Progetto Gomory Cuts

Applicato a Minimum Weighted Vertex Cover

Piani di taglio

- Modello PLI rilassato
- Risoluzione PL
- Aggiunta di disequazione che taglia la soluzione ottima non intera:
  - Vincolo non soddisfatto dalla soluzione x^*_{PL} ma soddisfatto da tutte le soluzioni ammissibili del PLI

Gomory Cuts (I)

- PLI: $z = min\{c^T x : x \in \Omega \subseteq \mathbb{Z}^n\}$ con $\Omega = \{x \in \mathbb{Z}_+^n : Ax = b\}$
- Rilassamento lineare: $z^c = min\{c^T x : Ax = b, x \in \mathbb{R}_+^n\}$
- B matrice di base ottima
- N matrice delle soluzioni fuori base

$$A = \begin{array}{|c|c|} \hline B & N \\ \leftarrow m \rightarrow & \leftarrow n-m \rightarrow \\ \hline \end{array} \begin{array}{l} \uparrow \\ m \\ \downarrow \end{array}$$

- $\hat{x} = \begin{pmatrix} B^{-1}b \\ 0^{n-m} \end{pmatrix}$ soluzione ottima del rilassamento

Gomory Cuts (II)

- $B^{-1}Ax = B^{-1}b$

I	$B^{-1}N$ $\leftarrow \varphi \rightarrow$	x	$= \overline{b}$

- φ colonne fuori base
- i-esimo vincolo: $x_i + \sum_{j \in \varphi} \overline{a}_{ij} x_j = \overline{b}_i$
- Taglio intero: $x_i + \sum_{j \in \varphi} \lfloor \overline{a}_{ij} \rfloor x_j \leq \lfloor \overline{b}_i \rfloor$
- Taglio frazionario: $\sum_{j \in \varphi} \alpha_{ij} x_j \geq \beta_i$
  - Con $\alpha_{ij} = \overline{a}_{ij} - \lfloor \overline{a}_{ij} \rfloor$ e $\beta_i = \overline{b}_i - \lfloor \overline{b}_i \rfloor$

Minimum weighted vertex cover

- Dato un grafo $G = (V, E)$
- $C \subseteq V$ è un vertex cover se $\forall (u, v) \in E,\ u \in C\ o\ v \in C$
- PLI:
  - $min \sum_{u \in V} c_u x_u$
  - Subject to $x_u + x_v \geq 1,\ \forall (u, v) \in E$
  - $x_u \in \{0,1\}, \forall u \in V$

Scelte implementative (I)

- Uso delle API Java di GUROBI
- Libreria Apache Commons Math per il calcolo matriciale
- Libreria JFreeChart per i grafici relativi all'andamento della funzione obiettivo
- Uso di Java Swing per l'interfaccia grafica
- Libreria JGraphX per i grafi relativi alle istanze

Scelte implementative (II)

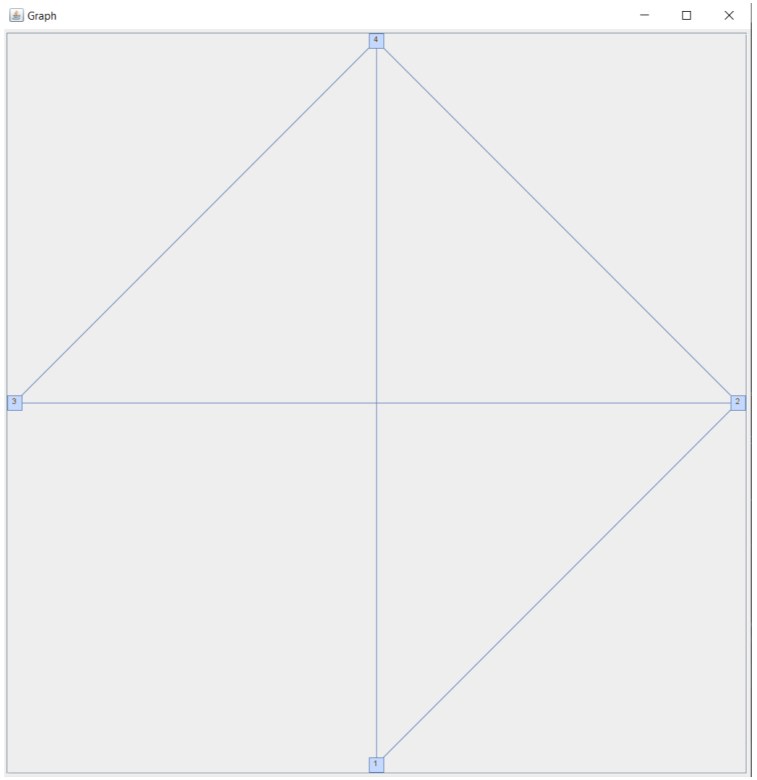
- Import delle istanze da file
- Generazione e risoluzione del problema di PLI
- Generazione e risoluzione del problema rilassato
- Calcolo delle varie matrici per la derivazione dei tagli di Gomory
- Varie modalità di taglio:
  - Singolo in forma intera
  - Singolo in forma frazionaria
  - Singolo in forma intera e frazionaria
  - Multiplo in forma intera
  - Multiplo in forma frazionaria
  - Multiplo in forma intera e frazionaria

Scelte implementative (III)

- Creazione e risoluzione nuovo modello con aggiunta dei tagli generati, nel caso singolo scegliendo quello con parte frazionaria maggiore.
- Visualizzazione del grafo dell'istanza
- Visualizzazione dei grafici dell'andamento delle funzioni obiettivo
- Visualizzazione delle tempistiche di calcolo e dei file di output

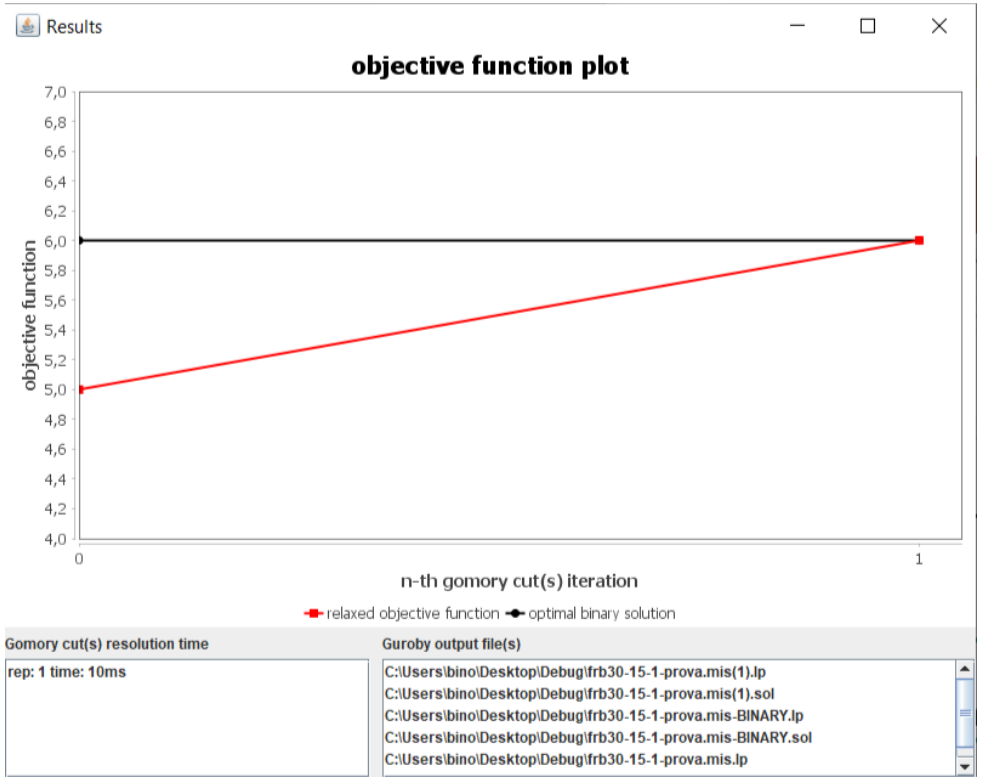
Validazione

Grafo istanza di prova



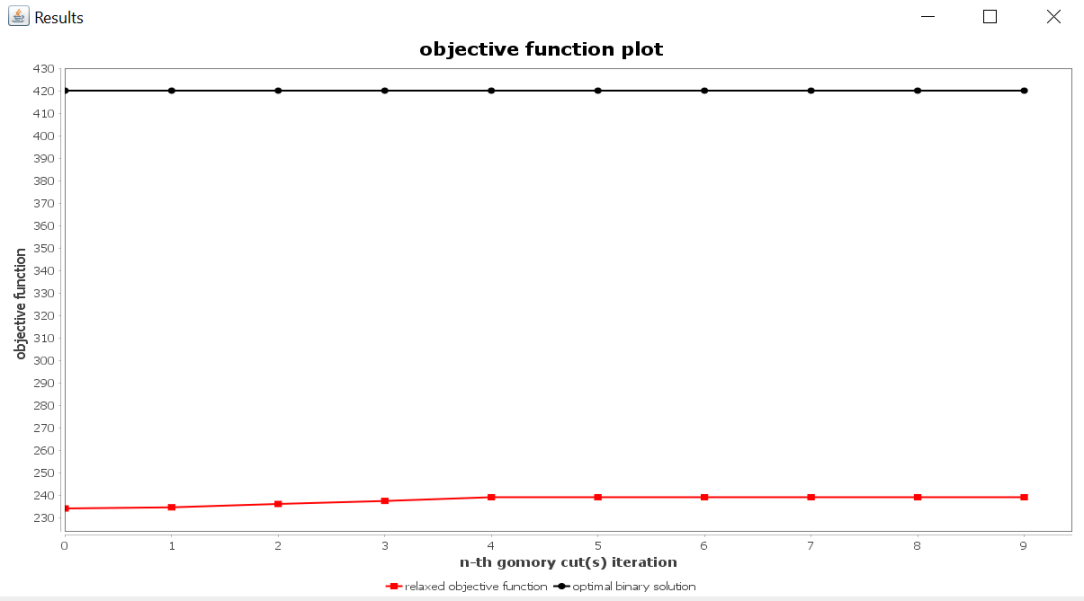
X=(0,1,0,1)

Andamento soluzione rilassata con tagli

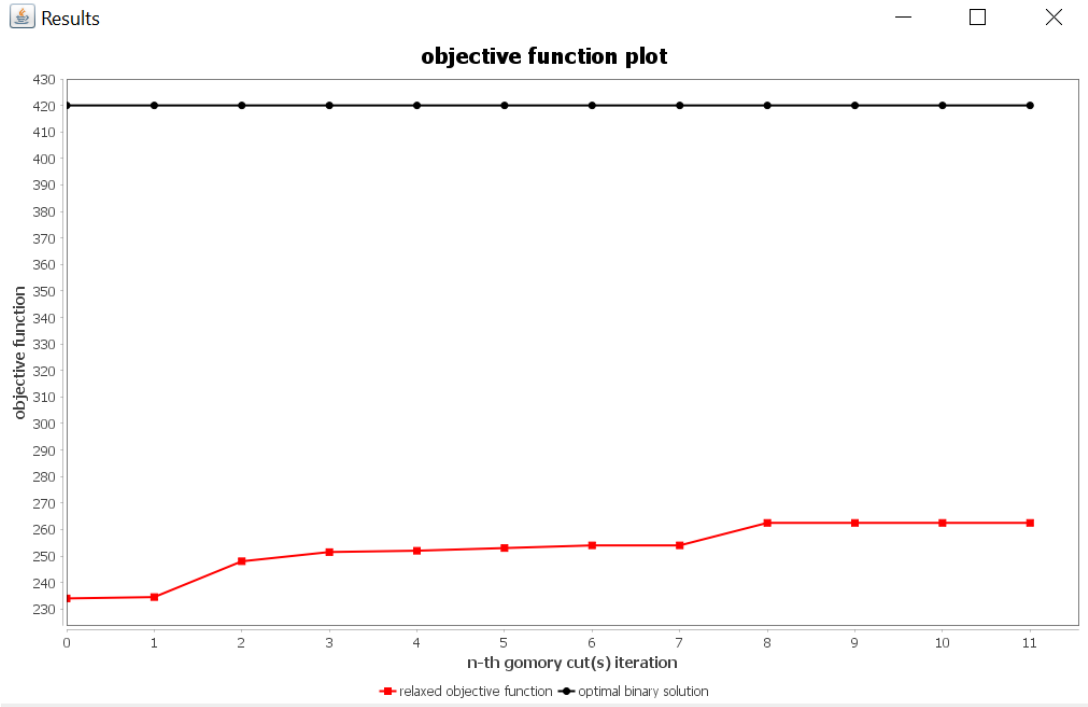


Grafici e tempi per istanze prova

Istanza 1: tagli interi



Istanza 1: tagli frazionari



Grafici e tempi per istanze prova

Istanza 1: tagli interi

Gomory cut(s) resolution time

rep: 1 time: 109ms
rep: 2 time: 78ms
rep: 3 time: 63ms
rep: 4 time: 62ms
rep: 5 time: 47ms
rep: 6 time: 31ms
rep: 7 time: 32ms
rep: 8 time: 47ms
rep: 9 time: 31ms
rep: 10 time: 47ms

Istanza 1: tagli frazionari

Gomory cut(s) resolution time

rep: 1 time: 109ms
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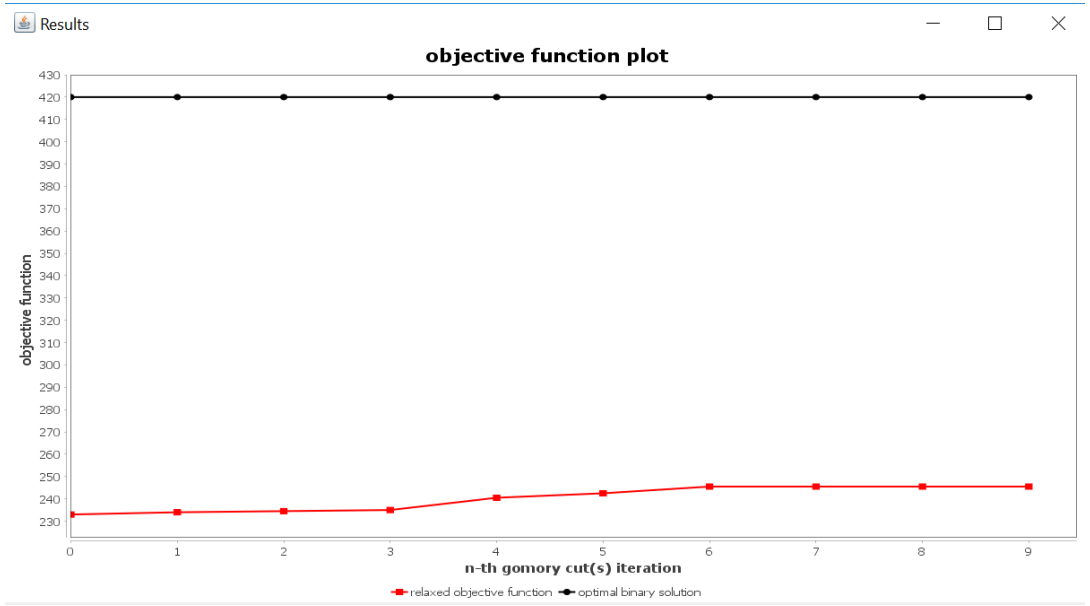
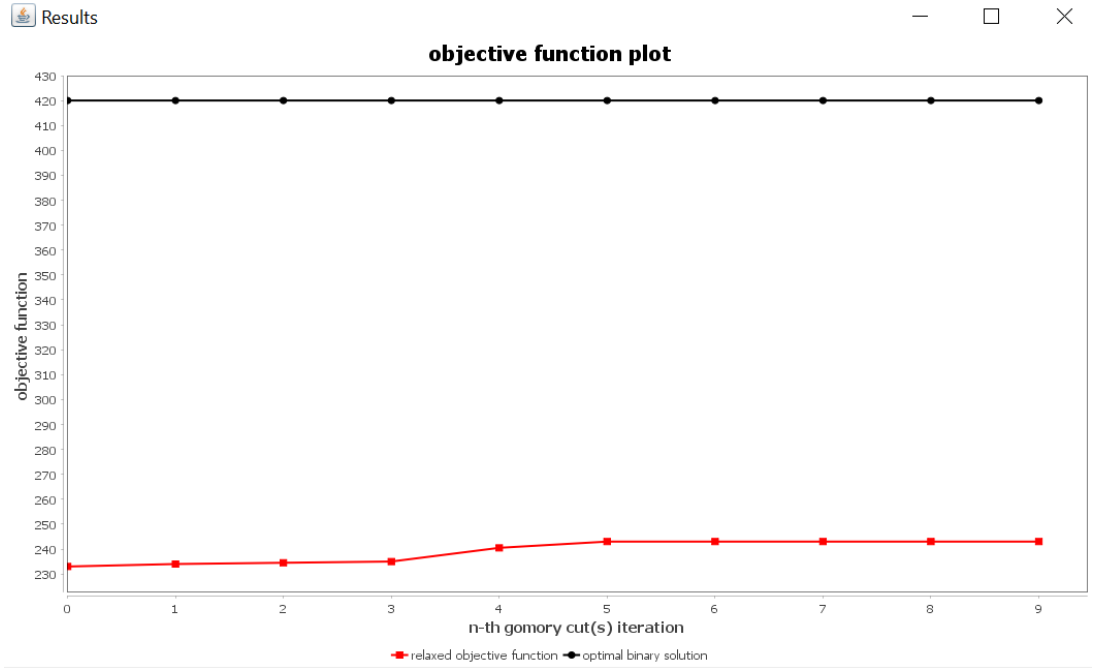
rep: 7 time: 31ms
rep: 8 time: 32ms
rep: 9 time: 47ms
rep: 10 time: 31ms
rep: 11 time: 31ms
rep: 12 time: 47ms

Grafici e tempi per istanze prova

Istanza 2: tagli interi



Istanza 2: tagli frazionari



Grafici e tempi per istanze prova

Istanza 2: tagli interi

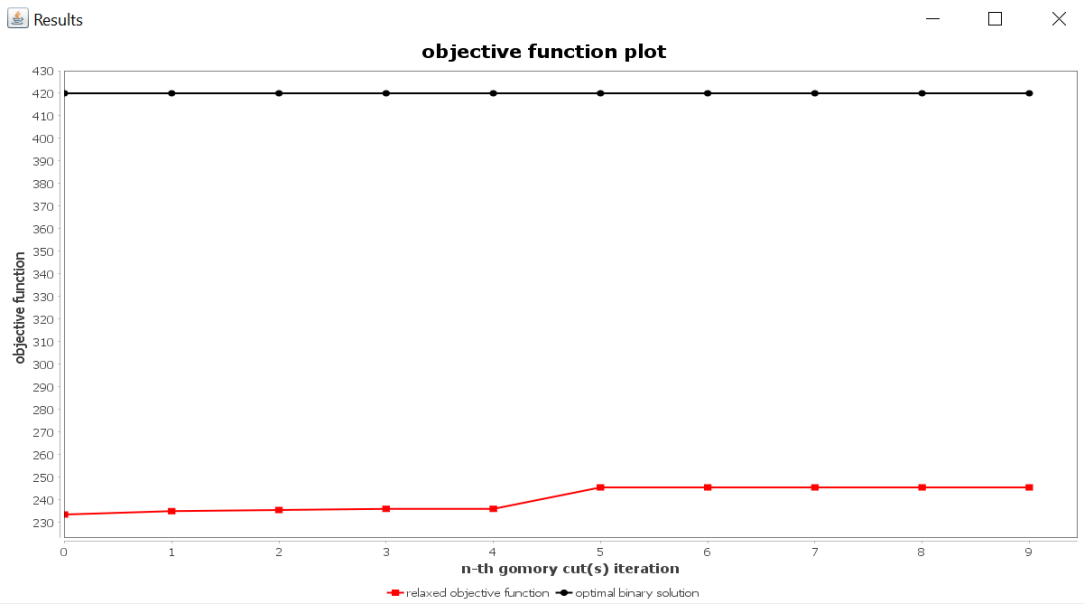
Gomory cut(s) resolution time
rep: 1 time: 47ms
rep: 2 time: 16ms
rep: 3 time: 15ms
rep: 4 time: 32ms
rep: 5 time: 15ms
rep: 6 time: 16ms
rep: 7 time: 15ms
rep: 8 time: 32ms
rep: 9 time: 31ms
rep: 10 time: 16ms

Istanza 2: tagli frazionari

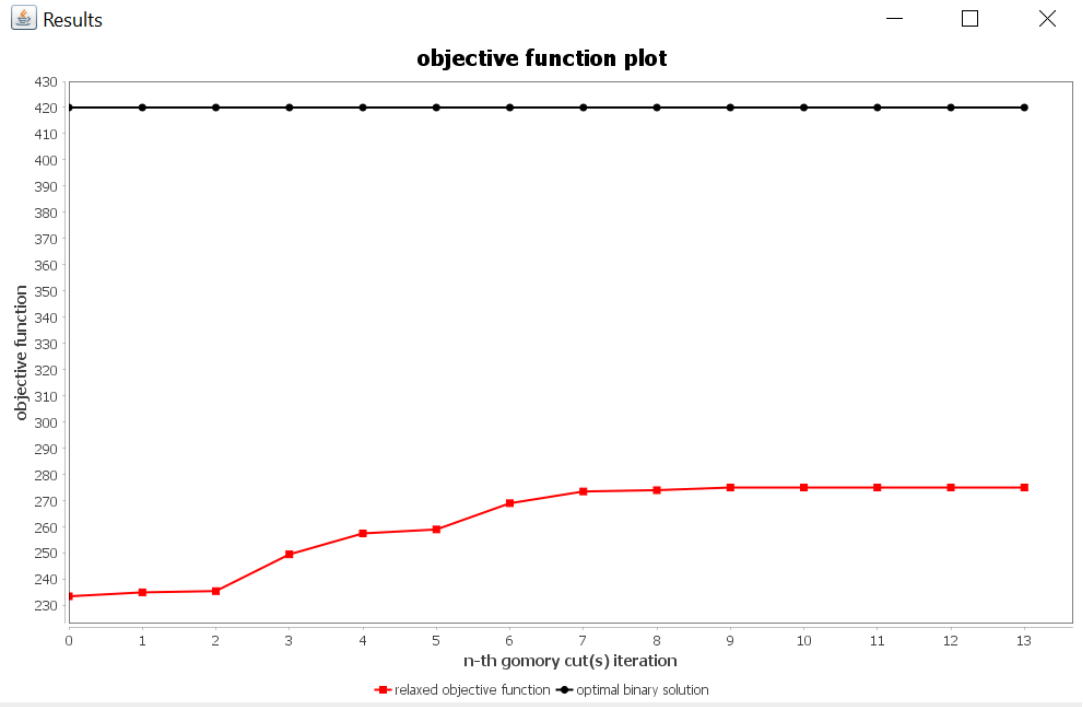
Gomory cut(s) resolution time
rep: 1 time: 47ms
rep: 2 time: 31ms
rep: 3 time: 31ms
rep: 4 time: 32ms
rep: 5 time: 31ms
rep: 6 time: 31ms
rep: 7 time: 31ms
rep: 8 time: 32ms
rep: 9 time: 31ms
rep: 10 time: 31ms

Grafici e tempi per istanze prova

Istanza 3: tagli interi



Istanza 3: tagli frazionari



Grafici e tempi per istanze prova

Istanza 3: tagli interi

Gomory cut(s) resolution time

rep: 1 time: 32ms
rep: 2 time: 31ms
rep: 3 time: 31ms
rep: 4 time: 47ms
rep: 5 time: 31ms
rep: 6 time: 31ms
rep: 7 time: 63ms
rep: 8 time: 31ms
rep: 9 time: 31ms
rep: 10 time: 47ms

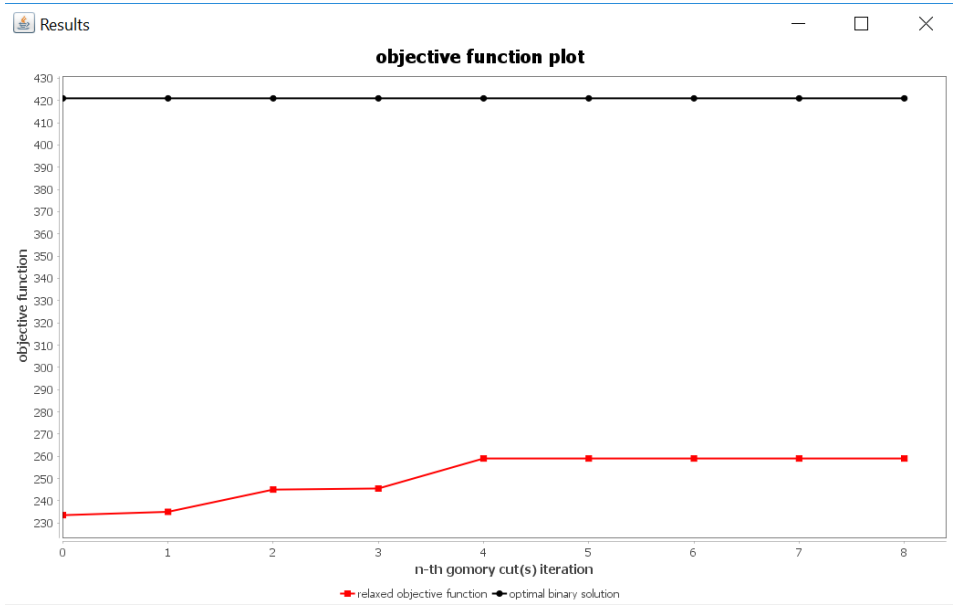
Istanza 3: tagli frazionari

Gomory cut(s) resolution time

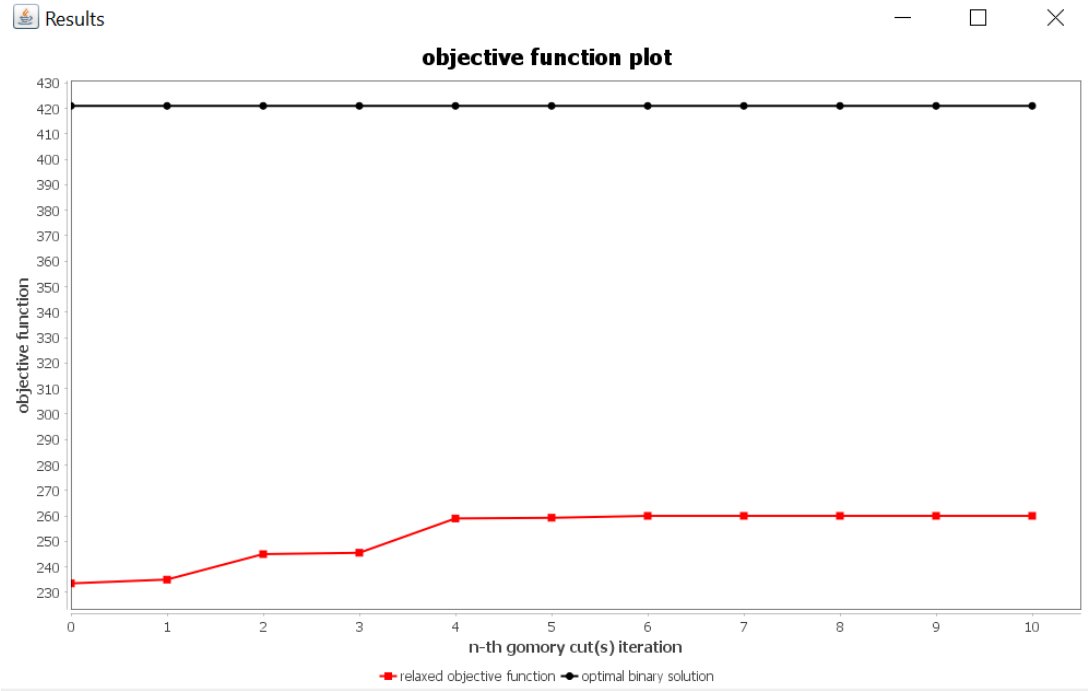
rep: 1 time: 47ms
rep: 2 time: 32ms
rep: 3 time: 31ms
rep: 4 time: 31ms
rep: 5 time: 47ms
rep: 6 time: 47ms
rep: 7 time: 47ms
rep: 8 time: 46ms
rep: 9 time: 47ms
rep: 10 time: 32ms
rep: 11 time: 31ms
rep: 12 time: 47ms
rep: 13 time: 31ms
rep: 14 time: 31ms

Grafici e tempi per istanze prova

Istanza 4: tagli interi



Istanza 4: tagli frazionari



Grafici e tempi per istanze prova

Istanza 4: tagli interi

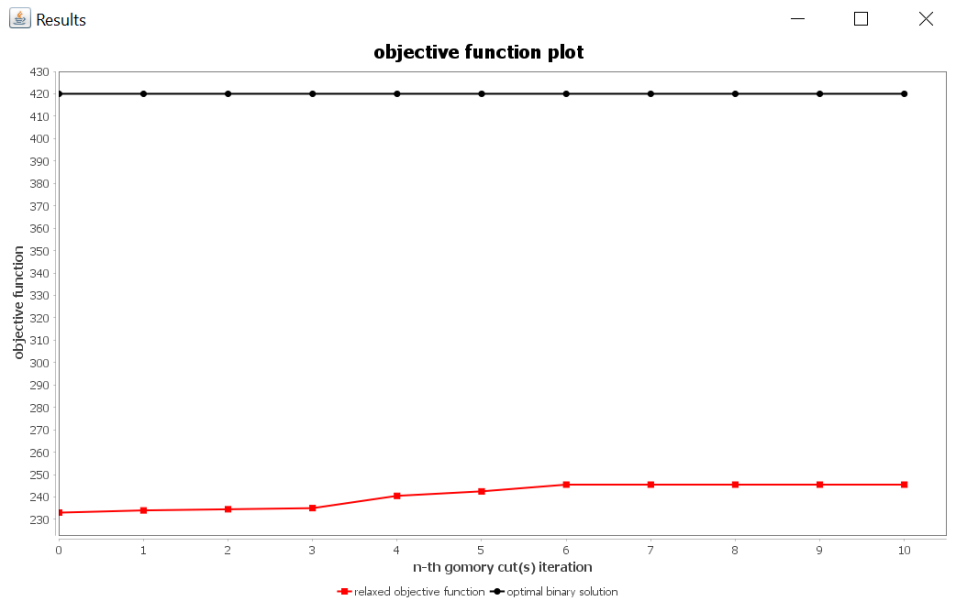
Gomory cut(s) resolution time
rep: 1 time: 93ms
rep: 2 time: 94ms
rep: 3 time: 47ms
rep: 4 time: 78ms
rep: 5 time: 47ms
rep: 6 time: 31ms
rep: 7 time: 47ms
rep: 8 time: 32ms
rep: 9 time: 31ms

Istanza 4: tagli frazionari

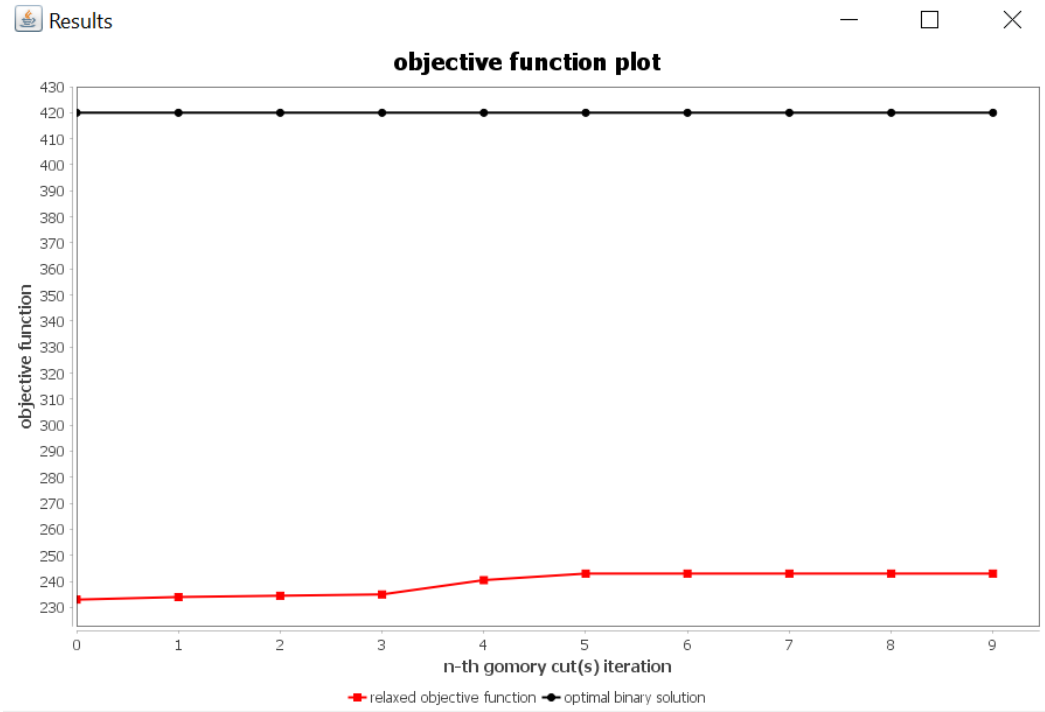
Gomory cut(s) resolution time
rep: 1 time: 47ms
rep: 2 time: 47ms
rep: 3 time: 31ms
rep: 4 time: 31ms
rep: 5 time: 31ms
rep: 6 time: 32ms
rep: 7 time: 31ms
rep: 8 time: 47ms
rep: 9 time: 31ms
rep: 10 time: 31ms
rep: 11 time: 31ms

Grafici e tempi per istanze prova

Istanza 5: tagli interi



Istanza 5: tagli frazionari



Grafici e tempi per istanze prova

Istanza 5: tagli interi

Gomory cut(s) resolution time
rep: 1 time: 31ms
rep: 2 time: 31ms
rep: 3 time: 16ms
rep: 4 time: 31ms
rep: 5 time: 16ms
rep: 6 time: 15ms
rep: 7 time: 32ms
rep: 8 time: 15ms
rep: 9 time: 16ms
rep: 10 time: 31ms
rep: 11 time: 16ms

Istanza 5: tagli frazionari

Gomory cut(s) resolution time
rep: 1 time: 15ms
rep: 2 time: 16ms
rep: 3 time: 15ms
rep: 4 time: 16ms
rep: 5 time: 31ms
rep: 6 time: 31ms
rep: 7 time: 32ms
rep: 8 time: 31ms
rep: 9 time: 31ms
rep: 10 time: 31ms