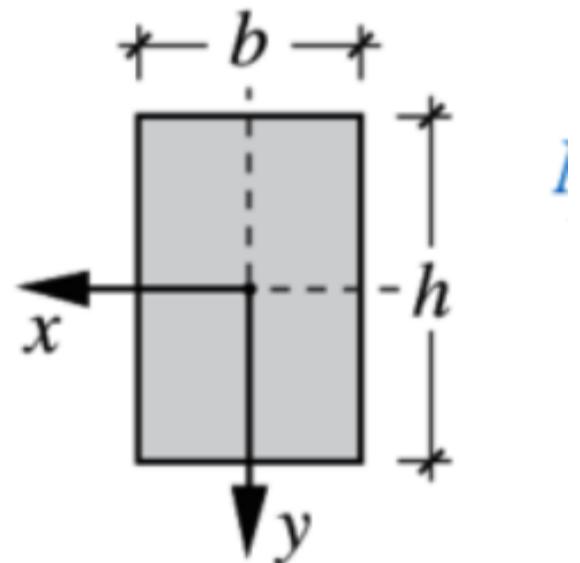
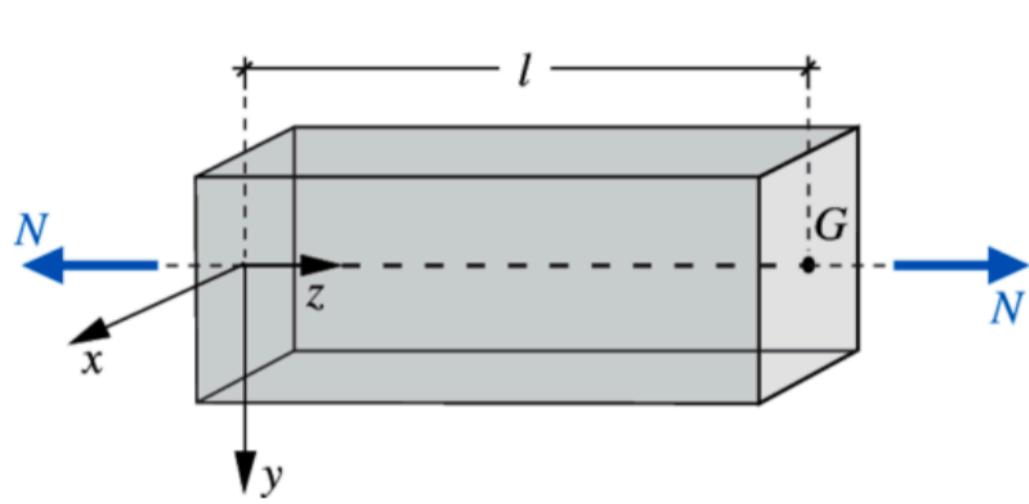
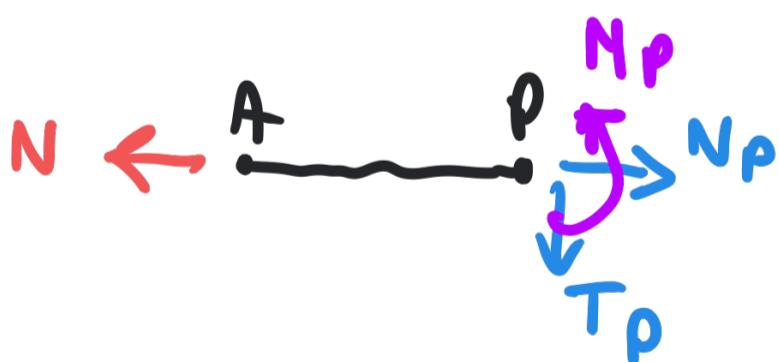
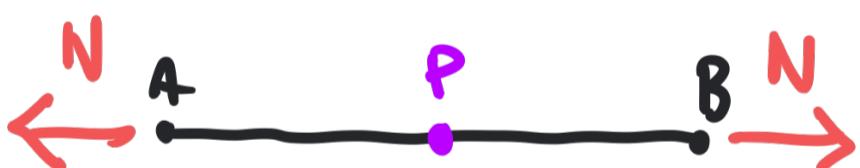


Stato tensionale in una trave tesa/compressa



Schema strutturale

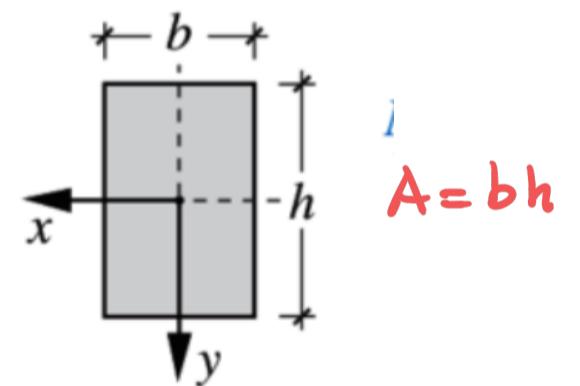
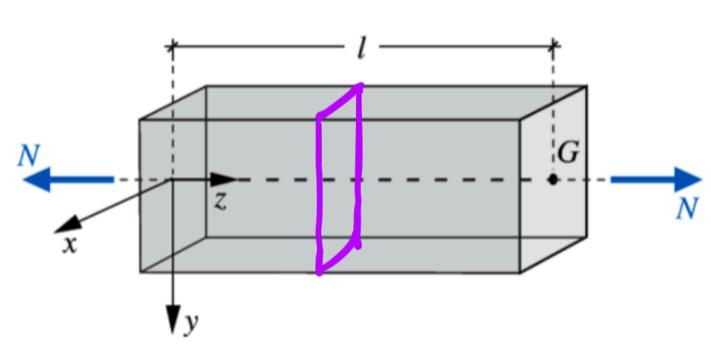


$$\sum F_x = 0 \Rightarrow N_p = N$$

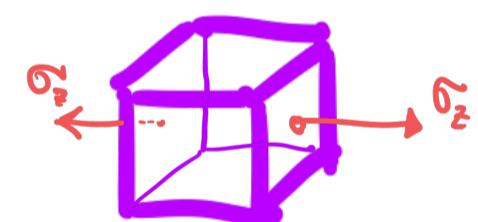
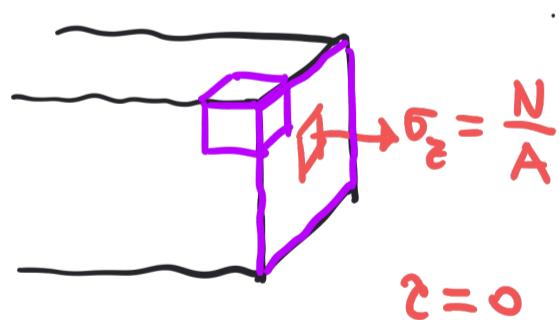
$$\sum F_y = 0 \Rightarrow T_p = 0$$

$$\sum M = 0 \Rightarrow M_p = 0$$

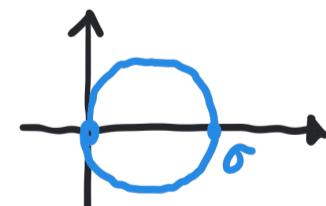
Stato tensionale in una trave tesa/compressa



Tensione



Cerchio di Mohr (piani $z-x$ e $z-y$)



FLESSIONE UNIFORME → il momento flettente è costante.

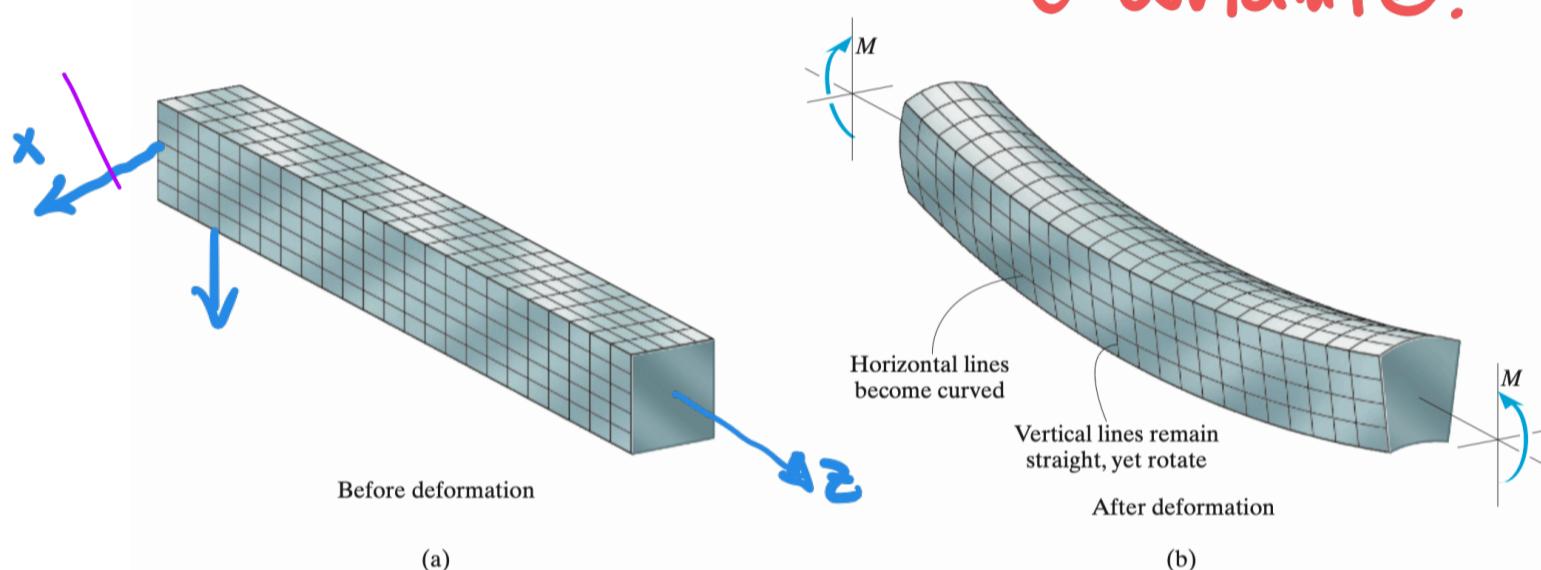
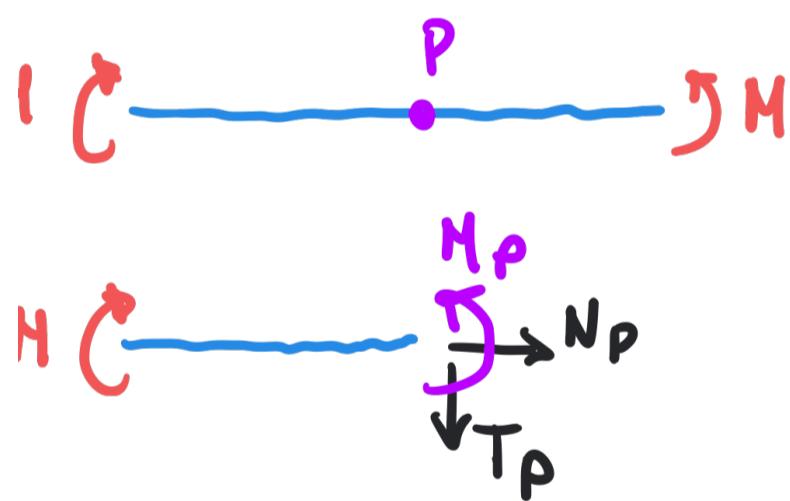
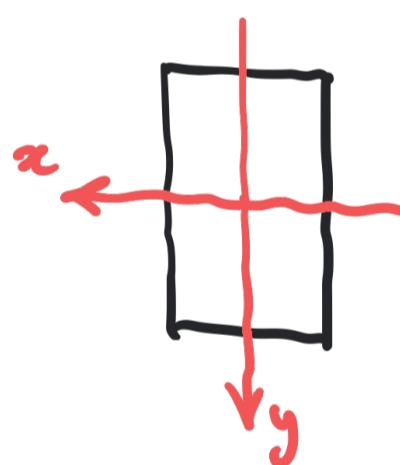


Fig. 6-19



$$M_p = M \text{ (costante)}$$

$$N_p = T_p = 0$$

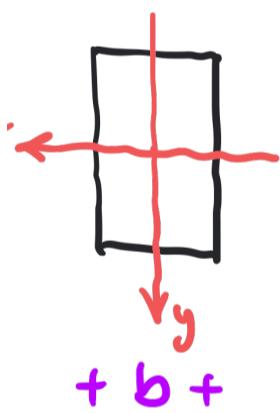
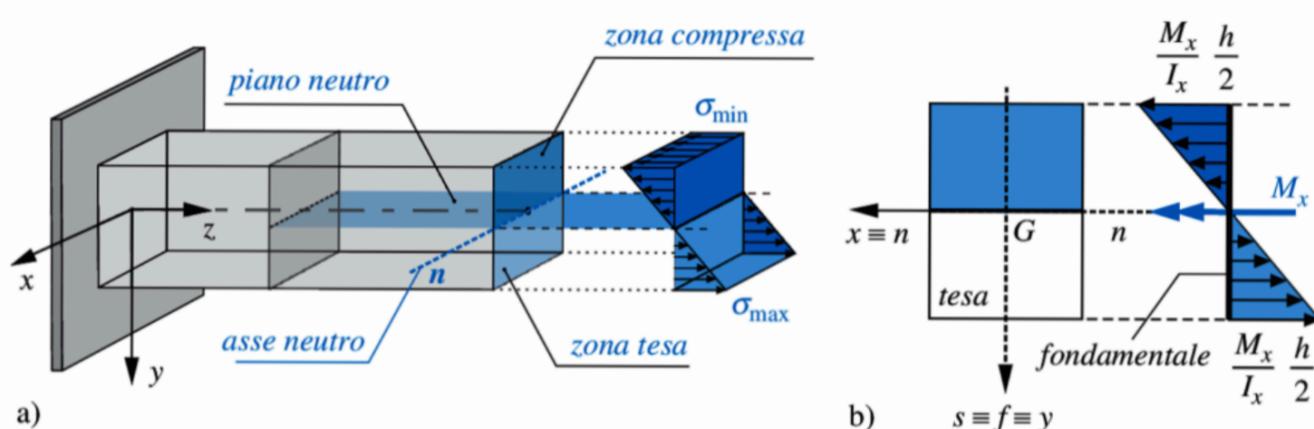


$$\theta_z = \frac{M_x}{I_x} y$$

$$I_x = \int_A y^2 dA$$

momento
d'inerzia

Stato tensionale in una trave inflessa



momento
d'inerzia rispetto
all'asse x .

$$I_x = \frac{M_x}{I_x} y$$

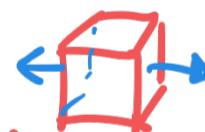
$$I_x = \int_A y^2 dA$$

Sezione rettangolare

$$I_x = \frac{1}{12} b h^3$$

$\sigma_z = 0$ per $y=0$ (asse neutro)

- Lo stato tensionale è uniaxiale



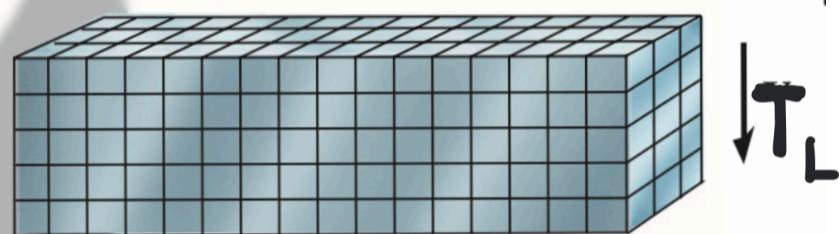
- I punti più sollecitati sono quelli più lontani dall'asse neutro

Rif: Casini-Vasta pag. 18.10

Hibbeler, pag. 10.7.

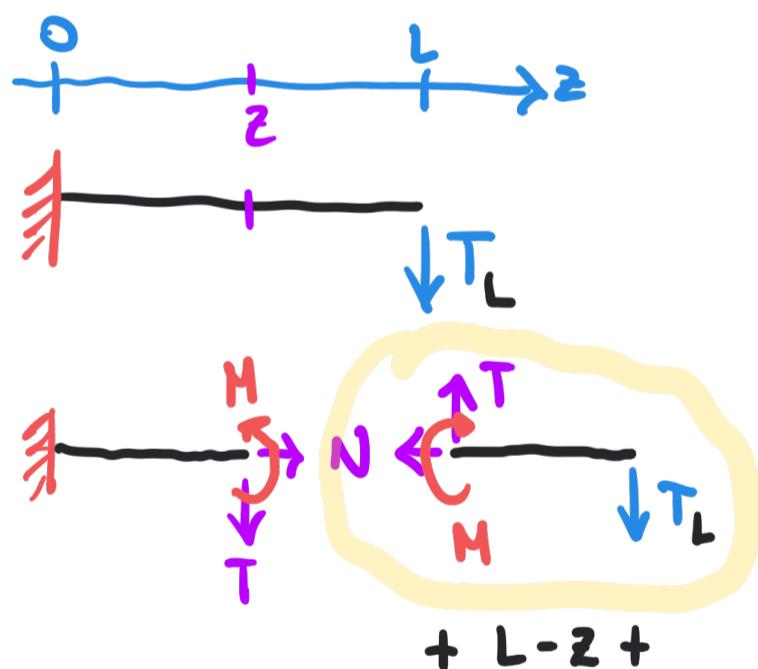
A differenza: Hibbeler orienta l'asse y verso l'alto,
e dunque nella sua formula
cominciano un segno -.

Flessione e taglio



(a) Before deformation

Schema strutturale



$$\sum F_x = 0 \quad N = 0$$

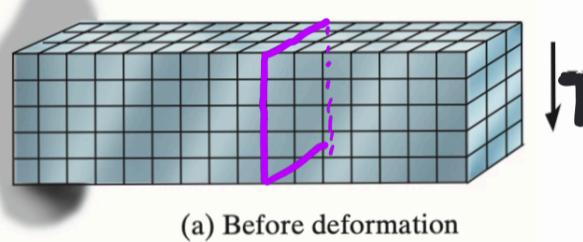
$$\sum F_y = 0 \quad T = T_L$$

$$\sum M_x = 0 \quad -T_L(L-z) - M = 0$$

$$M = -T_L(L-z)$$

NB: una sollecitazione di taglio è sempre accompagnata da un momento flettente.

Flessione e taglio



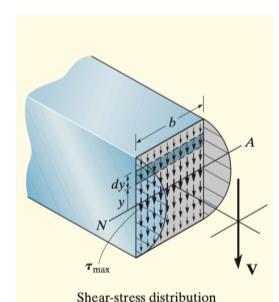
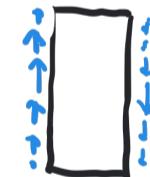
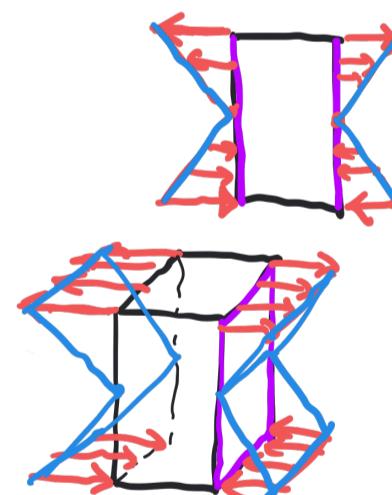
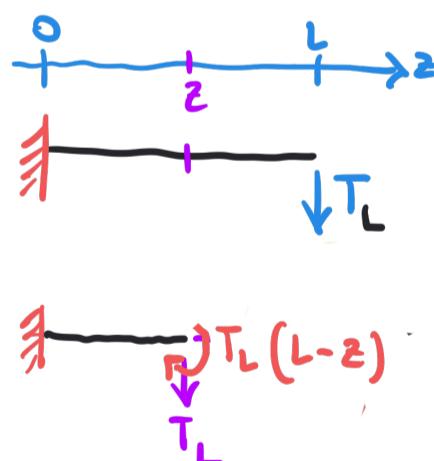
$$T_L(L-z + \Delta z) = T_L(L-z) + T_L \Delta z$$

$T_L(L-z + \Delta z) = T_L(L-z) + T_L \Delta z$

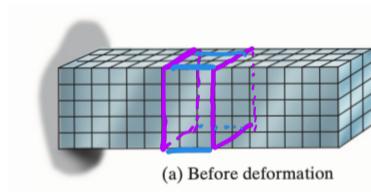
$z - \Delta z$ z

$+ \Delta z +$

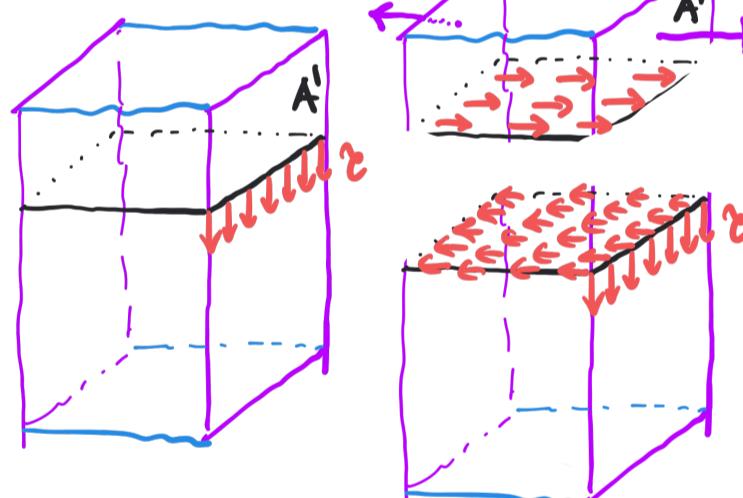
Schema strutturale



Flessione e taglio

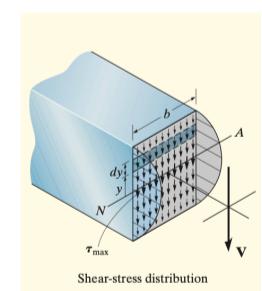
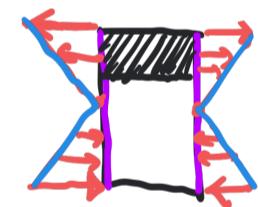


$$N'(z - \Delta z) = \int_{A'} -\frac{T_L(L-z)}{I_x} y dA$$

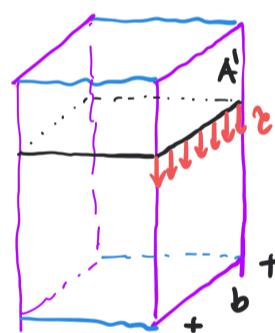
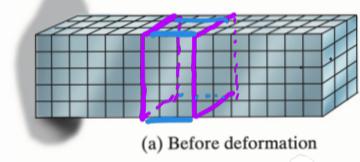


$$T_L(z - \Delta z + \Delta z) = T_L(z) + T_L \Delta z$$

reciprocity



FORMULA DI JOURAVSKY



$$N'(z) = \frac{1}{A'} \int_{A'} -T_L(L-z) y \, dA$$

Equilibrio lungo z :

$$\rightarrow \sum F_z = 0 \quad z b \Delta \varepsilon - \frac{T_L(L-z)}{I_x} \int_{A'} y \, dA + \frac{T_L(L-z)}{I_x} \int_{A'} y \, dA + \frac{T_L \Delta \varepsilon}{I_x} \int_{A'} y \, dA = 0$$

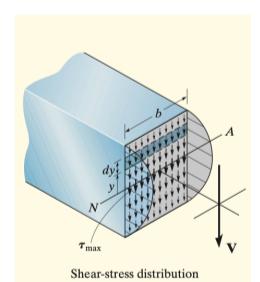
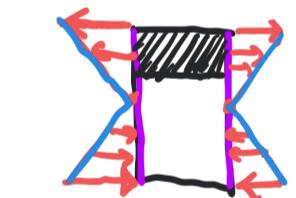
$$z = - \frac{T_L}{b I_x} A' y'$$

area delle regioni sopra la corde

$$\int_{A'} y \, dA = y' \text{ baricentro di } A'$$

$$T_L(L-z+\Delta z) = T_L(L-z) + T_L \Delta \varepsilon$$

$T_L(L-z+\Delta z) = T_L(L-z) + T_L \Delta \varepsilon$

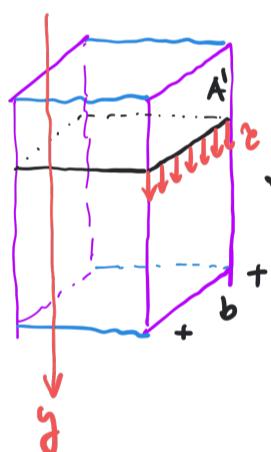


FORMULA DI JOURANSY

CASINI-VASTA (21.11)

$$c = - \frac{T_y S^x}{S I_x}$$

$S^* = A' y'$



$$c = - \frac{T_y}{b I_x} A' y'$$

area delle regioni
sopra la corda

$$T_y \int_{A'} y dA = y' \text{ baricentro di } A'$$

HIBBELER
(11.3)

$$c = \frac{V Q}{I t} \quad Q = \bar{y}' A'$$

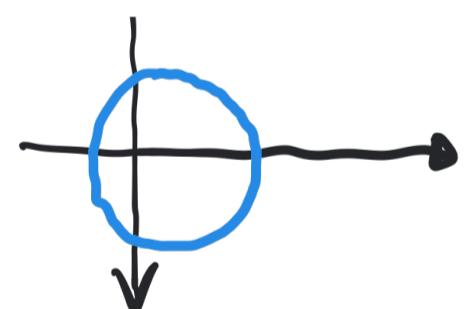
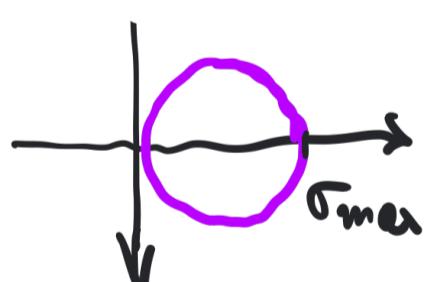
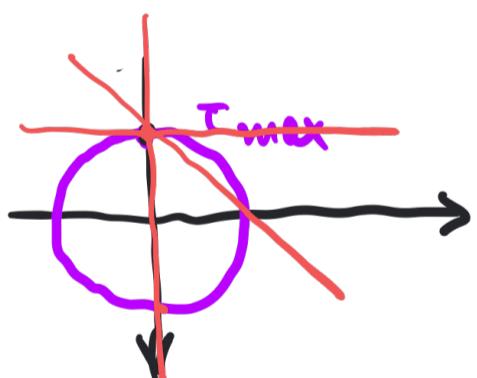
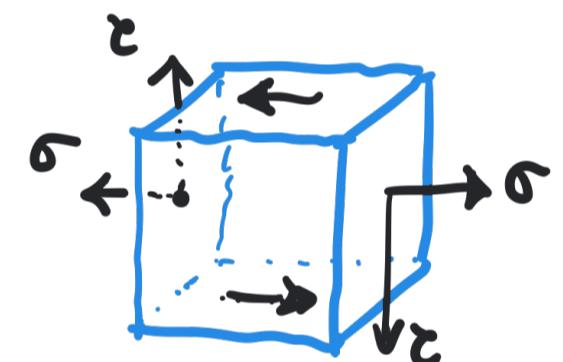
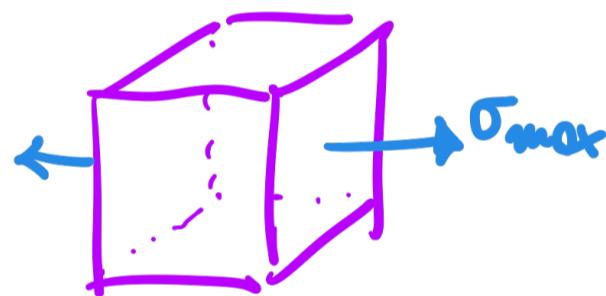
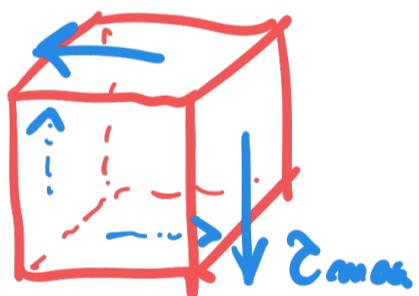
↑ spessore
corda

LA DISCEPANZA di segno è dovuta
al fatto che H. orienta l'asse y
verso l'alto

Stato tensionale

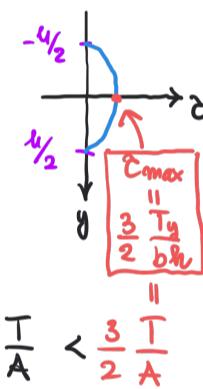


(a) Before deformation



TENSIONI TANGENZIALI NELLA SEZIONE RETTANGOLARE

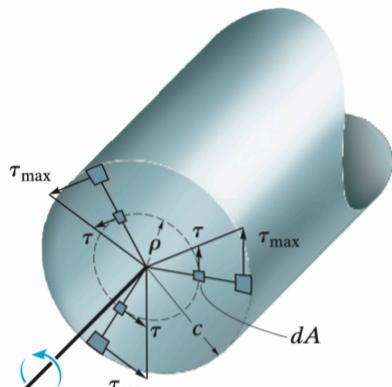
A'
 $z = -\frac{h}{2}$
 $\int y dA = y'$ baricentro di A'
 $A' = \left(\frac{h}{2} + y\right) b$
 $y' = \frac{1}{2} \left[y + \left(-\frac{h}{2}\right)\right] = \frac{y}{2} - \frac{h}{4}$
 $I_x = \frac{1}{12} b h^3$
 $\tau = -\frac{T_y}{b \frac{1}{12} b h^3} \left(\frac{h}{2} + y\right) \left(\frac{y}{2} - \frac{h}{4}\right) b = \frac{6 T_y}{b h^3} \left(\frac{h^2}{4} - y^2\right)$



In una sezione rettangolare le tensioni tangenziali mostrano un andamento parabolico, e attingono il loro valore massimo in corrispondenza della corda media

Stimare τ_{max} con la formula $\tau_{max} = T/A$ produce una sofferstima di τ_{max} del 30%.

TORSIONE DI BARRE A SEZIONE CIRCOLARE



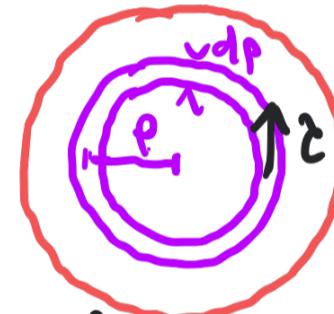
- Le tensioni tangenziali sono perpendicolari alla direzione radiale
- La loro intensità è proporzionale alla distanza dal centro.

$$\tau(\rho) = \frac{\tau_{\max}}{R} \rho$$

$$dM_t = \tau 2\pi \rho d\rho = \tau_{\max} \frac{2\pi}{R} \rho^2 d\rho$$

$$M_t = \int dM_t = 2\pi / R \tau_{\max} \int_0^R \rho^2 d\rho = \\ = \frac{\tau_{\max}}{R} I_p = I_p \frac{\tau(\rho)}{\rho}$$

$$\boxed{\tau(\rho) = \rho \frac{M_t}{I_p}}$$



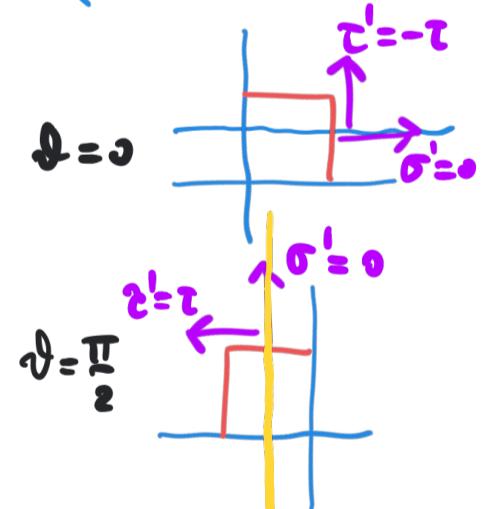
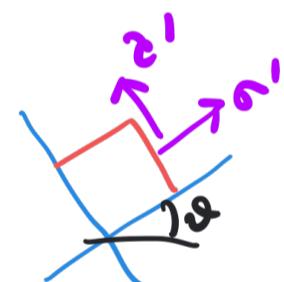
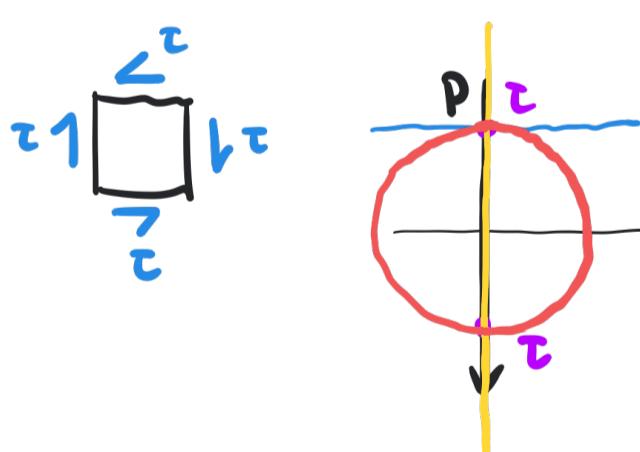
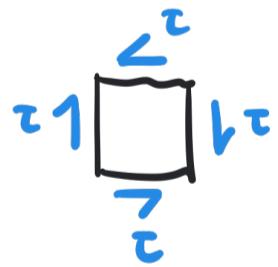
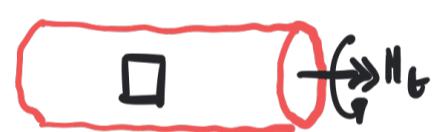
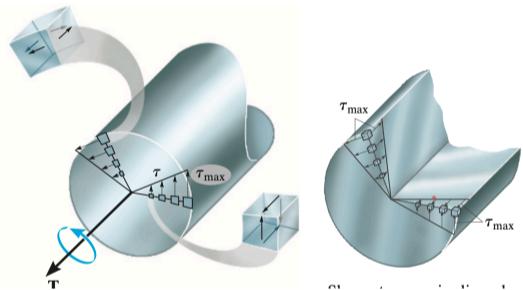
$$2\pi \int_0^R \rho^2 d\rho = I_p$$

momento d'inerzia polare

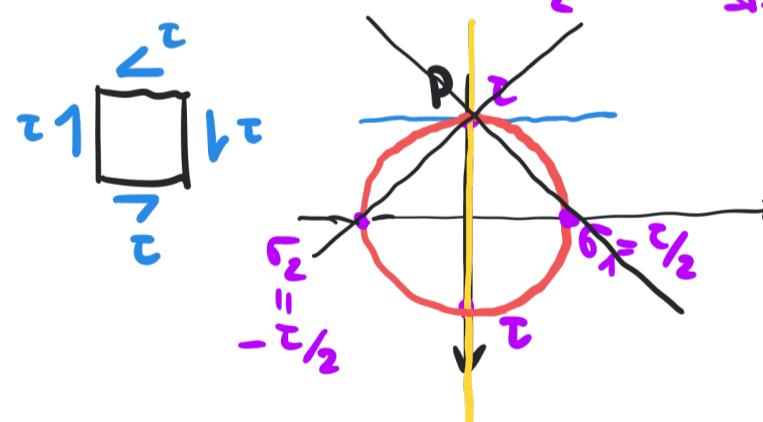
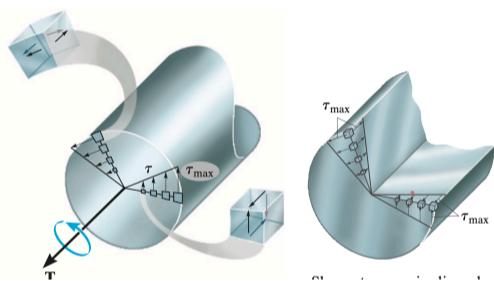
Rif: Casini-Vasta eq. (20.14)

Hibbeler eq. (9-6).

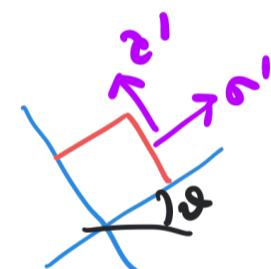
STATO TENSIONALE IN UNA BARRA DI TORSIONE



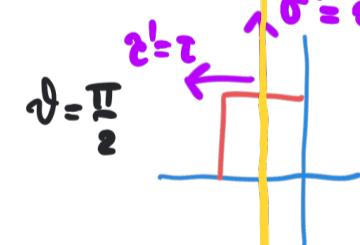
STATO TENSIONALE IN UNA BARRA DI TORSIONE



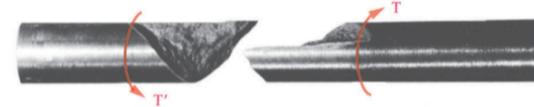
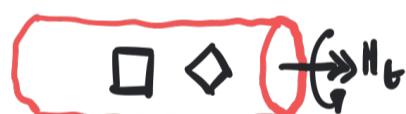
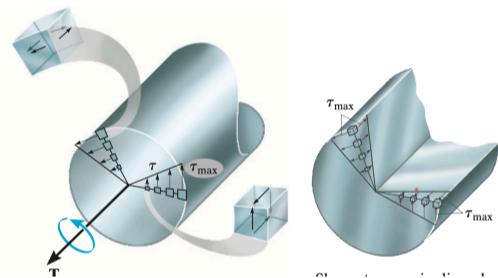
$$\theta = \varphi \quad \sigma' = -\frac{\tau}{2}$$



$$\theta = \varphi \quad \tau' = -\tau \quad \sigma' = 0$$



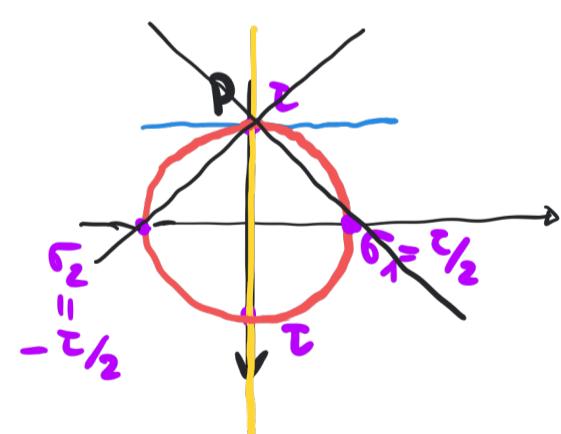
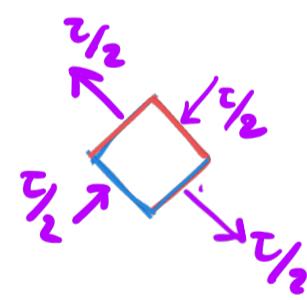
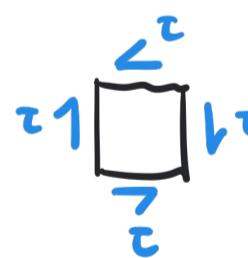
STATO TENSIONALE IN UNA BARRA DI TORSIONE



(b) Brittle Failure



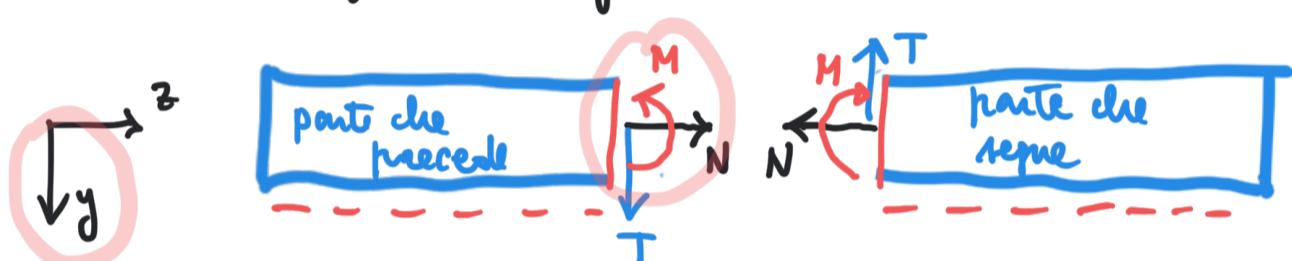
Failure of a wooden shaft due to torsion.



LE CARATTERISTICHE DELLA SOLLECITA-
ZIONE NELLE TRAVI PIANE

LE CARATTERISTICHE DELLA SOLLECITAZIONE NELLE TRAVI PIANE

Convenzione dei segni delle CdS.



$$R = N_k + T_j$$

$$M = M_i$$



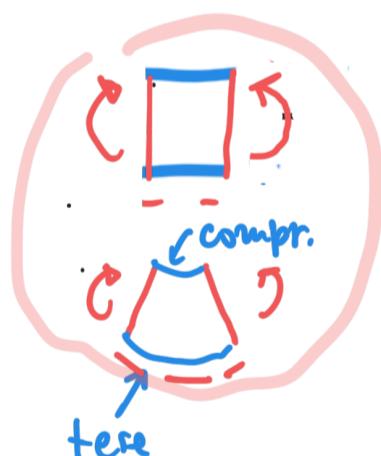
$N > 0$ se di TRAZIONE

$T > 0$ se tende a far ruotare in senso ORARIO la posizione di trave alle quell'è applicata

$M > 0$ se tende a far ruotare in senso antiorario la parte che PRECEDE.

(equiv. in senso orario la parte che SEGUO).

equiv: se "tende" la punteggiata



TESTO

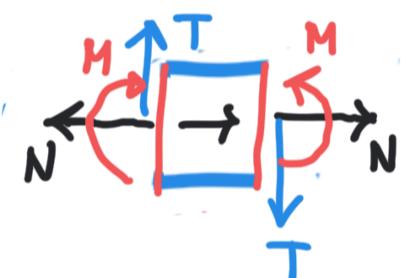
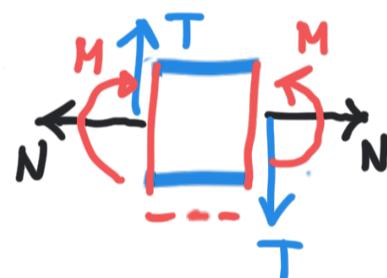
ATTENZIONE: il segno di M dipende da come è stata orientata la trave

Cambiando l'orientazione, la parte che precede diventa parte che segue e viceversa, e cambia la disposizione delle punteggiata!!!

LE CARATTERISTICHE DELLA SOLLECITAZIONE NELLE TRAVI PIANE

Mattonelle dei regni.

si trova
in alcuni
esercizi.



ATT: le mattonelle deve comunque essere orientata, altrimenti il regno di N non è univoco !!

