Distributed Gradient Swarming

combined effects onto attractant/repellent profiles

Master Degree in Data Science

Giuseppe Di Poce (2072371)

Academic Year 2023/2024

SAPIENZA
UNIVERSITÀ DI ROMA

- This project has been developed based on Stability Analysis of Social Foraging Swarms from Veysel Gazi.

- Its derivation is intended to understand, in a *gradient distributed manner*, multi-agent systems foraging behaviour given a balance between inter-individual interactions.

Table of Contents

Agents motion

We consider the equation of motion of each i^{th} individual (from which we can retrieve the *center motion* $\dot{\bar{x}}$) described by:

$$\dot{x}^i = -\nabla_{x^i}\sigma\left(x^i\right) + \sum_{j=1, j\neq i}^{M} g\left(x^i - x^j\right), \{i = 1, \ldots, M\}$$

- $g()$ represents the function of mutual attraction and repulsion between the individuals and it's an odd function. In our implementation it is given by :

$$g(y) = -y\left[a - b\exp\left(-\frac{\|y\|^2}{c}\right)\right]$$

- $\sigma()$ represent the profile onto which the swarm is moving for foraging:
  we treat various profiles with *linearity* (plane or convex one) and non-linearity of the gradient $\nabla_x\sigma()$ such as Gaussian or multimodal mixture of Gaussian.

Agents motion

1 Swarm Model Development

- Iteratively individual motion can be written as :

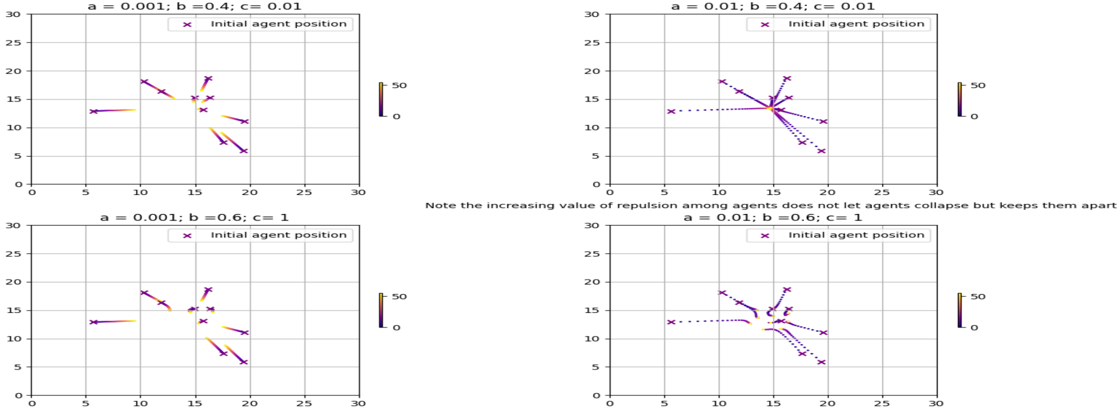
$$x^{[k+1]} = x^{[k]} - \nabla_{x_i}\sigma\left(x^i\right) - \epsilon \sum_{j=1, j\neq i}^{M} g\left(x_j^{[k]} - x_i^{[k]}\right)$$

  due to the fact that attraction-repulsion function it's an odd function $(g(y) = -g(-y))$;

- According with above definition, swarm center motion can be computed as:

$$\dot{\bar{x}} = -\frac{1}{M}\sum_{i=1}^{M} \nabla_{x^i}\sigma\left(x^i\right)$$

Attraction-Repulsion implementation



- Interactions without profile effect: size of the swarm strictly depends by $\pi = \{a, b\}$.

Cohesion Assumption

- Assumption 1: There exists a constant $\bar{\bar{\sigma}} > 0$ such that

$$\left\| \nabla_y \sigma(y) \right\| \leq \bar{\sigma}$$

- Assumption 2

$$\dot{V}_i = -aM \left\| e^i \right\|^2 + \sum_{j=1, j \neq i}^{M} g_r \left(\left\| x^i - x^j \right\| \right) \left(x^i - x^j \right)^\top e^i - \left[\nabla_{x^i} \sigma \left(x^i \right) - \frac{1}{M} \sum_{j=1}^{M} \nabla_{x^j} \sigma \left(x^j \right) \right]^\top e^i$$

There exists a constant $A_\sigma > -aM$ such that

$$\left[\nabla_{x^i} \sigma \left(x^i \right) - \frac{1}{M} \sum_{j=1}^{M} \nabla_{x^j} \sigma \left(x^j \right) \right]^\top e^i \geq A_\sigma \left\| e^i \right\|^2$$

for all x^i and x^j.

Basically the effect of the profile does not prevent swarm cohesion;

Table of Contents

The plane profile is described by equation of the form:

$$\sigma(y) = a_\sigma^\top y + b_\sigma,$$

and gradient of the profile is linear and constant and is given by:

$$\nabla_y \sigma(y) = a_\sigma$$

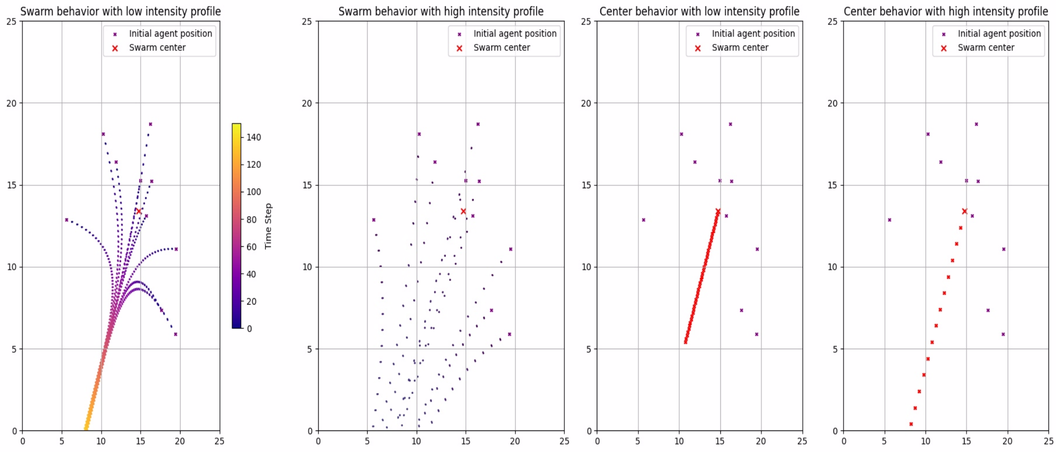
In our simulation we use $\mathrm{n} = 2$ dimensional space, $M = 10$ individuals and used a region $[0, 30] \times [0, 30]$.

Parameter	Low Intensity Profile	High Intensity Profile
a_σ	$[0.1, 0.2]$	$[0.5, 1]$
b_σ	1	1
$\pi = \{a, b, c\}$	$0.09, 0.4, 0.01$	$0.09, 0.4, 0.01$
ϵ	0.1	0.1

Table: Parameter values for each intensity profile scenario.

Motion along Plane Profile

2 Profile Interactions Analysis



Quadratic Profile

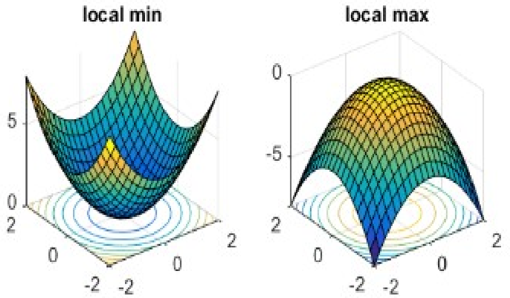
The profile and its gradient are described by:

$$\sigma(y) = \frac{A_\sigma}{2}\,\|y - c_\sigma\|^2 + b_\sigma$$

$$\nabla_y \sigma(y) = A_\sigma\,(y - c_\sigma)$$

For the center of the swarm, holds:

$$\bar{x}^{[k+1]} = \bar{x}^{[k]} - \frac{1}{M}\sum_{i=1}^{M} \nabla_{x_i}\sigma\left(x_i\right)$$



- $A_\sigma > 0$, the quadratic function is convex.
- $A_\sigma < 0$, the quadratic function is concave.

```
A_sigma, c_sigma = 0.02, np.array([20 ,20])
inter_values = [0.091,0.4,0.01] #attraction_value, repulsion_value, c_value
learning_rate = .1
```

Quadratic profile

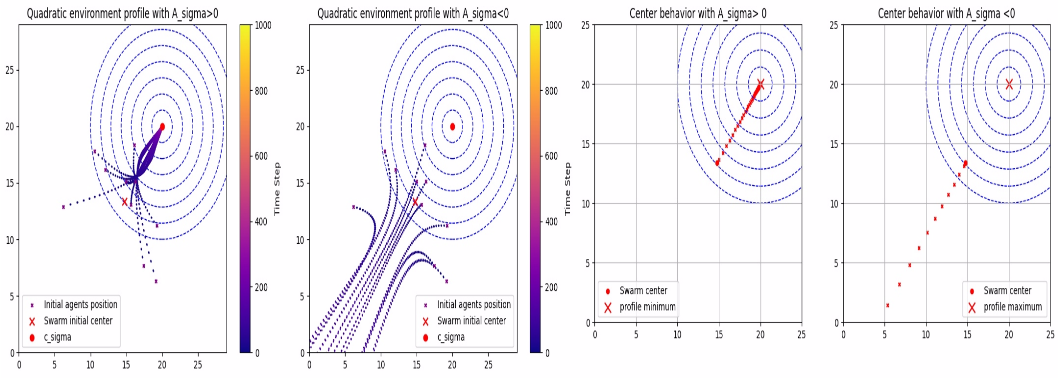
Defining the distance between the center $\bar{x}$ and the extremum point c_σ as $e_\sigma = \bar{x} - c_\sigma$, we have

$$\dot{e}_\sigma = -A_\sigma e_\sigma,$$

- as $t \to \infty$ we have $e_\sigma(t) \to 0 \quad$ if $A_\sigma > 0$
- as $t \to \infty$ happen that $e_\sigma(t) \to \infty$ if $A_\sigma < 0$ and $e_\sigma(0) \neq 0$.

Therefore the center of the swarm converges to the global minimum c_σ of the profile only in the case $A_\sigma > 0$, otherwise it will diverge by the maximum of the quadratic profile;

Quadratic Profile



Gaussian Profile

In this section, we consider profiles that are described by a Gaussian-type of equation s.t. :

$$\sigma(y) = -\frac{A_\sigma}{2} \exp\left(-\frac{\|y - c_\sigma\|^2}{l_\sigma}\right) + b_\sigma,$$

where $A_\sigma \in \mathbb{R}, b_\sigma \in \mathbb{R}, l_\sigma \in \mathbb{R}^+$, and $c_\sigma \in \mathbb{R}^n$. Note that this profile also has the unique extremum minimum or a global maximum, depending on the sign of A_σ, at $y = c_\sigma$. Its gradient is given by:

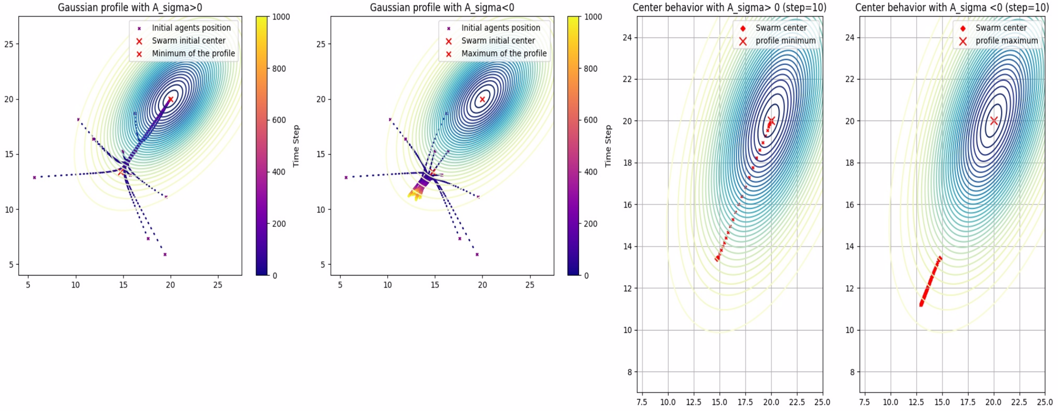
$$\nabla_y \sigma(y) = \frac{A_\sigma}{l_\sigma}(y - c_\sigma) \exp\left(-\frac{\|y - c_\sigma\|^2}{l_\sigma}\right).$$

Calculating the time derivative of the center of the swarm one can obtain

$$\dot{\bar{x}} = -\frac{A_\sigma}{M l_\sigma} \sum_{i=1}^{M} (x^i - c_\sigma) \exp\left(-\frac{\|x^i - c_\sigma\|^2}{l_\sigma}\right)$$

Gaussian Profile

- Compared to the quadratic case, here we cannot write $\dot{\bar{x}}$ as a function of $e_\sigma = \bar{x} - c_\sigma$ because of the *non-linearity* of the gradient of the profile.



Multi-modal Gaussian profile

Now we consider a profile which is a combination of N Gaussian profiles ($N = 8$ in our simulations) given by:

$$\sigma(y) = -\sum_{i=1}^{N} \frac{A_\sigma^i}{2} \exp\left(-\frac{\left\|y - c_\sigma^i\right\|^2}{l_\sigma^i}\right) + b_\sigma,$$

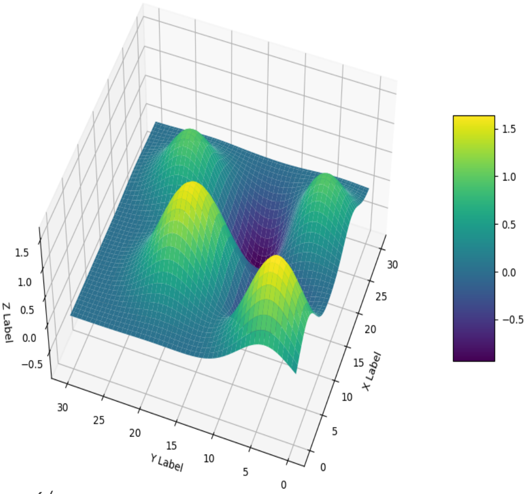
where $c_\sigma^i \in \mathbb{R}^n, l_\sigma^i \in \mathbb{R}^+, A_\sigma^i \in \mathbb{R}$ for all $i = 1, \ldots, N$, and $b_\sigma \in \mathbb{R}$. Note that since the A_σ^i 's can be positive or negative there can be both hills and valleys leading to a "more irregular" profile. The multiplicity of "depressions" in the surface is equal to 2 in simulations.

The gradient of the profile at a point y is given by

$$\nabla_y \sigma(y) = \sum_{i=1}^{N} \frac{A_\sigma^i}{l_\sigma^i} \left(y - c_\sigma^i\right) \exp\left(-\frac{\left\|y - c_\sigma^i\right\|^2}{l_\sigma^i}\right).$$

Multi-modal Gaussian profile

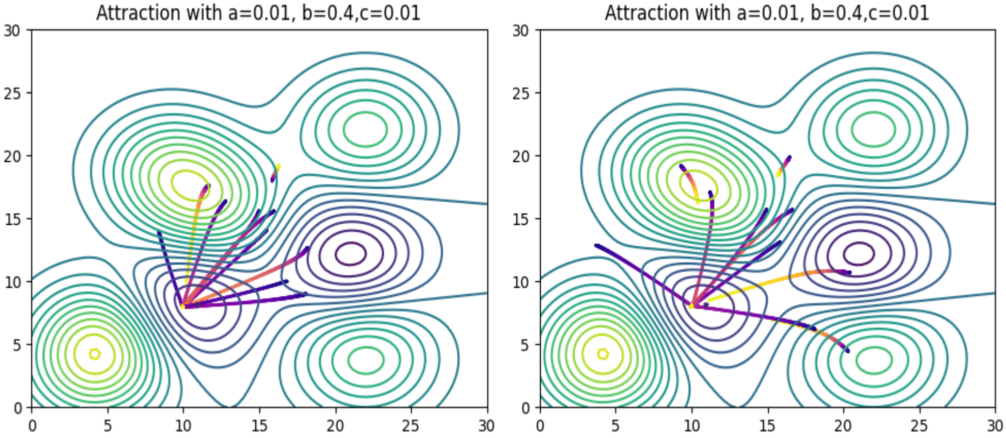
2 Profile Interactions Analysis




```
[(2, array([ 9, 19]), 20),
 (2, array([22,  4]), 20),
 (2, array([4, 3]), 20),
 (2, array([5, 6]), 20),
 (2, array([22, 22]), 20),
 (2, array([12, 16]), 20),
 (-2, array([21, 12]), 20),
 (-2, array([10,  8]), 20)]
```

- $A_\sigma, c_\sigma, l_\sigma$ values

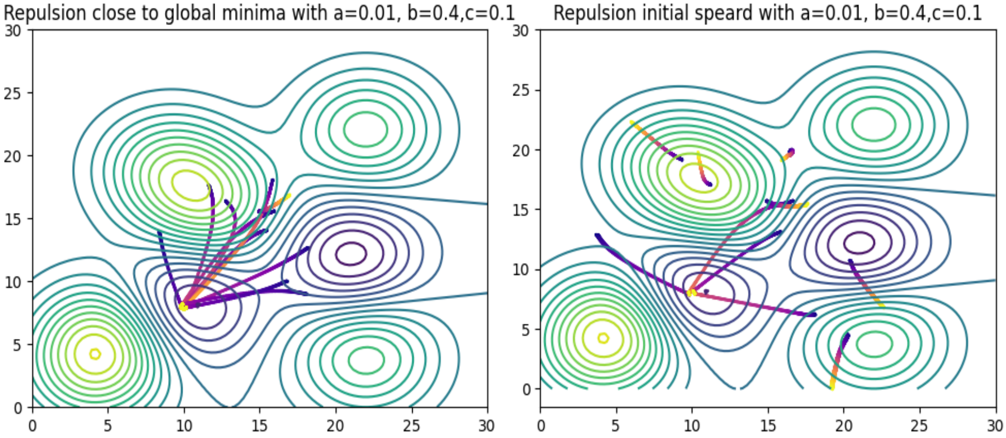
Multi-modal Gaussian profile



- Attraction scenario: Notice that agents when able to aggregate perform foraging better than alone;

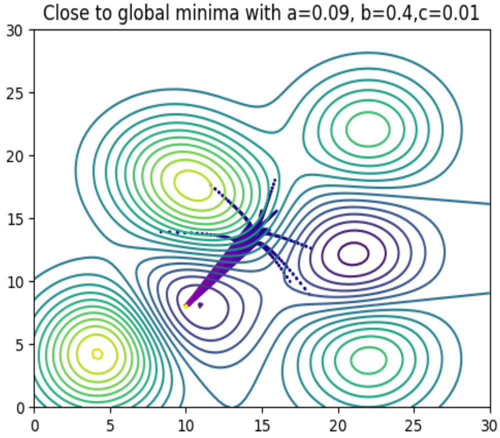
Multi-modal Gaussian profile

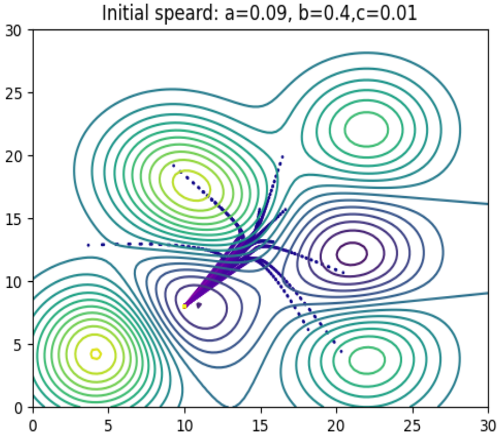
2 Profile Interactions Analysis



- Repulsion scenario;

Multi-modal Gaussian profile





- Increasing attraction parameter swarm tend to aggregate with more strength, also in a multi modal profile;

Table of Contents

Conclusions

We treat and show swarm convergence in the presence of an attractant/repellent for different profiles. The swarm is able to achieve collective convergence to more favorable regions of the profile and diverge from unfavorable regions (i.e. maximum points in the simulated envoiromments).
Please notice that we made two fundamental assumption:

- *SYNCRONOUS* motion of the agents;
- swarm members (or agents) have *unlimited* sensing range;

Further improvements

- From DG Swarming to Federated Swarming;

$$\dot{x}^i = -\nabla_{x^i}\sigma\left(x^i\right) + \sum_{j=1, j\neq i}^{M} g\left(x^i - x^j\right), \{i = 1, \ldots, M\}$$

- Relax assumption of asynchronous update of agents position;
- Run on top of swarm architecture learning function $\phi() \in C$ to drive into ERM in a Federated learning fashion.
- Explore deeper cohesion analysis of the swarm and stability theory of dynamic systems.