Introduction to Machine Learning feat. TensorFlow

Peter Goldsborough

July 9, 2016

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Table of Catents

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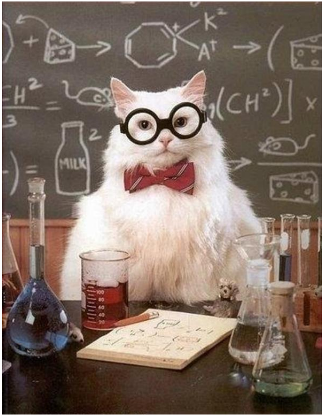
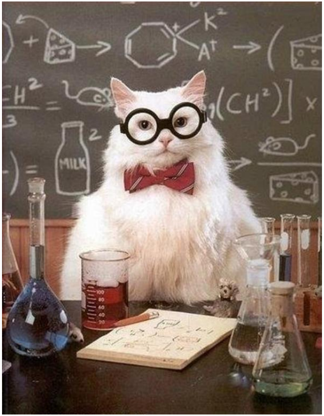


Theory

Table of Catents



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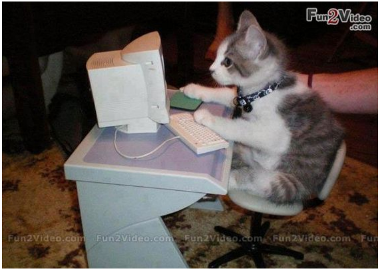

Practice

Background

Background

- CS Student @ TUM

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- CS Student @ TUM
- Google & Bloomberg Intern

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- I like cats

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Seminar Topic: *Deep Learning With TensorFlow*

`github.com/peter-can-write/tensorflow-paper` github.com/peter-can-talk/python-meetup-munich-2016

What is Machine Learning?

What is Machine Learning?

(and can I eat it?)

Definition I

Definition I

Machine Learning is cool.

Definition II

Machine Learning is *really* cool.

Definition III

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Machine learning is not magic; it can't get something from nothing.

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What it does is get more from less.

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What it does is get more from less.

Learning is like farming, which lets nature do most of the work. Farmers combine seeds with nutrients to grow crops. Learners combine knowledge with data to grow programs.

[**?**]

Definition IV

A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P, if its performance at tasks in T, as measured by P, improves with experience E.

[**?**]

Definition III

$$f : \text{Image} \rightarrow \{\text{cat, banana, spaceship, } \dots\}$$

Definition III

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$$f^\star(x) \approx f(x)$$

The Task, T

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- $\mathbf{x}$ contains features, such as (lunch, dinner)

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- $\mathbf{x}$ contains features, such as (lunch, dinner)
- The output $\mathbf{y}$ can take on various forms

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Classification

$$f : \mathbb{R}^n \to \{1, \ldots, k\}$$

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- Image classification

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- Image classification
- Recognizing handwritten digits

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- Image classification
- Recognizing handwritten digits
- Predictive Policing

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- Predicting the market price of a house
- Predicting the amount of rain in a season

The Task, T

Transcription

f : Language $\rightarrow$ Text

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Transcription

$$f : \text{Language} \rightarrow \text{Text}$$

- Transform language from an unstructured representation into text

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- Similar to *Translation*

The Task, T

Synthesis

f : Stuff $\rightarrow$ More Stuff

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- Generate new data similar to the input
- Reproduce patterns of the sky for video games
- Synthesize speech
- Generate more data to train our algorithm to generate more data to train our algorithm to generate more data to train our algorithm to generate more data to train our algorithm to generate more data ...

How do we make our algorithm learn?

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Unsupervised Learning

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Unsupervised Learning

Supervised Learning

How do I Machine Learning?

Methods

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$$\mathbf{b} = (R, G, B)^\top$$

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$$f : \mathbf{b} \mapsto \text{market price} \in \mathbb{R}$$

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$$0 \xrightarrow{R} 255$$

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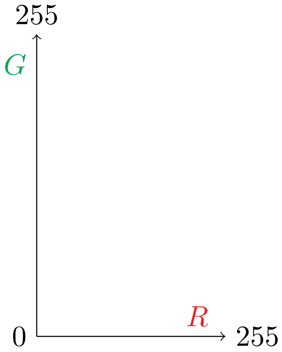

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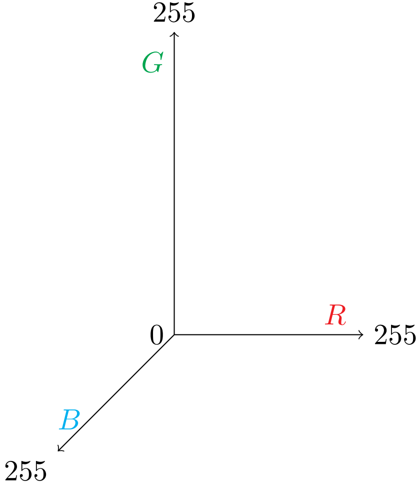

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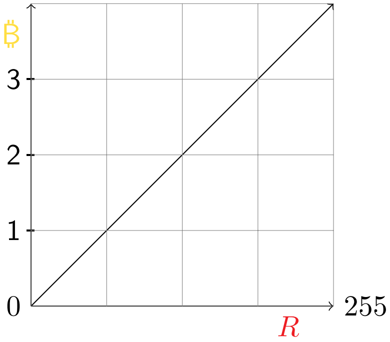

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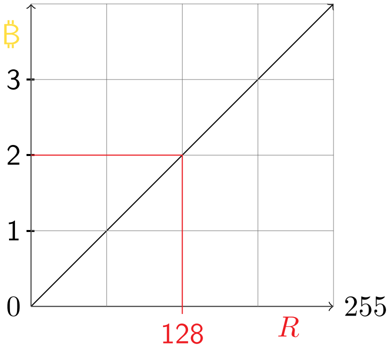

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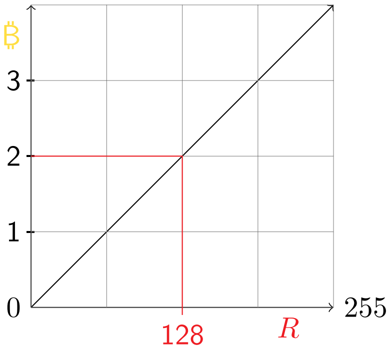
- The components of each vector $\mathbf{b}$ are our **features**
- Each feature represents one axis in space
- Each feature should contribute to some extent to the output value (market price)



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- Positive values of w_i *encourage* positive values of b_i
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- We add a bias c as an offset
- We expect our algorithm to learn $\mathbf{w}$ and c

$$f(\mathbf{b}) = \mathbf{w}^\top \mathbf{b} + c$$

Methods: Data

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$$\begin{array}{cc} & \begin{array}{ccc} \text{R} & \text{G} & \text{B} \end{array} \\ \text{n} & \begin{bmatrix} 34 & 147 & 73 \\ 247 & 69 & 13 \\ 66 & 66 & 66 \end{bmatrix} \end{array}$$

Design Matrix

Methods: Data

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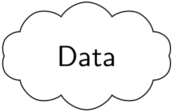
- In practice, we organize our data into a matrix $\mathbf{D} \in \mathbb{R}^{n \times k}$

```
D = np.array([[34, 147, 37], [247, 69, 13], [66, 66, 66]])
w = np.random.rand(3, 1)
b = np.random.randn()
y = D.dot(w) + b
```

Methods: Data



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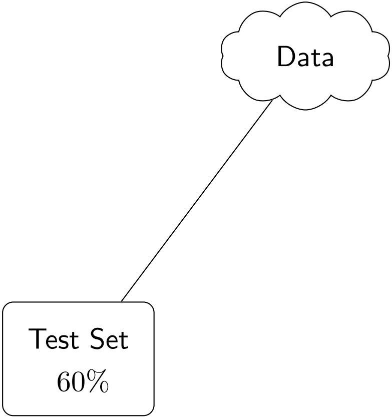

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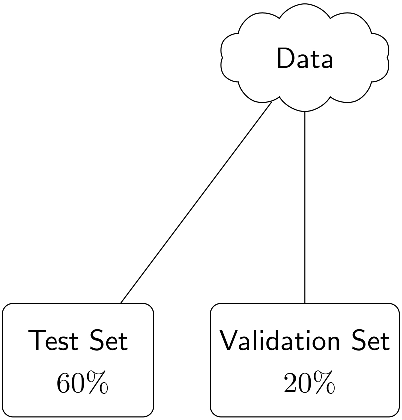

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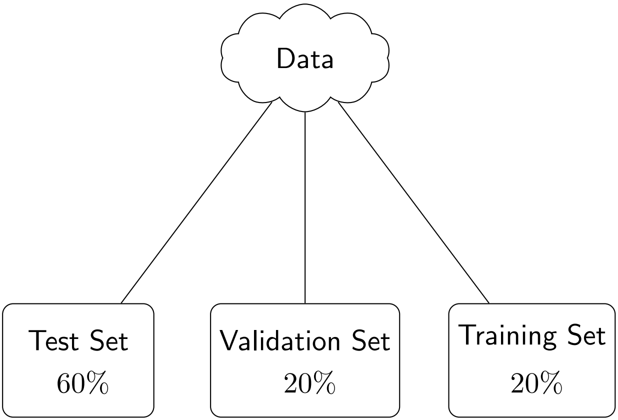

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$$L(\mathbf{y}, \hat{\mathbf{y}}) \in \mathbb{R}$$

Methods: Measuring Performance

Regression

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Regression

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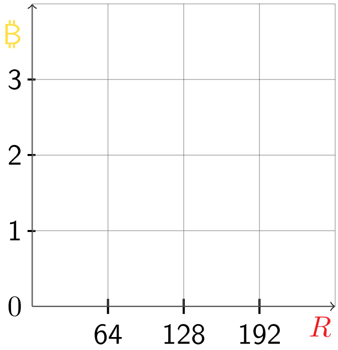
$$L(\mathbf{y}, \hat{\mathbf{y}}) = MSE(\mathbf{y}, \hat{\mathbf{y}}) = \frac{1}{k} \sum_{i=1}^{k} (y_i - \hat{y_i})^2$$

Methods: Measuring Performance

Regression



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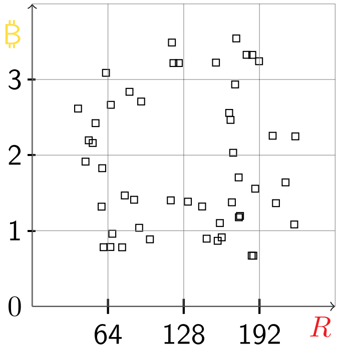

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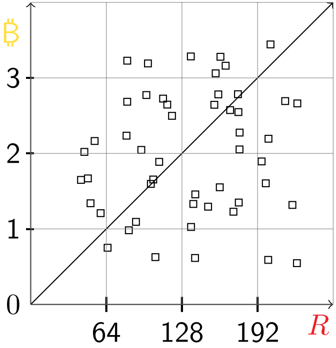

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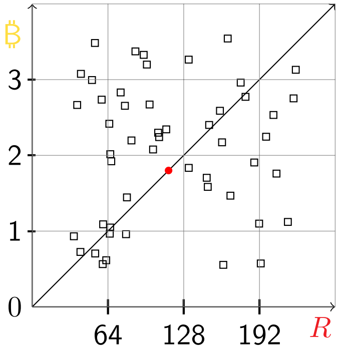

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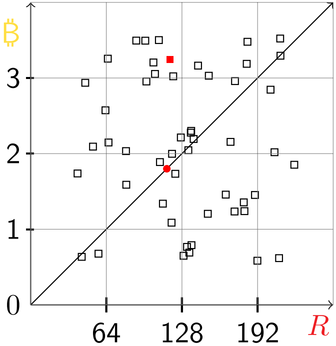

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This is the core idea behind Machine Learning

Methods: Weight Update

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- $L(\mathbf{y}, \hat{\mathbf{y}})$ depends on $\mathbf{y}$ and $\hat{\mathbf{y}}$, but *also* $\mathbf{w}$: $L(\mathbf{y}, \hat{\mathbf{y}}; \mathbf{w})$

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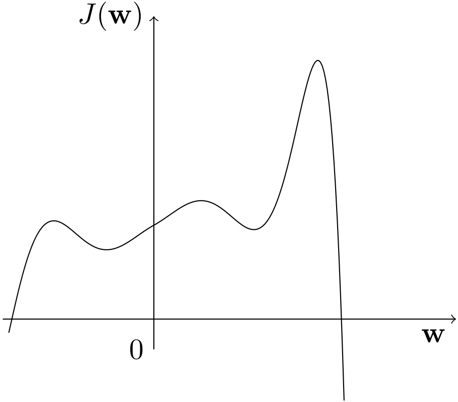
We can think of $\mathbf{Y}, \hat{\mathbf{Y}}$ as being *constant* and write:

$$J(w) = \frac{1}{n} \sum_{i=1}^{n} L(\mathbf{Y}_i, \hat{\mathbf{Y}}_i; w)$$

Methods: Weight Update



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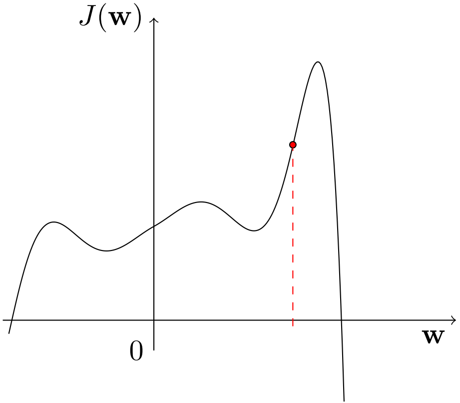

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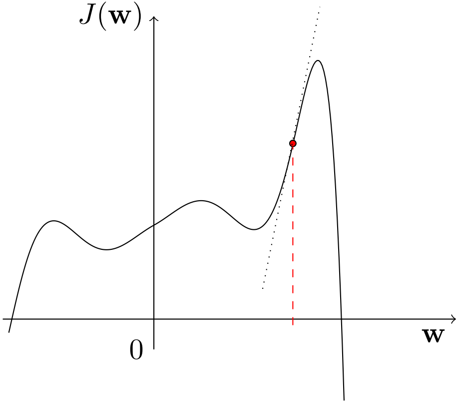

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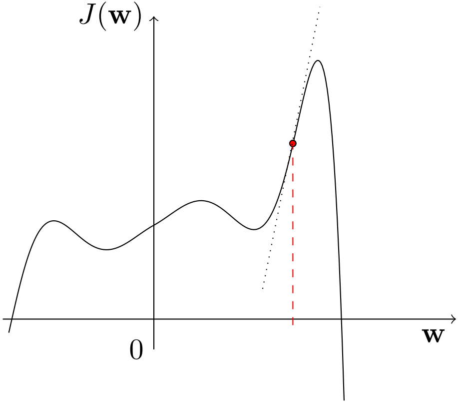

$$\mathbf{w} \leftarrow \mathbf{w} - \text{rate of change at } \mathbf{w}$$

Digression: Calculus Recap

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$$f'(x) = \frac{\mathrm{d}f}{\mathrm{d}x}$$

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- It tells us how f is changing in dependence of x

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- In this view, all variables but x are constant
- Thus, we can extract the change of f w.r.t. to f

Digression: Calculus Recap

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- Let $f : \mathbb{R}^n \to \mathbb{R}$ be a scalar-valued function *taking* a vector $\mathbf{x}$ as input.

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$$\nabla f = \left(\frac{\partial f}{\partial v_1}, \ldots, \frac{\partial f}{\partial v_n}\right)^\top$$

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- ∇f the vector containing the partial derivatives of w:
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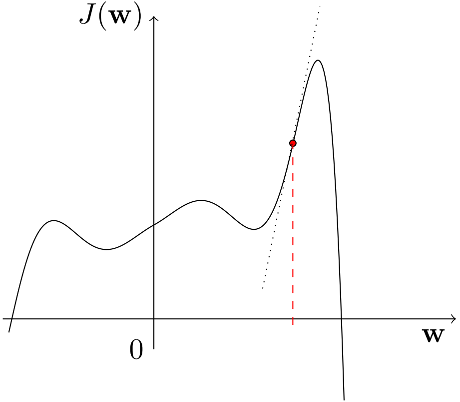
- ∇f is called the *gradient* of f

Methods: Weight Update



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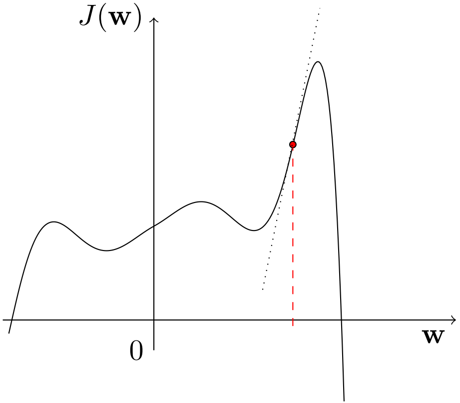

$$\nabla J(w) = \frac{1}{n} \sum_{i=1}^{n} \nabla_w L(\mathbf{Y}, \hat{\mathbf{Y}}_i; w)$$

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- α is called the *learning rate*
- Often, we will apply a decay to it

Methods: Weight Update

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$$\nu = \gamma\nu + \alpha\nabla J(w)$$

$$w \leftarrow w - \nu$$

Momentum Optimizer

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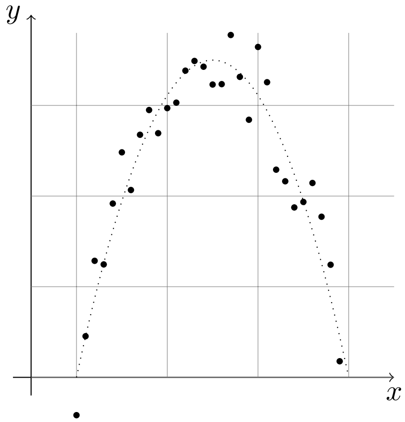
- We try to influence a model's *capacity*

Concepts: Model Capacity



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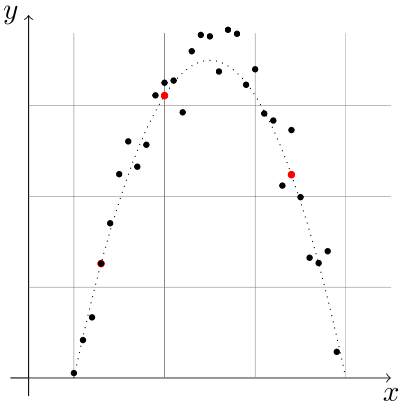

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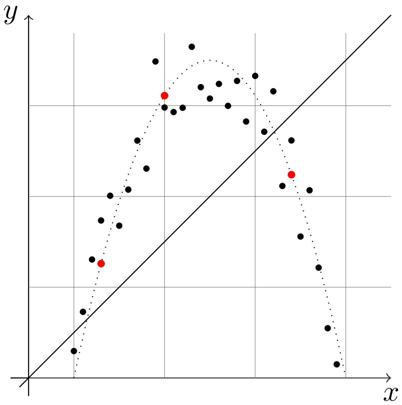

Underfitting (low capacity)

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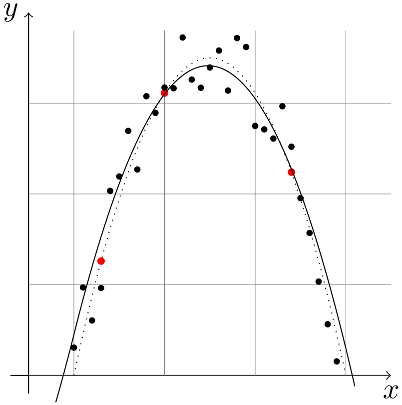
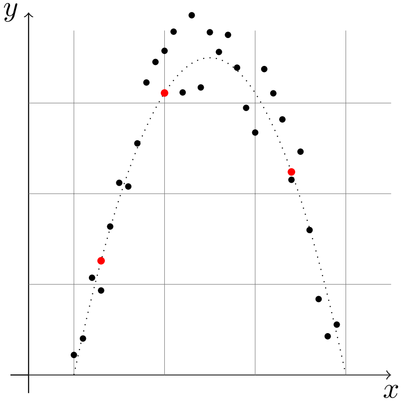


Just Right (optimal capacity)

Concepts: Model Capacity



Overfitting (too high capacity)

Methods: Regularization

$$
\begin{aligned}
f(x) &= 0x^8 + 0x^7 + 0x^6 \\
&+ 0x^5 + 0x^4 + 0x^3 \\
&- 1.56x^2 + 4.67x + 0
\end{aligned}
$$

$$
\begin{aligned}
g(x) &= -0.69x^8 + 10.90x^7 - 69.52x^6 \\
&+ 229.12x^5 - 413.77x^4 + 399.50x^3 \\
&- 185.95x^2 + 33.51x + 7.17 \cdot 10^{-8}
\end{aligned}
$$

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- We thus reduce the capacity by *favoring* certain functions over others

References