

# Exercise 1

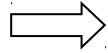
Specify the LR(1) parsing table relevant to the language defined by the following grammar:

$decl \rightarrow type\ var\text{-}list$   
 $type \rightarrow \mathbf{int} \mid \mathbf{float}$   
 $var\text{-}list \rightarrow \mathbf{id}, var\text{-}list \mid \mathbf{id}$

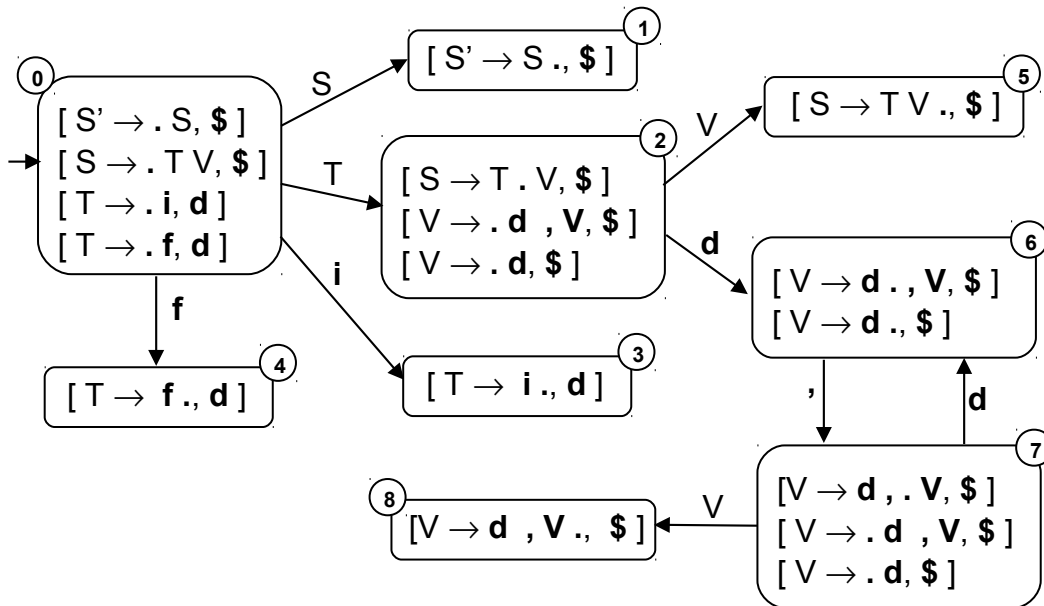
# Exercise 1

Specify the LR(1) parsing table relevant to the language defined by the following grammar:

$decl \rightarrow type \ var\text{-}list$   
 $type \rightarrow \mathbf{int} \mid \mathbf{float}$   
 $var\text{-}list \rightarrow \mathbf{id} \mid \mathbf{id} \ , \ var\text{-}list \mid \mathbf{id}$



$S' \rightarrow S$   
 $S \rightarrow T \ V$   
 $T \rightarrow \mathbf{i} \mid \mathbf{f}$   
 $V \rightarrow \mathbf{d} \ , \ V \mid \mathbf{d}$



State	Input					Goto		
	i	f	d	,	\$	S	T	V
0	s3	s4				1	2	
1					accept			
2			s6					5
3			$T \rightarrow \mathbf{i}$					
4			$T \rightarrow \mathbf{f}$					
5					$S \rightarrow T V$			
6				s7	$V \rightarrow \mathbf{d}$			
7			s6					8
8					$V \rightarrow \mathbf{d} \ , \ V$			

## Exercise 2

Check whether the following grammar is LR(1).

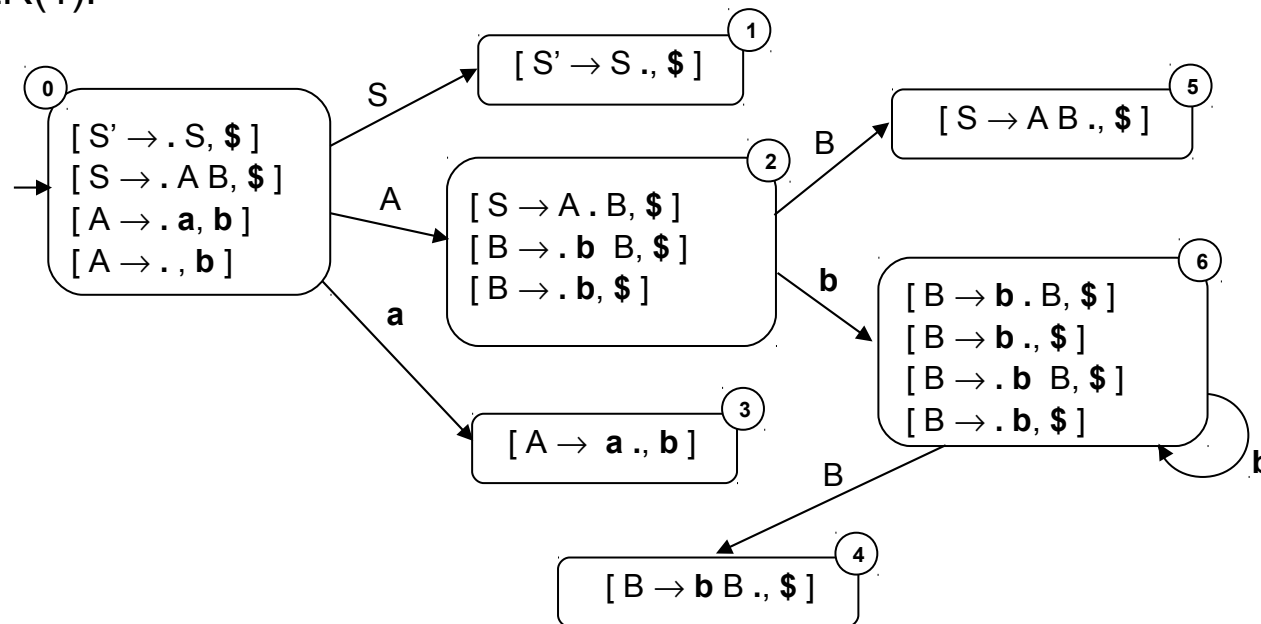
$$\begin{aligned} S &\rightarrow A B \\ A &\rightarrow \mathbf{a} \mid \varepsilon \\ B &\rightarrow \mathbf{b} B \mid \mathbf{b} \end{aligned}$$

## Exercise 2

Check whether the following grammar is LR(1).

$S \rightarrow A B$   
 $A \rightarrow \mathbf{a} \mid \epsilon$   
 $B \rightarrow \mathbf{b} B \mid \mathbf{b}$

The grammar is LR(1):



## Exercise 3

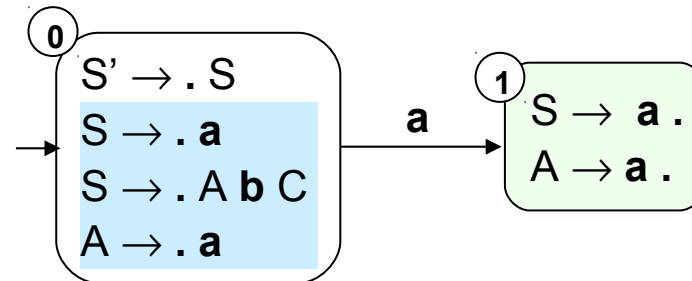
Check whether the following grammar is SLR(1):

$$S \rightarrow \mathbf{a} \mid A \mathbf{b} C$$
$$A \rightarrow \mathbf{a}$$
$$C \rightarrow A \mid \mathbf{c}$$

## Exercise 3

Check whether the following grammar is SLR(1):

$S \rightarrow \mathbf{a} \mid A \mathbf{b} C$   
 $A \rightarrow \mathbf{a}$   
 $C \rightarrow A \mid \mathbf{c}$



$FOLLOW(S') = \{ \$ \}$   
 $FOLLOW(S) = \{ \$ \}$   
 $FOLLOW(C) = \{ \$ \}$   
 $FOLLOW(A) = \{ \$, \mathbf{b} \}$

$$FOLLOW(S) \cap FOLLOW(A) = \{ \$ \} \neq \emptyset$$

↓  
**Not SLR(1)!**

# Exercise 4

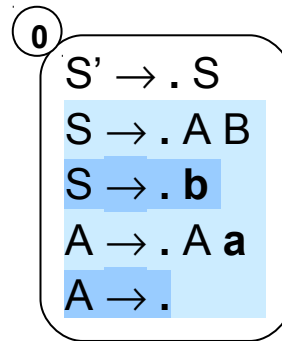
Check whether the following grammar is SLR(1):

$$\begin{aligned} S &\rightarrow A B \mid \mathbf{b} \\ A &\rightarrow A \mathbf{a} \mid \epsilon \\ B &\rightarrow B \mathbf{a} \mid \mathbf{b} \mid \epsilon \end{aligned}$$

## Exercise 4

Check whether the following grammar is SLR(1):

$S \rightarrow A B \mid \mathbf{b}$   
 $A \rightarrow A \mathbf{a} \mid \epsilon$   
 $B \rightarrow B \mathbf{a} \mid \mathbf{b} \mid \epsilon$



$FOLLOW(S') = \{ \$ \}$

$FOLLOW(S) = \{ \$ \}$

$FIRST(B) = \{ \mathbf{a}, \mathbf{b}, \epsilon \}$

$FOLLOW(B) = \{ \mathbf{a}, \$ \}$

$FOLLOW(A) = \{ \mathbf{a}, \mathbf{b}, \$ \}$



$\mathbf{b} \in FOLLOW(A)$



Not SLR(1)!



# Exercise 5

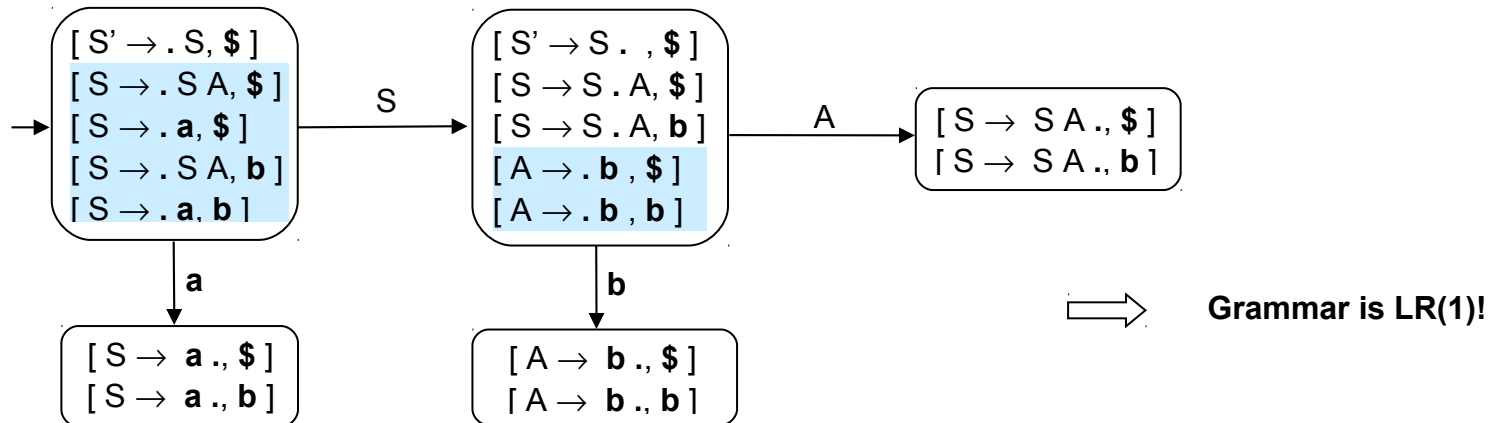
After generating the relevant parsing automaton, determine (with motivation) whether the following grammar is LR(1):

$$\begin{aligned} S &\rightarrow SA \mid \mathbf{a} \\ A &\rightarrow \mathbf{b} \end{aligned}$$

## Exercise 5

After generating the relevant parsing automaton, determine (with motivation) whether the following grammar is LR(1):

$S \rightarrow SA \mid a$   
 $A \rightarrow b$



# Exercise 6

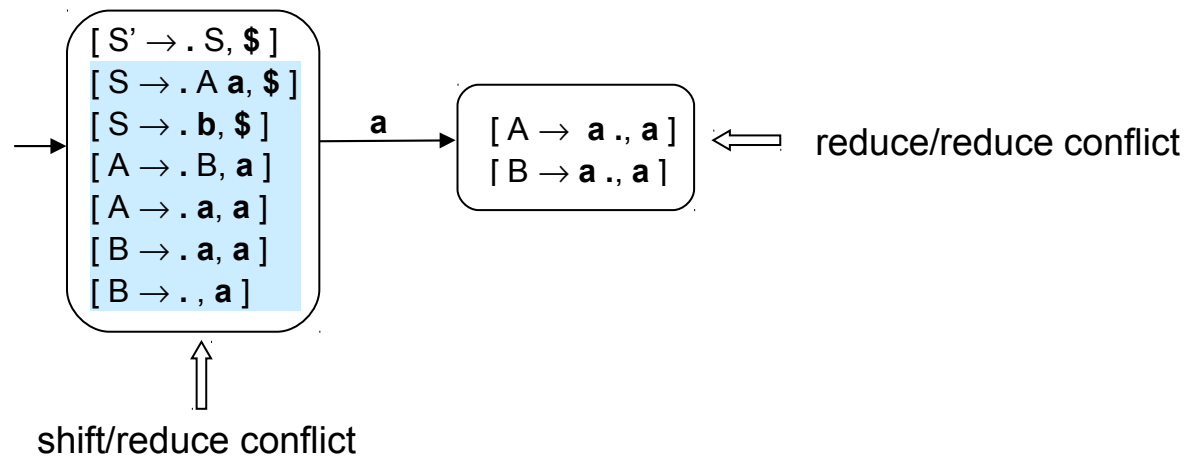
Prove that the following grammar is not LR(1):

$$S \rightarrow A \mathbf{a} \mid \mathbf{b}$$
$$A \rightarrow B \mid \mathbf{a}$$
$$B \rightarrow \mathbf{a} \mid \epsilon$$

## Exercise 6

Prove that the following grammar is not LR(1):

$S \rightarrow A a \mid b$   
 $A \rightarrow B \mid a$   
 $B \rightarrow a \mid \epsilon$



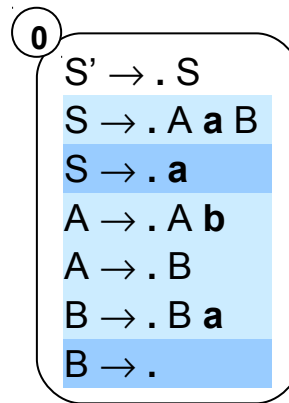
# Exercise 7

Check whether the following grammar is SLR(1):

$$\begin{aligned} S &\rightarrow A \mathbf{a} B \mid \mathbf{a} \\ A &\rightarrow A \mathbf{b} \mid B \\ B &\rightarrow B \mathbf{a} \mid \epsilon \end{aligned}$$

## Exercise 7

Check whether the following grammar is SLR(1):

$$\begin{aligned} S &\rightarrow A \mathbf{a} B \mid \mathbf{a} \\ A &\rightarrow A \mathbf{b} \mid B \\ B &\rightarrow B \mathbf{a} \mid \epsilon \end{aligned}$$

$$FOLLOW(S') = \{ \$ \}$$
$$FOLLOW(S) = \{ \$ \}$$
$$FOLLOW(A) = \{ \mathbf{a}, \mathbf{b} \}$$
$$FOLLOW(B) = \{ \mathbf{a}, \mathbf{b}, \$ \}$$

$\Rightarrow \mathbf{a} \in FOLLOW(B) \Rightarrow \text{Not SLR(1)!}$

# Exercise 8

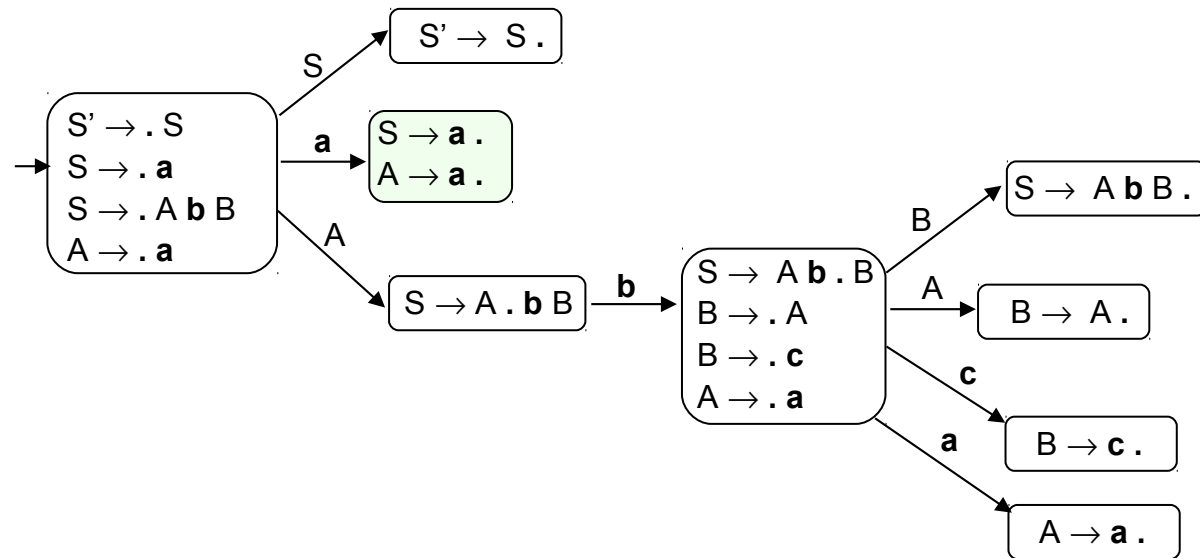
After generating the relevant parsing automaton, determine whether the following grammar is SLR(1):

$$S \rightarrow \mathbf{a} \mid A \mathbf{b} B$$
$$A \rightarrow \mathbf{a}$$
$$B \rightarrow A \mid \mathbf{c}$$

## Exercise 8

After generating the relevant parsing automaton, determine whether the following grammar is SLR(1):

$S \rightarrow \mathbf{a} \mid A \mathbf{b} B$   
 $A \rightarrow \mathbf{a}$   
 $B \rightarrow A \mid \mathbf{c}$



$FOLLOW(S) = \{ \$ \}$   
 $FOLLOW(B) = \{ \$ \}$   
 $FOLLOW(A) = \{ \mathbf{b}, \$ \}$

$\Rightarrow FOLLOW(A) \cap FOLLOW(S) = \{ \$ \} \neq \emptyset \Rightarrow \text{Not SLR(1)!}$



# Exercise 9

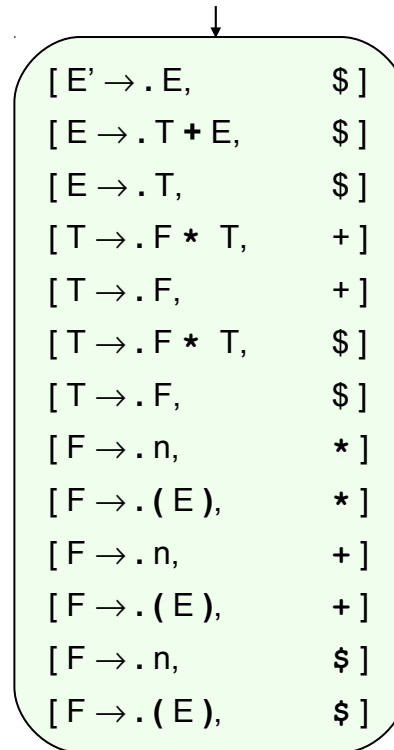
Generate the initial state of the LR(1) parsing (deterministic) automaton relevant to the following BNF:

$$E \rightarrow T + E \mid T$$
$$T \rightarrow F * T \mid F$$
$$F \rightarrow \mathbf{n} \mid ( E )$$

## Exercise 9

Generate the initial state of the LR(1) parsing (deterministic) automaton relevant to the following BNF:

$E \rightarrow T + E \mid T$   
 $T \rightarrow F * T \mid F$   
 $F \rightarrow n \mid ( E )$



$[ E' \rightarrow . E,$	$\$ ]$
$[ E \rightarrow . T + E,$	$\$ ]$
$[ E \rightarrow . T,$	$\$ ]$
$[ T \rightarrow . F * T,$	$+ ]$
$[ T \rightarrow . F,$	$+ ]$
$[ T \rightarrow . F * T,$	$\$ ]$
$[ T \rightarrow . F,$	$\$ ]$
$[ F \rightarrow . n,$	$* ]$
$[ F \rightarrow . ( E ),$	$* ]$
$[ F \rightarrow . n,$	$+ ]$
$[ F \rightarrow . ( E ),$	$+ ]$
$[ F \rightarrow . n,$	$\$ ]$
$[ F \rightarrow . ( E ),$	$\$ ]$

# Exercise 10

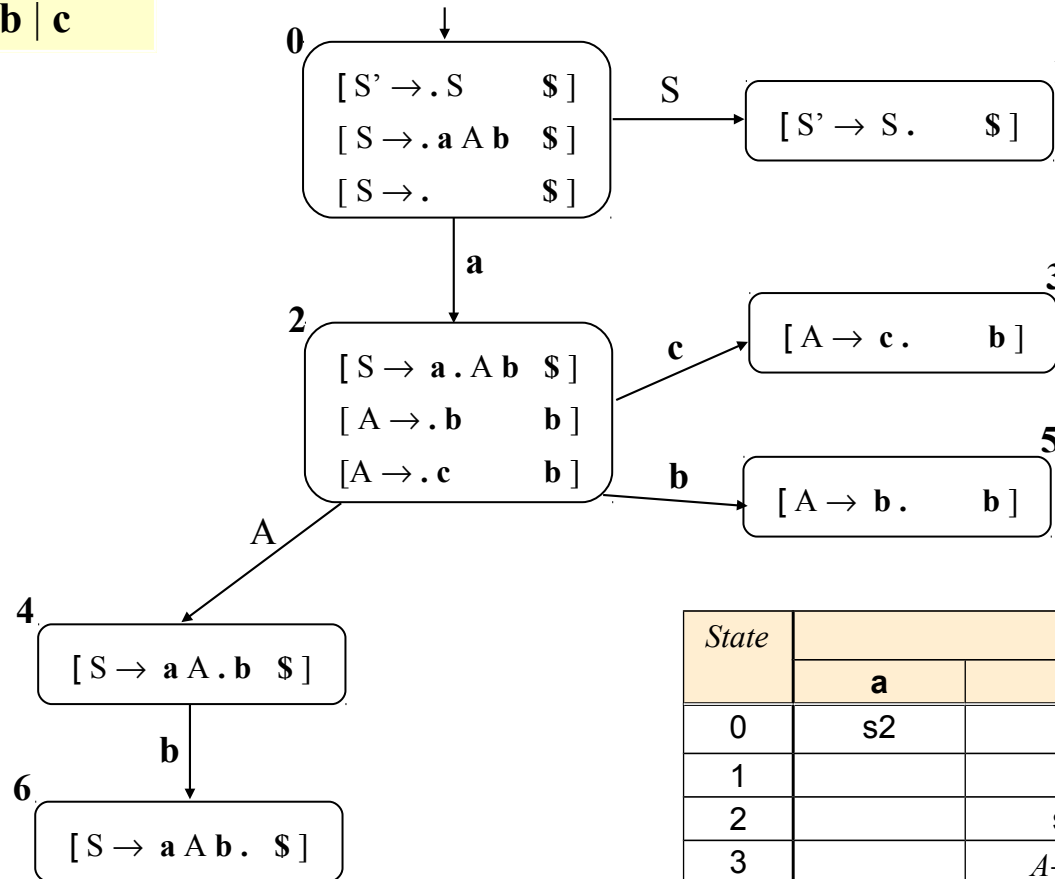
Specify the LALR(1) parsing table relevant to the following BNF:

$$S \rightarrow \mathbf{a} A \mathbf{b} \mid \epsilon$$
$$A \rightarrow \mathbf{b} \mid \mathbf{c}$$

# Exercise 10

Specify the LALR(1) parsing table relevant to the following BNF:

$S \rightarrow a A b \mid \epsilon$   
 $A \rightarrow b \mid c$



State	Input				Goto	
	a	b	c	\$	A	S
0	s2			$S \rightarrow \epsilon$		1
1				accept		
2		s5	s3		4	
3		$A \rightarrow c$				
4		s6				
5		$A \rightarrow b$				
6				$S \rightarrow a A b$		

# Exercise 11

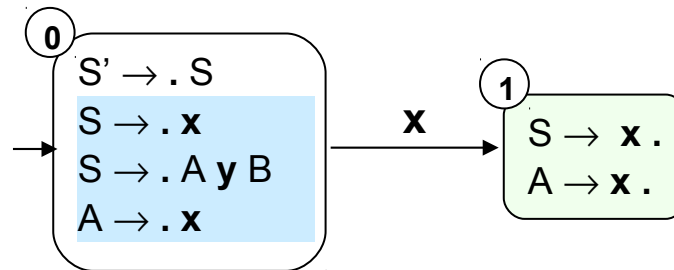
Check whether the following grammar is SLR(1):

$$\begin{aligned} S &\rightarrow \mathbf{x} \mid A \mathbf{y} B \\ A &\rightarrow \mathbf{x} \\ B &\rightarrow A \mid \mathbf{z} \mid \mathbf{w} B \end{aligned}$$

# Exercise 11

Check whether the following grammar is SLR(1):

$S \rightarrow x \mid A y B$   
 $A \rightarrow x$   
 $B \rightarrow A \mid z \mid w B$



$FOLLOW(S') = \{ \$ \}$   
 $FOLLOW(S) = \{ \$ \}$   
 $FOLLOW(B) = \{ \$ \}$   
 $FOLLOW(A) = \{ y, \$ \}$

$$FOLLOW(S) \cap FOLLOW(A) = \{ \$ \} \neq \emptyset$$



**Not SLR(1)!**

# Exercise 12

Generate the initial state of the LR(1) parsing (deterministic) automaton relevant to the following BNF:

$$\begin{aligned} A &\rightarrow B \mathbf{a} B \mid B \\ B &\rightarrow C \mathbf{b} B \mid C \\ C &\rightarrow \mathbf{c} \mid \mathbf{d} A \end{aligned}$$

## Exercise 12

Generate the initial state of the LR(1) parsing (deterministic) automaton relevant to the following BNF:

$A \rightarrow B \mathbf{a} B \mid B$   
 $B \rightarrow C \mathbf{b} B \mid C$   
 $C \rightarrow \mathbf{c} \mid \mathbf{d} A$

↓

$[A' \rightarrow \cdot A,$	$\$]$
$[A \rightarrow \cdot B \mathbf{a} B,$	$\$]$
$[A \rightarrow \cdot B,$	$\$]$
$[B \rightarrow \cdot C \mathbf{b} B,$	$\mathbf{a}]$
$[B \rightarrow \cdot C,$	$\mathbf{a}]$
$[B \rightarrow \cdot C \mathbf{b} B,$	$\$]$
$[B \rightarrow \cdot C,$	$\$]$
$[C \rightarrow \cdot \mathbf{c},$	$\mathbf{b}]$
$[C \rightarrow \cdot \mathbf{d} A,$	$\mathbf{b}]$
$[C \rightarrow \cdot \mathbf{c},$	$\mathbf{a}]$
$[C \rightarrow \cdot \mathbf{d} A,$	$\mathbf{a}]$
$[C \rightarrow \cdot \mathbf{c},$	$\$]$
$[C \rightarrow \cdot \mathbf{d} A,$	$\$]$



# Exercise 13

Check whether the following grammar is SLR(1):

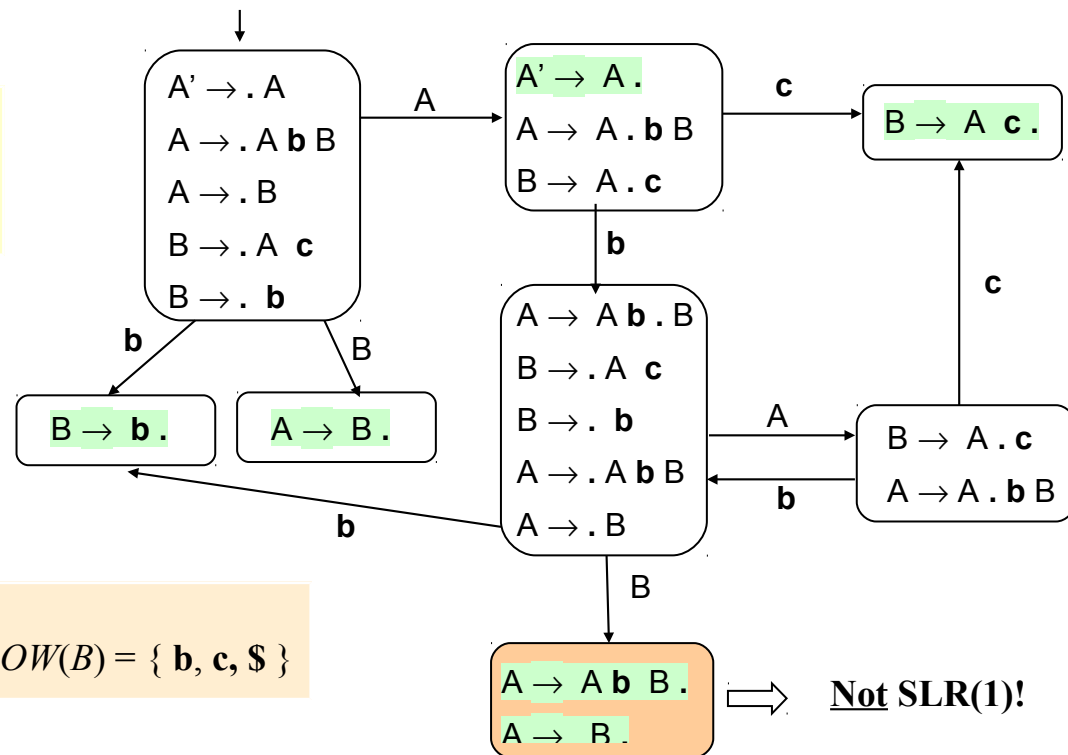
$$\begin{aligned} A &\rightarrow A \mathbf{b} B \mid B \\ B &\rightarrow A \mathbf{c} \mid \mathbf{b} \end{aligned}$$

# Exercise 13

Check whether the following grammar is SLR(1):

$A \rightarrow A \mathbf{b} B \mid B$   
 $B \rightarrow A \mathbf{c} \mid \mathbf{b}$

$A' \rightarrow A$   
 $A \rightarrow A \mathbf{b} B \mid B$   
 $B \rightarrow A \mathbf{c} \mid \mathbf{b}$



$FOLLOW(A') = \{ \$ \}$   
 $FOLLOW(A) = FOLLOW(B) = \{ \mathbf{b}, \mathbf{c}, \$ \}$

# Exercise 14

Outline the initial state of the LR(1) parsing (deterministic) automaton relevant to the following BNF:

$$\begin{aligned} S &\rightarrow S \mathbf{a} T \mid T \\ T &\rightarrow S \mathbf{b} \mid \mathbf{a} \end{aligned}$$

## Exercise 14

Outline the initial state of the LR(1) parsing (deterministic) automaton relevant to the following BNF:

$S \rightarrow S \mathbf{a} T \mid T$   
 $T \rightarrow S \mathbf{b} \mid \mathbf{a}$

$[S' \rightarrow . S,$	$\$]$
$[S \rightarrow . S \mathbf{a} T,$	$\$]$
$[S \rightarrow . T,$	$\$]$
$[S \rightarrow . S \mathbf{a} T,$	$\mathbf{a}]$
$[S \rightarrow . T,$	$\mathbf{a}]$
$[T \rightarrow . S \mathbf{b},$	$\$]$
$[T \rightarrow . \mathbf{a},$	$\$]$
$[T \rightarrow . S \mathbf{b},$	$\mathbf{a}]$
$[T \rightarrow . \mathbf{a},$	$\mathbf{a}]$
$[S \rightarrow . S \mathbf{a} T,$	$\mathbf{b}]$
$[S \rightarrow . T,$	$\mathbf{b}]$
$[T \rightarrow . S \mathbf{b},$	$\mathbf{b}]$
$[T \rightarrow . \mathbf{a},$	$\mathbf{b}]$

# Exercise 15

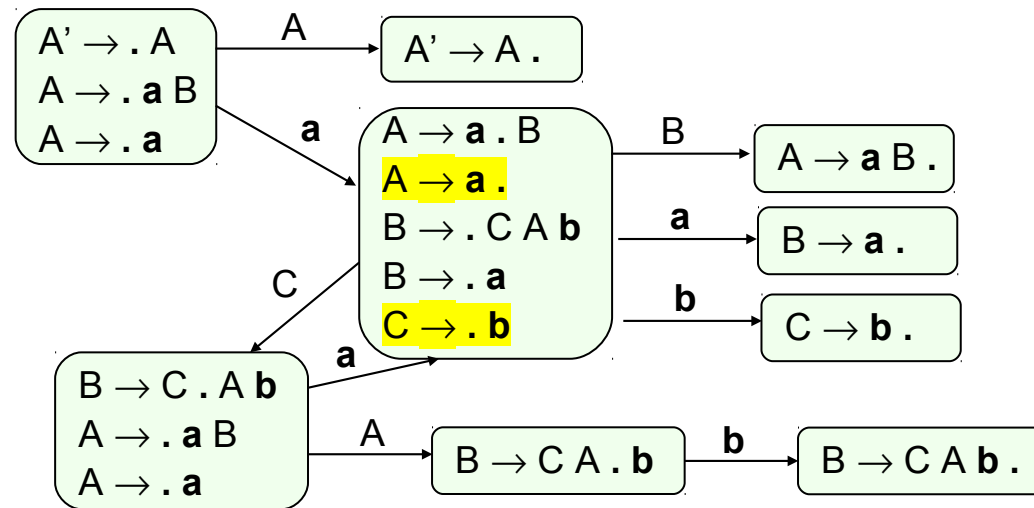
After constructing the complete parsing automaton for the following BNF, determine whether the grammar is SLR(1):

$$\begin{aligned} A &\rightarrow \mathbf{a} B \mid \mathbf{a} \\ B &\rightarrow C A \mathbf{b} \mid \mathbf{a} \\ C &\rightarrow \mathbf{b} \end{aligned}$$

## Exercise 15

After constructing the complete parsing automaton for the following BNF, determine whether the grammar is SLR(1):

$A \rightarrow a B \mid a$   
 $B \rightarrow C A b \mid a$   
 $C \rightarrow b$



$b \in \text{FOLLOW}(A) = \{ \$, b \} \Rightarrow \text{BNF not SLR(1)} !$

# Exercise 16

Given the following BNF:

$$\begin{aligned} A &\rightarrow B \mathbf{a} C \mid B \\ B &\rightarrow C \mathbf{b} B \mid C \\ C &\rightarrow \mathbf{a} \end{aligned}$$

We ask to:

- Generate the initial state of the LR(1) (deterministic) parsing automaton.
- Check whether such a state presents any parsing conflict.

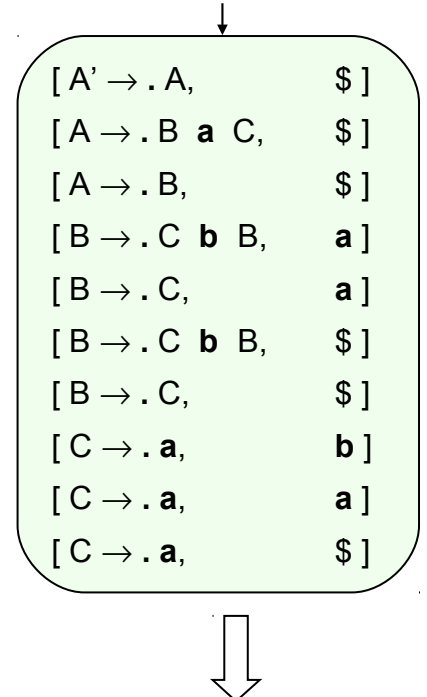
## Exercise 16

Given the following BNF:

$$\begin{aligned} A &\rightarrow B \mathbf{a} C \mid B \\ B &\rightarrow C \mathbf{b} B \mid C \\ C &\rightarrow \mathbf{a} \end{aligned}$$

We ask to:

- Generate the initial state of the LR(1) (deterministic) parsing automaton.
- Check whether such a state presents any parsing conflict.



[ A' → . A,	\$ ]
[ A → . B <b>a</b> C,	\$ ]
[ A → . B,	\$ ]
[ B → . C <b>b</b> B,	<b>a</b> ]
[ B → . C,	<b>a</b> ]
[ B → . C <b>b</b> B,	\$ ]
[ B → . C,	\$ ]
[ C → . <b>a</b> ,	<b>b</b> ]
[ C → . <b>a</b> ,	<b>a</b> ]
[ C → . <b>a</b> ,	\$ ]

*No parsing conflict*



# Exercise 17

After constructing the parsing automaton for the following grammar,

$$\begin{aligned} A &\rightarrow A \mathbf{a} B \mid B \\ B &\rightarrow A \mathbf{b} \mid \mathbf{a} \end{aligned}$$

determine whether the grammar is SLR(1).

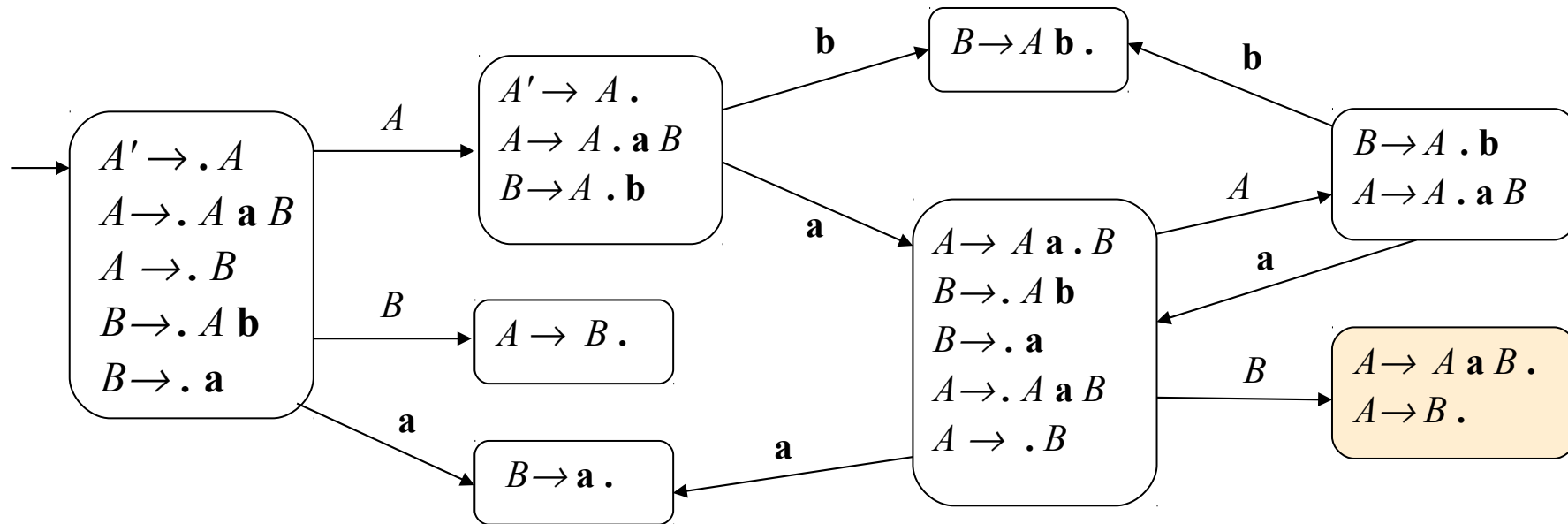
## Exercise 17

After constructing the parsing automaton for the following grammar,

$$A \rightarrow A \mathbf{a} B \mid B$$

$$B \rightarrow A \mathbf{b} \mid \mathbf{a}$$

determine whether the grammar is SLR(1).



# Exercise 18

After constructing the complete parsing automaton, check whether the following grammar is LR(1).

$A \rightarrow B \mid \text{id} \mid \text{num}$

$B \rightarrow C \text{ num} \mid \text{id}$

$C \rightarrow \text{id} \mid \text{num} \mid \epsilon$

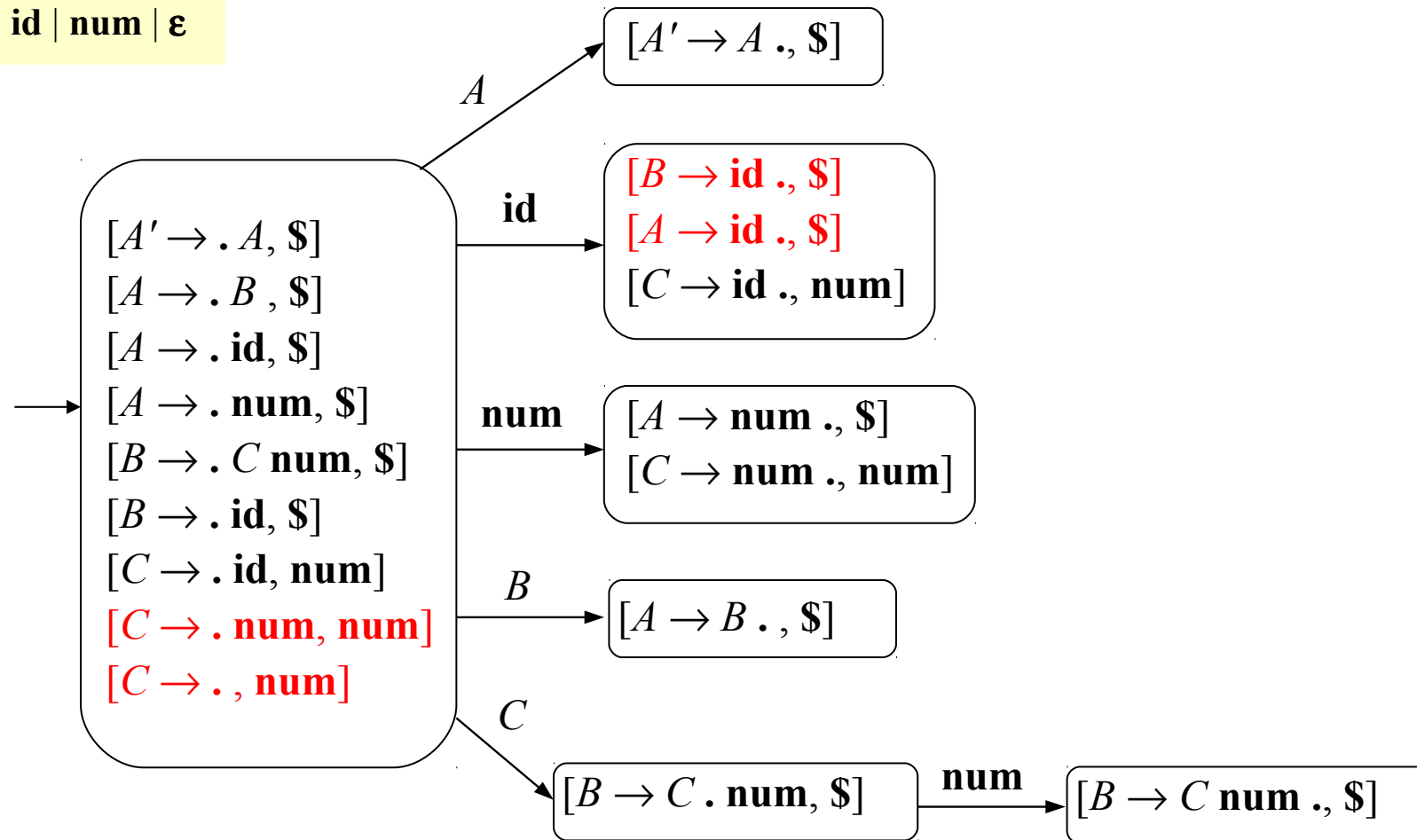
## Exercise 18

After constructing the complete parsing automaton, check whether the following grammar is LR(1).

$A \rightarrow B \mid \text{id} \mid \text{num}$

$B \rightarrow C \text{ num} \mid \text{id}$

$C \rightarrow \text{id} \mid \text{num} \mid \epsilon$



# Exercise 19

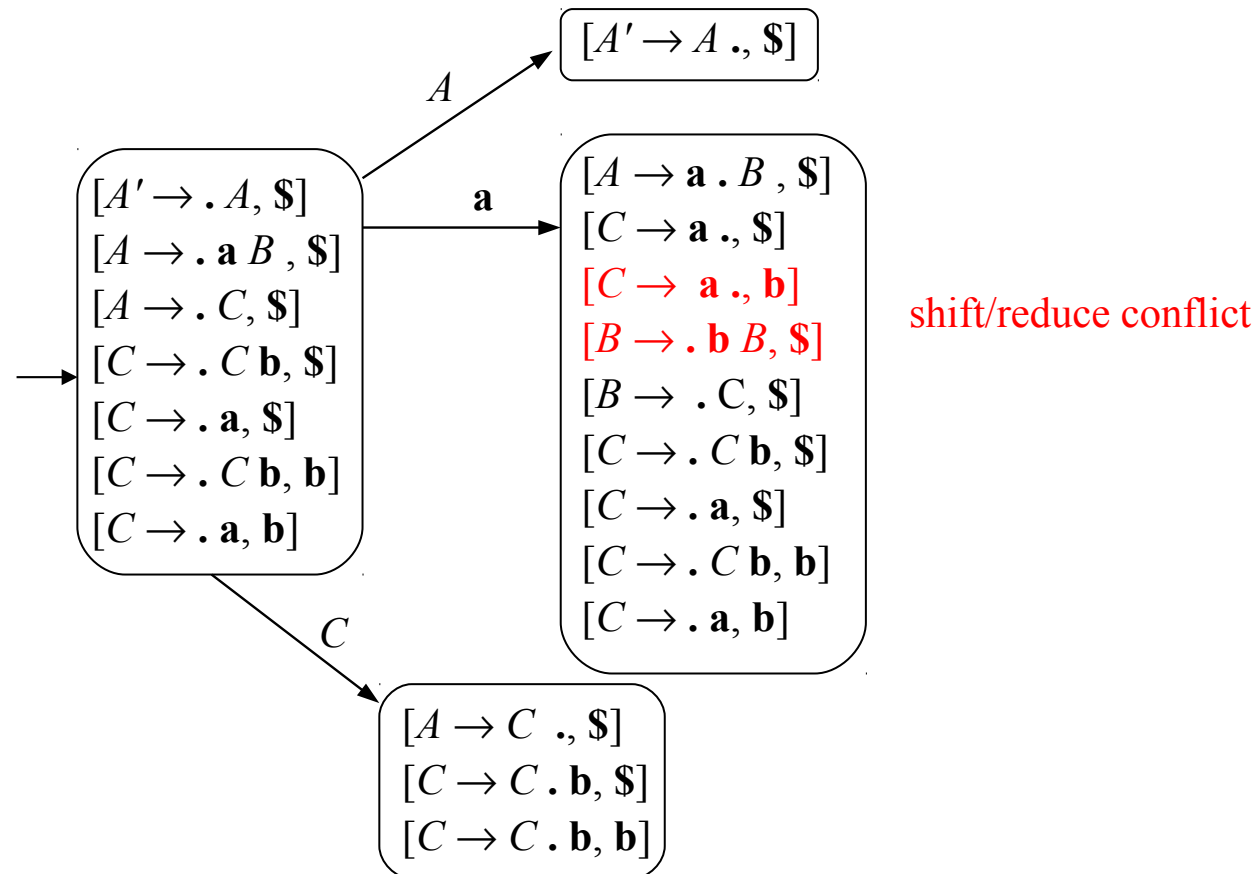
With reference to the following BNF, after constructing the portion of the LR(1) parsing automaton up to the states reached by a single transition from the initial state, check whether this portion of automaton includes any conflict:

$$A \rightarrow \mathbf{a} B \mid C$$
$$B \rightarrow \mathbf{b} B \mid C$$
$$C \rightarrow C \mathbf{b} \mid \mathbf{a}$$

## Exercise 19

With reference to the following BNF, after constructing the portion of the LR(1) parsing automaton up to the states reached by a single transition from the initial state, check whether this portion of automaton includes any conflict:

$A \rightarrow \mathbf{a} B \mid C$   
 $B \rightarrow \mathbf{b} B \mid C$   
 $C \rightarrow C \mathbf{b} \mid \mathbf{a}$



## Exercise 20

After constructing the complete parsing automaton for the following grammar, determine whether it is SLR(1), providing relevant explanation.

$$A' \rightarrow A$$
$$A \rightarrow B \mathbf{a} C \mid C$$
$$B \rightarrow C \mathbf{b} \mid \mathbf{b}$$
$$C \rightarrow A \mathbf{b} \mid \mathbf{a}$$

## Exercise 20

After constructing the complete parsing automaton for the following grammar, determine whether it is SLR(1), providing relevant explanation.

$A' \rightarrow A$

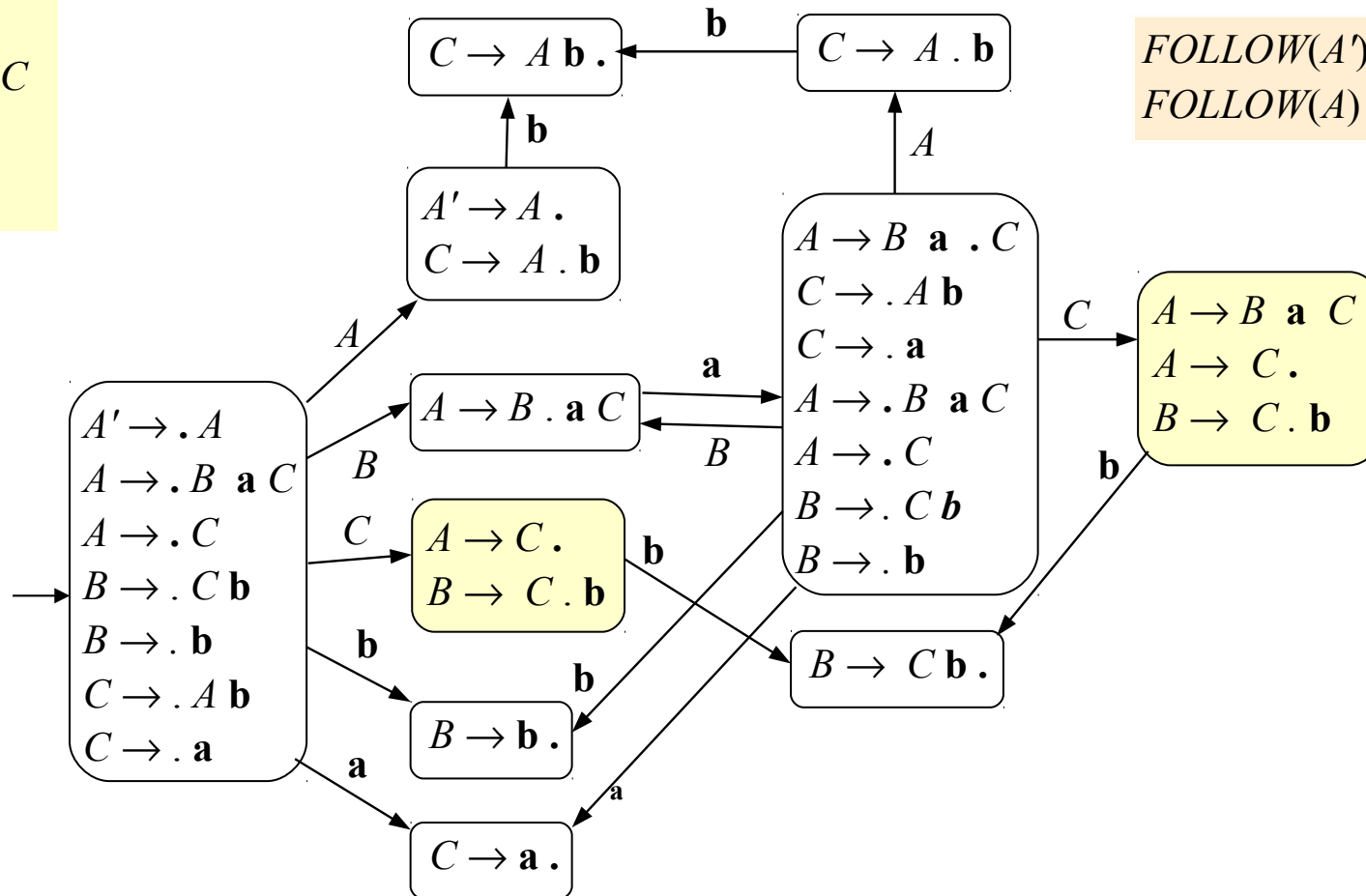
$A \rightarrow B a C \mid C$

$B \rightarrow C b \mid b$

$C \rightarrow A b \mid a$

$FOLLOW(A') = \{ \$ \}$

$FOLLOW(A) = \{ \$, b \}$





# Exercise 21

Outline the LR(1) parsing table relevant to the following BNF.

$$\begin{aligned} A &\rightarrow A \mathbf{a} B \mid \epsilon \\ B &\rightarrow \mathbf{b} \mid \epsilon \end{aligned}$$

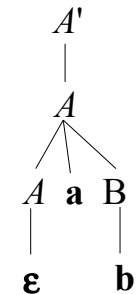
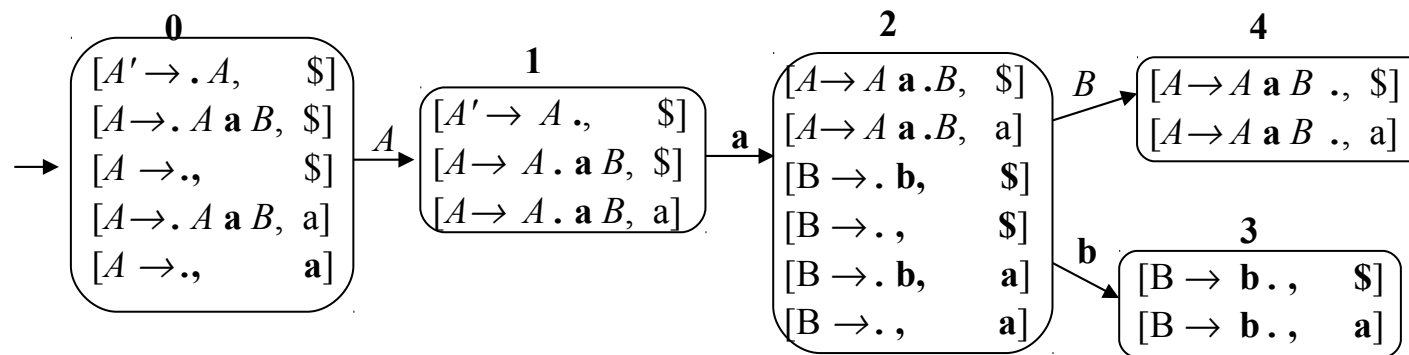
Then, trace the LR(1) parsing of phrase **a b**. Finally, draw the corresponding syntax tree based on the traced parsing actions.

# Exercise 21

Outline the LR(1) parsing table relevant to the following BNF.

$A \rightarrow A \mathbf{a} B \mid \epsilon$   
 $B \rightarrow \mathbf{b} \mid \epsilon$

Then, trace the LR(1) parsing of phrase **a b**. Finally, draw the corresponding syntax tree based on the traced parsing actions.



State	Input			Goto	
	a	b	\$	A	B
0	$A \rightarrow \epsilon$		$A \rightarrow \epsilon$	1	
1	s2		accept		
2	$B \rightarrow \epsilon$	s3	$B \rightarrow \epsilon$		4
3	$B \rightarrow \mathbf{b}$		$B \rightarrow \mathbf{b}$		
4	$A \rightarrow A \mathbf{a} B$		$A \rightarrow A \mathbf{a} B$		

Stack	Input	Action
\$0	a b \$	$A \rightarrow \epsilon$
\$0 A1	a b \$	shift
\$0 A1 a2	b \$	shift
\$0 A1 a2 b3	\$	$B \rightarrow \mathbf{b}$
\$0 A1 a2 B4	\$	$A \rightarrow A \mathbf{a} B$
\$0 A1	\$	accept

## Exercise 22

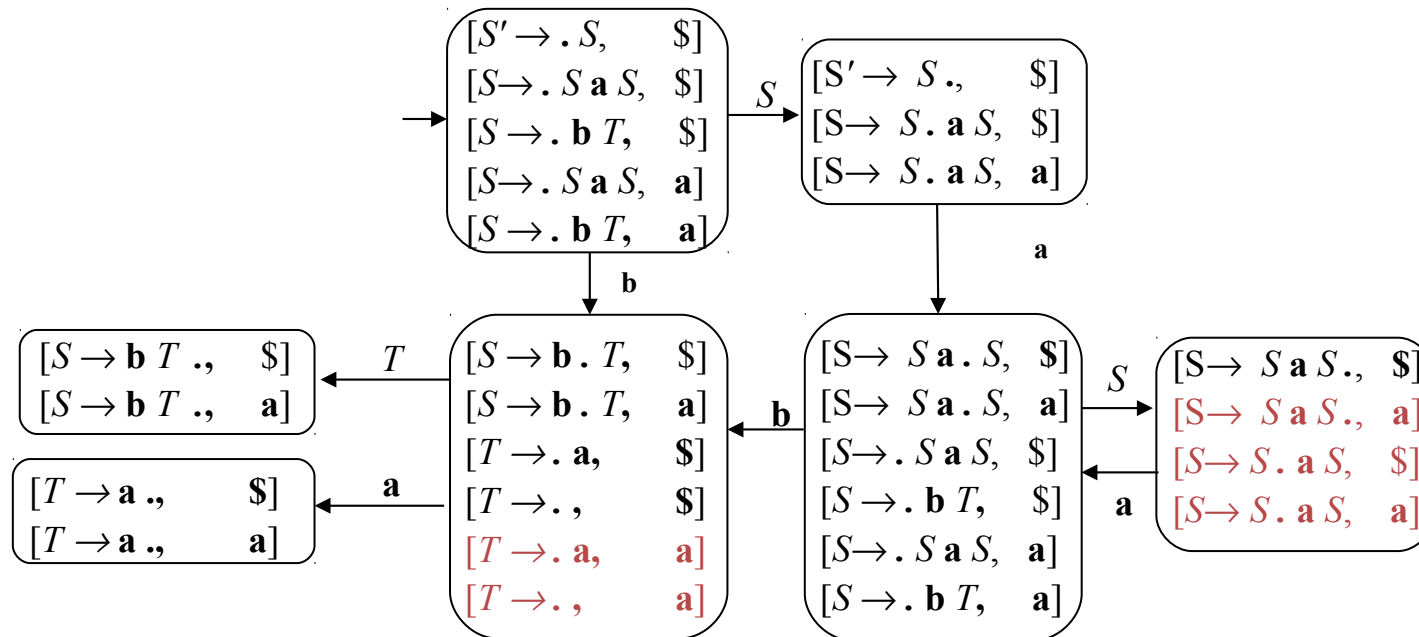
After constructing the parsing automaton for the following BNF, discuss whether the BNF is LR(1) .

$$S \rightarrow S \mathbf{a} S \mid \mathbf{b} T$$
$$T \rightarrow \mathbf{a} \mid \epsilon$$

## Exercise 22

After constructing the parsing automaton for the following BNF, discuss whether the BNF is LR(1) .

$S \rightarrow S a S \mid b T$   
 $T \rightarrow a \mid \epsilon$



## Exercise 23

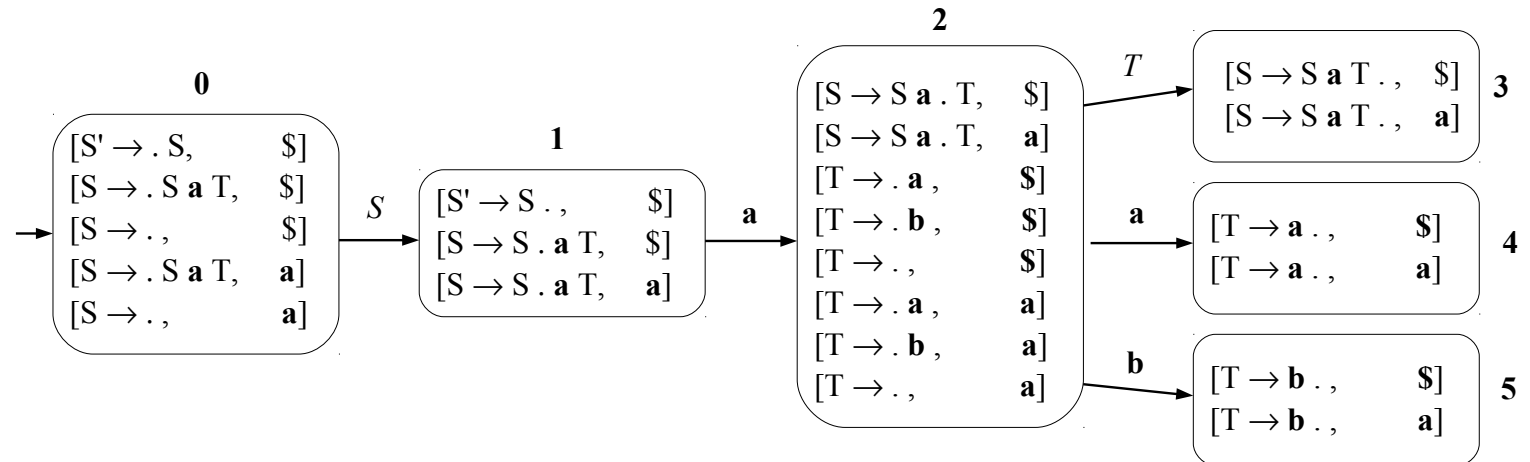
Given the following BNF, we ask to generate the complete LR(1) parsing table and to establish whether the grammar is LR(1).

$$\begin{aligned} S &\rightarrow S \mathbf{a} T \mid \epsilon \\ T &\rightarrow \mathbf{a} \mid \mathbf{b} \mid \epsilon \end{aligned}$$

## Exercise 23

Given the following BNF, we ask to generate the complete LR(1) parsing table and to establish whether the grammar is LR(1).

$S \rightarrow S a T \mid \epsilon$   
 $T \rightarrow a \mid b \mid \epsilon$



State	Input			Goto	
	a	b	\$	S	T
0	$S \rightarrow \epsilon$		$S \rightarrow \epsilon$	1	
1	s2		accept		
2	$T \rightarrow \epsilon$ s4	s5	$T \rightarrow \epsilon$		3
3	$S \rightarrow S a$ $T$		$S \rightarrow S a T$		
4	$T \rightarrow a$		$T \rightarrow a$		
5	$T \rightarrow b$		$T \rightarrow b$		

## Exercise 24

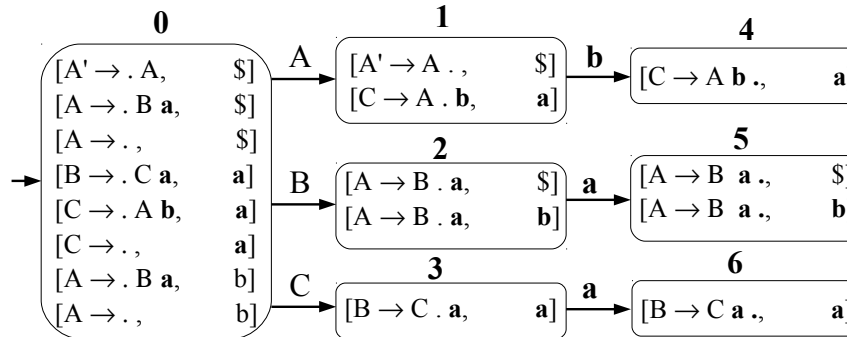
Given the following BNF, we ask to (i) outline the corresponding LR(1) parsing table, (ii) trace the LR(1) parsing of phrase **baa**, and (iii) based on the trace, outline the syntax tree of the given phrase.

$$A \rightarrow B \mathbf{a} \mid \epsilon$$
$$B \rightarrow C \mathbf{a}$$
$$C \rightarrow A \mathbf{b} \mid \epsilon$$

## Exercise 24

Given the following BNF, we ask to (i) outline the corresponding LR(1) parsing table, (ii) trace the LR(1) parsing of phrase **baa**, and (iii) based on the trace, outline the syntax tree of the given phrase.

$A \rightarrow B a \mid \epsilon$   
 $B \rightarrow C a$   
 $C \rightarrow A b \mid \epsilon$



State	Input			Goto		
	a	b	\$	A	B	C
0	$C \rightarrow \epsilon$	$A \rightarrow \epsilon$	$A \rightarrow \epsilon$	1	2	3
1		s4	accept			
2	s5					
3	s6					
4	$C \rightarrow A b$					
5		$A \rightarrow B a$	$A \rightarrow B a$			
6	$B \rightarrow C a$					

Stack	Input	Action
\$0	<b>b a a \$</b>	$A \rightarrow \epsilon$
\$0 A1	<b>b a a \$</b>	shift
\$0 A1 b4	<b>a a \$</b>	$C \rightarrow A b$
\$0 C3	<b>a a \$</b>	shift
\$0 C3 a6	<b>a \$</b>	$B \rightarrow C a$
\$0 B2	<b>a \$</b>	shift
\$0 B2 a5	<b>\$</b>	$A \rightarrow B a$
\$0 A1	<b>\$</b>	accept

