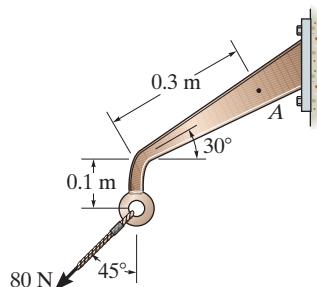


- 1-1.** A force of 80 N is supported by the bracket as shown. Determine the resultant internal loadings acting on the section through point A.



## SOLUTION

*Equations of Equilibrium:*

$$+\nearrow \sum F_x' = 0; \quad N_A - 80 \cos 15^\circ = 0$$

$$N_A = 77.3 \text{ N}$$

**Ans.**

$$\nwarrow \sum F_y' = 0; \quad V_A - 80 \sin 15^\circ = 0$$

$$V_A = 20.7 \text{ N}$$

**Ans.**

$$\zeta + \sum M_A = 0; \quad M_A + 80 \cos 45^\circ(0.3 \cos 30^\circ)$$

$$- 80 \sin 45^\circ(0.1 + 0.3 \sin 30^\circ) = 0$$

$$M_A = -0.555 \text{ N} \cdot \text{m}$$

**Ans.**

or

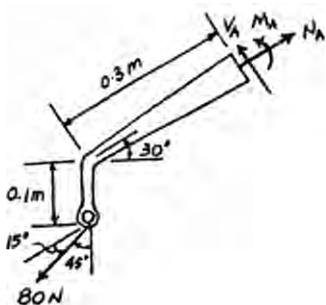
$$\zeta + \sum M_A = 0; \quad M_A + 80 \sin 15^\circ(0.3 + 0.1 \sin 30^\circ)$$

$$- 80 \cos 15^\circ(0.1 \cos 30^\circ) = 0$$

$$M_A = -0.555 \text{ N} \cdot \text{m}$$

**Ans.**

Negative sign indicates that  $M_A$  acts in the opposite direction to that shown on FBD.



These solutions represent a preliminary version of the Instructors' Solutions Manual (ISM). It is possible and even likely that at this preliminary stage of preparing the ISM there are some omissions and errors in the draft solutions.

**Ans:**

$$N_A = 77.3 \text{ N}, V_A = 20.7 \text{ N}, M_A = -0.555 \text{ N} \cdot \text{m}$$

**1-2.**

Determine the resultant internal loadings on the cross section at point D.

### SOLUTION

**Support Reactions:** Member BC is the two force member.

$$\zeta + \sum M_A = 0; \quad \frac{4}{5}F_{BC}(1.5) - 1.875(0.75) = 0$$

$$F_{BC} = 1.1719 \text{ kN}$$

$$+\uparrow \sum F_y = 0; \quad A_y + \frac{4}{5}(1.1719) - 1.875 = 0$$

$$A_y = 0.9375 \text{ kN}$$

$$\pm \sum F_x = 0; \quad \frac{3}{5}(1.1719) - A_x = 0$$

$$A_x = 0.7031 \text{ kN}$$

**Equations of Equilibrium:** For point D

$$\pm \sum F_x = 0; \quad N_D - 0.7031 = 0$$

$$N_D = 0.703 \text{ kN}$$

**Ans.**

$$+\uparrow \sum F_y = 0; \quad 0.9375 - 0.625 - V_D = 0$$

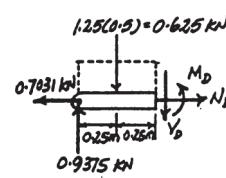
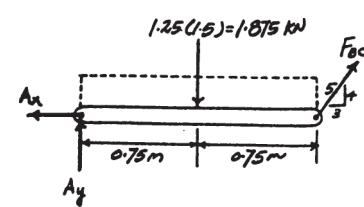
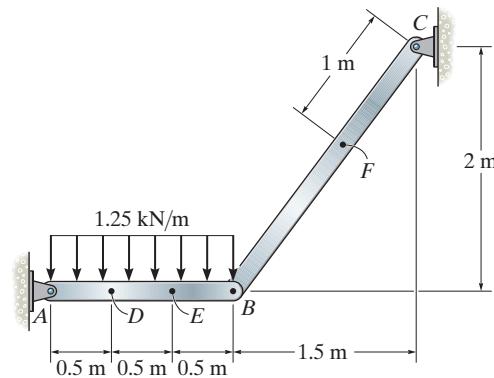
$$V_D = 0.3125 \text{ kN}$$

**Ans.**

$$\zeta + \sum M_D = 0; \quad M_D + 0.625(0.25) - 0.9375(0.5) = 0$$

$$M_D = 0.3125 \text{ kN} \cdot \text{m}$$

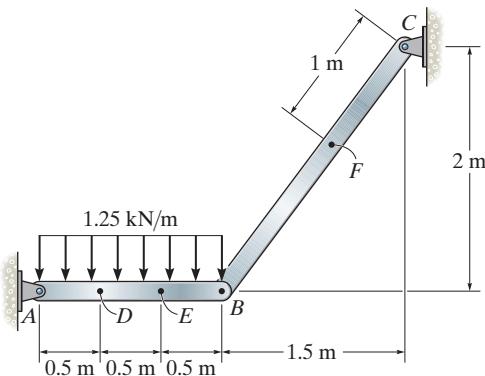
**Ans.**



**Ans:**  
 $N_D = 0.703 \text{ kN}$ ,  
 $V_D = 0.3125 \text{ kN}$ ,  
 $M_D = 0.3125 \text{ kN} \cdot \text{m}$

**1–3.**

Determine the resultant internal loadings at cross sections at points *E* and *F* on the assembly.



## SOLUTION

**Support Reactions:** Member *BC* is the two-force member.

$$\zeta + \sum M_A = 0; \quad \frac{4}{5}F_{BC}(1.5) - 1.875(0.75) = 0$$

$$F_{BC} = 1.1719 \text{ kN}$$

$$+\uparrow \sum F_y = 0; \quad A_y + \frac{4}{5}(1.1719) - 1.875 = 0$$

$$A_y = 0.9375 \text{ kN}$$

$$\pm \sum F_x = 0; \quad \frac{3}{5}(1.1719) - A_x = 0$$

$$A_x = 0.7031 \text{ kN}$$

**Equations of Equilibrium:** For point *F*

$$+\angle \sum F_{x'} = 0; \quad N_F - 1.1719 = 0$$

$$N_F = 1.17 \text{ kN}$$

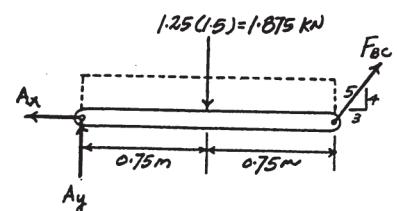
**Ans.**

$$\nwarrow \sum F_{y'} = 0; \quad V_F = 0$$

**Ans.**

$$\zeta + \sum M_F = 0; \quad M_F = 0$$

**Ans.**



**Equations of Equilibrium:** For point *E*

$$\leftarrow \sum F_x = 0; \quad N_E - \frac{3}{5}(1.1719) = 0$$

$$N_E = 0.703 \text{ kN}$$

**Ans.**

$$+\uparrow \sum F_y = 0; \quad V_E - 0.625 + \frac{4}{5}(1.1719) = 0$$

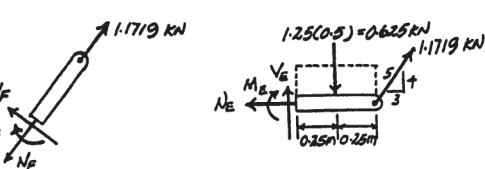
$$V_E = -0.3125 \text{ kN}$$

**Ans.**

$$\zeta + \sum M_E = 0; \quad -M_E - 0.625(0.25) + \frac{4}{5}(1.1719)(0.5) = 0$$

$$M_E = 0.3125 \text{ kN} \cdot \text{m}$$

**Ans.**



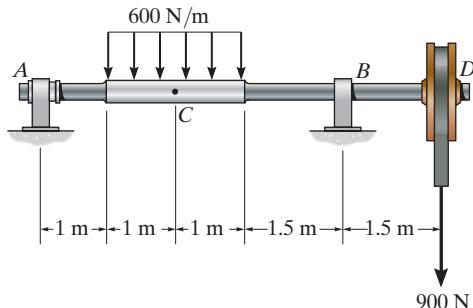
Negative sign indicates that  $V_E$  acts in the opposite direction to that shown on FBD.

**Ans:**

$$\begin{aligned} N_F &= 1.17 \text{ kN}, \\ V_F &= 0, \\ M_F &= 0, \\ N_E &= 0.703 \text{ kN}, \\ V_E &= -0.3125 \text{ kN}, \\ M_E &= 0.3125 \text{ kN} \cdot \text{m} \end{aligned}$$

\*1-4.

The shaft is supported by a smooth thrust bearing at *A* and a smooth journal bearing at *B*. Determine the resultant internal loadings acting on the cross section at *C*.



## SOLUTION

**Support Reactions:** We will only need to compute  $B_y$  by writing the moment equation of equilibrium about *A* with reference to the free-body diagram of the entire shaft, Fig. *a*.

$$\zeta + \sum M_A = 0; \quad B_y(4.5) - 600(2)(2) - 900(6) = 0 \quad B_y = 1733.33 \text{ N}$$

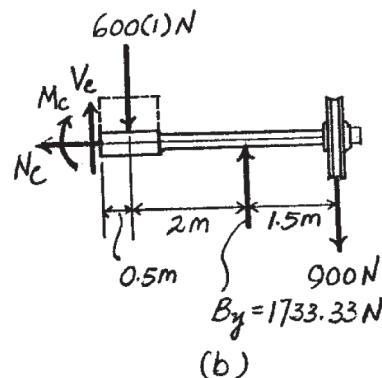
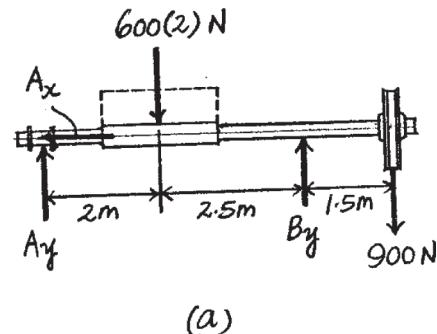
**Internal Loadings:** Using the result of  $B_y$ , section *CD* of the shaft will be considered. Referring to the free-body diagram of this part, Fig. *b*,

$$\pm \sum F_x = 0; \quad N_C = 0 \quad \text{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad V_C - 600(1) + 1733.33 - 900 = 0 \quad V_C = -233 \text{ N} \quad \text{Ans.}$$

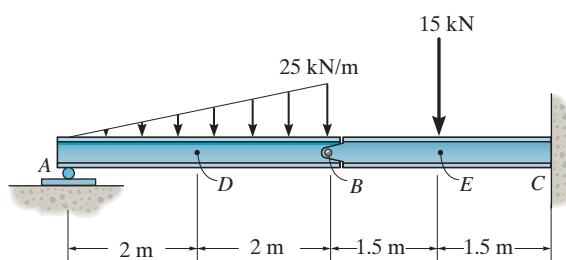
$$\zeta + \sum M_C = 0; \quad 1733.33(2.5) - 600(1)(0.5) - 900(4) - M_C = 0 \\ M_C = 433 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

The negative sign indicates that  $V_C$  acts in the opposite sense to that shown on the free-body diagram.



**Ans:**  
 $N_C = 0$ ,  
 $V_C = -233 \text{ N}$ ,  
 $M_C = 433 \text{ N} \cdot \text{m}$

- 1-5.** Determine the resultant internal loadings in the beam at cross sections through points *D* and *E*. Point *E* is just to the right of the 15-kN load.



## SOLUTION

**Support Reactions:** For member *AB*

$$\zeta + \sum M_B = 0; \quad 50(4/3) - A_y(4) = 0 \quad A_y = 16.67 \text{ kN}$$

$$\stackrel{+}{\rightarrow} \sum F_x = 0; \quad B_x = 0$$

$$+\uparrow \sum F_y = 0; \quad B_y + 16.67 - 50 = 0 \quad B_y = 33.33 \text{ kN}$$

**Equations of Equilibrium:** For point *D*

$$\stackrel{+}{\rightarrow} \sum F_x = 0; \quad N_D = 0 \quad \text{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad 16.67 - 12.5 - V_D = 0$$

$$V_D = 4.17 \text{ kN} \quad \text{Ans.}$$

$$\zeta + \sum M_D = 0; \quad M_D + 12.25(\frac{2}{3}) - 16.67(2) = 0$$

$$M_D = 25.17 \text{ kN} \cdot \text{m} \quad \text{Ans.}$$

**Equations of Equilibrium:** For point *E*

$$\stackrel{+}{\rightarrow} \sum F_x = 0; \quad N_E = 0 \quad \text{Ans.}$$

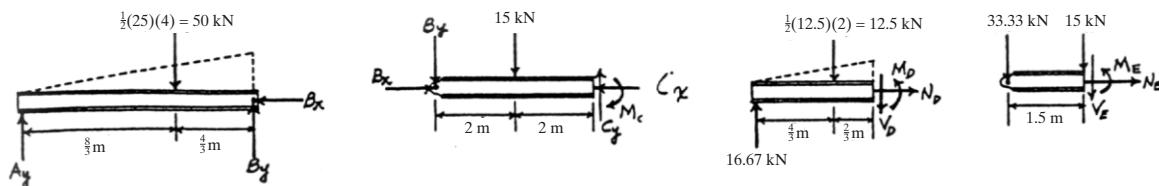
$$+\uparrow \sum F_y = 0; \quad -33.33 - 15 - V_E = 0$$

$$V_E = -48.33 \text{ kN} \quad \text{Ans.}$$

$$\zeta + \sum M_E = 0; \quad M_E + 33.33(1.5) = 0$$

$$M_E = -50.00 \text{ kN} \cdot \text{m} \quad \text{Ans.}$$

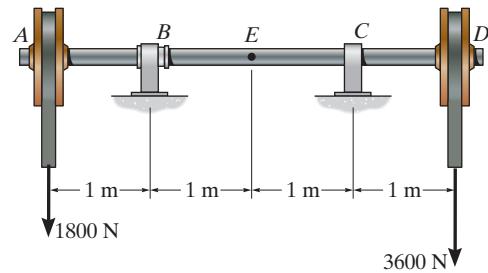
Negative signs indicate that  $M_E$  and  $V_E$  act in the opposite direction to that shown on FBD.



**Ans:**

$$\begin{aligned} N_D &= 0, V_D = 4.17 \text{ kN}, \\ M_D &= 25.0 \text{ kN} \cdot \text{m}, N_E = 0, V_E = -48.3 \text{ kN}, \\ M_E &= -50.0 \text{ kN} \cdot \text{m} \end{aligned}$$

- 1-6.** The shaft is supported by a smooth thrust bearing at *B* and a journal bearing at *C*. Determine the resultant internal loadings acting on the cross section at *E*.



## SOLUTION

**Support Reactions:** We will only need to compute  $C_y$  by writing the moment equation of equilibrium about *B* with reference to the free-body diagram of the entire shaft, Fig. *a*.

$$\zeta + \sum M_B = 0; \quad C_y(2) + 1800(1) - 3600(3) = 0 \quad C_y = 4500 \text{ N}$$

**Internal Loadings:** Using the result for  $C_y$ , section *DE* of the shaft will be considered. Referring to the free-body diagram, Fig. *b*,

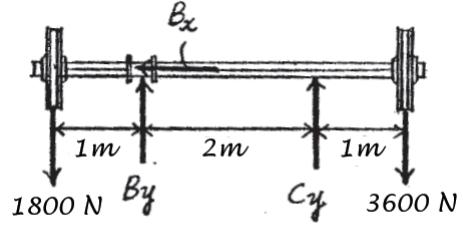
$$\stackrel{+}{\rightarrow} \sum F_x = 0; \quad N_E = 0 \quad \text{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad V_E + 4500 - 3600 = 0 \quad V_E = -900 \text{ N} \quad \text{Ans.}$$

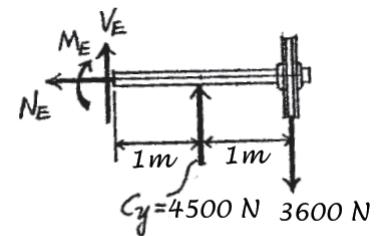
$$\zeta + \sum M_E = 0; 4500(1) - 3600(2) - M_E = 0$$

$$M_E = -2700 \text{ N}\cdot\text{m} = -2.70 \text{ kN}\cdot\text{m} \quad \text{Ans.}$$

The negative signs indicates that  $V_E$  and  $M_E$  act in the opposite sense to that shown on the free-body diagram.



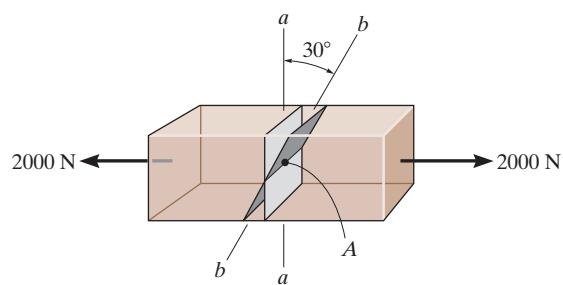
(a)



(b)

**Ans:**  
 $N_E = 0, V_E = -900 \text{ N}, M_E = -2.7 \text{ kN}\cdot\text{m}$

- 1–7.** Determine the resultant internal normal and shear force in the member at (a) section *a*–*a* and (b) section *b*–*b*, each of which passes through point *A*. The 2000-N load is applied along the centroidal axis of the member.



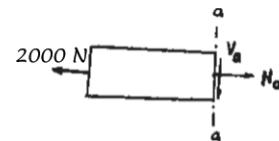
(a)

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \quad N_a - 2000 = 0$$

$$N_a = 2000 \text{ N}$$

$$+\downarrow \Sigma F_y = 0; \quad V_a = 0$$

**Ans.**



(b)

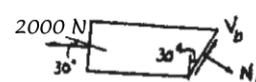
$$\stackrel{+}{\searrow} \Sigma F_x = 0; \quad N_b - 2000 \cos 30^\circ = 0$$

$$N_b = 1732 \text{ N}$$

$$\stackrel{+}{\nearrow} \Sigma F_y = 0; \quad V_b - 2000 \sin 30^\circ = 0$$

$$V_b = 1000 \text{ N}$$

**Ans.**



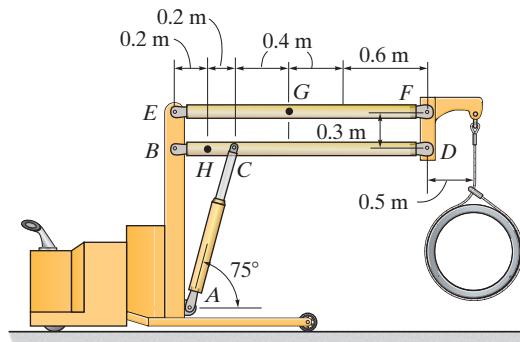
**Ans.**

**Ans:**

$$N_a = 2000 \text{ N}, V_a = 0, \\ N_b = 1732 \text{ N}, V_b = 1000 \text{ N}$$

**\*1-8.**

The floor crane is used to lift a 600-kg concrete pipe. Determine the resultant internal loadings acting on the cross section at G.



**SOLUTION**

**Support Reactions:** We will only need to compute  $F_{EF}$  by writing the moment equation of equilibrium about D with reference to the free-body diagram of the hook, Fig. a.

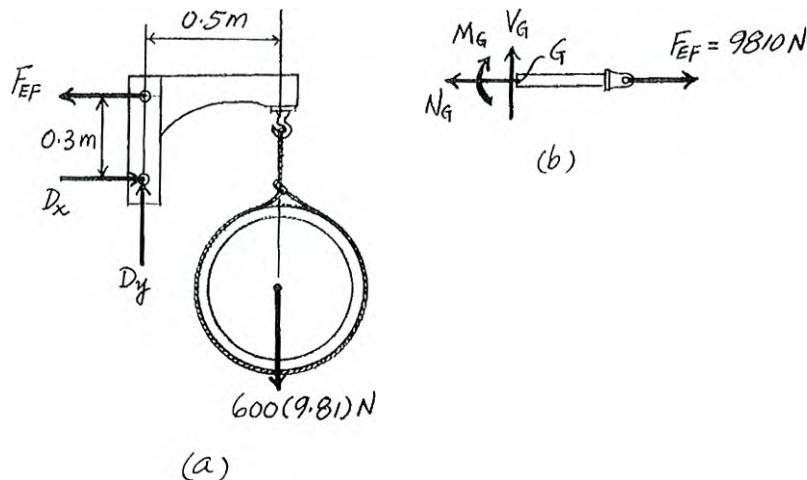
$$\zeta + \sum M_D = 0; \quad F_{EF}(0.3) - 600(9.81)(0.5) = 0 \quad F_{EF} = 9810 \text{ N}$$

**Internal Loadings:** Using the result for  $F_{EF}$ , section FG of member EF will be considered. Referring to the free-body diagram, Fig. b,

$$\stackrel{+}{\rightarrow} \sum F_x = 0; \quad 9810 - N_G = 0 \quad N_G = 9810 \text{ N} = 9.81 \text{ kN} \quad \text{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad V_G = 0 \quad \text{Ans.}$$

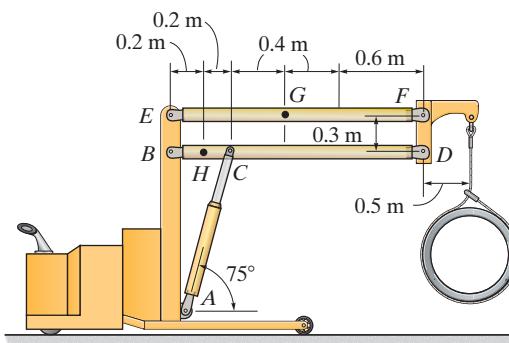
$$\zeta + \sum M_G = 0; \quad M_G = 0 \quad \text{Ans.}$$



**Ans:**

$$N_G = 9.81 \text{ kN}, \quad V_G = 0, \quad M_G = 0$$

- 1-9.** The floor crane is used to lift a 600-kg concrete pipe. Determine the resultant internal loadings acting on the cross section at  $H$ .



## SOLUTION

**Support Reactions:** Referring to the free-body diagram of the hook, Fig. a.

$$\zeta + \sum M_F = 0; \quad D_x(0.3) - 600(9.81)(0.5) = 0 \quad D_x = 9810 \text{ N}$$

$$+\uparrow \sum F_y = 0; \quad D_y - 600(9.81) = 0 \quad D_y = 5886 \text{ N}$$

Subsequently, referring to the free-body diagram of member  $BCD$ , Fig. b,

$$\zeta + \sum M_B = 0; \quad F_{AC} \sin 75^\circ(0.4) - 5886(1.8) = 0 \quad F_{AC} = 27421.36 \text{ N}$$

$$\rightarrow \sum F_x = 0; \quad B_x + 27421.36 \cos 75^\circ - 9810 = 0 \quad B_x = 2712.83 \text{ N}$$

$$+\uparrow \sum F_y = 0; \quad 27421.36 \sin 75^\circ - 5886 - B_y = 0 \quad B_y = 20601 \text{ N}$$

**Internal Loadings:** Using the results of  $B_x$  and  $B_y$ , section  $BH$  of member  $BCD$  will be considered. Referring to the free-body diagram of this part shown in Fig. c,

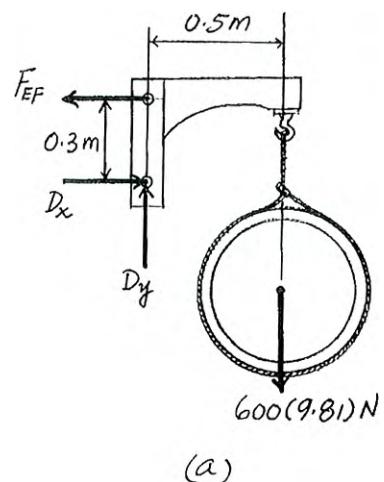
$$\rightarrow \sum F_x = 0; \quad N_H + 2712.83 = 0 \quad N_H = -2712.83 \text{ N} = -2.71 \text{ kN} \quad \text{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad -V_H - 20601 = 0 \quad V_H = -20601 \text{ N} = -20.6 \text{ kN} \quad \text{Ans.}$$

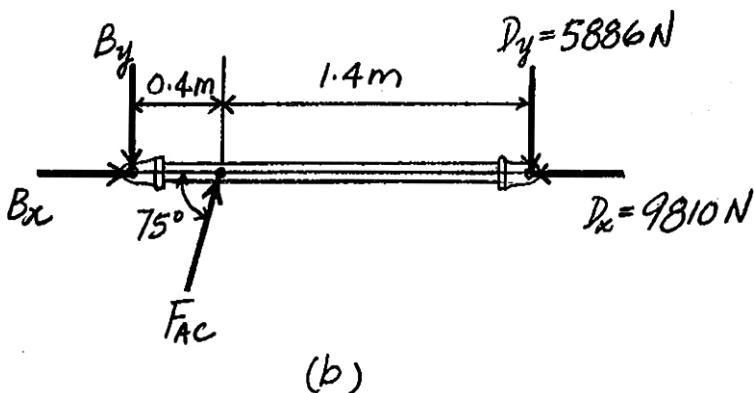
$$\zeta + \sum M_D = 0; \quad M_H + 20601(0.2) = 0 \quad M_H = -4120.2 \text{ N}\cdot\text{m}$$

$$= -4.12 \text{ kN}\cdot\text{m} \quad \text{Ans.}$$

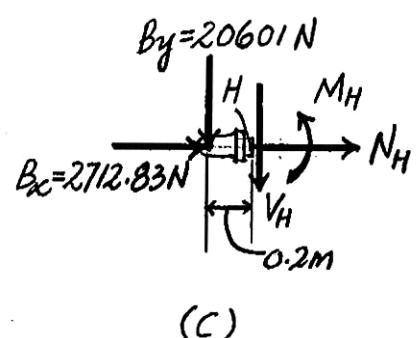
The negative signs indicates that  $N_H$ ,  $V_H$ , and  $M_H$  act in the opposite sense to that shown on the free-body diagram.



(a)



(b)



(c)

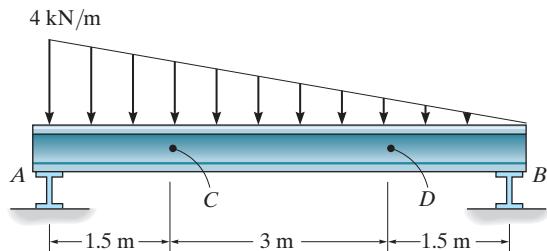
**Ans:**

$$N_H = -2.71 \text{ kN}, V_H = -20.6 \text{ kN},$$

$$M_H = -4.12 \text{ kN}\cdot\text{m}$$

**1–10.**

The beam supports the distributed load shown. Determine the resultant internal loadings acting on the cross section at point C. Assume the reactions at the supports A and B are vertical.



**SOLUTION**

**Support Reactions:** Referring to the FBD of the entire beam, Fig. a,

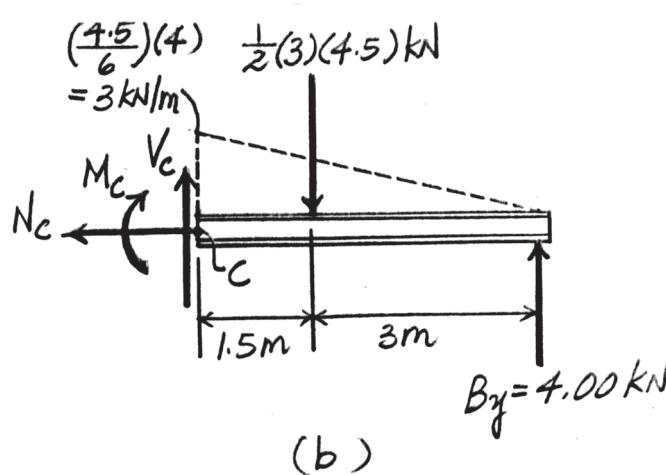
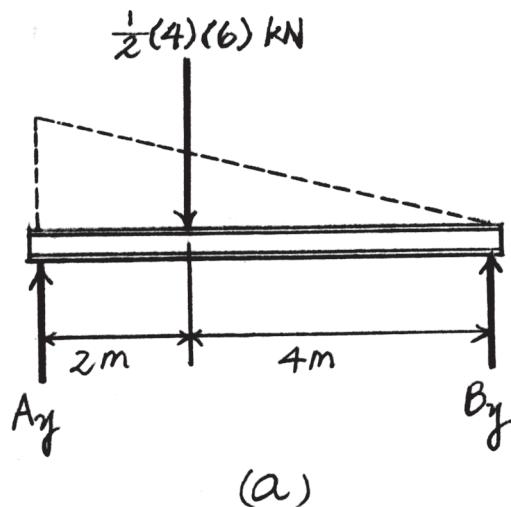
$$\zeta + \sum M_A = 0; \quad B_y(6) - \frac{1}{2}(4)(6)(2) = 0 \quad B_y = 4.00 \text{ kN}$$

**Internal Loadings:** Referring to the FBD of the right segment of the beam sectioned through C, Fig. b,

$$\pm \sum F_x = 0; \quad N_C = 0 \quad \text{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad V_C + 4.00 - \frac{1}{2}(3)(4.5) = 0 \quad V_C = 2.75 \text{ kN} \quad \text{Ans.}$$

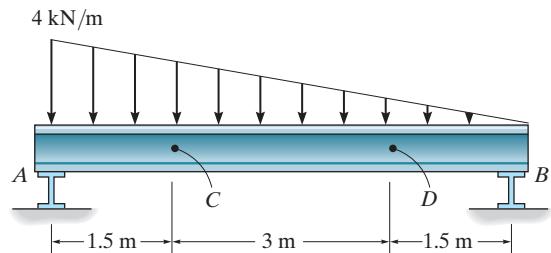
$$\zeta + \sum M_C = 0; \quad 4.00(4.5) - \frac{1}{2}(3)(4.5)(1.5) - M_C = 0 \quad M_C = 7.875 \text{ kN} \cdot \text{m} \quad \text{Ans.}$$



**Ans:**  
 $N_C = 0$ ,  
 $V_C = 2.75 \text{ kN}$ ,  
 $M_C = 7.875 \text{ kN} \cdot \text{m}$

**1-11.**

The beam supports the distributed load shown. Determine the resultant internal loadings acting on the cross section at point D. Assume the reactions at the supports A and B are vertical.



**SOLUTION**

**Support Reactions:** Referring to the FBD of the entire beam, Fig. a,

$$\zeta + \sum M_A = 0; \quad B_y(6) - \frac{1}{2}(4)(6)(2) = 0 \quad B_y = 4.00 \text{ kN}$$

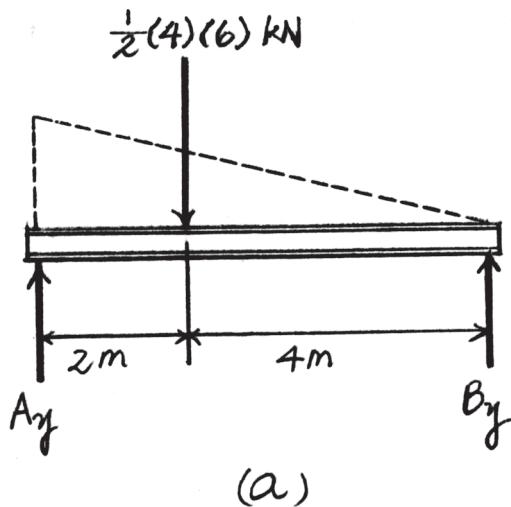
**Internal Loadings:** Referring to the FBD of the right segment of the beam sectioned through D, Fig. b,

$$\stackrel{+}{\rightarrow} \sum F_x = 0; \quad N_D = 0 \quad \text{Ans.}$$

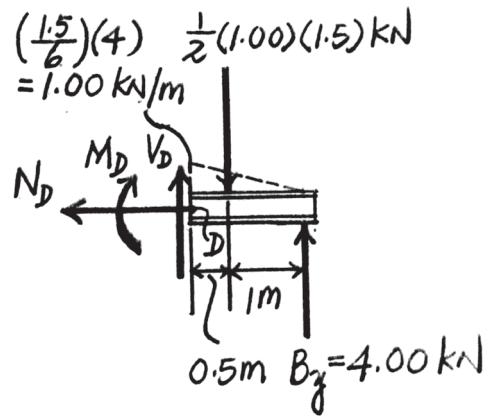
$$+\uparrow \sum F_y = 0; \quad V_D + 4.00 - \frac{1}{2}(1.00)(1.5) = 0 \quad V_D = -3.25 \text{ kN} \quad \text{Ans.}$$

$$\zeta + \sum M_D = 0; \quad 4.00(1.5) - \frac{1}{2}(1.00)(1.5)(0.5) - M_D = 0 \\ M_D = 5.625 \text{ kN}\cdot\text{m} \quad \text{Ans.}$$

The negative sign indicates that  $V_D$  acts in the sense opposite to that shown on the FBD.



(a)

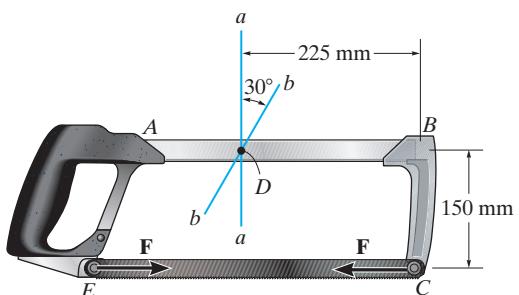


(b)

**Ans:**  
 $N_D = 0$ ,  
 $V_D = -3.25 \text{ kN}$ ,  
 $M_D = 5.625 \text{ kN}\cdot\text{m}$

**\*1-12.**

The blade of the hacksaw is subjected to a pretension force of  $F = 100 \text{ N}$ . Determine the resultant internal loadings acting on section  $a-a$  that passes through point  $D$ .



**SOLUTION**

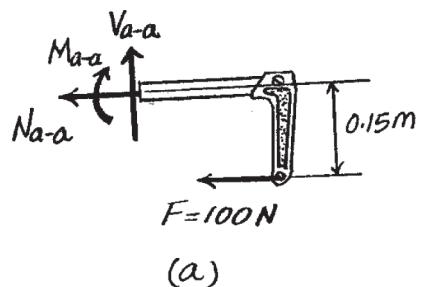
**Internal Loadings:** Referring to the free-body diagram of the section of the hacksaw shown in Fig. *a*,

$$\leftarrow \sum F_x = 0; \quad N_{a-a} + 100 = 0 \quad N_{a-a} = -100 \text{ N} \quad \text{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad V_{a-a} = 0 \quad \text{Ans.}$$

$$\zeta + \sum M_D = 0; \quad -M_{a-a} - 100(0.15) = 0 \quad M_{a-a} = -15 \text{ N}\cdot\text{m} \quad \text{Ans.}$$

The negative sign indicates that  $N_{a-a}$  and  $M_{a-a}$  act in the opposite sense to that shown on the free-body diagram.

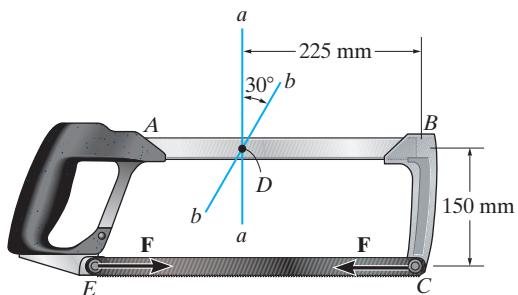


**Ans:**

$$N_{a-a} = -100 \text{ N}, V_{a-a} = 0, M_{a-a} = -15 \text{ N}\cdot\text{m}$$

**1-13.**

The blade of the hacksaw is subjected to a pretension force of  $F = 100 \text{ N}$ . Determine the resultant internal loadings acting on section  $b-b$  that passes through point  $D$ .



**SOLUTION**

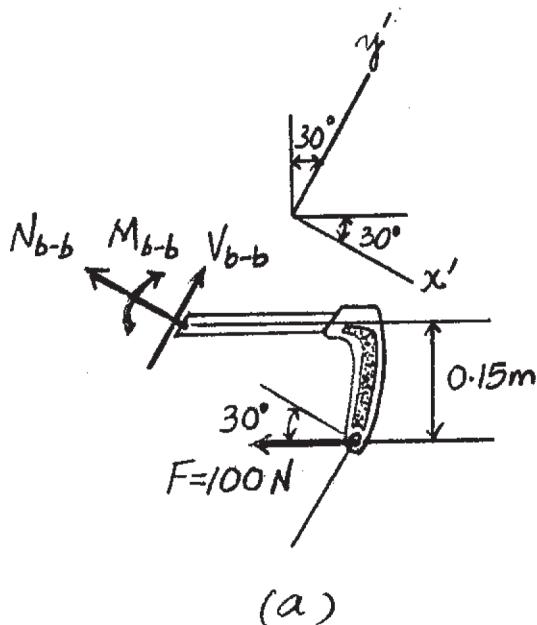
**Internal Loadings:** Referring to the free-body diagram of the section of the hacksaw shown in Fig. *a*,

$$\sum F_x' = 0; \quad N_{b-b} + 100 \cos 30^\circ = 0 \quad N_{b-b} = -86.6 \text{ N} \quad \text{Ans.}$$

$$\sum F_y' = 0; \quad V_{b-b} - 100 \sin 30^\circ = 0 \quad V_{b-b} = 50 \text{ N} \quad \text{Ans.}$$

$$\zeta + \sum M_D = 0; \quad -M_{b-b} - 100(0.15) = 0 \quad M_{b-b} = -15 \text{ N}\cdot\text{m} \quad \text{Ans.}$$

The negative sign indicates that  $N_{b-b}$  and  $M_{b-b}$  act in the opposite sense to that shown on the free-body diagram.



**Ans:**

$$N_{b-b} = -86.6 \text{ N}, V_{b-b} = 50 \text{ N}, M_{b-b} = -15 \text{ N}\cdot\text{m}$$

- 1-14.** The boom  $DF$  of the jib crane and the column  $DE$  have a uniform weight of  $750 \text{ N/m}$ . If the hoist and load weigh  $1500 \text{ N}$ , determine the resultant internal loadings in the crane on cross sections through points  $A$ ,  $B$ , and  $C$ .

## SOLUTION

**Equations of Equilibrium:** For point  $A$

$$\leftarrow \sum F_x = 0; \quad N_A = 0$$

**Ans.**

$$+\uparrow \sum F_y = 0; \quad V_A - 0.675 - 1.500 = 0$$

**Ans.**

$$V_A = 2.175 \text{ kN}$$

$$\zeta + \sum M_A = 0; \quad -M_A - 0.675(0.45) - 1.500(0.9) = 0$$

**Ans.**

$$M_A = -1.65 \text{ kN} \cdot \text{m}$$

Negative sign indicates that  $M_A$  acts in the opposite direction to that shown on FBD.

**Equations of Equilibrium:** For point  $B$

$$\leftarrow \sum F_x = 0; \quad N_B = 0$$

**Ans.**

$$+\uparrow \sum F_y = 0; \quad V_B - 2.475 - 1.5 = 0$$

**Ans.**

$$V_B = 3.975 \text{ kN}$$

$$\zeta + \sum M_B = 0; \quad -M_B - 2.475(1.65) - 1.500(3.3) = 0$$

**Ans.**

$$M_B = -9.03 \text{ kN} \cdot \text{m}$$

Negative sign indicates that  $M_B$  acts in the opposite direction to that shown on FBD.

**Equations of Equilibrium:** For point  $C$

$$\leftarrow \sum F_x = 0; \quad V_C = 0$$

**Ans.**

$$+\uparrow \sum F_y = 0; \quad -N_C - 1.125 - 2.925 - 1.500 = 0$$

**Ans.**

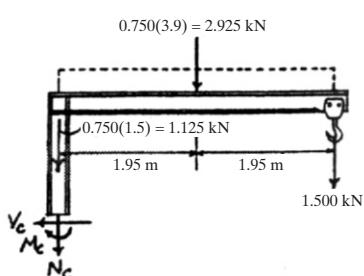
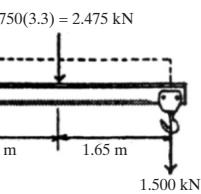
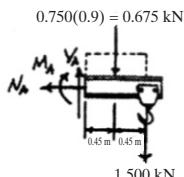
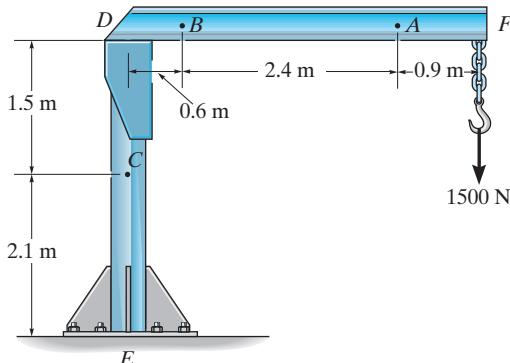
$$N_C = -5.55 \text{ kN}$$

$$\zeta + \sum M_C = 0; \quad -M_C - 2.925(1.95) - 1.500(3.9) = 0$$

**Ans.**

$$M_C = -11.6 \text{ kN} \cdot \text{m}$$

Negative signs indicate that  $N_C$  and  $M_C$  act in the opposite direction to that shown on FBD.



**Ans:**

$$\begin{aligned} N_A &= 0, V_A = 2.175 \text{ kN}, M_A = -1.65 \text{ kN} \cdot \text{m}, \\ N_B &= 0, V_B = 3.975 \text{ kN}, M_B = -9.03 \text{ kN} \cdot \text{m}, \\ V_C &= 0, N_C = -5.55 \text{ kN}, M_C = -11.6 \text{ kN} \cdot \text{m} \end{aligned}$$

### 1-15.

The metal stud punch is subjected to a force of 120 N on the handle. Determine the magnitude of the reactive force at the pin *A* and in the short link *BC*. Also, determine the resultant internal loadings acting on the cross section at point *D*.

### SOLUTION

Member:

$$\zeta + \sum M_A = 0; \quad F_{BC} \cos 30^\circ(50) - 120(500) = 0$$

$$F_{BC} = 1385.6 \text{ N} = 1.39 \text{ kN}$$

$$+ \uparrow \sum F_y = 0; \quad A_y - 1385.6 - 120 \cos 30^\circ = 0$$

$$A_y = 1489.56 \text{ N}$$

$$+ \leftarrow \sum F_x = 0; \quad A_x - 120 \sin 30^\circ = 0; \quad A_x = 60 \text{ N}$$

$$F_A = \sqrt{1489.56^2 + 60^2}$$

$$= 1491 \text{ N} = 1.49 \text{ kN}$$

Segment:

$$\nwarrow + \sum F_{x'} = 0; \quad N_D - 120 = 0$$

$$N_D = 120 \text{ N}$$

**Ans.**

$$+\nearrow \sum F_{y'} = 0; \quad V_D = 0$$

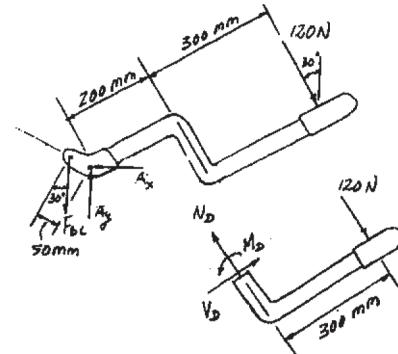
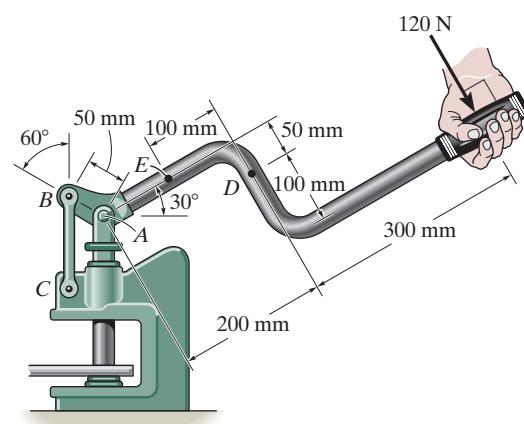
**Ans.**

$$\zeta + \sum M_D = 0; \quad M_D - 120(0.3) = 0$$

**Ans.**

$$M_D = 36.0 \text{ N} \cdot \text{m}$$

**Ans.**



**Ans:**

$F_{BC} = 1.39 \text{ kN}$ ,  $F_A = 1.49 \text{ kN}$ ,  $N_D = 120 \text{ N}$ ,  
 $V_D = 0$ ,  $M_D = 36.0 \text{ N} \cdot \text{m}$

**\*1-16.**

Determine the resultant internal loadings acting on the cross section at point  $E$  of the handle arm, and on the cross section of the short link  $BC$ .

**SOLUTION**

Member:

$$\zeta + \sum M_A = 0; \quad F_{BC} \cos 30^\circ(50) - 120(500) = 0$$

$$F_{BC} = 1385.6 \text{ N} = 1.3856 \text{ kN}$$

Segment:

$$\nabla \sum F_x = 0; \quad N_E = 0$$

$$\nwarrow \sum F_y = 0; \quad V_E - 120 = 0; \quad V_E = 120 \text{ N}$$

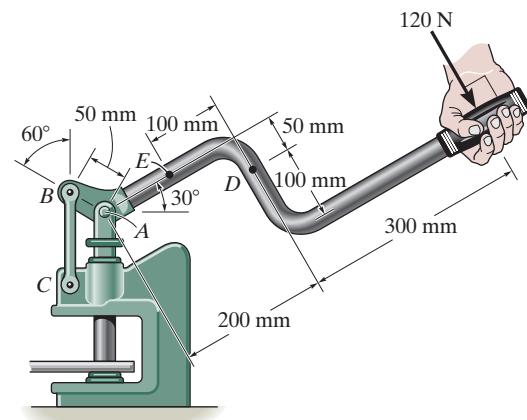
$$\zeta + \sum M_E = 0; \quad M_E - 120(0.4) = 0; \quad M_E = 48.0 \text{ N}\cdot\text{m}$$

Short link:

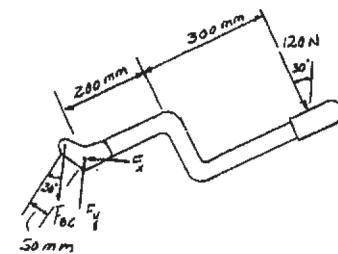
$$\pm \sum F_x = 0; \quad V = 0$$

$$+\uparrow \sum F_y = 0; \quad 1.3856 - N = 0; \quad N = 1.39 \text{ kN}$$

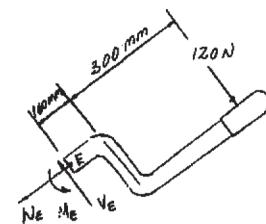
$$\zeta + \sum M_H = 0; \quad M = 0$$



**Ans.**



**Ans.**



**Ans.**

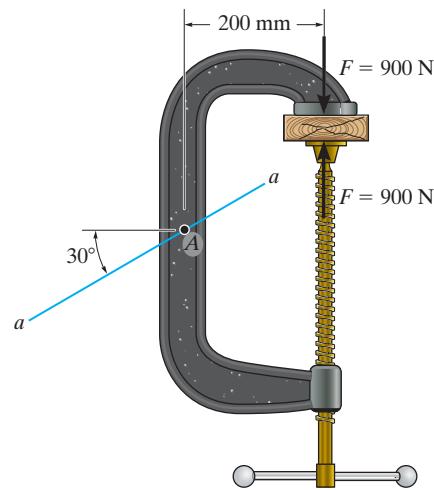
**Ans.**

**Ans.**

**Ans:**

$N_E = 0, V_E = 120 \text{ N}, M_E = 48.0 \text{ N}\cdot\text{m}$ ,  
Short link:  $V = 0, N = 1.39 \text{ kN}, M = 0$

- 1-17.** The forged steel clamp exerts a force of  $F = 900 \text{ N}$  on the wooden block. Determine the resultant internal loadings acting on section  $a-a$  passing through point  $A$ .



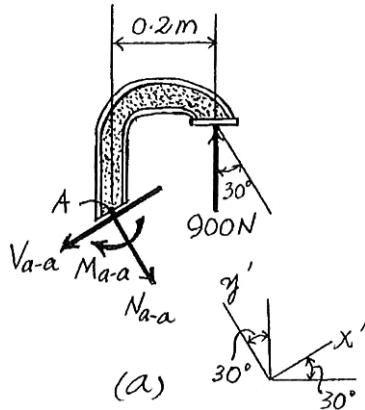
### SOLUTION

**Internal Loadings:** Referring to the free-body diagram of the section of the clamp shown in Fig.  $a$ ,

$$\sum F_{y'} = 0; \quad 900 \cos 30^\circ - N_{a-a} = 0 \quad N_{a-a} = 779 \text{ N} \quad \text{Ans.}$$

$$\sum F_{x'} = 0; \quad V_{a-a} - 900 \sin 30^\circ = 0 \quad V_{a-a} = 450 \text{ N} \quad \text{Ans.}$$

$$\zeta + \sum M_A = 0; \quad 900(0.2) - M_{a-a} = 0 \quad M_{a-a} = 180 \text{ N}\cdot\text{m} \quad \text{Ans.}$$

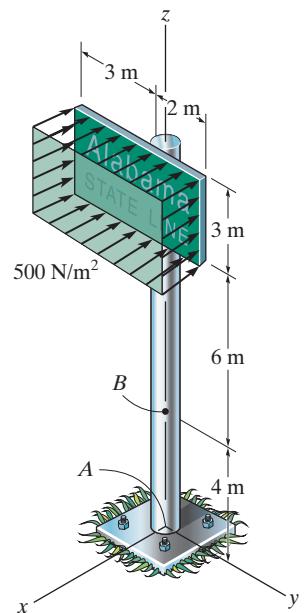


**Ans:**

$$N_{a-a} = 779 \text{ N}, V_{a-a} = 450 \text{ N},$$

$$900(0.2) - M_{a-a} = 0, M_{a-a} = 180 \text{ N}\cdot\text{m}$$

- 1-18.** Determine the resultant internal loadings acting on the cross section through point *B* of the signpost. The post is fixed to the ground and a uniform pressure of  $500 \text{ N/m}^2$  acts perpendicular to the face of the sign.



### SOLUTION

$$\Sigma F_x = 0; \quad (V_B)_x - 7500 = 0; \quad (V_B)_x = 7500 \text{ N} = 7.5 \text{ kN}$$

**Ans.**

$$\Sigma F_y = 0; \quad (V_B)_y = 0$$

**Ans.**

$$\Sigma F_z = 0; \quad (N_B)_z = 0$$

**Ans.**

$$\Sigma M_x = 0; \quad (M_B)_x = 0$$

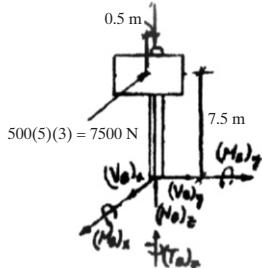
**Ans.**

$$\Sigma M_y = 0; \quad (M_B)_y - 7500(7.5) = 0; \quad (M_B)_y = 56250 \text{ N} \cdot \text{m} = 56.25 \text{ kN} \cdot \text{m}$$

**Ans.**

$$\Sigma M_z = 0; \quad (T_B)_z - 7500(0.5) = 0; \quad (T_B)_z = 3750 \text{ N} \cdot \text{m} = 3.75 \text{ kN} \cdot \text{m}$$

**Ans.**

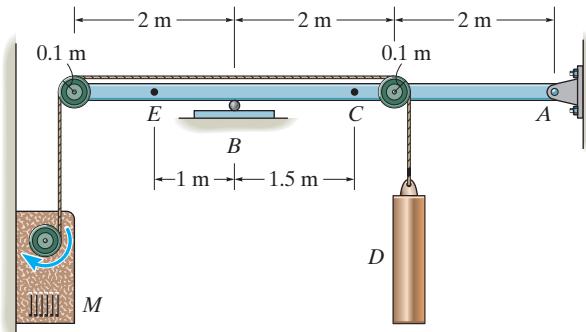


**Ans:**

$$(V_B)_x = 7.5 \text{ kN}, (V_B)_y = 0, (N_B)_z = 0, \\ (M_B)_x = 0, (M_B)_y = 56.25 \text{ kN} \cdot \text{m}, \\ (T_B)_z = 3.75 \text{ kN} \cdot \text{m}$$

**1-19.**

Determine the resultant internal loadings acting on the cross section at point C in the beam. The load D has a mass of 300 kg and is being hoisted by the motor M with constant velocity.



**SOLUTION**

$$\leftarrow \sum F_x = 0; \quad N_C + 2.943 = 0; \quad N_C = -2.94 \text{ kN}$$

$$+\uparrow \sum F_y = 0; \quad V_C - 2.943 = 0; \quad V_C = 2.94 \text{ kN}$$

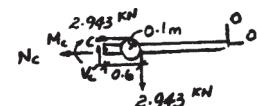
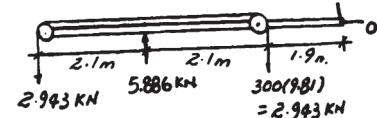
$$\zeta + \sum M_C = 0; \quad -M_C - 2.943(0.6) + 2.943(0.1) = 0$$

$$M_C = -1.47 \text{ kN} \cdot \text{m}$$

**Ans.**

**Ans.**

**Ans.**



**Ans:**

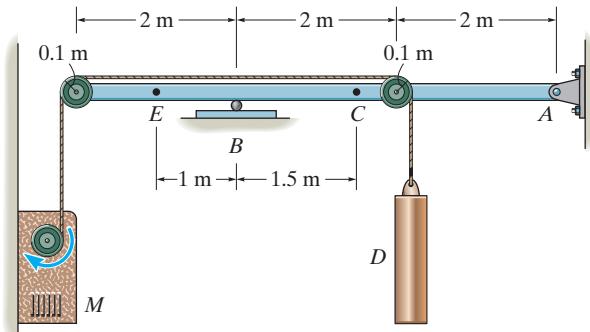
$$N_C = -2.94 \text{ kN},$$

$$V_C = 2.94 \text{ kN},$$

$$M_C = -1.47 \text{ kN} \cdot \text{m}$$

**\*1-20.**

Determine the resultant internal loadings acting on the cross section at point *E*. The load *D* has a mass of 300 kg and is being hoisted by the motor *M* with constant velocity.



**SOLUTION**

$$\pm \sum F_x = 0; \quad N_E + 2943 = 0$$

$$N_E = -2.94 \text{ kN}$$

**Ans.**

$$+\uparrow \sum F_y = 0; \quad -2943 - V_E = 0$$

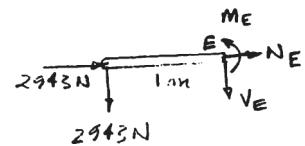
$$V_E = -2.94 \text{ kN}$$

**Ans.**

$$\zeta + \sum M_E = 0; \quad M_E + 2943(1) = 0$$

$$M_E = -2.94 \text{ kN} \cdot \text{m}$$

**Ans.**



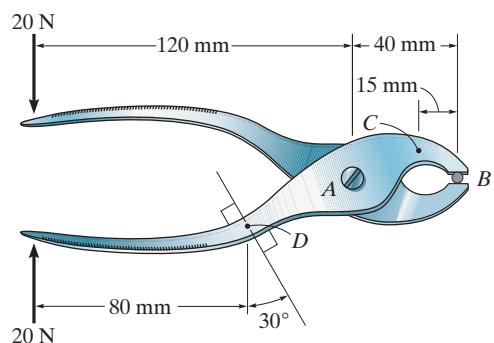
**Ans:**

$$N_E = -2.94 \text{ kN}$$

$$V_E = -2.94 \text{ kN}$$

$$M_E = -2.94 \text{ kN} \cdot \text{m}$$

- 1-21.** Determine the resultant internal loading on the cross section through point C of the pliers. There is a pin at A, and the jaws at B are smooth.

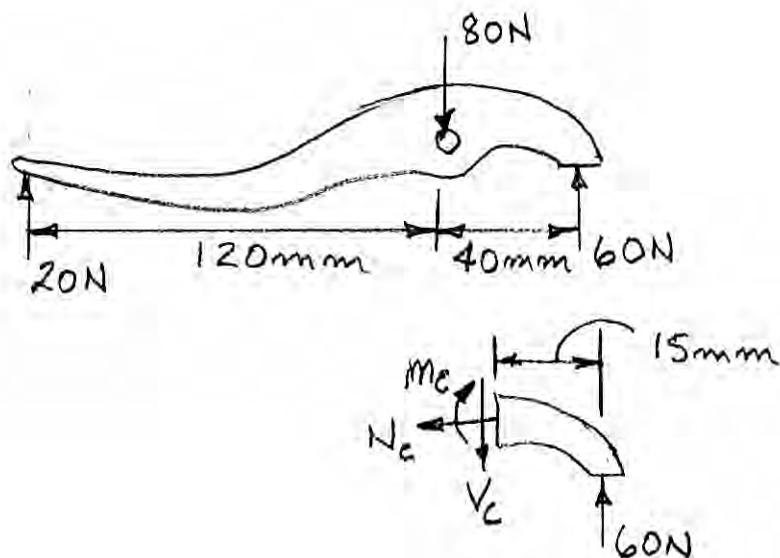


### SOLUTION

$$+\uparrow \sum F_y = 0; \quad -V_C + 60 = 0; \quad V_C = 60 \text{ N} \quad \text{Ans.}$$

$$\stackrel{+}{\rightarrow} \sum F_x = 0; \quad N_C = 0 \quad \text{Ans.}$$

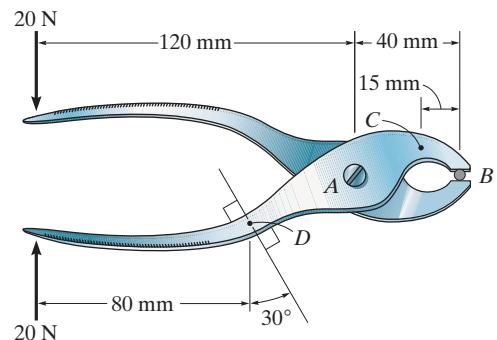
$$+\circlearrowleft \sum M_C = 0; \quad -M_C + 60(0.015) = 0; \quad M_C = 0.9 \text{ N.m} \quad \text{Ans.}$$



**Ans:**

$$V_C = 60 \text{ N}, N_C = 0, M_C = 0.9 \text{ N.m}$$

- 1-22.** Determine the resultant internal loading on the cross section through point D of the pliers.

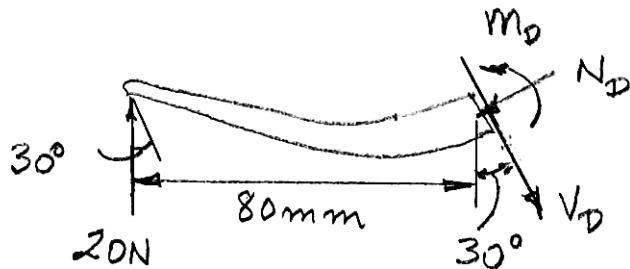


### SOLUTION

$$\nabla + \sum F_y = 0; \quad V_D - 20 \cos 30^\circ = 0; \quad V_D = 17.3 \text{ N} \quad \text{Ans.}$$

$$+\checkmark \sum F_x = 0; \quad N_D - 20 \sin 30^\circ = 0; \quad N_D = 10 \text{ N} \quad \text{Ans.}$$

$$+\circlearrowleft \sum M_D = 0; \quad M_D - 20(0.08) = 0; \quad M_D = 1.60 \text{ N.m} \quad \text{Ans.}$$

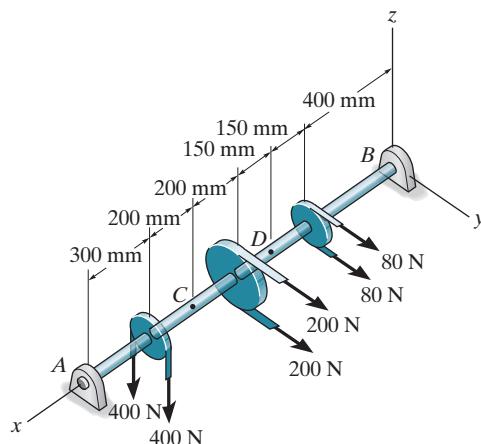


**Ans:**

$$V_D = 17.3 \text{ N}, N_D = 10 \text{ N}, M_D = 1.60 \text{ N.m}$$

**1–23.**

The shaft is supported at its ends by two bearings *A* and *B* and is subjected to the forces applied to the pulleys fixed to the shaft. Determine the resultant internal loadings acting on the cross section at point *C*. The 400-N forces act in the  $-z$  direction and the 200-N and 80-N forces act in the  $+y$  direction. The journal bearings at *A* and *B* exert only  $y$  and  $z$  components of force on the shaft.



**SOLUTION**

**Support Reactions:**

$$\Sigma M_z = 0; \quad 160(0.4) + 400(0.7) - A_y(1.4) = 0$$

$$A_y = 245.71 \text{ N}$$

$$\Sigma F_y = 0; \quad -245.71 - B_y + 400 + 160 = 0$$

$$B_y = 314.29 \text{ N}$$

$$\Sigma M_y = 0; \quad 800(1.1) - A_z(1.4) = 0 \quad A_z = 628.57 \text{ N}$$

$$\Sigma F_z = 0; \quad B_z + 628.57 - 800 = 0 \quad B_z = 171.43 \text{ N}$$

**Equations of Equilibrium: For point *C***

$$\Sigma F_x = 0; \quad (N_C)_x = 0$$

$$\Sigma F_y = 0; \quad -245.71 + (V_C)_y = 0$$

$$(V_C)_y = -246 \text{ N}$$

$$\Sigma F_z = 0; \quad 628.57 - 800 + (V_C)_z = 0$$

$$(V_C)_z = -171 \text{ N}$$

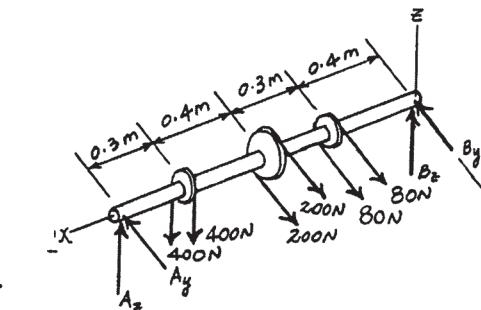
$$\Sigma M_x = 0; \quad (T_C)_x = 0$$

$$\Sigma M_y = 0; \quad (M_C)_y - 628.57(0.5) + 800(0.2) = 0$$

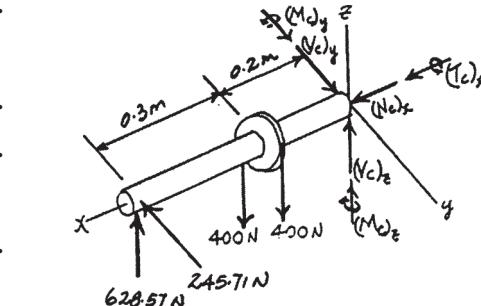
$$(M_C)_y = -154 \text{ N} \cdot \text{m}$$

$$\Sigma M_z = 0; \quad (M_C)_z - 245.71(0.5) = 0$$

$$(M_C)_z = -123 \text{ N} \cdot \text{m}$$



**Ans.**



**Ans.**

**Ans.**

**Ans.**

**Ans.**

**Ans:**

$$(N_C)_x = 0,$$

$$(V_C)_y = -246 \text{ N},$$

$$(V_C)_z = -171 \text{ N},$$

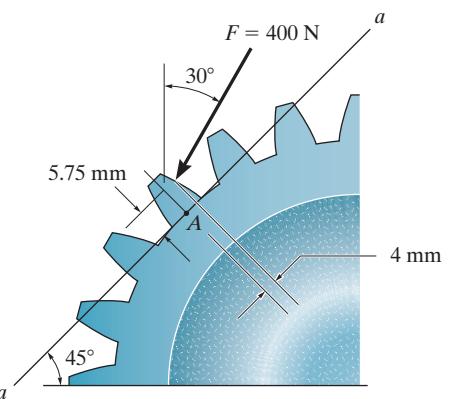
$$(T_C)_x = 0,$$

$$(M_C)_y = -154 \text{ N} \cdot \text{m},$$

$$(M_C)_z = -123 \text{ N} \cdot \text{m}$$

\* 1.24.

The force  $F = 400 \text{ N}$  acts on the gear tooth. Determine the resultant internal loadings on the root of the tooth, i.e., at the centroid point  $A$  of section  $a-a$ .



**SOLUTION**

**Equations of Equilibrium:** For section  $a-a$

$$+ \sum F_{x'} = 0; \quad V_A - 400 \cos 15^\circ = 0$$

$$V_A = 386.37 \text{ N}$$

**Ans.**

$$\nwarrow + \sum F_{y'} = 0; \quad N_A - 400 \sin 15^\circ = 0$$

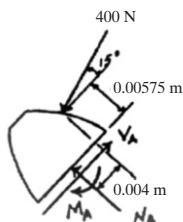
$$N_A = 103.53 \text{ N}$$

**Ans.**

$$\zeta + \sum M_A = 0; \quad -M_A - 400 \sin 15^\circ(0.004) + 400 \cos 15^\circ(0.00575) = 0$$

$$M_A = 1.808 \text{ N} \cdot \text{m}$$

**Ans.**



|

**Ans:**

$$V_A = 386.37 \text{ N}, \quad N_A = 103.53 \text{ N},$$

$$M_A = 1.808 \text{ N} \cdot \text{m}$$

**1-25.**

The shaft is supported at its ends by two bearings *A* and *B* and is subjected to the forces applied to the pulleys fixed to the shaft. Determine the resultant internal loadings acting on the cross section at point *D*. The 400-N forces act in the  $-z$  direction and the 200-N and 80-N forces act in the  $+y$  direction. The journal bearings at *A* and *B* exert only  $y$  and  $z$  components of force on the shaft.

## SOLUTION

### Support Reactions:

$$\Sigma M_z = 0; \quad 160(0.4) + 400(0.7) - A_y(1.4) = 0$$

$$A_y = 245.71 \text{ N}$$

$$\Sigma F_y = 0; \quad -245.71 - B_y + 400 + 160 = 0$$

$$B_y = 314.29 \text{ N}$$

$$\Sigma M_y = 0; \quad 800(1.1) - A_z(1.4) = 0 \quad A_z = 628.57 \text{ N}$$

$$\Sigma F_z = 0; \quad B_z + 628.57 - 800 = 0 \quad B_z = 171.43 \text{ N}$$

### Equations of Equilibrium: For point *D*

$$\Sigma F_x = 0; \quad (N_D)_x = 0$$

**Ans.**

$$\Sigma F_y = 0; \quad (V_D)_y - 314.29 + 160 = 0$$

$$(V_D)_y = 154 \text{ N}$$

**Ans.**

$$\Sigma F_z = 0; \quad 171.43 + (V_D)_z = 0$$

$$(V_D)_z = -171 \text{ N}$$

**Ans.**

$$\Sigma M_x = 0; \quad (T_D)_x = 0$$

**Ans.**

$$\Sigma M_y = 0; \quad 171.43(0.55) + (M_D)_y = 0$$

**Ans.**

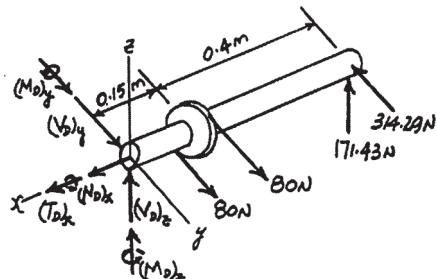
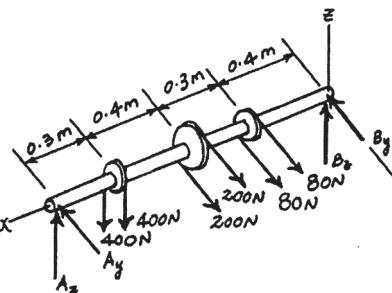
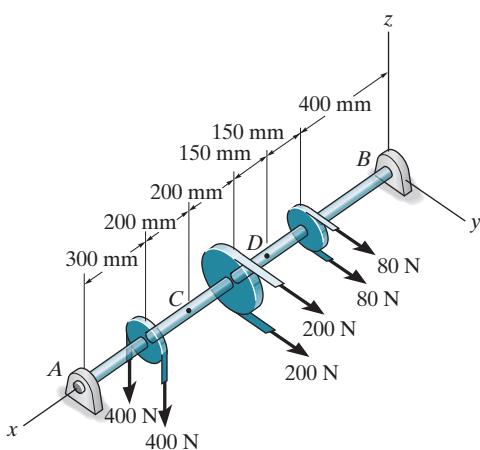
$$(M_D)_y = -94.3 \text{ N} \cdot \text{m}$$

**Ans.**

$$\Sigma M_z = 0; \quad 314.29(0.55) - 160(0.15) + (M_D)_z = 0$$

**Ans.**

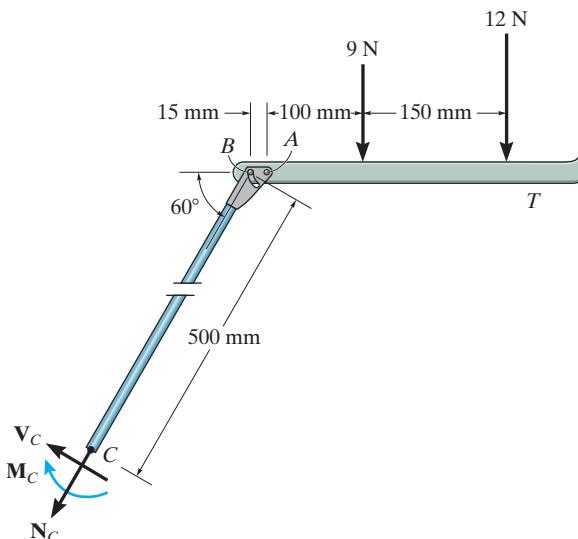
$$(M_D)_z = -149 \text{ N} \cdot \text{m}$$



**Ans:**

$$\begin{aligned} (N_D)_x &= 0, \\ (V_D)_y &= 154 \text{ N}, \\ (V_D)_z &= -171 \text{ N}, \\ (T_D)_x &= 0, \\ (M_D)_y &= -94.3 \text{ N} \cdot \text{m}, \\ (M_D)_z &= -149 \text{ N} \cdot \text{m} \end{aligned}$$

**1–26.** The serving tray  $T$  used on an airplane is supported on *each side* by an arm. The tray is pin connected to the arm at  $A$ , and at  $B$  there is a smooth pin. (The pin can move within the slot in the arms to permit folding the tray against the front passenger seat when not in use.) Determine the resultant internal loadings acting on the cross section of the arm through point  $C$  when the tray arm supports the loads shown.



$$\curvearrowleft + \sum F_x = 0; \quad N_C + 9 \cos 30^\circ + 12 \cos 30^\circ = 0; \quad N_C = -18.2 \text{ N}$$

**Ans.**

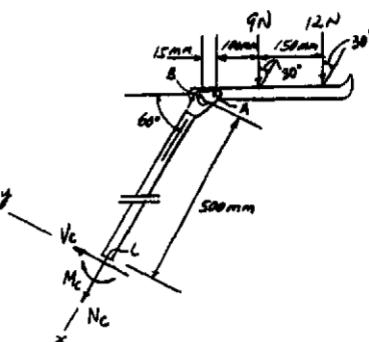
$$\uparrow + \sum F_y = 0; \quad V_C - 9 \sin 30^\circ - 12 \sin 30^\circ = 0; \quad V_C = 10.5 \text{ N}$$

**Ans.**

$$\zeta + \sum M_C = 0; \quad -M_C - 9(0.5 \cos 60^\circ + 0.115) - 12(0.5 \cos 60^\circ + 0.265) = 0$$

**Ans.**

$$M_C = -9.46 \text{ N} \cdot \text{m}$$



**Ans:**

$$N_C = -18.2 \text{ N}$$

$$V_C = 10.5 \text{ N}$$

$$M_C = -9.46 \text{ N} \cdot \text{m}$$

**1-27.**

The pipe has a mass of 12 kg/m. If it is fixed to the wall at A, determine the resultant internal loadings acting on the cross section at B.

### SOLUTION

**Internal Loadings:** Referring to the FBD of the right segment of the pipe assembly sectioned through B, Fig. a,

$$\Sigma F_x = 0; \quad (V_B)_x + 300 = 0 \quad (V_B)_x = -300 \text{ N} \quad \text{Ans.}$$

$$\Sigma F_y = 0; \quad (N_B)_y + 400 + 500\left(\frac{4}{5}\right) = 0 \quad (N_B)_y = -800 \text{ N} \quad \text{Ans.}$$

$$\Sigma F_z = 0; \quad (V_B)_z - 2[12(2)(9.81)] - 500\left(\frac{3}{5}\right) = 0 \quad (V_B)_z = 770.88 \text{ N} = 771 \text{ N} \quad \text{Ans.}$$

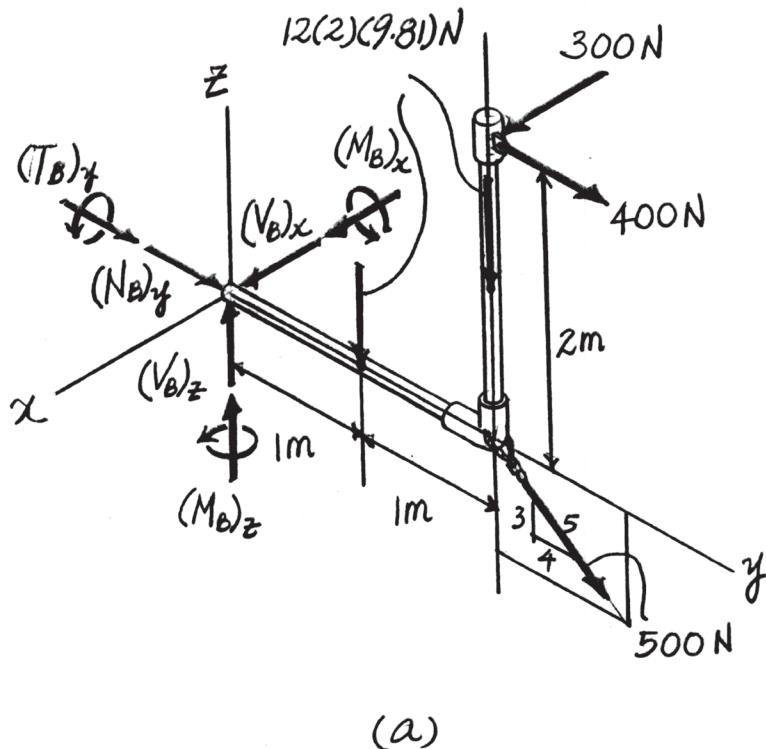
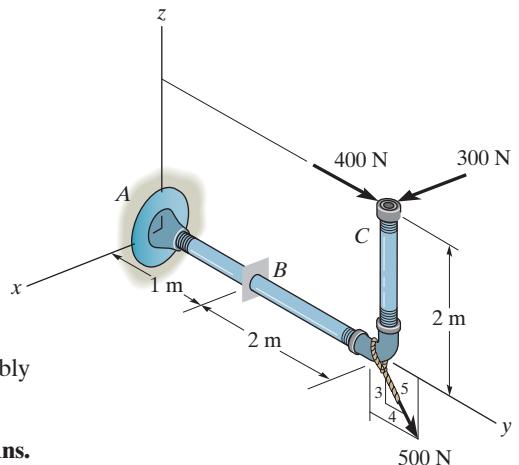
$$\Sigma M_x = 0; \quad (M_B)_x - 12(2)(9.81)(1) - 12(2)(9.81)(2) - 500\left(\frac{3}{5}\right)(2) \\ - 400(2) = 0$$

$$(M_B)_x = 2106.32 \text{ N} \cdot \text{m} = 2.11 \text{ kN} \cdot \text{m} \quad \text{Ans.}$$

$$\Sigma M_y = 0; \quad (T_B)_y + 300(2) = 0 \quad (T_B)_y = -600 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

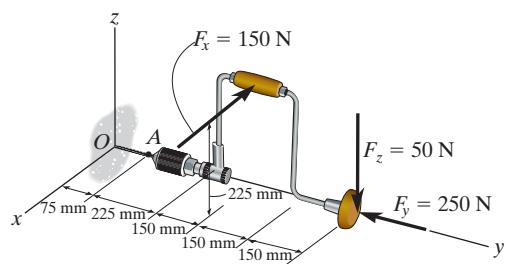
$$\Sigma M_z = 0; \quad (M_B)_z - 300(2) = 0 \quad (M_B)_z = 600 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

The negative signs indicates that  $(V_B)_x$ ,  $(N_B)_y$ , and  $(T_B)_y$  act in the sense opposite to those shown in the FBD.



**Ans:**  
 $(V_B)_x = -300 \text{ N}$ ,  
 $(N_B)_y = -800 \text{ N}$ ,  
 $(V_B)_z = 771 \text{ N}$ ,  
 $(M_B)_x = 2.11 \text{ kN} \cdot \text{m}$ ,  
 $(T_B)_y = -600 \text{ N} \cdot \text{m}$ ,  
 $(M_B)_z = 600 \text{ N} \cdot \text{m}$

- \*1–28.** The brace and drill bit is used to drill a hole at  $O$ . If the drill bit jams when the brace is subjected to the forces shown, determine the resultant internal loadings acting on the cross section of the drill bit at  $A$ .

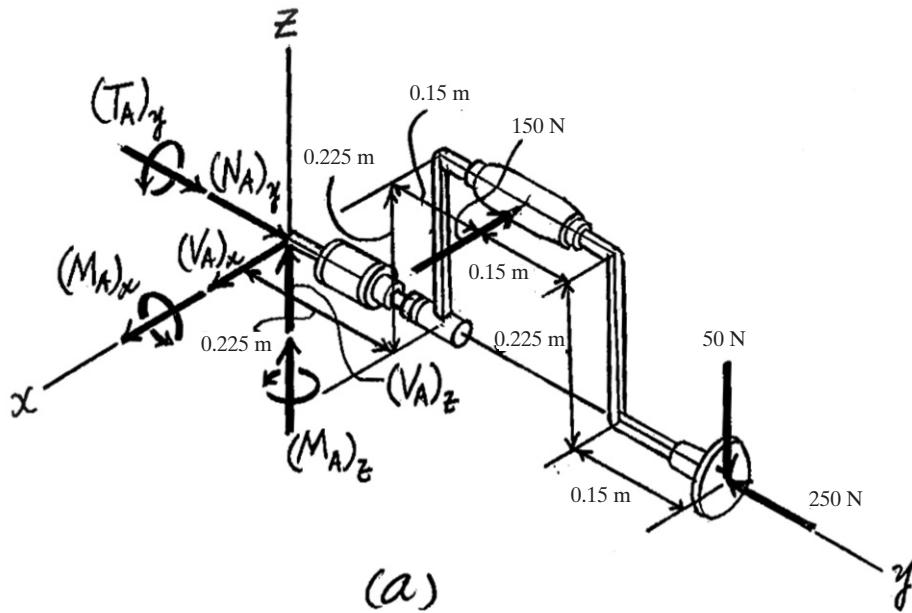


## SOLUTION

**Internal Loading:** Referring to the free-body diagram of the section of the drill and brace shown in Fig. *a*,

$$\begin{array}{lll} \Sigma F_x = 0; & (V_A)_x - 150 = 0 & (V_A)_x = 150 \text{ N} \\ \Sigma F_y = 0; & (N_A)_y - 250 = 0 & (N_A)_y = 250 \text{ N} \\ \Sigma F_z = 0; & (V_A)_z - 50 = 0 & (V_A)_z = 50 \text{ N} \\ \Sigma M_x = 0; & (M_A)_x - 50(0.675) = 0 & (M_A)_x = 33.75 \text{ N} \cdot \text{m} \\ \Sigma M_y = 0; & (T_A)_y - 150(0.225) = 0 & (T_A)_y = 33.75 \text{ N} \cdot \text{m} \\ \Sigma M_z = 0; & (M_A)_z + 150(0.375) = 0 & (M_A)_z = -56.25 \text{ N} \cdot \text{m} \end{array} \quad \text{Ans.}$$

The negative sign indicates that  $(M_A)_z$  acts in the opposite sense to that shown on the free-body diagram.

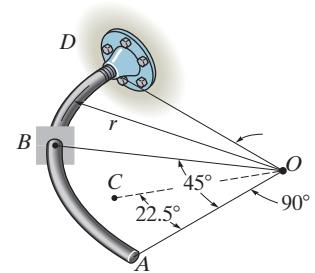


**Ans:**

$$(V_A)_x = 150 \text{ N}, (N_A)_y = 250 \text{ N}, (V_A)_z = 50 \text{ N}, \\ (M_A)_x = 33.75 \text{ N} \cdot \text{m}, (T_A)_y = 33.75 \text{ N} \cdot \text{m}, \\ (M_A)_z = -56.25 \text{ N} \cdot \text{m}$$

**1-29.**

The curved rod  $AD$  of radius  $r$  has a weight per length of  $w$ . If it lies in the horizontal plane, determine the resultant internal loadings acting on the cross section at point  $B$ .  
*Hint:* The distance from the centroid  $C$  of segment  $AB$  to point  $O$  is  $CO = 0.9745r$ .



**SOLUTION**

$$\Sigma F_z = 0; \quad V_B - \frac{\pi}{4} rw = 0; \quad V_B = 0.785 w r$$

**Ans.**

$$\Sigma F_x = 0; \quad N_B = 0$$

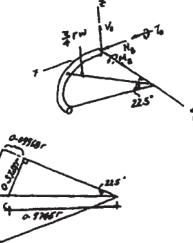
**Ans.**

$$\Sigma M_x = 0; \quad T_B - \frac{\pi}{4} rw(0.09968r) = 0; \quad T_B = 0.0783 w r^2$$

**Ans.**

$$\Sigma M_y = 0; \quad M_B + \frac{\pi}{4} rw(0.3729 r) = 0; \quad M_B = -0.293 w r^2$$

**Ans.**

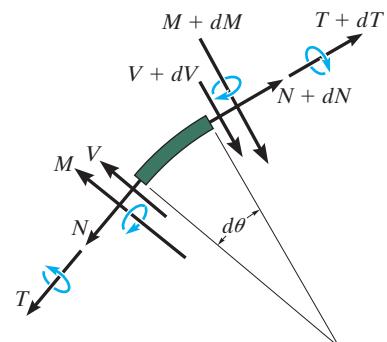


**Ans:**

$$V_B = 0.785wr, \\ N_B = 0, \\ T_B = 0.0783wr^2, \\ M_B = -0.293wr^2$$

**1–30.**

A differential element taken from a curved bar is shown in the figure. Show that  $dN/d\theta = V$ ,  $dV/d\theta = -N$ ,  $dM/d\theta = -T$ , and  $dT/d\theta = M$ .



**SOLUTION**

$$\Sigma F_x = 0;$$

$$N \cos \frac{d\theta}{2} + V \sin \frac{d\theta}{2} - (N + dN) \cos \frac{d\theta}{2} + (V + dV) \sin \frac{d\theta}{2} = 0 \quad (1)$$

$$\Sigma F_y = 0;$$

$$N \sin \frac{d\theta}{2} - V \cos \frac{d\theta}{2} + (N + dN) \sin \frac{d\theta}{2} + (V + dV) \cos \frac{d\theta}{2} = 0 \quad (2)$$

$$\Sigma M_x = 0;$$

$$T \cos \frac{d\theta}{2} + M \sin \frac{d\theta}{2} - (T + dT) \cos \frac{d\theta}{2} + (M + dM) \sin \frac{d\theta}{2} = 0 \quad (3)$$

$$\Sigma M_y = 0;$$

$$T \sin \frac{d\theta}{2} - M \cos \frac{d\theta}{2} + (T + dT) \sin \frac{d\theta}{2} + (M + dM) \cos \frac{d\theta}{2} = 0 \quad (4)$$

Since  $\frac{d\theta}{2}$  is small, we can add, then  $\sin \frac{d\theta}{2} = \frac{d\theta}{2}$ ,  $\cos \frac{d\theta}{2} = 1$

$$\text{Eq. (1) becomes } Vd\theta - dN + \frac{dVd\theta}{2} = 0$$

Neglecting the second order term,  $Vd\theta - dN = 0$

$$\frac{dN}{d\theta} = V \quad \text{QED}$$

$$\text{Eq. (2) becomes } Nd\theta + dV + \frac{dNd\theta}{2} = 0$$

Neglecting the second order term,  $Nd\theta + dV = 0$

$$\frac{dV}{d\theta} = -N \quad \text{QED}$$

$$\text{Eq. (3) becomes } Md\theta - dT + \frac{dTd\theta}{2} = 0$$

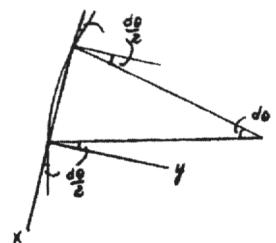
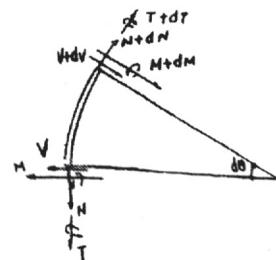
Neglecting the second order term,  $Md\theta - dT = 0$

$$\frac{dT}{d\theta} = M \quad \text{QED}$$

$$\text{Eq. (4) becomes } Td\theta + dM + \frac{dTd\theta}{2} = 0$$

Neglecting the second order term,  $Td\theta + dM = 0$

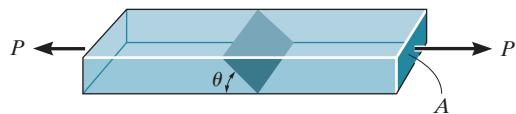
$$\frac{dM}{d\theta} = -T \quad \text{QED}$$



**Ans:**  
N/A

**1-31.**

The bar has a cross-sectional area  $A$  and is subjected to the axial load  $P$ . Determine the average normal and average shear stresses acting over the shaded section, which is oriented at  $\theta$  from the horizontal. Plot the variation of these stresses as a function of  $\theta$  ( $0 \leq \theta \leq 90^\circ$ ).



**SOLUTION**

**Equations of Equilibrium:**

$$\nabla + \sum F_x = 0; \quad V - P \cos \theta = 0 \quad V = P \cos \theta$$

$$\nearrow + \sum F_y = 0; \quad N - P \sin \theta = 0 \quad N = P \sin \theta$$

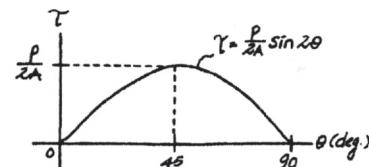
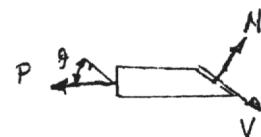
**Average Normal Stress and Shear Stress:** Area at  $\theta$  plane,  $A' = \frac{A}{\sin \theta}$ .

$$\sigma_{\text{avg}} = \frac{N}{A'} = \frac{P \sin \theta}{\frac{A}{\sin \theta}} = \frac{P}{A} \sin^2 \theta$$

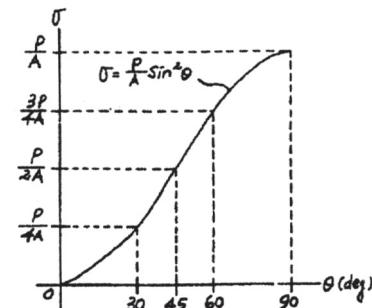
$$\tau_{\text{avg}} = \frac{V}{A'} = \frac{P \cos \theta}{\frac{A}{\sin \theta}}$$

$$= \frac{P}{A} \sin \theta \cos \theta = \frac{P}{2A} \sin 2\theta$$

**Ans.**



**Ans.**

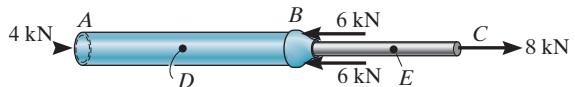


**Ans:**

$$\sigma_{\text{avg}} = \frac{P}{A} \sin^2 \theta, \quad \tau_{\text{avg}} = \frac{P}{2A} \sin 2\theta$$

**\*1–32.**

The built-up shaft consists of a pipe *AB* and solid rod *BC*. The pipe has an inner diameter of 20 mm and outer diameter of 28 mm. The rod has a diameter of 12 mm. Determine the average normal stress at points *D* and *E* and represent the stress on a volume element located at each of these points.

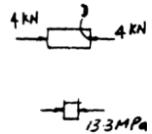


**SOLUTION**

At *D*:

$$\sigma_D = \frac{P}{A} = \frac{4(10^3)}{\frac{\pi}{4}(0.028^2 - 0.02^2)} = 13.3 \text{ MPa} \quad (\text{C})$$

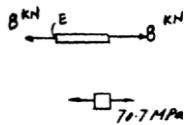
**Ans.**



At *E*:

$$\sigma_E = \frac{P}{A} = \frac{8(10^3)}{\frac{\pi}{4}(0.012^2)} = 70.7 \text{ MPa} \quad (\text{T})$$

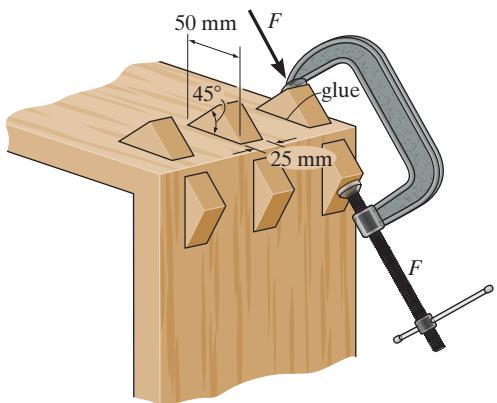
**Ans.**



**Ans:**

$$\sigma_D = 13.3 \text{ MPa}, \sigma_E = 70.7 \text{ MPa}$$

- 1-33.** The triangular blocks are glued along each side of the joint. A C-clamp placed between two of the blocks is used to draw the joint tight. If the glue can withstand a maximum average shear stress of 800 kPa, determine the maximum allowable clamping force  $F$ .



### SOLUTION

**Internal Loadings:** The shear force developed on the glued shear plane can be obtained by writing the force equation of equilibrium along the  $x$  axis with reference to the free-body diagram of the triangular block, Fig. *a*.

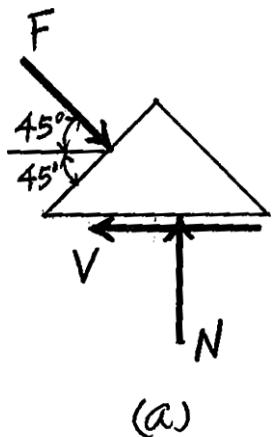
$$\stackrel{+}{\rightarrow} \sum F_x = 0; \quad F \cos 45^\circ - V = 0 \quad V = \frac{\sqrt{2}}{2} F$$

**Average Normal and Shear Stress:** The area of the glued shear plane is  $A = 0.05(0.025) = 1.25(10^{-3})\text{m}^2$ . We obtain

$$\tau_{\text{avg}} = \frac{V}{A}; \quad 800(10^3) = \frac{\frac{\sqrt{2}}{2} F}{1.25(10^{-3})}$$

$$F = 1414 \text{ N} = 1.41 \text{ kN}$$

Ans.

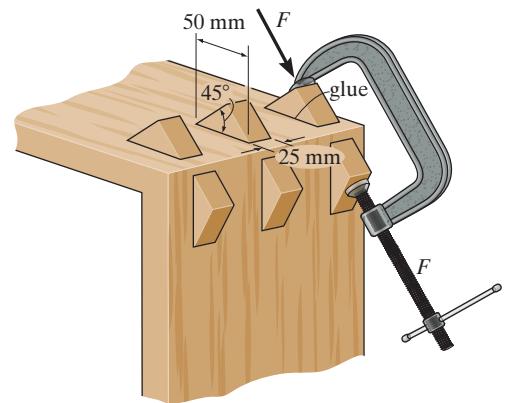


(a)

Ans:

$$F = 1.41 \text{ kN}$$

- 1-34.** The triangular blocks are glued along each side of the joint. A C-clamp placed between two of the blocks is used to draw the joint tight. If the clamping force is  $F = 900 \text{ N}$ , determine the average shear stress developed in the glued shear plane.



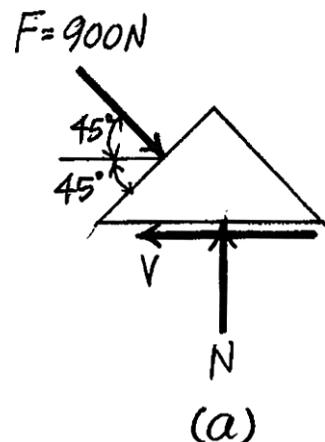
## SOLUTION

**Internal Loadings:** The shear force developed on the glued shear plane can be obtained by writing the force equation of equilibrium along the  $x$  axis with reference to the free-body diagram of the triangular block, Fig. a.

$$\rightarrow \sum F_x = 0; \quad 900 \cos 45^\circ - V = 0 \quad V = 636.40 \text{ N}$$

**Average Normal and Shear Stress:** The area of the glued shear plane is  $A = 0.05(0.025) = 1.25(10^{-3})\text{m}^2$ . We obtain

$$\tau_{\text{avg}} = \frac{V}{A} = \frac{636.40}{1.25(10^{-3})} = 509 \text{ kPa} \quad \text{Ans.}$$

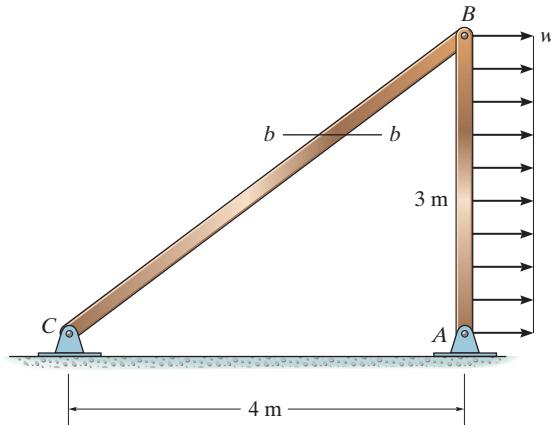


**Ans.**

$$V = 636.40 \text{ N}, \tau_{\text{avg}} = 509 \text{ kPa}$$

**1–35.**

Determine the largest intensity  $w$  of the uniform loading that can be applied to the frame without causing either the average normal stress or the average shear stress at section  $b-b$  to exceed  $\sigma = 15 \text{ MPa}$  and  $\tau = 16 \text{ MPa}$ , respectively. Member  $CB$  has a square cross section of 30 mm on each side.



**SOLUTION**

**Support Reactions:** FBD(a)

$$\zeta + \sum M_A = 0; \quad \frac{4}{5}F_{BC}(3) - 3w(1.5) = 0 \quad F_{BC} = 1.875w$$

**Equations of Equilibrium:** For section  $b-b$ , FBD(b)

$$\pm \sum F_x = 0; \quad \frac{4}{5}(1.875w) - V_{b-b} = 0 \quad V_{b-b} = 1.50w$$

$$+ \uparrow \sum F_y = 0; \quad \frac{3}{5}(1.875w) - N_{b-b} = 0 \quad N_{b-b} = 1.125w$$

**Average Normal Stress and Shear Stress:** The cross-sectional area of section  $b-b$ ,  $A' = \frac{5A}{3}$ ; where  $A = (0.03)(0.03) = 0.90(10^{-3}) \text{ m}^2$ .

$$\text{Then } A' = \frac{5}{3}(0.90)(10^{-3}) = 1.50(10^{-3}) \text{ m}^2.$$

**Assume failure due to normal stress.**

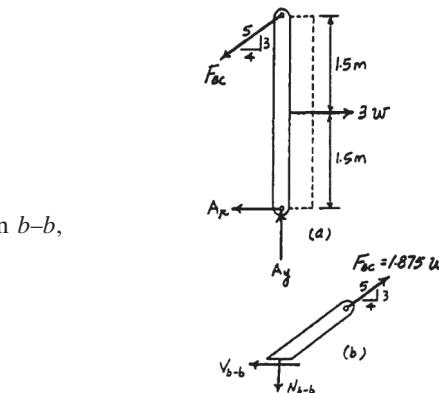
$$(\sigma_{b-b})_{\text{Allow}} = \frac{N_{b-b}}{A'}; \quad 15(10^6) = \frac{1.125w}{1.50(10^{-3})}$$

$$w = 20000 \text{ N/m} = 20.0 \text{ kN/m}$$

**Assume failure due to shear stress.**

$$(\tau_{b-b})_{\text{Allow}} = \frac{V_{b-b}}{A'}; \quad 16(10^6) = \frac{1.50w}{1.50(10^{-3})}$$

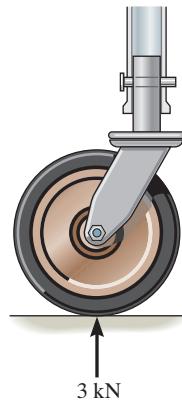
$$w = 16000 \text{ N/m} = 16.0 \text{ kN/m} (\text{Controls !})$$



**Ans:**  
 $w = 16.0 \text{ kN/m} (\text{Controls !})$

**\*1–36.**

The supporting wheel on a scaffold is held in place on the leg using a 4-mm-diameter pin. If the wheel is subjected to a normal force of 3 kN, determine the average shear stress in the pin. Assume the pin only supports the vertical 3-kN load.



**SOLUTION**

$$+\uparrow \sum F_y = 0; \quad 3 \text{ kN} \cdot 2V = 0; \quad V = 1.5 \text{ kN}$$

$$\tau_{\text{avg}} = \frac{V}{A} = \frac{1.5(10^3)}{\frac{\pi}{4}(0.004)^2} = 119 \text{ MPa}$$

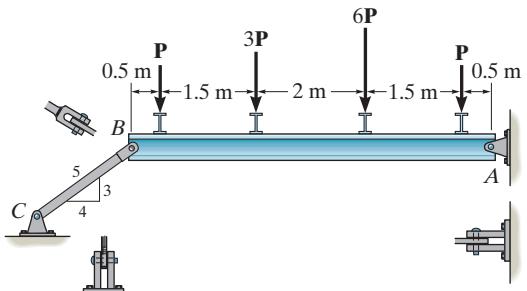
**Ans.**



**Ans:**  
 $\tau_{\text{avg}} = 119 \text{ MPa}$

**1-37.**

If  $P = 5 \text{ kN}$ , determine the average shear stress in the pins at  $A$ ,  $B$ , and  $C$ . All pins are in double shear, and each has a diameter of 18 mm.



## SOLUTION

**Support Reactions:** Referring to the FBD of the entire beam, Fig. *a*,

$$\zeta + \sum M_A = 0; \quad 5(0.5) + 30(2) + 15(4) + 5(5.5) - F_{BC} \left( \frac{3}{5} \right)(6) = 0$$

$$F_{BC} = 41.67 \text{ kN}$$

$$\zeta + \sum M_B = 0; \quad A_y(6) - 5(0.5) - 15(2) - 30(4) - 5(5.5) = 0 \quad A_y = 30.0 \text{ kN}$$

$$\pm \sum F_x = 0; \quad 41.67 \left( \frac{4}{5} \right) - A_x = 0 \quad A_x = 33.33 \text{ kN}$$

Thus,

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{33.33^2 + 30.0^2} = 44.85 \text{ kN}$$

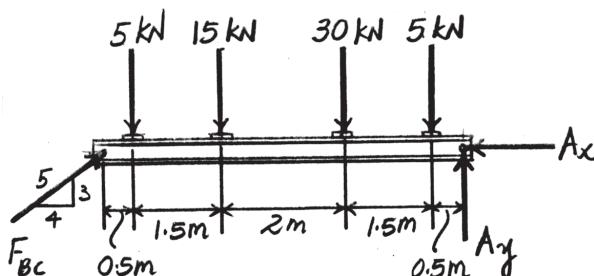
**Average Shear Stress:** Since all the pins are subjected to double shear,

$$V_B = V_C = \frac{F_{BC}}{2} = \frac{41.67}{2} \text{ kN} = 20.83 \text{ kN} \text{ (Fig. } b\text{) and } V_A = 22.42 \text{ kN} \text{ (Fig. } c\text{)}$$

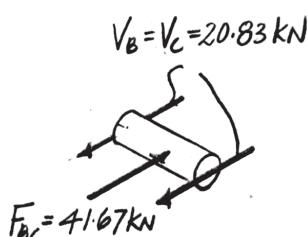
For pins  $B$  and  $C$

$$\tau_B = \tau_C = \frac{V_C}{A} = \frac{20.83(10^3)}{\frac{\pi}{4}(0.018^2)} = 81.87 \text{ MPa} = 81.9 \text{ MPa} \quad \text{Ans.}$$

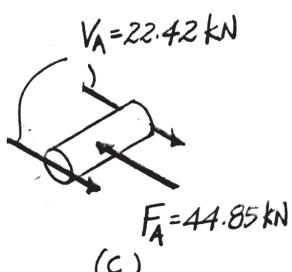
$$\tau_A = \frac{V_A}{A} = \frac{22.42(10^3)}{\frac{\pi}{4}(0.018^2)} = 88.12 \text{ MPa} = 88.1 \text{ MPa} \quad \text{Ans.}$$



(a)



(b)

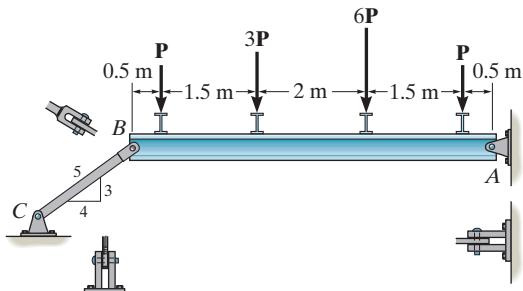


(c)

**Ans:**  
 $\tau_B = \tau_C = 81.9 \text{ MPa}$ ,  
 $\tau_A = 88.1 \text{ MPa}$

**1-38.**

Determine the maximum magnitude  $P$  of the loads the beam can support if the average shear stress in each pin is not to exceed 80 MPa. All pins are in double shear, and each has a diameter of 18 mm.



**SOLUTION**

**Support Reactions:** Referring to the FBD of the entire beam, Fig. a,

$$\zeta + \sum M_A = 0; \quad P(0.5) + 6P(2) + 3P(4) + P(5.5) - F_{BC} \left( \frac{3}{5} \right)(6) = 0$$

$$F_{BC} = 8.3333P$$

$$\zeta + \sum M_B = 0; \quad A_y(6) - P(0.5) - 3P(2) - 6P(4) - P(5.5) = 0 \quad A_y = 6.00P$$

$$\pm \sum F_x = 0; \quad 8.3333P \left( \frac{4}{5} \right) - A_x = 0 \quad A_x = 6.6667P$$

Thus,

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{(6.6667P)^2 + (6.00P)^2} = 8.9691P$$

**Average Shear Stress:** Since all the pins are subjected to double shear,

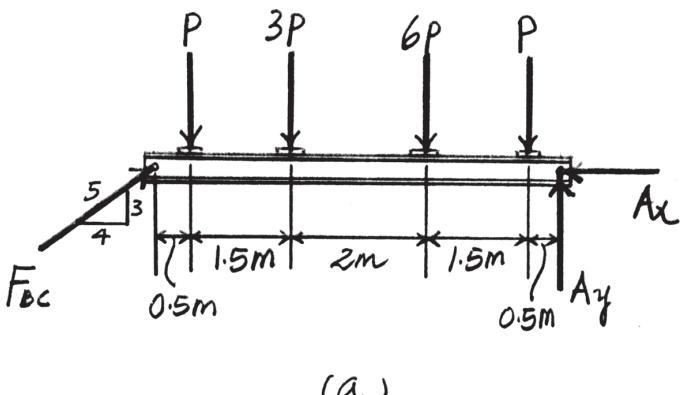
$$V_B = V_C = \frac{F_{BC}}{2} = \frac{8.3333P}{2} = 4.1667P \text{ (Fig. b)}$$

and  $V_A = 4.4845P$  (Fig. c). Since pin A is subjected to a larger shear force, it is critical. Thus

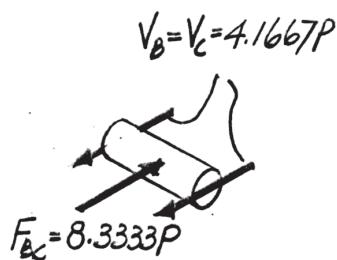
$$\tau_{\text{allow}} = \frac{V_A}{A}; \quad 80(10^6) = \frac{4.4845P}{\frac{\pi}{4}(0.018^2)}$$

$$P = 4.539(10^3) \text{ N} = 4.54 \text{ kN}$$

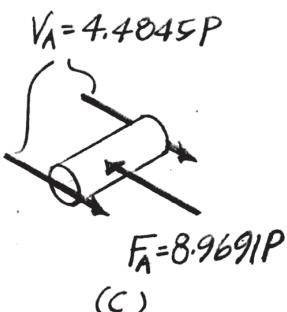
**Ans.**



(a)



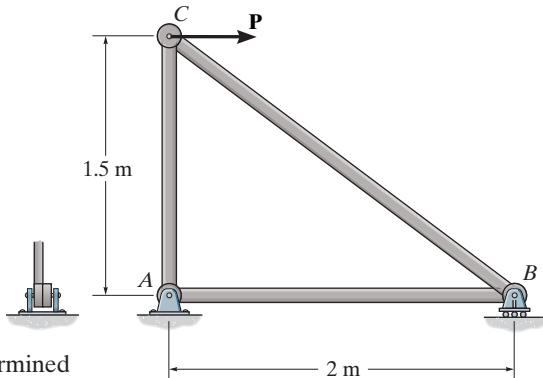
(b)



**Ans:**  
 $P = 4.54 \text{ kN}$

**1-39.**

Determine the average normal stress in each of the 20-mm-diameter bars of the truss. Set  $P = 40 \text{ kN}$ .



**SOLUTION**

**Internal Loadings:** The force developed in each member of the truss can be determined by using the method of joints. First, consider the equilibrium of joint C, Fig. a,

$$\pm \sum F_x = 0; \quad 40 - F_{BC} \left( \frac{4}{5} \right) = 0 \quad F_{BC} = 50 \text{ kN (C)}$$

$$+ \uparrow \sum F_y = 0; \quad 50 \left( \frac{3}{5} \right) - F_{AC} = 0 \quad F_{AC} = 30 \text{ kN (T)}$$

Subsequently, the equilibrium of joint B, Fig. b, is considered

$$\pm \sum F_x = 0; \quad 50 \left( \frac{4}{5} \right) - F_{AB} = 0 \quad F_{AB} = 40 \text{ kN (T)}$$

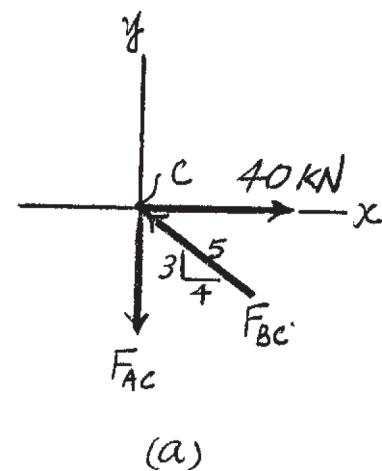
**Average Normal Stress:** The cross-sectional area of each of the bars is

$$A = \frac{\pi}{4} (0.02^2) = 0.3142(10^{-3}) \text{ m}^2. \text{ We obtain,}$$

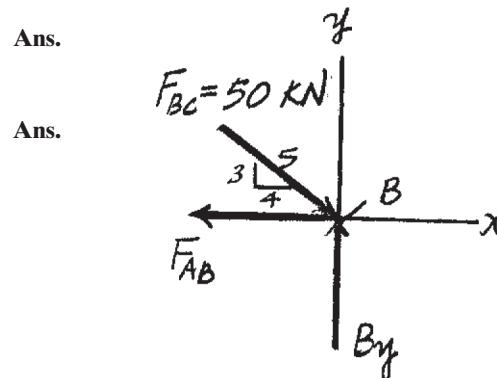
$$(\sigma_{\text{avg}})_{BC} = \frac{F_{BC}}{A} = \frac{50(10^3)}{0.3142(10^{-3})} = 159 \text{ MPa} \quad \text{Ans.}$$

$$(\sigma_{\text{avg}})_{AC} = \frac{F_{AC}}{A} = \frac{30(10^3)}{0.3142(10^{-3})} = 95.5 \text{ MPa} \quad \text{Ans.}$$

$$(\sigma_{\text{avg}})_{AB} = \frac{F_{AB}}{A} = \frac{40(10^3)}{0.3142(10^{-3})} = 127 \text{ MPa} \quad \text{Ans.}$$



(a)



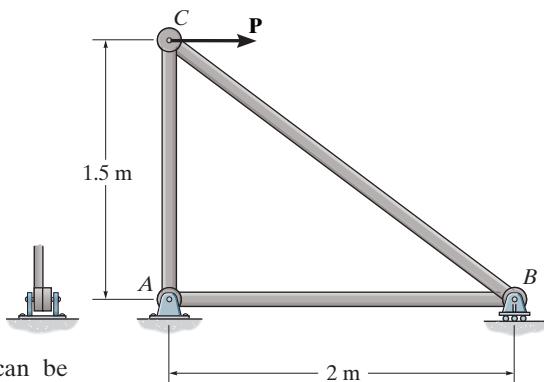
(b)

**Ans:**

$$(\sigma_{\text{avg}})_{BC} = 159 \text{ MPa}, \\ (\sigma_{\text{avg}})_{AC} = 95.5 \text{ MPa}, \\ (\sigma_{\text{avg}})_{AB} = 127 \text{ MPa}$$

**\*1-40.**

If the average normal stress in each of the 20-mm-diameter bars is not allowed to exceed 150 MPa, determine the maximum force  $\mathbf{P}$  that can be applied to joint  $C$ .



**SOLUTION**

**Internal Loadings:** The force developed in each member of the truss can be determined by using the method of joints. First, consider the equilibrium of joint  $C$ , Fig. a.

$$\pm \sum F_x = 0; \quad P - F_{BC} \left( \frac{4}{5} \right) = 0 \quad F_{BC} = 1.25P(C)$$

$$+ \uparrow \sum F_y = 0; \quad 1.25P \left( \frac{3}{5} \right) - F_{AC} = 0 \quad F_{AC} = 0.75P(T)$$

Subsequently, the equilibrium of joint  $B$ , Fig. b, is considered.

$$\pm \sum F_x = 0; \quad 1.25P \left( \frac{4}{5} \right) - F_{AB} = 0 \quad F_{AB} = P(T)$$

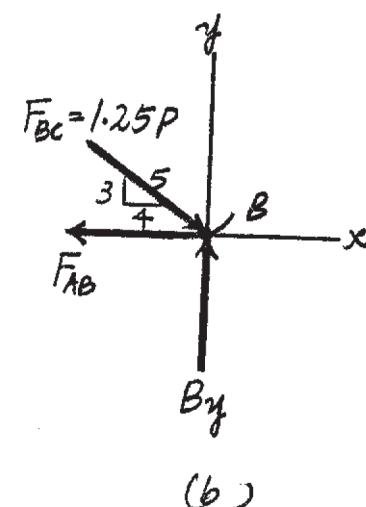
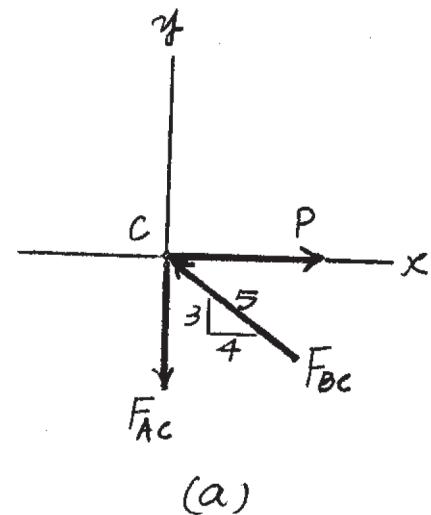
**Average Normal Stress:** Since the cross-sectional area and the allowable normal stress of each bar are the same, member  $BC$ , which is subjected to the maximum normal force, is the critical member. The cross-sectional area of each of the bars is

$$A = \frac{\pi}{4} (0.02)^2 = 0.3142(10^{-3}) \text{ m}^2. \text{ We have,}$$

$$(\sigma_{\text{avg}})_{\text{allow}} = \frac{F_{BC}}{A}; \quad 150(10^6) = \frac{1.25P}{0.3142(10^{-3})}$$

$$P = 37\,699 \text{ N} = 37.7 \text{ kN}$$

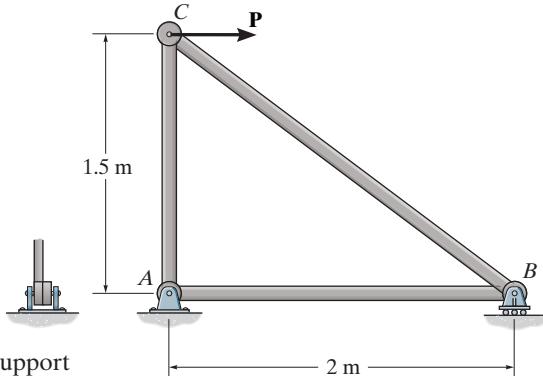
**Ans.**



**Ans:**  
 $P = 37.7 \text{ kN}$

**1-41.**

Determine the maximum average shear stress in pin A of the truss. A horizontal force of  $P = 40 is applied to joint C. Each pin has a diameter of 25 mm and is subjected to double shear.$



**SOLUTION**

**Internal Loadings:** The forces acting on pins A and B are equal to the support reactions at A and B. Referring to the free-body diagram of the entire truss, Fig. a,

$$\sum M_A = 0; \quad B_y(2) - 40(1.5) = 0$$

$$B_y = 30 \text{ kN}$$

$$\pm \sum F_x = 0; \quad 40 - A_x = 0$$

$$A_x = 40 \text{ kN}$$

$$+\uparrow \sum F_y = 0; \quad 30 - A_y = 0$$

$$A_y = 30 \text{ kN}$$

Thus,

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{40^2 + 30^2} = 50 \text{ kN}$$

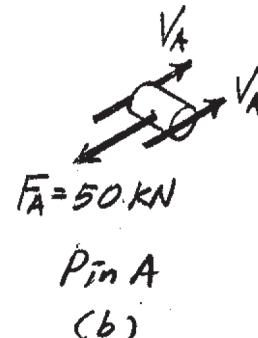
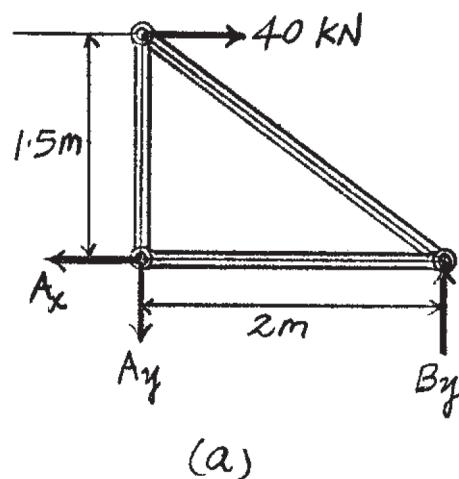
Since pin A is in double shear, Fig. b, the shear forces developed on the shear planes of pin A are

$$V_A = \frac{F_A}{2} = \frac{50}{2} = 25 \text{ kN}$$

**Average Shear Stress:** The area of the shear plane for pin A is  $A_A = \frac{\pi}{4}(0.025^2) = 0.4909(10^{-3}) \text{ m}^2$ . We have

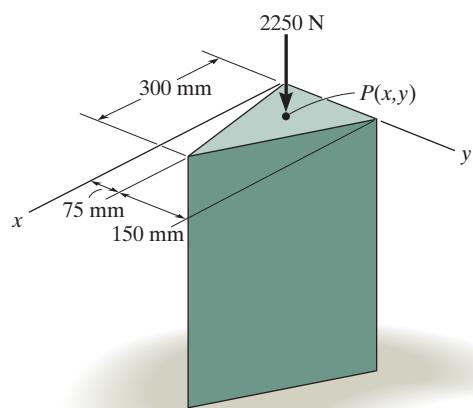
$$(\tau_{\text{avg}})_A = \frac{V_A}{A_A} = \frac{25(10^3)}{0.4909(10^{-3})} = 50.9 \text{ MPa}$$

**Ans.**



**Ans:**  
 $(\tau_{\text{avg}})_A = 50.9 \text{ MPa}$

**1-42.** The pedestal has a triangular cross section as shown. If it is subjected to a compressive force of 2250 N, specify the  $x$  and  $y$  coordinates for the location of point  $P(x, y)$ , where the load must be applied on the cross section, so that the average normal stress is uniform. Compute the stress and sketch its distribution acting on the cross section at a location removed from the point of load application.



## SOLUTION

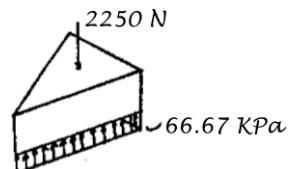
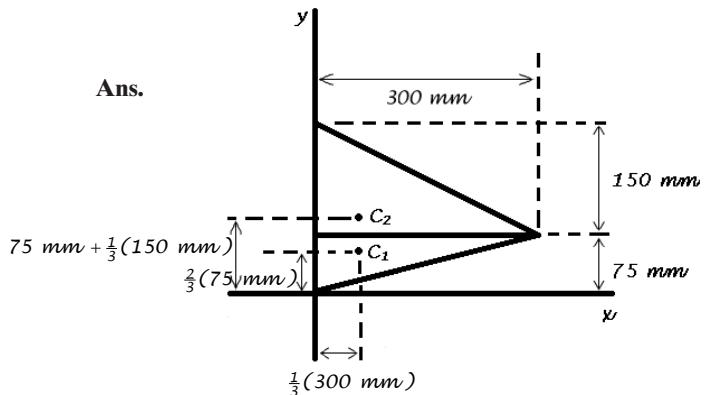
$$x = \frac{\left[\frac{2}{3}(75)\right]\left[\frac{1}{2}(75)(300)\right] + \left[75 + \frac{1}{3}(150)\right]\left[\frac{1}{2}(150)(300)\right]}{\frac{1}{2}(75)(300) + \frac{1}{2}(150)(300)} = 100 \text{ mm} \quad \text{Ans.}$$

$$y = \frac{1}{3}(300 \text{ mm}) = 100 \text{ mm} \quad \text{Ans.}$$

$$\begin{aligned} \sigma &= \frac{P}{A} = \frac{2250}{\frac{1}{2}(0.225)(0.3)} \\ &= 66.67(10^3) \text{ N/m}^2 \\ &= 66.7 \text{ kPa} \end{aligned}$$

**Ans.**

**Ans.**

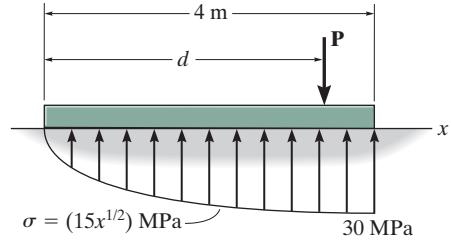


**Ans:**

$$x = 100 \text{ mm}, y = 100 \text{ mm}, \sigma = 66.7 \text{ kPa}$$

**1-43.**

The plate has a width of 0.5 m. If the stress distribution at the support varies as shown, determine the force  $\mathbf{P}$  applied to the plate and the distance  $d$  to where it is applied.



**SOLUTION**

The resultant force  $dF$  of the bearing pressure acting on the plate of area  $dA = b dx = 0.5 dx$ , Fig. a,

$$dF = \sigma_b dA = (15x^{1/2})(10^6)(0.5dx) = 7.5(10^6)x^{1/2}dx$$

$$+\uparrow \sum F_y = 0; \quad \int dF - P = 0$$

$$\int_0^{4\text{m}} 7.5(10^6)x^{1/2}dx - P = 0$$

$$P = 40(10^6) \text{ N} = 40 \text{ MN}$$

**Ans.**

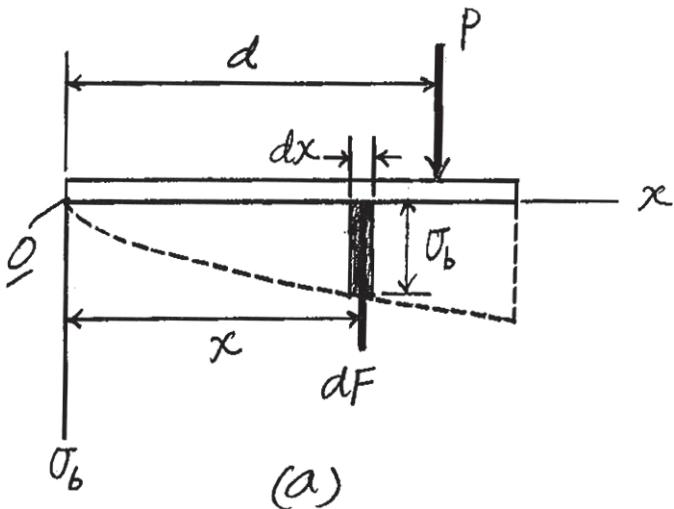
Equilibrium requires

$$\zeta + \sum M_O = 0; \quad \int x dF - Pd = 0$$

$$\int_0^{4\text{m}} x[7.5(10^6)x^{1/2}dx] - 40(10^6)d = 0$$

$$d = 2.40 \text{ m}$$

**Ans.**

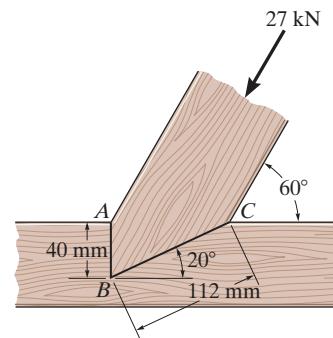


**Ans:**

$P = 40 \text{ MN}, d = 2.40 \text{ m}$

\*1-44.

The joint is subjected to the axial member force of 27 kN. Determine the average normal stress acting on sections AB and BC. Assume the member is smooth and is 40 mm thick.



### SOLUTION

$$+\uparrow \Sigma F_y = 0; \quad -27 \sin 60^\circ + N_{BC} \cos 20^\circ = 0$$

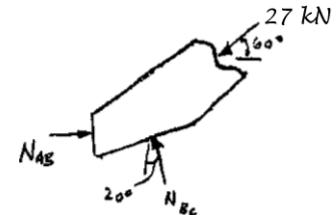
$$N_{BC} = 24.88 \text{ kN}$$

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \quad N_{AB} - 27 \cos 60^\circ - 24.88 \sin 20^\circ = 0$$

$$N_{AB} = 22.01 \text{ kN}$$

$$\sigma_{AB} = \frac{N_{AB}}{A_{AB}} = \frac{22.01 (10^3)}{(0.04)(0.04)} = 13.76(10^6) \text{ N/m}^2 = 13.8 \text{ MPa} \quad \text{Ans.}$$

$$\sigma_{BC} = \frac{N_{BC}}{A_{BC}} = \frac{24.88 (10^3)}{(0.04)(0.112)} = 5.554(10^6) \text{ N/m}^2 = 5.55 \text{ MPa} \quad \text{Ans.}$$

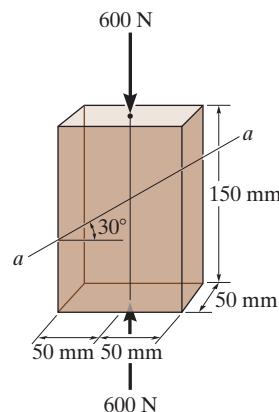


**Ans:**

$$\sigma_{AB} = 13.8 \text{ MPa}, \sigma_{BC} = 5.55 \text{ MPa}$$

**1–45.**

The plastic block is subjected to an axial compressive force of 600 N. Assuming that the caps at the top and bottom distribute the load uniformly throughout the block, determine the average normal and average shear stress acting along section  $a-a$ .



**SOLUTION**

Along  $a-a$ :

$$+\not\sum F_x = 0; \quad V - 600 \sin 30^\circ = 0$$

$$V = 300 \text{ N}$$

$$+\not\sum F_y = 0; \quad -N + 600 \cos 30^\circ = 0$$

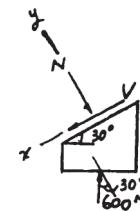
$$N = 519.6 \text{ N}$$

$$\sigma_{a-a} = \frac{519.6}{(0.05)\left(\frac{0.1}{\cos 30^\circ}\right)} = 90.0 \text{ kPa}$$

**Ans.**

$$\tau_{a-a} = \frac{300}{(0.05)\left(\frac{0.1}{\cos 30^\circ}\right)} = 52.0 \text{ kPa}$$

**Ans.**



**Ans:**  
 $\sigma_{a-a} = 90.0 \text{ kPa}$ ,  
 $\tau_{a-a} = 52.0 \text{ kPa}$

**1-46.**

The column is made of concrete having a density of  $2.30 \text{ Mg/m}^3$ . At its top  $B$  it is subjected to an axial compressive force of  $15 \text{ kN}$ . Determine the average normal stress in the column as a function of the distance  $z$  measured from its base.

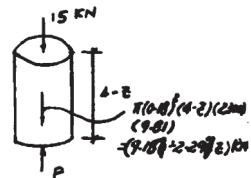
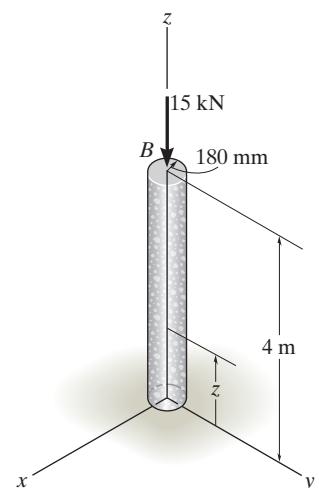
**SOLUTION**

$$+\uparrow \sum F_y = 0 \quad P - 15 - 9.187 + 2.297z = 0$$

$$P = 24.187 - 2.297z$$

$$\sigma = \frac{P}{A} = \frac{24.187 - 2.297z}{\pi(0.18)^2} = (238 - 22.6z) \text{ kPa}$$

**Ans.**

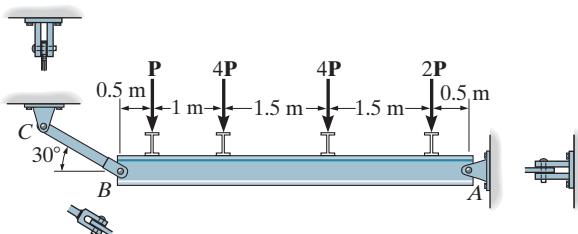


**Ans:**

$$\sigma = (238 - 22.6z) \text{ kPa}$$

**1-47.**

If  $P = 15 \text{ kN}$ , determine the average shear stress in the pins at  $A$ ,  $B$ , and  $C$ . All pins are in double shear, and each has a diameter of 18 mm.

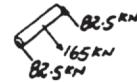


## SOLUTION

For pins  $B$  and  $C$ :

$$\tau_B = \tau_C = \frac{V}{A} = \frac{82.5 (10^3)}{\frac{\pi}{4} (\frac{18}{1000})^2} = 324 \text{ MPa}$$

**Ans.**



For pin  $A$ :

$$F_A = \sqrt{(82.5)^2 + (142.9)^2} = 165 \text{ kN}$$

$$\tau_A = \frac{V}{A} = \frac{82.5 (10^3)}{\frac{\pi}{4} (\frac{18}{1000})^2} = 324 \text{ MPa}$$

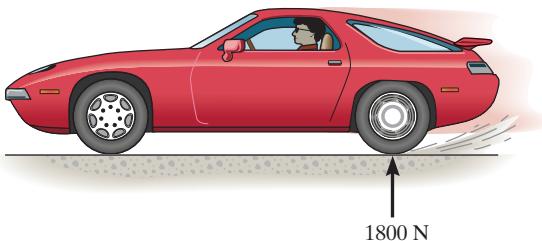
**Ans.**



**Ans:**

$$\begin{aligned}\tau_B &= 324 \text{ MPa}, \\ \tau_A &= 324 \text{ MPa}\end{aligned}$$

\* 1-48. The driver of the sports car applies his rear brakes and causes the tires to slip. If the normal force on each rear tire is 1800 N and the coefficient of kinetic friction between the tires and the pavement is  $\mu_k = 0.5$ , determine the average shear stress developed by the friction force on the tires. Assume the rubber of the tires is flexible and each tire is filled with an air pressure of 225 KPa



## SOLUTION

From the tire pressure,

$$p = \frac{N}{A}; \quad 225(10^3) = \frac{1800}{A} \quad A = 0.008 \text{ m}^2$$

The friction is

$$F = \mu_k N = 0.5(1800) = 900 \text{ N}$$

Then the shear stress is

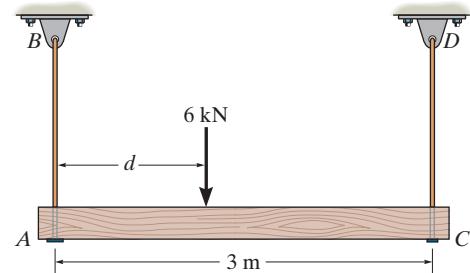
$$\tau_{\text{avg}} = \frac{F}{A} = \frac{900}{0.008} = 112.5(10^3) \text{ N/m}^2 = 112.5 \text{ KPa}$$

**Ans.**

**Ans:**  
 $\tau_{\text{avg}} = 112.5 \text{ KPa}$

**1-49.**

The beam is supported by two rods  $AB$  and  $CD$  that have cross-sectional areas of  $12 \text{ mm}^2$  and  $8 \text{ mm}^2$ , respectively. If  $d = 1 \text{ m}$ , determine the average normal stress in each rod.



**SOLUTION**

$$\zeta + \sum M_A = 0; \quad F_{CD}(3) - 6(1) = 0$$

$$F_{CD} = 2 \text{ kN}$$

$$+\uparrow \sum F_y = 0; \quad F_{AB} - 6 + 2 = 0$$

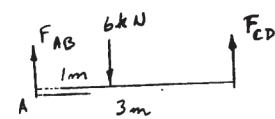
$$F_{AB} = 4 \text{ kN}$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{4(10^3)}{12(10^{-6})} = 333 \text{ MPa}$$

**Ans.**

$$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{2(10^3)}{8(10^{-6})} = 250 \text{ MPa}$$

**Ans.**

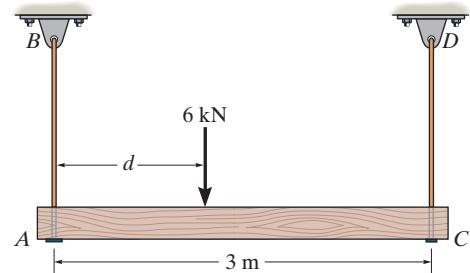


**Ans:**

$$\sigma_{AB} = 333 \text{ MPa}, \\ \sigma_{CD} = 250 \text{ MPa}$$

**1-50.**

The beam is supported by two rods *AB* and *CD* that have cross-sectional areas of  $12 \text{ mm}^2$  and  $8 \text{ mm}^2$ , respectively. Determine the position *d* of the 6-kN load so that the average normal stress in each rod is the same.



**SOLUTION**

$$\zeta + \sum M_O = 0; \quad F_{CD}(3 - d) - F_{AB}(d) = 0 \quad (1)$$

$$\sigma = \frac{F_{AB}}{12} = \frac{F_{CD}}{8}$$

$$F_{AB} = 1.5 F_{CD} \quad (2)$$

From Eqs. (1) and (2),

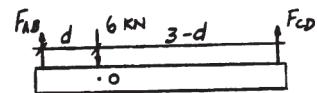
$$F_{CD}(3 - d) - 1.5 F_{CD}(d) = 0$$

$$F_{CD}(3 - d - 1.5 d) = 0$$

$$3 - 2.5 d = 0$$

$$d = 1.20 \text{ m}$$

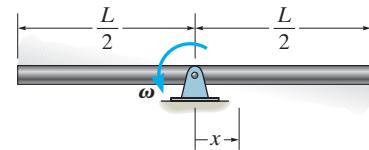
**Ans.**



**Ans:**  
 $d = 1.20 \text{ m}$

**1-51.**

The uniform bar, having a cross-sectional area of  $A$  and mass per unit length of  $m$ , is pinned at its center. If it is rotating in the horizontal plane at a constant angular rate of  $\omega$ , determine the average normal stress in the bar as a function of  $x$ .



## SOLUTION

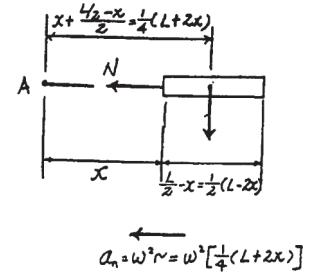
**Equation of Motion:**

$$\begin{aligned} \sum F_x &= ma_N; \quad N = m \left[ \frac{1}{2}(L - 2x) \right] \omega^2 \left[ \frac{1}{4}(L + 2x) \right] \\ &= \frac{m\omega^2}{8} (L^2 - 4x^2) \end{aligned}$$

**Average Normal Stress:**

$$\sigma = \frac{N}{A} = \frac{m\omega^2}{8A} (L^2 - 4x^2)$$

**Ans.**



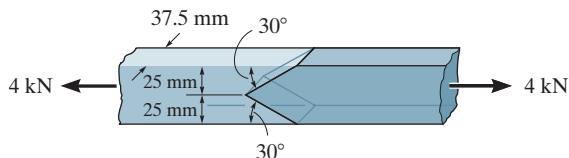
$$\sigma_n = \omega^2 r = \omega^2 \left[ \frac{1}{4}(L + 2x) \right]$$

**Ans:**  

$$\sigma = \frac{m\omega^2}{8A} (L^2 - 4x^2)$$

**\*1–52.**

The two members used in the construction of an aircraft fuselage are joined together using a  $30^\circ$  fish-mouth weld. Determine the average normal and average shear stress on the plane of each weld. Assume each inclined plane supports a horizontal force of 2 kN.



**SOLUTION**

$$\nabla + \sum F_y = 0; \quad N - 2 \sin 30^\circ = 0; \quad N = 1 \text{ kN}$$

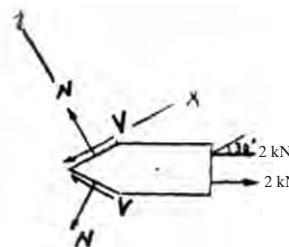
$$+\not\! \sum F_x = 0; \quad 2 \cos 30^\circ - V = 0; \quad V = 1.732 \text{ kN}$$

$$A' = \frac{(0.0375)(0.025)}{\sin 30^\circ} = 1.875(10^{-3}) \text{ m}^2$$

$$\sigma = \frac{N}{A'} = \frac{1(10^3)}{1.875(10^{-3}) \text{ m}^2} = 533 \text{ kPa}$$

$$\tau = \frac{V}{A'} = \frac{1.732(10^3)}{1.875(10^{-3}) \text{ m}^2} = 924 \text{ kPa}$$

**Ans.**



**Ans.**

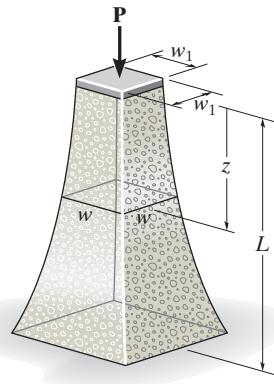
**Ans:**

$$\sigma = 533 \text{ kPa}$$

$$\tau = 924 \text{ kPa}$$

**1-53.**

The pier is made of material having a specific weight  $\gamma$ . If it has a square cross section, determine its width  $w$  as a function of  $z$  so that the average normal stress in the pier remains constant. The pier supports a constant load  $\mathbf{P}$  at its top where its width is  $w_1$ .



## SOLUTION

Assume constant stress  $\sigma_1$ , then at the top,

$$\sigma_1 = \frac{P}{w_1^2} \quad (1)$$

For an increase in  $z$  the area must increase,

$$dA = \frac{dW}{\sigma_1} = \frac{\gamma A dz}{\sigma_1} \quad \text{or} \quad \frac{dA}{A} = \frac{\gamma}{\sigma_1} dz$$

For the top section:

$$\int_{A_1}^A \frac{dA}{A} = \frac{\gamma}{\sigma_1} \int_0^z dz$$

$$\ln \frac{A}{A_1} = \frac{\gamma}{\sigma_1} z$$

$$A = A_1 e^{(\frac{\gamma}{\sigma_1})z}$$

$$A = w^2$$

$$A_1 = w_1^2$$

$$w = w_1 e^{(\frac{\gamma}{\sigma_1})z}$$

From Eq. (1),

$$w = w_1 e^{\left[ \frac{w_1^2 \gamma}{2P} \right] z}$$

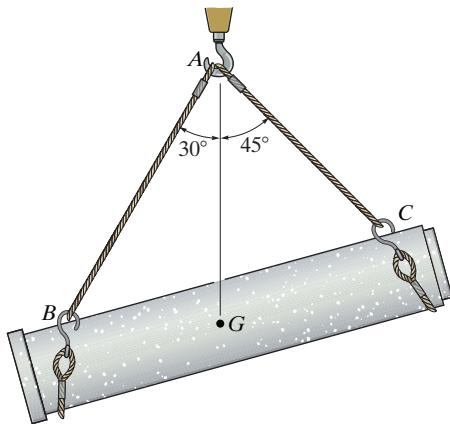
**Ans.**

**Ans:**

$$w = w_1 e^{\left[ \frac{w_1^2 \gamma}{2P} \right] z}$$

**1-54.**

The 2-Mg concrete pipe has a center of mass at point *G*. If it is suspended from cables *AB* and *AC*, determine the average normal stress in the cables. The diameters of *AB* and *AC* are 12 mm and 10 mm, respectively.



**SOLUTION**

**Internal Loadings:** The normal force developed in cables *AB* and *AC* can be determined by considering the equilibrium of the hook for which the free-body diagram is shown in Fig. *a*.

$$\sum F_x' = 0; \quad 2000(9.81) \cos 45^\circ - F_{AB} \cos 15^\circ = 0 \quad F_{AB} = 14\,362.83 \text{ N (T)}$$

$$\sum F_y' = 0; \quad 2000(9.81) \sin 45^\circ - 14\,362.83 \sin 15^\circ - F_{AC} = 0 \quad F_{AC} = 10\,156.06 \text{ N (T)}$$

**Average Normal Stress:** The cross-sectional areas of cables *AB* and *AC* are

$$A_{AB} = \frac{\pi}{4}(0.012^2) = 0.1131(10^{-3}) \text{ m}^2 \quad \text{and} \quad A_{AC} = \frac{\pi}{4}(0.01^2) = 78.540(10^{-6}) \text{ m}^2.$$

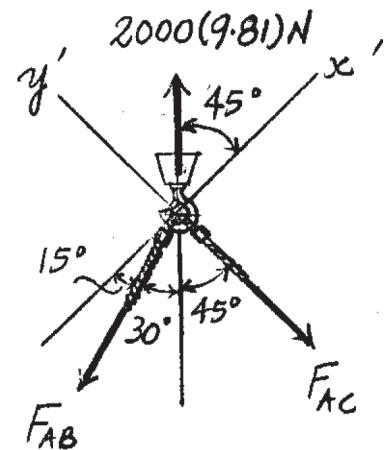
We have

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{14\,362.83}{0.1131(10^{-3})} = 127 \text{ MPa}$$

**Ans.**

$$\sigma_{AC} = \frac{F_{AC}}{A_{AC}} = \frac{10\,156.06}{78.540(10^{-6})} = 129 \text{ MPa}$$

**Ans.**



(a)

**Ans:**

$$\sigma_{AB} = 127 \text{ MPa}, \sigma_{AC} = 129 \text{ MPa}$$

**1–55.**

The 2-Mg concrete pipe has a center of mass at point *G*. If it is suspended from cables *AB* and *AC*, determine the diameter of cable *AB* so that the average normal stress in this cable is the same as in the 10-mm-diameter cable *AC*.

### SOLUTION

**Internal Loadings:** The normal force in cables *AB* and *AC* can be determined by considering the equilibrium of the hook for which the free-body diagram is shown in Fig. *a*.

$$\Sigma F_x' = 0; 2000(9.81) \cos 45^\circ - F_{AB} \cos 15^\circ = 0 \quad F_{AB} = 14\,362.83 \text{ N (T)}$$

$$\Sigma F_y' = 0; 2000(9.81) \sin 45^\circ - 14\,362.83 \sin 15^\circ - F_{AC} = 0 \quad F_{AC} = 10\,156.06 \text{ N (T)}$$

**Average Normal Stress:** The cross-sectional areas of cables *AB* and *AC* are

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 \text{ and } A_{AC} = \frac{\pi}{4} (0.01^2) = 78.540(10^{-6}) \text{ m}^2.$$

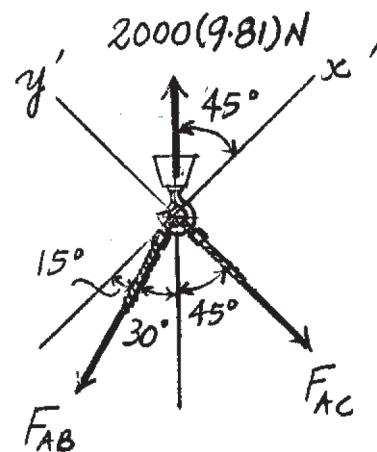
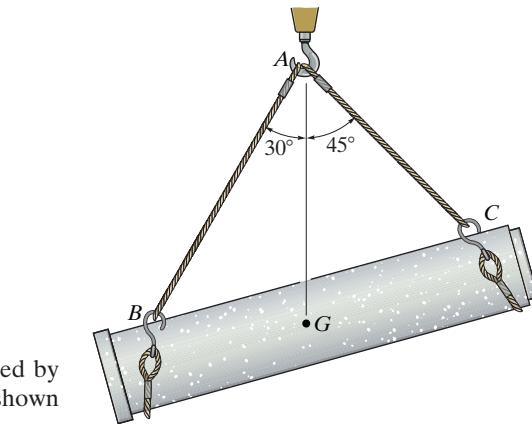
Here, we require

$$\sigma_{AB} = \sigma_{AC}$$

$$\frac{F_{AB}}{A_{AB}} = \frac{F_{AC}}{A_{AC}}$$

$$\frac{14\,362.83}{\frac{\pi}{4} d_{AB}^2} = \frac{10\,156.06}{78.540(10^{-6})}$$

$$d_{AB} = 0.01189 \text{ m} = 11.9 \text{ mm}$$



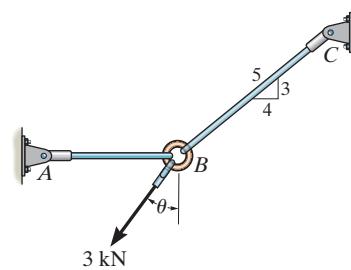
**Ans.**

(a)

**Ans:**  
 $d_{AB} = 11.9 \text{ mm}$

**\*1–56.**

Rods  $AB$  and  $BC$  have diameters of 4 mm and 6 mm, respectively. If the 3 kN force is applied to the ring at  $B$ , determine the angle  $\theta$  so that the average normal stress in each rod is equivalent. What is this stress?



**SOLUTION**

**Method of Joints:** Referring to the FBD of joint  $B$ , Fig. *a*,

$$+\uparrow \sum F_y = 0; \quad F_{BC} \left( \frac{3}{5} \right) - 3 \cos \theta = 0 \quad F_{BC} = 5 \cos \theta \text{ kN}$$

$$\pm \sum F_x = 0; \quad (5 \cos \theta) \left( \frac{4}{5} \right) - 3 \sin \theta - F_{AB} = 0 \quad F_{AB} = (4 \cos \theta - 3 \sin \theta) \text{ kN}$$

**Average Normal Stress:**

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{[4 \cos \theta - 3 \sin \theta](10^3)}{\frac{\pi}{4}(0.004)^2} = \frac{250(10^6)}{\pi} (4 \cos \theta - 3 \sin \theta)$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{(5 \cos \theta)(10^3)}{\frac{\pi}{4}(0.006)^2} = \left[ \frac{555.56(10^6)}{\pi} \right] \cos \theta$$

It is required that

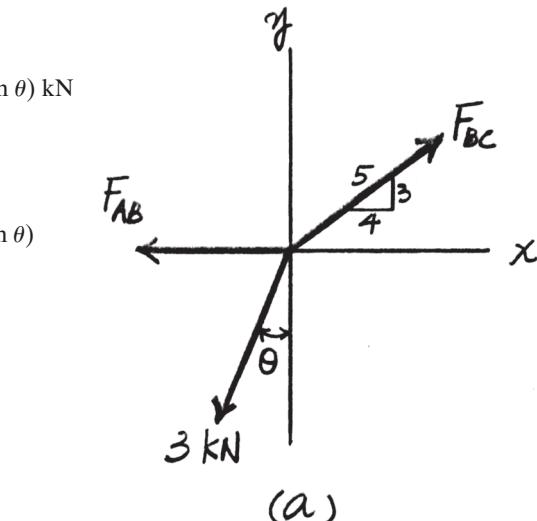
$$\sigma_{AB} = \sigma_{BC}$$

$$\frac{250(10^6)}{\pi} (4 \cos \theta - 3 \sin \theta) = \left[ \frac{555.56(10^6)}{\pi} \right] \cos \theta$$

$$1.7778 \cos \theta - 3 \sin \theta = 0$$

$$\tan \theta = \frac{1.7778}{3}$$

$$\theta = 30.65^\circ = 30.7^\circ$$



(a)

**Ans.**

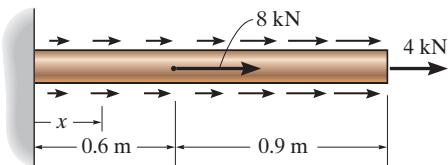
Then

$$\sigma = \sigma_{BC} = \left[ \frac{555.56(10^6)}{\pi} \right] \cos 30.65^\circ = 152.13(10^6) \text{ Pa} = 152 \text{ MPa} \quad \text{Ans.}$$

**Ans:**  
 $\theta = 30.7^\circ$ ,  
 $\sigma = 152 \text{ MPa}$

**1-57.**

The bar has a cross-sectional area of  $400(10^{-6}) \text{ m}^2$ . If it is subjected to a triangular axial distributed loading along its length which is 0 at  $x = 0$  and  $9 \text{ kN/m}$  at  $x = 1.5 \text{ m}$ , and to two concentrated loads as shown, determine the average normal stress in the bar as a function of  $x$  for  $0 \leq x < 0.6 \text{ m}$ .



## SOLUTION

**Internal Loading:** Referring to the FBD of the right segment of the bar sectioned at  $x$ , Fig. *a*,

$$\pm \sum F_x = 0; \quad 8 + 4 + \frac{1}{2}(6x + 9)(1.5 - x) = 0$$

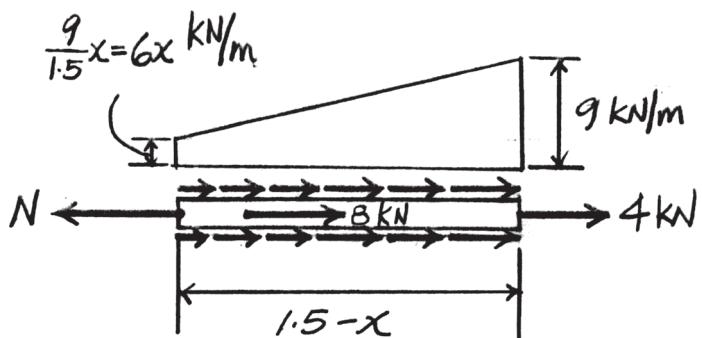
$$N = \{18.75 - 3x^2\} \text{ kN}$$

**Average Normal Stress:**

$$\sigma = \frac{N}{A} = \frac{(18.75 - 3x^2)(10^3)}{400(10^{-6})}$$

$$= \{46.9 - 7.50x^2\} \text{ MPa}$$

**Ans.**



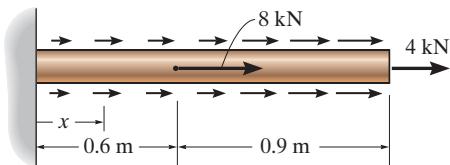
(a)

**Ans:**

$$\sigma = \{46.9 - 7.50x^2\} \text{ MPa}$$

**1-58.**

The bar has a cross-sectional area of  $400(10^{-6}) \text{ m}^2$ . If it is subjected to a uniform axial distributed loading along its length of  $9 \text{ kN/m}$ , and to two concentrated loads as shown, determine the average normal stress in the bar as a function of  $x$  for  $0.6 \text{ m} < x \leq 1.5 \text{ m}$ .



**SOLUTION**

**Internal Loading:** Referring to a FBD of the right segment of the bar sectioned at  $x$ ,

$$\pm \sum F_x = 0; \quad 4 + 9(1.5 - x) - N = 0$$

$$N = \{17.5 - 9x\} \text{ kN}$$

**Average Normal Stress:**

$$\begin{aligned}\sigma &= \frac{N}{A} = \frac{(17.5 - 9x)(10^3)}{400(10^{-6})} \\ &= \{43.75 - 22.5x\} \text{ MPa}\end{aligned}$$

**Ans.**

**Ans:**

$$\sigma = \{43.75 - 22.5x\} \text{ MPa}$$

**1-59.**

The two steel members are joined together using a  $30^\circ$  scarf weld. Determine the average normal and average shear stress resisted in the plane of the weld.

**SOLUTION**

**Internal Loadings:** Referring to the FBD of the upper segment of the member sectioned through the scarf weld, Fig. *a*,

$$\Sigma F_x = 0; \quad N - 15 \sin 30^\circ = 0 \quad N = 7.50 \text{ kN}$$

$$\Sigma F_y = 0; \quad V - 15 \cos 30^\circ = 0 \quad V = 12.99 \text{ kN}$$

**Average Normal and Shear Stress:** The area of the scarf weld is

$$A = 0.02 \left( \frac{0.04}{\sin 30^\circ} \right) = 1.6(10^{-3}) \text{ m}^2$$

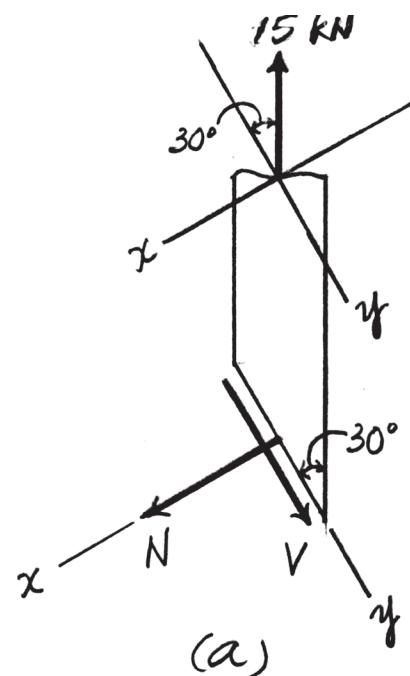
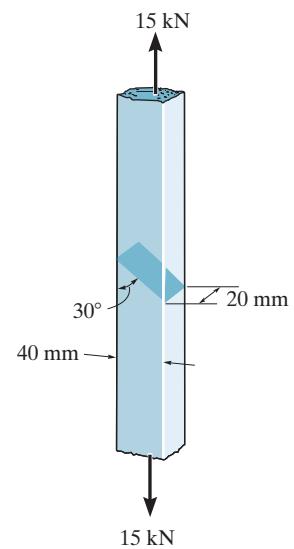
Thus,

$$\sigma = \frac{N}{A_n} = \frac{7.50(10^3)}{1.6(10^{-3})} = 4.6875(10^6) \text{ Pa} = 4.69 \text{ MPa}$$

**Ans.**

$$\tau = \frac{V}{A_v} = \frac{12.99(10^3)}{1.6(10^{-3})} = 8.119(10^6) \text{ Pa} = 8.12 \text{ MPa}$$

**Ans.**

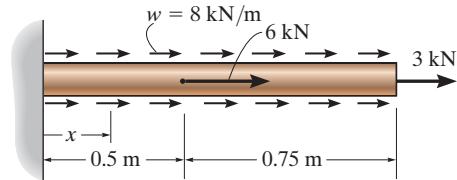


**Ans:**

$$\sigma = 4.69 \text{ MPa}, \quad \tau = 8.12 \text{ MPa}$$

**\*1–60.**

The bar has a cross-sectional area of  $400(10^{-6}) \text{ m}^2$ . If it is subjected to a uniform axial distributed loading along its length and to two concentrated loads, determine the average normal stress in the bar as a function of  $x$  for  $0 < x \leq 0.5 \text{ m}$ .



## SOLUTION

### Equation of Equilibrium:

$$\pm \sum F_x = 0; \quad -N + 3 + 6 + 8(1.25 - x) = 0$$

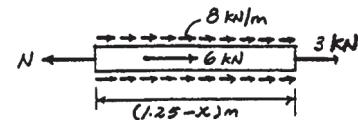
$$N = (19.0 - 8.00x) \text{ kN}$$

### Average Normal Stress:

$$\sigma = \frac{N}{A} = \frac{(19.0 - 8.00x)(10^3)}{400(10^{-6})}$$

$$= (47.5 - 20.0x) \text{ MPa}$$

**Ans.**

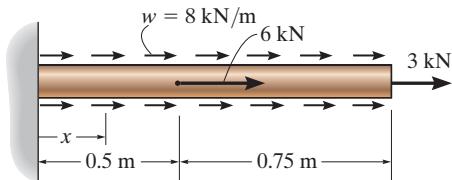


**Ans:**

$$\sigma = (47.5 - 20.0x) \text{ MPa}$$

**1-61.**

The bar has a cross-sectional area of  $400(10^{-6}) \text{ m}^2$ . If it is subjected to a uniform axial distributed loading along its length and to two concentrated loads, determine the average normal stress in the bar as a function of  $x$  for  $0.5 \text{ m} < x \leq 1.25 \text{ m}$ .



**SOLUTION**

**Equation of Equilibrium:**

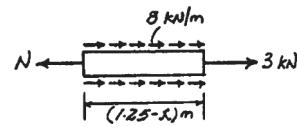
$$\pm \sum F_x = 0; \quad -N + 3 + 8(1.25 - x) = 0$$

$$N = (13.0 - 8.00x) \text{ kN}$$

**Average Normal Stress:**

$$\sigma = \frac{N}{A} = \frac{(13.0 - 8.00x)(10^3)}{400(10^{-6})}$$
$$= (32.5 - 20.0x) \text{ MPa}$$

**Ans.**

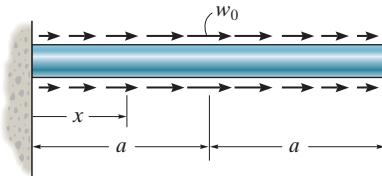


**Ans:**

$$\sigma = (32.5 - 20.0x) \text{ MPa}$$

**1–62.**

The prismatic bar has a cross-sectional area  $A$ . If it is subjected to a distributed axial loading that increases linearly from  $w = 0$  at  $x = 0$  to  $w = w_0$  at  $x = a$ , and then decreases linearly to  $w = 0$  at  $x = 2a$ , determine the average normal stress in the bar as a function of  $x$  for  $0 \leq x < a$ .



**SOLUTION**

**Equation of Equilibrium:**

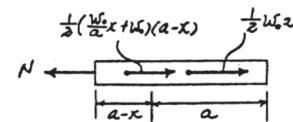
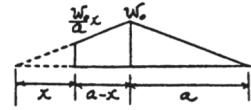
$$\pm \sum F_x = 0; \quad -N + \frac{1}{2} \left( \frac{w_0}{a} x + w_0 \right) (a - x) + \frac{1}{2} w_0 a = 0$$

$$N = \frac{w_0}{2a} (2a^2 - x^2)$$

**Average Normal Stress:**

$$\sigma = \frac{N}{A} = \frac{\frac{w_0}{2a} (2a^2 - x^2)}{A} = \frac{w_0}{2aA} (2a^2 - x^2)$$

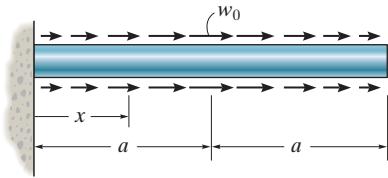
**Ans.**



**Ans:**  
 $\sigma = \frac{w_0}{2aA} (2a^2 - x^2)$

**1-63.**

The prismatic bar has a cross-sectional area  $A$ . If it is subjected to a distributed axial loading that increases linearly from  $w = 0$  at  $x = 0$  to  $w = w_0$  at  $x = a$ , and then decreases linearly to  $w = 0$  at  $x = 2a$ , determine the average normal stress in the bar as a function of  $x$  for  $a < x \leq 2a$ .



**SOLUTION**

**Equation of Equilibrium:**

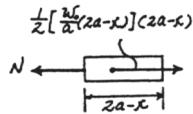
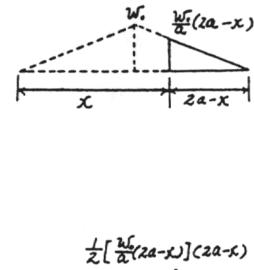
$$\pm \sum F_x = 0; \quad -N + \frac{1}{2} \left[ \frac{w_0}{a} (2a - x) \right] (2a - x) = 0$$

$$N = \frac{w_0}{2a} (2a - x)^2$$

**Average Normal Stress:**

$$\sigma = \frac{N}{A} = \frac{\frac{w_0}{2a} (2a - x)^2}{A} = \frac{w_0}{2aA} (2a - x)^2$$

**Ans.**

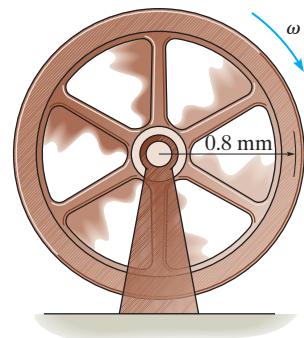


**Ans:**

$$\sigma = \frac{w_0}{2aA} (2a - x)^2$$

\*1-64.

Determine the greatest constant angular velocity  $\omega$  of the flywheel so that the average normal stress in its rim does not exceed  $\sigma = 15 \text{ MPa}$ . Assume the rim is a thin ring having a thickness of 3 mm, width of 20 mm, and a mass of  $30 \text{ kg/m}$ . Rotation occurs in the horizontal plane. Neglect the effect of the spokes in the analysis. Hint: Consider a free-body diagram of a semicircular segment of the ring. The center of mass for this segment is located at  $\hat{r} = 2r/\pi$  from the center.



**SOLUTION**

$$+\downarrow \sum F_n = m(a_G)_n;$$

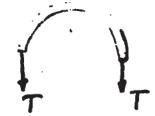
$$2T = m(\bar{r})\omega^2$$

$$2\sigma A = m\left(\frac{2r}{\pi}\right)\omega^2$$

$$2(15(10^6))(0.003)(0.020) = \pi(0.8)(30)\left(\frac{2(0.8)}{\pi}\right)\omega^2$$

$$\omega = 6.85 \text{ rad/s}$$

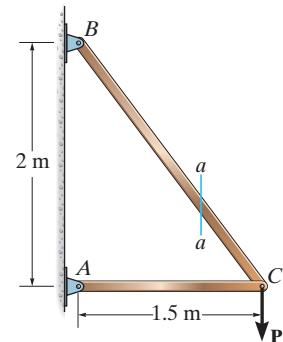
**Ans.**



**Ans:**  
 $\omega = 6.85 \text{ rad/s}$

**1–65.**

Determine the largest load  $\mathbf{P}$  that can be applied to the frame without causing either the average normal stress or the average shear stress at section  $a-a$  to exceed  $\sigma = 150 \text{ MPa}$  and  $\tau = 60 \text{ MPa}$ , respectively. Member  $CB$  has a square cross section of 25 mm on each side.



**SOLUTION**

Analyze the equilibrium of joint  $C$  using the FBD shown in Fig. *a*,

$$+\uparrow\sum F_y = 0; \quad F_{BC} \left(\frac{4}{5}\right) - P = 0 \quad F_{BC} = 1.25P$$

Referring to the FBD of the cut segment of member  $BC$  Fig. *b*.

$$\pm\sum F_x = 0; \quad N_{a-a} - 1.25P \left(\frac{3}{5}\right) = 0 \quad N_{a-a} = 0.75P$$

$$+\uparrow\sum F_y = 0; \quad 1.25P \left(\frac{4}{5}\right) - V_{a-a} = 0 \quad V_{a-a} = P$$

The cross-sectional area of section  $a-a$  is  $A_{a-a} = (0.025) \left(\frac{0.025}{3/5}\right) = 1.0417(10^{-3}) \text{ m}^2$ . For Normal stress,

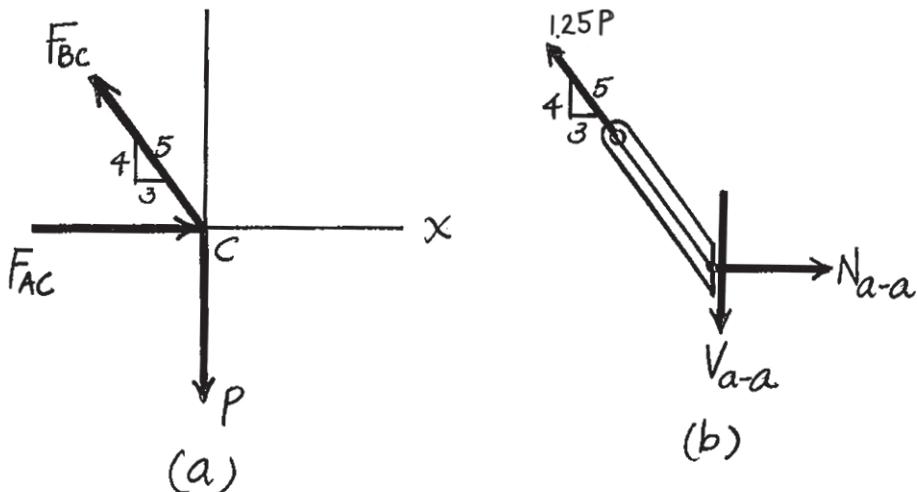
$$\sigma_{\text{allow}} = \frac{N_{a-a}}{A_{a-a}}; \quad 150(10^6) = \frac{0.75P}{1.0417(10^{-3})}$$

$$P = 208.33(10^3) \text{ N} = 208.33 \text{ kN}$$

For Shear Stress

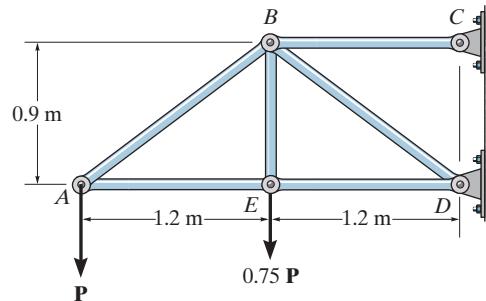
$$\tau_{\text{allow}} = \frac{V_{a-a}}{A_{a-a}}; \quad 60(10^6) = \frac{P}{1.0417(10^{-3})}$$

$$P = 62.5(10^3) \text{ N} = 62.5 \text{ kN} \text{ (Controls!)} \quad \text{Ans.}$$



**Ans:**  
 $P = 62.5 \text{ kN}$

- 1-66.** The bars of the truss each have a cross-sectional area of  $780 \text{ mm}^2$ . Determine the average normal stress in each member due to the loading  $P = 40 \text{ kN}$ . State whether the stress is tensile or compressive.



## SOLUTION

Joint A:

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{66.67(10^3)}{780(10^{-6})} = 85.5 \text{ MPa} \quad (\text{T})$$

**Ans.**

$$\sigma_{AE} = \frac{F_{AE}}{A_{AE}} = \frac{53.33(10^3)}{780(10^{-6})} = 68.4 \text{ MPa} \quad (\text{C})$$

**Ans.**

Joint E:

$$\sigma_{ED} = \frac{F_{ED}}{A_{ED}} = \frac{53.33(10^3)}{780(10^{-6})} = 68.4 \text{ MPa} \quad (\text{C})$$

**Ans.**

$$\sigma_{EB} = \frac{F_{EB}}{A_{EB}} = \frac{30(10^3)}{780(10^{-6})} = 38.5 \text{ MPa} \quad (\text{T})$$

**Ans.**

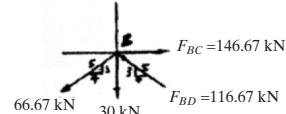
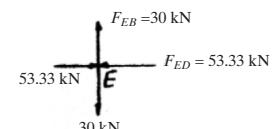
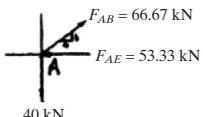
Joint B:

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{146.67(10^3)}{780(10^{-6})} = 188 \text{ MPa} \quad (\text{T})$$

**Ans.**

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}} = \frac{116.67(10^3)}{780(10^{-6})} = 150 \text{ MPa} \quad (\text{C})$$

**Ans.**



**Ans:**

Joint A:  $\sigma_{AB} = 85.5 \text{ MPa}$  (T),  
 $\sigma_{AE} = 68.4 \text{ MPa}$  (C)

Joint E:  $\sigma_{ED} = 68.4 \text{ MPa}$  (C),  
 $\sigma_{EB} = 38.5 \text{ MPa}$  (T)

Joint B:  $\sigma_{BC} = 188 \text{ MPa}$  (T),

- 1–67.** The bars of the truss each have a cross-sectional area of  $780 \text{ mm}^2$ . If the maximum average normal stress in any bar is not to exceed  $140 \text{ MPa}$ , determine the maximum magnitude  $P$  of the loads that can be applied to the truss.

### SOLUTION

Joint A:

$$+\uparrow \sum F_y = 0; \quad -P + \frac{3}{5} F_{AB} = 0$$

$$F_{AB} = 1.6667P \quad (\text{T})$$

$$+\rightarrow \sum F_x = 0; \quad -F_{AE} + (1.6667P)\left(\frac{4}{5}\right) = 0$$

$$F_{AE} = 1.3333P \quad (\text{C})$$

Joint E:

$$+\uparrow \sum F_y = 0; \quad F_{EB} - 0.75P = 0$$

$$F_{EB} = 0.75P \quad (\text{T})$$

$$+\rightarrow \sum F_x = 0; \quad 1.3333P - F_{ED} = 0$$

$$F_{ED} = 1.3333P \quad (\text{C})$$

Joint B:

$$+\uparrow \sum F_y = 0; \quad \frac{3}{5}F_{BD} - 0.75P - (1.6667P)\left(\frac{3}{5}\right) = 0$$

$$F_{BD} = 2.9167P \quad (\text{C})$$

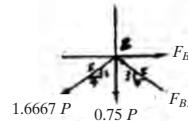
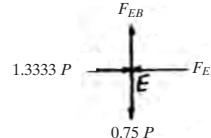
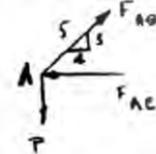
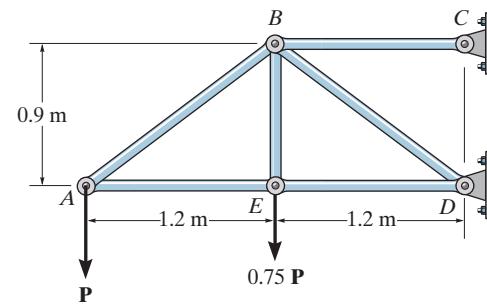
$$+\rightarrow \sum F_x = 0; \quad F_{BC} - (2.9167P)\left(\frac{4}{5}\right) - (1.6667P)\left(\frac{4}{5}\right) = 0$$

$$F_{BC} = 3.6667P \quad (\text{T})$$

The highest stressed member is  $BC$ :

$$\sigma_{BC} = \sigma_{\max}; \quad \frac{3.6667P}{780(10^{-6})} = 140(10^6)$$

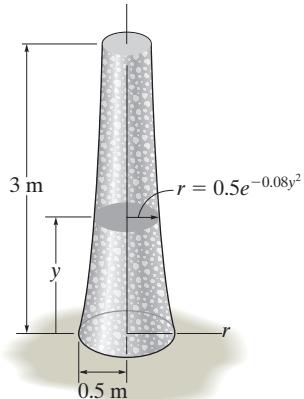
$$P = 29.78(10^3) \text{ N} = 29.8 \text{ kN}$$



**Ans:**  
 $P = 29.8 \text{ kN}$

**\*1–68.**

The radius of the pedestal is defined by  $r = (0.5e^{-0.08y^2})$  m, where  $y$  is in meters. If the material has a density of  $2.5 \text{ Mg/m}^3$ , determine the average normal stress at the support.



**SOLUTION**

$$A = \pi(0.5)^2 = 0.7854 \text{ m}^2$$

$$dV = \pi(r^2) dy = \pi(0.5)^2 (e^{-0.08y^2})^2 dy$$

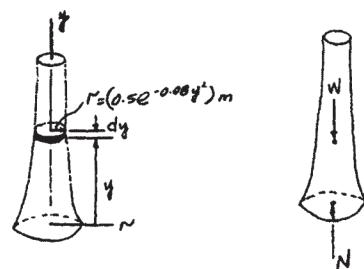
$$V = \int_0^3 \pi(0.5)^2 (e^{-0.08y^2})^2 dy = 0.7854 \int_0^3 (e^{-0.08y^2})^2 dy$$

$$W = \rho g V = (2500)(9.81)(0.7854) \int_0^3 (e^{-0.08y^2})^2 dy$$

$$W = 19.262(10^3) \int_0^3 (e^{-0.08y^2})^2 dy = 38.849 \text{ kN}$$

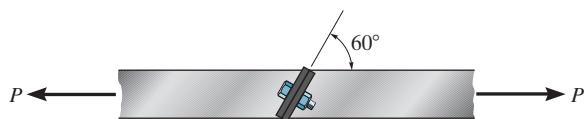
$$\sigma = \frac{W}{A} = \frac{38.849}{0.7854} = 49.5 \text{ kPa}$$

**Ans.**



**Ans:**  
 $\sigma = 49.5 \text{ kPa}$

- 1–69.** The tension member is fastened together using *two* bolts, one on each side of the member as shown. Each bolt has a diameter of 7.5 mm. Determine the maximum load  $P$  that can be applied to the member if the allowable shear stress for the bolts is  $\tau_{\text{allow}} = 84 \text{ MPa}$  and the allowable average normal stress is  $\sigma_{\text{allow}} = 140 \text{ MPa}$ .



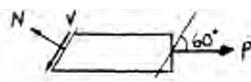
## SOLUTION

$$\nabla + \sum F_y = 0; \quad N - P \sin 60^\circ = 0$$

$$N = 0.8660 P$$

$$\checkmark + \sum F_x = 0; \quad V - P \cos 60^\circ = 0$$

$$V = 0.5P$$



Assume failure due to shear:

$$\tau_{\text{allow}} = \frac{V}{2A_b}; \quad 84(10^6) = \frac{0.5P}{2\left[\frac{\pi}{4}(0.0075^2)\right]}$$

$$P = 14.84(10^3) \text{ N} = 14.84 \text{ kN}$$

Assume failure due to normal force:

$$\sigma_{\text{allow}} = \frac{N}{2A_b}; \quad 140(10^6) = \frac{0.8660P}{2\left[\frac{\pi}{4}(0.0075^2)\right]}$$

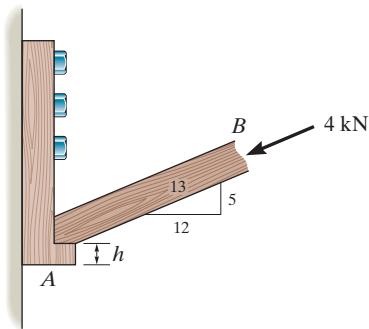
$$P = 14.28(10^3) \text{ N} = 14.3 \text{ kN} \text{ (controls)}$$

**Ans.**

**Ans:**

$P = 14.3 \text{ kN}$  (controls)

**1–70.** Member *B* is subjected to a compressive force of 4 kN. If *A* and *B* are both made of wood and are 10 mm thick, determine to the nearest multiples of 5 mm the smallest dimension *h* of the horizontal segment so that it does not fail in shear. The average shear stress for the segment is  $\tau_{\text{allow}} = 2.1 \text{ MPa}$ .



### SOLUTION

$$\tau_{\text{allow}} = \frac{V}{A}; 2.1(10^6) = \frac{1.538(10^3)}{(0.01)h}$$

$$h = 0.07326 \text{ m} = 73.26 \text{ mm}$$

Use  $h = 75 \text{ mm}$

**Ans.**

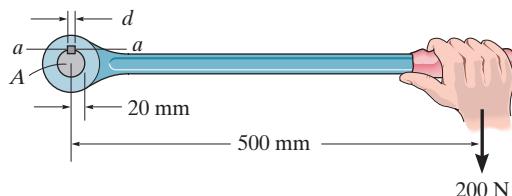


**Ans:**

$h = 0.07326 \text{ m}$ , use  $h = 75 \text{ mm}$

**1-71.**

The lever is attached to the shaft *A* using a key that has a width *d* and length of 25 mm. If the shaft is fixed and a vertical force of 200 N is applied perpendicular to the handle, determine the dimension *d* if the allowable shear stress for the key is  $\tau_{\text{allow}} = 35 \text{ MPa}$ .



**SOLUTION**

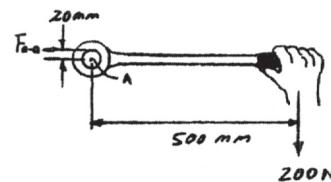
$$\zeta + \sum M_A = 0; \quad F_{a-a}(20) - 200(500) = 0$$

$$F_{a-a} = 5000 \text{ N}$$

$$\tau_{\text{allow}} = \frac{F_{a-a}}{A_{a-a}}; \quad 35(10^6) = \frac{5000}{d(0.025)}$$

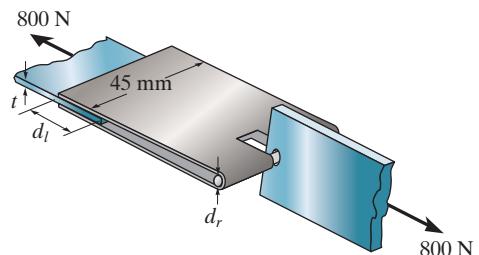
$$d = 0.00571 \text{ m} = 5.71 \text{ mm}$$

**Ans.**



**Ans:**  
 $d = 5.71 \text{ mm}$

**\*1-72.** The lapbelt assembly is to be subjected to a force of 800 N. Determine (a) the required thickness  $t$  of the belt if the allowable tensile stress for the material is  $(\sigma_t)_{allow} = 10 \text{ MPa}$ , (b) the required lap length  $d_l$  if the glue can sustain an allowable shear stress of  $(\tau_{allow})_g = 0.75 \text{ MPa}$ , and (c) the required diameter  $d_r$  of the pin if the allowable shear stress for the pin is  $(\tau_{allow})_p = 30 \text{ MPa}$ .



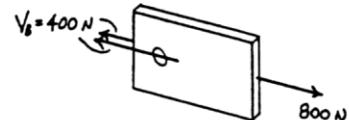
## SOLUTION

**Allowable Normal Stress:** Design of belt thickness.

$$(\sigma_t)_{allow} = \frac{P}{A}; \quad 10(10^6) = \frac{800}{(0.045)t}$$

$$t = 0.001778 \text{ m} = 1.78 \text{ mm}$$

**Ans.**

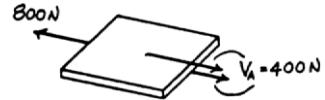


**Allowable Shear Stress:** Design of lap length.

$$(\tau_{allow})_g = \frac{V_A}{A}; \quad 0.750(10^6) = \frac{400}{(0.045)d_l}$$

$$d_l = 0.01185 \text{ m} = 11.9 \text{ mm}$$

**Ans.**



**Allowable Shear Stress:** Design of pin size.

$$(\tau_{allow})_p = \frac{V_B}{A}; \quad 30(10^6) = \frac{400}{\frac{\pi}{4} d_r^2}$$

$$d_r = 0.004120 \text{ m} = 4.12 \text{ mm}$$

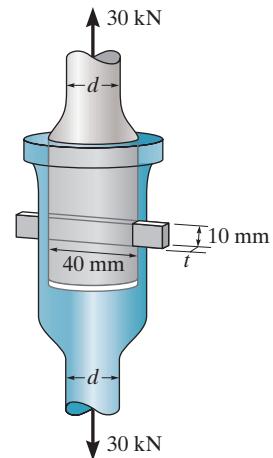
**Ans.**

**Ans:**

$$t = 1.78 \text{ mm}, d_l = 11.9 \text{ mm}, d_r = 4.12 \text{ mm}$$

**1–73.**

The cotter is used to hold the two rods together. Determine the smallest thickness  $t$  of the cotter and the smallest diameter  $d$  of the rods. All parts are made of steel for which the failure normal stress is  $\sigma_{\text{fail}} = 500 \text{ MPa}$  and the failure shear stress is  $\tau_{\text{fail}} = 375 \text{ MPa}$ . Use a factor of safety of  $(\text{F.S.})_t = 2.50$  in tension and  $(\text{F.S.})_s = 1.75$  in shear.



**SOLUTION**

**Allowable Normal Stress:** Design of rod size

$$\sigma_{\text{allow}} = \frac{\sigma_{\text{fail}}}{\text{F.S.}} = \frac{P}{A}; \quad \frac{500(10^6)}{2.5} = \frac{30(10^3)}{\frac{\pi}{4}d^2}$$

$$d = 0.01382 \text{ m} = 13.8 \text{ mm}$$

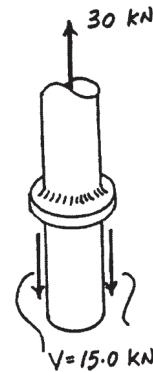
**Ans.**

**Allowable Shear Stress:** Design of cotter size.

$$\tau_{\text{allow}} = \frac{\tau_{\text{fail}}}{\text{F.S.}} = \frac{V}{A}; \quad \frac{375(10^6)}{1.75} = \frac{15.0(10^3)}{(0.01)t}$$

$$t = 0.0070 \text{ m} = 7.00 \text{ mm}$$

**Ans.**



**Ans:**

$d = 13.8 \text{ mm}$ ,  
 $t = 7.00 \text{ mm}$

- 1–74.** The truss is used to support the loading shown. Determine the required cross-sectional area of member *BC* if the allowable normal stress is  $\sigma_{\text{allow}} = 165 \text{ MPa}$ .

### SOLUTION

For the entire truss,

$$\zeta + \sum M_A = 0; \quad -2000(2) - 4000(2.8284) + 2(2.8284)(D_y) = 0$$

$$D_y = 2707.11 \text{ N}$$

For method of section,

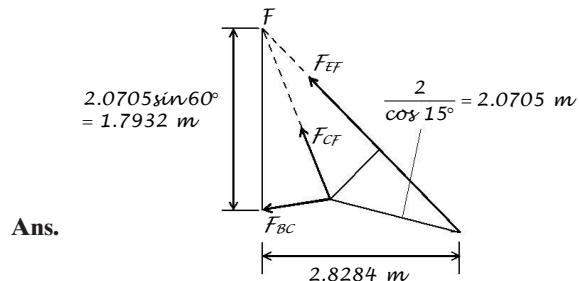
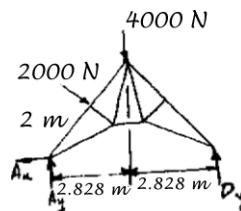
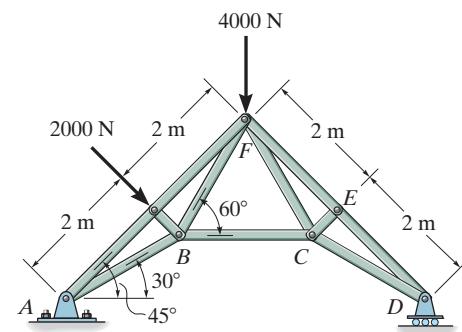
$$\zeta + \sum M_F = 0; \quad 2707.11(2.8284) - F_{BC}(1.7932) = 0$$

$$F_{BC} = 4270.06 \text{ N}$$

Using the given allowable stress,

$$\sigma_{\text{allow}} = \frac{F_{BC}}{A}; \quad 165(10^6) = \frac{4270.06}{A}$$

$$A = 25.88(10^{-6}) \text{ m}^2 = 25.9 \text{ mm}^2$$

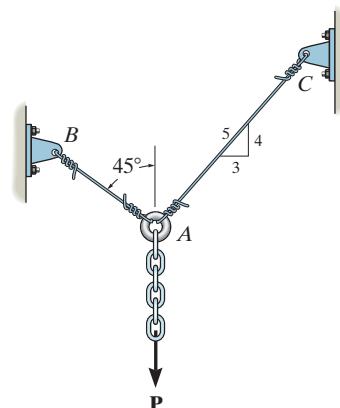


**Ans:**

$$A = 25.9 \text{ mm}^2$$

**1–75.**

If the allowable tensile stress for wires  $AB$  and  $AC$  is  $\sigma_{\text{allow}} = 200 \text{ MPa}$ , determine the required diameter of each wire if the applied load is  $P = 6 \text{ kN}$ .



**SOLUTION**

**Normal Forces:** Analyzing the equilibrium of joint  $A$ , Fig. *a*,

$$\pm \sum F_x = 0; \quad F_{AC} \left( \frac{3}{5} \right) - F_{AB} \sin 45^\circ = 0 \quad (1)$$

$$+ \uparrow \sum F_y = 0; \quad F_{AC} \left( \frac{4}{5} \right) + F_{AB} \cos 45^\circ - 6 = 0 \quad (2)$$

Solving Eqs. (1) and (2)

$$F_{AC} = 4.2857 \text{ kN} \quad F_{AB} = 3.6365 \text{ kN}$$

**Average Normal Stress:** For wire  $AB$ ,

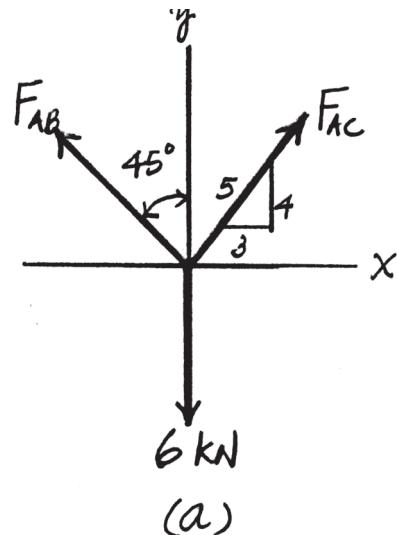
$$\sigma_{\text{allow}} = \frac{F_{AB}}{A_{AB}}; \quad 200(10^6) = \frac{3.6365(10^3)}{\frac{\pi}{4} d_{AB}^2}$$

$$d_{AB} = 0.004812 \text{ m} = 4.81 \text{ mm} \quad \text{Ans.}$$

For wire  $AC$ ,

$$\sigma_{\text{allow}} = \frac{F_{AC}}{A_{AC}}; \quad 200(10^6) = \frac{4.2857(10^3)}{\frac{\pi}{4} d_{AC}^2}$$

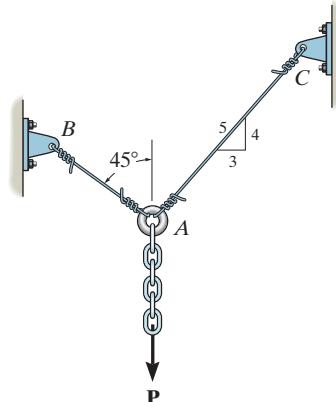
$$d_{AC} = 0.005223 \text{ m} = 5.22 \text{ mm} \quad \text{Ans.}$$



**Ans:**  
 $d_{AB} = 4.81 \text{ mm}$ ,  
 $d_{AC} = 5.22 \text{ mm}$

**\*1-76.**

If the allowable tensile stress for wires  $AB$  and  $AC$  is  $\sigma_{\text{allow}} = 180 \text{ MPa}$ , and wire  $AB$  has a diameter of 5 mm and  $AC$  has a diameter of 6 mm, determine the greatest force  $P$  that can be applied to the chain.



**SOLUTION**

**Normal Forces:** Analyzing the equilibrium of joint  $A$ , Fig.  $a$ ,

$$\rightarrow \sum F_x = 0; \quad F_{AC} \left( \frac{3}{5} \right) - F_{AB} \sin 45^\circ = 0 \quad (1)$$

$$+\uparrow \sum F_y = 0; \quad F_{AC} \left( \frac{4}{5} \right) + F_{AB} \cos 45^\circ - P = 0 \quad (2)$$

Solving Eqs. (1) and (2)

$$F_{AC} = 0.7143P \quad F_{AB} = 0.6061P$$

**Average Normal Stress:** Assuming failure of wire  $AB$ ,

$$\sigma_{\text{allow}} = \frac{F_{AB}}{A_{AB}}; \quad 180(10^6) = \frac{0.6061P}{\frac{\pi}{4}(0.005^2)}$$

$$P = 5.831(10^3) \text{ N} = 5.83 \text{ kN}$$

Assume the failure of wire  $AC$ ,

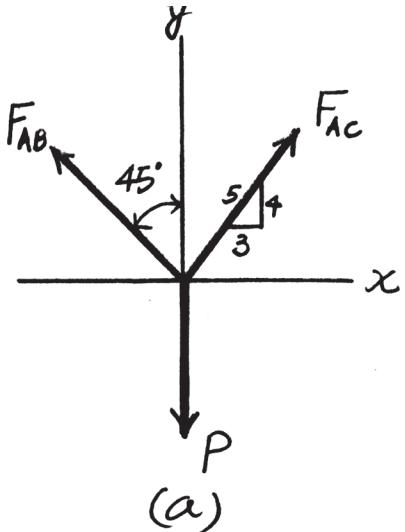
$$\sigma_{\text{allow}} = \frac{F_{AC}}{A_{AC}}; \quad 180(10^6) = \frac{0.7143P}{\frac{\pi}{4}(0.006^2)}$$

$$P = 7.125(10^3) \text{ N} = 7.13 \text{ kN}$$

Choose the smaller of the two values of  $P$ ,

$$P = 5.83 \text{ kN}$$

**Ans.**



**Ans:**  
 $P = 5.83 \text{ kN}$

**1-77.**

The spring mechanism is used as a shock absorber for a load applied to the drawbar *AB*. Determine the force in each spring when the 50-kN force is applied. Each spring is originally unstretched and the drawbar slides along the smooth guide posts *CG* and *EF*. The ends of all springs are attached to their respective members. Also, what is the required diameter of the shank of the bolts *CG* and *EF* if the allowable stress for the bolts is  $\sigma_{\text{allow}} = 150 \text{ MPa}$ ?

## SOLUTION

### Equations of Equilibrium:

$$\zeta + \sum M_H = 0; -F_{BF}(200) + F_{AG}(200) = 0$$

$$F_{BF} = F_{AG} = F$$

$$+\uparrow \sum F_y = 0; 2F + F_H - 50 = 0$$

(1)

Required,

$$\Delta_H = \Delta_B; \frac{F_H}{80} = \frac{F}{60}$$

$$F = 0.75 F_H$$

(2)

Solving Eqs. (1) and (2) yields,

$$F_H = 20.0 \text{ kN}$$

Ans.

$$F_{BF} = F_{AG} = F = 15.0 \text{ kN}$$

Ans.

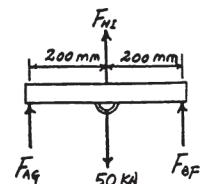
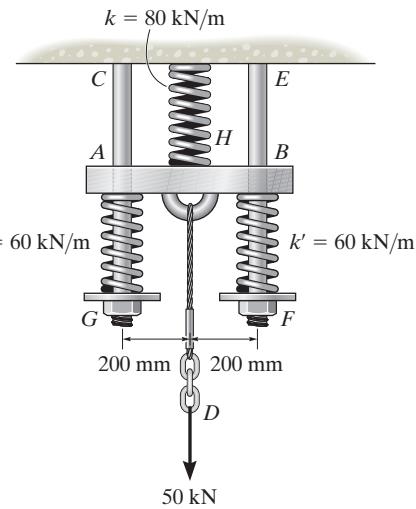
**Allowable Normal Stress:** Design of bolt shank size.

$$\sigma_{\text{allow}} = \frac{P}{A}; 150(10^6) = \frac{15.0(10^3)}{\frac{\pi}{4}d^2}$$

$$d = 0.01128 \text{ m} = 11.3 \text{ mm}$$

$$d_{EF} = d_{CG} = 11.3 \text{ mm}$$

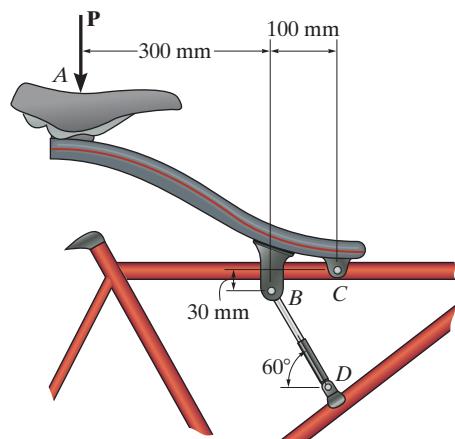
Ans.



**Ans:**

$$F_H = 20.0 \text{ kN}, \\ F_{BF} = F_{AG} = 15.0 \text{ kN}, \\ d_{EF} = d_{CG} = 11.3 \text{ mm}$$

**1-78.** The soft-ride suspension system of the mountain bike is pinned at *C* and supported by the shock absorber *BD*. If it is designed to support a load  $P = 1500 \text{ N}$ , determine the required minimum diameter of pins *B* and *C*. Use a factor of safety of 2 against failure. The pins are made of material having a failure shear stress of  $\tau_{\text{fail}} = 150 \text{ MPa}$ , and each pin is subjected to double shear.



### SOLUTION

**Internal Loadings:** The forces acting on pins *B* and *C* can be determined by considering the equilibrium of the free-body diagram of the soft-ride suspension system shown in Fig. *a*.

$$\zeta + \sum M_C = 0; \quad 1500(0.4) - F_{BD} \sin 60^\circ(0.1) - F_{BD} \cos 60^\circ(0.03) = 0$$

$$F_{BD} = 5905.36 \text{ N}$$

$$\stackrel{+}{\rightarrow} \sum F_x = 0; \quad C_x - 5905.36 \cos 60^\circ = 0 \quad C_x = 2952.68 \text{ N}$$

$$+\uparrow \sum F_y = 0; \quad 5905.36 \sin 60^\circ - 1500 - C_y = 0 \quad C_y = 3614.20 \text{ N}$$

Thus,

$$F_B = F_{BD} = 5905.36 \text{ N} \quad F_C = 2 \sqrt{C_x^2 + C_y^2} = 2 \sqrt{2952.68^2 + 3614.20^2} \\ = 4666.98 \text{ N}$$

Since **both** pins are in double shear,

$$V_B = \frac{F_B}{2} = \frac{5905.36}{2} = 2952.68 \text{ N} \quad V_C = \frac{F_C}{2} = \frac{4666.98}{2} = 2333.49 \text{ N}$$

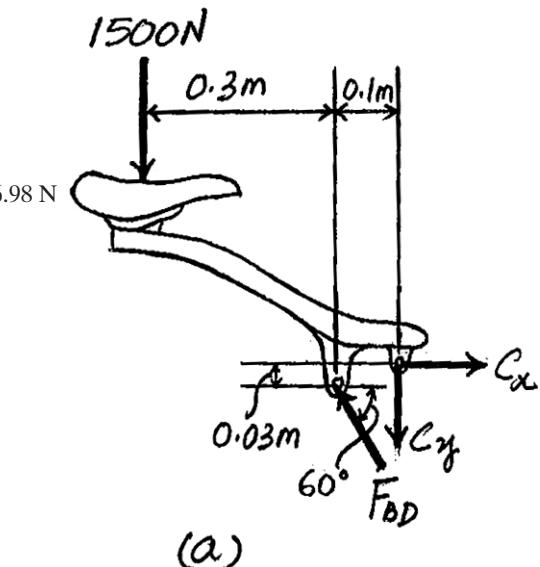
**Allowable Shear Stress:**

$$\tau_{\text{allow}} = \frac{\tau_{\text{fail}}}{\text{F.S.}} = \frac{150}{2} = 75 \text{ MPa}$$

Using this result,

$$\tau_{\text{allow}} = \frac{V_B}{A_B}; \quad 75(10^6) = \frac{2952.68}{\frac{\pi}{4} d_B^2}$$

$$d_B = 0.007080 \text{ m} = 7.08 \text{ mm}$$



Ans.

$$\tau_{\text{allow}} = \frac{V_C}{A_C}; \quad 75(10^6) = \frac{2333.49}{\frac{\pi}{4} d_C^2}$$

$$d_C = 0.006294 \text{ m} = 6.29 \text{ mm}$$

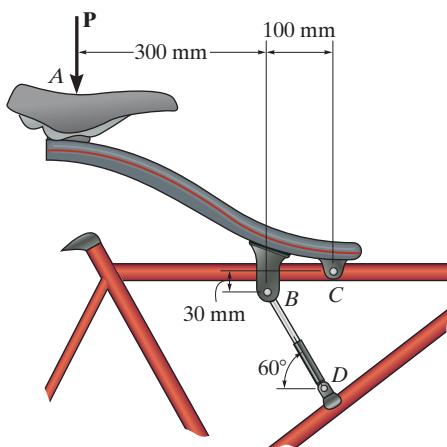
Ans.

Ans:

$$d_B = 7.08 \text{ mm}, d_C = 6.29 \text{ mm}$$

**1-79.** The soft-ride suspension system of the mountain bike is pinned at *C* and supported by the shock absorber *BD*. If it is designed to support a load of  $P = 1500 \text{ N}$ , determine the factor of safety of pins *B* and *C* against failure if they are made of a material having a shear failure stress of  $\tau_{\text{fail}} = 150 \text{ MPa}$ . Pin *B* has a diameter of 7.5 mm, and pin *C* has a diameter of 6.5 mm. Both pins are subjected to double shear.

### SOLUTION



**Internal Loadings:** The forces acting on pins *B* and *C* can be determined by considering the equilibrium of the free-body diagram of the soft-ride suspension system shown in Fig. *a*.

$$+\sum M_C = 0; \quad 1500(0.4) - F_{BD} \sin 60^\circ(0.1) - F_{BD} \cos 60^\circ(0.03) = 0$$

$$F_{BD} = 5905.36 \text{ N}$$

$$\stackrel{+}{\rightarrow} \sum F_x = 0; \quad C_x - 5905.36 \cos 60^\circ = 0 \quad C_x = 2952.68 \text{ N}$$

$$+\uparrow \sum F_y = 0; \quad 5905.36 \sin 60^\circ - 1500 - C_y = 0 \quad C_y = 3614.20 \text{ N}$$

Thus,

$$F_B = F_{BD} = 5905.36 \text{ N} \quad F_C = 2 \sqrt{C_x^2 + C_y^2} = 2 \sqrt{2952.68^2 + 3614.20^2} \\ = 4666.98 \text{ N}$$

Since both pins are in double shear,

$$V_B = \frac{F_B}{2} = \frac{5905.36}{2} = 2952.68 \text{ N} \quad V_C = \frac{F_C}{2} = \frac{4666.98}{2} = 2333.49 \text{ N}$$

**Allowable Shear Stress:** The areas of the shear plane for pins *B* and *C* are  $A_B = \frac{\pi}{4}(0.0075^2) = 44.179(10^{-6}) \text{ m}^2$  and  $A_C = \frac{\pi}{4}(0.0065^2) = 33.183(10^{-6}) \text{ m}^2$ .

We obtain

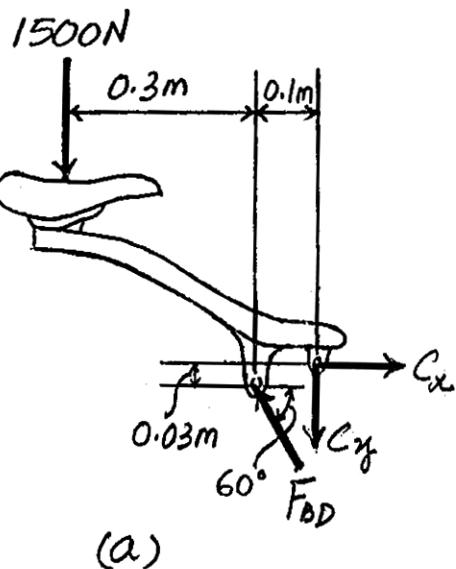
$$(\tau_{\text{avg}})_B = \frac{V_B}{A_B} = \frac{2952.68}{44.179(10^{-6})} = 66.84 \text{ MPa}$$

$$(\tau_{\text{avg}})_C = \frac{V_C}{A_C} = \frac{2333.49}{33.183(10^{-6})} = 70.32 \text{ MPa}$$

Using these results,

$$(\text{F.S.})_B = \frac{\tau_{\text{fail}}}{(\tau_{\text{avg}})_B} = \frac{150}{66.84} = 2.24 \quad \text{Ans.}$$

$$(\text{F.S.})_C = \frac{\tau_{\text{fail}}}{(\tau_{\text{avg}})_C} = \frac{150}{70.32} = 2.13 \quad \text{Ans.}$$

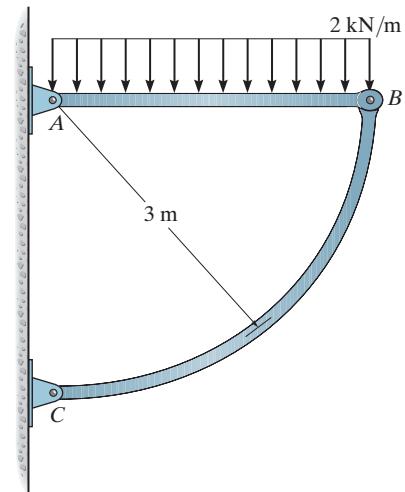


Ans:

$$(\text{F.S.})_B = 2.24, (\text{F.S.})_C = 2.13$$

\*1–80.

Determine the required diameter of the pins at *A* and *B* if the allowable shear stress for the material is  $\tau_{\text{allow}} = 100 \text{ MPa}$ . Both pins are subjected to double shear.



## SOLUTION

**Support Reactions:** Member *BC* is a two force member.

$$\zeta + \sum M_A = 0; \quad F_{BC} \sin 45^\circ(3) - 6(1.5) = 0 \\ F_{BC} = 4.243 \text{ kN}$$

$$+\uparrow \sum F_y = 0; \quad A_y + 4.243 \sin 45^\circ - 6 = 0 \\ A_y = 3.00 \text{ kN}$$

$$\pm \sum F_x = 0; \quad A_x - 4.243 \cos 45^\circ = 0 \\ A_x = 3.00 \text{ kN}$$

**Allowable Shear Stress:** Pin *A* and pin *B* are subjected to double shear.

$$F_A = \sqrt{3.00^2 + 3.00^2} = 4.243 \text{ kN} \text{ and}$$

$$F_B = F_{BC} = 4.243 \text{ kN}.$$

Therefore,

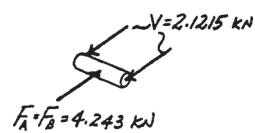
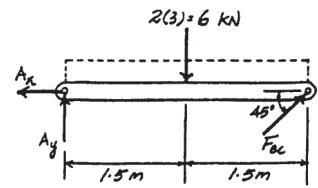
$$V_A = V_B = \frac{4.243}{2} = 2.1215 \text{ kN}$$

$$\tau_{\text{allow}} = \frac{V}{A}; \quad 100(10^6) = \frac{2.1215(10^3)}{\frac{\pi}{4}d^2}$$

$$d = 0.005197 \text{ m} = 5.20 \text{ mm}$$

$$d_A = d_B = d = 5.20 \text{ mm}$$

**Ans.**

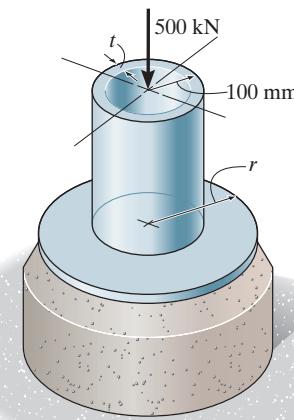


**Ans:**

$$d_A = d_B = 5.20 \text{ mm}$$

**1–81.**

The steel pipe is supported on the circular base plate and concrete pedestal. If the thickness of the pipe is  $t = 5 \text{ mm}$  and the base plate has a radius of  $150 \text{ mm}$ , determine the factors of safety against failure of the steel and concrete. The applied force is  $500 \text{ kN}$ , and the normal failure stresses for steel and concrete are  $(\sigma_{\text{fail}})_{\text{st}} = 350 \text{ MPa}$  and  $(\sigma_{\text{fail}})_{\text{con}} = 25 \text{ MPa}$ , respectively.



**SOLUTION**

**Average Normal and Bearing Stress:** The cross-sectional area of the steel pipe and the bearing area of the concrete pedestal are  $A_{\text{st}} = \pi(0.1^2 - 0.095^2) = 0.975(10^{-3})\pi \text{ m}^2$  and  $(A_{\text{con}})_b = \pi(0.15^2) = 0.0225\pi \text{ m}^2$ . We have

$$(\sigma_{\text{avg}})_{\text{st}} = \frac{P}{A_{\text{st}}} = \frac{500(10^3)}{0.975(10^{-3})\pi} = 163.24 \text{ MPa}$$

$$(\sigma_{\text{avg}})_{\text{con}} = \frac{P}{(A_{\text{con}})_b} = \frac{500(10^3)}{0.0225\pi} = 7.074 \text{ MPa}$$

Thus, the factor of safety against failure of the steel pipe and concrete pedestal are

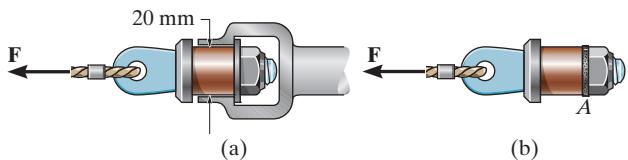
$$(\text{F.S.})_{\text{st}} = \frac{(\sigma_{\text{fail}})_{\text{st}}}{(\sigma_{\text{avg}})_{\text{st}}} = \frac{350}{163.24} = 2.14 \quad \text{Ans.}$$

$$(\text{F.S.})_{\text{con}} = \frac{(\sigma_{\text{fail}})_{\text{con}}}{(\sigma_{\text{avg}})_{\text{con}}} = \frac{25}{7.074} = 3.53 \quad \text{Ans.}$$

**Ans:**

$$(\text{F.S.})_{\text{st}} = 2.14, (\text{F.S.})_{\text{con}} = 3.53$$

- 1–82.** The steel swivel bushing in the elevator control of an airplane is held in place using a nut and washer as shown in Fig. (a). Failure of the washer *A* can cause the push rod to separate as shown in Fig. (b). If the average shear stress is  $\tau_{\text{avg}} = 145 \text{ MPa}$ , determine the force  $\mathbf{F}$  that must be applied to the bushing that will cause this to happen. The washer is 1.5 mm thick.



## SOLUTION

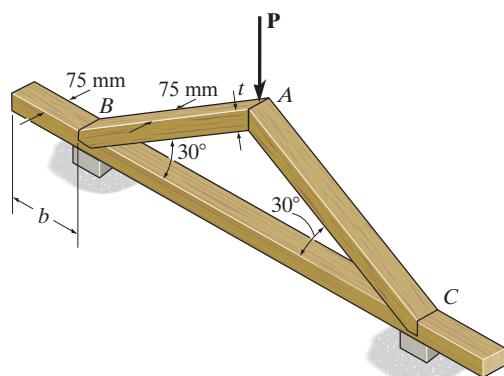
From the given average shear stress,

$$\tau_{\text{avg}} = \frac{V}{A}; \quad 145(10^6) = \frac{F}{2\pi(0.01)(0.0015)}$$

$$F = 13.67(10^3) \\ = 13.7 \text{ kN} \quad \text{Ans.}$$

**Ans:**  
 $F = 13.7 \text{ kN}$

- 1-83.** Determine the required minimum thickness  $t$  of member  $AB$  and edge distance  $b$  of the frame if  $P = 40 \text{ kN}$  and the factor of safety against failure is 2. The wood has a normal failure stress of  $\sigma_{\text{fail}} = 42 \text{ MPa}$ , and shear failure stress of  $\tau_{\text{fail}} = 10.5 \text{ MPa}$ .



## SOLUTION

**Internal Loadings:** The normal force developed in member  $AB$  can be determined by considering the equilibrium of joint  $A$ , Fig. *a*.

$$\begin{aligned} \stackrel{+}{\rightarrow} \sum F_x &= 0; & F_{AB} \cos 30^\circ - F_{AC} \cos 30^\circ &= 0 & F_{AC} &= F_{AB} \\ \stackrel{+}{\uparrow} \sum F_y &= 0; & 2F_{AB} \sin 30^\circ - 40 &= 0 & F_{AB} &= 40 \text{ kN} \end{aligned}$$

Subsequently, the horizontal component of the force acting on joint  $B$  can be determined by analyzing the equilibrium of member  $AB$ , Fig. *b*.

$$\stackrel{+}{\rightarrow} \sum F_x = 0; \quad (F_B)_x - 40 \cos 30^\circ = 0 \quad (F_B)_x = 34.64 \text{ kN}$$

Referring to the free-body diagram shown in Fig. *c*, the shear force developed on the shear plane  $a-a$  is

$$\stackrel{+}{\rightarrow} \sum F_x = 0; \quad V_{a-a} - 34.64 = 0 \quad V_{a-a} = 34.64 \text{ kN}$$

### Allowable Normal Stress:

$$\sigma_{\text{allow}} = \frac{\sigma_{\text{fail}}}{\text{F.S.}} = \frac{42}{2} = 21 \text{ MPa}$$

$$\tau_{\text{allow}} = \frac{\tau_{\text{fail}}}{\text{F.S.}} = \frac{10.5}{2} = 5.25 \text{ MPa}$$

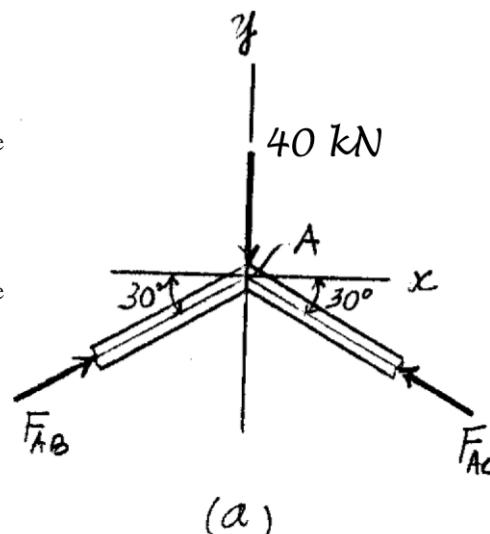
Using these results,

$$\sigma_{\text{allow}} = \frac{F_{AB}}{A_{AB}}; \quad 21(10^6) = \frac{40(10^3)}{(0.075)t} \quad t = 0.02540 \text{ m} = 25.4 \text{ mm}$$

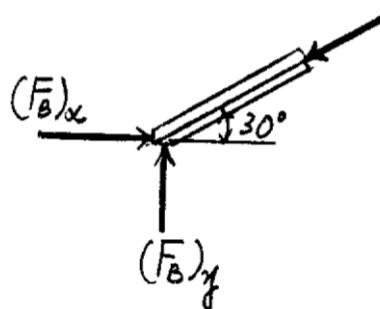
Ans.

$$\tau_{\text{allow}} = \frac{V_{a-a}}{A_{a-a}}; \quad 5.25(10^6) = \frac{34.64(10^3)}{(0.075)b} \quad b = 0.08798 \text{ m} = 88.0 \text{ mm}$$

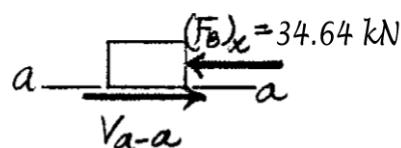
Ans.



$$(F_B)_x = 40 \text{ kN}$$



(b)

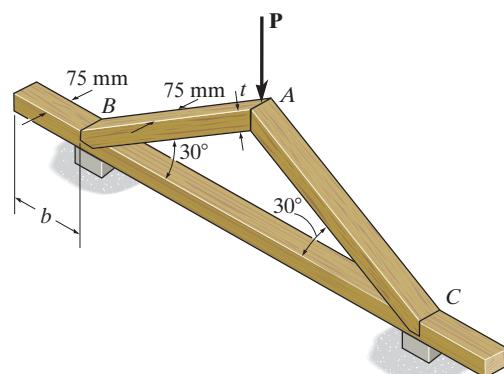


(c)

Ans:

$$t = 25.4 \text{ mm}, b = 88.0 \text{ mm}$$

**\*1-84.** Determine the maximum allowable load  $P$  that can be safely supported by the frame if  $t = 30 \text{ mm}$  and  $b = 90 \text{ mm}$ . The wood has a normal failure stress of  $\sigma_{\text{fail}} = 42 \text{ MPa}$ , and shear failure stress of  $\tau_{\text{fail}} = 10.5 \text{ MPa}$ . Use a factor of safety against failure of 2.



## SOLUTION

**Internal Loadings:** The normal force developed in member  $AB$  can be determined by considering the equilibrium of joint  $A$ , Fig.  $a$ .

$$\stackrel{+}{\rightarrow} \sum F_x = 0; \quad F_{AB} \cos 30^\circ - F_{AC} \cos 30^\circ = 0 \quad F_{AC} = F_{AB}$$

$$\stackrel{+}{\uparrow} \sum F_y = 0; \quad 2F_{AB} \sin 30^\circ - P = 0 \quad F_{AB} = P$$

Subsequently, the horizontal component of the force acting on joint  $B$  can be determined by analyzing the equilibrium of member  $AB$ , Fig.  $b$ .

$$\stackrel{+}{\rightarrow} \sum F_x = 0; \quad (F_B)_x - P \cos 30^\circ = 0 \quad (F_B)_x = 0.8660P$$

Referring to the free-body diagram shown in Fig.  $c$ , the shear force developed on the shear plane  $a-a$  is

$$\stackrel{+}{\rightarrow} \sum F_x = 0; \quad V_{a-a} - 0.8660P = 0 \quad V_{a-a} = 0.8660P$$

### Allowable Normal and Shear Stress:

$$\sigma_{\text{allow}} = \frac{\sigma_{\text{fail}}}{\text{F.S.}} = \frac{42}{2} = 21 \text{ MPa}$$

$$\tau_{\text{allow}} = \frac{\tau_{\text{fail}}}{\text{F.S.}} = \frac{10.5}{2} = 5.25 \text{ MPa}$$

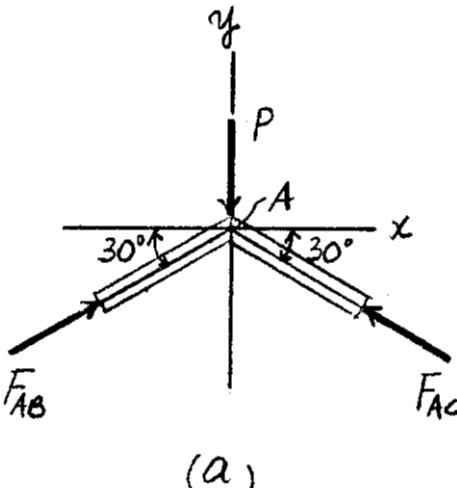
Using these results,

$$\sigma_{\text{allow}} = \frac{F_{AB}}{A_{AB}}; \quad 21(10^6) = \frac{P}{0.075(0.03)}$$

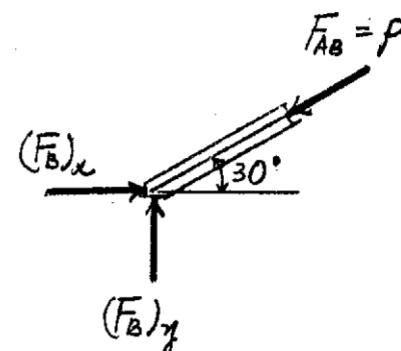
$$P = 47.25(10^3) \text{ N} = 47.25 \text{ kN}$$

$$\tau_{\text{allow}} = \frac{V_{a-a}}{A_{a-a}}; \quad 5.25(10^6) = \frac{0.8660P}{0.075(0.09)}$$

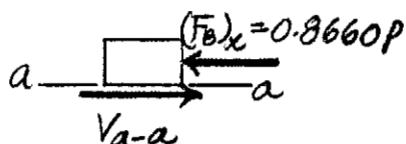
$$P = 40.92(10^3) \text{ N} = 40.9 \text{ kN} \text{ (controls)} \quad \text{Ans.}$$



(a)



(b)



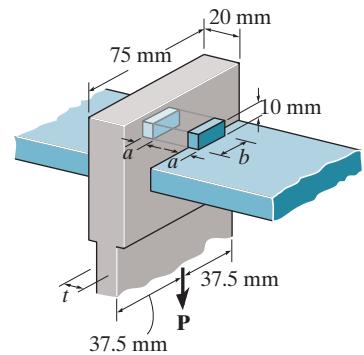
(c)

Ans:

$$P = 40.9 \text{ kN}$$

**1-85.**

The hanger is supported using the rectangular pin. Determine the magnitude of the allowable suspended load  $P$  if the allowable bearing stress is  $(\sigma_b)_{\text{allow}} = 220 \text{ MPa}$ , the allowable tensile stress is  $(\sigma_t)_{\text{allow}} = 150 \text{ MPa}$ , and the allowable shear stress is  $\tau_{\text{allow}} = 130 \text{ MPa}$ . Take  $t = 6 \text{ mm}$ ,  $a = 5 \text{ mm}$  and  $b = 25 \text{ mm}$ .



**SOLUTION**

**Allowable Normal Stress:** For the hanger

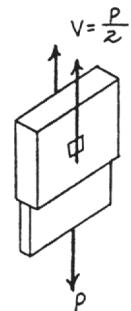
$$(\sigma_t)_{\text{allow}} = \frac{P}{A}; \quad 150(10^6) = \frac{P}{(0.075)(0.006)}$$

$$P = 67.5 \text{ kN}$$

**Allowable Shear Stress:** The pin is subjected to double shear. Therefore,  $V = \frac{P}{2}$

$$\tau_{\text{allow}} = \frac{V}{A}; \quad 130(10^6) = \frac{P/2}{(0.01)(0.025)}$$

$$P = 65.0 \text{ kN}$$



**Allowable Bearing Stress:** For the bearing area

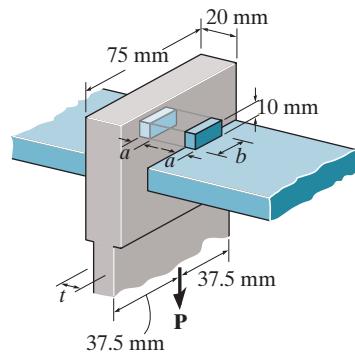
$$(\sigma_b)_{\text{allow}} = \frac{P}{A}; \quad 220(10^6) = \frac{P/2}{(0.005)(0.025)}$$

$$P = 55.0 \text{ kN} \text{ (Controls!)} \qquad \text{Ans.}$$

**Ans:**  
 $P = 55.0 \text{ kN}$

**1–86.**

The hanger is supported using the rectangular pin. Determine the required thickness  $t$  of the hanger, and dimensions  $a$  and  $b$  if the suspended load is  $P = 60 \text{ kN}$ . The allowable tensile stress is  $(\sigma_t)_{\text{allow}} = 150 \text{ MPa}$ , the allowable bearing stress is  $(\sigma_b)_{\text{allow}} = 290 \text{ MPa}$ , and the allowable shear stress is  $\tau_{\text{allow}} = 125 \text{ MPa}$ .



**SOLUTION**

**Allowable Normal Stress:** For the hanger

$$(\sigma_t)_{\text{allow}} = \frac{P}{A}; \quad 150(10^6) = \frac{60(10^3)}{(0.075)t}$$

$$t = 0.005333 \text{ m} = 5.33 \text{ mm}$$

**Ans.**

**Allowable Shear Stress:** For the pin

$$\tau_{\text{allow}} = \frac{V}{A}; \quad 125(10^6) = \frac{30(10^3)}{(0.01)b}$$

$$b = 0.0240 \text{ m} = 24.0 \text{ mm}$$

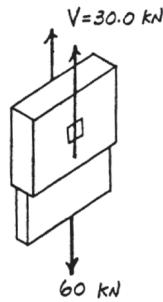
**Ans.**

**Allowable Bearing Stress:** For the bearing area

$$(\sigma_b)_{\text{allow}} = \frac{P}{A}; \quad 290(10^6) = \frac{30(10^3)}{(0.0240)a}$$

$$a = 0.00431 \text{ m} = 4.31 \text{ mm}$$

**Ans.**



**Ans:**

$$t = 5.33 \text{ mm}, b = 24.0 \text{ mm}, a = 4.31 \text{ mm}$$

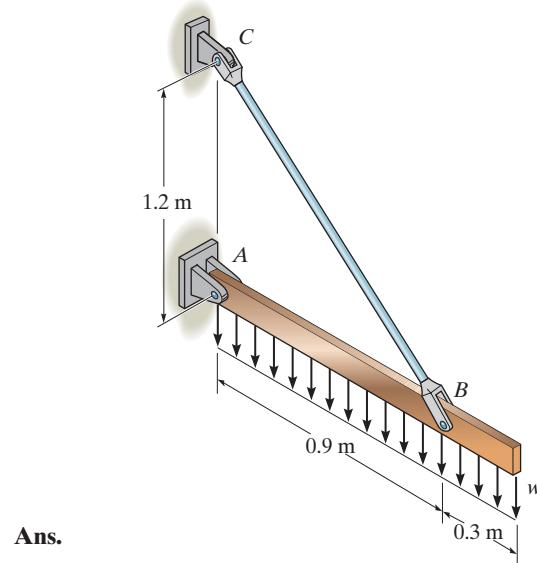
**1-87.** The assembly is used to support the distributed loading of  $w = 10 \text{ kN/m}$ . Determine the factor of safety with respect to yielding for the steel rod  $BC$  and the pins at  $A$  and  $B$  if the yield stress for the steel in tension is  $\sigma_y = 250 \text{ MPa}$  and in shear  $\tau_y = 125 \text{ MPa}$ . The rod has a diameter of 13 mm, and the pins each have a diameter of 10 mm.

## SOLUTION

For rod  $BC$ :

$$\sigma = \frac{P}{A} = \frac{10(10^3)}{\frac{\pi}{4}(0.013^2)} = 75.34 \text{ MPa}$$

$$\text{F. S.} = \frac{\sigma_y}{\sigma} = \frac{250}{75.34} = 3.32$$



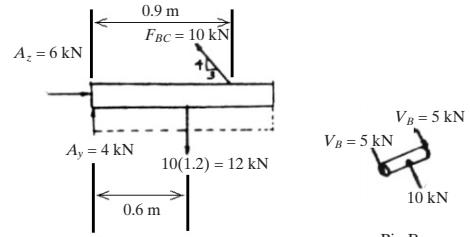
**Ans.**

For pin  $B$ :

$$\tau_B = \frac{V_B}{A} = \frac{5(10^3)}{\frac{\pi}{4}(0.01^2)} = 63.66 \text{ MPa}$$

$$\text{F. S.} = \frac{\tau_y}{\tau_B} = \frac{125}{63.66} = 1.96$$

**Ans.**



Pin B

For pin  $A$ :

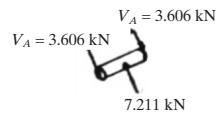
$$F_A = \sqrt{4^2 + 6^2} = 7.211 \text{ kN}$$

$$V_A = \frac{7.211 \text{ kN}}{2} = 3.606 \text{ kN}$$

$$\tau_A = \frac{V_A}{A} = \frac{3.606(10^3)}{\frac{\pi}{4}(0.01^2)} = 45.91 \text{ MPa}$$

$$\text{F. S.} = \frac{\tau_y}{\tau_A} = \frac{125}{45.91} = 2.72$$

**Ans.**



Pin A

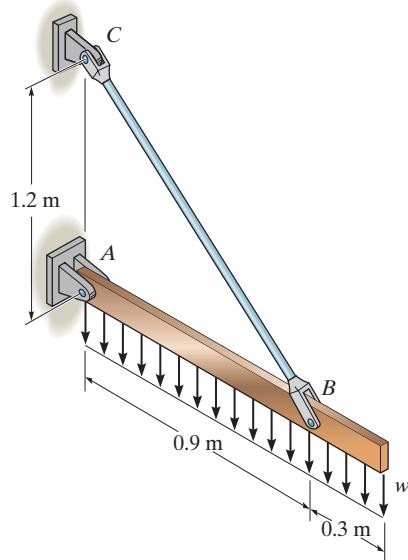
**Ans:**

$$(\text{F.S.})_{\text{rod}} = 3.32,$$

$$(\text{F.S.})_{\text{pinB}} = 1.96$$

$$(\text{F.S.})_{\text{pinA}} = 2.72$$

- \*1–88.** If the allowable shear stress for each of the 10-mm-diameter steel pins at *A*, *B*, and *C* is  $\tau_{\text{allow}} = 90 \text{ MPa}$ , and the allowable normal stress for the 13-mm-diameter rod is  $\sigma_{\text{allow}} = 150 \text{ MPa}$ , determine the largest intensity *w* of the uniform distributed load that can be suspended from the beam.



## SOLUTION

Assume failure of pins *B* and *C*:

$$\tau_{\text{allow}} = 90 = \frac{0.5w(10^3)}{\frac{\pi}{4}(10^2)}$$

$$w = 14.14 \text{ kN/m} \quad (\text{controls})$$

**Ans.**

Assume failure of pins *A*:

$$F_A = \sqrt{(0.6w)^2 + (0.4w)^2} = 0.721w$$

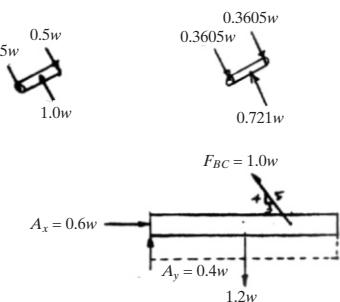
$$\tau_{\text{allow}} = 90 = \frac{0.3605w(10^3)}{\frac{\pi}{4}(10^2)}$$

$$w = 19.61 \text{ kN/m}$$

Assume failure of rod *BC*:

$$\sigma_{\text{allow}} = 150 = \frac{1.0w(10^3)}{\frac{\pi}{4}(13^2)}$$

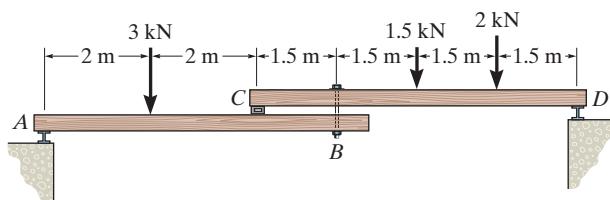
$$w = 19.91 \text{ kN/m}$$



**Ans:**  
 $w = 14.14 \text{ kN/m}$  (controls)

**1–89.**

The compound wooden beam is connected together by a bolt at *B*. Assuming that the connections at *A*, *B*, *C*, and *D* exert only vertical forces on the beam, determine the required diameter of the bolt at *B* and the required outer diameter of its washers if the allowable tensile stress for the bolt is  $(\sigma_t)_{\text{allow}} = 150 \text{ MPa}$  and the allowable bearing stress for the wood is  $(\sigma_b)_{\text{allow}} = 28 \text{ MPa}$ . Assume that the hole in the washers has the same diameter as the bolt.



## SOLUTION

From FBD (a):

$$\zeta + \sum M_D = 0; \quad F_B(4.5) + 1.5(3) + 2(1.5) - F_C(6) = 0$$

$$4.5 F_B - 6 F_C = -7.5 \quad (1)$$

From FBD (b):

$$\zeta + \sum M_A = 0; \quad F_B(5.5) - F_C(4) - 3(2) = 0$$

$$5.5 F_B - 4 F_C = 6 \quad (2)$$

Solving Eqs. (1) and (2) yields

$$F_B = 4.40 \text{ kN}; \quad F_C = 4.55 \text{ kN}$$

For bolt:

$$\sigma_{\text{allow}} = 150(10^6) = \frac{4.40(10^3)}{\frac{\pi}{4}(d_B)^2}$$

$$d_B = 0.00611 \text{ m}$$

$$= 6.11 \text{ mm}$$

**Ans.**

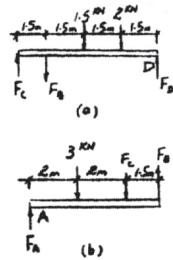


For washer:

$$\sigma_{\text{allow}} = 28(10^4) = \frac{4.40(10^3)}{\frac{\pi}{4}(d_w^2 - 0.00611^2)}$$

$$d_w = 0.0154 \text{ m} = 15.4 \text{ mm}$$

**Ans.**

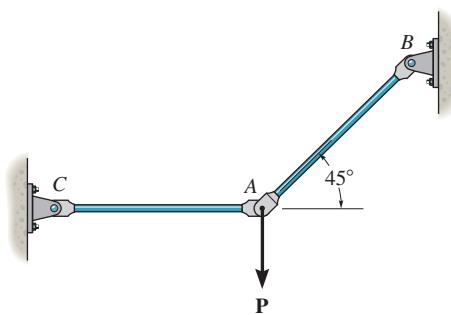


**Ans:**

$$d_B = 6.11 \text{ mm}, d_w = 15.4 \text{ mm}$$

**1–90.**

The two aluminum rods support the vertical force of  $P = 20 \text{ kN}$ . Determine their required diameters if the allowable tensile stress for the aluminum is  $\sigma_{\text{allow}} = 150 \text{ MPa}$ .



**SOLUTION**

$$+\uparrow \sum F_y = 0; \quad F_{AB} \sin 45^\circ - 20 = 0; \quad F_{AB} = 28.284 \text{ kN}$$

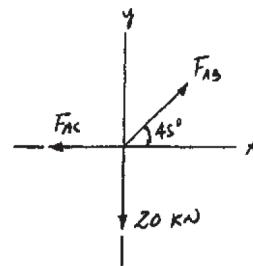
$$\pm \rightarrow \sum F_x = 0; \quad 28.284 \cos 45^\circ - F_{AC} = 0; \quad F_{AC} = 20.0 \text{ kN}$$

For rod  $AB$ :

$$\sigma_{\text{allow}} = \frac{F_{AB}}{A_{AB}}, \quad 150(10^6) = \frac{28.284(10^3)}{\frac{\pi}{4} d_{AB}^2}$$

$$d_{AB} = 0.0155 \text{ m} = 15.5 \text{ mm}$$

**Ans.**



For rod  $AC$ :

$$\sigma_{\text{allow}} = \frac{F_{AC}}{A_{AC}}, \quad 150(10^6) = \frac{20.0(10^3)}{\frac{\pi}{4} d_{AC}^2}$$

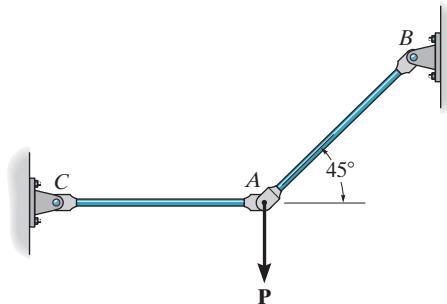
$$d_{AC} = 0.0130 \text{ m} = 13.0 \text{ mm}$$

**Ans.**

**Ans:**  
 $d_{AB} = 15.5 \text{ mm}, d_{AC} = 13.0 \text{ mm}$

**1-91.**

The two aluminum rods  $AB$  and  $AC$  have diameters of 10 mm and 8 mm, respectively. Determine the largest vertical force  $P$  that can be supported. The allowable tensile stress for the aluminum is  $\sigma_{\text{allow}} = 150 \text{ MPa}$ .



**SOLUTION**

$$+\uparrow \sum F_y = 0; \quad F_{AB} \sin 45^\circ - P = 0; \quad P = F_{AB} \sin 45^\circ \quad (1)$$

$$\pm \sum F_x = 0; \quad F_{AB} \cos 45^\circ - F_{AC} = 0 \quad (2)$$

Assume failure of rod  $AB$ :

$$\sigma_{\text{allow}} = \frac{F_{AB}}{A_{AB}}, \quad 150(10^6) = \frac{F_{AB}}{\frac{\pi}{4}(0.01)^2}$$

$$F_{AB} = 11.78 \text{ kN}$$

From Eq. (1),

$$P = 8.33 \text{ kN}$$

Assume failure of rod  $AC$ :

$$\sigma_{\text{allow}} = \frac{F_{AC}}{A_{AC}}, \quad 150(10^6) = \frac{F_{AC}}{\frac{\pi}{4}(0.008)^2}$$

$$F_{AC} = 7.540 \text{ kN}$$

Solving Eqs. (1) and (2) yields:

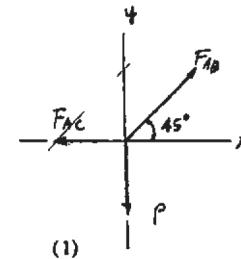
$$F_{AB} = 10.66 \text{ kN}; \quad P = 7.54 \text{ kN}$$

Choose the smallest value

$$P = 7.54 \text{ kN}$$

**(1)**

**(2)**



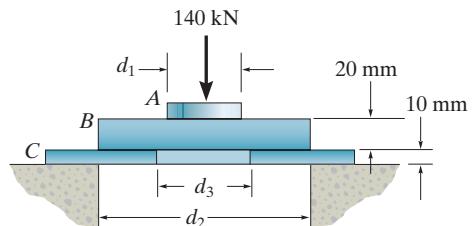
**(1)**

**Ans.**

**Ans:**  
 $P = 7.54 \text{ kN}$

**\*1-92.**

The assembly consists of three disks *A*, *B*, and *C* that are used to support the load of 140 kN. Determine the smallest diameter  $d_1$  of the top disk, the largest diameter  $d_2$  of the opening, and the largest diameter  $d_3$  of the hole in the bottom disk. The allowable bearing stress for the material is  $(\sigma_b)_{\text{allow}} = 350 \text{ MPa}$  and allowable shear stress is  $\tau_{\text{allow}} = 125 \text{ MPa}$ .



## SOLUTION

**Allowable Shear Stress:** Assume shear failure for disk *C*.

$$\tau_{\text{allow}} = \frac{V}{A}; \quad 125(10^6) = \frac{140(10^3)}{\pi d_2(0.01)}$$

$$d_2 = 0.03565 \text{ m} = 35.7 \text{ mm}$$

**Ans.**

**Allowable Bearing Stress:** Assume bearing failure for disk *C*.

$$(\sigma_b)_{\text{allow}} = \frac{P}{A}; \quad 350(10^6) = \frac{140(10^3)}{\frac{\pi}{4}(0.03565^2 - d_3^2)}$$

$$d_3 = 0.02760 \text{ m} = 27.6 \text{ mm}$$

**Ans.**

**Allowable Bearing Stress:** Assume bearing failure for disk *B*.

$$(\sigma_b)_{\text{allow}} = \frac{P}{A}; \quad 350(10^6) = \frac{140(10^3)}{\frac{\pi}{4}d_1^2}$$

$$d_1 = 0.02257 \text{ m} = 22.6 \text{ mm}$$

Since  $d_3 = 27.6 \text{ mm} > d_1 = 22.6 \text{ mm}$ , disk *B* might fail due to shear.

$$\tau = \frac{V}{A} = \frac{140(10^3)}{\pi(0.02257)(0.02)} = 98.7 \text{ MPa} < \tau_{\text{allow}} = 125 \text{ MPa} (\text{O.K!})$$

Therefore,

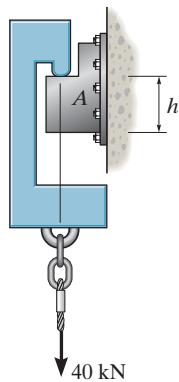
$$d_1 = 22.6 \text{ mm}$$

**Ans.**

**Ans:**

$d_2 = 35.7 \text{ mm}$ ,  
 $d_3 = 27.6 \text{ mm}$ ,  
 $d_1 = 22.6 \text{ mm}$

**1-93.** The aluminium bracket *A* is used to support the centrally applied load of 40 kN. If it has a constant thickness of 12 mm, determine the smallest height *h* in order to prevent a shear failure. The failure shear stress is  $\tau_{\text{fail}} = 160 \text{ MPa}$ . Use a factor of safety for shear of F.S. = 2.5.



## SOLUTION

### Equation of Equilibrium:

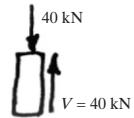
$$+\uparrow \sum F_y = 0; \quad V - 40 = 0 \quad V = 40 \text{ kN}$$

**Allowable Shear Stress:** Design of the support size

$$\tau_{\text{allow}} = \frac{\tau_{\text{fail}}}{\text{F.S.}} = \frac{V}{A}; \quad \frac{160(10^6)}{2.5} = \frac{40(10^3)}{h(0.012)}$$

$$h = 0.05208 \text{ m} = 52.1 \text{ mm}$$

**Ans.**

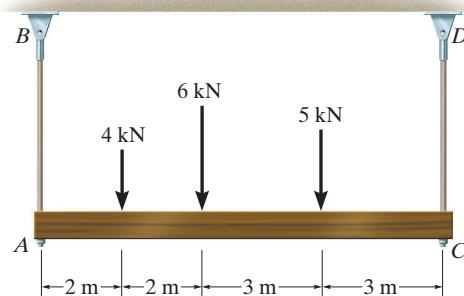


**Ans:**

$$h = 52.1 \text{ mm}$$

**1-94.**

The rods  $AB$  and  $CD$  are made of steel. Determine their smallest diameter so that they can support the dead loads shown. The beam is assumed to be pin connected at  $A$  and  $C$ . Use the LRFD method, where the resistance factor for steel in tension is  $\phi = 0.9$ , and the dead load factor is  $\gamma_D = 1.4$ . The failure stress is  $\sigma_{fail} = 345 \text{ MPa}$ .



**SOLUTION**

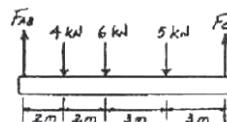
**Support Reactions:**

$$\zeta + \sum M_A = 0; \quad F_{CD}(10) - 5(7) - 6(4) - 4(2) = 0$$

$$F_{CD} = 6.70 \text{ kN}$$

$$\zeta + \sum M_C = 0; \quad 4(8) + 6(6) + 5(3) - F_{AB}(10) = 0$$

$$F_{AB} = 8.30 \text{ kN}$$



**Factored Loads:**

$$F_{CD} = 1.4(6.70) = 9.38 \text{ kN}$$

$$F_{AB} = 1.4(8.30) = 11.62 \text{ kN}$$

**For rod  $AB$**

$$0.9[345(10^6)] \pi \left(\frac{d_{AB}}{2}\right)^2 = 11.62(10^3)$$

$$d_{AB} = 0.00690 \text{ m} = 6.90 \text{ mm}$$

**Ans.**

**For rod  $CD$**

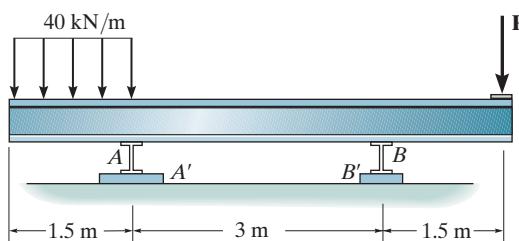
$$0.9[345(10^6)] \pi \left(\frac{d_{CD}}{2}\right)^2 = 9.38(10^3)$$

$$d_{CD} = 0.00620 \text{ m} = 6.20 \text{ mm}$$

**Ans.**

**Ans:**  
 $d_{AB} = 6.90 \text{ mm}, d_{CD} = 6.20 \text{ mm}$

- 1-95.** If the allowable bearing stress for the material under the supports at  $A$  and  $B$  is  $(\sigma_b)_{allow} = 1.5 \text{ MPa}$ , determine the size of *square* bearing plates  $A'$  and  $B'$  required to support the load. Dimension the plates to the nearest mm. The reactions at the supports are vertical. Take  $P = 100 \text{ kN}$ .



## SOLUTION

Referring to the FBD of the beam, Fig. *a*

$$\zeta + \sum M_A = 0; \quad N_B(3) + 40(1.5)(0.75) - 100(4.5) = 0 \quad N_B = 135 \text{ kN}$$

$$\zeta + \sum M_B = 0; \quad 40(1.5)(3.75) - 100(1.5) - N_A(3) = 0 \quad N_A = 25.0 \text{ kN}$$

For plate  $A'$ ,

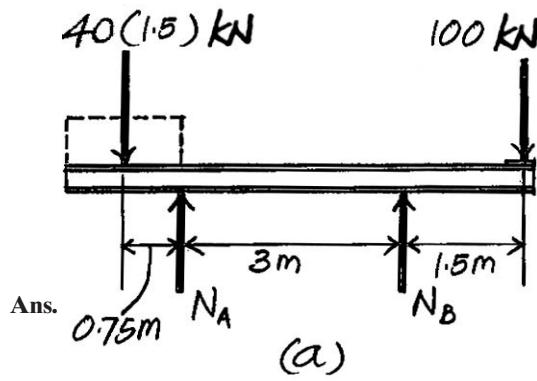
$$(\sigma_b)_{allow} = \frac{N_A}{A_{A'}}; \quad 1.5(10^6) = \frac{25.0(10^3)}{a_{A'}^2}$$

$$a_{A'} = 0.1291 \text{ m} = 130 \text{ mm}$$

For plate  $B'$ ,

$$\sigma_{allow} = \frac{N_B}{A_{B'}}; \quad 1.5(10^6) = \frac{135(10^3)}{a_{B'}^2}$$

$$a_{B'} = 0.300 \text{ m} = 300 \text{ mm}$$

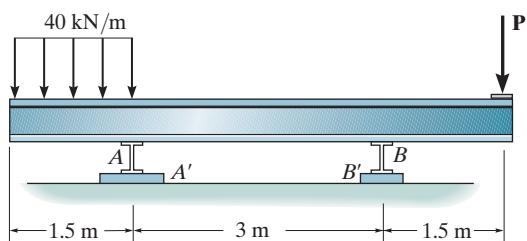


Ans.

Ans:

$$a_{A'} = 130 \text{ mm}, a_{B'} = 300 \text{ mm}$$

**\*1-96.** If the allowable bearing stress for the material under the supports at  $A$  and  $B$  is  $(\sigma_b)_{\text{allow}} = 1.5 \text{ MPa}$ , determine the maximum load  $P$  that can be applied to the beam. The bearing plates  $A'$  and  $B'$  have square cross sections of 150 mm  $\times$  150 mm and 250 mm  $\times$  250 mm, respectively.



## SOLUTION

Referring to the FBD of the beam, Fig. a,

$$\zeta + \sum M_A = 0; \quad N_B(3) + 40(1.5)(0.75) - P(4.5) = 0 \quad N_B = 1.5P - 15$$

$$\zeta + \sum M_B = 0; \quad 40(1.5)(3.75) - P(1.5) - N_A(3) = 0 \quad N_A = 75 - 0.5P$$

For plate  $A'$ ,

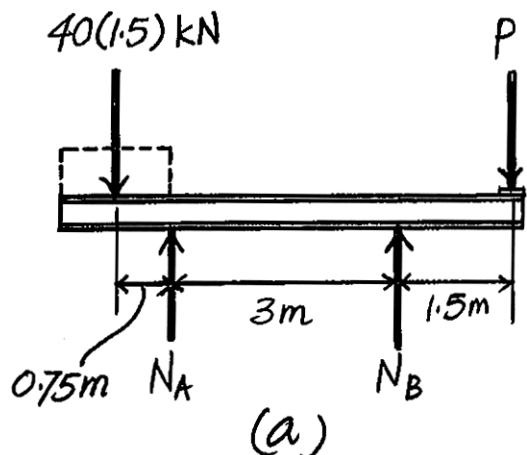
$$(\sigma_b)_{\text{allow}} = \frac{N_A}{A_{A'}}; \quad 1.5(10^6) = \frac{(75 - 0.5P)(10^3)}{0.15(0.15)}$$

$$P = 82.5 \text{ kN}$$

For plate  $B'$ ,

$$(\sigma_b)_{\text{allow}} = \frac{N_B}{A_{B'}}; \quad 1.5(10^6) = \frac{(1.5P - 15)(10^3)}{0.25(0.25)}$$

$$P = 72.5 \text{ kN} \quad (\text{Controls!})$$



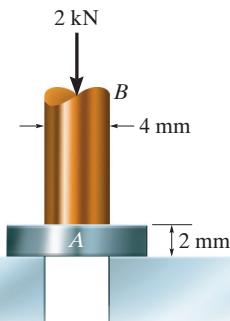
Ans.

Ans:

$P = 72.5 \text{ kN} \quad (\text{Controls!})$

**R1-1.**

The circular punch *B* exerts a force of 2 kN on the top of the plate *A*. Determine the average shear stress in the plate due to this loading.



**SOLUTION**

**Average Shear Stress:** The shear area  $A = \pi(0.004)(0.002) = 8.00(10^{-6})\pi \text{ m}^2$

$$\tau_{\text{avg}} = \frac{V}{A} = \frac{2(10^3)}{8.00(10^{-6})\pi} = 79.6 \text{ MPa}$$

**Ans.**

**Ans:**

$$\tau_{\text{avg}} = 79.6 \text{ MPa}$$

### R1-2.

Determine the required thickness of member  $BC$  and the diameter of the pins at  $A$  and  $B$  if the allowable normal stress for member  $BC$  is  $\sigma_{\text{allow}} = 200 \text{ MPa}$  and the allowable shear stress for the pins is  $\tau_{\text{allow}} = 70 \text{ MPa}$ .

### SOLUTION

Referring to the FBD of member  $AB$ , Fig. *a*,

$$\zeta + \sum M_A = 0; \quad 30(2.4)(1.2) - F_{BC} \sin 60^\circ (2.4) = 0 \quad F_{BC} = 41.57 \text{ kN}$$

$$\rightarrow \sum F_x = 0; \quad 41.57 \cos 60^\circ - A_x = 0 \quad A_x = 20.785 \text{ kN}$$

$$+ \uparrow \sum F_y = 0; \quad 41.57 \sin 60^\circ - 30(2.4) + A_y = 0 \quad A_y = 36.00 \text{ kN}$$

Thus, the force acting on pin  $A$  is

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{20.785^2 + 36.00^2} = 41.57 \text{ kN}$$

Pin  $A$  is subjected to single shear, Fig. *c*, while pin  $B$  is subjected to double shear, Fig. *b*.

$$V_A = F_A = 41.57 \text{ kN} \quad V_B = \frac{F_{BC}}{2} = \frac{41.57}{2} = 20.785 \text{ kN}$$

For member  $BC$

$$\sigma_{\text{allow}} = \frac{F_{BC}}{A_{BC}}; \quad 200(10^6) = \frac{41.57(10^3)}{(0.04)t} \quad t = 0.005196 \text{ m} = 5.196 \text{ mm}$$

$$\text{Use } t = 6 \text{ mm} \quad \text{Ans.}$$

For pin  $A$ ,

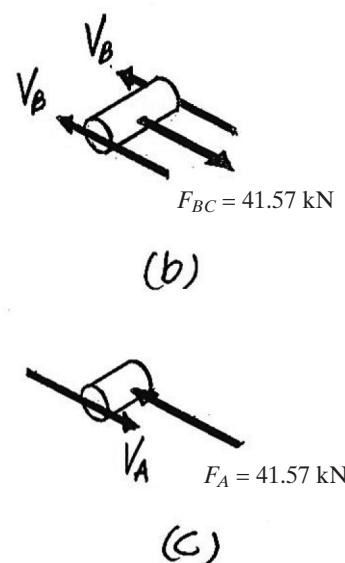
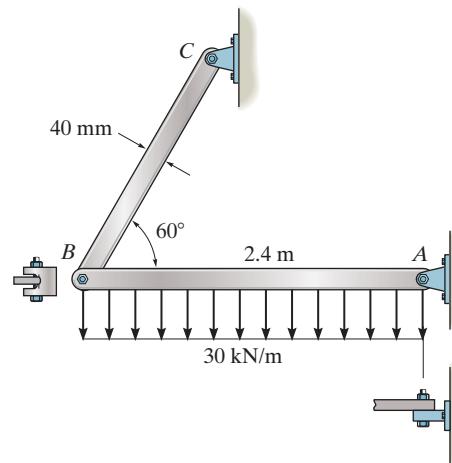
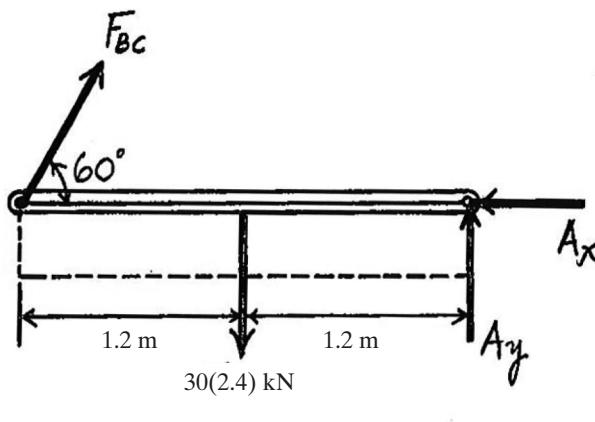
$$\tau_{\text{allow}} = \frac{V_A}{A_A}; \quad 70(10^6) = \frac{41.57(10^3)}{\frac{\pi}{4}d_A^2} \quad d_A = 0.02750 \text{ m} = 27.50 \text{ mm}$$

$$\text{Use } d_A = 28 \text{ mm} \quad \text{Ans.}$$

For pin  $B$ ,

$$\tau_{\text{allow}} = \frac{V_B}{A_B}; \quad 70(10^6) = \frac{20.785(10^3)}{\frac{\pi}{4}d_B^2} \quad d_B = 0.01944 \text{ m} = 19.44 \text{ mm}$$

$$\text{Use } d_B = 20 \text{ mm} \quad \text{Ans.}$$

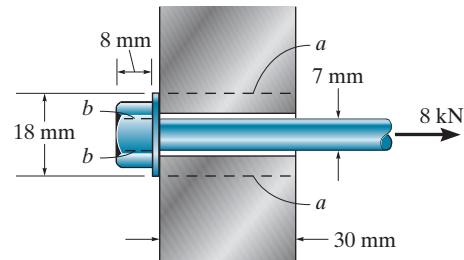


**Ans:**

$$t = 6 \text{ mm}, d_A = 28 \text{ mm}, d_B = 20 \text{ mm}$$

**R1-3.**

The long bolt passes through the 30-mm-thick plate. If the force in the bolt shank is 8 kN, determine the average normal stress in the shank, the average shear stress along the cylindrical area of the plate defined by the section lines *a-a*, and the average shear stress in the bolt head along the cylindrical area defined by the section lines *b-b*.



**SOLUTION**

$$\sigma_s = \frac{P}{A} = \frac{8(10^3)}{\frac{\pi}{4}(0.007)^2} = 208 \text{ MPa}$$

**Ans.**

$$(\tau_{\text{avg}})_a = \frac{V}{A} = \frac{8(10^3)}{\pi(0.018)(0.030)} = 4.72 \text{ MPa}$$

**Ans.**

$$(\tau_{\text{avg}})_b = \frac{V}{A} = \frac{8(10^3)}{\pi(0.007)(0.008)} = 45.5 \text{ MPa}$$

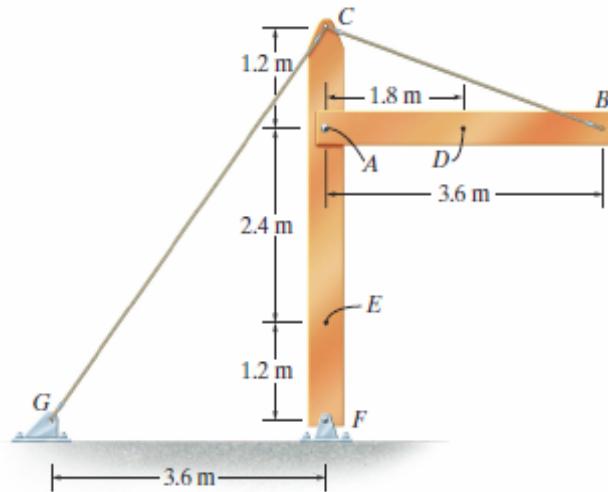
**Ans.**

**Ans:**  
 $\sigma_s = 208 \text{ MPa}$ ,  $(\tau_{\text{avg}})_a = 4.72 \text{ MPa}$ ,  
 $(\tau_{\text{avg}})_b = 45.5 \text{ MPa}$

**\*R1-4.**

The beam  $AB$  is pin supported at  $A$  and supported by a cable  $BC$ . A separate cable  $CG$  is used to hold up the frame. If  $AB$  weighs  $2.0 \text{ kN/m}$  and the column  $FC$  has a weight of  $3.0 \text{ kN/m}$ , determine the resultant internal loadings acting on cross sections located at points  $D$  and  $E$ . Neglect the thickness of both the beam and column in the calculation.

**Given:**  $w_b := 2.0 \frac{\text{kN}}{\text{m}}$   $L := 3.6\text{m}$   $d := 1.8\text{m}$   
 $w_c := 3.0 \frac{\text{kN}}{\text{m}}$   $H := 4.8\text{m}$   $e := 1.2\text{m}$   
 $a := 3.6\text{m}$   $b := 3.6\text{m}$   $c := 1.2\text{m}$



**SOLUTION**

**Beam  $AB$ :**  $L_c := \sqrt{L^2 + c^2}$   $v_b := \frac{c}{L_c}$   $h_b := \frac{L}{L_c}$

$\sum M_A = 0; B_y \cdot (L) - w_{AB} \cdot (L)(0.5 \cdot L) = 0$

$$B_y := w_b \cdot (L) \cdot \left(0.5 \frac{L}{L}\right)$$

$$B_y = 7936.64 \frac{\text{m}}{\text{s}^2} \text{ lb}$$

$\sum F_y = 0; -A_y - B_y + w_b \cdot (L) = 0 \quad A_y := -B_y + w_b \cdot (L) \quad A_y = 3.6 \text{ kN}$

$$B_y = F_{BC} \cdot (v_b)$$

$$F_{BC} := \frac{B_y}{v_b}$$

$$F_{BC} = 11.38 \text{ kN}$$

$\sum F_x = 0; -F_{BC} \cdot (h) + A_x = 0 \quad A_x := F_{BC} \cdot (h_b) \quad A_x = 10.8 \text{ kN}$

**Segment  $AD$ :**

$\sum F_x = 0; N_D + A_x = 0 \quad N_D := -A_x \quad N_D = -10.8 \text{ kN} \quad \text{Ans.}$

$\sum F_y = 0; -A_y + w_b \cdot (d) + V_D = 0 \quad V_D := A_y - w_b \cdot (d) \quad V_D = 0 \text{ kN} \quad \text{Ans.}$

$\sum M_D = 0; M_D + [w_b \cdot (d)] \cdot (0.5 \cdot d) - A_y \cdot (d) = 0$

$$M_D := -[w_b \cdot (d)] \cdot (0.5 \cdot d) + A_y \cdot (d) \quad M_D = 3.24 \text{ kN} \cdot \text{m} \quad \text{Ans.}$$

\*R1-4. Continued

**Member CG:**  $H_b := \sqrt{H^2 + b^2}$     $v_c := \frac{H}{H_b}$     $h_c := \frac{b}{H_b}$

**Column FC:**

$$\zeta + \sum M_C = 0; \quad F_x \cdot (H) - A_x \cdot c = 0 \quad F_x := A_x \cdot \left( \frac{c}{H} \right)$$

$$F_x = 2.7 \text{ kN}$$

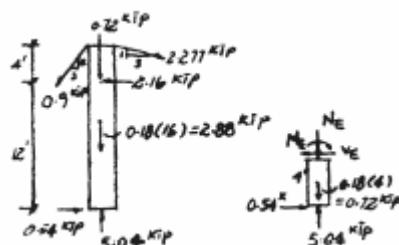
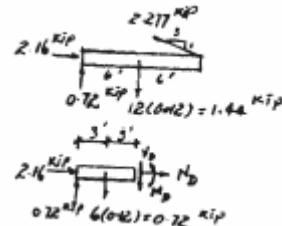
$$\pm \rightarrow \sum F_x = 0; \quad F_{BC} \cdot (h_b) - A_x + F_x - F_{CG} \cdot (h_c) = 0$$

$$\pm \downarrow \sum F_y = 0; \quad F_{CG} := \frac{F_{BC} \cdot (h_b) - A_x + F_x}{h_c} \quad F_{CG} = 4.5 \text{ kN}$$

$$-F_y + B_y + w_c \cdot (H) + F_{BC} \cdot (v_b) + F_{CG} \cdot (v_c) = 0$$

$$F_y := B_y + w_c \cdot (H) + F_{BC} \cdot (v_b) + F_{CG} \cdot (v_c)$$

$$F_y = 25.2 \text{ kN}$$



**Segment FE:**

$$\leftarrow \sum F_x = 0; \quad V_E - F_x = 0 \quad V_E := F_x \quad V_E = 2.7 \text{ kN} \quad \text{Ans.}$$

$$\downarrow \sum F_y = 0; \quad N_E + w_c \cdot (e) - F_y = 0 \quad N_E := -w_c \cdot (e) + F_y \quad N_E = 21.6 \text{ kN} \quad \text{Ans.}$$

$$\zeta + \sum M_E = 0; \quad -M_E + F_y \cdot (e) = 0 \quad M_E := F_y \cdot (e) \quad M_E = 30.24 \text{ kN}\cdot\text{m} \quad \text{Ans.}$$

**Ans:**

$$N_D = -10.8 \text{ kN}, V_D = 0 \text{ kN},$$

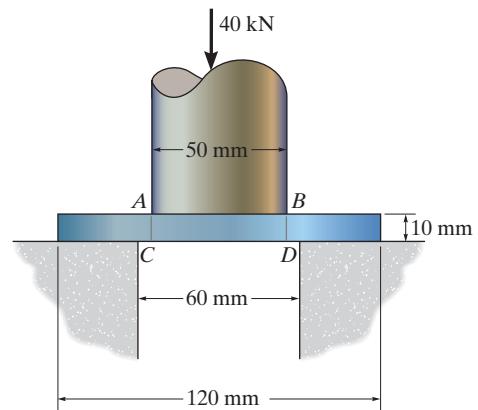
$$M_D = 3.24 \text{ kN}\cdot\text{m}$$

$$N_E = 21.6 \text{ kN}, V_E = 2.7 \text{ kN},$$

$$M_E = 30.24 \text{ kN}\cdot\text{m}$$

**R1-5.**

Determine the average punching shear stress the circular shaft creates in the metal plate through section *AC* and *BD*. Also, what is the average bearing stress developed on the surface of the plate under the shaft?



**SOLUTION**

**Average Shear and Bearing Stress:** The area of the shear plane and the bearing area on the punch are  $A_V = \pi(0.05)(0.01) = 0.5(10^{-3})\pi \text{ m}^2$  and  $A_b = \frac{\pi}{4}(0.12^2 - 0.06^2) = 2.7(10^{-3})\pi \text{ m}^2$ . We obtain

$$\tau_{\text{avg}} = \frac{P}{A_V} = \frac{40(10^3)}{0.5(10^{-3})\pi} = 25.5 \text{ MPa} \quad \text{Ans.}$$

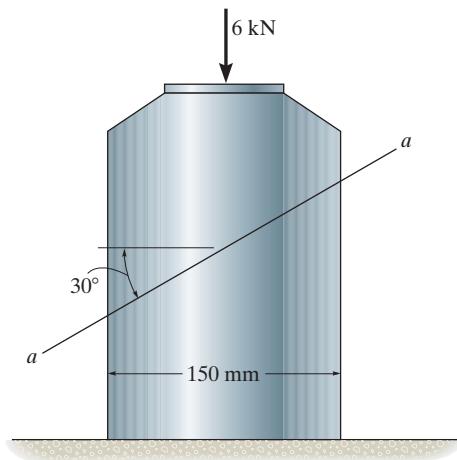
$$\sigma_b = \frac{P}{A_b} = \frac{40(10^3)}{2.7(10^{-3})\pi} = 4.72 \text{ MPa} \quad \text{Ans.}$$

**Ans:**

$\tau_{\text{avg}} = 25.5 \text{ MPa}$ ,  $\sigma_b = 4.72 \text{ MPa}$

**R1-6.**

The 150 mm by 150 mm block of aluminum supports a compressive load of 6 kN. Determine the average normal and shear stress acting on the plane through section *a-a*. Show the results on a differential volume element located on the plane.



**SOLUTION**

**Equation of Equilibrium:**

$$+\nearrow \sum F_x = 0; \quad V_{a-a} - 6 \cos 60^\circ = 0 \quad V_{a-a} = 3.00 \text{ kN}$$

$$\nwarrow + \sum F_y = 0; \quad N_{a-a} - 6 \sin 60^\circ = 0 \quad N_{a-a} = 5.196 \text{ kN}$$

**Average Normal Stress and Shear Stress:** The cross sectional Area at section *a-a* is

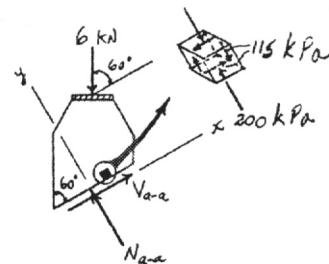
$$A = \left( \frac{0.15}{\sin 60^\circ} \right) (0.15) = 0.02598 \text{ m}^2.$$

$$\sigma_{a-a} = \frac{N_{a-a}}{A} = \frac{5.196(10^3)}{0.02598} = 200 \text{ kPa}$$

$$\tau_{a-a} = \frac{V_{a-a}}{A} = \frac{3.00(10^3)}{0.02598} = 115 \text{ kPa}$$

**Ans.**

**Ans.**

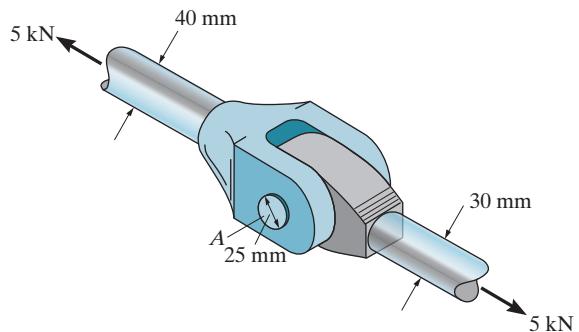


**Ans:**

$$\sigma_{a-a} = 200 \text{ kPa}, \tau_{a-a} = 115 \text{ kPa}$$

**R1-7.**

The yoke-and-rod connection is subjected to a tensile force of 5 kN. Determine the average normal stress in each rod and the average shear stress in the pin *A* between the members.

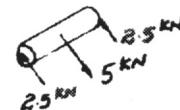


**SOLUTION**

For the 40 - mm - dia. rod:

$$\sigma_{40} = \frac{P}{A} = \frac{5(10^3)}{\frac{\pi}{4}(0.04)^2} = 3.98 \text{ MPa}$$

**Ans.**



For the 30 - mm - dia. rod:

$$\sigma_{30} = \frac{V}{A} = \frac{5(10^3)}{\frac{\pi}{4}(0.03)^2} = 7.07 \text{ MPa}$$

**Ans.**

Average shear stress for pin *A*:

$$\tau_{\text{avg}} = \frac{P}{A} = \frac{2.5(10^3)}{\frac{\pi}{4}(0.025)^2} = 5.09 \text{ MPa}$$

**Ans.**

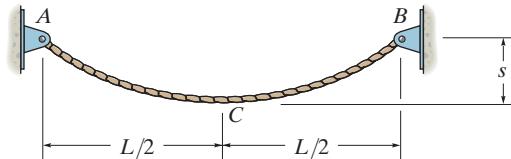


**Ans:**

$$\sigma_{40} = 3.98 \text{ MPa}, \sigma_{30} = 7.07 \text{ MPa}, \tau_{\text{avg}} = 5.09 \text{ MPa}$$

**\*R1-8.**

The cable has a specific weight  $\gamma$  (weight/volume) and cross-sectional area  $A$ . Assuming the sag  $s$  is small, so that the cable's length is approximately  $L$  and its weight can be distributed uniformly along the horizontal axis, determine the average normal stress in the cable at its lowest point  $C$ .



## SOLUTION

### Equation of Equilibrium:

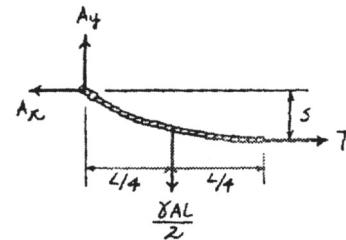
$$\zeta + \sum M_A = 0; \quad Ts - \frac{\gamma AL}{2} \left( \frac{L}{4} \right) = 0$$

$$T = \frac{\gamma AL^2}{8s}$$

### Average Normal Stress:

$$\sigma = \frac{T}{A} = \frac{\frac{\gamma AL^2}{8s}}{A} = \frac{\gamma L^2}{8s}$$

**Ans.**



**Ans:**

$$\sigma = \frac{\gamma L^2}{8s}$$