Tarefa 5 - Prof Sávio

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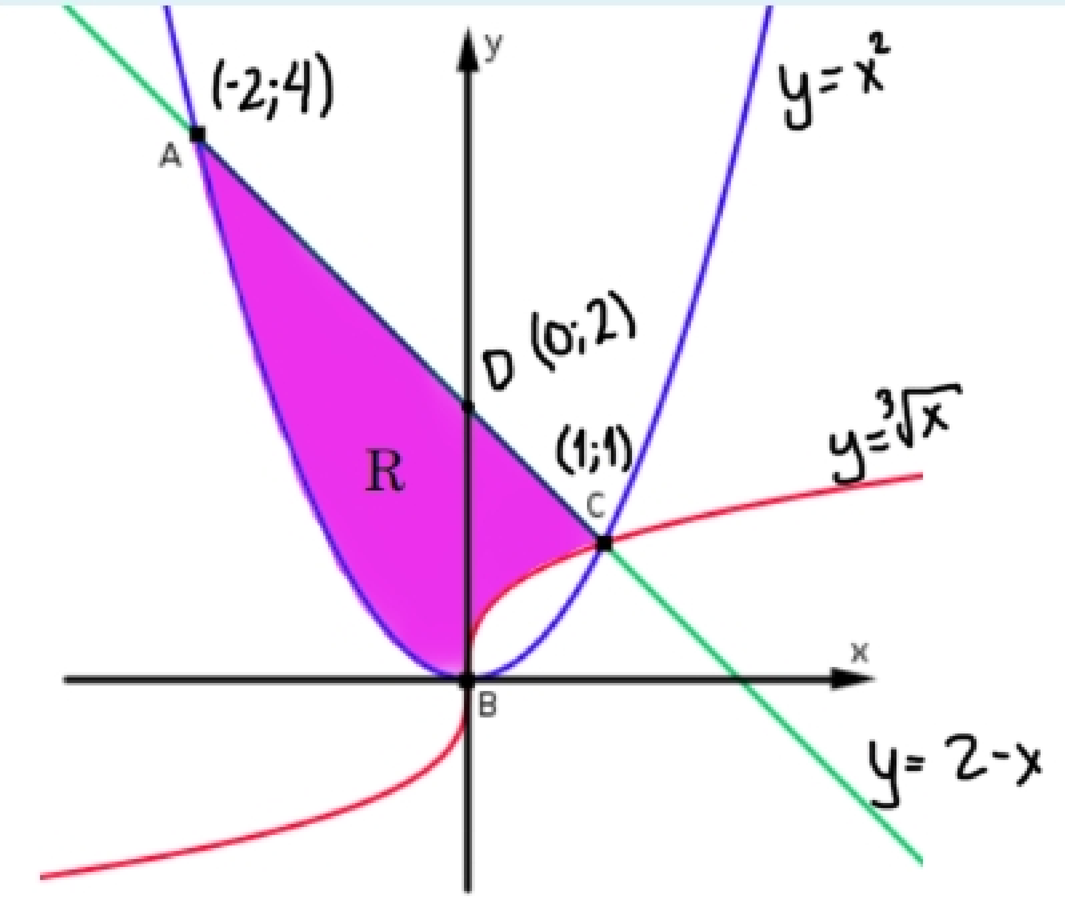
Considere as curvas $C_1 : \ y = x^2, C_2 : \ x = y^3$ e $C_3 : \ x + y = 2$, as quais estão esboçadas abaixo



$$2 - x = x^2$$
$$x^2 + x - 2 = 0$$
$$\Delta = 1 + 8$$
$$x = \frac{-1 \pm 3}{2} \begin{cases} x_1 = 1 & y_1 = 1 \\ x_2 = -2 & y_2 = 4 \end{cases}$$

$$R = \int_{-2}^{0} (2 - x - x^2)\,dx + \int_{0}^{1} (2 - x - x^{\frac{1}{3}})\,dx$$
$$R = \int_{-2}^{0} 2\,dx - \int_{-2}^{0} x\,dx - \int_{-2}^{0} x^2\,dx + \int_{0}^{1} 2\,dx - \int_{0}^{1} x\,dx - \int_{0}^{1} x^{\frac{1}{3}}\,dx$$
$$R = 4 - (-2) - \frac{8}{3} + 2 - \frac{1}{2} - \frac{3}{4} = \frac{49}{12}$$

$$\int_{-2}^{0} 2\,dx = 2x - 2x_0 = 0 - (-4) = 4$$
$$\int_{-2}^{0} x^2\,dx = \frac{x^3}{3} - \frac{x_0^3}{3} = 0 - \frac{(-8)}{3} = \frac{8}{3}$$
$$\int_{0}^{1} x\,dx = \frac{x^2}{2} - \frac{x_0^2}{2} = \frac{1}{2} - 0 = \frac{1}{2}$$

$$\int_{-2}^{0} x\,dx = \frac{x^2}{2} - \frac{x_0^2}{2} = 0 - \frac{4}{2} = -2$$
$$\int_{0}^{1} 2\,dx = 2x - 2x_0 = 2 - 0 = 2$$
$$\int_{0}^{1} x^{\frac{1}{3}}\,dx = \frac{3x^{\frac{4}{3}}}{4} - \frac{3x_0^{\frac{4}{3}}}{4} = \frac{3}{4}$$

Calcule o volume do sólido cuja base é um triângulo equilátero de lado 5, e cujas secções perpendiculares a um dos lados são quadrados.

$$V = \int_0^5 A\,dx = \int_0^5 \frac{x^2\sqrt{3}}{4}\,dx$$

$$\int_0^5 \frac{\sqrt{3}}{4}\cdot x^2\,dx = \frac{\sqrt{3}}{12}x^3 - \frac{\sqrt{3}}{12}x_0^3 = \frac{\sqrt{3}\cdot 125}{12}$$

$$V = \frac{125\sqrt{3}}{12}$$

Calcule o volume do sólido obtido por rotação, em torno do eixo x, da região plana delimitada pela hipociclóide $x^{2/3} + y^{2/3} = 1$. (Escreva sua resposta aproximada, com erro $\leq 0,01$.

$$y = (1 - x^{\frac{2}{3}})^{\frac{3}{2}}$$

$$1 - 3x^{\frac{2}{3}} + 3x^{\frac{4}{3}} - x^2$$

$$x > 0$$
$$y > 0$$

$$V_2 = \frac{V}{2} = \int_0^1 \pi\,(1 - x^{\frac{2}{3}})^3\,dx$$

$$V_2 = \int_0^1 \pi - 3x^{\frac{2}{3}}\pi + 3x^{\frac{4}{3}}\pi - x^2\pi\,dx$$

$$V_2 = \left(\int_0^1 \pi - \int_0^1 3x^{\frac{2}{3}}\pi + \int_0^1 3x^{\frac{4}{3}}\pi - \int_0^1 x^2\pi\right)dx$$

$$V_2 = \pi - \frac{9\pi}{5} + \frac{9\pi}{7} - \frac{\pi}{3} = \frac{16\pi}{105}$$

$$V = \frac{32\pi}{105}$$

$$\int_0^1 \pi\,dx = \pi x - \pi x_0 = \pi$$

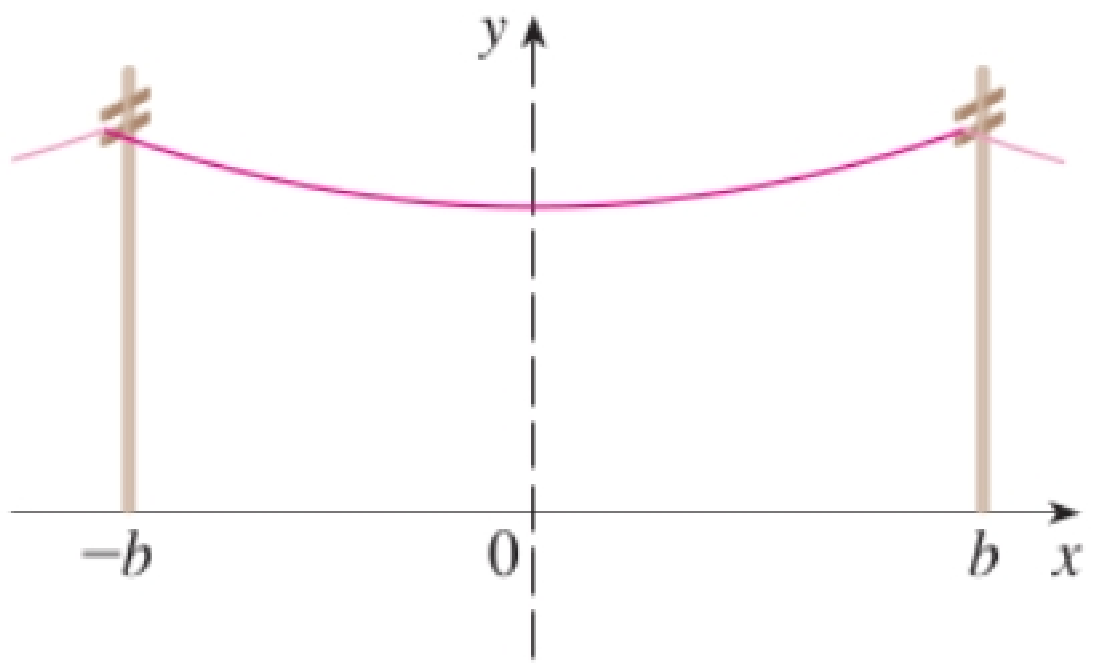
$$\int_0^1 x^2\pi\,dx = \frac{x^3\pi}{3} - \frac{x_0^3\pi}{3} = \frac{\pi}{3}$$

$$\int_0^1 3x^{\frac{2}{3}}\pi\,dx = \frac{3\cdot 3x^{\frac{5}{3}}\pi}{5} - \frac{3\cdot 3x_0^{\frac{5}{3}}\pi}{5} = \frac{9\pi}{5}$$

$$\int_0^1 3x^{\frac{4}{3}}\pi\,dx = \frac{3\cdot 3x^{\frac{7}{3}}\pi}{7} - \frac{3\cdot 3x_0^{\frac{7}{3}}\pi}{7} = \frac{9\pi}{7}$$

A figura mostra um fio de telefone pendurado entre dois postes em $x = -b$ e $x = b$. Ele tem o formato de uma catenária, ou seja, uma curva com equação,

$$y = -20 + 29\left(\frac{e^{\frac{x}{29}} + e^{-\frac{x}{29}}}{2}\right)$$



$$y = -20 + 29\left(\frac{e^{\frac{x}{29}} + e^{\frac{-x}{29}}}{2}\right)$$

$$y' = 0 + \frac{29}{2}\left(\frac{e^{\frac{x}{29}}}{29} - \frac{e^{\frac{-x}{29}}}{29}\right)$$

$$y' = \frac{e^{\frac{x}{29}} - e^{\frac{-x}{29}}}{2}$$

$$1 + \left(\frac{e^{\frac{x}{29}} - e^{\frac{-x}{29}}}{2}\right)^2 = 1 + e^{\frac{2x}{29}} - 2 + e^{\frac{-2x}{29}} = e^{\frac{2x}{29}} + e^{\frac{-2x}{29}} - 1$$

$$L = \int_{-10}^{10} \sqrt{e^{\frac{2x}{29}} + e^{\frac{-2x}{29}} - 1}\, dx = 20{,}39$$