

Exercise 3 - Singular Value Decomposition

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1 Problem 1

1.1 1

The size is $AA^T \in \mathbb{N}^{6 \times 6}$ and this matrix suggest us the similarities between movies, in fact we are multiplying by rows.

1.2 2

The size is $A^T A \in \mathbb{N}^{5 \times 5}$ and this matrix suggest us the similarities between users and their preferences. In fact we are multiplying by columns.

1.3 3

$$A = UDV^T$$
$$A \in \mathbb{N}^{5 \times 6}, U \in \mathbb{N}^{6 \times 6}, D \in \mathbb{N}^{6 \times 5}, V^T \in \mathbb{N}^{5 \times 6}$$

1.4 4

By adding some noise as values (5.5) we want to find the underlying structure in order to predict the mssing values. In addition we find the correlation between users and concepts (i.e.: genre of the movie), and between movies and concepts.

1.5 5

Columns 1 – 3 are the more reasonable to keep, since their values is higher also the affecting factor is higher.

1.6 6

Matrix U maps movies-to-concepts.

1.7 7

Matrix V maps users-to-concepts.

1.8 8+9

Solution on the sheet.

1.9 10

29.7 is the strength of comedies.
10.00 is the strength of action movies.
7.09 is the strength of sci-fi movies.

1.10 11

$$A_3 = U_3 D_3 V_3^T$$
$$U[:, 1 : 3] D[1 : 3, 1 : 3] V[:, 1 : 3]^T$$

1.11 12

Error under euclidian norm:

$$\|A - A_3\|_2 = 2.7599$$

Error under Frobenius norm:

$$\|A - A_3\|_2^F = 2.8415$$

1.12 13

1.12.1 a.

$$\begin{aligned} A &= UDV^T \\ V^T &= D^{-1}U^T A \\ V^T [-0.460.35]^T \end{aligned}$$

1.12.2 b.

The genres that Bob may like are action movies and sci-fi movies.

1.13 14

Bob's rating does not affect our prediction system.