In[187]:=
```
A = {{2, 1, 1, 3, 2}, {1, 2, 2, 1, 1}, {1, 2, 9, 1, 5}, {3, 1, 1, 7, 1}, {2, 1, 5, 1, 8}} ;
lu = LUDecomposition [A][[1]];
lower = LowerTriangularize [lu, -1] + IdentityMatrix [Dimensions [A]];
upper = UpperTriangularize [lu, 0];
(*Doolittle*)
l = Table[0, {i, 1, 5}, {j, 1, 5}];
u = l;
Doolittle[A_] := For[k = 1, k < 6, k++, l[[k, k]] = 1;
    For[j = k, j < 6, j++, u[[k, j]] = A[[k, j]] - Sum[l[[k, m]] * u[[m, j]], {m, 1, k - 1}]];
    For[i = k + 1, i < 6, i++,
     l[[i, k]] = (1 / u[[k, k]]) * (A[[i, k]] - Sum[l[[i, m]] * u[[m, k]], {m, 1, k - 1}])]];
Doolittle[A];
A == l.u
Print[MatrixForm[A] , " = " , MatrixForm[l] , " * ", MatrixForm[u]]
```

Out[195]= True

$$\begin{pmatrix} 2 & 1 & 1 & 3 & 2 \\ 1 & 2 & 2 & 1 & 1 \\ 1 & 2 & 9 & 1 & 5 \\ 3 & 1 & 1 & 7 & 1 \\ 2 & 1 & 5 & 1 & 8 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 1 & 0 & 0 \\ \frac{3}{2} & -\frac{1}{3} & 0 & 1 & 0 \\ 1 & 0 & \frac{4}{7} & -\frac{6}{7} & 1 \end{pmatrix} * \begin{pmatrix} 2 & 1 & 1 & 3 & 2 \\ 0 & \frac{3}{2} & \frac{3}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 7 & 0 & 4 \\ 0 & 0 & 0 & \frac{7}{3} & -2 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

In[197]:=
```
(*Crout*)
l = Table[0, {i, 1, 5}, {j, 1, 5}];
u = l;
Crout[A_] := For[k = 1, k < 6, k++,
l[[k, k]] = A[[k, k]] - Sum[l[[k, m]] * u[[m, k]], {m, 1, k - 1}];
For[j = k, j < 6, j++,
u[[k, j]] = (1 / l[[k, k]]) * (A[[k, j]] - Sum[l[[k, m]] * u[[m, j]], {m, 1, k - 1}])];
   For[i = k + 1, i < 6, i++,
        l[[i, k]] = (1 / u[[k, k]]) * (A[[i, k]] - Sum[l[[i, m]] * u[[m, k]], {m, 1, k - 1}])]];
Crout[A];
A == l.u
Print[MatrixForm[A] , " = " , MatrixForm[l] , " * ", MatrixForm[u]]
(*We notice that in this factorisation , L and U are the transposed
 matrices of U and L from the Doolittle Factorisation Method*)
```

Out[201]= True

$$\begin{pmatrix} 2 & 1 & 1 & 3 & 2 \\ 1 & 2 & 2 & 1 & 1 \\ 1 & 2 & 9 & 1 & 5 \\ 3 & 1 & 1 & 7 & 1 \\ 2 & 1 & 5 & 1 & 8 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 1 & \frac{3}{2} & 0 & 0 & 0 \\ 1 & \frac{3}{2} & 7 & 0 & 0 \\ 3 & -\frac{1}{2} & 0 & \frac{7}{3} & 0 \\ 2 & 0 & 4 & -2 & 2 \end{pmatrix} * \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{3}{2} & 1 \\ 0 & 1 & 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & \frac{4}{7} \\ 0 & 0 & 0 & 1 & -\frac{6}{7} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[207]:=
```
(*Cholesky*)
l = Table[0, {i, 1, 5}, {j, 1, 5}];
Cholesky[A_] := For[k = 1, k < 6, k++,
l[[k, k]] = Sqrt[A[[k, k]] - Sum[l[[k, m]] ^ 2, {m, 1, k - 1}]];
For[i = k + 1, i < 6, i++,
l[[i, k]] = (1 / l[[k, k]]) * (A[[i, k]] - Sum[l[[i, m]] * l[[k, m]], {m, 1, k - 1}])]]
Cholesky[A];
u = Transpose[l];
A == l.Transpose[l]
Print[MatrixForm[A] , " = " , MatrixForm[l] , " * ", MatrixForm[u]]
```

Out[211]= True

$$\begin{pmatrix} 2 & 1 & 1 & 3 & 2 \\ 1 & 2 & 2 & 1 & 1 \\ 1 & 2 & 9 & 1 & 5 \\ 3 & 1 & 1 & 7 & 1 \\ 2 & 1 & 5 & 1 & 8 \end{pmatrix} = \begin{pmatrix} \sqrt{2} & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & \sqrt{\frac{3}{2}} & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & \sqrt{\frac{3}{2}} & \sqrt{7} & 0 & 0 \\ \frac{3}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & 0 & \sqrt{\frac{7}{3}} & 0 \\ \sqrt{2} & 0 & \frac{4}{\sqrt{7}} & -2\sqrt{\frac{3}{7}} & \sqrt{2} \end{pmatrix} * \begin{pmatrix} \sqrt{2} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{3}{\sqrt{2}} & \sqrt{2} \\ 0 & \sqrt{\frac{3}{2}} & \sqrt{\frac{3}{2}} & -\frac{1}{\sqrt{6}} & 0 \\ 0 & 0 & \sqrt{7} & 0 & \frac{4}{\sqrt{7}} \\ 0 & 0 & 0 & \sqrt{\frac{7}{3}} & -2\sqrt{\frac{3}{7}} \\ 0 & 0 & 0 & 0 & \sqrt{2} \end{pmatrix}$$

In[235]:=
```
n  = 6;
q  = Table[RandomReal[{-1, 1}], {n}, {n}];
mat  = Transpose[q].q;
Print["Matrix  " MatrixForm[mat],
 "is positive  definite : ", PositiveDefiniteMatrixQ [mat]]
Doolittle[mat];
Print[MatrixForm[mat] , " = " , MatrixForm[l] , " * ", MatrixForm[u]]
Crout[mat];
Print[MatrixForm[mat] , " = " , MatrixForm[l] , " * ", MatrixForm[u]]
Cholesky[mat];
u = Transpose[l];
Print[MatrixForm[mat] , " = " , MatrixForm[l] , " * ", MatrixForm[u]]
```

Matrix $\begin{pmatrix} 2.02517 & 1.41931 & 0.0111831 & 0.779106 & 1.17791 & -0.805382 \\ 1.41931 & 2.21854 & -0.139334 & 1.17401 & 1.68696 & 0.0705233 \\ 0.0111831 & -0.139334 & 2.23813 & -0.532258 & -0.116335 & -1.59165 \\ 0.779106 & 1.17401 & -0.532258 & 1.57073 & 0.644567 & 0.195319 \\ 1.17791 & 1.68696 & -0.116335 & 0.644567 & 2.32913 & -0.406229 \\ -0.805382 & 0.0705233 & -1.59165 & 0.195319 & -0.406229 & 2.14172 \end{pmatrix}$

is positive definite : True

$$\begin{pmatrix} 2.02517 & 1.41931 & 0.0111831 & 0.779106 & 1.17791 & -0.805382 \\ 1.41931 & 2.21854 & -0.139334 & 1.17401 & 1.68696 & 0.0705233 \\ 0.0111831 & -0.139334 & 2.23813 & -0.532258 & -0.116335 & -1.59165 \\ 0.779106 & 1.17401 & -0.532258 & 1.57073 & 0.644567 & 0.195319 \\ 1.17791 & 1.68696 & -0.116335 & 0.644567 & 2.32913 & -0.406229 \\ -0.805382 & 0.0705233 & -1.59165 & 0.195319 & -0.406229 & 2.14172 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.700834 & 1 & 0 & 0 & 0 \\ 0.00552206 & -0.120255 & 1 & 0 & 0 \\ 0.384711 & 0.513125 & -0.207642 & 1 & 0 \\ 0.581633 & 0.703892 & -0.00866812 & -0.29848 & 1 \end{pmatrix} *$$

$$\begin{pmatrix} 2.02517 & 1.41931 & 0.0111831 & 0.779106 & 1.17791 \\ 0 & 1.22384 & -0.147172 & 0.627981 & 0.861448 \\ 0 & 0 & 2.22037 & -0.461042 & -0.0192465 \\ 0 & 0 & 0 & 0.853035 & -0.254614 \\ 0 & 0 & 0 & 0 & 0.961491 \end{pmatrix}$$

$$\begin{pmatrix} 2.02517 & 1.41931 & 0.0111831 & 0.779106 & 1.17791 & -0.805382 \\ 1.41931 & 2.21854 & -0.139334 & 1.17401 & 1.68696 & 0.0705233 \\ 0.0111831 & -0.139334 & 2.23813 & -0.532258 & -0.116335 & -1.59165 \\ 0.779106 & 1.17401 & -0.532258 & 1.57073 & 0.644567 & 0.195319 \\ 1.17791 & 1.68696 & -0.116335 & 0.644567 & 2.32913 & -0.406229 \\ -0.805382 & 0.0705233 & -1.59165 & 0.195319 & -0.406229 & 2.14172 \end{pmatrix}$$

$$= \begin{pmatrix} 2.02517 & 0 & 0 & 0 & 0 \\ 1.41931 & 1.22384 & 0 & 0 & 0 \\ 0.0111831 & -0.147172 & 2.22037 & 0 & 0 \\ 0.779106 & 0.627981 & -0.461042 & 0.853035 & 0 \\ 1.17791 & 0.861448 & -0.0192465 & -0.254614 & 0.961491 \end{pmatrix}$$

$$* \begin{pmatrix} 1. & 0.700834 & 0.00552206 & 0.384711 & 0.581633 \\ 0 & 1. & -0.120255 & 0.513125 & 0.703892 \\ 0 & 0 & 1. & -0.207642 & -0.00866812 \\ 0 & 0 & 0 & 1. & -0.29848 \\ 0 & 0 & 0 & 0 & 1. \end{pmatrix}$$

$$\begin{pmatrix} 2.02517 & 1.41931 & 0.0111831 & 0.779106 & 1.17791 & -0.805382 \\ 1.41931 & 2.21854 & -0.139334 & 1.17401 & 1.68696 & 0.0705233 \\ 0.0111831 & -0.139334 & 2.23813 & -0.532258 & -0.116335 & -1.59165 \\ 0.779106 & 1.17401 & -0.532258 & 1.57073 & 0.644567 & 0.195319 \\ 1.17791 & 1.68696 & -0.116335 & 0.644567 & 2.32913 & -0.406229 \\ -0.805382 & 0.0705233 & -1.59165 & 0.195319 & -0.406229 & 2.14172 \end{pmatrix}$$

$$= \begin{pmatrix} 1.42309 & 0 & 0 & 0 & 0 \\ 0.997346 & 1.10627 & 0 & 0 & 0 \\ 0.00785836 & -0.133034 & 1.49009 & 0 & 0 \\ 0.547476 & 0.567656 & -0.309405 & 0.923599 & 0 \\ 0.827713 & 0.778695 & -0.0129163 & -0.275676 & 0.980556 \end{pmatrix}$$

$$* \begin{pmatrix} 1.42309 & 0.997346 & 0.00785836 & 0.547476 & 0.827713 \\ 0 & 1.10627 & -0.133034 & 0.567656 & 0.778695 \\ 0 & 0 & 1.49009 & -0.309405 & -0.0129163 \\ 0 & 0 & 0 & 0.923599 & -0.275676 \\ 0 & 0 & 0 & 0 & 0.980556 \end{pmatrix}$$