Systems Theory Laboratory Assignment 3: Analysis of linear continuous systems. First and second-order systems. Steady-state error

Exercise 1. Consider the systems given by the block diagrams from Figure 1 and Figure 2. Determine the overall transfer functions for each system.

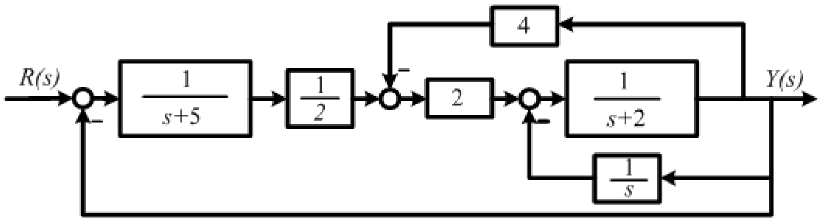


Figure 1:

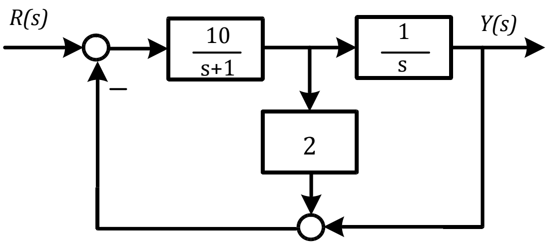
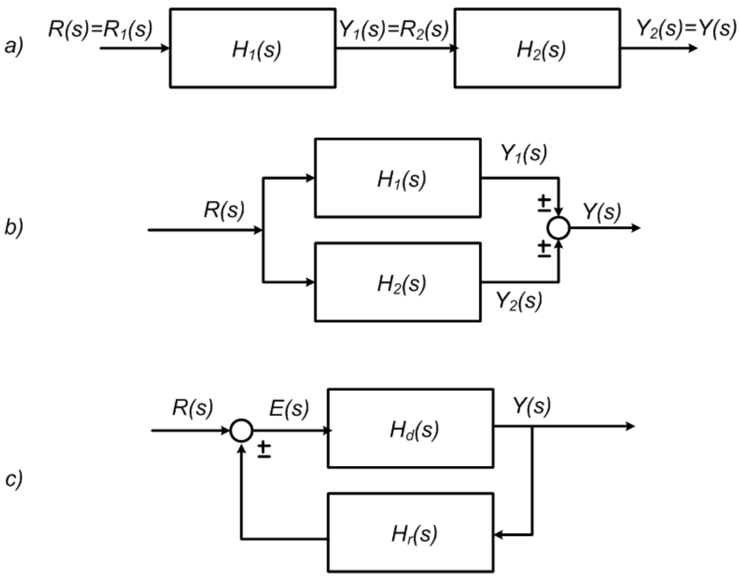


Figure 2:

Hint. Figure 1: Find the blocks connected in series or feedback loop and simplify. Start with the inner loops. Remember the rules for basic connections:



$$H_{series}(s) = H_1(s) \cdot H_2(s)$$

$$H_{parallel}(s) = \pm H_1(s) \pm H_2(s)$$

$$H_{feedback}(s) = \frac{H_d(s)}{1 \mp H_d(s) H_r(s)}$$

Hint. Figure 2: See the example from the lecture notes - Chap2.pdf, page 42, Example 2.8.1

Exercise 2. Consider a first-order system with the transfer function:

$$H(s) = \frac{K}{Ts + 1}$$

1. Use the Matlab functions *tf* and *step* or Simulink to plot the step response of the system when:
   (i) $K = 1$, $T = 1$
   (ii) $K = 3$, $T = 1$
   (iii) $K = 1$, $T = 3$
   (iv) $K = 1$, $T = 6$
2. Compare the plots and discuss the influence of the gain K and the time constant T on the system response.
3. Determine the settling time for all cases.

Solution

Simulation using Matlab functions. Write the next script and run it.

Listing 1: ex2.m

```
close all
clear all
clc

% input the transfer functions
K = 1; T = 1; sys1=tf(K, [T 1]);
K = 3; T = 1; sys2=tf(K, [T 1]);
K = 1; T = 3; sys3=tf(K, [T 1]);
K = 1; T = 6; sys4=tf(K, [T 1]);

% simulate the step responses for 25 seconds
step(sys1,sys2,sys3,sys4,25), grid on
legend('H1: K = 1; T = 1', 'H2: K = 3; T = 1', 'H3: K = 1; T = 3', 'H4: K = 1; T = 6')
```

Simulation using Simulink. Watch the MathWorks tutorials:

- Getting Started with Simulink, Part 1
- Getting Started with Simulink, Part 2

or follow the steps:

(a) In the command window write *simulink* at the command prompt, or press the *Simulink* button from the Home tab.
(b) In Simulink interface choose *Blank model* → *Create a blank model*
(c) In the model window, open the block library by pressing the *Library browser* button.
(d) Arrange the Library browser window and the model window so that you can drag blocks from the library into your model.
(e) Open the library *Sources* by clicking on it.
(f) Notice that all libraries and blocks are arranged in alphabetical order. Select *Step* and drag the block into your model window.
(g) Double click on the *Step* block from the model window and set *Step time* to 0.
(h) Open the library *Continuous* and drag a *Transfer fcn* block into your model window. Place it to the right of the *Step*.
(i) Change the name of the block (in the model window) by changing the text below the block. For example you can write *H1*.
(j) Double click on the block *H1* and fill in the numerator and denominator coefficients.
(k) From the *Sinks* library drag a *Scope* and place it to the right of the *H1*.
(l) Connect the output of *H1* to the *Scope*.

(m) Copy-paste the *H1* block another three times, or drag another 3 transfer functions into your model.

(n) Change the names into *H2*, *H3* and *H4*, then change numerator and denominator coefficients for each block, according to the values given in the exercise.

(o) Connect all inputs to the step source and all outputs to the *Scope*. The model should be similar to Figure 3

(p) Change the *Stop time* - in the upper part of the model window - to 25 (i.e. 25 seconds),

(q) Save the model.

(r) Press *Run*

(s) Double click on *Scope* to see the simulation results. In the *Scope* window, find *View* in the menu and check *Legend*.

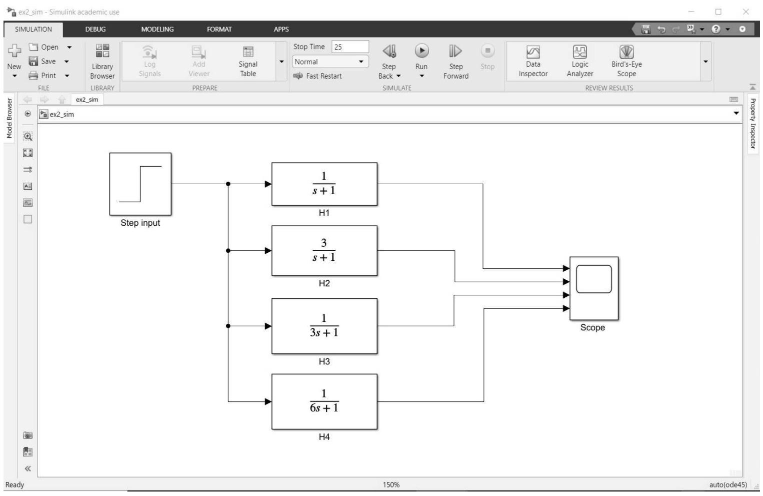


Figure 3: Simulink model

Compare the plots. Compare all plots obtained and discuss the influence of the gain and the time constant

Settling time. Remember that the settling time, for a first-order system is equal to 4 time constants, $t_s = 4T$. Determine the settling time for all cases and find it in the plots.

Exercise 3. Consider a second-order system with the transfer function:

$$H(s) = \frac{K}{\dfrac{1}{\omega_n^2}s^2 + \dfrac{2\zeta}{\omega_n}s + 1} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

1. Use the Matlab functions *tf* and *step* or Simulink to plot the step response of the system when:
   (i) $K = 1$, $\omega_n = 1$, $\zeta = 0$
   (ii) $K = 1$, $\omega_n = 3$, $\zeta = 0$
   (iii) $K = 1$, $\omega_n = 1$, $\zeta = 0.1$
   (iv) $K = 1$, $\omega_n = 1$, $\zeta = 0.6$
   (v) $K = 1$, $\omega_n = 1$, $\zeta = 1$
   (vi) $K = 1$, $\omega_n = 1$, $\zeta = 2$
   (vii) $K = 3$, $\omega_n = 1$, $\zeta = 0.6$
2. Compare the plots (i) and (ii) and discuss the influence of the natural frequency ω_n on the system response.
3. Compare the plots (iii), (iv), (v) and (vi) and discuss the influence of the damping factor ζ on the system response.

4. Compare the plots (iv) and (vii) and discuss the influence of the gain K on the system response. Determine the settling time from the plot.

Hint. If you choose to plot the step responses using Matlab functions, write a script using the previous exercise as an example. Create 3 figures: one for (i) and (ii), the second for (iii), (iv), (v) and (vi) and the third for (iv) and (vii).

If you choose the use Simulink, the model should be similar to Figure 4.

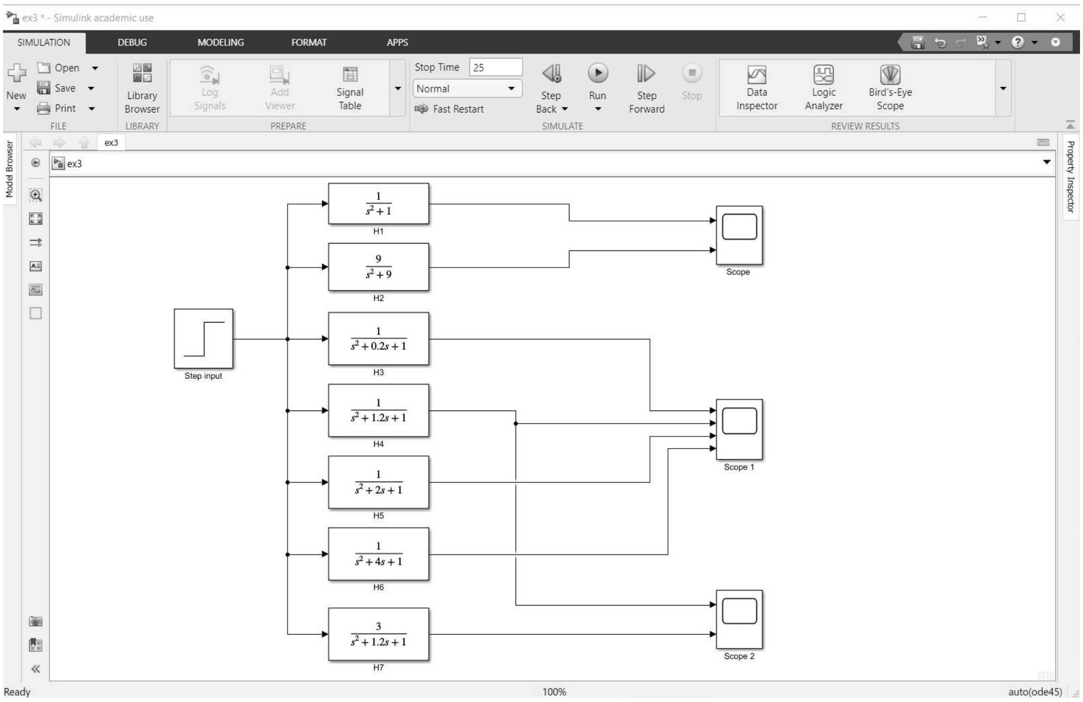


Figure 4: Simulink model

Exercise 4. Consider the plots in Figure 5 representing the unit step responses of eight systems. Match the step response plots to the following transfer functions:

$$H_1(s) = \frac{0.5}{s+0.5}, \qquad H_2(s) = \frac{2}{s+2}, \qquad H_3(s) = \frac{4}{s+2}$$

$$H_4(s) = \frac{1}{s^2+1}, \qquad H_5(s) = \frac{9}{s^2+9}$$

$$H_6(s) = \frac{9}{s^2+0.9s+9}, \quad H_7(s) = \frac{9}{s^2+3s+9}, \quad H_8(s) = \frac{18}{s^2+3s+9}$$

Exercise 5. A feedback control system is shown in Figure 6.

Suppose that our design objective is to find a controller of minimal complexity such that the closed-loop system can track a unit step with zero steady-state error.

1. As a first try, consider a simple proportional controller

$$G_C(s) = 2$$

   (a) Compute the steady-state error.
   (b) Plot the unit step response and determine the steady-state error from the plot.

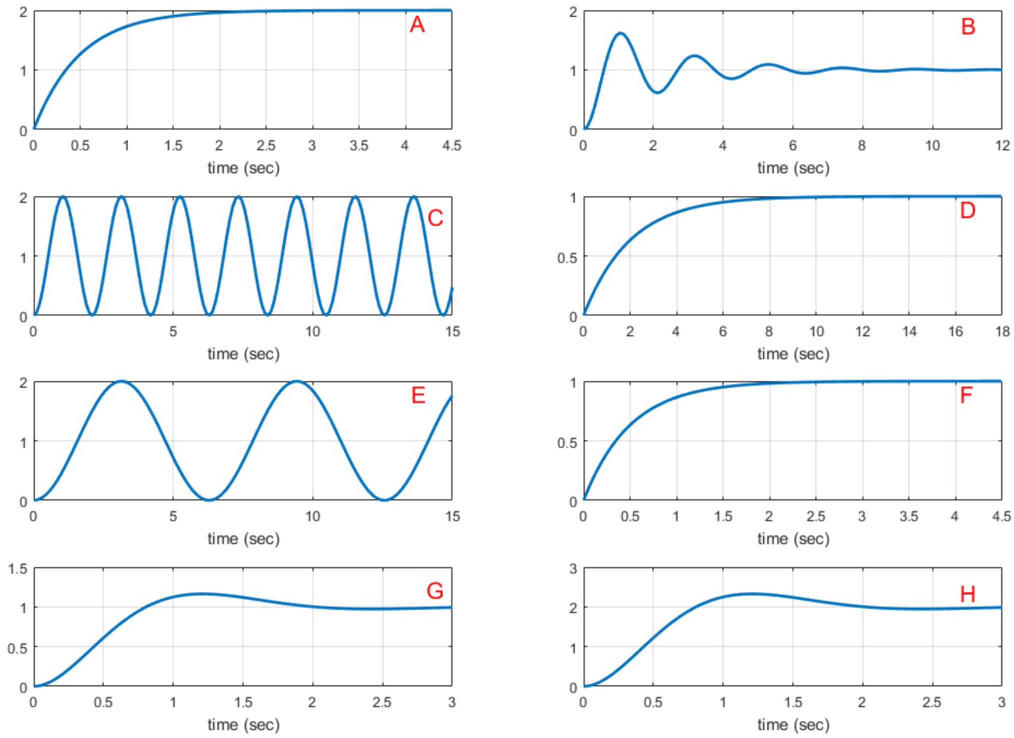


Figure 5: Step responses

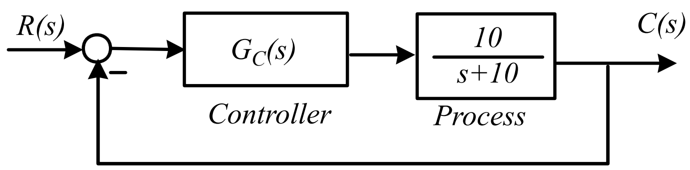


Figure 6: Closed-loop control system

2. Consider a more complex proportional-integral (PI) controller, with the transfer function:

$$G_C(s) = 2 + \frac{20}{s} = \frac{2s+20}{s}$$

(a) Compute the steady-state error.

(b) Plot the unit step response and determine the steady-state error from the plot.

Hint 1. Remember the steady-state error:

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s), \quad E(s) = R(s) - C(s)$$

Find $E(s)$ and compute the steady-state error.

Hint 2. A Simulink model showing the system response and the step input on the same plot can be similar to Figure 7 (case 2). Find the Sum *block in the* Math operations *library. Double click on the block and change the list of signs to +-, to obtain a negative feedback.*

Exercise 6. For the systems represented by the block diagrams shown in Figure 8.

1. Calculate the steady-state error for a ramp input, $r(t) = t,\ t \geq 0$

2. Choose a value for k and plot the system response for a ramp input. Place the input ramp on the same plot and determine the steady-state error in the plot.

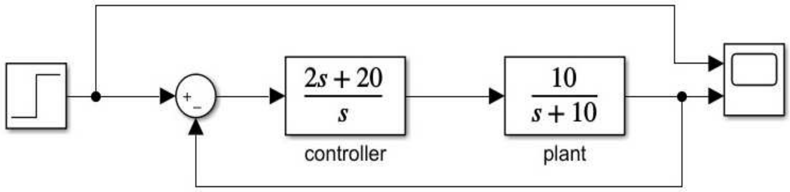


Figure 7: Simulink model

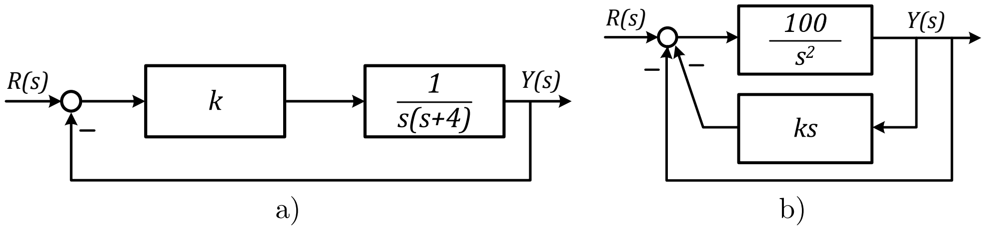


Figure 8: Closed-loop systems

3. Determine the range of values for k, ($k > 0$) for which the step response is overdamped (with no overshoot).

Hints

1. *The steady-state error will depend on k.*
2. *If you choose to plot the system response using Matlab functions, use* lsim. *See* help lsim *for details. If you choose to use Simulink, find a* Gain *block in* Commonly used blocks *and use it for k. For Figure b), the block ks will be replaced by a* Gain *and a* Derivative *(*Continuous *library)*
3. *Remember that a system is overdamped if all the poles are real, or the damping factor $\zeta \geq 1$.*

Exercise 7. Predator-prey model, [1]. The predator-prey problem, refers to an ecological system in which we have two species, one of which feeds on the other. This type of system has been studied for decades and is known to exhibit interesting dynamics. A simple model for this situation can be constructed by keeping track of the rate of births and deaths of each species. Let $H(t)$ represent the number of hares (prey) and let $L(t)$ represent the number of lynxes (predator). The input u corresponds to the growth rate for hares, which we might modulate by controlling a food source for the hares. The dynamics of the system are modeled as:

$$\begin{aligned} \frac{dH(t)}{dt} &= (1.6 + u(t))H(t)\left(1 - \frac{H(t)}{125}\right) - \frac{3.2H(t)L(t)}{50 + H(t)}, \quad H \geq 0, \\ \frac{dL(t)}{dt} &= 0.6\frac{3.2H(t)L(t)}{50 + H(t)} - 0.56L(t), \quad L \geq 0 \end{aligned}$$

We first linearize the system around the equilibrium point of the system (H_e, L_e, u_e) which can be determined numerically to be $H_e = 20.6$, $L_e = 29.5$ for $u_e = 0$. This yields a linear dynamical system:

$$\frac{dz_1(t)}{dt} = 0.13z_1(t) - 0.93z_2(t) + 17.2u(t) \tag{1}$$

$$\frac{dz_2(t)}{dt} = 0.57z_1(t) \tag{2}$$

where $z_1(t) = H(t) - H_e$ and $z_2(t) = L(t) - L_e$ (i.e. the variation of the number of hares and lynxes around the equilibrium values).

The block diagram for this system is shown in Figure 9, where $Z_1(s) = \mathscr{L}\{z_1(t)\}$, $Z_2(s) = \mathscr{L}\{z_2(t)\}$, $U(s) = \mathscr{L}\{u(t)\}$.

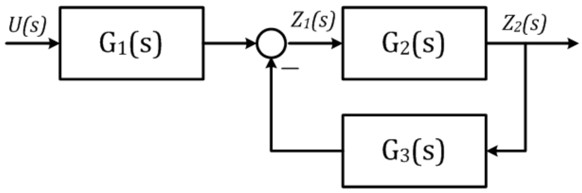


Figure 9: Block diagram of the predator-prey system

1. Apply the Laplace transform of relations (1) and (2) and determine the transfer functions $G_1(s)$, $G_2(s)$ and $G_3(s)$, as shown in Figure 9.
2. Build the block diagram in Simulink.
3. Simulate the system for a step input, plot the evolution of $z_1(t)$ and $z_2(t)$ and explain the result.

References

[1] Karl Astrom, Richard Murray, *Feedback Systems: An Introduction for Scientists and Engineers*, Princeton University Press, 2008, $http://www.cds.caltech.edu/\ murray/amwiki/index.php/Main_Page$