



**POLITECNICO**  
MILANO 1863

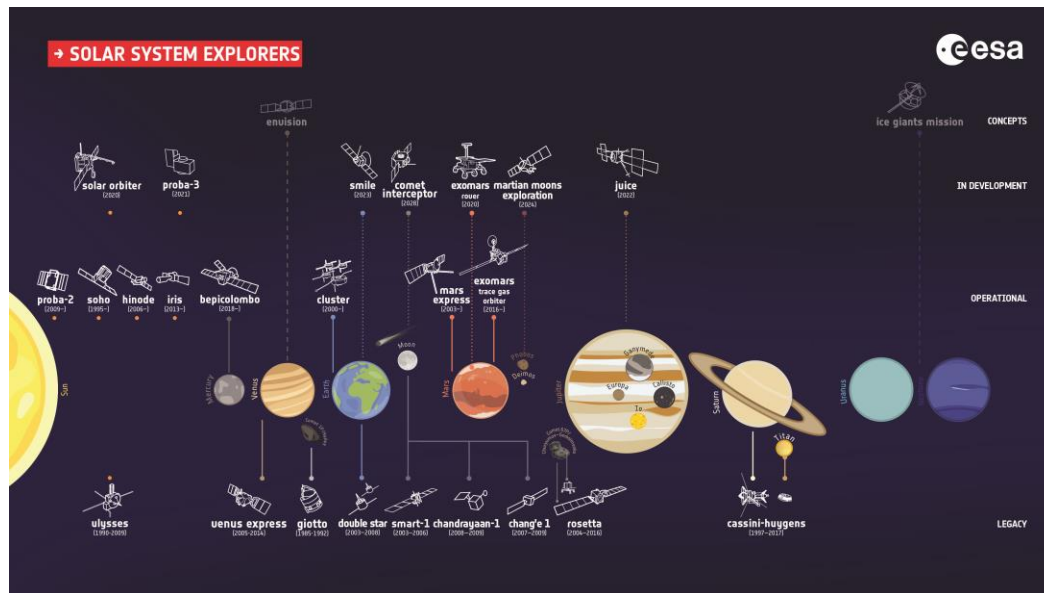
## **Orbital Mechanics**

# **ASSIGNMENT PRESENTATION**

**Professor: Camilla Colombo**  
**Juan Luis Gonzalo Gomez**

**Group 5:**  
**Lyle Campbell**  
**Giulio Pacifici**  
**Luca Rizzieri**  
**Davide Sisana**

# INTERPLANETARY EXPLORER MISSION



*Solar System Explorers  
Image from ESA*

## MISSION:

- Departure from Neptune
- Flyby at Mars
- Arrive at Earth

## FIGURE OF MERIT:

- $\Delta v_{tot}$

# DESIGN PROCESS

## CONSTRAINTS:

- Earliest Departure  
 $1^{st} \text{ Jan } 2020$
- Latest Arrival  
 $1^{st} \text{ Jan } 2060$
- Flyby Minimum Altitude  
 $h_{min} = 250 \text{ km}$



*Rosetta's flyby at Mars  
Image from ESA*

# DESIGN PROCESS

## **ASSUMPTIONS:**

- Patched Conics Method
- Other planets ignored
- *SRP* neglected
- Neptune departure and Earth arrival neglected

# PRELIMINARY ESTIMATIONS

Planet	Orbital period in Earth years		Planets	Synodic Period in Earth years
Neptune	164,8		Neptune & Mars	1,90
Mars	1,88		Mars & Earth	2,13
Earth	1		Neptune & Earth	1,01

#	Neptune - Mars	Mars - Earth
Parabolic Transfer Time	13.36 <i>years</i>	24.14 <i>days</i>
Hohmann Transfer Time	31.47 <i>years</i>	258.87 <i>days</i>

Neptune-Earth Hohmann Transfer	$\Delta v = 15.71 \frac{km}{s}$
--------------------------------	---------------------------------

# FIRST STRATEGY

## GRID SEARCH

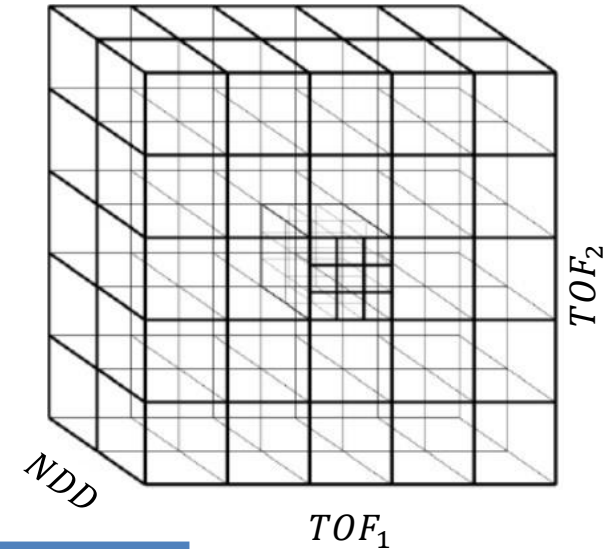
3 degrees of freedom:

- Departure time  $NDD$
- First time of flight  $TOF_1$
- Second time of flight  $TOF_2$

3 nested loop cycles evaluate the  $\Delta v_{tot}$

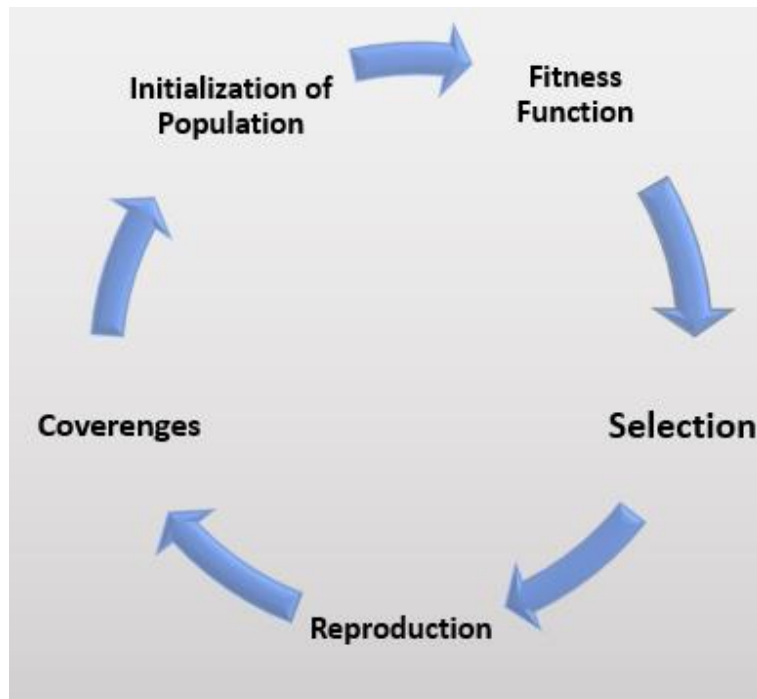
## Array Boundaries

$dof$	Lower Boundary	Upper Boundary	Time Step [days]
$NDD$	$Earliest\ Dep.$	$ED + T_{synME}$	165
$TOF_1$	$TOF_{par}^{N \rightarrow M}$	$120\% TOF_H^{N \rightarrow M}$	1
$TOF_2$	$TOF_{par}^{M \rightarrow E}$	$120\% TOF_H^{M \rightarrow E}$	1



Cube grid  
Image from Lab Notes  
Orbital Mechanics

# SECOND STRATEGY



*Genetic Algorithm scheme  
Image from educba.com*

## GENETIC ALGORITHM

Heuristic approach applied to a wider domain:

- Number of runs  $N_{runs} = 5$
- Populations  $N_{pop} = 2000$
- Generations  $N_{gen} = 300$
- Fitness function

# SECOND STRATEGY

## Boundaries

<i>dof</i>	Lower Boundary	Upper Boundary
<i>NDD</i>	<i>ED</i>	$ED + T_{synME}$
<i>TOF</i> <sub>1</sub>	$TOF_{par}^{NM}$	$120\% TOF_H^{NM}$
<i>TOF</i> <sub>2</sub>	$TOF_{par}^{ME}$	12 years

## Constraints:

- $NDD + TOF_1 + TOF_2 < LA;$
- $r_p(NDD, TOF_1, TOF_2) > R_{Mars} + h_{min}$

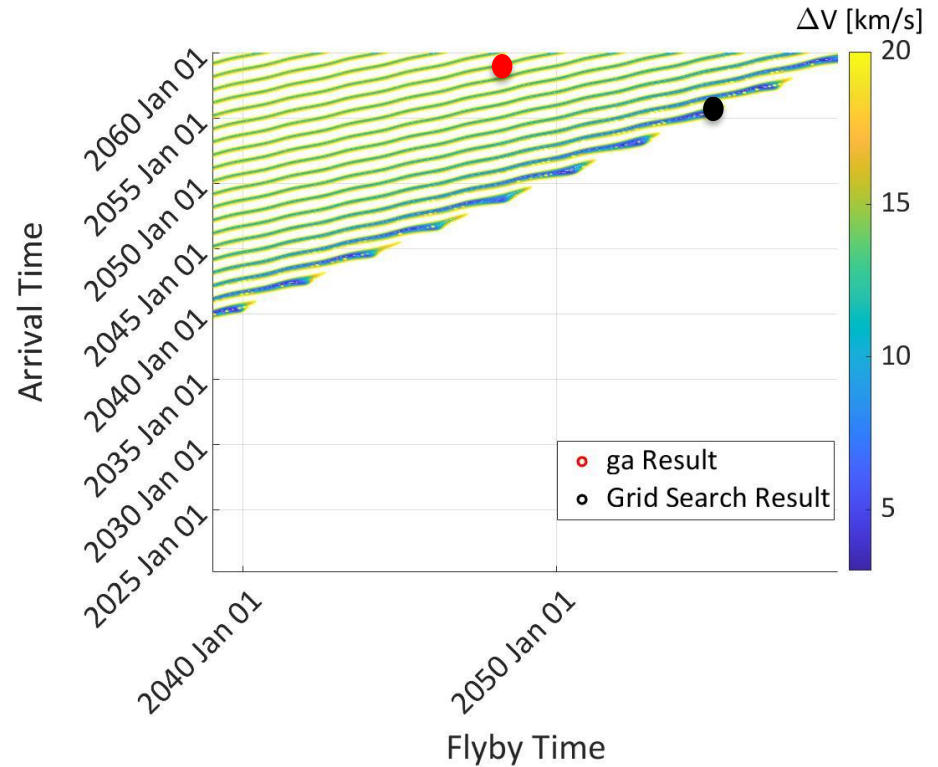
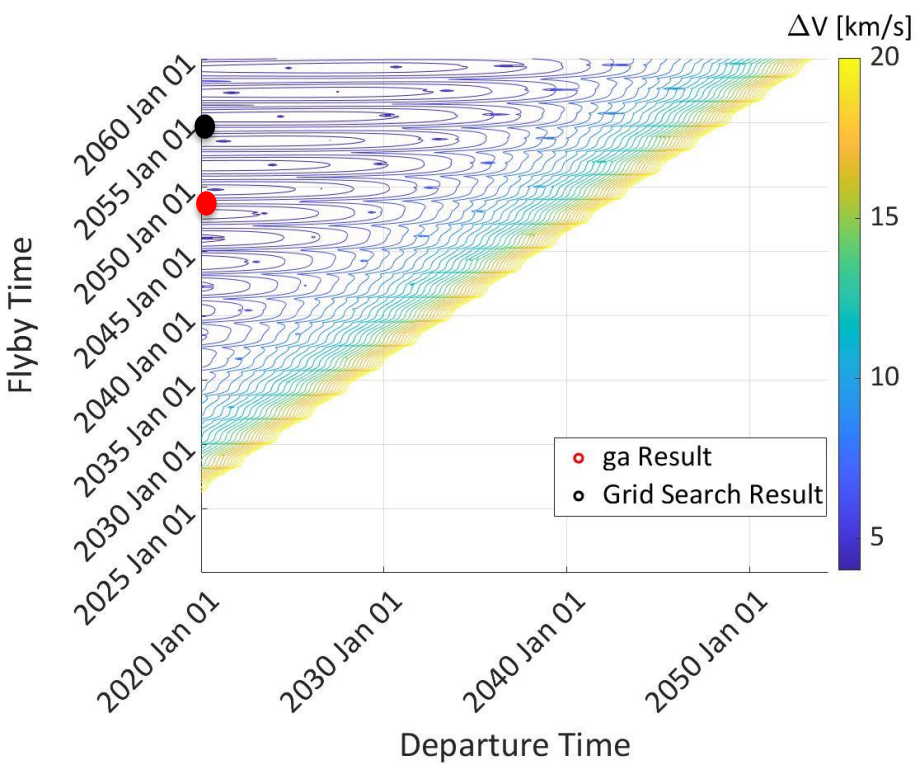


# RESULTS

#	$NDD[MJD]$	$TOF_1[days]$	$TOF_2[days]$	Normalised comp. time
Grid Search	7304,50	12777	85,1	0,66
G.A.	7401,07	10257	3952	1,00

#	$\Delta v_1 [km/s]$	$\Delta v_2 [km/s]$	$\Delta v_3 [km/s]$	$\Delta v_{tot} [km/s]$
Grid Search	4,199	0,071	12,569	16,840
G.A.	4,098	1,705	10,092	15,895

# PORKCHOP PLOTS



# FINAL TRAJECTORY CHARACTERIZATION

Departure	Flyby	Arrival
2020 April 06 at 13: 40: 10,30	2048 May 07 at 13: 14: 25,53	2059 March 03 at 18: 37: 05,26

## Transfer Arcs

#	$a$ [km]	$e$ [—]	$i$ [deg]	$\Omega$ [deg]	$\omega$ [deg]	$\theta_i$ [deg]	$\theta_f$ [deg]
Neptune - Mars	$2,33 * 10^9$	0,935	1,236	45,18	121,23	181,58	72,93
Mars - Earth	$7,42 * 10^8$	0,800	0,311	343,19	178,16	77,99	1,84

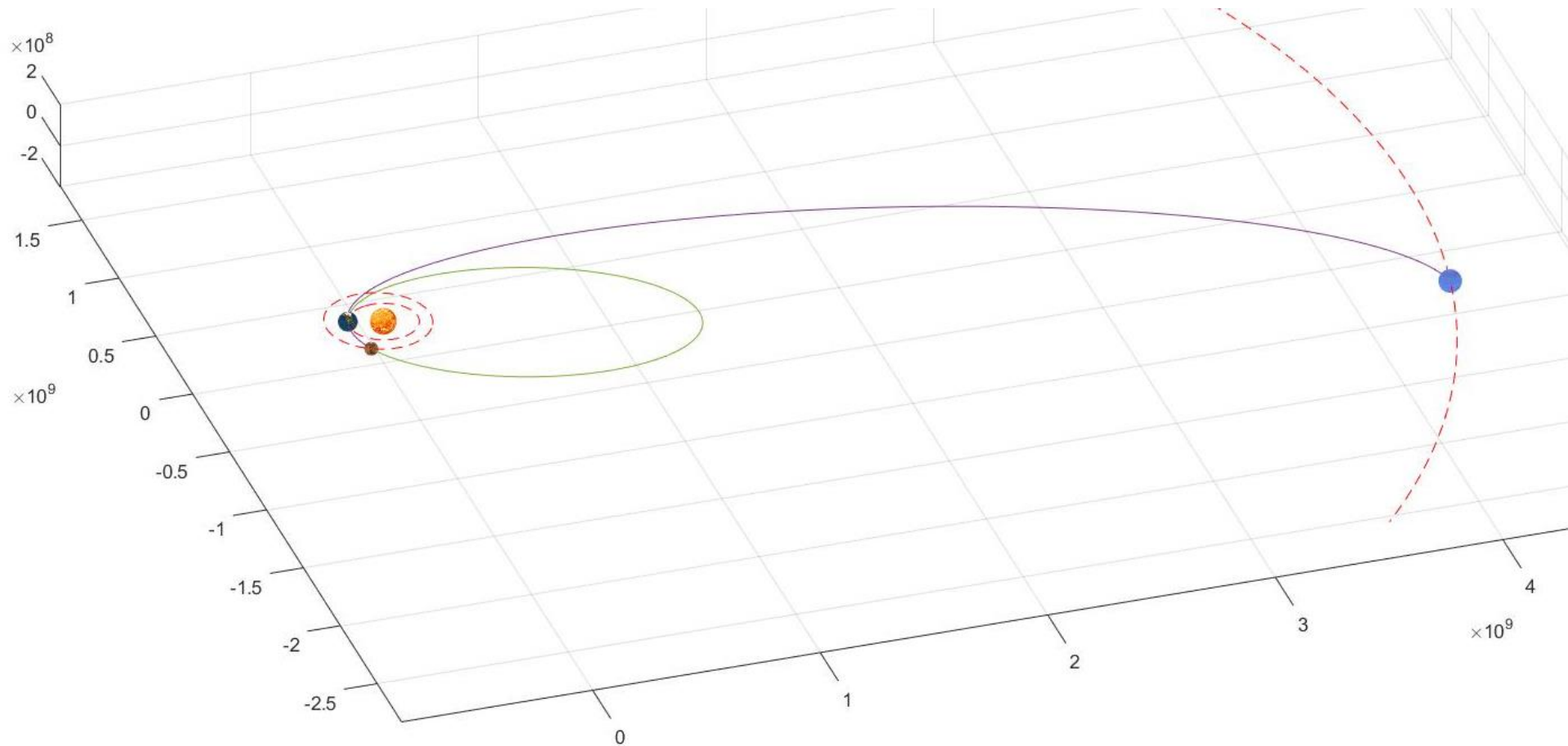
## Flyby Hyperbolas

#	$h_p$ [km]	$v_p$ [km/s]	$v_\infty$ [km/s]	$e$ [—]	$TOF_f$ [h]
Incoming	250,006	22,109	21,571	40,545	7,455
Outgoing	250,006	20,404	19,819	34,384	8,113

$$V^- = [13,6593 \quad -30,2687 \quad -0,6695]'$$

$$V^+ = [13,4802 \quad -28,2751 \quad -0,1260]'$$

# FINAL TRAJECTORY



# DISCUSSION

- Mars not ideal for gravity assist
- Mission time/cost trade-off
- Improvement Possibilities
  - Alternate flybys
  - Earth aero-capture

Spacecraft	Flyby Planet	Planet Earth Masses	Flyby Altitude [ <i>km</i> ]	$\Delta v_f$ [ <i>km/s</i> ]
Voyager	Jupiter	318	570000	16
Cassini Huygens	Venus	0,82	250	7
PoliMi	Mars	0,11	250	2*

# PLANETARY EXPLORER MISSION

## Initial Keplerian Elements:

$a$ [km]	$e$ [–]	$i$ [deg]	$\Omega$ [deg]	$\omega$ [deg]	$\theta$ [deg]
18302	0,6158	71,5679	20	70	0

**Orbital Period:** 6.8447 *hours*

### Spacecraft Data:

- $C_d = 2.2$
- $\frac{A}{m} = 0.06$

### Groundtrack repetition:

- $m = 2$
- $k = 7$

### Perturbations:

- $J_2$
- Drag

# DRAG

## Atmospheric Density

Wertz exponential model:

$$\rho(h,t) = \rho_0 \exp\left[-\frac{h-h_0}{H}\right]$$

Maximum altitude considered: 1700 km

- Machine epsilon cutoff



*Upper atmosphere  
Image from NASA*

# INTEGRATION METHODS

## Integration in Cartesian Coordinates

$$\frac{d}{dt} \begin{Bmatrix} \vec{r} \\ \vec{v} \end{Bmatrix} = \begin{Bmatrix} \vec{v} \\ -\frac{\mu}{r^3} \vec{r} + \vec{a}_{pert} \end{Bmatrix}$$

## Integration of Gauss Planetary Equations

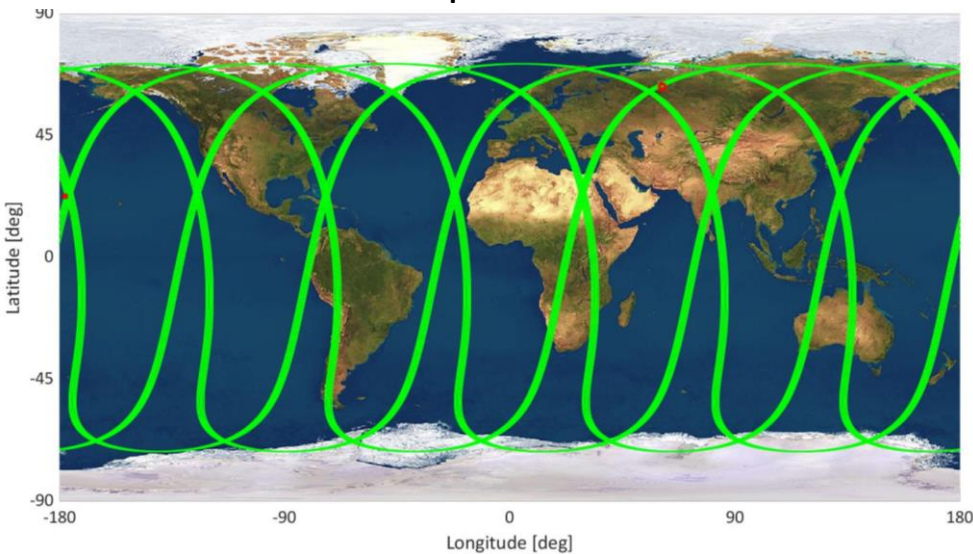
$$\frac{d}{dt} \vec{K} = f \left( \vec{K}, \vec{a}_{pert}^{(tnh)} \right)$$



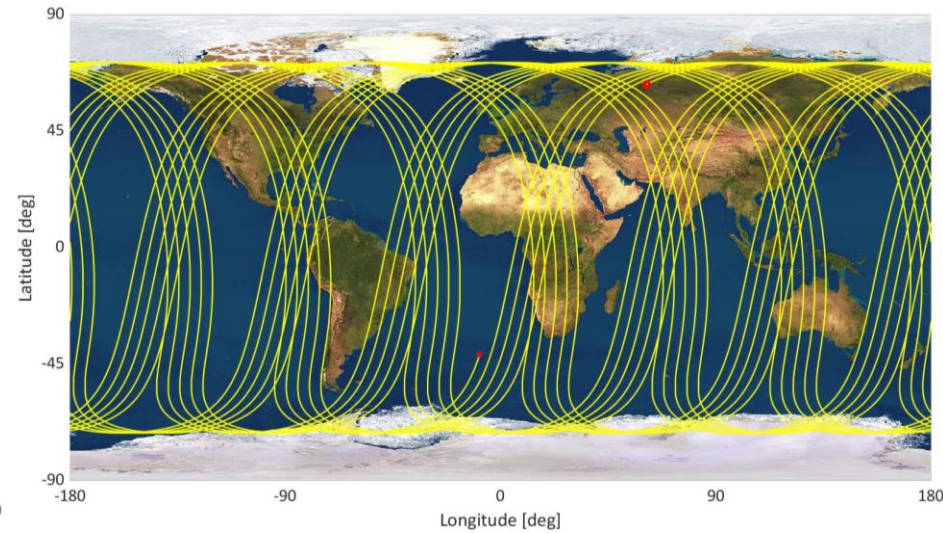
# GROUND TRACK

After 10 Days

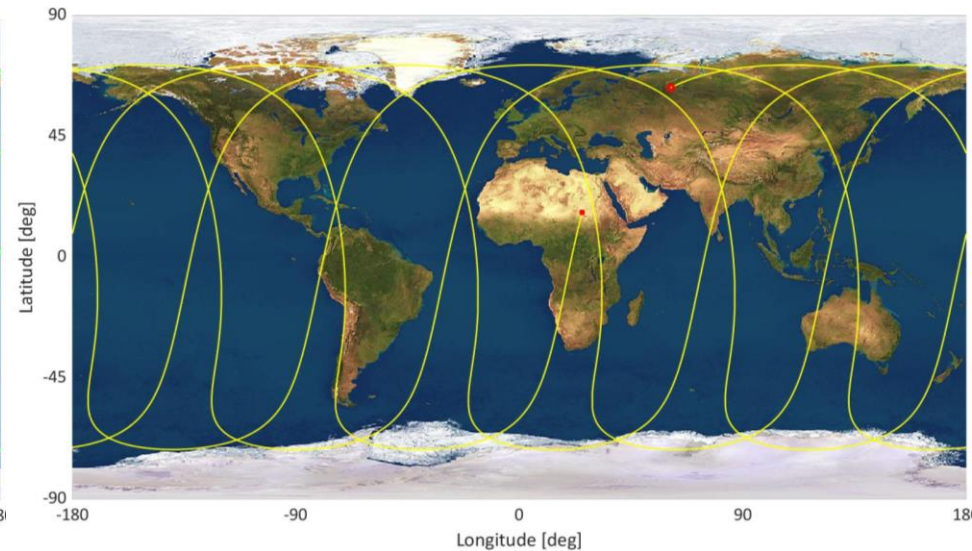
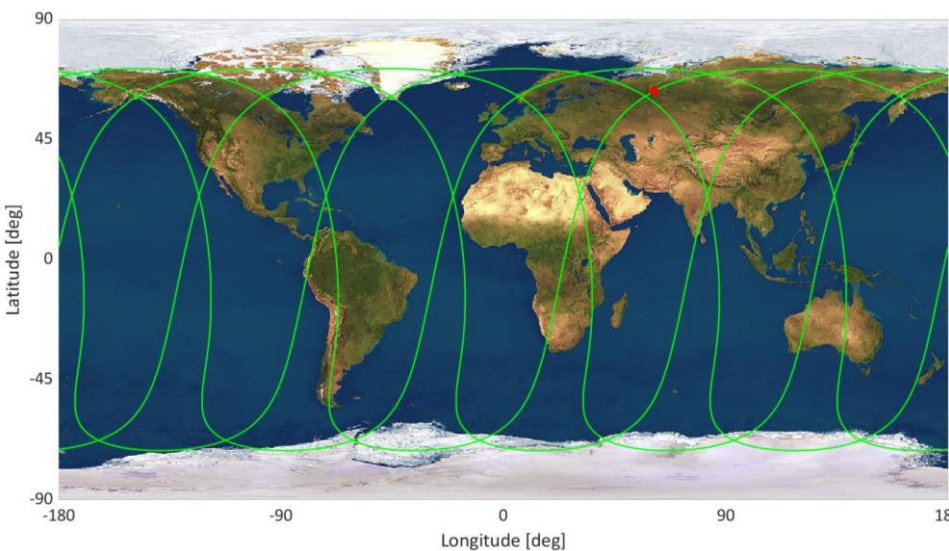
Unperturbed



With J2 effects



# REPEATING GROUND TRACK



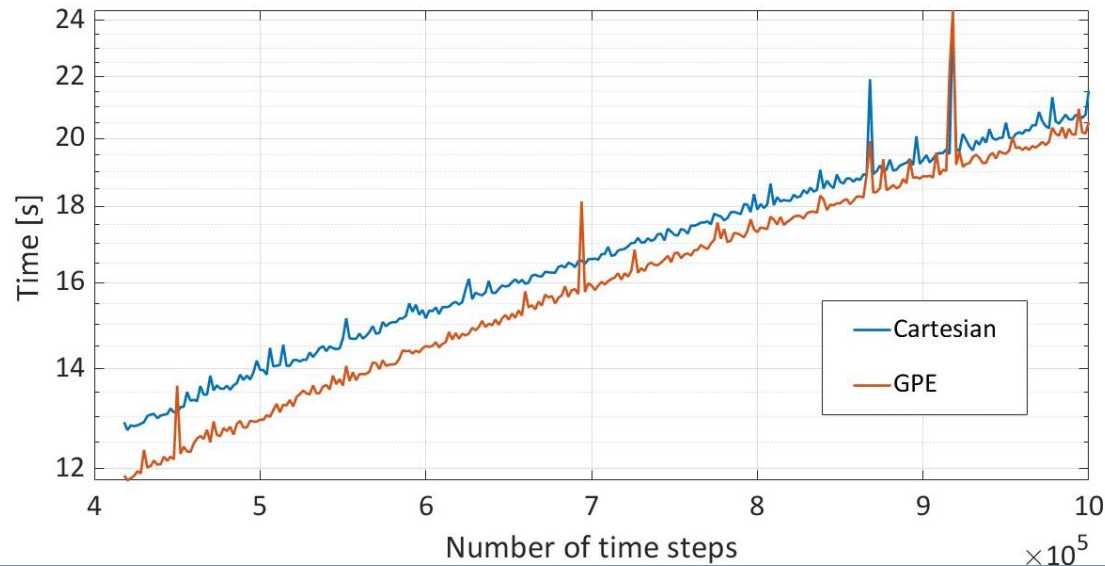
#	Original Orbit	Unperturbed orbit	$J_2$ perturbed orbit
$a$ [km]	18302	18284	18290

# METHOD COMPARISON

After 100 orbit, 500 sec. time steps

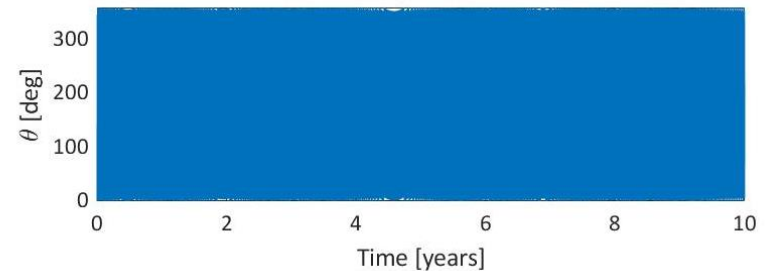
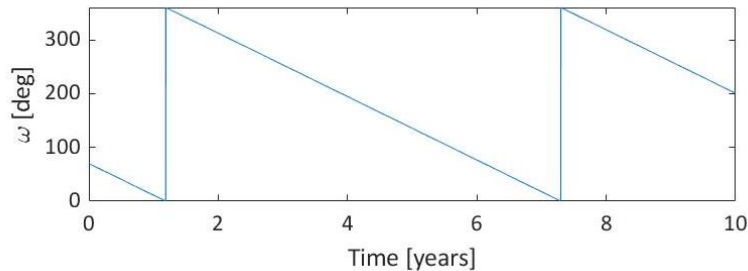
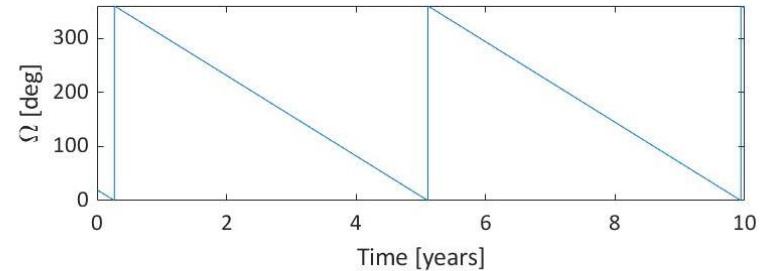
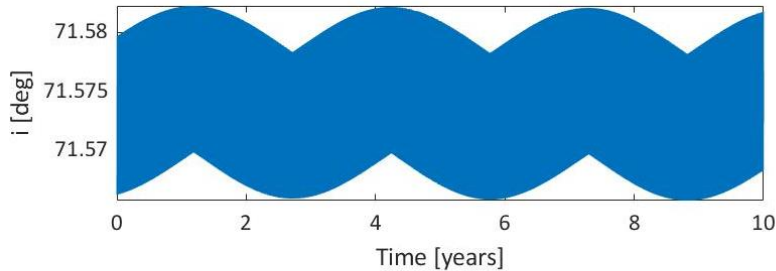
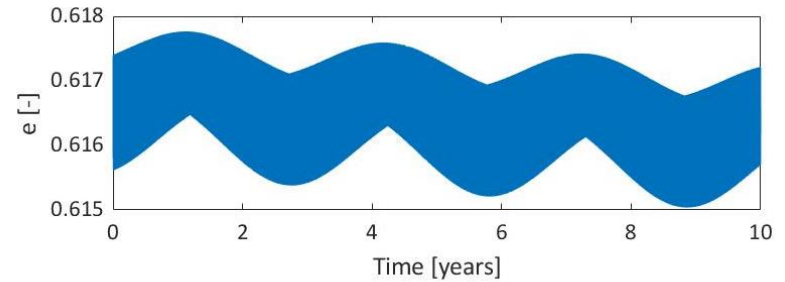
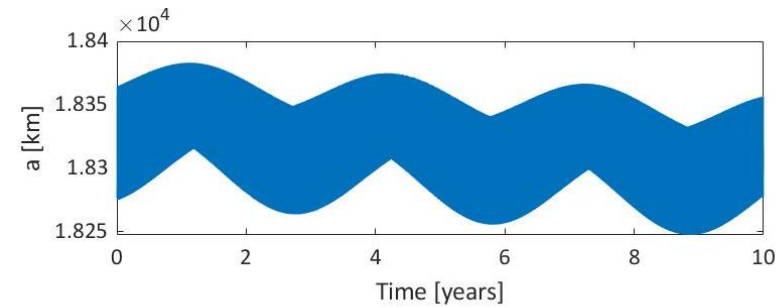
$a$ [km]	$e$ [—]	$i$ [deg]	$\Omega$ [deg]	$\omega$ [deg]	$\theta$ [deg]
$2,0019 \cdot 10^{-4}$	$4,2074 \cdot 10^{-9}$	$2,2819 \cdot 10^{-8}$	$4,7728 \cdot 10^{-8}$	$3,0187 \cdot 10^{-7}$	$9,4882 \cdot 10^{-5}$

Absolute error between Cartesian and GPE integrations



# TIME EVOLUTION

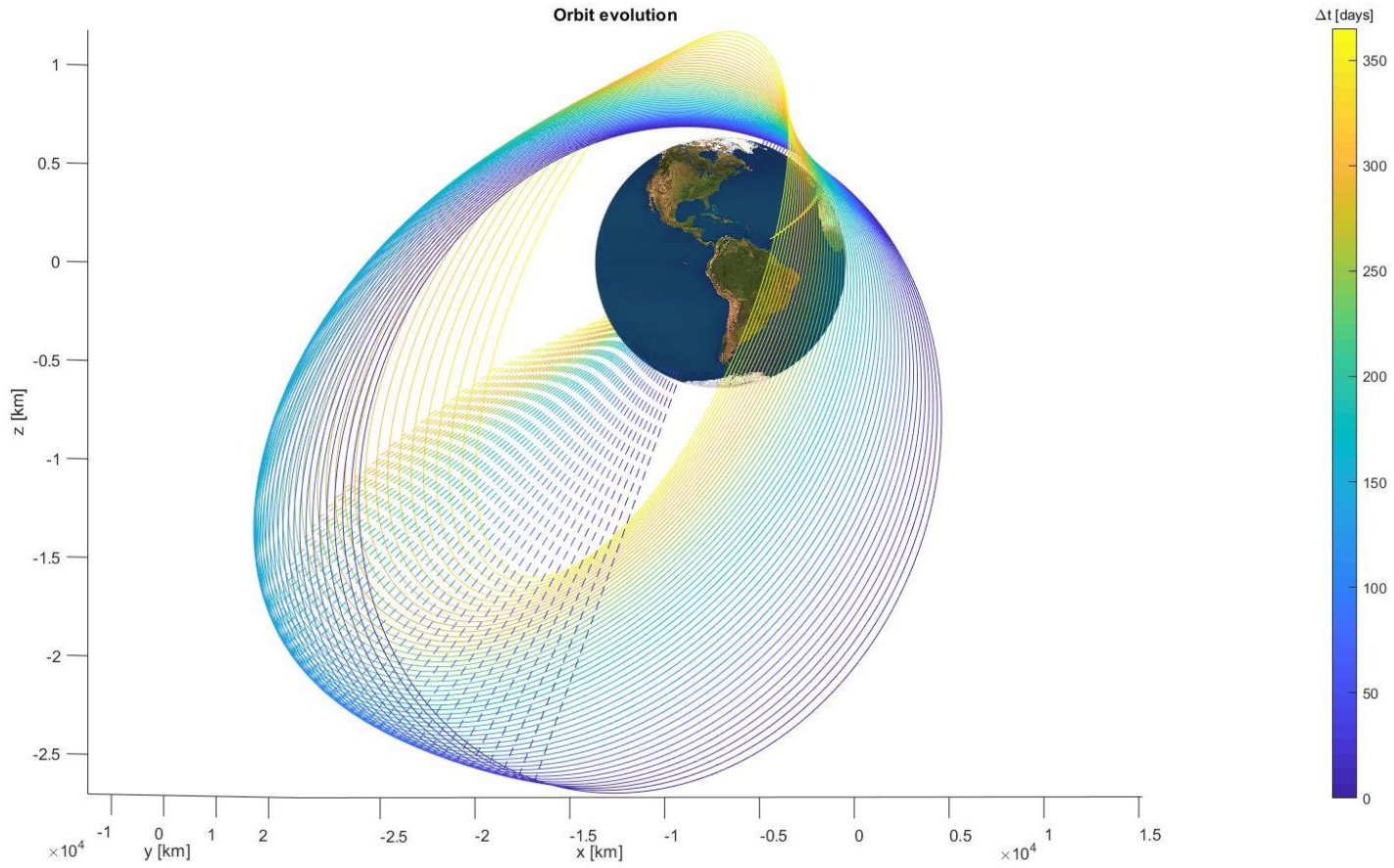
10 year propagation, 500 sec. time steps



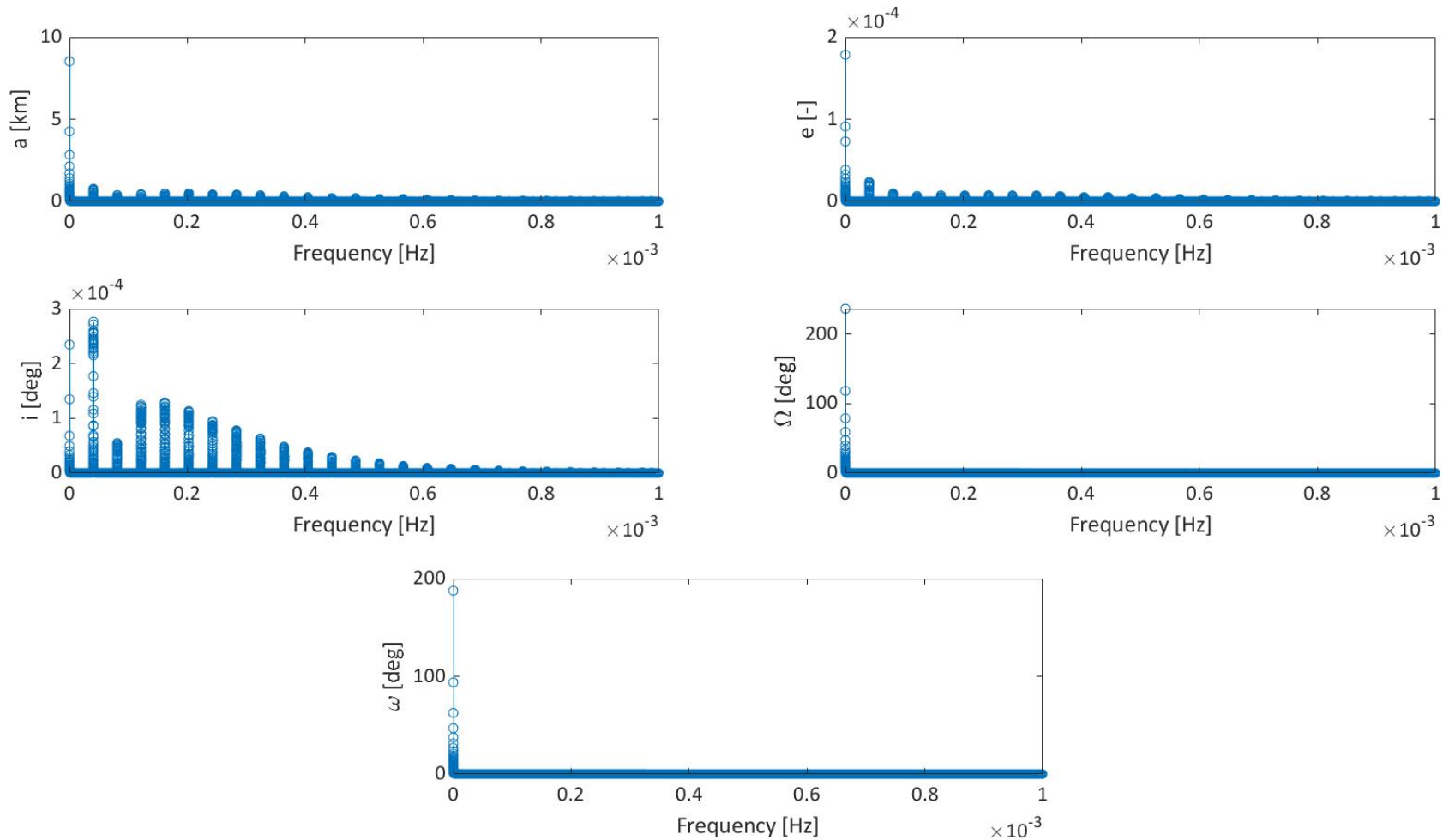


# TIME EVOLUTION

1 year propagation, plotted weekly

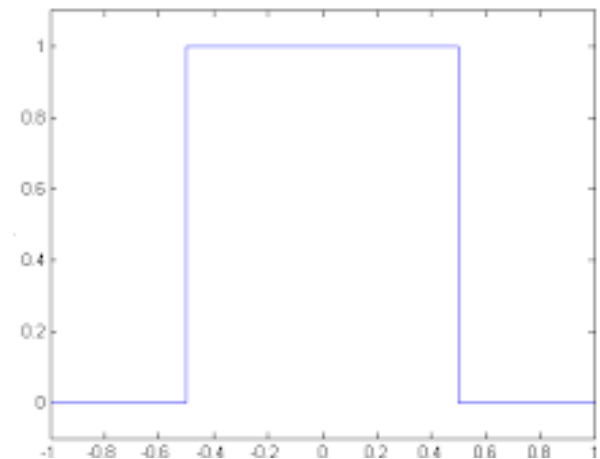
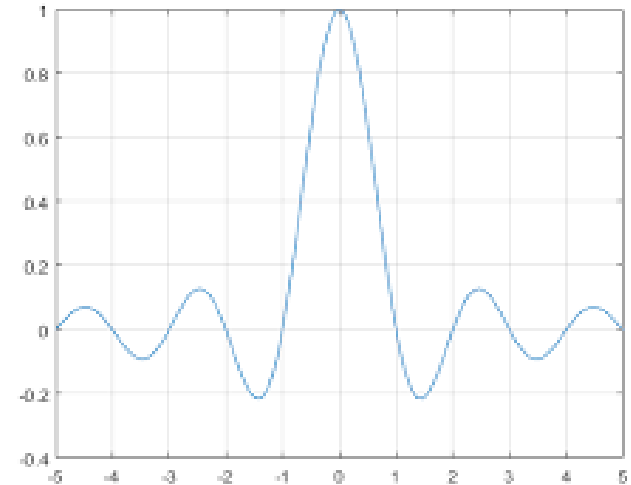


# FREQUENCY ANALYSIS

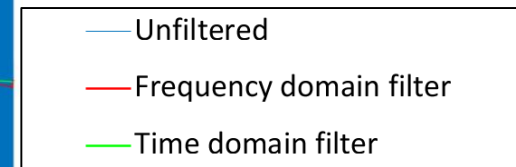
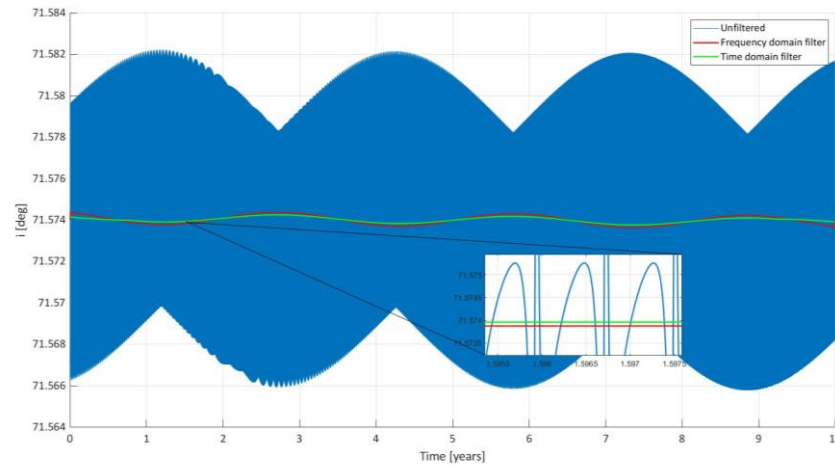
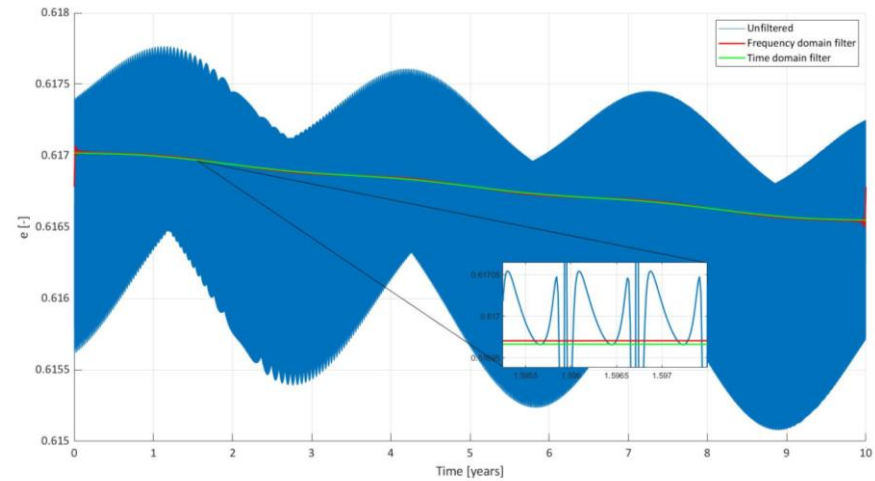
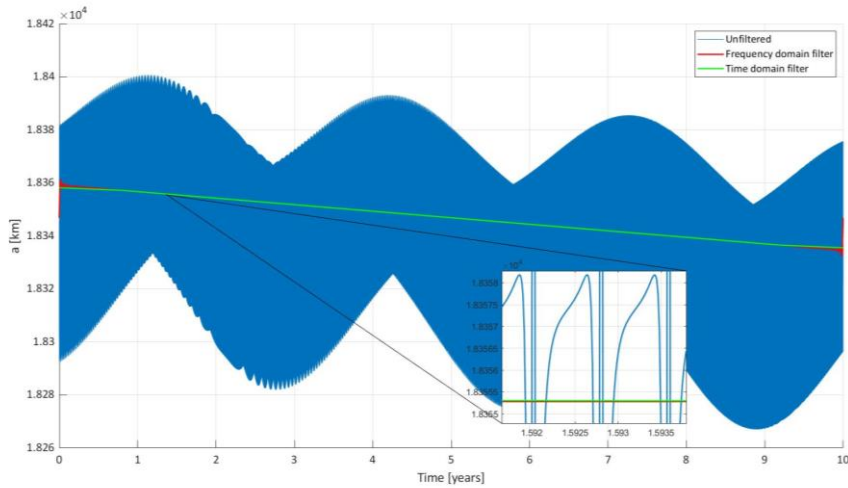


# FILTERING METHODS

- Time domain filter
  - Centred moving mean
- Frequency domain filter
  - Rectangular low-pass filter



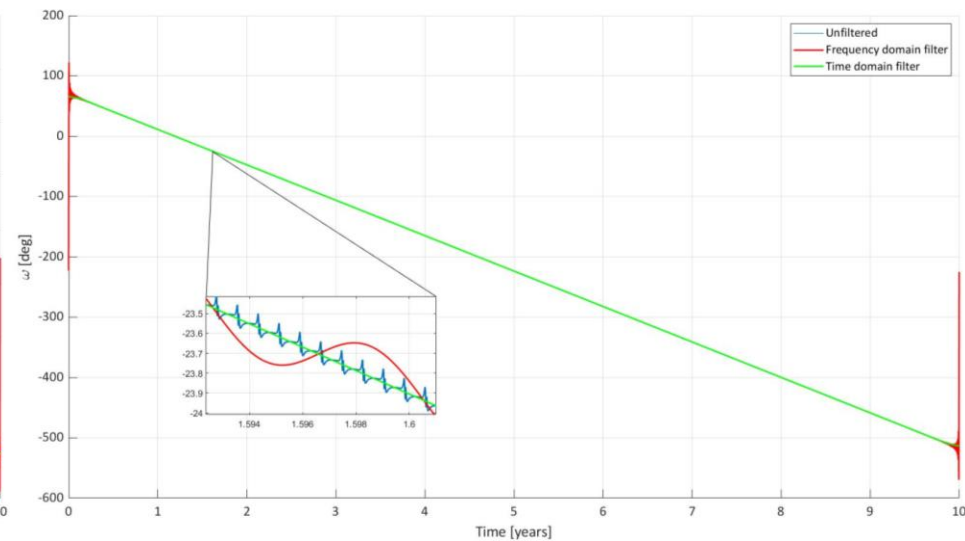
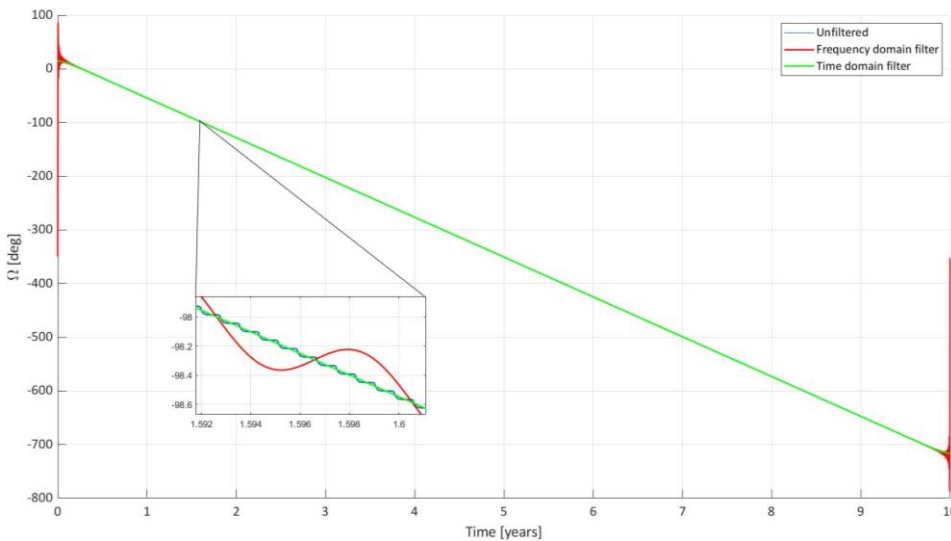
# FILTERING RESULTS





# FILTERING RESULTS

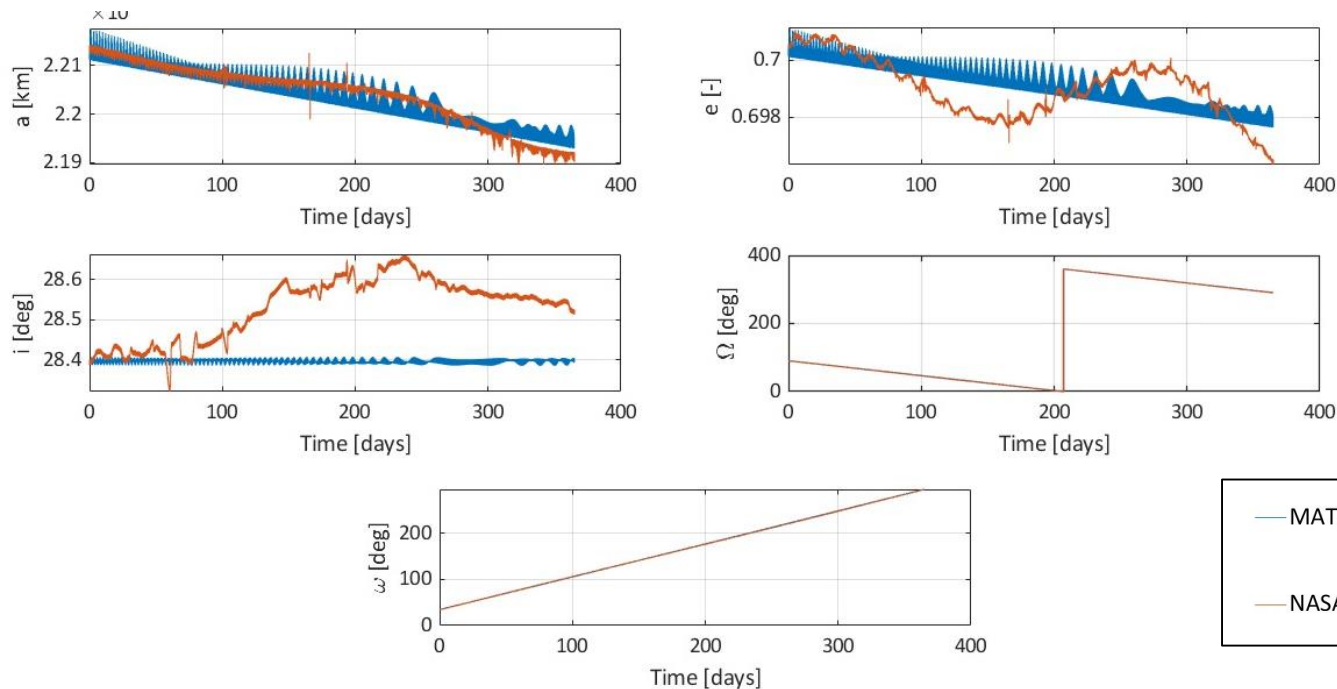
— Unfiltered  
— Frequency domain filter  
— Time domain filter



# REAL SATELLITE COMPARISON

## Satellite KIKU-3(ETS-IV)

$h_p$	$h_a$	$i$	$D$	Mass	$A/m$	$c_d$
225 km	36000 km	28,5°	2,1 m	638 kg	0,0054 m <sup>2</sup> /kg	2,2



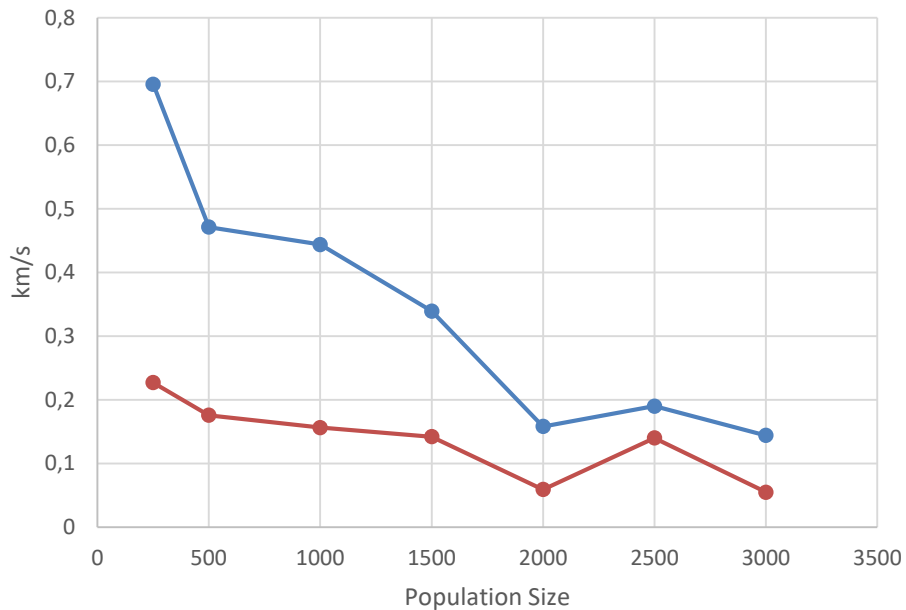
# QUESTIONS



# QUESTIONS

## GA Population Size Selection

—●— Maximum Difference    —●— Standard Deviation



## Solution Time

