Systems Theory
Homework 1

1) Solve the following equations:

a) $x^2 + 5x + 2 = 0$,

b) $x^2 + 4x + 5 = 0$,

c) $x^2 + 2 = 0$,

d) $x^2 - 3 = 0$,

e) $x^3 + 2x^2 - x - 2 = 0$

2) For the given matrices

$$M = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 0 \end{bmatrix}, N = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, P = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}, Q = \begin{bmatrix} 1 & 4 \\ -2 & 5 \end{bmatrix}, R = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix},$$

calculate:

a) the determinant of M, the inverse of M and the rank of N,

b) M*N and N*M, M*P and P*M,

c) eigenvalues of Q and R.

3) Write the following systems of linear equations in matrix form **Ax**=**b**:

a) $\begin{cases} -x_1 + 2x_2 = 4 \\ 2x_1 + 3x_2 = 8, \end{cases}$ b) $\begin{cases} 2x_1 + 3x_2 + 4x_3 = 5 \\ -x_1 + x_3 = 2 \\ x_1 + x_2 - x_3 = 3 \end{cases}$

4) Write the following systems of linear differential equations in matrix form $\dot{\boldsymbol{x}}(\boldsymbol{t}) = \boldsymbol{A}\boldsymbol{x}(\boldsymbol{t})$:

a) $\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -x_1(t) - 2x_2(t) \end{cases}$ b) $\begin{cases} \dot{x}_1(t) = x_1(t) + x_2(t) \\ \dot{x}_2(t) = 2x_1(t) + 3x_3(t) \\ \dot{x}_3(t) = -x_1(t) - x_2(t) - 2x_3(t) \end{cases}$

5) For the following complex numbers:

$z_1 = 3+4j$, $z_2 = -1-j$, $z_3 = \frac{j-2}{2j+1}$, $z_4 = \frac{1}{j+1}$

a) Compute the real and imaginary part.

b) Compute the absolute value and the argument.

c) Represent the numbers in the complex plane.

6) Calculate the Laplace transform for:

a) $f(t) = t+2$

b) $f(t) = e^{-2t}$

c) $f(t) = \sin(t)$

7) Calculate the inverse Laplace transform for:

a) $F(s) = \frac{1}{s}$; b) $F(s) = \frac{1}{s+1}$, c) $F(s) = \frac{1}{s(s+1)}$, d) $F(s) = \frac{1}{s^2 + 2s + 2}$