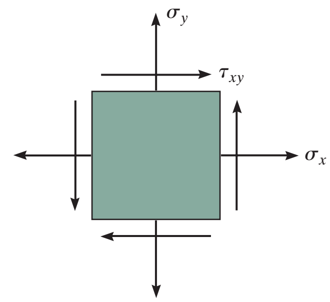
14–1.

A material is subjected to a general state of plane stress. Express the strain energy density in terms of the elastic constants E, G, and ν and the stress components σ_x, σ_y, and τ_{xy}.



SOLUTION

Strain Energy Due to Normal Stresses: We will consider the application of normal stresses on the element in two successive stages. For the first stage, we apply only σ_x on the element. Since σ_x is a constant,

$$(U_i)_1 = \int_v \frac{\sigma_x^2}{2E} dV = \frac{\sigma_x^2 V}{2E}$$

When σ_y is applied in the second stage, the normal strain ϵ_x will be strained by $\epsilon_x{}' = -\nu\epsilon_y = -\frac{v\sigma_y}{E}$. Therefore, the strain energy for the second stage is

$$(U_i)_2 = \int_v \left(\frac{\sigma_y^2}{2E} + \sigma_x \epsilon_x{}'\right) dV$$

$$= \int_v \left[\frac{\sigma_y^2}{2E} + \sigma_x\left(-\frac{\nu\sigma_y}{E}\right)\right] dV$$

Since σ_x and σ_y are constants,

$$(U_i)_2 = \frac{V}{2E}(\sigma_y^2 - 2\nu\sigma_x\sigma_y)$$

Strain Energy Due to Shear Stresses: The application of τ_{xy} does not strain the element in a normal direction. Thus, from Eq. 14–11, we have

$$(U_i)_3 = \int_v \frac{\tau_{xy}^2}{2G} dV = \frac{\tau_{xy}^2 V}{2G}$$

The total strain energy is

$$U_i = (U_i)_1 + (U_i)_2 + (U_i)_3$$

$$= \frac{\sigma_x^2 V}{2E} + \frac{V}{2E}(\sigma_y^2 - 2\nu\sigma_x\sigma_y) + \frac{\tau_{xy}^2 V}{2G}$$

$$= \frac{V}{2E}(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y) + \frac{\tau_{xy}^2 V}{2G}$$

and the strain energy density is

$$\frac{U_i}{V} = \frac{1}{2E}(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y) + \frac{\tau_{xy}^2}{2G} \qquad \textbf{Ans.}$$

Ans:

$$\frac{U_i}{V} = \frac{1}{2E}(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y) + \frac{\tau_{xy}^2}{2G}$$

14–2.

The strain-energy density for plane stress must be the same whether the state of stress is represented by σ_x, σ_y, and τ_{xy}, or by the principal stresses σ_1 and σ_2. This being the case, equate the strain–energy expressions for each of these two cases and show that $G = E/[2(1 + \nu)]$.

SOLUTION

$$U = \int_\nu \left[\frac{1}{2\,E}(\sigma_x^2 + \sigma_y^2) - \frac{\nu}{E}\sigma_x\sigma_y + \frac{1}{2\,G}\tau_{xy}^2\right] dV$$

$$U = \int_\nu \left[\frac{1}{2\,E}(\sigma_1^2 + \sigma_2^2) - \frac{\nu}{E}\sigma_1\,\sigma_2\right] dV$$

Equating the above two equations yields.

$$\frac{1}{2\,E}(\sigma_x^2 + \sigma_y^2) - \frac{\nu}{E}\sigma_x\sigma_y + \frac{1}{2\,G}\tau_{xy}^2 = \frac{1}{2\,E}(\sigma_1^2 + \sigma_2^2) - \frac{\nu}{E}\sigma_1\,\sigma_2 \qquad \textbf{(1)}$$

However, $\sigma_{1,2} = \dfrac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Thus, $\left(\sigma_1^2 + \sigma_2^2\right) = \sigma_x^2 + \sigma_y^2 + 2\,\tau_{xy}^2$

and also $\qquad \sigma_1\,\sigma_2 = \sigma_x\sigma_y - \tau_{xy}^2$

Substitute into Eq. (1)

$$\frac{1}{2\,E}\left(\sigma_x^2 + \sigma_y^2\right) - \frac{\nu}{E}\sigma_x\sigma_y + \frac{1}{2\,G}\tau_{xy}^2 = \frac{1}{2\,E}(\sigma_x^2 + \sigma_y^2 + 2\tau_{xy}^2) - \frac{\nu}{E}\sigma_x\sigma_y + \frac{\nu}{E}\tau_{xy}^2$$

$$\frac{1}{2\,G}\tau_{xy}^2 = \frac{\tau_{xy}^2}{E} + \frac{\nu}{E}\tau_{xy}^2$$

$$\frac{1}{2\,G} = \frac{1}{E} + \frac{\nu}{E}$$

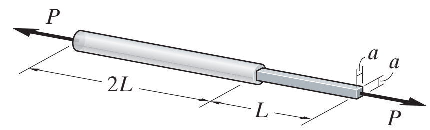
$$\frac{1}{2\,G} = \frac{1}{E}(1 + \nu)$$

$$G = \frac{E}{2(1 + \nu)} \qquad \textbf{QED}$$

Ans:
N/A

14–3.

The A-36 steel bar consists of two segments, one of circular cross section of radius r, and one of square cross section. If the bar is subjected to the axial loading of P, determine the dimensions a of the square segment so that the strain energy within the square segment is the same as in the circular segment.



SOLUTION

Axial Strain Energy: Applying Eq. 14–16 to the circular segment gives

$$(U_i)_c = \frac{N^2 L_c}{2AE} = \frac{P^2(2L)}{2(\pi r^2)E} = \frac{P^2 L}{\pi r^2 E}$$

Applying Eq. 14–16 to the square segment gives

$$(U_i)_s = \frac{N^2 L_s}{2AE} = \frac{P^2 L}{2(a^2)E} = \frac{P^2 L}{2a^2 E}$$

Require

$$(U_i)_c = (U_i)_s$$

$$\frac{P^2 L}{\pi r^2 E} = \frac{P^2 L}{2a^2 E}$$

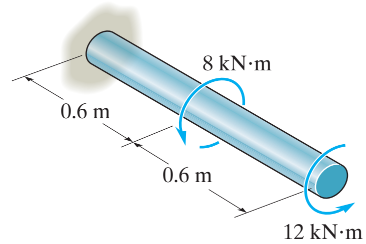
$$a = \sqrt{\frac{\pi}{2}}\, r \qquad \textbf{Ans.}$$

Ans:

$a = \sqrt{\frac{\pi}{2}}\, r$

***14–4.**

Determine the torsional strain energy in the A992 steel shaft. The shaft has a radius of 50 mm.



SOLUTION

Internal Torque: As shown on the FBD in Fig. *a* and *b*.

Torsional Strain Energy: With polar moment of inertia $J = \frac{\pi}{2}(0.05^4) = 3.125(10^{-6})\pi \text{ m}^4$ and $G = 75$ GPa for A992 steel,

$$U_i = \Sigma \frac{T^2 L}{2GJ}$$

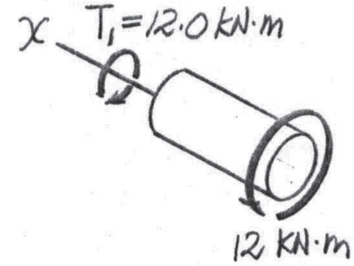
$$= \frac{1}{2GJ}\left\{\left[12.0(10^3)\right]^2(0.6) + \left[20.0(10^3)\right]^2(0.6)\right\}$$

$$= \frac{0.1632(10^9)\ \text{N}^2\cdot\text{m}^3}{GJ}$$

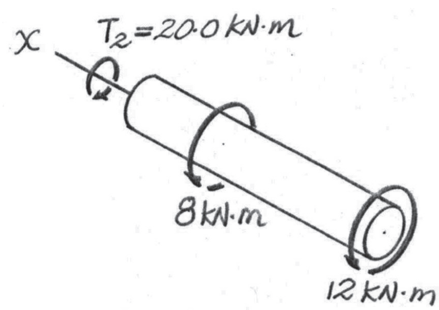
$$= \frac{0.1632(10^9)}{75(10^9)\left[3.125(10^{-6})\pi\right]}$$

$$= 221.65 \text{ J}$$

$$= 222 \text{ J}$$ **Ans.**



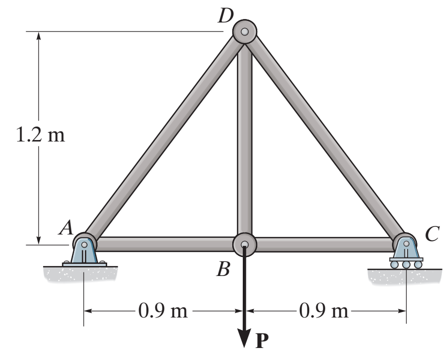
(a)



(b)

Ans:
$U_i = 222$ J

14–5. If $P = 50$ kN, determine the total strain energy stored in the truss. Each member has a diameter of 50 mm and is made of A992 steel.



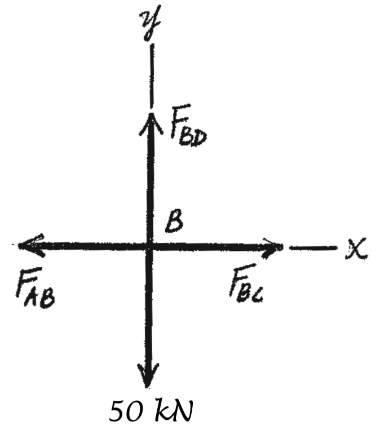
SOLUTION

Normal Forces. The normal forces developed in each member of the truss can be determined using the method of joints.

Joint *B* (Fig. *a*)

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{BC} - F_{AB} = 0 \qquad (1)$

$+\uparrow \Sigma F_y = 0; \quad F_{BD} - 50 = 0 \qquad F_{BD} = 50 \text{ kN (T) (max)}$

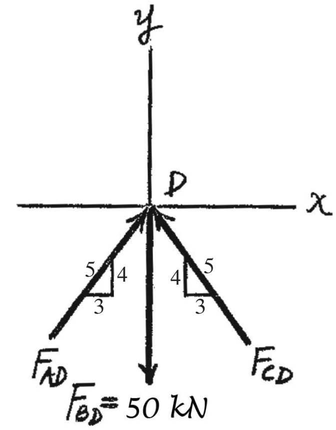


(a)

Joint *D* (Fig. *b*)

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AD} = \left(\frac{3}{5}\right) - F_{CD}\left(\frac{3}{5}\right) = 0 \qquad F_{AD} = F_{CD} = F$

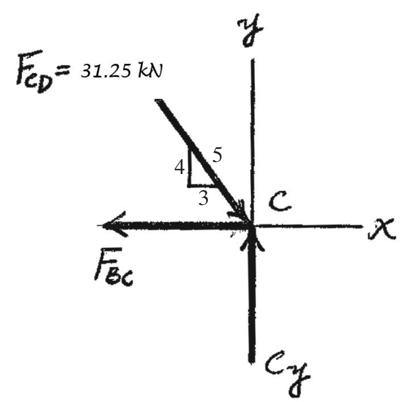
$+\uparrow \Sigma F_y = 0; \quad 2\left[F\left(\frac{4}{5}\right)\right] - 50 = 0 \qquad F_{AD} = F_{CD} = F = 31.25 \text{ kN (C)}$



(b)

Joint *C* (Fig. *c*)

$\xrightarrow{+} \Sigma F_x = 0; \quad 31.25\left(\frac{3}{5}\right) - F_{BC} = 0 \qquad F_{BC} = 18.75 \text{ kN (T)}$



(c)

Using the result of $\mathbf{F}_{BC}$, Eq. (1) gives

$$F_{AB} = 18.75 \text{ kN (T)}$$

Axial Strain Energy. $A = \frac{\pi}{4}(0.05^2) = 0.625(10^{-3})\pi \text{m}^2$ and $L_{CD} = \sqrt{0.9^2 + 1.2^2} = 1.5$ m

$$(U_i)_a = \Sigma \frac{N^2 L}{2AE}$$

$$= \frac{1}{2[0.625(10^{-3})\pi][200(10^9)]}\left\{[50(10^3)]^2(1.2) + 2[31.25(10^3)]^2(1.5) + 2[18.75(10^3)]^2(0.9)\right\}$$

$$= 8.356 \text{ N}\cdot\text{m} = 8.36 \text{ J}$$

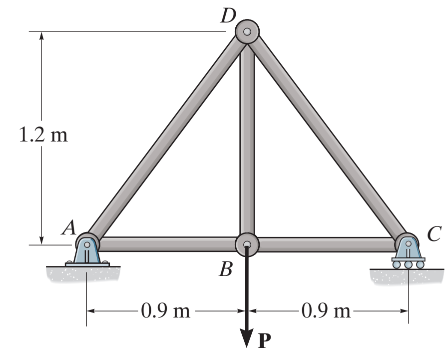
Ans.

This result is only valid if $\sigma < \sigma_Y$. We only need to check member *BD* since it is subjected to the greatest normal force

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{50(10^3)}{0.625(10^{-3})\pi} = 25.46(10^6) \text{ N/m}^2 = 25.46 \text{ MPa} < \sigma_Y$$

$$= 345 \text{ MPa} \qquad \text{(O.K.)}$$

14–6. Determine the maximum force **P** and the corresponding maximum total strain energy that can be stored in the truss without causing any of the members to have permanent deformation. Each member of the truss has a diameter of 50 mm and is made of A-36 steel.



SOLUTION

Normal Forces. The normal force developed in each member of the truss can be determined using the method of joints.

Joint *B* (Fig. *a*)

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{BC} - F_{AB} = 0 \qquad (1)$

$+\uparrow \Sigma F_y = 0; \quad F_{BD} - P = 0 \qquad F_{BD} = P\text{(T)}$

Joint *D* (Fig. *b*)

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AD} = \left(\frac{3}{5}\right) - F_{CD}\left(\frac{3}{5}\right) = 0 \qquad F_{AD} = F_{CD} = F$

$+\uparrow \Sigma F_y = 0; \quad 2\left[F\left(\frac{4}{5}\right)\right] - P = 0 \qquad F_{AD} = F_{CD} = F = 0.625P\text{ (C)}$

Joint *C* (Fig. *c*)

$\xrightarrow{+} \Sigma F_x = 0; \quad 0.625\left(\frac{3}{5}\right) - F_{BC} = 0 \qquad F_{BC} = 0.375P\text{ (T)}$

Using the result of $\mathbf{F}_{BC}$, Eq. (1) gives

$$F_{AB} = 0.375P\text{ (T)}$$

Axial Strain Energy. $A = \frac{\pi}{4}(0.05^2) = 0.625(10^{-3})\pi\text{m}^2$ and $L_{CD} = \sqrt{0.9^2 + 1.2^2} = 1.5\text{ m}$

Member *BD* is critical since it is subjected to greatest normal force. Thus,

$$\sigma_Y = \frac{F_{BD}}{A}$$

$$250(10^6) = \frac{P}{0.625(10^{-3})\pi}$$

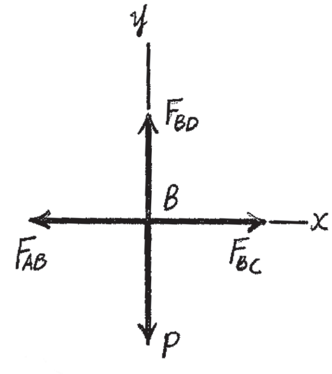
$$P = 490.87(10^3)\text{ N} = 491\text{ kN} \qquad \textbf{Ans.}$$

Using the result of *P*,

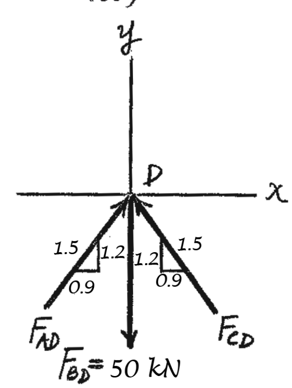
$$F_{BD} = 490.87(10^3)\text{ N} \quad F_{AD} = F_{CD} = 306.80(10^3)\text{ N} \quad F_{BC} = F_{AB} = 184.08(10^3)\text{ N}$$

$$(U_i)_a = \Sigma\frac{N^2L}{2AE} = \frac{1}{2[0.625(10^{-3})\pi][200(10^9)]}\left\{[490.87(10^3)]^2(1.2) + 2[306.80(10^3)]^2(1.5) + 2[184.08(10^3)]^2(0.9)\right\}$$

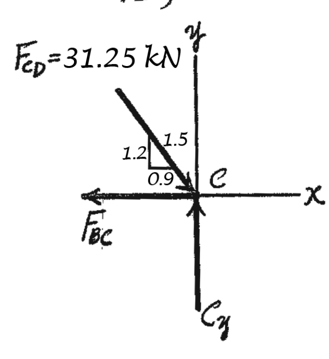
$$= 805.34\text{ N}\cdot\text{m} = 805\text{ J} \qquad \textbf{Ans.}$$



(a)



(b)

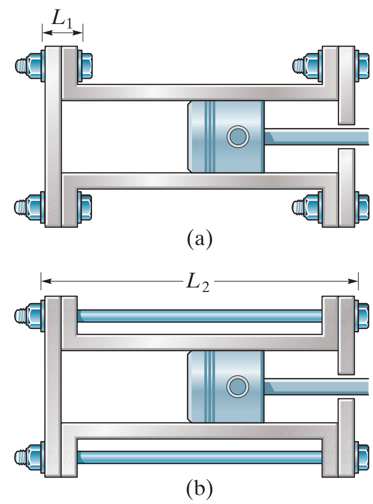


(c)

Ans:
$P = 491\text{ kN}, U_i = 0.805\text{ J}$

14–7.

Using bolts of the same material and cross-sectional area, two possible attachments for a cylinder head are shown. Compare the strain energy developed in each case, and then explain which design is better for resisting an axial shock or impact load.



SOLUTION

Case (a)

$$U_a = \frac{N^2 L_1}{2AE}$$ **Ans.**

Case (b)

$$U_b = \frac{N^2 L_2}{2AE}$$ **Ans.**

Since $U_b > U_a$, i.e., $L_2 > L_1$, the design for case (b) is better able to absorb energy.

Case (b) **Ans.**

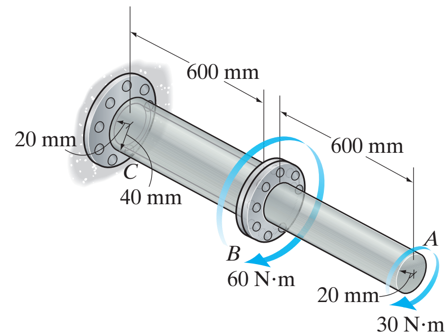
Ans:

$U_a = \dfrac{N^2 L_1}{2AE}$, $U_b = \dfrac{N^2 L_2}{2AE}$

Since $U_b > U_a$, i.e., $L_2 > L_1$, the design for case (b) is better able to absorb energy.

***14–8.**

The shaft assembly is fixed at C. The hollow segment BC has an inner radius of 20 mm and outer radius of 40 mm, while the solid segment AB has a radius of 20 mm. Determine the torsional strain energy stored in the shaft. The shaft is made of 2014-T6 aluminum alloy. The coupling at B is rigid.



SOLUTION

Internal Torque: Referring to the free-body diagram of segment AB, Fig. a,

$\Sigma M_x = 0; \qquad T_{AB} + 30 = 0 \qquad T_{AB} = -30\,\mathrm{N \cdot m}$

Referring to the free-body diagram of segment BC, Fig. b,

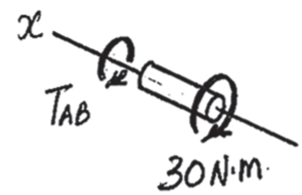
$\Sigma M_x = 0; \qquad T_{BC} + 30 + 60 = 0 \qquad T_{AB} = -90\,\mathrm{N \cdot m}$

Torsional Strain Energy: Here, $J_{AB} = \frac{\pi}{2}\left(0.02^4\right) = 80\left(10^{-9}\right)\pi\ \mathrm{m}^4$ and $J_{BC} = \frac{\pi}{2}\left(0.04^4 - 0.02^4\right) = 1200\left(10^{-9}\right)\pi\ \mathrm{m}^4$,

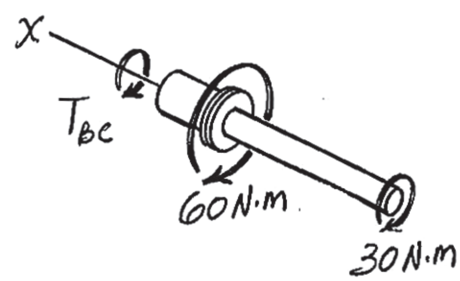
$$(U_i)_t = \Sigma\frac{T^2L}{2GJ} = \frac{{T_{AB}}^2L_{AB}}{2GJ_{AB}} + \frac{{T_{BC}}^2L_{BC}}{2GJ_{BC}}$$

$$= \frac{(-30)^2(0.6)}{2\left[27\left(10^9\right)\right]\left[80\left(10^{-9}\right)\pi\right]} + \frac{(-90)^2(0.6)}{2\left[27\left(10^9\right)\right]\left[1200\left(10^{-9}\right)\pi\right]}$$

$$= 0.06379\ \mathrm{J} = 0.0638\ \mathrm{J}$$ **Ans.**



(a)

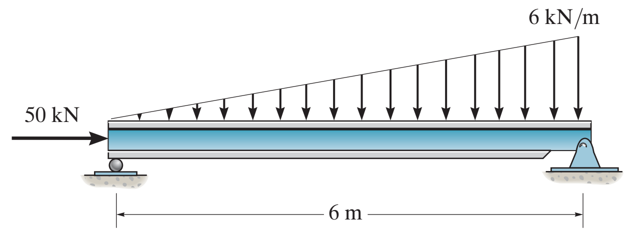


(b)

Ans:
$U_i = 0.0638\ \mathrm{J}$

14–9.

Determine the total axial and bending strain energy in the A992 steel beam. $A = 2850\ \text{mm}^2$, $I = 28.9(10^6)\ \text{mm}^4$.



SOLUTION

Support Reactions and Internal Loadings: The support reactions and the necessary moment function are shown on the FBD in Figs. *a* and *b*, respectively.

Strain Energy: For the axial load and bending,

$$U_i = (U_a)_i + (U_b)_i$$

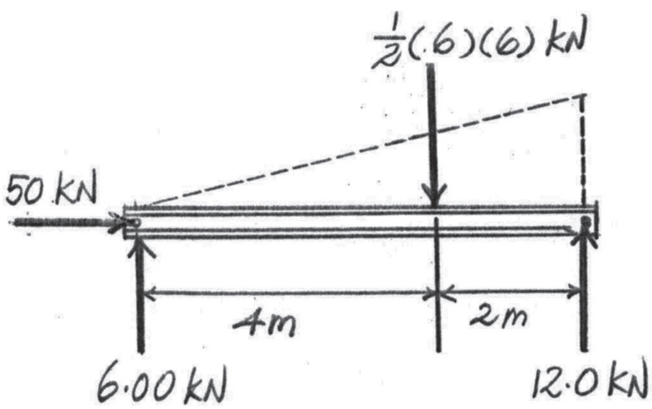
$$= \frac{N^2L}{2EA} + \int_0^L \frac{M^2dx}{2EI}$$

$$= \frac{\left[-50(10^3)\right]^2(6)}{2EA} + \int_0^{6\text{ m}} \frac{\left[\left(6.00x - \frac{1}{6}x^3\right)(10^3)\right]^2dx}{2EI}$$

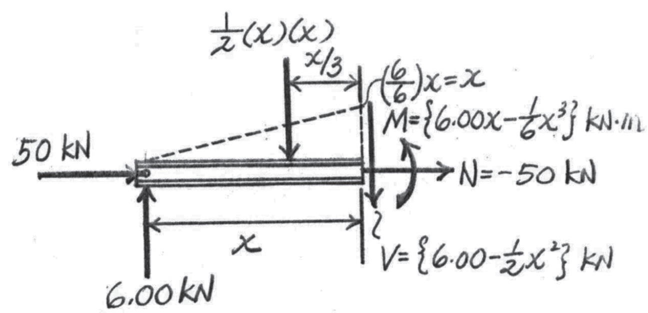
$$= \frac{7.50(10^9)\ \text{N}^2\cdot\text{m}}{EA} + \frac{0.29623(10^9)\ \text{N}^2\cdot\text{m}^3}{EI}$$

$$= \frac{7.50(10^9)}{200(10^9)\left[2.85(10^{-3})\right]} + \frac{0.29623(10^9)}{200(10^9)\left[28.9(10^{-6})\right]}$$

$$= 64.41\ \text{J} = 64.4\ \text{J}$$ **Ans.**



(a)

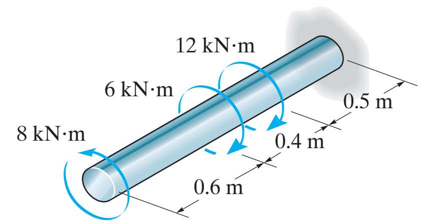


(b)

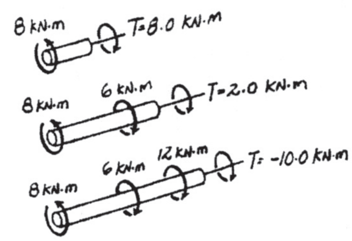
Ans:
$U_i = 64.4\ \text{J}$

14–10.

Determine the torsional strain energy in the A-36 steel shaft. The shaft has a radius of 40 mm.



SOLUTION



Internal Torsional Moment: As shown on FBD.

Torsional Strain Energy: With polar moment of inertia $J = \frac{\pi}{2}\left(0.04^4\right) = 1.28\left(10^{-6}\right)\pi\ \text{m}^4$. Applying Eq. 14–22 gives

$$U_i = \sum \frac{T^2 L}{2GJ}$$

$$= \frac{1}{2GJ}\left[8000^2\,(0.6) + 2000^2\,(0.4) + \left(-10000^2\right)(0.5)\right]$$

$$= \frac{45.0(10^6)\ \text{N}^2\cdot\text{m}^3}{GJ}$$

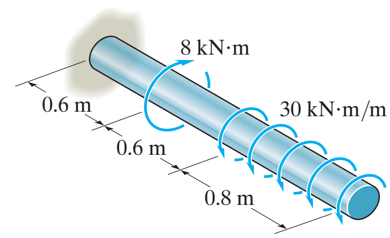
$$= \frac{45.0(10^6)}{75(10^9)[1.28(10^{-6})\,\pi]}$$

$$= 149\ \text{J}$$ **Ans.**

Ans:
$U_i = 149$ J

14–11.

Determine the torsional strain energy in the A992 steel shaft. The shaft has a radius of 40 mm.



SOLUTION

Internal Torque: As shown on the FBD in Fig. *a*, *b* and *c*.

Torsional Strain Energy: With polar moment of inertia $J = \frac{\pi}{2}(0.04^4) = 1.28(10^{-6})\pi$ m^4 and $G = 75$ GPa for A992 steel,

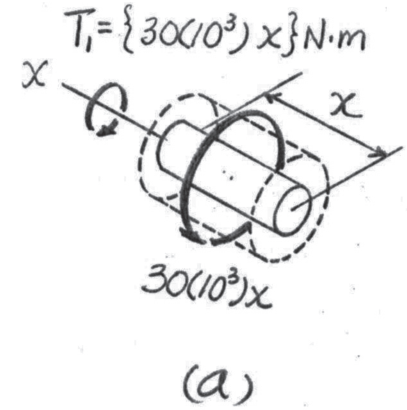
$$U_i = \int_0^L \frac{T^2 dx}{2GJ}$$

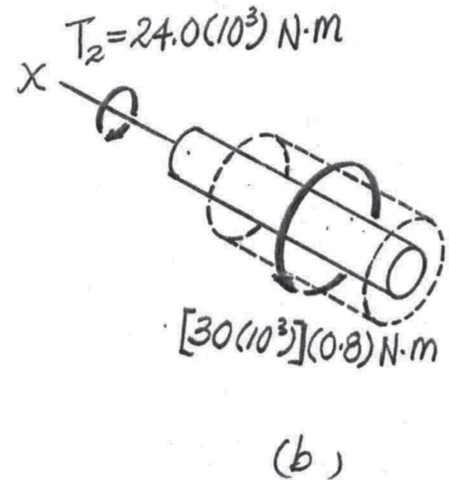
$$= \frac{1}{2GJ}\left\{\int_0^{0.8\text{ m}} \left[30(10^3)x\right]^2 dx + \left[24.0(10^3)\right]^2(0.6) + \left[16.0(10^3)\right]^2(0.6)\right\}$$

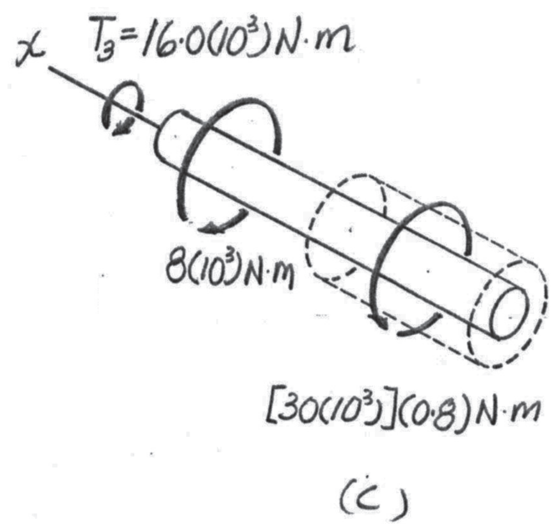
$$= \frac{0.3264(10^9)\text{ N}^2\cdot\text{m}^3}{GJ}$$

$$= \frac{0.3264(10^9)}{75(10^9)\left[1.28(10^{-6})\pi\right]}$$

$$= 1082.25\text{ J} = 1.08\text{ kJ} \qquad \textbf{Ans.}$$



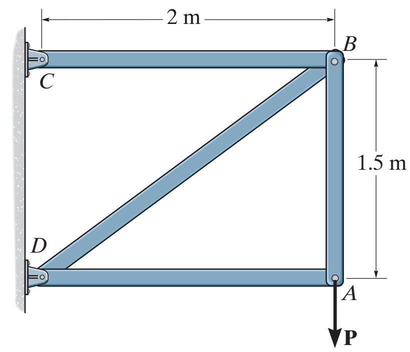




Ans:
$U_i = 1.08$ kJ

***14–12.**

If $P = 60$ kN, determine the total strain energy stored in the truss. Each member has a cross-sectional area of $2.5(10^3)$ mm^2 and is made of A-36 steel.



SOLUTION

Normal Forces: The normal force developed in each member of the truss can be determined using the method of joints.

Joint *A* (Fig. *a*)

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AD} = 0$

$+\uparrow \Sigma F_y = 0; \qquad F_{AB} - 60 = 0 \qquad F_{AB} = 60 \text{ kN (T)}$

Joint *B* (Fig. *b*)

$+\uparrow \Sigma F_y = 0; \qquad F_{BD}\left(\frac{3}{5}\right) - 60 = 0 \qquad F_{BD} = 100\,\text{kN (C)}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad 100\left(\frac{4}{5}\right) - F_{BC} = 0 \qquad F_{BC} = 80\,\text{kN (T)}$

Axial Strain Energy: $A = 2.5\left(10^3\right)\text{ mm}^2 = 2.5\left(10^{-3}\right)\text{ m}^2$ and $L_{BD} = \sqrt{2^2 + 1.5^2} = 2.5$ m

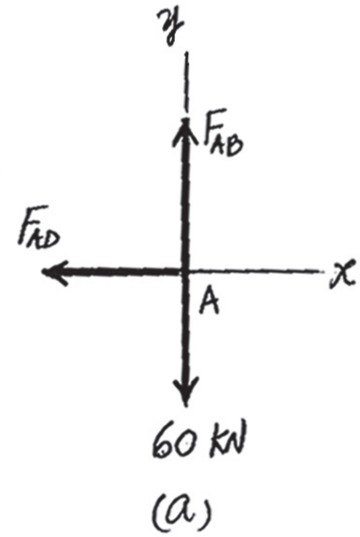
$$(U_i)_a = \Sigma \frac{N^2L}{2AE}$$

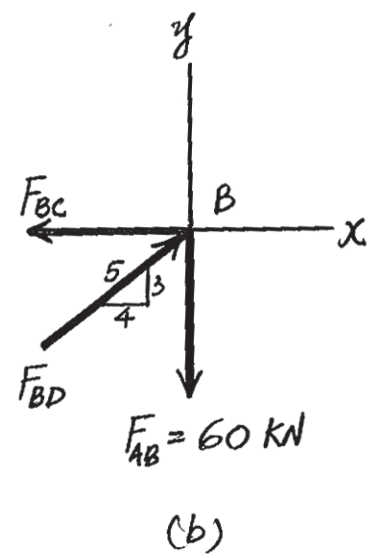
$$= \frac{1}{2\left[2.5\left(10^{-3}\right)\right]\left[200\left(10^9\right)\right]}\Big[\left[60\left(10^3\right)\right]^2(1.5) + \left[100\left(10^3\right)\right]^2(2.5) + \left[80\left(10^3\right)\right]^2(2)\Big]$$

$$= 43.2 \text{ J} \qquad \textbf{Ans.}$$

This result is only valid if $\sigma < \sigma_Y$. We only need to check member BD since it is subjected to the greatest normal force

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{100\left(10^3\right)}{2.5\left(10^{-3}\right)} = 40 \text{ MPa} < \sigma_Y = 250 \text{ MPa} \qquad \textbf{O.K.}$$

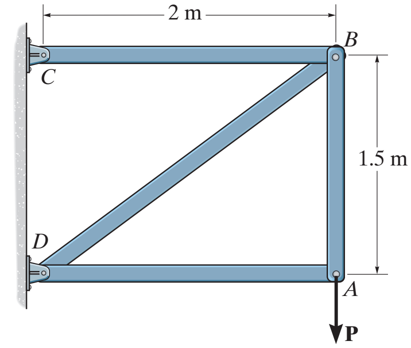




Ans:
$U_i = 43.2$ J

14–13.

Determine the maximum force **P** and the corresponding maximum total strain energy stored in the truss without causing any of the members to have permanent deformation. Each member has the cross-sectional area of $2.5(10^3)$ mm^2 and is made of A-36 steel.



SOLUTION

Normal Forces: The normal force developed in each member of the truss can be determined using the method of joints.

Joint *A* (Fig. *a*)

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad F_{AD} = 0$

$+\uparrow\Sigma F_y = 0;\qquad F_{AB} - P = 0\qquad F_{AB} = P\ (T)$

Joint *B* (Fig. *b*)

$+\uparrow\Sigma F_y = 0;\qquad F_{BD}\left(\frac{3}{5}\right) - P = 0\qquad F_{BD} = 1.6667P\ \text{(C)}$

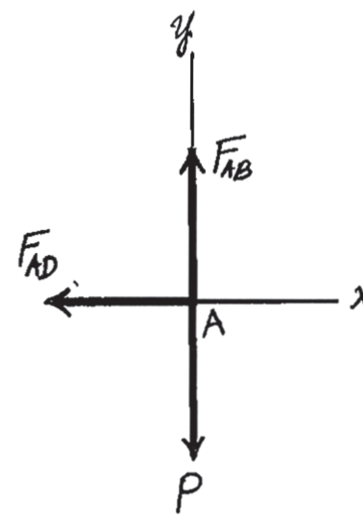
$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad 1.6667P\left(\frac{4}{5}\right) - F_{BC} = 0\qquad F_{BC} = 1.3333P\text{(T)}$

Axial Strain Energy: $A = 2.5\left(10^3\right)\text{mm}^2 = 2.5\left(10^{-3}\right)\text{m}^2$. Member BD is critical since it is subjected to the greatest force. Thus,

$$\sigma_Y = \frac{F_{BD}}{A}$$

$$250\left(10^6\right) = \frac{1.6667P}{2.5\left(10^{-3}\right)}$$

$$P = 375\,\text{kN}\qquad\textbf{Ans.}$$



(a)

Using the result of P

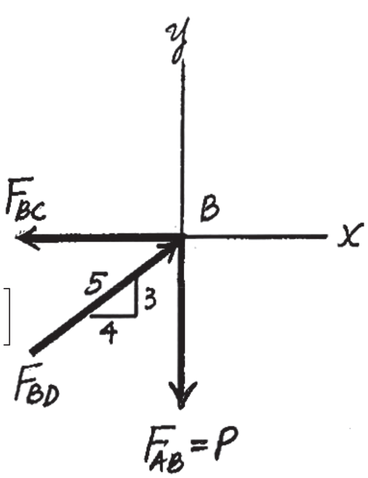
$$F_{AB} = 375\,\text{kN}\qquad F_{BD} = 625\,\text{kN}\qquad F_{BC} = 500\,\text{kN}$$

Here, $L_{BD} = \sqrt{1.5^2 + 2^2} = 2.5$ m.

$$(U_i)_a = \Sigma\frac{N^2L}{2AE}$$

$$= \frac{1}{2\left[2.5\left(10^{-3}\right)\right]\left[200\left(10^9\right)\right]}\left[\left[375\left(10^3\right)\right]^2(1.5) + \left[625\left(10^3\right)\right]^2(2.5) + \left[500\left(10^3\right)\right]^2(2)\right]$$

$$= 1687.5\,\text{J} = 1.69\text{ kJ}\qquad\textbf{Ans.}$$



(b)

Ans:
$P = 375$ kN, $U_i = 1.69$ kJ

14–14.

Consider the thin-walled tube of Fig. 5–26. Use the formula for shear stress, $\tau_{avg} = T/2tA_m$, Eq. 5–18, and the general equation of shear strain energy, Eq. 14–11, to show that the twist of the tube is given by Eq. 5–20. *Hint*: Equate the work done by the torque T to the strain energy in the tube, determined from integrating the strain energy for a differential element, Fig. 14–4, over the volume of material.

SOLUTION

$$U_i = \int_v \frac{\tau^2\, dV}{2\,G} \quad \text{but } \tau = \frac{T}{2\,t\,A_m}$$

Thus,

$$U_i = \int_v \frac{T^2}{8\,t^2\,A_m^2 G} dV$$

$$= \frac{T^2}{8\,A_m^2\,G}\int_v \frac{dV}{t^2} = \frac{T^2}{8\,A_m^2\,G}\int_A \frac{dV}{t^2}\int_0^L dx = \frac{T^2 L}{8\,A_m^2 G}\int_A \frac{dA}{t^2}$$

However, $dA = t\,ds$. Thus,

$$U_i = \frac{T^2 L}{8\,A_m^2 G}\int \frac{ds}{t}$$

$$U_e = \frac{1}{2}T\phi$$

$$U_e = U_i$$

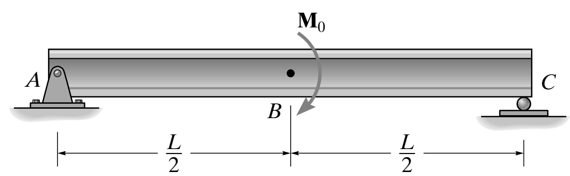
$$\frac{1}{2}\,T\,\phi = \frac{T^2 L}{8\,A_m^2 G}\int \frac{ds}{t}$$

$$\phi = \frac{T\,L}{4\,A_m^2 G}\int \frac{ds}{t}$$

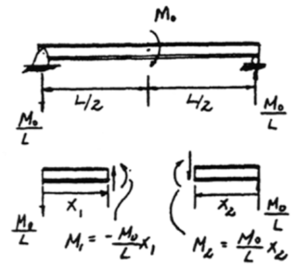
QED

Ans:
N/A

14–15. Determine the bending strain-energy in the beam due to the loading shown. EI is constant.



SOLUTION



$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{1}{2EI}\left[\int_0^{L/2}\left(\frac{-M_0}{L}x_1\right)^2 dx_1 + \int_0^{L/2}\left(\frac{M_0}{L}x_2\right)^2 dx_2\right]$$

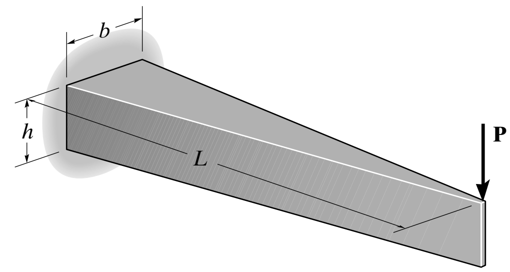
$$= \frac{M_0^2 L}{24EI}$$ **Ans.**

Note: Strain energy is always positive regardless of the sign of the moment function.

Ans:

$$U_i = \frac{M_0^2 L}{24EI}$$

***14–16.** The beam shown is tapered along its width. If a force **P** is applied to its end, determine the strain energy in the beam and compare this result with that of a beam that has a constant rectangular cross section of width b and height h.



SOLUTION

Moment of Inertia: For the beam with the uniform section,

$$I = \frac{bh^3}{12} = I_0$$

For the beam with the tapered section,

$$I = \frac{1}{12}\left(\frac{b}{L}x\right)\left(h^3\right) = \frac{bh^3}{12L}x = \frac{I_0}{L}x$$

Internal Moment Function: As shown on FBD.

Bending Strain Energy: For the beam with the tapered section, applying Eq. 14–17 gives

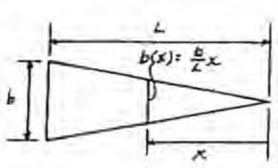
$$\begin{aligned} U_I &= \int_0^L \frac{M^2\,dx}{2EI} \\ &= \frac{1}{2E}\int_0^L \frac{(-Px)^2}{\frac{I_0}{L}x}dx \\ &= \frac{P^2L}{2EI_0}\int_0^L x\,dx \\ &= \frac{P^2L^3}{4EI_0} = \frac{3P^2L^3}{bh^3E} \end{aligned}$$

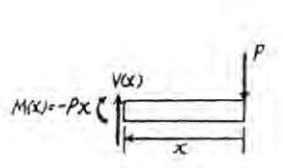
Ans.

For the beam with the uniform section,

$$\begin{aligned} U_i &= \int_0^L \frac{M^2dx}{2EI} \\ &= \frac{1}{2EI_0}\int_0^L (-Px)^2\,dx \\ &= \frac{P^3L^3}{6EI_0} \end{aligned}$$

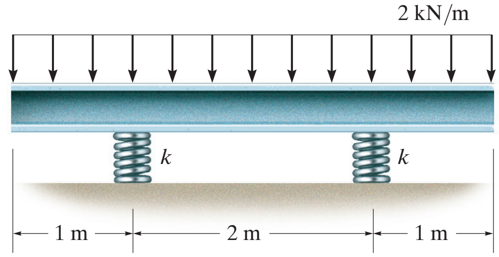
The strain energy in the capered beam is 1.5 times as great as that in the beam having a uniform cross section. **Ans.**





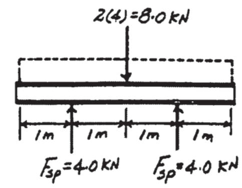
14–17.

The steel beam is supported on two springs, each having a stiffness of $k = 8$ MN/m. Determine the strain energy in each of the springs and the bending strain energy in the beam. $E_{st} = 200$ GPa, $I = 5(10^6)$ mm^4.

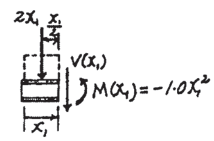


SOLUTION

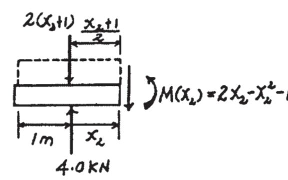
Spring Strain Energy: The spring deforms $\delta_{sp} = \dfrac{F_{sp}}{k} = \dfrac{4.00(10^3)}{8(10^6)} = 0.500(10^{-3})$ m under the applied load.



$$(U_i)_{sp} = \frac{1}{2}k\delta_{sp}^2$$
$$= \frac{1}{2}\left[8(10^6)\right]\left[0.500(10^{-})\right]^2$$
$$= 1.00 \text{ J} \qquad \textbf{Ans.}$$



Bending Strain Energy: Applying Eq. 14–17 gives

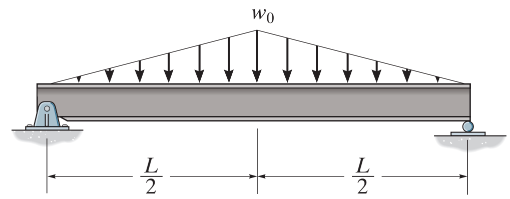


$$(U_i)_b = \int_0^L \frac{M^2 dx}{2EI}$$
$$= \frac{1}{2EI}\left[2\int_0^{1\text{m}} (-1.00x_1^2)^2\,dx_1 + \int_0^{2\text{m}} (2x_2 - x_2^2 - 1)^2\,dx_2\right]$$
$$= \frac{0.400 \text{ kN}^2\cdot\text{m}^3}{EI}$$
$$= \frac{0.400(10^6)}{200(10^9)\left[5(10^{-6})\right]}$$
$$= 0.400 \text{ J} \qquad \textbf{Ans.}$$

Ans:
$(U_i)_{sp} = 1.00$ J
$(U_i)_b = 0.400$ J

14–18.

Determine the bending strain energy in the simply supported beam. EI is constant.



SOLUTION

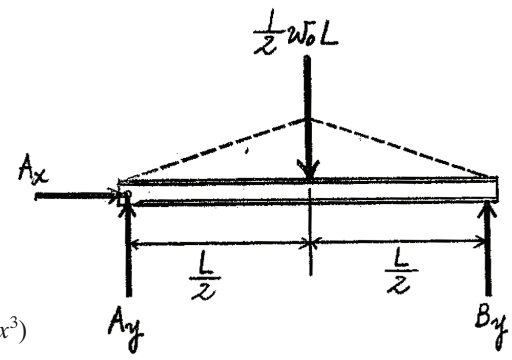
Support Reactions: Referring to the FBD of the entire beam, Fig. a,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad \frac{1}{2} w_0 L\left(\frac{L}{2}\right) - A_y(L) = 0 \qquad A_y = \frac{w_0 L}{4}$$

Internal Moment: Referring to the FBD of the beam's left cut segment, Fig. b,

$$\circlearrowleft + \Sigma M_0 = 0; \quad M + \left[\frac{1}{2}\left(\frac{2w_0}{L}x\right)x\right]\left(\frac{x}{3}\right) - \frac{w_0 L}{4}x = 0 \quad M = \frac{w_0}{12L}(3L^2x - 4x^3)$$



(a)

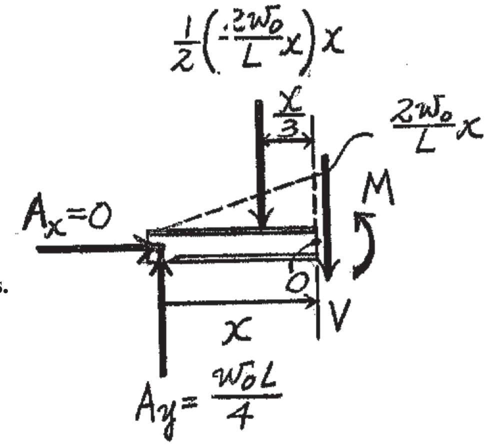
Bending Strain Energy:

$$(U_i)_b = \Sigma \int_0^L \frac{M^2 dx}{2EI} = 2\int_0^{L/2} \frac{\left[\frac{w_0}{12L}(3L^2x - 4x^3)\right]^2 dx}{2EI}$$

$$= \frac{w_0^2}{144\,EIL^2}\int_0^{L/2} (9L^4x^2 + 16x^6 - 24L^2x^4)\,dx.$$

$$= \frac{w_0^2}{144\,EIL^2}\left(3L^4x^3 + \frac{16}{7}x^7 - \frac{24}{5}L^2x^5\right)\Bigg|_0^{L/2}$$

$$= \frac{17w_0^2L^5}{10\,080\,EI} \qquad \textbf{Ans.}$$



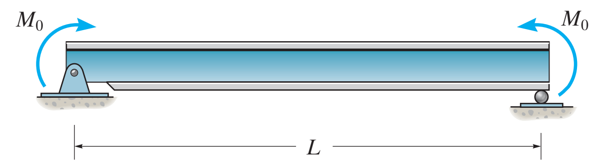
(b)

Ans:

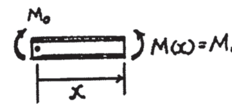
$$U_i = \frac{17w_0^2L^5}{10\,080\,EI}$$

14–19.

Determine the bending strain energy in the beam. EI is constant.



SOLUTION



Support Reactions: As shown on FBD(a).

Internal Moment Function: As shown on FBD(b).

Bending Strain Energy:

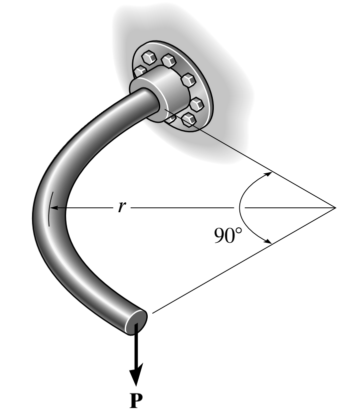
$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{M_0}{2EI}\int_0^L dx$$

$$= \frac{M_0 L}{2EI}$$ **Ans.**

Ans:
$U_i = \dfrac{M_0 L}{2EI}$

***14–20.** Determine the strain energy in the *horizontal* curved bar due to torsion. There is a *vertical* force **P** acting at its end. JG is constant.



SOLUTION

$T = Pr(1 - \cos\theta)$

Strain energy:

$$U_i = \int_0^L \frac{T^2\,ds}{2JG}$$

However,

$$s = r\theta; \qquad ds = r\,d\theta$$

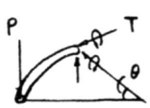
$$U_i = \int_0^\theta \frac{T^2 r\,d\theta}{2JG} = \frac{r}{2JG}\int_0^{\pi/2} [Pr(1 - \cos\theta)]^2 d\theta$$

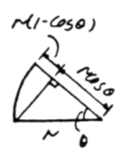
$$= \frac{P^2r^3}{2JG}\int_0^{\pi/2} (1 - \cos\theta)^2\,d\theta$$

$$= \frac{P^2r^3}{2JG}\int_0^{\pi/2} (1 + \cos^2\theta - 2\cos\theta)d\theta$$

$$= \frac{P^2r^3}{2JG}\int_0^{\pi/2} \left(1 + \frac{\cos 2\theta + 1}{2} - 2\cos\theta\right) d\theta$$

$$= \frac{P^2r^3}{JG}\left(\frac{3\pi}{8} - 1\right)$$ **Ans.**



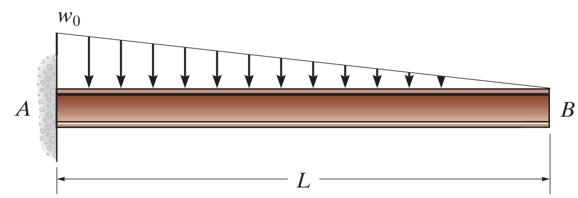


Ans:

$$U_i = \frac{P^2r^3}{JG}\left(\frac{3\pi}{8} - 1\right)$$

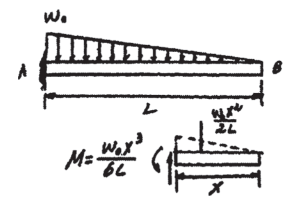
14–21.

Determine the bending strain energy in the beam. EI is constant.



SOLUTION

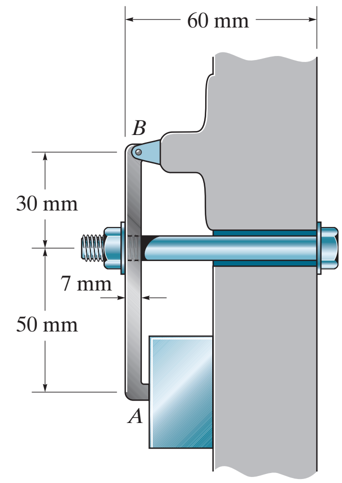
$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\int_0^L \left(\frac{w_0 x^3}{6L}\right)^2 dx = \frac{w_0^2 L^5}{504\ EI}$$ **Ans.**



Ans:
$U_i = \dfrac{w_0^2 L^5}{504\ EI}$

14–22.

The bolt has a diameter of 10 mm, and the arm AB has a rectangular cross section that is 12 mm wide by 7 mm thick. Determine the strain energy in the arm due to bending and in the bolt due to axial force. The bolt is tightened so that it has a tension of 500 N. Both members are made of A-36 steel. Neglect the hole in the arm.



SOLUTION

Axial Strain Energy: Applying Eq. 14–16 gives

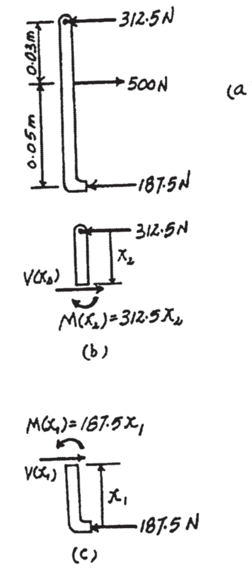
$$(U_i)_b = \frac{N^2L}{2AE}$$

$$= \frac{500^2(0.06)}{2AE}$$

$$= \frac{7500\ \text{N}^2\cdot\text{m}}{AE}$$

$$= \frac{7500}{\frac{\pi}{4}(0.01^2)\left[200(10^9)\right]}$$

$$= 0.477(10^{-3})\ \text{J} \qquad \textbf{Ans.}$$



Bending Strain Energy: Applying Eq. 14–17 gives

$$(U_i)_l = \int_0^L \frac{M^2dx}{2EI}$$

$$= \frac{1}{2EI}\left[\int_0^{0.05\text{ m}} (187.5x_1)^2dx_1 + \int_0^{0.03\text{ m}} (312.5x_2)^2dx_2\right]$$

$$= \frac{1.171875\ \text{N}^2\cdot\text{m}^3}{EI}$$

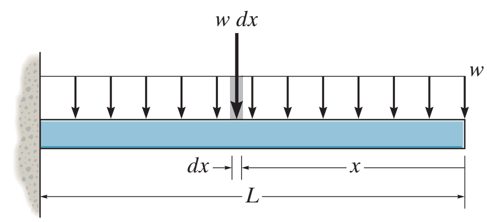
$$= \frac{1.171875}{200(10^9)\left[\frac{1}{12}(0.012)(0.007^3)\right]}$$

$$= 0.0171\ \text{J} \qquad \textbf{Ans.}$$

Ans:
$(U_i)_b = 0.477(10^{-3})$ J
$(U_i)_l = 0.0171$ J

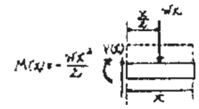
14–23.

Determine the bending strain energy in the cantilevered beam. Solve the problem two ways. (a) Apply Eq. 14–17. (b) The load $w\ dx$ acting on a segment dx of the beam is displaced a distance y, where $y = w(-x^4 + 4L^3x - 3L^4)/(24EI)$, the equation of the elastic curve. Hence the internal strain energy in the differential segment dx of the beam is equal to the external work, i.e., $dU_i = \frac{1}{2}(w\ dx)(-y)$. Integrate this equation to obtain the total strain energy in the beam. EI is constant.



SOLUTION

Internal Moment Function: As shown on FBD.



Bending Strain Energy: a) Applying Eq. 14–17 gives

$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{1}{2EI}\left[\int_0^L \left[-\frac{w}{2}x^2\right]^2 dx\right]$$

$$= \frac{w^2}{8EI}\left[\int_0^L x^4 dx\right]$$

$$= \frac{w^2 L^5}{40EI} \qquad \textbf{Ans.}$$

b) Integrating $dU_i = \dfrac{1}{2}(wdx)(-y)$

$$dU_i = \frac{1}{2}(wdx)\left[-\frac{w}{24EI}\left(-x^4 + 4L^3x - 3L^4\right)\right]$$

$$dU_i = \frac{w^2}{48EI}\left(x^4 - 4L^3x + 3L^4\right) dx$$

$$U_i = \frac{w^2}{48EI}\int_0^L \left(x^4 - 4L^3x + 3L^4\right) dx$$

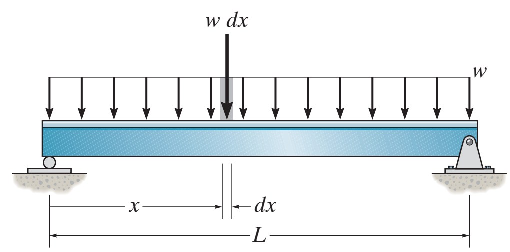
$$= \frac{w^2 L^5}{40EI} \qquad \textbf{Ans.}$$

Ans:

$$U_i = \frac{w^2 L^5}{40\,EI}$$

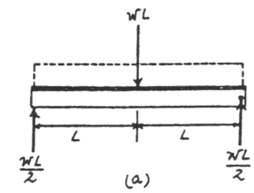
***14–24.**

Determine the bending strain energy in the simply supported beam. Solve the problem two ways. (a) Apply Eq. 14–17. (b) The load $w\ dx$ acting on the segment dx of the beam is displaced a distance y, where $y = w(-x^4 + 2Lx^3 - L^3x)/(24EI)$, the equation of the elastic curve. Hence the internal strain energy in the differential segment dx of the beam is equal to the external work, i.e., $dU_i = \frac{1}{2}(w\ dx)(-y)$. Integrate this equation to obtain the total strain energy in the beam. EI is constant.



SOLUTION

Support Reactions: As shown on FBD(a).



Internal Moment Function: As shown on FBD(b).

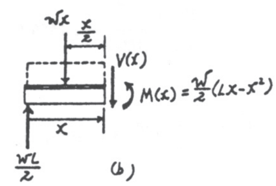
Bending Strain Energy: a) Applying Eq. 14–17 gives

$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{1}{2EI}\left[\int_0^L \left[\frac{w}{2}(Lx - x^2)\right]^2 dx\right]$$

$$= \frac{w^2}{8EI}\left[\int_0^L (L^2x^2 + x^4 - 2Lx^3)\, dx\right]$$

$$= \frac{w^2L^5}{240EI} \qquad \textbf{Ans.}$$



b) Integrating $dU_i = \dfrac{1}{2}(wdx)(-y)$

$$dU_i = \frac{1}{2}(wdx)\left[-\frac{w}{24EI}(-x^4 + 2Lx^3 - L^3x)\right]$$

$$dU_i = \frac{w^2}{48EI}(x^4 - 2Lx^3 + L^3x)\, dx$$

$$U_i = \frac{w^2}{48EI}\int_0^L (x^4 - 2Lx^3 + L^3x)\, dx$$

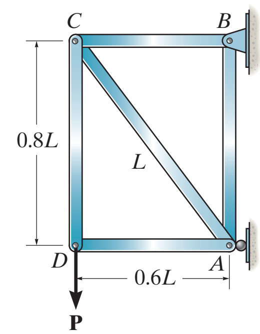
$$= \frac{w^2L^5}{240EI} \qquad \textbf{Ans.}$$

Ans:

$$U_i = \frac{w^2L^5}{240EI}$$

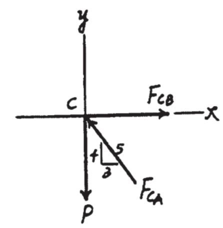
14–25.

Determine the vertical displacement of joint D. AE is constant.



SOLUTION

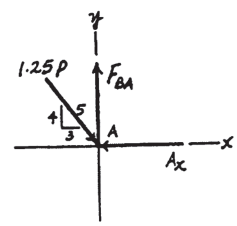
Member Forces: By inspection of joint D, member AD is a zero-force member and $F_{CD} = P(T)$. Applying the method of joints at C, we have



$$+\uparrow \Sigma F_y = 0; \qquad \frac{4}{5}F_{CA} - P = 0 \qquad F_{CA} = 1.25P \text{ (C)}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{CB} - \frac{3}{5}(1.25P) = 0 \qquad F_{CB} = 0.750P \text{ (T)}$$

At joint A



$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} - \frac{4}{5}(1.25P) = 0 \qquad F_{BA} = 1.00P \text{ (T)}$$

Axial Strain Energy: Applying Eq. 14–16, we have

$$U_i = \sum \frac{N^2 L}{2AE}$$

$$= \frac{1}{2AE}\left[P^2(0.8L) + (-1.25P)^2(L) + (0.750P)^2(0.6L) + (1.00P)^2(0.8L)\right]$$

$$= \frac{1.750P^2 L}{AE}$$

External Work: The external work done by force P is

$$U_e = \frac{1}{2}(P)(\Delta_D)_v$$

Conservation of Energy:

$$U_e = U_i$$

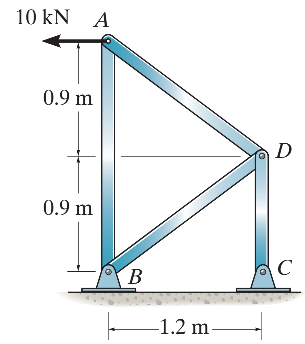
$$\frac{1}{2}(P)(\Delta_D)_v = \frac{1.750P^2 L}{AE}$$

$$(\Delta_D)_v = \frac{3.50PL}{AE} \qquad \textbf{Ans.}$$

Ans:

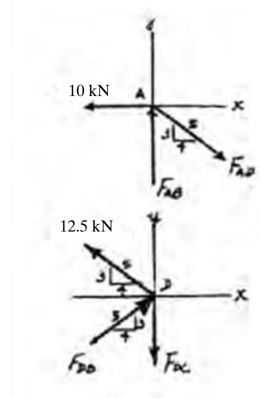
$$(\Delta_D)_v = \frac{3.50PL}{AE}$$

14–26. Determine the horizontal displacement of joint A. Each bar is made of A-36 steel and has a cross-sectional area of 950 mm^2.



SOLUTION

Member Forces: Applying the method of joints to joint at A, we have



$$\xrightarrow{+} \Sigma F_x = 0; \quad \frac{4}{5}F_{AD} - 10 = 0 \qquad F_{AD} = 12.5 \text{ kN (T)}$$

$$+\uparrow \Sigma F_y = 0; \quad F_{AB} - \frac{3}{5}(12.5) = 0 \qquad F_{AB} = 7.5 \text{ kN (C)}$$

At joint D

$$\xrightarrow{+} \Sigma F_x = 0; \quad \frac{4}{5}F_{DB} - \frac{4}{5}(12.5) = 0 \qquad F_{DB} = 12.5 \text{ kN (C)}$$

$$+\uparrow \Sigma F_y = 0; \quad \frac{3}{5}(12.5) + \frac{3}{5}(12.5) - F_{DC} = 0$$

$$F_{DC} = 15 \text{ kN (T)}$$

Axial Strain Energy: Applying Eq. 14–16, we have

$$U_i = \sum \frac{N^2 L}{2AE}$$

$$= \frac{1}{2AE}[(12.5)^2(1.5) + (-7.5)^2(1.8) + (-12.5)^2(1.5) + (15)^2(0.9)]$$

$$= \frac{386.25 \text{ kN}^2 \cdot \text{m}}{AE}$$

$$= \frac{386.25(10^3)^2 \text{ N}^2 \cdot \text{m}}{[0.95(10^{-3}) \text{ m}^2][200(10^9) \text{ N/m}^2]} = 2.0329 \text{ N} \cdot \text{m}$$

External Work: The external work done by 10-kN force is

$$U_e = \frac{1}{2}(10)(10^3)(\Delta_A)_h = 5(10^3)(\Delta_A)_h$$

Conservation of Energy:

$$U_e = U_i$$

$$(\Delta_A)_h = \frac{2.0329}{5(10^3)}$$

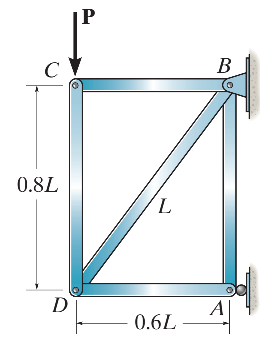
$$= 0.4066(10^{-3}) \text{ m} = 0.407 \text{ mm} \qquad \textbf{Ans.}$$

Ans:

$$(\Delta_D)_v = \frac{3.50PL}{AE}$$

14–27.

Determine the vertical displacement of joint C. AE is constant.

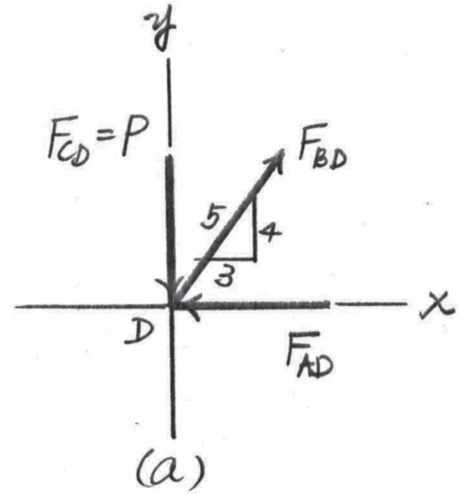


SOLUTION

Member Forces: By inspecting joint C, we notice that member BC is a zero-force member and $F_{CD} = P$ (C). Subsequently, consider the equilibrium of joint D, Fig. a.

$$+\uparrow \Sigma F_y = 0; \qquad F_{BD}\left(\frac{4}{5}\right) - P = 0 \qquad F_{BD} = \frac{5}{4}P \text{ (T)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad \frac{5}{4}P\left(\frac{3}{5}\right) - F_{AD} = 0 \qquad F_{AD} = \frac{3}{4}P \text{ (C)}$$



Axial Strain Energy:

$$(U_i)_a = \sum \frac{N^2 L}{2AE}$$

$$= \frac{1}{2AE}\left[P^2\left(\frac{4}{5}L\right) + \left(\frac{5}{4}P\right)^2(L) + \left(\frac{3}{4}P\right)^2\left(\frac{3}{5}L\right)\right]$$

$$= \frac{27P^2L}{20AE}$$

External Work: The external work done by force **P** is

$$U_e = \frac{1}{2}P\,(\Delta_C)_v$$

Using the concept of conservation of energy,

$$U_e = (U_i)_a$$

$$\frac{1}{2}P\,(\Delta_C)_v = \frac{27P^2L}{20AE}$$

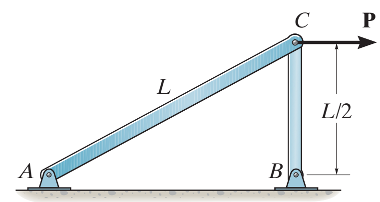
$$(\Delta_C)_v = \frac{27PL}{10AE} \qquad \textbf{Ans.}$$

Ans:

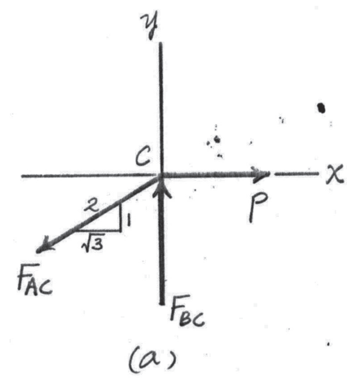
$$(\Delta_C)_v = \frac{27PL}{10AE}$$

***14–28.**

Determine the horizontal displacement of joint C. AE is constant.



SOLUTION



Member Forces: Consider the equilibrium of joint C, Fig. *a*.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad P - F_{AC}\left(\frac{\sqrt{3}}{2}\right) = 0 \qquad F_{AC} = \frac{2}{\sqrt{3}} P \text{ (T)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC} - \left(\frac{2}{\sqrt{3}} P\right)\left(\frac{1}{2}\right) = 0 \qquad F_{BC} = \frac{1}{\sqrt{3}} P \text{ (C)}$$

Axial Strain Energy:

$$(U_i)_a = \sum \frac{N^2 L}{2AE}$$

$$= \frac{1}{2AE}\left[\left(\frac{2}{\sqrt{3}} P\right)^2 (L) + \left(\frac{1}{\sqrt{3}} P\right)^2 \left(\frac{L}{2}\right)\right]$$

$$= \frac{3P^2 L}{4AE}$$

External Work: The external work done by force **P** is

$$U_e = \frac{1}{2} P\,(\Delta_C)_h$$

Using the concept of conservation of energy,

$$U_e = (U_i)_a$$

$$\frac{1}{2} P\,(\Delta_C)_h = \frac{3P^2 L}{4AE}$$

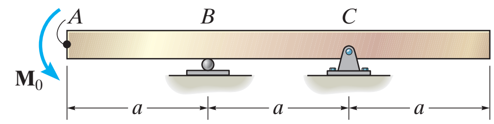
$$(\Delta_C)_h = \frac{3PL}{2AE} \qquad \textbf{Ans.}$$

Ans:

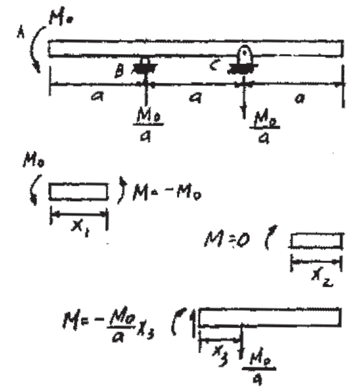
$$(\Delta_C)_h = \frac{3PL}{2AE}$$

14–29.

Determine the slope at point A of the beam. EI is constant.



SOLUTION



$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[\int_0^a (-M_0)^2 dx_1 + \int_0^a (0) dx_2 + \int_0^b \left(-\frac{M_0}{a}x_3\right)^2 dx_3\right]$$

$$= \frac{2M_0^2 a}{3EI}$$

$$U_e = \frac{1}{2}M\theta = \frac{1}{2}M_0\,\theta_A$$

Conservation of Energy:

$$U_e = U_i$$

$$\frac{1}{2}M_0\,\theta_A = \frac{2M_0^2 a}{3EI}$$

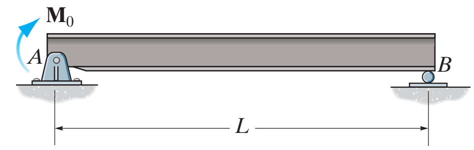
$$\theta_A = \frac{4M_0\,a}{3EI} \qquad \textbf{Ans.}$$

Ans:

$$\theta_A = \frac{4M_0\,a}{3EI}$$

14–30.

Determine the slope of the beam at the pin support A. Consider only bending strain energy. EI is constant.



SOLUTION

Support Reactions: Referring to the FBD of the entire beam, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y L - M_0 = 0 \qquad B_y = \frac{M_0}{L}$$

Internal Moment: Referring to the FBD of the beam's right cut segment, Fig. *b*,

$$\circlearrowleft + \Sigma M_B = 0; \qquad \frac{M_0}{L}x - M = 0 \qquad M = \frac{M_0}{L}x$$

Bending Strain Energy:

$$(U_i)_b = \int_0^L \frac{M^2 dx}{2EI} = \int_0^L \frac{\left(\frac{M_0}{L}x\right)^2 dx}{2EI} = \frac{M_0^2}{2EIL^2}\int_0^L x^2 dx = \frac{M_0^{\,2}}{2EIL^2}\left(\frac{x^3}{3}\right)\Big|_0^L$$

$$= \frac{M_0^{\,2}L}{6EI}$$

External Work: The external work done by $\mathbf{M}_0$ is

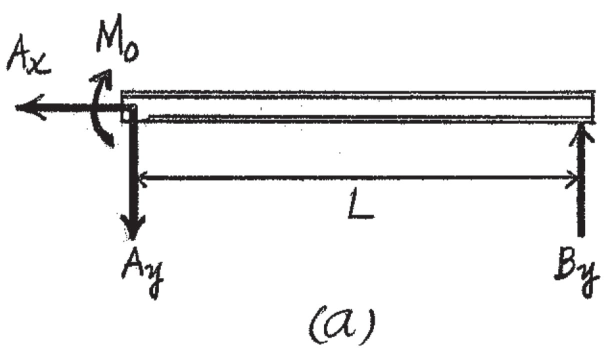
$$U_e = \frac{1}{2}M_0\theta_A$$

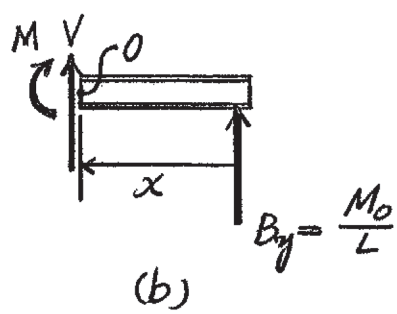
Conservation of Energy:

$$U_e = (U_i)_b$$

$$\frac{1}{2}M_0\theta_A = \frac{M_0^2\, L}{6EI}$$

$$\theta_A = \frac{M_0\, L}{3EI} \circlearrowright \qquad \textbf{Ans.}$$



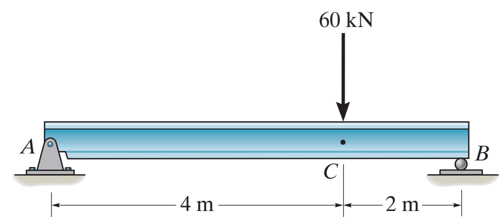


Ans:

$\theta_A = -\dfrac{M_0 L}{3EI}$

14–31.

Determine the vertical displacement of point C of the A992 steel beam. $I = 80(10^6)\ \text{mm}^4$.



SOLUTION

Support Reactions and Internal Loadings: The support reactions and the necessary moment functions are shown on the FBD in Figs. *a* and *b*, respectively,

Bending Strain Energy:

$$(U_i)_b = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{1}{2EI}\left[\int_0^{4\text{ m}} (20.0x_1)^2\, dx_1 + \int_0^{2\text{ m}} \left(40.0\, x_2^2\right) dx_2\right]$$

$$= \frac{1}{2EI}\left[\left(\frac{400}{3}x_1^3\right)\Bigg|_0^{4\text{ m}} + \left(\frac{1600}{3}x_2^3\right)\Bigg|_0^{2\text{ m}}\right]$$

$$= \frac{6400\ \text{kN}^2\cdot\text{m}^3}{EI}$$

For A992 steel, $E = 200$ GPa. Then

$$(U_i)_b = \frac{6400\left(1000^2\right)}{200\left(10^9\right)\left[80\left(10^{-6}\right)\right]} = 400\ \text{J}$$

External Work: The external work done by 60 kN force is

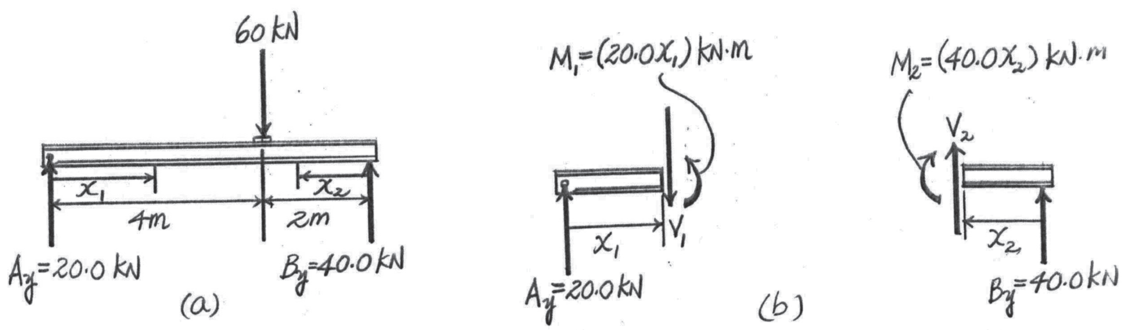
$$U_e = \frac{1}{2}P\,(\Delta_C)_v = \frac{1}{2}\left[60\left(10^3\right)(\Delta_C)_v = 30\left(10^3\right)(\Delta_C)_v\right.$$

Using the concept of conservation of energy,

$$U_e = (U_i)_b$$

$$30\left(10^3\right)(\Delta_C)_v = 400$$

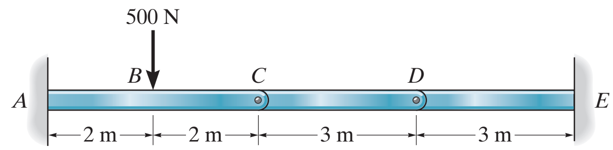
$$(\Delta_C)_v = 0.013333\ \text{m} = 13.3\ \text{mm} \qquad \textbf{Ans.}$$



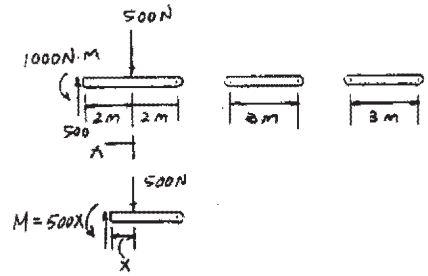
Ans:
$(\Delta_C)_v = 13.3$ mm

***14–32.**

The A992 steel bars are pin connected at C and D. If they each have the same rectangular cross section, with a height of 200 mm and a width of 100 mm, determine the vertical displacement at B. Neglect the axial load in the bars.



SOLUTION



Internal Strain Energy:

$$U_i = \int_0^L \frac{M^2\,dx}{2\,EI} = \frac{1}{2\,EI}\int_0^{2\text{ m}} [500\,x]^2\,dx = \frac{0.3333\,(10^6)}{EI}$$

External Work:

$$U_e = \frac{1}{2}P\,\Delta_B = \frac{1}{2}(500)\,\Delta_B = 250\,\Delta_B$$

Conservation of Energy:

$$U_e = U_i$$

$$250\,\Delta_B = \frac{0.3333\,(10^6)}{EI}$$

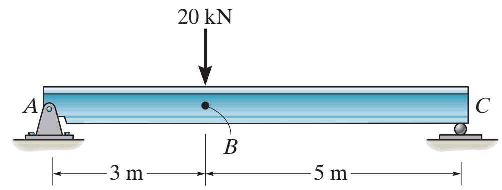
$$\Delta_B = \frac{1333.33}{EI} = \frac{1333.33}{200\,(10^9)\left(\frac{1}{12}\right)(0.1)(0.2^3)}$$

$$= 0.1\,(10^{-3})\text{ m} = 0.100\text{ mm}$$ **Ans.**

Ans:
$\Delta_B = 0.100$ mm

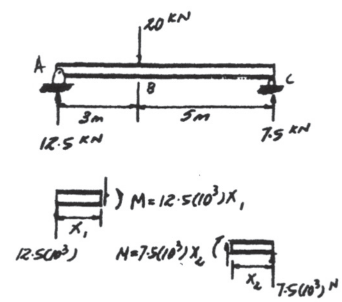
14–33.

Determine the vertical displacement of point B on the A992 steel beam. $I = 80(10^6)\ \text{mm}^4$.



SOLUTION

$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[\int_0^3 [(12.5)(10^3)(x_1)]^2 dx_1 + \int_0^3 [(7.5)(10^3)(x_2)]^2 dx_2\right] = \frac{1.875(10^9)}{EI}$$



$$U_e = \frac{1}{2}P\Delta = \frac{1}{2}(20)(10^3)\Delta_B = 10(10^3)\Delta_B$$

Conservation of Energy:

$$U_e = U_i$$

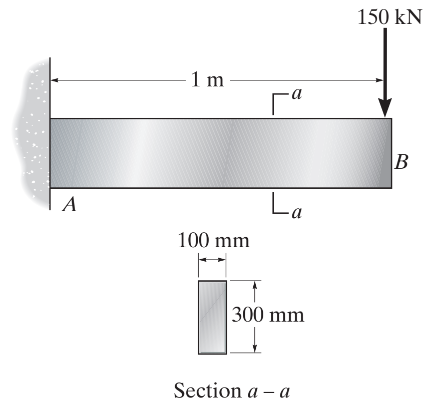
$$10(10^3)\Delta_B = \frac{1.875(10^9)}{EI}$$

$$\Delta_B = \frac{187500}{EI} = \frac{187500}{200(10^9)(80)(10^{-6})} = 0.0117\ \text{m} = 11.7\ \text{mm}$$ **Ans.**

Ans:
$\Delta_B = 11.7$ mm

14–34.

Determine the vertical displacement of end B of the cantilevered 6061-T6 aluminum alloy rectangular beam. Consider both shearing and bending strain energy.



SOLUTION

Internal Loadings: Referring to the FBD of beam's right cut segment, Fig. *a*,

$+\uparrow \Sigma F_y = 0; \qquad V - 150(10^3) = 0 \qquad V = 150(10^3)\text{ N}$

$\circlearrowleft + \Sigma M_0 = 0; \qquad -M - 150(10^3)\,x = 0 \qquad M = -150(10^3)x$

Shearing Strain Energy: For the rectangular beam, the form factor is $f_s = \dfrac{6}{5}$.

$$(U_i)_v = \int_0^L \frac{f_s V^2 dx}{2GA} = \int_0^{1\text{ m}} \frac{\frac{6}{5}[150(10^3)]^2 dx}{2[26(10^9)][0.1(0.3)]} = 17.31\text{ J}$$

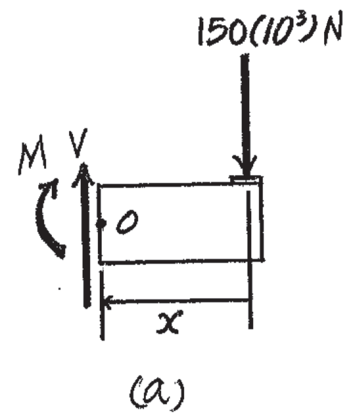
Bending Strain Energy: $I = \dfrac{1}{12}(0.1)(0.3^3) = 0.225(10^{-3})\text{ m}^4$. We obtain

$$(U_i)_b = \int_0^L \frac{M^2 dx}{2EI} = \int_0^{1\text{ m}} \frac{[-150(10^3)x]^2 dx}{2[68.9(10^9)][0.225(10^{-3})]}$$

$$= 725.689 \int_0^{1\text{ m}} x^2\,dx$$

$$= 725.689\left(\frac{x^3}{3}\right)\Bigg|_0^{1\text{ m}}$$

$$= 241.90\text{ J}$$



(a)

Thus, the strain energy stored in the beam is

$$U_i = (U_i)_v + (U_i)_b$$
$$= 17.31 + 241.90$$
$$= 259.20\text{ J}$$

External Work: The work done by the external force $P = 150$ kN is

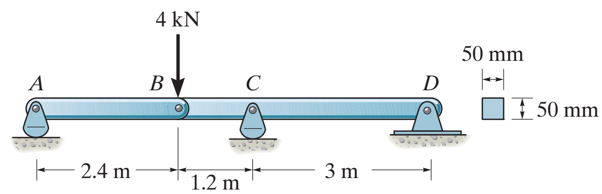
$$U_e = \frac{1}{2}P\Delta = \frac{1}{2}[150\,(10^3)]\,\Delta_B = 75(10^3)\,\Delta_B$$

Conservation of Energy:

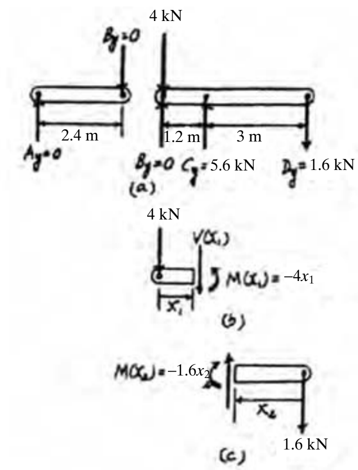
$$U_e = U_i$$
$$75(10^3)\,\Delta_B = 259.20$$
$$\Delta_B = 3.456\,(10^{-3})\text{ m} = 3.46\text{ mm}$$ **Ans.**

Ans:
$\Delta_B = 3.46$ mm

14–35. The A-36 steel bars are pin connected at B. If each has a square cross section, determine the vertical displacement at B.



SOLUTION



Support Reactions: As shown on FBD(a).

Moment Functions: As shown on FBD(b) and (c).

Bending Strain Energy: Applying 14–17, we have

$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{1}{2EI}\left[\int_0^{1.2\text{ m}} (-4x_1)^2\, dx_1 + \int_0^{3\text{ m}} (-1.6x_2)^2\, dx_2\right]$$

$$= \frac{16.128 \text{ kN}^2 \cdot \text{m}^3}{EI}$$

$$= \frac{16.128(10^3\text{ N})^2 \cdot \text{m}^3}{[200(10^9)\text{ N/m}^2]\left[\frac{1}{12}(0.05\text{ m})(0.05\text{ m})^3\right]} = 154.829\text{ J}$$

External Work: The external work done by 4 kN force is

$$U_e = \frac{1}{2}[4(10^3)](\Delta_B) = 2000\Delta_B$$

Conservation of Energy:

$$U_e = U_i$$

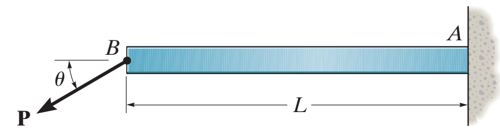
$$2000\Delta_B = 154.829$$

$$\Delta_B = 0.07741 \text{ m} = 77.41 \text{ mm}$$ **Ans.**

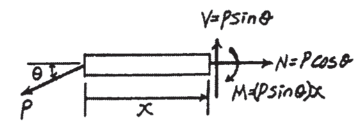
Ans:
$\Delta_B = 77.4$ mm

***14–36.**

The cantilevered beam has a rectangular cross-sectional area A, a moment of inertia I, and a modulus of elasticity E. If a load $\mathbf{P}$ acts at point B as shown, determine the displacement at B in the direction of $\mathbf{P}$, accounting for bending, axial force, and shear.



SOLUTION



Strain Energy: Applying Eq. 14–15, 14–17 and 14–19, we have

$$U_i = \int_0^L \frac{N^2 dx}{2AE} + \int_0^L \frac{M^2 dx}{2EI} + \int_0^L \frac{f_s V^2 dx}{2GA}$$

However, $f_s = \dfrac{6}{5}$ for a rectangular section.

$$U_i = \int_0^L \frac{(P\cos\theta)^2\,dx}{2AE} + \int_0^L \frac{[(P\sin\theta)\,x]^2\,dx}{2EI} + \frac{6}{5}\int_0^L \frac{(P\sin\theta)^2\,dx}{2GA}$$

$$= \frac{P^2 L}{30}\left(\frac{15\cos^2\theta}{AE} + \frac{5L^2\sin^2\theta}{EI} + \frac{18\sin^2\theta}{GA}\right)$$

External Work: The external work done by force P is

$$U_e = \frac{1}{2}(P)(\Delta_B)$$

Conservation of Energy:

$$U_e = U_i$$

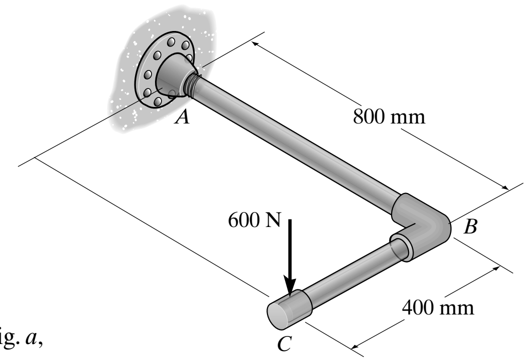
$$\frac{1}{2}(P)(\Delta_B) = \frac{P^2 L}{30}\left(\frac{15\cos^2\theta}{AE} + \frac{5L^2\sin^2\theta}{EI} + \frac{18\sin^2\theta}{GA}\right)$$

$$\Delta_B = \frac{PL}{15}\left(\frac{15\cos^2\theta}{AE} + \frac{5L^2\sin^2\theta}{EI} + \frac{18\sin^2\theta}{GA}\right) \qquad \textbf{Ans.}$$

Ans:

$$\Delta_B = \frac{PL}{15}\left(\frac{15\cos^2\theta}{AE} + \frac{5L^2\sin^2\theta}{EI} + \frac{18\sin^2\theta}{GA}\right)$$

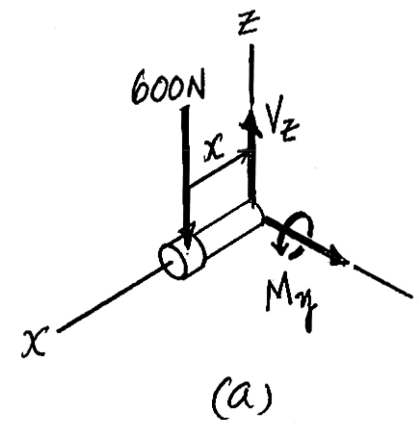
14–37. The pipe assembly is fixed at A. Determine the vertical displacement of end C of the assembly. The pipe has an inner diameter of 40 mm and outer diameter of 60 mm and is made of A-36 steel. Neglect the shearing strain energy.



SOLUTION

Internal Loading: Referring to the free-body diagram of the cut segment BC, Fig. a,

$\Sigma M_y = 0;\ M_y + 600x = 0 \qquad M_y = -600x$



Referring to the free-body diagram of the cut segment AB, Fig. b,

$\Sigma M_x = 0;\ M_x - 600y = 0 \qquad M_x = 600y$

$\Sigma M_y = 0;\ 600(0.4) - T_y = 0 \qquad T_y = 240\,\text{N}\cdot\text{m}$

Torsional Strain Energy. $J = \frac{\pi}{2}\left(0.03^4 - 0.02^4\right) = 0.325\left(10^{-6}\right)\pi\,\text{m}^4$. We obtain

$$(U_i)_t = \int_0^L \frac{T^2dx}{2GJ} = \int_0^{0.8\text{ m}} \frac{240^2dx}{2\left[75\left(10^9\right)\right]\left[0.325\left(10^{-6}\right)\pi\right]} = 0.3009\text{ J}$$

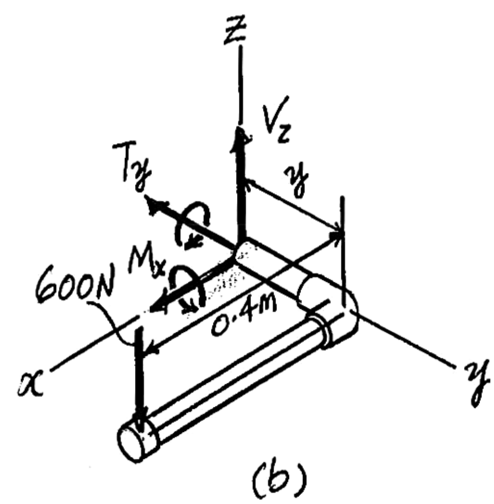
Bending Strain Energy. $I = \frac{\pi}{4}\left(0.03^4 - 0.02^4\right) = 0.1625\left(10^{-6}\right)\pi\,\text{m}^4$. We obtain

$$(U_i)_b = \int_0^L \frac{M^2dx}{2EI} = \frac{1}{2EI}\left[\int_0^{0.4\text{ m}}(-600x)^2\,dx + \int_0^{0.8\text{ m}}(600y)^2\,dy\right]$$

$$= \frac{1}{2EI}\left[120\left(10^3\right)x^3\Big|_0^{0.4\text{ m}} + 120\left(10^3\right)y^3\Big|_0^{0.8\text{ m}}\right]$$

$$= \frac{34\,560\text{ N}^2\cdot\text{m}^3}{EI}$$

$$= \frac{34\,560}{200\left(10^9\right)\left[0.1625\left(10^{-6}\right)\pi\right]} = 0.3385\text{ J}$$



Thus, the strain energy stored in the pipe is

$$U_i = (U_i)_t + (U_i)_b$$
$$= 0.3009 + 0.3385$$
$$= 0.6394\text{ J}$$

External Work. The work done by the external force $P = 600$ N is

$$U_e = \frac{1}{2}P\Delta = \frac{1}{2}(600)\Delta_C = 300\Delta_C$$

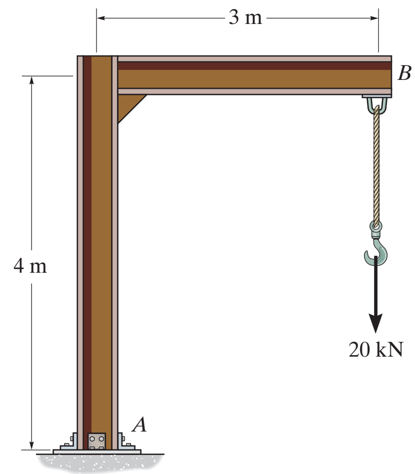
Conservation of Energy.

$$U_e = U_t$$
$$300\Delta_C = 0.6394$$
$$\Delta_C = 2.1312\left(10^{-3}\right) = 2.13\text{ mm} \qquad \textbf{Ans.}$$

Ans:
$\Delta_C = 2.13$ mm

14–38.

Determine the vertical displacement of end B of the frame. Consider only bending strain energy. The frame is made using two A-36 steel W460 × 68 wide-flange sections.



SOLUTION

Internal Loading: Using the coordinates x_1 and x_2, the free-body diagrams of the frame's segments in Figs. a and b are drawn. For coordinate x_1,

$$\circlearrowleft + \Sigma M_O = 0; \qquad -M_1 - 20(10^3)x_1 = 0 \qquad M_1 = -20(10^3)x_1$$

For coordinate x_2,

$$\circlearrowleft + \Sigma M_O = 0; \qquad M_2 - 20(10^3)(3) = 0 \qquad M_2 = 60(10^3)\,\text{N}\cdot\text{m}$$

Bending Strain Energy:

$$(U_b)_i = \int_0^L \frac{M^2dx}{2EI} = \frac{1}{2EI}\left[\int_0^{3\text{ m}}\left[-20(10^3)x_1\right]^2 dx_1 + \int_0^{4\text{ m}}\left[60(10^3)\right]^2 dx_2\right]$$

$$= \frac{1}{2EI}\left[\left(\frac{400(10^6)}{3}x_1{}^3\right)\Bigg|_0^{3\text{ m}} + 3.6(10^9)x\Big|_0^{4\text{ m}}\right]$$

$$= \frac{9(10^9)\,\text{N}^2\cdot\text{m}^3}{EI}$$

For a W460 × 68, $I = 297(10^6)\,\text{mm}^4 = 297(10^{-6})\,\text{m}^4$. Then

$$(U_b)_i = \frac{9(10^9)}{200(10^9)(297)(10^{-6})} = 151.52\text{ J}$$

External Work: The work done by the external force $P = 20$ kN is

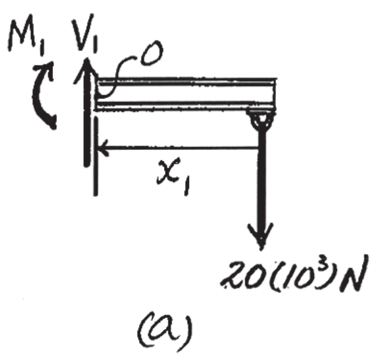
$$U_e = \frac{1}{2}P\Delta = \frac{1}{2}\left[20(10^3)\right]\Delta_B = 10(10^3)\Delta_B$$

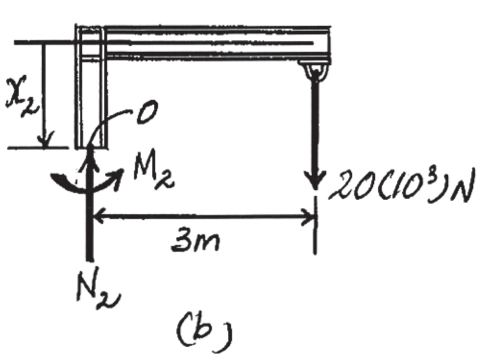
Conservation of Energy:

$$U_e = U_i$$

$$10(10^3)\Delta_B = 151.52$$

$$\Delta_B = 0.01515\text{ m} = 15.2\text{ mm} \qquad \textbf{Ans.}$$

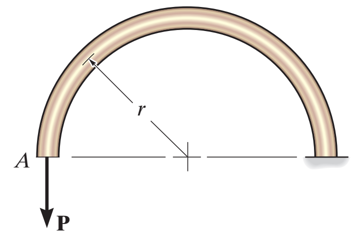




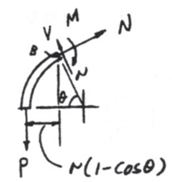
Ans:
$\Delta_B = 15.2$ mm

14–39.

The rod has a circular cross section with a moment of inertia I. If a vertical force $\mathbf{P}$ is applied at A, determine the vertical displacement at this point. Only consider the strain energy due to bending. The modulus of elasticity is E.



SOLUTION



Moment Function:

$$\zeta+ \Sigma M_B = 0; \qquad P[r(1 - \cos\theta)] - M = 0; \qquad M = Pr(1 - \cos\theta)$$

Bending Strain Energy:

$$U_i = \int_0^S \frac{M^2\,ds}{2EI} \qquad ds = r\,d\theta$$

$$= \int_0^\theta \frac{M^2 r\,d\theta}{2EI} = \frac{r}{2EI}\int_0^\pi [P\,r(1 - \cos\theta)]^2 d\theta$$

$$= \frac{P^2r^3}{2EI}\int_0^\pi (1 + \cos^2\theta - 2\cos\theta)\,d\theta$$

$$= \frac{P^2r^3}{2EI}\int_0^\pi \left(1 + \frac{1}{2} + \frac{\cos 2\theta}{2} - 2\cos\theta\right)d\theta$$

$$= \frac{P^2r^3}{2EI}\int_0^\pi \left(\frac{3}{2} + \frac{\cos 2\theta}{2} - 2\cos\theta\right)d\theta = \frac{P^2r^3}{2EI}\left(\frac{3}{2}\pi\right) = \frac{3\pi P^2r^3}{4EI}$$

Conservation of Energy:

$$U_e = U_i; \qquad \frac{1}{2}P\,\Delta_A = \frac{3\pi P^2r^3}{4EI}$$

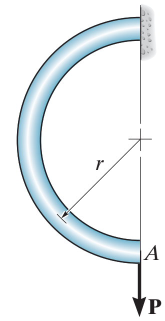
$$\Delta_A = \frac{3\pi Pr^3}{2EI}$$ **Ans.**

Ans:

$$\Delta_A = \frac{3\pi Pr^3}{2EI}$$

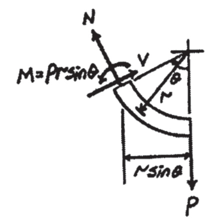
***14–40.**

The rod has a circular cross section with a moment of inertia I. If a vertical force **P** is applied at A, determine the vertical displacement at this point. Only consider the strain energy due to bending. The modulus of elastcity is E.



SOLUTION

Bending Strain Energy: Applying 14–17 with $ds = rd\theta$, we have



$$U_i = \int_0^s \frac{M^2 ds}{2EI}$$

$$= \frac{1}{2EI}\int_0^\pi (Pr\sin\theta)^2\, rd\theta$$

$$= \frac{P^2r^3}{2EI}\int_0^\pi \sin^2\theta d\theta$$

$$= \frac{P^2r^3}{4EI}\int_0^\pi (1 - \cos 2\theta)\, d\theta$$

$$= \frac{\pi P^2 r^3}{4EI}$$

External Work: The external work done by force P is

$$U_e = \frac{1}{2}(P)(\Delta_A)$$

Conservation of Energy:

$$U_e = U_i$$

$$\frac{1}{2}(P)(\Delta_A) = \frac{\pi P^2 r^3}{4EI}$$

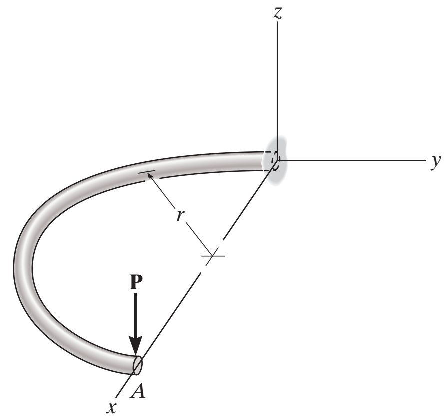
$$\Delta_A = \frac{\pi P r^3}{2EI}$$ **Ans.**

Ans:

$$\Delta_A = \frac{\pi P r^3}{2EI}$$

14–41.

The rod has a circular cross section with a polar moment of inertia J and moment of inertia I. If a vertical force $\mathbf{P}$ is applied at A, determine the vertical displcement at this point. Consider the strain energy due to bending and torsion. The material constants are E and G.



SOLUTION

$T = Pr(1 - \cos\theta); \quad M = Pr\sin\theta$

Torsion Strain Energy:

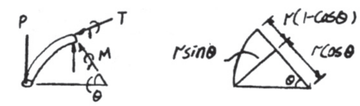
$$U_i = \int_0^s \frac{T^2\,ds}{2GJ} = \int_0^\theta \frac{T^2\,r d\theta}{2GJ}$$

$$= \frac{r}{2GJ}\int_0^\pi [Pr(1 - \cos\theta)]^2\,d\theta$$

$$= \frac{P^2r^3}{2GJ}\int_0^\pi (1 + \cos^2\theta - 2\cos\theta)d\theta$$

$$= \frac{P^2r^3}{2GJ}\int_0^\pi \left(1 + \frac{\cos 2\theta + 1}{2} - 2\cos\theta\right)d\theta$$

$$= \frac{3P^2r^3\pi}{4GJ}$$



T= Pr(1-cosθ)
M= Pr sinθ

Bending Strain Energy:

$$U_i = \int_0^s \frac{M^2ds}{2EI}$$

$$= \int_0^\theta \frac{M^2r\,d\theta}{2EI} = \frac{r}{2EI}\int_0^\pi [Pr\sin\theta]^2\,d\theta$$

$$= \frac{P^2r^3}{2EI}\int_0^\pi \left(\frac{1 - \cos 2\theta}{2}\right)d\theta = \frac{P^2r^3\pi}{4EI}$$

Conservation of Energy:

$$U_e = U_i$$

$$\frac{1}{2}P\Delta_A = \frac{3P^2r^3\pi}{4GJ} + \frac{P^2r^3\pi}{4EI}$$

$$\Delta_A = \frac{Pr^3\pi}{2}\left(\frac{3}{GJ} + \frac{1}{EI}\right)$$ **Ans.**

Ans:

$$\Delta_A = \frac{Pr^3\pi}{2}\left(\frac{3}{GJ} + \frac{1}{EI}\right)$$

14–42.

A bar is 4 m long and has a diameter of 30 mm. Determine the total amount of elastic energy that it can absorb from an impact loading if (a) it is made of steel for which $E_{st} = 200$ GPa, $\sigma_Y = 800$ MPa, and (b) it is made from an aluminum alloy for which $E_{al} = 70$ GPa, $\sigma_Y = 405$ MPa.

SOLUTION

a) $\epsilon_Y = \dfrac{\sigma_Y}{E} = \dfrac{800(10^6)}{200(10^9)} = 4(10^{-3})\ \text{m/m}$

$$u_r = \frac{1}{2}(\sigma_Y)(\epsilon_Y) = \frac{1}{2}(800)(10^6)(\text{N/m}^2)(4)(10^{-3})\ \text{m/m} = 1.6\ \text{MJ/m}^3$$

$$V = \frac{\pi}{4}(0.03)^2(4) = 0.9(10^{-3})\pi\ \text{m}^2$$

$$U_i = 1.6(10^6)(0.9)(10^{-3})\pi = 4.52\ \text{kJ}$$ **Ans.**

b)

$$\epsilon_Y = \frac{\sigma_Y}{E} = \frac{405(10^6)}{70(10^9)} = 5.786(10^{-3})\ \text{m/m}$$

$$U_r = \frac{1}{2}(\sigma_Y)(\epsilon_Y) = \frac{1}{2}(405)(10^6)(\text{N/m}^2)(5.786)(10^{-3})\ \text{m/m} = 1.172\ \text{MJ/m}^3$$

$$V = \frac{\pi}{4}(0.03)^2(4) = 0.9(10^{-3})\pi\ \text{m}^3$$

$$U_i = 1.172(10^6)(0.9)(10^{-3})\pi = 3.31\ \text{kJ}$$ **Ans.**

Ans:
(a) $U_i = 4.52$ kJ
(b) $U_i = 3.31$ kJ

14–43.

Determine the diameter of a red brass C83400 bar that is 8 ft long if it is to be used to absorb 800 ft·lb of energy in tension from an impact loading. No yielding occurs.

SOLUTION

Elastic Strain Energy: The yielding axial force is $P_Y = \sigma_Y A$.

$$U_i = \frac{N^2 L}{2AE} = \frac{(\sigma_Y A)^2 L}{2AE} = \frac{\sigma_Y^2 A L}{2E}$$

Substituting, we have

$$U_i = \frac{\sigma_Y^2 A L}{2E}$$

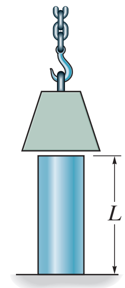
$$0.8(12) = \frac{11.4^2\left[\frac{\pi}{4}(d^2)\right](8)(12)}{2[14.6(10^3)]}$$

$$d = 5.35 \text{ in.}$$ **Ans.**

Ans:
$d = 5.35$ in.

***14–44.**

Determine the speed v of the 50-Mg mass when it is just over the top of the steel post, if after impact, the maximun stress developed in the post is 550 MPa. The post has a length of $L = 1$ m and a cross-sectional area of 0.01 m^2. $E_{st} = 200$ GPa, $\sigma_Y = 600$ MPa.



SOLUTION

The Maximum Stress:

$$\sigma_{max} = \frac{P_{max}}{A}$$

$$550(10^6) = \frac{P_{max}}{0.01}; \qquad P_{max} = 5500 \text{ kN}$$

$$\Delta_{max} = \frac{P_{max}}{k} \qquad \text{Here } k = \frac{AE}{L} = \frac{0.01(200)(10^9)}{1} = 2(10^9) \text{ N/m}$$

$$= \frac{5500(10^3)}{2(10^9)} = 2.75(10^{-3}) \text{ m}$$

Conservation of Energy:

$$U_e = U_i$$

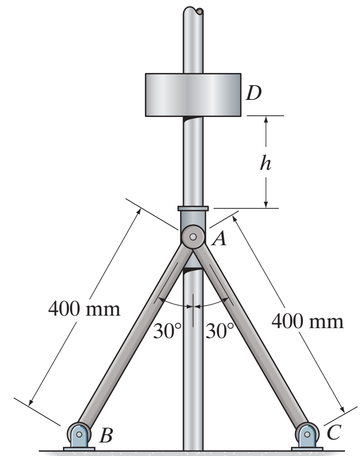
$$\frac{1}{2}mv^2 + W\,\Delta_{max} = \frac{1}{2}k\,\Delta_{max}^2$$

$$\frac{1}{2}(50)(10^3)(v^2) + 50(10^3)(9.81)[2.75(10^{-3})] = \frac{1}{2}(2)(10^9)[2.75(10^{-3})]^2$$

$v = 0.499$ m/s **Ans.**

Ans:
$v = 0.499$ m/s

14–45. Rods AB and AC have a diameter of 20 mm and are made of 6061-T6 aluminum alloy. They are connected to the rigid collar which slides freely along the vertical guide rod. Determine the maximum height h from which the 50-kg block D can be dropped without causing yielding in the rods when the block strikes the collar.

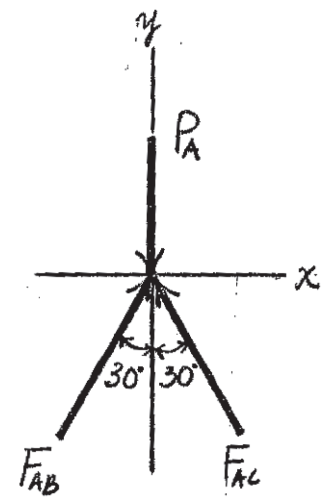


SOLUTION

Equilibrium. Referring to the free-body diagram of joint A, Fig. a

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AB} \sin 30° - F_{AC} \sin 30° = 0 \qquad F_{AB} = F_{AC} = F$$

$$+\uparrow \Sigma F_y = 0; \quad 2F \cos 30° - P_A = 0 \qquad P_A = 1.732F \qquad (1)$$



Compatibility Equation. From the geometry shown in Fig. b

$$\delta_F = \Delta_A \cos 30°$$

$$\frac{F(0.4)}{\frac{\pi}{4}(0.02^2)[68.9(10^9)]} = \Delta_A \cos 30°$$

$$\Delta_A = 21.3383(10^{-9})F$$

Thus, the equivalent spring constant for the system can be determined from

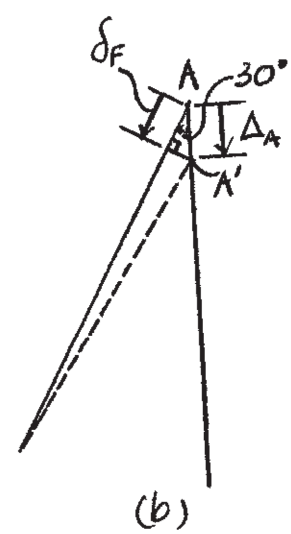
$$P_A = k\Delta_A$$

$$1.732F = k[21.3383(10^{-9})F]$$

$$k = 81.171(10^6) \text{ N/m}$$

Maximum Stress. The maximum force that can be developed in members AB and AC is

$$F_{max} = \sigma_Y A = 255(10^6)\left[\frac{\pi}{4}(0.02^2)\right] = 80.11(10^3) \text{ N}$$



From Eq. (1),

$$(P_A)_{max} = 1.732F_{max} = 1.732[80.11(10^3)] = 138.76(10^3) \text{ N}$$

Then,

$$(\Delta_A)_{max} = \frac{(P_A)_{max}}{k} = \frac{138.76(10^3)}{81.171(10^6)} = 1.7094(10^{-3}) \text{ m}$$

Conservation of Energy.

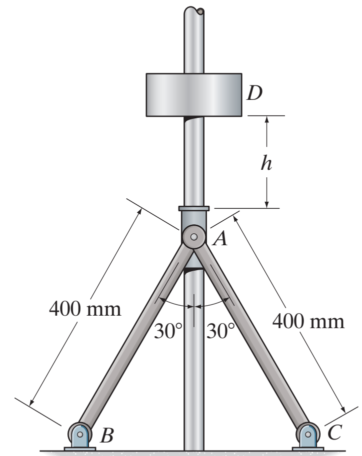
$$mg[h + (\Delta_A)_{max}] = \frac{1}{2}k(\Delta_A)_{max}^2$$

$$50(9.81)[h + 1.7094(10^{-3})] = \frac{1}{2}[81.171(10^6)][1.7094(10^{-3})]^2$$

$$h = 0.240 \text{ m}$$ **Ans.**

Ans:
$h = 0.240$ m

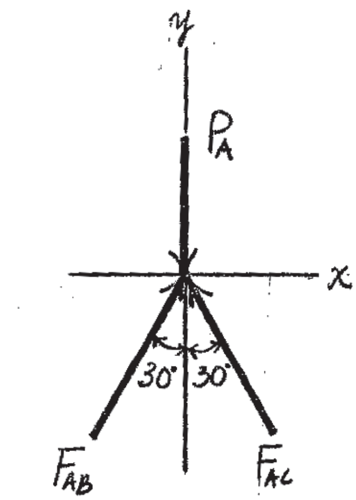
14–46. Rods AB and AC have a diameter of 20 mm and are made of 6061-T6 aluminum alloy. They are connected to the rigid collar A which slides freely along the vertical guide rod. If the 50-kg block D is dropped from height $h = 200$ mm above the collar, determine the maximum normal stress developed in the rods.



Equilibrium. Referring to the free-body diagram of joint A, Fig. a

$\xrightarrow{+}\Sigma F_x = 0;\qquad F_{AB}\sin 30° - F_{AC}\sin 30° = 0 \qquad F_{AB} = F_{AC} = F$

$+\uparrow\Sigma F_y = 0;\qquad 2F\cos 30° - P_A = 0 \qquad P_A = 1.732F \qquad (1)$



(a)

Compatibility Equation. From the geometry shown in Fig. b

$$\delta_F = \Delta_A \cos 30°$$

$$\frac{F(0.4)}{\frac{\pi}{4}(0.02^2)[68.9(10^9)]} = \Delta_A \cos 30°$$

$$\Delta_A = 21.3383(10^{-9})F$$

Thus, the equivalent spring constant for the system can be determined from

$$P_A = k\Delta_A$$

$$1.732F = k[21.3383(10^{-9})F]$$

$$k = 81.171(10^6)\text{ N/m}$$

Conservation of Energy.

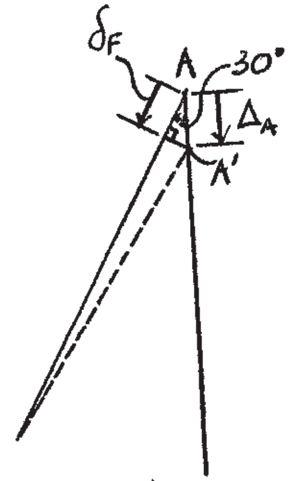
$$mg[h + (\Delta_A)_{max}] = \frac{1}{2}k(\Delta_A)_{max}^2$$

$$50(9.81)[0.2 + (\Delta_A)_{max}] = \frac{1}{2}[81.171(10^6)](\Delta_A)_{max}^2$$

$$40.5855(10^6)(\Delta_A)_{max}^2 - 490.5(\Delta_A)_{max} - 98.1 = 0$$

Solving for the positive root,

$$(\Delta_A)_{max} = 1.5608(10^{-3})\text{ m}$$



(b)

Then,

$$(P_A)_{max} = k(\Delta_A)_{max} = 81.171(10^6)[1.5608(10^{-3})] = 126.69(10^3)\text{ N}$$

Maximum Stress. From Eq. (1),

$$(P_A)_{max} = 1.732F_{max}$$

$$F_{max} = 73.143(10^3)\text{ N}$$

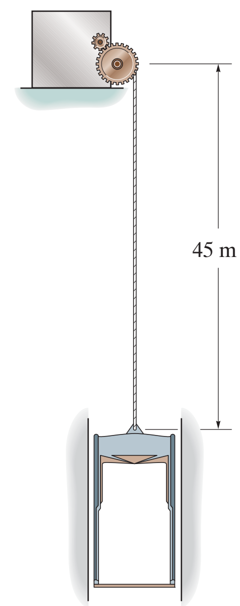
Thus, the maximum normal stress developed in members AB and AC is

$$(\sigma_{max})_{AB} = (\sigma_{max})_{AC} = \frac{F_{max}}{A} = \frac{73.143(10^3)}{\frac{\pi}{4}(0.02^2)} = 232.82\text{ MPa} = 233\text{ MPa} \qquad \textbf{Ans.}$$

Since $(\sigma_{max})_{AB} = (\sigma_{max})_{AC} < \sigma_Y = 255$ MPa, this result is valid.

Ans:
$(\sigma_{max})_{AB} = (\sigma_{max})_{AC} = 233$ MPa

14–47. A steel cable having a diameter of 16 mm wraps over a drum and is used to lower an elevator having a mass of 400 kg. The elevator is 45 m below the drum and is descending at the constant rate of 0.6 m/s when the drum suddenly stops. Determine the maximum stress developed in the cable when this occurs. $E_{st} = 200$ GPa, when $\sigma_Y = 350$ MPa.



SOLUTION

$$k = \frac{AE}{L} = \frac{\left[\frac{\pi}{4}(0.016^2)\right][200(10^9)]}{45} = 893.61(10^3)\text{ N/m}$$

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + mg\,\Delta_{max} = \frac{1}{2}k\Delta_{max}^2$$

$$\frac{1}{2}(400)(0.6^2) + 400(9.81)\Delta_{max} = \frac{1}{2}[893.61(10^3)]\Delta_{max}^2$$

$$446.80(10^3)\Delta_{max}^2 - 3924\Delta_{max} - 72 = 0$$

$$\Delta_{max} = 0.017823\text{ m}$$

$$P_{max} = k\Delta_{max} = [893.61(10^3)](0.017823) = 15.927(10^3)\text{ N}$$

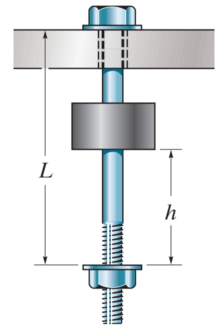
$$\sigma_{max} = \frac{P_{max}}{A} = \frac{15.927(10^3)}{\frac{\pi}{4}(0.016^2)} = 79.22(10^6)\text{ N/m}^2$$

$$= 79.22\text{ MPa} < \sigma_Y \quad \text{O.K.}$$ **Ans.**

Ans:
$\sigma_{max} = 79.2$ MPa

***14–48.**

The A-36 steel bolt is required to absorb the energy of a 2-kg mass that falls $h = 30$ mm. If the bolt has a diameter of 4 mm, determine its required length L so the stress in the bolt does not exceed 150 MPa.



SOLUTION

Maximum Stress: With $\Delta_{st} = \dfrac{WL}{AE} = \dfrac{2(9.81)(L)}{\frac{\pi}{4}(0.004^2)\left[200(10^9)\right]}$

$= 7.80655\left(10^{-6}\right)L$ and $\sigma_{st} = \dfrac{W}{A} = \dfrac{2(9.81)}{\frac{\pi}{4}(0.004^2)} = 1.56131$ MPa, we have

$$\sigma_{max} = n\sigma_{st} \quad \text{where } n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}$$

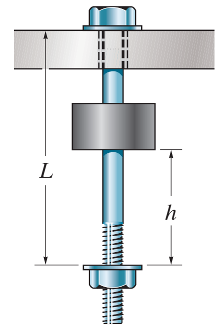
$$150\left(10^6\right) = \left[1 + \sqrt{1 + 2\left(\frac{0.03}{7.80655\left(10^{-6}\right)L}\right)}\right]\left[1.56131\left(10^6\right)\right]$$

$$L = 0.8504 \text{ m} = 850 \text{ mm}$$ **Ans.**

Ans:
$L = 850$ mm

14–49.

The A-36 steel bolt is required to absorb the energy of a 2-kg mass that falls $h = 30$ mm. If the bolt has a diameter of 4 mm and a length of $L = 200$ mm, determine if the stress in the bolt will exceed 175 MPa.



SOLUTION

Maximum Stress: With

$$\Delta_{st} = \frac{WL}{AE} = \frac{2(9.81)(0.2)}{\frac{\pi}{4}(0.004^2)\left[200(10^9)\right]} = 1.56131(10^{-6})\text{ m}$$

$$\sigma_{st} = \frac{W}{A} = \frac{2(9.81)}{\frac{\pi}{4}(0.004^2)} = 1.56131\text{ MPa}$$

Applying Eq. 14–34, we have

$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)} = 1 + \sqrt{1 + 2\left(\frac{0.03}{1.56131(10^{-6})}\right)} = 197.04$$

Thus,

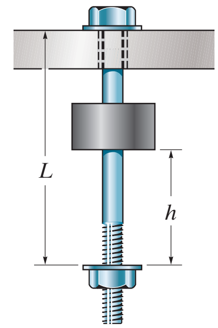
$$\sigma_{max} = n\sigma_{st} = 197.04(1.56131) = 307.6\text{ MPa}$$

Yes, σ_{max} exceeded 175 MPa. **Ans.**

Ans:
Yes

14–50.

The A-36 steel bolt is required to absorb the energy of a 2-kg mass that falls along the 4-mm-diameter bolt shank that is 150 mm long. Determine the maximum height h of release so the stress in the bolt does not exceed 150 MPa.



SOLUTION

Maximum Stress: With $\Delta_{st} = \dfrac{WL}{AE} = \dfrac{2(9.81)(0.15)}{\frac{\pi}{4}(0.004^2)\left[200(10^9)\right]}$

$= 1.17098(10^{-6})$ m and $\sigma_{st} = \dfrac{W}{A} = \dfrac{2(9.81)}{\frac{\pi}{4}(0.004^2)} = 1.56131$ MPa,

we have

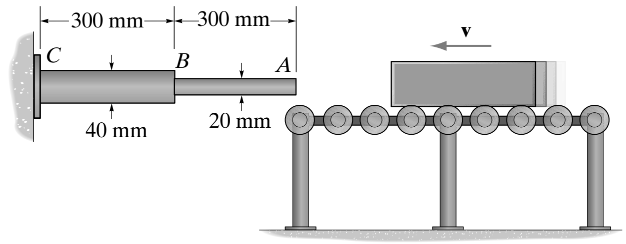
$$\sigma_{max} = n\sigma_{st} \quad \text{where } n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}$$

$$150(10^6) = \left[1 + \sqrt{1 + 2\left(\frac{h}{1.17098(10^{-6})}\right)}\right]\left[1.56131(10^6)\right]$$

$$h = 5.292(10^{-3})\text{ m} = 5.29\text{ mm} \qquad \textbf{Ans.}$$

Ans:
$h = 5.29$ mm

14–51. The 5-kg block is traveling with the speed of $v = 4$ m/s just before it strikes the 6061-T6 aluminum stepped cylinder. Determine the maximum normal stress developed in the cylinder.



SOLUTION

Equilibrium. The equivalent spring constant for segments AB and BC are

$$k_{AB} = \frac{A_{AB}E}{L_{AB}} = \frac{\frac{\pi}{4}(0.02^2)\left[68.9(10^9)\right]}{0.3} = 72.152(10^6)\text{ N/m}$$

$$k_{BC} = \frac{A_{BC}E}{L_{BC}} = \frac{\frac{\pi}{4}(0.04^2)\left[68.9(10^9)\right]}{0.3} = 288.608(10^6)\text{N/m}$$

Equilibrium requires

$$F_{AB} = F_{BC}$$

$$k_{AB}\,\Delta_{AB} = k_{BC}\Delta_{BC}$$

$$72.152(10^6)\Delta_{AB} = 288.608(10^6)\Delta_{BC}$$

$$\Delta_{BC} = \frac{1}{4}\,\Delta_{AB} \qquad \textbf{(1)}$$

Conservation of Energy.

$$U_e = U_i$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k_{AB}\Delta_{AB}{}^2 + \frac{1}{2}k_{BC}\Delta_{BC}{}^2 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2),

$$\frac{1}{2}\,mv^2 = \frac{1}{2}\,k_{AB}\Delta_{AB}{}^2 + \frac{1}{2}\,k_{BC}\left(\frac{1}{4}\Delta_{AB}\right)^2$$

$$\frac{1}{2}\,mv^2 = \frac{1}{2}\,k_{AB}\Delta_{AB}{}^2 + \frac{1}{32}\,k_{BC}\Delta_{AB}{}^2$$

$$\frac{1}{2}(5)(4^2) = \frac{1}{2}\left[72.152(10^6)\right]\Delta_{AB}{}^2 + \frac{1}{32}\left[288.608(10^6)\right]\Delta_{AB}{}^2$$

$$\Delta_{AB} = 0.9418(10^{-3})\text{ m}$$

Maximum Stress. The force developed in segment AB is $F_{AB} = k_{AB}\Delta_{AB} = 72.152(10^6)\left[0.9418(10^{-3})\right] = 67.954(10^3)$N.

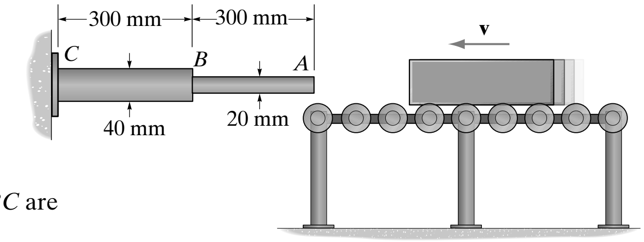
Thus,

$$\sigma_{\max} = \sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{67.954(10^3)}{\frac{\pi}{4}\left(0.02^2\right)} = 216.30\text{ MPa} = 216\text{ MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{\max} < \sigma_Y = 255$ MPa, this result is valid.

Ans:
$\sigma_{\max} = 216$ MPa

***14–52.** Determine the maximum speed v of the 5-kg block without causing the 6061-T6 aluminum stepped cylinder to yield after it is struck by the block.



SOLUTION

Equilibrium. The equivalent spring constant for segments AB and BC are

$$k_{AB} = \frac{A_{AB}E}{L_{AB}} = \frac{\frac{\pi}{4}\left(0.02^2\right)\left[68.9\left(10^9\right)\right]}{0.3} = 72.152\left(10^6\right)\text{ N/m}$$

$$k_{BC} = \frac{A_{BC}E}{L_{BC}} = \frac{\frac{\pi}{4}\left(0.04^2\right)\left[68.9\left(10^9\right)\right]}{0.3} = 288.608\left(10^6\right)\text{ N/m}$$

Equilibrium requires

$$F_{AB} = F_{BC}$$

$$k_{AB}\Delta_{AB} = k_{BC}\Delta_{BC}$$

$$72.152\left(10^6\right)\Delta_{AB} = 288.608\left(10^6\right)\Delta_{BC}$$

$$\Delta_{BC} = \frac{1}{4}\Delta_{AB} \qquad \textbf{(1)}$$

Conservation of Energy.

$$U_e = U_i$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k_{AB}\Delta_{AB}{}^2 + \frac{1}{2}k_{BC}\Delta_{BC}{}^2 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2),

$$\frac{1}{2}mv^2 = \frac{1}{2}k_{AB}\Delta_{AB}{}^2 + \frac{1}{2}k_{BC}\left(\frac{1}{4}\Delta_{AB}\right)^2$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k_{AB}\Delta_{AB}{}^2 + \frac{1}{32}k_{BC}\Delta_{AB}{}^2$$

$$\frac{1}{2}(5)v^2 = \frac{1}{2}\left[72.152\left(10^6\right)\right]\Delta_{AB}{}^2 + \frac{1}{32}\left[288.608\left(10^6\right)\right]\Delta_{AB}{}^2$$

$$\Delta_{AB} = 0.23545\left(10^{-3}\right)v$$

Maximum Stress. The force developed in segment AB is $F_{AB} = k_{AB}\Delta_{AB} = 72.152\left(10^6\right)\left[0.23545\left(10^{-3}\right)v\right] = 16988.46v$.

Thus,

$$\sigma_{\max} = \sigma_{AB} = \frac{F_{AB}}{A_{AB}}$$

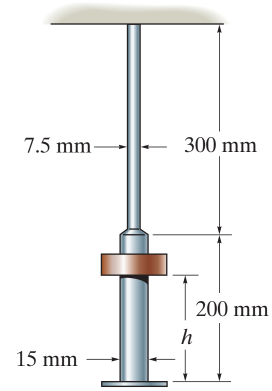
$$255\left(10^6\right) = \frac{16988.46v}{\frac{\pi}{4}\left(0.02^2\right)}$$

$$v = 4.716\text{ m/s} = 4.72\text{ m/s} \qquad \textbf{Ans.}$$

Ans.
$v = 4.72$ m/s

14–53.

The composite aluminum 2014-T6 bar is made from two segments having diameters of 7.5 mm and 15 mm. Determine the maximum axial stress developed in the bar if the 10-kg collar is dropped from a height of $h = 100$ mm.



SOLUTION

$$\Delta_{st} = \Sigma \frac{WL}{AE} = \frac{10(9.81)(0.3)}{\frac{\pi}{4}(0.0075)^2(73.1)(10^9)} + \frac{10(9.81)(0.2)}{\frac{\pi}{4}(0.015)^2(73.1)(10^9)}$$

$$= 10.63181147\,(10^{-6})\text{ m}$$

$$n = \left[1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}\right] = \left[1 + \sqrt{1 + 2\left(\frac{0.1}{10.63181147(10^{-6})}\right)}\right] = 138.16$$

$$\sigma_{max} = n\,\sigma_{st} \quad \text{Here } \sigma_{st} = \frac{W}{A} = \frac{10\,(9.81)}{\frac{\pi}{4}(0.0075^2)} = 2.22053\text{ MPa}$$

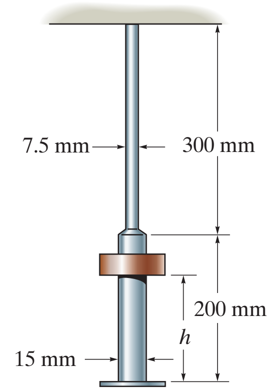
$$\sigma_{max} = 138.16\,(2.22053)$$

$$= 307\text{ MPa} < \sigma_Y = 414\text{ MPa} \qquad \textbf{OK} \qquad \textbf{Ans.}$$

Ans:
$\sigma_{max} = 307$ MPa

14–54.

The composite aluminum 2014-T6 bar is made from two segments having diameters of 7.5 mm and 15 mm. Determine the maximum height h from which the 10-kg collar should be dropped so that it produces a maximum axial stress in the bar of $\sigma_{max} = 300$ MPa.



SOLUTION

$$\Delta_{st} = \Sigma\frac{WL}{AE} = \frac{10(9.81)(0.3)}{\frac{\pi}{4}(0.0075)^2(73.1)(10^9)} + \frac{10(9.81)(0.2)}{\frac{\pi}{4}(0.015)^2(73.1)(10^9)}$$

$$= 10.63181147\,(10^{-6})\text{ m}$$

$$n = \left[1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}\right] = \left[1 + \sqrt{1 + 2\left(\frac{h}{10.63181147(10^{-6})}\right)}\right]$$

$$= \left[1 + \sqrt{1 + 188114.7\,h}\right]$$

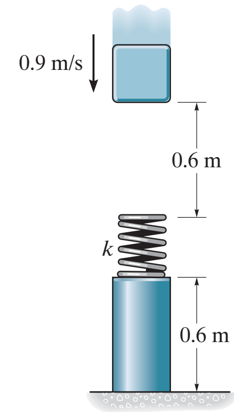
$$\sigma_{max} = n\,\sigma_{st} \qquad \text{Here } \sigma_{st} = \frac{W}{A} = \frac{10\,(9.81)}{\frac{\pi}{4}(0.0075^2)} = 2.22053\text{ MPa}$$

$$300\,(10^6) = \left[1 + \sqrt{1 + 188114.7\,h}\right](2220530)$$

$$h = 0.09559\text{ m} = 95.6\text{ mm}$$ **Ans.**

Ans:
$h = 95.6$ mm

14–55. The 25-kg block is failing at 0.9 m/s at the instant it is 0.6 m above the spring and post assembly. Determine the maximum stress in the post if the spring has a stiffness of $k = 40$ MN/m. The post has a diameter of 75 mm and a modulus of elasticity of $E = 48$ GPa. Assume the material will not yield.



SOLUTION

Equilibrium: This requires $F_{sp} = F_P$. Hence

$$k_{sp}\Delta_{sp} = k_P\Delta_P \qquad \text{and} \qquad \Delta_{sp} = \frac{k_p}{k_{sp}}\Delta_P \tag{1}$$

Conservation of Energy: The equivalent spring constant for the post is $k_p = \dfrac{AE}{L} = \dfrac{\left[\frac{\pi}{4}(0.075^2)\right][48(10^9)]}{0.6} = 353.43(10^6)$ N/m

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + W(h + \Delta_{\max}) = \frac{1}{2}k_P\Delta_P^2 + \frac{1}{2}k_{sp}\Delta_{sp}^2 \tag{2}$$

However, $\Delta_{\max} = \Delta_P + \Delta_{sp}$. Then, Eq. [2] becomes

$$\frac{1}{2}mv^2 + W\left(h + \Delta_P + \Delta_{sp}\right) = \frac{1}{2}k_P\Delta_P^2 + \frac{1}{2}k_{sp}\Delta_{sp}^2 \tag{3}$$

Substituting Eq. [1] into [3] yields

$$\frac{1}{2}mv^2 + W\left(h + \Delta_P + \frac{k_p}{k_{sp}}\Delta_P\right) = \frac{1}{2}k_p\Delta_P^2 + \frac{1}{2}\left(\frac{k_P^2}{k_{sp}}\Delta_P^2\right)$$

$$\frac{1}{2}(25)(0.9^2) + [25(9.81)]\left[0.6 + \Delta_p + \frac{353.43(10^6)}{40(10^6)}\Delta_p\right]$$

$$= \frac{1}{2}[353.43(10^6)]\Delta_p^2 + \frac{1}{2}\left\{\frac{[353.43(10^6)]^2}{40(10^6)}\right\}\Delta_p^2$$

$$1.73812(10^9)\Delta_p^2 - 2412.21\Delta_p - 157.275 = 0$$

Solving for positive root, we have

$$\Delta_P = 0.301503(10^{-3})\ \text{m}$$

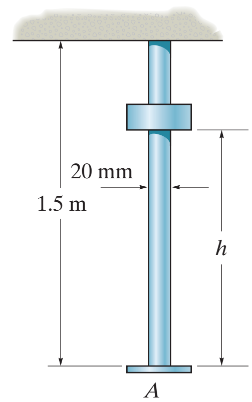
Maximum Stress: The maximum axial force for the post is $P_{\max} = k_p\Delta_p = [353.43(10^6)][0.301503(10^{-3})] = 106.56(10^3)$

$$\sigma_{\max} = \frac{P_{\max}}{A} = \frac{106.56(10^3)}{\frac{\pi}{4}(0.075^2)} = 24.12(10^6)\ \text{N/m}^2 = 24.1\ \text{MPa}$$ **Ans.**

Ans:
$\sigma_{\max} = 24.1$ MPa

***14–56.**

The collar has a mass of 5 kg and falls down the titanium Ti-6A1-4V bar. If the bar has a diameter of 20 mm, determine the maximum stress developed in the bar if the weight is (a) dropped from a height of $h = 1$ m, (b) released from a height $h \approx 0$, and (c) placed slowly on the flange at A.



SOLUTION

Maximum Stress: With $\Delta_{st} = \dfrac{WL}{AE} = \dfrac{5(9.81)(1.5)}{\frac{\pi}{4}(0.02^2)\left[120(10^9)\right]} = 1.9516(10^{-6})$ m

and $\sigma_{st} = \dfrac{W}{A} = \dfrac{5(9.81)}{\frac{\pi}{4}(0.02^2)} = 0.156131$ MPa and Applying Eq. 14–34, we have

a)

$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)} = 1 + \sqrt{1 + 2\left(\frac{1}{1.9516(10^{-6})}\right)} = 1013.31$$

Thus,

$$\sigma_{max} = n\sigma_{st} = 1013.31(0.156131) = 158 \text{ MPa} \qquad \textbf{Ans.}$$

b)

$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)} = 1 + \sqrt{1 + 2\left(\frac{0}{1.9516(10^{-6})}\right)} = 2$$

Thus,

$$\sigma_{max} = n\sigma_{st} = 2(0.156131) = 0.312 \text{ MPa} \qquad \textbf{Ans.}$$

c)

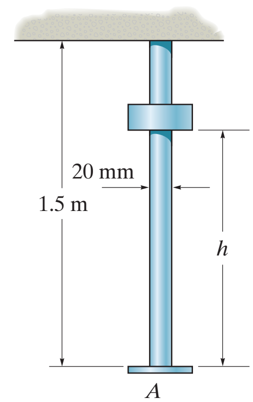
$$\sigma_{max} = \sigma_{st} = 0.156 \text{ MPa} \qquad \textbf{Ans.}$$

Since all of the $\sigma_{max} < \sigma_Y = 924$ MPa, the above analysis is valid.

Ans:
a) $\sigma_{max} = 158$ MPa,
b) $\sigma_{max} = 0.312$ MPa,
c) $\sigma_{max} = 0.156$ MPa

14–57.

The collar has a mass of 5 kg and falls down the titanium Ti-6A1-4V bar. If the bar has a diameter of 20 mm, determine if the weight can be released from rest at any point along the bar and not permanently damage the bar after striking the flange at A.



SOLUTION

Maximum Stress: With $\Delta_{st} = \dfrac{WL}{AE} = \dfrac{5(9.81)(1.5)}{\frac{\pi}{4}(0.02^2)\left[120(10^9)\right]} = 1.9516(10^{-6})$ m, $\sigma_{st} = \dfrac{W}{A} = \dfrac{5(9.81)}{\frac{\pi}{4}(0.02^2)} = 0.156131$ MPa and $h = h_{max} = 1.5$ m. Applying Eq. 14–34, we have

$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)} = 1 + \sqrt{1 + 2\left(\frac{1.5}{1.9516(10^{-6})}\right)} = 1240.83$$

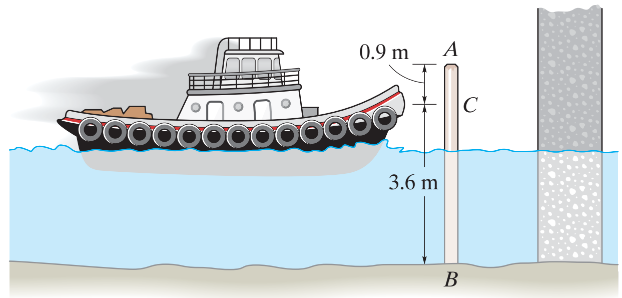
Thus,

$$\sigma_{max} = n\sigma_{st} = 1240.83(0.156131) = 193.7 \text{ MPa}$$

Since $\sigma_{max} < \sigma_Y = 924$ MPa, the weight can be released from rest at any position along the bar without causing permanent damage to the bar. **Ans.**

Ans:
Yes, from any position

14–58. The tugboat has a mass of 60 tonnes and is traveling forward at 0.6 m/s when it strikes the 300-mm-diameter fender post AB used to protect a bridge pier. If the post is made from treated white spruce and is assumed fixed at the river bed, determine the maximum horizontal distance the top of the post will move due to the impact. Assume the tugboat is rigid and neglect the effect of the water.



SOLUTION

From Appendix C:

$$P_{max} = \frac{3EI(\Delta_C)_{max}}{(L_{BC})^3}$$

Conservation of energy:

$$\frac{1}{2}mv^2 = \frac{1}{2}P_{max}(\Delta_C)_{max}$$

$$\frac{1}{2}mv^2 = \frac{1}{2}\left[\frac{3EI(\Delta_C)^2_{max}}{(L_{BC})^3}\right]$$

$$(\Delta_C)_{max} = \sqrt{\frac{mv^2L^3_{BC}}{3EI}}$$

$$(\Delta_C)_{max} = \sqrt{\frac{60(10^3)(0.6)^2(3.6)^3}{3(9.65)(10^9)(\frac{\pi}{64})(0.3)^4}} = 0.29589 \text{ m} = 295.89 \text{ mm.}$$

$$P_{max} = \left[\frac{3(9.65)(10^9)\left(\frac{\pi}{64}\right)(0.3)^4}{(3.6)^3}\right](0.29589) = 73.00(10^3) \text{ N}$$

$$\theta_C = \frac{P_{max}L^2_{BC}}{2EI} = \frac{73.00(10^3)(3.6)^2}{2(9.65)(10^9)\left(\frac{\pi}{64}\right)(0.3)^4} = 0.12329 \text{ πad}$$

$$(\Delta_A)_{max} = (\Delta_C)_{max} + \theta_C(L_{CA})$$

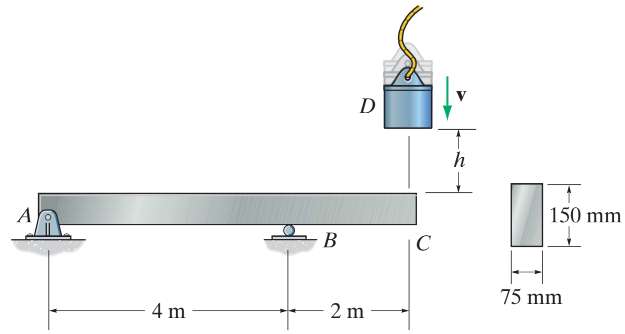
$$(\Delta_A)_{max} = 295.89 + 0.12329(0.9)(10^3) = 406.74 \text{ mm} = 407 \text{ mm}$$ **Ans.**

Ans:

$(\Delta_A)_{max} = 407$ mm

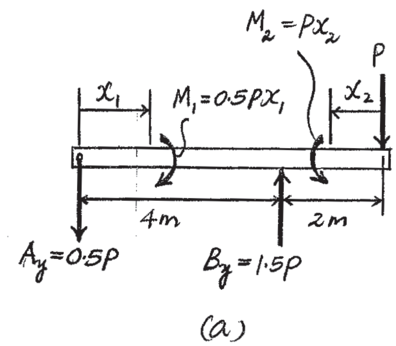
14–59.

The overhang beam is made of 2014-T6 aluminum. If the 75-kg block has a speed of $v = 3$ m/s at $h = 0.75$ m, determine the maximum bending stress in the beam.



SOLUTION

Equilibrium: The support reactions and the moment function for regions AB and BC of the beam under static conditions are indicated on the free-body diagram of the beam, Fig. a.



(a)

$$U_e = U_i$$

$$\frac{1}{2}P\Delta_{st} = \Sigma\int_0^L \frac{M^2dx}{2EI}$$

$$\frac{1}{2}P\Delta_{st} = \frac{1}{2EI}\left[\int_0^{4\text{ m}}(0.5Px_1)^2dx_1 + \int_0^{2\text{ m}}(Px_2)^2dx_2\right]$$

$$\frac{1}{2}P\Delta_{st} = \frac{1}{2EI}\left[\left(\frac{0.25}{3}P^2x_1^3\right)\Bigg|_0^{4\text{ m}} + \frac{P^2}{3}x_2^3\Bigg|_0^{2\text{ m}}\right]$$

$$\Delta_{st} = \frac{8P}{EI}$$

$I = \frac{1}{12}(0.075)(0.15^3) = 21.09375(10^{-6})\text{ m}^4$ and $E = E_{al} = 73.1$ GPa. Then, the equivalent spring constant can be determined from

$$P = k\Delta_{st}$$

$$P = k\left(\frac{8P}{EI}\right)$$

$$k = \frac{EI}{8} = \frac{73.1(10^9)[21.09375(10^{-6})]}{8}$$

$$= 192.74(10^3)\text{ N/m}$$

Conservation of Energy:

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + mg(h + \Delta_{max}) = \frac{1}{2}k\Delta_{max}{}^2$$

$$\frac{1}{2}(75)(3^2) + 75(9.81)(0.75 + \Delta_{max}) = \frac{1}{2}\left[192.74(10^3)\right]\Delta_{max}{}^2$$

$$96372.07\Delta_{max}{}^2 - 735.75\Delta_{max} - 889.3125 = 0$$

$$\Delta_{max} = 0.09996\text{ m}$$

Maximum Bending Stress: The maximum force on the beam is $P_{max} = k\Delta_{max} = 192.74(10^3)[0.09996] = 19.266(10^3)$ N. The maximum moment occurs at support B. Thus, $M_{max} = P_{max}(2) = 19.266(10^3)(2) = 38.531(10^3)\text{ N}\cdot\text{m}$.

Applying the flexure formula,

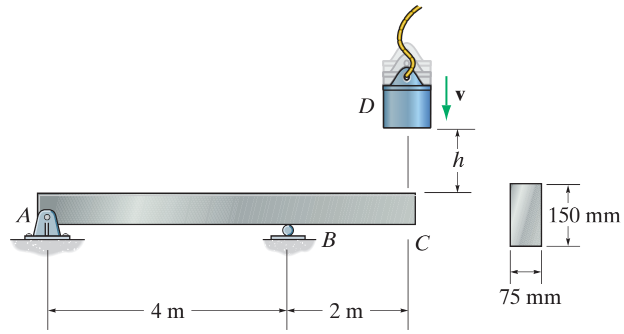
$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{38.531(10^3)(0.15/2)}{21.09375(10^{-6}))} = 137\text{ MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{max} < \sigma_Y = 414$ MPa, this result is valid.

Ans:
$\sigma_{max} = 137$ MPa

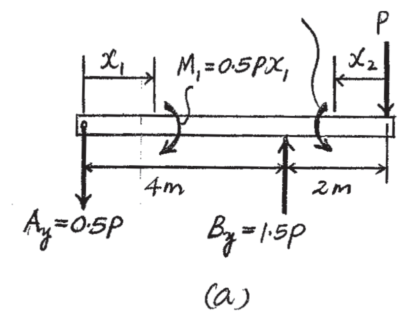
***14–60.**

The overhang beam is made of 2014-T6 aluminum. Determine the maximum height h from which the 100-kg block can be dropped from rest ($v = 0$), without causing the beam to yield.



SOLUTION

Equilibrium: The support reactions and the moment function for regions AB and BC of the beam under static conditions are indicated on the free-body diagram of the beam, Fig. a.



$$U_e = U_i$$

$$\frac{1}{2}P\Delta_{st} = \Sigma \int_0^L \frac{M^2 dx}{2EI}$$

$$\frac{1}{2}P\Delta_{st} = \frac{1}{2EI}\left[\int_0^{4\text{ m}} (0.5Px_1)^2 dx_1 + \int_0^{2\text{ m}} (Px_2)^2 dx_2\right]$$

$$\frac{1}{2}P\Delta_{st} = \frac{1}{2EI}\left[\left(\frac{0.25}{3}P^2x_1^3\right)\Big|_0^{4\text{ m}} + \frac{P^2}{3}x_2^3\Big|_0^{2\text{ m}}\right]$$

$$\Delta_{st} = \frac{8P}{EI}$$

$I = \frac{1}{12}(0.075)(0.15^3) = 21.09375(10^{-6})\text{ m}^4$, $P = 100(9.81) = 981\text{N}$, and $E = E_{al} = 73.1$ GPa. Then,

$$\Delta_{st} = \frac{8(981)}{73.1(10^9)[21.09375(10^{-6})]} = 5.0896(10^{-3})\text{ m}$$

Maximum Bending Stress: The maximum force on the beam is $P_{max} = nP = 981n$. The maximum moment occurs at support B. Thus, $M_{max} = P_{max}(2) = (981n)(2) = 1962n$. Applying the flexure formula,

$$\sigma_{max} = \frac{M_{max}c}{I}$$

$$414(10^6) = \frac{1962n(0.15/2)}{21.09375(10^{-6})}$$

$$n = 59.35$$

Impact Factor:

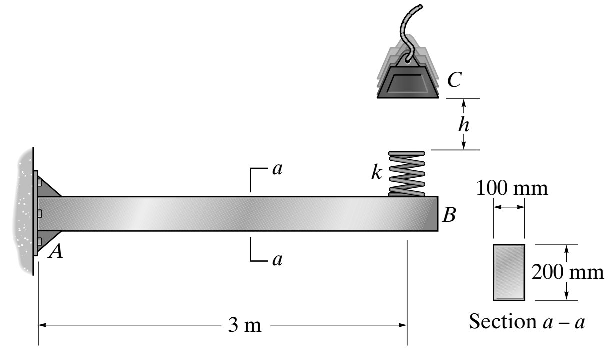
$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}$$

$$59.35 = 1 + \sqrt{1 + 2\left[\frac{h}{5.0896(10^{-3})}\right]}$$

$h = 8.66$ m **Ans.**

Ans:
$h = 8.66$ m

14–61. Block C of mass 50 kg is dropped from height $h = 0.9$ m onto the spring of stiffness $k = 150$ kN/m mounted on the end B of the 6061-T6 aluminum cantilever beam. Determine the maximum bending stress developed in the beam.



SOLUTION

Conservation of Energy. From the table listed in the appendix, the displacement of end B under static conditions is $\Delta_{st} = \dfrac{PL^3}{3EI}$. Thus, the equivalent spring constant for the beam is $k_b = \dfrac{3EI}{L^3}$, where $I = \dfrac{1}{12}(0.1)\left(0.2^3\right) = 66.6667\left(10^{-6}\right)\text{m}^4$, $L = 3$ m, and $E = E_{al} = 68.9$ GPa. Thus,

$$k_b = \frac{3EI}{L^3} = \frac{3\left[68.9\left(10^9\right)\right]\left[66.6667\left(10^{-6}\right)\right]}{3^3} = 510.37\left(10^3\right)\text{N/m}$$

Equilibrium requires,

$$F_{sp} = P$$

$$k_{sp}\Delta_{sp} = k_b\Delta_b$$

$$150\left(10^3\right)\Delta_{sp} = 510.37\left(10^3\right)\Delta_b$$

$$\Delta_{sp} = 3.4025\Delta_b \qquad \textbf{(1)}$$

We have,

$$U_e = U_i$$

$$mg\left(h + \Delta_{sp} + \Delta_b\right) = \frac{1}{2}k_b\Delta_b^{\,2} + \frac{1}{2}k_{sp}\Delta_{sp}^{\,2}$$

Substituting Eq. (1) into this equation,

$$50(9.81)(0.9 + 3.4025\Delta_b + \Delta_b) = \frac{1}{2}\left[510.37\left(10^3\right)\right]\Delta_b^{\,2} + \frac{1}{2}\left[150\left(10^3\right)\right](3.4025\Delta_b)^2$$

$$1123444.90\Delta_b^{\,2} - 2159.41\Delta_b - 441.45 = 0$$

Solving for the positive root,

$$\Delta_b = 0.020807 \text{ m}$$

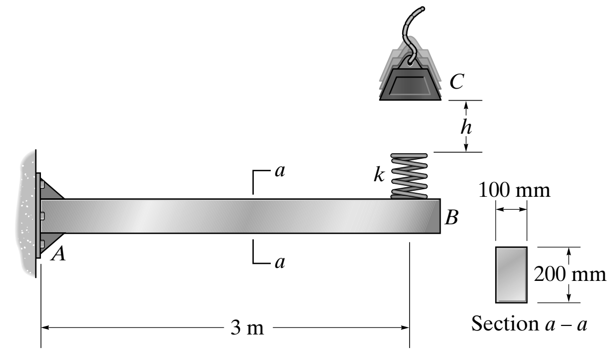
Maximum Stress. The maximum force on the beam is $P_{\max} = k_b\Delta_b = 510.37\left(10^3\right)(0.020807) = 10.619\left(10^3\right)$ N. The maximum moment occurs at fixed support A, where $M_{\max} = P_{\max}L = 10.619\left(10^3\right)(3) = 31.858\left(10^3\right)$ N·m. Applying the flexure formula with $c = \dfrac{0.2}{2} = 0.1$ m,

$$\sigma_{\max} = \frac{M_{\max}c}{I} = \frac{31.858\left(10^3\right)(0.1)}{66.6667\left(10^{-6}\right)} = 47.79 \text{ MPa} = 47.8 \text{ MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{\max} < \sigma_Y = 255$ MPa, this result is valid.

Ans:
$\sigma_{\max} = 47.8$ MPa

14–62. Determine the maximum height h from which 200-kg block C can be dropped without causing the 6061-T6 aluminum cantilever beam to yield. The spring mounted on the end B of the beam has a stiffness of $k = 150\ \text{kN/m}$.



SOLUTION

Maximum Stress. From the table listed in the appendix, the displacement of end B under static conditions is $\Delta_{st} = \dfrac{PL^3}{3EI}$. Thus, the equivalent spring constant for the beam is $k_b = \dfrac{3EI}{L^3}$, where $I = \dfrac{1}{12}(0.1)\left(0.2^3\right) = 66.6667\left(10^{-6}\right)\ \text{m}^4$, $L = 3\ \text{m}$, and $E = E_{al} = 68.9\ \text{GPa}$. Thus,

$$k_b = \frac{3EI}{L^3} = \frac{3\left[68.9\left(10^9\right)\right]\left[66.6667\left(10^{-6}\right)\right]}{3^3} = 510.37\left(10^3\right)\text{N/m}$$

The maximum force on the beam is $P_{\max} = k_b\Delta_b = 510.37\left(10^3\right)\Delta_b$. The maximum moment occurs at the fixed support A, where $M_{\max} = P_{\max}\,L = 510.37\left(10^3\right)\Delta_b(3) = 1.5311\left(10^6\right)\Delta_b$. Applying the flexure formula with $\sigma_{\max} = \sigma_Y = 255\ \text{MPa}$ and $c = \dfrac{0.2}{2} = 0.1\ \text{m}$,

$$\sigma_{\max} = \sigma_Y = \frac{M_{\max}\,c}{I}$$

$$255\left(10^6\right) = \frac{1.5311\left(10^6\right)\Delta_b(0.1)}{66.6667\left(10^{-6}\right)}$$

$$\Delta_b = 0.11103\ \text{m}$$

Equilibrium requires,

$$F_{sp} = P$$

$$k_{sp}\Delta_{sp} = k_b\Delta_b$$

$$150\left(10^3\right)\Delta_{sp} = 510.37\left(10^3\right)(0.11103)$$

$$\Delta_{sp} = 0.37778\ \text{m}$$

Conservation of Energy.

$$U_e = U_i$$

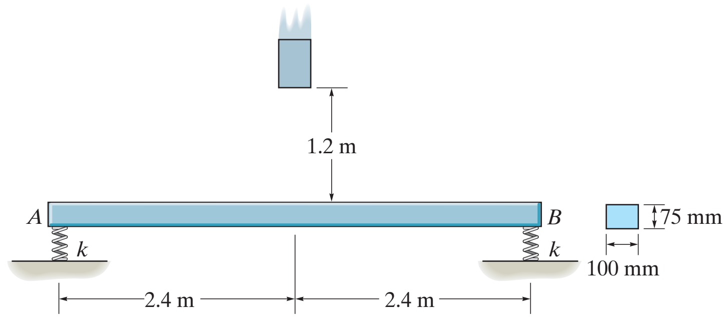
$$mg\left(h + \Delta_{sp} + \Delta_b\right) = \frac{1}{2}k_b\,\Delta_b{}^2 + \frac{1}{2}k_{sp}\,\Delta_{sp}{}^2$$

$$200(9.81)(h + 0.37778 + 0.11103) = \frac{1}{2}\left[510.37\left(10^3\right)\right](0.11103)^2 + \frac{1}{2}\left[150\left(10^3\right)\right](0.37778)^2$$

$h = 6.57\ \text{m}$ **Ans.**

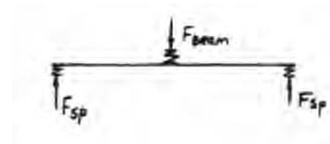
Ans:
$h = 6.57\ \text{m}$

14–63. The weight of 90 kg is dropped from a height of 1.2 m from the top of the A-36 steel beam. Determine the maximum deflection and maximum stress in the beam if the supporting springs at A and B each have a stiffness of $k = 100$ kN/m. The beam is 75 mm thick and 100 mm wide.



SOLUTION

From Appendix C:



$$\Delta_{\text{beam}} = \frac{PL^3}{48EI}$$

$$k_{\text{beam}} = \frac{48EI}{L^3} = \frac{48[200(10^9)]\left[\frac{1}{12}(0.1)(0.075^3)\right]}{4.8^3} = 305.18(10^3)\ \text{N}\cdot\text{m}$$

From equilibrium (equivalent system):

$$2F_{\text{sp}} = F_{\text{beam}}$$

$$2k_{\text{sp}}\Delta_{\text{sp}} = k_{\text{beam}}\Delta_{\text{beam}}$$

$$\Delta_{\text{sp}} = \frac{305.18(10^3)}{2[100(10^3)]}\Delta_{\text{beam}}$$

$$\Delta_{\text{sp}} = 1.5259\ \Delta_{\text{beam}} \qquad \textbf{(1)}$$

Conservation of energy:

$$U_e = U_i$$

$$W(h + \Delta_{\text{sp}} + \Delta_{\text{beam}}) = \frac{1}{2}k_{\text{beam}}\Delta_{\text{beam}}^2 + 2\left(\frac{1}{2}\right)k_{\text{sp}}\ \Delta_{\text{sp}}^2$$

From Eq. (1):

$$90(9.81)[1.2 + 1.5259\Delta_{\text{beam}} + \Delta_{\text{beam}}] = \frac{1}{2}[305.18(10^3)]\Delta_{\text{beam}}^2 + [100(10^3)](1.5259\Delta_{\text{beam}})^2$$

$$385.4185(10^3)\Delta_{\text{beam}}^2 - 2230.0985\Delta_{\text{beam}} - 1059.48 = 0$$

$$\Delta_{\text{beam}} = 0.05540\ \text{m} = 55.40\ \text{mm}$$

From Eq. (1):

$$\Delta_{\text{sp}} = 84.54\ \text{mm}$$

$$\Delta_{\max} = \Delta_{\text{sp}} + \Delta_{\text{beam}}$$

$$= 84.54 + 55.40 = 139.94\ \text{mm} = 140\ \text{mm} \qquad \textbf{Ans.}$$

$$F_{\text{beam}} = k_{\text{beam}}\Delta_{\text{beam}}$$

$$= [305.18(10^3)](0.05540) = 16.91(10^3)$$

$$M_{\max} = \frac{F_{\text{beam}}L}{4} = \frac{[16.91(10^3)](4.8)}{4} = 20.29(10^3)\ \text{N}\cdot\text{m}$$

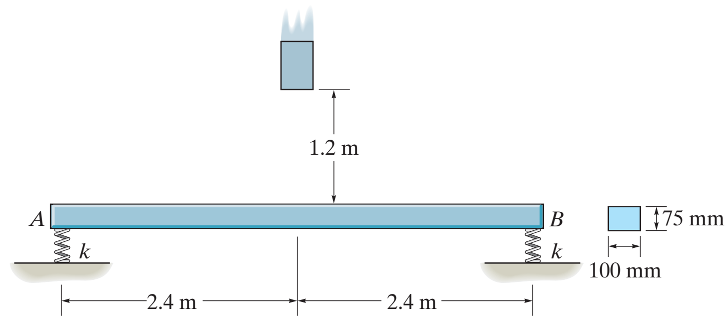
$$\sigma_{\max} = \frac{M_{\max}c}{I} = \frac{[20.29(10^3)](0.0375)}{\frac{1}{12}(0.1)(0.075^3)}$$

$$= 216.42(10^6)\ \text{N}\cdot\text{m}^2 = 216\ \text{MPa} < \sigma_Y \quad \text{O.K.} \qquad \textbf{Ans.}$$

Ans:

$\sigma_{\max} = 216$ MPa

***14–64.** The weight of 90 kg is dropped from a height of 1.2 m from the top of the A-36 steel beam. Determine the load factor n if the supporting springs at A and B each have a stiffness of $k = 60$ kN/m. The beam is 75 mm thick and 100 mm wide.



SOLUTION

From Appendix C:

$$\Delta_{\text{beam}} = \frac{PL^3}{48EI}$$

$$k_{\text{beam}} = \frac{48EI}{L^3} = \frac{48[200(10^9)]\left[\frac{1}{12}(0.1)(0.075^3)\right]}{4.8^3} = 305.18(10^3)\text{ N/m}$$

From equilibrium (equivalent system):

$$2F_{\text{sp}} = F_{\text{beam}}$$

$$2k_{\text{sp}}\,\Delta_{\text{sp}} = k_{\text{beam}}\,\Delta_{\text{beam}}$$

$$\Delta_{\text{sp}} = \frac{305.18(10^3)}{2[60(10^3)]}\Delta_{\text{beam}}$$

$$\Delta_{\text{sp}} = 2.5431\,\Delta_{\text{beam}} \qquad \textbf{(1)}$$

Conservation of energy:

$$U_e = U_i$$

$$W(h + \Delta_{\text{beam}} + \Delta_{\text{sp}}) = \frac{1}{2}k_{\text{beam}}\,\Delta_{\text{beam}}^2 + 2\left(\frac{1}{2}\right)k_{\text{sp}}\,\Delta_{\text{sp}}^2$$

From Eq. (1):

$$90(9.81)[1.2 + \Delta_{\text{beam}} + 2.5431\Delta_{\text{beam}}] = \frac{1}{2}[305.18(10^3)]\Delta_{\text{beam}}^2 + [60(10^3)](2.5431\Delta_{\text{beam}})^2$$

$$540.6359(10^3)\Delta_{\text{beam}}^2 - 3128.23\Delta_{\text{beam}} - 1059.48 = 0$$

$$\Delta_{\text{beam}} = 0.047256\text{ m}$$

$$F_{\text{beam}} = k_{\text{beam}}\Delta_{\text{beam}}$$

$$= [305.18(10^3)][0.047256] = 14.42(10^3)\text{ N}$$

$$n = \frac{14.42(10^3)}{90(9.81)} = 16.33 = 16.3 \qquad \textbf{Ans.}$$

$$\sigma_{\max} = n(\sigma_{\text{st}})_{\max} = n\left(\frac{Mc}{I}\right)$$

$$M = \frac{90(9.81)(4.8)}{4} = 1059.48\text{ N}\cdot\text{m}$$

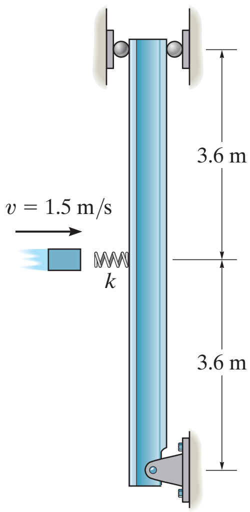
$$\sigma_{\max} = 16.33\left[\frac{1059.48(0.0375)}{\frac{1}{12}(0.1)(0.075^3)}\right]$$

$$= 184.59(10^6)\text{ N/m}^2 = 184.59\text{ MPa} < \sigma_Y \quad \text{O.K.}$$

Ans:

$\sigma_{\max} = 184.59$ MPa

14–65. The simply supported W250 × 22 structural A-36 steel beam lies in the horizontal plane and acts as a shock absorber for the 250-kg block which is traveling toward it at 1.5 m/s. Determine the maximum deflection of the beam and the maximum stress in the beam during the impact. The spring has a stiffness of $k = 200$ kN/m.



For W250 × 22: $I = 28.8(10^6)\ \text{mm}^4 = 28.8(10^{-6})\ \text{m}^4 \qquad d = 254\ \text{mm} = 0.254\ \text{m}$

From Appendix C:

$$\Delta_{\text{beam}} = \frac{PL^3}{48EI}$$

$$k_{\text{beam}} = \frac{48EI}{L^3} = \frac{48[200(10^9)][28.8(10^{-6})]}{7.2^3} = 740.74(10^3)\ \text{N/m}$$

Equilibrium (equivalent system):

$$F_{\text{sp}} = F_{\text{beam}}$$

$$k_{\text{sp}}\Delta_{\text{sp}} = k_{\text{beam}}\Delta_{\text{beam}}$$

$$\Delta_{\text{sp}} = \frac{740.74(10^3)}{200(10^3)}\Delta_{\text{beam}}$$

$$\Delta_{\text{sp}} = 3.7037\,\Delta_{\text{beam}} \qquad \textbf{(1)}$$

Conservation of energy:

$$U_e = U_i$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k_{\text{beam}}\,\Delta_{\text{beam}}^2 + \frac{1}{2}k_{\text{sp}}\,\Delta_{\text{sp}}^2$$

From Eq. (1):

$$\frac{1}{2}(250)(1.5^2) = \frac{1}{2}[740.74(10^3)]\Delta_{\text{beam}}^2 + \frac{1}{2}[200(10^3)](3.7037\Delta_{\text{beam}})^2$$

$$1.74211(10^6)\Delta_{\text{beam}}^2 = 281.25$$

$$\Delta_{\text{beam}} = 0.01271\ \text{m} = 12.7\ \text{mm} \qquad \textbf{Ans.}$$

$$F_{\text{beam}} = k_{\text{beam}}\,\Delta_{\text{beam}}$$

$$= [740.74(10^3)][0.01271] = 9.4118(10^3)\ \text{N} = 9.41\ \text{kN}$$

$$M_{\max} = \frac{[9.4118(10^3)](7.2)}{4} = 16.941(10^3)\ \text{N}\cdot\text{m}$$

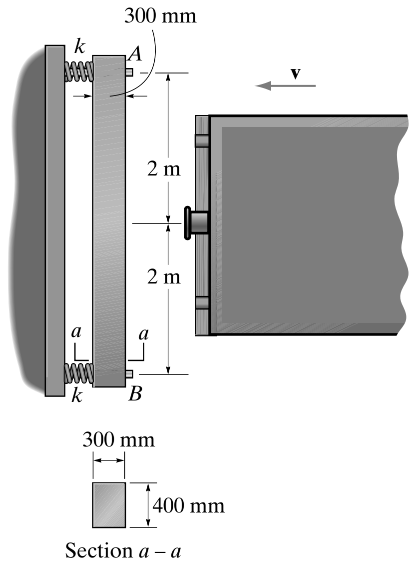
$$\sigma_{\max} = \frac{M_{\max}c}{I} = \frac{[16.941(10^3)](0.25412)}{28.8(10^{-6})}$$

$$= 74.71(10^6)\ \text{N/m}^2 = 74.7\ \text{MPa} < \sigma_Y \quad \text{O.K.} \qquad \textbf{Ans.}$$

Ans:

$\Delta_{\text{beam}} = 12.7$ mm, $\sigma_{\max} = 74.7$ MPa

14–66. The 2014-T6 aluminum bar AB can slide freely along the guides mounted on the rigid crash barrier. If the railcar of mass 10 Mg is traveling with a speed of $v = 1.5$ m/s, determine the maximum bending stress developed in the bar. The springs at A and B have a stiffness of $k = 15$ MN/m.



SOLUTION

Equilibrium. Referring to the free-body diagram of the bar for static conditions, Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 2F_{sp} - P = 0 \qquad F_{sp} = \frac{P}{2} \qquad \textbf{(1)}$$

Referring to the table listed in the appendix, the displacement of the bar at the position where $\mathbf{P}$ is applied under static conditions is $\Delta_{st} = \dfrac{PL^3}{48EI}$. Thus, the equivalent spring constant for the bar is $k_b = \dfrac{48EI}{L^3}$, where $I = \dfrac{1}{12}(0.4)\left(0.3^3\right) = 0.9\left(10^{-3}\right)\text{ m}^4$, $L = 4$ m, and $E = E_{al} = 73.1$ GPa. Thus,

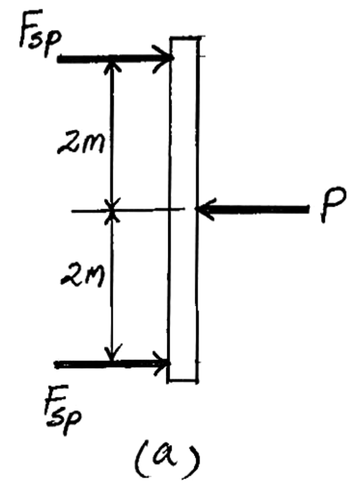
$$k_b = \frac{48\left[73.1\left(10^9\right)\right]\left[0.9\left(10^{-3}\right)\right]}{4^3} = 49.3425\left(10^6\right)\text{ N/m}$$

Using Eq. (1)

$$F_{sp} = \frac{P}{2}$$

$$k_{sp}\Delta_{sp} = \frac{1}{2}k_b\Delta_b$$

$$\Delta_{sp} = \frac{1}{2}\left(\frac{k_b}{k_{sp}}\right)\Delta_b = \frac{1}{2}\left[\frac{49.3425\left(10^6\right)}{15\left(10^6\right)}\right]\Delta_b = 1.64475\Delta_b \qquad (2)$$



Conservation of Energy.

$$\frac{1}{2}mv^2 = \frac{1}{2}k_b{\Delta_b}^2 + 2\left[\frac{1}{2}k_{sp}{\Delta_{sp}}^2\right]$$

Substituting Eq. (2) into this equation,

$$\frac{1}{2}mv^2 = \frac{1}{2}k_b{\Delta_b}^2 + k_{sp}(1.64475\Delta_b)^2$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k_b{\Delta_b}^2 + 2.7052k_{sp}{\Delta_b}^2$$

$$\frac{1}{2}\left[10\left(10^3\right)\right]\left(1.5^2\right) = \frac{1}{2}\left[49.3425\left(10^6\right)\right]{\Delta_b}^2 + 2.7052\left[15\left(10^6\right)\right]{\Delta_b}^2$$

$$\Delta_b = 0.01313\text{ m}$$

14–66. Continued

SOLUTION

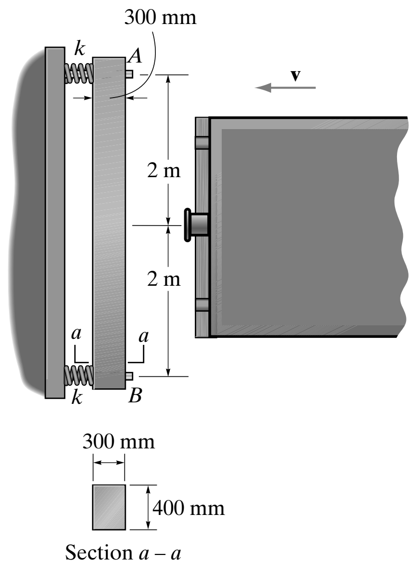
Maximum Stress. The maximum force on the bar is $(P_b)_{\max} = k_b\,\Delta_b = 49.3425\left(10^6\right)(0.01313) = 647.90\left(10^3\right)$ N. The maximum moment occurs at the midspan of the bar, where $M_{\max} = \dfrac{(P_b)_{\max}\,L}{4} = \dfrac{647.90\left(10^3\right)(4)}{4}$ $= 647.90\left(10^3\right)$ N · m. Applying the flexure formula,

$$\sigma_{\max} = \frac{M_{\max}\,c}{I} = \frac{647.90\left(10^3\right)(0.15)}{0.9\left(10^{-3}\right)} = 107.98 \text{ MPa} = 108 \text{ MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{\max} < \sigma_Y = 414$ MPa, this result is valid.

Ans:
$\sigma_{\max} = 108$ MPa

14–67. The 2014-T6 aluminum bar AB can slide freely along the guides mounted on the rigid crash barrier. Determine the maximum speed v the 10-Mg railcar without causing the bar to yield when it is struck by the railcar. The springs at A and B have a stiffness of $k = 15$ MN/m.



SOLUTION

Equilibrium. Referring to the free-body diagram of the bar for static conditions, Fig. a,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 2F_{sp} - P = 0 \qquad F_{sp} = \frac{P}{2} \qquad \textbf{(1)}$$

Referring to the table listed in the appendix, the displacement of the bar at the position where $\mathbf{P}$ is applied under static conditions is $\Delta_{st} = \dfrac{PL^3}{48EI}$. Thus, the equivalent spring constant for the bar is $k_b = \dfrac{48EI}{L^3}$, where $I = \dfrac{1}{12}(0.4)\left(0.3^3\right) = 0.9\left(10^{-3}\right)$ m^4, $L = 4$ m, and $E = E_{al} = 73.1$ GPa. Thus,

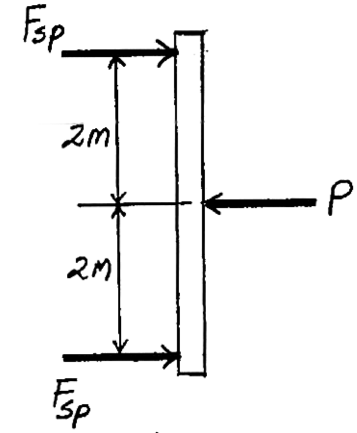
$$k_b = \frac{48\left[73.1\left(10^9\right)\right]\left[0.9\left(10^{-3}\right)\right]}{4^3} = 49.3425\left(10^6\right)\text{ N/m}$$

Using Eq. (1)

$$F_{sp} = \frac{P}{2}$$

$$k_{sp}\Delta_{sp} = \frac{1}{2}k_b\Delta_b$$

$$\Delta_{sp} = \frac{1}{2}\left(\frac{k_b}{k_{sp}}\right)\Delta_b = \frac{1}{2}\left[\frac{49.3425\left(10^6\right)}{15\left(10^6\right)}\right]\Delta_b = 1.64475\Delta_b \qquad \textbf{(2)}$$



(a)

Maximum Stress. The maximum force on the bar is $(P_b)_{\max} = k_b\Delta_b = 49.3425\left(10^6\right)\Delta_b$. The maximum moment occurs at the midspan of the bar, where $M_{\max} = \dfrac{(P_b)_{\max}L}{4} = \dfrac{49.3425\left(10^6\right)\Delta_b(4)}{4} = 49.3425\left(10^6\right)\Delta_b$. Applying the flexure formula with $\sigma_{\max} = \sigma_Y = 414$ MPa,

$$\sigma_{\max} = \frac{M_{\max}c}{I}$$

$$414\left(10^6\right) = \frac{49.3425\left(10^6\right)\Delta_b(0.15)}{0.9\left(10^{-3}\right)}$$

$$\Delta_b = 0.050342\text{ m}$$

14–67. Continued

SOLUTION

Substituting this result into Eq. (2),

$$\Delta_{sp} = 0.0828 \text{ m}$$

Conservation of Energy.

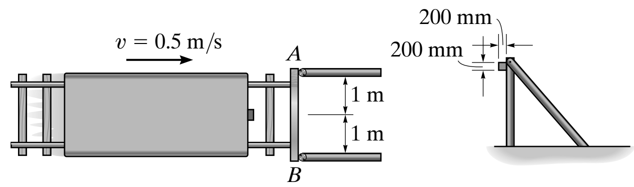
$$\frac{1}{2}mv^2 = \frac{1}{2}k_b\Delta_b{}^2 + 2\left[\frac{1}{2}k_{sp}\Delta_{sp}{}^2\right]$$

$$\frac{1}{2}\left[10\left(10^3\right)\right]v^2 = \frac{1}{2}\left[49.3425\left(10^6\right)\right]\left(0.050342^2\right) + 2\left[\frac{1}{2}\left[15\left(10^6\right)\right]\left(0.0828^2\right)\right]$$

$v = 5.75$ m/s **Ans.**

Ans:
$k_b = 49.3425\ (10^6)$ N/m, $\Delta_b = 0.050342$ m,
$v = 5.75$ m/s

*14–68. The steel beam AB acts to stop the oncoming railroad car, which has a mass of 10 Mg and is coasting towards it at $v = 0.5$ m/s. Determine the maximum stress developed in the beam if it is struck at its center by the car. The beam is simply supported and only horizontal forces occur at A and B. Assume that the railroad car and the supporting framework for the beam remains rigid. Also, compute the maximum deflection of the beam. $E_{st} = 200$ GPa, $\sigma_Y = 250$ MPa.



SOLUTION

From Appendix C:

$$\Delta_{st} = \frac{PL^3}{48EI} = \frac{10(10^3)(9.81)(2^3)}{48(200)(10^4)(\frac{1}{12})(0.2)(0.2^3)} = 0.613125(10^{-3})\text{ m}$$

$$k = \frac{W}{\Delta_{st}} = \frac{10(10^3)(9.81)}{0.613125(10^{-3})} = 160(10^6)\text{ N/m}$$

$$\Delta_{max} = \sqrt{\frac{\Delta_{st} v^2}{g}} = \sqrt{\frac{0.613125(10^{-3})(0.5^2)}{9.81}} = 3.953(10^{-3})\text{ m} = 3.95\text{ mm} \qquad \textbf{Ans.}$$

$$W' = k\Delta_{max} = 160(10^6)(3.953)(10^{-3}) = 632455.53\text{ N}$$

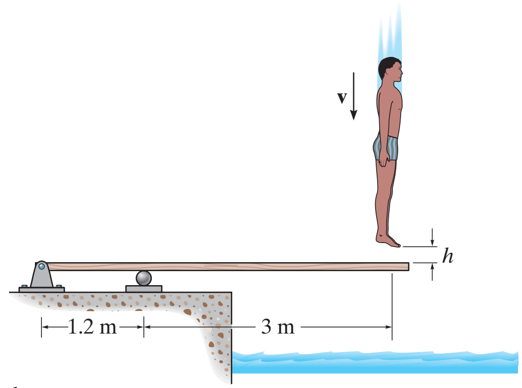
$$M' = \frac{w'L}{4} = \frac{632455.53(2)}{4} = 316228\text{ N}\cdot\text{m}$$

$$\sigma_{max} = \frac{M'c}{I} = \frac{316228(0.1)}{\frac{1}{12}(0.2)(0.2^3)} = 237\text{ MPa} < \sigma_\gamma \qquad \text{O.K.} \qquad \textbf{Ans.}$$

Ans:

$\Delta_{max} = 3.95$ mm, $\sigma_{max} = 237$ MPa

14–69. The diver weighs 750 N and, while holding himself rigid, strikes the end of a wooden diving board ($h = 0$) with a downward velocity of 1.2 m/s. Determine the maximum bending stress developed in the board. The board has a thickness of 40 mm and width of 450 mm. $E_w = 12.6$ GPa, $\sigma_Y = 56$ MPa.



SOLUTION

Static Displacement: The static displacement at the end of the diving board can be determined using the conservation of energy.

$$\frac{1}{2}P\Delta = \int_0^L \frac{M^2\,dx}{2EI}$$

$$\frac{1}{2}(750)\Delta_{st} = \frac{1}{2EI}\left[\int_0^{1.2\text{ m}} (-1875x_1)^2\,dx_1 + \int_0^{3\text{ m}} (-750x_2)^2\,dx_2\right]$$

$$\Delta_{st} = \frac{9450\text{ N}\cdot\text{m}^3}{EI}$$

$$= \frac{9450}{[12.6(10^9)]\left[\frac{1}{12}(0.45)(0.04^3)\right]}$$

$$= 0.3125\text{ m}$$

Conservation of Energy: The equivalent spring constant for the board is $k = \frac{W}{\Delta_{st}} = \frac{750}{0.3125} = 2.40(10^3)$ N/m,

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + W\Delta_{max} = \frac{1}{2}k\Delta_{max}^2$$

$$\frac{1}{2}\left(\frac{750}{9.81}\right)(1.2^2) + 750\Delta_{max} = \frac{1}{2}[2.40(10^3)]\Delta_{max}^2$$

$$1.20(10^3)\Delta_{max}^2 - 750\Delta_{max} - 55.05 = 0$$

Solving for the positive root, we have

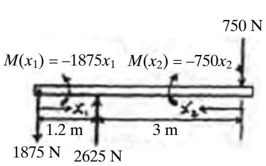
$$\Delta_{max} = 0.69135\text{ m}$$

Maximum Stress: The maximum force on to the beam is $P_{max} = k\Delta_{max}$ $[2.40(10^3)](0.69135) = 1.6592(10^3)$ N. The maximum moment occurs at the middle support support $M_{max} = [1.6592(10^3)](3) = 4.9777(10^3)$ N·m

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{4.9777(10^3)(0.02)}{\frac{1}{12}(0.45)(0.04^3)}$$

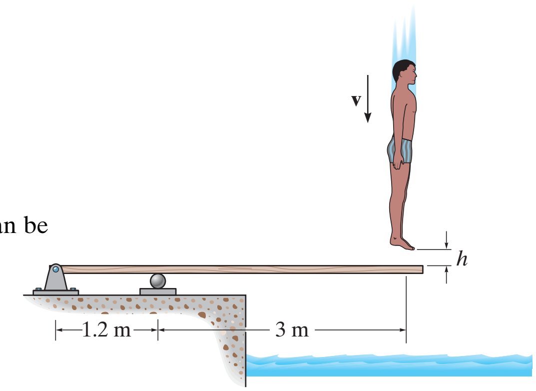
$$= 41.48(10^6)\text{ N/m}^2 = 41.5\text{ MPa} < \sigma_Y \quad \text{O.K.} \qquad \textbf{Ans.}$$

Note: The result will be somewhat inaccurate since the static displacement is so large.



Ans:
$\sigma_{max} = 41.5$ MPa

14–70. The diver weighs 750 N and, while holding himself rigid, strikes the end of the wooden diving board. Determine the maximum height h from which he can jump onto the board so that the maximum bending stress in the wood does not exceed 42 MPa. The board has a thickness of 40 mm and width of 450 mm. $E_w = 12.6$ GPa.



Static Displacement: The static displacement at the end of the diving board can be determined using the conservation of energy.

$$\frac{1}{2}P\Delta = \int_0^L \frac{M^2\,dx}{2EI}$$

$$\frac{1}{2}(750)\Delta_{st} = \frac{1}{2EI}\left[\int_0^{1.2\text{ m}}(-1875x_1)^2\,dx_1 + \int_0^{3\text{ m}}(-750x_2)^2\,dx_2\right]$$

$$\Delta_{st} = \frac{9450\text{ N}\cdot\text{m}^3}{EI}$$

$$= \frac{9450}{[12.6(10^9)]\left[\frac{1}{12}(0.45)(0.04^3)\right]}$$

$$= 0.3125\text{ m}$$

Maximum Stress: The maximum force on the beam is P_{max}. The maximum moment occurs at the middle support $M_{max} = P_{max}(3) = 3P_{max}$.

$$\sigma_{max} = \frac{M_{max}\,c}{I}$$

$$12.6(10^9) = \frac{(3P_{max})(0.02)}{\frac{1}{12}(0.45)(0.04^3)}$$

$$P_{max} = 1680\text{ N}$$

Conservation of Energy: The equivalent spring constant for the board is $k = \frac{W}{\Delta_{st}} = \frac{750}{0.3125} = 2.40(10^3)$ N/m. The maximum displacement at the end of the board is $\Delta_{max} = \frac{P_{max}}{k} = \frac{1680}{2.40(10^3)} = 0.700$ m.

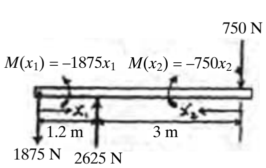
$$U_e = U_i$$

$$W(h + \Delta_{max}) = \frac{1}{2}k\Delta_{max}^2$$

$$750(h + 0.700) = \frac{1}{2}[2.40(10^3)](0.700^2)$$

$$h = 0.0840\text{ m} = 84.0\text{ mm.}$$ **Ans.**

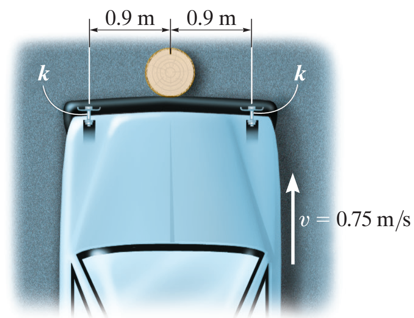
Note: The result will be somewhat at inaccurate since the static displacement is so large.



Ans:
$h = 84.0$ mm

14–71.

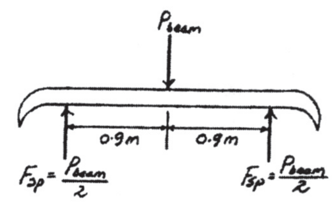
The car bumper is made of polycarbonate-polybutylene terephthalate. If $E = 2.0$ GPa, determine the maximum deflection and maximum stress in the bumper if it strikes the rigid post when the car is coasting at $v = 0.75$ m/s. The car has a mass of 1.80 Mg, and the bumper can be considered simply supported on two spring supports connected to the rigid frame of the car. For the bumper take $I = 300(10^6)$ mm^4, $c = 75$ mm, $\sigma_Y = 30$ MPa and $k = 1.5$ MN/m.



SOLUTION

Equilibrium: This requires $F_{sp} = \dfrac{P_{beam}}{2}$. Then

$$k_{sp}\,\Delta_{sp} = \frac{k\Delta_{beam}}{2} \qquad \text{or} \qquad \Delta_{sp} = \frac{k}{2k_{sp}}\,\Delta_{beam} \qquad \textbf{(1)}$$



Conservation of Energy: The equivalent spring constant for the beam can be determined using the deflection table listed in the Appendix C.

$$k = \frac{48EI}{L^3} = \frac{48\left[2(10^9)\right]\left[300(10^{-6})\right]}{1.8^3} = 4\,938\,271.6 \text{ N/m}$$

Thus,

$$U_e = U_i$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k\Delta_{beam}^2 + 2\left(\frac{1}{2}k_{sp}\,\Delta_{sp}^2\right) \qquad \textbf{(2)}$$

Substitute Eq. (1) into (2) yields

$$\frac{1}{2}mv^2 = \frac{1}{2}k\Delta_{beam}^2 + \frac{k^2}{4k_{sp}}\,\Delta_{beam}^2$$

$$\frac{1}{2}(1800)\left(0.75^2\right) = \frac{1}{2}(493\,8271.6)\,\Delta_{beam}^2 + \frac{(4\,93\,8271.6)^2}{4[1.5(10^6)]}\,\Delta_{beam}^2$$

$$\Delta_{beam} = 8.8025\left(10^{-3}\right) \text{ m}$$

Maximum Displacement: From Eq. (1) $\Delta_{sp} = \dfrac{4\,938\,271.6}{2[1.5(10^6)]}\left[8.8025\left(10^{-3}\right)\right] = 0.014490$ m.

$$\Delta_{max} = \Delta_{sp} + \Delta_{beam}$$
$$= 0.014490 + 8.8025\left(10^{-3}\right)$$
$$= 0.02329 \text{ m} = 23.3 \text{ mm} \qquad \textbf{Ans.}$$

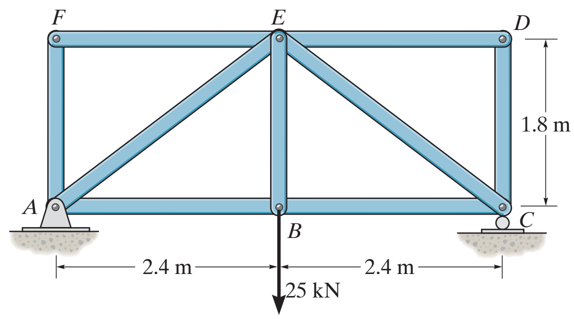
Maximum Stress: The maximum force on the beam is $P_{beam} = k\Delta_{beam} = 4\,938\,271.6\left[8.8025\left(10^{-3}\right)\right] = 43\,469.3$ N. The maximum moment occurs at mid-span. $M_{max} = \dfrac{P_{beam}\,L}{4} = \dfrac{43\,469.3(1.8)}{4} = 19\,561.2 \text{ N}\cdot\text{m}$.

$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{19\,561.2(0.075)}{300(10^{-6})} = 4.89 \text{ MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{max} < \sigma_Y = 30$ MPa, the above analysis is valid.

Ans:
$\Delta_{max} = 23.3$ mm, $\sigma_{max} = 4.89$ MPa

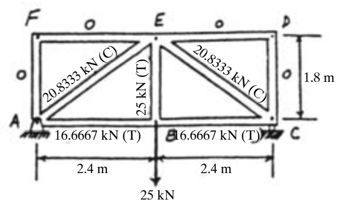
***14-72.** Determine thevertical displacement of point E. Each A-36 steel member has a cross-sectional area of 2800 mm^2.



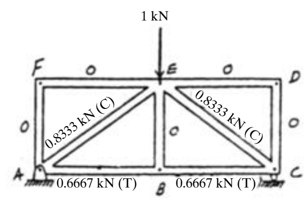
SOLUTION

Virtual-Work Equation: Applying Eq. 14–39, we have

Member	n (kN)	N (kN)	L (m)	nNL
AB	0.6667	16.6667	2.4	26.667
BC	0.6667	16.6667	2.4	26.667
CD	0	0	1.8	0
DE	0	0	2.4	0
EF	0	0	2.4	0
AF	0	0	1.8	0
AE	–0.8333	–20.8333	3.0	52.083
CE	–0.8333	–20.8333	3.0	52.083
BE	0	25.00	1.8	0
				$\Sigma 157.5\,\text{kN}^2\cdot\text{m}$



(a)



(c)

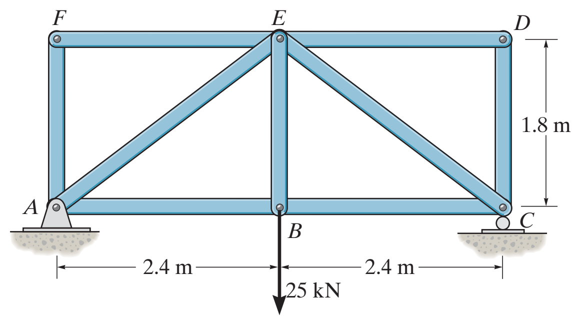
$$1 \cdot \Delta = \sum \frac{nNL}{AE}$$

$$1\text{ kN}\cdot(\Delta_E)_v = \frac{157.5\text{ kN}\cdot\text{m}^2}{AE}$$

$$(\Delta_E)_v = \frac{157.5(10^3)}{[2.80(10^{-3})][200(10^9)]} = 0.28125(10^{-3})\text{ m} = 0.281\text{ mm}\downarrow \qquad \textbf{Ans.}$$

Ans:
$(\Delta_E)_v = 0.281$ mm $\downarrow$

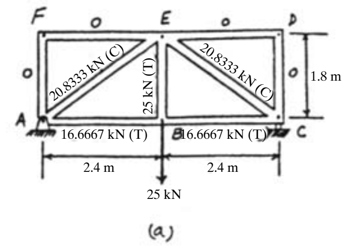
14–73. Determine the vertical displacement of point B. Each A-36 steel member has a cross-sectional area of 2800 mm^2.

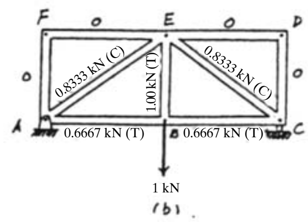


SOLUTION

Virtual-Work Equation: Applying Eq. 14–39, we have

Member	n (kN)	N (kN)	L (m)	nNL
AB	0.6667	16.6667	2.4	26.667
BC	0.6667	16.6667	2.4	26.667
CD	0	0	1.8	0
DE	0	0	2.4	0
EF	0	0	2.4	0
AF	0	0	1.8	0
AE	–0.8333	–20.8333	3.0	52.083
CE	–0.8333	–20.8333	3.0	52.083
BE	1.00	25.00	1.8	45.00
				$\Sigma 202.5\,\text{kN}^2\cdot\text{m}$





$$1 \cdot \Delta = \sum \frac{nNL}{AE}$$

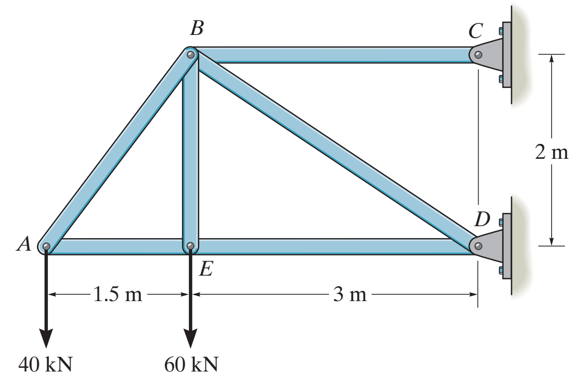
$$1\ \text{kN} \cdot (\Delta_B)_v = \frac{202.5\ \text{kN}\cdot\text{m}^2}{AE}$$

$$(\Delta_B)_v = \frac{202.5(10^3)}{[2.80(10^{-3})][200(10^9)]} = 0.3616(10^{-3})\ \text{m} = 0.362\ \text{mm} \downarrow \qquad \textbf{Ans.}$$

Ans:
$(\Delta_B)_v = 0.362$ mm

14–74.

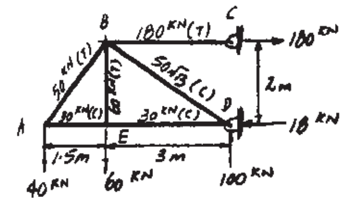
Determine the vertical displacement of joint A. Each A992 steel member has a cross-sectional area of 400 mm^2.



SOLUTION

Member	n	N	L	nNL
AB	1.25	50	2.5	156.25
AE	−0.75	−30	1.5	33.75
BC	2.25	180	3.0	1215.00
BD	$-0.5\sqrt{13}$	$-50\sqrt{13}$	$\sqrt{13}$	1171.80
BE	0	60	2.0	0
DE	−0.75	−30	3.0	67.5

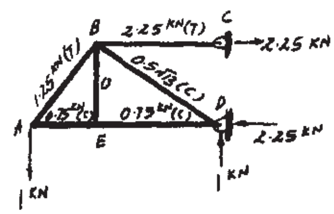
$\Sigma = 2644.30$



$$1 \cdot \Delta_{A_v} = \Sigma \frac{nNL}{AE}$$

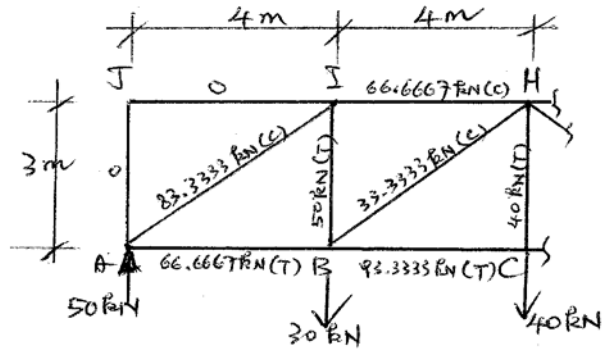
$$(\Delta_A)_v = \frac{2644.30(10^3)}{400(10^{-6})(200)(10^9)} = 0.0331 \text{ m} = 33.1 \text{ mm}\downarrow$$

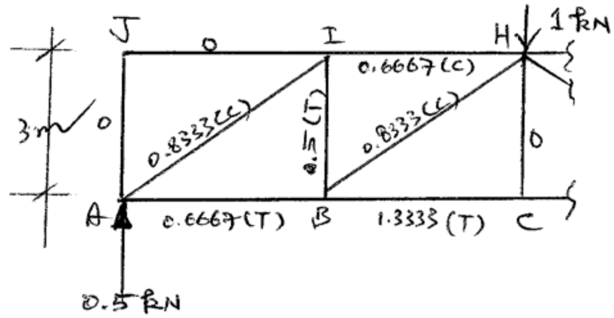
Ans.



Ans:
$(\Delta_A)_v = 33.1 \text{ mm}\downarrow$

***14–75.** Determine the vertical displacement of joint H. Each A-36 steel member has a cross-sectional area of $2800\ \text{mm}^2$.

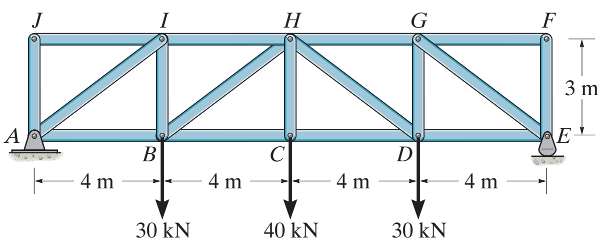




SOLUTION

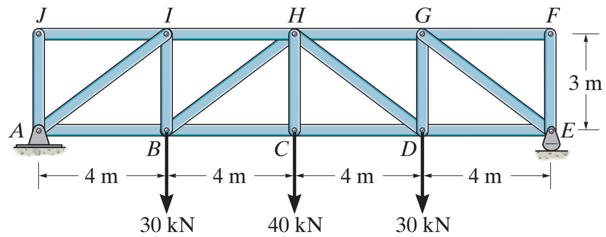
$$1 \cdot \Delta_{N_v} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{H_v} = \frac{2828.89(10^3)}{[2.80(10^{-3})][200(10^9)]} = 5.052(10^{-3})\ \text{m} = 5.05\ \text{mm} \downarrow \qquad \textbf{Ans.}$$



Ans:
$\Delta_{H_v} = 5.05$ mm

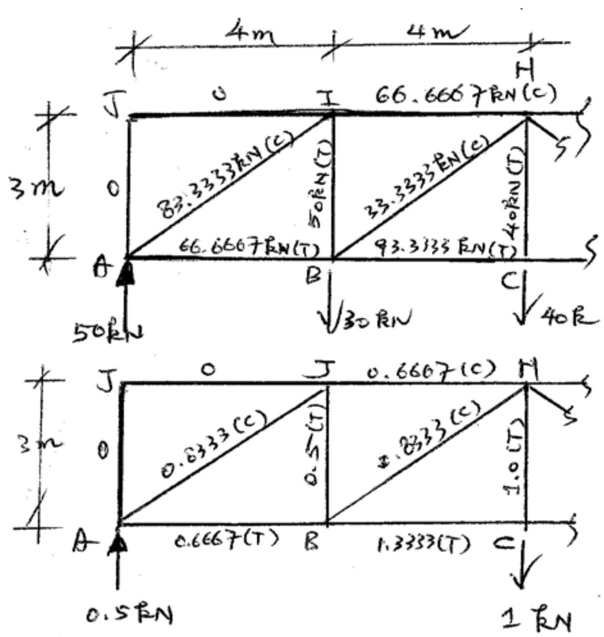
***14–76.** Determine the vertical displacement of joint *C*. Each A-36 steel member has a cross-sectional area of 2800 mm².



SOLUTION

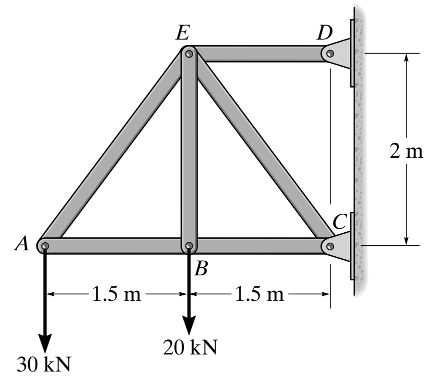
$$1 \cdot \Delta_{C_v} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{C_v} = \frac{2948.89(10^3)}{[2.80(10^{-3})][200(10^9)]} = 5.266(10^{-3})\text{ m} = 5.27\text{ mm}\downarrow \qquad \textbf{Ans.}$$



Ans:
$\Delta_{C_v} = 5.27$ mm

14–77. Determine the vertical displacement of point B. Each A-36 steel member has a cross-sectional area of 400 mm^2.



SOLUTION

Virtual-Work Equation:

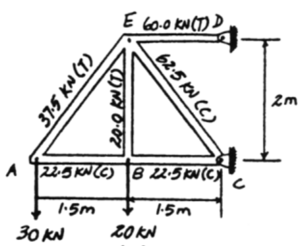
Member	n	N	L	nNL
AB	0	$-22.5(10^3)$	1.5	0
BC	0	$-22.5(10^3)$	1.5	0
AE	0	$37.5(10^3)$	2.5	0
CE	-1.25	$-62.5(10^3)$	2.5	$195.3125(10^3)$
BE	1.00	$22.0(10^3)$	2	$40.0(10^3)$
DE	0.750	$60.0(10^3)$	1.5	$67.5(10^3)$
				$\Sigma\ 302.8125(10^3)\ \text{N}^2\cdot\text{m}$

$$1 \cdot \Delta = \sum \frac{nNL}{AE}$$

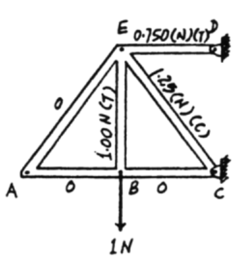
$$1\ \text{N} \cdot (\Delta_B)_v = \frac{302.8125(10^3)\ \text{N}^2\cdot\text{m}}{AE}$$

$$(\Delta_B)_v = \frac{302.8125(10^3)}{0.400(10^{-3})[200(10^9)]}$$

$$= 3.785(10^{-3})\ \text{m} = 3.79\ \text{mm} \downarrow$$ **Ans.**



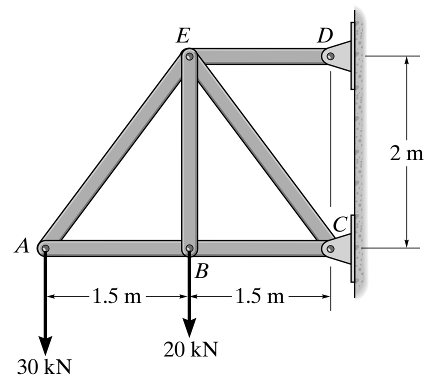
(a)



(b)

Ans:
$(\Delta_B)_v = 3.79\ \text{mm}\downarrow$

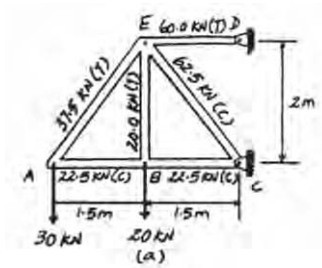
14–78. Determine the vertical displacement of point A. Each A-36 steel member has a cross-sectional area of 400 mm^2.



SOLUTION

Virtual-Work Equation:

Member	n	N	L	nNL
AB	–0.750	$-22.5(10^3)$	1.5	$25.3125(10^3)$
BC	–0.750	$-22.5(10^3)$	1.5	$25.3125(10^3)$
AE	1.25	$37.5(10^3)$	2.5	$117.1875(10^3)$
CE	–1.25	$-62.5(10^3)$	2.5	$195.3125(10^3)$
BE	0	$22.0(10^3)$	2	0
DE	1.50	$60.0(10^3)$	1.5	$135.00(10^3)$
				$\Sigma\ 498.125(10^3)\ \text{N}^2\cdot\text{m}$



$$1\cdot\Delta = \sum\frac{nNL}{AE}$$

$$1\ \text{N}\cdot(\Delta_A)_v = \frac{498.125(10^3)\ \text{N}^2\cdot\text{m}}{AE}$$

$$(\Delta_A)_v = \frac{498.125(10^3)}{0.400(10^{-3})[200(10^9)]}$$

$$= 6.227(10^{-3})\ \text{m} = 6.23\ \text{mm}\downarrow \qquad \textbf{Ans.}$$

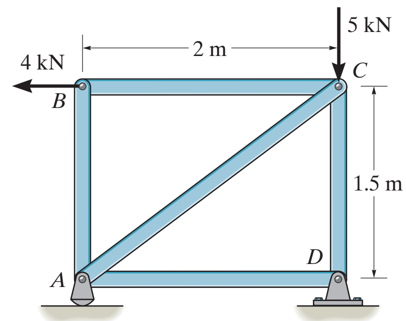
Ans:

$$1\ \text{N}\cdot(\Delta_A)_v = \frac{498.125(10^3)\ \text{N}^2\cdot\text{m}}{AE},$$

$$(\Delta_A)_v = 6.23\ \text{mm}\downarrow$$

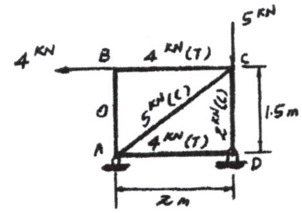
14–79.

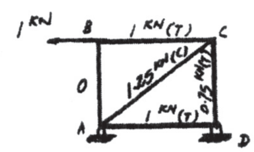
Determine the horizontal displacement of joint B of the truss. Each A992 steel member has a cross-sectional area of 400 mm^2.



SOLUTION

Member	n	N	L	nNL
AB	0	0	1.5	0
AC	–1.25	–5.00	2.5	15.625
AD	1.00	4.00	2.0	8.000
BC	1.00	4.00	2.0	8.000
CD	0.75	–2.00	1.5	–2.25
				$\Sigma = 29.375$





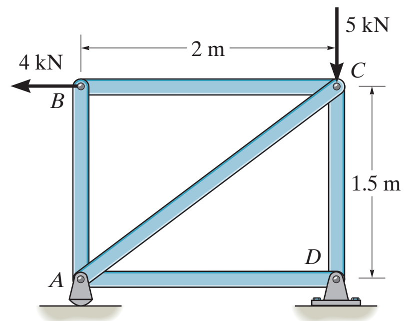
$$1 \cdot (\Delta_B)_h = \Sigma \frac{nNL}{AE}$$

$$(\Delta_B)_h = \frac{29.375(10^3)}{400(10^{-6})(200)(10^9)} = 0.3672(10^{-3})\text{ m} = 0.367\text{ mm} \leftarrow \qquad \textbf{Ans.}$$

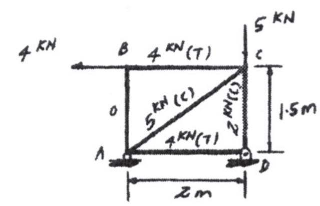
Ans:
$(\Delta_B)_h = 0.367\text{ mm} \leftarrow$

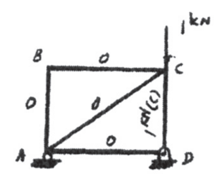
***14–80.**

Determine the vertical displacement of joint C of the truss. Each A992 steel member has a cross-sectional area of 400 mm^2.



SOLUTION





Member	n	N	L	nNL
AB	0	0	1.5	0
AC	0	–5.00	2.5	0
AD	0	4.00	2.0	0
BC	0	4.00	2.0	0
CD	–1.00	–2.00	1.5	3.00
				$\Sigma = 3.00$

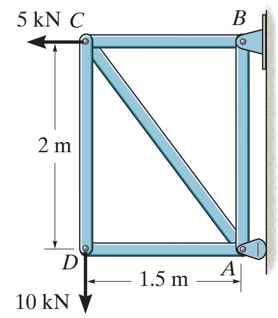
$$1 \cdot (\Delta_C)_v = \Sigma \frac{nNL}{AE}$$

$$(\Delta_C)_v = \frac{3.00\,(10^3)}{400(10^{-6})(200)(10^9)} = 37.5(10^{-6}) \text{ m} = 0.0375 \text{ mm}\downarrow \qquad \textbf{Ans.}$$

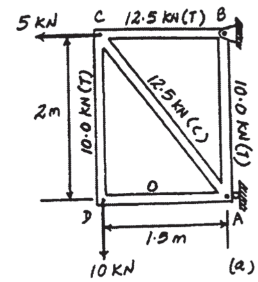
Ans:
$(\Delta_C)_v = 0.0375 \text{ mm}\downarrow$

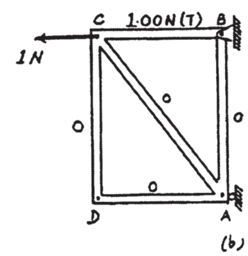
14–81.

Determine the horizontal displacement of joint C. Each A-36 steel member has a cross-sectional area of 400 mm^2.



SOLUTION





Member Real Forces N: As shown on Figure (a).

Member Virtual Forces n: As shown on Figure (b).

Virtual-Work Equation: Applying Eq. 14–39, we have

Member	n	N	L	nNL
AB	0	$10.0(10^3)$	2	0
BC	1.00	$12.5(10^3)$	1.5	$18.75(10^3)$
CD	0	$10.0(10^3)$	2	0
AD	0	0	1.5	0
AC	0	$-12.5(10^3)$	2.5	0
				$\Sigma 18.7(10^3)\ \mathrm{N^2 \cdot m}$

$$1 \cdot \Delta = \Sigma \frac{nNL}{AE}$$

$$1\ \mathrm{N} \cdot (\Delta_C)_h = \frac{18.75(10^3)\ \mathrm{N^2 \cdot m}}{AE}$$

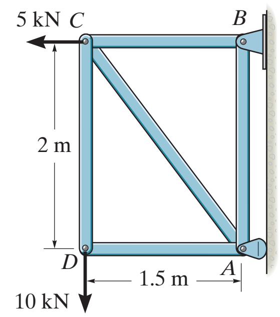
$$(\Delta_C)_h = \frac{18.75(10^3)}{0.400(10^{-3})\left[200(10^9)\right]}$$

$$= 0.2344(10^{-3})\ \mathrm{m} = 0.234\ \mathrm{mm} \quad \leftarrow \qquad \textbf{Ans.}$$

Ans:
$(\Delta_C)_h = 0.234$ mm $\leftarrow$

14–82.

Determine the vertical displacement of joint D. Each A-36 steel member has a cross-sectional area of 400 mm^2.



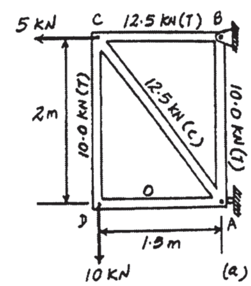
SOLUTION

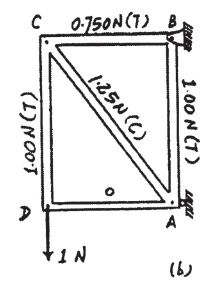
Member Real Forces *N*: As shown on Figure (a).

Member Virtual Forces *n*: As shown on Figure (b).

Virtual-Work Equation: Applying Eq. 14–39, we have

Member	n	N	L	nNL
AB	1.00	$10.0(10^3)$	2	$20.0(10^3)$
BC	0.750	$12.5(10^3)$	1.5	$14.0625(10^3)$
CD	1.00	$10.0(10^3)$	2	$20.0(10^3)$
AD	0	0	1.5	0
AC	–1.25	$-12.5(10^3)$	2.5	$39.0625(10^3)$
				$\Sigma 93.125(10^3)\ \text{N}^2\cdot\text{m}$





$$1\cdot\Delta = \Sigma\frac{nNL}{AE}$$

$$1\ \text{N}\cdot(\Delta_D)_v = \frac{93.125(10^3)\ \text{N}^2\cdot\text{m}}{AE}$$

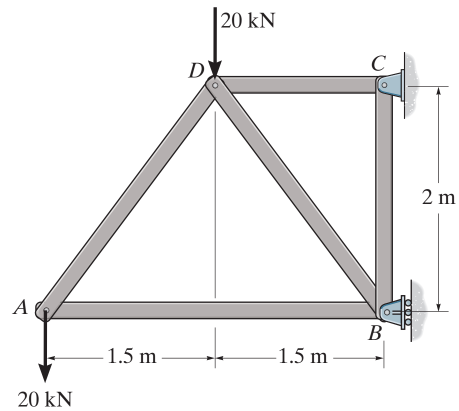
$$(\Delta_D)_v = \frac{93.125(10^3)}{0.400(10^{-3})\left[200(10^9)\right]}$$

$$= 1.164(10^{-3})\ \text{m} = 1.16\ \text{mm}\quad\downarrow$$ **Ans.**

Ans:
$(\Delta_D)_v = 1.16\ \text{mm}\quad\downarrow$

14–83.

Determine the vertical displacement of joint A. The truss is made from A992 steel rods having a diameter of 30 mm.



SOLUTION

Members Real Force N: As indicated in Fig. *a*.

Members Virtual Force n: As indicated in Fig. *b*.

Virtual Work Equation: Since $\sigma_{\max} = \dfrac{F_{BD}}{A} = \dfrac{50(10^3)}{\frac{\pi}{4}(0.03^2)} = 70.74 \text{ MPa} < \sigma_Y = 345 \text{ MPa}$,

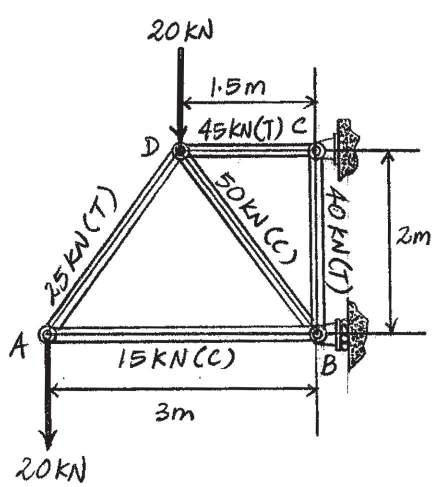
Member	n (N)	N (N)	L(m)	$nNL(\text{N}^2 \cdot \text{m})$
AB	-0.75	$-15(10^3)$	3	$33.75(10^3)$
AD	1.25	$25(10^3)$	2.5	$78.125(10^3)$
BC	1	$40(10^3)$	2	$80(10^3)$
BD	-1.25	$-50(10^3)$	2.5	$156.25(10^3)$
CD	1.5	$45(10^3)$	1.5	$101.25(10^3)$
				$\Sigma\ 449.375(10^3)$

Then

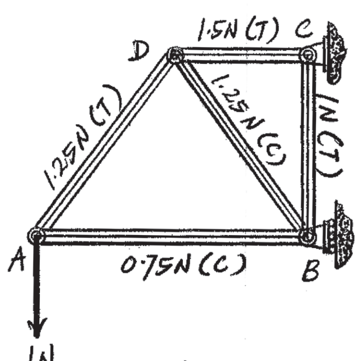
$$1 \cdot \Delta = \Sigma \frac{nNL}{AE}$$

$$1 \text{ N} \cdot (\Delta_A)_v = \frac{449.375(10^3)}{\frac{\pi}{4}(0.03^2)[200(10^9)]}$$

$$(\Delta_A)_v = 3.179(10^{-3}) \text{ m} = 3.18 \text{ mm} \downarrow$$ **Ans.**



(a)

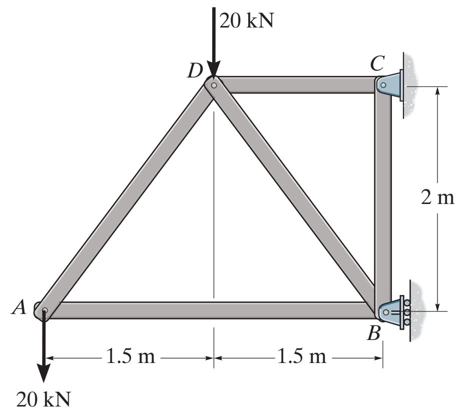


(b)

Ans:
$(\Delta_A)_v = 3.18 \text{ mm} \downarrow$

***14–84.**

Determine the vertical displacement of joint D. The truss is made from A992 steel rods having a diameter of 30 mm.



SOLUTION

Members Real Force N: As indicated in Fig. a.

Members Virtual Force n: As indicated in Fig. b.

Virtual Work Equation: Since $\sigma_{\max} = \dfrac{F_{BD}}{A} = \dfrac{50(10^3)}{\frac{\pi}{4}(0.03^2)} = 70.74 \text{ MPa} < \sigma_Y = 345 \text{ MPa}$,

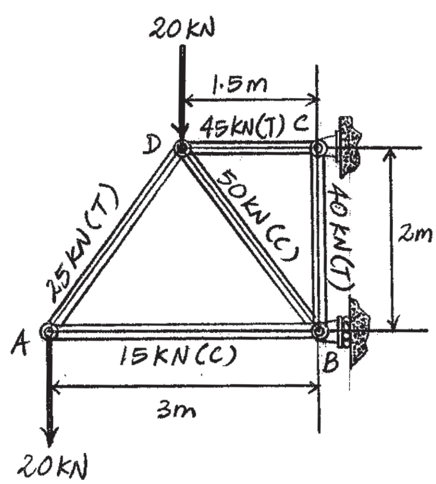
Member	n (N)	N (N)	L(m)	$nNL(\text{N}^2\cdot\text{m})$
AB	0	$-15(10^3)$	3	0
AD	0	$25(10^3)$	2.5	0
BC	1	$40(10^3)$	2	$80(10^3)$
BD	-1.25	$-50(10^3)$	2.5	$156.25(10^3)$
CD	0.75	$45(10^3)$	1.5	$50.625(10^3)$
				$\Sigma\ 286.875(10^3)$

Then

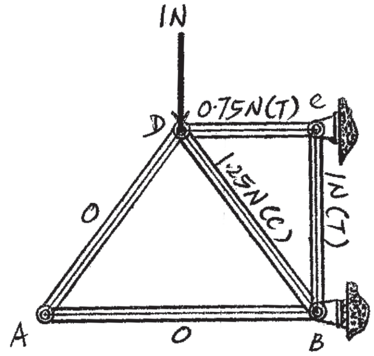
$$1 \cdot \Delta = \Sigma \frac{nNL}{AE}$$

$$1 \text{ N} \cdot (\Delta_D)_v = \frac{286.875(10^3)}{\frac{\pi}{4}(0.03^2)[200(10^9)]}$$

$$(\Delta_D)_v = 2.029(10^{-3}) \text{ m} = 2.03 \text{ mm} \downarrow$$ **Ans.**



(a)

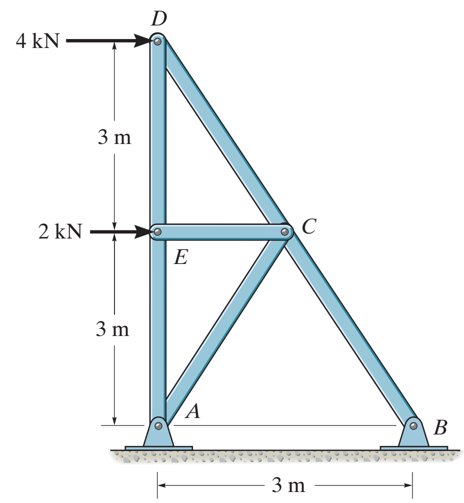


(b)

Ans:
$(\Delta_D)_v = 2.03 \text{ mm} \downarrow$

14–85.

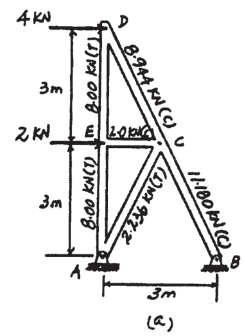
Determine the horizontal displacement of joint D. Each A-36 steel member has a cross-sectional area of 300 mm^2.



SOLUTION

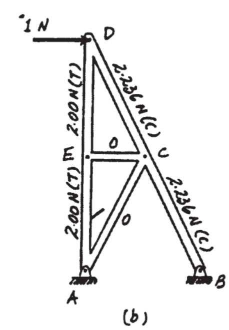
Virtual-Work Equation: Applying Eq. 14–39, we have

Member	n	N	L	nNL
AE	2.00	$800(10^3)$	3	$48.0(10^3)$
ED	2.00	$800(10^3)$	3	$48.0(10^3)$
CD	-2.236	$-8.944(10^3)$	3.354	$67.082(10^3)$
BC	-2.236	$-11.180(10^3)$	3.354	$83.853(10^3)$
CE	0	$-2.00(10^3)$	1.5	0
AC	0	$2.236(10^3)$	3.354	0
				$\Sigma 246.935(10^3)\ \text{N}^2\cdot\text{m}$



$$1 \cdot \Delta = \Sigma \frac{nNL}{AE}$$

$$1\ \text{N} \cdot (\Delta_D)_h = \frac{246.935(10^3)\ \text{N}^2\cdot\text{m}}{AE}$$



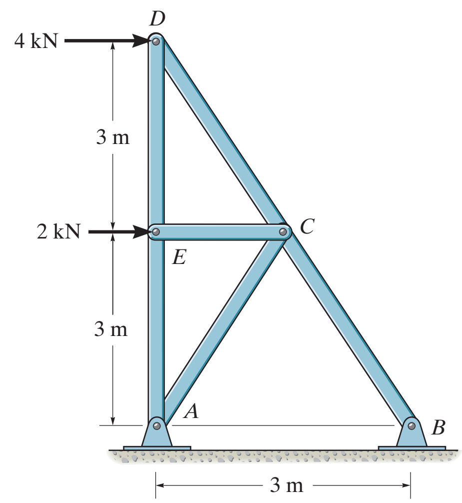
$$(\Delta_D)_h = \frac{246.935(10^3)}{0.300(10^{-3})\left[200(10^9)\right]}$$

$$= 4.116(10^{-3})\ \text{m} = 4.12\ \text{mm} \quad \rightarrow \qquad \textbf{Ans.}$$

Ans:
$(\Delta_D)_h = 4.12$ mm $\rightarrow$

14–86.

Determine the horizontal displacement of joint E. Each A-36 steel member has a cross-sectional area of 300 mm^2.



SOLUTION

Virtual-Work Equation: Applying Eq. 14–39, we have

Member	n	N	L	nNL
AE	0	$800(10^3)$	3	0
ED	0	$800(10^3)$	3	0
CD	0	$-8.944(10^3)$	3.354	0
BC	-1.118	$-11.180(10^3)$	3.354	$41.926(10^3)$
CE	-1.00	$-2.00(10^3)$	1.5	$300(10^3)$
AC	1.118	$2.236(10^3)$	3.354	$8.385(10^3)$
				$\Sigma 53.312(10^3)\ \text{N}^2\cdot\text{m}$

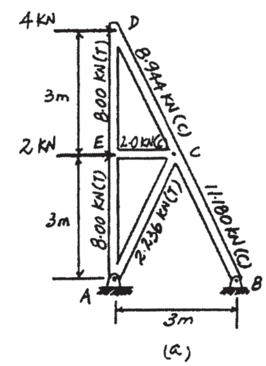
$$1 \cdot \Delta = \Sigma \frac{nNL}{AE}$$

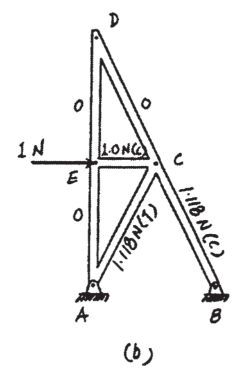
$$1\ \text{N} \cdot (\Delta_E)_h = \frac{53.312(10^3)\ \text{N}^2\cdot\text{m}}{AE}$$

$$(\Delta_E)_h = \frac{53.312(10^3)}{0.300(10^{-3})\left[200(10^9)\right]}$$

$$= 0.8885(10^{-3})\ \text{m} = 0.889\ \text{mm} \quad \rightarrow$$

Ans.

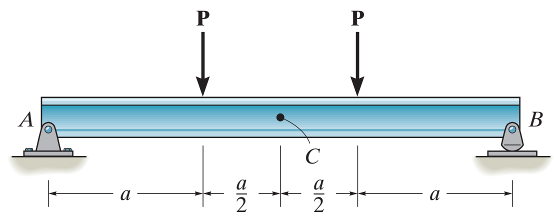




Ans:
$(\Delta_E)_h = 0.889\ \text{mm} \quad \rightarrow$

14–87.

Determine the displacement at point C. EI is constant.

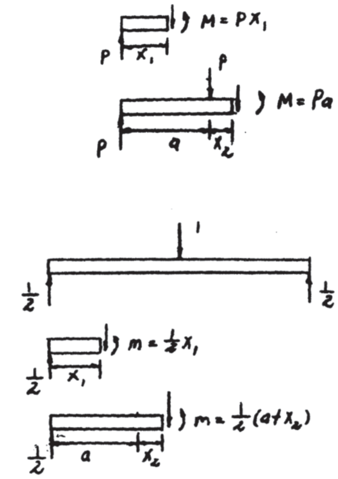


SOLUTION

$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI}\,dx$$

$$\Delta_C = 2\left(\frac{1}{EI}\right)\left[\int_0^a \left(\frac{1}{2}x_1\right)(Px_1)dx_1 + \int_0^{a/2} \frac{1}{2}(a + x_2)(Pa)dx_2\right]$$

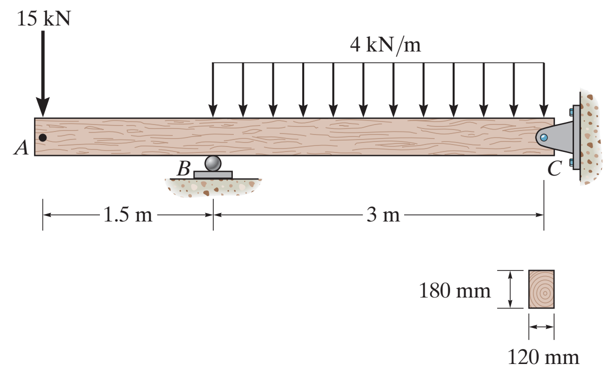
$$= \frac{23Pa^3}{24EI}$$ **Ans.**



Ans:
$\Delta_C = \dfrac{23Pa^3}{24EI}$

***14–88.**

The beam is made of southern pine for which $E_p = 13$ GPa. Determine the displacement at A.

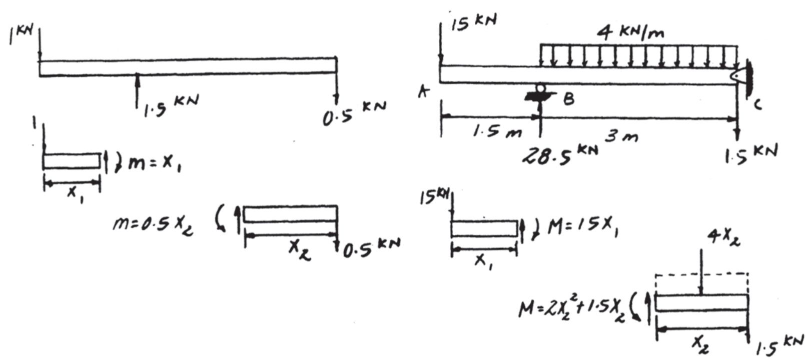


SOLUTION

$$1 \cdot \Delta_A = \int_0^L \frac{mM}{EI}$$

$$\Delta_A = \frac{1}{EI}\left[\int_0^{1.5}(x_1)(15x_1)dx_1 + \int_0^{3}(0.5x_2)(2x_2^2 + 1.5x_2)dx_2\right]$$

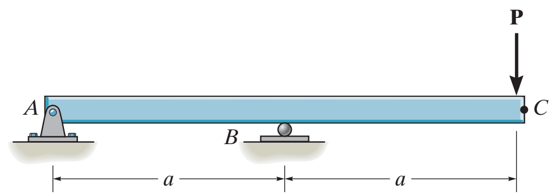
$$= \frac{43.875\text{ kN}\cdot\text{m}^3}{EI} = \frac{43.875(10^3)}{13(10^9)(\frac{1}{12})(0.12)(0.18)^3} = 0.0579\text{ m} = 57.9\text{ mm} \quad \textbf{Ans.}$$



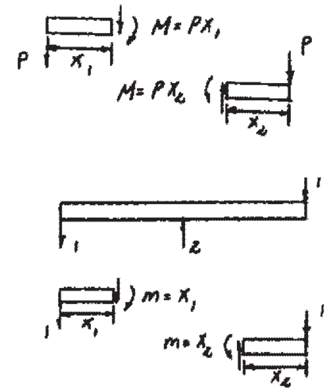
Ans:
$\Delta_A = 57.9$ mm

14–89.

Determine the displacement at point C. EI is constant.



SOLUTION



$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI} dx$$

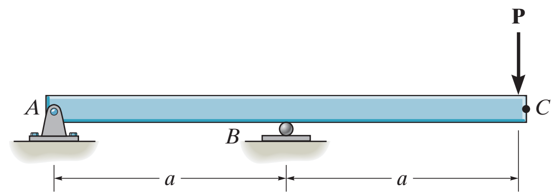
$$\Delta_C = \frac{1}{EI}\left[\int_0^a (x_1)(Px_1)dx_1 + \int_0^a (x_2)(Px_2)dx_2\right]$$

$$= \frac{2Pa^3}{3EI}$$ **Ans.**

Ans:
$\Delta_C = \dfrac{2Pa^3}{3EI}$

14–90.

Determine the slope at point C. EI is constant.



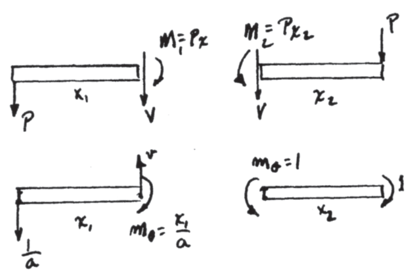
SOLUTION

$$1 \cdot \theta_C = \int_0^L \frac{m_\theta M dx}{EI}$$

$$\theta_C = \int_0^a \frac{\left(\frac{x_1}{a}\right) P x_1 \, dx_1}{EI} + \int_0^a \frac{(1) P x_2 \, dx_2}{EI}$$

$$= \frac{Pa^2}{3EI} + \frac{Pa^2}{2EI} = \frac{5Pa^2}{6EI}$$

Ans.

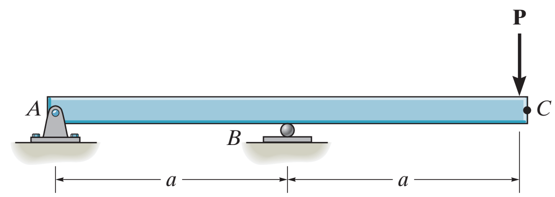


Ans:

$$\theta_C = -\frac{5Pa^2}{6EI}$$

14–91.

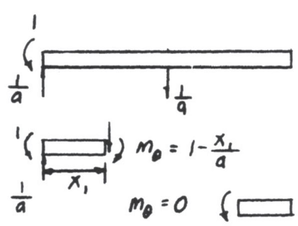
Determine the slope at point A. EI is constant.

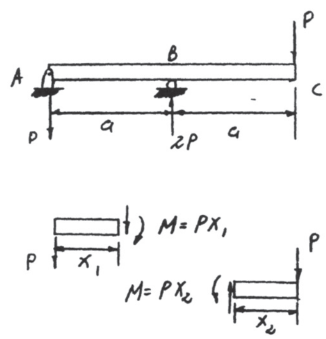


SOLUTION

$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI} dx$$

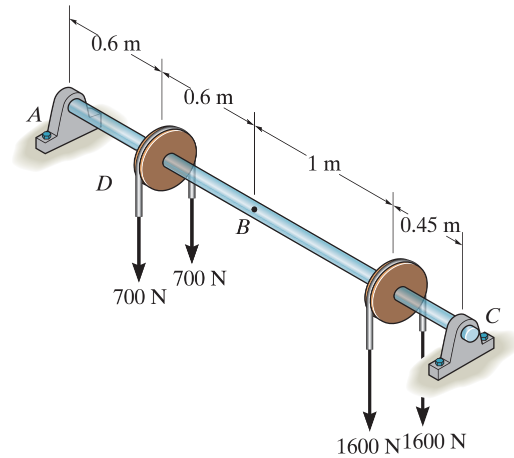
$$\theta_A = \frac{1}{EI}\left[\int_0^a \left(1 - \frac{x_1}{a}\right)(Px_1)\, dx_1 + \int_0^a (0)\,(Px_2)\, dx_2\right] = \frac{Pa^2}{6EI} \qquad \textbf{Ans.}$$





Ans:
$\theta_A = \frac{Pa^2}{6EI}$

***14–92.** Determine the displacement at B of the 30-mm-diameter A-36 steel shaft.



SOLUTION

$$1 \cdot \Delta_B = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_B = \frac{1}{EI}\left[\int_0^{0.6\text{m}} (0.5472x_1)(1.6264x_1)dx_1\right.$$

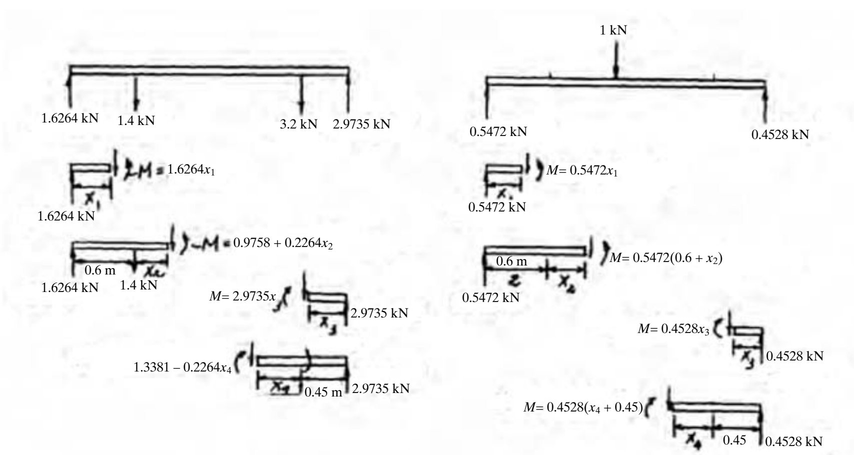
$$+ \int_0^{0.6\text{m}} (0.5472)(0.6 + x_2)(0.9758 + 0.2264x_2)dx_2$$

$$+ \int_0^{0.45\text{m}} (0.4528x_3)(2.9731x_3)dx_3$$

$$\left. + \int_0^{1\text{m}} 0.4528(x_4 + 0.45)(1.3381 - 0.2264x_4)dx_4\right]$$

$$= \frac{0.93401 \text{ kN}\cdot\text{m}^3}{EI} = \frac{0.93401(10^3)}{[200(10^9)]\left[\frac{\pi}{4}(0.015^4)\right]}$$

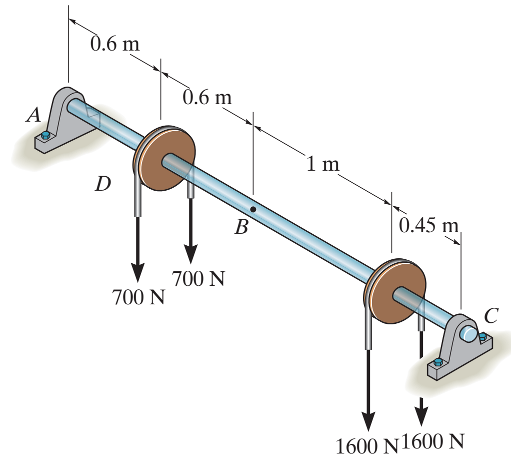
$$= 0.11745 \text{ m} = 117 \text{ mm}\downarrow$$ **Ans.**



Ans:

$\Delta_B = 117 \text{ mm}\downarrow$

14–93. Determine the slope of the 30-mm-diameter A-36 steel shaft at the bearing support A.



SOLUTION

$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI} dx$$

$$\theta_A = \frac{1}{EI}\left[\int_0^{0.6\text{m}} (1 - 0.3774x_1)(1.6264x_1)dx_1\right.$$

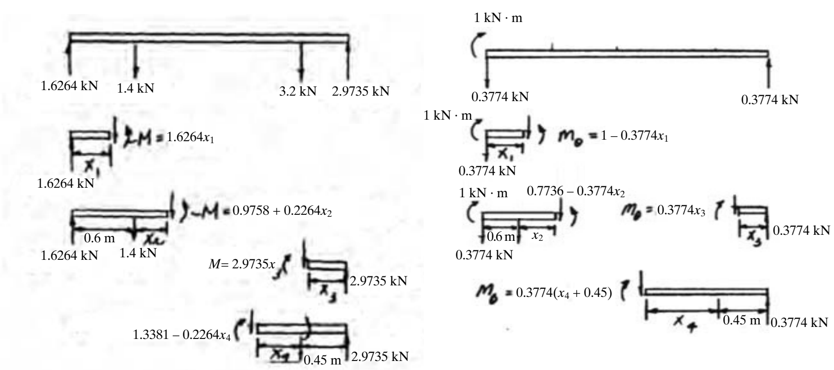
$$+ \int_0^{0.6\text{m}} (0.7736 - 0.3774x_2)(0.9758 + 0.2264x_2)dx_2$$

$$+ \int_0^{0.45\text{m}} 0.3774x_3(2.9735x_3)dx_3$$

$$\left.+ \int_0^{1\text{m}} 0.3774(x_4 + 0.45)(1.3381 - 0.2264x_4)dx_4\right]$$

$$= \frac{1.12668 \text{ kN}\cdot\text{m}^2}{EI} = \frac{1.12668(10^3)}{[200(10^9)]\left[\frac{\pi}{4}(0.015^4)\right]}$$

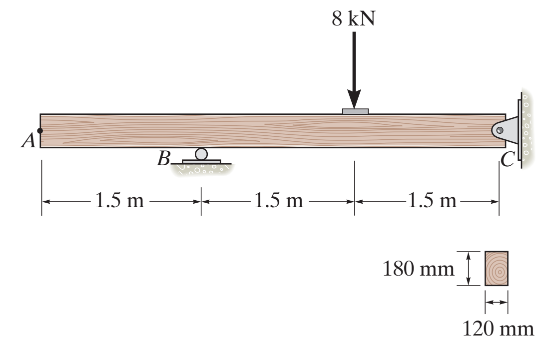
$$= 0.141682 \text{ rad} = 8.12° \text{ (clockwise)}$$ **Ans.**



Ans:
$\theta_A = 8.12°$ (clockwise)

14–94.

The beam is made of Douglas fir. Determine the slope at C.



SOLUTION

Virtual Work Equation: For the slope at point C,

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\, dx$$

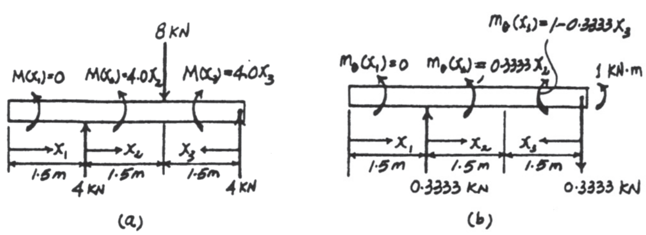
$$1\ \text{kN} \cdot \text{m} \cdot \theta_C = 0 + \frac{1}{EI}\int_0^{1.5\ \text{m}} (0.3333x_2)(4.00x_2)\, dx_2$$

$$+ \frac{1}{EI}\int_0^{1.5\ \text{m}} (1 - 0.3333x_3)(4.00x_3)dx_3$$

$$\theta_C = \frac{4.50\ \text{kN} \cdot \text{m}^3}{EI}$$

$$= \frac{4.50(1000)}{13.1(10^9)\left[\frac{1}{12}(0.12)(0.18^3)\right]}$$

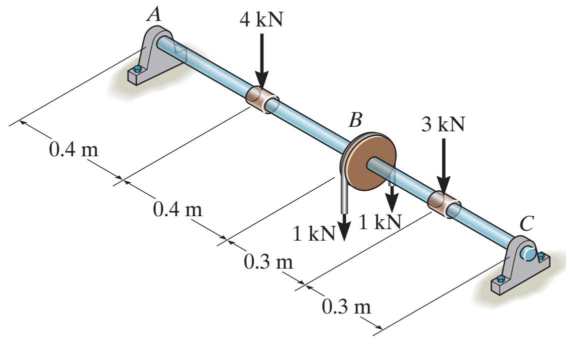
$$= 5.89\left(10^{-3}\right)\ \text{rad} = 0.337° \qquad \textbf{Ans.}$$



Ans:
$\theta_C = 0.337°$

14–95.

Determine the displacement at pulley B. The A992 steel shaft has a diameter of 30 mm.



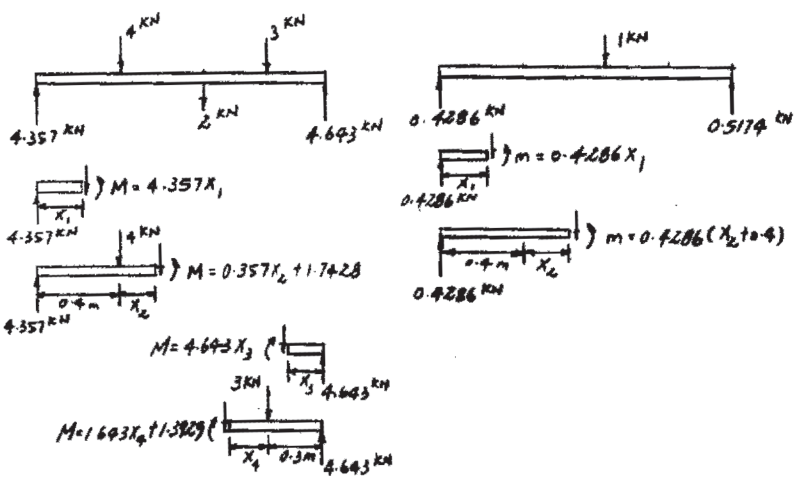
SOLUTION

$$1 \cdot \Delta_B = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_B = \frac{1}{EI}\left[\int_0^{0.4} (0.4286x_1)(4.357x_1)\, dx_1 + \int_0^{0.4} 0.4286(x_2 + 0.4)(0.357x_2 + 1.7428)\, dx_2\right.$$

$$\left.\int_0^{0.3} (0.5714x_3)(4.643x_3)\, dx_3 + \int_0^{0.3} 0.5714(x_4 + 0.3)(1.643x_4 + 1.3929)\, dx_4\right]$$

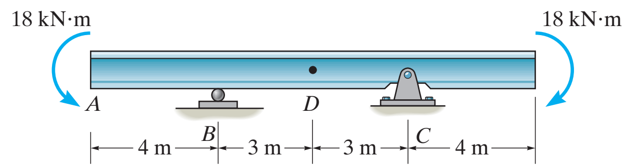
$$= \frac{0.37972 \text{ kN} \cdot \text{m}^3}{EI} = \frac{0.37972(10^3)}{200(10^9)\left(\frac{\pi}{4}\right)(0.015^4)} = 0.0478 \text{ m} = 47.8 \text{ mm} \downarrow \quad \textbf{Ans.}$$



Ans:
$\Delta_B = 47.8$ mm $\downarrow$

***14–96.**

The A992 steel beam has a moment of inertia of $I = 125(10^6)\text{ mm}^4$. Determine the displacement at point D.

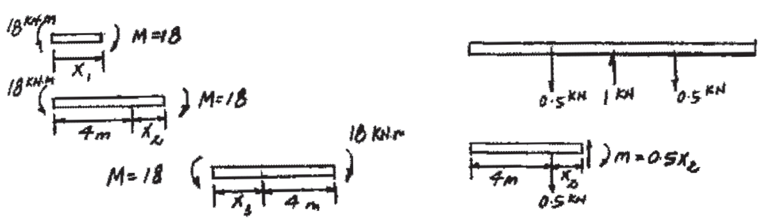


SOLUTION

$$1 \cdot \Delta_D = \int_0^L \frac{mM}{EI}\,dx$$

$$\Delta_D = (2)\frac{1}{EI}\left[\int_0^3 (0.5x_2)(18)\,dx_2\right] = \frac{81\text{ kN}\cdot\text{m}^3}{EI} = \frac{81(10^3)}{200(10^9)(125)(10^{-6})}$$

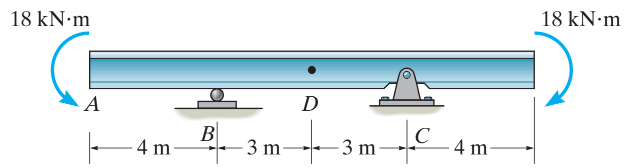
$$= 3.24(10^{-3}) = 3.24\text{ mm} \downarrow$$ **Ans.**



Ans:
$\Delta_D = 3.24\text{ mm} \downarrow$

14–97.

The A992 steel beam has a moment of inertia of $I = 125(10^6)\ \text{mm}^4$. Determine the slope at A.

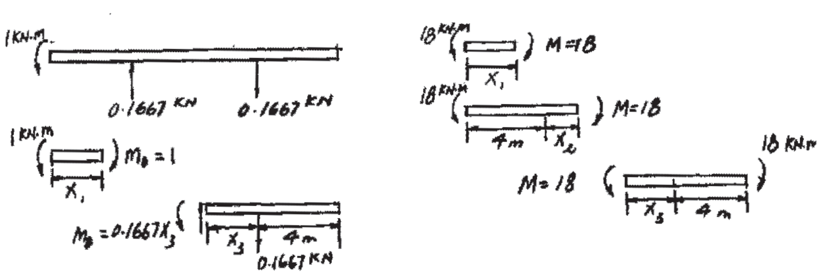


SOLUTION

$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI} dx$$

$$\theta_A = \frac{1}{EI}\left[\int_0^4 (1)(18)(dx_1) + \int_0^6 (0.1667x_3)(18)\,dx_3\right] = \frac{126\ \text{kN}\cdot\text{m}^2}{EI}$$

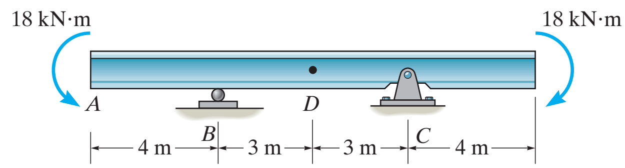
$$= \frac{126(10^3)}{200(10^9)(125)(10^{-6})} = 5.04(10^{-3})\ \text{rad} = 0.289° \qquad \textbf{Ans.}$$



Ans:
$\theta_A = 0.289°$

14–98.

The A992 structural steel beam has a moment of inertia of $I = 125(10^6)\ \text{mm}^4$. Determine the slope at B.

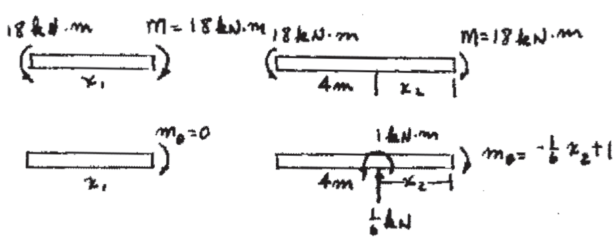


SOLUTION

$$1 \cdot \theta_B = \int_0^L \frac{m_\theta M}{EI} dx$$

$$\theta_B = 0 + \frac{1}{EI}\int_0^6 \frac{\left(-\frac{1}{6}x_2 + 1\right)(18)dx}{EI}$$

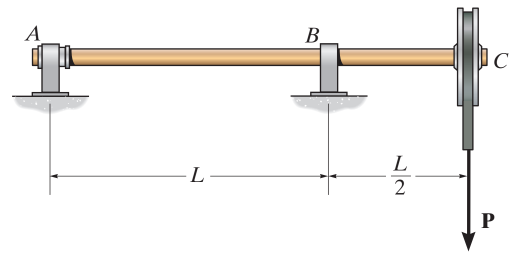
$$= \frac{54}{EI} = \frac{54(10^3)}{200(10^9)(125(10^{-6}))} = 0.00216 \text{ rad} = 0.124°$$ **Ans.**



Ans:
$\theta_B = 0.124°$

14–99.

Determine the displacement at point C of the shaft. EI is constant.



SOLUTION

Real Moment Function M: As indicated in Fig. *a*.

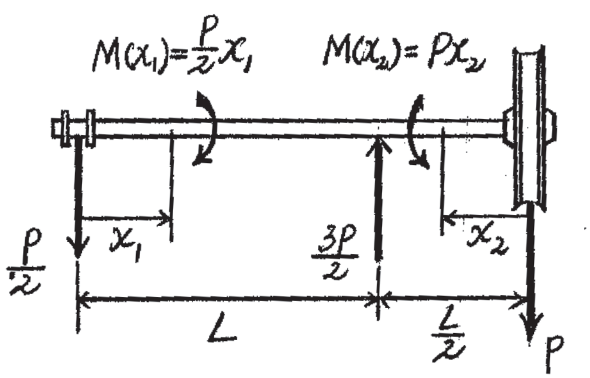
Virtual Moment Function m: As indicated in Fig. *b*.

Virtual Work Equation:

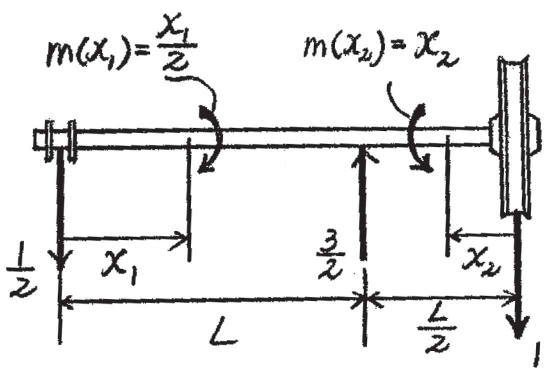
$$1 \cdot \Delta = \int_0^L \frac{mM}{EI} dx$$

$$1 \cdot \Delta_C = \frac{1}{EI}\left[\int_0^L \left(\frac{x_1}{2}\right)\left(\frac{P}{2}x_1\right) dx_1 + \int_0^{L/2} x_2(Px_2)\, dx_2\right]$$

$$\Delta_C = \frac{PL^3}{8EI} \downarrow$$ **Ans.**



(a)

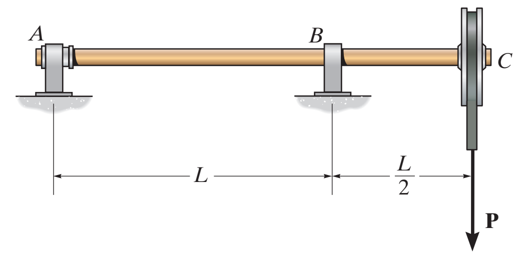


(b)

Ans:
$\Delta_C = \frac{PL^3}{8EI} \downarrow$

***14–100.**

Determine the slope at A of the shaft. EI is constant.



SOLUTION

Real Moment Function M: As indicated in Fig. *a*.

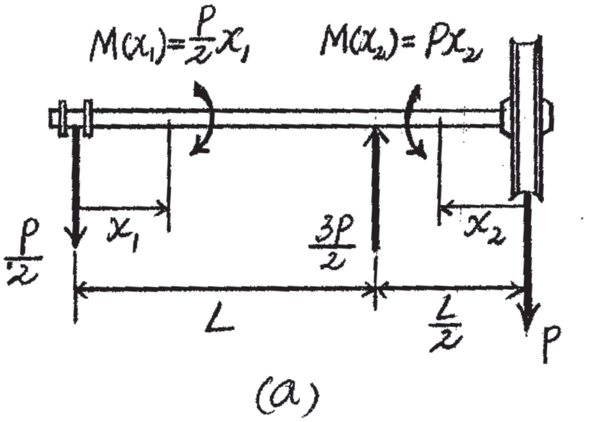
Virtual Moment Function M: As indicated in Fig. *b*.

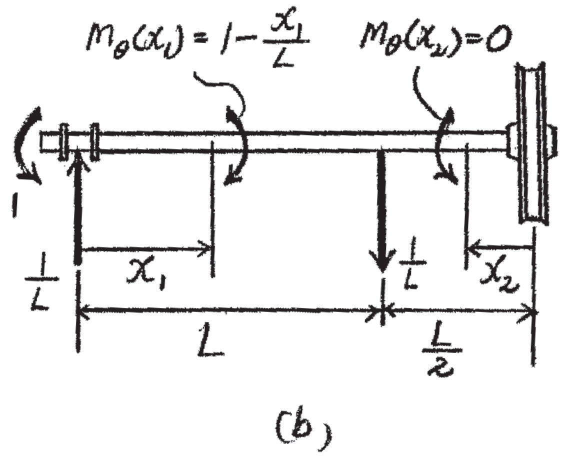
Virtual Work Equation:

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\, dx$$

$$1 \cdot \theta_A = \frac{1}{EI}\left[\int_0^L \left(1 - \frac{x_1}{L}\right)\left(\frac{P}{2}x_1\right) dx_1 + \int_0^{L/2} (0)\,(Px_2)\, dx_2\right]$$

$$\theta_A = \frac{PL^2}{12EI}$$ **Ans.**

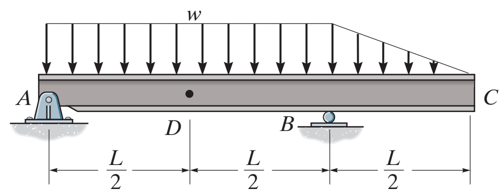




Ans:
$\theta_A = \dfrac{PL^2}{12EI}$

14–101.

Determine the slope of end C of the overhang beam. EI is constant.



SOLUTION

Real Moment Function M: As indicated in Fig. *a*.

Virtual Moment Function m_θ: As indicated in Fig. *b*.

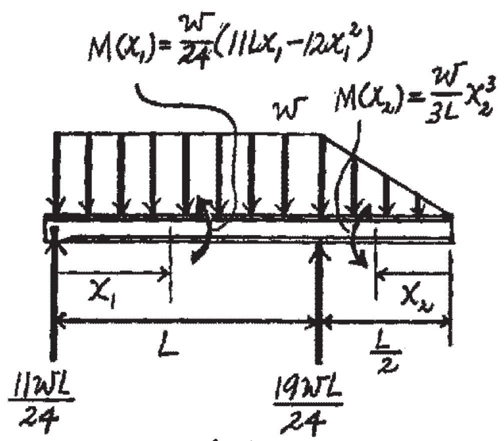
Virtual Work Equation:

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI} dx$$

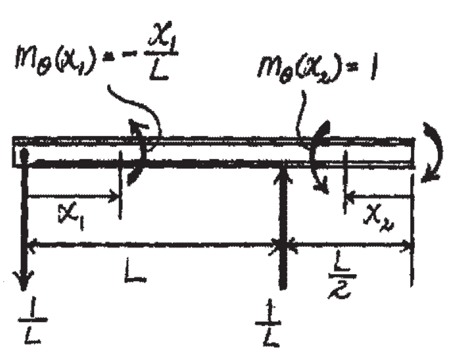
$$1 \cdot \theta_C = \frac{1}{EI}\left[\int_0^L \left(-\frac{x_1}{L}\right)\left[\frac{w}{24}\left(11Lx_1 - 12x_1^{\,2}\right)\right]dx_1 + \int_0^{L/2} (1)\left(\frac{w}{3L}x_2^{\,3}\right)dx_2\right]$$

$$\theta_C = \frac{1}{EI}\left[\frac{w}{24L}\int_0^L \left(12x_1^{\,3} - 11Lx_1^{\,2}\right)dx_1 + \frac{w}{3L}\int_0^{L/2} x_2^{\,3}\,dx_2\right]$$

$$\theta_C = -\frac{13wL^3}{576EI} = \frac{13wL^3}{576EI} \circlearrowright$$ **Ans.**



(a)

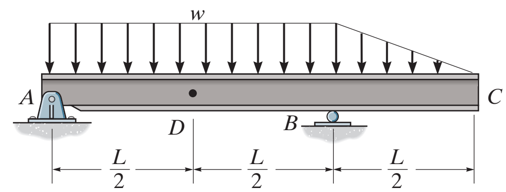


(b)

Ans:
$\theta_C = -\dfrac{13wL^3}{576\,EI}$

14–102.

Determine the displacement of point D of the overhang beam. EI is constant.



SOLUTION

Real Moment Function M: As indicated in Fig. *a*.

Virtual Moment Function m: As indicated in Fig. *b*.

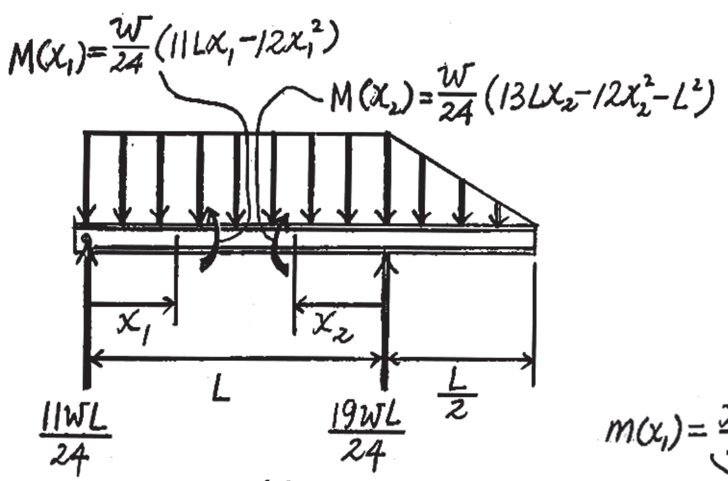
Virtual Work Equation:

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$

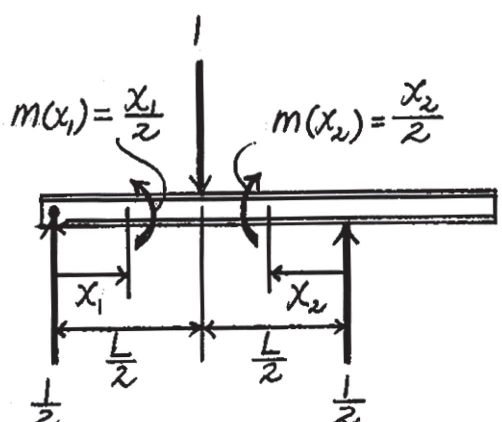
$$1 \cdot \Delta_D = \frac{1}{EI}\left[\int_0^{L/2}\left(\frac{x_1}{2}\right)\left[\frac{w}{24}\left(11Lx_1 - 12x_1^{\,2}\right)\right]dx_1 + \int_0^{L/2}\left(\frac{x_2}{2}\right)\left[\frac{w}{24}\left(13Lx_2 - 12x_2^{\,2} - L^2\right)\right]dx_2\right]$$

$$\Delta_D = \frac{w}{48EI}\left[\int_0^{L/2}\left(11Lx_1^{\,2} - 12x_1^{\,3}\right)dx_1 + \int_0^{L/2}\left(13Lx_2^{\,2} - 12x_2^{\,3} - L^2x_2\right)dx_2\right]$$

$$\Delta_D = \frac{wL^4}{96EI}\downarrow$$ **Ans.**



(a)

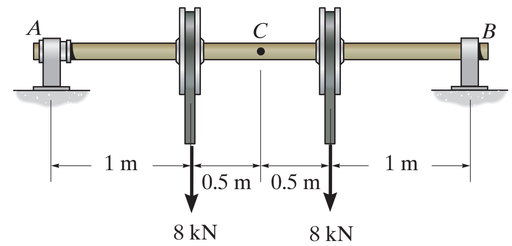


(b)

Ans:
$\Delta_D = \dfrac{wL^4}{96\,EI}\downarrow$

14–103.

Determine the slope at A of the 2014-T6 aluminum shaft having a diameter of 100 mm.



SOLUTION

Real Moment Function M: As indicated in Fig. *a*.

Virtual Moment Function m: As indicated in Fig. *b*.

Virtual Work Equation:

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI} dx$$

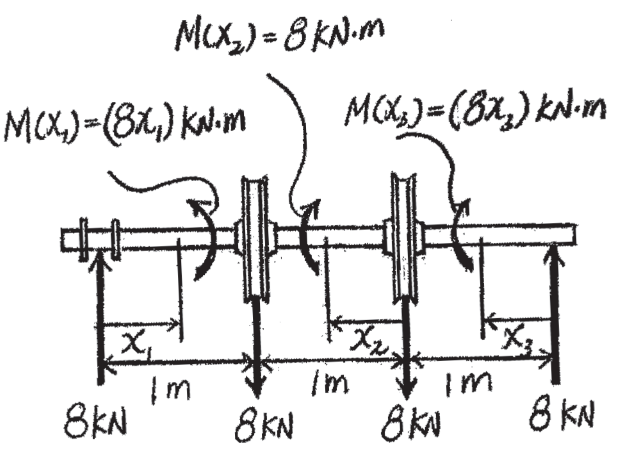
$$1 \text{ kN} \cdot \text{m} \cdot \theta_A = \frac{1}{EI}\left[\int_0^{1\text{ m}} (1 - 0.3333x_1)(8x_1)\, dx_1 + \int_0^{1\text{ m}} [0.3333(x_2+1)]\, 8dx_2 + \int_0^{1\text{ m}} (0.3333x_3)(8x_3)\, dx_3 \right]$$

$$\theta_A = \frac{1}{EI}\left[8\int_0^{1\text{ m}} (1 - 0.3333x_1)\, dx_1 + 2.6667\int_0^{1\text{ m}} (x_2 + 1)\, dx_2 + 2.6667\int_0^{1\text{ m}} x_3^2\, dx_3 \right]$$

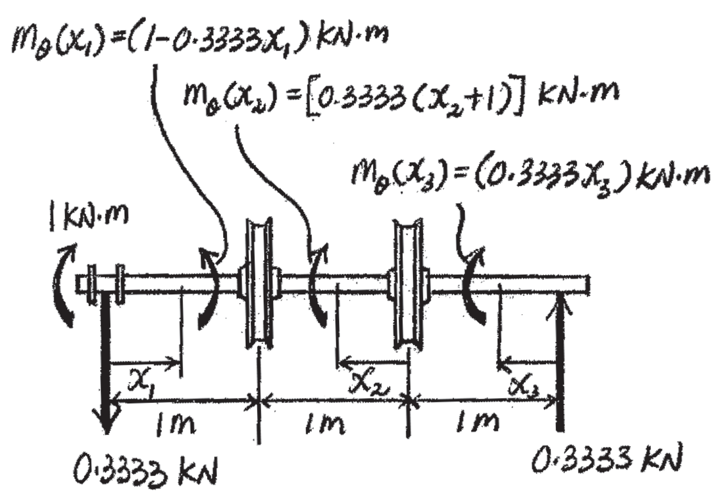
$$= \frac{8 \text{ kN} \cdot \text{m}^2}{EI}$$

$$= \frac{8(10^3)}{73.1(10^9)\left[\frac{\pi}{4}(0.05^4)\right]}$$

$$= 0.02229 \text{ rad} = 1.28°$$ **Ans.**



(a)

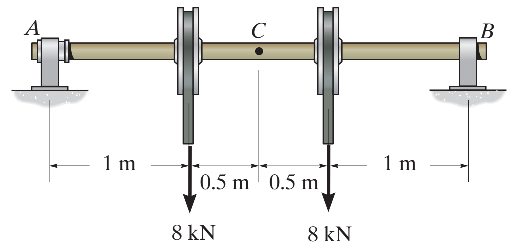


(b)

Ans:
$\theta_A = -1.28°$

***14–104.**

Determine the displacement at point C of the 2014-T6 aluminum shaft having a diameter of 100 mm.



SOLUTION

Real Moment Function M: As indicated in Fig. *a*.

Virtual Moment Function m: As indicated in Fig. *b*.

Virtual Work Equation:

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$

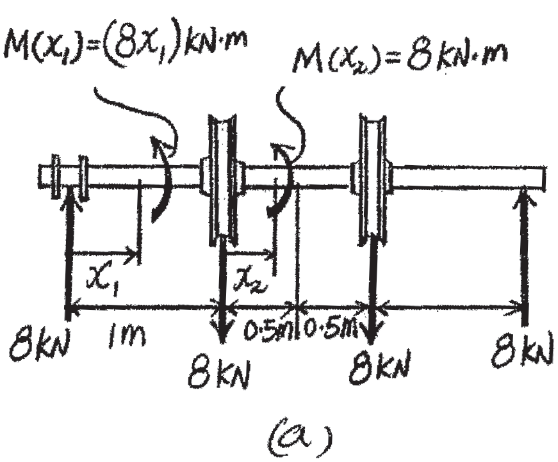
$$1 \cdot \Delta_C = 2\left[\frac{1}{EI}\left[\int_0^{1\text{ m}} (0.5x_1)(8x_1)\,dx_1 + \int_0^{0.5\text{ m}} [0.5(x_2 + 1)](8)\,dx_2\right]\right]$$

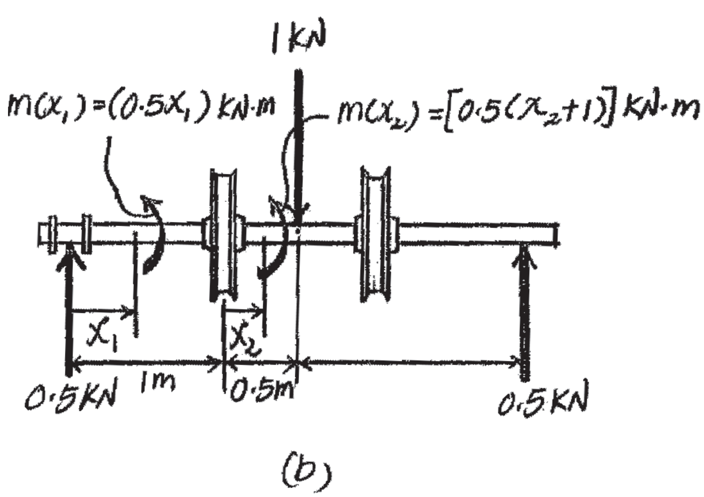
$$\Delta_C = \frac{2}{EI}\left[\int_0^{1\text{ m}} 4x_1^2\,dx_1 + \int_0^{0.5\text{ m}} 4(x_2 + 1)\,dx_2\right]$$

$$= \frac{7.6667\text{ kN}\cdot\text{m}^3}{EI}$$

$$= \frac{7.6667(10^3)}{73.1(10^9)\left[\frac{\pi}{4}(0.05^4)\right]}$$

$$= 0.02137\text{ m} = 21.4\text{ mm} \downarrow \qquad \textbf{Ans.}$$

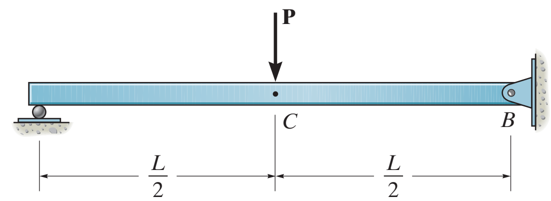




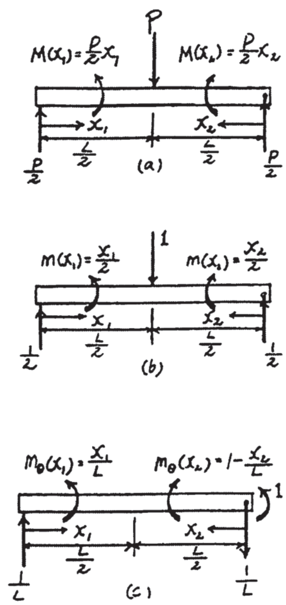
Ans:
$\Delta_C = 21.4\text{ mm} \downarrow$

14–105.

Determine the displacement at point C and the slope at B. EI is constant.



SOLUTION



Real Moment Function $M(x)$: As shown on Figure (a).

Virtual Moment Functions $m(x)$ and $m_\theta(x)$: As shown on Figure (b) and (c).

Virtual Work Equation: For the displacement at point C, apply Eq. 14–42.

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$

$$1 \cdot \Delta_C = 2\left[\frac{1}{EI}\int_0^{\frac{L}{2}}\left(\frac{x_1}{2}\right)\left(\frac{P}{2}x_1\right)dx_1\right]$$

$$\Delta_C = \frac{PL^3}{48EI}\downarrow$$ **Ans.**

For the slope at B, apply Eq. 14–43.

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\,dx$$

$$1 \cdot \theta_B = \frac{1}{EI}\left[\int_0^{\frac{L}{2}}\left(\frac{x_1}{L}\right)\left(\frac{P}{2}x_1\right)dx_1 + \int_0^{\frac{L}{2}}\left(1-\frac{x_2}{L}\right)\left(\frac{P}{2}x_2\right)dx_2\right]$$

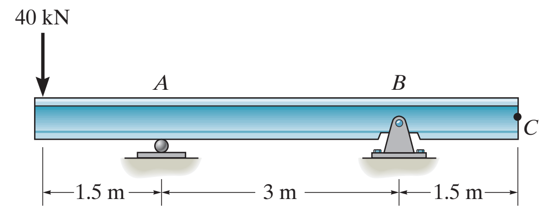
$$\theta_B = \frac{PL^2}{16EI}$$ **Ans.**

Ans:

$$\Delta_C = \frac{PL^3}{48EI}\downarrow$$

$$\theta_B = \frac{PL^2}{16EI}$$

14–106. Determine the displacement of point C of the beam made from A992 steel and having a moment of inertia of $I = 22.3(10^6)$ mm^4.



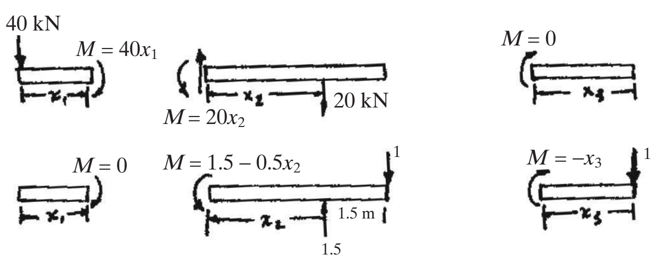
SOLUTION

$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_C = \frac{1}{EI}\left[0 + \int_0^{3\text{m}} (1.5 - 0.5x_2)(20x_2)dx_2 + 0\right]$$

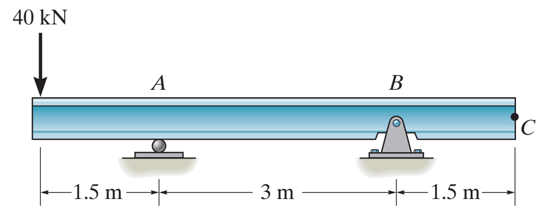
$$= \frac{45 \text{ kN} \cdot \text{m}^2}{EI} = \frac{45(10^3)}{[200(10^9)][22.3(10^{-6})]}$$

$$= 0.01009 \text{ m} = 10.1 \text{ mm} \downarrow$$ **Ans.**



Ans:
$\Delta_C = 10.1$ mm

14–107. Determine the slope at B of the beam made from A992 steel and having a moment of inertia of $I = 22.3(10^6)$ mm^4.



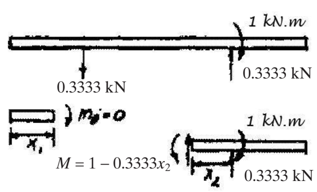
SOLUTION

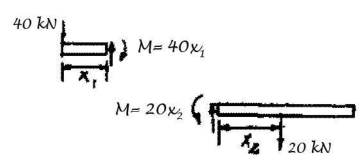
$$1 \cdot \theta_B = \int_0^L \frac{m_\theta M}{EI} dx$$

$$\theta_B = \frac{1}{EI}\left[\int_0^{1.5\text{m}} (0)(40x_1)dx_1 + \int_0^{3\text{m}} (1 - 0.3333x_2)(20x_2)dx_2 + 0\right]$$

$$= \frac{30 \text{ kN}\cdot\text{m}^2}{EI} = \frac{30(10^3)}{[200(10^9)][22.3(10^{-6})]}$$

$$= 6.7265(10^{-3}) \text{ rad} = 0.385° \quad \measuredangle\theta_B$$ **Ans.**

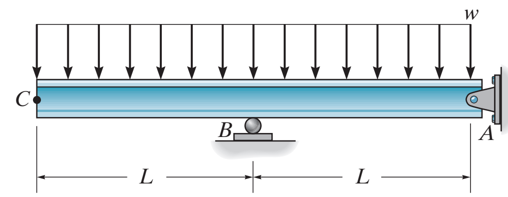




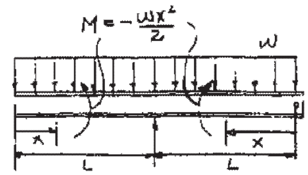
Ans:
$\theta_B = 0.385° \quad \measuredangle\theta_B$

***14–108.**

Determine the slope at A. EI is constant.



SOLUTION

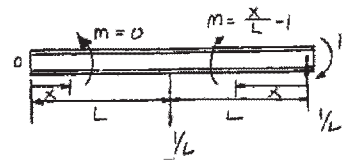


$$\theta_A = \int_0^L \frac{m_\theta M}{EI} dx$$

$$= 0 + \int_0^L \frac{\left(\frac{x}{L} - 1\right)\left(\frac{-wx^2}{2}\right)}{EI} dx$$

$$= \frac{-\frac{wL^4}{8L} + \frac{wL^3}{6}}{EI} = -\frac{wL^3}{24\,EI}$$

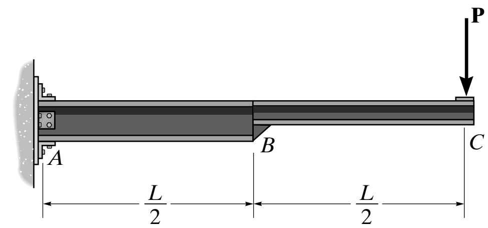
Ans.



Ans:

$\theta_A = -\dfrac{wL^3}{24\,EI}$

14–109. Determine the slope and displacement of end C of the cantilevered beam. The beam is made of a material having a modulus of elasticity of E. The moments of inertia for segments AB and BC of the beam are $2I$ and I, respectively.



Real Moment Function M. As indicated in Fig. a.

Virtual Moment Functions m_θ **and** M. As indicated in Figs. b and c.

Virtual Work Equation. For the slope at C,

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI} dx$$

$$1 \cdot \theta_C = \frac{1}{EI}\int_0^{L/2} 1(Px_1)dx_1 + \frac{1}{2\,EI}\int_0^{L/2} 1\left[P\left(x_2 + \frac{L}{2}\right)\right]dx_2$$

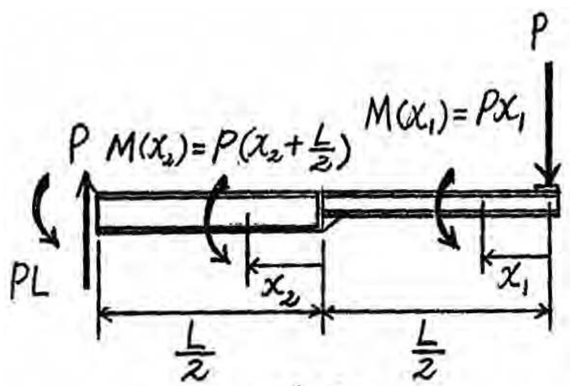
$$\theta_C = \frac{5PL^2}{16\,EI}$$ **Ans.**

For the displacement at C,

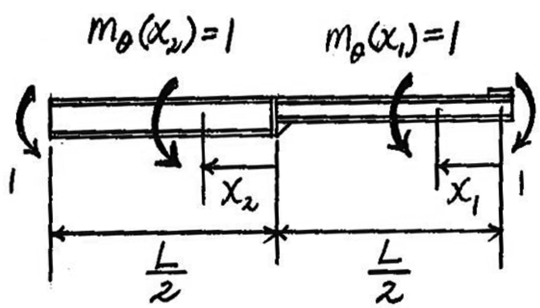
$$1 \cdot \Delta = \int_0^L \frac{mM}{EI} dx$$

$$1 \cdot \Delta_C = \frac{1}{EI}\int_0^{L/2} x_1(Px_1)dx_1 + \frac{1}{2EI}\int_0^{L/2}\left(x_2 + \frac{L}{2}\right)\left[P\left(x_2 + \frac{L}{2}\right)\right]dx_2$$

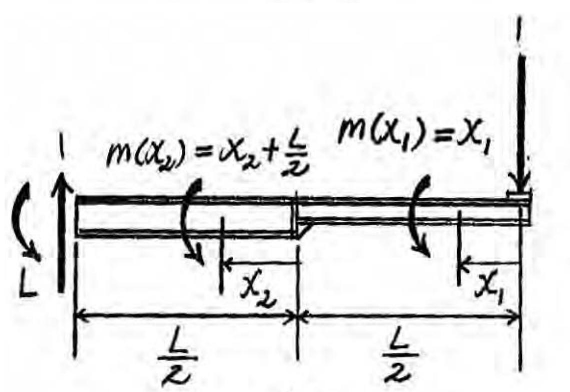
$$\Delta_C = \frac{3PL}{16EI} \downarrow$$ **Ans.**



(a)



(b)

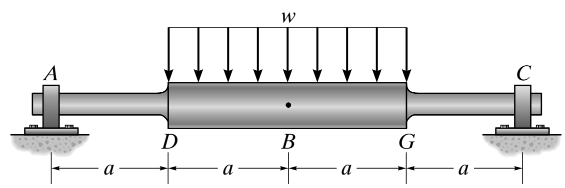


(c)

Ans:

$\theta_C = \frac{5PL^2}{16EI}$, $\Delta_C = \frac{3PL}{16EI} \downarrow$

14–110. Determine the displacement at point B. The moment of inertia of the center portion DG of the shaft is $2I$, whereas the end segments AD and GC have a moment of inertia I. The modulus of elasticity for the material is E.



SOLUTION

Real Moment Function M(x): As shown on Fig. a.

Virtual Moment Functions m(x): As shown on Fig. b.

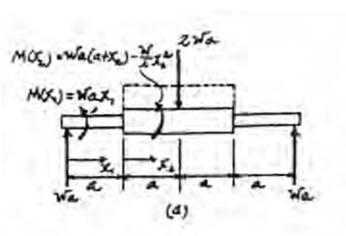
Virtual Work Equation: For the slope at point B, apply Eq. 14–42.

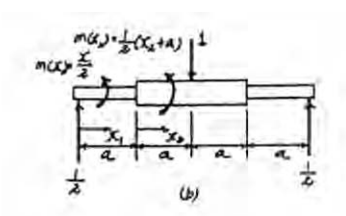
$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$

$$1 \cdot \Delta_B = 2\left[\frac{1}{EI}\int_0^a \left(\frac{x_1}{2}\right)(wax_1)dx_1\right]$$

$$+\,2\left[\frac{1}{2EI}\int_0^a \frac{1}{2}(x_2 + a)\left[wa(a + x_2) - \frac{w}{2}x_2^2\right]dx_2\right]$$

$$\Delta_B = \frac{65wa^4}{48EI} \quad \downarrow$$ **Ans.**





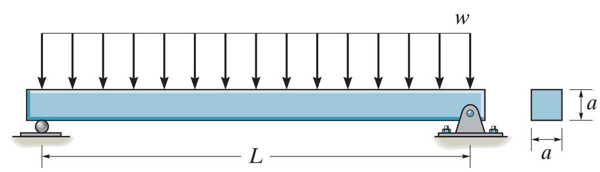
Ans:

$M(x_2) = wa(a + x_2) - \frac{w}{2}x_2^2$, $M(x_1) = wax_1$,

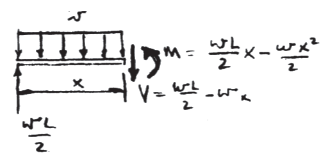
$m(x_2) = \frac{1}{2}(x_2 + a)$, $m(x_1) = \frac{x_1}{2}$, $\Delta_B = \frac{65wa^4}{48EI} \downarrow$

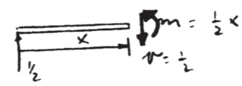
14–111.

Determine the maximum deflection of the beam caused only by bending, and caused by bending and shear. Take $E = 3G$.



SOLUTION





For bending and shear,

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx + \int_0^L \frac{f_s vV}{GA}\,dx$$

$$\Delta = 2\int_0^{L/2} \frac{\left(\frac{1}{2}x\right)\left(\frac{wL}{2}x - w\frac{x^2}{2}\right)dx}{EI} + 2\int_0^{L/2} \frac{\left(\frac{6}{5}\right)\left(\frac{1}{2}\right)\left(\frac{wL}{2} - wx\right)dx}{GA}$$

$$= \frac{1}{EI}\left(\frac{wL}{6}x^3 - \frac{wx^4}{8}\right)\Bigg|_0^{L/2} + \frac{\left(\frac{6}{5}\right)}{GA}\left(\frac{wL}{2}x - \frac{wx^2}{2}\right)\Bigg|_0^{L/2}$$

$$= \frac{5wL^4}{384EI} + \frac{3wL^2}{20\,GA}$$

$$\Delta = \frac{5wL^4}{384(3G)\left(\frac{1}{12}\right)a^4} + \frac{3wL^2}{20(G)a^2}$$

$$= \frac{20wL^4}{384Ga^4} + \frac{3wL^2}{20Ga^2}$$

$$= \left(\frac{w}{G}\right)\left(\frac{L}{a}\right)^2\left[\left(\frac{5}{96}\right)\left(\frac{L}{a}\right)^2 + \frac{3}{20}\right]$$ **Ans.**

For bending only,

$$\Delta = \frac{5w}{96G}\left(\frac{L}{a}\right)^4$$ **Ans.**

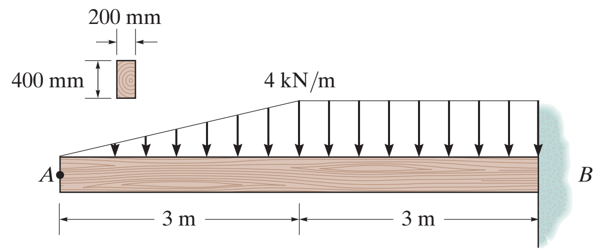
Ans:

$$\Delta_{\text{tot}} = \left(\frac{w}{G}\right)\left(\frac{L}{a}\right)^2\left[\left(\frac{5}{96}\right)\left(\frac{L}{a}\right)^2 + \frac{3}{20}\right],$$

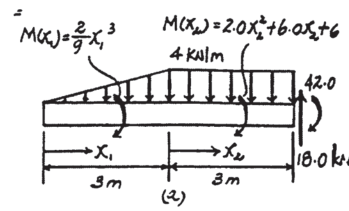
$$\Delta_b = \frac{5w}{96G}\left(\frac{L}{a}\right)^4$$

***14–112.**

The beam is made of oak, for which $E_o = 11$ GPa. Determine the slope and displacement at point A.



SOLUTION



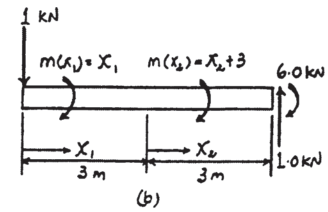
Virtual Work Equation: For the displacement at point A, apply Eq. 14–42.

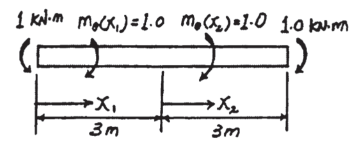
$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$

$$1\text{ kN} \cdot \Delta_A = \frac{1}{EI}\int_0^{3\text{ m}} x_1\left(\frac{2}{9}x_1^3\right)dx_1 + \frac{1}{EI}\int_0^{3\text{ m}} (x_2 + 3)\left(2.00x_2^2 + 6.00x_2 + 6.00\right)dx_2$$

$$\Delta_A = \frac{321.3\text{ kN}\cdot\text{m}^3}{EI} = \frac{321.3\left(10^3\right)}{11\left(10^9\right)\left[\frac{1}{12}(0.2)\left(0.4^3\right)\right]}$$

$$= 0.02738\text{ m} = 27.4\text{ m}\downarrow$$ **Ans.**





For the slope at A, apply Eq. 14–43.

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\,dx$$

$$1\text{ kN}\cdot\text{m}\cdot\theta_A = \frac{1}{EI}\int_0^{3\text{ m}} 1.00\left(\frac{2}{9}x_1^3\right)dx_1 + \int_0^{3\text{ m}} 1.00\left(2.00x_2^2 + 6.00x_2 + 6.00\right)dx_2$$

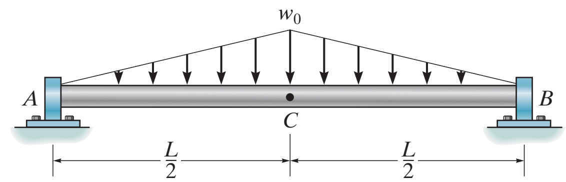
$$\theta_A = \frac{67.5\text{ kN}\cdot\text{m}^2}{EI} = \frac{67.5\,(1000)}{11\left(10^9\right)\left[\frac{1}{12}(0.2)\left(0.4^3\right)\right]}$$

$$= 5.75\left(10^{-3}\right)\text{ rad}$$ **Ans.**

Ans:
$\Delta_A = 27.4\text{ m}\downarrow$,
$\theta_A = 5.75\left(10^{-3}\right)$ rad

14–113.

Determine the slope of the shaft at the bearing support A. EI is constant.

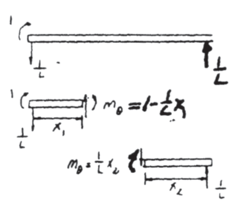


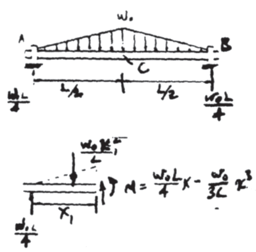
SOLUTION

$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI} dx$$

$$\theta_A = \frac{1}{EI}\left[\int_0^{\frac{L}{2}} \left(1 - \frac{1}{L}x_1\right)\left(\frac{w_0 L}{4}x_1 - \frac{w_0}{3L}x_1^3\right)dx_1 + \int_0^{\frac{L}{2}} \left(\frac{1}{L}x_2\right)\left(\frac{w_0 L}{4}x_2 - \frac{w_0}{3L}x_2^3\right)dx_2\right.$$

$$= \frac{5w_0 L^3}{192EI} \circlearrowright \qquad \textbf{Ans.}$$

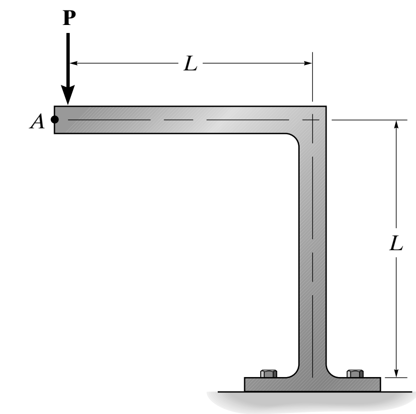




Ans:

$$\theta_A = -\frac{5w_0 L^3}{192\,EI}$$

14–114. Determine the vertical displacement of point A on the angle bracket due to the concentrated force **P**. The bracket is fixed connected to its support. EI is constant. Consider only the effect of bending.

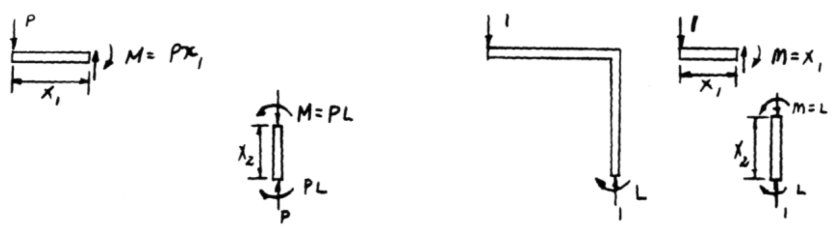


SOLUTION

$$1 \cdot \Delta_{A_v} = \int_0^L \frac{mM}{EI}\,dx$$

$$\Delta_{A_v} = \frac{1}{EI}\left[\int_0^L (x_1)(Px_1)dx_1 + \int_0^L (1L)(PL)dx_2\right]$$

$$= \frac{4PL^3}{3EI}$$ **Ans.**

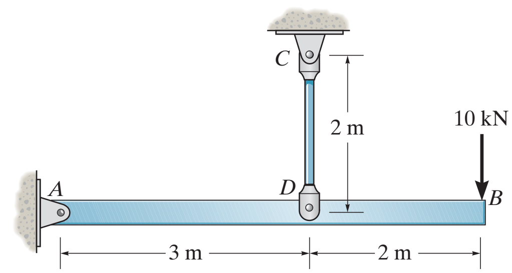


Ans:

$$\Delta_{A_v} = \frac{4PL^3}{3EI}$$

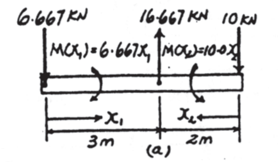
14–115.

Beam AB has a square cross section of 100 mm by 100 mm. Bar CD has a diameter of 10 mm. If both members are made of A992 steel, determine the vertical displacement of point B due to the loading of 10 kN.

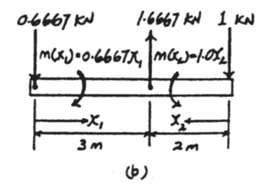


SOLUTION

Real Moment Function $M(x)$: As shown in Figure *a*.



Virtual Moment Functions $m(x)$: As shown in Figure *b*.



Virtual Work Equation: For the displacement at point B,

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx + \frac{nNL}{AE}$$

$$1\text{ kN} \cdot \Delta_B = \frac{1}{EI}\int_0^{3\text{ m}} (0.6667x_1)(6.667x_1)dx_1 + \frac{1}{EI}\int_0^{2\text{ m}} (1.00x_2)(10.0x_2)dx_2 + \frac{1.667(16.667)(2)}{AE}$$

$$\Delta_B = \frac{66.667\text{ kN}\cdot\text{m}^3}{EI} + \frac{55.556\text{ kN}\cdot\text{m}}{AE}$$

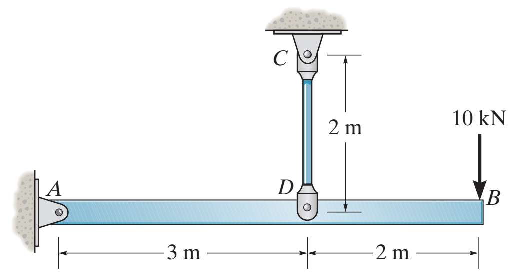
$$= \frac{66.667(1000)}{200(10^9)\left[\frac{1}{12}(0.1)\left(0.1^3\right)\right]} + \frac{55.556(1000)}{\left[\frac{\pi}{4}\left(0.01^2\right)\right]\left[200\left(10^9\right)\right]}$$

$$= 0.04354\text{ m} = 43.5\text{ mm} \quad \downarrow$$ **Ans.**

Ans:
$\Delta_B = 43.5\text{ mm} \downarrow$

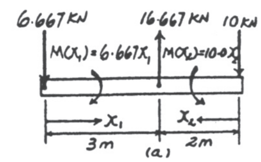
***14–116.**

Beam AB has a square cross section of 100 mm by 100 mm. Bar CD has a diameter of 10 mm. If both members are made of A992 steel, determine the slope at A due to the loading of 10 kN.

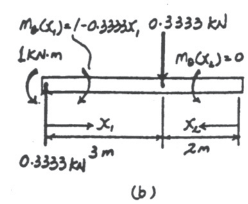


SOLUTION

Real Moment Function $M(x)$: As shown in Figure *a*.



Virtual Moment Functions $m_\theta(x)$: As shown in Figure *b*.



Virtual Work Equation: For the slope at point A,

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\,dx + \frac{nNL}{AE}$$

$$1\ \text{kN}\cdot\text{m}\cdot\theta_A = \frac{1}{EI}\int_0^{3\text{ m}} (1 - 0.3333x_1)(6.667x_1)dx_1$$

$$+ \frac{1}{EI}\int_0^{2\text{ m}} 0(10.0x_2)dx_2 + \frac{(-0.3333)(16.667)(2)}{AE}$$

$$\theta_A = \frac{10.0\ \text{kN}\cdot\text{m}^2}{EI} - \frac{11.111\ \text{kN}}{AE}$$

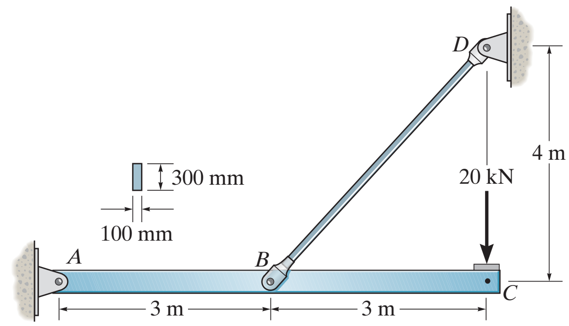
$$= \frac{10.0(1000)}{200\left(10^9\right)\left[\frac{1}{12}(0.1)\left(0.1^3\right)\right]} - \frac{11.111(1000)}{\left[\frac{\pi}{4}\left(0.01^2\right)\right]\left[200\left(10^9\right)\right]}$$

$$= 0.00529\ \text{rad} = 0.303°$$ **Ans.**

Ans:
$\theta_A = -0.303°$

14–117.

Bar *ABC* has a rectangular cross section of 300 mm by 100 mm. Attached rod *DB* has a diameter of 20 mm. If both members are made of A-36 steel, determine the vertical displacement of point *C* due to the loading. Consider only the effect of bending in A*BC* and axial force in *DB*.



SOLUTION

Real Moment Function $M(x)$: As shown in Figure *a*.

Virtual Moment Functions $m(x)$: As shown in Figure *b*.

Virtual Work Equation: For the displacement at point *C*,

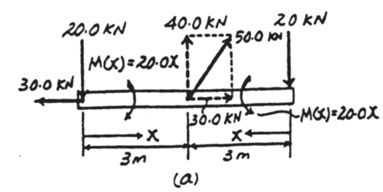
$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx + \frac{nNL}{AE}$$

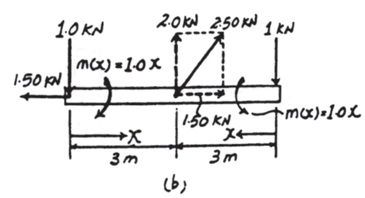
$$1\text{ kN} \cdot \Delta_C = 2\left[\frac{1}{EI}\int_0^{3\text{ m}} (1.00x)(20.0x)\,dx\right] + \frac{2.50(50.0)\,(5)}{AE}$$

$$\Delta_C = \frac{360\text{ kN}\cdot\text{m}^3}{EI} + \frac{625\text{ kN}\cdot\text{m}}{AE}$$

$$= \frac{360(1000)}{200\left(10^9\right)\left[\frac{1}{12}(0.1)\left(0.3^3\right)\right]} + \frac{625(1000)}{\left[\frac{\pi}{4}\left(0.02^2\right)\right]\left[200\left(10^9\right)\right]}$$

$$= 0.017947\text{ m} = 17.9\text{ mm} \downarrow$$ **Ans.**

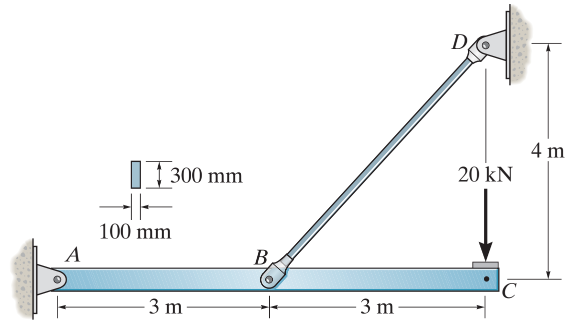




Ans:
$\Delta_C = 17.9\text{ mm}\downarrow$

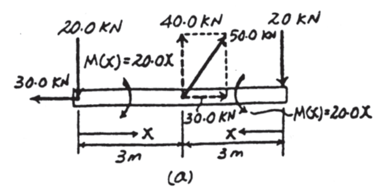
14–118.

Bar ABC has a rectangular cross section of 300 mm by 100 mm. Attached rod DB has a diameter of 20 mm. If both members are made of A-36 steel, determine the slope at A due to the loading. Consider only the effect of bending in ABC and axial force in DB.



SOLUTION

Real Moment Function $M(x)$: As shown in Figure *a*.



Virtual Moment Functions $m_\theta(x)$: As shown in Figure *b*.

Virtual Work Equation: For the slope at point A,

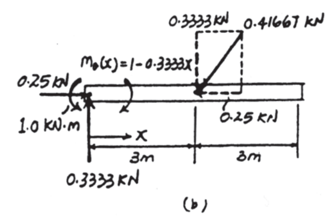
$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}dx + \frac{nNL}{AE}$$

$$1\text{ kN}\cdot\text{m}\cdot\theta_A = \frac{1}{EI}\int_0^{3\text{ m}}(1 - 0.3333x)(20.0x)dx + \frac{(-0.41667)(50.0)(5)}{AE}$$

$$\theta_A = \frac{30.0\text{ kN}\cdot\text{m}^2}{EI} - \frac{104.167\text{ kN}}{AE}$$

$$= \frac{30.0(1000)}{200\left(10^9\right)\left[\frac{1}{12}(0.1)\left(0.3^3\right)\right]} - \frac{104.167(1000)}{\left[\frac{\pi}{4}\left(0.02^2\right)\right]\left[200\left(10^9\right)\right]}$$

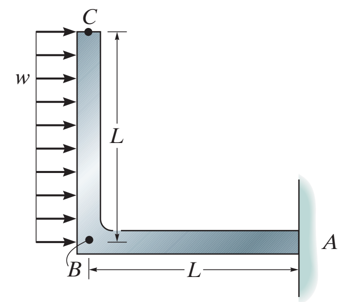
$$= -0.991\left(10^{-3}\right)\text{ rad} = 0.0568^\circ$$ **Ans.**



Ans:
$\theta_A = -0.0568^\circ$

14–119.

The L-shaped frame is made from two segments, each of length L and flexural stiffness EI. If it is subjected to the uniform distributed load, determine the horizontal displacement of point C.

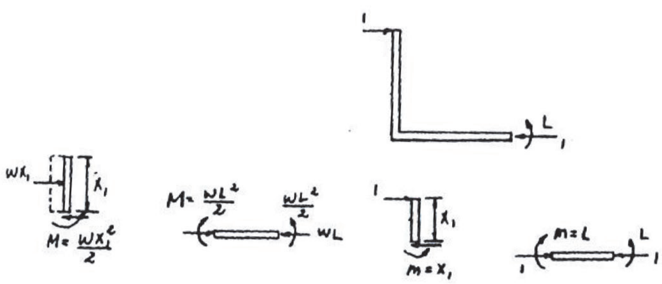


SOLUTION

$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_C = \frac{1}{EI}\left[\int_0^L (1x_1)\left(\frac{wx_1^2}{2}\right)dx_1 + \int_0^L (1L)\left(\frac{wL^2}{2}\right)dx_2\right]$$

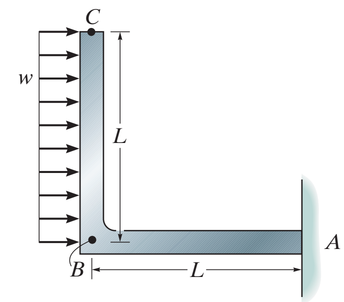
$$= \frac{5wL^4}{8EI} \rightarrow$$ **Ans.**



Ans:
$$(\Delta_C)_h = \frac{5wL^4}{8EI} \rightarrow$$

***14–120.**

The L-shaped frame is made from two segments, each of length L and flexural stiffness EI. If it is subjected to the uniform distributed load, determine the vertical displacement of point B.



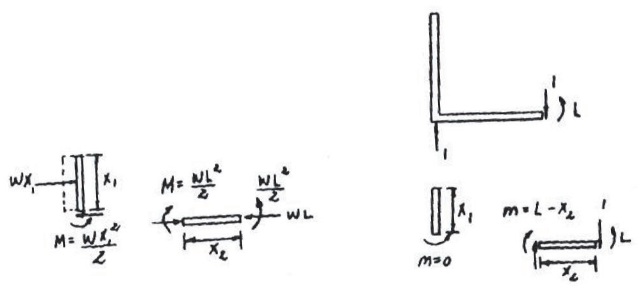
SOLUTION

$$1 \cdot \Delta_B = \int_0^L \frac{mM}{EI}\,dx$$

$$\Delta_B = \frac{1}{EI}\left[\int_0^L (0)\left(\frac{wx_1^2}{2}\right)dx_1 + \int_0^L (L - x_2)\left(\frac{wL^2}{2}\right)dx_2\right]$$

$$= \frac{wL^4}{4EI}\uparrow$$

Ans.

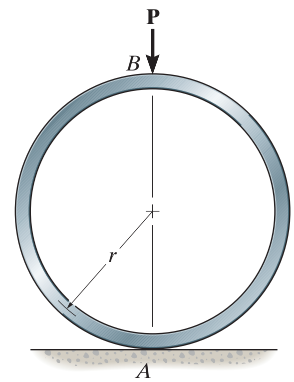


Ans:

$(\Delta_B)_v = \dfrac{wL^4}{4EI}\uparrow$

14–121.

Determine the vertical displacement of the ring at point B. EI is constant.



SOLUTION

Model: The ring can be modeled as a half ring as shown in Figure (a).

Real Moment Function $M(x)$: As shown on Figure (a).

Virtual Moment Functions $m(x)$ and $m_\theta(x)$: As shown on Figure (b) and (c).

Virtual Work Equation: Due to symmetry, the slope at B remains horizontal, i.e., equal to zero. Applying Eq. 14–43, we have

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\, ds \qquad \text{Where } ds = r d\theta$$

$$1 \cdot \theta_B = 0 = \frac{1}{EI}\int_0^\pi 1.00\left(\frac{Pr}{2}\sin\theta - M_0\right) r d\theta$$

$$M_0 = \frac{Pr}{\pi}$$

For the vertical displacement at B, apply Eq. 14–42.

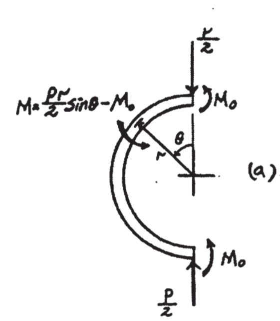
$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\, ds$$

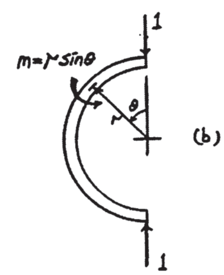
$$1 \cdot (\Delta_B)_v = \frac{1}{EI}\int_0^\pi (r\sin\theta)\left(\frac{Pr}{2}\sin\theta - \frac{Pr}{\pi}\right) r d\theta$$

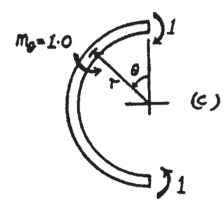
$$= \frac{Pr^3}{2\pi EI}\int_0^\pi \left(\pi \sin^2\theta - 2\sin\theta\right) d\theta$$

$$= \frac{Pr^3}{4\pi EI}\int_0^\pi \left[\pi(1 - \cos 2\theta) - 4\sin\theta\right] d\theta$$

$$(\Delta_B)_v = \frac{Pr^3}{4\pi EI}\left(\pi^2 - 8\right)\downarrow \qquad \textbf{Ans.}$$





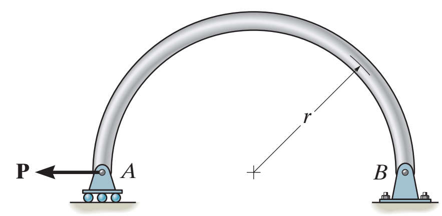


Ans:

$$(\Delta_B)_v = \frac{Pr^3}{4\pi EI}\left(\pi^2 - 8\right)\downarrow$$

14–122.

Determine the horizontal displacement at the roller at A due to the loading. EI is constant.



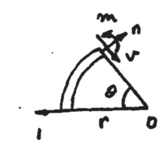
SOLUTION

$\Sigma F_x = 0;$ $\quad N - P\sin\theta = 0$

$$N = P\sin\theta$$

$\Sigma M_o = 0;$ $\quad M - P\sin\theta\, r = 0$

$$M = Pr\sin\theta$$



$\Sigma F_x = 0;$ $\quad n - 1\sin\theta = 0$

$$n = 1\sin\theta$$

$\circlearrowleft + \Sigma M_o = 0;$ $\quad m - (1\sin\theta)\,r = 0$

$$m = r\sin\theta$$

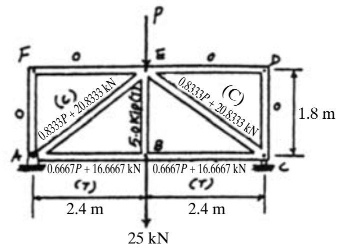
$$1 \cdot (\Delta_A)_h = \int_0^L \frac{mM}{EI}\,dx = \frac{Pr^2}{EI}\int_0^\pi \sin^2\theta\,(r\,d\theta) = \frac{Pr^3}{EI}\left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)\Big|_0^\pi$$

$$(\Delta_A)_h = \frac{\pi Pr^3}{2EI} \leftarrow$$ **Ans.**

Ans:

$(\Delta_A)_h = \dfrac{\pi Pr^3}{2EI} \leftarrow$

14–123. Solve Prob. 14–72 using Castigliano's theorem.



SOLUTION

Member Forces N: Member forces due to external force **P** and external applied forces are shown on the figure.

Castigliano's Second Theorem:

Member	N	$\dfrac{\partial N}{\partial P}$	$N(P = 0)$	L	$N\left(\dfrac{\partial N}{\partial P}\right)L$
AB	$0.6667P + 3.333$	0.6667	16.6667	2.4	26.667
BC	$0.6667P + 3.333$	0.6667	16.6667	2.4	26.667
CD	0	0	0	1.8	0
DE	0	0	0	2.4	0
EF	0	0	0	2.4	0
AF	0	0	0	1.8	0
AE	$-(0.8333P + 4.167)$	-0.8333	-20.8333	3.0	52.083
CE	$-(0.8333P + 4.167)$	-0.8333	-20.8333	3.0	52.083
BE	5.0	0	25.00	1.8	0
					$\Sigma 157.5\ \text{kN}\cdot\text{m}$

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_E)_v = \frac{157.5\ \text{kN}\cdot\text{m}}{AE}$$

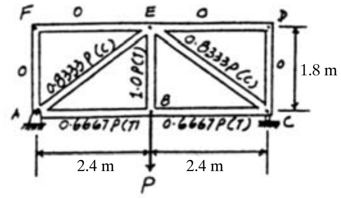
$$= \frac{157.5(10^3)}{[2.80(10^{-3})][200(10^9)]} = 0.28125(10^{-3})\ \text{m} = 0.281\ \text{mm}\ \downarrow \qquad \textbf{Ans.}$$

Ans:

$(\Delta_E)_v = 0.281\ \text{mm}$

***14–124.** Solve Prob. 14–73 using Castigliano's theorem.

SOLUTION



Member Forces N: Member forces due to external force **P** and external applied forces are shown on the figure.

Castigliano's Second Theorem:

Member	N	$\dfrac{\partial N}{\partial P}$	$N(P = 25\text{ kN})$	L	$N\left(\dfrac{\partial N}{\partial P}\right)L$
AB	$0.6667P$	0.6667	16.6667	2.4	26.667
BC	$0.6667P$	0.6667	16.6667	2.4	26.667
CD	0	0	0	1.8	0
DE	0	0	0	2.4	0
EF	0	0	0	2.4	0
AF	0	0	0	1.8	0
AE	$-0.8333P$	-0.8333	–20.8333	3.0	52.083
CE	$-0.8333P$	-0.8333	–20.8333	3.0	52.083
BE	$1.00P$	1.00	25.00	1.8	45.0
					$\Sigma 202.5\text{ kN}\cdot\text{m}$

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_B)_v = \frac{202.5\text{ kN}\cdot\text{m}}{AE}$$

$$= \frac{202.5(10^3)}{[2.80(10^{-3})][200(10^9)]} = 0.3616(10^{-3})\text{ m} = 0.362\text{ mm}\downarrow \qquad \textbf{Ans.}$$

Ans:

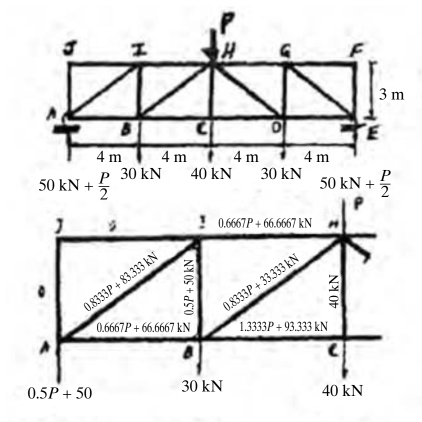
$(\Delta_B)_v = 0.362\text{ mm}$

14–125. Solve Prob. 14–75 using Castigliano's theorem.

SOLUTION

$$\Delta_{Hv} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{2828.89\ \text{kN}\cdot\text{m}}{AE} = \frac{2828.89(10^3)}{[2.80(10^{-3})][200(10^9)]}$$

$$= 5.052(10^{-3})\ \text{m} = 5.05\ \text{mm}$$ **Ans.**



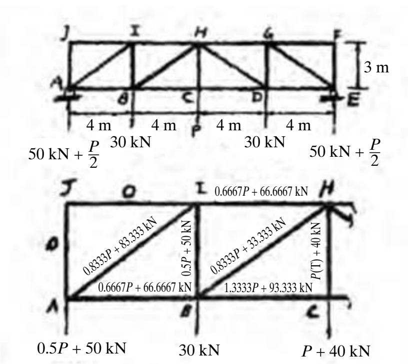
Ans:
$\Delta_{H_v} = 5.05$ mm

14–126. Solve Prob. 14–76 using Castigliano's theorem.

SOLUTION

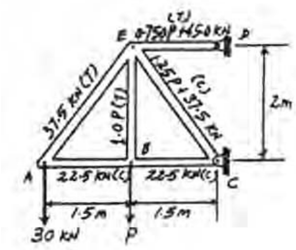
$$\Delta_{C_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{2948.89\ \text{kN}\cdot\text{m}}{AE} = \frac{2848.89(10^3)}{[2.80(10^{-3})][200(10^9)]}$$

$$= 5.266(10^{-3})\ \text{m} = 5.27\ \text{mm} \downarrow$$ **Ans.**



Ans:
$\Delta_{C_v} = 5.27$ mm

14–127. Solve Prob. 14–77 using Castigliano's theorem.



SOLUTION

Member Forces N: Member forces due to external force **P** and external applied forces are shown on the figure.

Castigliano's Second Theorem:

Member	N	$\dfrac{\partial N}{\partial P}$	$N(P = 20\text{ kN})$	L	$N\left(\dfrac{\partial N}{\partial P}\right)L$
AB	-22.5	0	-22.5	1.5	0
BC	-22.5	0	-22.5	1.5	0
AE	37.5	0	37.5	2.5	0
CE	$-(1.25P + 37.5)$	-1.25	-62.5	2.5	195.3125
BE	$1.00P$	1.00	20.0	2	40.0
DE	$0.750P + 45$	0.750	60.0	1.5	67.50
					$\sum 302.8125\text{ kN}\cdot\text{m}$

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_B)_v = \frac{302.8125\text{ kN}\cdot\text{m}}{AE}$$

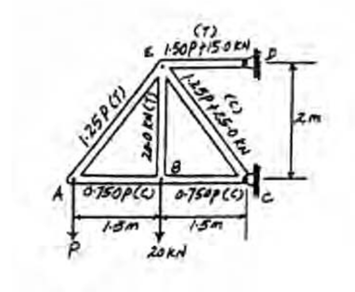
$$= \frac{302.8125\left(10^3\right)}{0.400\left(10^{-3}\right)\left[200\left(10^9\right)\right]}$$

$$= 3.785\left(10^{-3}\right)\text{ m} = 3.79\text{ mm} \downarrow$$ **Ans.**

Ans:

$(\Delta_B)_h = 0.699\ (10^{-3})$ in. $\rightarrow$

***14–128.** Solve Prob. 14–78 using Castigliano's theorem.



SOLUTION

Member Forces N: Member forces due to external force **P** and external applied forces are shown on the figure.

Castigliano's Second Theorem:

Member	N	$\dfrac{\partial N}{\partial P}$	$N(P = 30\text{ kN})$	L	$N\left(\dfrac{\partial N}{\partial P}\right)L$
AB	$-0.750P$	-0.750	-22.5	1.5	25.3125
BC	$-0.750P$	-0.750	-22.5	1.5	25.3125
AE	$1.25P$	1.25	37.5	2.5	117.1875
CE	$-(1.25P + 25.0)$	-1.25	-62.5	2.5	195.3125
BE	20.0	0	20.0	2	0
DE	$1.50P+15.0$	1.50	60.0	1.5	135.00
					$\sum 498.125\text{ kN}\cdot\text{m}$

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_A)_v = \frac{498.125\text{ kN}\cdot\text{m}}{AE}$$

$$= \frac{498.125\left(10^3\right)}{0.400\left(10^{-3}\right)\left[200\left(10^9\right)\right]}$$

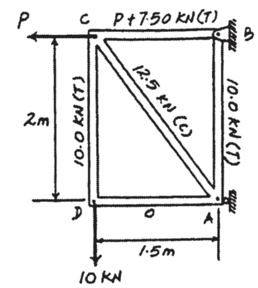
$$= 6.227\left(10^{-3}\right)\text{ m} = 6.23\text{ mm}\downarrow$$ **Ans.**

Ans:
$(\Delta_A)_v = 6.23$ mm

14–129.

Solve Prob. 14–81 using Castigliano's theorem.

SOLUTION



Member Forces *N*: Member forces due to external force **P** and external applied forces are shown on the figure.

Castigliano's Second Theorem: Applying Eq. 14–48, we have

Member	N	$\dfrac{\partial N}{\partial P}$	$N(P = 5\text{ kN})$	L	$N\left(\dfrac{\partial N}{\partial P}\right)L$
AB	10.0	0	10.0	2	0
BC	$1.00P + 7.50$	1.00	12.5	1.5	18.75
CD	10.0	0	10.0	2	0
AD	0	0	0	1.5	0
AC	-12.5	0	-12.5	2.5	0
					Σ 18.75 kN·m

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_C)_h = \frac{18.75\text{ kN}\cdot\text{m}}{AE}$$

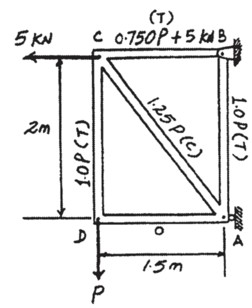
$$= \frac{18.75(10^3)}{0.400(10^{-3})[200(10^9)]}$$

$$= 0.2344(10^{-3})\text{ m} = 0.234\text{ mm} \leftarrow \qquad \textbf{Ans.}$$

Ans:
$(\Delta_C)_h = 0.234\text{ mm} \leftarrow$

14–130.

Solve Prob. 14–82 using Castigliano's theorem.



SOLUTION

Member Forces *N*: Member forces due to external force **P** and external applied forces are shown on the figure.

Castigliano's Second Theorem: Applying Eq. 14–48, we have

Member	N	$\frac{\partial N}{\partial P}$	$N(P = 10\text{ kN})$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AB	$1.00P$	1.00	10.0	2	20.00
BC	$0.750P + 5.00$	0.750	12.5	1.5	14.0625
CD	$1.00P$	1.00	10.0	2	20.00
AD	0	0	0	1.5	0
AC	$-1.25P$	-1.25	-12.5	2.5	39.0625
					Σ 93.125 kN · m

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_D)_v = \frac{93.125\text{ kN}\cdot\text{m}}{AE}$$

$$= \frac{93.125(10^3)}{0.400(10^{-3})[200(10^9)]}$$

$$= 1.164(10^{-3})\text{ m} = 1.16\text{ mm}\downarrow \qquad \textbf{Ans.}$$

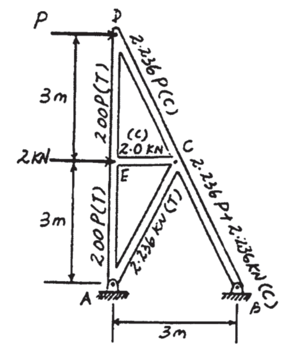
Ans:
$(\Delta_D)_v = 1.16\text{ mm}\downarrow$

14–131.

Solve Prob. 14–85 using Castigliano's theorem.

SOLUTION

Member Forces *N*: Member forces due to external force **P** and external applied forces are shown on the figure.



Castigliano's Second Theorem: Applying Eq. 14–48, we have

Member	N	$\dfrac{\partial N}{\partial P}$	$N(P = 4\text{ kN})$	L	$N\left(\dfrac{\partial N}{\partial P}\right)L$
AE	$2.00P$	2.00	8.00	3	48.00
ED	$2.00P$	2.00	8.00	3	48.00
CD	$-2.236P$	-2.236	-8.944	3.354	67.082
BC	$-(2.236P + 2.236)$	-2.236	-11.180	3.354	83.853
CE	-2.00	0	-2.00	1.5	0
AC	2.236	0	2.236	3.354	0
					Σ 246.935 kN·m

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_D)_h = \frac{246.935\text{ kN}\cdot\text{m}}{AE}$$

$$= \frac{246.935(10^3)}{0.300(10^{-3})[200(10^9)]}$$

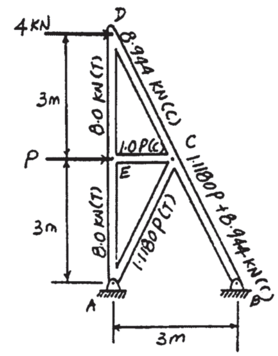
$$= 4.116(10^{-3})\text{ m} = 4.12\text{ mm} \rightarrow \qquad \textbf{Ans.}$$

Ans:
$(\Delta_D)_h = 4.12\text{ mm} \rightarrow$

***14–132.**

Solve Prob. 14–86 using Castigliano's theorem.

SOLUTION



Member Forces N: Member forces due to external force **P** and external applied forces are shown on the figure.

Castigliano's Second Theorem: Applying Eq. 14–48, we have

Member	N	$\dfrac{\partial N}{\partial P}$	$N(P = 2\text{ kN})$	L	$N\left(\dfrac{\partial N}{\partial P}\right)L$
AE	8.00	0	8.00	3	0
ED	8.00	0	8.00	3	0
CD	−8.944	0	−8.944	3.354	0
BC	$-(1.118P + 8.944)$	−1.118	−11.180	3.354	41.926
CE	$-1.00P$	−1.00	−2.00	1.5	3.00
AC	$1.118P$	1.118	2.236	3.354	8.385
					Σ 53.312 kN·m

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_E)_h = \frac{53.312\text{ kN}\cdot\text{m}}{AE}$$

$$= \frac{53.312(10^3)}{0.300(10^{-3})[200(10^9)]}$$

$$= 0.8885(10^{-3})\text{ m} = 0.889\text{ mm} \rightarrow \qquad \textbf{Ans.}$$

Ans:
$(\Delta_E)_h = 0.889\text{ mm} \rightarrow$

14–133.

Solve Prob. 14–90 using Castigliano's theorem.

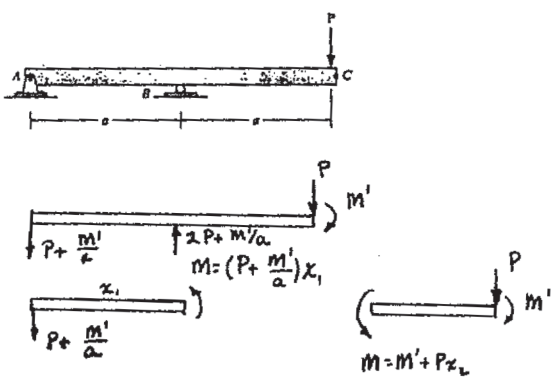
SOLUTION

Set $M' = 0$

$$\theta_C = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

$$= \int_0^a \frac{(Px_1)(\frac{1}{a}x_1)dx_1}{EI} + \int_0^a \frac{(Px_2)(1)dx_2}{EI}$$

$$= \frac{Pa^2}{3EI} + \frac{Pa^2}{2EI} = \frac{5Pa^2}{6EI}$$ **Ans.**



Ans:
$\theta_C = -\dfrac{5Pa^2}{6EI}$

14–134.

Solve Prob. 14–91 using Castigliano's theorem.

SOLUTION

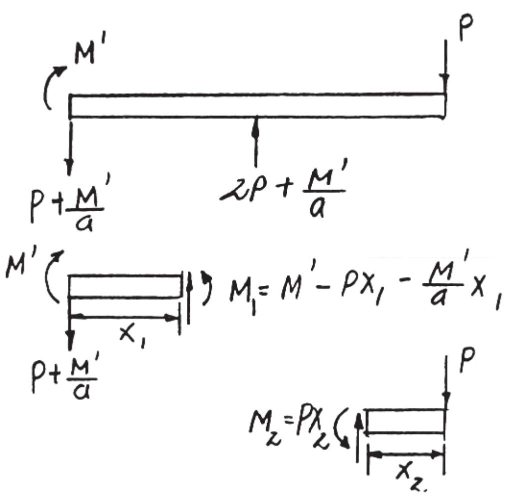
$$\frac{\partial M_1}{\partial M'} = 1 - \frac{x_1}{a} \qquad \frac{\partial M_2}{\partial M'} = 0$$

Set $M' = 0$

$$M_1 = -Px_1 \qquad M_2 = Px_2$$

$$\theta_A = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \frac{1}{EI}\left[\int_0^a (-Px_1)\left(1 - \frac{x_1}{a}\right)dx_1 + \int_0^a (Px_2)(0)dx_2\right] = \frac{-Pa^2}{6EI}$$

$$= \frac{Pa^2}{6EI}$$ **Ans.**



Ans:
$\theta_A = \dfrac{Pa^2}{6EI}$

14–135. Solve Prob. 14–106 using Castigliano's theorem.

SOLUTION

$$\frac{\partial M_1}{\partial p} = 0 \quad \frac{\partial M_2}{\partial p} = 1.5 - 0.5x_2 \quad \frac{\partial M_3}{\partial p} = x_3$$

Set $p = 0$

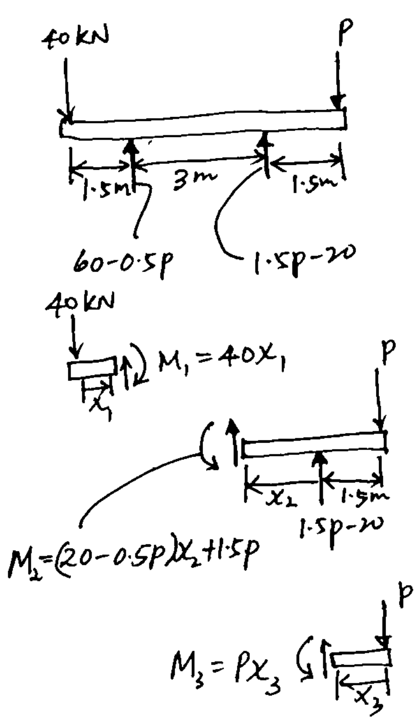
$$M_1 = 40x_1 \quad M_2 = 20x_2 \quad M_3 = 0$$

$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial p}\right)\frac{dx}{EI}$$

$$= \frac{1}{EI}\left[\int_0^{1.5\text{m}} 40x_1(0)dx_1 + \int_0^{3\text{m}} 20x_2(1.5 - 0.5x_2)dx_2 + \int_0^{3\text{m}} (0)(x_3)dx_3\right]$$

$$= \frac{45\ \text{kN}\cdot\text{m}^3}{EI} = \frac{45(10^3)}{[200(10^9)][22.3(10^{-6})]}$$

$$= 0.01009\ \text{m} = 10.1\ \text{mm}\ \downarrow$$ **Ans.**



Ans:
$\Delta_C = 10.1$ mm

***14–136.** Solve Prob. 14–107 using Castigliano's theorem.

SOLUTION

$$\frac{\partial M_1}{\partial M'} = 0 \qquad \frac{\partial M_2}{\partial M'} = 1 - 0.3333x_2 \qquad \frac{\partial M_3}{\partial M'} = 0$$

Set $M' = 0$

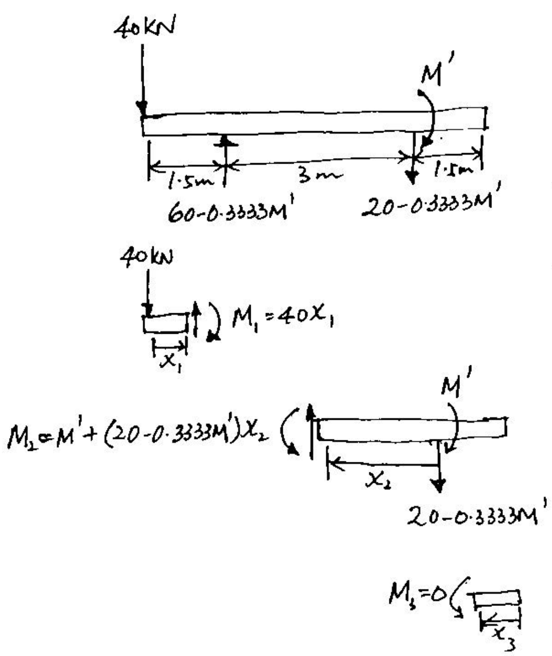
$$M_1 = 40x_1 \qquad M_2 = 20x_2 \qquad M_3 = 0$$

$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

$$= \frac{1}{EI}\left[\int_0^{1.5\text{m}} 40x_1(0)dx_1 + \int_0^{3\text{m}} 20x_2(1 - 0.3333x_2)dx_2 + 0\right]$$

$$= \frac{30 \text{ kN}\cdot\text{m}^2}{EI} = \frac{30(10^3)}{[200(10^9)][22.3(10^{-6})]}$$

$$= 6.7265(10^{-3}) \text{ rad} = 0.385° \quad \theta_B$$ **Ans.**

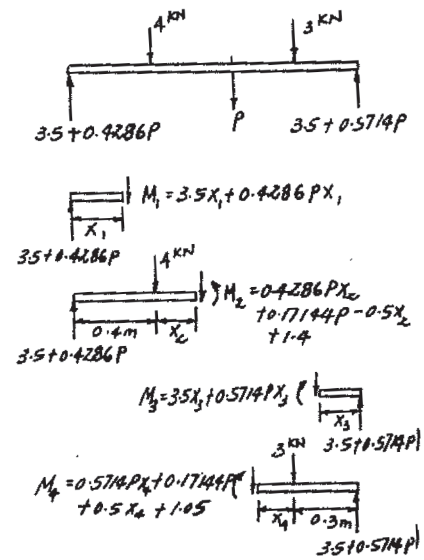


Ans:
$\theta_B = 0.385°$

14–137.

Solve Prob. 14–95 using Castigliano's theorem.

SOLUTION



$$\frac{\partial M_1}{\partial P} = 0.4286x_1 \qquad \frac{\partial M_2}{\partial P} = 0.4286x_2 + 0.17144$$

$$\frac{\partial M_3}{\partial P} = 0.5714x_3 \qquad \frac{\partial M_4}{\partial P} = 0.5714x_4 + 0.17144$$

Set $P = 2$ kN

$$M_1 = 4.3572x_1 \qquad M_2 = 0.3572x_2 + 1.7429$$

$$M_3 = 4.6428x_3 \qquad M_4 = 1.6428x_4 + 1.3929$$

$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

$$= \frac{1}{EI}\left[\int_0^{0.4}(4.3572x_1)(0.4286x_1)\,dx_1 + \int_0^{0.4}(0.3572x_2 + 1.7429)(0.4286x_2 + 0.17144)dx_2 + \int_0^{0.3}(4.6428x_3)(0.5714x_3)dx_3 + \int_0^{0.3}(1.6428x_4 + 1.3929)(0.5714x_4 + 0.17144)dx_4\right]$$

$$= \frac{0.37944\text{ kN}\cdot\text{m}^3}{EI} = \frac{0.37944(10^3)}{200(10^9)\frac{\pi}{4}(0.015)^4} = 0.0478\text{ m} = 47.8\text{ mm}$$ **Ans.**

Ans:
$\Delta_B = 47.8$ mm

14–138.

Solve Prob. 14–96 using Castigliano's theorem.

SOLUTION

$$\frac{\partial M_1}{\partial P} = 0 \qquad \frac{\partial M_2}{\partial P} = -0.5x_2$$

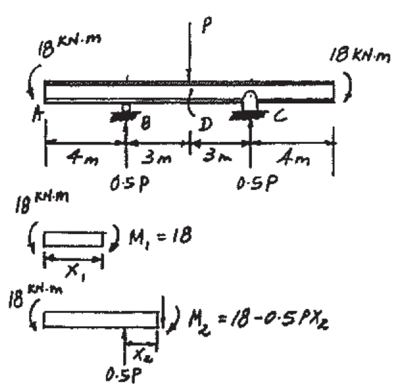
Set $P = 0$

$M_1 = 18 \qquad M_2 = 18$

$$\Delta_D = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

$$= (2)\frac{1}{EI}\left[\int_0^4 (18)(0)dx_1 + \int_0^3 (18)(-0.5x_2)dx_2\right]$$

$$= \frac{81 \text{ kN}\cdot\text{m}^3}{EI} = \frac{81(10)^3}{200(10^9)(125)(10^{-6})} = 3.24(10^{-3}) \text{ m} = 3.24 \text{ mm}$$ **Ans.**



Ans:
$\Delta_D = 3.24$ mm

14–139.

Solve Prob. 14–97 using Castigliano's theorem.

SOLUTION

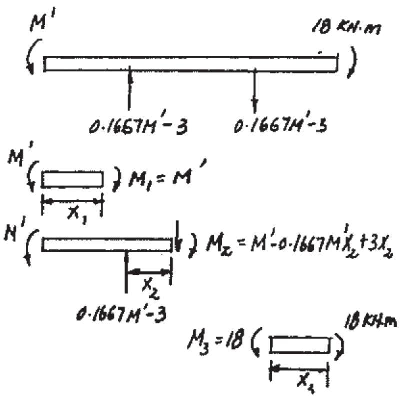
$$\frac{\partial M_1}{\partial M'} = 1 \qquad \frac{\partial M_2}{\partial M'} = 1 - 0.1667x_2 \qquad \frac{\partial M_3}{\partial M'} = 0$$

Set $M' = 18\ \text{kN} \cdot \text{m}$

$$M_1 = 18\ \text{kN} \cdot \text{m} \qquad M_2 = 18\ \text{kN} \cdot \text{m} \qquad M_3 = 18\ \text{kN} \cdot \text{m}$$

$$\theta_A = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \frac{1}{EI}\left[\int_0^4 (18)(1)dx_1 + \int_0^6 18(1 - 0.1667x_2)dx_2 + \int_0^4 (18)(0)dx_3\right]$$

$$= \frac{126\ \text{kN} \cdot \text{m}^2}{EI} = \frac{126(10^3)}{200(10^9)(125)(10^{-6})} = 5.04(10^{-3})\ \text{rad} = 0.289° \qquad \textbf{Ans.}$$



Ans:
$\theta_A = 0.289°$

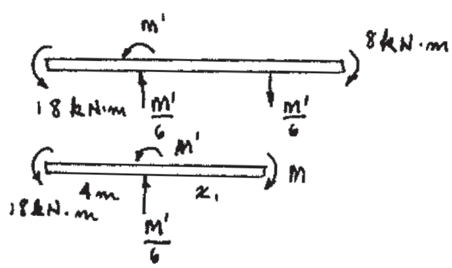
***14–140.**

Solve Prob. 14–98 using Castigliano's theorem.

SOLUTION

$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \int_0^6 \frac{(-18)\left(-\frac{1}{6}x\right)dx(10^3)}{EI}$$

$$= \frac{18(6^2)(10^3)}{6(2)(200)(10^9)(125)(10^{-6})} = 0.00216 \text{ rad} = 0.124°$$ **Ans.**



Ans:
$\theta_B = 0.124°$

14–141.

Solve Prob. 14–108 using Castigliano's theorem.

SOLUTION

M' does not influence the moment within the overhang.

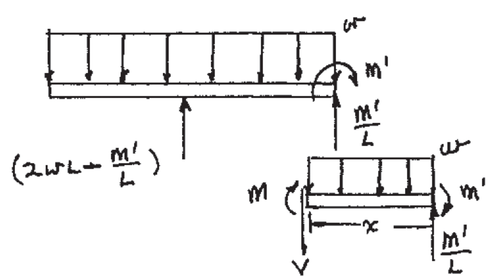
$$M = \frac{M'}{L}x - M' - \frac{wx^2}{2}$$

$$\frac{\partial M}{\partial M'} = \frac{x}{L} - 1$$

Setting $M' = 0$,

$$\theta_A = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \frac{1}{EI}\int_0^L\left(-\frac{wx^2}{2}\right)\left(\frac{x}{L} - 1\right)dx = \frac{-w}{2EI}\left[\frac{L^3}{4} - \frac{L^3}{3}\right]$$

$$= \frac{wL^3}{24EI}$$ **Ans.**



Ans:
$\theta_A = \frac{wL^3}{24EI}$

14–142.

Solve Prob. 14–119 using Castigliano's theorem.

SOLUTION

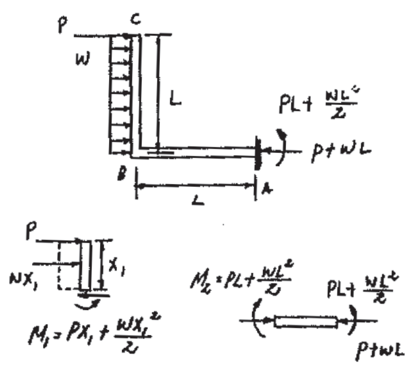
$$M_1 = Px_1 + \frac{wx_1^2}{2}$$

$$\frac{\partial M_1}{\partial P} = x_1 \qquad \frac{\partial M_2}{\partial P} = L$$

Setting $P = 0$

$$M_1 = \frac{wx_1^2}{2} \qquad M_2 = \frac{wL^2}{2}$$

$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \frac{1}{EI}\left[\int_0^L \frac{wx_1^2}{2}(x_1)dx_1 + \int_0^L \frac{wL^2}{2}Ldx_2\right] = \frac{5wL^4}{8EI} \qquad \textbf{Ans.}$$



Ans:

$$\Delta_C = \frac{5wL^4}{8EI}$$

14–143.

Solve Prob. 14–120 using Castigliano's theorem.

SOLUTION

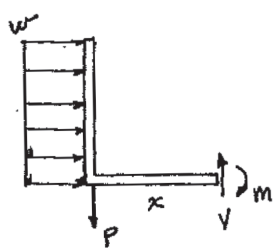
P does not influence moment within segment.

$$M = Px = \frac{wL^2}{2}$$

$$\frac{\partial M}{\partial P} = x$$

Set $P = 0$

$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^L \left(-\frac{wL^2}{2}\right)(x)\frac{dx}{EI} = \frac{wL^4}{4EI}$$ **Ans.**



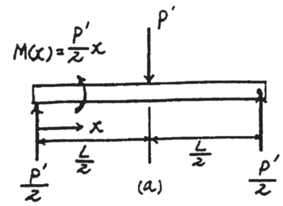
Ans:

$\Delta_B = \dfrac{wL^4}{4EI}$

***14–144.**

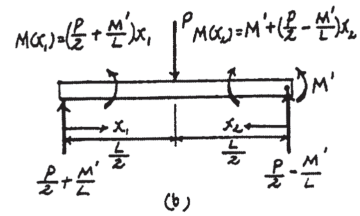
Solve Prob. 14–105 using Castigliano's theorem.

SOLUTION



Internal Moment Function $M(x)$: The internal moment function in terms of the load $\mathbf{P}'$ and couple of moment $\mathbf{M}'$ and externally applied load are shown on figures (a) and (b), respectively.

Castigliano's Second Theorem: The displacement at C can be determined using Eq. 14–49 with $\dfrac{\partial M(x)}{\partial P'} = \dfrac{x}{2}$ and set $P' = P$.



$$\Delta = \int_0^L M\left(\frac{\partial M}{\partial P'}\right)\frac{dx}{EI}$$

$$\Delta_C = 2\left[\frac{1}{EI}\int_0^{\frac{L}{2}}\left(\frac{P}{2}x\right)\left(\frac{x}{2}\right)dx\right]$$

$$= \frac{PL^3}{48EI}\downarrow \qquad \textbf{Ans.}$$

To determine the slope at B, we apply Eq. 14–50 with $\dfrac{\partial M(x_1)}{\partial M'} = \dfrac{x_1}{L}, \dfrac{\partial M(x_2)}{\partial M'} = 1 - \dfrac{x_2}{L}$ and setting $M' = 0$.

$$\theta = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

$$\theta_B = \frac{1}{EI}\int_0^{\frac{L}{2}}\left(\frac{P}{2}x_1\right)\left(\frac{x_1}{L}\right)dx_1 + \frac{1}{EI}\int_0^{\frac{L}{2}}\left(\frac{P}{2}x_2\right)\left(1 - \frac{x_2}{L}\right)dx_2$$

$$= \frac{PL^2}{16EI} \qquad \textbf{Ans.}$$

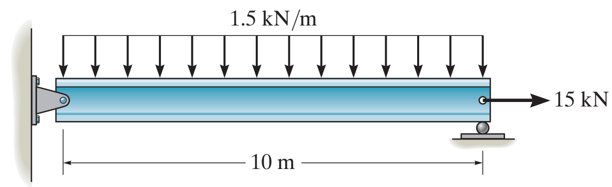
Ans:

$$\Delta_C = \frac{PL^3}{48EI}\downarrow$$

$$\theta_B = \frac{PL^2}{16EI}$$

R14–1.

Determine the total axial and bending strain energy in the A992 steel beam. $A = 2300\text{ mm}^2$, $I = 9.5(10^6)\text{ mm}^4$.



SOLUTION

Axial Load:

$$(U_e)_i = \int_0^L \frac{N^2 dx}{2EA} = \frac{N^2 L}{2EA}$$

$$(U_e)_i = \frac{((15)(10^3))^2(10)}{2(200)(10^9)(2.3)(10^{-3})} = 2.4456\text{ J}$$

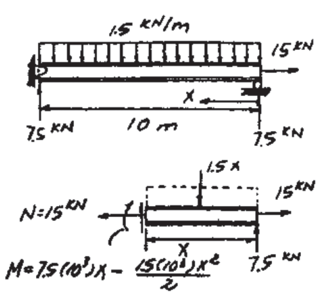
Bending:

$$(U_b)_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\int_0^{10} [(7.5)(10^3)x - 0.75(10^3)x^2]^2 dx$$

$$= \frac{1}{2EI}\int_0^{10} [56.25(10^6)x^2 + 562.5(10^3)x^4 - 11.25(10^6)x^3]\, dx$$

$$(U_b)_i = \frac{0.9375(10^9)}{200(10^9)(9.5)(10^{-6})} = 493.4210\text{ J}$$

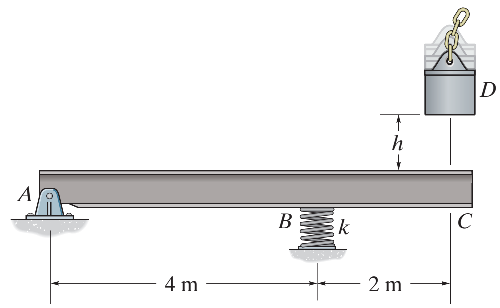
$$U_i = (U_a)_i + (U_b)_i = 2.4456 + 493.4210 = 496\text{ J}$$ **Ans.**



Ans:
$U_i = 496\text{ J}$

R14–2.

The 200-kg block D is dropped from a height $h = 1$ m onto end C of the A992 steel W200 × 36 overhang beam. If the spring at B has a stiffness $k = 200$ kN/m, determine the maximum bending stress developed in the beam.



SOLUTION

Equilibrium: The support reactions and the moment functions for regions AB and BC of the beam under static conditions are indicated on the free-body diagram of the beam, Fig. a.

$$U_e = U_i$$

$$\frac{1}{2}P\Delta_{st} = \Sigma\int_0^L \frac{M^2dx}{2EI}$$

$$\frac{1}{2}P\Delta_{st} = \frac{1}{2EI}\left[\int_0^{4\text{ m}}\left(\frac{P}{2}x_2\right)^2 dx + \int_0^{2\text{ m}}(Px_1)^2\,dx\right]$$

$$\Delta_{st} = \frac{8P}{EI}$$

Here, $I = 34.4\left(10^6\right)\text{ mm}^4 = 34.4\left(10^{-6}\right)\text{ m}^4$ (see the appendix) and $E = E_{st} = 200$ GPa. Then, the equivalent spring constant can be determined from

$$P = k_b\,\Delta_{st}$$

$$P = k_b\left(\frac{8P}{EI}\right)$$

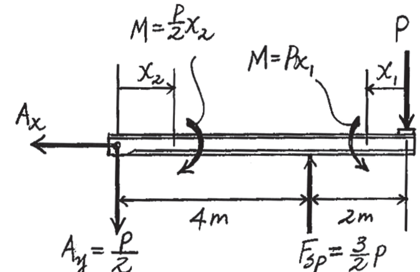
$$k_b = \frac{EI}{8} = \frac{200\left(10^9\right)\left[34.4\left(10^{-6}\right)\right]}{8} = 860\left(10^3\right)\text{ N/m}$$

From the free-body diagram,

$$F_{sp} = \frac{3}{2}P$$

$$k_{sp}\Delta_{sp} = \frac{3}{2}(k_b\Delta_b)$$

$$\Delta_{sp} = \frac{3}{2}\left(\frac{k_b}{k_{sp}}\right)\Delta_b = \frac{3}{2}\left(\frac{860\left(10^3\right)}{200\left(10^3\right)}\right)\Delta_b = 6.45\,\Delta_b \qquad \textbf{(1)}$$



(a)

Conservation of Energy:

$$mg\left(h + \Delta_b + \frac{3}{2}\Delta_{sp}\right) = \frac{1}{2}k_{sp}\Delta_{sp}^{\,2} + \frac{1}{2}k_b\Delta_b^{\,2}$$

R14–2. Continued

SOLUTION

Substiuting Eq. (1) into this equation,

$$200(9.81)\left[1 + \Delta_b + \frac{3}{2}(6.45\Delta_b)\right] = \frac{1}{2}\left[200\left(10^3\right)\right](6.45\Delta_b)^2 + \frac{1}{2}\left[860\left(10^3\right)\right]\Delta_b{}^2$$

$$4590.25\left(10^3\right)\Delta_b{}^2 - 20944.35\Delta_b - 1962 = 0$$

Solving for the positive root

$$\Delta_b = 0.02308 \text{ m}$$

Maximum Stress: The maximum force on the beam is $P_{\max} = k_b\Delta_b = 860\left(10^3\right)(0.02308) = 19.85\left(10^3\right)$ N. The maximum moment occurs at the supporting spring, where $M_{\max} = P_{\max}L = 19.85\left(10^3\right)(2) = 39.70\left(10^3\right)$ N·m. Applying the flexure formula with $c = \dfrac{d}{2} = \dfrac{0.201}{2} = 0.1005$ m,

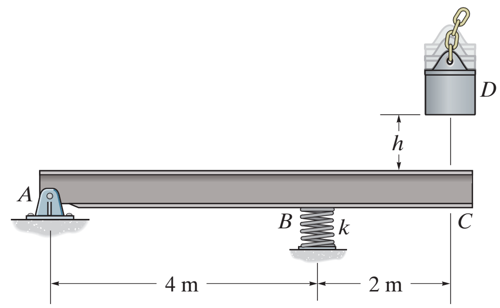
$$\sigma_{\max} = \frac{M_{\max}c}{I} = \frac{39.70\left(10^3\right)(0.1005)}{34.4\left(10^{-6}\right)} = 115.98 \text{ MPa} = 116 \text{ MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{\max} < \sigma_Y = 345$ MPa, this result is valid.

Ans:
$\sigma_{\max} = 116$ MPa

R14–3.

Determine the maximum height h from which the 200-kg block D can be dropped without causing the A992 steel W200 × 36 overhang beam to yield. The spring at B has a stiffness $k = 200$ kN/m.



SOLUTION

Equilibrium: The support reactions and the moment functions for regions AB and BC of the beam under static conditions are indicated on the free-body diagram of the beam, Fig. *a*.

$$U_e = U_i$$

$$\frac{1}{2} P\Delta_{st} = \Sigma \int_0^L \frac{M^2 dx}{2EI}$$

$$\frac{1}{2} P\Delta_{st} = \frac{1}{2EI}\left[\int_0^{4\text{ m}} \left(\frac{P}{2}x_2\right)^2 dx + \int_0^{2\text{ m}} (Px_1)^2\, dx\right]$$

$$\Delta_{st} = \frac{8P}{EI}$$

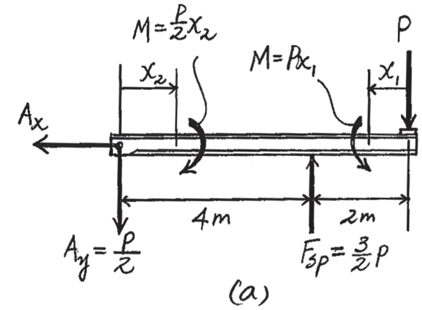
Here, $I = 34.4\left(10^6\right)\text{ mm}^4 = 34.4\left(10^{-6}\right)\text{ m}^4$ (see the appendix) and $E = E_{st} = 200$ GPa. Then, the equivalent spring constant can be determined from

$$P = k_b\, \Delta_{st}$$

$$P = k_b\left(\frac{8P}{EI}\right)$$

$$k_b = \frac{EI}{8} = \frac{200\left(10^9\right)\left[34.4\left(10^{-6}\right)\right]}{8} = 860\left(10^3\right)\text{ N/m}$$

From the free-body diagram,



$$F_{sp} = \frac{3}{2}P$$

$$k_{sp}\Delta_{sp} = \frac{3}{2}(k_b\Delta_b)$$

$$\Delta_{sp} = \frac{3}{2}\left(\frac{k_b}{k_{sp}}\right)\Delta_b = \frac{3}{2}\left(\frac{860\left(10^3\right)}{200\left(10^3\right)}\right)\Delta_b = 6.45\Delta_b \qquad \textbf{(1)}$$

R14–3. Continued

SOLUTION

Maximum Stress: The maximum force on the beam is $P_{\max} = k_b\Delta_b = 860\left(10^3\right)\Delta_b$. The maximum moment occurs at the supporting spring, where $M_{\max} = P_{\max}L = 860\left(10^3\right)\Delta_b(2) = 1720\left(10^3\right)\Delta_b$. Applying the flexure formula with $c = \dfrac{d}{2} = \dfrac{0.201}{2} = 0.1005$ m,

$$\sigma_{\max} = \frac{M_{\max}c}{I}$$

$$345\left(10^6\right) = \frac{1720\left(10^3\right)\Delta_b(0.1005)}{34.4\left(10^{-6}\right)}$$

$$\Delta_b = 0.06866 \text{ m}$$

Substituting this result into Eq. (1),

$$\Delta_{sp} = 0.44284 \text{ m}$$

Conservation of Energy:

$$mg\left(h + \Delta_b + \frac{3}{2}\Delta_{sp}\right) = \frac{1}{2}k_{sp}\Delta_{sp}^{\,2} + \frac{1}{2}k_b\Delta_b^{\,2}$$

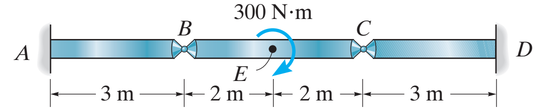
$$200(9.81)\left[h + 0.06866 + \frac{3}{2}(0.44284)\right] = \frac{1}{2}\left[200\left(10^3\right)\right](0.44284)^2 + \frac{1}{2}\left[860\left(10^3\right)\right](0.06866)^2$$

$$h = 10.3 \text{ m}$$ **Ans.**

Ans:
$h = 10.3$ m

***R14–4.**

The A992 steel bars are pin connected at B and C. If they each have a diameter of 30 mm, determine the slope at E.



SOLUTION

$$U_i = \int_0^L \frac{M^2 dx}{2EI} = (2)\frac{1}{2EI}\int_0^3 (75x_1)^2 dx_1 + (2)\frac{1}{2EI}\int_0^2 (-75x_2)^2 dx_2 = \frac{65625}{EI}$$

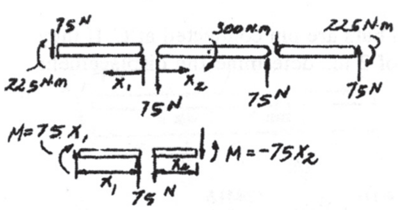
$$U_e = \frac{1}{2}(M')\theta = \frac{1}{2}(300)\theta_E = 150\theta_E$$

Conservation of Energy:

$$U_e = U_i$$

$$150\theta_E = \frac{65625}{EI}$$

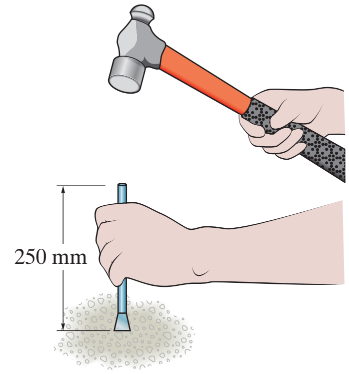
$$\theta_E = \frac{473.5}{EI} = \frac{473.5}{(200)(10^9)(\frac{\pi}{4})(0.015^4)} = 0.0550 \text{ rad} = 3.15° \circlearrowright$$ **Ans.**



Ans:
$\theta_E = -3.15°$

R14–5.

The steel chisel has a diameter of 12 mm and a length of 250 mm. It is struck by a hammer of mass 1.5 kg, and at the instant of impact it is moving at 3.6 m/s. Determine the maximum compressive stress in the chisel, assuming that 80% of the impacting energy goes into the chisel. $E_{st} = 200$ GPa, $\sigma_Y = 700$ MPa.



SOLUTION

$$k = \frac{AE}{L} = \frac{\left[\frac{\pi}{4}(0.012^2)\right][200(10^9)]}{0.25} = 90.4779(10^6)\ \text{N/m}$$

$$0.8U_e = U_i$$

$$0.8\left[\frac{1}{2}(1.5)(3.6^2) + 1.5(9.81)\Delta_{\max}\right] = \frac{1}{2}[90.4779(10^6)]\Delta_{\max}^2$$

$$45.2389(10^6)\Delta_{\max}^2 - 11.772\Delta_{\max} - 7.776 = 0$$

$$\Delta_{\max} = 0.41472(10^{-3})\ \text{m}$$

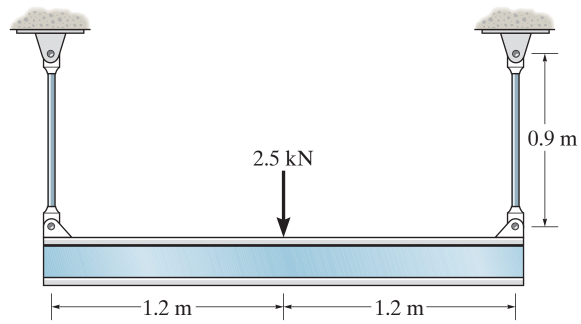
$$P_{\max} = k\Delta_{\max} = [90.4779(10^6)][0.41472(10^{-3})] = 37.5233(10^3)\ \text{N}$$

$$\sigma_{\max} = \frac{P_{\max}}{A} = \frac{37.5233(10^3)}{\frac{\pi}{4}(0.012^2)} = 331.78(10^6)\ \text{N/m}^2 = 332\ \text{MPa} < \sigma_Y \quad \text{O.K.} \qquad \textbf{Ans.}$$

Ans:
$\sigma_{\max} = 332$ MPa

R14–6.

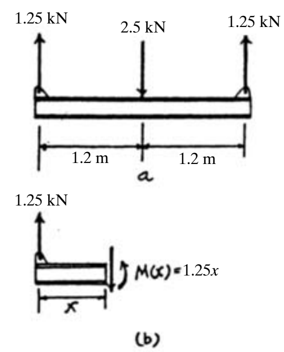
Determine the total strain energy in the A-36 steel assembly. Consider the axial strain energy in the two 12-mm-diameter rods and the bending strain energy in the beam for which $I = 17.0(10^6)\ \text{mm}^4$.



SOLUTION

Support Reactions: As shown FBD(a).

Internal Moment Function: As shown on FBD(b).



Total Strain Energy:

$$(U_i)_T = \int_0^L \frac{M^2\,dx}{2EI} + \frac{N^2L}{2AE}$$

$$= 2\left[\frac{1}{2EI}\int_0^{1.2\text{ m}}(1.25x)^2dx\right] + 2\left[\frac{1.25^2(0.9)}{2AE}\right]$$

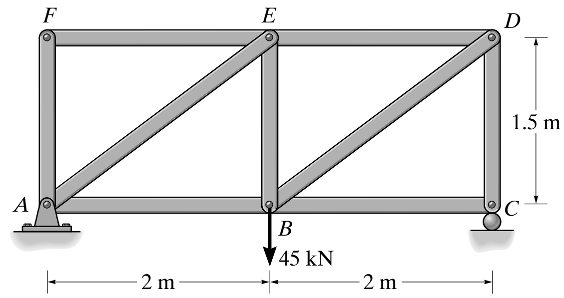
$$= \frac{0.9\ \text{kN}^2\cdot\text{m}^3}{EI} + \frac{1.40625\ \text{kN}^2\cdot\text{m}^3}{AE}$$

$$= \frac{0.9\ \text{kN}^2\cdot\text{m}^3}{[200(10^9)\ \text{N}/\text{m}^2][17.0(10^{-6})\ \text{m}^4]} + \frac{1.40625(10^3\ \text{N})^2\cdot\text{m}^3}{\left[\frac{\pi}{4}(0.012\ \text{m})^2\right][200(10^9)\ \text{N}/\text{m}^2]}$$

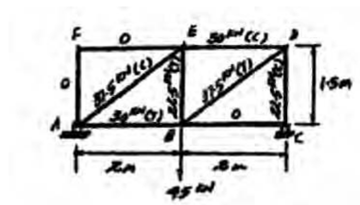
$$= 0.3269\ \text{N}\cdot\text{m} = 0.327\ \text{J}$$ **Ans.**

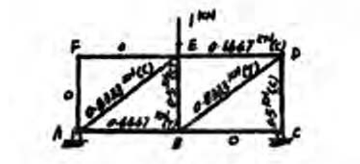
Ans:
$(U_i)_T = 0.327\ \text{J}$

R14–7. Determine the vertical displacement of joint E. For each member $A = 400\ \text{mm}^2$, $E = 200$ GPa. Use the method of virtual work.



SOLUTION





Member	n	N	L	nNL
AF	0	0	1.5	0
AE	−0.8333	−37.5	2.5	78.125
AB	0.6667	30.0	2.0	40.00
EF	0	0	2.0	0
EB	−0.50	22.5	1.5	−16.875
ED	−0.6667	−30.0	2.0	40.00
BC	0	0	2.0	0
BD	0.8333	37.5	2.5	78.125
CD	−0.5	−22.5	1.5	16.875
				$\Sigma = 236.25$

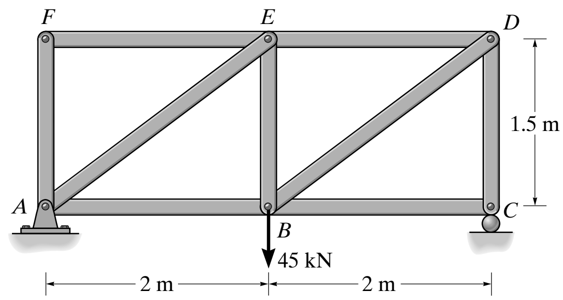
$$1 \cdot \Delta_{B_v} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{B_v} = \frac{236.25\left(10^3\right)}{400\left(10^{-6}\right)(200)\left(10^9\right)} = 2.95\left(10^{-3}\right) = 2.95\text{ mm}$$ **Ans.**

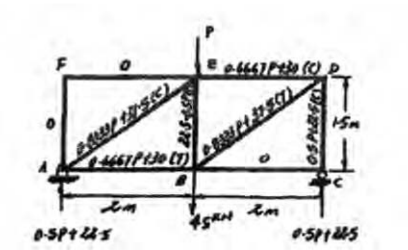
Ans:

$\Delta_{B_v} = 2.95$ mm

***R14–8.** Solve Prob. 14–152 using Castigliano's theorem.



SOLUTION



Member	N	$\partial N/\partial P$	$N(P = 45)$	L	$N(\partial N/\partial P)L$
AF	0	0	0	1.5	0
AE	$-(0.8333P + 37.5)$	-0.8333	-37.5	2.5	78.125
AB	$0.6667P + 30$	0.6667	30.0	2.0	40.00
BE	$22.5–0.5P$	-0.5	22.5	1.5	-16.875
BD	$0.8333P + 37.5$	0.8333	37.5	2.5	78.125
BC	0	0	0	2.0	0
CD	$-(0.5P + 22.5)$	-0.5	-22.5	1.5	16.875
DE	$-(0.6667P + 30)$	-0.6667	-30.0	2.0	40.00
EF	0	0	0	2.0	0
					$\Sigma = 236.25$

$$\Delta_{B_v} = \Sigma N \frac{\partial N}{\partial P} \frac{L}{AE} = \frac{236.25}{AE}$$

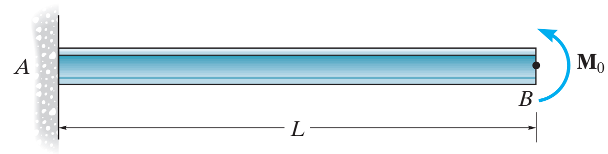
$$= \frac{236.25\left(10^3\right)}{400\left(10^{-6}\right)(200)\left(10^9\right)} = 2.95\left(10^{-3}\right)\text{m} = 2.95 \text{ mm} \qquad \textbf{Ans.}$$

Ans:

$\Delta_{B_v} = 2.95$ mm

R14–9.

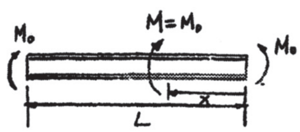
The cantilevered beam is subjected to a couple moment $\mathbf{M}_0$ applied at its end. Determine the slope of the beam at B. EI is constant. Use the method of virtual work.

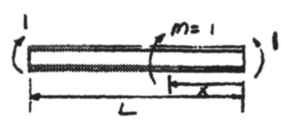


SOLUTION

$$\theta_B = \int_0^L \frac{m_\theta M}{EI}\,dx = \int_0^L \frac{(1)\,M_0}{EI}\,dx$$

$$= \frac{M_0 L}{EI} \qquad \textbf{Ans.}$$



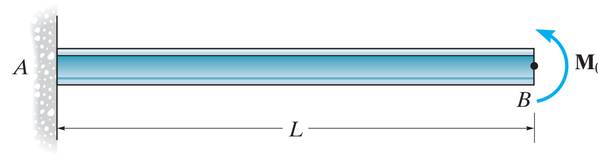


Ans:

$\theta_B = \dfrac{M_0 L}{EI}$

R14–10.

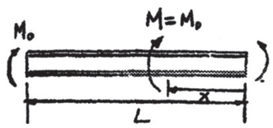
Solve Prob. R14–9 using Castigliano's theorem.



SOLUTION

$$\theta_B = \int_0^L m\left(\frac{dm}{dm'}\right)\frac{dy}{EI} = \int_0^L \frac{M_0(1)}{EI}\,dx$$

$$= \frac{M_0 L}{EI} \qquad \textbf{Ans.}$$

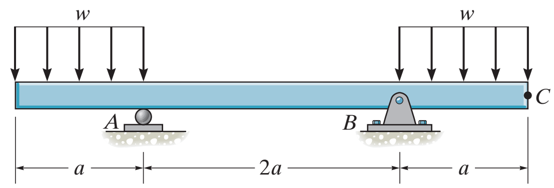


Ans:

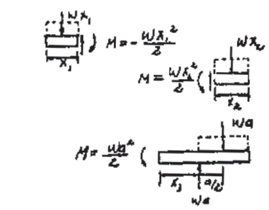
$\theta_B = \dfrac{M_0 L}{EI}$

R14–11.

Determine the slope and displacement at point C. EI is constant.



SOLUTION

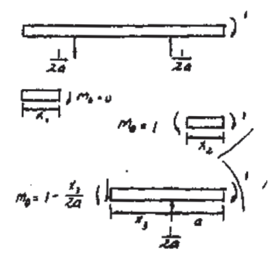


$$\theta_C = \int_0^L \frac{m_\theta M}{EI} dx$$

$$= \frac{1}{EI}\left[\int_0^a (0)\left(\frac{wx_1^2}{2}\right)dx_1 + \int_0^a (1)\left(\frac{wx_2^2}{2}\right)dx_2 + \int_0^{2a}\left(1 - \frac{x_3}{2a}\right)\left(\frac{wa^2}{2}\right)dx_3\right]$$

$$= -\frac{2\,w\,a^3}{3\,E\,I}$$

Ans.

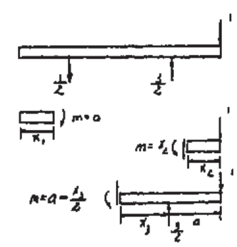


$$\Delta_C = \int_0^L \frac{m\,M}{EI} dx$$

$$= \frac{1}{EI}\left[\int_0^a (0)\left(\frac{wx_1^2}{2}\right)dx_1 + \int_0^a (x_2)\left(\frac{wx_2^2}{2}\right)dx_2 + \int_0^{2a}\left(a - \frac{x_3}{2}\right)\left(\frac{wa^2}{2}\right)dx_3\right]$$

$$= \frac{5\,w\,a^4}{8\,E\,I}\downarrow$$

Ans.



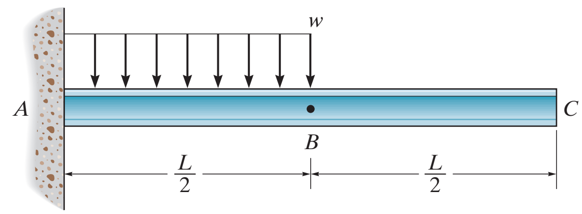
Ans:

$$\theta_C = -\frac{2wa^3}{3EI}$$

$$\Delta_C = \frac{5wa^4}{8EI}\downarrow$$

***R14–12.**

Determine the displacement at B. EI is constant.



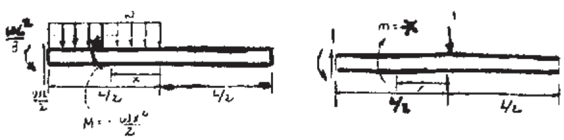
SOLUTION

$$1 \cdot \Delta_B = \int_0^L \frac{mM}{EI}\,dx$$

$$\Delta_B = \int_0^{\frac{L}{2}} \frac{(-1x)\left(\frac{-wx^2}{2}\right)}{EI}\,dx = \frac{w\left(\frac{L}{2}\right)^4}{8EI}$$

$$= \frac{wL^4}{128\,EI}\downarrow$$

Ans.



Ans:

$$\Delta_B = \frac{wL^4}{128\,EI}\downarrow$$