Esercitazione in preparazione esame scritto di matematica

Disegnare la funzione

$$y = (4 - x)(e^x - 1)$$

(a)

Disegnare la funzione

$$y = (4-x)(e^x - 1)$$

1) $\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to \infty} \underset{\substack{\downarrow \\ -\infty}}{(4-x)} \underset{\substack{\downarrow \\ +\infty}}{(e^x - 1)} = +\infty \times -\infty = -\infty$

2) Asintoto obliquo per $x \to -\infty$

$$m = \lim_{x \to -\infty} \frac{(4-x)(e^x-1)}{x} = \lim_{x \to -\infty} \underset{\substack{\downarrow \\ -1}}{\frac{(4-x)}{x}} \underset{\substack{\downarrow \\ -1}}{(e^x - 1)}$$

$m = 1$

$$q = \lim_{x \to -\infty} f(x) - \cancel{m}x = \lim_{x \to -\infty} \underset{\substack{\downarrow \\ 0}}{4e^x} - 4 - xe^x + \cancel{x} - \cancel{x}$$

$$= \lim_{x \to -\infty} -4 - xe^x$$

6

me

$$\lim_{x \to -\infty} x e^{x} = 0 \cdot -\infty$$

applico l'Hopital

$$\lim_{x \to -\infty} \frac{x}{\frac{1}{e^{x}}} = \lim_{x \to -\infty} \frac{f(x)}{g(x)} = \lim_{x \to -\infty} \frac{f'(x)}{g'(x)}$$

$$\lim_{x \to -\infty} \frac{1}{-\frac{e^{x}}{e^{2x}}} = \frac{1}{-\frac{1}{e^{x}}} = -e^{x} = \emptyset$$

$$q = \lim_{x \to -\infty} -4 - \underbrace{x e^{x}}_{0} = -4$$

asintoto obliquo

$$\boxed{y = mx + q = x - 4}$$

(c)

<u>Intersezione con gli assi</u>

$$\begin{cases} y = 0 \\ (4-x)(e^{x}-1) = 0 \end{cases} \Longrightarrow \begin{array}{l} e^{x} = 1 \Rightarrow \boxed{x=0} \\ 4-x=0 \Rightarrow \boxed{x=4} \end{array}$$

$A \equiv (0,0) \qquad B \equiv (4,0)$

06

$$y = (4-x)(e^x - 1)$$

$$y' = -1\,(e^x - 1) + (4-x)\,e^x$$

$$= -e^x + 1 + 4e^x - xe^x$$

$$= 3e^x - xe^x + 1$$

$$= e^x(3-x) + 1$$

$$y' = e^x(3-x) + 1$$

$y' > 0$

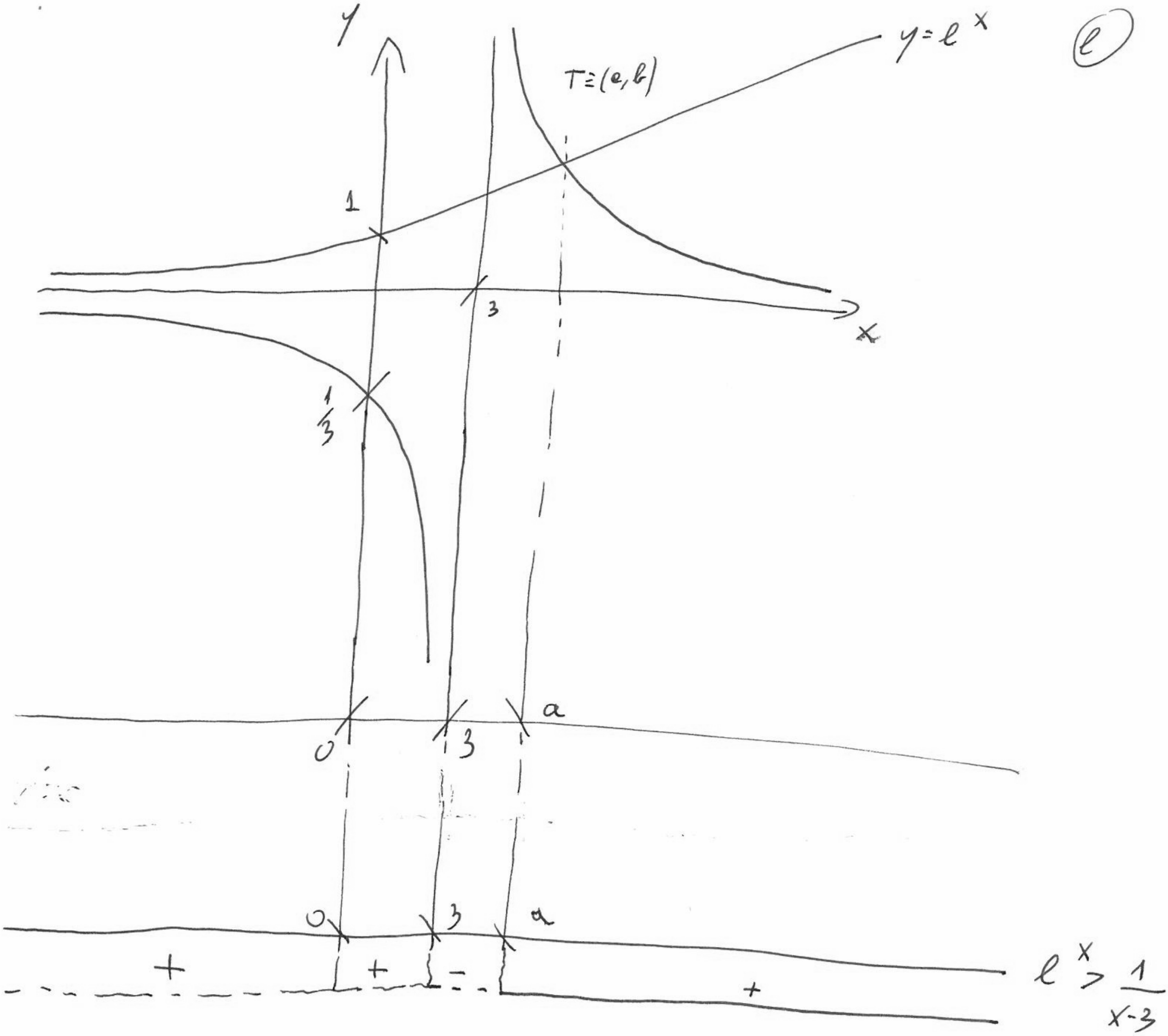
1 caso) $\boxed{\text{PER}}$ $3 - x > 0 \Rightarrow \boxed{x < 3}$ PER

$$e^x > \frac{1}{x-3}$$

2 caso) $\boxed{\text{PER}}$ $3 - x < 0 \Rightarrow \boxed{\begin{array}{c}\text{PER}\\ x > 3\end{array}}$

$$e^x < \frac{1}{x-3}$$

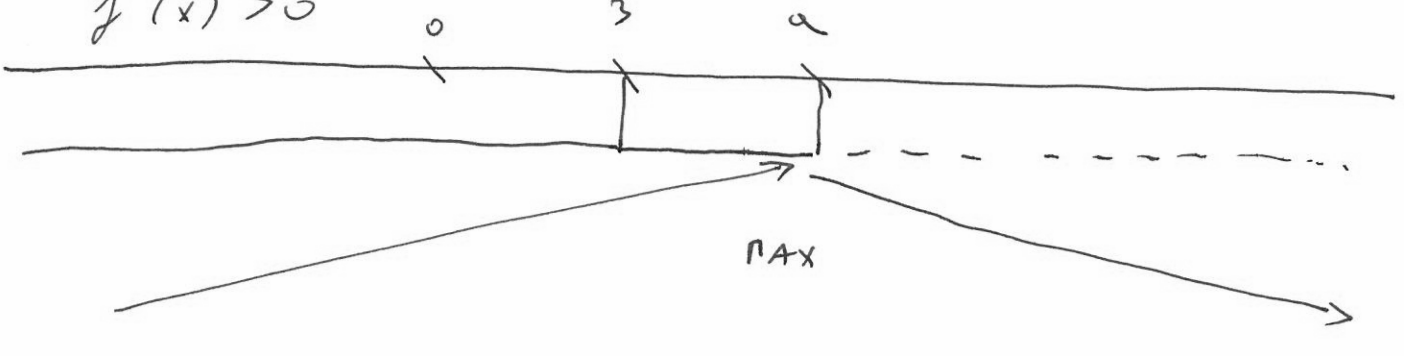
e



per $x > 3$ devo prendere il valore $-$

per $x < 3$ devo prendere il valore $+$

$f'(x) > 0$



$\cancel{y''(x)=}$

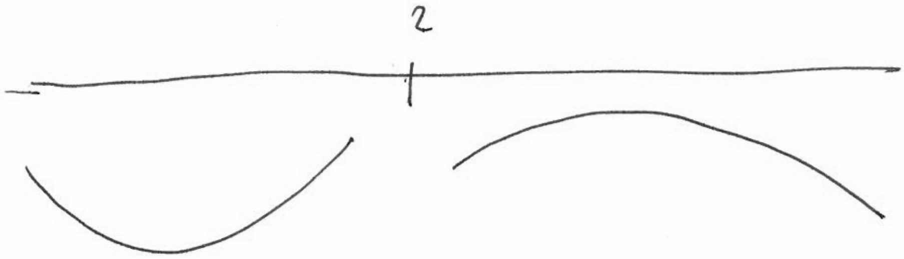
$$y'(x) = e^x(3-x)+1$$

$$y''(x) = e^x(-1) + e^x(3-x)$$

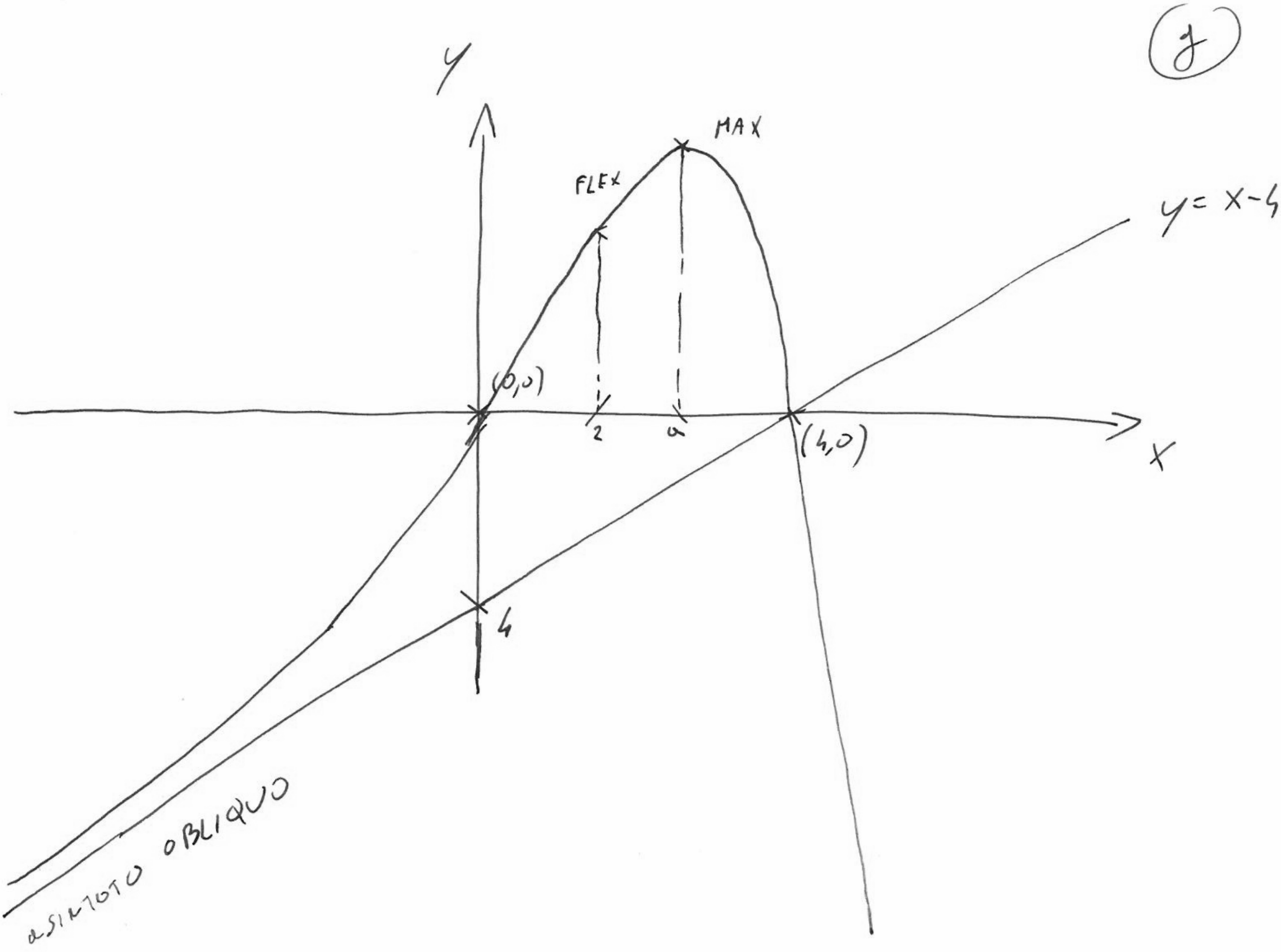
$$y''(x) = -e^x + 3e^x - xe^x = 2e^x - xe^x$$

$$y''(x) > 0$$

$$2-x>0 \Rightarrow x<2$$



$f''(x)>0$



g
y
MAX
FLEX
y = x-4
(0,0)
2
a
(4,0)
x
4
asintoto OBLIQUO

Disegnare la funzione

$$f(x) = |x| \left(\log |x| \right)^2 + \frac{e x^2}{2} + 1$$

Disegno la funzione

$$f(x) = |x| \left(\log |x| \right)^2 + \frac{e\, x^2}{2} + 1$$

$$f(x) = \begin{cases} x \left(\log x \right)^2 + \frac{e\, x^2}{2} + 1 & \text{per } x > 0 \\ -x \left(\log (-x) \right)^2 + \frac{e\, x^2}{2} + 1 & \text{per } x < 0 \end{cases}$$

$$f(x) = \begin{cases} x \left(\log x \right)^2 + \frac{e\, x^2}{2} + 1 & \text{per } x > 0 \\ -x \left(\log (-x) \right)^2 + \frac{e\, (-x)^2}{2} + 1 & \text{per } x < 0 \end{cases}$$

Simmetrie di $f(x)$

La funzione è pari perché

$$\begin{gathered} f(x) \text{ per } x > 0 \\ \parallel \\ f(-x) \text{ per } x < 0 \end{gathered}$$

Campo di esistenza

f è definita in $\mathbb{R} - \{0\}$

$\boxed{\text{Studio } x(\log x)^2 + \frac{e x^2}{2} + 1}$ per $x > 0$

poi sfrutto la simmetria.

$$\boxed{\lim_{x \to +\infty} x(\log x)^2 + \frac{e x^2}{2} + 1 = +\infty}$$

calcolo $\lim_{x \to 0^+} x(\log x)^2 = 0 \infty$

applico l'Hospital

$$\lim_{x \to 0^+} \frac{(\log x)^2}{\frac{1}{x}} = \lim_{x \to 0^+} \frac{f(x)}{g(x)} = \lim_{x \to 0^+} \frac{f'(x)}{g'(x)}$$

$$\lim_{x\to 0^+} \frac{(\log x)^2}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{2\log(x)\,\frac{1}{x}}{-\frac{1}{x^2}} =$$

$$= \lim_{x\to 0^+} \frac{2\log(x)}{x}\left(-x^2\right) = -2\log(x)\,x$$

riapplico Hospital

$$\lim_{x\to 0^+} -2\,\frac{\log(x)}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{-2\,\frac{1}{x}}{-\frac{1}{x^2}} =$$

$$= \lim_{x\to 0^+} \frac{2}{x}\,x^2 = 2x = 0$$

$$\boxed{\lim_{x\to 0^+} \underbrace{x(\log x)^2}_{\downarrow\, 0} + \underbrace{\frac{e x^2}{2}}_{\downarrow\, 0} + 1 = 1}$$

$\boxed{\text{Intersezione con l'asse } x}$

$$\frac{e x^2}{2} + x\left(\log(x)\right)^2 + 1 = 0$$

La funzione non interseca l'asse x essendo sempre positiva

$$y'(x) = \frac{\cancel{2} e x}{\cancel{2}} + \left(\log(x)\right)^2 + \cancel{x}\, 2 \log(x) \frac{1}{\cancel{x}}$$

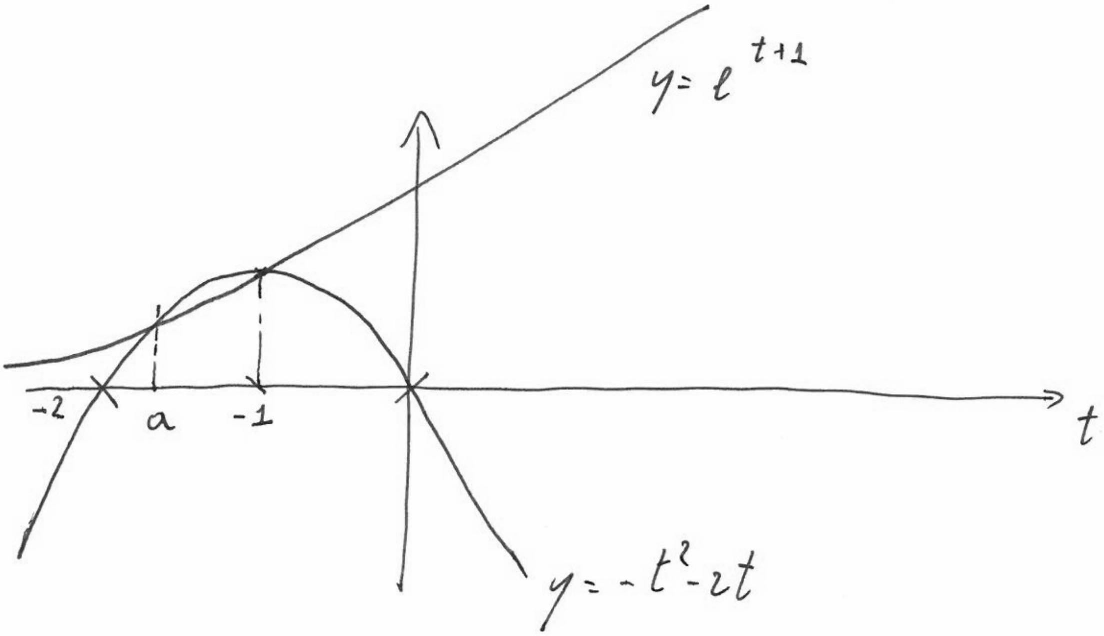
$$= e x + \left(\log(x)\right)^2 + 2\left(\log(x)\right)$$

pongo $\begin{cases} \log(x) = t \\ e^{\log(x)} = e^t \Rightarrow x = e^t \end{cases}$

$$\boxed{y'(t) = e^{t+1} + t^2 + 2t}$$

$y'(t) > 0$

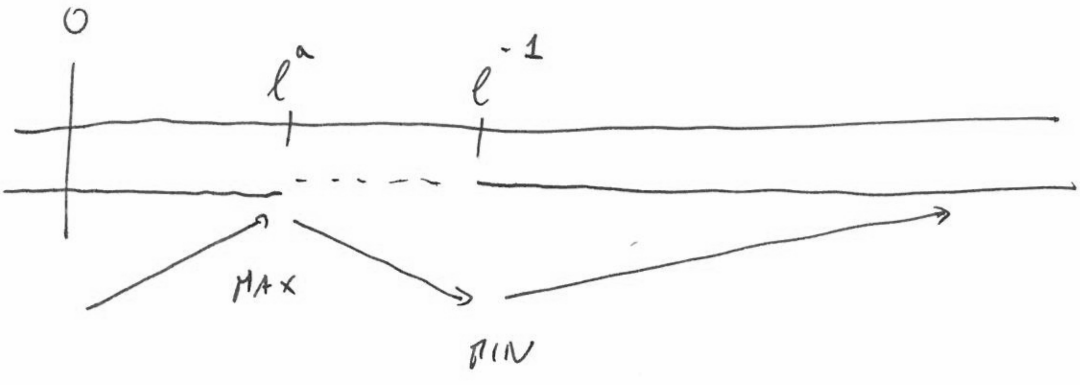
$$e^{t+1} + t^2 + 2t > 0 \Rightarrow \boxed{e^{t+1} > -t^2 - 2t}$$



$y'(t) > 0$

$$\begin{cases} t > -1 \\ t < a \end{cases} \Rightarrow \begin{cases} \log x > -1 \\ \log x < a \end{cases} =$$

$$\Rightarrow \begin{cases} x > e^{-1} \\ x < e^{a} \end{cases}$$



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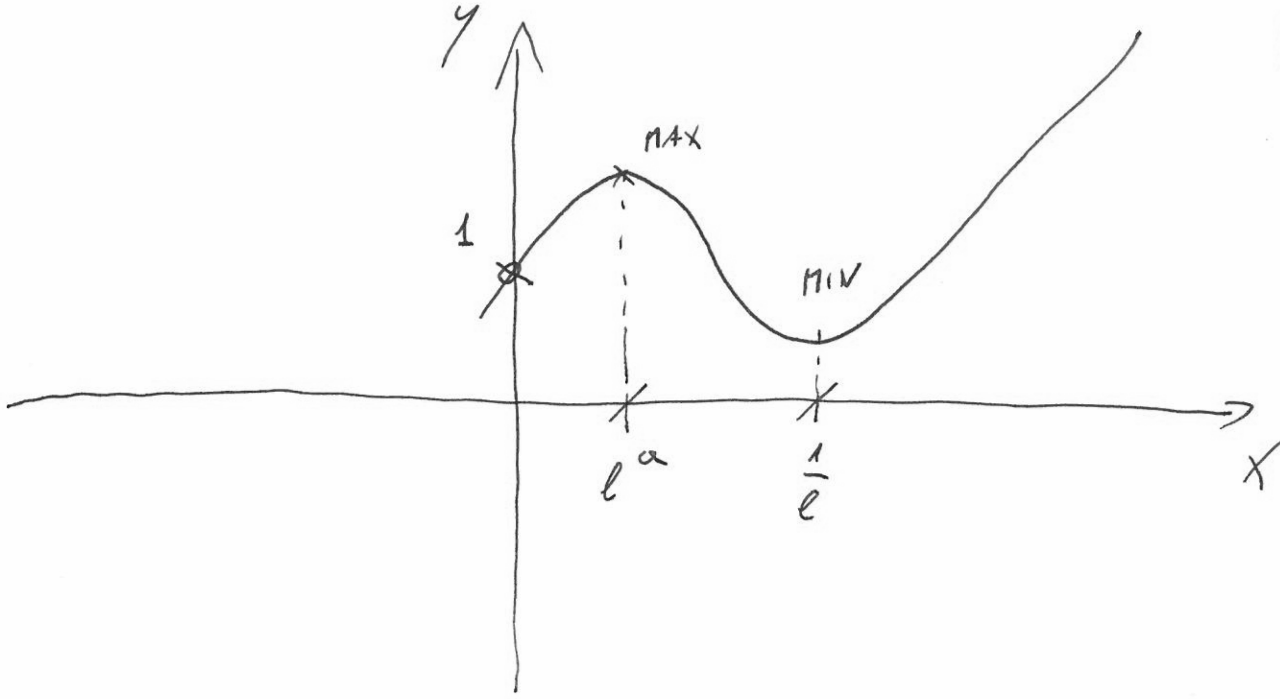
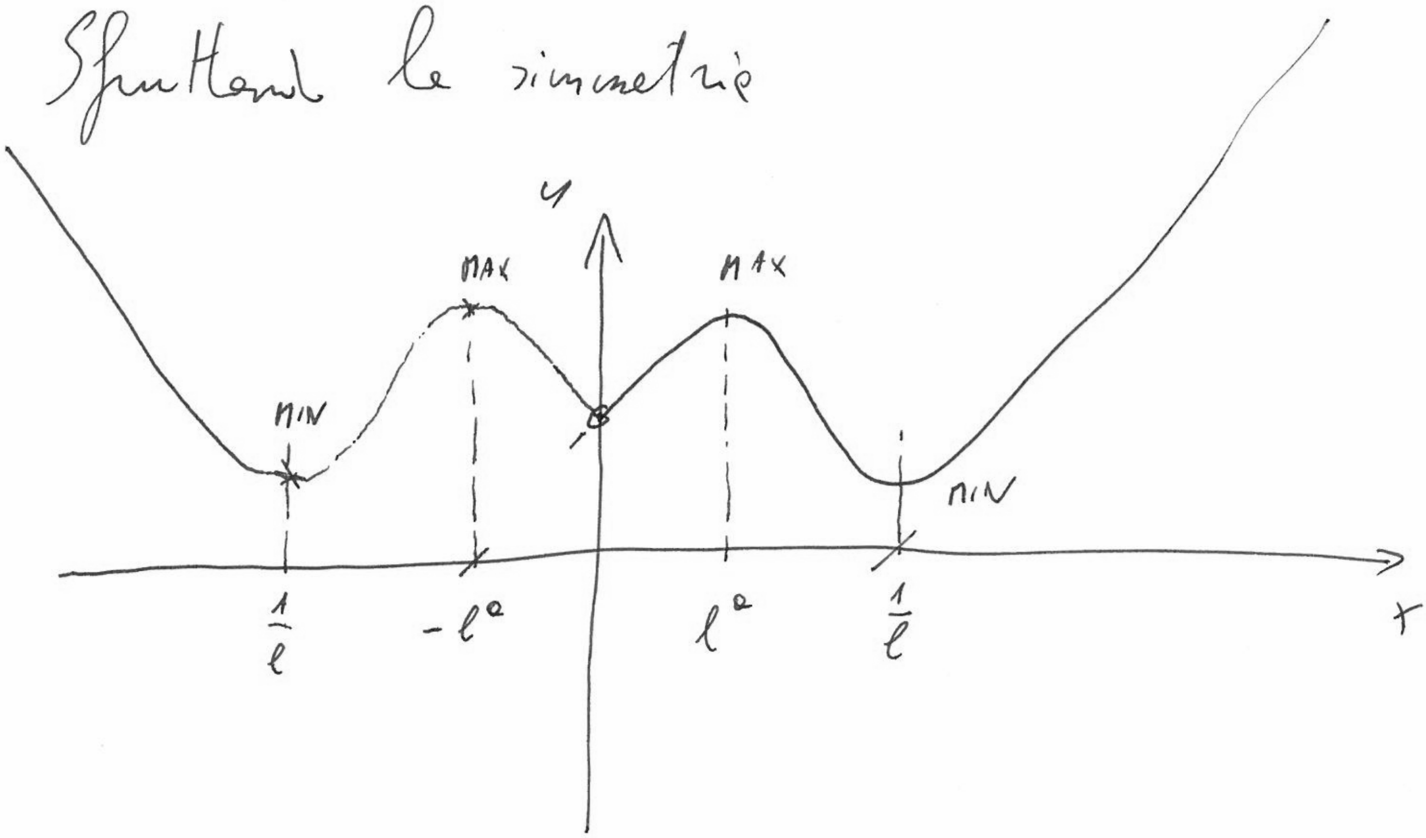


Sfruttando le simmetrie



Calcolare l'integrale

$$\int \sin\left(\log(x)\right) dx$$

Calcolare il seguente integrale

$$\int \operatorname{sen}(\log(x))\,dx = \int \frac{d}{dx}(x)\operatorname{sen}(\log(x))\,dx$$

integrando per parti

$$\int \operatorname{sen}(\log(x))\,dx = x\operatorname{sen}(\log(x)) +$$

$$-\int x\frac{d}{dx}\left[\operatorname{sen}(\log(x))\right]dx$$

$$\Downarrow$$

$$\int \operatorname{sen}(\log(x))\,dx = x\operatorname{sen}(\log(x)) +$$

$$-\int \cancel{x}\cos(\log(x))\frac{1}{\cancel{x}}\,dx$$

$$\int \operatorname{sen}(\log(x))\,dx = x\operatorname{sen}(\log(x)) +$$

$$- \int \cos(\log(x))\,dx$$

$$\int \operatorname{sen}(\log(x))\,dx = x\operatorname{sen}(\log(x)) +$$

$$- \int \frac{d}{dx}(x)\,\cos(\log(x))\,dx$$

integrando per parti

$$\int \operatorname{sen}(\log(x))\,dx = x\operatorname{sen}(\log(x)) +$$

$$- x\cos(\log(x)) + \int x\,\frac{d}{dx}\left(\cos(\log(x))\right)dx$$

$$\int \operatorname{sen}(\log(x))\, dx = x \operatorname{sen}(\log(x))$$

$$- x \cos(\log(x)) - \int \cancel{x} \operatorname{sen}(\log(x)) \frac{1}{\cancel{x}}\, dx$$

$$2 \int \operatorname{sen}(\log(x)) = x \operatorname{sen}(\log(x)) +$$

$$- x \cos(\log(x))$$

$$\Downarrow$$

$$\int \operatorname{sen}(\log(x)) = \frac{x}{2} \operatorname{sen}(\log(x)) - \frac{x}{2} \cos(\log(x))$$