Performance Analysis of 2D Airfoils and Finite Wings

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1. Hess-Smith method

In Figure 1 it is shown the comparison between the results obtained with XFoil (1a) and Hess-Smith method implemented on MATLAB (1b), on the NACA0012 airfoil at $\alpha = 2^o$. The slight differences between the values of C_l and C_m obtained with the two methods may be related to a different choice of panels to describe the profile. However, the overall result can be considered coherent.

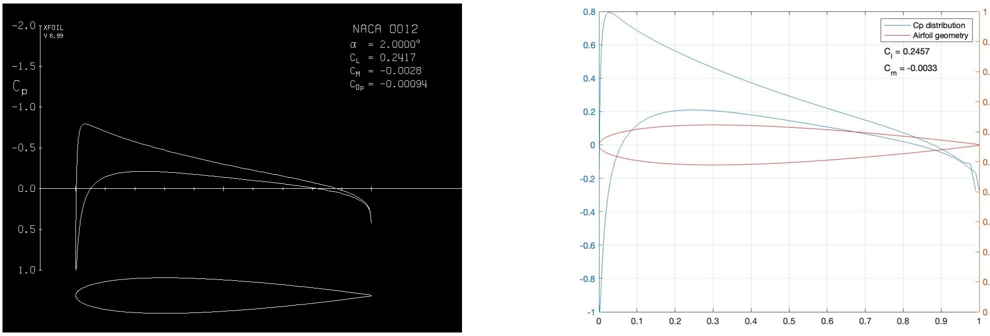
2. Estimation of Theodorsen's angle

In order to evaluate the design angle of attack for the Grumman K-2 and NACA0012 airfoils, four different definitions were used:

- α_{Th} as the angle at which there is no singularity on the leading edge, hence the angle that minimizes the said velocity. For this definition, we employed the previously implemented Hess-Smith method.
- α_{Th} as the angle that reduces the $-C_p$ peak.
- α_{Th} as the angle *at which the flow enters the leading edge smoothly or, more precisely, as the angle of attack at which the lift at the leading edge equals zero*. Clearly, it is not possible to have a local distribution of C_l, however, the lift is generated from a difference between pressure, hence C_p, on the suction and pressure sides of the airfoil. The AoA at which the difference in C_p straddling the leading edge is minimum (null), is the ideal angle at which no lift is generated at the leading edge.
- α_{Th} calculated through numerical integration, using its definition:

$$\alpha_{Th} = \frac{1}{\pi} \int_{-L/2}^{L/2} \frac{1}{\sqrt{\frac{L^2}{4} - s^2}} y'(s)\, \mathrm{d}s \tag{2.1}$$

For NACA0012, all four methods were effective, with consistent outputs. For Grumman K-2, the first three methods give almost identical results when accounting for panel distribution, while the best approximation obtained with the numerical integration still differs by about 3-5%. The main issue with the fourth method is the estimation of the mean camber line and its derivative, that tends to oscillate and diverge. In addition, the function is very sensitive to the ε variable that defines the limits of integration. It is to be noted that the Hess-Smith method was used with a lower number of panels in order to assess the α_{Th} sensitivity to the airfoil's panel discretization of the airfoil and the comparison with the other methods was consistent albeit returning a higher α_{Th} value.



(a) XFoil (b) MATLAB

Figure 1: Hess-Smith implementation

In conclusion, $\alpha_{Th_{NACA0012}} = 0^o$ and $\alpha_{Th_{K-2}} = -1.2190^o$ (for the maximum amount of panels that XFoil allows, that is 364).

3. Separation and Transition

For the analysis of the performance of the two airfoils in terms of separation and transition, we decided to compare their performance by studying the profiles at 2^o AoA, and at their respective design angles. The study was conducted by varying the Reynolds number from 100,000 to 1,000,000 and the value of N_{cr} from 5 to 13, to determine the sensitivity of the profile to incoming turbulence.

3.1. *Low Reynolds Numbers*

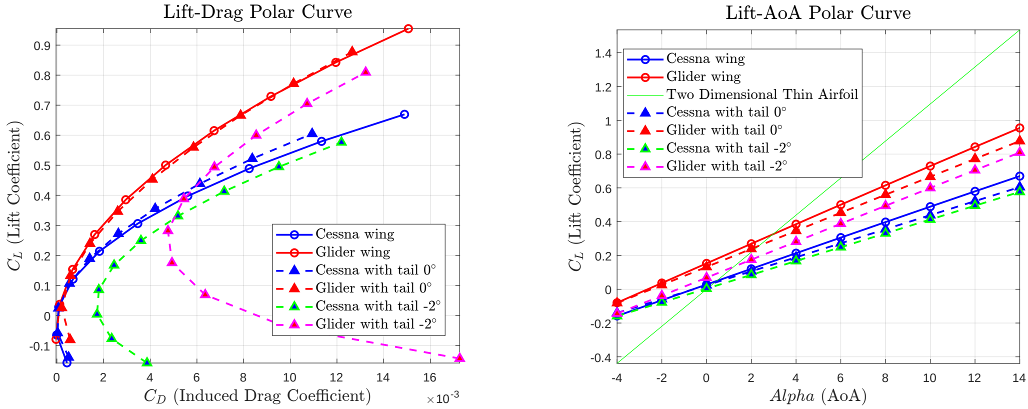
For low Reynolds numbers, the Grumman K-2 airfoil shows little sensitivity to incoming turbulence and to variations in the angle of attack. In contrast, the NACA0012 airfoil is much more sensitive to variations in these two parameters, while still maintaining decent performance in terms of friction coefficient.

3.2. *High Reynolds Numbers*

For high Reynolds numbers, both profiles perform best when operating at their design AoA. In particular, the flat back of the Grumman K-2 airfoil is typical of transonic profiles; therefore, the transition occurs on the pressure side and at around 70% of the chord, as expected for more modern transonic profiles. Moreover, the Grumman K-2 airfoil is designed for long periods of cruise flight, making it essential for it to exhibit excellent performance around the design angle. Placing the profiles at 2^o significantly worsens the performance of both, making them highly sensitive to incoming turbulence. At this AoA, Grumman K-2 and NACA0012 airfoils present similar maximum C_f values, but the Grumman K-2 transitions much earlier. The results indicate that transonic profiles like Grumman K-2 are designed to operate in low AoA conditions. However, in this analysis, the Mach number, an extremely relevant parameter for dealing with high speeds and profiles of this type, has been entirely neglected.

4. Analysis and comparison of Cessna 172 Skyhawk and Glider K8

Three-dimensional analysis has been performed computing an efficient Weissinger algorithm written from scratch on MATLAB that can handle geometric twist, taper,



(a) Polar (b) $C_{l/\alpha}$

Figure 2: Aerodynamic diagrams

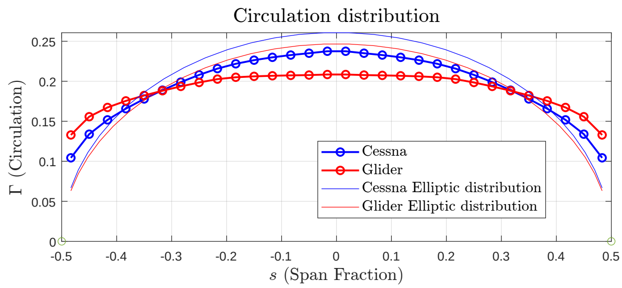


Figure 3: Γ distribution

airfoil mean line shape, sweep and dihedral angles. Glider K8 was the aircraft selected to be compared with Cessna 172. The main criteria in choosing the airplane were the necessity for its wing to comply with the limits imposed by the method, as well as aiming for notable differences between aerodynamic performances of the two wings.

4.1. *Examination of the wings characteristics*

Results, represented in Figures 2, 3, clearly show differences between Cessna and Glider wings as the second one presents a very high aspect ratio, hence circulation distribution fits better the elliptical one. The high aerodynamic performance-oriented wing shape of the glider benefits lift-to-induced drag ratio, and lift over α. The lift over angle of attack gain over the Cessna wing increases for high AoA, reflecting its operating point.

4.2. *Tail planes effect*

We assume preliminarily that the tail planes are downforce-generating. Introducing these tail aerodynamic effects, data show a significant loss in $C_{l/\alpha}$ and lift-to-drag ratio, which obviously decreases when the tail is tilted by -2^o. Despite the aerodynamic surface tails being small, they are also responsible for induced drag growth because of their significantly low aspect ratio.