Foundations of High Performance Computing

Lecture 11: Using High Performance Libraries

2021-2022 Stefano Cozzini



"Foundation of HPC" course

DATA SCIENCE & SCIENTIFIC COMPUTING

Agenda

- Intro:
 - What are we using HPC for ?
 - Where should we start optimizing?
- High Performance Libraries
- Linear Algebra libraries
- Using HP libraries: some examples.

The seven dwarfs of HPC



- Phil Colella (LBL) identified 7 kernels of which most simulation and data analysis program are composed:
- Dense Linear Algebra
 - Ex: solve Ax=B or Ax=lambdax where A is a dense matrix
- Sparse Linear Algebra
 - solve Ax=B or Ax=lambdax where A is a sparse matrix (mostly zero)
- Operation on structured Grids:
 - ANEWj()=4*(A(i,j)-A(i-1,j)-A(i+1,j)-A(i,j-1)-A(i,j+1)
- Operation on unstructured Grids:
 - similar but list of neighbours varies from entry to entry
- Spectral Methods
 - Fast Fourier Transform (FFT)
- Particle Methods
 - Compute electrostatic forces on n-particles
- Monte Carlo
 - many independent simulation using different inputs

Is this the real picture in 2021?

We are missing ALL data analytics working load..

• But...

A lot of them relies on highly optimized libraries.

Where should you start optimizing your application?

Optimization techniques

- There are basically three different categories:
 - Improve memory performance (the most important)
 - Improve CPU performance



Use already highly optimized libraries/subroutines

The easiest and more efficient way...

What are High Performance Libraries ?

- Routines for common (math) functions written in a specific way to take advantage of all capabilities of the CPU.
- Each CPU type normally has its own version of the library specifically written or compiled to maximally exploit that architecture

Why using High Performance Libraries?

- Compilers can optimize code only to a certain point. Effective programming needs deep knowledge of the platform
- Performance libraries are designed to use the CPU in the most efficient way, which is not necessarily the most straightforward way.
- It is normally best to use the libraries supplied by or recommended by the CPU vendor
- On modern hardware they are hugely important, as they most efficiently exploit caches, special instructions and parallelism
- Parallelism (at least on single node) comes for free..

Any other reason apart from performance?

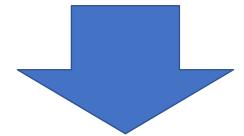
- Usage of libraries makes coding easier.
 Complicated math operations can be used from existing routines
- Increase portability of code as standard (and well optimized) libraries exist for ALL computing platforms.
- Lego approach: build your own code using already available bricks..

What is available?

- Linear Algebra:
 - BLAS/LAPACK/SCALAPACK
- FFT:
 - FFTW
- ODE/PDE
 - PETSC
- Machine Learning:
 - Tensorflow / Caffe etc..

Should I write my own algorithm for L. A. ?

- 99.99% of time: NO!
- Tons of libraries out there
- Well tested
- Extremely efficient in 99.99% of the case
- With some "de facto" standard implemented



PORTABILITY IS COMING (almost) FOR FREE

Why Linear Algebra?

- Basic Linear Algebra Problems:
 - Linear Equations: Solve Ax=b for x
 - Least Squares: Find x that minimizes $||\mathbf{r}||_2 \equiv \sqrt{\Sigma} \, \mathbf{r}_{\mathbf{i}^2}$ where $\mathbf{r} = \mathbf{A} \mathbf{x} \cdot \mathbf{b}$
 - Statistics: Fitting data with simple functions
 - Eigenvalues: Find λ and x where $Ax = \lambda x$
 - Vibration analysis, Quantum Simulations, etc...
 - Singular Value Decomposition: $A^TAx = \sigma^2x$
 - Data fitting, Information retrieval

Lots of variations depending on structure of A

Why dense Linear Algebra?

- Many large matrices are sparse, but ...
- Dense algorithms easier to understand
- Some applications yields large dense matrices
- Large sparse matrix algorithms often yield smaller (but still large) dense problems
- LINPACK Benchmark (<u>www.top500.org</u>)

"How fast is your computer?"
–

"How fast can you solve dense Ax=b?"

BLAS Basic Linear Algebra Subprograms

BLAS history

- In the beginning it was libraries like EISPACK (for eigenvalue problems)
- Then the BLAS-1 were invented (1973-1977)
 - Create a standard library of 15 operations (mostly) on vectors "AXPY" ($y = \alpha \cdot x + y$), dot product, scale ($x = \alpha \cdot x$), etc
 - Up to 4 versions of each (S/D/C/Z), 46 routines, 3300 LOC
 - Language: FORTRAN
- GOALS
 - Common "pattern" to ease programming, readability
 - Robustness, via careful coding (avoiding over/underflow) --> Accuracy Portability (common interface)
 - Efficiency via machine specific implementations
 - Maintaibility
- Why BLAS-1? They do O(n) ops on O(n) data
 - Used in libraries like LINPACK (for linear systems)
 - Source of the name "LINPACK Benchmark" (not the code!)

BLAS history

- But the BLAS-1 weren't enough
 - Consider AXPY ($y = \alpha \cdot x + y$): 2n flops on 3n read/writes
 - Computational intensity = (2n)/(3n) = 2/3
 - Too low to run near peak speed (read/write dominates)
- So the BLAS-2 were developed (1984-1986)
 - Standard library of 25 operations (mostly) on matrix/vector pairs
 - "GEMV": $y = \alpha \cdot A \cdot x + \beta \cdot x$, "GER": $A = A + \alpha \cdot x \cdot yT$, $x = T-1 \cdot x$
 - Up to 4 versions of each (S/D/C/Z), 66 routines, 18K LOC
- Why BLAS-2?
 - They do O(n²) ops on O(n²) data
 - So computational intensity still just $\sim (2 \text{ n}^2)/(\text{n}^2) = 2$
 - OK for vector machines, but not for machine with caches

BLAS history

- The next step: BLAS-3 (1987-1988)
 - Standard library of 9 operations (mostly) on matrix/matrix pairs
 - "GEMM": $C = \alpha \cdot A \cdot B + \beta \cdot C$, $C = \alpha \cdot A \cdot AT + \beta \cdot C$, $C = T-1 \cdot B$
 - Up to 4 versions of each (S/D/C/Z), 30 routines, 10K LOC
- Why BLAS 3?
 - They do O(n³) ops on O(n²) data
 - So computational intensity $(2 n^3)/(4 n^2) = n/2 big$ at last!
 - Good for machines with caches, other mem. hierarchy levels
 - Performing implementations left to others...

Where can I get BLAS?

www.netlib.org/blas

- Source: 142 routines, 31K LOC,
- Testing: 28K LOC
- Reference (unoptimized) implementation only!
- http://www.netlib.org/blas/# reference blas versi on 3 8 0
- Ex: 3 nested loops for GEMM

BLAS list

```
Level 1 BLAS
                        dim scalar vector
                                                                                        5-element array
                                                                                                                                                                                                   prefixes
SUBROUTINE MRDTG (
                                                                       A. B. C. S )
                                                                                                                    Generate plane rotation
                                                                                                                                                                                                   S, D
                                                             D1. D2, A. B.
SUBROUTINE *ROTMG(
                                                                                        PARAN )
                                                                                                                    Generate modified plane rotation
                                                                                                                                                                                                   S, D
SUBROUTINE MROT ( N.
                                                                               c. s )
                                     X, INCX, Y, INCY,
                                                                                                                    Apply plane rotation
                                                                                                                                                                                                   S. D
SUBROUTINE MADTH ( N.
                                      I, INCK, Y. INCY.
                                                                                                                    Apply modified plane rotation
                                                                                                                                                                                                   S. D
SUBROUTINE ASWAP ( N.
                                      I. INCE, Y. INCY )
                                                                                                                    x\leftrightarrow y
                                                                                                                                                                                                   S. D. C. Z.
SUBROUTINE *SCAL ( N. ALPHA, X. INCX )
                                                                                                                                                                                                    S, D, C, Z, CS, ZD
SUBROUTINE *COPY ( N.
                                      M. INCK, Y. INCY )
                                                                                                                    y \leftarrow x
                                                                                                                                                                                                   S. D. C. Z.
SUBROUTINE MAXPY ( N.
                             ALPHA, X, INCX, Y, INCY )
                                                                                                                    y \leftarrow \alpha x + y
                                                                                                                                                                                                   S. D. C. Z
FUNCTION MOOT ( N.
                                     M. INCM. Y. INCY )
                                                                                                                    dot \leftarrow x^T y
                                                                                                                                                                                                   S. D. DS
                                                                                                                    dot \leftarrow x^T y
FUNCTION
             *DOTU ( N.
                                      M. INCM. Y. INCY )
                                                                                                                                                                                                   C. Z
                                                                                                                    dot \leftarrow x^H y
FUNCTION
             MOOTO ( N.
                                      I, INCI, Y, INCY )
                                                                                                                                                                                                   C. Z
                                                                                                                    dot \leftarrow \alpha + x^T y
FUNCTION
             MEDOT ( N.
                                      M. INCM. Y. INCY )
                                                                                                                                                                                                   SDS
FUNCTION
             HERRYZ ( N.
                                      I, INCE )
                                                                                                                    nrm2 \leftarrow |x|_2
                                                                                                                                                                                                   S. D. SC. DZ
                                      I, INCE )
             MASUM ( N.
                                                                                                                    asum \leftarrow ||re(x)||_1 + ||im(x)||_1
FUNCTION
                                                                                                                                                                                                   S, D, SC, DZ
                                     I, INCE )
                                                                                                                    amax \leftarrow 1^{st}k \ni |re(x_k)| + |im(x_k)|
                                                                                                                                                                                                   S, D, C, Z
FUNCTION
             INAMARC M.
                                                                                                                                   = max(|re(x_i)| + |im(x_i)|)
Level 2 BLAS
                                  dim b-width scalar matrix vector scalar vector
XCEMY (
                   TRANS.
                                  8. N.
                                                    ALPHA, A, LDA, X, INCX, BETA, Y, INCY )
                                                                                                                    y \leftarrow \alpha Ax + \beta y, y \leftarrow \alpha A^T x + \beta y, y \leftarrow \alpha A^H x + \beta y, A - m \times n
                                                                                                                                                                                                   S, D, C, Z
                                  M, N, ML, NU, ALPHA, A, LDA, X, INCX, BETA, Y, INCY )
                                                                                                                    y \leftarrow \alpha Ax + \beta y, y \leftarrow \alpha A^T x + \beta y, y \leftarrow \alpha A^H x + \beta y, A - m \times n
*CSMY (
                   TRANS.
                                                                                                                                                                                                   S, D, C, Z
XHENV ( UPLO.
                                      x.
                                                    ALPHA, A, LDA, X, INCX, BETA, Y, INCY )
                                                                                                                    y \leftarrow \alpha Ax + \beta y
                                                                                                                                                                                                   C. Z
XHBMV ( UPLO,
                                      N. K.
                                                    ALPHA, A. LDA, X. INCX, BETA, Y. INCY )
                                                                                                                    y \leftarrow \alpha Ax + \beta y
                                                                                                                                                                                                   C. Z
*HPMV ( UPLO.
                                      ×.
                                                    ALPHA, AP. X, INCX, BETA, Y, INCY )
                                                                                                                    y \leftarrow \alpha Ax + \beta y
                                                                                                                                                                                                   C.Z
                                                    ALPHA, A. LDA, X. INCX, BETA, Y. INCY )
                                                                                                                    y \leftarrow \alpha Ax + \beta y
ESYMP ( UPLO.
                                      N.
                                                                                                                                                                                                   S, D
ESBMY ( UPLO.
                                                    ALPRA, A. LDA, X. INCX, BETA, Y. INCY )
                                                                                                                    y \leftarrow \alpha Ax + \beta y
                                                                                                                                                                                                   S, D
                                      N. K.
ESPHY ( UPLO.
                                      N.
                                                    ALPHA, AP. X, INCX, BETA, Y, INCY )
                                                                                                                    y \leftarrow \alpha Ax + \beta y
                                                                                                                                                                                                   S. D.
                                                           A, LDA, X, INCX )
XTRMV ( UPLO, TRANS, DIAG,
                                      x.
                                                                                                                    x \leftarrow Ax, x \leftarrow A^Tx, x \leftarrow A^Hx
                                                                                                                                                                                                   S, D, C, Z
                                                                                                                    x \leftarrow Ax, x \leftarrow A^Tx, x \leftarrow A^Hx
XTBMV ( UPLO, TRANS, DIAG,
                                                             A, LDA, X, INCX )
                                                                                                                                                                                                   S, D, C, Z
                                                                                                                    x \leftarrow Ax, x \leftarrow A^Tx, x \leftarrow A^Hx
XTPMV ( UPLO, TRANS, DIAG,
                                                             AP, X, INCX )
                                     N.
                                                                                                                                                                                                   S. D. C. Z.
                                                                                                                    x \leftarrow A^{-1}x, x \leftarrow A^{-T}x, x \leftarrow A^{-H}x
XTRSV ( UPLO, TRANS, DIAG,
                                                             A, LDA, X, INCX )
                                                                                                                                                                                                   S. D. C. Z.
                                                                                                                    x \leftarrow A^{-1}x, x \leftarrow A^{-T}x, x \leftarrow A^{-H}x
xTBSV ( UPLO, TRANS, DIAG.
                                                             A. LDA. X. INCX )
                                                                                                                                                                                                   S. D. C. Z.
                                      S. K.
                                                                                                                    x \leftarrow A^{-1}x, x \leftarrow A^{-T}x, x \leftarrow A^{-H}x
XTPSV ( UPLO, TRANS, DIAG,
                                                            AP. X, INCX )
                                                                                                                                                                                                   S, D, C, Z
                                  dim scalar vector vector matrix
          options
*CER
                                  M. H. ALPHA, X. INCX, Y. INCY, A. LDA )
                                                                                                                    A \leftarrow \alpha x y^T + A, A - m \times n
                                                                                                                                                                                                   S. D
                                                                                                                    A \leftarrow \alpha x y^T + A, A - m \times n
XCERU (
                                  H, N, ALPHA, X, INCX, Y, INCY, A, LDA )
                                                                                                                                                                                                   C. Z
                                                                                                                    A \leftarrow \alpha x y^H + A, A - m \times n
XCERC (
                                  M. N. ALPHA, E. INCE, Y. INCY, A. LDA )
                                                                                                                                                                                                   C. Z
XHER ( UPLO.
                                     N. ALPHA, N. INCK.
                                                                         A. LDA )
                                                                                                                    A \leftarrow \alpha x x^H + A
                                                                                                                                                                                                   C. Z
                                                                                                                    A \leftarrow \alpha xx^{H} + A
xHPR ( UPLO,
                                      N. ALPHA, X. INCX.
                                                                          AP 3
                                                                                                                                                                                                   C. Z
                                                                                                                    A \leftarrow \alpha x y^H + y(\alpha x)^H + A
                                      N, ALPHA, X, INCX, Y, INCY, A, LDA )
xHER2 ( UPLO.
                                                                                                                                                                                                   C. Z
                                                                                                                    A \leftarrow \alpha x y^H + y(\alpha x)^M + A
xHPR2 ( UPLO,
                                      N, ALPHA, N, INCE, Y, INCY, AP )
                                                                                                                                                                                                   C, Z
xSYR ( UPLO,
                                      N. ALPHA, X. INCX.
                                                                          A. LDA )
                                                                                                                    A \leftarrow \alpha x x^T + A
                                                                                                                                                                                                   S. D
                                                                                                                    A \leftarrow oxx^T + A
xSPR ( UPLO,
                                      N. ALPHA, X. INCX.
                                                                          AP )
                                                                                                                                                                                                   S. D
                                                                                                                   A \leftarrow \alpha x y^T + \alpha y x^T + A
                                                                                                                                                                                                   S, D
*SYR2 ( UPLO.
                                     N. ALPRA, E. INCE, Y. INCY, A. LDA )
                                                                                                                    A \leftarrow \alpha x y^T + \alpha y x^T + A
xSPR2 ( UPLO.
                                      N. ALPHA, X. INCX, Y. INCY, AP )
Level 3 BLAS
                                                               scalar matrix matrix scalar matrix
         options
                                                                                                                   C \leftarrow aop(A)op(B) + \beta C, op(X) = X, X^T, X^H, C - m \times n
*C3000 (
                         TRANSA, TRANSB,
                                                    H, N, K, ALPHA, A, LDA, B, LDB, BETA, C, LDC )
                                                                                                                                                                                                   S. D. C. Z.
XSYMM ( SIDE, UPLO,
                                                              ALPHA, A. LDA, B. LDB, BETA, C. LDC )
                                                                                                                    C \leftarrow \alpha AB + \beta C, C \leftarrow \alpha BA + \beta C, C - m \times n, A = A^T
                                                                                                                                                                                                   S. D. C. Z.
                                                                                                                   C \leftarrow \alpha AB + \beta C, C \leftarrow \alpha BA + \beta C, C - m \times n, A = A^{H}
XHEMM ( SIDE, UPLO,
                                                              ALPHA, A. LDA, B. LDB, BETA, C. LDC )
                                                                                                                                                                                                   C, Z
                                                                                                                    C \leftarrow \alpha A A^T + \beta C, C \leftarrow \alpha A^T A + \beta C, C - n \times n
xSYRK (
                 UPLO, TRANS.
                                                        N. K. ALPHA, A. LDA.
                                                                                             BETA, C. LDC )
                                                                                                                                                                                                   S, D, C, Z
                                                                                                                    C \leftarrow \alpha A A^H + \beta C, C \leftarrow \alpha A^H A + \beta C, C - n \times n
                 UPLO, TRANS.
                                                        N. K. ALPHA, A. LDA.
*HERK (
                                                                                             BETA, C. LDC )
                                                                                                                                                                                                   C.Z
                                                                                                                    C \leftarrow \alpha A B^T + \tilde{\alpha} B A^T + \beta C, C \leftarrow \alpha A^T B + \tilde{\alpha} B^T A + \beta C, C - n \times n
XSYR2X (
                 UPLO, TRANS,
                                                        N. K. ALPHA, A. LDA, B. LDB, BETA, C. LDC )
                                                                                                                                                                                                   S. D. C. Z.
                                                                                                                    C \leftarrow \alpha A B^H + \tilde{\alpha} B A^H + \beta C, C \leftarrow \alpha A^H B + \tilde{\alpha} B^H A + \beta C, C - n \times n
*HER2K (
                 UPLO, TRANS,
                                                        N, K, ALPHA, A, LDA, B, LDB, BETA, C, LDC )
                                                                                                                                                                                                   C, Z
                                                                                                                    B \leftarrow \alpha op(A)B, B \leftarrow \alpha Bop(A), op(A) = A, A^T, A^H, B - m \times n
XTRMM ( SIDE, UPLD, TRANSA,
                                            DIAG, M. N. ALPHA, A. LDA, B. LDB )
                                                                                                                                                                                                   S. D. C. Z.
                                                                                                                    B \leftarrow \text{coop}(A^{-1})B, B \leftarrow \alpha Bop(A^{-1}), op(A) = A, A^T, A^H, B - m \times n
XTRSM ( SIDE, UPLO, TRANSA,
                                            DIAG. M. N.
                                                              ALPHA, A, LDA, B, LDB )
```

BLAS summary

Basic Linear Algebra Subroutines

Name	Description	Examples
Level-1 BLAS	Vector Operations	$C = \sum X_i Y_i$
Level-2 BLAS	Matrix-Vector Operations	$\boldsymbol{B}_i = \sum_k \boldsymbol{A}_{ik} \boldsymbol{X}_k$
Level-3 BLAS	Matrix-Matrix Operations	$C_{ij} = \sum_{k} A_{ik} B_{kj}$

Why BLAS are important?

- Because the BLAS are efficient, portable, parallel, and widely available, they are commonly used in the development of high quality linear algebra software.
- Performance of lot of applications depends a lot on the performance of the underlying BLAS
- Lot of applications include ML/DL stuff as well....
 - https://petewarden.com/2015/04/20/why-gemm-is-at-the-heartof-deep-learning/

Standardization: BLAS example

- Each BLAS
 Subroutines have a standardized layout
- BLAS is documented in the source code
- Man pages exist
- Vendor supplied docs
- Different BLAS implementations have the same calling sequence

```
TRANSA, TRANSB
                     M. N. K. LDA. LDB. LDC
                     ALPHA. BETA
                     A( LDA, * ), B( LDB, * ), C( LDC, * )
      PERFORMS ONE OF THE MATRIX-MATRIX OPERATIONS
  C := ALPHA*OP( A )*OP( B ) * BETA*C.
WHERE OP(X) IS ONE OF
ALPHA AND BETA ARE SCALARS. AND A. B AND C ARE MATRICES. WITH OP( A )
AN M BY K MATRIX. OP(B) A K BY N MATRIX AND CAN M BY N
TRAKSA - CHARACTER*1.
        ON ENTRY, TRANSA SPECIFIES THE FORM OF OP( A ) TO BE USED IN
         THE MATRIX MULTIPLICATION AS FOLLOWS:
           TRANSA = 'N' OR 'N'. OP(A) = A.
           TRANSA = 'T' OR 'T' OP(A) = A'.
           TRANSA = "C" OR "C". OP(A) = A".
        UNCHANGED ON EXIT.
TRAMSB - CHARACTER*1.
        ON ENTRY, TRANSB SPECIFIES THE FORM OF OP( B ) TO BE USED IN
         THE MATRIX MULTIPLICATION AS FOLLOWS:
            TRANSB = "N" OR "N" OP(B) = B.
           TRAKSB = 'T' OR 'T', OP(B) = B'.
```

Vendor/Optimized BLAS libraries

ACML

 The AMD Core Math Library, supporting the AMD processors

ATLAS

 Automatically Tuned Linear Algebra, an open source implementation of BLAS APIs for C and Fortran 77

Intel MKL

 The Intel Math Kernel Library supporting x86 32-bits and 64-bits. Includes optimizations for Intel Pentium, Core and Intel Xeon CPUs and Intel Xeon Phi; suppor for Linux, Windows and Mac OS X

cuBLAS

 Optimized BLAS for NVIDIA based GPU cards

ESSL

 IBM's Engineering and Scientific Subroutine Library, supporting the PowerPC architecture under AIX and Linux

GotoBLAS

 Kazushige Goto's BSD-licensed implementation of BLAS, tuned in particular for Intel, VIA Nanoprocessor, AMD Opteron

OpenBLAS

- Optimized BLAS based on Goto BLAS hosted at GitHub, supporting Intel platform and other
- And many others...

What about C/C++ program?

- BLAS routines are Fortran-style, when calling them from C language programs, follow the Fortran-style calling conventions:
 - Pass variables by address, not by value.
 - Store your data in Fortran style, that is, column-major rather than row-major order.
- Be aware that because the Fortran language is caseinsensitive, the routine names can be both upper-case or lower-case, with or without the trailing underscore.
- For example, the following names are equivalent:
 - dgemm, DGEMM, dgemm_, and DGEMM_

Use CBLAS!

C-style interface to the BLAS routines

www.netlib.org/blas/blast-forum/cblas.tgz

- •You can call CBLAS routines using regular C- style calls.
- •The header file specifies enumerated values and prototypes of all the functions.
- Details and examples here:

https://software.intel.com/en-us/mkl-tutorial-c-multiplying-matrices-using-dgemm

Q parameter: aka computational efficiency...

Table 2: Basic Linear Algebra Subroutines (BLAS)

Operation	Definition	Floating	Memory	q
		point	references	
		operations		
saxpy	$y_i = \alpha x_i + y_i, \ i = 1,, n$	2n	3n + 1	2/3
Matrix-vector mult	$y_i = \sum_{j=1}^n A_{ij}x_j + y_i$	$2n^2$	$n^2 + 3n$	2
Matrix-matrix mult	$C_{ij} = \sum_{k=1}^{n} A_{ik} B_{kj} + C_{ij}$	$2n^3$	$4n^2$	n/2

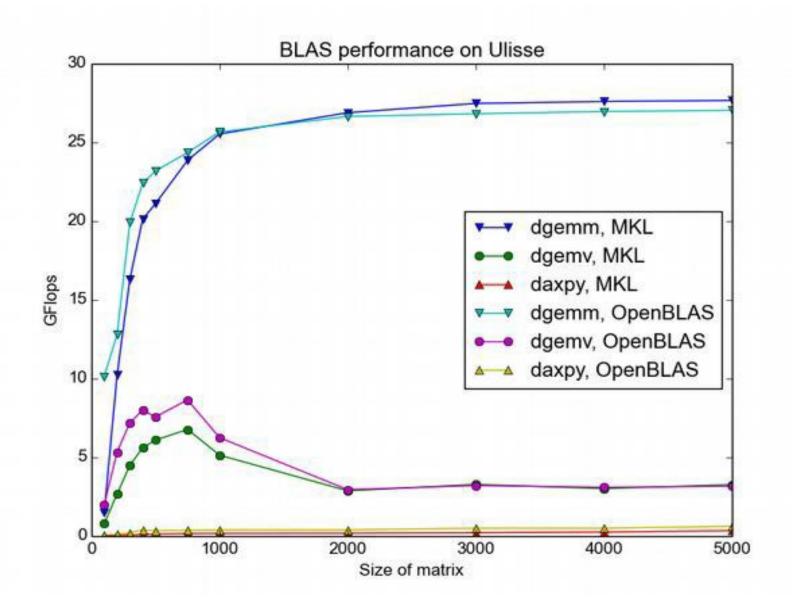
The parameter q is the ratio of flops to memory references. Generally:

- 1.Larger values of q maximize useful work to time spent moving data.
- 2. The higher the level of the BLAS, the larger q.

It follows...

- BLAS1 are memory bounded !(for each computation a memory transfer is required)
- BLAS2 are not so memory bounded (can have good performance on super-scalar architecture)
- BLAS3 can be very efficient on super-scalar computers because not memory bounded

BLAS performance on SandyBridge (1core)



Proposed exercise/tutorial

 Create the same graph for ORFEO cores using MKL and OpenBLAS

• STEPS:

- Install OpenBLAS libraries in your directory
- Write a small program to call the three routines
 - Dgemm/dgemv/daxypi
- Write a script to collect all sizes of interest
- Make nice plots

Linking optimized libraries...

OpenBLAS:

- In comes with cblas bundled in so no problem with C/C++
- Automatically includes lapack reference implementation
- Compilation is straightforward:

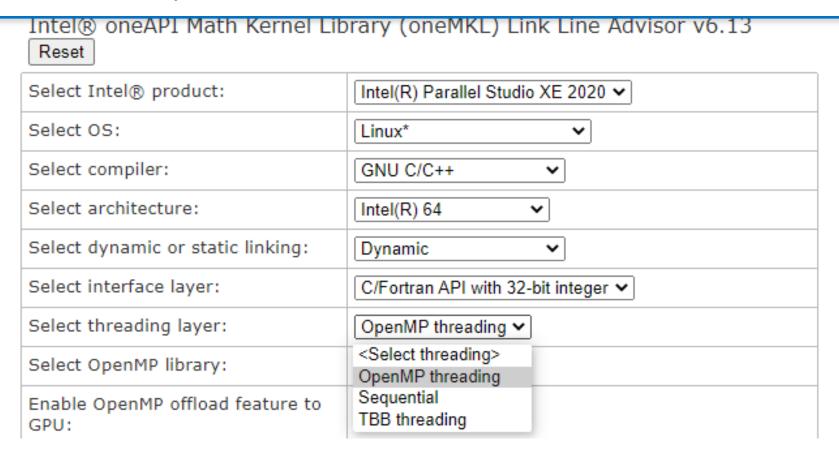
```
gcc -o test test.c -I
/your_path/OpenBLAS/include/
-L/your_path/OpenBLAS/lib -lopenblas
```

•MKL

- Generally complex and highly dependent on version and/or HW/SW implementation
- •https://software.intel.com/en-us/articles/intel-mkl-link-line-advisor

Are this libraries multithreaded?

- MKL
 - Both sequential and multithreaded version available



MKL: how to control number of threads?

- OpenMP threading ? → OMP_NUM_THREADS
- Other threading ? → MKL_NUM_THREDS
- Define yourself the number of threads:
 - Place mkl_set_num_thread(N) routine in your code.
- All MKL routines call takes precedence over any environment variables. and MKL environment Variables will take precedence over the OpenMP* environments.

More details

here:ttps://software.intel.com/content/www/us/en/develop/articles/recommended-settings-for-calling-intel-mkl-routines-from-multi-threaded-applications.html

OpenBLAS:

- By default: multithreaded version, maximum number of threads established by cores available on the machine when compilation is performed;
- In our case:

```
OpenBLAS build complete. (BLAS CBLAS LAPACK LAPACKE)

OS ... Linux
Architecture ... x86_64
BINARY ... 64bit
C compiler ... GCC (cmd & version : cc (GCC) 4.8.5 20150623 (Red Hat 4.8.5-39))
Fortran compiler ... GFORTRAN (cmd & version : GNU Fortran (GCC) 9.3.0)
Library Name ... libopenblas_haswellp-r0.3.13.a (Multi-threading; Max num-threads is 48)

To install the library, you can run "make PREFIX=/path/to/your/installation install".
```

OpenBLAS: caveat

- If your application is already multi-threaded, it will conflict with OpenBLAS multi-threading. Thus, you must set OpenBLAS to use single thread as following.
 - export OPENBLAS_NUM_THREADS=1 in the environment variables.
 - call openblas_set_num_threads(1) in the application on runtime.
- You can compile the library itself in sequential mode:
 - make USE_THREAD=0 USE_LOCKING=1*(see comment below)
 - If your application is parallelized by OpenMP, please build OpenBLAS with USE_OPENMP=1

^{*} the thread management provided by OpenMP is not sufficient to prevent race conditions when OpenBLAS was built single-threaded by USE_THREAD=0 and there are concurrent calls from multiple threads to OpenBLAS functions. In this case, it is vital to also specify USE_LOCKING=1