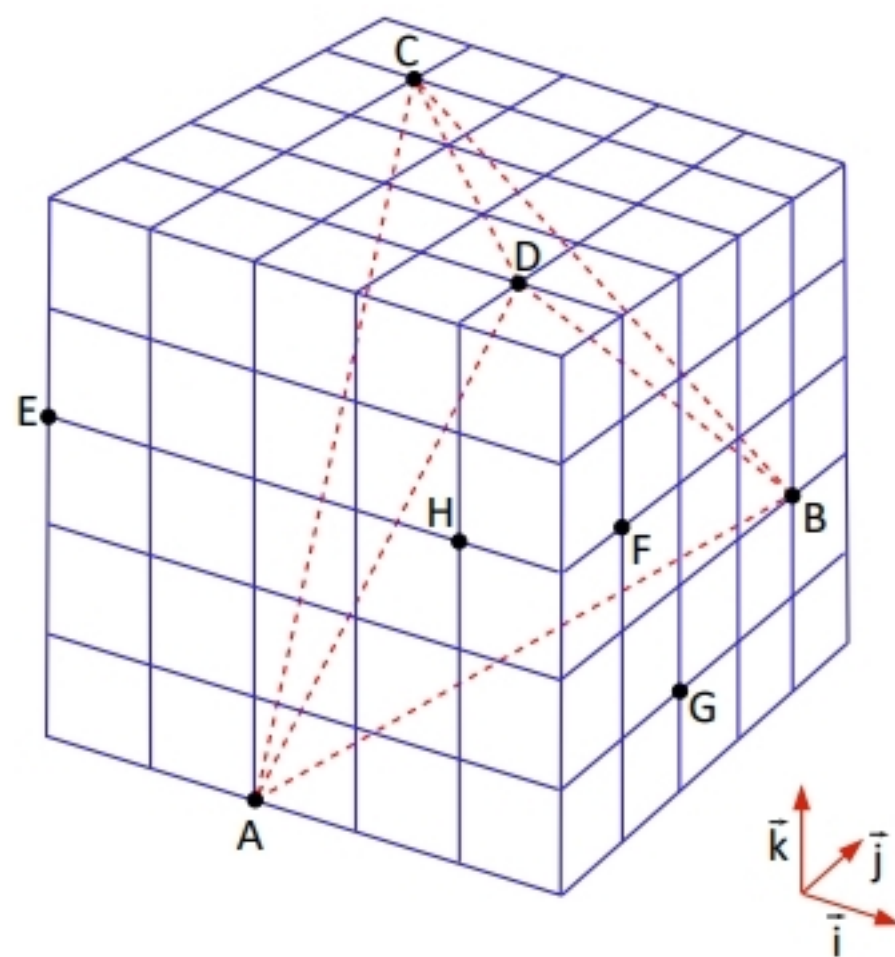


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Tarefa 2 - Geometria Analítica



1. Determine o ângulo formado pelos segmentos AB e AD.

$$\vec{v} = \overrightarrow{AB} = (3, 4, 2)$$

$$\vec{u} = \overrightarrow{AD} = (2, 1, 5)$$

$$\cos \theta = \frac{\vec{v} \cdot \vec{u}}{|\vec{v}| \cdot |\vec{u}|}$$

$$|\vec{v}| = \sqrt{(3)^2 + (4)^2 + (2)^2}$$

$$|\vec{v}| = \sqrt{9 + 16 + 4} = \sqrt{29}$$

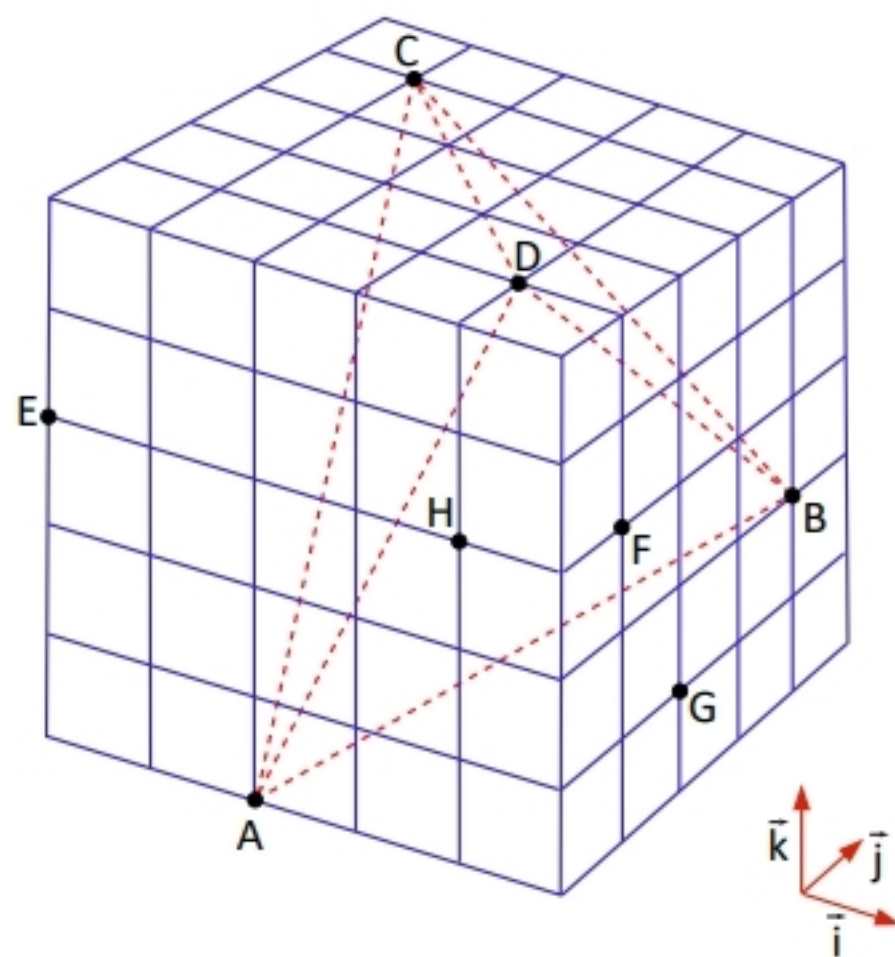
$$|\vec{u}| = \sqrt{(2)^2 + (1)^2 + (5)^2}$$

$$|\vec{u}| = \sqrt{4 + 1 + 25} = \sqrt{30}$$

$$\cos \theta = \frac{(3 \cdot 2) + (4 \cdot 1) + (2 \cdot 5)}{\sqrt{29} \cdot \sqrt{30}}$$

$$\cos \theta = \frac{20}{\sqrt{870}}$$

$$\theta = \arccos \frac{20}{\sqrt{870}} = 47,30$$



2. Calcule a área do triângulo ADC.

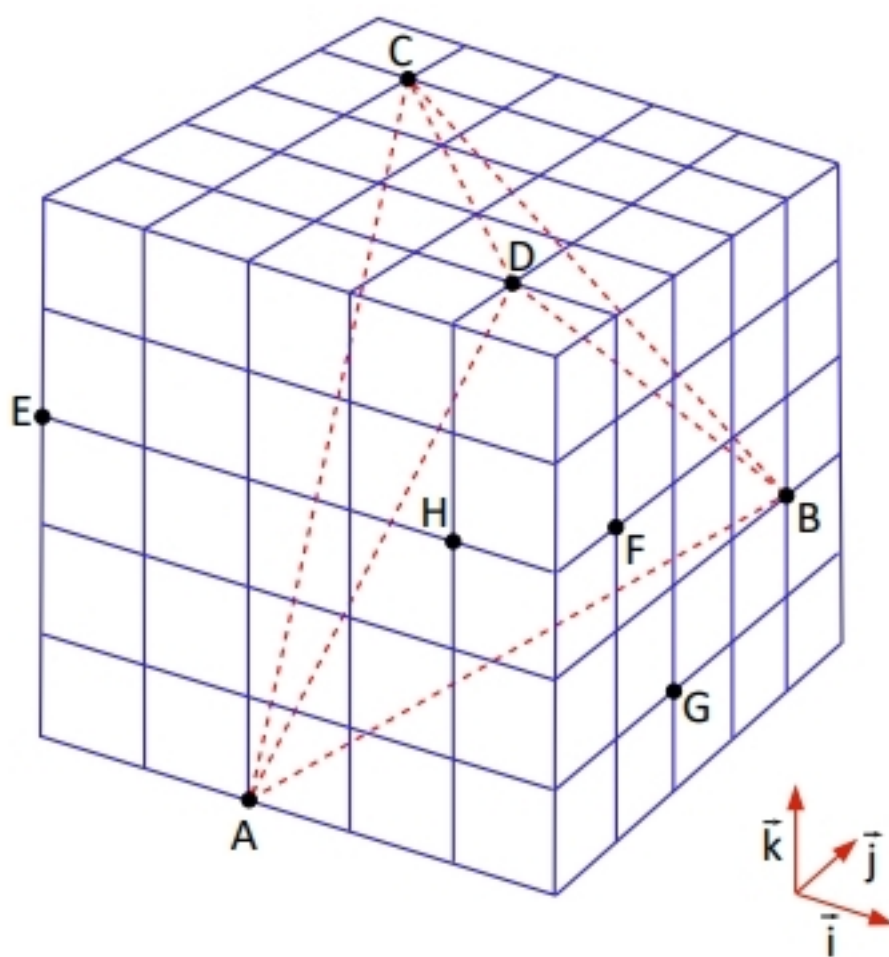
$$|\vec{v}| = |\vec{AC}| = (-1, 4, 5)$$

$$|\vec{u}| = |\vec{AD}| = (2, 1, 5)$$

$$|\vec{u} \cdot \vec{v}| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 5 \\ -1 & 4 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 4 & 5 \end{vmatrix} \vec{i} - \begin{vmatrix} 2 & 5 \\ -1 & 5 \end{vmatrix} \vec{j} + \begin{vmatrix} 2 & 1 \\ -1 & 4 \end{vmatrix} \vec{k}$$

$$-15\vec{i} \quad -15\vec{j} \quad -9\vec{k}$$

$$\frac{1}{2} \cdot |\vec{v} \cdot \vec{u}| = \frac{\sqrt{(-15)^2 + (-15)^2 + (-9)^2}}{2} = \frac{\sqrt{531}}{2} = 11,52 \text{ cm}^2$$



3. Calcule a distância do ponto D ao segmento AC.

$$A = \frac{b \cdot h}{2}$$

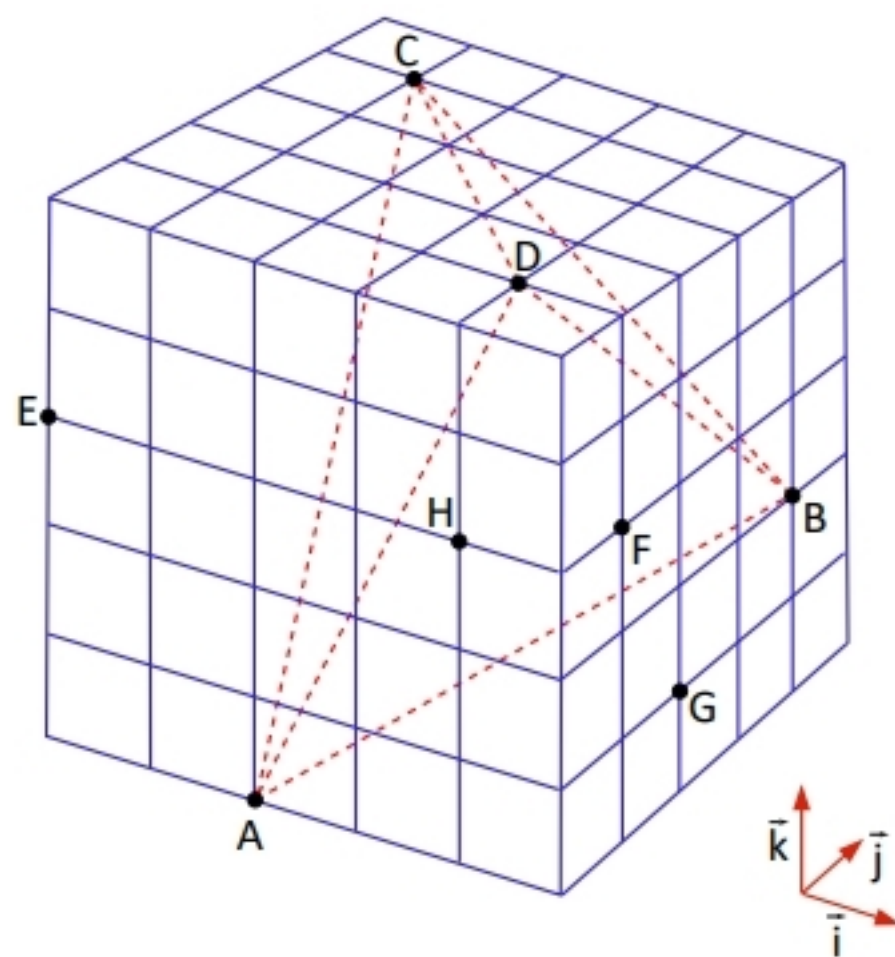
$$\vec{v} = \vec{AC}$$

$$\vec{u} = \vec{AD}$$

$$|\vec{v} \cdot \vec{u}| = \frac{|\vec{v}| \cdot h}{2}$$

$$2 \cdot 11,52 = \sqrt{42} \cdot h$$

$$h = \frac{23,04}{\sqrt{42}} = 3,55 \text{ cm}$$



4. Calcule o volume do tetraedro ABCD.

$$\vec{v} = \overrightarrow{AB} = (3, 4, 2)$$

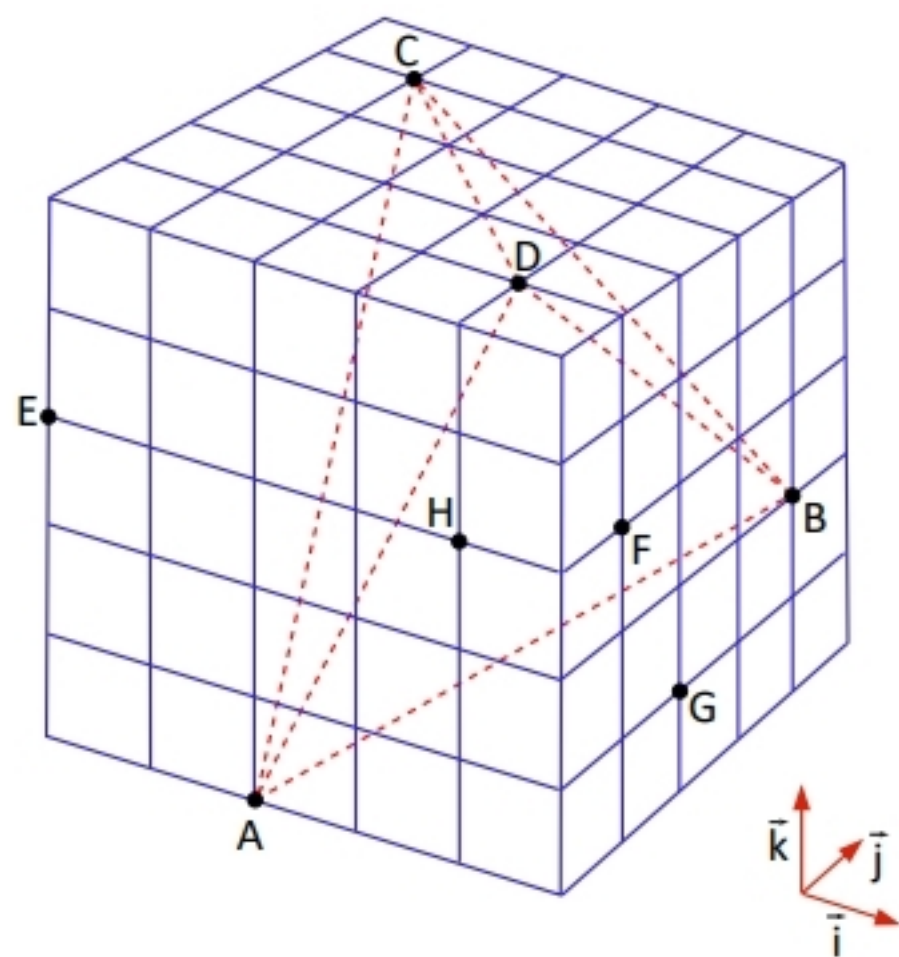
$$\vec{u} = \overrightarrow{AC} = (-1, 4, 5)$$

$$\vec{\theta} = \overrightarrow{AD} = (2, 1, 5)$$

$$\det \begin{vmatrix} 3 & 4 & 2 \\ -1 & 4 & 5 \\ 2 & 1 & 5 \end{vmatrix} = 60 + 40 - 2 - 16 - 15 + 20 = 87$$

$$\frac{1}{6} \cdot |\vec{v}, \vec{u}, \vec{\theta}| = \frac{|87|}{6} = 14,5 \text{ cm}^3$$





5. Calcule a distância do ponto B ao plano que contém o triângulo ADC.

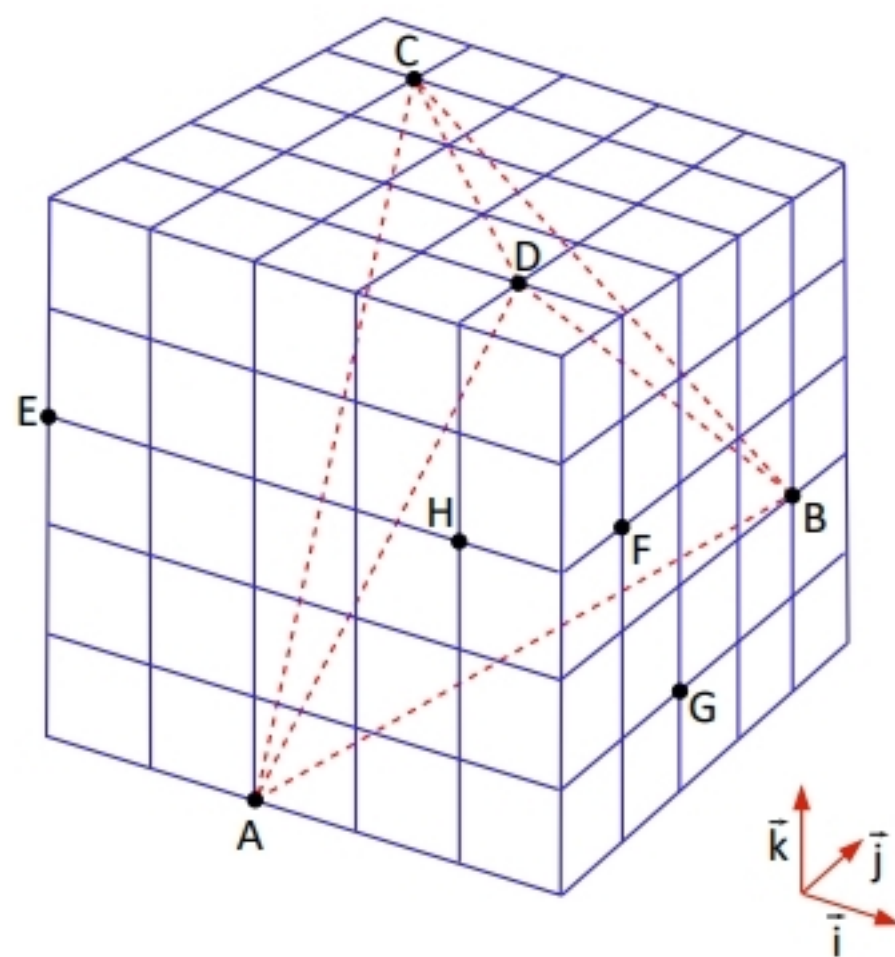
$$A_{ADC} = 11,52 \text{ un}$$

$$V = 14,5 \text{ un}^3$$

$$V = \frac{A_{ADC} \cdot h}{3}$$

$$14,5 \cdot 3 = 11,52 \cdot h$$

$$h = 3,78 \text{ un}$$

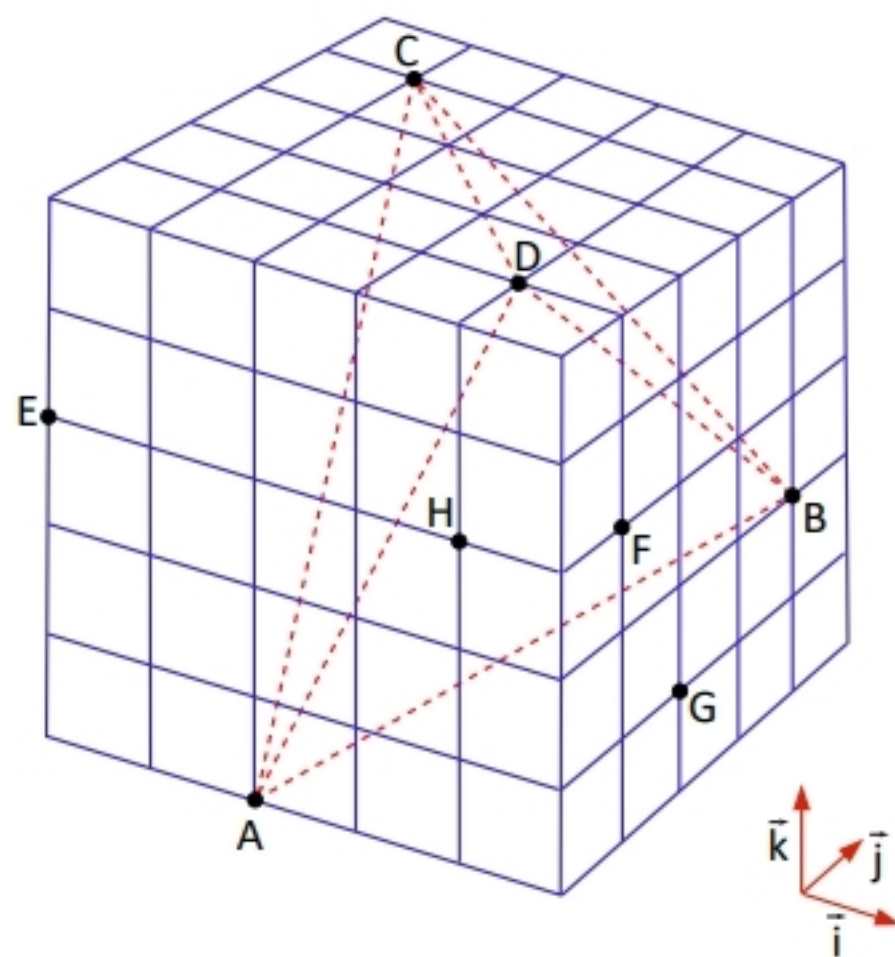


6. Calcule o comprimento da projeção ortogonal do segmento AB sobre o segmento AC.

$$\vec{v} = \vec{AB} = (3, 4, 2)$$

$$\vec{u} = \vec{AC} = (-1, 4, 5)$$

$$P_{\vec{u}} \vec{v} = \left( \frac{\vec{v} \cdot \vec{u}}{|\vec{u}|^2} \right) \vec{u} = \left( \frac{23}{(\sqrt{42})^2} \right) \cdot (-1, 4, 5) = \frac{-23 + 92 + 115}{42} = \frac{184}{42} = 4,38 \text{ um}$$



7. Mostre que os pontos E, A, G e B não são coplanares.

$$A = (2, 0, 0)$$

$$B = (5, 4, 2)$$

$$E = (0, 0, 3)$$

$$G = (3, 2, 1)$$

$$\vec{AB} = (3, 4, 2)$$

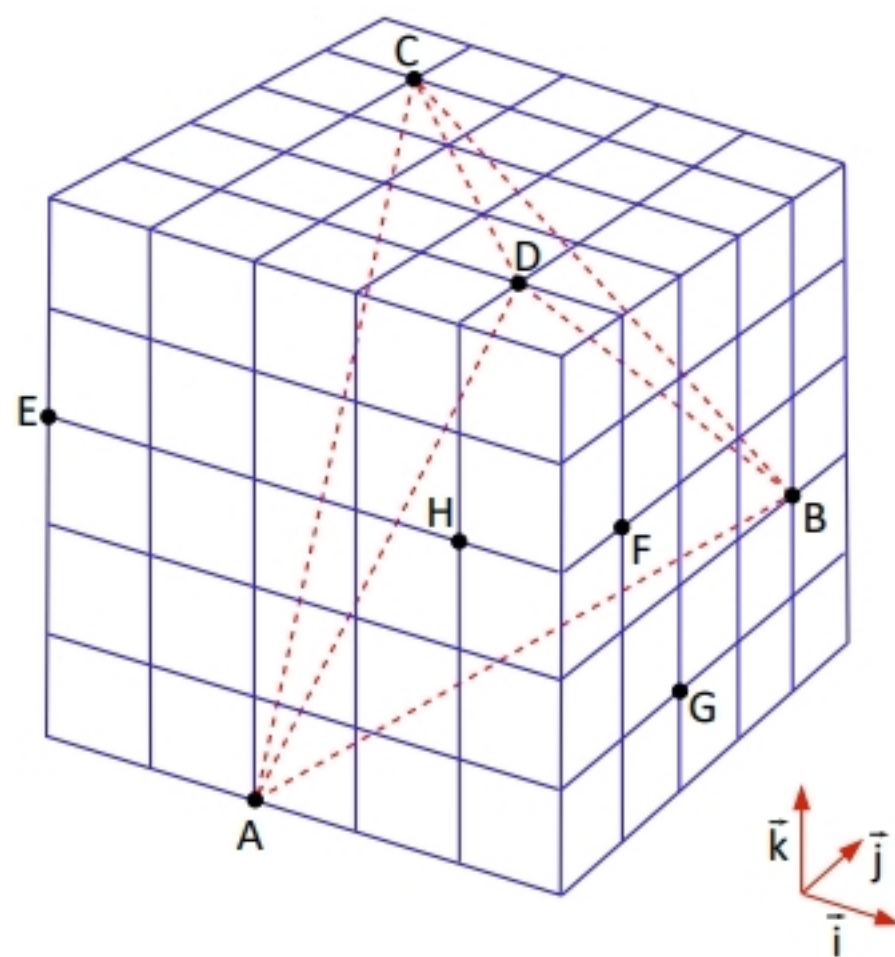
$$\vec{AE} = (-2, 0, 3)$$

$$\vec{AG} = (3, 2, 1)$$

$$(\vec{AB}, \vec{AE}, \vec{AG}) = \begin{vmatrix} 3 & 4 & 2 \\ -2 & 0 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0 + 36 - 8 - 0 - 18 + 8 = 18$$

Os pontos E, A, G, B não são coplanares, pois  $(\vec{AB}, \vec{AE}, \vec{AG}) \neq 0$





8. Existem quatro pontos no diagrama (dentre os oito pontos destacados) que são coplanares. Identifique-os e prove, algebricamente, que são coplanares.

$$F = (5, 1, 3) \quad \vec{CF} = (4, -3, -2)$$

$$D = (4, 1, 5) \quad \vec{CD} = (3, -3, 0)$$

$$C = (1, 4, 5) \quad \vec{CG} = (4, -2, -4)$$

$$G = (5, 2, 1)$$

$$(\vec{CD}, \vec{CF}, \vec{CG}) = \begin{vmatrix} 3 & -3 & 0 \\ 4 & -3 & -2 \\ 4 & -2 & -4 \end{vmatrix} = 36 + 24 + 0 - 0 - 12 - 48 = 0$$

\*Aparentemente não os pontos\*  
F, D, C, G

Os pontos F, D, C, G não coplanares, pois o produto misto dos vetores é 0