initalize helper variables
          loss_dict = {'train':[],'test':[]}
          estimators = [2,5,10,100,200,500,1000]
          #Iterate over the estimators
          for n in estimators:
              #Helper Print Statement to let me know it isn't dead
              print(f'Now fiitting GBC - {n} Estimators used')
              #Initliaze GBC Estimator
              gbc = GradientBoostingClassifier(n estimators=n, loss='deviance', max dep
              #Fit the GBC estimator
              gbc.fit(X train, y train)
              #Append results
              loss_dict['train'].append(gbc.score(X_train, y_train)) #Append Train Loss
              loss_dict['test'].append(gbc.score(X_test,y_test)) #Append Test Loss
          #Plot our results
          plt.plot(estimators, loss_dict['train'], label = 'train accuracy') #Plot Trai
          plt.plot(estimators, loss_dict['test'], label = 'test accuracy') #Plot Test
          plt.legend()
          plt.plot()
```

fitting gbc with 2 estimators fitting gbc with 5 estimators fitting gbc with 10 estimators fitting gbc with 100 estimators fitting gbc with 200 estimators fitting gbc with 500 estimators fitting gbc with 1000 estimators []

Out[32]:



Problem 11

Random Forest Classifier

```
In [36]:
          #Initalize helper variables
          loss dict = {'train':[],'test':[]}
          estimators = [2,5,10,100,200,500,1000]
          #Iterate over our estimators
          for n in estimators:
              #Initalize and fit a Random Forest Classifier from sklearn
              gbrf = RandomForestClassifier(n estimators=n, criterion = 'entropy', max
              gbrf.fit(X_train, y_train)
              #Append our train / test loss
              loss_dict['train'].append(gbrf.score(X_train, y_train))
              loss dict['test'].append(gbrf.score(X test,y test))
          #Plot our results
          plt.plot(estimators, loss_dict['train'], label = 'train accuracy') #Plot Trai
          plt.plot(estimators, loss_dict['test'], label = 'test accuracy') #Plot Test
          plt.legend()
          plt.plot()
```

Out[36]: []

