Partial Sum

Vogliamo implementare una struttura dati che supporti le seguenti operazioni:

`Init` che prende una lista A e restituisce una rappresentazione `Repr` di A.

`Lookup` che prende in input $i < j$ e `Repr` restituisce la somma $A[i] + A[i+1] + \ldots + A[j-1]$.

`Set` che prende in input k e `val` ed aggiorna la rappresentazione `Repr`.

Partial Sum

Descriviamo un'implementazione che, per una lista di N elementi, usa spazio $O(N)$ e le operazioni prendono tempo

`Init`	$O(N)$
`Lookup`	$O(\log N)$
`Set`	$O(\log N)$

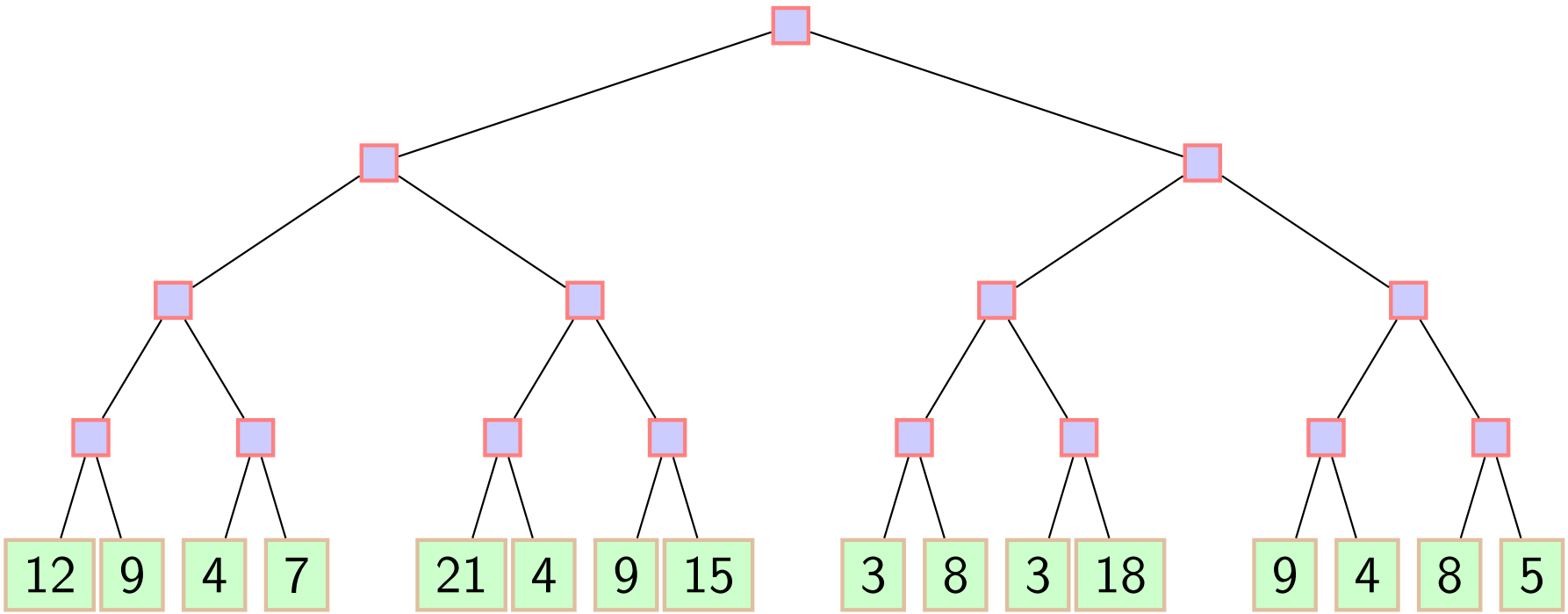
Init: costruire la rappresentazione

Precomputiamo le risposte alle seguenti query:

- $(0, 1), (1, 2), \ldots, (N-1, N)$:
  tutte le query $(i, i+1)$ *di lunghezza* 1, *per* i *multiplo di* 1.
- $(0, 2), (2, 4), \ldots, (N-2, N)$:
  tutte le query $(i, i+2)$ *di lunghezza* 2, *per* i *multiplo di* 2.
- $(0, 4), (4, 8), \ldots (N-4, N)$:
  tutte le query $(i, i+4)$ *di lunghezza* 4, *per* i *multiplo di* 4.
- .....
- $(0, N)$:
  tutte le query $(i, i+N)$ *di lunghezza* 4, *per* i *multiplo di* N.

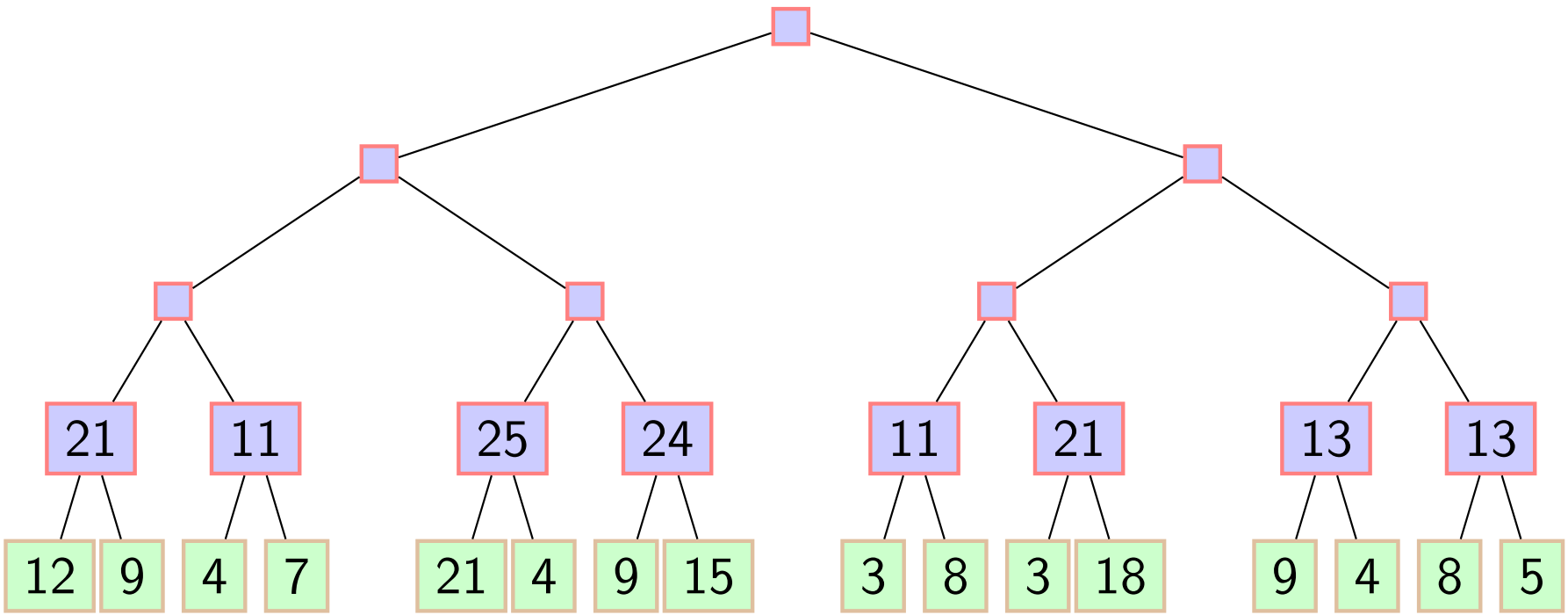
$[12, 9, 4, 7, 21, 4, 9, 15, 3, 8, 3, 18, 9, 4, 8, 5]$

Risposte a Lookup$(i, i+1)$, per i multiplo di 1



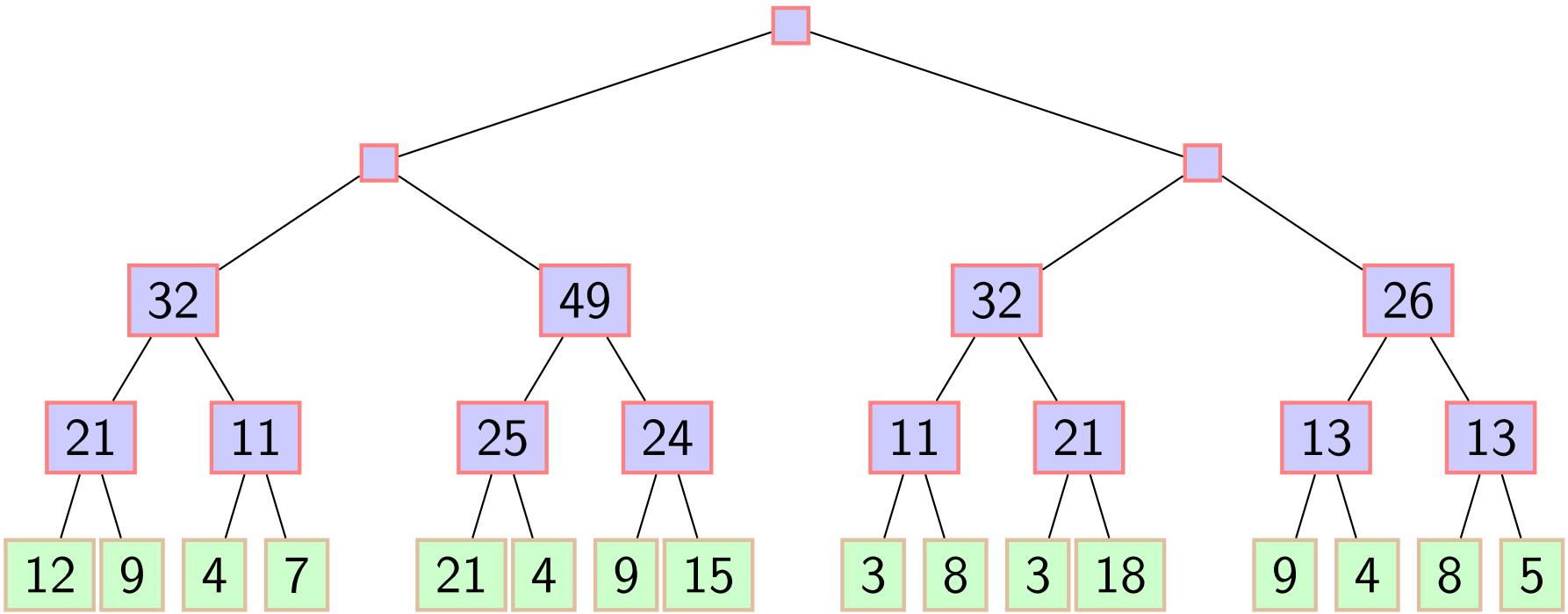
$[12, 9, 4, 7, 21, 4, 9, 15, 3, 8, 3, 18, 9, 4, 8, 5]$

Risposte a Lookup$(i, i+2)$, per i multiplo di 2



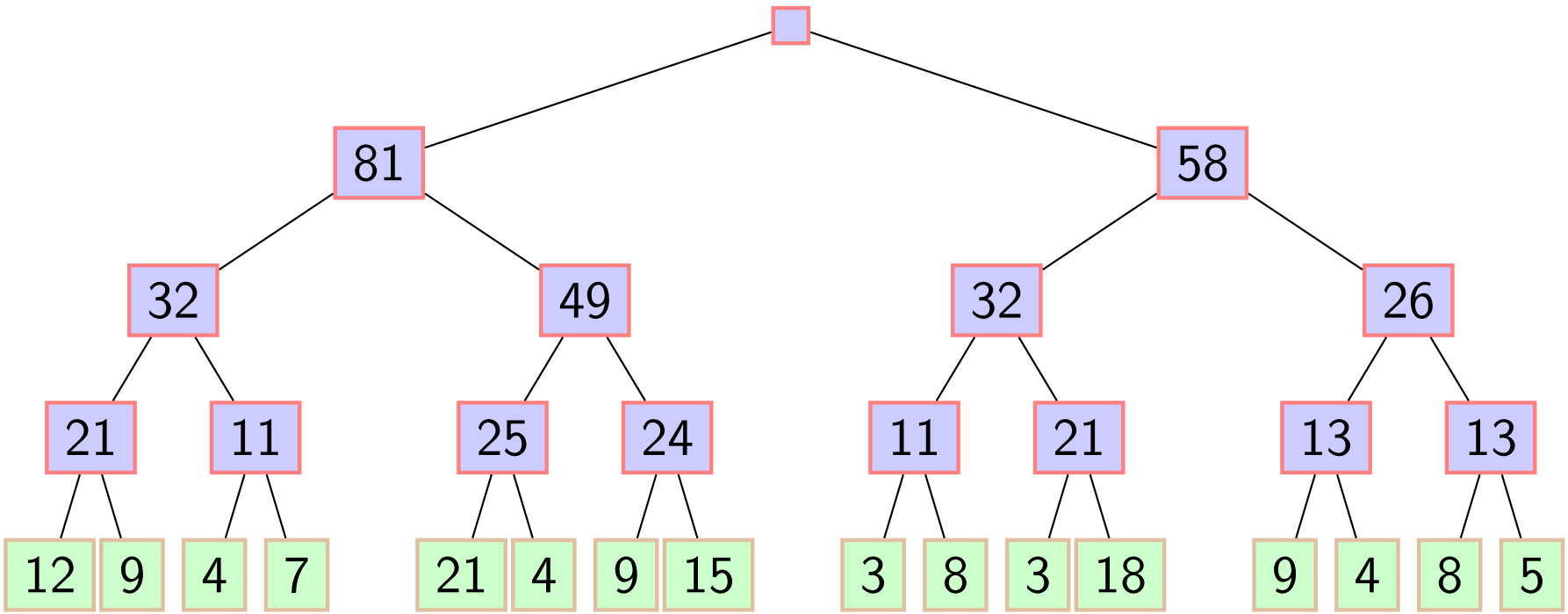
$[12, 9, 4, 7, 21, 4, 9, 15, 3, 8, 3, 18, 9, 4, 8, 5]$

Risposte a Lookup$(i, i + 4)$, per i multiplo di 4



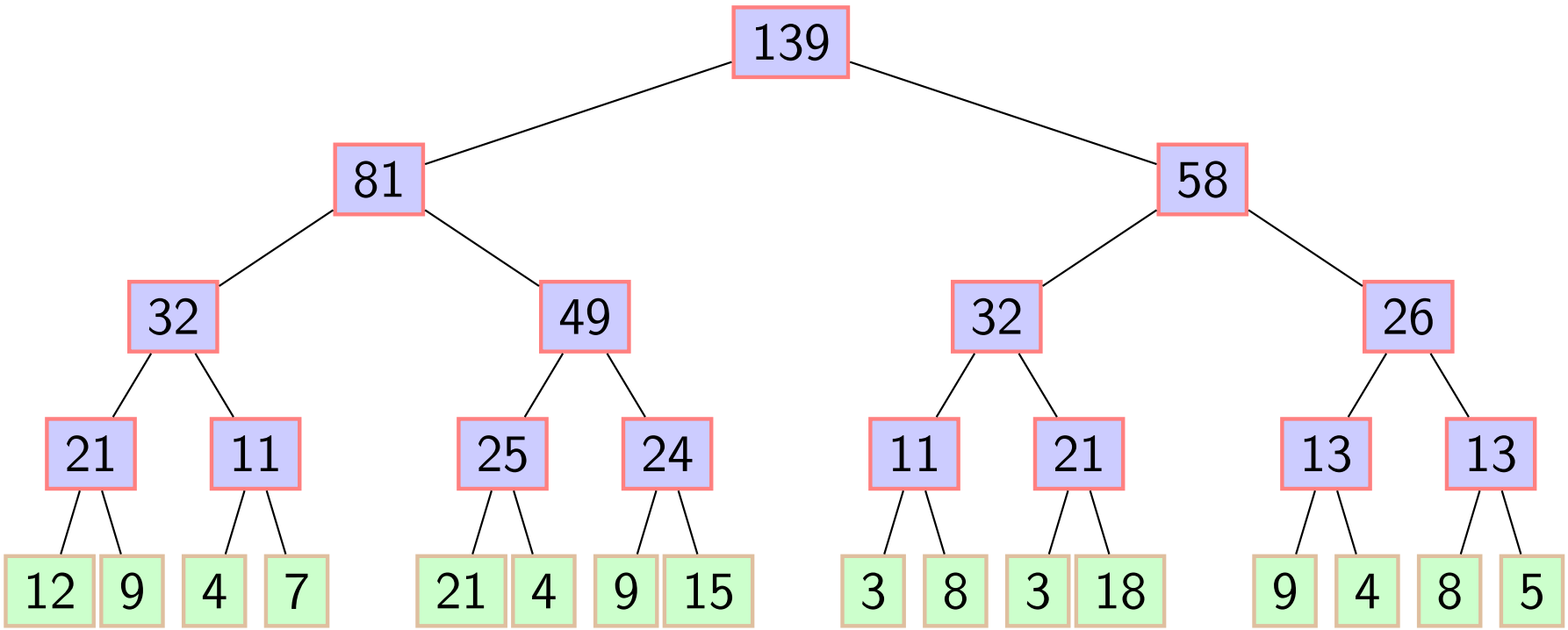
$[12, 9, 4, 7, 21, 4, 9, 15, 3, 8, 3, 18, 9, 4, 8, 5]$

Risposte a Lookup$(i, i + 8)$, per i multiplo di 8

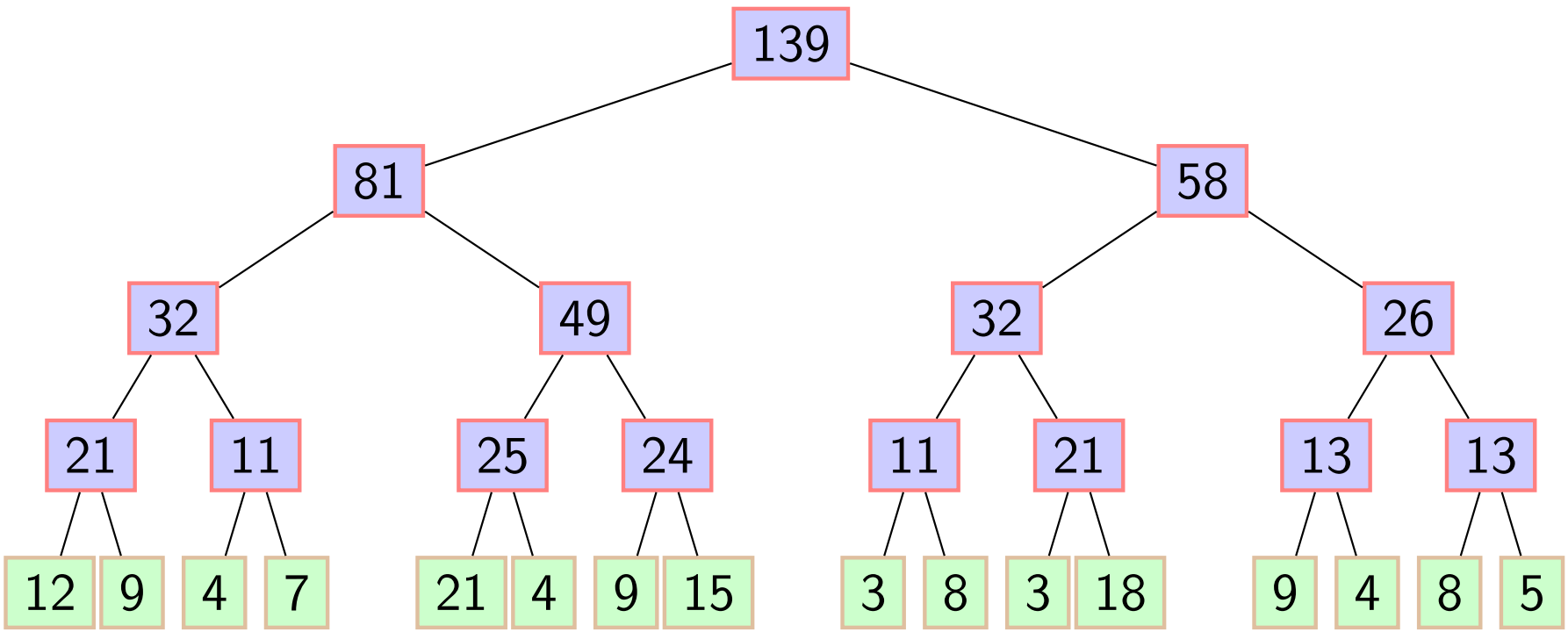


$[12, 9, 4, 7, 21, 4, 9, 15, 3, 8, 3, 18, 9, 4, 8, 5]$

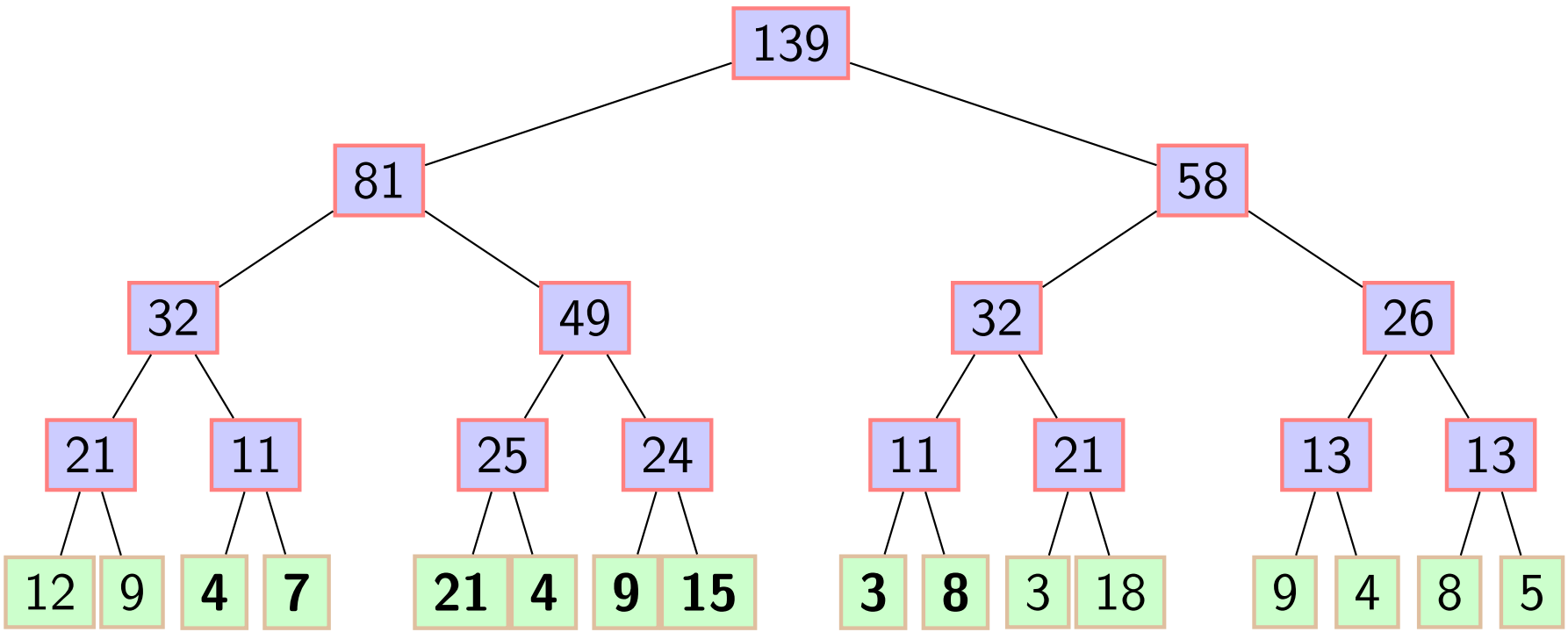
Risposte a Lookup$(i, i + 16)$, per i multiplo di 16



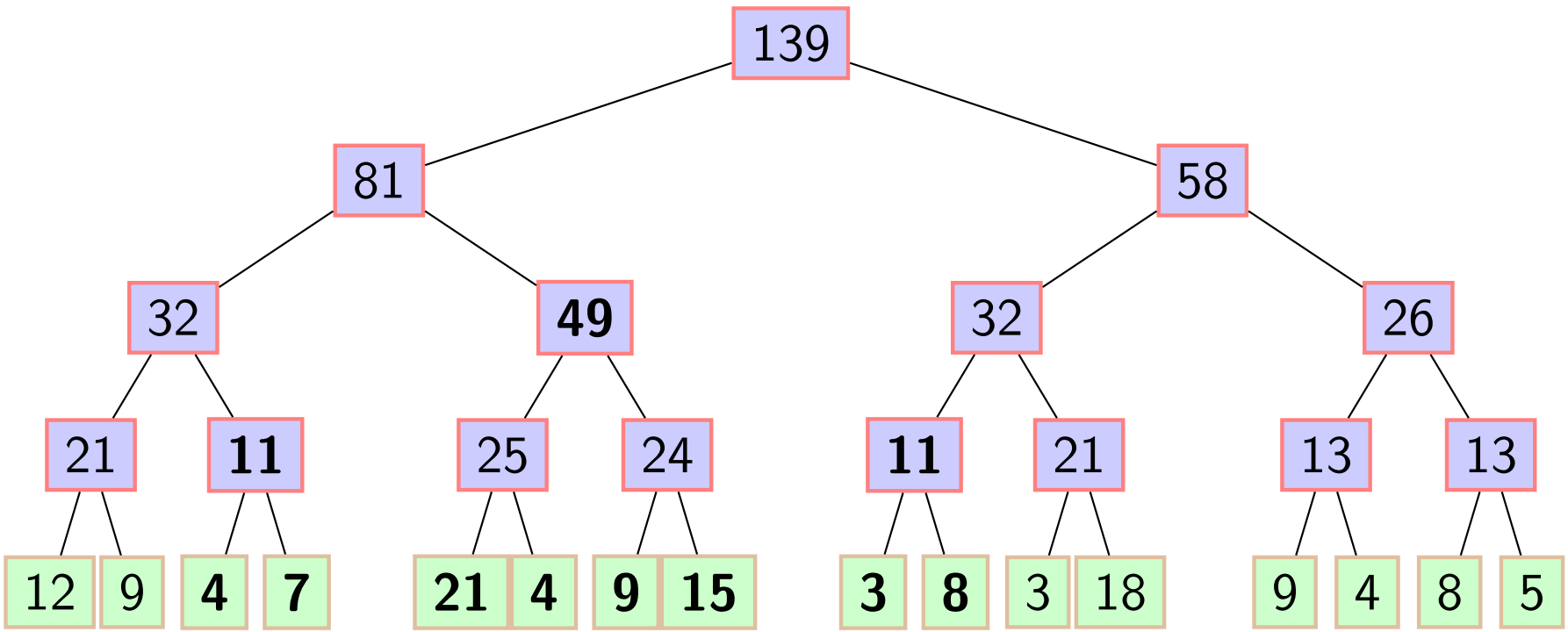
$$\text{Lookup}(2, 10) = \text{Lookup}(2, 4) + \text{Lookup}(4, 8) + \text{Lookup}(8, 10)$$



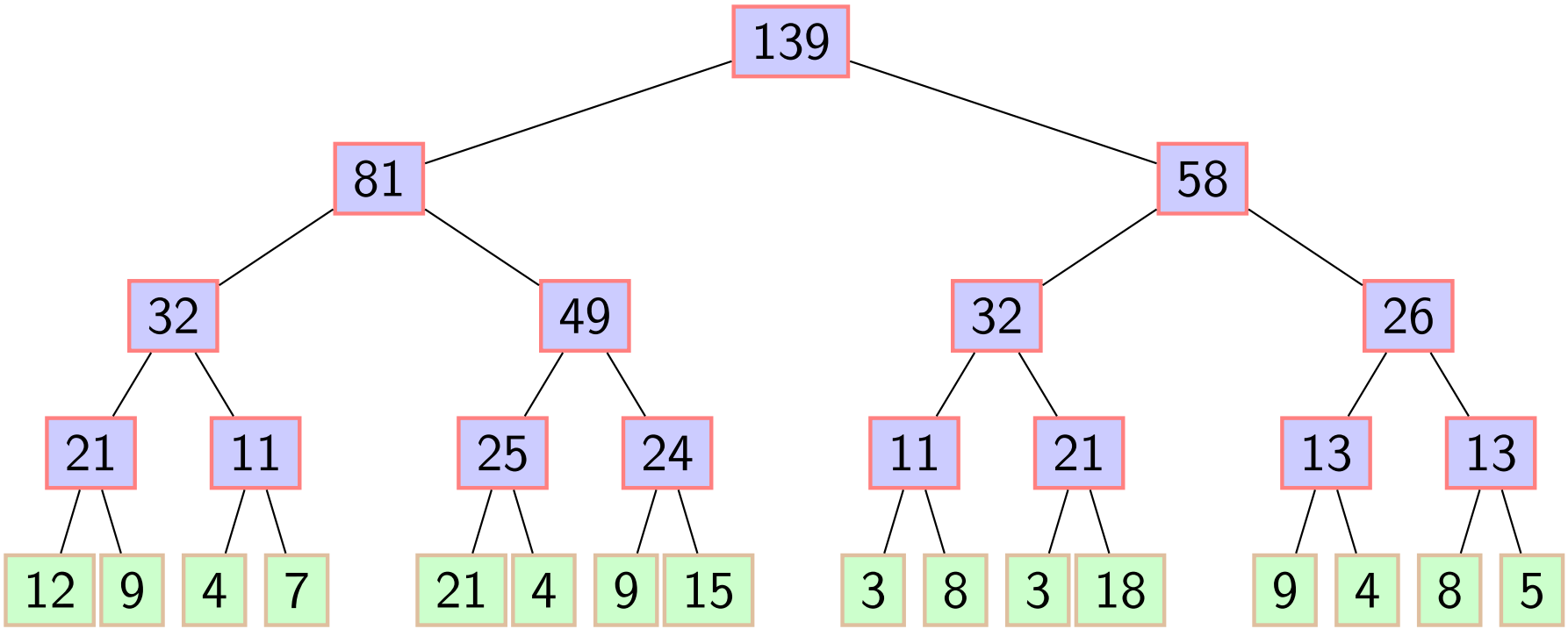
$$\text{Lookup}(2, 10) = \text{Lookup}(2, 4) + \text{Lookup}(4, 8) + \text{Lookup}(8, 10)$$



$$\text{Lookup}(2, 10) = \text{Lookup}(2, 4) + \text{Lookup}(4, 8) + \text{Lookup}(8, 10)$$



Set(12, 16)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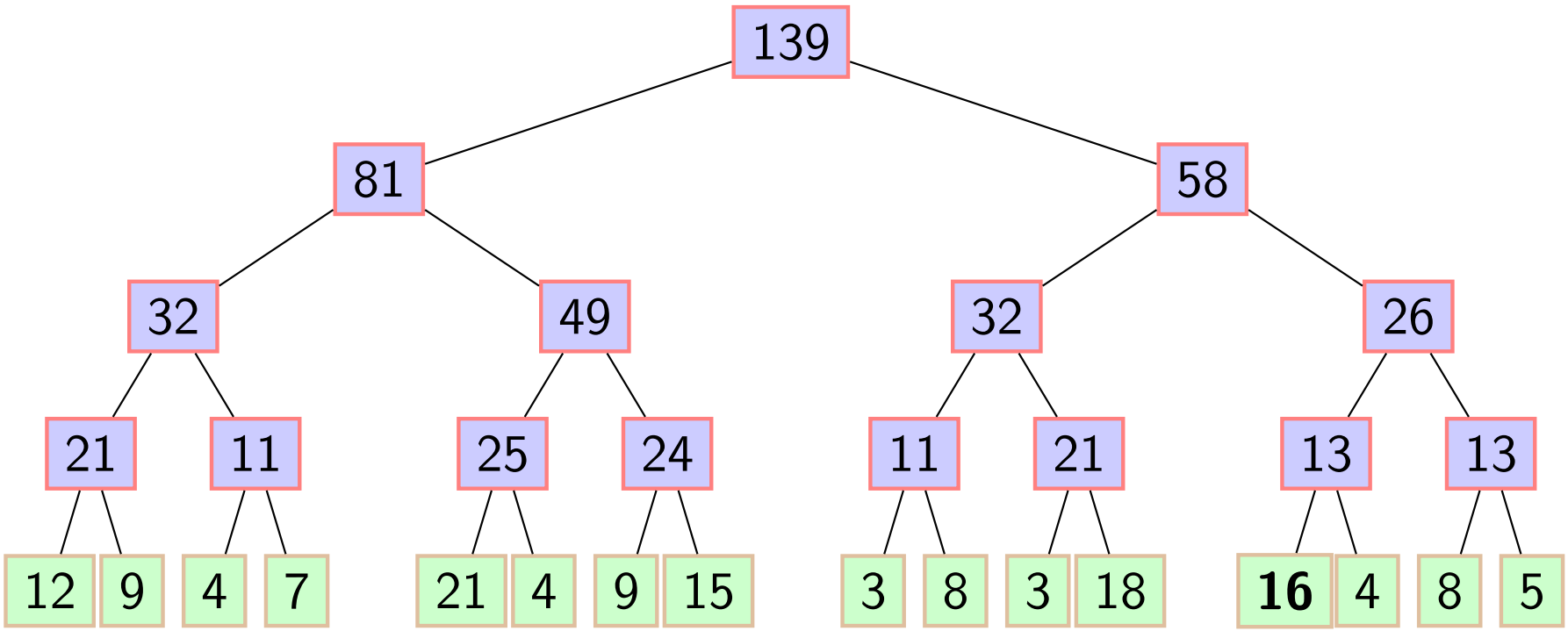
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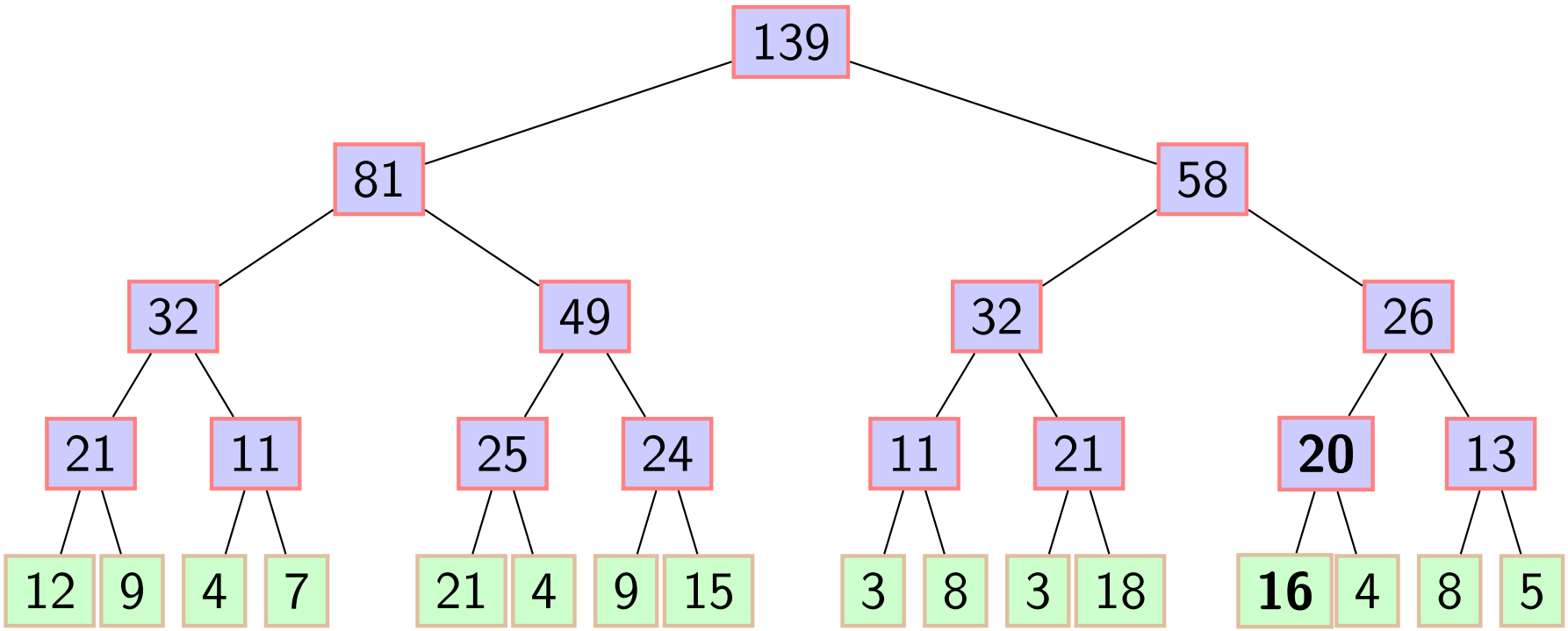
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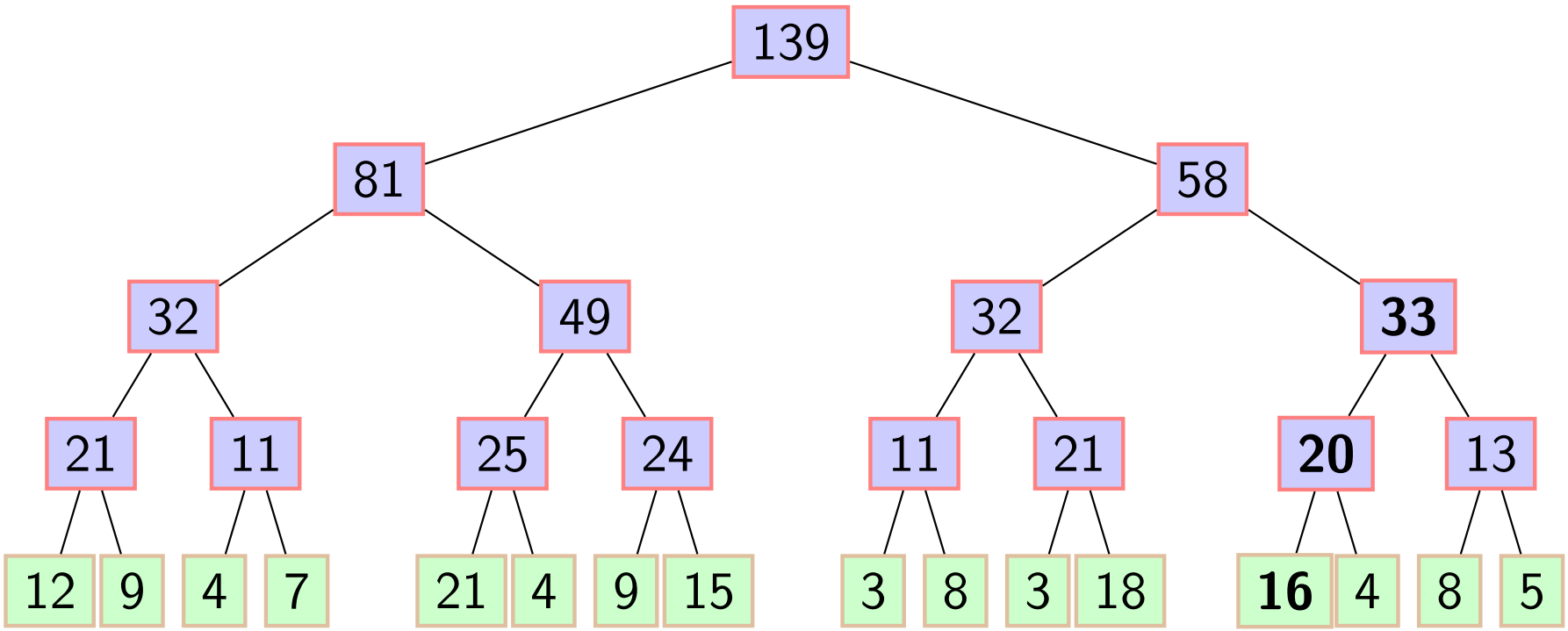
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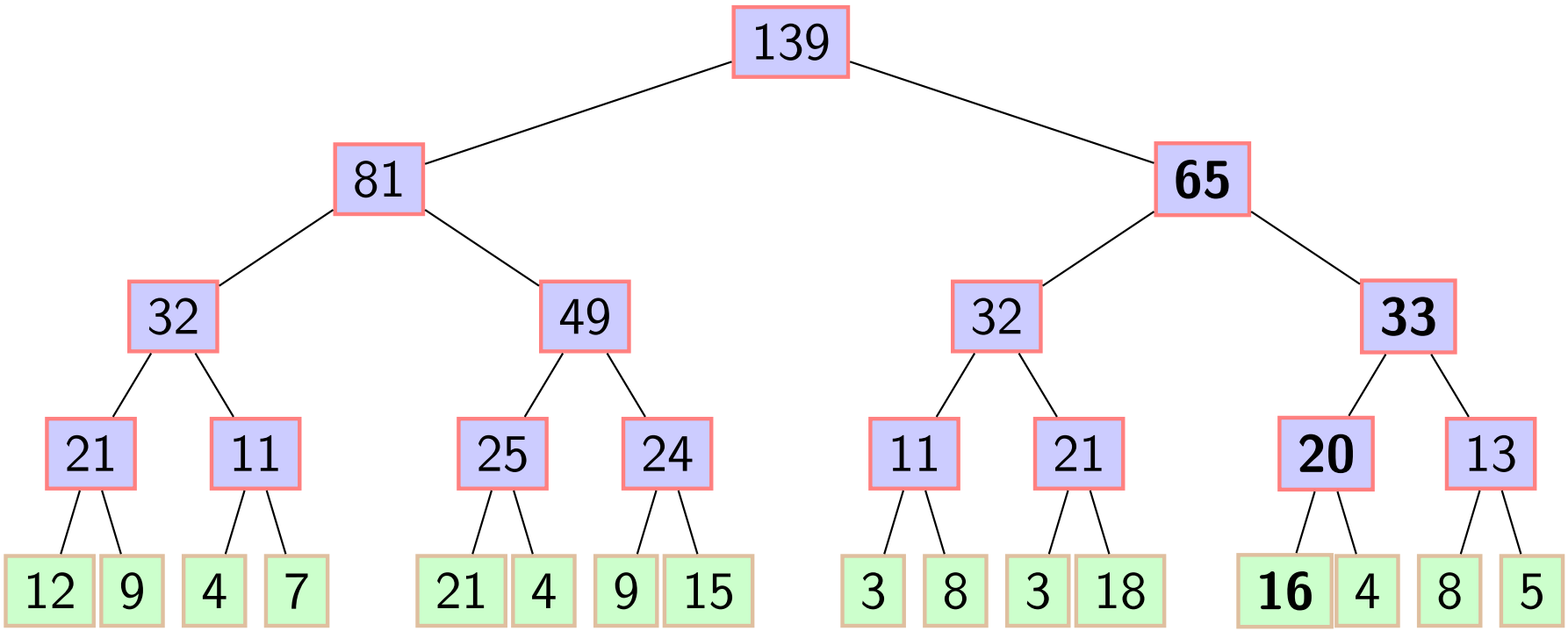
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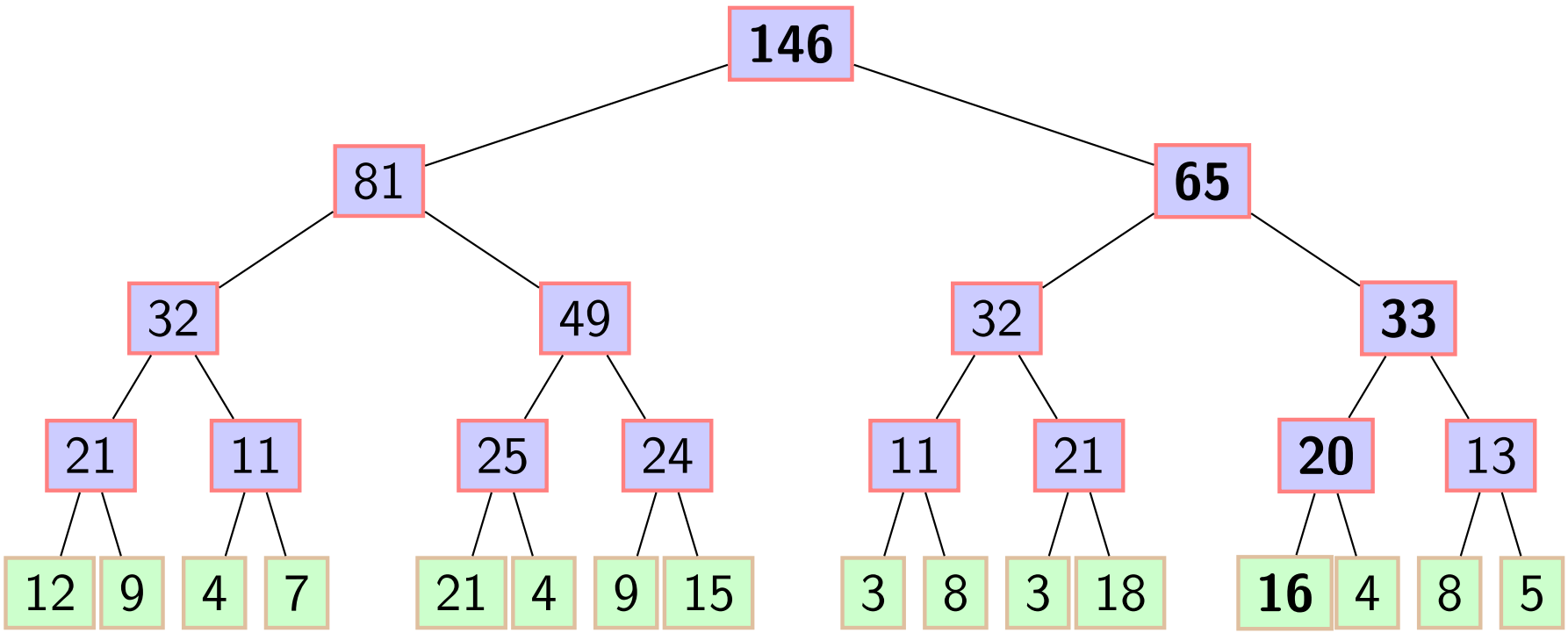
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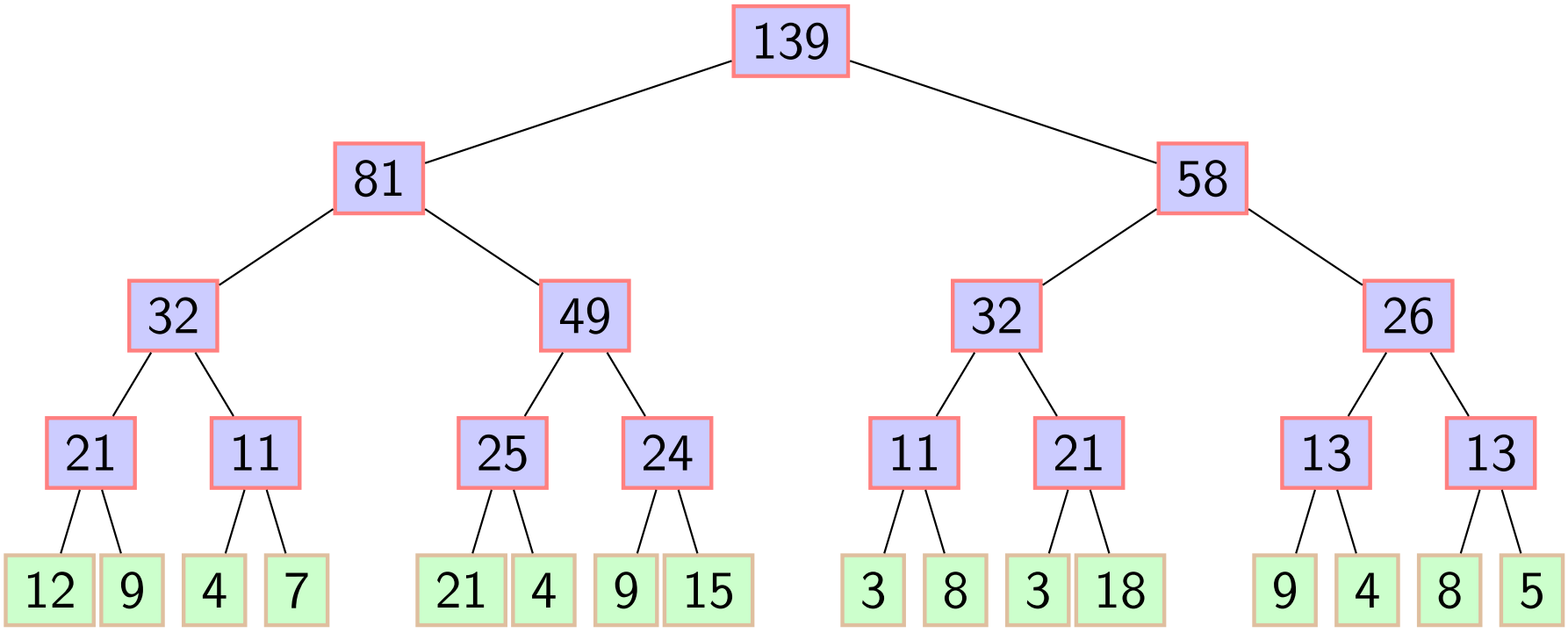
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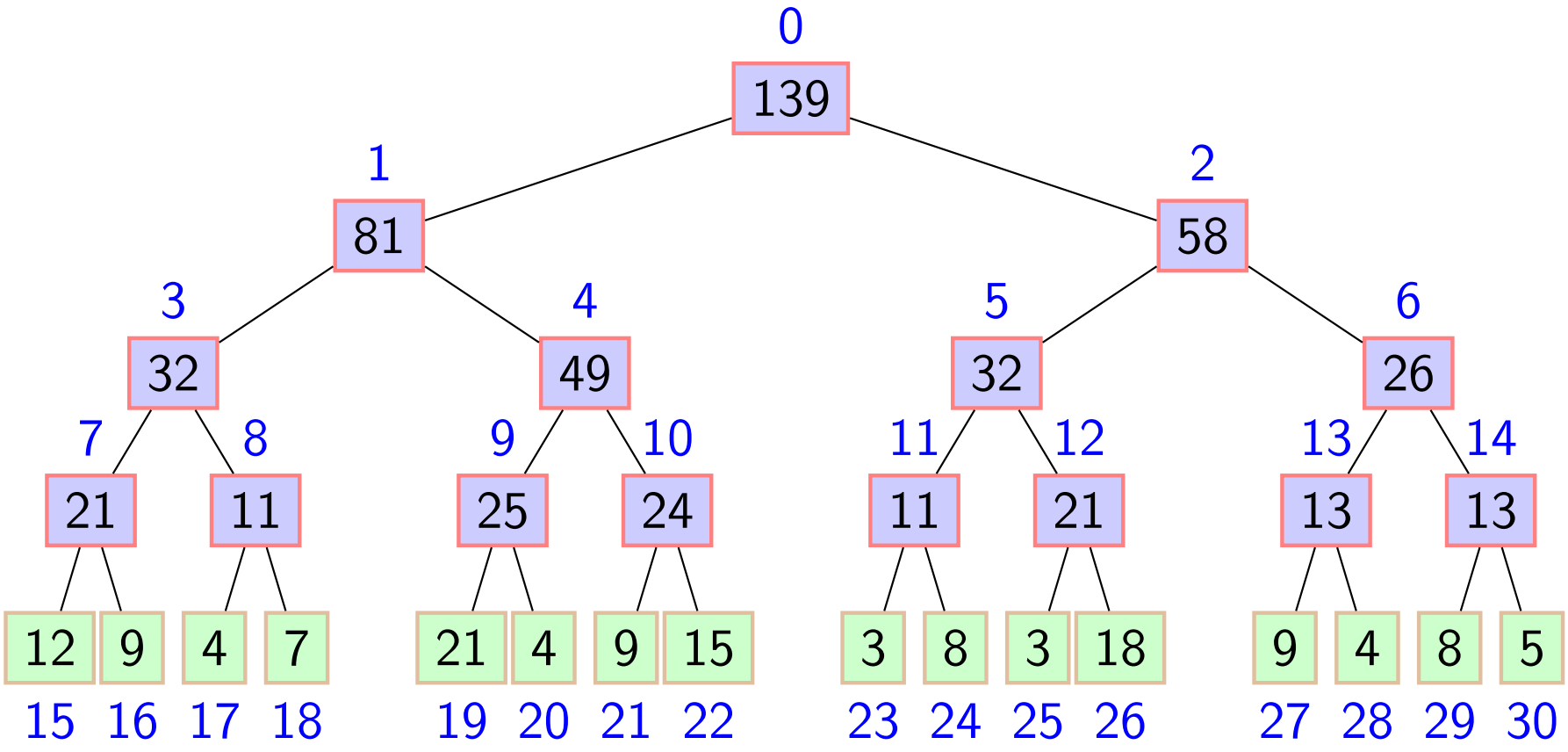
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