Scan Conversion Algorithms (2)

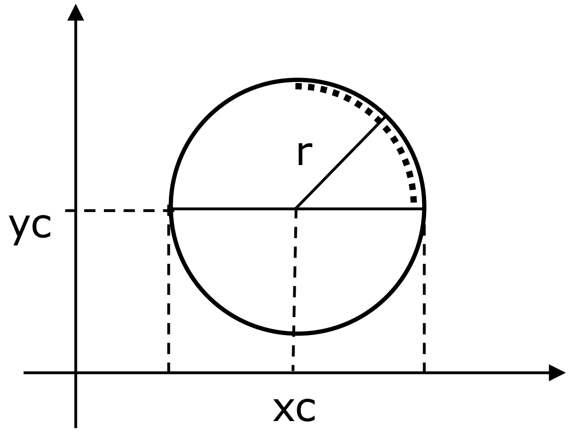
Contents

- Circle scan conversion
- Circle drawing
- Circle drawing by 4 and 8 way symmetry
- Midpoint circle
  - Basic algorithm
  - Algorithm by integer arithmetic
  - Algorithm by 2^{nd} order differences

Circle Drawing

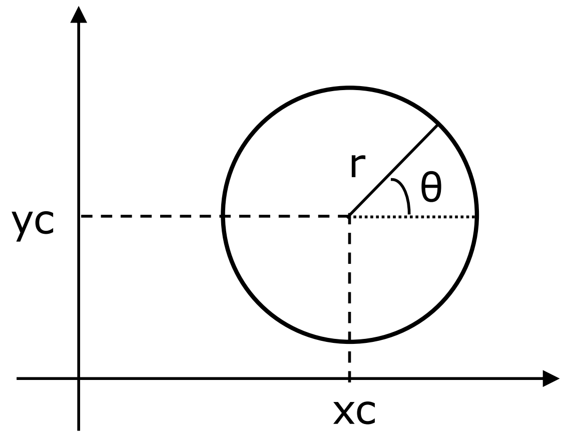
$(x-xc)^2 + (y-yc)^2 = r^2$

$y = yc \pm \text{sqrt}((r^2) - (x-xc)^2)$

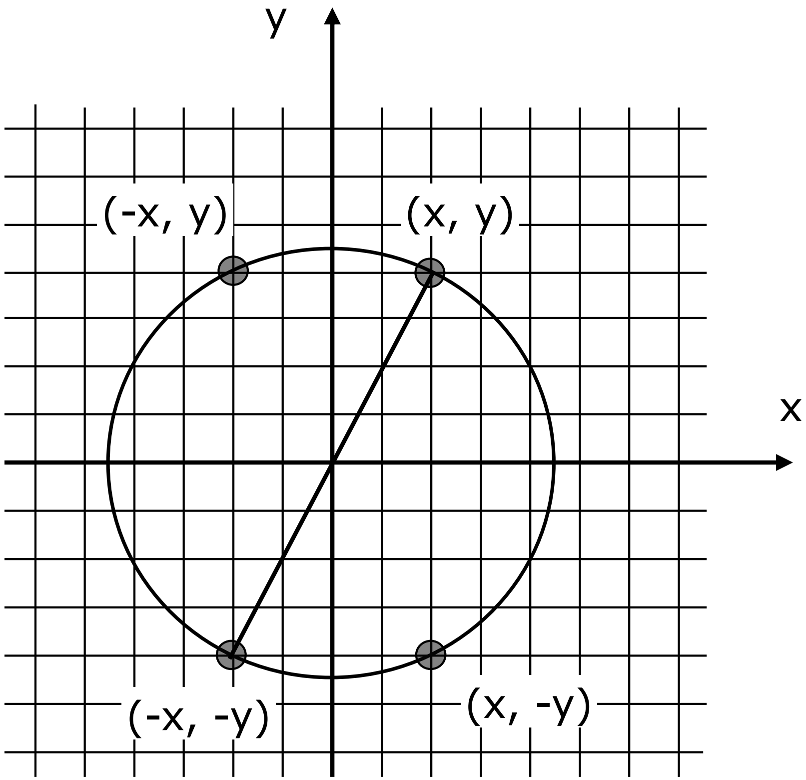


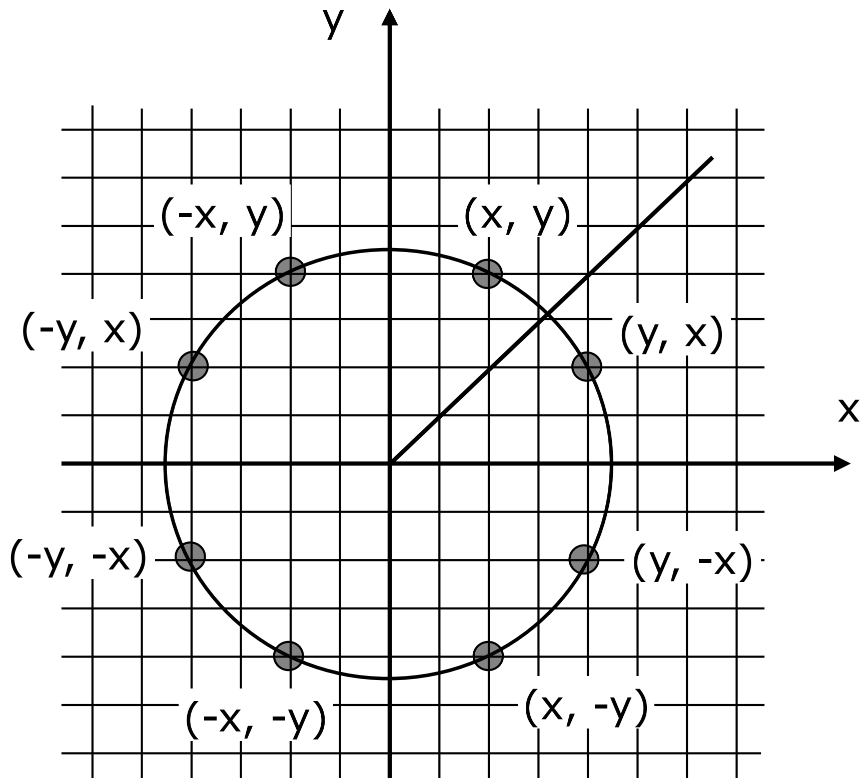
$x = xc + r\cos\theta$

$y = yc + r\sin\theta$



Circle rendering by 4 and 8 way symmetry





4 way symmetry algorithm

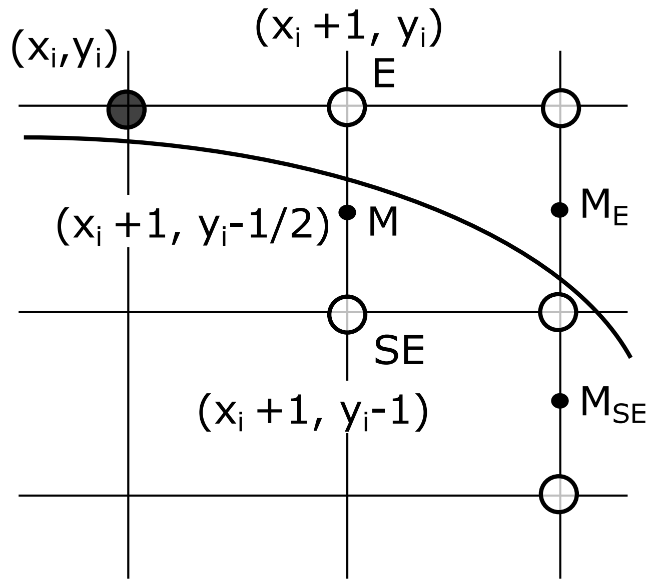
```
public void circleSym4 (int xCenter, int yCenter, int radius, Color c)
    {
       int pix = c.getRGB();
       int x, y, r2;

       r2 = radius * radius;
       raster.setPixel(pix, xCenter, yCenter + radius);
       raster.setPixel(pix, xCenter, yCenter - radius);
       for (x = 1; x <= radius; x++) {
          y = (int) (Math.sqrt(r2 - x*x) + 0.5);
          raster.setPixel(pix, xCenter + x, yCenter + y);
          raster.setPixel(pix, xCenter + x, yCenter - y);
          raster.setPixel(pix, xCenter - x, yCenter + y);
          raster.setPixel(pix, xCenter - x, yCenter - y);
       }
    }
```

Midpoint circle algorithm

$x^2 + y^2 = R^2$

Let us consider $F(x,y) = x^2 + y^2 - R^2$



$F(x,y) > 0$ for $P(x,y)$ outside of the circle

$F(x,y) = 0$ for P on the circle

$F(x,y) < 0$ inside of the circle

Therefore:

if $F(M) < 0$, next point $P(x_{i+1},y_{i+1}) = E(x_i+1,y_i)$

otherwise next point $P(x_{i+1},y_{i+1}) = SE(x_i+1,y_i-1)$

Midpoint circle algorithm - computation

Let us consider the decision variable:

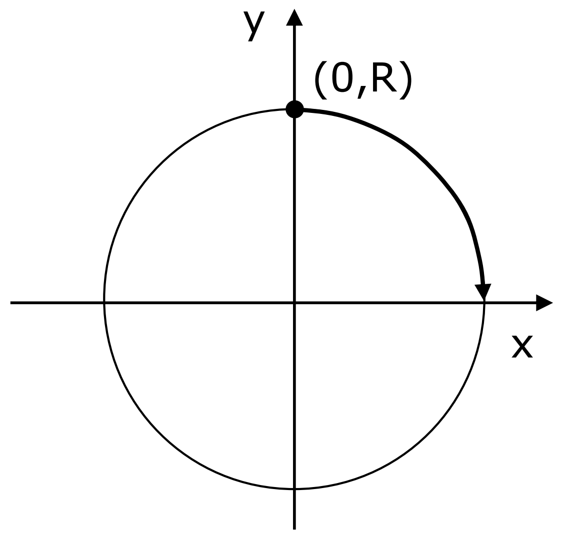
$d_i = F(x_i+1, y_i-1/2)=(x_i+1)^2 + (y_i-1/2)^2 - R^2$

$d_{i+1} = F(x_{i+1}+1, y_{i+1}-1/2)=(x_i+2)^2 + (y_{i+1}-1/2)^2 - R^2$

where $x_{i+1} = x_i + 1$

if $d_i < 0$, $y_{i+1}= y_i$

otherwise $y_{i+1}= y_i-1$



$$d_{i+1} = \begin{cases} d_i + 2x_i+3, & \text{if } d_i<0 \\ d_i + 2(x_i-y_i)+5, & \text{otherwise} \end{cases}$$

The starting point $(x_0, y_0) = (0, R)$

$d_0 = F(x_0+1, y_0-1/2)=(0+1)^2 + (R-1/2)^2 - R^2 = 5/4 - R$

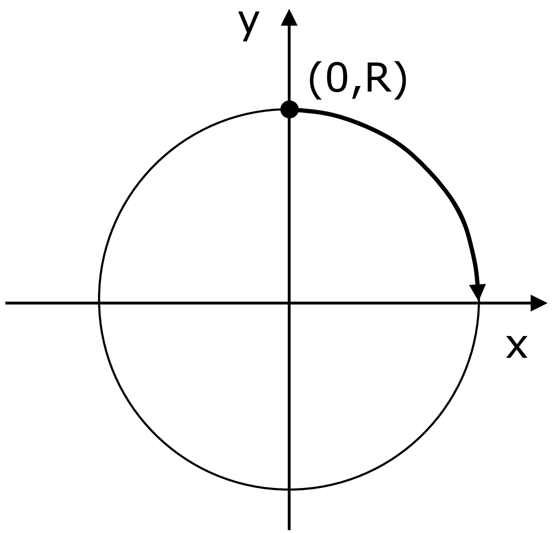
Midpoint circle algorithm - summary

Next point $P(x_{i+1},y_{i+1})$:

$$x_{i+1} = x_i + 1$$

$$y_{i+1} = \begin{cases} y_i & \text{if } d_i < 0, \\ y_i - 1 & \text{otherwise} \end{cases}$$

$$d_{i+1} = \begin{cases} d_i + 2x_i + 3, & \text{if } d_i < 0 \\ d_i + 2(x_i - y_i) + 5, & \text{otherwise} \end{cases}$$



Initial decision variable value:

$d_0 = F(0, R) = 5/4 - R$

Midpoint circle algorithm - improvement

- The next decision variable value is computed as a function of:
  - Current decision variable value
  - Current coordinate values x and y

$$d_{i+1} = \begin{cases} d_i + 2x_i + 3, & \text{if } d_i < 0 \\ d_i + 2(x_i - y_i) + 5, & \text{otherwise} \end{cases}$$

- Aim:
  - Next decision variable value does not depend on current coordinates x and y

$$d_{i+1} = d_i + \text{const}$$

Improvement by 2nd order differences

$d_{i+1} = d_i + \Delta d_i$

$\Delta d_i = d_{i+1} - d_i = 2x_i+3$ if $d_i<0$ (Δd_i^E)

$2(x_i-y_i)+5$ otherwise (Δd_i^{SE})

Passing to the next position:

$(x_i,y_i) \rightarrow (x_i+1,y_i)$

$\Delta d_{i+1}^E = 2(x_i+1)+3= \Delta d_i^E+2$

$\Delta d_{i+1}^{SE} = 2(x_i+1-y_i)+5= \Delta d_i^{SE}+2$

$(x_i,y_i) \rightarrow (x_i+1,y_i-1)$

$\Delta d_{i+1}^E = 2(x_i+1)+3= \Delta d_i^E+2$

$\Delta d_{i+1}^{SE} = 2(x_i+1-y_i+1)+5= \Delta d_i^{SE}+4$

Initially, on (x_0,y_0)

$\Delta d_0 = 3$ if $d_0<0$ (Δd_0^E)

$-2R+5$ otherwise (Δd_0^{SE})

Questions and proposed problems

1. Demonstrate what values take the function $F(x,y)= x^2 + y^2 - R^2$ inside and outside the circle, of center O and radius R.
2. Explain why the circle rendering is only computed in the second octant?
3. Modify the Midpoint circle algorithm to work in the first octant (under the first bisection).
4. Modify the Midpoint circle algorithm to work in the first quadrant (x, y positive).
5. Extend the Midpoint circle algorithm to render a filled circle.
6. Explain the improvement of the Midpoint circle algorithm by the second order differences.
7. Explain a method to render the following shapes of figure 7, composed of a line and a quarter of circle, by the Bresenham approach:

Questions and proposed problems

Figure 7.1.

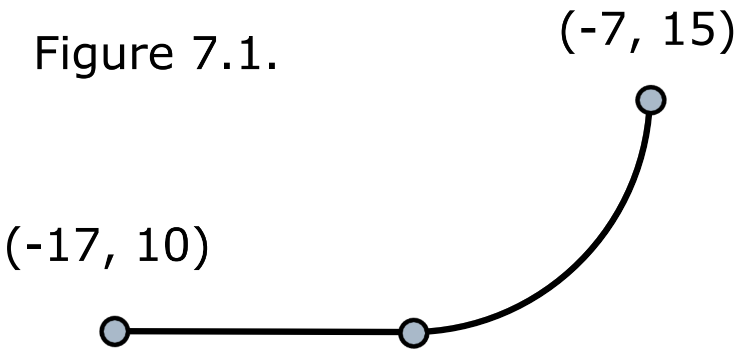


Figure 7.3.

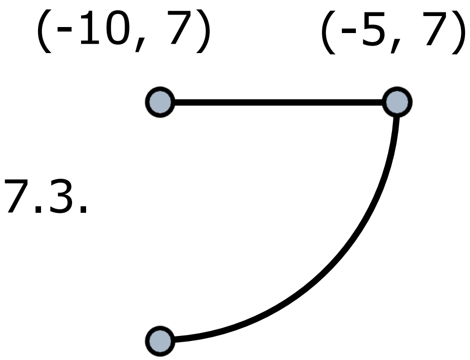


Figure 2.

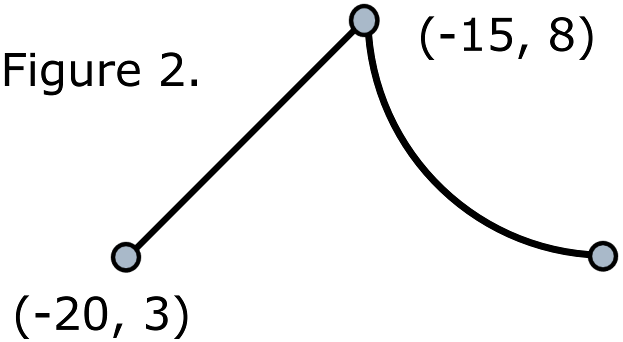


Figure 7.4.