Hiding Access Patterns

Obliviousness and Differential Privacy

Giuseppe Persiano

Università di Salerno

February 12, 2019

Describing joint work with:
Sarvar Patel, Mariana Raykova and Kevin Yeo (Google LLC)

Cloud Storage (simplified)

The perfect marriage of two parties

- The Data Owner $\mathcal{O}$:
  owns large amount of data and not enough local storage
- The Storage Manager $\mathcal{M}$:
  owns large amount of storage and not enough data

Cloud Storage (simplified)

The perfect marriage of two parties

- The Data Owner $\mathcal{O}$:
  owns large amount of data and not enough local storage
- The Storage Manager $\mathcal{M}$:
  owns large amount of storage and not enough data

If $\mathcal{O}$ and $\mathcal{M}$ trust each other

Cloud Storage (simplified)

The perfect marriage of two parties

- The Data Owner $\mathcal{O}$:
  owns large amount of data and not enough local storage
- The Storage Manager $\mathcal{M}$:
  owns large amount of storage and not enough data

If $\mathcal{O}$ and $\mathcal{M}$ trust each other no problem. we can go home now.

Cloud Storage (simplified)

The perfect marriage of two parties

- The Data Owner $\mathcal{O}$:
  owns large amount of data and not enough local storage
- The Storage Manager $\mathcal{M}$:
  owns large amount of storage and not enough data

If $\mathcal{O}$ and $\mathcal{M}$ trust each other no problem. we can go home now.

Lack of trust is much more interesting.

Enter Encryption

$\mathcal{O}$ does not trust $\mathcal{M}$ because $\mathcal{O}$'s data contain personal data.

Enter Encryption

$\mathcal{O}$ should not trust $\mathcal{M}$ because $\mathcal{O}$'s data contain personal data.

Enter Encryption

$\mathcal{O}$ should not trust $\mathcal{M}$ because $\mathcal{O}$'s data contain personal data.

Use Encryption

- Private Key: if $\mathcal{O}$ is the source of data
- Public Key: if data come from various sources

Enter Encryption

$\mathcal{O}$ should not trust $\mathcal{M}$ because $\mathcal{O}$'s data contain personal data.

Use Encryption

- Private Key: if $\mathcal{O}$ is the source of data
- Public Key: if data come from various sources

Data is

- encrypted before being uploaded to $\mathcal{M}$

Enter Encryption

$\mathcal{O}$ should not trust $\mathcal{M}$ because $\mathcal{O}$'s data contain personal data.

Use Encryption

- Private Key: if $\mathcal{O}$ is the source of data
- Public Key: if data come from various sources

Data is

- encrypted before being uploaded to $\mathcal{M}$
- decrypted when downloaded from $\mathcal{M}$

Are we done?

What if $\mathcal{O}$ wants to run an algorithm on the encrypted data?

Are we done?

What if $\mathcal{O}$ wants to run an algorithm on the encrypted data?

Running an algorithm might reveal information on the data.

Are we done?

What if $\mathcal{O}$ wants to run an algorithm on the encrypted data?

Running an algorithm might reveal information on the data.

Suppose $\mathcal{O}$ wants to sort the data.

Are we done?

What if $\mathcal{O}$ wants to run an algorithm on the encrypted data?

Running an algorithm might reveal information on the data.

Suppose $\mathcal{O}$ wants to sort the data.

Example

4 customers must be sorted according to revenue.

Are we done?

What if $\mathcal{O}$ wants to run an algorithm on the encrypted data?

Running an algorithm might reveal information on the data.

Suppose $\mathcal{O}$ wants to sort the data.

Example

4 customers must be sorted according to revenue.

1. download 1 and 3. decrypt, swap if out of order, re-encrypt, upload.

C;100	D;150	A;200	B;300

Are we done?

What if $\mathcal{O}$ wants to run an algorithm on the encrypted data?

Running an algorithm might reveal information on the data.

Suppose $\mathcal{O}$ wants to sort the data.

Example

4 customers must be sorted according to revenue.

1. download 1 and 3. decrypt, swap if out of order, re-encrypt, upload.
2. download 2 and 4. decrypt, swap if out of order, re-encrypt, upload.

C;100	D;150	A;200	B;300

Are we done?

What if $\mathcal{O}$ wants to run an algorithm on the encrypted data?

Running an algorithm might reveal information on the data.

Suppose $\mathcal{O}$ wants to sort the data.

Example

4 customers must be sorted according to revenue.

1. download 1 and 3. decrypt, swap if out of order, re-encrypt, upload.
2. download 2 and 4. decrypt, swap if out of order, re-encrypt, upload.
3. download 1 and 2. decrypt, swap if out of order, re-encrypt, upload.

C;100	D;150	A;200	B;300

Are we done?

What if $\mathcal{O}$ wants to run an algorithm on the encrypted data?

Running an algorithm might reveal information on the data.

Suppose $\mathcal{O}$ wants to sort the data.

Example

4 customers must be sorted according to revenue.

1. download 1 and 3. decrypt, swap if out of order, re-encrypt, upload.
2. download 2 and 4. decrypt, swap if out of order, re-encrypt, upload.
3. download 1 and 2. decrypt, swap if out of order, re-encrypt, upload.
4. download 3 and 4. decrypt, swap if out of order, re-encrypt, upload.

C;100 D;150 A;200 B;300

Are we done?

What if $\mathcal{O}$ wants to run an algorithm on the encrypted data?

Running an algorithm might reveal information on the data.

Suppose $\mathcal{O}$ wants to sort the data.

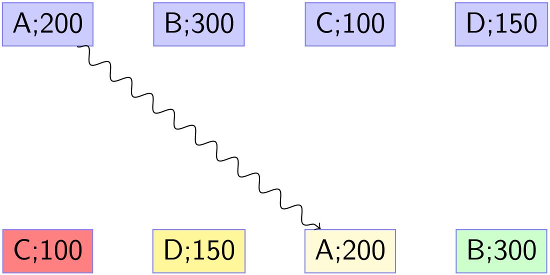
Example

4 customers must be sorted according to revenue.

1. download 1 and 3. decrypt, swap if out of order, re-encrypt, upload.
2. download 2 and 4. decrypt, swap if out of order, re-encrypt, upload.
3. download 1 and 2. decrypt, swap if out of order, re-encrypt, upload.
4. download 3 and 4. decrypt, swap if out of order, re-encrypt, upload.
5. download 2 and 3. decrypt, swap if out of order, re-encrypt, upload.

Security

Can $\mathcal{M}$ link the first record in the starting configuration to its position in the last configuration?



Two Concepts

Indistinguishability of *Swap or Not*

- Download, Decrypt, Swap or Not, Re-encrypt, Upload

Two Concepts

Indistinguishability of *Swap or Not*

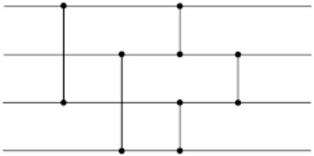
- Download, Decrypt, Swap or Not, Re-encrypt, Upload

Chosen-Ciphertext Security: Standard notion of security for encryption guarantee that $\mathcal{M}$ is unable to deduce if a swap has happened.

Enter Obliviousness

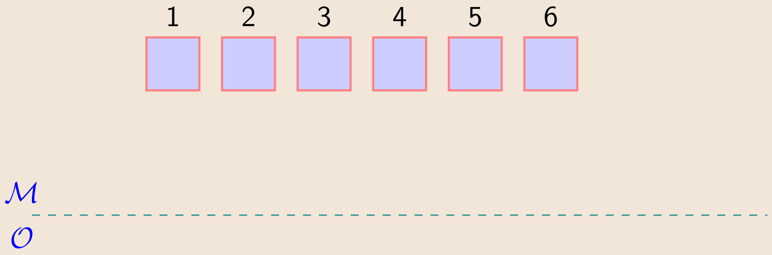
Definition (Weak Obliviousness)

An algorithm is *weakly oblivious* if the *access pattern* to data is the same for all possible inputs of the same length.

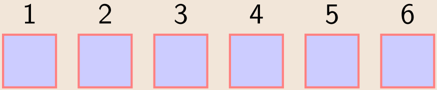


Thanks to Wikipedia for the image

The adversarial setting



The adversarial setting

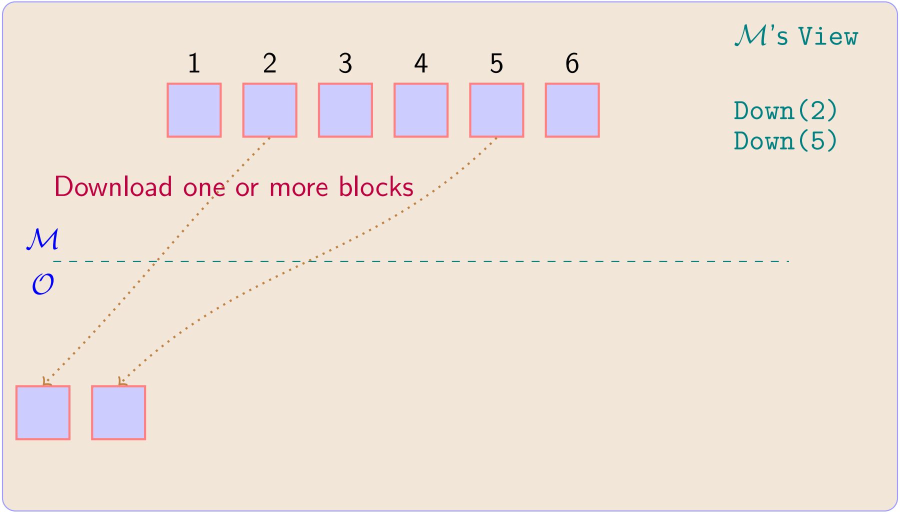


Download one or more blocks

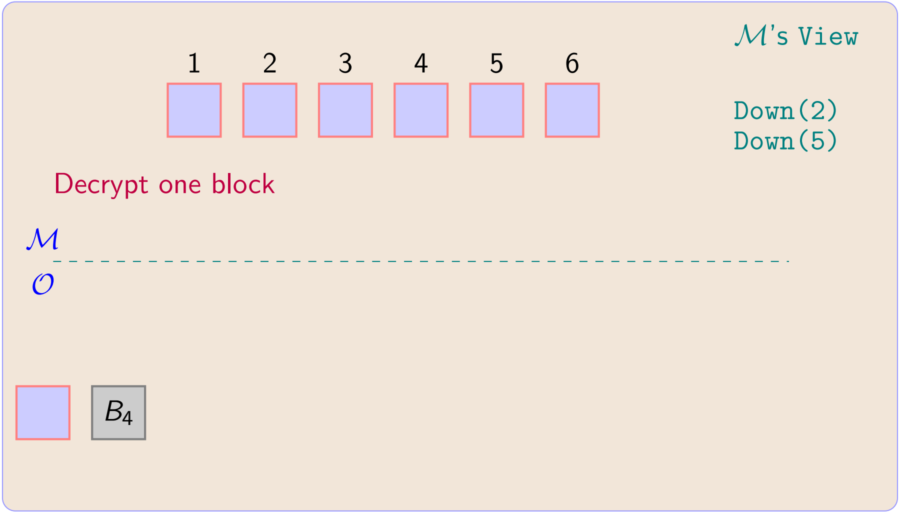
$\mathcal{M}$

$\mathcal{O}$

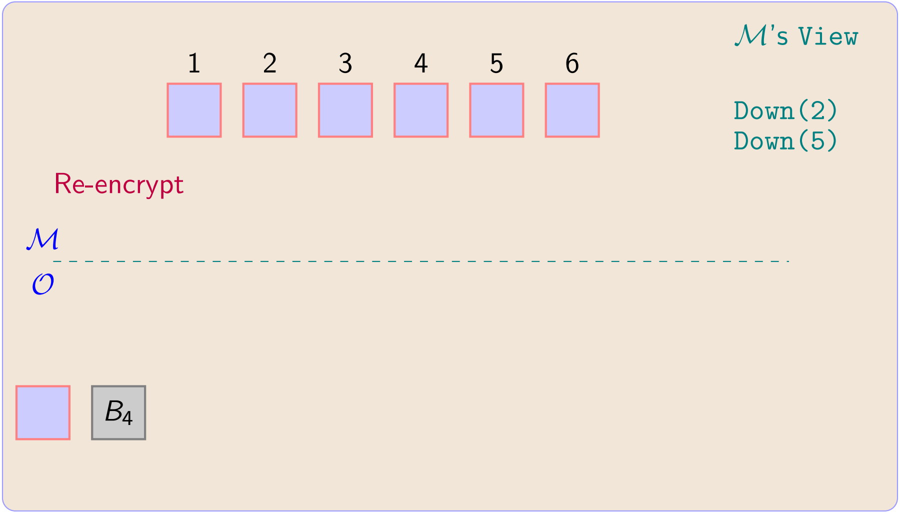
The adversarial setting



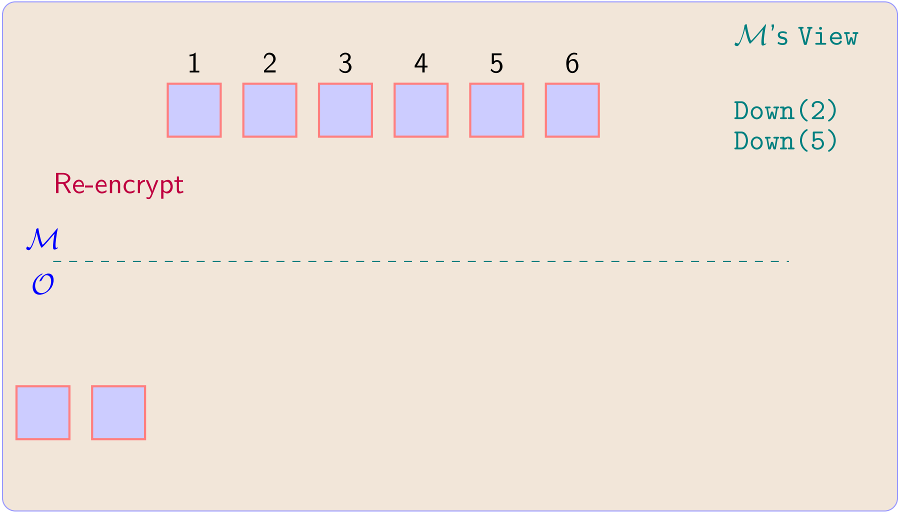
The adversarial setting



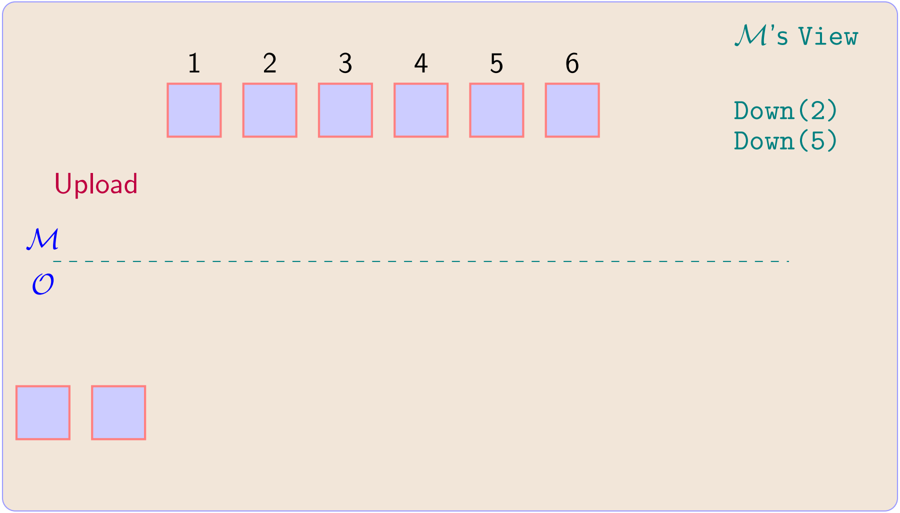
The adversarial setting



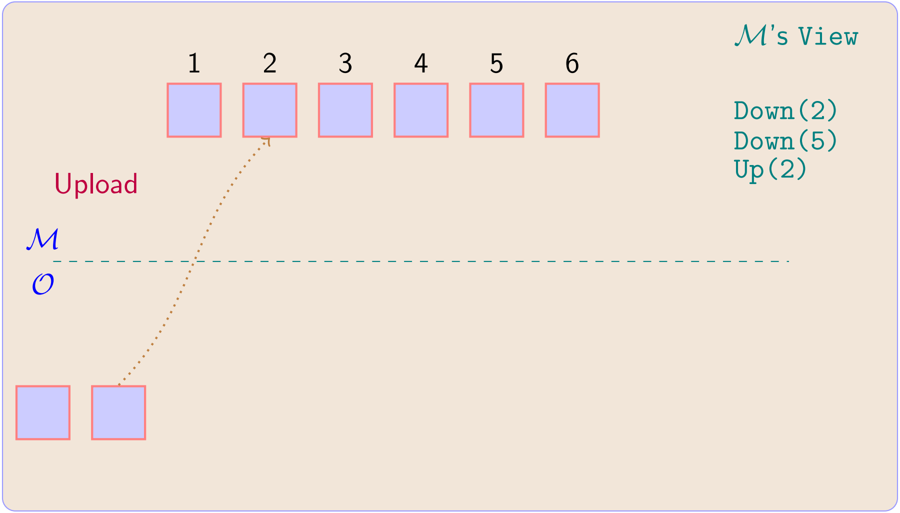
The adversarial setting



The adversarial setting



The adversarial setting



A new industry

Job Opportunities for Algorithmists

- Re-design all algorithms to be oblivious!
- Remove all *if*s, and *while*s
- Insertion Sort is not oblivious:
  - when the last element of the array is inserted, $\mathcal{M}$ sees where it lands

A new industry

Job Opportunities for Algorithmists

- Re-design all algorithms to be oblivious!
- Remove all *if*s, and *while*s
- Insertion Sort is not oblivious:
  - when the last element of the array is inserted, $\mathcal{M}$ sees where it lands

Abbiamo abolito la povertà

A new industry

Job Opportunities for Algorithmists

- Re-design all algorithms to be oblivious!
- Remove all *if*s, and *while*s
- Insertion Sort is not oblivious:
  - when the last element of the array is inserted, $\mathcal{M}$ sees where it lands

Abbiamo abolito la povertà ... per gli algoritmisti

A new industry

Job Opportunities for Algorithmists

- Re-design all algorithms to be oblivious!
- Remove all *if*s, and *while*s
- Insertion Sort is not oblivious:
  - when the last element of the array is inserted, $\mathcal{M}$ sees where it lands

What? Just move on to the next slide and stop talking politics

Hiding the Algorithm

A new threat

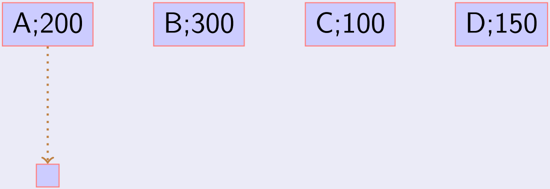
- which algorithm is being run should also be private information

A;200	B;300	C;100	D;150

Hiding the Algorithm

A new threat

- which algorithm is being run should also be private information



Hiding the Algorithm

A new threat

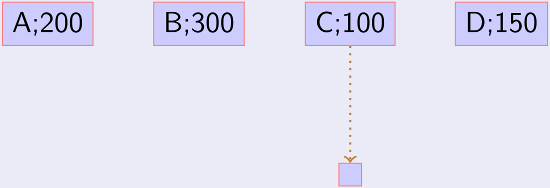
- which algorithm is being run should also be private information

A;200 B;300 C;100 D;150

Hiding the Algorithm

A new threat

- which algorithm is being run should also be private information



Hiding the Algorithm

A new threat

- which algorithm is being run should also be private information

A;200 B;300 C;100 D;150

Enter Oblivious RAM

Differential Privacy

ORAM [Goldreich-Ostrovsky]

- $\mathcal{M}$ stores n blocks of memory.
- Every time $\mathcal{O}$ wants a block, he asks $\mathcal{M}$ one or more blocks.
- Security notion:
  - For any two block sequences $\mathbb{B} = B_1, \ldots, B_n$ and $\mathbb{C} = C_1, \ldots, C_n$
  - For any two access sequences $I = (i_1, \ldots, i_l)$ and $J = (j_1, \ldots, j_l)$
    - performing accesses $i_1, \ldots, i_l$ on $\mathbb{B} = B_1, \ldots, B_n$;
    - performing access $j_1, \ldots, j_l$ on $\mathbb{C} = C_1, \ldots, C_n$

  generate the same distribution of accesses to the data stored by $\mathcal{M}$

Enter Oblivious RAM

Differential Privacy

ORAM [Goldreich-Ostrovsky]

- $\mathcal{M}$ stores n blocks of memory.
- Every time $\mathcal{O}$ wants a block, he asks $\mathcal{M}$ one or more blocks.
- Security notion:
  - For any two block sequences $\mathbb{B} = B_1, \ldots, B_n$ and $\mathbb{C} = C_1, \ldots, C_n$
  - For any two access sequences $I = (i_1, \ldots, i_l)$ and $J = (j_1, \ldots, j_l)$
    - performing accesses $i_1, \ldots, i_l$ on $\mathbb{B} = B_1, \ldots, B_n$;
    - performing access $j_1, \ldots, j_l$ on $\mathbb{C} = C_1, \ldots, C_n$

generate the same distribution of accesses to the data stored by $\mathcal{M}$

For every predicate A

$$\mathrm{Prob}[\mathtt{view} \leftarrow \mathtt{View}(I, \mathbb{B}) : A(\mathtt{view}) = 1]$$
$$\leq e^0 \cdot \mathrm{Prob}[\mathtt{view} \leftarrow \mathtt{View}(J, \mathbb{C}) : A(\mathtt{view}) = 1] + \mathsf{negl}(n)$$

ORAM makes all Algorithms Oblivious

Composing ORAM and Non-Oblivious Algorithms

- $\mathcal{O}$ runs the algorithm
- when a block of memory is requested, $\mathcal{O}$ retrieves it from $\mathcal{M}$ using ORAM.

ORAM makes all Algorithms Oblivious

Composing ORAM and Non-Oblivious Algorithms

- $\mathcal{O}$ runs the algorithm
- when a block of memory is requested, $\mathcal{O}$ retrieves it from $\mathcal{M}$ using ORAM.

Is ORAM possible at all?

Yes! This is possible!

A Trivial ORAM

Jump ahead

- All blocks are uploaded to $\mathcal{M}$ in encrypted form.

B_1	B_2	B_3	B_4	B_5	B_6

- Every time $\mathcal{O}$ needs to access block B_i, all the blocks are downloaded and all except for B_i are discarded.

Yes! This is possible!

A Trivial ORAM

Jump ahead

- All blocks are uploaded to $\mathcal{M}$ in encrypted form.

B_1 B_2 B_3 B_4 B_5 B_6

- Every time $\mathcal{O}$ needs to access block B_i, all the blocks are downloaded and all except for B_i are discarded.

Accessing block B_3

B_1 B_2 B_3 B_4 B_5 B_6

Yes! This is possible!

A Trivial ORAM

▸ Jump ahead

- All blocks are uploaded to $\mathcal{M}$ in encrypted form.

B_1 B_2 B_3 B_4 B_5 B_6

- Every time $\mathcal{O}$ needs to access block B_i, all the blocks are downloaded and all except for B_i are discarded.

Accessing block B_3

B_1 B_2 B_3 B_4 B_5 B_6

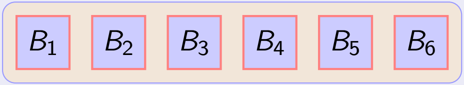
B_1

Yes! This is possible!

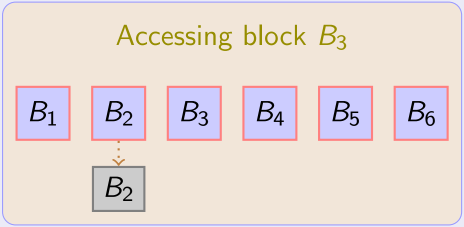
A Trivial ORAM

Jump ahead

- All blocks are uploaded to $\mathcal{M}$ in encrypted form.



- Every time $\mathcal{O}$ needs to access block B_i, all the blocks are downloaded and all except for B_i are discarded.



Yes! This is possible!

A Trivial ORAM

Jump ahead

- All blocks are uploaded to $\mathcal{M}$ in encrypted form.

B_1 B_2 B_3 B_4 B_5 B_6

- Every time $\mathcal{O}$ needs to access block B_i, all the blocks are downloaded and all except for B_i are discarded.

Accessing block B_3

B_1 B_2 B_3 B_4 B_5 B_6

B_3

Yes! This is possible!

A Trivial ORAM

Jump ahead

- All blocks are uploaded to $\mathcal{M}$ in encrypted form.

B_1 B_2 B_3 B_4 B_5 B_6

- Every time $\mathcal{O}$ needs to access block B_i, all the blocks are downloaded and all except for B_i are discarded.

Accessing block B_3

B_1 B_2 B_3 B_4 B_5 B_6

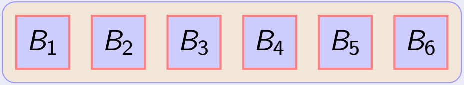
B_3 B_4

Yes! This is possible!

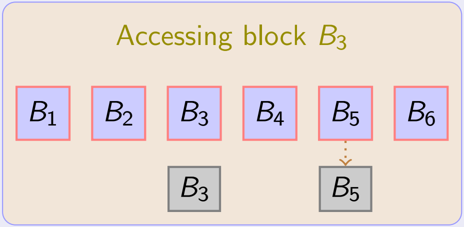
A Trivial ORAM

Jump ahead

- All blocks are uploaded to $\mathcal{M}$ in encrypted form.



- Every time $\mathcal{O}$ needs to access block B_i, all the blocks are downloaded and all except for B_i are discarded.

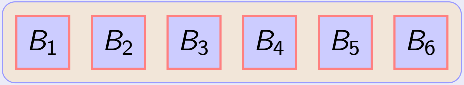


Yes! This is possible!

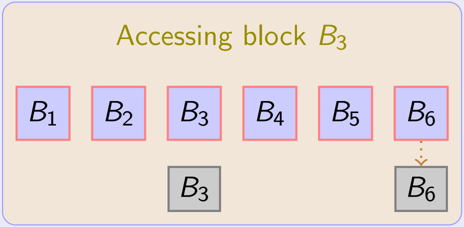
A Trivial ORAM

Jump ahead

- All blocks are uploaded to $\mathcal{M}$ in encrypted form.



- Every time $\mathcal{O}$ needs to access block B_i, all the blocks are downloaded and all except for B_i are discarded.



Yes! This is possible!

A Trivial ORAM

Jump ahead

- All blocks are uploaded to $\mathcal{M}$ in encrypted form.

B_1	B_2	B_3	B_4	B_5	B_6

- Every time $\mathcal{O}$ needs to access block B_i, all the blocks are downloaded and all except for B_i are discarded.

Accessing block B_3

B_1	B_2	B_3	B_4	B_5	B_6
		B_3			

Access pattern independent from the block accessed but...

Yes! This is possible!

A Trivial ORAM

Jump ahead

- All blocks are uploaded to $\mathcal{M}$ in encrypted form.

B_1 B_2 B_3 B_4 B_5 B_6

- Every time $\mathcal{O}$ needs to access block B_i, all the blocks are downloaded and all except for B_i are discarded.

Accessing block B_3

B_1 B_2 B_3 B_4 B_5 B_6

B_3

Access pattern independent from the block accessed

but... **linear slowdown!!**

First try

Can this be made efficient?

First try

Can this be made efficient?

First try: Initialization

- permute blocks according to permutation π
  - an encryption of B_i is uploaded in position $\pi(i)$;

$$\boxed{B_1}\ \boxed{B_2}\ \boxed{B_3}\ \boxed{B_4}\ \boxed{B_5}\ \boxed{B_6}$$

- $\mathcal{O}$ keeps π private;

First try

Can this be made efficient?

First try: Initialization

- permute blocks according to permutation π
  - an encryption of B_i is uploaded in position $\pi(i)$;

$$\boxed{B_2}\ \boxed{B_4}\ \boxed{B_3}\ \boxed{B_6}\ \boxed{B_1}\ \boxed{B_5}$$

- $\mathcal{O}$ keeps π private;

First try

Can this be made efficient?

First try: Reading block i

- ask $\mathcal{M}$ for block in position $\pi(i)$;
- decrypt to obtain B_i;
- re-encrypt and upload in position $\pi(i)$;

Accessing block B_3

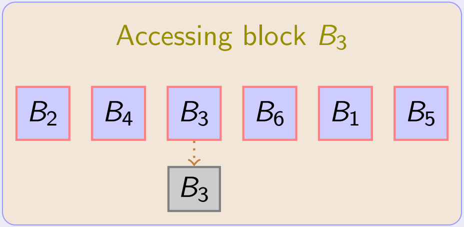
B_2	B_4	B_3	B_6	B_1	B_5

First try

Can this be made efficient?

First try: Reading block i

- ask $\mathcal{M}$ for block in position $\pi(i)$;
- decrypt to obtain B_i;
- re-encrypt and upload in position $\pi(i)$;



First try: Security

Access sequence: B_1, B_2, B_3

Accessing block B_1

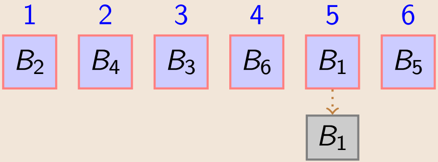
1	2	3	4	5	6
B_2	B_4	B_3	B_6	B_1	B_5

Access pattern seen by $\mathcal{M}$:

First try: Security

Access sequence: B_1, B_2, B_3

Accessing block B_1



Access pattern seen by $\mathcal{M}$: 5

First try: Security

Access sequence: B_1, B_2, B_3

Accessing block B_2

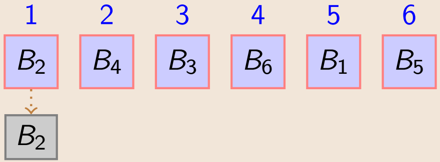
1	2	3	4	5	6
B_2	B_4	B_3	B_6	B_1	B_5

Access pattern seen by $\mathcal{M}$: 5

First try: Security

Access sequence: B_1, B_2, B_3

Accessing block B_2



Access pattern seen by $\mathcal{M}$: $5, 1$

First try: Security

Access sequence: B_1, B_2, B_3

Accessing block B_3

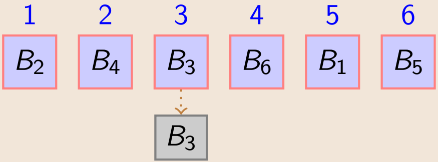
1	2	3	4	5	6
B_2	B_4	B_3	B_6	B_1	B_5

Access pattern seen by $\mathcal{M}$: $5, 1$

First try: Security

Access sequence: B_1, B_2, B_3

Accessing block B_3



Access pattern seen by $\mathcal{M}$: $5, 1, 3$

First try: Security

Access sequence: B_1, B_2, B_3

1	2	3	4	5	6
B_2	B_4	B_3	B_6	B_1	B_5

Access pattern seen by $\mathcal{M} : x, y, z$

First try: Security

Access sequence: B_1, B_2, B_1

Accessing block B_1

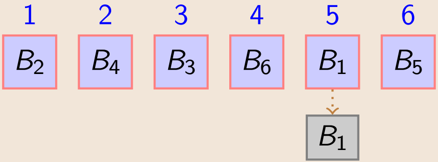
1	2	3	4	5	6
B_2	B_4	B_3	B_6	B_1	B_5

Access pattern seen by $\mathcal{M}$:

First try: Security

Access sequence: B_1, B_2, B_1

Accessing block B_1



Access pattern seen by $\mathcal{M}$: 5

First try: Security

Access sequence: B_1, B_2, B_1

Accessing block B_2

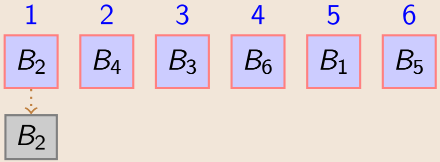
1	2	3	4	5	6
B_2	B_4	B_3	B_6	B_1	B_5

Access pattern seen by $\mathcal{M}$: 5

First try: Security

Access sequence: B_1, B_2, B_1

Accessing block B_2



Access pattern seen by $\mathcal{M}$: $5, 1$

First try: Security

Access sequence: B_1, B_2, B_1

Accessing block B_1

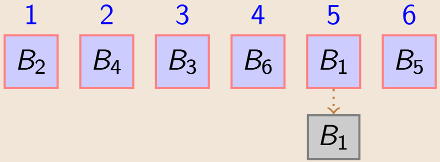
1	2	3	4	5	6
B_2	B_4	B_3	B_6	B_1	B_5

Access pattern seen by $\mathcal{M}$: $5, 1$

First try: Security

Access sequence: B_1, B_2, B_1

Accessing block B_1



Access pattern seen by $\mathcal{M}$: $5, 1, 5$

First try: Security

Access sequence: B_1, B_2, B_1

1	2	3	4	5	6
B_2	B_4	B_3	B_6	B_1	B_5

Access pattern seen by $\mathcal{M}: x, y, x$

Oblivious RAM

Obliviousness

For any two *access sequences* $O_1 = (i_1^1, \ldots, i_l^1)$ $O_2 = (i_1^2, \ldots, i_l^2)$ of the same length, the distribution of the positions requested by $\mathcal{O}$ to $\mathcal{M}$ is the same.

Oblivious RAM

Obliviousness

For any two *access sequences* $O_1 = (i_1^1, \ldots, i_l^1)$ $O_2 = (i_1^2, \ldots, i_l^2)$ of the same length, the distribution of the positions requested by $\mathcal{O}$ to $\mathcal{M}$ is the same.

Oblivious for Non-repeating sequences

- $k_1 \neq k_2$ implies $i_{k_1}^1 \neq i_{k_2}^1$ and $i_{k_1}^2 \neq i_{k_2}^1$;
- $\mathcal{M}$ sees requests for l different randomly chosen blocks both for O_1 and for O_2.

Repetition Pattern is leaked

Repetition Pattern

If the same block is requested twice by $\mathcal{O}$ then $\mathcal{M}$ sees the same position accessed twice.

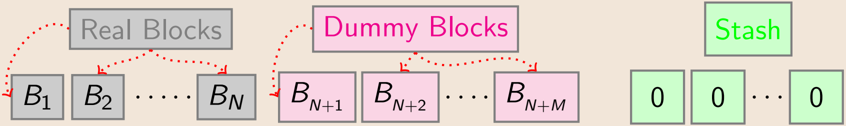
Block	3	4	7	8	4	2	4	10	12	8	6
Position	12	2	9	3	2	6	2	10	1	3	5

Hiding the Repetition Pattern

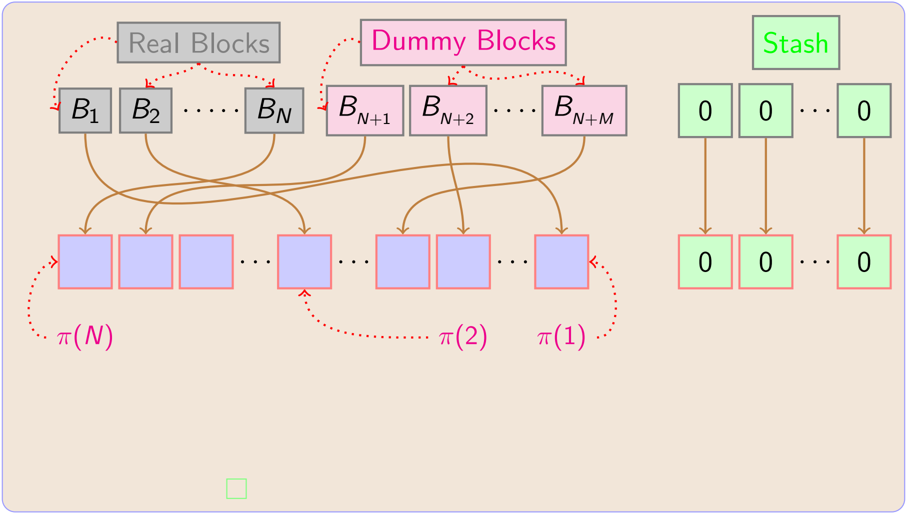
Initialization for N blocks

1. N *real* blocks $B_1, \ldots, B_N$;
2. create M *dummy* blocks $B_{N+1}, \ldots, B_{N+M}$;
3. create M *stash* blocks $S_1, \ldots, S_M$ initialized to 0;
4. pick a random permutation π over $[N + M]$;
5. permute *real* and *dummy* blocks according to permutation π
   - an encryption of B_i is uploaded in position $\pi(i)$;
6. upload all stash blocks in encrypted form;
7. initialize $\mathsf{nxt} = 1, \mathsf{cnt} = 1$;
8. π is kept private;

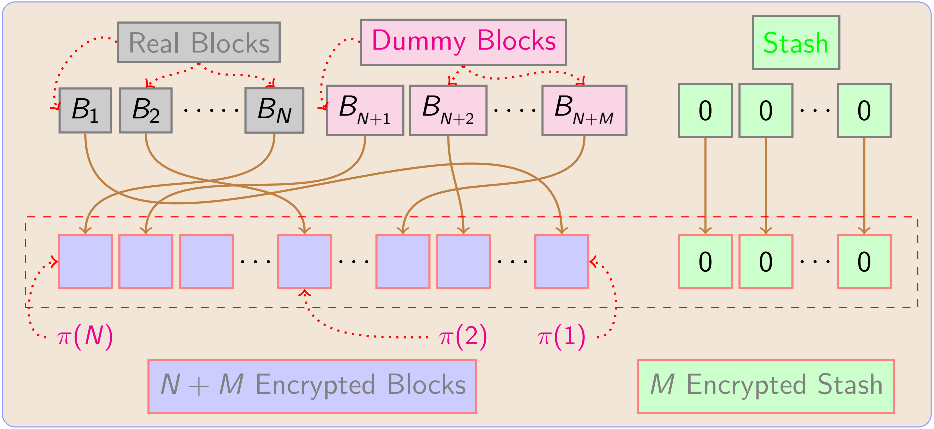
Initial Configuration



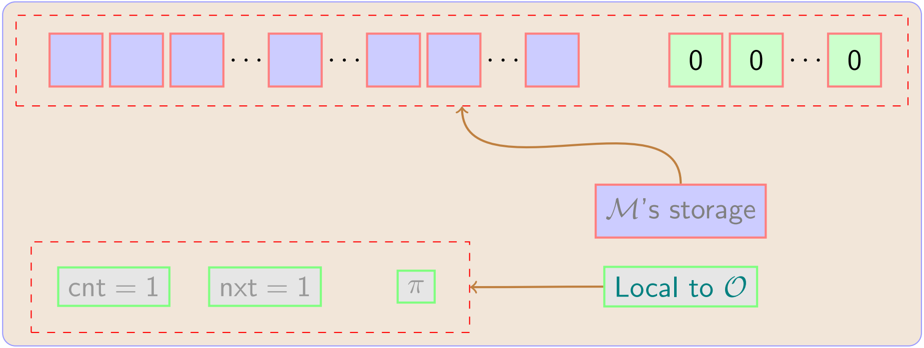
Initial Configuration



Initial Configuration



Initial Configuration



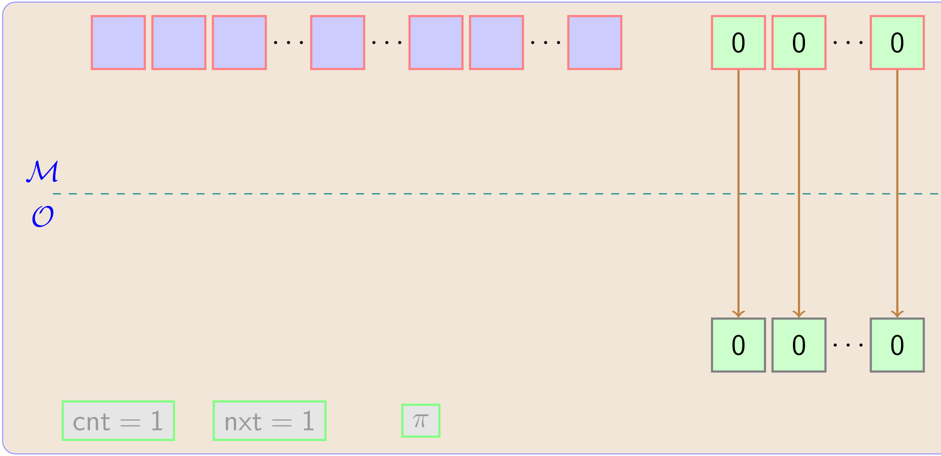
Reading Block B_i

1. download and decrypt all M blocks in the Stash;
2. if B_i is found in the Stash then
   - download dummy block $\pi(N + \mathrm{cnt})$;
   - set $\mathrm{cnt} = \mathrm{cnt} + 1$;

   else
   - download encrypted real block in position $\pi(i)$;
   - decrypt and obtain real block B_i;
   - set next available Stash block $S_{\mathrm{nxt}} = B_i$;
   - set $\mathrm{nxt} = \mathrm{nxt} + 1$;
3. re-encrypt and upload all blocks in the Stash;

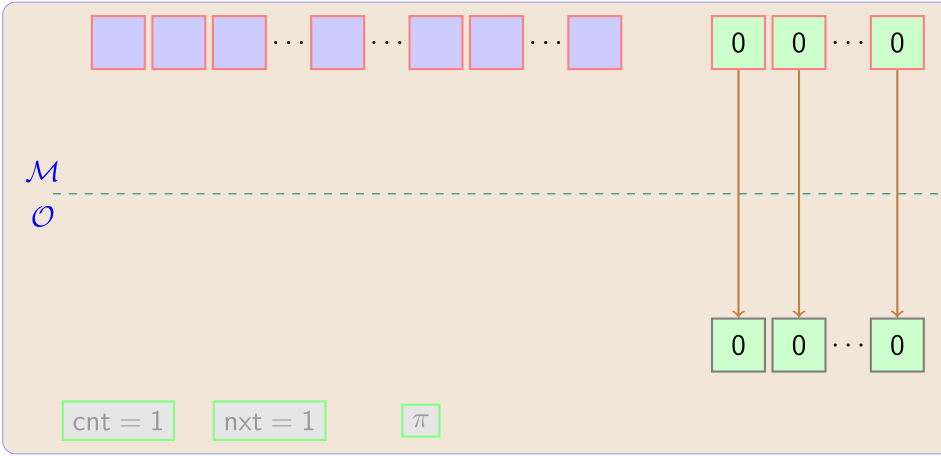
Reading Block B_1

Download and decrypt all blocks from Stash



Reading Block B_1

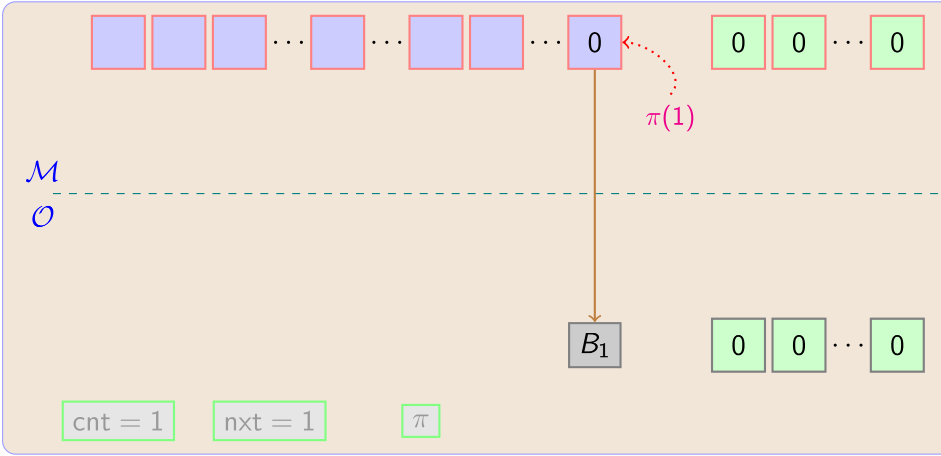
Download and decrypt all blocks from Stash



B_1 is not found in the stash

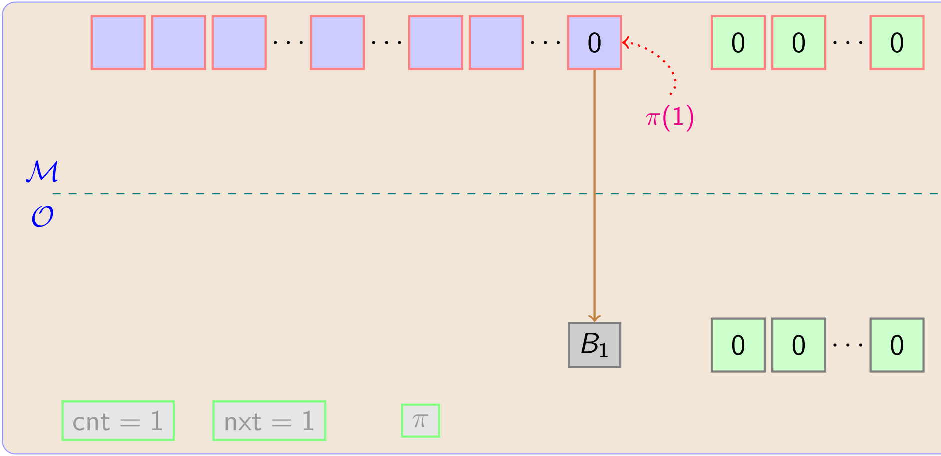
Reading Block B_1

Download block in position $\pi(1)$



Reading Block B_1

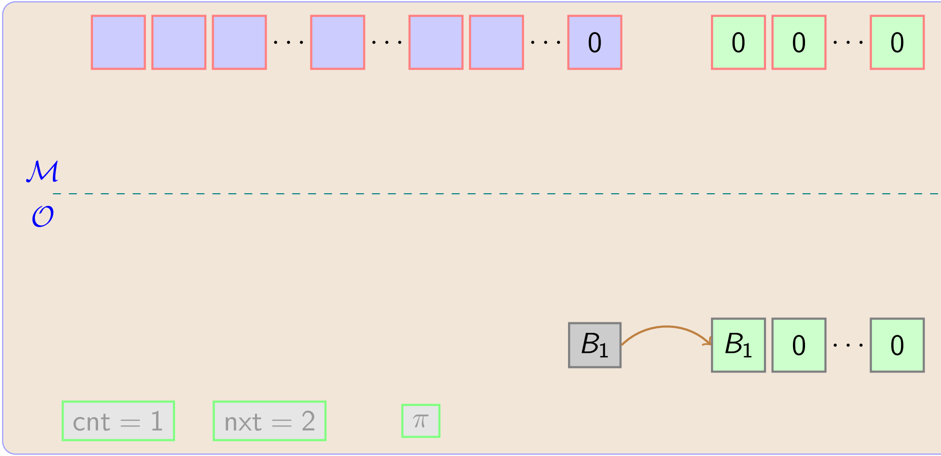
Download block in position $\pi(1)$



Decrypt and obtain B_1

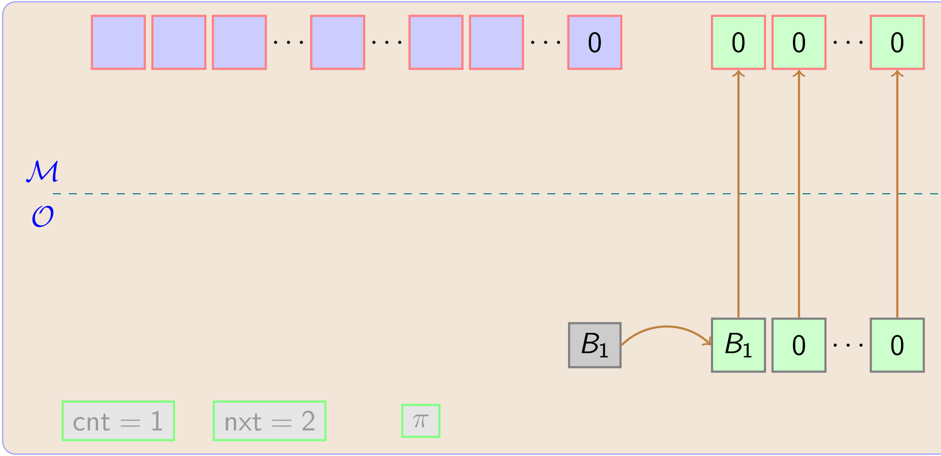
Reading Block B_1

Copy B_1 in the Stash at position nxt



Reading Block B_1

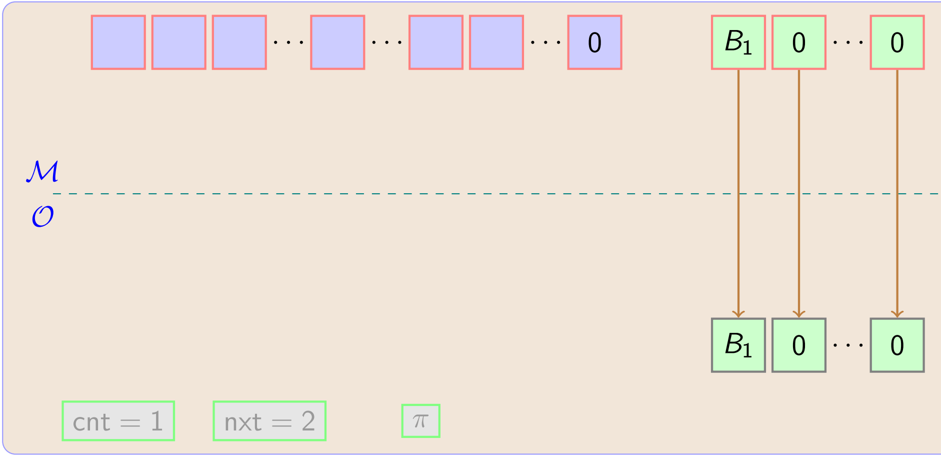
Copy B_1 in the Stash at position nxt



Encrypt and Upload the Stash

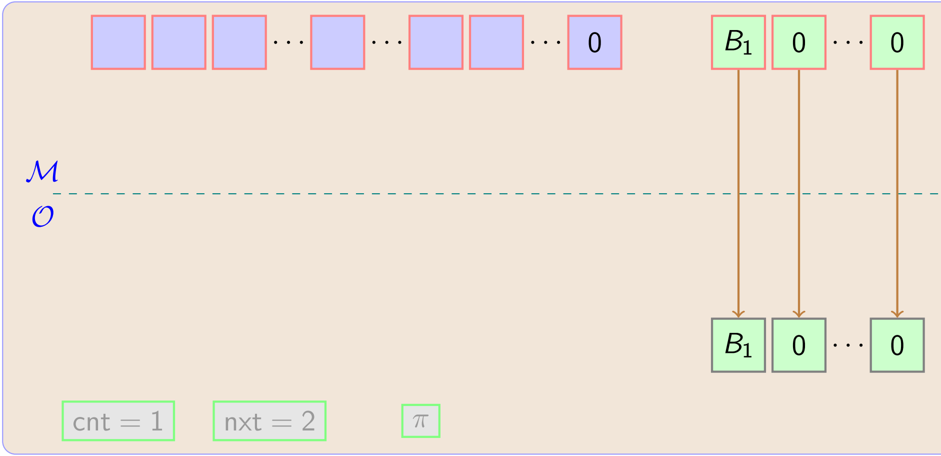
Reading Block B_2

Download and decrypt all blocks from Stash



Reading Block B_2

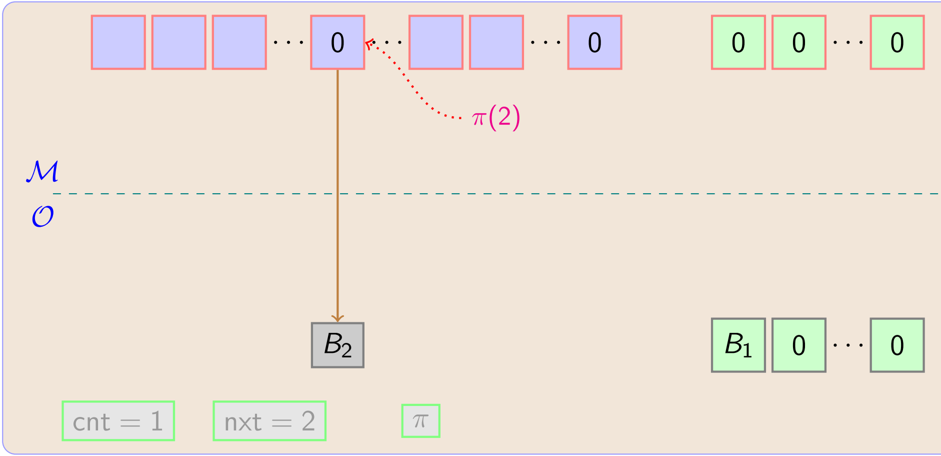
Download and decrypt all blocks from Stash



B_2 is not found in the Stash

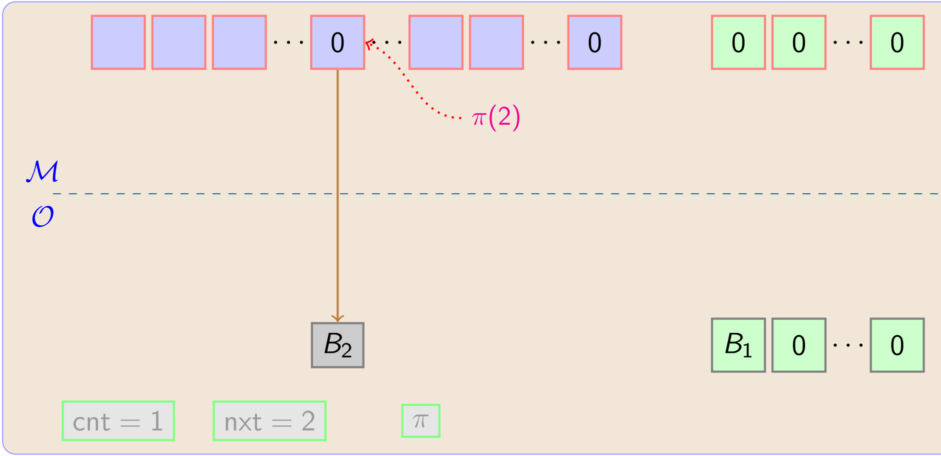
Reading Block B_2

Download block in position $\pi(2)$



Reading Block B_2

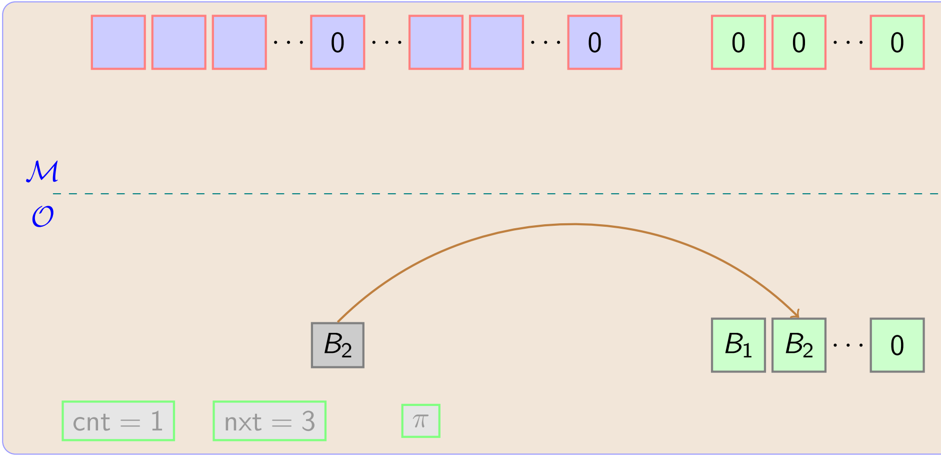
Download block in position $\pi(2)$



Decrypt and obtain B_2

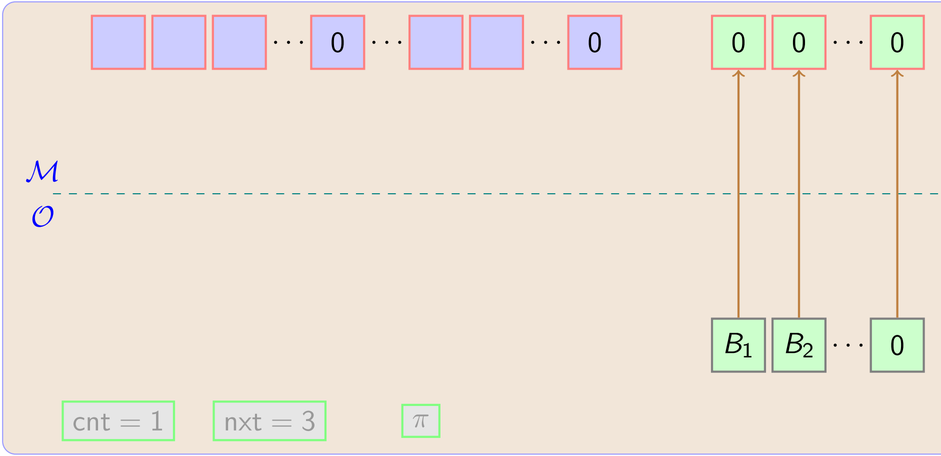
Reading Block B_2

Copy B_2 in the Stash at position nxt



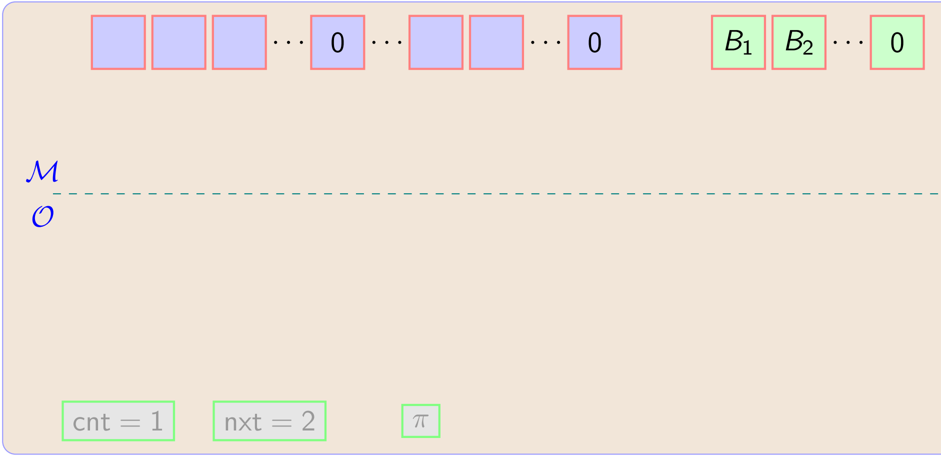
Reading Block B_2

Copy B_2 in the Stash at position nxt



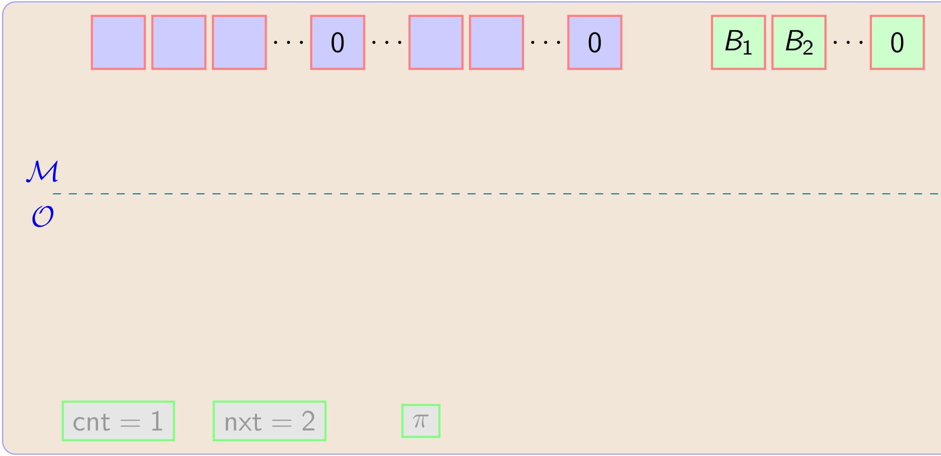
Encrypt and Upload the Stash

Status after reading B_1 and B_2



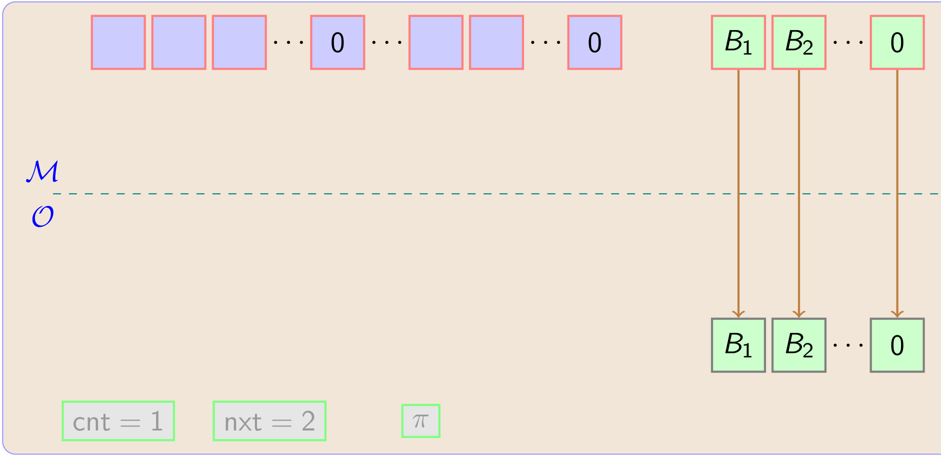
Status after reading B_1 and B_2

Now read B_1 again



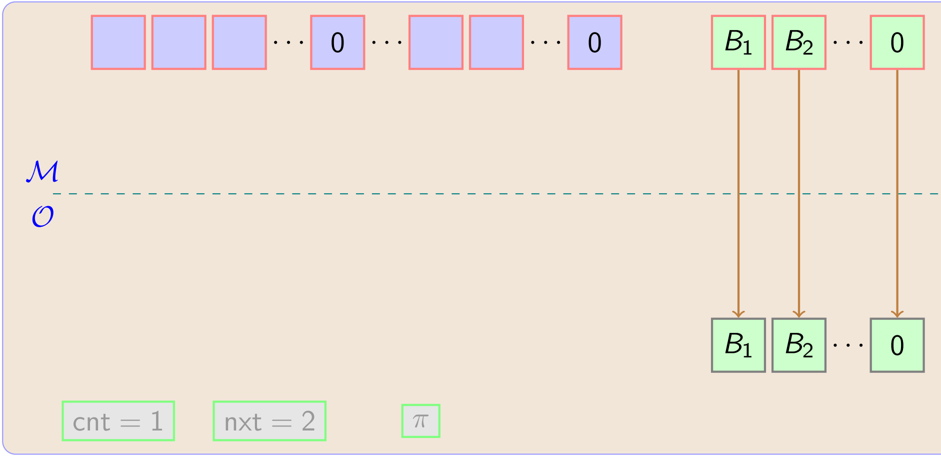
Status after reading B_1 and B_2

Download and decrypt all blocks from Stash



Status after reading B_1 and B_2

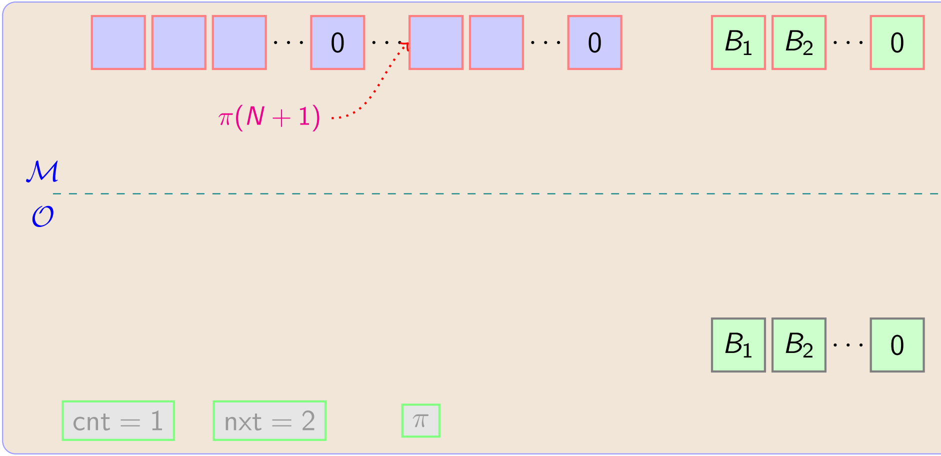
Download and decrypt all blocks from Stash



B_1 is found in the Stash

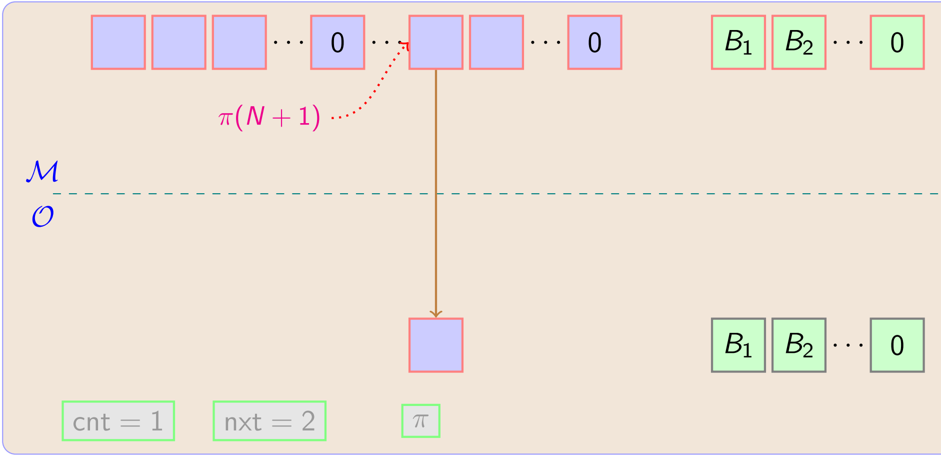
Reading Block B_1 (again)

Download block in position $\pi(N + \mathrm{cnt})$



Reading Block B_1 (again)

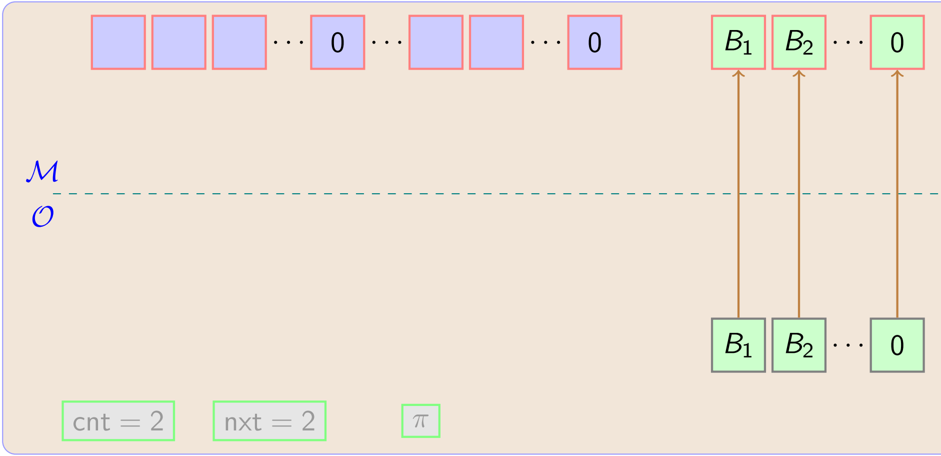
Download block in position $\pi(N + \mathrm{cnt})$



No need to decrypt

Reading Block B_1 (again)

Download block in position $\pi(N + \mathrm{cnt})$



Encrypt and Upload Stash

Insert slide in which we argue obliviousness

Two issues to be dealt with

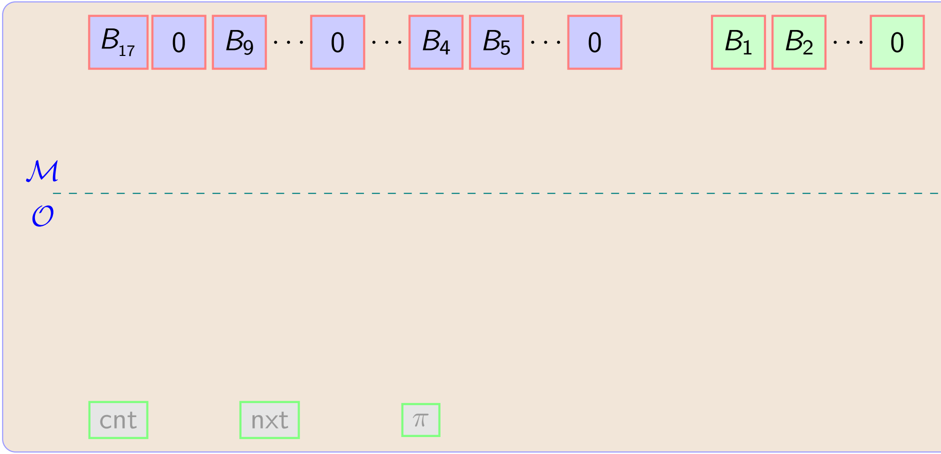
- What happens when the Stash is full?

Two issues to be dealt with

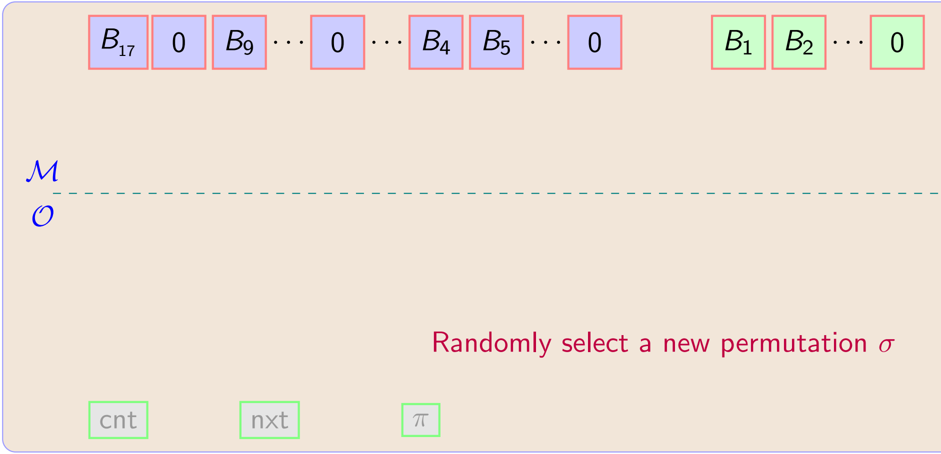
- What happens when the Stash is full?

- How much memory does $\mathcal{O}$ need?
  - needs to store cnt and nxt: $\Theta(1)$ memory;
  - π needs $O(N)$ memory.

Overflowing the Stash

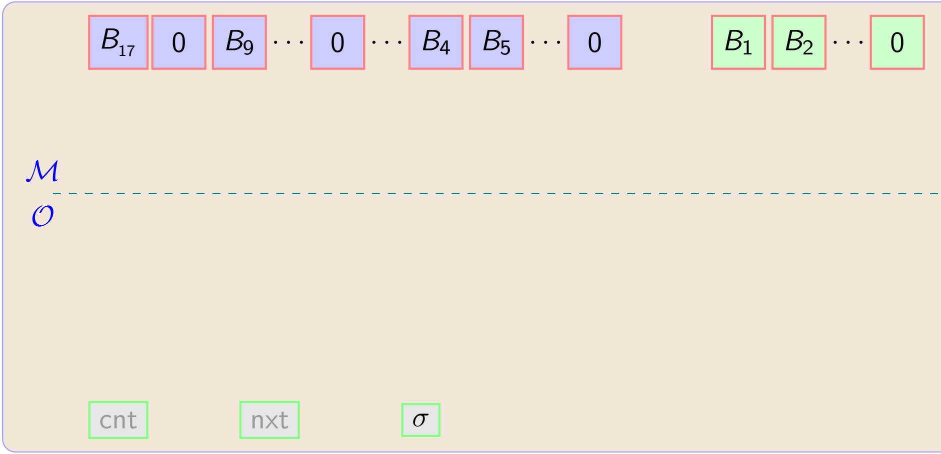


Overflowing the Stash

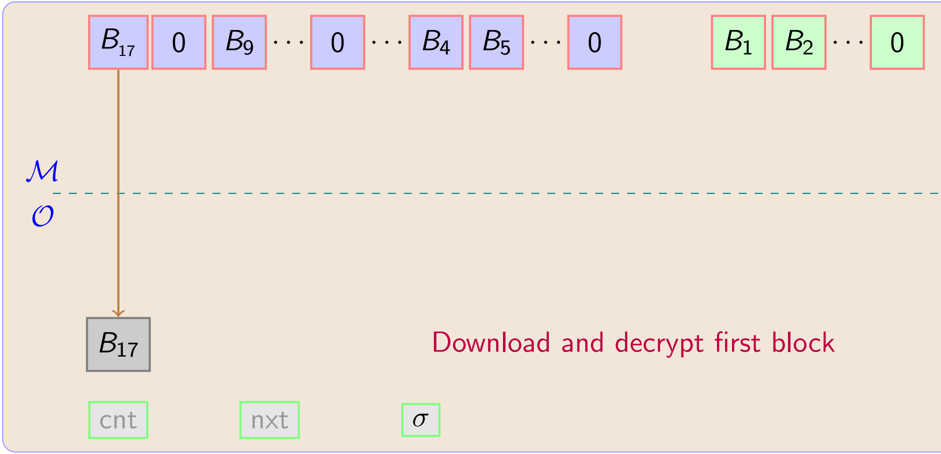


Randomly select a new permutation σ

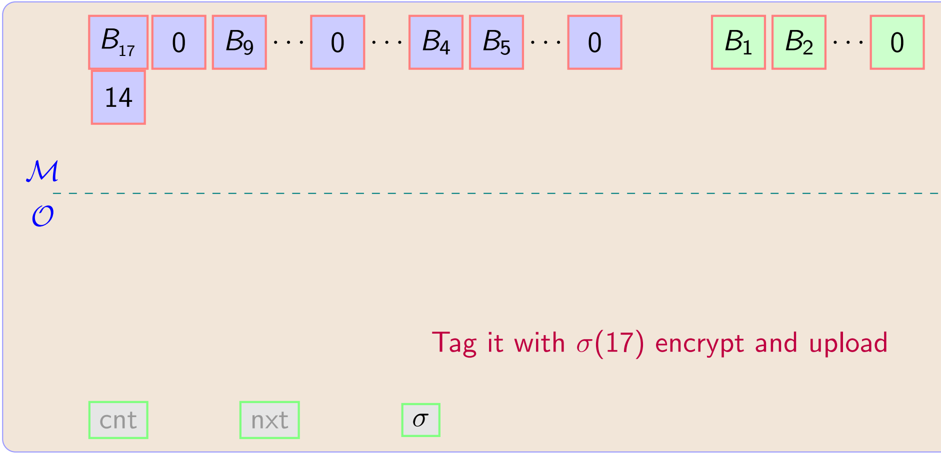
Overflowing the Stash



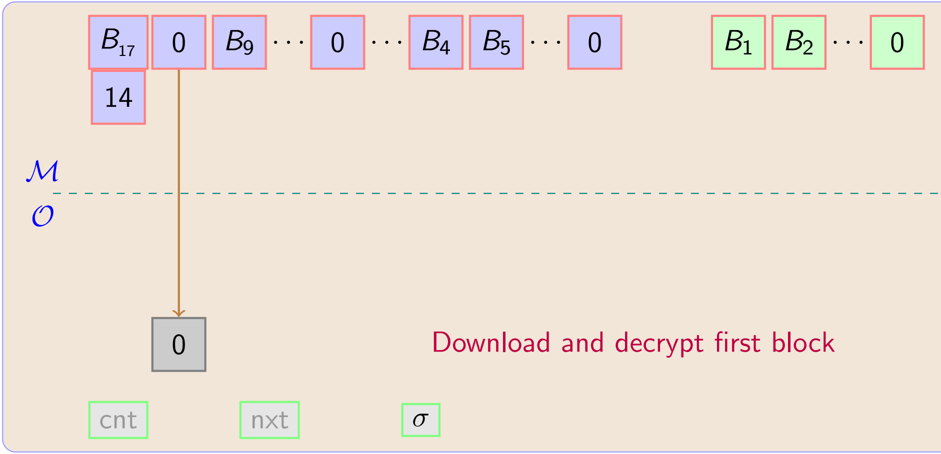
Overflowing the Stash



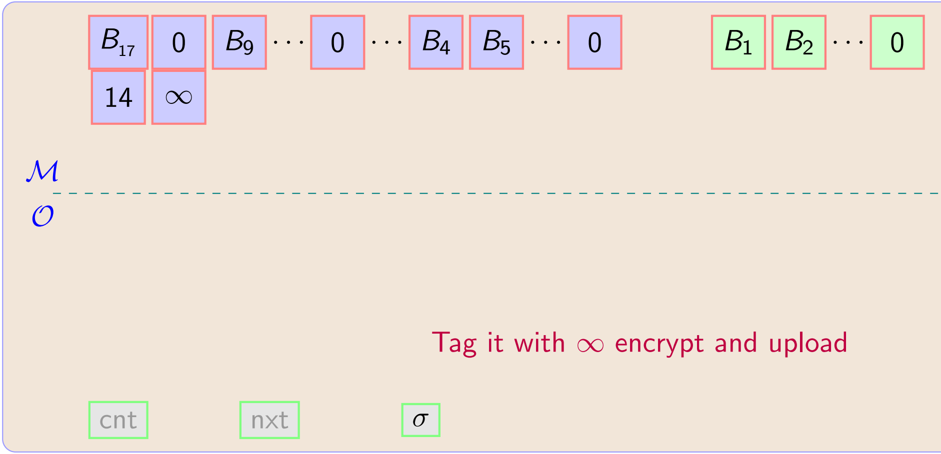
Overflowing the Stash



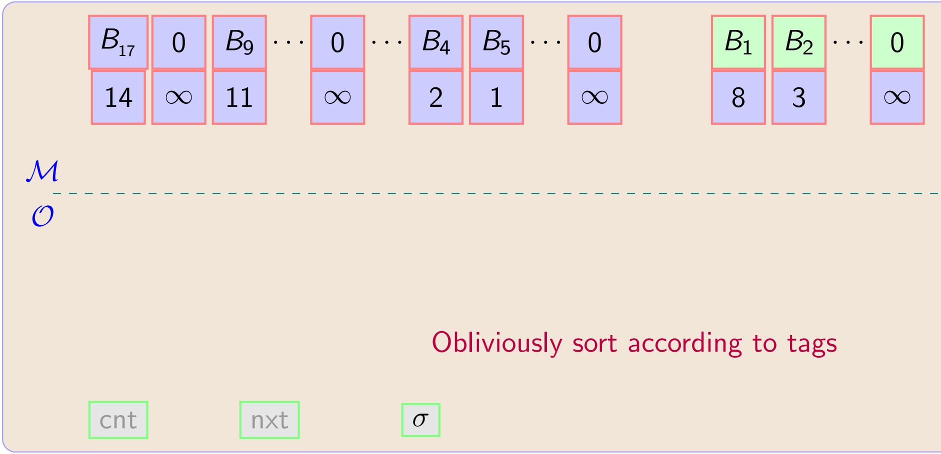
Overflowing the Stash



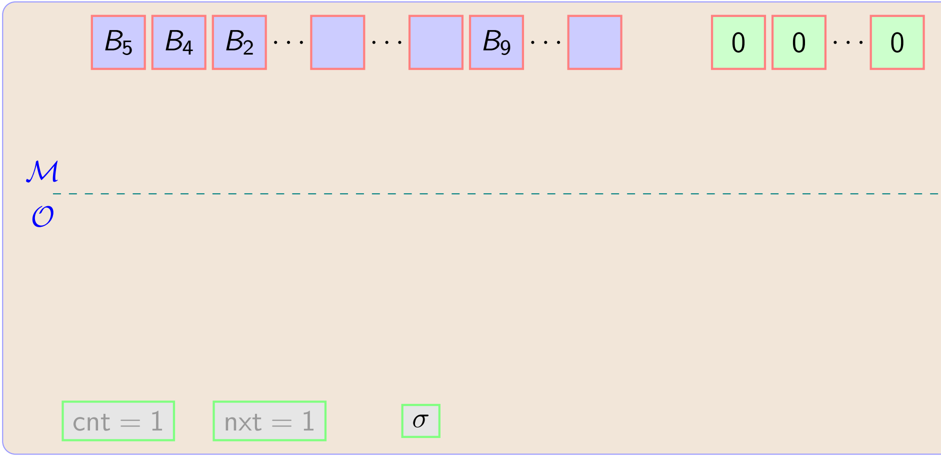
Overflowing the Stash



Overflowing the Stash



Overflowing the Stash



Amortized cost per read operation

Let us count:

Amortized cost per read operation

Let us count:

- each read costs

Amortized cost per read operation

Let us count:

- each read costs
  - $\Theta(M)$ blocks of bandwidth for the stash;

Amortized cost per read operation

Let us count:

- each read costs
  - $\Theta(M)$ blocks of bandwidth for the stash;
  - $\Theta(1)$ blocks of bandwidth for real/dummy blocks;

Amortized cost per read operation

Let us count:

- each read costs
  - $\Theta(M)$ blocks of bandwidth for the stash;
  - $\Theta(1)$ blocks of bandwidth for real/dummy blocks;
- after M reads, we shuffle

Amortized cost per read operation

Let us count:

- each read costs
  - $\Theta(M)$ blocks of bandwidth for the stash;
  - $\Theta(1)$ blocks of bandwidth for real/dummy blocks;
- after M reads, we shuffle
  - $\Theta(M + N) = \Theta(N)$ blocks of bandwidth for tagging;

Amortized cost per read operation

Let us count:

- each read costs
  - $\Theta(M)$ blocks of bandwidth for the stash;
  - $\Theta(1)$ blocks of bandwidth for real/dummy blocks;
- after M reads, we shuffle
  - $\Theta(M + N) = \Theta(N)$ blocks of bandwidth for tagging;
  - $\Theta((M + N)\log(M + N)) = \Theta(N \log N)$ blocks of bandwidth for sorting;

Amortized cost per read operation

Let us count:

- each read costs
  - $\Theta(M)$ blocks of bandwidth for the stash;
  - $\Theta(1)$ blocks of bandwidth for real/dummy blocks;
- after M reads, we shuffle
  - $\Theta(M+N) = \Theta(N)$ blocks of bandwidth for tagging;
  - $\Theta((M+N)\log(M+N)) = \Theta(N \log N)$ blocks of bandwidth for sorting;

Amortized cost per read operation

Let us count:

- each read costs
  - $\Theta(M)$ blocks of bandwidth for the stash;
  - $\Theta(1)$ blocks of bandwidth for real/dummy blocks;
- after M reads, we shuffle
  - $\Theta(M+N)=\Theta(N)$ blocks of bandwidth for tagging;
  - $\Theta((M+N)\log(M+N))=\Theta(N\log N)$ blocks of bandwidth for sorting;

for an amortized cost of

$$\Theta\left(M+\frac{N\log N}{M}\right)$$

Amortized cost per read operation

Let us count:

- each read costs
  - $\Theta(M)$ blocks of bandwidth for the stash;
  - $\Theta(1)$ blocks of bandwidth for real/dummy blocks;
- after M reads, we shuffle
  - $\Theta(M+N) = \Theta(N)$ blocks of bandwidth for tagging;
  - $\Theta((M+N)\log(M+N)) = \Theta(N\log N)$ blocks of bandwidth for sorting;

for an amortized cost of

$$\Theta\left(M + \frac{N\log N}{M}\right)$$

for $M = \sqrt{N}$ we have

$$\sqrt{N} \cdot \log N.$$

Amortized cost per read operation

Let us count:

- each read costs
  - $\Theta(M)$ blocks of bandwidth for the stash;
  - $\Theta(1)$ blocks of bandwidth for real/dummy blocks;
- after M reads, we shuffle
  - $\Theta(M+N)=\Theta(N)$ blocks of bandwidth for tagging;
  - $\Theta((M+N)\log(M+N))=\Theta(N\log N)$ blocks of bandwidth for sorting;

for an amortized cost of

$$\Theta\left(M+\frac{N\log N}{M}\right)$$

for $M=\sqrt{N}$ we have

$$\sqrt{N}\cdot\log N.$$

Using AKS to sort.

Amortized cost per read operation

Let us count:

- each read costs
  - $\Theta(M)$ blocks of bandwidth for the stash;
  - $\Theta(1)$ blocks of bandwidth for real/dummy blocks;
- after M reads, we shuffle
  - $\Theta(M+N)=\Theta(N)$ blocks of bandwidth for tagging;
  - $\Theta((M+N)\log(M+N))=\Theta(N\log N)$ blocks of bandwidth for sorting;

for an amortized cost of

$$\Theta\left(M+\frac{N\log N}{M}\right)$$

for $M=\sqrt{N}$ we have

$$\sqrt{N}\cdot\log N.$$

Using AKS to sort. Huge constant

Amortized cost per read operation

Let us count:

- each read costs
  - $\Theta(M)$ blocks of bandwidth for the stash;
  - $\Theta(1)$ blocks of bandwidth for real/dummy blocks;
- after M reads, we shuffle
  - $\Theta(M+N)=\Theta(N)$ blocks of bandwidth for tagging;
  - $\Theta((M+N)\log(M+N))=\Theta(N\log N)$ blocks of bandwidth for sorting;

for an amortized cost of

$$\Theta\left(M+\frac{N\log N}{M}\right)$$

for $M=\sqrt{N}$ we have

$$\sqrt{N}\cdot\log N.$$

Using AKS to sort. Huge constant

In practice $\sqrt{N}\cdot\log^2 N$.

In practice...

Jump ahead

One possible setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

In practice...

Jump ahead

One possible setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

In practice...

Jump ahead

One possible setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

- $\mathcal{M}$'s storage: $N + M = 10^6 + 10^3$ blocks.

In practice...

Jump ahead

One possible setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

- $\mathcal{M}$'s storage: $N + M = 10^6 + 10^3$ blocks.
- Cost of shuffling amortized per read operation:

$$1/2 \cdot 6^2 \cdot 10^3 \approx 18000$$

using Batcher's sort

In practice...

Jump ahead

One possible setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

- $\mathcal{M}$'s storage: $N + M = 10^6 + 10^3$ blocks.
- Cost of shuffling amortized per read operation:

$$1/2 \cdot 6^2 \cdot 10^3 \approx 18000$$

  using Batcher's sort
- Online cost

$$2 \cdot 10^3 \approx 2000$$

In practice...

Jump ahead

One possible setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

- $\mathcal{M}$'s storage: $N + M = 10^6 + 10^3$ blocks.
- Cost of shuffling amortized per read operation:

$$1/2 \cdot 6^2 \cdot 10^3 \approx 18000$$

  using Batcher's sort
- Online cost

$$2 \cdot 10^3 \approx 2000$$

- $\mathcal{O}$'s storage

In practice...

Jump ahead

One possible setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

- $\mathcal{M}$'s storage: $N + M = 10^6 + 10^3$ blocks.
- Cost of shuffling amortized per read operation:

$$1/2 \cdot 6^2 \cdot 10^3 \approx 18000$$

  using Batcher's sort
- Online cost

$$2 \cdot 10^3 \approx 2000$$

- $\mathcal{O}$'s storage
  - cnt and nxt use constant storage

In practice...

Jump ahead

One possible setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

- $\mathcal{M}$'s storage: $N + M = 10^6 + 10^3$ blocks.
- Cost of shuffling amortized per read operation:

$$1/2 \cdot 6^2 \cdot 10^3 \approx 18000$$

  using Batcher's sort
- Online cost

$$2 \cdot 10^3 \approx 2000$$

- $\mathcal{O}$'s storage
  - cnt and nxt use constant storage
  - π requires storing 10^6 4 bytes integers=4 Megabytes

Keep the stash in $\mathcal{O}$'s memory

Jump ahead

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Keep the stash in $\mathcal{O}$'s memory

Jump ahead

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

Keep the stash in $\mathcal{O}$'s memory

Jump ahead

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

- $\mathcal{M}$'s storage: $N + M = 10^6 + 10^3$ blocks

Keep the stash in $\mathcal{O}$'s memory

Jump ahead

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

- $\mathcal{M}$'s storage: $N + M = 10^6 + 10^3$ blocks
- Cost of shuffling amortized per read operation:

$$1/2 \cdot 6^2 \cdot 10^3 \approx 18000$$

Keep the stash in $\mathcal{O}$'s memory

Jump ahead

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

- $\mathcal{M}$'s storage: $N + M = 10^6 + 10^3$ blocks
- Cost of shuffling amortized per read operation:

$$1/2 \cdot 6^2 \cdot 10^3 \approx 18000$$

- Online cost: 2 blocks per read

Keep the stash in $\mathcal{O}$'s memory

▸ Jump ahead

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

- $\mathcal{M}$'s storage: $N + M = 10^6 + 10^3$ blocks
- Cost of shuffling amortized per read operation:

$$1/2 \cdot 6^2 \cdot 10^3 \approx 18000$$

- Online cost: 2 blocks per read
- $\mathcal{O}$'s storage

Keep the stash in $\mathcal{O}$'s memory

Jump ahead

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

- $\mathcal{M}$'s storage: $N + M = 10^6 + 10^3$ blocks
- Cost of shuffling amortized per read operation:

$$1/2 \cdot 6^2 \cdot 10^3 \approx 18000$$

- Online cost: 2 blocks per read
- $\mathcal{O}$'s storage
  - cnt and nxt use constant storage

Keep the stash in $\mathcal{O}$'s memory

Jump ahead

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

- $\mathcal{M}$'s storage: $N + M = 10^6 + 10^3$ blocks
- Cost of shuffling amortized per read operation:

$$1/2 \cdot 6^2 \cdot 10^3 \approx 18000$$

- Online cost: 2 blocks per read
- $\mathcal{O}$'s storage
  - cnt and nxt use constant storage
  - π requires storing 10^6 4-byte integers=4 Megabytes

Keep the stash in $\mathcal{O}$'s memory

Jump ahead

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

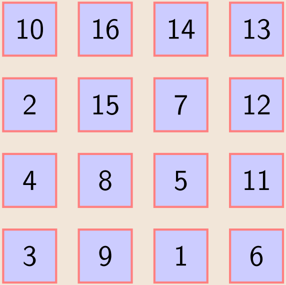
- $\mathcal{M}$'s storage: $N + M = 10^6 + 10^3$ blocks
- Cost of shuffling amortized per read operation:

$$1/2 \cdot 6^2 \cdot 10^3 \approx 18000$$

- Online cost: 2 blocks per read
- $\mathcal{O}$'s storage
  - cnt and nxt use constant storage
  - π requires storing 10^6 4-byte integers=4 Megabytes
  - 1000 blocks of stash for a total of 4 Megabytes

Shuffling without Sorting

- Input: N blocks stored in $S[1, \ldots, N]$ according to π



Blocks in S according to π

Shuffling without Sorting

- Input: N blocks stored in $S[1, \ldots, N]$ according to π
  - Block B_l is found in $[\pi(l)]$

10	16	14	13
2	15	7	12
4	8	5	11
3	9	1	6

Blocks in S according to π

Shuffling without Sorting

- Input: N blocks stored in $S[1, \ldots, N]$ according to π
  - Block B_I is found in $[\pi(I)]$
- Output: N blocks stored in $D[1, \ldots, N]$ according to σ

10	16	14	13
2	15	7	12
4	8	5	11
3	9	1	6

Blocks in S according to π

Shuffling without Sorting

- Input: N blocks stored in $S[1, \ldots, N]$ according to π
  - Block B_l is found in $[\pi(l)]$
- Output: N blocks stored in $D[1, \ldots, N]$ according to σ
  - Block B_l will be in $[\sigma(l)]$

10	16	14	13
2	15	7	12
4	8	5	11
3	9	1	6

Blocks in S according to π

16	15	5	1
14	2	4	9
10	7	11	6
13	12	8	3

Blocks in D according to σ

Shuffling $N = 16$

An easy case:

Partition the N blocks in $\sqrt{N}$ groups of $\sqrt{N}$

Blocks in S according to π

10	16	14	13
2	15	7	12
4	8	5	11
3	9	1	6

$\mathcal{M}$

$\mathcal{O}$

Shuffling $N = 16$

An easy case:

Partition the N destinations in $\sqrt{N}$ groups of $\sqrt{N}$

Blocks in S according to π

10	16	14	13	0	0	0	0
2	15	7	12	0	0	0	0
4	8	5	11	0	0	0	0
3	9	1	6	0	0	0	0

$\mathcal{M}$

$\mathcal{O}$

Shuffling $N = 16$

An easy case:

Download first source group

Blocks in S according to π

	10	16	14	13	0	0	0	0
	2	15	7	12	0	0	0	0
	4	8	5	11	0	0	0	0
$\mathcal{M}$	3	9	1	6	0	0	0	0

$\mathcal{O}$

Shuffling $N = 16$

An easy case:

Download first source group

Blocks in S according to π

$\mathcal{M}$

10	16	14	13	0	0	0	0
2	15	7	12	0	0	0	0
4	8	5	11	0	0	0	0
3	9	1	6	0	0	0	0

$\mathcal{O}$

3	4	2	10

Shuffling $N = 16$

An easy case:

One block to each destination group

Blocks in S according to π

10	16	14	13		0	0	0	0
2	15	7	12		0	0	0	0
4	8	5	11		0	0	0	0
3	9	1	6		10	2	4	3

$\mathcal{M}$

$\mathcal{O}$

Shuffling $N = 16$

An easy case:

Download second source group

Blocks in S according to π

10	16	14	13	0	0	0	0
2	15	7	12	0	0	0	0
4	8	5	11	0	0	0	0
3	9	1	6	10	2	4	3

$\mathcal{M}$

$\mathcal{O}$

Shuffling $N = 16$

An easy case:

Download second source group

Blocks in S according to π

	10	16	14	13	0	0	0	0
	2	15	7	12	0	0	0	0
	4	8	5	11	0	0	0	0
$\mathcal{M}$	3	9	1	6	10	2	4	3

$\mathcal{O}$	9	8	15	16

Shuffling $N = 16$

An easy case:

One block to each destination group

Blocks in S according to π

10	16	14	13	0	0	0	0
2	15	7	12	0	0	0	0
4	8	5	11	16	15	8	9
3	9	1	6	10	2	4	3

$\mathcal{M}$

$\mathcal{O}$

Shuffling $N = 16$

An easy case:

Download second source group

Blocks in S according to π

10	16	14	13	0	0	0	0
2	15	7	12	0	0	0	0
4	8	5	11	16	15	8	9
3	9	1	6	10	2	4	3

$\mathcal{M}$

$\mathcal{O}$

Shuffling $N = 16$

An easy case:

Download second source group

Blocks in S according to π

$\mathcal{M}$

10	16	14	13	0	0	0	0
2	15	7	12	0	0	0	0
4	8	5	11	16	15	8	9
3	9	1	6	10	2	4	3

$\mathcal{O}$

1	5	7	14

Shuffling $N = 16$

An easy case:

One block to each destination group

Blocks in S according to π

10	16	14	13	0	0	0	0
2	15	7	12	14	7	5	1
4	8	5	11	16	15	8	9
3	9	1	6	10	2	4	3

$\mathcal{M}$

$\mathcal{O}$

Shuffling $N = 16$

An easy case:

Download second source group

Blocks in S according to π

	10	16	14	13	0	0	0	0
	2	15	7	12	14	7	5	1
	4	8	5	11	16	15	8	9
$\mathcal{M}$	3	9	1	6	10	2	4	3

$\mathcal{O}$

Shuffling $N = 16$

An easy case:

Download second source group

Blocks in S according to π

$\mathcal{M}$

10	16	14	13	0	0	0	0
2	15	7	12	14	7	5	1
4	8	5	11	16	15	8	9
3	9	1	6	10	2	4	3

$\mathcal{O}$

6	11	12	13

Shuffling $N = 16$

An easy case:

One block to each destination group

Blocks in S according to π

10	16	14	13
2	15	7	12
4	8	5	11
3	9	1	6

13	12	11	6
14	7	5	1
16	15	8	9
10	2	4	3

$\mathcal{M}$

$\mathcal{O}$

Shuffling $N = 16$

An easy case:

Each block in the right destination group

Blocks in S according to π

10	16	14	13
2	15	7	12
4	8	5	11
3	9	1	6

13	12	11	6
14	7	5	1
16	15	8	9
10	2	4	3

$\mathcal{M}$

$\mathcal{O}$

Shuffling $N = 16$

An easy case:

Download each group and upload in correct position

Blocks in S according to π

10	16	14	13
2	15	7	12
4	8	5	11
3	9	1	6

13	12	11	6
14	7	5	1
16	15	8	9
10	2	4	3

$\mathcal{M}$

$\mathcal{O}$

Shuffling $N = 16$

An easy case:

Download each group and upload in correct position

Blocks in S according to π

$\mathcal{M}$

10	16	14	13	13	12	11	6
2	15	7	12	14	7	5	1
4	8	5	11	16	15	8	9
3	9	1	6	10	2	4	3

$\mathcal{O}$

10	16	14	13

Shuffling $N = 16$

An easy case:

Download each group and upload in correct position

Blocks in S according to π

10	16	14	13	16	12	11	6
2	15	7	12	14	7	5	1
4	8	5	11	10	15	8	9
3	9	1	6	13	2	4	3

$\mathcal{M}$

$\mathcal{O}$

Shuffling $N = 16$

An easy case:

Download each group and upload in correct position

Blocks in S according to π

$\mathcal{M}$

10	16	14	13	16	12	11	6
2	15	7	12	14	7	5	1
4	8	5	11	10	15	8	9
3	9	1	6	13	2	4	3

$\mathcal{O}$

2	15	7	12

Shuffling $N = 16$

An easy case:

Download each group and upload in correct position

Blocks in S according to π

$\mathcal{M}$

10	16	14	13	16	15	11	6
2	15	7	12	14	2	5	1
4	8	5	11	10	7	8	9
3	9	1	6	13	12	4	3

$\mathcal{O}$

4	8	5	11

Shuffling $N = 16$

An easy case:

Download each group and upload in correct position

Blocks in S according to π

10	16	14	13	16	15	5	6
2	15	7	12	14	2	4	1
4	8	5	11	10	7	11	9
3	9	1	6	13	12	8	3

$\mathcal{M}$

$\mathcal{O}$

Shuffling $N = 16$

An easy case:

Download each group and upload in correct position

Blocks in S according to π

$\mathcal{M}$

10	16	14	13
2	15	7	12
4	8	5	11
3	9	1	6

16	15	5	6
14	2	4	1
10	7	11	9
13	12	8	3

$\mathcal{O}$

3	9	1	6

Shuffling $N = 16$

An easy case:

Download each group and upload in correct position

Blocks in S according to π

10	16	14	13
2	15	7	12
4	8	5	11
3	9	1	6

16	15	5	1
14	2	4	9
10	7	11	6
13	12	8	3

$\mathcal{M}$

$\mathcal{O}$

Shuffling $N = 16$

An easy case:

Download each group and upload in correct position

Blocks in S according to π

10	16	14	13
2	15	7	12
4	8	5	11
3	9	1	6

Blocks in D according to σ

16	15	5	1
14	2	4	9
10	7	11	6
13	12	8	3

$\mathcal{M}$

$\mathcal{O}$

Analysis of Shuffling Algorithm

- Obliviousness: access pattern independent of π, σ

Analysis of Shuffling Algorithm

- Obliviousness: access pattern independent of π, σ
  - download each source group

Analysis of Shuffling Algorithm

- Obliviousness: access pattern independent of π, σ
  - download each source group
  - upload one block to each destination group in the next available empty location

Analysis of Shuffling Algorithm

- Obliviousness: access pattern independent of π, σ
  - download each source group
  - upload one block to each destination group in the next available empty location
  - download each destination group

Analysis of Shuffling Algorithm

- Obliviousness: access pattern independent of π, σ
  - download each source group
  - upload one block to each destination group in the next available empty location
  - download each destination group
  - upload each destination group

Analysis of Shuffling Algorithm

- Obliviousness: access pattern independent of π, σ
  - download each source group
  - upload one block to each destination group in the next available empty location
  - download each destination group
  - upload each destination group

- bandwidth: $4N$
  each block

Analysis of Shuffling Algorithm

- Obliviousness: access pattern independent of π, σ
  - download each source group
  - upload one block to each destination group in the next available empty location
  - download each destination group
  - upload each destination group

- bandwidth: $4N$
  each block
  - downloaded exactly twice

Analysis of Shuffling Algorithm

- Obliviousness: access pattern independent of π, σ
  - download each source group
  - upload one block to each destination group in the next available empty location
  - download each destination group
  - upload each destination group

- bandwidth: $4N$
  each block
  - downloaded exactly twice
  - uploaded exactly twice

Analysis of Shuffling Algorithm

- Obliviousness: access pattern independent of π, σ
  - download each source group
  - upload one block to each destination group in the next available empty location
  - download each destination group
  - upload each destination group
- bandwidth: $4N$
  each block
  - downloaded exactly twice
  - uploaded exactly twice
- Luck: so much!!!

Analysis of Shuffling Algorithm

- Obliviousness: access pattern independent of π, σ
  - download each source group
  - upload one block to each destination group in the next available empty location
  - download each destination group
  - upload each destination group
- bandwidth: $4N$
  each block
  - downloaded exactly twice
  - uploaded exactly twice
- Luck: so much!!!
  - each source group contains exactly one block for each destination group under σ

Efficient Shuffling: CacheShuffleRoot

when you know you are not going to be lucky, just randomize

Efficient Shuffling: CacheShuffleRoot

when you know you are not going to be lucky, just randomize

- Randomly partition destination array in $(1 + 2\epsilon)\sqrt{N}$ groups

Efficient Shuffling: CacheShuffleRoot

when you know you are not going to be lucky, just randomize

- Randomly partition destination array in $(1+2\epsilon)\sqrt{N}$ groups
  - Each group has size at most $(1-\epsilon)\sqrt{N}$, except with negligible probability

Efficient Shuffling: CacheShuffleRoot

when you know you are not going to be lucky, just randomize

- Randomly partition destination array in $(1 + 2\epsilon)\sqrt{N}$ groups
  - Each group has size at most $(1 - \epsilon)\sqrt{N}$, except with negligible probability
- Spray phase

Efficient Shuffling: CacheShuffleRoot

when you know you are not going to be lucky, just randomize

- Randomly partition destination array in $(1 + 2\epsilon)\sqrt{N}$ groups
  - Each group has size at most $(1 - \epsilon)\sqrt{N}$, except with negligible probability
- Spray phase
  - Download each source group and spray exactly one block to each destination group

Efficient Shuffling: CacheShuffleRoot

when you know you are not going to be lucky, just randomize

- Randomly partition destination array in $(1+2\epsilon)\sqrt{N}$ groups
  - Each group has size at most $(1-\epsilon)\sqrt{N}$, except with negligible probability
- Spray phase
  - Download each source group and spray exactly one block to each destination group
  - if none available, spray a dummy block

Efficient Shuffling: CacheShuffleRoot

when you know you are not going to be lucky, just randomize

- Randomly partition destination array in $(1+2\epsilon)\sqrt{N}$ groups
  - Each group has size at most $(1-\epsilon)\sqrt{N}$, except with negligible probability
- Spray phase
  - Download each source group and spray exactly one block to each destination group
  - if none available, spray a dummy block
  - if more than one available, spray exactly one and store the extra blocks in local cache

Efficient Shuffling: CacheShuffleRoot

when you know you are not going to be lucky, just randomize

- Randomly partition destination array in $(1+2\epsilon)\sqrt{N}$ groups
  - Each group has size at most $(1-\epsilon)\sqrt{N}$, except with negligible probability
- Spray phase
  - Download each source group and spray exactly one block to each destination group
  - if none available, spray a dummy block
  - if more than one available, spray exactly one and store the extra blocks in local cache
- Recalibrate phase

Efficient Shuffling: CacheShuffleRoot

when you know you are not going to be lucky, just randomize

- Randomly partition destination array in $(1+2\epsilon)\sqrt{N}$ groups
  - Each group has size at most $(1-\epsilon)\sqrt{N}$, except with negligible probability
- Spray phase
  - Download each source group and spray exactly one block to each destination group
  - if none available, spray a dummy block
  - if more than one available, spray exactly one and store the extra blocks in local cache
- Recalibrate phase
  - download each destination group

Efficient Shuffling: CacheShuffleRoot

when you know you are not going to be lucky, just randomize

- Randomly partition destination array in $(1+2\epsilon)\sqrt{N}$ groups
  - Each group has size at most $(1-\epsilon)\sqrt{N}$, except with negligible probability
- Spray phase
  - Download each source group and spray exactly one block to each destination group
  - if none available, spray a dummy block
  - if more than one available, spray exactly one and store the extra blocks in local cache
- Recalibrate phase
  - download each destination group
  - remove dummies

Efficient Shuffling: CacheShuffleRoot

when you know you are not going to be lucky, just randomize

- Randomly partition destination array in $(1 + 2\epsilon)\sqrt{N}$ groups
  - Each group has size at most $(1 - \epsilon)\sqrt{N}$, except with negligible probability
- Spray phase
  - Download each source group and spray exactly one block to each destination group
  - if none available, spray a dummy block
  - if more than one available, spray exactly one and store the extra blocks in local cache
- Recalibrate phase
  - download each destination group
  - remove dummies
  - add blocks found in local cache

Efficient Shuffling: CacheShuffleRoot

when you know you are not going to be lucky, just randomize

- Randomly partition destination array in $(1+2\epsilon)\sqrt{N}$ groups
  - Each group has size at most $(1-\epsilon)\sqrt{N}$, except with negligible probability
- Spray phase
  - Download each source group and spray exactly one block to each destination group
  - if none available, spray a dummy block
  - if more than one available, spray exactly one and store the extra blocks in local cache
- Recalibrate phase
  - download each destination group
  - remove dummies
  - add blocks found in local cache
  - upload in correct order

Analysis of CacheShuffleRoot

- bandwidth: $(4+\epsilon)N$ blocks

Analysis of CacheShuffleRoot

- bandwidth: $(4 + \epsilon)N$ blocks
  - each real block is downloaded exactly twice and uploaded exactly twice

Analysis of CacheShuffleRoot

- bandwidth: $(4 + \epsilon)N$ blocks
  - each real block is downloaded exactly twice and uploaded exactly twice
  - each dummy block is uploaded exactly once and downloaded exactly once

Analysis of CacheShuffleRoot

- bandwidth: $(4+\epsilon)N$ blocks
  - each real block is downloaded exactly twice and uploaded exactly twice
  - each dummy block is uploaded exactly once and downloaded exactly once
  - number of dummy blocks is at most $\epsilon \cdot N$ except with negligible probability

Analysis of CacheShuffleRoot

- bandwidth: $(4 + \epsilon)N$ blocks
  - each real block is downloaded exactly twice and uploaded exactly twice
  - each dummy block is uploaded exactly once and downloaded exactly once
  - number of dummy blocks is at most $\epsilon \cdot N$ except with negligible probability
- Obliviousness: access pattern is fixed for all executions

Analysis of CacheShuffleRoot

- bandwidth: $(4+\epsilon)N$ blocks
  - each real block is downloaded exactly twice and uploaded exactly twice
  - each dummy block is uploaded exactly once and downloaded exactly once
  - number of dummy blocks is at most $\epsilon \cdot N$ except with negligible probability
- Obliviousness: access pattern is fixed for all executions
  - download each source group

Analysis of CacheShuffleRoot

- bandwidth: $(4+\epsilon)N$ blocks
  - each real block is downloaded exactly twice and uploaded exactly twice
  - each dummy block is uploaded exactly once and downloaded exactly once
  - number of dummy blocks is at most $\epsilon \cdot N$ except with negligible probability
- Obliviousness: access pattern is fixed for all executions
  - download each source group
  - upload one block to each destination group

Analysis of CacheShuffleRoot

- bandwidth: $(4 + \epsilon)N$ blocks
  - each real block is downloaded exactly twice and uploaded exactly twice
  - each dummy block is uploaded exactly once and downloaded exactly once
  - number of dummy blocks is at most $\epsilon \cdot N$ except with negligible probability
- Obliviousness: access pattern is fixed for all executions
  - download each source group
  - upload one block to each destination group
  - when done, download and upload each destination group

Analysis of CacheShuffleRoot

- bandwidth: $(4+\epsilon)N$ blocks
  - each real block is downloaded exactly twice and uploaded exactly twice
  - each dummy block is uploaded exactly once and downloaded exactly once
  - number of dummy blocks is at most $\epsilon \cdot N$ except with negligible probability
- Obliviousness: access pattern is fixed for all executions
  - download each source group
  - upload one block to each destination group
  - when done, download and upload each destination group
- $\mathcal{M}$'s storage: $\Theta(N)$

Analysis of CacheShuffleRoot

- bandwidth: $(4+\epsilon)N$ blocks
  - each real block is downloaded exactly twice and uploaded exactly twice
  - each dummy block is uploaded exactly once and downloaded exactly once
  - number of dummy blocks is at most $\epsilon \cdot N$ except with negligible probability
- Obliviousness: access pattern is fixed for all executions
  - download each source group
  - upload one block to each destination group
  - when done, download and upload each destination group
- $\mathcal{M}$'s storage: $\Theta(N)$
- $\mathcal{O}$'s storage

Analysis of CacheShuffleRoot

- bandwidth: $(4+\epsilon)N$ blocks
  - each real block is downloaded exactly twice and uploaded exactly twice
  - each dummy block is uploaded exactly once and downloaded exactly once
  - number of dummy blocks is at most $\epsilon \cdot N$ except with negligible probability
- Obliviousness: access pattern is fixed for all executions
  - download each source group
  - upload one block to each destination group
  - when done, download and upload each destination group
- $\mathcal{M}$'s storage: $\Theta(N)$
- $\mathcal{O}$'s storage
  - next slide

Analysis of CacheShuffleRoot: $\mathcal{O}$'s storage

Intuition

- $\mathcal{O}$'s keeps a cache for each of the $(1 + 2\epsilon)\sqrt{N}$ destination groups

Analysis of CacheShuffleRoot: $\mathcal{O}$'s storage

Intuition

- $\mathcal{O}$'s keeps a cache for each of the $(1 + 2\epsilon)\sqrt{N}$ destination groups
- each cache has arrival rate $\approx (1 - 2\epsilon)$

Analysis of CacheShuffleRoot: $\mathcal{O}$'s storage

Intuition

- $\mathcal{O}$'s keeps a cache for each of the $(1+2\epsilon)\sqrt{N}$ destination groups
- each cache has arrival rate $\approx (1-2\epsilon)$
- departure rate 1

Analysis of CacheShuffleRoot: $\mathcal{O}$'s storage

Intuition

- $\mathcal{O}$'s keeps a cache for each of the $(1+2\epsilon)\sqrt{N}$ destination groups
- each cache has arrival rate $\approx (1-2\epsilon)$
- departure rate 1
- each cache is expected to hold a constant number of blocks

Analysis of CacheShuffleRoot: $\mathcal{O}$'s storage

Intuition

- $\mathcal{O}$'s keeps a cache for each of the $(1 + 2\epsilon)\sqrt{N}$ destination groups
- each cache has arrival rate $\approx (1 - 2\epsilon)$
- departure rate 1
- each cache is expected to hold a constant number of blocks
- total $\mathcal{O}$'s storage is $\Theta(\sqrt{N})$ except with negligible probability

Analysis of CacheShuffleRoot: $\mathcal{O}$'s storage

Intuition

- $\mathcal{O}$'s keeps a cache for each of the $(1+2\epsilon)\sqrt{N}$ destination groups
- each cache has arrival rate $\approx (1-2\epsilon)$
- departure rate 1
- each cache is expected to hold a constant number of blocks
- total $\mathcal{O}$'s storage is $\Theta(\sqrt{N})$ except with negligible probability

Analysis of CacheShuffleRoot: $\mathcal{O}$'s storage

Intuition

- $\mathcal{O}$'s keeps a cache for each of the $(1 + 2\epsilon)\sqrt{N}$ destination groups
- each cache has arrival rate $\approx (1 - 2\epsilon)$
- departure rate 1
- each cache is expected to hold a constant number of blocks
- total $\mathcal{O}$'s storage is $\Theta(\sqrt{N})$ except with negligible probability

Formal argument

- cache sizes are not independent so cannot use Chernoff bound

Analysis of CacheShuffleRoot: $\mathcal{O}$'s storage

Intuition

- $\mathcal{O}$'s keeps a cache for each of the $(1+2\epsilon)\sqrt{N}$ destination groups
- each cache has arrival rate $\approx (1-2\epsilon)$
- departure rate 1
- each cache is expected to hold a constant number of blocks
- total $\mathcal{O}$'s storage is $\Theta(\sqrt{N})$ except with negligible probability

Formal argument

- cache sizes are not independent so cannot use Chernoff bound
- prove negative association and then use Chernoff bound

Keep the stash in $\mathcal{O}$'s memory and use CacheShuffleRoot

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Keep the stash in $\mathcal{O}$'s memory and use CacheShuffleRoot

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

Keep the stash in $\mathcal{O}$'s memory and use CacheShuffleRoot

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

- $\mathcal{M}$'s storage: $N + M = 10^6 + 10^3$ blocks

Keep the stash in $\mathcal{O}$'s memory and use CacheShuffleRoot

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

- $\mathcal{M}$'s storage: $N + M = 10^6 + 10^3$ blocks
- Cost of shuffling amortized per read operation:

$$4 \cdot 10^3 \approx 4000$$

Keep the stash in $\mathcal{O}$'s memory and use CacheShuffleRoot

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

- $\mathcal{M}$'s storage: $N + M = 10^6 + 10^3$ blocks
- Cost of shuffling amortized per read operation:

$$4 \cdot 10^3 \approx 4000$$

- Online cost: 2 blocks per read

Keep the stash in $\mathcal{O}$'s memory and use CacheShuffleRoot

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

- $\mathcal{M}$'s storage: $N + M = 10^6 + 10^3$ blocks
- Cost of shuffling amortized per read operation:

$$4 \cdot 10^3 \approx 4000$$

- Online cost: 2 blocks per read
- $\mathcal{O}$'s storage

Keep the stash in $\mathcal{O}$'s memory and use CacheShuffleRoot

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

- $\mathcal{M}$'s storage: $N + M = 10^6 + 10^3$ blocks
- Cost of shuffling amortized per read operation:

$$4 \cdot 10^3 \approx 4000$$

- Online cost: 2 blocks per read
- $\mathcal{O}$'s storage
  - cnt and nxt use constant storage

Keep the stash in $\mathcal{O}$'s memory and use CacheShuffleRoot

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

- $\mathcal{M}$'s storage: $N + M = 10^6 + 10^3$ blocks
- Cost of shuffling amortized per read operation:

$$4 \cdot 10^3 \approx 4000$$

- Online cost: 2 blocks per read
- $\mathcal{O}$'s storage
  - cnt and nxt use constant storage
  - π requires storing 10^6 4-byte integers=4 Megabytes

Keep the stash in $\mathcal{O}$'s memory and use CacheShuffleRoot

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash

Resources needed:

- $\mathcal{M}$'s storage: $N + M = 10^6 + 10^3$ blocks
- Cost of shuffling amortized per read operation:

$$4 \cdot 10^3 \approx 4000$$

- Online cost: 2 blocks per read
- $\mathcal{O}$'s storage
  - cnt and nxt use constant storage
  - π requires storing 10^6 4-byte integers=4 Megabytes
  - 1000 blocks of stash for a total of 4 Megabytes

Where are we now?

Construction -1 — Keep It

- $\mathcal{M}$'s storage: 0
- $\mathcal{O}$'s storage: N
- bandwidth 0

Construction 0 — Download It

- $\mathcal{M}$'s storage: N
- $\mathcal{O}$'s storage: 1
- bandwidth N

Construction 1 — Download Stash

- $\mathcal{M}$'s storage: $N + \sqrt{N}$
- $\mathcal{O}$'s storage: 1
- Online Comm. $O(\sqrt{N})$
- Am. Comm. $O(\sqrt{N} \cdot \log N)$

Construction 2 — Keep Stash

- $\mathcal{M}$'s storage: $N + \sqrt{N}$
- $\mathcal{O}$'s storage: $\sqrt{N}$
- Online Comm. 1
- Am. Comm. $O(\sqrt{N} \cdot \log N)$

Download and Keep Stash

- Use a cache (the Stash) to hide the repetition pattern

Download and Keep Stash

- Use a cache (the Stash) to hide the repetition pattern
- How does $\mathcal{O}$ hide access to the Stash?

Download and Keep Stash

- Use a cache (the Stash) to hide the repetition pattern
- How does $\mathcal{O}$ hide access to the Stash?
  - Use an ORAM!

Download and Keep Stash

- Use a cache (the Stash) to hide the repetition pattern
- How does $\mathcal{O}$ hide access to the Stash?
  - Use an ORAM!
  - We only have two possible ORAMs:

Download and Keep Stash

- Use a cache (the Stash) to hide the repetition pattern
- How does $\mathcal{O}$ hide access to the Stash?
  - Use an ORAM!
  - We only have two possible ORAMs:
  - Download Stash uses the Download It ORAM to manage the Stash

Download and Keep Stash

- Use a cache (the Stash) to hide the repetition pattern
- How does $\mathcal{O}$ hide access to the Stash?
  - Use an ORAM!
  - We only have two possible ORAMs:
  - Download Stash uses the Download It ORAM to manage the Stash
  - Keep Stash uses the Keep It ORAM to manage the Stash

Download and Keep Stash

- Use a cache (the Stash) to hide the repetition pattern
- How does $\mathcal{O}$ hide access to the Stash?
  - Use an ORAM!
  - We only have two possible ORAMs:
  - Download Stash uses the Download It ORAM to manage the Stash
  - Keep Stash uses the Keep It ORAM to manage the Stash

Download and Keep Stash

- Use a cache (the Stash) to hide the repetition pattern
- How does $\mathcal{O}$ hide access to the Stash?
  - Use an ORAM!
  - We only have two possible ORAMs:
  - Download Stash uses the Download It ORAM to manage the Stash
  - Keep Stash uses the Keep It ORAM to manage the Stash

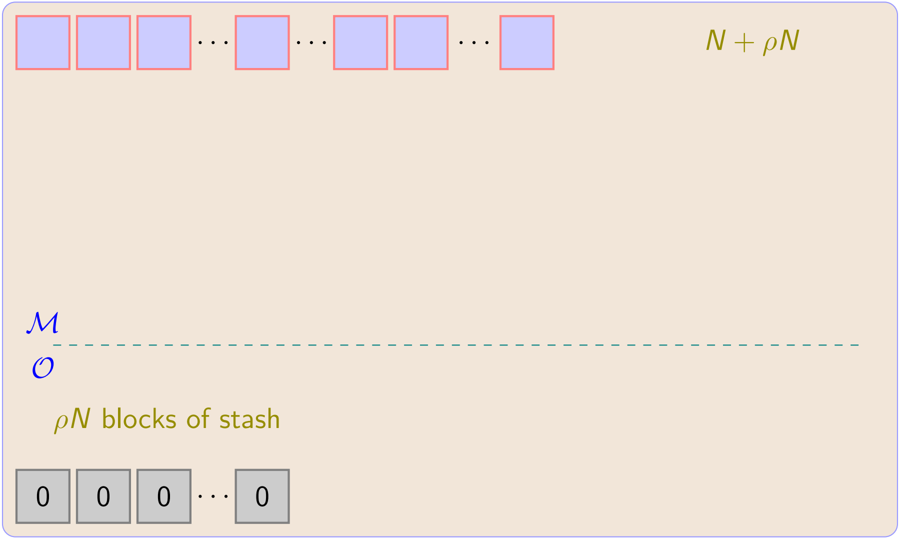
But now we have more ORAMs!!!

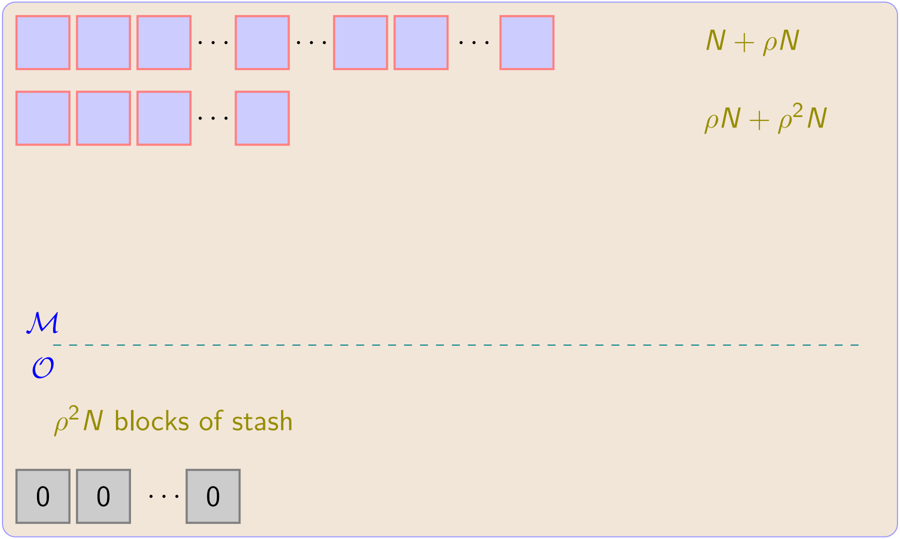
$\mathcal{M}$

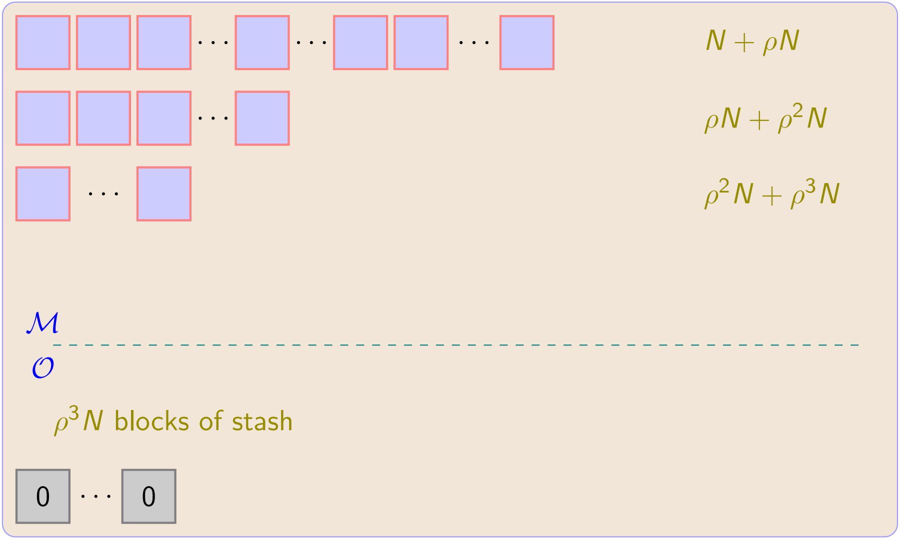
$\mathcal{O}$

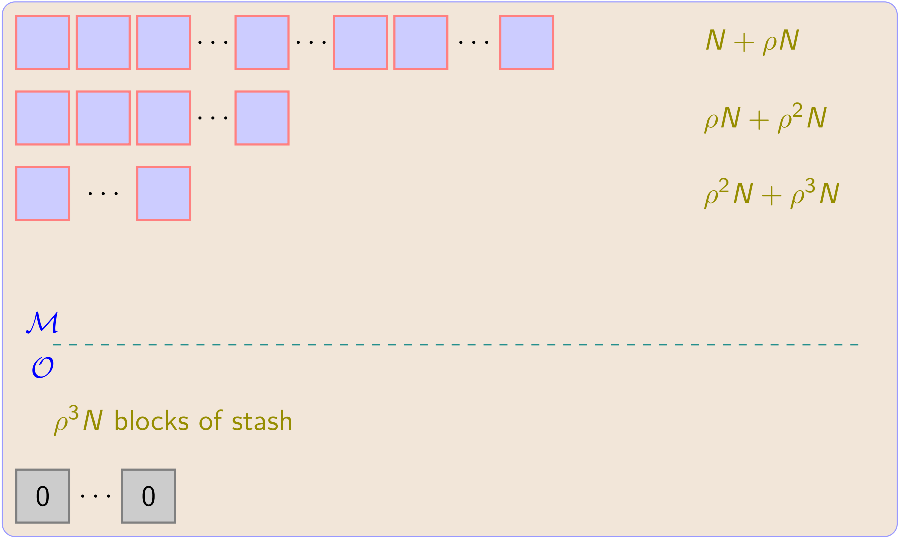
N blocks of data

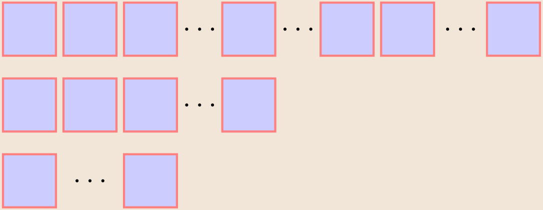
B_1 B_2 B_3 $\cdots$ B_N


$N + \rho N$
$\mathcal{M}$
$\mathcal{O}$
ρN blocks of stash
0 0 0 $\cdots$ 0


$\cdots$ $\cdots$ $\cdots$
$N + \rho N$
$\cdots$
$\rho N + \rho^2 N$
$\mathcal{M}$
$\mathcal{O}$
$\rho^2 N$ blocks of stash
0 0 $\cdots$ 0


$N + \rho N$
$\rho N + \rho^2 N$
$\rho^2 N + \rho^3 N$
$\mathcal{M}$
$\mathcal{O}$
$\rho^3 N$ blocks of stash
0 ... 0


$N + \rho N$
$\rho N + \rho^2 N$
$\rho^2 N + \rho^3 N$
$\mathcal{M}$
$\mathcal{O}$
$\rho^3 N$ blocks of stash
0 ⋯ 0



$N + \rho N$

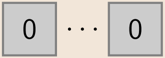
$\rho N + \rho^2 N$

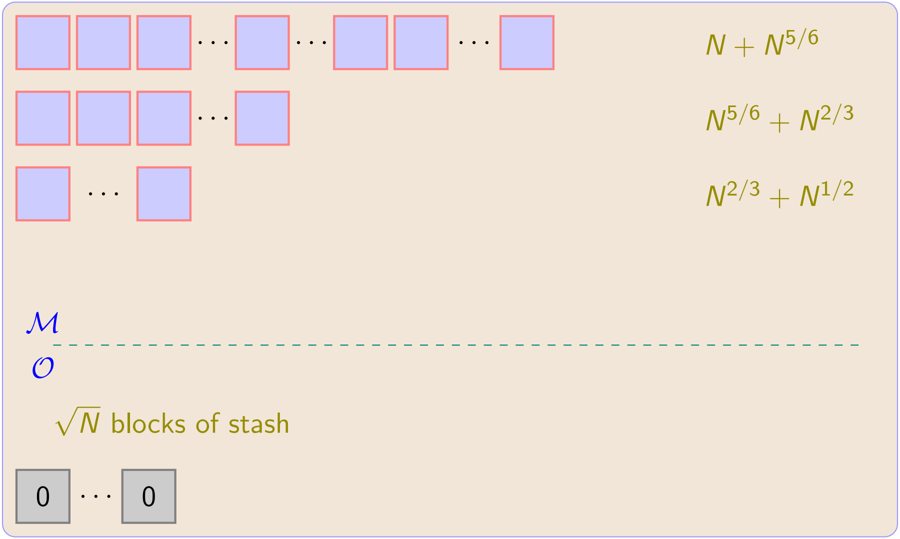
$\rho^2 N + \rho^3 N$

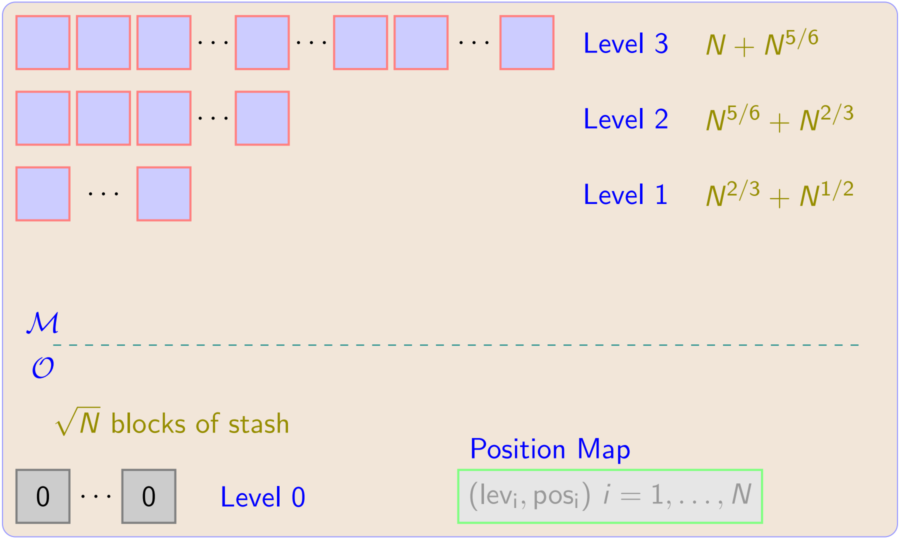
$\mathcal{M}$

$\mathcal{O}$

$\sqrt{N}$ blocks of stash $\qquad \rho = N^{-1/6}$




$N + N^{5/6}$
$N^{5/6} + N^{2/3}$
$N^{2/3} + N^{1/2}$
$\mathcal{M}$
$\mathcal{O}$
$\sqrt{N}$ blocks of stash
0 ⋯ 0


Level 3 $N + N^{5/6}$
Level 2 $N^{5/6} + N^{2/3}$
Level 1 $N^{2/3} + N^{1/2}$
$\mathcal{M}$
$\mathcal{O}$
$\sqrt{N}$ blocks of stash
Position Map
0 ⋯ 0
Level 0
$(\mathsf{lev_i}, \mathsf{pos_i})$ $i = 1, \ldots, N$

3-level ORAM: Querying

Querying B_q

- retrieve $(\mathrm{lev_q}, \mathrm{pos_q})$ from local memory;

3-level ORAM: Querying

Querying B_q

- retrieve $(\mathsf{lev}_\mathsf{q}, \mathsf{pos}_\mathsf{q})$ from local memory;
- if $\mathsf{lev}_\mathsf{q} \neq 0$

3-level ORAM: Querying

Querying B_q

- retrieve $(\mathsf{lev_q}, \mathsf{pos_q})$ from local memory;
- if $\mathsf{lev_q} \neq 0$
  - for all $l \neq \mathsf{lev_q}$, $\mathcal{O}$ asks for the next dummy in level l;

3-level ORAM: Querying

Querying B_q

- retrieve $(\mathsf{lev_q}, \mathsf{pos_q})$ from local memory;
- if $\mathsf{lev_q} \neq 0$
  - for all $l \neq \mathsf{lev_q}$, $\mathcal{O}$ asks for the next dummy in level l;
  - asks for block in $\mathsf{pos_q}$ from level $\mathsf{lev_q}$;

3-level ORAM: Querying

Querying B_q

- retrieve $(\mathsf{lev_q}, \mathsf{pos_q})$ from local memory;
- if $\mathsf{lev_q} \neq 0$
  - for all $l \neq \mathsf{lev_q}$, $\mathcal{O}$ asks for the next dummy in level l;
  - asks for block in $\mathsf{pos_q}$ from level $\mathsf{lev_q}$;
  - B_q is then stored in level 0 and $(\mathsf{lev_q}, \mathsf{pos_q})$ is updated;

3-level ORAM: Querying

Querying B_q

- retrieve $(\mathsf{lev_q}, \mathsf{pos_q})$ from local memory;
- if $\mathsf{lev_q} \neq 0$
  - for all $l \neq \mathsf{lev_q}$, $\mathcal{O}$ asks for the next dummy in level l;
  - asks for block in $\mathsf{pos_q}$ from level $\mathsf{lev_q}$;
  - B_q is then stored in level 0 and $(\mathsf{lev_q}, \mathsf{pos_q})$ is updated;
- else

3-level ORAM: Querying

Querying B_q

- retrieve $(\mathsf{lev_q}, \mathsf{pos_q})$ from local memory;
- if $\mathsf{lev_q} \neq 0$
  - for all $l \neq \mathsf{lev_q}$, $\mathcal{O}$ asks for the next dummy in level l;
  - asks for block in $\mathsf{pos_q}$ from level $\mathsf{lev_q}$;
  - B_q is then stored in level 0 and $(\mathsf{lev_q}, \mathsf{pos_q})$ is updated;
- else
  - for $l = 1, 2, 3$, $\mathcal{O}$ asks for the next dummy in level l;

3-level ORAM: Querying

Querying B_q

- retrieve $(\mathrm{lev_q}, \mathrm{pos_q})$ from local memory;
- if $\mathrm{lev_q} \neq 0$
  - for all $l \neq \mathrm{lev_q}$, $\mathcal{O}$ asks for the next dummy in level l;
  - asks for block in $\mathrm{pos_q}$ from level $\mathrm{lev_q}$;
  - B_q is then stored in level 0 and $(\mathrm{lev_q}, \mathrm{pos_q})$ is updated;
- else
  - for $l = 1, 2, 3$, $\mathcal{O}$ asks for the next dummy in level l;
  - block B_q is retrieved from local stash (level 0);

3-level ORAM: Analysis

- at the start only level 3 contains real blocks;

3-level ORAM: Analysis

- at the start only level 3 contains real blocks;
- the local stash is full after $N^{1/2}$ queries

3-level ORAM: Analysis

- at the start only level 3 contains real blocks;
- the local stash is full after $N^{1/2}$ queries
  - it is shuffled with the level 1 of size $N^{2/3}$;

3-level ORAM: Analysis

- at the start only level 3 contains real blocks;
- the local stash is full after $N^{1/2}$ queries
  - it is shuffled with the level 1 of size $N^{2/3}$;
  - each shuffle costs $4N^{2/3}$

3-level ORAM: Analysis

- at the start only level 3 contains real blocks;
- the local stash is full after $N^{1/2}$ queries
  - it is shuffled with the level 1 of size $N^{2/3}$;
  - each shuffle costs $4N^{2/3}$
  - over N queries, it happens $N^{1/2}$ times

3-level ORAM: Analysis

- at the start only level 3 contains real blocks;
- the local stash is full after $N^{1/2}$ queries
  - it is shuffled with the level 1 of size $N^{2/3}$;
  - each shuffle costs $4N^{2/3}$
  - over N queries, it happens $N^{1/2}$ times
  - total cost: $4 \cdot N^{7/6}$

3-level ORAM: Analysis

- at the start only level 3 contains real blocks;
- the local stash is full after $N^{1/2}$ queries
  - it is shuffled with the level 1 of size $N^{2/3}$;
  - each shuffle costs $4N^{2/3}$
  - over N queries, it happens $N^{1/2}$ times
  - total cost: $4 \cdot N^{7/6}$
- level 1 is full after $N^{2/3}$ queries

3-level ORAM: Analysis

- at the start only level 3 contains real blocks;
- the local stash is full after $N^{1/2}$ queries
  - it is shuffled with the level 1 of size $N^{2/3}$;
  - each shuffle costs $4N^{2/3}$
  - over N queries, it happens $N^{1/2}$ times
  - total cost: $4 \cdot N^{7/6}$
- level 1 is full after $N^{2/3}$ queries
  - it is shuffled with the level 2 of size $N^{5/6}$;

3-level ORAM: Analysis

- at the start only level 3 contains real blocks;
- the local stash is full after $N^{1/2}$ queries
  - it is shuffled with the level 1 of size $N^{2/3}$;
  - each shuffle costs $4N^{2/3}$
  - over N queries, it happens $N^{1/2}$ times
  - total cost: $4 \cdot N^{7/6}$
- level 1 is full after $N^{2/3}$ queries
  - it is shuffled with the level 2 of size $N^{5/6}$;
  - each shuffle costs $4N^{5/6}$

3-level ORAM: Analysis

- at the start only level 3 contains real blocks;
- the local stash is full after $N^{1/2}$ queries
  - it is shuffled with the level 1 of size $N^{2/3}$;
  - each shuffle costs $4N^{2/3}$
  - over N queries, it happens $N^{1/2}$ times
  - total cost: $4 \cdot N^{7/6}$
- level 1 is full after $N^{2/3}$ queries
  - it is shuffled with the level 2 of size $N^{5/6}$;
  - each shuffle costs $4N^{5/6}$
  - over N queries, it happens $N^{1/3}$ times

3-level ORAM: Analysis

- at the start only level 3 contains real blocks;
- the local stash is full after $N^{1/2}$ queries
  - it is shuffled with the level 1 of size $N^{2/3}$;
  - each shuffle costs $4N^{2/3}$
  - over N queries, it happens $N^{1/2}$ times
  - total cost: $4 \cdot N^{7/6}$
- level 1 is full after $N^{2/3}$ queries
  - it is shuffled with the level 2 of size $N^{5/6}$;
  - each shuffle costs $4N^{5/6}$
  - over N queries, it happens $N^{1/3}$ times
  - total cost: $4 \cdot N^{7/6}$

3-level ORAM: Analysis

- at the start only level 3 contains real blocks;
- the local stash is full after $N^{1/2}$ queries
  - it is shuffled with the level 1 of size $N^{2/3}$;
  - each shuffle costs $4N^{2/3}$
  - over N queries, it happens $N^{1/2}$ times
  - total cost: $4 \cdot N^{7/6}$
- level 1 is full after $N^{2/3}$ queries
  - it is shuffled with the level 2 of size $N^{5/6}$;
  - each shuffle costs $4N^{5/6}$
  - over N queries, it happens $N^{1/3}$ times
  - total cost: $4 \cdot N^{7/6}$
- level 2 is full after $N^{5/6}$ queries

3-level ORAM: Analysis

- at the start only level 3 contains real blocks;
- the local stash is full after $N^{1/2}$ queries
  - it is shuffled with the level 1 of size $N^{2/3}$;
  - each shuffle costs $4N^{2/3}$
  - over N queries, it happens $N^{1/2}$ times
  - total cost: $4 \cdot N^{7/6}$
- level 1 is full after $N^{2/3}$ queries
  - it is shuffled with the level 2 of size $N^{5/6}$;
  - each shuffle costs $4N^{5/6}$
  - over N queries, it happens $N^{1/3}$ times
  - total cost: $4 \cdot N^{7/6}$
- level 2 is full after $N^{5/6}$ queries
  - it is shuffled with the level 3 of size N;

3-level ORAM: Analysis

- at the start only level 3 contains real blocks;
- the local stash is full after $N^{1/2}$ queries
  - it is shuffled with the level 1 of size $N^{2/3}$;
  - each shuffle costs $4N^{2/3}$
  - over N queries, it happens $N^{1/2}$ times
  - total cost: $4 \cdot N^{7/6}$
- level 1 is full after $N^{2/3}$ queries
  - it is shuffled with the level 2 of size $N^{5/6}$;
  - each shuffle costs $4N^{5/6}$
  - over N queries, it happens $N^{1/3}$ times
  - total cost: $4 \cdot N^{7/6}$
- level 2 is full after $N^{5/6}$ queries
  - it is shuffled with the level 3 of size N;
  - each shuffle costs $4N$

3-level ORAM: Analysis

- at the start only level 3 contains real blocks;
- the local stash is full after $N^{1/2}$ queries
  - it is shuffled with the level 1 of size $N^{2/3}$;
  - each shuffle costs $4N^{2/3}$
  - over N queries, it happens $N^{1/2}$ times
  - total cost: $4 \cdot N^{7/6}$
- level 1 is full after $N^{2/3}$ queries
  - it is shuffled with the level 2 of size $N^{5/6}$;
  - each shuffle costs $4N^{5/6}$
  - over N queries, it happens $N^{1/3}$ times
  - total cost: $4 \cdot N^{7/6}$
- level 2 is full after $N^{5/6}$ queries
  - it is shuffled with the level 3 of size N;
  - each shuffle costs $4N$
  - over N queries, it happens $N^{1/6}$ times

3-level ORAM: Analysis

- at the start only level 3 contains real blocks;
- the local stash is full after $N^{1/2}$ queries
  - it is shuffled with the level 1 of size $N^{2/3}$;
  - each shuffle costs $4N^{2/3}$
  - over N queries, it happens $N^{1/2}$ times
  - total cost: $4 \cdot N^{7/6}$
- level 1 is full after $N^{2/3}$ queries
  - it is shuffled with the level 2 of size $N^{5/6}$;
  - each shuffle costs $4N^{5/6}$
  - over N queries, it happens $N^{1/3}$ times
  - total cost: $4 \cdot N^{7/6}$
- level 2 is full after $N^{5/6}$ queries
  - it is shuffled with the level 3 of size N;
  - each shuffle costs $4N$
  - over N queries, it happens $N^{1/6}$ times
  - total cost: $4 \cdot N^{7/6}$

3-level ORAM: Analysis

- at the start only level 3 contains real blocks;
- the local stash is full after $N^{1/2}$ queries
  - it is shuffled with the level 1 of size $N^{2/3}$;
  - each shuffle costs $4N^{2/3}$
  - over N queries, it happens $N^{1/2}$ times
  - total cost: $4 \cdot N^{7/6}$
- level 1 is full after $N^{2/3}$ queries
  - it is shuffled with the level 2 of size $N^{5/6}$;
  - each shuffle costs $4N^{5/6}$
  - over N queries, it happens $N^{1/3}$ times
  - total cost: $4 \cdot N^{7/6}$
- level 2 is full after $N^{5/6}$ queries
  - it is shuffled with the level 3 of size N;
  - each shuffle costs $4N$
  - over N queries, it happens $N^{1/6}$ times
  - total cost: $4 \cdot N^{7/6}$
- Over N queries, the cost is $12 \cdot N^{7/6}$

3-level ORAM: Analysis

- at the start only level 3 contains real blocks;
- the local stash is full after $N^{1/2}$ queries
  - it is shuffled with the level 1 of size $N^{2/3}$;
  - each shuffle costs $4N^{2/3}$
  - over N queries, it happens $N^{1/2}$ times
  - total cost: $4 \cdot N^{7/6}$
- level 1 is full after $N^{2/3}$ queries
  - it is shuffled with the level 2 of size $N^{5/6}$;
  - each shuffle costs $4N^{5/6}$
  - over N queries, it happens $N^{1/3}$ times
  - total cost: $4 \cdot N^{7/6}$
- level 2 is full after $N^{5/6}$ queries
  - it is shuffled with the level 3 of size N;
  - each shuffle costs $4N$
  - over N queries, it happens $N^{1/6}$ times
  - total cost: $4 \cdot N^{7/6}$
- Over N queries, the cost is $12 \cdot N^{7/6}$
  - each query has an amortized cost of $12N^{1/6}$ blocks;

3-level ORAM: in practice

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash for a total of 4 Megabytes

3-level ORAM: in practice

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash for a total of 4 Megabytes

Resources needed:

3-level ORAM: in practice

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash for a total of 4 Megabytes

Resources needed:

- $\mathcal{M}$'s storage: $10^6 + 2 \cdot 10^5 + 2 \cdot 10^4 + 10^3 = 1,221,000$ blocks

3-level ORAM: in practice

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash for a total of 4 Megabytes

Resources needed:

- $\mathcal{M}$'s storage: $10^6 + 2 \cdot 10^5 + 2 \cdot 10^4 + 10^3 = 1,221,000$ blocks
- Cost of shuffling amortized per read operation:

$$4 \cdot 10^3 \approx 120$$

3-level ORAM: in practice

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash for a total of 4 Megabytes

Resources needed:

- $\mathcal{M}$'s storage: $10^6 + 2 \cdot 10^5 + 2 \cdot 10^4 + 10^3 = 1,221,000$ blocks
- Cost of shuffling amortized per read operation:

$$4 \cdot 10^3 \approx 120$$

- Online cost: 3 blocks downloaded

3-level ORAM: in practice

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash for a total of 4 Megabytes

Resources needed:

- $\mathcal{M}$'s storage: $10^6 + 2 \cdot 10^5 + 2 \cdot 10^4 + 10^3 = 1,221,000$ blocks
- Cost of shuffling amortized per read operation:

$$4 \cdot 10^3 \approx 120$$

- Online cost: 3 blocks downloaded
- $\mathcal{O}$'s storage

3-level ORAM: in practice

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash for a total of 4 Megabytes

Resources needed:

- $\mathcal{M}$'s storage: $10^6 + 2 \cdot 10^5 + 2 \cdot 10^4 + 10^3 = 1,221,000$ blocks
- Cost of shuffling amortized per read operation:

$$4 \cdot 10^3 \approx 120$$

- Online cost: 3 blocks downloaded
- $\mathcal{O}$'s storage
  - cnt and nxt use constant storage

3-level ORAM: in practice

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash for a total of 4 Megabytes

Resources needed:

- $\mathcal{M}$'s storage: $10^6 + 2 \cdot 10^5 + 2 \cdot 10^4 + 10^3 = 1,221,000$ blocks
- Cost of shuffling amortized per read operation:

$$4 \cdot 10^3 \approx 120$$

- Online cost: 3 blocks downloaded
- $\mathcal{O}$'s storage
  - cnt and nxt use constant storage
  - position map requires storing 10^6 6-byte integers $\approx$ 4 Megabytes

3-level ORAM: in practice

Same setting:

- $N = 10^6$ blocks of 4K each for a total of 4 Gigabytes
- $M = 10^3$ blocks of stash for a total of 4 Megabytes

Resources needed:

- $\mathcal{M}$'s storage: $10^6 + 2 \cdot 10^5 + 2 \cdot 10^4 + 10^3 = 1,221,000$ blocks
- Cost of shuffling amortized per read operation:

$$4 \cdot 10^3 \approx 120$$

- Online cost: 3 blocks downloaded
- $\mathcal{O}$'s storage
  - cnt and nxt use constant storage
  - position map requires storing 10^6 6-byte integers $\approx$ 4 Megabytes
  - 1000 blocks of stash for a total of 4 Megabytes

Why stop at 3 levels?

$\mathcal{O}$'s memory $\approx \sqrt{N}$

- set $\rho = 1/2$

Why stop at 3 levels?

$\mathcal{O}$'s memory $\approx \sqrt{N}$

- set $\rho = 1/2$
- $1/2 \log_2 N$ levels with

Why stop at 3 levels?

$\mathcal{O}$'s memory $\approx \sqrt{N}$

- set $\rho = 1/2$
- $1/2 \log_2 N$ levels with
  - $N + N/2$ blocks

Why stop at 3 levels?

$\mathcal{O}$'s memory $\approx \sqrt{N}$

- set $\rho = 1/2$
- $1/2 \log_2 N$ levels with
  - $N + N/2$ blocks
  - $N/2 + N/4$ blocks

Why stop at 3 levels?

$\mathcal{O}$'s memory $\approx \sqrt{N}$

- set $\rho = 1/2$
- $1/2 \log_2 N$ levels with
  - $N + N/2$ blocks
  - $N/2 + N/4$ blocks
  - $\ldots$

Why stop at 3 levels?

$\mathcal{O}$'s memory $\approx \sqrt{N}$

- set $\rho = 1/2$
- $1/2 \log_2 N$ levels with
  - $N + N/2$ blocks
  - $N/2 + N/4$ blocks
  - $\ldots$
  - $3\sqrt{N}$ blocks

Why stop at 3 levels?

$\mathcal{O}$'s memory $\approx \sqrt{N}$

- set $\rho = 1/2$
- $1/2 \log_2 N$ levels with
  - $N + N/2$ blocks
  - $N/2 + N/4$ blocks
  - $\ldots$
  - $3\sqrt{N}$ blocks
- Amortized bandwidth $0.55 \cdot \log_2 N$

Why stop at 3 levels?

$\mathcal{O}$'s memory $\approx \sqrt{N}$

- set $\rho = 1/2$
- $1/2 \log_2 N$ levels with
  - $N + N/2$ blocks
  - $N/2 + N/4$ blocks
  - $\ldots$
  - $3\sqrt{N}$ blocks
- Amortized bandwidth $0.55 \cdot \log_2 N$
  - For $N = 10^6$, 21 Blocks

Why stop at 3 levels?

$\mathcal{O}$'s memory $\approx \sqrt{N}$

- set $\rho = 1/2$
- $1/2 \log_2 N$ levels with
  - $N + N/2$ blocks
  - $N/2 + N/4$ blocks
  - $\ldots$
  - $3\sqrt{N}$ blocks
- Amortized bandwidth $0.55 \cdot \log_2 N$
  - For $N = 10^6$, 21 Blocks
- OnLine bandwidth: 1 Block

Why stop at 3 levels?

$\mathcal{O}$'s memory $\approx \sqrt{N}$

- set $\rho = 1/2$
- $1/2 \log_2 N$ levels with
  - $N + N/2$ blocks
  - $N/2 + N/4$ blocks
  - $\ldots$
  - $3\sqrt{N}$ blocks
- Amortized bandwidth $0.55 \cdot \log_2 N$
  - For $N = 10^6$, 21 Blocks
- OnLine bandwidth: 1 Block

Why stop at 3 levels?

$\mathcal{O}$'s memory $\approx \sqrt{N}$

- set $\rho = 1/2$
- $1/2 \log_2 N$ levels with
  - $N + N/2$ blocks
  - $N/2 + N/4$ blocks
  - $\ldots$
  - $3\sqrt{N}$ blocks
- Amortized bandwidth $0.55 \cdot \log_2 N$
  - For $N = 10^6$, 21 Blocks
- OnLine bandwidth: 1 Block

Techniques to reduce bandwidth

- XOR Technique
- Homomorphic Selection
- Compression via Polynomial Interpolation

The XOR technique to reduce bandwidth

l-level ORAM

- when asking for B_q, $\mathcal{O}$ asks one block for each level

The XOR technique to reduce bandwidth

l-level ORAM

- when asking for B_q, $\mathcal{O}$ asks one block for each level
- at most one block is real

The XOR technique to reduce bandwidth

l-level ORAM

- when asking for B_q, $\mathcal{O}$ asks one block for each level
- at most one block is real
- suppose $\mathcal{O}$ can compute locally the $l-1$ dummy blocks requested

The XOR technique to reduce bandwidth

l-level ORAM

- when asking for B_q, $\mathcal{O}$ asks one block for each level
- at most one block is real
- suppose $\mathcal{O}$ can compute locally the $l-1$ dummy blocks requested
  - $\mathcal{M}$ instead of sending all l blocks individually, xors them together and sends the result to $\mathcal{O}$

The XOR technique to reduce bandwidth

l-level ORAM

- when asking for B_q, $\mathcal{O}$ asks one block for each level
- at most one block is real
- suppose $\mathcal{O}$ can compute locally the $l-1$ dummy blocks requested
  - $\mathcal{M}$ instead of sending all l blocks individually, xors them together and sends the result to $\mathcal{O}$
  - $\mathcal{O}$ computes the $l-1$ dummy blocks and xors them with the block received from $\mathcal{M}$

Assumption:

suppose $\mathcal{O}$ can compute any dummy block without interacting with $\mathcal{M}$

- each block uniquely identified by $(\mathsf{l}, \mathsf{pos})$
- a dummy block is an AES-ECB encryption of 0^{len}
- using key $\mathcal{F}(K, (\mathsf{l}, \mathsf{pos}))$
  - $\mathcal{F}$ is a pseudorandom function
  - K is a randomly chosen seed private to $\mathcal{O}$

Some Theory

A Taxonomy

- **OnLine** vs **OffLine ORAM**
  - In an **OnLine** ORAM, all requests come one at the time and must be satisfied before the next one
  - in an **OffLine** ORAM, all requests come together
- **BallsAndBins**
  - Blocks are atomic and opaque blobs of data
- **Passive** vs **Active** $\mathcal{M}$
  - A **Passive** $\mathcal{M}$ only moves data
  - An **Active** $\mathcal{M}$ can perform computation on data
    - ⋆ The **XOR technique** requires an **Active** $\mathcal{M}$

What we know in the asymptotic world

- All the following require $\mathbf{\Omega}(\log \mathbf{N})$

What we know in the asymptotic world

- All the following require $\mathbf{\Omega}(\log \mathbf{N})$
  - **BallsAndBins** and **OffLine** with **Passive** $\mathcal{M}$

What we know in the asymptotic world

- All the following require $\mathbf{\Omega}(\log \mathbf{N})$
  - **BallsAndBins** and **OffLine** with **Passive** $\mathcal{M}$
  - **NonBallsAndBins** and **OnLine** with **Passive** $\mathcal{M}$

What we know in the asymptotic world

- All the following require $\mathbf{\Omega}(\log \mathbf{N})$
  - **BallsAndBins** and **OffLine** with **Passive** $\mathcal{M}$
  - **NonBallsAndBins** and **OnLine** with **Passive** $\mathcal{M}$
- There are $\tilde{\mathbf{O}}(\log \mathbf{N})$ **OnLine** ORAM in the Balls and Bins model with **Passive** $\mathcal{M}$

What we know in the asymptotic world

- All the following require $\mathbf{\Omega}(\log \mathbf{N})$
  - **BallsAndBins** and **OffLine** with **Passive** $\mathcal{M}$
  - **NonBallsAndBins** and **OnLine** with **Passive** $\mathcal{M}$
- There are $\tilde{\mathbf{O}}(\log \mathbf{N})$ **OnLine** ORAM in the Balls and Bins model with **Passive** $\mathcal{M}$
- There are $\mathbf{o}(\log \mathbf{N})$ **OnLine** ORAM with **Active** $\mathcal{M}$

What we know in the asymptotic world

- All the following require $\mathbf{\Omega}(\log \mathbf{N})$
  - **BallsAndBins** and **OffLine** with **Passive** $\mathcal{M}$
  - **NonBallsAndBins** and **OnLine** with **Passive** $\mathcal{M}$
- There are $\tilde{\mathbf{O}}(\log \mathbf{N})$ **OnLine** ORAM in the Balls and Bins model with **Passive** $\mathcal{M}$
- There are $\mathbf{o}(\log \mathbf{N})$ **OnLine** ORAM with **Active** $\mathcal{M}$
- Proving lower bound for **Non-BallsAndBins** and **OffLine** with **Passive** $\mathcal{M}$ would give a superlinear lower bound for sorting circuits.

Differential Privacy

ORAM

(ϵ, δ)-Differential Privacy

- $\mathcal{M}$ stores n blocks of memory.
- Every time $\mathcal{O}$ wants a block, he asks $\mathcal{M}$ one or more blocks.
- Security notion:
  - For any two block sequences $\mathbb{B} = B_1, \ldots, B_n$ and $\mathbb{C} = C_1, \ldots, C_n$
  - For any two access sequences $i_1, \ldots, i_l$ and $j_1, \ldots, j_l$ **that differ in one position**
    - performing access $i_1, \ldots, i_l$ on $\mathbb{B} = B_1, \ldots, B_n$;
    - performing access $j_1, \ldots, j_l$ on $\mathbb{C} = C_1, \ldots, C_n$

  generate the same distribution of accesses to the data stored by $\mathcal{M}$

Differential Privacy

▸ ORAM

(ϵ, δ)-Differential Privacy

- $\mathcal{M}$ stores n blocks of memory.
- Every time $\mathcal{O}$ wants a block, he asks $\mathcal{M}$ one or more blocks.
- Security notion:
  - For any two block sequences $\mathbb{B} = B_1, \ldots, B_n$ and $\mathbb{C} = C_1, \ldots, C_n$
  - For any two access sequences $i_1, \ldots, i_l$ and $j_1, \ldots, j_l$ **that differ in one position**
    - performing access $i_1, \ldots, i_l$ on $\mathbb{B} = B_1, \ldots, B_n$;
    - performing access $j_1, \ldots, j_l$ on $\mathbb{C} = C_1, \ldots, C_n$

  generate the same distribution of accesses to the data stored by $\mathcal{M}$

For every predicate A

$$\begin{aligned} &\mathrm{Prob}[\mathtt{view} \leftarrow \mathtt{View}(I, \mathbb{B}) : A(\mathtt{view}) = 1] \\ &\qquad \leq e^{\epsilon} \cdot \mathrm{Prob}[\mathtt{view} \leftarrow \mathtt{View}(J, \mathbb{C}) : A(\mathtt{view}) = 1] + \delta \end{aligned}$$

Differential Privacy

- it protects only individual accesses

Differential Privacy

- it protects only individual accesses
- if the probability of an access is larger than $1/e^{\epsilon}$ no guarantee is given

Differential Privacy

- it protects only individual accesses
- if the probability of an access is larger than $1/e^{\epsilon}$ no guarantee is given
- it protects unusual accesses

Differential Privacy

- it protects only individual accesses
- if the probability of an access is larger than $1/e^{\epsilon}$ no guarantee is given
- it protects unusual accesses
- how unusual?

Differential Privacy

- it protects only individual accesses
- if the probability of an access is larger than $1/e^{\epsilon}$ no guarantee is given
- it protects unusual accesses
- how unusual?
  - it depends on ϵ

Differential Privacy

- it protects only individual accesses
- if the probability of an access is larger than $1/e^{\epsilon}$ no guarantee is given
- it protects unusual accesses
- how unusual?
  - it depends on ϵ
- no protection is offered for high-probability accesses

Differential Privacy

- it protects only individual accesses
- if the probability of an access is larger than $1/e^{\epsilon}$ no guarantee is given
- it protects unusual accesses
- how unusual?
  - it depends on ϵ
- no protection is offered for high-probability accesses
  - nothing is lost: my daily routine is public

Differential Privacy

- it protects only individual accesses
- if the probability of an access is larger than $1/e^{\epsilon}$ no guarantee is given
- it protects unusual accesses
- how unusual?
  - it depends on ϵ
- no protection is offered for high-probability accesses
  - nothing is lost: my daily routine is public
- I really want to protect unusual accesses to documents that might reveal something

Differential Privacy

- it protects only individual accesses
- if the probability of an access is larger than $1/e^{\epsilon}$ no guarantee is given
- it protects unusual accesses
- how unusual?
  - it depends on ϵ
- no protection is offered for high-probability accesses
  - nothing is lost: my daily routine is public
- I really want to protect unusual accesses to documents that might reveal something
  - I am checking my medical records from some time ago...

$\mathcal{M}$

$\mathcal{O}$

N blocks of data

$\boxed{B_1}\ \boxed{B_2}\ \boxed{B_3} \cdots \boxed{B_N}$

B_1 B_2 B_3 $\cdots$ B_N

N encrypted blocks

probability p

$\mathcal{M}$

$\mathcal{O}$

B_1 B_2 B_3 $\cdots$ B_N

N encrypted blocks

probability p

$\mathcal{M}$

$\mathcal{O}$

Stash B_7 B_9 $\cdot\cdot$ B_{N-2}

pN expected blocks

B_1 B_2 B_3 $\cdots$ B_N

N encrypted blocks

probability p

$\mathcal{M}$

$\mathcal{O}$

Stash B_7 B_9 $\cdot\cdot$ B_{N-2}

pN expected blocks

B_1 B_2 B_3 $\cdots$ B_N

N encrypted blocks

probability p

$\mathcal{M}$

$\mathcal{O}$

Stash B_7 B_9 $\cdots$ B_{N-2}

pN expected blocks

B_1 B_2 B_3 $\cdots$ B_N

N encrypted blocks

probability p

$\mathcal{M}$

$\mathcal{O}$

Stash B_7 B_9 $\cdot\cdot$ B_{N-2}

pN expected blocks

Querying for B_i

Download Phase

- if B_i is found in the stash:

Overwrite Phase

Querying for B_i

Download Phase

- if B_i is found in the stash:
  - return it

Overwrite Phase

Querying for B_i

Download Phase

- if B_i is found in the stash:
  - return it
  - remove it from the stash

Overwrite Phase

Querying for B_i

Download Phase

- if B_i is found in the stash:
  - return it
  - remove it from the stash
  - ask $\mathcal{M}$ for a random block and then discard it

Overwrite Phase

Querying for B_i

Download Phase

- if B_i is found in the stash:
  - return it
  - remove it from the stash
  - ask $\mathcal{M}$ for a random block and then discard it
- if B_i is not found in the stash:

Overwrite Phase

Querying for B_i

Download Phase

- if B_i is found in the stash:
  - return it
  - remove it from the stash
  - ask $\mathcal{M}$ for a random block and then discard it
- if B_i is not found in the stash:
  - ask $\mathcal{M}$ for B_i

Overwrite Phase

Querying for B_i

Download Phase

- if B_i is found in the stash:
  - return it
  - remove it from the stash
  - ask $\mathcal{M}$ for a random block and then discard it
- if B_i is not found in the stash:
  - ask $\mathcal{M}$ for B_i

Overwrite Phase

- Toss a coin with probability $(p, 1 - p)$

Querying for B_i

Download Phase

- if B_i is found in the stash:
  - return it
  - remove it from the stash
  - ask $\mathcal{M}$ for a random block and then discard it
- if B_i is not found in the stash:
  - ask $\mathcal{M}$ for B_i

Overwrite Phase

- Toss a coin with probability $(p, 1 - p)$
- if head

Querying for B_i

Download Phase

- if B_i is found in the stash:
  - return it
  - remove it from the stash
  - ask $\mathcal{M}$ for a random block and then discard it
- if B_i is not found in the stash:
  - ask $\mathcal{M}$ for B_i

Overwrite Phase

- Toss a coin with probability $(p, 1 - p)$
- if head
  - add B_i to stash

Querying for B_i

Download Phase

- if B_i is found in the stash:
  - return it
  - remove it from the stash
  - ask $\mathcal{M}$ for a random block and then discard it
- if B_i is not found in the stash:
  - ask $\mathcal{M}$ for B_i

Overwrite Phase

- Toss a coin with probability $(p, 1-p)$
- if head
  - add B_i to stash
  - ask $\mathcal{M}$ for a randomly selected block, re-encrypt it and upload to the same location

Querying for B_i

Download Phase

- if B_i is found in the stash:
  - return it
  - remove it from the stash
  - ask $\mathcal{M}$ for a random block and then discard it
- if B_i is not found in the stash:
  - ask $\mathcal{M}$ for B_i

Overwrite Phase

- Toss a coin with probability $(p, 1 - p)$
- if head
  - add B_i to stash
  - ask $\mathcal{M}$ for a randomly selected block, re-encrypt it and upload to the same location
- if tail

Querying for B_i

Download Phase

- if B_i is found in the stash:
  - return it
  - remove it from the stash
  - ask $\mathcal{M}$ for a random block and then discard it
- if B_i is not found in the stash:
  - ask $\mathcal{M}$ for B_i

Overwrite Phase

- Toss a coin with probability $(p, 1 - p)$
- if head
  - add B_i to stash
  - ask $\mathcal{M}$ for a randomly selected block, re-encrypt it and upload to the same location
- if tail
  - download B_i from $\mathcal{M}$

Querying for B_i

Download Phase

- if B_i is found in the stash:
  - return it
  - remove it from the stash
  - ask $\mathcal{M}$ for a random block and then discard it
- if B_i is not found in the stash:
  - ask $\mathcal{M}$ for B_i

Overwrite Phase

- Toss a coin with probability $(p, 1 - p)$
- if head
  - add B_i to stash
  - ask $\mathcal{M}$ for a randomly selected block, re-encrypt it and upload to the same location
- if tail
  - download B_i from $\mathcal{M}$
  - decrypt and re-encrypt and upload it to the same location

Analysis

- Two blocks of communication

Analysis

- Two blocks of communication
- $p = \omega(\log n/n)$ with $\omega(\log n)$ client memory

Analysis

- Two blocks of communication
- $p = \omega(\log n/n)$ with $\omega(\log n)$ client memory
- $\epsilon = \Theta(\log n)$

Analysis

- Two blocks of communication
- $p = \omega(\log n/n)$ with $\omega(\log n)$ client memory
- $\epsilon = \Theta(\log n)$

Analysis

- Two blocks of communication
- $p = \omega(\log n/n)$ with $\omega(\log n)$ client memory
- $\epsilon = \Theta(\log n)$

Theorem

For any $\epsilon \geq 0$, any DP-RAM *with error probability $\alpha \geq 0$ in the balls and bins model and a client that stores at most c blocks must operate on*

$$\Omega\left(\log_c\left(\frac{(1-\alpha)\cdot n}{e^{\epsilon}}\right)\right)$$

records.

Conclusions

- **Leaking the access pattern can be dangerous**

Conclusions

- **Leaking the access pattern can be dangerous**
- Obliviousness: a strong security notion

Conclusions

- **Leaking the access pattern can be dangerous**
- Obliviousness: a strong security notion
  - requires $>$ 20x overhead for 4 Giga of data

Conclusions

- **Leaking the access pattern can be dangerous**
- Obliviousness: a strong security notion
  - requires $>$ 20x overhead for 4 Giga of data
- Differential Privacy: a weaker security notion

Conclusions

- **Leaking the access pattern can be dangerous**
- Obliviousness: a strong security notion
  - requires $>$ 20x overhead for 4 Giga of data
- Differential Privacy: a weaker security notion
  - very efficient

Conclusions

- **Leaking the access pattern can be dangerous**
- Obliviousness: a strong security notion
  - requires $>$ 20x overhead for 4 Giga of data
- Differential Privacy: a weaker security notion
  - very efficient
  - suitable only for specific applications

Conclusions

- **Leaking the access pattern can be dangerous**
- Obliviousness: a strong security notion
  - requires $>$ 20x overhead for 4 Giga of data
- Differential Privacy: a weaker security notion
  - very efficient
  - suitable only for specific applications
- Other security notions?

Conclusions

- **Leaking the access pattern can be dangerous**
- Obliviousness: a strong security notion
  - requires $>$ 20x overhead for 4 Giga of data
- Differential Privacy: a weaker security notion
  - very efficient
  - suitable only for specific applications
- Other security notions?
- Better analysis under reasonable assumptions for access distributions? Zipf?

Conclusions

- **Leaking the access pattern can be dangerous**
- Obliviousness: a strong security notion
  - requires $>$ 20x overhead for 4 Giga of data
- Differential Privacy: a weaker security notion
  - very efficient
  - suitable only for specific applications
- Other security notions?
- Better analysis under reasonable assumptions for access distributions? Zipf?

Conclusions

- **Leaking the access pattern can be dangerous**
- Obliviousness: a strong security notion
  - requires $>$ 20x overhead for 4 Giga of data
- Differential Privacy: a weaker security notion
  - very efficient
  - suitable only for specific applications
- Other security notions?
- Better analysis under reasonable assumptions for access distributions? Zipf?

Conclusions

- **Leaking the access pattern can be dangerous**
- Obliviousness: a strong security notion
  - requires $>$ 20x overhead for 4 Giga of data
- Differential Privacy: a weaker security notion
  - very efficient
  - suitable only for specific applications
- Other security notions?
- Better analysis under reasonable assumptions for access distributions? Zipf?

`https://github.com/giuper/talks/nonTechnical/oramTutorial.pdf`

The original papers

O. Goldreich.
Towards a Theory of Software Protection and Simulation by Oblivious RAMs.
In *STOC*, 1987.

R. Ostrovsky.
Efficient computation on oblivious RAMs.
In *STOC*, pages 514–523, 1990.

O. Goldreich and R. Ostrovsky.
Software Protection and Simulation on Oblivious RAMs.
J. ACM, 43(3), 1996.

Asymptotics

G. Asharov, I. Komargodski, W.-K. Lin, K. Nayak, E. Peserico, and E. Shi.
OptORAMa: Optimal oblivious RAM.
Cryptology ePrint Archive, Report 2018/892.

S. Patel, G. Persiano, M. Raykova, and K. Yeo.
PanORAMa: Oblivious RAM with logarithmic overhead.
Cryptology ePrint Archive, Report 2018/373, 2018.
https://eprint.iacr.org/2018/373.
FOCS '18.

Constant client memory

Efficient constructions for large blocks

E. Stefanov, M. van Dijk, E. Shi, C. Fletcher, L. Ren, X. Yu, and S. Devadas.
Path ORAM: An Extremely Simple Oblivious RAM Protocol.
In *CCS '13*, pages 299–310, 2013.

S. Devadas, M. van Dijk, C. W. Fletcher, L. Ren, E. Shi, and D. Wichs.
Onion ORAM: A constant bandwidth blowup oblivious RAM.
In *TCC*, pages 145–174, 2015.

T. Moataz, T. Mayberry, and E.-O. Blass.
Constant communication ORAM with small blocksize.
In *CCS*, pages 862–873, 2015.

polylog(n)-bit blocks

Efficient constructions for large client memory

L. Ren, C. Fletcher, A. Kwon, E. Stefanov, E. Shi, M. van Dijk, and S. Devadas.
Constants count: Practical improvements to oblivious RAM.
In *USENIX Security*, pages 415–430, 2015.

E. Stefanov, E. Shi, and D. X. Song.
Towards practical oblivious RAM.
In *NDSS 2012*, 2012.

S. Patel, G. Persiano, and K. Yeo.
Recursive ORAMs with practical constructions.
Cryptology ePrint Archive, Report 2017/964, 2017.

$O(\sqrt{n})$ **client memory**

Lower bounds

E. Boyle and M. Naor.
Is There an Oblivious RAM Lower Bound?
In *ITCS*, pages 357–368, 2016.

K. G. Larsen and J. B. Nielsen.
Yes, there is an oblivious RAM lower bound!
Cryptology ePrint Archive, Report 2018/423, 2018.
CRYPTO '18

M. Weiss and D. Wichs.
Is there an Oblivious RAM lower bound for online reads?
Cryptology ePrint Archive, Report 2018/619, 2018.
TCC '18

G. Persiano and K. Yeo.
Lower bounds for differentially private RAMs.
Cryptology ePrint Archive, Report 2018/1051, 2018.
Eurocrypt '19

Oblivious Sorting and Shuffling

M. T. Goodrich.
Zig-Zag Sort: A Simple Deterministic Data-oblivious Sorting Algorithm Running in $O(N \log N)$ Time.
In *STOC*, pages 684–693, 2014.

O. Ohrimenko, M. T. Goodrich, R. Tamassia, and E. Upfal.
The melbourne shuffle: Improving oblivious storage in the cloud.
In *ICALP '14*, pages 556–567. Springer, 2014.

S. Patel, G. Persiano, and K. Yeo.
CacheShuffle: A Family of Oblivious Shuffles.
In *ICALP*, pages 161:1–161:13, 2018.