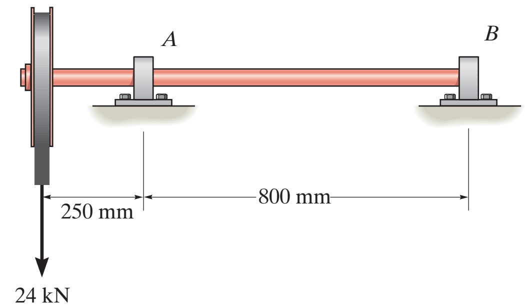
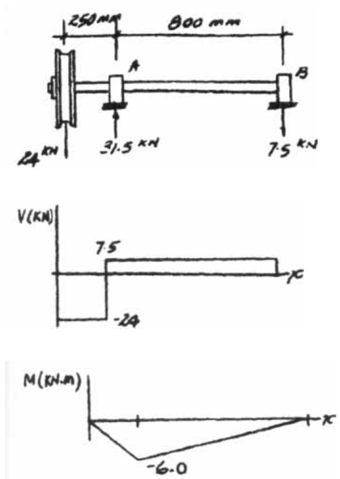
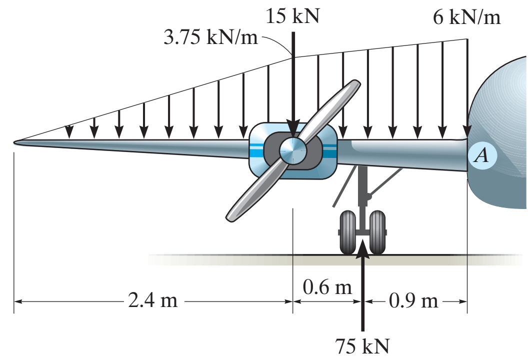
6–1. Draw the shear and moment diagrams for the shaft. The bearings at A and B exert only vertical reactions on the shaft.





Ans:
$x = 0.25^{-}, V = -24, M = -6$

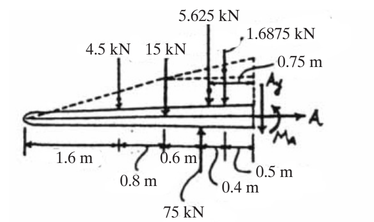
6–2. The dead-weight loading along the centerline of the airplane wing is shown. If the wing is fixed to the fuselage at A, determine the reactions at A, and then draw the shear and moment diagram for the wing.



Support Reactions:

$+\uparrow \Sigma F_y = 0; \qquad -4.5 - 15 + 75 - 5.625 - 1.6875 - A_y = 0$

$A_y = 48.1875 \text{ kN} = 48.2 \text{ kN}$ **Ans.**



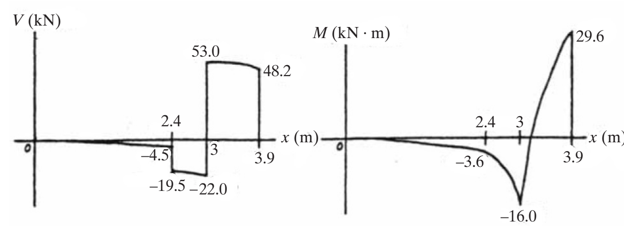
$\circlearrowleft +\Sigma M_A = 0; \qquad 4.5(2.3) + 15(1.5) - 75(0.9)$

$+ 5.625(0.75) + 1.6875(0.5) + M_A = 0$

$M_A = 29.5875 \text{ kN} \cdot \text{m} = 29.6 \text{ kN} \cdot \text{m}$ **Ans.**

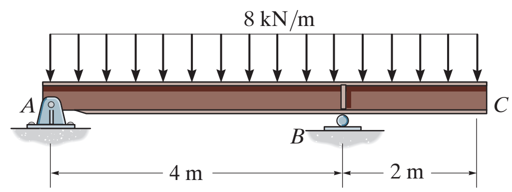
$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$ **Ans.**

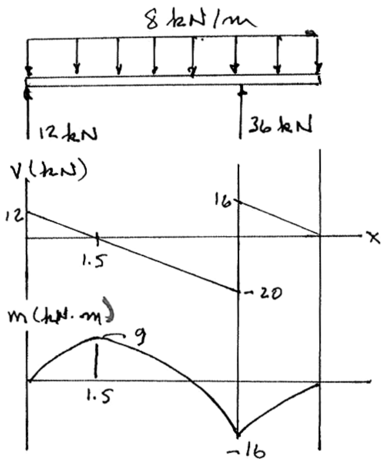
Shear and Moment Diagram:



Ans:
$A_y = 48.2 \text{ kN}, A_x = 0$

6–3. Draw the shear and moment diagrams for the overhang beam.

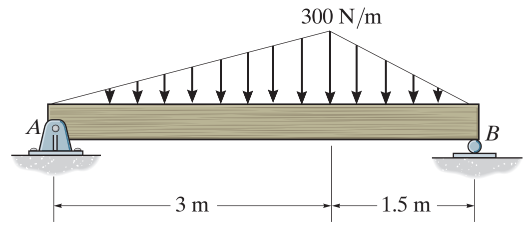




Ans:
$x = 1.5, V = 0, M = 9, x = 4^-,$
$V = -20, M = -16$

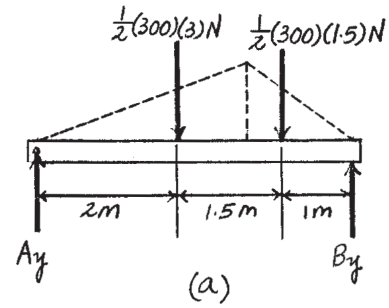
***6–4.**

Express the shear and moment in terms of x for $0 < x < 3$ m and $3\text{ m} < x < 4.5$ m, and then draw the shear and moment diagrams for the simply supported beam.



SOLUTION

Support Reactions: Referring to the free-body diagram of the entire beam shown in Fig. a,



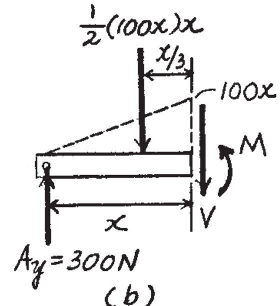
$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(4.5) - \frac{1}{2}(300)(3)(2) - \frac{1}{2}(300)(1.5)(3.5) = 0$$

$$B_y = 375 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + 375 - \frac{1}{2}(300)(3) - \frac{1}{2}(300)(1.5) = 0$$

$$A_y = 300 \text{ N}$$

Shear and Moment Function: For $0 \le x < 3$ m, we refer to the free-body diagram of the beam segment shown in Fig. b.

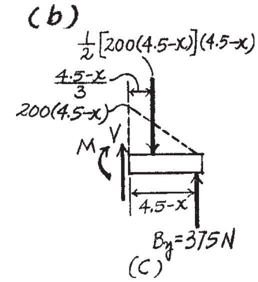


$$+\uparrow \Sigma F_y = 0; \qquad 300 - \frac{1}{2}(100x)x - V = 0$$

$$V = \{300 - 50x^2\} \text{ N} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M = 0; \quad M + \frac{1}{2}(100x)x\left(\frac{x}{3}\right) - 300x = 0$$

$$M = \left\{300x - \frac{50}{3}x^3\right\} \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

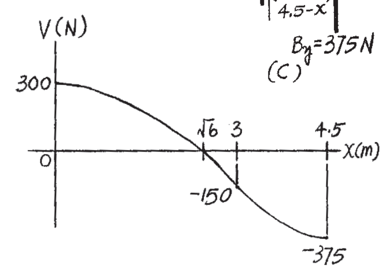


When $V = 0$, from the shear function,

$$0 = 300 - 50x^2 \qquad x = \sqrt{6}\text{ m}$$

Substituting this result into the moment equation,

$$M|_{x=\sqrt{6}\text{ m}} = 489.90 \text{ N}\cdot\text{m}$$

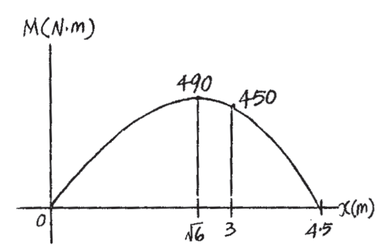


For $3\text{ m} < x \le 4.5$ m, we refer to the free-body diagram of the beam segment shown in Fig. c.

$$+\uparrow \Sigma F_y = 0; \qquad V + 375 - \frac{1}{2}[200(4.5 - x)](4.5 - x) = 0$$

$$V = \left\{100(4.5 - x)^2 - 375\right\} \text{N} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M = 0; \quad 375(4.5 - x) - \frac{1}{2}[200(4.5 - x)](4.5 - x)\left(\frac{4.5 - x}{3}\right) - M = 0$$

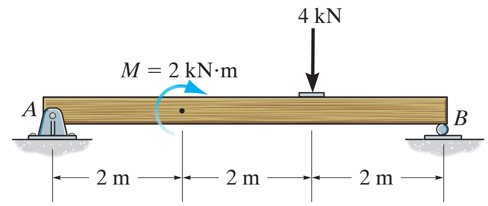


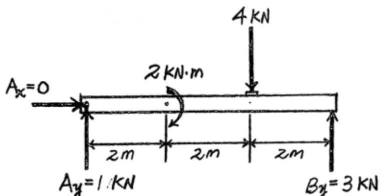
$$M = \left\{375(4.5 - x) - \frac{100}{3}(4.5 - x)^3\right\} \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

Shear and Moment Diagrams: As shown in Figs. d and e.

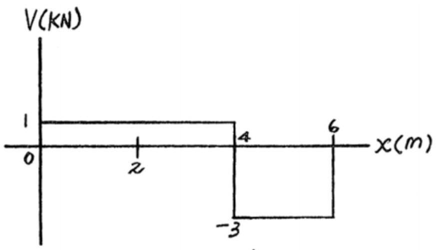
Ans:
For $0 \le x < 3$ m, $V = \{300 - 50x^2\}$ N,
$M = \left\{300x - \frac{50}{3}x^3\right\}$ N·m,
For $3\text{ m} < x \le 4.5$ m, $V = \left\{100(4.5 - x)^2 - 375\right\}$ N,
$M = \left\{375(4.5 - x) - \frac{100}{3}(4.5 - x)^3\right\}$ N·m

6–5. Draw the shear and moment diagrams for the simply supported beam.



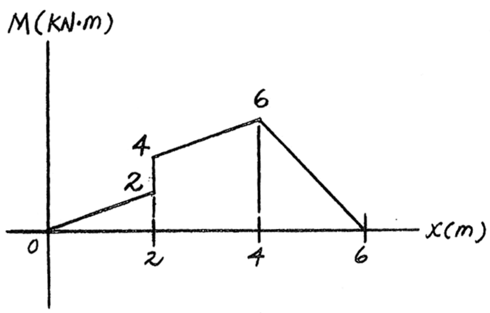


(a)



shear diagram

(b)



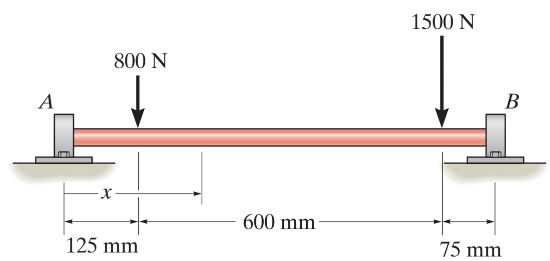
moment diagram

(c)

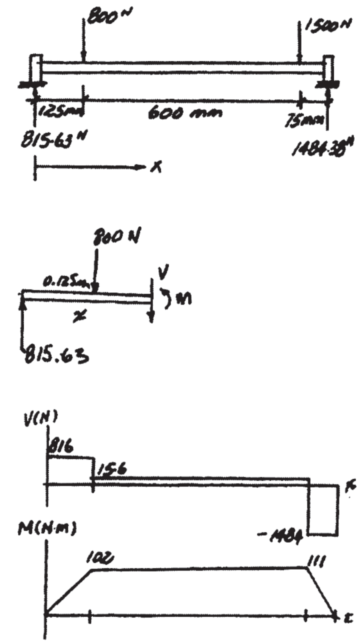
Ans:
$x = 2^-, V = 1, M = 2, x = 4^-, V = 1, M = 6$

6–6.

Draw the shear and moment diagrams for the shaft. The bearings at A and B exert only vertical reactions on the shaft. Also, express the shear and moment in the shaft as a function of x within the region 125 mm $< x <$ 725 mm.



SOLUTION



$+\uparrow\Sigma F_y = 0;\qquad 815.63 - 800 - V = 0$

$$V = 15.6\ \text{N}$$ **Ans.**

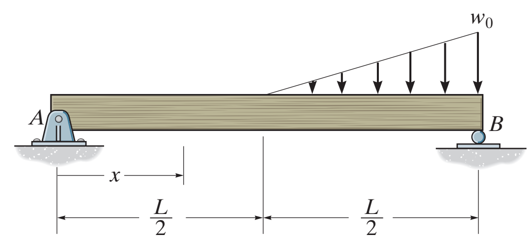
$\circlearrowleft + \Sigma M = 0;\qquad M + 800(x - 0.125) - 815.63\,x = 0$

$$M = (15.6x + 100)\ \text{N}\cdot\text{m}$$ **Ans.**

Ans:
$V = 15.6$ N,
$M = (15.6x + 100)$ N·m

6–7.

Express the internal shear and moment in terms of x for $0 \le x < L/2$, and $L/2 < x < L$, and then draw the shear and moment diagrams.



SOLUTION

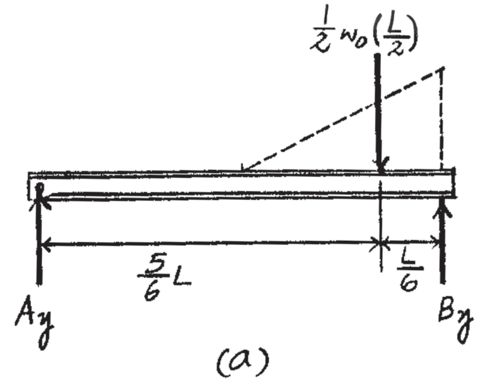
Support Reactions: Referring to the free-body diagram of the entire beam shown in Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(L) - \frac{1}{2} w_0\left(\frac{L}{2}\right)\left(\frac{5}{6}L\right) = 0$$

$$B_y = \frac{5}{24} w_0 L$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + \frac{5}{24} w_0 L - \frac{1}{2} w_0\left(\frac{L}{2}\right) = 0$$

$$A_y = \frac{w_0 L}{24}$$



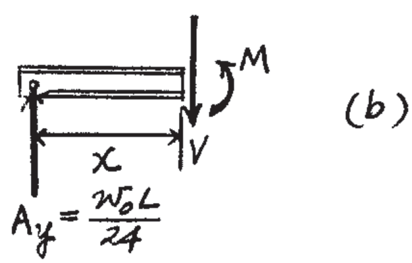
Shear and Moment Function: For $0 \le x < \frac{L}{2}$, we refer to the free-body diagram of the beam segment shown in Fig. *b*.

$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0 L}{24} - V = 0$$

$$V = \frac{w_0 L}{24} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M = 0; \qquad M - \frac{w_0 L}{24} x = 0$$

$$M = \frac{w_0 L}{24} x \qquad \textbf{Ans.}$$



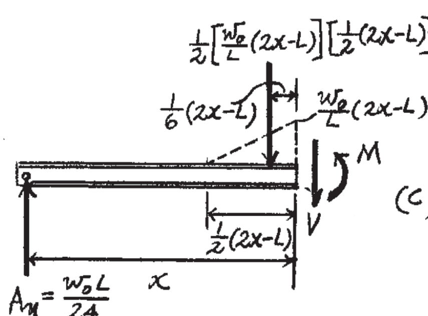
For $\frac{L}{2} < x \le L$, we refer to the free-body diagram of the beam segment shown in Fig. *c*.

$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0 L}{24} - \frac{1}{2}\left[\frac{w_0}{L}(2x - L)\right]\left[\frac{1}{2}(2x - L)\right] - V = 0$$

$$V = \frac{w_0}{24L}\left[L^2 - 6(2x - L)^2\right] \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M = 0; \qquad M + \frac{1}{2}\left[\frac{w_0}{L}(2x - L)\right]\left[\frac{1}{2}(2x - L)\right]\left[\frac{1}{6}(2x - L)\right] - \frac{w_0}{24L}x = 0$$

$$M = \frac{w_0}{24L}\left[L^2 x - (2x - L)^3\right] \qquad \textbf{Ans.}$$



6–7. Continued

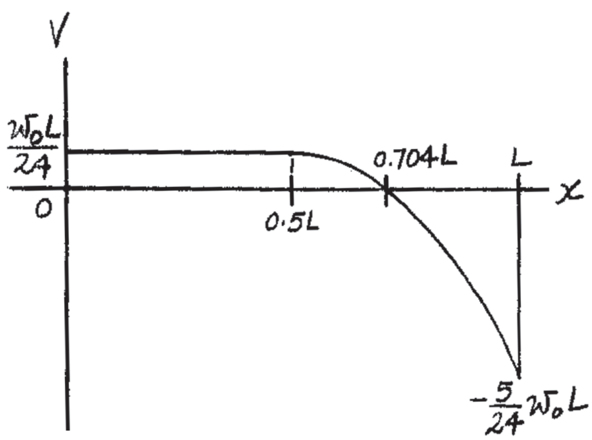
When $V = 0$, the shear function gives

$$0 = L^2 - 6(2x - L)^2 \qquad x = 0.7041L$$

Substituting this result into the moment equation,

$$M|_{x=0.7041L} = 0.0265 w_0 L^2$$

Shear and Moment Diagrams: As shown in Figs. *d* and *e*.



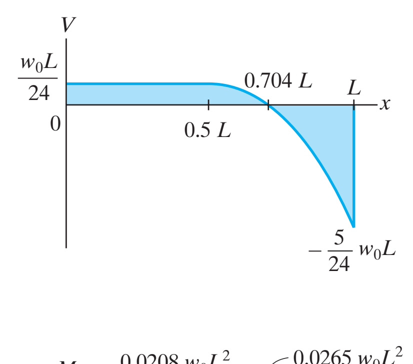
(d)

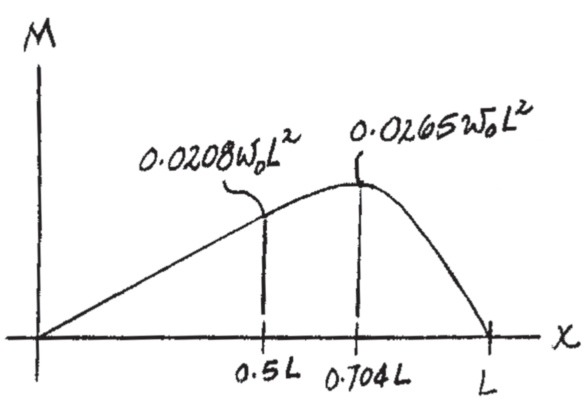
Ans:

For $0 \le x < \frac{L}{2}$: $V = \frac{w_0 L}{24}$, $M = \frac{w_0 L}{24}x$,

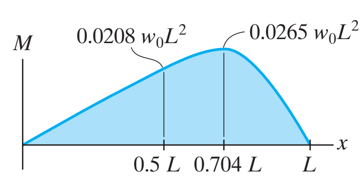
For $\frac{L}{2} < x \le L$: $V = \frac{w_0}{24L}\left[L^2 - 6(2x - L)^2\right]$,

$M = \frac{w_0}{24L}\left[L^2 x - (2x - L)^3\right]$

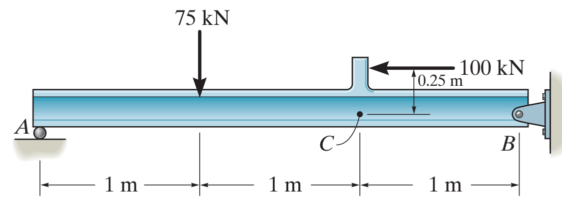


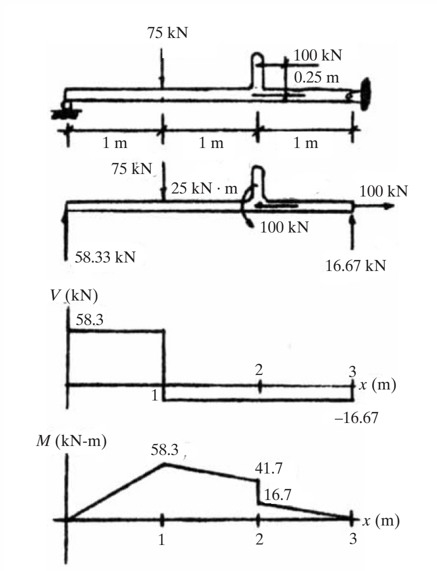


(e)

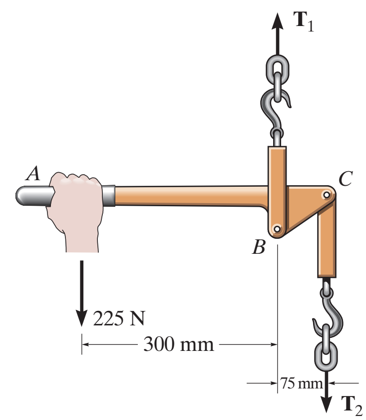


***6–8.** Draw the shear and moment diagrams for the beam. *Hint:* The 100-kN load must be replaced by equivalent loadings at point C on the axis of the beam.



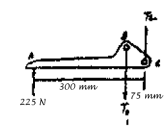


6–9. The load binder is used to support a load. If the force applied to the handle is 225 N, determine the tensions T_1 and T_2 in each end of the chain and then draw the shear and moment diagrams for the arm ABC.



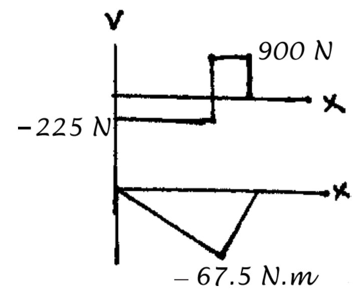
$\circlearrowleft + \Sigma M_C = 0;\quad 225(375) - T_1\ (75) = 0$

$T_1 = 1125\text{ N}$ **Ans.**

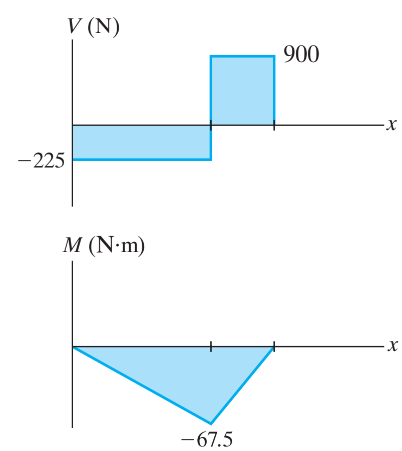


$+\downarrow \Sigma F_y = 0;\quad 225 - 1125 + T_2 = 0$

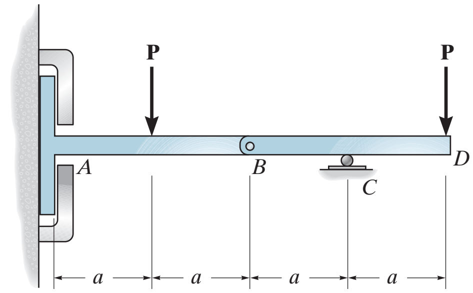
$T_2 = 900\text{ N}$ **Ans.**



Ans:
$T_1 = 1125\text{ N},\ T_2 = 900\text{ N}$



6–10. Draw the shear and moment diagrams for the compound beam. It is supported by a smooth plate at A which slides within the groove and so it cannot support a vertical force, although it can support a moment and axial load.



Support Reactions:

From the FBD of segment BD

$\zeta + \Sigma M_C = 0; \quad B_y(a) - P(a) = 0 \quad B_y = P$

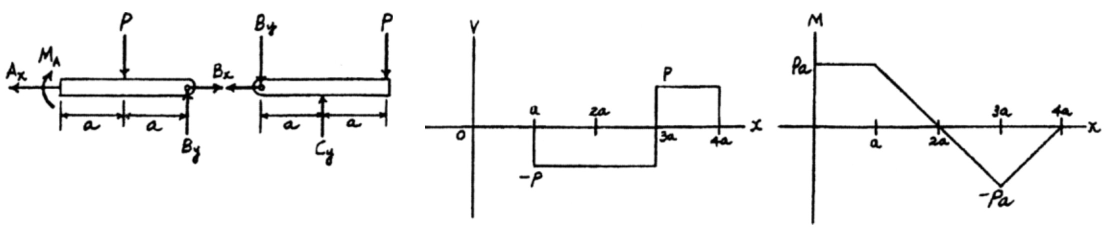
$+\uparrow \Sigma F_y = 0; \quad C_y - P - P = 0 \quad C_y = 2P$

$\xrightarrow{+} \Sigma F_x = 0; \quad B_x = 0$

From the FBD of segment AB

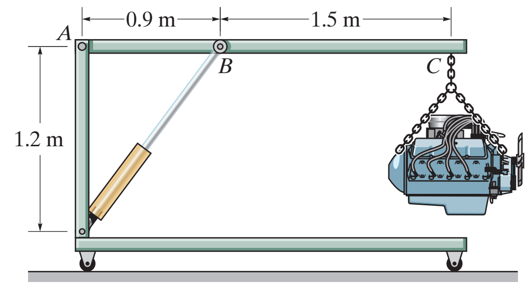
$\zeta + \Sigma M_A = 0; \quad P(2a) - P(a) - M_A = 0 \quad M_A = Pa$

$+\uparrow \Sigma F_y = 0; \quad P - P = 0$ (equilibrium is statisfied!)



Ans:
$x = 3a^-, V = -P, M = -Pa$

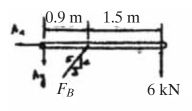
6–11. The engine crane is used to support the engine, which has a weightof 6 kN. Draw the shear and moment diagrams of the boom ABC when it is in the horizontal position shown.

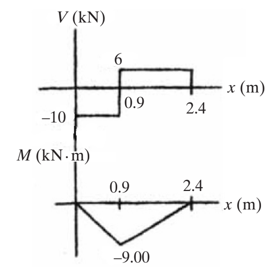


$\zeta + \Sigma M_A = 0;\quad \frac{4}{5}F_B(0.9) - 6(2.4) = 0 \qquad F_B = 20\text{ kN}$

$+\uparrow \Sigma F_y = 0;\quad -A_y + \frac{4}{5}(20) - 6 = 0; \qquad A_y = 10\text{ kN}$

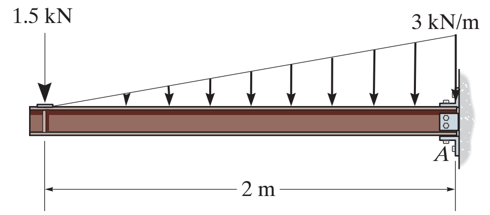
$\overset{+}{\leftarrow} \Sigma F_x = 0;\quad A_x - \frac{3}{5}(20) = 0; \qquad A_x = 12\text{ kN}$

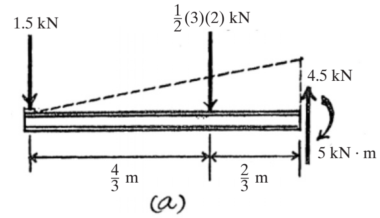




Ans:
$x = 0.9^{-}\text{m}, V = -10\text{ kN} \cdot x = 0.9^{+}\text{m}, V = 6\text{ kN}$
$x = 0.9\text{ m}, M = -9.00\text{ kN}\cdot\text{m}$

***6–12.** Draw the shear and moment diagrams for the cantilevered beam.

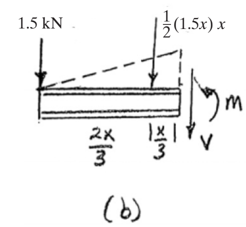




The free-body diagram of the beam's left segment sectioned through an arbitrary point shown in Fig. *b* will be used to write the shear and moment equations. The intensity of the triangular distributed load at the point of sectioning is

$$w = 3\left(\frac{x}{2}\right) = 1.5x$$

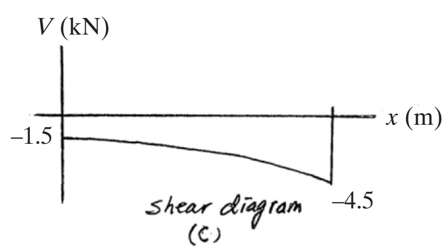
Referring to Fig. *b*,

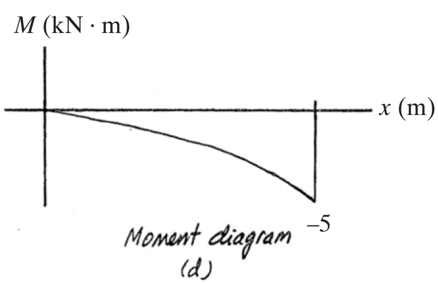


$$+\uparrow \Sigma F_y = 0; \qquad -1.5 - \frac{1}{2}(1.5x)(x) - V = 0 \qquad V = \{-1.5 - 0.75x^2\}\text{ kN} \qquad (1)$$

$$\circlearrowleft + \Sigma M = 0; \qquad M + \frac{1}{2}(1.5x)(x)\left(\frac{x}{3}\right) + 1.5x = 0 \qquad M = \{-1.5x - 0.25x^3\}\text{ kN}\cdot\text{m} \qquad (2)$$

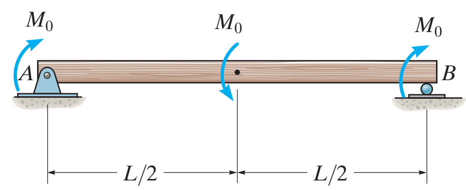
The shear and moment diagrams shown in Figs. *c* and *d* are plotted using Eqs. (1) and (2), respectively.





6–13.

Draw the shear and moment diagrams for the beam.



SOLUTION

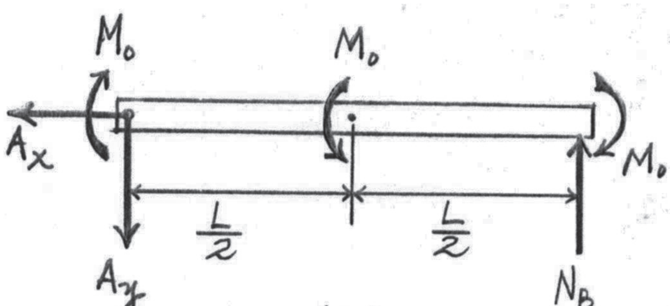
Support Reactions: Referring to the FBD of the beam, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad N_B(L) + M_0 - M_0 - M_0 = 0 \quad N_B = \frac{M_0}{L}$$

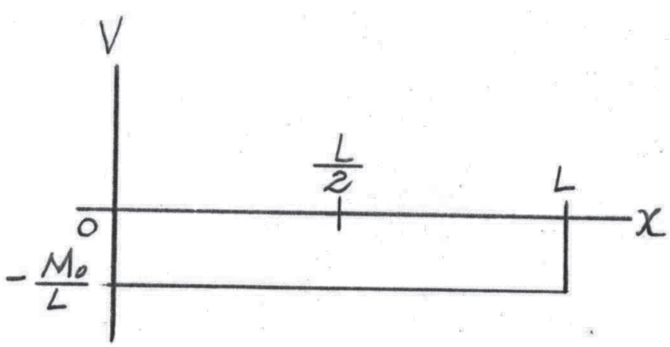
$$\circlearrowleft + \Sigma M_B = 0; \qquad A_y(L) + M_0 - M_0 - M_0 = 0 \quad A_y = \frac{M_0}{L}$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad A_x = 0.$$

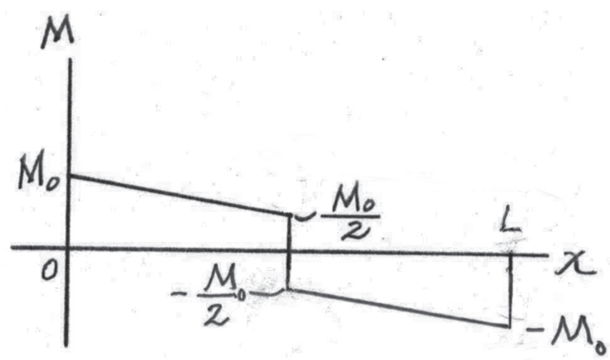
Shear and Moment Diagram: Using the results of the support reaction, the shear and moment diagrams shown in Fig. *b* and *c* respectively can be plotted.



(a)



(b)



(C)

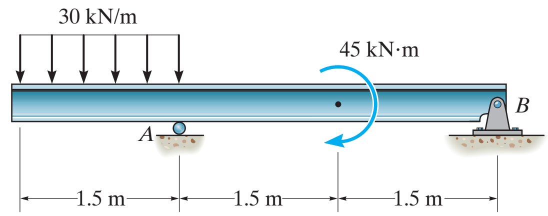
Ans:

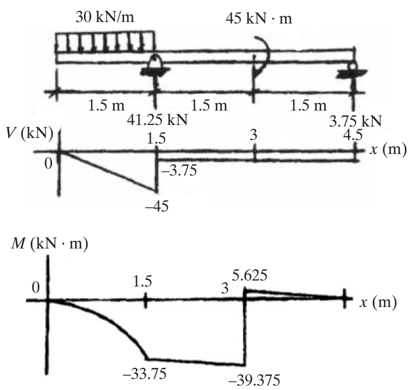
$V = -\dfrac{M_0}{L}$,

For $0 \le x < \dfrac{L}{2}$, $M = M_0 - \left(\dfrac{M_0}{L}\right)x$,

For $\dfrac{L}{2} < x \le L$, $M = -\left(\dfrac{M_0}{L}\right)x$

6–14. Draw the shear and moment diagrams for the beam.

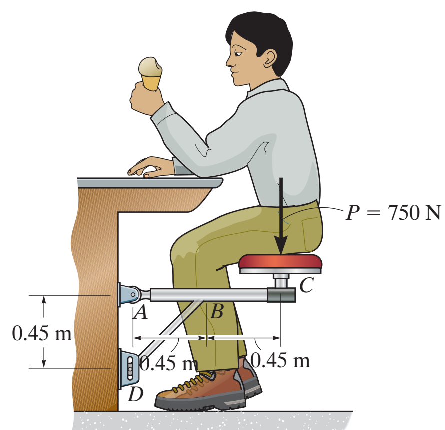




Ans:

$x = 1.5^{-}\,\text{m}, V = -45\text{ kN} \cdot x = 1.5^{+}\text{m},$
$V = -3.75\text{ kN}$
$x = 1.5\text{ m}, M = -33.75\text{ kN}\cdot\text{m} \cdot x = 3^{-}\,\text{m}$
$M = -39.375\text{ kN}\cdot\text{m}$
$x = 3^{+}\text{m}, M = 5.625\text{ kN}\cdot\text{m}$

6–15. Members ABC and BD of the counter chair are rigidly connected at B and the smooth collar at D is allowed to move freely along the vertical slot. Draw the shear and moment diagrams for member ABC.



Equations of Equilibrium: Referring to the free-body diagram of the frame shown in Fig. *a*,

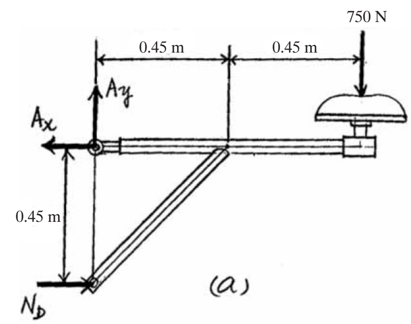
$+\uparrow \Sigma F_y = 0; \qquad A_y - 750 = 0$

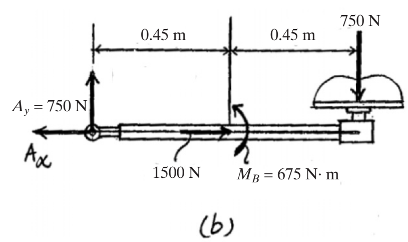
$A_y = 750\text{ N}$

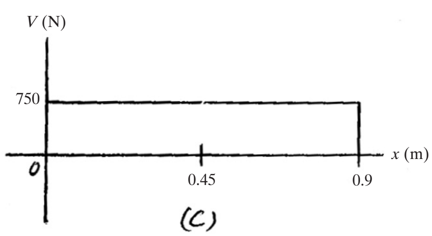
$\circlearrowleft + \Sigma M_A = 0; \qquad N_D(0.45) - 750(0.9) = 0$

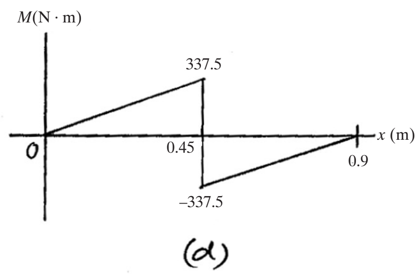
$N_D = 1500\text{ N}$

Shear and Moment Diagram: The couple moment acting on B due to N_D is $M_B = 1500(0.45) = 675\text{ N}\cdot\text{m}$. The loading acting on member ABC is shown in Fig. *b* and the shear and moment diagrams are shown in Figs. *c* and *d*.







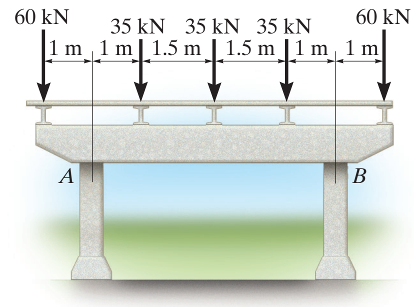


Ans:

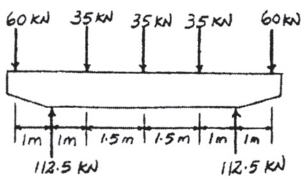
$0 \le x \le 0.9\text{ m}, V = 750\text{ N} \cdot x = 0.45^{-}\text{m},$
$M = -337.5\text{ N}\cdot\text{m} \cdot x = 0.45^{+}\text{m},$
$M = 337.5\text{ N}\cdot\text{m}$

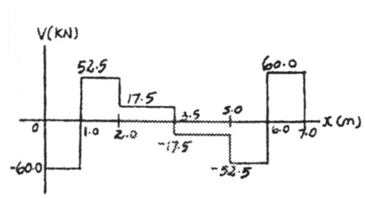
***6–16.**

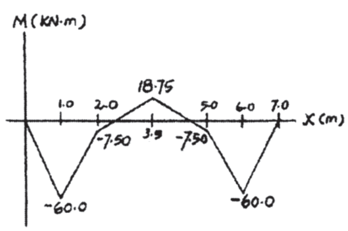
A reinforced concrete pier is used to support the stringers for a bridge deck. Draw the shear and moment diagrams for the pier. Assume the columns at A and B exert only vertical reactions on the pier.



SOLUTION

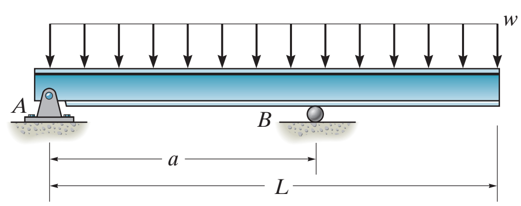






Ans:
N/A

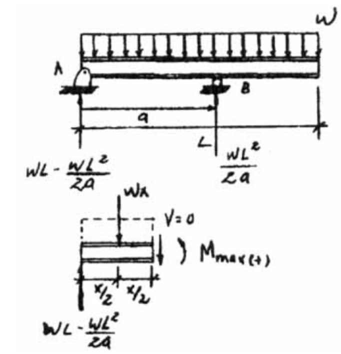
6–17. Determine the placement distance a of the roller support so that the largest absolute value of the moment is a minimum. Draw the shear and moment diagrams for this condition.



$$+\uparrow\Sigma F_y = 0; \qquad wL - \frac{wL^2}{2a} - wx = 0$$

$$x = L - \frac{L^2}{2a}$$

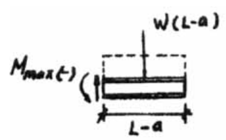
$$\circlearrowleft+\Sigma M = 0; \qquad M_{\max(+)} + wx\left(\frac{x}{2}\right) - \left(wL - \frac{wL^2}{2a}\right)x = 0$$



Substitute $x = L - \dfrac{L^2}{2a}$;

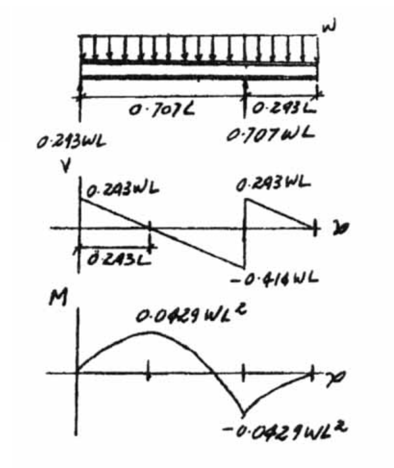
$$M_{\max(+)} = \left(wL - \frac{wL^2}{2a}\right)\left(L - \frac{L^2}{2a}\right) - \frac{w}{2}\left(L - \frac{L^2}{2a}\right)^2$$

$$= \frac{w}{2}\left(L - \frac{L^2}{2a}\right)^2$$



$$\Sigma M = 0; \qquad M_{\max(-)} - w(L-a)\frac{(L-a)}{2} = 0$$

$$M_{\max(-)} = \frac{w(L-a)^2}{2}$$



To get absolute minimum moment,

$$M_{\max(+)} = M_{\max(-)}$$

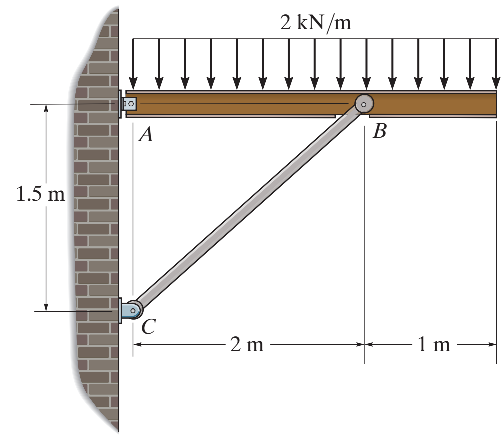
$$\frac{w}{2}\left(L - \frac{L^2}{2a}\right)^2 = \frac{w}{2}(L-a)^2$$

$$L - \frac{L^2}{2a} = L - a$$

$$a = \frac{L}{\sqrt{2}}, \qquad \textbf{Ans.}$$

Ans:
$a = \dfrac{L}{\sqrt{2}}$
$x = 0$, $V = 0.243\,w\mathrm{L} \cdot x = 0.243\,\mathrm{L}$, $V = 0 \cdot$
$x = 0.707\,\mathrm{L}^-$, $V = -0.414\,w\mathrm{L}$
$x = 0.707\,\mathrm{L}^+$, $V = 0.293\,w\mathrm{L}$
$x = 0.243\,\mathrm{L}$, $M = 0.0429\,w\mathrm{L}^2 \cdot x = 0.707\,\mathrm{L}$,
$M = -0.0429\,w\mathrm{L}^2$

6–18. The beam is subjected to the uniform distributed load shown. Draw the shear and moment diagrams for the beam.



Equations of Equilibrium: Referring to the free-body diagram of the beam shown in Fig. *a*,

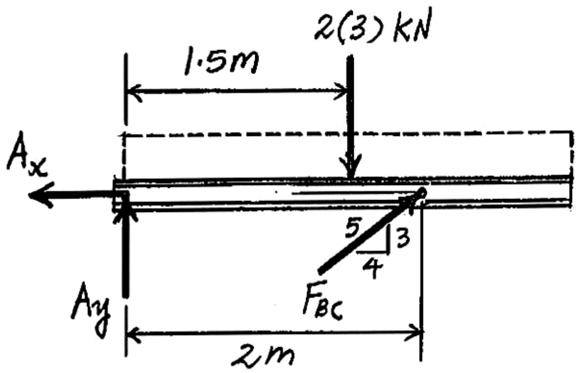
$$\circlearrowleft +\Sigma M_A = 0; \qquad F_{BC}\left(\frac{3}{5}\right)(2) - 2(3)(1.5) = 0$$

$$F_{BC} = 7.5\text{ kN}$$

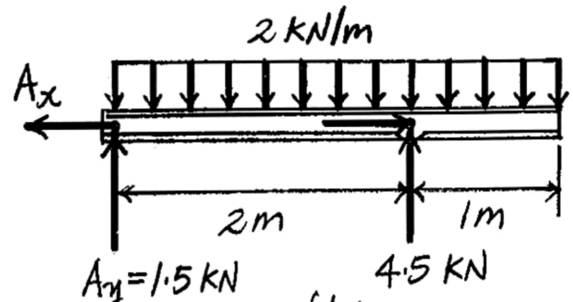
$$+\uparrow \Sigma F_y = 0; \qquad A_y + 7.5\left(\frac{3}{5}\right) - 2(3) = 0$$

$$A_y = 1.5\text{ kN}$$

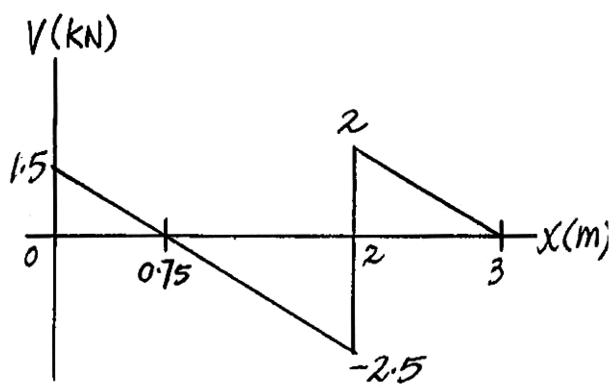
Shear and Moment Diagram: The vertical component of $\mathbf{F}_{BC}$ is $\left(F_{BC}\right)_y = 7.5\left(\frac{3}{5}\right) = 4.5$ kN. The shear and moment diagrams are shown in Figs. *c* and *d*.



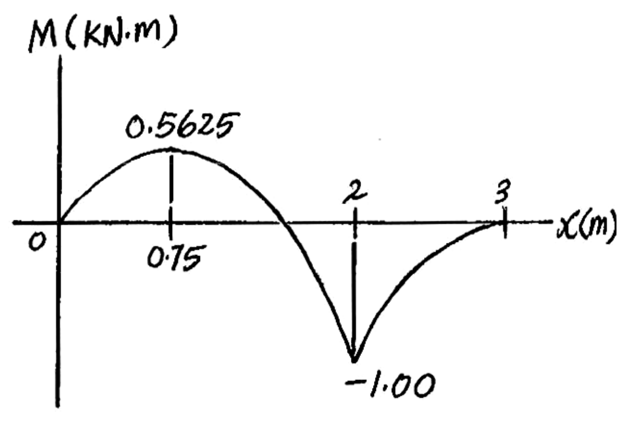
(a)



(b)



(c)

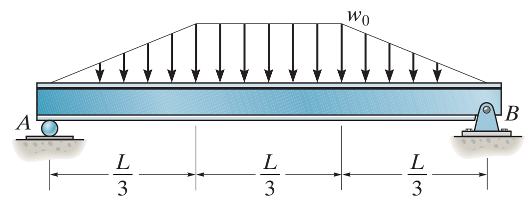


(d)

Ans:
$x = 0.75, V = 0, M = 0.5625,$
$F_{BC} = 7.5\text{ kN}, A_y = 1.5\text{ kN}$

6–19.

Draw the shear and moment diagrams for the beam.



SOLUTION

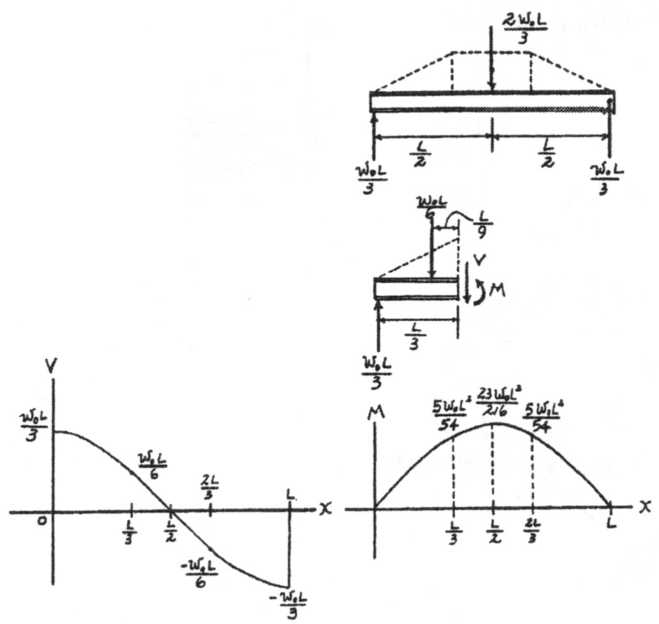
Support Reactions: As shown on FBD.

Shear and Moment Diagram: Shear and moment at $x = L/3$ can be determined using the method of sections.

$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0 L}{3} - \frac{w_0 L}{6} - V = 0 \qquad V = \frac{w_0 L}{6}$$

$$\zeta + \Sigma M_{NA} = 0; \qquad M + \frac{w_0 L}{6}\left(\frac{L}{9}\right) - \frac{w_0 L}{3}\left(\frac{L}{3}\right) = 0$$

$$M = \frac{5 w_0 L^2}{54}$$



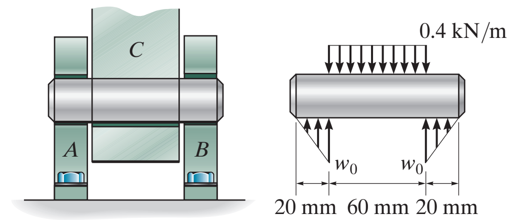
Ans:

$V_A = \frac{w_0 L}{3}$,

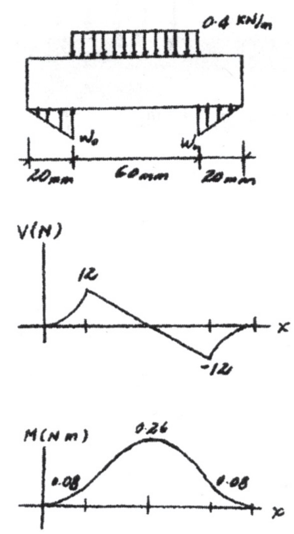
$M_{\max} = \frac{23\, w_0 L^2}{216}$

***6–20.**

The smooth pin is supported by two leaves A and B and subjected to a compressive load of 0.4 kN/m caused by bar C. Determine the intensity of the distributed load w_0 of the leaves on the pin and draw the shear and moment diagram for the pin.



SOLUTION

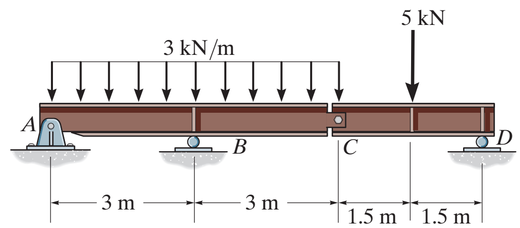


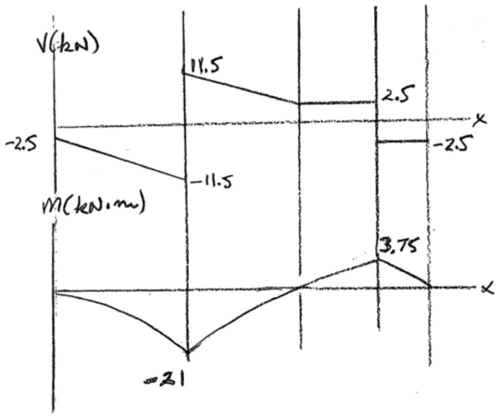
$$+\uparrow \Sigma F_y = 0; \qquad 2(w_0)(20)\left(\frac{1}{2}\right) - 60(0.4) = 0$$

$$w_0 = 1.2 \text{ kN/m} \qquad \textbf{Ans.}$$

Ans:
$w_0 = 1.2 \text{ kN/m}$

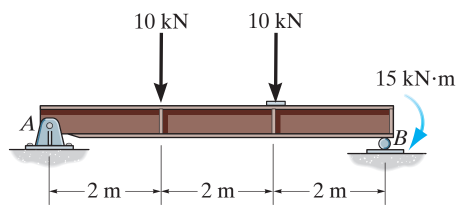
6–21. Draw the shear and moment diagrams for the compound beam.

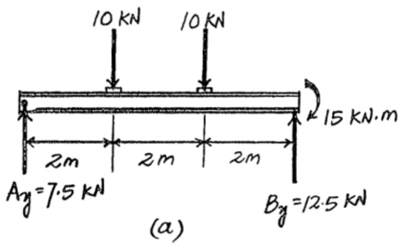




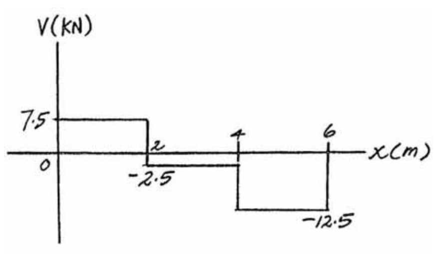
Ans:
$x = 3^-, V = -11.5, M = -21$

6–22. Draw the shear and moment diagrams for the simply supported beam.

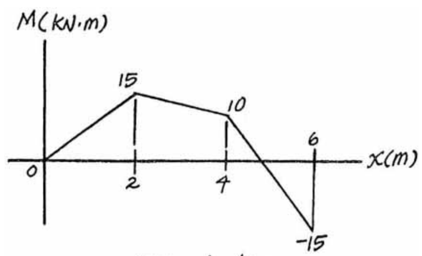




(a)



shear diagram
(b)



moment diagram
(c)

Ans:

$x = 0, V = 7.5\,\text{kN} \cdot x = 2^{-}\text{m}, V = 7.5\,\text{kN}$

$x = 2^{+}\text{m}, V = -2.5\,\text{kN}$

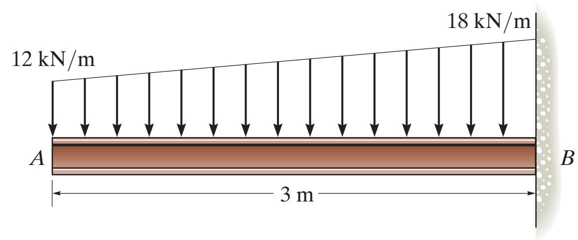
$x = 4^{+}\text{m}\ V = -12.5\,\text{kN}$

$x = 2\,\text{m}, M = 15\ \text{kN} \cdot \text{m} \cdot x = 4\,\text{m},$

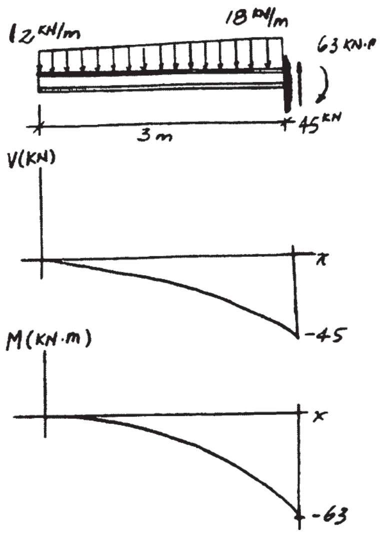
$M = 10\,\text{kN} \cdot \text{m} \cdot x = 6\,\text{m}, \text{M} = -15\,\text{kN} \cdot \text{m}$

6–23.

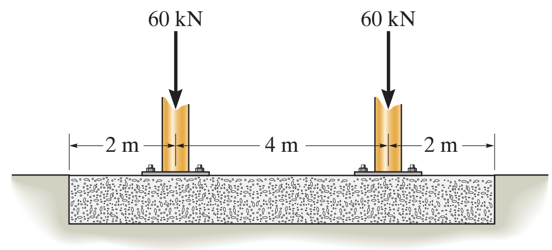
Draw the shear and moment diagrams for the beam.

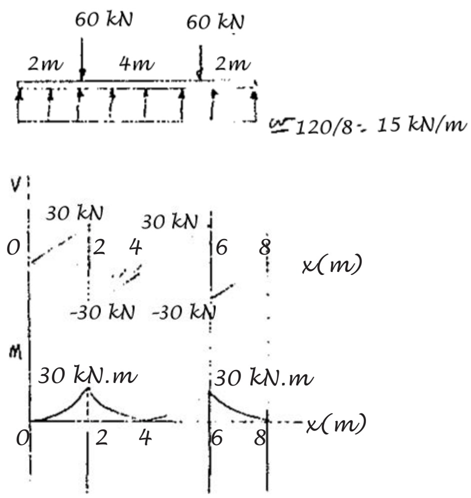


SOLUTION

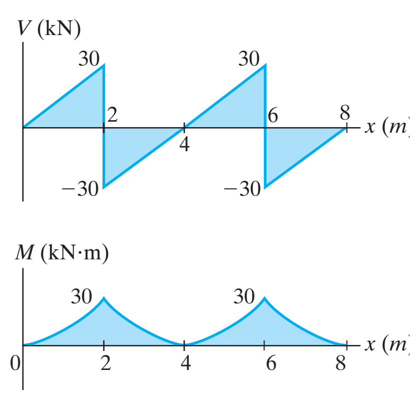


***6–24.** The footing supports the load transmitted by the two columns. Draw the shear and moment diagrams for the footing if the reaction of soil pressure on the footing is assumed to be uniform.

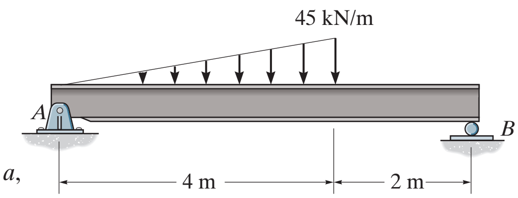




Ans:



6–25. Draw the shear and moment diagrams for the overhanging beam.



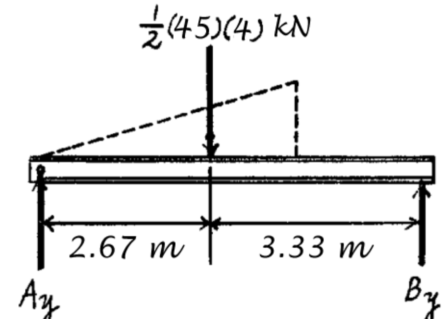
Support Reactions: Referring to the free-body diagram of the beam shown in Fig. *a*,

$\zeta + \Sigma M_A = 0; \qquad B_y(6) - \dfrac{1}{2}(45)(4)(2.67) = 0$

$B_y = 40.0 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \qquad A_y + 40.0 - \dfrac{1}{2}(45)(4) = 0$

$A_y = 50.0 \text{ kN}$



(a)

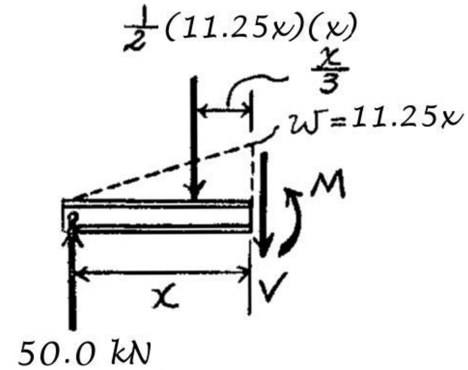
Shear and Moment Functions: For $0 \le x < 4$ m, we refer to the free-body diagram of the beam segment shown in Fig. *b*.

$+\uparrow \Sigma F_y = 0; \qquad 50.0 - \dfrac{1}{2}(11.25x)(x) - V = 0$

$V = \{50.0 - 5.625x^2\} \text{ kN}$ **Ans.**

$\zeta + \Sigma M = 0; \qquad M + \dfrac{1}{2}(11.25x)(x)\left(\dfrac{x}{3}\right) - 50.0x = 0$

$M = \{50.0x - 1.875x^3\} \text{ kN} \cdot \text{m}$ **Ans.**



(b)

When $V = 0$, from the shear function,

$0 = 50.0 - 5.625x^2 \qquad x = 2.981 \text{ m}$

Substituting this result into the moment function,

$M|_{x=2.981 \text{ m}} = 99.38 \text{ kN} \cdot \text{m}$

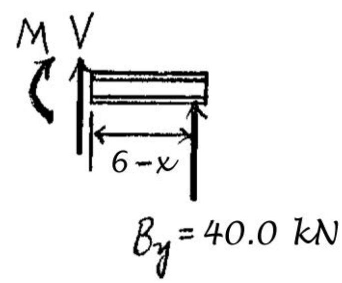
For $4 \text{ m} < x \le 6$ m, we refer to the free-body diagram of the beam segment shown in Fig. *c*.

$+\uparrow \Sigma F_y = 0; \qquad V + 40.0 \text{ kN} = 0$

$V = -40.0 \text{ kN}$ **Ans.**

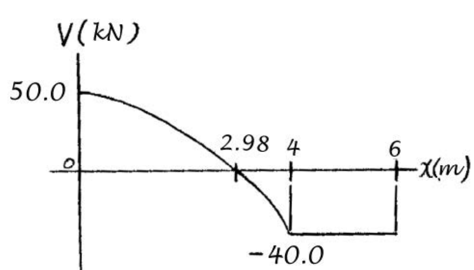
$\zeta + \Sigma M = 0; \; 40.05(6 - x) - M = 0$

$M = \{40.0(6 - x)\} \text{ kN} \cdot \text{m}$ **Ans.**

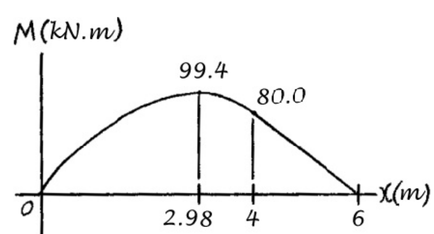


(c)

Shear and Moment Diagrams: As shown in Figs. *d* and *e*.



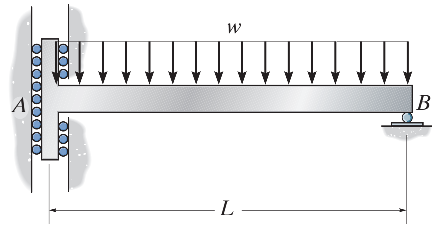
(d)



(e)

6–26.

The support at A allows the beam to slide freely along the vertical guide so that it cannot support a vertical force. Draw the shear and moment diagrams for the beam.



SOLUTION

Equations of Equilibrium: Referring to the free-body diagram of the beam shown in Fig. *a*,

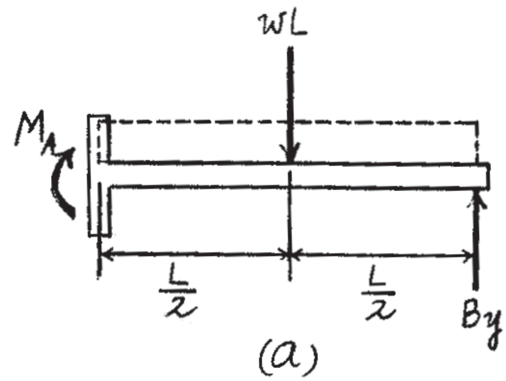
$$\circlearrowleft + \Sigma M_B = 0; \qquad wL\left(\frac{L}{2}\right) - M_A = 0$$

$$M_A = \frac{wL^2}{2}$$

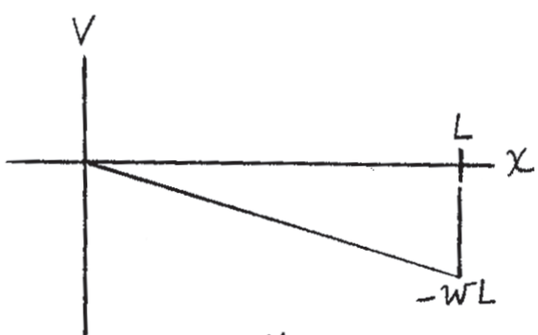
$$+\uparrow \Sigma F_y = 0; \qquad B_y - wL = 0$$

$$B_y = wL$$

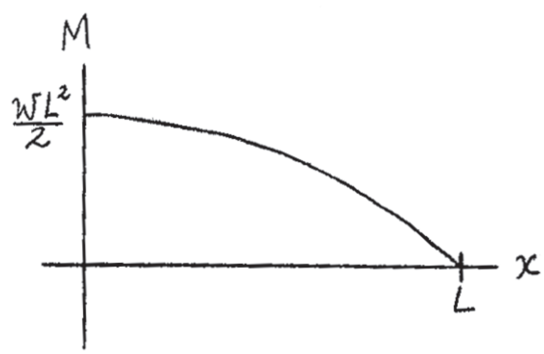
Shear and Moment Diagram: As shown in Figs. *b* and *c*.



(a)

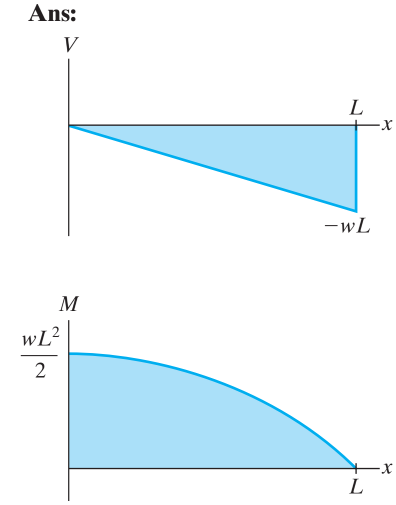


(b)



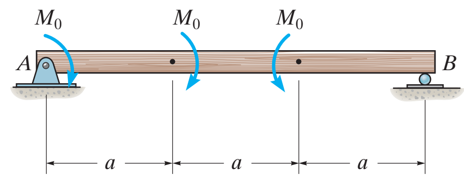
(c)

Ans:

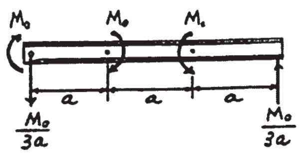


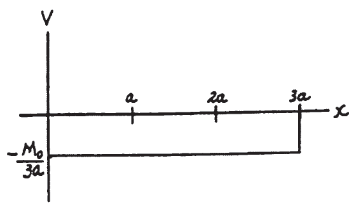
6–27.

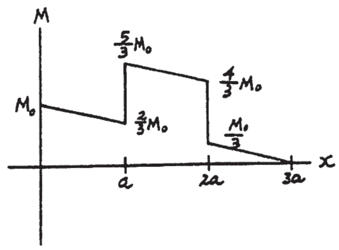
Draw the shear and moment diagrams for the beam.



SOLUTION

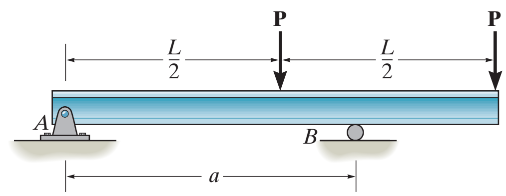






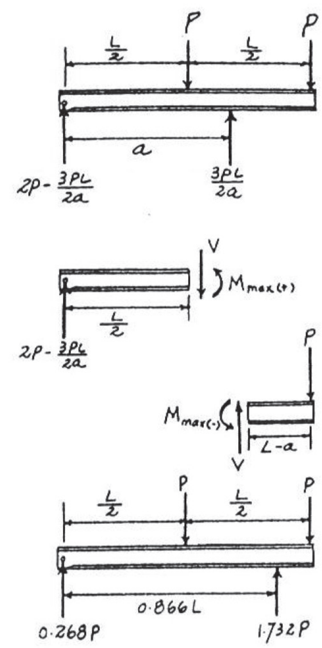
***6–28.**

Determine the placement distance a of the roller support so that the largest absolute value of the moment is a minimum. Draw the shear and moment diagrams for this condition.



SOLUTION

Support Reactions: As shown on FBD.



Absolute Minimum Moment: In order to get the absolute minimum moment, the maximum positive and maximum negative moment must be equal that is $M_{\max(+)} = M_{\max(-)}$.

For the positive moment:

$$\curvearrowleft + \Sigma M_{NA} = 0; \qquad M_{\max(+)} - \left(2P - \frac{3PL}{2a}\right)\left(\frac{L}{2}\right) = 0$$

$$M_{\max(+)} = PL - \frac{3PL^2}{4a}$$

For the negative moment:

$$\curvearrowleft + \Sigma M_{NA} = 0; \qquad M_{\max(-)} - P(L - a) = 0$$

$$M_{\max(-)} = P(L - a)$$

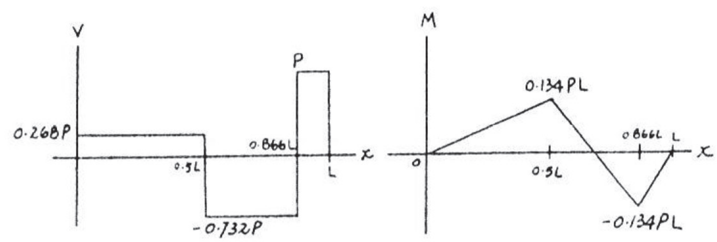
$$M_{\max(+)} = M_{\max(-)}$$

$$PL - \frac{3PL^2}{4a} = P(L - a)$$

$$4a\,L - 3L^2 = 4a\,L - 4a^2$$

$$a = \frac{\sqrt{3}}{2}L = 0.866L \qquad \textbf{Ans.}$$

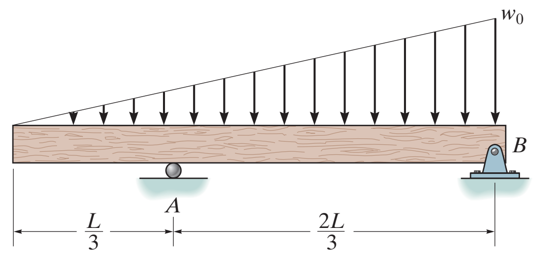
Shear and Moment Diagram:



Ans:
$a = 0.866L$

6–29.

Draw the shear and moment diagrams for the beam.



SOLUTION

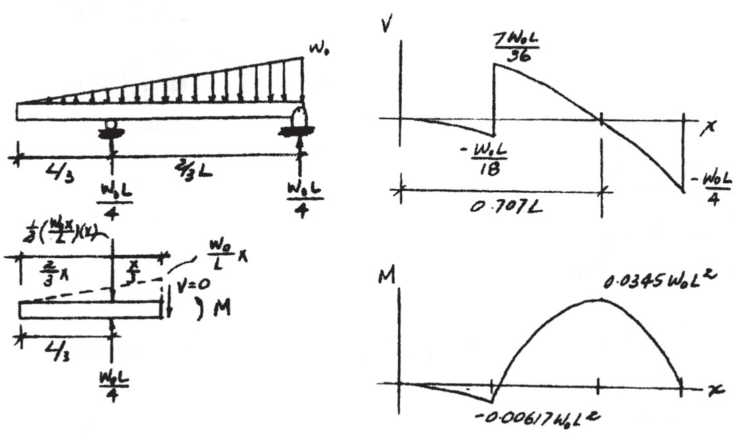
$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0 L}{4} - \frac{1}{2}\left(\frac{w_0 x}{L}\right)(x) = 0$$

$$x = 0.7071\, L$$

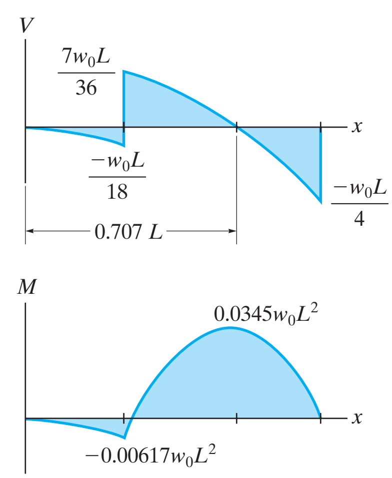
$$\zeta + \Sigma M_{NA} = 0; \qquad M + \frac{1}{2}\left(\frac{w_0 x}{L}\right)(x)\left(\frac{x}{3}\right) - \frac{w_0 L}{4}\left(x - \frac{L}{3}\right) = 0$$

Substitute $x = 0.7071L$,

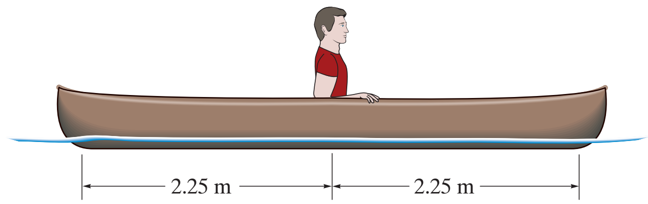
$$M = 0.0345\, w_0 L^2$$

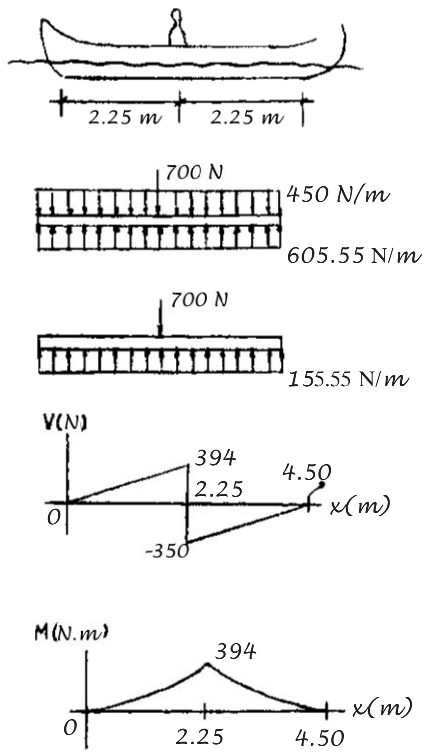


Ans:



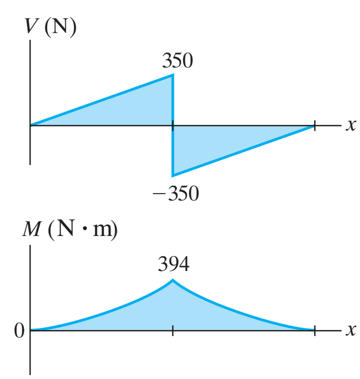
6–30. The 700-N man sits in the center of the boat, which has a uniform width and a weight per linear foot of 450 N/m. Determine the maximum bending moment exerted on the boat. Assume that the water exerts a uniform distributed load upward on the bottom of the boat.



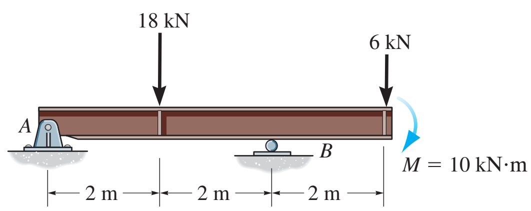


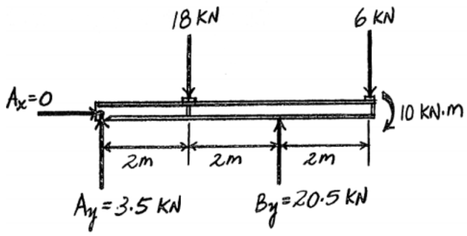
$M_{max} = 394\ \text{N} \cdot \text{m}$ **Ans.**

Ans:
$M_{max} = 394\ \text{N} \cdot \text{m}$

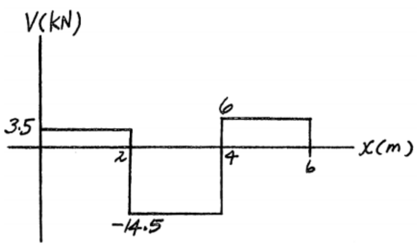


6–31. Draw the shear and moment diagrams for the overhang beam.

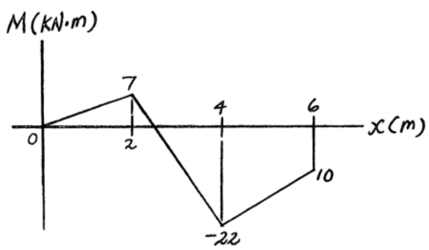




(a)



shear diagram
(b)

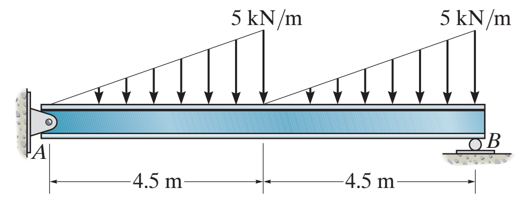


moment diagram
(c)

Ans:

$x = 0$, $V = 3.5\ \text{kN} \cdot x = 2^{+}\,\text{m}$, $V = -14.5\ \text{kN} \cdot$
$x = 4^{+}\,\text{m}$, $V = 6\ \text{kN}$
$x = 2\ \text{m}, M = 7\ \text{kN} \cdot \text{m}\, x = 4\ \text{m},$
$M = -22\ \text{kN} \cdot \text{m} \cdot x = 6\ \text{m}$
$M = 10\ \text{kN} \cdot \text{m}$

***6–32.** Draw the shear and moment diagrams for the beam.



From FBD(a)

$$+\uparrow \Sigma F_y = 0; \qquad 9.375 - 0.5556x^2 = 0 \qquad x = 4.108 \text{ m}$$

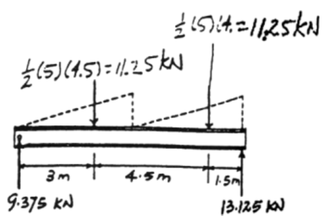
$$\zeta + \Sigma M_{NA} = 0; \qquad M + (0.5556)\left(4.108^2\right)\left(\frac{4.108}{3}\right) - 9.375(4.108) = 0$$

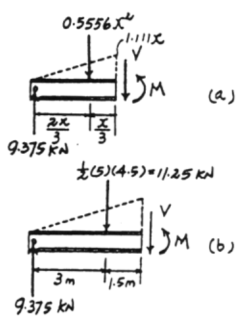
$$M = 25.67 \text{ kN} \cdot \text{m}$$

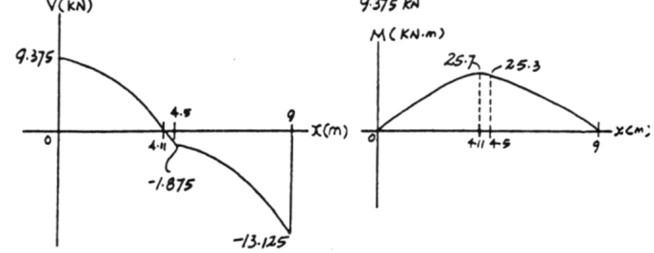
From FBD(b)

$$\zeta + \Sigma M_{NA} = 0; \qquad M + 11.25(1.5) - 9.375(4.5) = 0$$

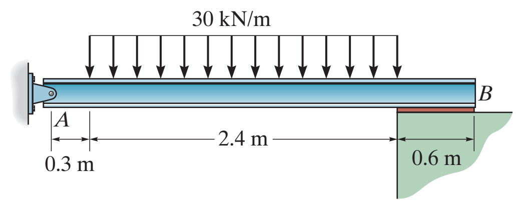
$$M = 25.31 \text{ kN} \cdot \text{m}$$

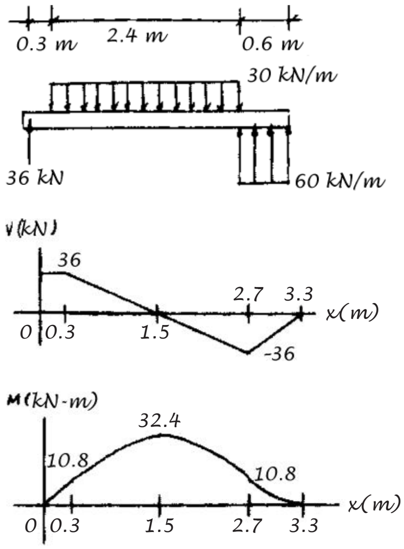




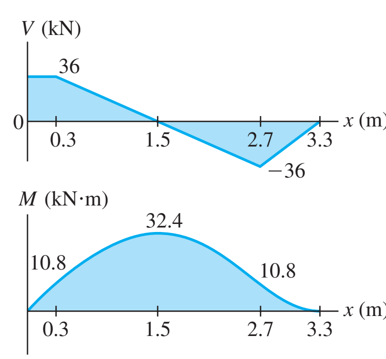


6–33. The beam is bolted or pinned at A and rests on a bearing pad at B that exerts a uniform distributed loading on the beam over its 0.6-m length. Draw the shear and moment diagrams for the beam if it supports a uniform loading of 30 kN/m.



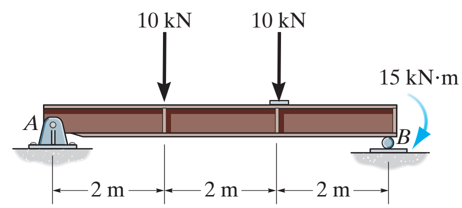


Ans:

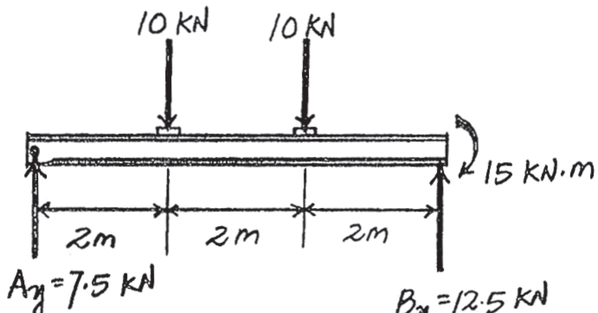


6–34.

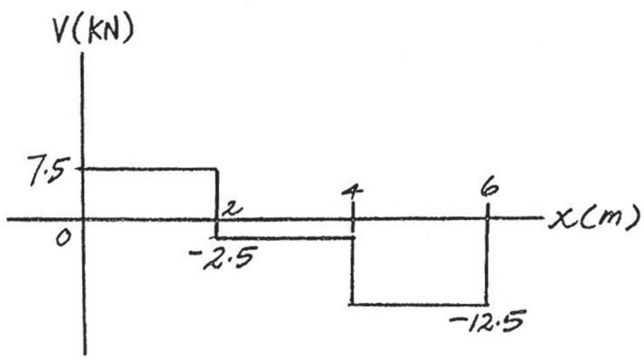
Draw the shear and moment diagrams for the simply supported beam.



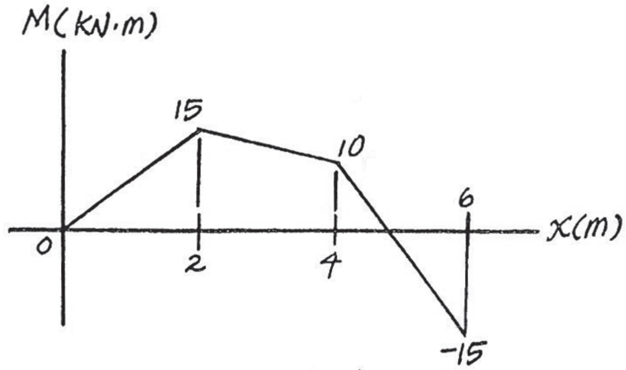
SOLUTION



(a)



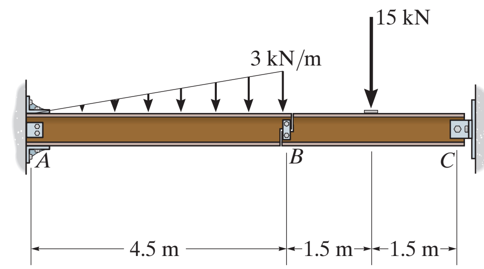
Shear diagram
(b)



moment diagram
(c)

6–35.

A short link at B is used to connect beams AB and BC to form the compound beam. Draw the shear and moment diagrams for the beam if the supports at A and C are considered fixed and pinned, respectively.



SOLUTION

Support Reactions: Referring to the free-body diagram of segment BC shown in Fig. a,

$$\circlearrowleft +\Sigma M_C = 0; \qquad 15(1.5) - F_B(3) = 0$$
$$F_B = 7.5 \text{ kN}$$

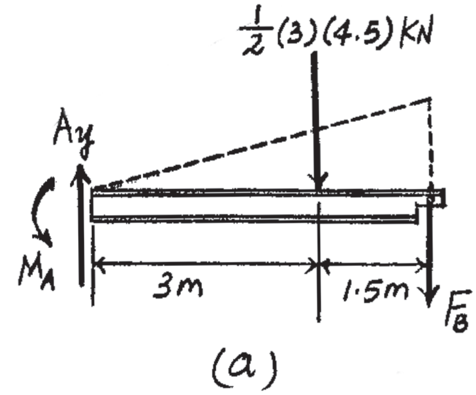
$$+\uparrow \Sigma F_y = 0; \qquad C_y + 7.5 - 15 = 0$$
$$C_y = 7.5 \text{ kN}$$

Using the result of $\mathbf{F}_B$ and referring to the free-body diagram of segment AB, Fig. b,

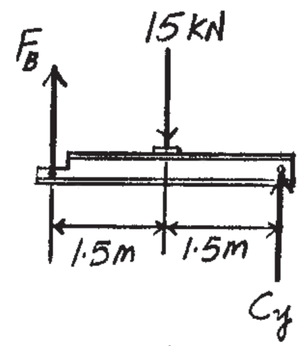
$$+\uparrow \Sigma F_y = 0; \qquad A_y - \frac{1}{2}(3)(4.5) - 7.5 = 0$$
$$A_y = 14.25 \text{ kN}$$

$$\circlearrowleft +\Sigma M_A = 0; \qquad M_A - \frac{1}{2}(3)(4.5)(3) - 7.5(4.5) = 0$$
$$M_A = 54 \text{ kN}\cdot\text{m}$$

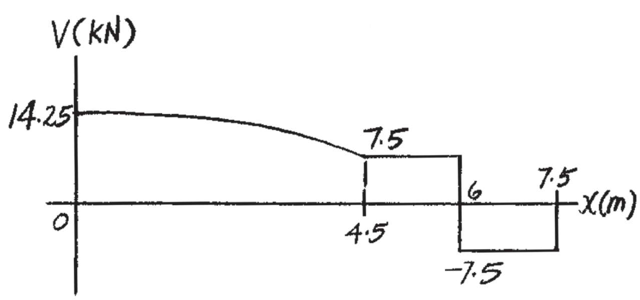
Shear and Moment Diagrams: As shown in Figs. c and d.



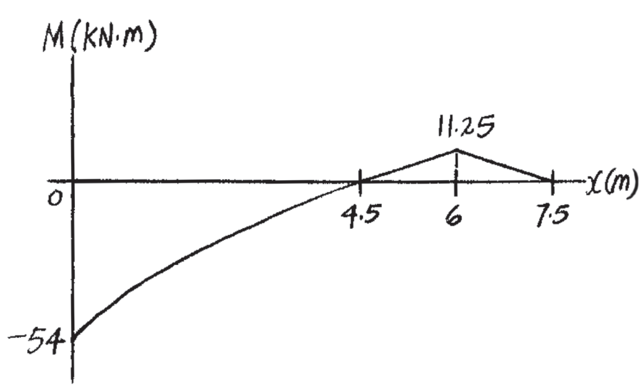
(a)



(b)

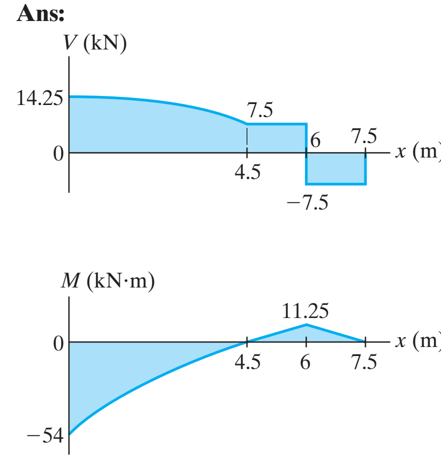


(c)



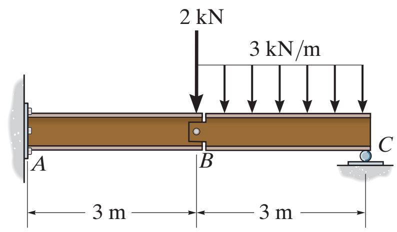
(d)

Ans:



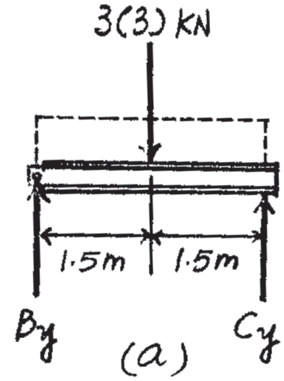
***6–36.**

The compound beam is fixed at A, pin connected at B, and supported by a roller at C. Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions: Referring to the free-body diagram of segment BC shown in Fig. a,



$\circlearrowleft + \Sigma M_B = 0;\qquad C_y(3) - 3(3)(1.5) = 0$

$C_y = 4.5\text{ kN}$

$+\uparrow \Sigma F_y = 0;\qquad B_y + 4.5 - 3(3) = 0$

$B_y = 4.5\text{ kN}$

Using the result of $\mathbf{B}_y$ and referring to the free-body diagram of segment AB, Fig. b,

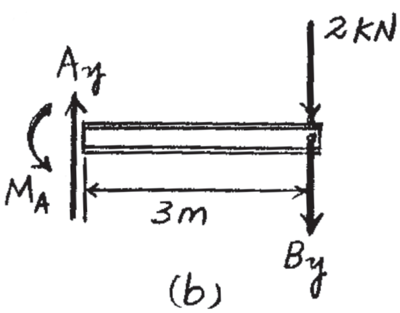
$+\uparrow \Sigma F_y = 0;\qquad A_y - 2 - 4.5 = 0$

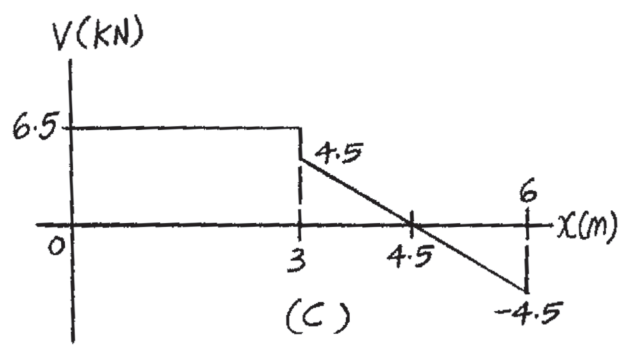
$A_y = 6.5\text{ kN}$

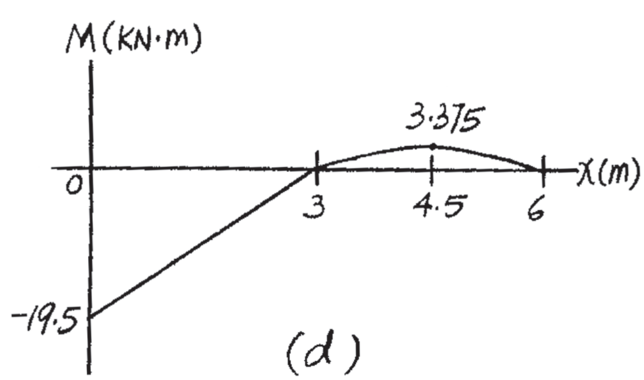
$\circlearrowleft + \Sigma M_A = 0;\qquad M_A - 2(3) - 4.5(3) = 0$

$M_A = 19.5\text{ kN}\cdot\text{m}$

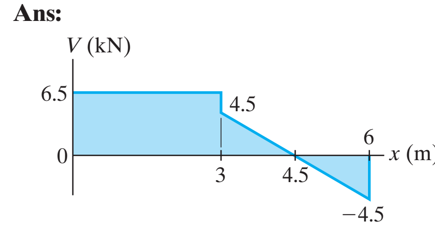
Shear and Moment Diagrams: As shown in Figs. c and d.

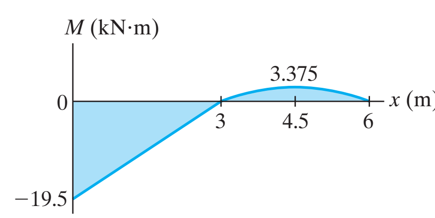






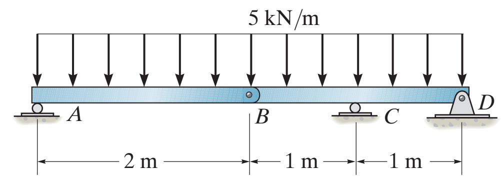
Ans:





6–37.

Draw the shear and moment diagrams for the compound beam.



SOLUTION

Support Reactions:

From the FBD of segment AB

$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(2) - 10.0(1) = 0 \qquad B_y = 5.00 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \qquad A_y - 10.0 + 5.00 = 0 \qquad A_y = 5.00 \text{ kN}$

From the FBD of segment BD

$\circlearrowleft + \Sigma M_C = 0; \qquad 5.00(1) + 10.0(0) - D_y(1) = 0$

$$D_y = 5.00 \text{ kN}$$

$+\uparrow \Sigma F_y = 0; \qquad C_y - 5.00 - 5.00 - 10.0 = 0$

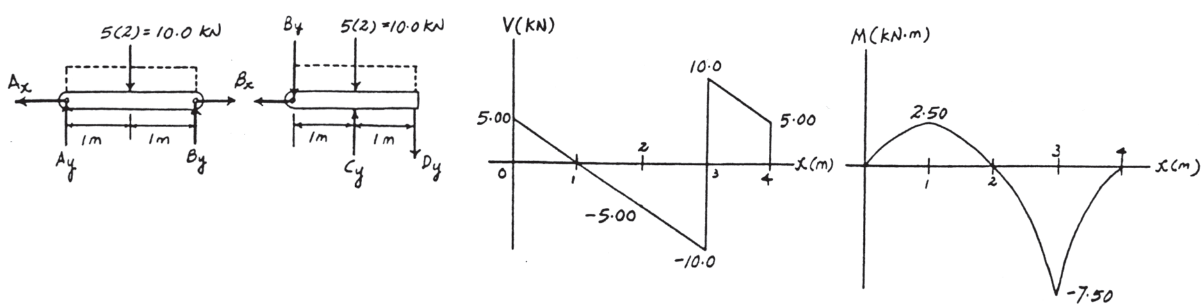
$$C_y = 20.0 \text{ kN}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad B_x = 0$

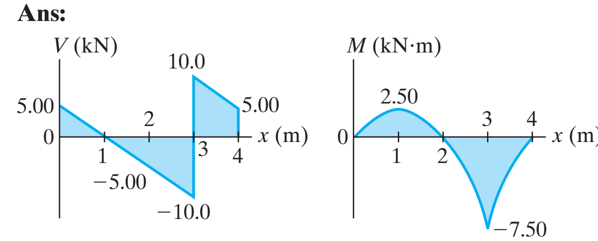
From the FBD of segment AB

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x = 0$

Shear and Moment Diagram:

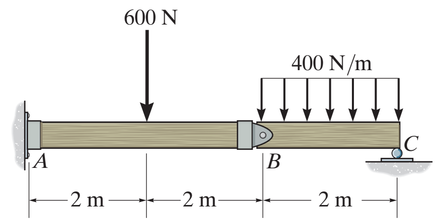


Ans:



6–38.

The compound beam is fixed at A, pin connected at B, and supported by a roller at C. Draw the shear and moment diagrams for the beam.



SOLUTION

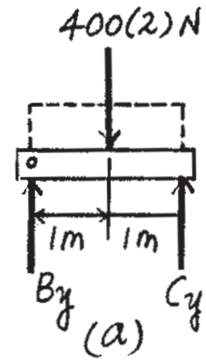
Support Reactions: Referring to the free-body diagram of segment BC shown in Fig. a,

$$\zeta + \Sigma M_B = 0; \qquad C_y(2) - 400(2)(1) = 0$$

$$C_y = 400 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y + 400 - 400(2) = 0$$

$$B_y = 400 \text{ N}$$



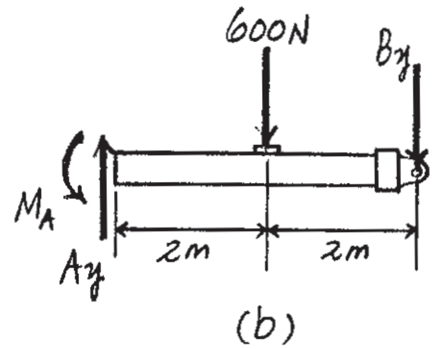
Using the result of $\mathbf{B}_y$ and referring to the free-body diagram of segment AB, Fig. b,

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 600 - 400 = 0$$

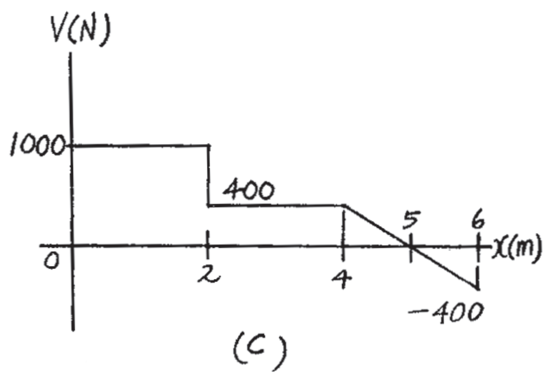
$$A_y = 1000 \text{ N}$$

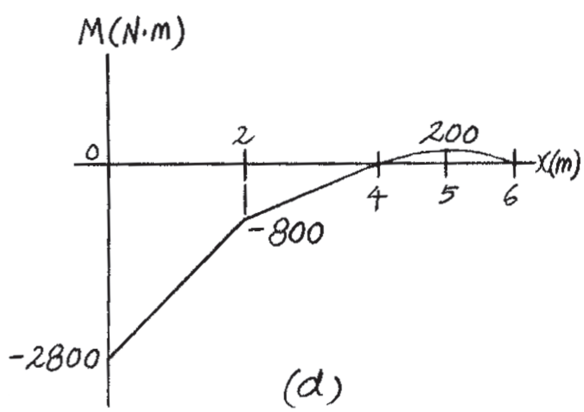
$$\zeta + \Sigma M_A = 0; \qquad M_A - 600(2) - 400(4) = 0$$

$$M_A = 2800 \text{ N}$$

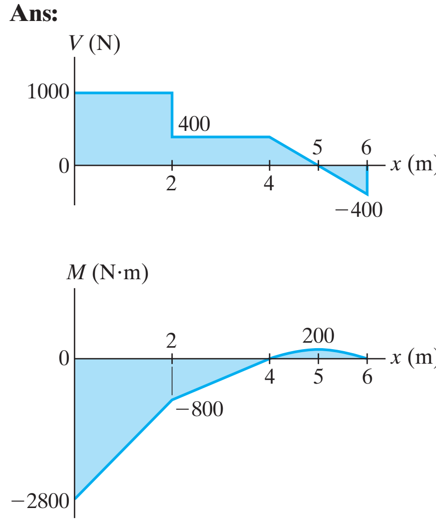


Shear and Moment Diagrams: As shown in Figs. c and d.



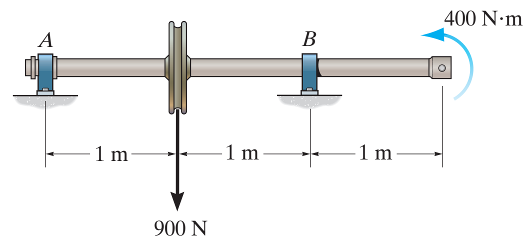


Ans:



6–39.

The shaft is supported by a smooth thrust bearing at A and smooth journal bearing at B. Draw the shear and moment diagrams for the shaft.



SOLUTION

Equations of Equilibrium: Referring to the free-body diagram of the shaft shown in Fig. a,

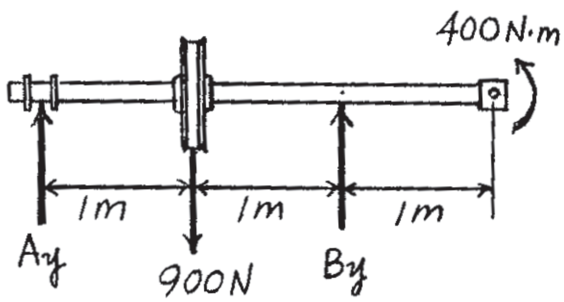
$\circlearrowleft + \Sigma M_A = 0;\qquad B_y(2) + 400 - 900(1) = 0$

$B_y = 250\text{ N}$

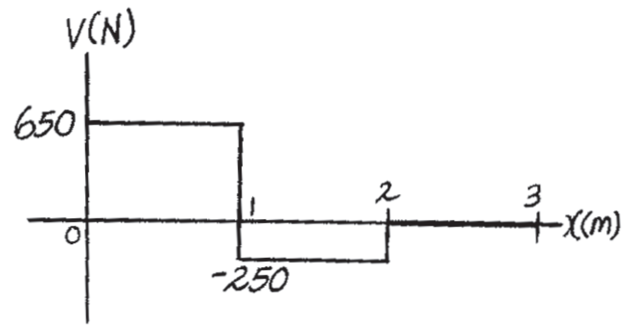
$+\uparrow \Sigma F_y = 0;\qquad A_y + 250 - 900 = 0$

$A_y = 650\text{ N}$

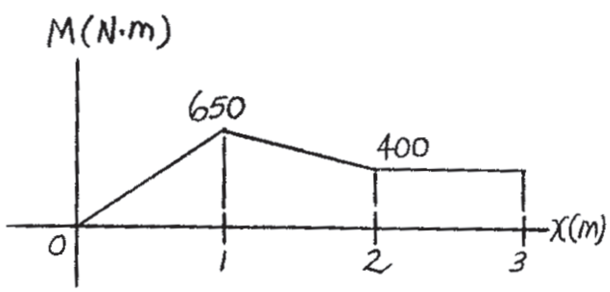
Shear and Moment Diagram: As shown in Figs. b and c.



(a)

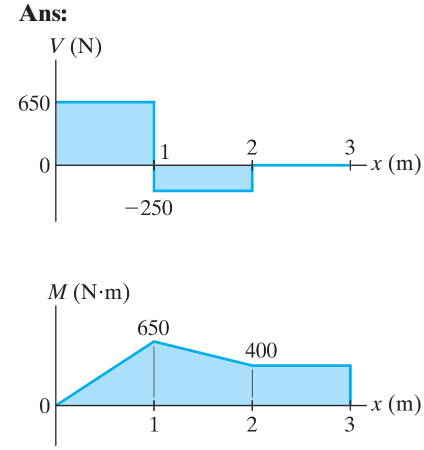


(b)



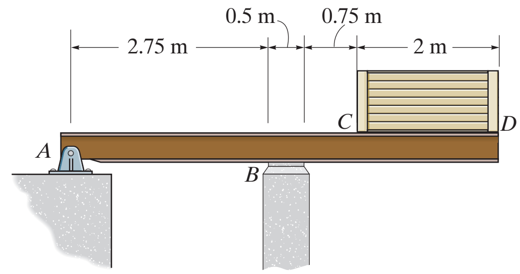
(c)

Ans:



***6–40.**

The beam is used to support a uniform load along *CD* due to the 6-kN weight of the crate. Also, the reaction at the bearing support *B* can be assumed uniformly distributed along its width. Draw the shear and moment diagrams for the beam.



SOLUTION

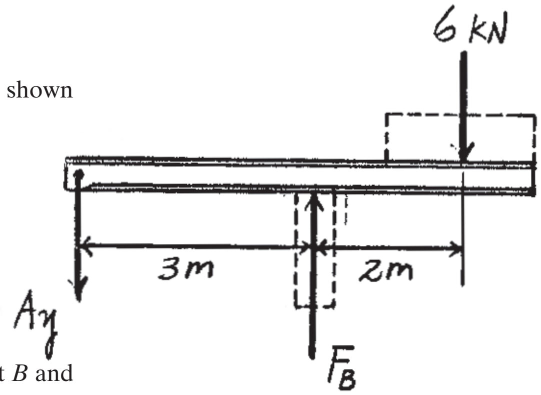
Equations of Equilibrium: Referring to the free-body diagram of the beam shown in Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0;$ $\quad F_B(3) - 6(5) = 0$

$F_B = 10\text{ kN}$

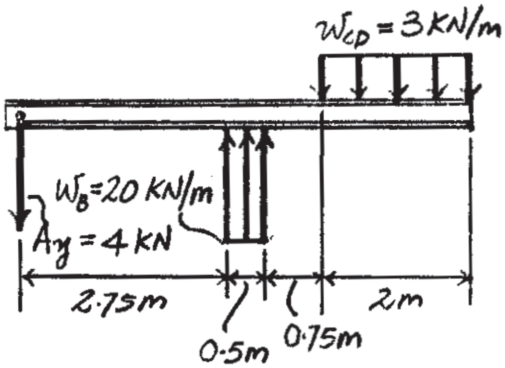
$+\uparrow \Sigma F_y = 0;$ $\quad 10 - 6 - A_y = 0$

$A_y = 4\text{ kN}$

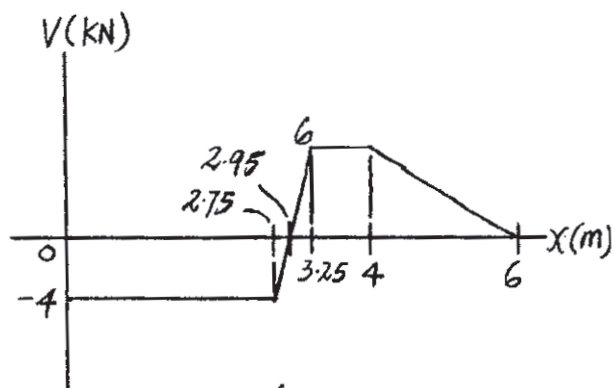


(a)

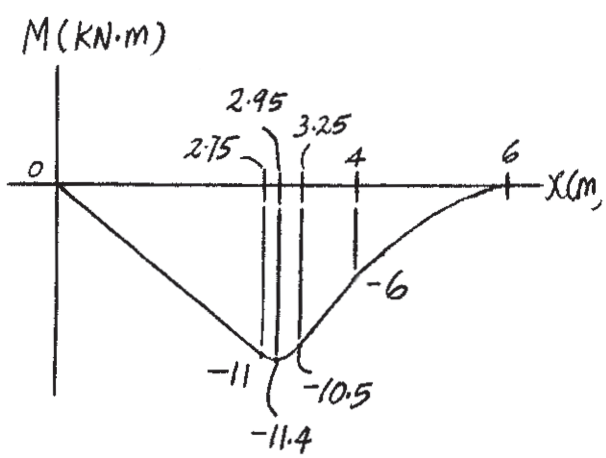
Shear and Moment Diagram: The intensity of the distributed load at support *B* and portion *CD* of the beam are $w_B = \dfrac{F_B}{0.5} = \dfrac{10}{0.5} = 20\text{ kN/m}$ and $w_{CD} = \dfrac{6}{2} = 3\text{ kN/m}$, Fig. *b*. The shear and moment diagrams are shown in Figs. *c* and *d*.



(b)

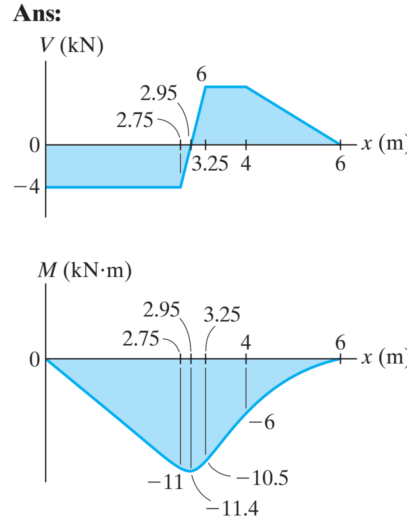


(c)



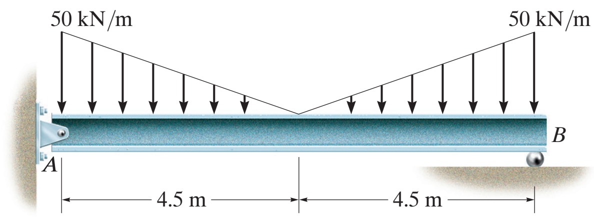
(d)

Ans:

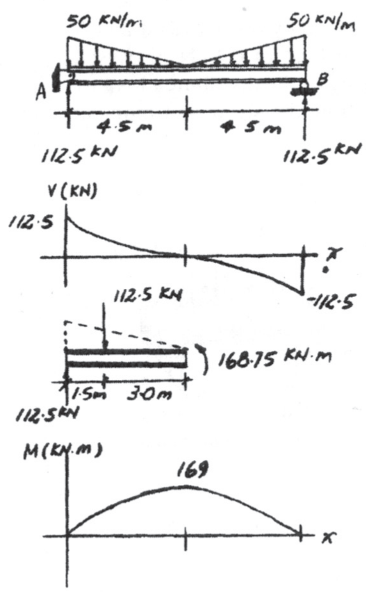


6–41.

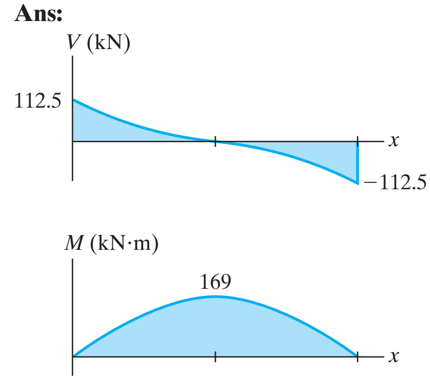
Draw the shear and moment diagrams for the beam.



SOLUTION

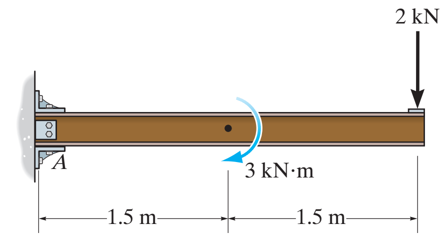


Ans:



6–42.

Draw the shear and moment diagrams for the cantilever beam.



SOLUTION

Equations of Equilibrium: Referring to the free-body diagram of the beam shown in Fig. *a*,

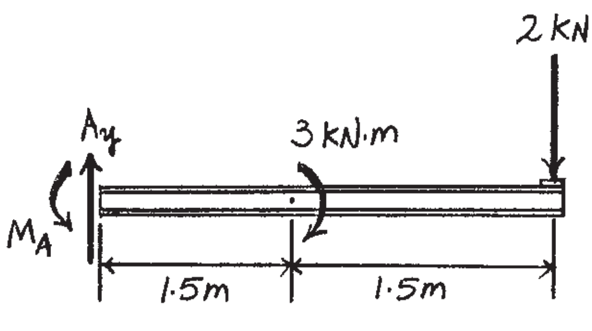
$$+\uparrow \Sigma F_y = 0; \qquad A_y - 2 = 0$$

$$A_y = 2\text{ kN}$$

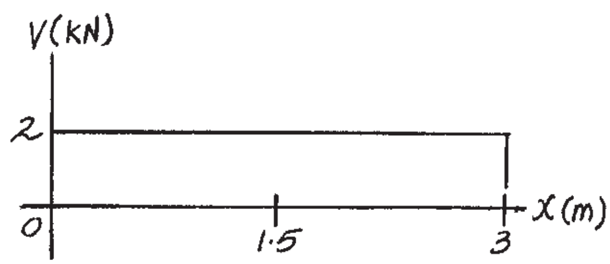
$$\circlearrowleft + \Sigma M_A = 0; \qquad M_A - 3 - 2(3) = 0$$

$$M_A = 9\text{ kN}\cdot\text{m}$$

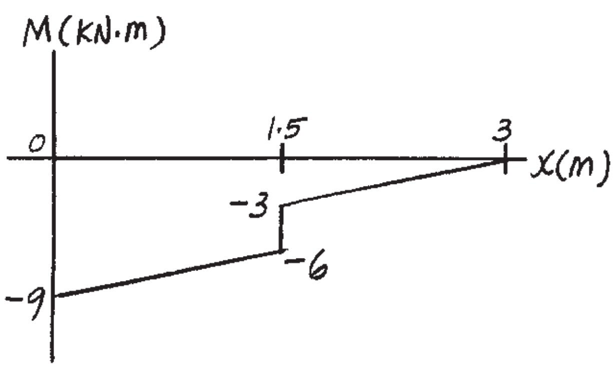
Shear and Moment Diagram: As shown in Figs. *b* and *c*.



(a)

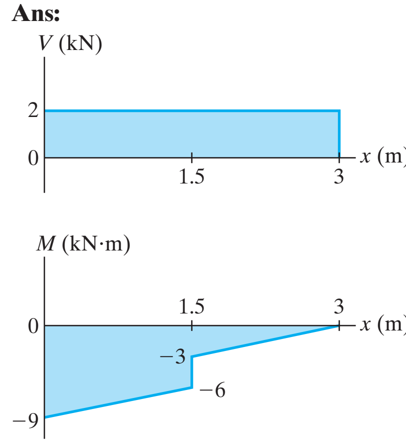


(b)



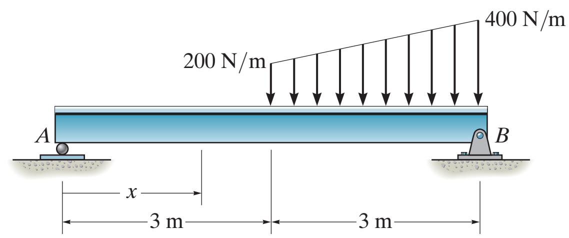
(c)

Ans:



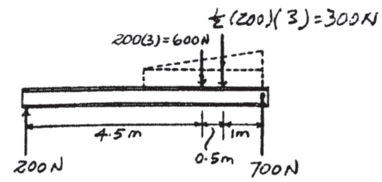
6–43.

Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions: As shown on FBD.



Shear and Moment Functions:

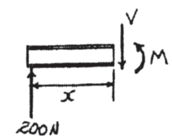
For $0 \le x < 3$ m:

$$+\uparrow \Sigma F_y = 0; \qquad 200 - V = 0 \qquad V = 200 \text{ N} \qquad \textbf{Ans.}$$

$$\zeta + \Sigma M_{NA} = 0; \qquad M - 200\,x = 0$$

$$M = \{200\,x\} \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$

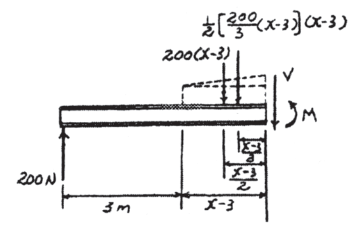
For $3 \text{ m} < x \le 6$ m:



$$+\uparrow \Sigma F_y = 0; \qquad 200 - 200(x-3) - \frac{1}{2}\left[\frac{200}{3}(x-3)\right](x-3) - V = 0$$

$$V = \left\{-\frac{100}{3}x^2 + 500\right\} \text{ N} \qquad \textbf{Ans.}$$

Set $V = 0, x = 3.873$ m

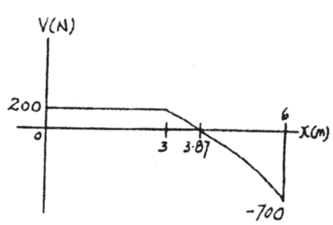


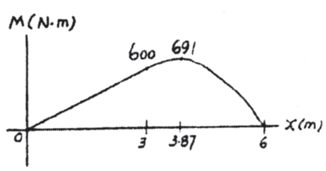
$$\zeta + \Sigma M_{NA} = 0; \qquad M + \frac{1}{2}\left[\frac{200}{3}(x-3)\right](x-3)\left(\frac{x-3}{3}\right)$$

$$+ 200(x-3)\left(\frac{x-3}{2}\right) - 200x = 0$$

$$M = \left\{-\frac{100}{9}x^3 + 500x - 600\right\} \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$

Substitute $x = 3.87$ m, $M = 691$ N $\cdot$ m

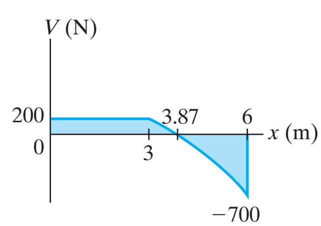


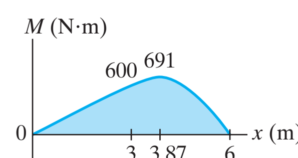


Ans:
For $0 \le x < 3$ m: $V = 200$ N, $M = \{200x\}$ N $\cdot$ m,

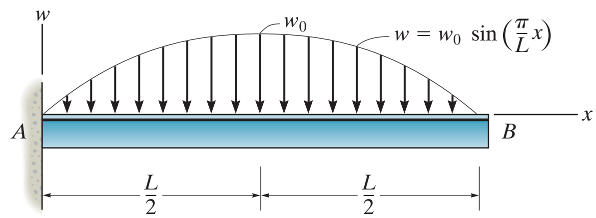
For $3 \text{ m} < x \le 6$ m: $V = \left\{-\frac{100}{3}x^2 + 500\right\}$ N,

$M = \left\{-\frac{100}{9}x^3 + 500x - 600\right\}$ N $\cdot$ m

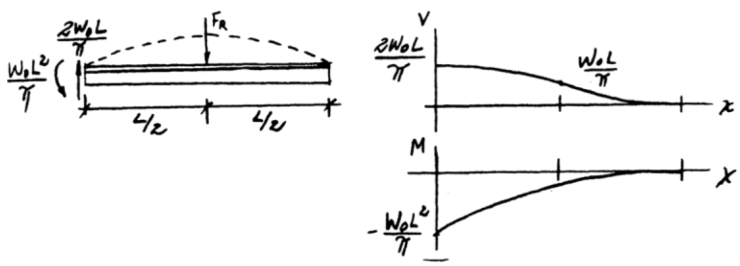




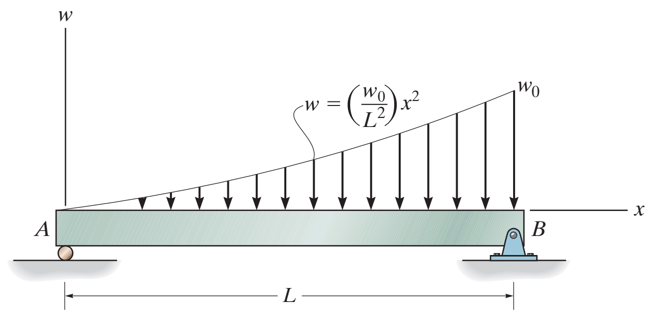
***6–44.** Draw the shear and moment diagrams for the beam.



$$F_R = \int_A dA = w_0 \int_0^L \sin\left(\frac{\pi}{L}x\right)dx = \frac{2w_0 L}{\pi}$$



6–45. Draw the shear and moment diagrams for the beam.



$$F_R = \int_A dA = \int_0^L w\,dx = \frac{w_0}{L^2}\int_0^L x^2\,dx = \frac{w_0 L}{3}$$

$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA} = \frac{\frac{w_0}{L^2}\int_0^L x^3\,dx}{\frac{w_0 L}{3}} = \frac{3L}{4}$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0 L}{12} - \frac{w_0 x^3}{3L^2} = 0$$

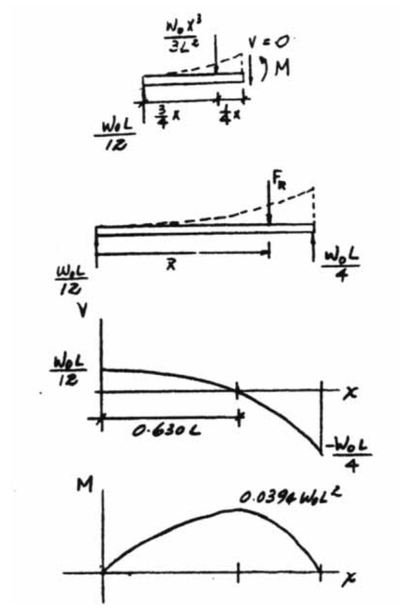
$$x = \left(\frac{1}{4}\right)^{1/3} L = 0.630\,L$$

$$\zeta + \Sigma M = 0; \qquad \frac{w_0 L}{12}(x) - \frac{w_0 x^3}{3L^2}\left(\frac{1}{4}x\right) - M = 0$$

$$M = \frac{w_0 L x}{12} - \frac{w_0 x^4}{12L^2}$$

Substitute $x = 0.630L$

$M = 0.0394\, w_0 L^2$

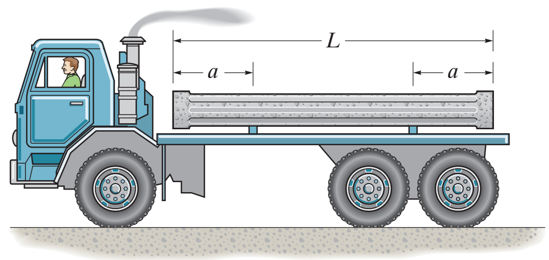


Ans:
$x = 0.630\,L,\ V = 0,\ M = 0.0394 w_0 L^2,$

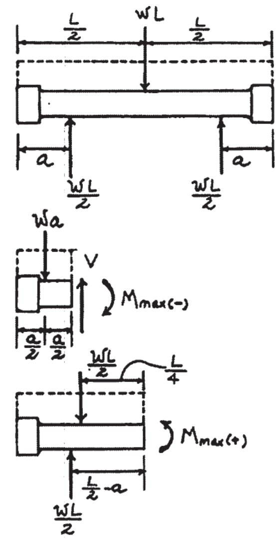
$$M = \frac{w_0 L x}{12} - \frac{w_0 x^4}{12L^2}$$

6–46.

The truck is to be used to transport the concrete column. If the column has a uniform weight of w (force/length), determine the equal placement a of the supports from the ends so that the absolute maximum bending moment in the column is as small as possible. Also, draw the shear and moment diagrams for the column.



SOLUTION



Support Reactions: As shown on FBD.

Absolute Minimum Moment: In order to get the absolute minimum moment, the maximum positive and maximum negative moment must be equal that is $M_{\max(+)} = M_{\min(-)}$.

For the positive moment:

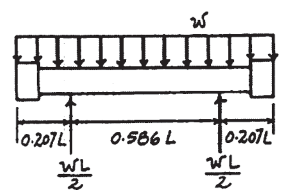
$$\circlearrowleft+\Sigma M_{NA} = 0; \qquad M_{\max(+)} + \frac{wL}{2}\left(\frac{L}{4}\right) - \frac{wL}{2}\left(\frac{L}{2} - a\right) = 0$$

$$M_{\max(+)} = \frac{wL^2}{8} - \frac{waL}{2}$$

For the negative moment:

$$\circlearrowleft+\Sigma M_{NA} = 0; \qquad wa\left(\frac{a}{2}\right) - M_{\max(-)} = 0$$

$$M_{\max(-)} = \frac{wa^2}{2}$$



$$M_{\max(+)} = M_{\max(-)}$$

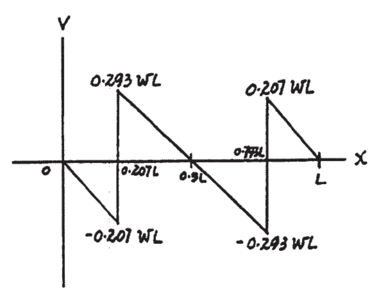
$$\frac{wL^2}{8} - \frac{wL}{2}a = \frac{wa^2}{2}$$

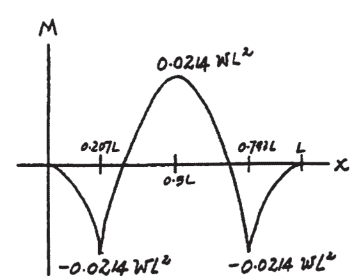
$$4a^2 + 4La - L^2 = 0$$

$$a = \frac{-4L \pm \sqrt{16L^2 - 4(4)(-L^2)}}{2(4)}$$

$$a = 0.207L$$ **Ans.**

Shear and Moment Diagram:

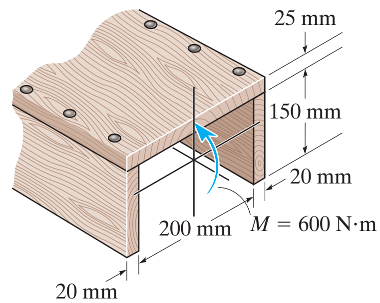




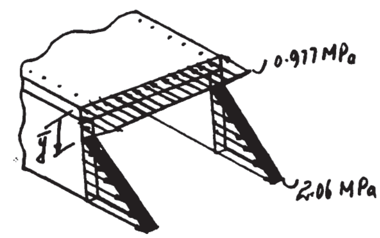
Ans:
$a = 0.207L$

6–47.

The beam is made from three boards nailed together as shown. If the moment acting on the cross section is $M = 600\ \text{N}\cdot\text{m}$, determine the maximum bending stress in the beam. Sketch a three-dimensional view of the stress distribution and cover the cross section.



SOLUTION



$$\bar{y} = \frac{(0.0125)(0.24)(0.025) + 2(0.1)(0.15)(0.02)}{0.24(0.025) + 2(0.15)(0.02)} = 0.05625\ \text{m}$$

$$I = \frac{1}{12}(0.24)\left(0.025^3\right) + (0.24)(0.025)\left(0.04375^2\right)$$

$$+2\left(\frac{1}{12}\right)(0.02)\left(0.15^3\right) + 2(0.15)(0.02)\left(0.04375^2\right)$$

$$= 34.53125\left(10^{-6}\right)\ \text{m}^4$$

$$\sigma_{\max} = \sigma_B = \frac{Mc}{I}$$

$$= \frac{600(0.175 - 0.05625)}{34.53125\left(10^{-6}\right)}$$

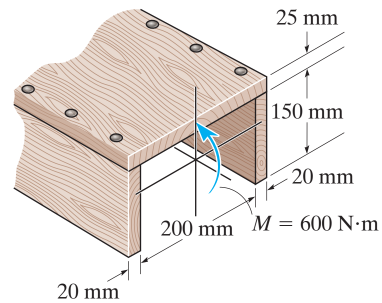
$$= 2.06\ \text{MPa}$$ **Ans.**

$$\sigma_c = \frac{My}{I} = \frac{600(0.05625)}{34.53125\left(10^{-6}\right)} = 0.977\ \text{MPa}$$

Ans:
$\sigma_{\max} = 2.06\ \text{MPa}$

***6–48.**

The beam is made from three boards nailed together as shown. If the moment acting on the cross section is $M = 600\ \text{N}\cdot\text{m}$, determine the resultant force the bending stress produces on the top board.



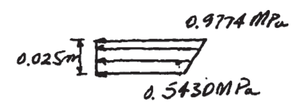
SOLUTION

$$\bar{y} = \frac{(0.0125)(0.24)(0.025) + 2(0.15)(0.1)(0.02)}{0.24(0.025) + 2(0.15)(0.02)} = 0.05625\ \text{m}$$

$$I = \frac{1}{12}(0.24)\left(0.025^3\right) + (0.24)(0.025)\left(0.04375^2\right)$$

$$+ 2\left(\frac{1}{12}\right)(0.02)\left(0.15^3\right) + 2(0.15)(0.02)\left(0.04375^2\right)$$

$$= 34.53125\left(10^{-6}\right)\ \text{m}^4$$



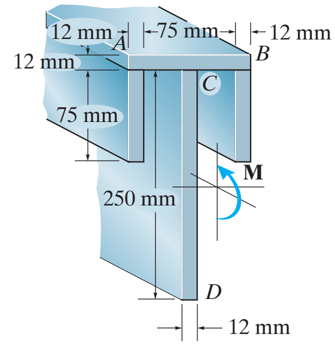
$$\sigma_t = \frac{My}{I} = \frac{600(0.05625)}{34.53125\left(10^{-6}\right)} = 0.9774\ \text{MPa}$$

$$\sigma_b = \frac{My}{I} = \frac{600(0.05625 - 0.025)}{34.53125\left(10^{-6}\right)} = 0.5430\ \text{MPa}$$

$$F = \frac{1}{2}(0.025)(0.9774 + 0.5430)\left(10^6\right)(0.240) = 4.56\ \text{kN}$$ **Ans.**

Ans:
$F = 4.56\ \text{kN}$

6–49. Determine the moment M that will produce a maximum stress of 70 MPa on the cross-section.



Section Properties:

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{0.006(0.099)(0.012) + 2[0.0495(0.012)(0.075)] + 0.137(0.012)(0.25)}{0.099(0.012) + 2(0.012)(0.075) + 0.012(0.25)} = 0.08471 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.099)(0.012^3) + (0.099)(0.012)(0.08471 - 0.006)^2$$

$$+ 2\left[\frac{1}{12}(0.012)(0.075^3) + (0.012)(0.075)(0.08471 - 0.0495)^2\right]$$

$$+ \frac{1}{12}(0.012)(0.25^3) + (0.012)(0.25)(0.137 - 0.08471)^2$$

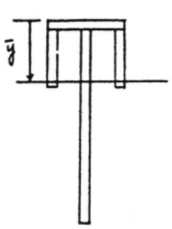
$$= 34.2773(10^{-6}) \text{ m}^4$$

Maximum Bending Stress: Applying the flexure formula

$$\sigma_{\max} = \frac{Mc}{I}$$

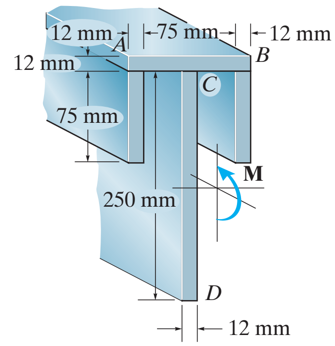
$$70(10^6) = \frac{M(0.262 - 0.08471)}{34.2773(10^{-6})}$$

$$M = 13.53(10^3) \text{ N} \cdot \text{m} = 13.5 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$



Ans:
$M = 13.5 \text{ kN} \cdot \text{m}$

6–50. Determine the maximum tensile and compressive bending stress in the beam if it is subjected to a moment of $M = 6\ \text{kN} \cdot \text{m}$.



Section Properties:

$$\bar{y} = \frac{\Sigma\,\bar{y}\,A}{\Sigma A}$$

$$= \frac{0.006(0.099)(0.012) + 2[0.0495(0.012)(0.075)] + 0.137(0.012)(0.25)}{0.099(0.012) + 2(0.012)(0.075) + 0.012(0.25)} = 0.08471\ \text{m}$$

$$I_{NA} = \frac{1}{12}(0.099)(0.012^3) + (0.099)(0.012)(0.08471 - 0.006)^2$$

$$+ 2\left[\frac{1}{12}(0.012)(0.075^3) + (0.012)(0.075)(0.08471 - 0.0495)^2\right]$$

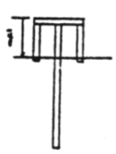
$$+ \frac{1}{12}(0.012)(0.25^3) + (0.012)(0.25)(0.137 - 0.08471)^2$$

$$= 34.2773(10^{-6})\ \text{m}^4$$

Maximum Bending Stress: Applying the flexure formula $\sigma_{\max} = \dfrac{Mc}{I}$

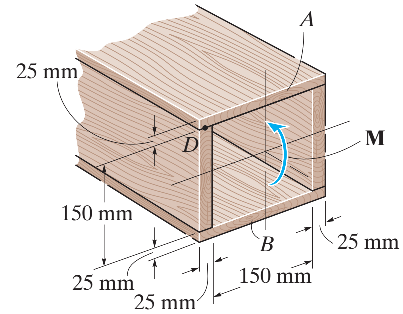
$$(\sigma_t)_{\max} = \frac{[6(10^3)](0.262 - 0.08471)}{34.2773(10^{-6})} = 31.03(10^6)\ \text{N/m}^2 = 31.0\ \text{MPa}$$ **Ans.**

$$(\sigma_c)_{\max} = \frac{[6(10^3)](0.08471)}{34.2773(10^{-6})} = 14.83(10^6)\ \text{N/m}^2 = 14.8\ \text{MPa}$$ **Ans.**



Ans:
$\bar{y} = 0.08471$ m,
$I_{NA} = 34.2773\,(10^{-6})\ \text{m}^4$,
$(\sigma_t)_{\max} = 31.0$ MPa, $(\sigma_c)_{\max} = 14.8$ MPa

6–51. The beam is subjected to a moment **M**. Determine the percentage of this moment that is resisted by the stresses acting on both the top and bottom boards, A and B, of the beam.

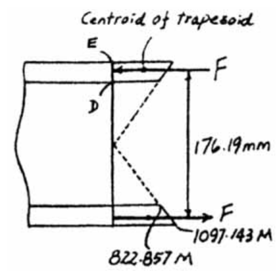


Section Property:

$$I = \frac{1}{12}(0.2)\left(0.2^3\right) - \frac{1}{12}(0.15)\left(0.15^3\right) = 91.14583\left(10^{-6}\right)\ \text{m}^4$$

Bending Stress: Applying the flexure formula

$$\sigma = \frac{My}{I}$$



$$\sigma_E = \frac{M(0.1)}{91.14583(10^{-6})} = 1097.143\ M$$

$$\sigma_D = \frac{M(0.075)}{91.14583(10^{-6})} = 822.857\ M$$

Resultant Force and Moment: For board A or B

$$F = 822.857M(0.025)(0.2) + \frac{1}{2}(1097.143M - 822.857M)(0.025)(0.2)$$

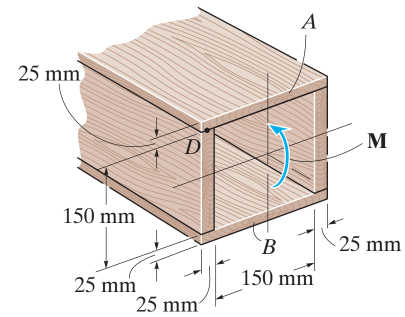
$$= 4.800\ M$$

$$M' = F(0.17619) = 4.80M(0.17619) = 0.8457\ M$$

$$\sigma_c\left(\frac{M'}{M}\right) = 0.8457(100\%) = 84.6\ \%$$ **Ans.**

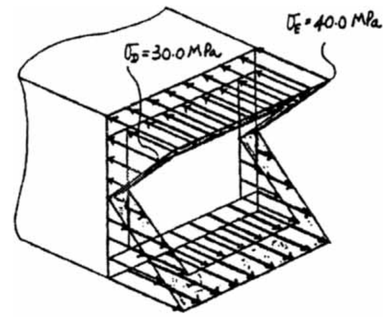
Ans:
84.6%

***6–52.** Determine the moment **M** that should be applied to the beam in order to create a compressive stress at point D of $\sigma_D = 30$ MPa. Also sketch the stress distribution acting over the cross section and compute the maximum stress developed in the beam.



Section Property:

$$I = \frac{1}{12}(0.2)\left(0.2^3\right) - \frac{1}{12}(0.15)\left(0.15^3\right) = 91.14583\left(10^{-6}\right)\text{ m}^4$$



Bending Stress: Applying the flexure formula

$$\sigma = \frac{My}{I}$$

$$30\left(10^6\right) = \frac{M(0.075)}{91.14583(10^{-6})}$$

$$M = 36458\text{ N}\cdot\text{m} = 36.5\text{ kN}\cdot\text{m}$$ **Ans.**

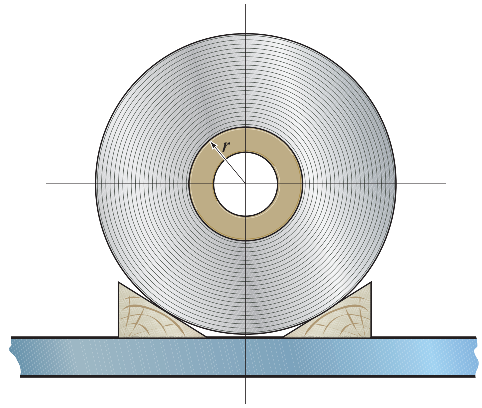
$$\sigma_{\max} = \frac{Mc}{I} = \frac{36458(0.1)}{91.14583(10^{-6})} = 40.0\text{ MPa}$$ **Ans.**

Ans:

$M = 36.5$ kN·m, $\sigma_{\max} = 40.0$ MPa

6–53.

An A-36 steel strip has an allowable bending stress of 165 MPa. If it is rolled up, determine the smallest radius r of the spool if the strip has a width of 10 mm and a thickness of 1.5 mm. Also, find the corresponding maximum internal moment developed in the strip.



SOLUTION

Bending Stress-Curvature Relation:

$$\sigma_{\text{allow}} = \frac{Ec}{\rho}; \qquad 165(10^6) = \frac{200(10^9)\left[0.75(10^{-3})\right]}{r}$$

$$r = 0.9091 \text{ m} = 909 \text{ mm} \qquad \textbf{Ans.}$$

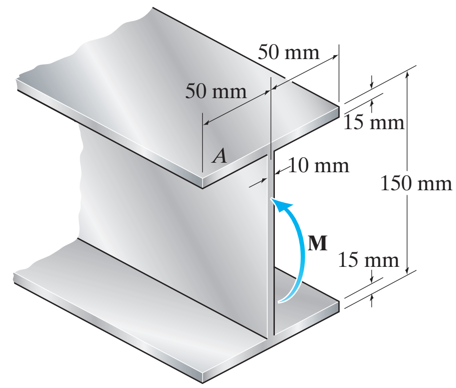
Moment Curvature Relation:

$$\frac{1}{\rho} = \frac{M}{EI}; \quad \frac{1}{0.9091} = \frac{M}{200(10^9)\left[\frac{1}{12}(1)(0.0015^3)\right]}$$

$$M = 61.875 \text{ N} \cdot \text{m} = 61.9 \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$

Ans:
$r = 909$ mm, $M = 61.9$ N · m

6–54. If the beam is subjected to an internal moment of $M = 30\ \text{kN}\cdot\text{m}$, determine the maximum bending stress in the beam. The beam is made from A992 steel. Sketch the bending stress distribution on the cross section.



Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.1)(0.15^3) - \frac{1}{12}(0.09)(0.12^3) = 15.165(10^{-6})\ \text{m}^4$$

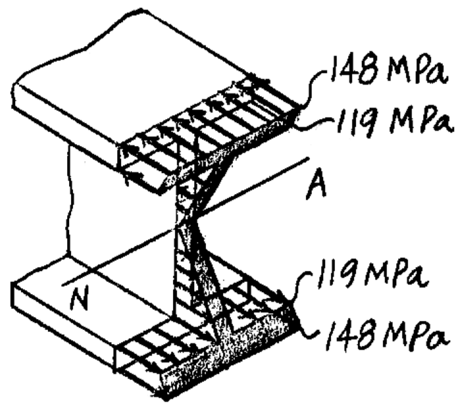
Maximum Bending Stress: The maximum bending stress occurs at the top and bottom surfaces of the beam since they are located at the furthest distance from the neutral axis. Thus, $c = 75\ \text{mm} = 0.075\ \text{m}$.

$$\sigma_{\max} = \frac{Mc}{I} = \frac{30(10^3)(0.075)}{15.165(10^{-6})} = 148\ \text{MPa}$$ **Ans.**

At $y = 60\ \text{mm} = 0.06\ \text{m}$,

$$\sigma|_{y=0.06\ \text{m}} = \frac{My}{I} = \frac{30(10^3)(0.06)}{15.165(10^{-6})} = 119\ \text{MPa}$$

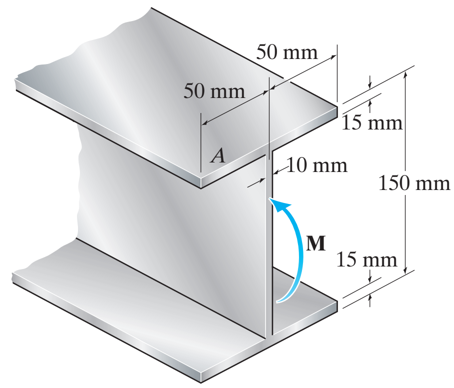
The bending stress distribution across the cross section is shown in Fig. *a*.



(a)

Ans:
$\sigma_{\max} = 148\ \text{MPa}$

6–55. If the beam is subjected to an internal moment of $M = 30$ kN·m, determine the resultant force caused by the bending stress distribution acting on the top flange A.



Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.1)(0.15^3) - \frac{1}{12}(0.09)(0.12^3) = 15.165(10^{-6})\ \text{m}^4$$

Bending Stress: The distance from the neutral axis to the top and bottom surfaces of flange A is $y_t = 75$ mm $= 0.075$ m and $y_b = 60$ mm $= 0.06$ m.

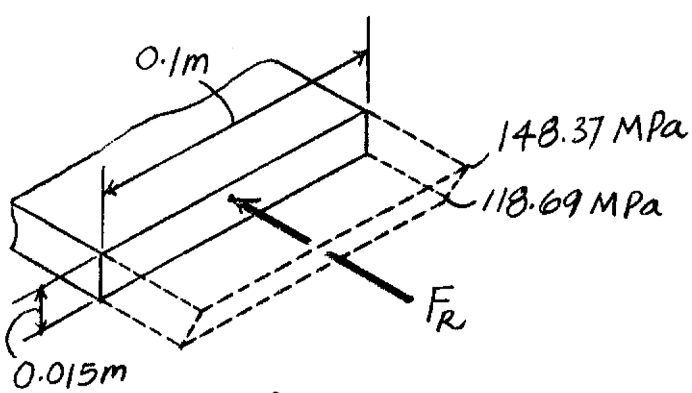
$$\sigma_t = \frac{My_t}{I} = \frac{30(10^3)(0.075)}{15.165(10^{-6})} = 148.37 = 148\ \text{MPa}$$

$$\sigma_b = \frac{My_b}{I} = \frac{30(10^3)(0.06)}{15.165(10^{-6})} = 118.69 = 119\ \text{MPa}$$

Resultant Force: The resultant force acting on flange A is equal to the volume of the trapezoidal stress block shown in Fig. a. Thus,

$$F_R = \frac{1}{2}(148.37 + 118.69)(10^6)(0.1)(0.015)$$

$$= 200\,296.74\ \text{N} = 200\ \text{kN}$$ **Ans.**

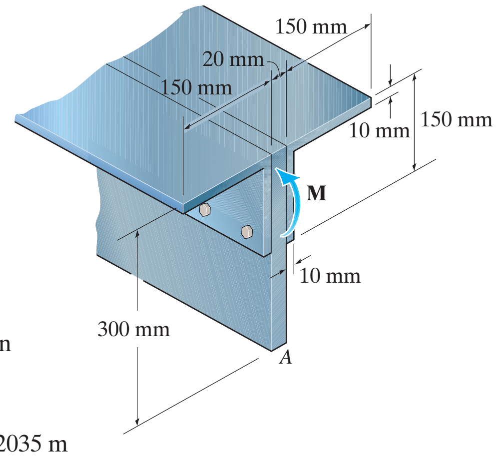


(a)

Ans:
$F_R = 200$ kN

***6–56.**

If the built-up beam is subjected to an internal moment of $M = 75\ \text{kN}\cdot\text{m}$, determine the maximum tensile and compressive stress acting in the beam.



SOLUTION

Section Properties: The neutral axis passes through centroid C of the cross section as shown in Fig. a. The location of C is

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{0.15(0.3)(0.02) + 2[0.225(0.15)(0.01)] + 2[0.295(0.01)(0.14)]}{0.3(0.02) + 2(0.15)(0.01) + 2(0.01)(0.14)} = 0.2035\ \text{m}$$

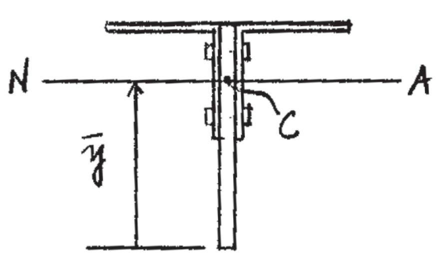
Thus, the moment of inertia of the cross section about the neutral axis is

$$\begin{aligned} I &= \Sigma\bar{I} + Ad^2 \\ &= \frac{1}{12}(0.02)(0.3^3) + 0.02(0.3)(0.2035 - 0.15)^2 \\ &\quad + 2\left[\frac{1}{12}(0.01)(0.15^3) + 0.01(0.15)(0.225 - 0.2035)^2\right] \\ &\quad + 2\left[\frac{1}{12}(0.14)(0.01^3) + 0.14(0.01)(0.295 - 0.2035)^2\right] \\ &= 92.6509(10^{-6})\ \text{m}^4 \end{aligned}$$

Maximum Bending Stress: The maximum compressive and tensile stress occurs at the top and bottom-most fiber of the cross section.

$$(\sigma_{\max})_c = \frac{My}{I} = \frac{75(10^3)(0.3 - 0.2035)}{92.6509(10^{-6})} = 78.1\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\sigma_{\max})_t = \frac{Mc}{I} = \frac{75(10^3)(0.2035)}{92.6509(10^{-6})} = 165\ \text{MPa} \qquad \textbf{Ans.}$$

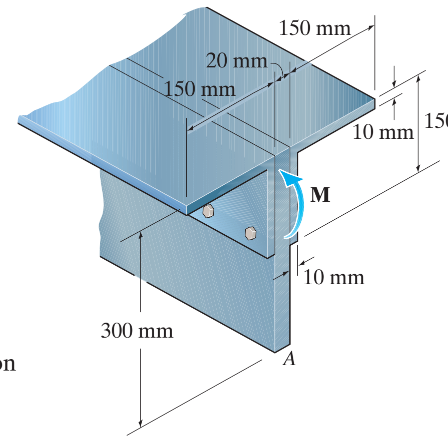


(a)

Ans:
$(\sigma_{\max})_c = 78.1$ MPa, $(\sigma_{\max})_t = 165$ MPa

6–57.

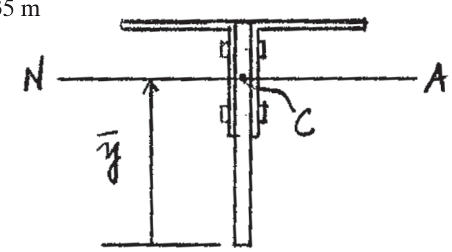
If the built-up beam is subjected to an internal moment of $M = 75\text{ kN}\cdot\text{m}$, determine the amount of this internal moment resisted by plate A.



SOLUTION

Section Properties: The neutral axis passes through centroid C of the cross section as shown in Fig. a. The location of C is

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{0.15(0.3)(0.02) + 2[0.225(0.15)(0.01)] + 2[0.295(0.01)(0.14)]}{0.3(0.02) + 2(0.15)(0.01) + 2(0.01)(0.14)} = 0.2035\text{ m}$$



(a)

Thus, the moment of inertia of the cross section about the neutral axis is

$$I = \bar{I} + Ad^2$$

$$= \frac{1}{12}(0.02)(0.3^3) + 0.02(0.3)(0.2035 - 0.15)^2$$

$$+ 2\left[\frac{1}{12}(0.01)(0.15^3) + 0.01(0.15)(0.225 - 0.2035)^2\right]$$

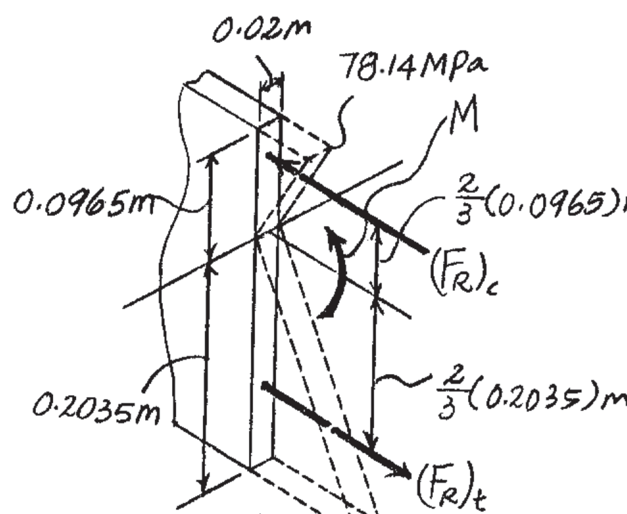
$$+ 2\left[\frac{1}{12}(0.14)(0.01^3) + 0.14(0.01)(0.295 - 0.2035)^2\right]$$

$$= 92.6509(10^{-6})\text{ m}^4$$

Bending Stress: The distance from the neutral axis to the top and bottom of plate A is $y_t = 0.3 - 0.2035 = 0.0965\text{ m}$ and $y_b = 0.2035\text{ m}$.

$$\sigma_t = \frac{My_t}{I} = \frac{75(10^3)(0.0965)}{92.6509(10^{-6})} = 78.14\text{ MPa (C)}$$

$$\sigma_b = \frac{My_b}{I} = \frac{75(10^3)(0.2035)}{92.6509(10^{-6})} = 164.71\text{ MPa (T)}$$



(b)

The bending stress distribution across the cross section of plate A is shown in Fig. b. The resultant forces of the tensile and compressive triangular stress blocks are

$$(F_R)_t = \frac{1}{2}(164.71)(10^6)(0.2035)(0.02) = 335\,144.46\text{ N}$$

$$(F_R)_c = \frac{1}{2}(78.14)(10^6)(0.0965)(0.02) = 75\,421.50\text{ N}$$

Thus, the amount of internal moment resisted by plate A is

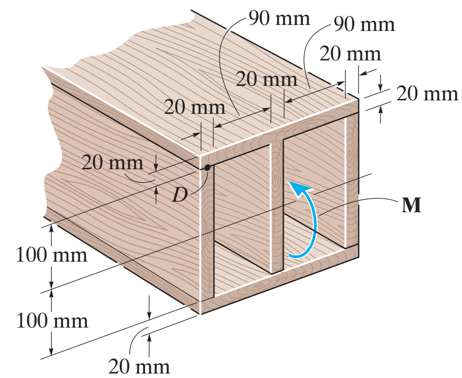
$$M = 335144.46\left[\frac{2}{3}(0.2035)\right] + 75421.50\left[\frac{2}{3}(0.0965)\right]$$

$$= 50315.65\text{ N}\cdot\text{m} = 50.3\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$M = 50.3\text{ kN}\cdot\text{m}$

6–58.

The beam is subjected to a moment M. Determine the percentage of this moment that is resisted by the stresses acting on both the top and bottom boards of the beam.

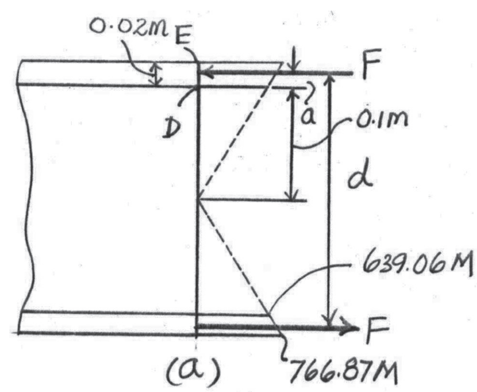


SOLUTION

Section Properties: The moment of inertia of the beam's cross-section about the neutral axis is

$$I = \frac{1}{12}(0.24)\left(0.24^3\right) - \frac{1}{12}(0.18)\left(0.2^3\right) = 0.15648\left(10^{-3}\right)\text{ m}^4$$

Bending Stress: Applying the flexure formula, $\sigma = \dfrac{My}{I}$, the bending stress on points D and E, Fig. a, is



$$\sigma_D = \frac{M(0.1)}{0.15648\left(10^{-3}\right)} = 639.06M$$

$$\sigma_E = \frac{M(0.12)}{0.15648\left(10^{-3}\right)} = 766.87M$$

Resultant Force and Moment: The resultant of the stress block acting on boards A and B, Fig. a, is

$$F = \frac{1}{2}(639.06M + 766.87M)(0.02)(0.24) = 3.3742M$$

The location of line of action of F is

$$a = \frac{1}{3}\left[\frac{2(766.87M) + 639.06M}{639.06M + 766.87M}\right](0.02) = 0.010303\text{ m}$$

Thus, the moment arm of F is

$$a' = 2(0.1 + 0.010303) = 0.22061\text{ m}$$

Then

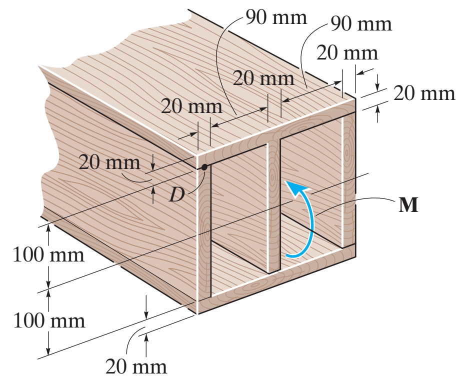
$$M' = Fd = 3.3742M(0.22061) = 0.74438M$$

$$\%\left(\frac{M'}{M}\right) = \left(\frac{0.74438M}{M}\right)(100) = 74.4\% \qquad \textbf{Ans.}$$

Ans:
$\dfrac{M'}{M} = 74.4\%$

6–59.

Determine the moment M that should be applied to the beam in order to create a compressive stress at point D of $\sigma_D = 10$ MPa. Also sketch the stress distribution acting over the cross section and calculate the maximum stress developed in the beam.



SOLUTION

Section Properties: The moment of inertia of the beam's cross-section about the neutral axis is

$$I = \frac{1}{12}(0.24)\left(0.24^3\right) - \frac{1}{12}(0.18)\left(0.2^3\right) = 0.15648\left(10^{-3}\right) M^4$$

Bending Stress: Applying the flexure formula,

$$\sigma_D = \frac{My_D}{I}$$

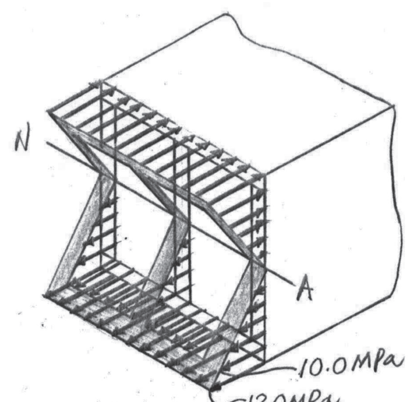
$$10\left(10^6\right) = \frac{M(0.1)}{0.15648\left(10^{-3}\right)}$$

$$M = 15.648\left(10^{-3}\right) \text{ N}\cdot\text{m} = 15.6 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

The maximum bending stress is

$$\sigma_{\max} = \frac{MC}{I} = \frac{[15.648\left(10^3\right)](0.12)}{0.15648\left(10^{-3}\right)} = 12.0\left(10^6\right) \text{ Pa} = 12.0 \text{ MPa} \qquad \textbf{Ans.}$$

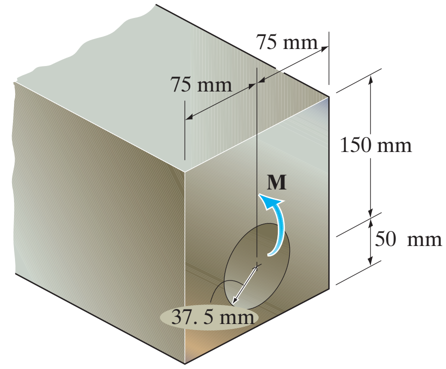
The sketch of Bending stress distribution on the beam's cross-section is shown in Fig. *a*.



(a)

Ans:
$M = 15.6$ kN · m,
$\sigma_{\max} = 12.0$ MPa

***6–60.** If the beam is subjected to an internal moment of of $M = 150$ kN · m, determine the maximum tensile and compressive bending stress in the beam.



Section Properties: The neutral axis passes through centroid C of the cross section as shown in Fig. a. The location of C is

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.1(0.15)(0.2) - 0.05[\pi(0.0375^2)]}{0.15(0.2) - \pi(0.0375^2)} = 0.10863 \text{ m}$$

Thus, the moment of inertia of the cross section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2$$

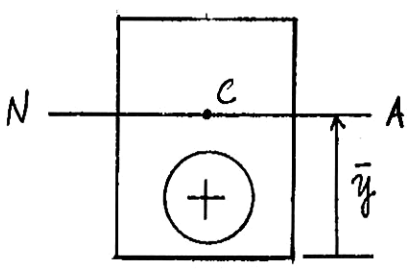
$$= \frac{1}{12}(0.15)(0.2^3) + 0.15(0.2)(0.10863 - 0.1)^2 - \left[\frac{\pi}{4}(0.0375^4) + \pi(0.0375^2)(0.10863 - 0.05)^2\right]$$

$$= 85.4948(10^{-6}) \text{ m}^4$$

Maximum Bending Stress: The maximum compressive and tensile bending stress occurs at the top and bottom edges of the cross section.

$$(\sigma_{\max})_T = \frac{MC}{I} = \frac{[150(10^3)](0.10863)}{85.4948(10^{-6})} = 190.60(10^6) \text{ N/m}^2 = 191 \text{ MPa} \qquad \textbf{Ans.}$$

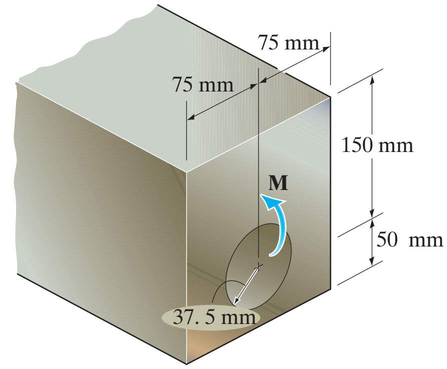
$$(\sigma_{\max})_C = \frac{MC}{I} = \frac{[150(10^3)](0.2 - 0.10863)}{85.4948(10^{-6})} = 160.30(10^6) \text{ N/m}^2 = 160 \text{ MPa} \qquad \textbf{Ans.}$$



(a)

Ans:
$(\sigma_{\max})_T = 191$ MPa,
$(\sigma_{\max})_C = 160$ MPa

6–61. If the beam is made of material having an allowable tensile and compressive stress of $(\sigma_{\text{allow}})_t = 168$ MPa and $(\sigma_{\text{allow}})_c = 154$ MPa, respectively, determine the maximum allowable internal moment **M** that can be applied to the beam.



Section Properties: The neutral axis passes through centroid C of the cross section as shown in Fig. *a*. The location of C is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.1(0.15)(0.2) - 0.05[\pi(0.0375^2)]}{0.15(0.2) - \pi(0.0375^2)} = 0.10863 \text{ m}$$

Thus, the moment of inertia of the cross section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2$$

$$= \frac{1}{12}(0.15)(0.2^3) + 0.15(0.2)(0.10863 - 0.1)^2 - \left[\frac{\pi}{4}(0.0375^4) + \pi(0.0375^2)(0.10863 - 0.05)^2\right]$$

$$= 85.4948(10^{-6}) \text{ m}^4$$

Allowable Bending Stress: The maximum compressive and tensile bending stress occurs at the top and bottom edges of the cross section. For the top edge,

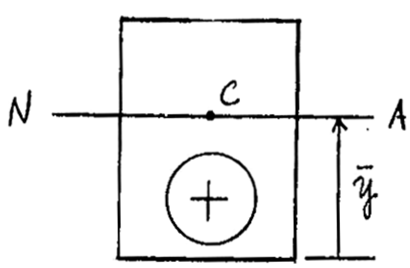
$$(\sigma_{\text{allow}})_c = \frac{My}{I}; \qquad 154(10^6) = \frac{M(0.2 - 0.10863)}{85.4948(10^{-6})}$$

$$M = 144.11(10^3) \text{ N}\cdot\text{m} = 144 \text{ kN}\cdot\text{m}$$

For the bottom edge,

$$(\sigma_{\max})_t = \frac{Mc}{I}; \qquad 154(10^6) = \frac{M(0.10863)}{85.4948(10^{-6})}$$

$$M = 132.22(10^3) \text{ N}\cdot\text{m} = 132 \text{ kN}\cdot\text{m} \; (\textit{controls!})$$ **Ans.**



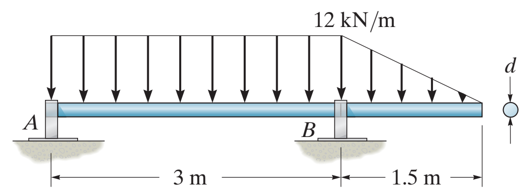
(a)

Ans:

$M = 132 \text{ kN}\cdot\text{m}$

6–62.

The shaft is supported by smooth journal bearings at A and B that only exert vertical reactions on the shaft. If $d = 90$ mm, determine the absolute maximum bending stress in the beam, and sketch the stress distribution acting over the cross section.



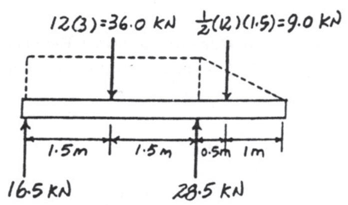
SOLUTION

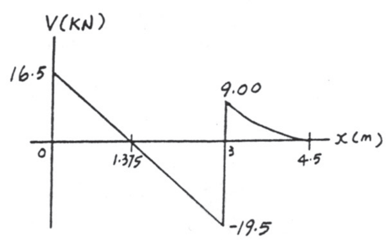
Absolute Maximum Bending Stress: The maximum moment is $M_{\max} = 11.34 \text{ kN} \cdot \text{m}$ as indicated on the moment diagram. Applying the flexure formula

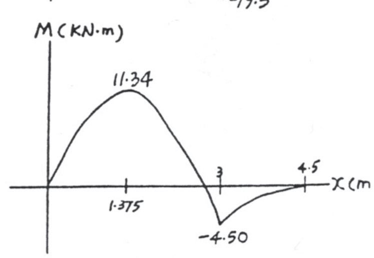
$$\sigma_{\max} = \frac{M_{\max}\, c}{I}$$

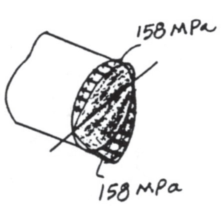
$$= \frac{11.34(10^3)(0.045)}{\frac{\pi}{4}(0.045^4)}$$

$$= 158 \text{ MPa}$$ **Ans.**





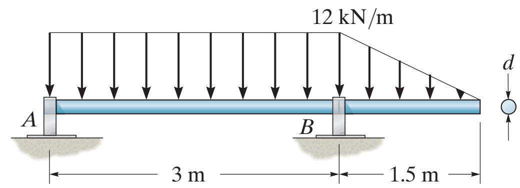




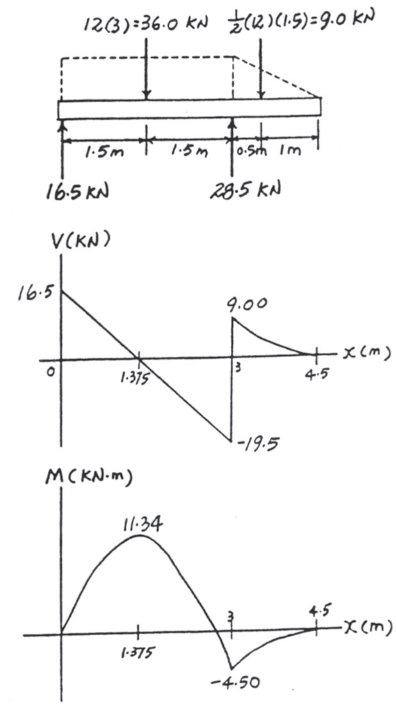
Ans:
$\sigma_{\max} = 158$ MPa

6–63.

The shaft is supported by smooth journal bearings at A and B that only exert vertical reactions on the shaft. Determine its smallest diameter d if the allowable bending stress is $\sigma_{\text{allow}} = 180$ MPa.



SOLUTION



Allowable Bending Stress: The maximum moment is $M_{\text{max}} = 11.34\ \text{kN}\cdot\text{m}$ as indicated on the moment diagram. Applying the flexure formula

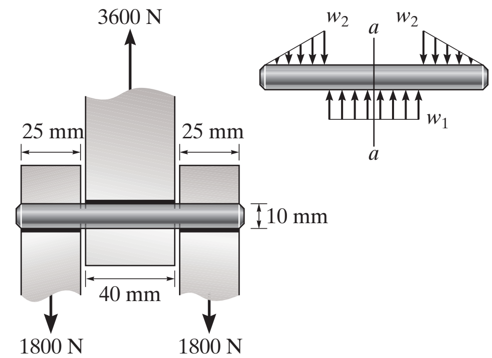
$$\sigma_{\text{max}} = \sigma_{\text{allow}} = \frac{M_{\text{max}}\, c}{I}$$

$$180\left(10^6\right) = \frac{11.34(10^3)\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

$$d = 0.08626\ \text{m} = 86.3\ \text{mm}$$ **Ans.**

Ans:
$d = 86.3$ mm

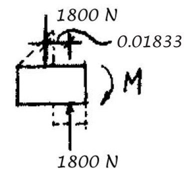
***6–64.** The pin is used to connect the three links together. Due to wear, the load is distributed over the top and bottom of the pin as shown on the free-body diagram. If the diameter of the pin is 10 mm, determine the maximum bending stress on the cross-sectional area at the center section *a–a*. For the solution it is first necessary to determine the load intensities w_1 and w_2.



$$\frac{1}{12}w_2(0.25) = 1800; \quad w_2 = 144(10^3)\ \text{N/m}$$

$$w_1(0.04) = 3600; \quad w_1 = 90(10^3)\ \text{N/m}$$

$$M = 1800(0.01833) = 33.0\ \text{N}\cdot\text{m}$$



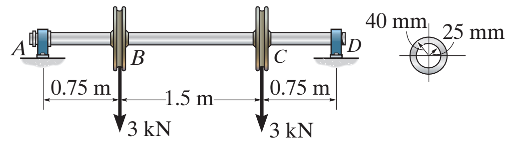
$$I = \frac{1}{4}\pi(0.005^4) = 0.49087(10^{-9})\ \text{m}^4$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{33.0(0.005)}{0.49087(10^{-9})} = 336.14(10^6)\ \text{N/m}^2 = 336\ \text{MPa}$$ **Ans.**

Ans:
$\sigma_{\max} = 324.45$ MPa

6–65.

The shaft is supported by a thrust bearing at A and journal bearing at D. If the shaft has the cross section shown, determine the absolute maximum bending stress in the shaft.



SOLUTION

Shear and Moment Diagrams: As shown in Fig. *a*.

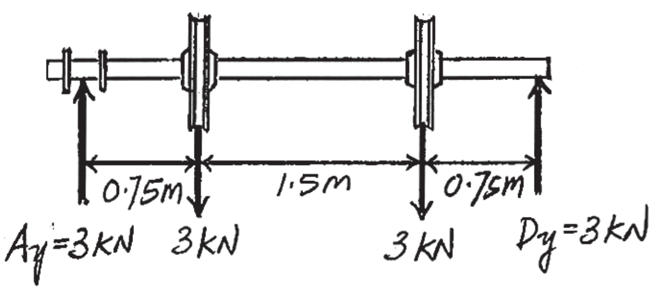
Maximum Moment: Due to symmetry, the maximum moment occurs in region BC of the shaft. Referring to the free-body diagram of the segment shown in Fig. *b*.

Section Properties: The moment of inertia of the cross section about the neutral axis is

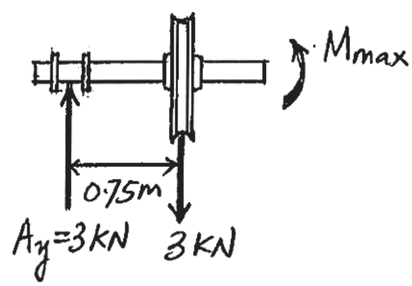
$$I = \frac{\pi}{4}(0.04^4 - 0.025^4) = 1.7038(10^{-6})\text{ m}^4$$

Absolute Maximum Bending Stress:

$$\sigma_{\max} = \frac{M_{\max}c}{I} = \frac{2.25(10^3)(0.04)}{1.7038(10^{-6})} = 52.8\text{ MPa} \qquad \textbf{Ans.}$$



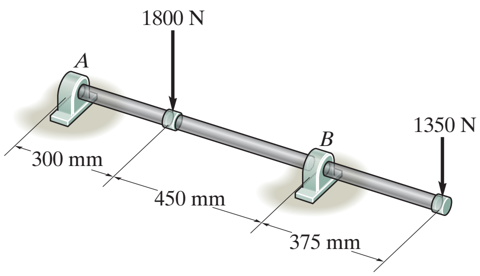
(a)



(b)

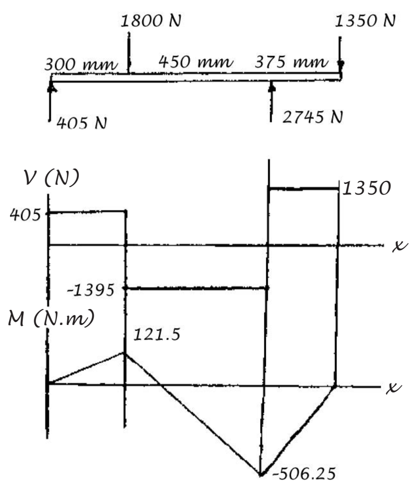
Ans:
$\sigma_{\max} = 52.8$ MPa

6–66. Determine the absolute maximum bending stress in the 40-mm-diameter shaft which is subjected to the concentrated forces. The sleeve bearings at A and B support only vertical forces.



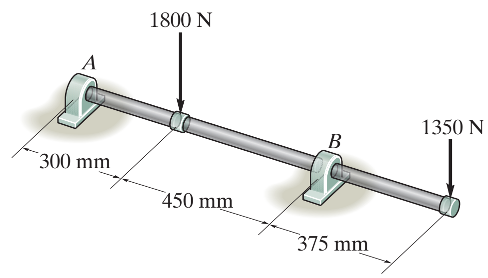
$$M_{\max} = 506.25\ \mathrm{N\cdot m}$$

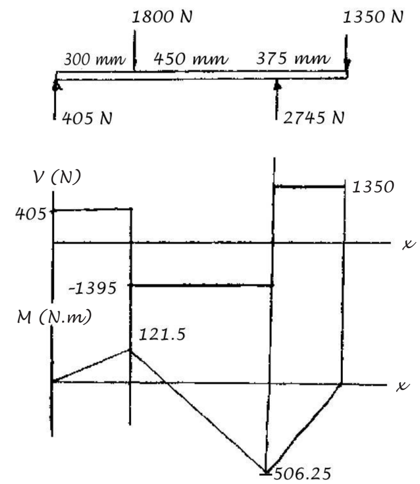
$$\sigma = \frac{Mc}{I} = \frac{506.25(0.02)}{\frac{\pi}{4}(0.02^4)} = 80.57(10^6)\ \mathrm{N/m^2} = 80.6\ \mathrm{MPa}$$ **Ans.**



Ans:
$\sigma_{\max} = 52.8\ \mathrm{MPa}$

6–63. Determine the smallest allowable diameter of the shaft which is subjected to the concentrated forces. The sleeve bearings at A and B support only vertical forces, and the allowable bending stress is $\sigma_{\text{allow}} = 154$ MPa.





$$M_{\text{max}} = 506.25 \text{ N} \cdot \text{m}$$

$$10.5(10^6) = \frac{(506.25)c}{\frac{\pi}{4}c^4}$$

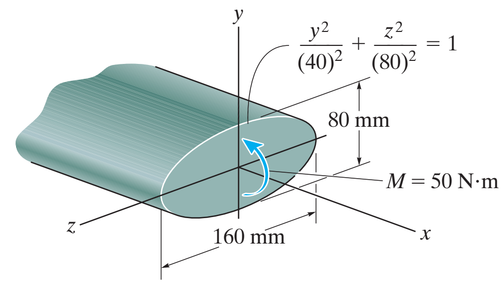
$$c = 0.01612 \text{ m}$$

$$d = 2c = 2(0.01612) = 0.03223 \text{ m} = 32.2 \text{ mm}$$ **Ans.**

Ans:
$d = 32.2$ mm

***6–68.**

A shaft is made of a polymer having an elliptical cross section. If it resists an internal moment of $M = 50\ \text{N}\cdot\text{m}$, determine the maximum bending stress in the material (a) using the flexure formula, where $I_z = \frac{1}{4}\pi(0.08\text{ m})(0.04\text{ m})^3$, (b) using integration. Sketch a three-dimensional view of the stress distribution acting over the cross-sectional area. Here $I_x = \frac{1}{4}\pi(0.08\text{ m})(0.04\text{ m})^3$.

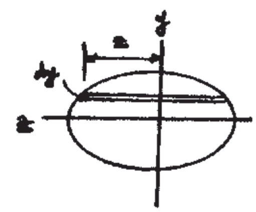


SOLUTION

(a)

$$I = \frac{1}{4}\pi\, ab^3 = \frac{1}{4}\pi(0.08)(0.04)^3 = 4.021238(10^{-6})\ \text{m}^4$$

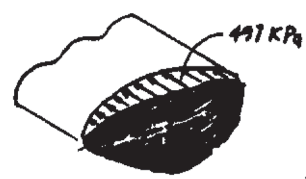
$$\sigma_{\max} = \frac{Mc}{I} = \frac{50(0.04)}{4.021238(10^{-6})} = 497\ \text{kPa}$$ **Ans.**



(b)

$$M = \frac{\sigma_{\max}}{c}\int_A y^2 dA$$

$$= \frac{\sigma_{\max}}{c}\int y^2 2z\,dy$$



$$z = \sqrt{0.0064 - 4y^2} = 2\sqrt{(0.04)^2 - y^2}$$

$$2\int_{-0.04}^{0.04} y^2 z\,dy = 4\int_{-0.04}^{0.04} y^2\sqrt{(0.04)^2 - y^2}\,dy$$

$$= 4\left[\frac{(0.04)^4}{8}\sin^{-1}\left(\frac{y}{0.04}\right) - \frac{1}{8}y\sqrt{(0.04)^2 - y^2}(0.04^2 - 2y^2)\right]\Bigg|_{-0.04}^{0.04}$$

$$= \frac{(0.04)^4}{2}\sin^{-1}\left(\frac{y}{0.04}\right)\Bigg|_{-0.04}^{0.04}$$

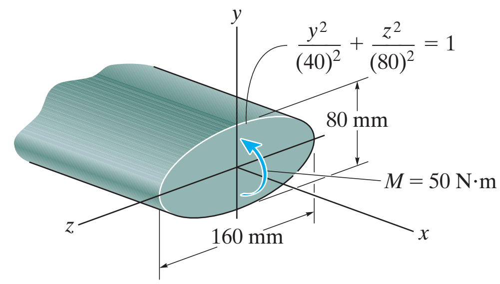
$$= 4.021238(10^{-6})\ \text{m}^4$$

$$\sigma_{\max} = \frac{50(0.04)}{4.021238(10^{-6})} = 497\ \text{kPa}$$ **Ans.**

Ans:
(a) $\sigma_{\max} = 497\ \text{kPa}$,
(b) $\sigma_{\max} = 497\ \text{kPa}$

6–69.

Solve Prob. 6–65 if the moment $M = 50\text{ N}\cdot\text{m}$ is applied about the y axis instead of the x axis. Here $I_y = \frac{1}{4}\pi\,(0.04\text{ m})(0.08\text{ m})^3$.

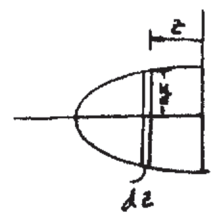


SOLUTION

(a)

$$I = \frac{1}{4}\pi\,ab^3 = \frac{1}{4}\pi(0.04)(0.08)^3 = 16.085(10^{-6})\text{ m}^4$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{50(0.08)}{16.085(10^{-6})} = 249\text{ kPa}$$ **Ans.**



(b)

$$M = \int_A z(\sigma\,dA) = \int_A z\left(\frac{\sigma_{max}}{0.08}\right)(z)(2y)dz$$

$$50 = 2\left(\frac{\sigma_{max}}{0.04}\right)\int_0^{0.08} z^2\left(1 - \frac{z^2}{(0.08)^2}\right)^{1/2}(0.04)dz$$

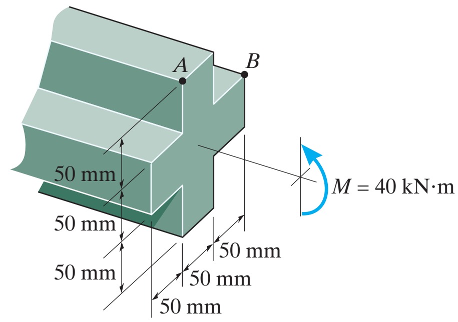
$$50 = 201.06(10^{-6})\sigma_{max}$$

$$\sigma_{max} = 249\text{ kPa}$$ **Ans.**

Ans:
(a) $\sigma_{max} = 249\text{ kPa}$,
(b) $\sigma_{max} = 249\text{ kPa}$

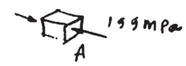
6–70.

The beam is subjected to a moment of $M = 40\ \text{kN}\cdot\text{m}$. Determine the bending stress at points A and B. Sketch the results on a volume element acting at each of these points.

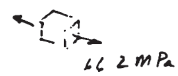


SOLUTION

$$I = \frac{1}{12}(0.150)(0.05)^3 + 2\left[\frac{1}{12}(0.05)(0.05)^3 + (0.05)(0.05)(0.05)^2\right] = 15.1042\left(10^{-6}\right)\ \text{m}^4$$



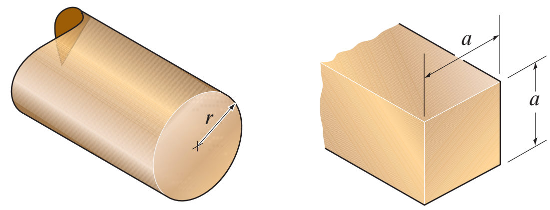
$$\sigma_A = \frac{Mc}{I} = \frac{40\left(10^3\right)(0.075)}{15.1042\left(10^{-6}\right)} = 199\ \text{MPa}$$ **Ans.**



$$\sigma_B = \frac{My}{I} = \frac{40\left(10^3\right)(0.025)}{15.1042\left(10^{-6}\right)} = 66.2\ \text{MPa}$$ **Ans.**

Ans:
$\sigma_A = 199$ MPa,
$\sigma_B = 66.2$ MPa

6–71. Determine the dimension a of a beam having a square cross section in terms of the radius r of a beam with a circular cross section if both beams are subjected to the same internal moment which results in the same maximum bending stress.



Section Properties: The moments of inertia of the square and circular cross sections about the neutral axis are

$$I_S = \frac{1}{12}a\left(a^3\right) = \frac{a^4}{12} \qquad I_C = \frac{1}{4}\pi r^4$$

Maximum Bending Stress: For the square cross section, $c = a/2$.

$$\left(\sigma_{\max}\right)_S = \frac{Mc}{I_S} = \frac{M(a/2)}{a^4/12} = \frac{6M}{a^3}$$

For the circular cross section, $c = r$.

$$\left(\sigma_{\max}\right)_c = \frac{Mc}{I_c} = \frac{Mr}{\frac{1}{4}\pi r^4} - \frac{4M}{\pi r^3}$$

It is required that

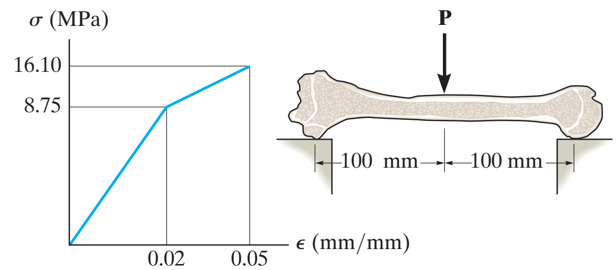
$$\left(\sigma_{\max}\right)_S = \left(\sigma_{\max}\right)_C$$

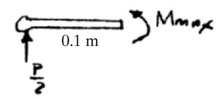
$$\frac{6M}{a^3} = \frac{4M}{\pi r^3}$$

$$a = 1.677r \qquad \textbf{Ans.}$$

Ans:
$a = 1.677r$

***6–72.** A portion of the femur can be modeled as a tube having an inner diameter of 9.5 mm and an outer diameter of 32 mm. Determine the maximum elastic static force P that can be applied to its center. Assume the bone to be roller supported at its ends. The $\sigma-\epsilon$ diagram for the bone mass is shown and is the same in tension as in compression.





$$I = \frac{\pi}{4}(0.016^4 - 0.00475^4) = 51.0720(10^{-9})\ \text{m}^4$$

$$M_{\max} = \frac{P}{2}(0.1) = 0.05P$$

Require $\sigma_{\max} = 8.75$ MPa

$$\sigma_{\max} = \frac{Mc}{I}$$

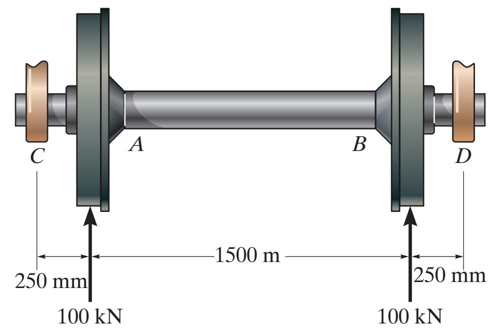
$$8.75(10^6) = \frac{(0.05P)(0.016)}{51.0720(10^{-9})}$$

$P = 558.6\ \text{N} = 559\ \text{N}$ **Ans.**

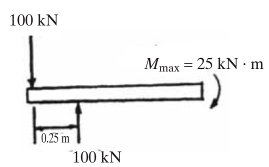
Ans:

$P = 559$ N

6–73. The axle of the freight car is subjected to wheel loadings of 100 kN. If it is supported by two journal bearings at C and D, determine the maximum bending stress developed at the center of the axle, where the diameter is 137.5 mm.

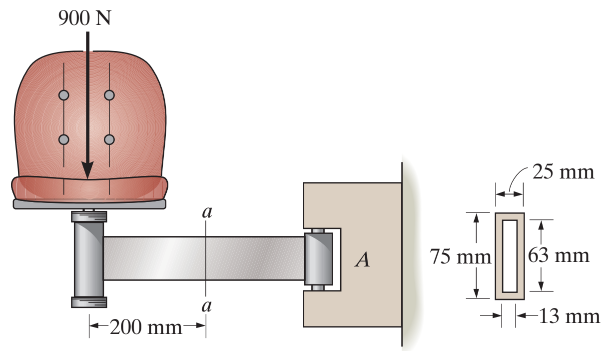


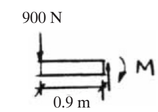
$$\sigma_{\max} = \frac{MC}{I} = \frac{[25(10^3)](0.06875)}{\frac{\pi}{4}(0.06875^4)} = 97.96(10^6)\ \text{N/m}^2 = 98\ \text{MPa}$$ **Ans.**



Ans:
$\sigma_{\max} = 98.0$ MPa

6–74. The chair is supported by an arm that is hinged so it rotates about the vertical axis at A. If the load on the chair is 900 N and the arm is a hollow tube section having the dimensions shown, determine the maximum bending stress at section a–a.





$\circlearrowleft + \Sigma M = 0; \qquad M - 900(0.9) = 0$

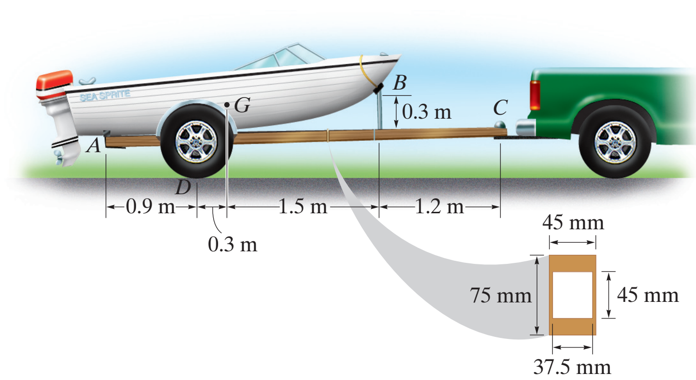
$M = 810 \text{ N} \cdot \text{m}$

$$I_x = \frac{1}{12}(0.025)(0.075^3) - \frac{1}{12}(0.013)(0.063^3) = 0.608022(10^{-6}) \text{ m}^4$$

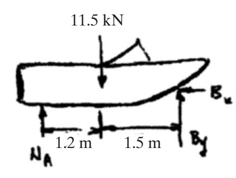
$$\sigma_{\max} = \frac{MC}{I} = \frac{810(0.0375)}{0.608022(10^{-6})} = 49.96(10^{-6}) \text{ N/m}^2 = 50.0 \text{ MPa}$$ **Ans.**

Ans:
$\sigma_{\max} = 11.1$ MPa

6–75. The boat has a weight of 11.5 kN and a center of gravity at G. If it rests on the trailer at the smooth contact A and can be considered pinned at B, determine the absolute maximum bending stress developed in the main strut of the trailer. Consider the strut to be a box-beam having the dimensions shown and pinned at C.



Boat:



$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 0$

$\circlearrowleft + \Sigma M_B = 0; \qquad -N_A(2.7) + 11.5(1.5) = 0$

$$N_A = 6.3889 \text{ kN}$$

$+\uparrow \Sigma F_y = 0; \qquad 6.3889 - 11.5 + B_y = 0$

$$B_y = 5.1111 \text{ kN}$$

Assembly:

$\circlearrowleft + \Sigma M_C = 0; \qquad -N_D(3.0) + 11.5(2.7) = 0$

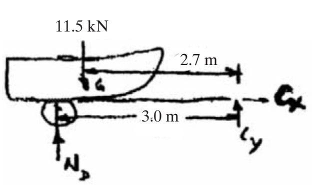
$$N_D = 10.35 \text{ kN}$$

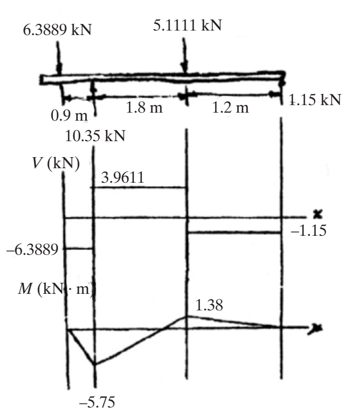
$+\uparrow \Sigma F_y = 0; \qquad C_y + 10.35 - 11.5 = 0$

$$C_y = 1.15 \text{ kN}$$

$$I = \frac{1}{12}(0.045)(0.075^3) - \frac{1}{12}(0.0375)(0.045^3) = 1.2973(10^{-6}) \text{ m}^4$$

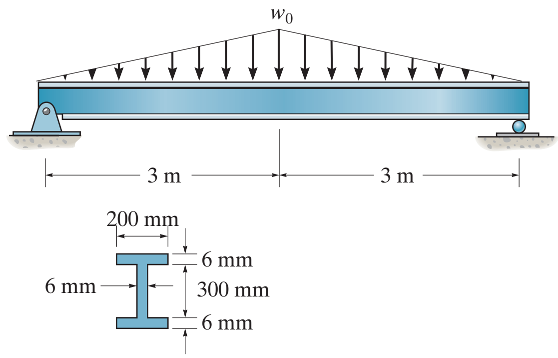
$$\sigma_{\max} = \frac{MC}{I} = \frac{[5.75(10^3)](0.0375)}{1.2973(10^{-6})} = 166.21(10^6) \text{ N-m}^2 = 166 \text{ MPa} \qquad \textbf{Ans.}$$





Ans:
$\sigma_{\max} = 166$ MPa

***6–76.** The steel beam has the cross-sectional area shown. Determine the largest intensity of the distributed load w_0 that it can support so that the maximum bending stress in the beam does not exceed $\sigma_{\text{allow}} = 160$ MPa.



Support Reactions. The FBD of the beam is shown in Fig. *a*.

The shear and moment diagrams are shown in Fig. *a* and *b*, respectively. As indicated on the moment diagram, $M_{\max} = 3w_o$.

The moment of inertia of the cross-section about the neutral axis is

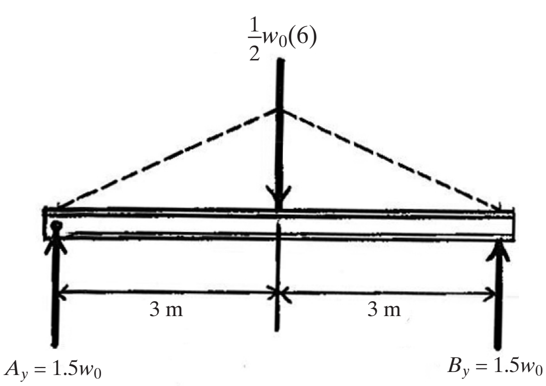
$$I = \frac{1}{12}(0.2)(0.312^3) - \frac{1}{12}(0.194)(0.3^3) = 69.6888(10^{-6})\ \text{m}^4$$

Hence, $c = 0.156$ m. Thus,

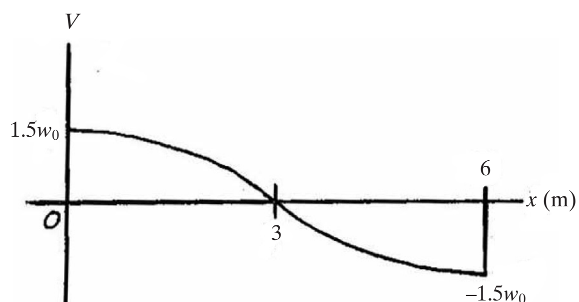
$$\sigma_{\text{allow}} = \frac{M_{\max}\, c}{I};$$

$$160(10^6) = \frac{(3w_o)(0.156)}{69.6888(10^{-6})}$$

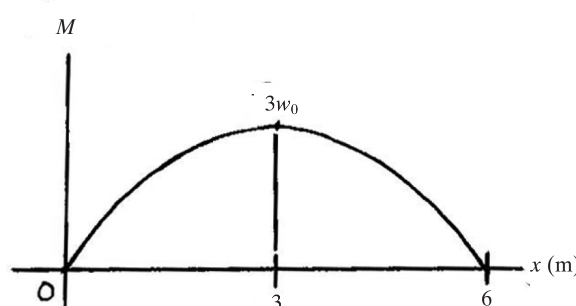
$$w_o = 23.83(10^3)\ \text{N/m} = 23.8\ \text{kN/m}$$ **Ans.**



(a)



(b)

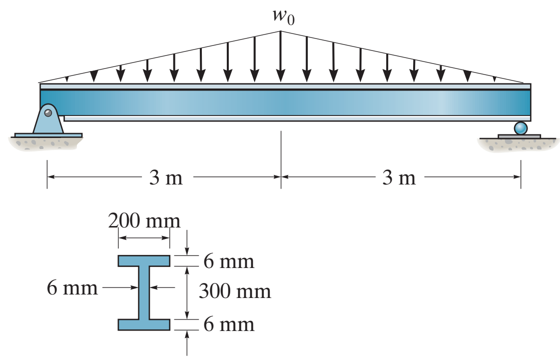


(c)

Ans:

$w_o = 23.8$ kN/m

6–77. The steel beam has the cross-sectional area shown. If $w_0 = 30$ kN/m, determine the maximum bending stress in the beam.



The FBD of the beam is shown in Fig. *a*

The shear and moment diagrams are shown in Fig. *b* and *c*, respectively. As indicated on the moment diagram, $M_{max} = 90$ kN · m.

The moment of inertia of the I cross-section about the bending axis is

$$I = \frac{1}{12}(0.2)(0.312^3) - \frac{1}{12}(0.194)(0.3^3)$$

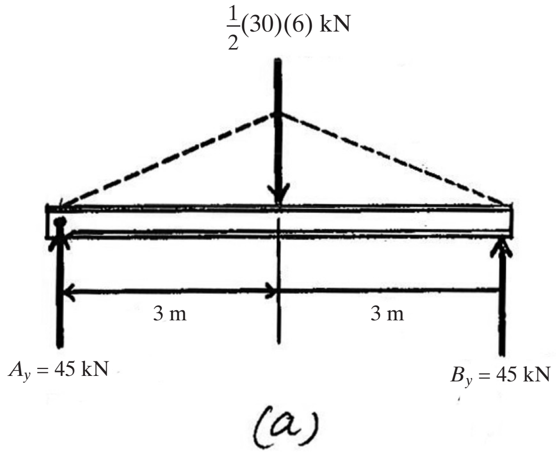
$$= 69.6888(10^{-6})\ \text{m}^4$$

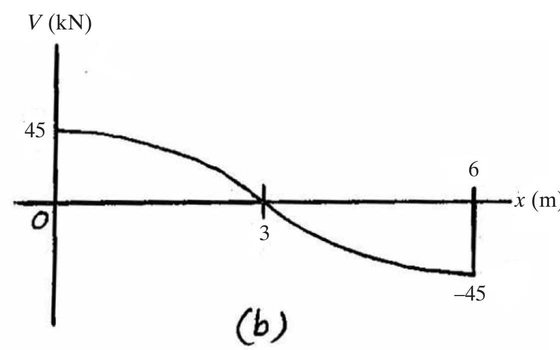
Hence, $c = 0.156$ m. Thus,

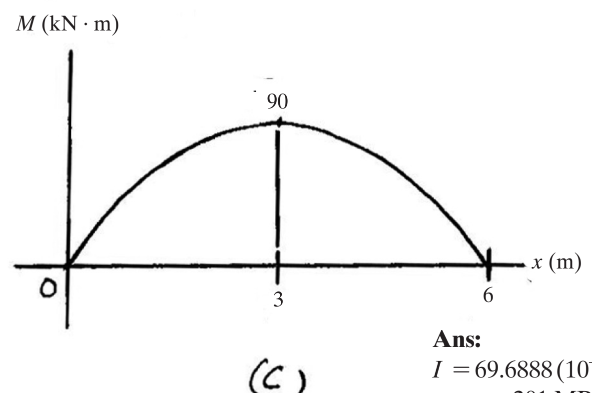
$$\sigma_{max} = \frac{M_{max}\,c}{I}$$

$$= \frac{[90(0.3^3)](0.156)}{69.6888(10^{-6})}$$

$$= 201.47(10^6)\ \text{N/m}^2 = 201\ \text{MPa}$$ **Ans.**



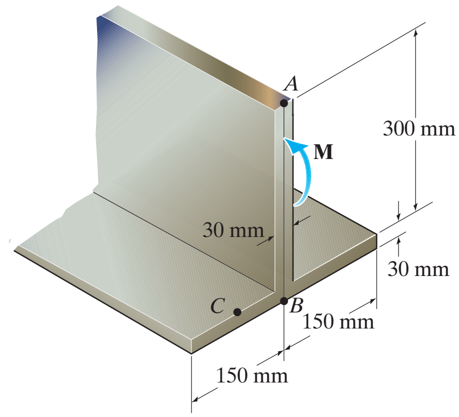




Ans:
$I = 69.6888\,(10^{-6})\ \text{m}^4$,
$\sigma_{max} = 201$ MPa

6–78.

If the beam is subjected to a moment of $M = 100\ \text{kN}\cdot\text{m}$, determine the bending stress at points A, B, and C. Sketch the bending stress distribution on the cross section.



SOLUTION

Section Properties: The neutral axis passes through centroid C of the cross section as shown in Fig. a. The location of C is

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.015(0.03)(0.3) + 0.18(0.3)(0.03)}{0.03(0.3) + 0.3(0.03)} = 0.0975\ \text{m}$$

Thus, the moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.3)(0.03^3) + 0.3(0.03)(0.0975 - 0.015)^2 + \frac{1}{12}(0.03)(0.3^3)$$

$$+ 0.03(0.3)(0.18 - 0.0975)^2$$

$$= 0.1907(10^{-3})\ \text{m}^4$$

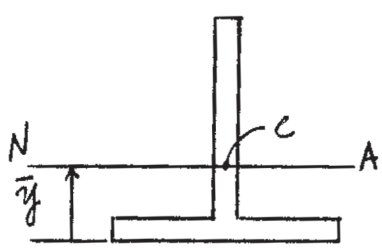
Bending Stress: The distance from the neutral axis to points A, B, and C is $y_A = 0.33 - 0.0975 = 0.2325$ m, $y_B = 0.0975$ m, and $y_C = 0.0975 - 0.03 = 0.0675$ m.

$$\sigma_A = \frac{My_A}{I} = \frac{100(10^3)(0.2325)}{0.1907(10^{-3})} = 122\ \text{MPa (C)}$$ **Ans.**

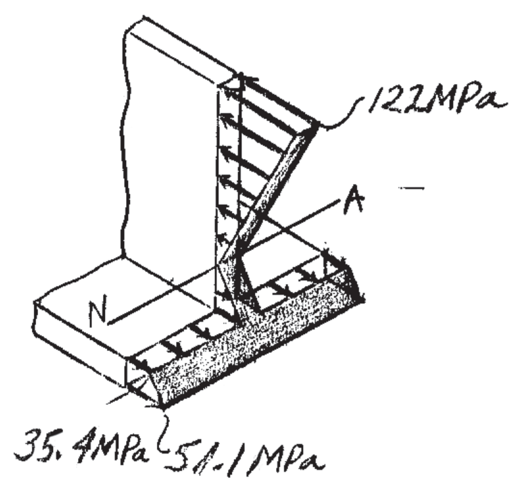
$$\sigma_B = \frac{My_B}{I} = \frac{100(10^3)(0.0975)}{0.1907(10^{-3})} = 51.1\ \text{MPa (T)}$$ **Ans.**

$$\sigma_C = \frac{My_C}{I} = \frac{100(10^3)(0.0675)}{0.1907(10^{-3})} = 35.4\ \text{MPa (T)}$$ **Ans.**

Using these results, the bending stress distribution across the cross section is shown in Fig. b.



(a)

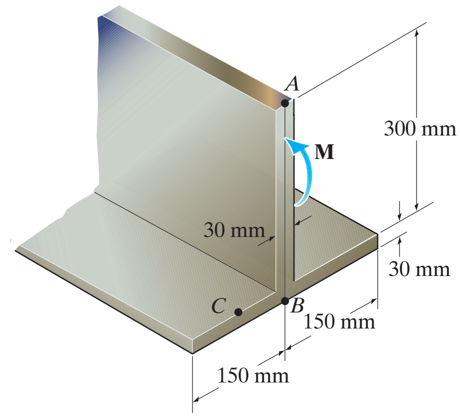


(b)

Ans:
$\sigma_A = 122$ MPa (C),
$\sigma_B = 51.1$ MPa (T),
$\sigma_C = 35.4$ MPa (T)

6–79.

If the beam is made of material having an allowable tensile and compressive stress of $(\sigma_{\text{allow}})_t = 125$ MPa and $(\sigma_{\text{allow}})_c = 150$ MPa, respectively, determine the maximum moment **M** that can be applied to the beam.



SOLUTION

Section Properties: The neutral axis passes through centroid C of the cross section as shown in Fig. a. The location of C is

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{0.015(0.03)(0.3) + 0.18(0.3)(0.03)}{0.03(0.3) + 0.3(0.03)} = 0.0975 \text{ m}$$

Thus, the moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.3)(0.03^3) + 0.3(0.03)(0.0975 - 0.015)^2 + \frac{1}{12}(0.03)(0.3^3)$$
$$+ 0.03(0.3)(0.18 - 0.0975)^2$$
$$= 0.1907(10^{-3})\text{ m}^4$$

Allowable Bending Stress: The maximum compressive and tensile stress occurs at the top and bottom-most fibers of the cross section. For the top-most fiber,

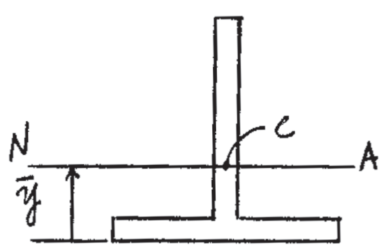
$$(\sigma_{\text{allow}})_c = \frac{Mc}{I} \qquad 150(10^6) = \frac{M(0.33 - 0.0975)}{0.1907(10^{-3})}$$

$$M = 123024.19 \text{ N}\cdot\text{m} = 123 \text{ kN}\cdot\text{m (controls)} \qquad \textbf{Ans.}$$

For the bottom-most fiber,

$$(\sigma_{\text{allow}})_t = \frac{My}{I} \qquad 125(10^6) = \frac{M(0.0975)}{0.1907(10^{-3})}$$

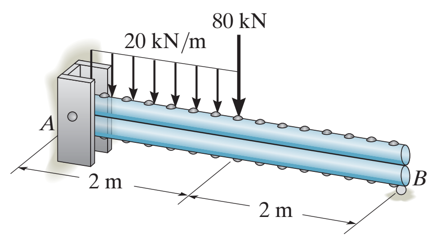
$$M = 244\,471.15 \text{ N}\cdot\text{m} = 244 \text{ kN}\cdot\text{m}$$

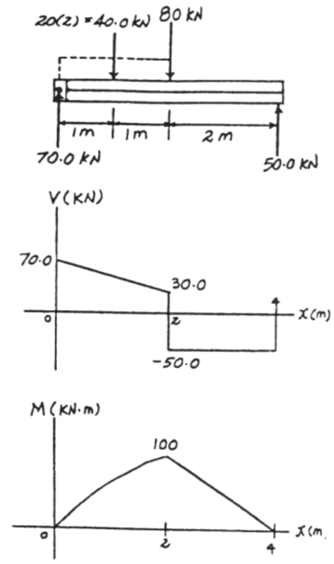


(a)

Ans:
$M = 123 \text{ kN}\cdot\text{m}$

***6–80.** The two solid steel rods are bolted together along their length and support the loading shown. Assume the support at A is a pin and B is a roller. Determine the required diameter d of each of the rods if the allowable bending stress is $\sigma_{\text{allow}} = 130$ MPa.





Section Property:

$$I = 2\left[\frac{\pi}{4}\left(\frac{d}{2}\right)^4 + \frac{\pi}{4}d^2\left(\frac{d}{2}\right)^2\right] = \frac{5\pi}{32}d^4$$

Allowable Bending Stress: The maximum moment is $M_{\max} = 100 \text{ kN}\cdot\text{m}$ as indicated on moment diagram. Applying the flexure formula

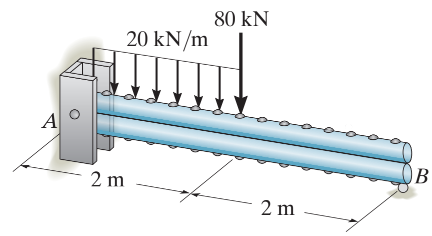
$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{M_{\max}\,c}{I}$$

$$130\left(10^6\right) = \frac{100(10^3)(d)}{\frac{5\pi}{32}d^4}$$

$$d = 0.1162 \text{ m} = 116 \text{ mm}$$ **Ans.**

Ans:
$d = 116$ mm

6–81. Solve Prob. 6–94 if the rods are rotated 90° so that both rods rest on the supports at A (pin) and B (roller).



Section Property:

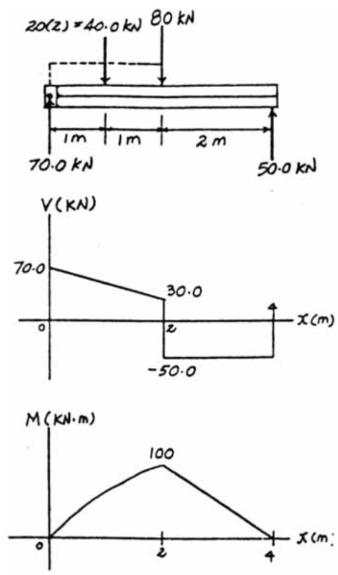
$$I = 2\left[\frac{\pi}{4}\left(\frac{d}{2}\right)^4\right] = \frac{\pi}{32}d^4$$

Allowable Bending Stress: The maximum moment is $M_{max} = 100\ \text{kN}\cdot\text{m}$ as indicated on the moment diagram. Applying the flexure formula

$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$130\left(10^6\right) = \frac{100(10^3)(d)}{\frac{\pi}{32}d^4}$$

$$d = 0.1986\ \text{m} = 199\ \text{mm}$$ **Ans.**



Ans:
$d = 199$ mm

6–82.

If the compound beam in Prob. 6–42 has a square cross section of side length a, determine the minimum value of a if the allowable bending stress is $\sigma_{\text{allow}} = 150$ MPa.

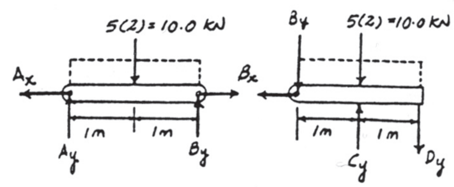
SOLUTION

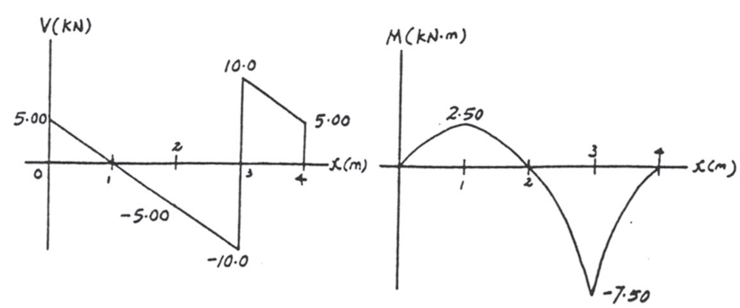
Allowable Bending Stress: The maximum moment is $M_{\max} = 7.50\ \text{kN} \cdot \text{m}$ as indicated on moment diagram. Applying the flexure formula

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{M_{\max}\, c}{I}$$

$$150\left(10^6\right) = \frac{7.50(10^3)\left(\frac{a}{2}\right)}{\frac{1}{12}a^4}$$

$$a = 0.06694\ \text{m} = 66.9\ \text{mm} \qquad \textbf{Ans.}$$





Ans:
$a = 66.9$ mm

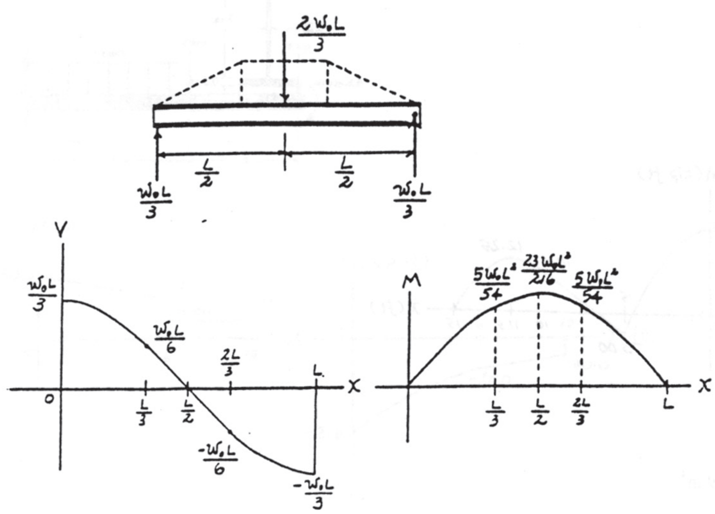
6–83.

If the beam in Prob. 6–28 has a rectangular cross section with a width b and a height h, determine the absolute maximum bending stress in the beam.

SOLUTION

Absolute Maximum Bending Stress: The maximum moment is $M_{max} = \dfrac{23w_0 L^2}{216}$ as indicated on the moment diagram. Applying the flexure formula

$$\sigma_{max} = \frac{M_{max}\, c}{I} = \frac{\frac{23w_0 L^2}{216}\left(\frac{h}{2}\right)}{\frac{1}{12}\, bh^3} = \frac{23w_0 L^2}{36bh^2} \qquad \textbf{Ans.}$$

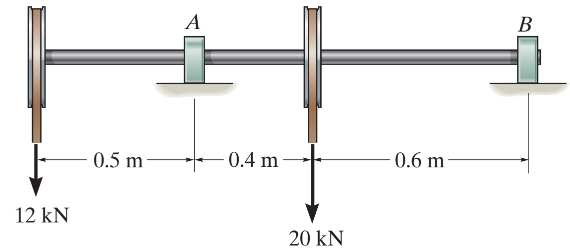


Ans:

$\sigma_{max} = \dfrac{23w_0 L^2}{36\, bh^2}$

***6–84.**

Determine the absolute maximum bending stress in the 80-mm-diameter shaft which is subjected to the concentrated forces. There is a journal bearing at A and a thrust bearing at B.



SOLUTION

The FBD of the shaft is shown in Fig. a.

The shear and moment diagrams are shown in Fig. b and c, respectively. As indicated on the moment diagram, $|M_{max}| = 6\ \text{kN}\cdot\text{m}$.

The moment of inertia of the cross section about the neutral axis is

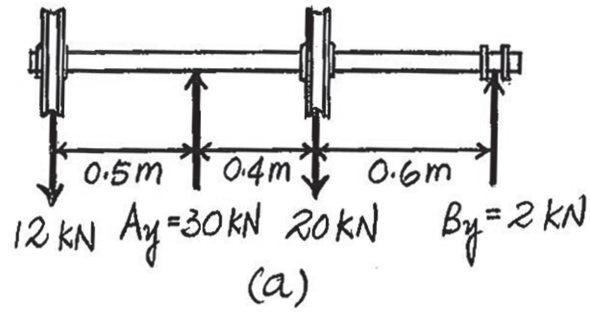
$$I = \frac{\pi}{4}(0.04^4) = 0.64(10^{-6})\pi\ \text{m}^4$$

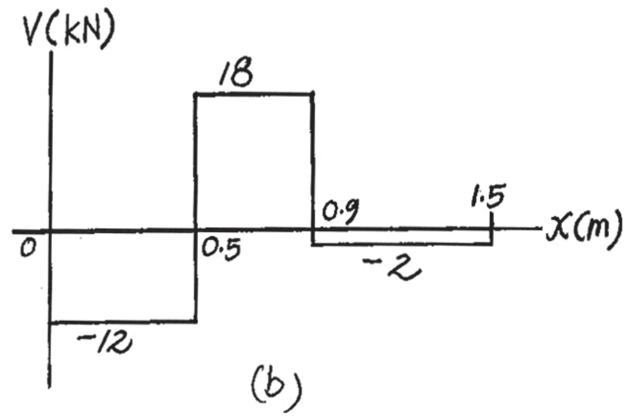
Here, $c = 0.04$ m. Thus

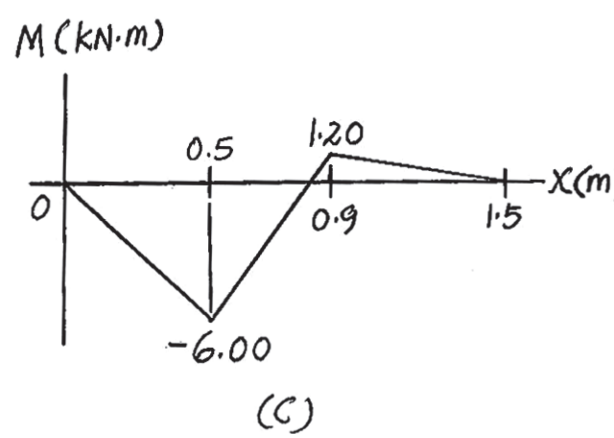
$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{6(10^3)(0.04)}{0.64(10^{-6})\pi}$$

$$= 119.37(10^6)\ \text{Pa}$$

$$= 119\ \text{MPa}$$ **Ans.**



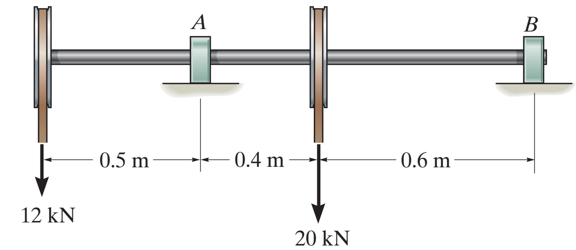




Ans:
$\sigma_{max} = 119$ MPa

6–85.

Determine, to the nearest millimeter, the smallest allowable diameter of the shaft which is subjected to the concentrated forces. There is a journal bearing at A and a thrust bearing at B. The allowable bending stress is $\sigma_{\text{allow}} = 150$ MPa.



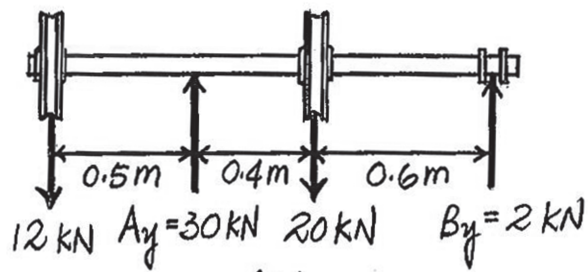
SOLUTION

The FBD of the shaft is shown in Fig. *a*.

The shear and moment diagrams are shown in Fig. *b* and *c*, respectively. As indicated on the moment diagram, $|M_{\max}| = 6\ \text{kN}\cdot\text{m}$.

The moment of inertia of the cross section about the neutral axis is

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}$$

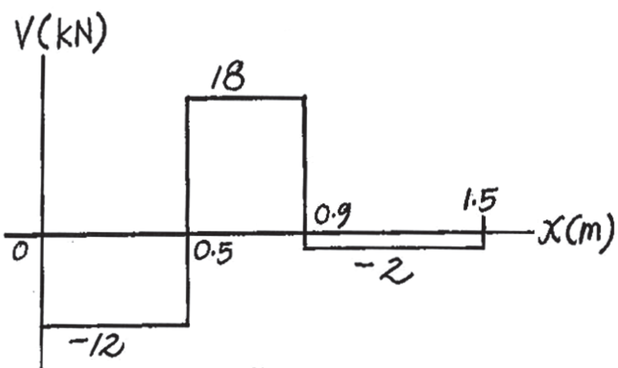


(a)

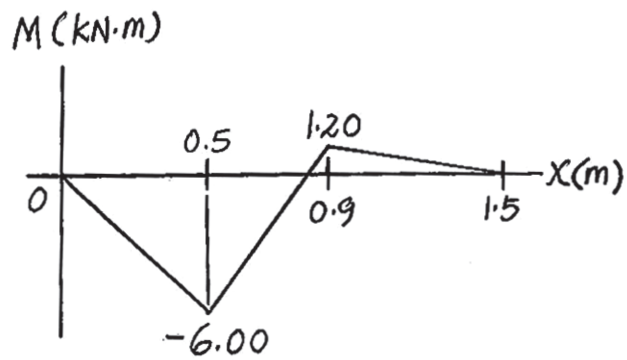
Here, $c = d/2$. Thus

$$\sigma_{\text{allow}} = \frac{M_{\max}\, c}{I}; \qquad 150(10^6) = \frac{6(10^3)(d/2)}{\pi d^4/64}$$

$$d = 0.07413\ \text{m} = 74.13\ \text{mm} = 75\ \text{mm} \qquad \textbf{Ans.}$$



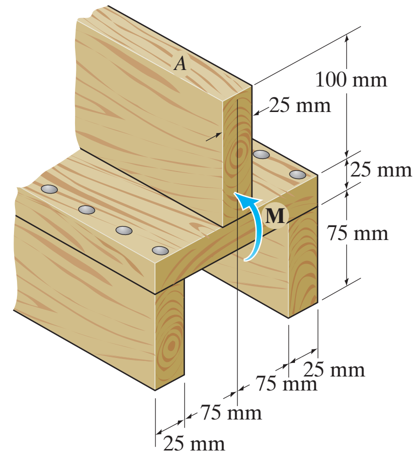
(b)



(c)

Ans:
$d = 75$ mm

6–86. If the beam is subjected to an internal moment of $M = 3\text{ kN}\cdot\text{m}$, determine the maximum tensile and compressive stress in the beam. Also, sketch the bending stress distribution on the cross section.



Section Properties: The neutral axis passes through centroid C of the cross section as shown in Fig. a. The location of C is

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{2[0.0375(0.025)(0.075)] + 0.0875(0.2)(0.025) + 0.15(0.025)(0.1)}{2(0.025)(0.075) + 0.2(0.025) + 0.025(0.1)}$$
$$= 0.08472\text{ m}$$

Thus, the moment of inertia of the cross section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2$$
$$= 2\left[\frac{1}{12}(0.025)(0.075^3) + 0.025(0.075)(0.08472 - 0.0375)^2\right] + \frac{1}{12}(0.2)(0.025^3) + 0.2(0.025)(0.0875 - 0.08472)^2$$
$$+ \frac{1}{12}(0.025)(0.1^3) + 0.025(0.1)(0.15 - 0.08472)^2$$
$$= 23.1554(10^{-6})\text{ m}^4$$

Maximum Bending Stress: The maximum compressive and tensile stress occurs at the top and bottom-most fibers of the cross section.

$$(\sigma_{\max})_c = \frac{Mc}{I} = \frac{[3(10^3)](0.2 - 0.08472)}{23.1554(10^{-6})} = 14.94(10^6)\text{ N/m}^2 = 14.9\text{ MPa} \qquad \textbf{Ans.}$$

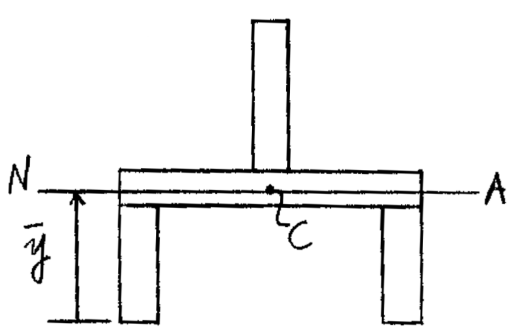
$$(\sigma_{\max})_t = \frac{My}{I} = \frac{[3(10^3)](0.08472)}{23.1554(10^{-6})} = 10.98(10^6)\text{ N/m}^2 = 11.0\text{ MPa} \qquad \textbf{Ans.}$$

The bending stresses at $y = 0.01528$ m and $y = -0.009722$ m are

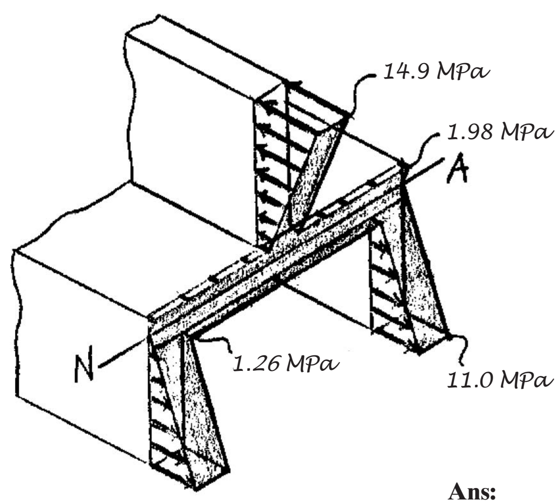
$$\sigma|_{y=0.1528\text{ m}} = \frac{My}{I} = \frac{[3(10^3)](0.01528)}{23.1554(10^{-6})} = 1.979(10^6)\text{ N/m}^2 = 1.98\text{ MPa (C)}$$

$$\sigma|_{y=-0.009722\text{ m}} = \frac{My}{I} = \frac{[3(10^3)](0.009722)}{23.1554(10^{-6})} = 1.260(10^6)\text{ N/m}^2 = 1.26\text{ MPa (T)}$$

The bending stress distribution across the cross section is shown in Fig. b.



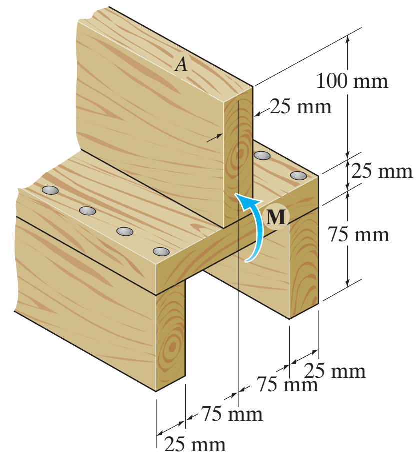
(a)



(b)

Ans:
$(\sigma_{\max})_c = 14.9\text{ MPa}$,
$(\sigma_{\max})_t = 11.0\text{ MPa}$

6–87. If the allowable tensile and compressive stress for the beam are $(\sigma_{\text{allow}})_t = 14$ MPa and $(\sigma_{\text{allow}})_c = 21$ MPa, respectively, determine the maximum allowable internal moment **M** that can be applied on the cross section.



Section Properties: The neutral axis passes through the centroid C of the cross section as shown in Fig. a. The location of C is given by

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{2[0.0375(0.025)(0.075)] + 0.0875(0.2)(0.025) + 0.15(0.025)(0.1)}{2(0.025)(0.075) + 0.2(0.025) + 0.025(0.1)}$$

$$= 0.08472 \text{ m}$$

Thus, the moment of inertia of the cross section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2$$

$$= 2\left[\frac{1}{12}(0.025)(0.075^3) + 0.025(0.075)(0.08472 - 0.0375)^2\right] + \frac{1}{12}(0.2)(0.025^3) + 0.2(0.025)(0.0875 - 0.08472)^2$$

$$+ \frac{1}{12}(0.025)(0.1^3) + 0.025(0.1)(0.15 - 0.08472)^2$$

$$= 23.1554(10^{-6}) \text{ m}^4$$

Allowable Bending Stress: The maximum compressive and tensile stress occurs at the top and bottom-most fibers of the cross section.

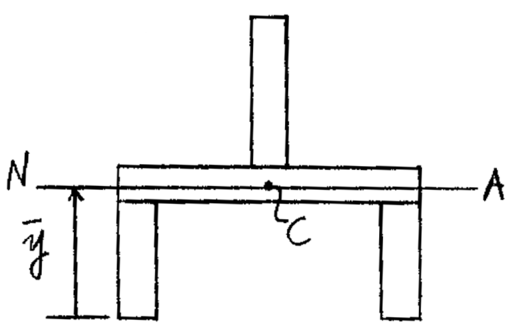
$$(\sigma_{\text{allow}})_c = \frac{Mc}{I}; \qquad 21(10^6) = \frac{M(0.2 - 0.08472)}{23.1554(10^{-6})}$$

$$M = 4.218(10^3) \text{ N} \cdot \text{m} = 4.22 \text{ kN} \cdot \text{m}$$

For the bottom-most fiber,

$$(\sigma_{\text{allow}})_t = \frac{My}{I}; \qquad 21(10^6) = \frac{M(0.08472)}{23.1554(10^{-6})}$$

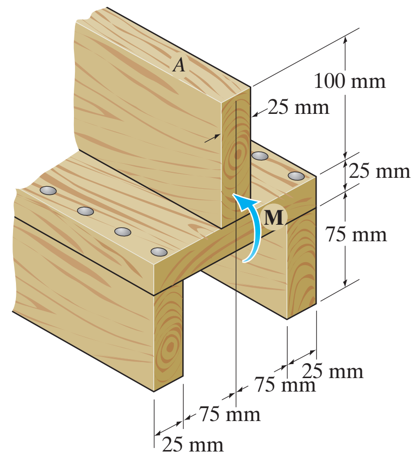
$$M = 3.826(10^3) \text{ N} \cdot \text{m} = 3.83 \text{ kN} \cdot \text{m (Controls)}$$ **Ans.**



(a)

Ans:
$M = 3.83 \text{ kN} \cdot \text{m}$

***6–88.** If the beam is subjected to an internal moment of $M = 3\ \text{kN}\cdot\text{m}$, determine the resultant force of the bending stress distribution acting on the top vertical board A.



Section Properties: The neutral axis passes through the centroid C of the cross section as shown in Fig. a. The location of C is given by

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{2[0.0375(0.025)(0.075)] + 0.0875(0.2)(0.025) + 0.15(0.025)(0.1)}{2(0.025)(0.075) + 0.2(0.025) + 0.025(0.1)}$$
$$= 0.08472\ \text{m}$$

Thus, the moment of inertia of the cross section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2$$
$$= 2\left[\frac{1}{12}(0.025)(0.075^3) + 0.025(0.075)(0.08472 - 0.0375)^2\right] + \frac{1}{12}(0.2)(0.025^3) + 0.2(0.025)(0.0875 - 0.08472)^2$$
$$+ \frac{1}{12}(0.025)(0.1^3) + 0.025(0.1)(0.15 - 0.08472)^2$$
$$= 23.1554(10^{-6})\ \text{m}^4$$

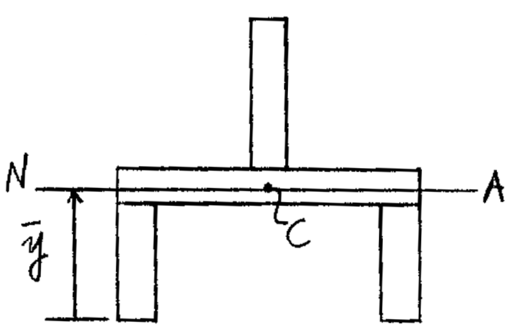
Bending Stress: The distance from the neutral axis to the top and bottom of board A is $y_t = 0.2 - 0.08472 = 0.11528$ m and $y_b = 0.1 - 0.08472 = 0.01528$ m. We have

$$\sigma_t = \frac{My_t}{I} = \frac{[3(10^3)](0.11528)}{23.1554(10^{-6})} = 14.94(10^6)\ \text{N/m}^2 = 14.94\ \text{MPa}$$

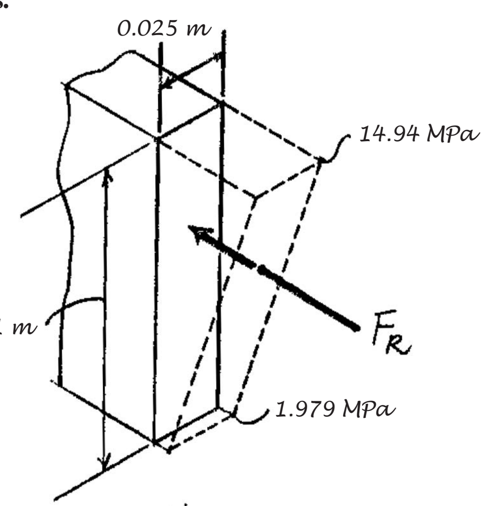
$$\sigma_b = \frac{My_b}{I} = \frac{[3(10^3)](0.01528)}{23.1554(10^{-6})} = 1.979(10^6)\ \text{N/m}^2 = 1.979\ \text{MPa}$$

Resultant Force: The resultant force acting on board A is equal to the volume of the trapezoidal stress block shown in Fig. b. Thus,

$$F_R = \frac{1}{2}[(14.94 + 1.979)(10^6)](0.025)(0.1) = 21.14(10^3)\ \text{N} = 21.1\ \text{kN}$$ **Ans.**



(a)

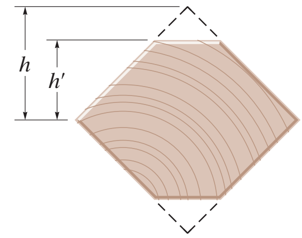


(b)

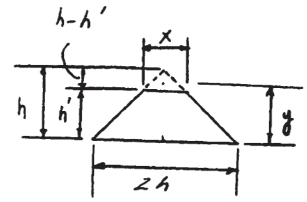
Ans:
$F_R = 21.1$ kN

6–89.

A timber beam has a cross section which is originally square. If it is oriented as shown, determine the dimension h' so that it can resist the maximum moment possible. By what factor is this moment greater than that of the beam without its top or bottom flattened?



SOLUTION



$$\frac{x}{h-h'} = \frac{2h}{h}; \qquad x = 2(h-h')$$

$$y = h' + \frac{h-h'}{3} = \frac{2h'+h}{3}$$

$$I = 2\left\{\frac{1}{12}(2h)\left(h^3\right) - \left[\frac{1}{36}(2)(h-h')(h-h')^3 + \frac{1}{2}(2)(h-h')(h-h')\left(\frac{2h'+h}{3}\right)^2\right]\right\}$$

$$= \frac{1}{3}h^4 - \frac{1}{9}(h-h')^4 - \frac{2}{9}(h-h')^2(2h'+h)^2$$

$$= \frac{1}{3}h^4 - \frac{1}{9}(h-h')^2\left[3h^2 + 9h'^2 + 6hh'\right]$$

$$= \frac{1}{3}h^4 - \frac{1}{9}\left(3h^4 + 9h'^4 - 12hh'^3\right)$$

$$= \frac{4}{3}hh'^3 - h'^4$$

$$\sigma_{max} = \frac{Mc}{I}$$

qui c e' y'

$$M = \frac{I}{c}\sigma_{max} \qquad \textbf{(1)}$$

$$= \frac{\frac{4}{3}hh'^3 - h'^4}{h'}\sigma_{max} = \left(\frac{4}{3}hh'^2 - h'^3\right)\sigma_{max}$$

$$\frac{dM}{dh'} = \left(\frac{8}{3}hh' - 3h'^2\right)\sigma_{max}$$

In-order to have maximum moment,

$$\frac{dM}{dh'} = 0 = \frac{8}{3}hh' - 3h'^2$$

$$h' = \frac{8}{9}h \qquad \textbf{Ans.}$$

For the square beam,

$$I = \bar{I} + Ad^2$$

$$I = 2\left[\frac{1}{36}(2h)(h)^3 + \frac{1}{12}(2h)(h)\left(\frac{h}{3}\right)^2\right] = \frac{h^4}{3}$$

6–89. Continued

From Eq. (1)

$$M = \frac{\frac{h^4}{3}}{h}\sigma_{\max} = \frac{h^3}{3}\sigma_{\max} = 0.3333h^3\,\sigma_{\max}$$

For the flattened beam:

$$I = \frac{4}{3}h\left(\frac{8}{9}h\right)^3 - \left(\frac{8}{9}h\right)^4 = 0.312147\,h^4$$

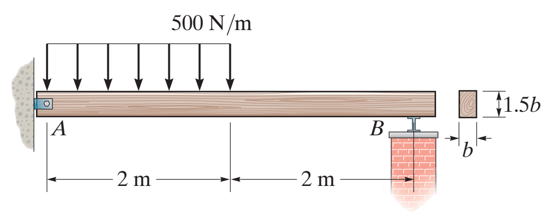
From Eq. (1)

$$M' = \frac{0.312147\,h^4}{\frac{8}{9}h}\sigma_{\max} = 0.35117\,h^3\,\sigma_{\max}$$

$$\text{Factor} = \frac{M'}{M'} = \frac{0.35117\,h^3\,\sigma_{\max}}{0.3333\,h^3\,\sigma_{\max}} = 1.05$$ **Ans.**

Ans:
$h' = \frac{8}{9}h,$
Factor $= 1.05$

6–90. The wood beam has a rectangular cross section in the proportion shown. Determine its required dimension b if the allowable bending stress is $\sigma_{\text{allow}} = 10$ MPa.

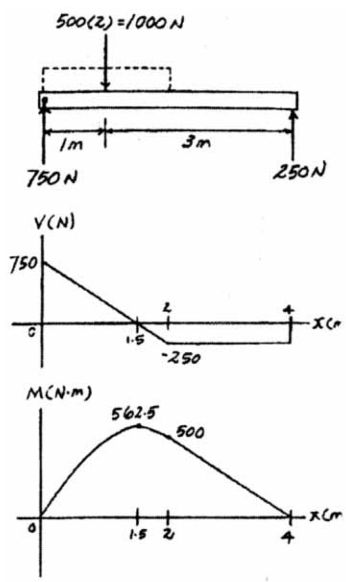


Allowable Bending Stress: The maximum moment is $M_{\max} = 562.5\ \text{N}\cdot\text{m}$ as indicated on the moment diagram. Applying the flexure formula

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{M_{\max}\,c}{I}$$

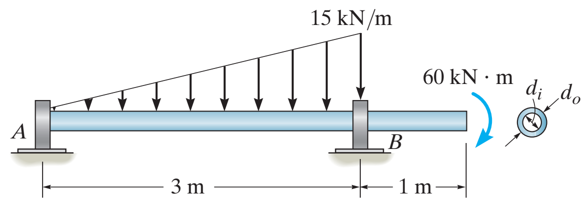
$$10\left(10^6\right) = \frac{562.5(0.75b)}{\frac{1}{12}(b)(1.5b)^3}$$

$$b = 0.05313 \text{ m} = 53.1 \text{ mm}$$ **Ans.**



Ans:
$b = 53.1$ mm

6–91. Determine the absolute maximum bending stress in the tubular shaft if $d_i = 160$ mm and $d_o = 200$ mm.



Section Property:

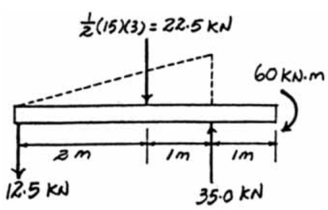
$$I = \frac{\pi}{4}\left(0.1^4 - 0.08^4\right) = 46.370\left(10^{-6}\right)\text{ m}^4$$

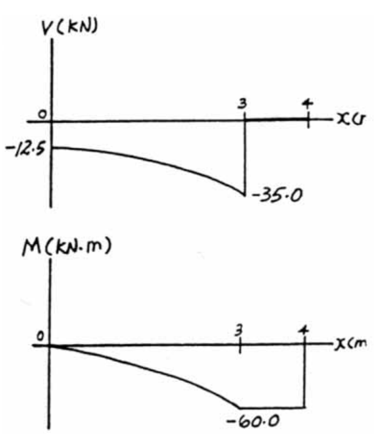
Absolute Maximum Bending Stress: The maximum moment is $M_{max} = 60.0$ kN · m as indicated on the moment diagram. Applying the flexure formula

$$\sigma_{max} = \frac{M_{max}c}{I}$$

$$= \frac{60.0(10^3)(0.1)}{46.370(10^{-6})}$$

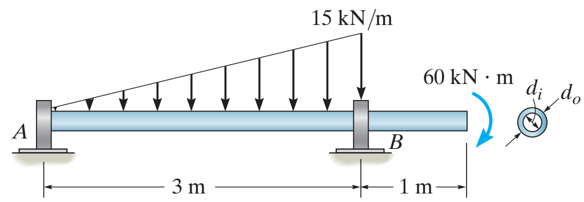
$$= 129 \text{ MPa}$$ **Ans.**





Ans:
$\sigma_{max} = 129$ MPa

***6–92.** The tubular shaft is to have a cross section such that its inner diameter and outer diameter are related by $d_i = 0.8d_o$. Determine these required dimensions if the allowable bending stress is $\sigma_{allow} = 155$ MPa.



Section Property:

$$I = \frac{\pi}{4}\left[\left(\frac{d_o}{2}\right)^4 - \left(\frac{d_l}{2}\right)^4\right] = \frac{\pi}{4}\left[\frac{d_o{}^4}{16} - \left(\frac{0.8d_o}{2}\right)^4\right] = 0.009225\pi d_o^4$$

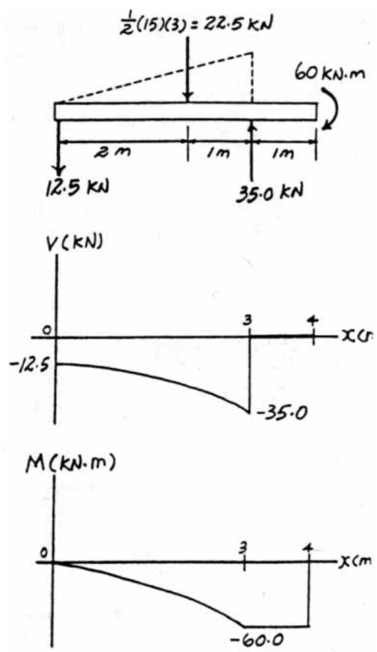
Allowable Bending Stress: The maximum moment is $M_{max} = 60.0\ \text{kN}\cdot\text{m}$ as indicated on the moment diagram. Applying the flexure formula

$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$155\left(10^6\right) = \frac{60.0(10^3)\left(\frac{d_o}{2}\right)}{0.009225\pi d_o^4}$$

$$d_o = 0.1883\ \text{m} = 188\ \text{mm}$$ **Ans.**

Thus, $$d_l = 0.8d_o = 151\ \text{mm}$$ **Ans.**

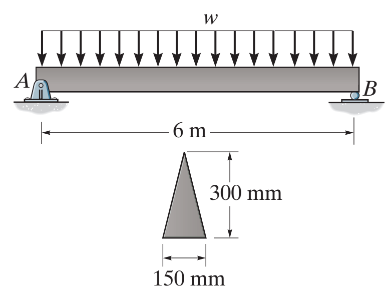


Ans:

$d_o = 0188$ mm, $d_l = 151$ mm

6–93.

If the intensity of the load $w = 15$ kN/m, determine the absolute maximum tensile and compressive stress in the beam.



SOLUTION

Support Reactions: Shown on the free-body diagram of the beam, Fig. *a*.

Maximum Moment: The maximum moment occurs when $V = 0$. Referring to the free-body diagram of the beam segment shown in Fig. *b*,

$+\uparrow \Sigma F_y = 0; \qquad 45 - 15x = 0 \qquad x = 3 \text{ m}$

$\circlearrowleft + \Sigma M = 0; \qquad M_{\max} + 15(3)\left(\frac{3}{2}\right) - 45(3) = 0 \qquad M_{\max} = 67.5 \text{ kN} \cdot \text{m}$

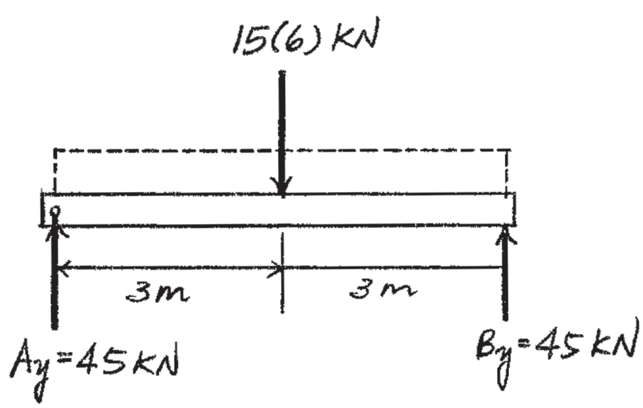
Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{36}(0.15)(0.3^3) = 0.1125(10^{-3}) \text{ m}^4$$

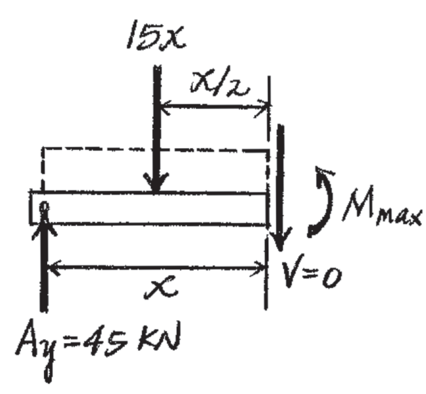
Absolute Maximum Bending Stress: The maximum compressive and tensile stresses occur at the top and bottom-most fibers of the cross section.

$$(\sigma_{\max})_c = \frac{Mc}{I} = \frac{67.5(10^3)(0.2)}{0.1125(10^{-3})} = 120 \text{ MPa (C)} \qquad \textbf{Ans.}$$

$$(\sigma_{\max})_t = \frac{My}{I} = \frac{67.5(10^3)(0.1)}{0.1125(10^{-3})} = 60 \text{ MPa (T)} \qquad \textbf{Ans.}$$



(a)

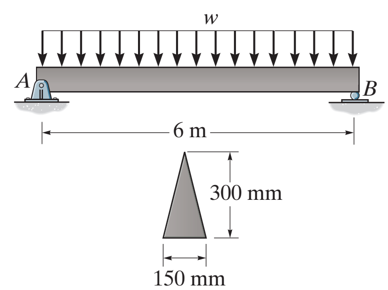


(b)

Ans:
$(\sigma_{\max})_c = 120$ MPa (C),
$(\sigma_{\max})_t = 60$ MPa (T)

6–94.

If the allowable bending stress is $\sigma_{\text{allow}} = 150$ MPa, determine the maximum intensity w of the uniform distributed load.



SOLUTION

Support Reactions: As shown on the free-body diagram of the beam, Fig. *a*.

Maximum Moment: The maximum moment occurs when $V = 0$. Referring to the free-body diagram of the beam segment shown in Fig. *b*,

$+\uparrow \Sigma F_y = 0;\qquad 3w - wx = 0 \qquad x = 3\text{ m}$

$\circlearrowleft + \Sigma M = 0;\qquad M_{\max} + w(3)\left(\dfrac{3}{2}\right) - 3w(3) = 0 \qquad M_{\max} = \dfrac{9}{2}w$

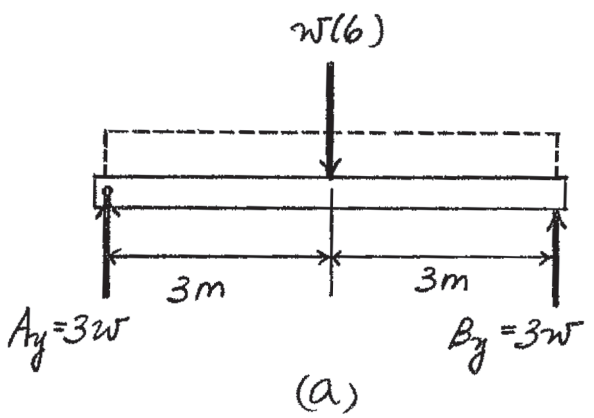
Section Properties: The moment of inertia of the cross section about the neutral axis is

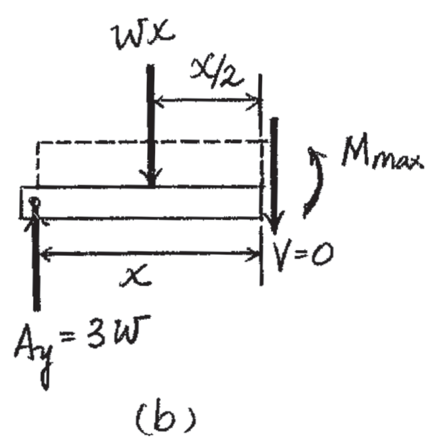
$$I = \frac{1}{36}(0.15)(0.3^3) = 0.1125(10^{-3})\,\text{m}^4$$

Absolute Maximum Bending Stress: Here, $c = \dfrac{2}{3}(0.3) = 0.2$ m.

$$\sigma_{\text{allow}} = \frac{Mc}{I};\qquad 150(10^6) = \frac{\frac{9}{2}w(0.2)}{0.1125\left(10^{-3}\right)}$$

$$w = 18750\text{ N/m} = 18.75\text{ kN/m}\qquad \textbf{Ans.}$$

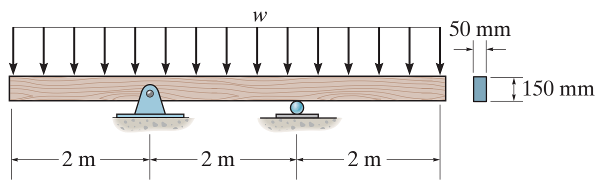




Ans:
$w = 18.75$ kN/m

6–95.

The beam has a rectangular cross section as shown. Determine the largest intensity w of the uniform distributed load so that the bending stress in the beam does not exceed $\sigma_{max} = 10$ MPa.

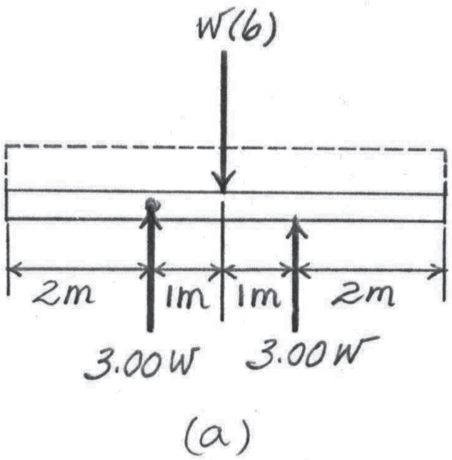


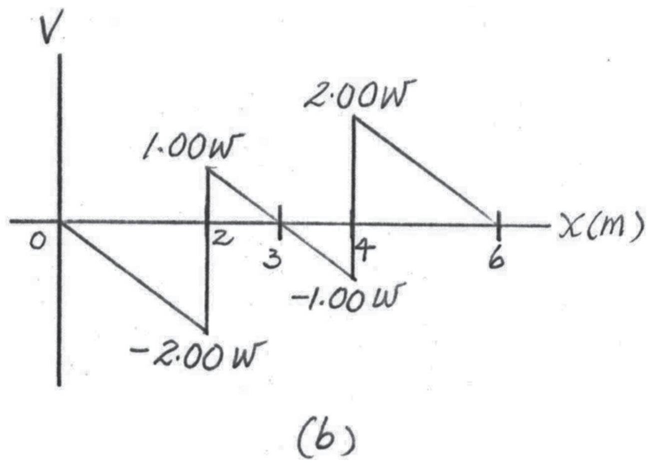
SOLUTION

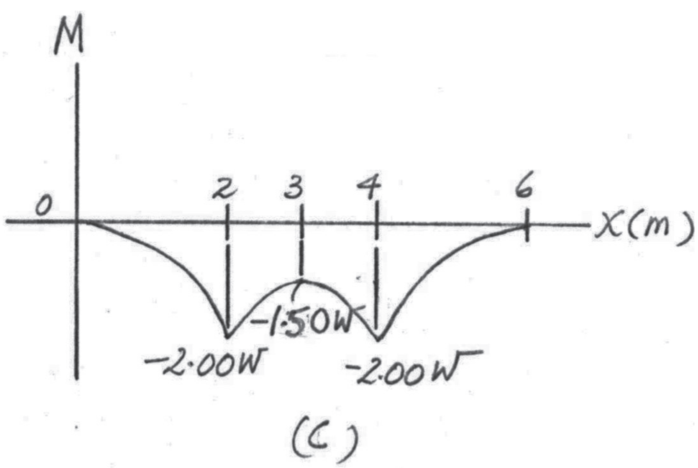
Absolute Maximum Bending Stress: The support reactions, shear diagram, and moment diagrams are shown in Figs. *a*, *b*, and *c*, respectively. From the moment diagram, the maximum moment is $M_{max} = 2.00\,w$, which occurs at the pin support and roller support. Applying the flexure formula,

$$\sigma_{\substack{abs \\ max}} = \frac{M_{max}C}{I}; \qquad 10(10^6) = \frac{(2.00\,w)(0.075)}{\frac{1}{12}(0.05)(0.15^3)}$$

$$w = 937.5\ \text{N/m} \qquad \textbf{Ans.}$$



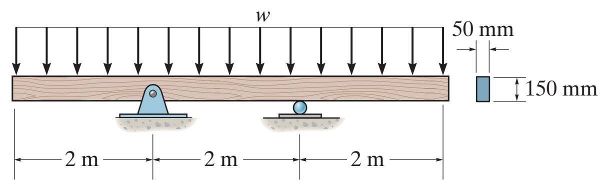




Ans:
$w = 937.5$ N/m

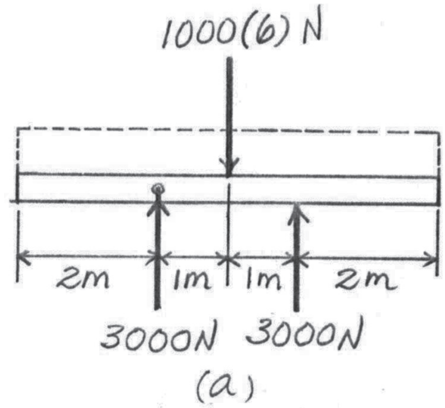
***6–96.**

The beam has the rectangular cross section shown. If $w = 1\ \text{kN/m}$, determine the maximum bending stress in the beam. Sketch the stress distribution acting over the cross section.



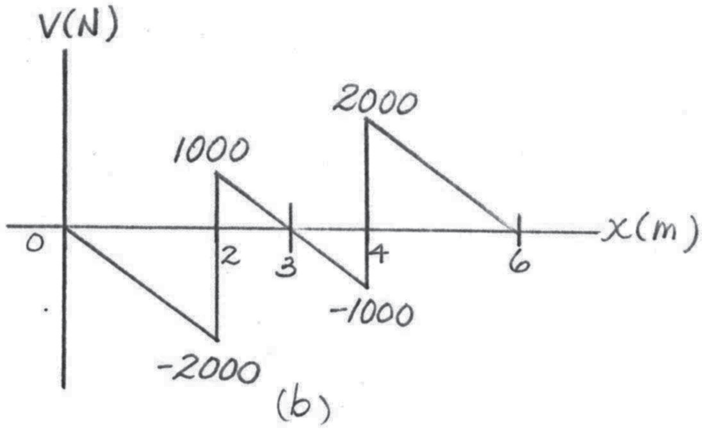
SOLUTION

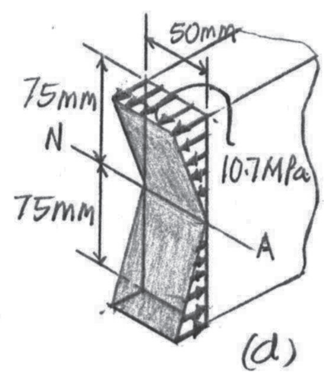
Absolute Maximum Bending Stress: The support reactions, shear diagram, and moment diagram are shown in Figs. *a*, *b*, and *c*, respectively. From the moment diagram, the maximum moment is $M_{\max} = 2000\ \text{N}\cdot\text{m}$, which occurs at the supports. Applying the flexure formula,

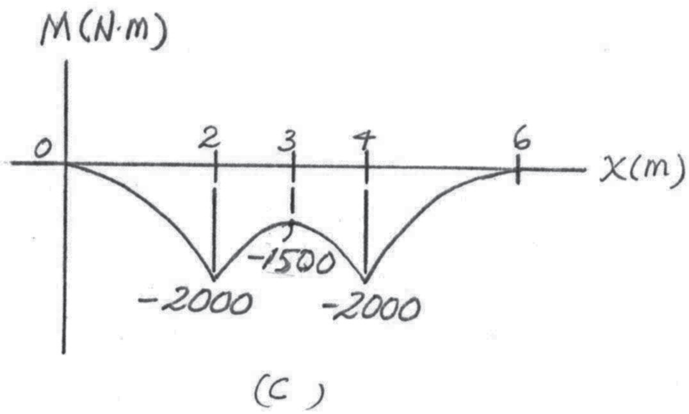


$$\sigma_{\substack{\text{abs}\\ \text{max}}} = \frac{M_{\max}C}{I} = \frac{2000(0.075)}{\frac{1}{12}(0.05)(0.15^3)} = 10.67\left(10^6\right)\ \text{MPa} = 10.7\ \text{MPa} \qquad \textbf{Ans.}$$

Using this result, the bending stress distribution on the beam's cross-section shown in Fig. *d* can be sketched.

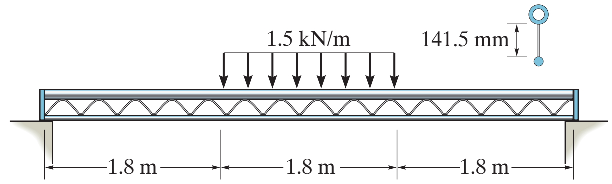




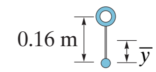


Ans:
$\sigma_{\substack{\text{abs}\\ \text{max}}} = 10.7\ \text{MPa}$

6–97. The simply supported truss is subjected to the central distributed load. Neglect the effect of the diagonal lacing and determine the absolute maximum bending stress in the truss. The top member is a pipe having an outer diameter of 25 mm and thickness of 5 mm, and the bottom member is a solid rod having a diameter of 12 mm.



$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{0 + 0.16[\pi(0.0125^2 - 0.0075^2)]}{\pi(0.006^2) + \pi(0.0125^2 - 0.0075^2)} = 0.11765 \text{ m}$$



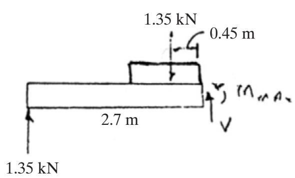
$$I = \frac{\pi}{4}(0.006^4) + \pi(0.006^2)(0.11765^2) + \frac{\pi}{4}(0.0125^4 - 0.0075^4)$$

$$+ \pi(0.0125^2 - 0.0075^2)(0.16 - 0.11765)^2 = 2.1466(10^{-6}) \text{ m}^4$$

$$M_{\max} = 1.35(2.7) - 1.35(0.45) = 3.0375 \text{ kN} \cdot \text{m}$$

$$\sigma_{\max} = \frac{MC}{I} = \frac{[3.0375(10^3)](0.11765 + 0.006)}{2.1466(10^{-6})}$$

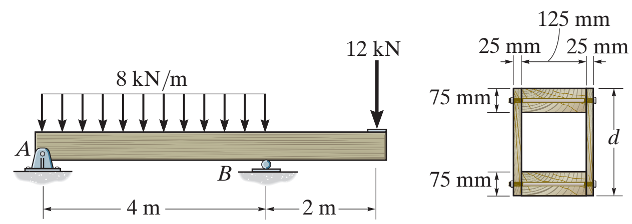
$$= 174.96(10^6) \text{ N}\cancel{\text{-m}}^2 = 175 \text{ MPa}$$ **Ans.**



Ans:
$(\sigma_{\max})_c = 120 \text{ MPa (C)}, (\sigma_{\max})_t = 60 \text{ MPa (T)}$

6–98.

If $d = 450$ mm, determine the absolute maximum bending stress in the overhanging beam.



SOLUTION

Support Reactions: Shown on the free-body diagram of the beam, Fig. *a*.

Maximum Moment: The shear and moment diagrams are shown in Figs. *b* and *c*. As indicated on the moment diagram, $M_{\max} = 24\,\text{kN}\cdot\text{m}$.

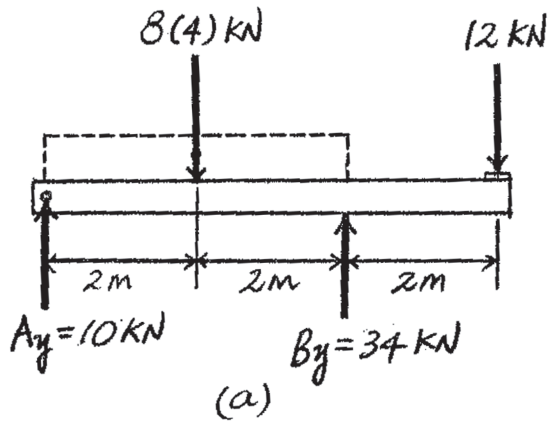
Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.175)(0.45^3) - \frac{1}{12}(0.125)(0.3^3)$$

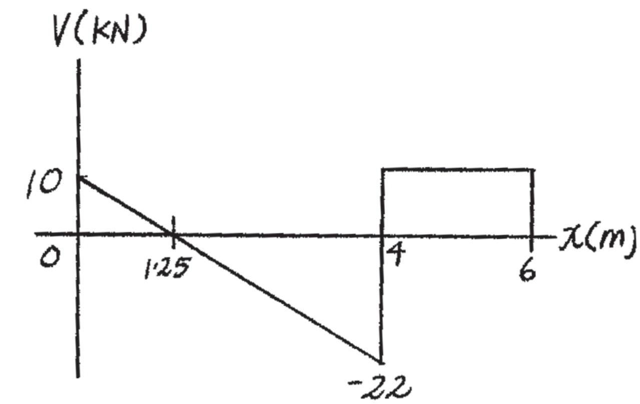
$$= 1.0477(10^{-3})\ \text{m}^4$$

Absolute Maximum Bending Stress: Here, $c = \dfrac{0.45}{2} = 0.225$ m.

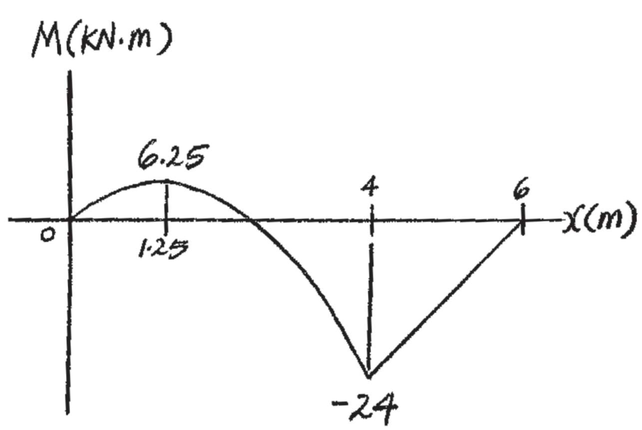
$$\sigma_{\max} = \frac{M_{\max}\,c}{I} = \frac{24(10^3)(0.225)}{1.0477(10^{-3})} = 5.15\ \text{MPa} \qquad \textbf{Ans.}$$



(a)



(b)

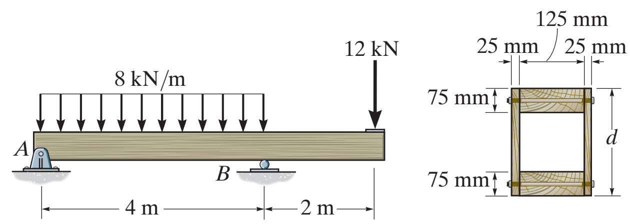


(C)

Ans:
$\sigma_{\max} = 5.15$ MPa

6–99.

If the allowable bending stress is $\sigma_{\text{allow}} = 6$ MPa, determine the minimum dimension d of the beam's cross-sectional area to the nearest mm.



SOLUTION

Support Reactions: Shown on the free-body diagram of the beam, Fig. *a*.

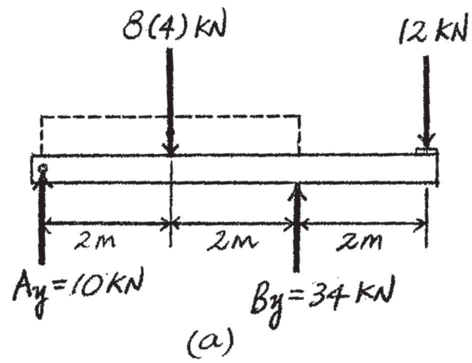
Maximum Moment: The shear and moment diagrams are shown in Figs. *b* and *c*. As indicated on the moment diagram, $M_{\max} = 24\,\text{kN}\cdot\text{m}$.

Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.175)d^3 - \frac{1}{12}(0.125)(d - 0.15)^3$$

$$= 4.1667(10^{-3})d^3 + 4.6875(10^{-3})d^2 - 0.703125(10^{-3})d + 35.15625(10^{-6})$$

Absolute Maximum Bending Stress: Here, $c = \dfrac{d}{2}$.



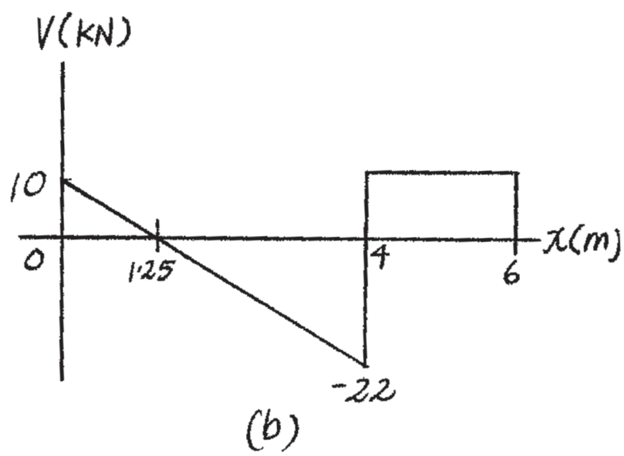
$$\sigma_{\text{allow}} = \frac{Mc}{I};$$

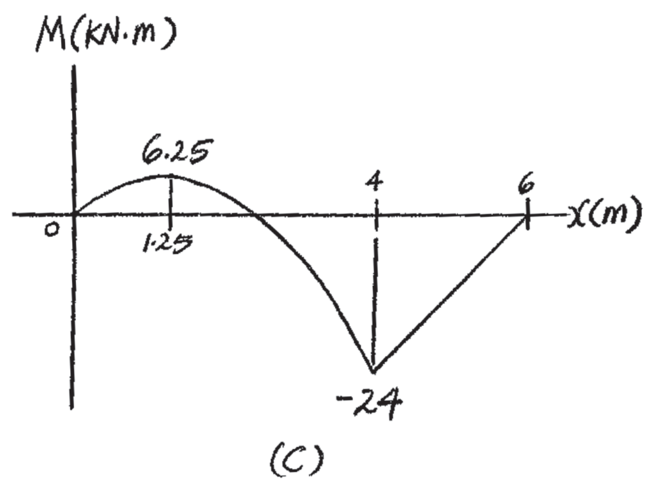
$$6(10^6) = \frac{24(10^3)\dfrac{d}{2}}{4.1667(10^{-3})d^3 + 4.6875(10^{-3})d^2 - 0.703125(10^{-3})d + 35.15625(10^{-6})}$$

$$4.1667(10^{-3})d^3 + 4.6875(10^{-3})d^2 - 2.703125(10^{-3})d + 35.15625(10^{-6}) = 0$$

Solving,

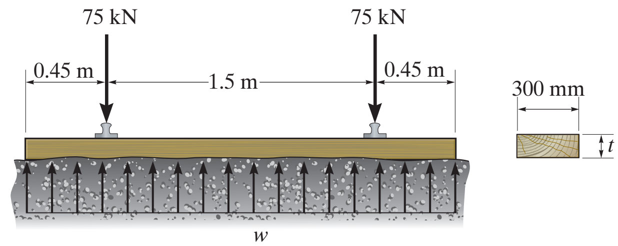
$d = 0.4094$ m $= 410$ mm **Ans.**





Ans:
$d = 410$ mm

***6–100.** If the reaction of the ballast on the railway tie can be assumed uniformly distributed over its length as shown, determine the maximum bending stress developed in the tie. The tie has the rectangular cross section with thickness $t = 150$ mm.



Support Reactions: Referring to the free - body diagram of the tie shown in Fig. *a*, we have

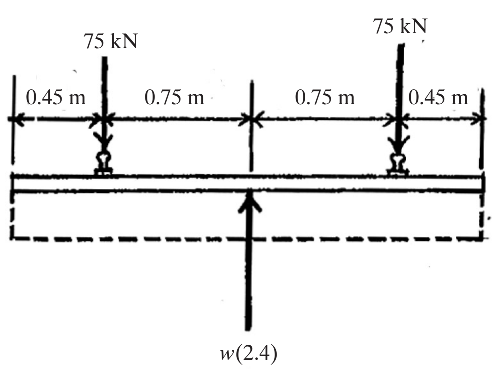
$$+\uparrow \Sigma F_y = 0; \qquad w(2.4) - 2(75) = 0$$

$$w = 62.5 \text{ kN/m}$$

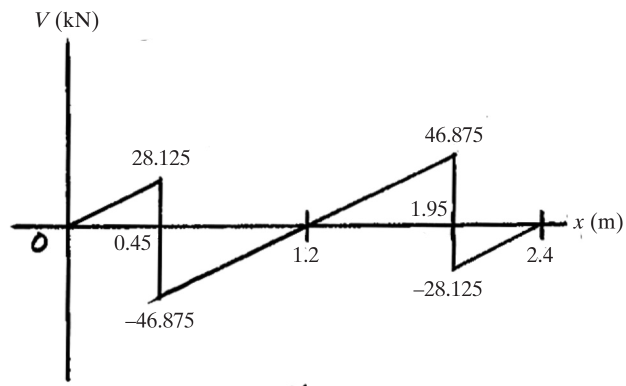
Maximum Moment: The shear and moment diagrams are shown in Figs. *b* and *c*. As indicated on the moment diagram, the maximum moment is $\left|M_{\max}\right| = 11.25 \text{ kN} \cdot \text{m}$

Absolute Maximum Bending Stress:

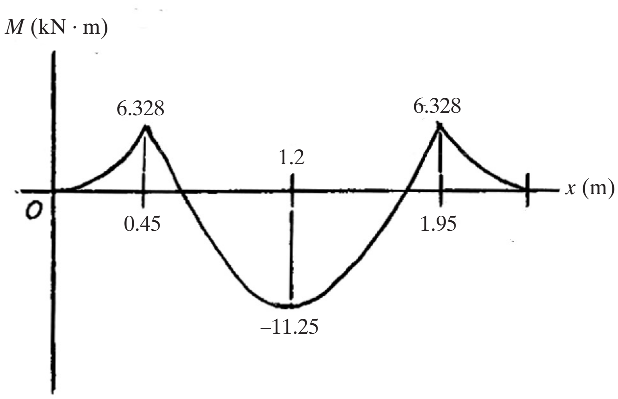
$$\sigma_{\max} = \frac{M_{\max} c}{I} = \frac{[11.25(10^3)](0.075)}{\frac{1}{12}(0.3)(0.15^3)} = 10.0(10^6) \text{ N/m}^2 = 10.0 \text{ MPa} \qquad \textbf{Ans.}$$



(a)



(b)

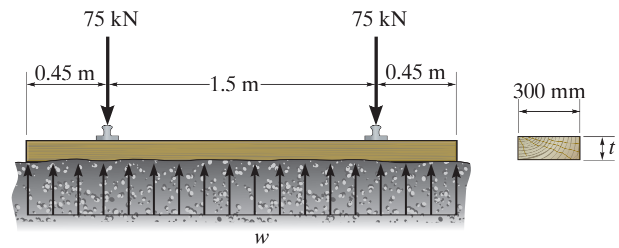


(c)

Ans:

$\sigma_{\max} = 10.0$ MPa

6–101. The reaction of the ballast on the railway tie can be assumed uniformly distributed over its length as shown. If the wood has an allowable bending stress of $\sigma_{allow} = 10.5$ MPa, determine the required minimum thickness t of the rectangular cross sectional area of the tie to the nearest multiples of 5 mm.

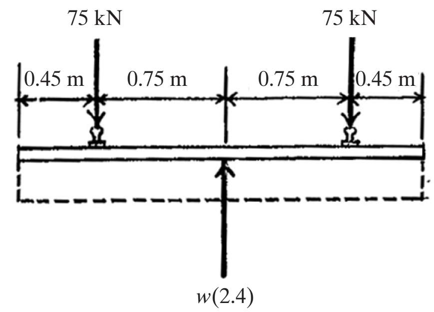


Support Reactions: Referring to the free-body diagram of the tie shown in Fig. *a*, we have

$$+\uparrow \Sigma F_y = 0; \qquad w = (2.4) - 2(75) = 0$$

$$w = 62.5 \text{ kN/m}$$

Maximum Moment: The shear and moment diagrams are shown in Figs. *b* and *c*. As indicated on the moment diagram, the maximum moment is $|M_{max}| = 11.25 \text{ kN} \cdot \text{m}$.



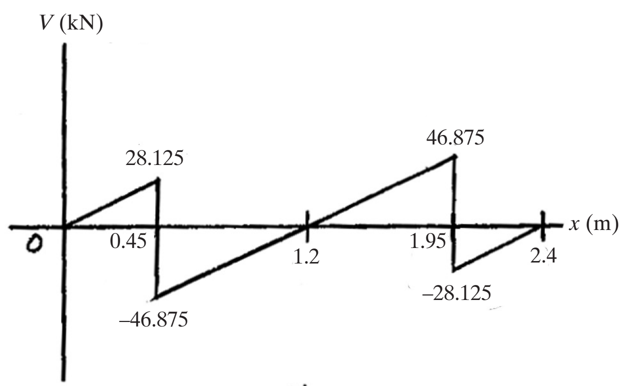
(a)

Absolute Maximum Bending Stress:

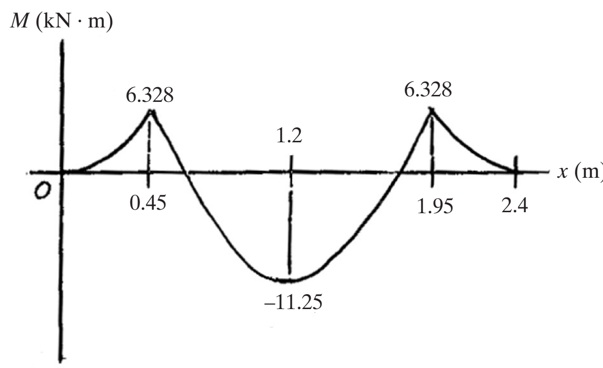
$$\sigma_{max} = \frac{Mc}{I}; \qquad 10.5(10^6) = \frac{[11.25(10^3)]\left(\frac{t}{2}\right)}{\frac{1}{12}(0.3)(t^3)}$$

$$t = 0.14638 \text{ m} = 146.38 \text{ mm}$$

Use $t = 150$ mm **Ans.**



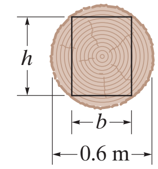
(b)

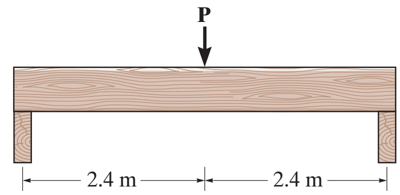


(c)

Ans:
Use $t = 150$ mm

6–102. A log that is 0.6 m in diameter is to be cut into a rectangular section for use as a simply supported beam. If the allowable bending stress for the wood is $\sigma_{\text{allow}} = 56$ MPa, determine the required width b and height h of the beam that will support the largest load possible. What is this load?

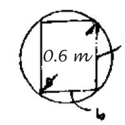




$$0.6^2 = b^2 + h^2$$

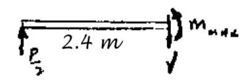
$$h^2 = 0.36 - b^2$$

$$M_{\max} = \frac{P}{2}(2.4) = 1.2P$$



$$\sigma_{\text{allow}} = \frac{Mc}{I} = \frac{M_{\max}\left(\frac{h}{2}\right)}{\frac{1}{12}(b)(h)^3}$$

$$\sigma_{\text{allow}} = \frac{6\,M_{\max}}{bh^2}$$



$$56(10^6) = \frac{6(1.2P)}{b(0.36 - b^2)}$$

$$P = 7.7778(10^6)(0.36b - b^3)$$

Set $\dfrac{dP}{db} = 0$ gives

$$\frac{dP}{db} = 7.7778(10^6)(0.36 - b^2) = 0$$

$b = 0.3464$ m

Thus, from the above equations, **Ans.**

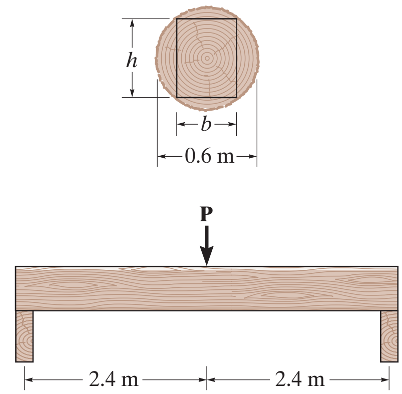
$b = 346.41$ mm $= 346$ mm **Ans.**

$h = 0.4899$ m $= 490$ mm **Ans.**

$P = 646.63(10^3)$ N $= 647$ kN

Ans:
$b = 346$ mm, $h = 490$ mm, $P = 647$ kN

6–103. A log that is 0.6 m in diameter is to be cut into a rectangular section for use as a simply supported beam. the allowable bending stress for the wood is $\sigma_{allow} = 56$ MPa, determine the largest load P that can be supported if the width of the beam is $b = 200$ mm.

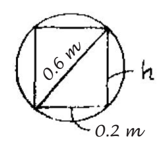


$$0.6^2 = h^2 + 0.2^2$$

$$h = 0.5657 \text{ m}$$

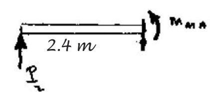
$$M_{max} = \frac{P}{2}(2.4) = 1.2P$$

$$\sigma_{allow} = \frac{M_{max}c}{I}$$



$$56(10^6) = \frac{(1.2P)(0.5657/2)}{\frac{1}{12}(0.2)(0.5657^3)}$$

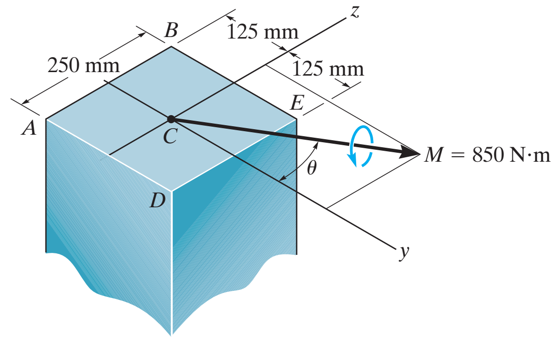
$$P = 497.78(10^3) = 498 \text{ kN}$$ **Ans.**



Ans:
$P = 498$ kN

***6–104.**

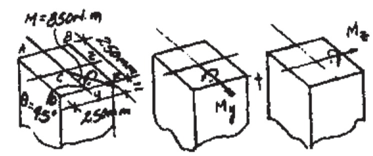
The member has a square cross section and is subjected to the moment $M = 850\ \text{N}\cdot\text{m}$. Determine the stress at each corner and sketch the stress distribution. Set $\theta = 45°$.



SOLUTION

$M_y = 850 \cos 45° = 601.04\ \text{N}\cdot\text{m}$

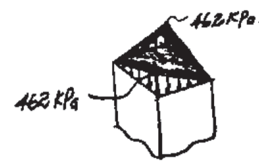
$M_z = 850 \sin 45° = 601.04\ \text{N}\cdot\text{m}$



$$I_z = I_y = \frac{1}{12}(0.25)(0.25)^3 = 0.3255208(10^{-3})\ \text{m}^4$$

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{601.04(-0.125)}{0.3255208(10^{-3})} + \frac{601.04(-0.125)}{0.3255208(10^{-3})} = 0 \qquad \textbf{Ans.}$$



$$\sigma_B = -\frac{601.04(-0.125)}{0.3255208(10^{-3})} + \frac{601.04(0.125)}{0.3255208(10^{-3})} = 462\ \text{kPa} \qquad \textbf{Ans.}$$

$$\sigma_D = -\frac{601.04(0.125)}{0.3255208(10^{-3})} + \frac{601.04(-0.125)}{0.3255208(10^{-3})} = -462\ \text{kPa} \qquad \textbf{Ans.}$$

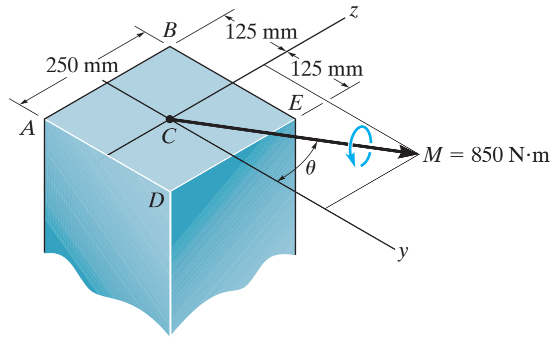
$$\sigma_E = -\frac{601.04(0.125)}{0.3255208(10^{-3})} + \frac{601.04(0.125)}{0.3255208(10^{-3})} = 0 \qquad \textbf{Ans.}$$

The negative sign indicates compressive stress.

Ans:
$\sigma_A = 0$,
$\sigma_B = 462\ \text{kPa}$,
$\sigma_D = -462\ \text{kPa}$,
$\sigma_E = 0$

6–105.

The member has a square cross section and is subjected to the moment $M = 850\ \text{N}\cdot\text{m}$ as shown. Determine the stress at each corner and sketch the stress distribution. Set $\theta = 30°$.



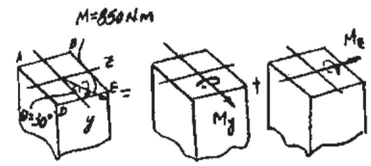
SOLUTION

$M_y = 850 \cos 30° = 736.12\ \text{N}\cdot\text{m}$

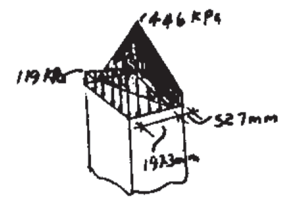
$M_z = 850 \sin 30° = 425\ \text{N}\cdot\text{m}$

$I_z = I_y = \frac{1}{12}(0.25)(0.25)^3 = 0.3255208(10^{-3})\ \text{m}^4$

$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$



$\sigma_A = -\frac{425(-0.125)}{0.3255208(10^{-3})} + \frac{736.12(-0.125)}{0.3255208(10^{-3})} = -119\ \text{kPa}$ **Ans.**



$\sigma_B = -\frac{425(-0.125)}{0.3255208(10^{-3})} + \frac{736.12(0.125)}{0.3255208(10^{-3})} = 446\ \text{kPa}$ **Ans.**

$\sigma_D = -\frac{425(0.125)}{0.3255208(10^{-3})} + \frac{736.12(-0.125)}{0.3255208(10^{-3})} = -446\ \text{kPa}$ **Ans.**

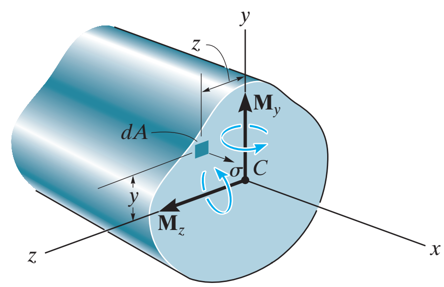
$\sigma_E = -\frac{425(0.125)}{0.3255208(10^{-3})} + \frac{736.12(0.125)}{0.3255208(10^{-3})} = 119\ \text{kPa}$ **Ans.**

The negative signs indicate compressive stress.

Ans:
$\sigma_A = -119\ \text{kPa}, \sigma_B = 446\ \text{kPa}, \sigma_D = -446\ \text{kPa}, \sigma_E = 119\ \text{kPa}$

6–106.

Consider the general case of a prismatic beam subjected to bending-moment components $\mathbf{M}_y$ and $\mathbf{M}_z$ when the x, y, z axes pass through the centroid of the cross section. If the material is linear elastic, the normal stress in the beam is a linear function of position such that $\sigma = a + by + cz$. Using the equilibrium conditions $0 = \int_A \sigma\, dA$, $M_y = \int_A z\sigma\, dA$, $M_z = \int_A -y\sigma\, dA$, determine the constants a, b, and c, and show that the normal stress can be determined from the equation $\sigma = [-(M_z I_y + M_y I_{yz})y + (M_y I_z + M_z I_{yz})z]/(I_y I_z - I_{yz}{}^2)$, where the moments and products of inertia are defined in Appendix A.



SOLUTION

Equilibrium Condition: $\sigma_x = a + by + cz$

$$0 = \int_A \sigma_x\, dA$$

$$0 = \int_A (a + by + cz)\, dA$$

$$0 = a\int_A dA + b\int_A y\, dA + c\int_A z\, dA \qquad \textbf{(1)}$$

$$M_y = \int_A z\,\sigma_x\, dA$$

$$= \int_A z(a + by + cz)\, dA$$

$$= a\int_A z\, dA + b\int_A yz\, dA + c\int_A z^2\, dA \qquad \textbf{(2)}$$

$$M_z = \int_A -y\,\sigma_x\, dA$$

$$= \int_A -y(a + by + cz)\, dA$$

$$= -a\int_A y dA - b\int_A y^2\, dA - c\int_A yz\, dA \qquad \textbf{(3)}$$

Section Properties: The integrals are defined in Appendix A. Note that $\int_A y\, dA = \int_A z\, dA = 0$. Thus,

From Eq. (1) $\qquad Aa = 0$

From Eq. (2) $\qquad M_y = bI_{yz} + cI_y$

From Eq. (3) $\qquad M_z = -bI_z - cI_{yz}$

Solving for a, b, c:

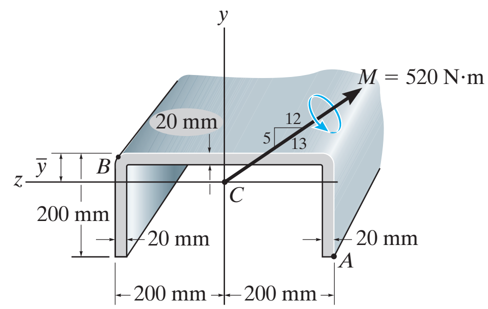
$a = 0$ (Since $A \neq 0$) **Ans.**

$$b = -\left(\frac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right) \qquad c = \frac{M_y I_z + M_z I_{yz}}{I_y I_z - I_{yz}^2} \qquad \textbf{Ans.}$$

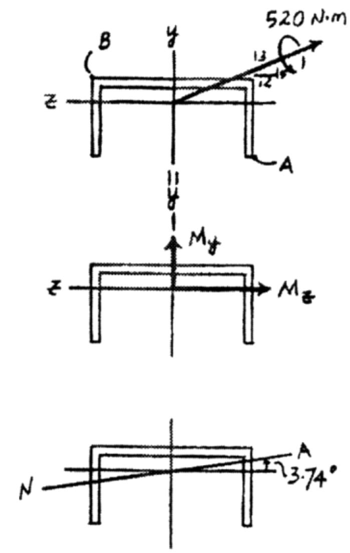
Thus, $$\sigma_x = -\left(\frac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right)y + \left(\frac{M_y I_y + M_z I_{yz}}{I_y I_z - I_{yz}^2}\right)z \qquad \textbf{(Q.E.D.)}$$

Ans:
$$a = 0, b = -\left(\frac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right), c = \frac{M_y I_z + M_z I_{yz}}{I_y I_z - I_{yz}^2}$$

6–107. If the resultant internal moment acting on the cross section of the aluminum strut has a magnitude of $M = 520\ \text{N}\cdot\text{m}$ and is directed as shown, determine the bending stress at points A and B. The location $\bar{y}$ of the centroid C of the strut's cross-sectional area must be determined. Also, specify the orientation of the neutral axis.



Internal Moment Components:



$$M_z = -\frac{12}{13}(520) = -480\ \text{N}\cdot\text{m} \qquad M_y = \frac{5}{13}(520) = 200\ \text{N}\cdot\text{m}$$

Section Properties:

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{0.01(0.4)(0.02) + 2[(0.110)(0.18)(0.02)]}{0.4(0.02) + 2(0.18)(0.02)}$$

$$= 0.057368\ \text{m} = 57.4\ \text{mm}$$ **Ans.**

$$I_z = \frac{1}{12}(0.4)\left(0.02^3\right) + (0.4)(0.02)(0.057368 - 0.01)^2$$

$$+ \frac{1}{12}(0.04)\left(0.18^3\right) + 0.04(0.18)(0.110 - 0.057368)^2$$

$$= 57.6014\left(10^{-6}\right)\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.2)\left(0.4^3\right) - \frac{1}{12}(0.18)\left(0.36^3\right) = 0.366827\left(10^{-3}\right)\ \text{m}^4$$

Maximum Bending Stress: Applying the flexure formula for biaxial at points A and B

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{-480(-0.142632)}{57.6014(10^{-6})} + \frac{200(-0.2)}{0.366827(10^{-3})}$$

$$= -1.298\ \text{MPa} = 1.30\ \text{MPa}\ (\mathbf{C})$$ **Ans.**

$$\sigma_B = -\frac{-480(0.057368)}{57.6014(10^{-6})} + \frac{200(0.2)}{0.366827(10^{-3})}$$

$$= 0.587\ \text{MPa}\ (\mathbf{T})$$ **Ans.**

Orientation of Neutral Axis:

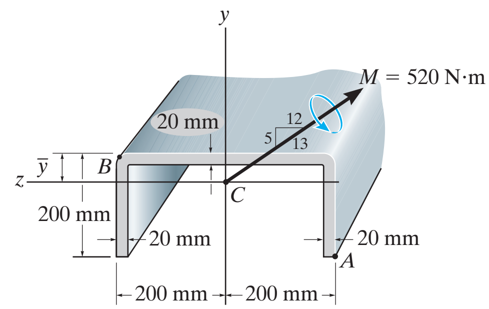
$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$

$$\tan\alpha = \frac{57.6014(10^{-6})}{0.366827(10^{-3})}\tan(-22.62^\circ)$$

$$\alpha = -3.74^\circ$$ **Ans.**

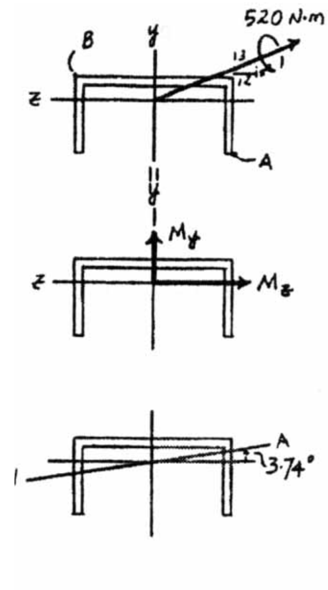
Ans:
$\bar{y} = 57.4\ \text{mm}$, $\sigma_A = 1.30\ \text{MPa (C)}$,
$\sigma_B = 0.587\ \text{MPa (T)}$,
$\alpha = -3.74^\circ$

***6–108.** The resultant internal moment acting on the cross section of the aluminum strut has a magnitude of $M = 520\ \text{N}\cdot\text{m}$ and is directed as shown. Determine maximum bending stress in the strut. The location $\bar{y}$ of the centroid C of the strut's cross-sectional area must be determined. Also, specify the orientation of the neutral axis.



Internal Moment Components:

$$M_z = -\frac{12}{13}(520) = -480\ \text{N}\cdot\text{m} \qquad M_y = \frac{5}{13}(520) = 200\ \text{N}\cdot\text{m}$$



Section Properties:

$$\bar{y} = \frac{\Sigma\,\tilde{y}\,A}{\Sigma A} = \frac{0.01(0.4)(0.02) + 2[(0.110)(0.18)(0.02)]}{0.4(0.02) + 2(0.18)(0.02)}$$

$$= 0.057368\ \text{m} = 57.4\ \text{mm} \qquad \textbf{Ans.}$$

$$I_z = \frac{1}{12}(0.4)\left(0.02^3\right) + (0.4)(0.02)(0.057368 - 0.01)^2$$

$$+ \frac{1}{12}(0.04)\left(0.18^3\right) + 0.04(0.18)(0.110 - 0.057368)^2$$

$$= 57.6014\left(10^{-6}\right)\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.2)\left(0.4^3\right) - \frac{1}{12}(0.18)\left(0.36^3\right) = 0.366827\left(10^{-3}\right)\ \text{m}^4$$

Maximum Bending Stress: By inspection, the maximum bending stress can occur at either point A or B. Applying the flexure formula for biaxial bending at points A and B

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{-480(-0.142632)}{57.6014(10^{-6})} + \frac{200(-0.2)}{0.366827(10^{-3})}$$

$$= -1.298\ \text{MPa} = 1.30\ \text{MPa (C) (Max)} \qquad \textbf{Ans.}$$

$$\sigma_B = -\frac{-480(0.057368)}{57.6014(10^{-6})} + \frac{200(0.2)}{0.366827(10^{-3})}$$

$$= 0.587\ \text{MPa (T)}$$

Orientation of Neutral Axis:

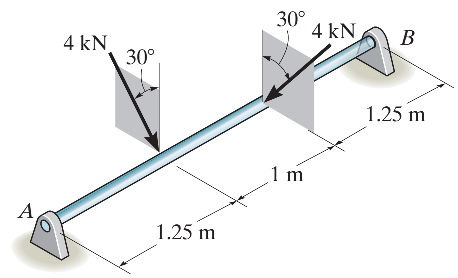
$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$

$$\tan\alpha = \frac{57.6014(10^{-6})}{0.366827(10^{-3})}\tan(-22.62°)$$

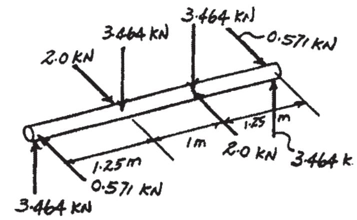
$$\alpha = -3.74° \qquad \textbf{Ans.}$$

6–109.

The steel shaft is subjected to the two loads. If the journal bearings at A and B do not exert an axial force on the shaft, determine the required diameter of the shaft if the allowable bending stress is $\sigma_{\text{allow}} = 180$ MPa.



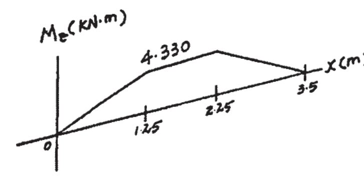
SOLUTION



Support Reactions: As shown on FBD.

Internal Moment Components: The shaft is subjected to two bending moment components M_y and M_z. The moment diagram for each component is drawn.

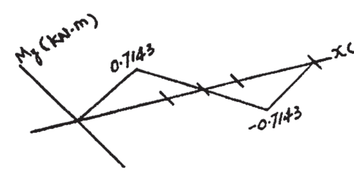
Allowable Bending Stress: Since all the axes through the circle's center for a circular shaft are principal axes, then the resultant moment $M = \sqrt{M_y^2 + M_z^2}$ can be used for the design. The maximum resultant moment is $M_{\max} = \sqrt{4.330^2 + 0.7143^2} = 4.389 \text{ kN} \cdot \text{m}$. Applying the flexure formula,



$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{M_{\max} c}{I}$$

$$180(10^6) = \frac{4.389(10^3)\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

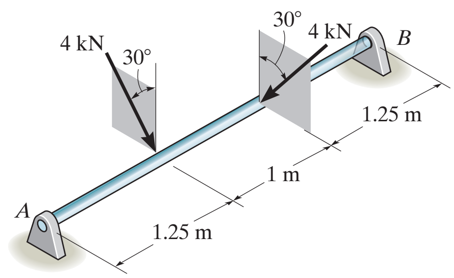
$d = 0.06286 \text{ m} = 62.9 \text{ mm}$ **Ans.**



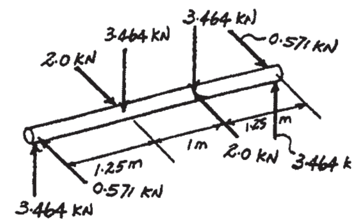
Ans:
$d = 62.9$ mm

6–110.

The 65-mm-diameter steel shaft is subjected to the two loads. If the journal bearings at A and B do not exert an axial force on the shaft, determine the absolute maximum bending stress developed in the shaft.



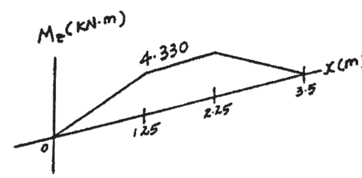
SOLUTION



Support Reactions: As shown on FBD.

Internal Moment Components: The shaft is subjected to two bending moment components M_y and M_z. The moment diagram for each component is drawn.

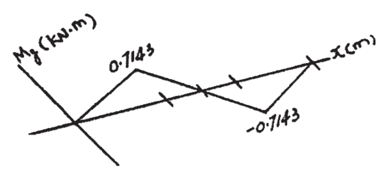
Maximum Bending Stress: Since all the axes through the circle's center for a circular shaft are principal axes, then the resultant moment $M = \sqrt{M_y^2 + M_z^2}$ can be used to determine the maximum bending stress. The maximum resultant moment is $M_{\max} = \sqrt{4.330^2 + 0.7143^2} = 4.389 \text{ kN} \cdot \text{m}$. Applying the flexure formula,



$$\sigma_{\max} = \frac{M_{\max}c}{I}$$

$$= \frac{4.389(10^3)(0.0325)}{\frac{\pi}{4}(0.0325^4)}$$

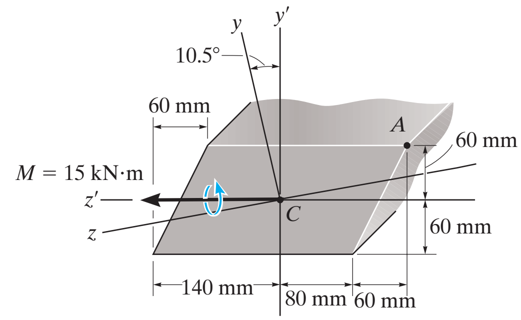
$$= 163 \text{ MPa} \qquad \textbf{Ans.}$$



Ans:
$\sigma_{\max} = 163 \text{ MPa}$

6–111.

For the section, $I_{z'} = 31.7(10^{-6})\ \text{m}^4$, $I_{y'} = 114(10^{-6})\ \text{m}^4$, $I_{y'z'} = -15.8(10^{-6})\ \text{m}^4$. Using the techniques outlined in Appendix A, the member's cross-sectional area has principal moments of inertia of $I_z = 28.8(10^{-6})\ \text{m}^4$ and $I_y = 117(10^{-6})\ \text{m}^4$, calculated about the principal axes of inertia y and z, respectively. If the section is subjected to the moment $M = 15\ \text{kN}\cdot\text{m}$, determine the stress at point A using Eq. 6–17.



SOLUTION

Internal Moment Components: The y and z components are

$$M_y = 15 \sin 10.5° = 2.7335\ \text{kN}\cdot\text{m} \qquad M_z = 15 \cos 10.5° = 14.7488\ \text{kN}\cdot\text{m}$$

Section Properties: Given that $I_y = 117\left(10^{-6}\right)\ \text{m}^4$ and $I_z = 28.8\left(10^{-6}\right)\ \text{m}^4$, the coordinates of point A with respect to the y and z axes are

$$y = 0.06 \cos 10.5° - 0.14 \sin 10.5° = 0.03348\ \text{m}$$

$$z = -(0.06 \sin 10.5° + 0.14 \cos 10.5°) = -0.14859\ \text{m}$$

Bending Stress: Applying the flexure formula for biaxial bending,

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{14.7488\left(10^3\right)(0.03348)}{28.8\left(10^{-6}\right)} + \frac{2.7335\left(10^3\right)(-0.14869)}{117\left(10^{-6}\right)}$$

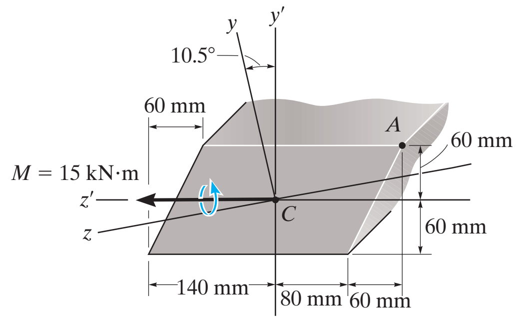
$$= -20.62\left(10^6\right)\ \text{Pa}$$

$$= 20.6\ \text{MPa (C)}$$ **Ans.**

Ans:
$\sigma_A = 20.6$ MPa (C)

***6–112.**

Solve Prob. 6–111 using the equation developed in Prob. 6–106.



SOLUTION

Internal Moment Components: The y' and z' components are

$$M_{z'} = 15\ \text{kN}\cdot\text{m} \qquad M_{y'} = 0$$

Section Properties: Given that $I_{y'} = 114\left(10^{-6}\right)\ \text{m}^4$, $I_{z'} = 31.7\left(10^{-6}\right)\ \text{m}^4$ and $I_{y'z'} = -15.8\left(10^{-6}\right)\ \text{m}^4$, the coordinates of point A with respect to the y' and z axes are

$$y' = 0.06\ \text{m} \qquad z' = -0.14\ \text{m}$$

Bending Stress: Using the formula,

$$\sigma = \frac{-\left(M_{z'}I_{y'} + M_{y'}I_{y'z'}\right)y' + \left(M_{y'}I_{z'} + M_{z'}I_{y'z'}\right)z'}{I_{y'}I_{z'} - I_{y'z'}^2}$$

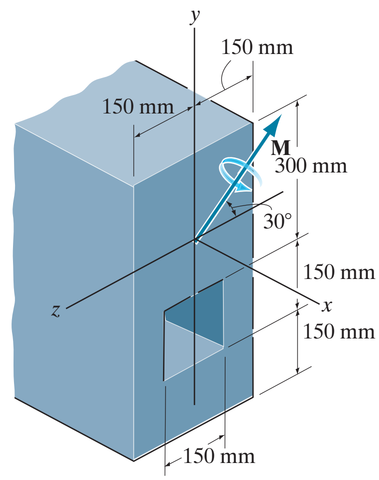
$$\sigma_A = \frac{-\left[15\left(10^3\right)(114)\left(10^{-6}\right) + 0\right](0.06) + \left[0 + 15\left(10^3\right)(-15.8)\left(10^{-6}\right)\right](-0.14)}{\left[114\left(10^{-6}\right)\right]\left[31.7\left(10^{-6}\right)\right] - \left[-15.8\left(10^{-6}\right)\right]^2}$$

$$= -20.64\left(10^6\right)\ \text{Pa}$$

$$= 20.6\ \text{MPa (C)} \qquad \textbf{Ans.}$$

Ans:
$\sigma_A = 20.6$ MPa (C)

6–113. If the beam is subjected to the internal moment of $M = 1200\ \text{kN}\cdot\text{m}$, determine the maximum bending stress acting on the beam and the orientation of the neutral axis.



Internal Moment Components: The y component of M is positive since it is directed towards the positive sense of the y axis, whereas the z component of **M**, which is directed towards the negative sense of the z axis, is negative, Fig. a. Thus,

$$M_y = 1200 \sin 30° = 600\ \text{kN}\cdot\text{m}$$

$$M_z = -1200 \cos 30° = -1039.23\ \text{kN}\cdot\text{m}$$

Section Properties: The location of the centroid of the cross-section is given by

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.3(0.6)(0.3) - 0.375(0.15)(0.15)}{0.6(0.3) - 0.15(0.15)} = 0.2893\ \text{m}$$

The moments of inertia of the cross section about the principal centroidal y and z axes are

$$I_y = \frac{1}{12}(0.6)\left(0.3^3\right) - \frac{1}{12}(0.15)\left(0.15^3\right) = 1.3078\left(10^{-3}\right)\ \text{m}^4$$

$$I_z = \frac{1}{12}(0.3)\left(0.6^3\right) + 0.3(0.6)(0.3 - 0.2893)^2$$

$$- \left[\frac{1}{12}(0.15)\left(0.15^3\right) + 0.15(0.15)(0.375 - 0.2893)^2\right]$$

$$= 5.2132\left(10^{-3}\right)\text{m}^4$$

Bending Stress: By inspection, the maximum bending stress occurs at either corner A or B.

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{\left[-1039.23\left(10^3\right)\right](0.2893)}{5.2132\left(10^{-3}\right)} + \frac{600\left(10^3\right)(0.15)}{1.3078\left(10^{-3}\right)}$$

$$= 126\ \text{MPa (T)}$$

$$\sigma_B = -\frac{\left[-1039.23\left(10^3\right)\right](-0.3107)}{5.2132\left(10^{-3}\right)} + \frac{600\left(10^3\right)(-0.15)}{1.3078\left(10^{-3}\right)}$$

$$= -131\ \text{MPa} = 131\ \text{MPa (C)(Max.)} \qquad \textbf{Ans.}$$

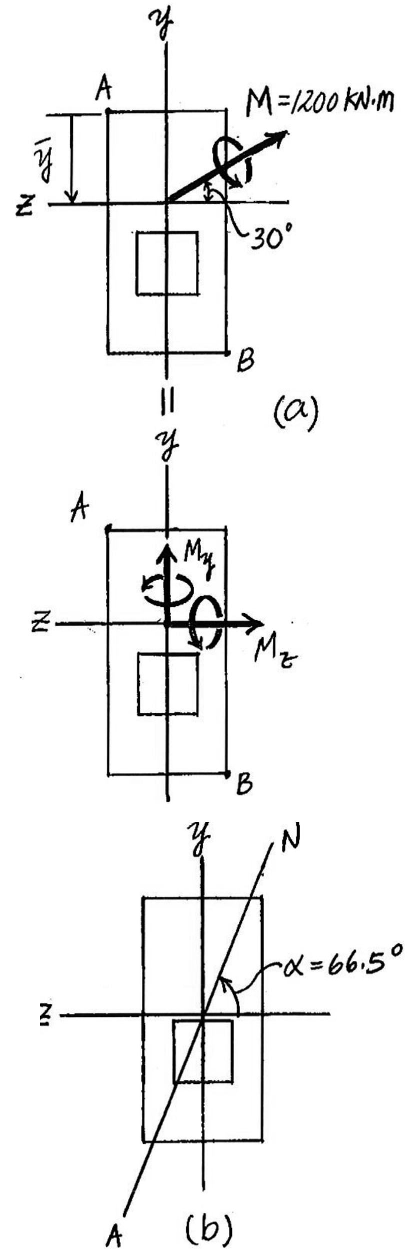
Orientation of Neutral Axis: Here, $\theta = -30°$.

$$\tan \alpha = \frac{I_z}{I_y} \tan \theta$$

$$\tan \alpha = \frac{5.2132\left(10^{-3}\right)}{1.3078\left(10^{-3}\right)} \tan(-30°)$$

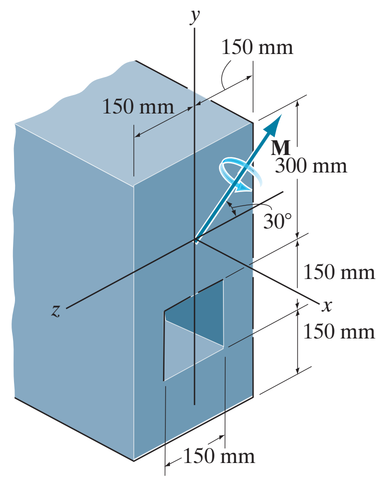
$$\alpha = -66.5° \qquad \textbf{Ans.}$$

The orientation of the neutral axis is shown in Fig. b.



Ans:
$\sigma_B = 131\ \text{MPa (C)}, \alpha = -66.5°$

6–114. If the beam is made from a material having an allowable tensile and compressive stress of $(\sigma_{\text{allow}})_t = 125$ MPa and $(\sigma_{\text{allow}})_c = 150$ MPa, respectively, determine the maximum allowable internal moment **M** that can be applied to the beam.



Internal Moment Components: The y component of **M** is positive since it is directed towards the positive sense of the y axis, whereas the z component of **M**, which is directed towards the negative sense of the z axis, is negative, Fig. *a*. Thus,

$$M_y = M \sin 30° = 0.5M$$

$$M_z = -M \cos 30° = -0.8660M$$

Section Properties: The location of the centroid of the cross section is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.3(0.6)(0.3) - 0.375(0.15)(0.15)}{0.6(0.3) - 0.15(0.15)} = 0.2893 \text{ m}$$

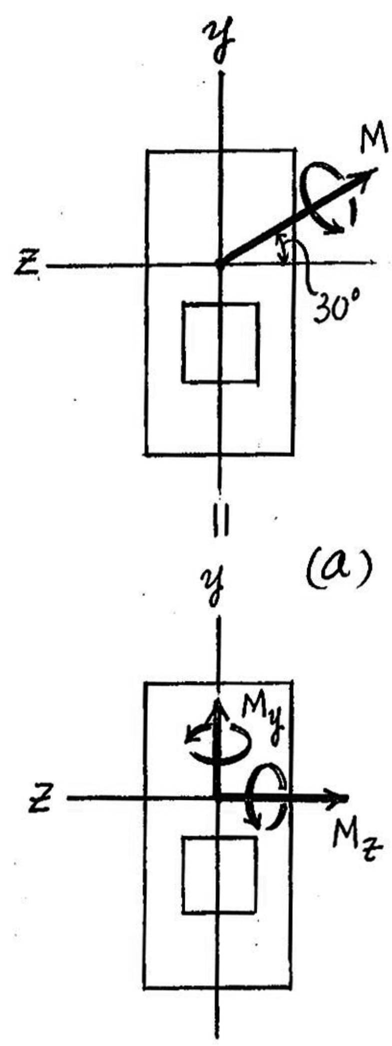
The moments of inertia of the cross section about the principal centroidal y and z axes are

$$I_y = \frac{1}{12}(0.6)\left(0.3^3\right) - \frac{1}{12}(0.15)\left(0.15^3\right) = 1.3078\left(10^{-3}\right) \text{ m}^4$$

$$I_z = \frac{1}{12}(0.3)\left(0.6^3\right) + 0.3(0.6)(0.3 - 0.2893)^2$$

$$- \left[\frac{1}{12}(0.15)\left(0.15^3\right) + 0.15(0.15)(0.375 - 0.2893)^2\right]$$

$$= 5.2132\left(10^{-3}\right)\text{m}^4$$



Bending Stress: By inspection, the maximum bending stress can occur at either corner A or B. For corner A which is in tension,

$$\sigma_A = (\sigma_{\text{allow}})_t = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

$$125\left(10^6\right) = -\frac{(-0.8660M)(0.2893)}{5.2132\left(10^{-3}\right)} + \frac{0.5M(0.15)}{1.3078\left(10^{-3}\right)}$$

$$M = 1185\,906.82 \text{ N}\cdot\text{m} = 1186 \text{ kN}\cdot\text{m (controls)}$$ **Ans.**

For corner B which is in compression,

$$\sigma_B = (\sigma_{\text{allow}})_c = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y}$$

$$-150\left(10^6\right) = -\frac{(-0.8660M)(-0.3107)}{5.2132\left(10^{-3}\right)} + \frac{0.5M(-0.15)}{1.3078\left(10^{-3}\right)}$$

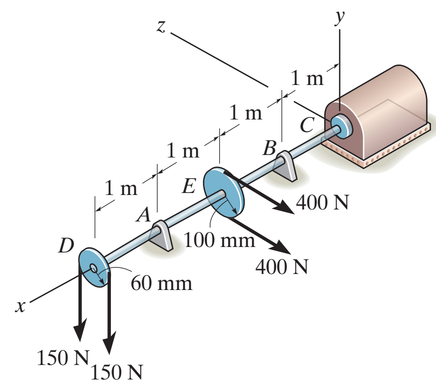
$$M = 1376\,597.12 \text{ N}\cdot\text{m} = 1377 \text{ kN}\cdot\text{m}$$

Ans:

$M = 1186 \text{ kN}\cdot\text{m}$

6–115.

The shaft is subjected to the vertical and horizontal loadings of two pulleys D and E as shown. It is supported on two journal bearings at A and B which offer no resistance to axial loading. Furthermore, the coupling to the motor at C can be assumed not to offer any support to the shaft. Determine the required diameter d of the shaft if the allowable bending stress is $\sigma_{allow} = 180$ MPa.



SOLUTION

Support Reactions: As shown on FBD.

Internal Moment Components: The shaft is subjected to two bending moment components M_y and M_z.

Allowable Bending Stress: Since all the axes through the circle's center for a circular shaft are principal axes, then the resultant moment $M = \sqrt{M_y^2 + M_z^2}$ can be used for the design. The maximum resultant moment is $M_{max} = \sqrt{400^2 + 150^2} = 427.2\ \text{N}\cdot\text{m}$. Applying the flexure formula

$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}c}{I}$$

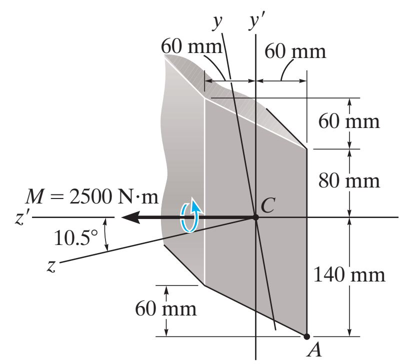
$$180(10^6) = \frac{427.2\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

$$d = 0.02891\ \text{m} = 28.9\ \text{mm}$$ **Ans.**

Ans:
$d = 28.9$ mm

***6–116.**

For the section, $I_{y'} = 31.7(10^{-6})\ \text{m}^4$, $I_{z'} = 114(10^{-6})\ \text{m}^4$, $I_{y'z'} = 15.8(10^{-6})\ \text{m}^4$. Using the techniques outlined in Appendix A, the member's cross-sectional area has principal moments of inertia of $I_y = 28.8(10^{-6})\ \text{m}^4$ and $I_z = 117(10^{-6})\ \text{m}^4$, calculated about the principal axes of inertia y and z, respectively. If the section is subjected to a moment of $M = 2500\ \text{N}\cdot\text{m}$, determine the stress produced at point A, using Eq. 6–17.



SOLUTION

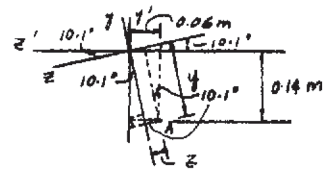
$I_z = 117(10^{-6})\ \text{m}^4 \qquad I_y = 28.8(10^{-6})\ \text{m}^4$

$M_y = 2500 \sin 10.5° = 455.59\ \text{N}\cdot\text{m}$

$M_z = 2500 \cos 10.5° = 2458.14\ \text{N}\cdot\text{m}$

$y = -0.06 \sin 10.5° - 0.14 \cos 10.5° = -0.14859\ \text{m}$

$z = 0.14 \sin 10.5° - 0.06 \cos 10.5° = -0.03348\ \text{m}$



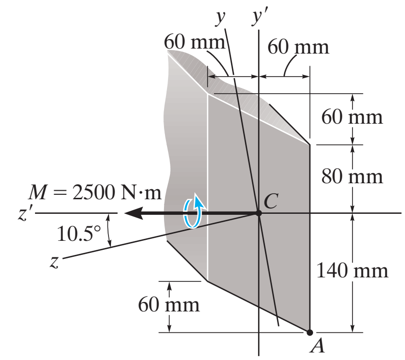
$$\sigma_A = \frac{-M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$= \frac{2458.14(-0.14859)}{117(10^{-6})} + \frac{455.59(-0.03348)}{28.8(10^{-6})} = 2.59\ \text{MPa (T)} \qquad \textbf{Ans.}$$

Ans:
$\sigma_A = 2.59\ \text{MPa (T)}$

6–117.

Solve Prob. 6–116 using the equation developed in Prob. 6–106.



SOLUTION

$$\sigma_A = \frac{-(M_{z'}I_{y'} + M_{y'}I_{y'z'})y' + (M_{y'}I_{z'} + M_{z'}I_{y'z'})z'}{I_{y'}I_{z'} - I_{y'z'}^{\,2}}$$

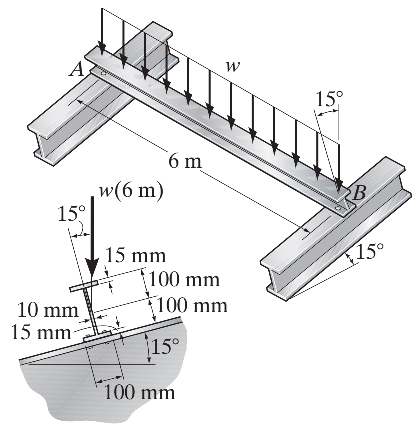
$$= \frac{-[2500(31.7)(10^{-6}) + 0](-0.14) + [0 + 2500(15.8)(10^{-6})](-0.06)}{31.7(10^{-6})(114)(10^{-6}) - [(15.8)(10^{-6})]^2}$$

$= 2.59$ MPa (T) **Ans.**

Ans:
$\sigma_A = 2.59$ MPa (T)

6–118.

If the applied distributed loading of $w = 4\ \text{kN/m}$ can be assumed to pass through the centroid of the beam's cross-sectional area, determine the absolute maximum bending stress in the joist and the orientation of the neutral axis. The beam can be considered simply supported at A and B.



SOLUTION

Internal Moment Components: The uniform distributed load **w** can be resolved into its y and z components as shown in Fig. a.

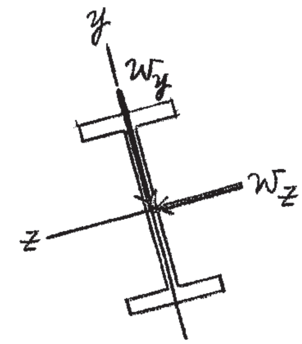
$$w_y = 4 \cos 15° = 3.864\ \text{kN/m}$$

$$w_z = 4 \sin 15° = 1.035\ \text{kN/m}$$

$\mathbf{w}_y$ and $\mathbf{w}_z$ produce internal moments in the beam about the z and y axes, respectively. For the simply supported beam subjected to the uniform distributed load, the maximum moment in the beam is $M_{\max} = \dfrac{wL^2}{8}$. Thus,

$$(M_z)_{\max} = \frac{w_y L^2}{8} = \frac{3.864(6^2)}{8} = 17.387\ \text{kN}\cdot\text{m}$$

$$(M_y)_{\max} = \frac{w_z L^2}{8} = \frac{1.035(6^2)}{8} = 4.659\ \text{kN}\cdot\text{m}$$



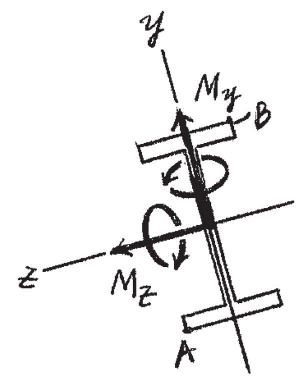
(a)

As shown in Fig. b, $(\mathbf{M}_z)_{\max}$ and $(\mathbf{M}_y)_{\max}$ are positive since they are directed towards the positive sense of their respective axes.

Section Properties: The moments of inertia of the cross section about the principal centroidal y and z axes are

$$I_y = 2\left[\frac{1}{12}(0.015)(0.1^3)\right] + \frac{1}{12}(0.17)(0.01^3) = 2.5142(10^{-6})\ \text{m}^4$$

$$I_z = \frac{1}{12}(0.1)(0.2^3) - \frac{1}{12}(0.09)(0.17^3) = 29.8192(10^{-6})\ \text{m}^4$$



(b)

Bending Stress: By inspection, the maximum bending stress occurs at points A and B.

$$\sigma = -\frac{(M_z)_{\max}\, y}{I_z} + \frac{(M_y)_{\max}\, z}{I_y}$$

$$\sigma_{\max} = \sigma_A = -\frac{17.387(10^3)(-0.1)}{29.8192\ (10^{-6})} + \frac{4.659(10^3)(0.05)}{2.5142(10^{-6})}$$

$$= 150.96\ \text{MPa} = 151\ \text{MPa (T)}$$ **Ans.**

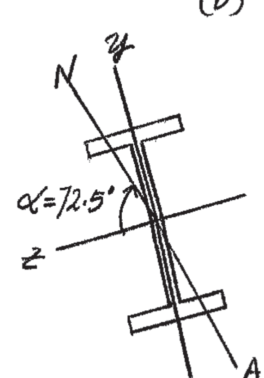
$$\sigma_{\max} = \sigma_B = -\frac{17.387(10^3)(0.1)}{29.8192\ (10^{-6})} + \frac{4.659(10^3)(-0.05)}{2.5142(10^{-6})}$$

$$= -150.96\ \text{MPa} = 151\ \text{MPa (C)}$$ **Ans.**

Orientation of Neutral Axis: Here, $\theta = \tan^{-1}\left[\dfrac{(M_y)_{\max}}{(M_z)_{\max}}\right] = \tan^{-1}\left(\dfrac{4.659}{17.387}\right) = 15°$.

$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$

$$\tan\alpha = \frac{29.8192(10^{-6})}{2.5142(10^{-6})}\tan 15°$$



(C)

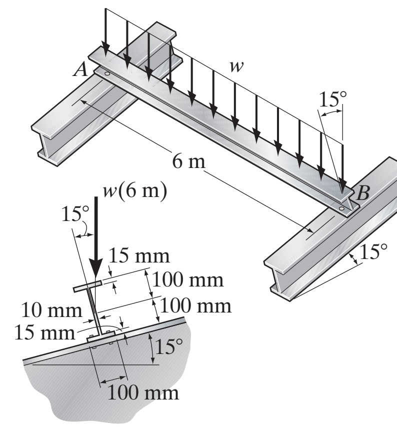
$$\alpha = 72.5°$$ **Ans.**

The orientation of the neutral axis is shown in Fig. c.

Ans:
$\sigma_{\max} = 151\ \text{MPa}$, $\alpha = 72.5°$

6–119.

Determine the maximum allowable intensity w of the uniform distributed load that can be applied to the beam. Assume w passes through the centroid of the beam's cross-sectional area, and the beam is simply supported at A and B. The allowable bending stress is $\sigma_{\text{allow}} = 165$ MPa.

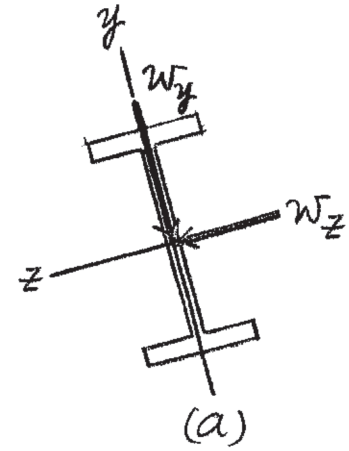


SOLUTION

Internal Moment Components: The uniform distributed load **w** can be resolved into its y and z components as shown in Fig. a.

$$w_y = w \cos 15° = 0.9659w$$

$$w_z = w \sin 15° = 0.2588w$$



$\mathbf{w}_y$ and $\mathbf{w}_z$ produce internal moments in the beam about the z and y axes, respectively. For the simply supported beam subjected to the a uniform distributed load, the maximum moment in the beam is $M_{\max} = \dfrac{wL^2}{8}$. Thus,

$$(M_z)_{\max} = \frac{w_y L^2}{8} = \frac{0.9659w(6^2)}{8} = 4.3476w$$

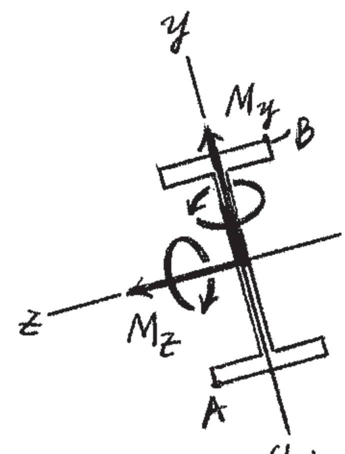
$$(M_y)_{\max} = \frac{w_z L^2}{8} = \frac{0.2588w(6^2)}{8} = 1.1647w$$

As shown in Fig. b, $(\mathbf{M}_z)_{\max}$ and $(\mathbf{M}_y)_{\max}$ are positive since they are directed towards the positive sense of their respective axes.

Section Properties: The moments of inertia of the cross section about the principal centroidal y and z axes are

$$I_y = 2\left[\frac{1}{12}(0.015)(0.1^3)\right] + \frac{1}{12}(0.17)(0.01^3) = 2.5142(10^{-6})\ \text{m}^4$$

$$I_z = \frac{1}{12}(0.1)(0.2^3) - \frac{1}{12}(0.09)(0.17^3) = 29.8192(10^{-6})\ \text{m}^4$$



Bending Stress: By inspection, the maximum bending stress occurs at points A and B. We will consider point A.

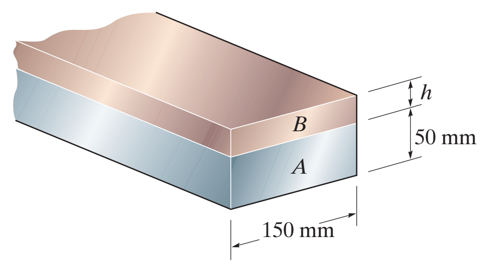
$$\sigma_A = \sigma_{\text{allow}} = -\frac{(M_z)_{\max} y_A}{I_z} + \frac{(M_y)_{\max} z_A}{I_y}$$

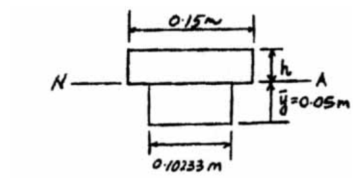
$$165(10^6) = -\frac{4.3467w(-0.1)}{29.8192(10^{-6})} + \frac{1.1647w(0.05)}{2.5142(10^{-6})}$$

$w = 4372.11$ N/m $= 4.37$ kN/m **Ans.**

Ans:
$w = 4.37$ kN/m

***6–120.** The composite beam is made of 6061-T6 aluminum (A) and C83400 red brass (B). Determine the dimension h of the brass strip so that the neutral axis of the beam is located at the seam of the two metals. What maximum moment will this beam support if the allowable bending stress for the aluminum is $(\sigma_{allow})_{al} = 128$ MPa and for the brass $(\sigma_{allow})_{br} = 35$ MPa?





Section Properties:

$$n = \frac{E_{al}}{E_{br}} = \frac{68.9(10^9)}{101(10^9)} = 0.68218$$

$$b_{br} = nb_{al} = 0.68218(0.15) = 0.10233 \text{ m}$$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$0.05 = \frac{0.025(0.10233)(0.05) + (0.05 + 0.5h)(0.15)h}{0.10233(0.05) + (0.15)h}$$

$$h = 0.04130 \text{ m} = 41.3 \text{ mm}$$ **Ans.**

$$I_{NA} = \frac{1}{12}(0.10233)\left(0.05^3\right) + 0.10233(0.05)(0.05 - 0.025)^2$$

$$+ \frac{1}{12}(0.15)\left(0.04130^3\right) + 0.15(0.04130)(0.070649 - 0.05)^2$$

$$= 7.7851\left(10^{-6}\right) \text{ m}^4$$

Allowable Bending Stress: Applying the flexure formula

Assume failure of red brass

$$(\sigma_{allow})_{br} = \frac{Mc}{I_{NA}}$$

$$35\left(10^6\right) = \frac{M(0.04130)}{7.7851(10^{-6})}$$

$$M = 6598 \text{ N}\cdot\text{m} = 6.60 \text{ kN}\cdot\text{m } (controls!)$$ **Ans.**

Assume failure of aluminium

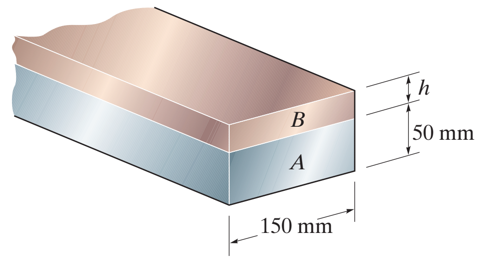
$$(\sigma_{allow})_{al} = n\frac{Mc}{I_{NA}}$$

$$128\left(10^6\right) = 0.68218\left[\frac{M(0.05)}{7.7851(10^{-6})}\right]$$

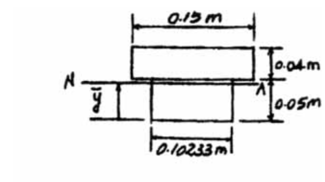
$$M = 29215 \text{ N}\cdot\text{m} = 29.2 \text{ kN}\cdot\text{m}$$

Ans:
$h = 41.3$ mm,
$M = 6.60$ kN·m (*controls!*)

6–121. The composite beam is made of 6061-T6 aluminum (A) and C83400 red brass (B). If the height $h = 40$ mm, determine the maximum moment that can be applied to the beam if the allowable bending stress for the aluminum is $(\sigma_{\text{allow}})_{\text{al}} = 128$ MPa and for the brass $(\sigma_{\text{allow}})_{\text{br}} = 35$ MPa.



Section Properties: For transformed section.



$$n = \frac{E_{\text{al}}}{E_{\text{br}}} = \frac{68.9(10^9)}{101.0(10^9)} = 0.68218$$

$$b_{\text{br}} = nb_{\text{al}} = 0.68218(0.15) = 0.10233 \text{ m}$$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{0.025(0.10233)(0.05) + (0.07)(0.15)(0.04)}{0.10233(0.05) + 0.15(0.04)}$$

$$= 0.049289 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.10233)\left(0.05^3\right) + 0.10233(0.05)(0.049289 - 0.025)^2$$

$$+ \frac{1}{12}(0.15)\left(0.04^3\right) + 0.15(0.04)(0.07 - 0.049289)^2$$

$$= 7.45799\left(10^{-6}\right) \text{ m}^4$$

Allowable Bending Stress: Applying the flexure formula

Assume failure of red brass

$$(\sigma_{\text{allow}})_{\text{br}} = \frac{Mc}{I_{NA}}$$

$$35\left(10^6\right) = \frac{M(0.09 - 0.049289)}{7.45799(10^{-6})}$$

$M = 6412 \text{ N}\cdot\text{m} = 6.41 \text{ kN}\cdot\text{m}$ (*controls!*) **Ans.**

Assume failure of aluminium

$$(\sigma_{\text{allow}})_{\text{al}} = n\frac{Mc}{I_{NA}}$$

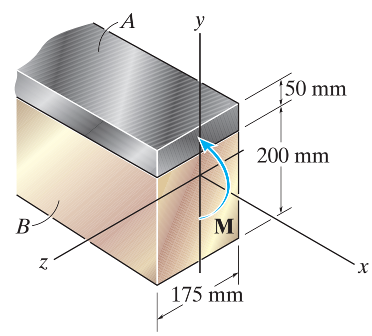
$$128\left(10^6\right) = 0.68218\left[\frac{M(0.049289)}{7.45799(10^{-6})}\right]$$

$$M = 28391 \text{ N}\cdot\text{m} = 28.4 \text{ kN}\cdot\text{m}$$

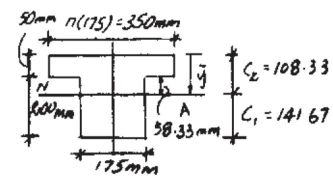
Ans:
$M = 6.41 \text{ kN}\cdot\text{m}$

6–122.

The composite beam is made of steel (A) bonded to brass (B) and has the cross section shown. If it is subjected to a moment of $M = 6.5\ \text{kN}\cdot\text{m}$, determine the maximum bending stress in the brass and steel. Also, what is the stress in each material at the seam where they are bonded together? $E_{br} = 100$ GPa, $E_{st} = 200$ GPa.



SOLUTION



$$n = \frac{E_{st}}{E_{br}} = \frac{200(10^9)}{100(10^9)} = 2$$

$$\bar{y} = \frac{(350)(50)(25) + (175)(200)(150)}{350(50) + 175(200)} = 108.33\ \text{mm}$$

$$I = \frac{1}{12}(0.35)(0.05^3) + (0.35)(0.05)(0.08333^2) + \frac{1}{12}(0.175)(0.2^3) +$$

$$(0.175)(0.2)(0.04167^2) = 0.3026042(10^{-3})\ \text{m}^4$$

Maximum stress in brass:

$$(\sigma_{br})_{max} = \frac{Mc_1}{I} = \frac{6.5(10^3)(0.14167)}{0.3026042(10^{-3})} = 3.04\ \text{MPa} \qquad \textbf{Ans.}$$

Maximum stress in steel:

$$(\sigma_{st})_{max} = \frac{nMc_2}{I} = \frac{(2)(6.5)(10^3)(0.10833)}{0.3026042(10^{-3})} = 4.65\ \text{MPa} \qquad \textbf{Ans.}$$

Stress at the junction:

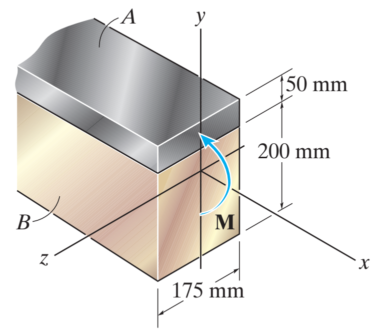
$$\sigma_{br} = \frac{M\rho}{I} = \frac{6.5(10^3)(0.05833)}{0.3026042(10^{-3})} = 1.25\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_{st} = n\sigma_{br} = 2(1.25) = 2.51\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$(\sigma_{br})_{max} = 3.04$ MPa,
$(\sigma_{st})_{max} = 4.65$ MPa,
$\sigma_{br} = 1.25$ MPa,
$\sigma_{st} = 2.51$ MPa

6–123.

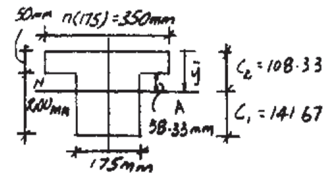
The composite beam is made of steel (A) bonded to brass (B) and has the cross section shown. If the allowable bending stress for the steel is $(\sigma_{\text{allow}})_{\text{st}} = 180$ MPa, and for the brass $(\sigma_{\text{allow}})_{\text{br}} = 60$ MPa, determine the maximum moment M that can be applied to the beam. $E_{\text{br}} = 100$ GPa, $E_{\text{st}} = 200$ GPa.



SOLUTION

$$n = \frac{E_{\text{st}}}{E_{\text{br}}} = \frac{200(10^9)}{100(10^9)} = 2$$

$$\bar{y} = \frac{(350)(50)(25) + (175)(200)(150)}{350(50) + 175(200)} = 108.33 \text{ mm}$$



$$I = \frac{1}{12}(0.35)(0.05^3) + (0.35)(0.05)(0.08333^2) + \frac{1}{12}(0.175)(0.2^3) +$$

$$(0.175)(0.2)(0.04167^2) = 0.3026042(10^{-3}) \text{ m}^4$$

$$(\sigma_{\text{st}})_{\text{allow}} = \frac{nMc_2}{I}; \qquad 180(10^6) = \frac{(2)\,M(0.10833)}{0.3026042(10^{-3})}$$

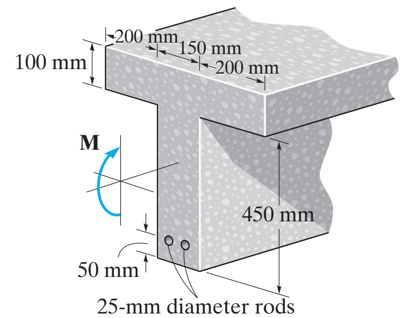
$$M = 251 \text{ kN}\cdot\text{m}$$

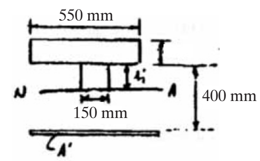
$$(\sigma_{\text{br}})_{\text{allow}} = \frac{Mc_1}{I}; \qquad 60(10^6) = \frac{M(0.14167)}{0.3026042(10^{-3})}$$

$$M = 128 \text{ kN}\cdot\text{m} \quad \text{(controls)}$$ **Ans.**

Ans:
$M = 128$ kN·m

***6–124.** The reinforced concrete beam is made using two steel reinforcing rods. If the allowable tensile stress for for the steel is $(\sigma_{st})_{allow} = 280$ MPa and the allowable compressive stress for the concrete is $(\sigma_{conc})_{allow} = 21$ MPa, determine the maximum moment M that can be applied to the section. Assume the concrete cannot support a tensile stress. $E_{st} = 200$ GPa, $E_{conc} = 26.5$ GPa.





$$A_{st} = 2[\pi(0.0125^2)] = 0.3125\pi(10^{-3})\ \text{m}^2$$

$$A' = nA_{st} = \left(\frac{200}{26.5}\right)[0.3125\pi(10^{-3})] = 7.4094(10^{-3})\ \text{m}^2$$

$$\Sigma\bar{y}A = 0; \qquad (h'+0.05)(0.55)(0.1) + \left(\frac{h'}{2}\right)(0.15)h' - 7.4094(10^{-3})(0.4-h') = 0$$

$$0.075h'^2 + 0.062409h' - 0.21377(10^{-3}) = 0$$

Solving for the positive root:

$$h' = 0.0034112\ \text{m}$$

$$I = \frac{1}{12}(0.55)(0.1^3) + (0.55)(0.1)(0.0034112+0.05)^2$$

$$+ \frac{1}{12}(0.15)(0.0034112^3) + 0.15(0.0034112)(0.0034112/2)^2$$

$$+ 7.4094(10^{-3})(0.4-0.0034112)^2$$

$$= 1.36811(10^{-3})\ \text{m}^4$$

Assume concrete fails:

$$(\sigma_{con})_{allow} = \frac{My}{I}; \qquad 21(10^6) = \left(\frac{200}{26.5}\right)\left[\frac{M(0.4-0.0034112)}{1.36811(10^{-3})}\right]$$

$$M = 277.83(10^3)\ \text{N}\cdot\text{m} = 278\ \text{kN}\cdot\text{m}$$

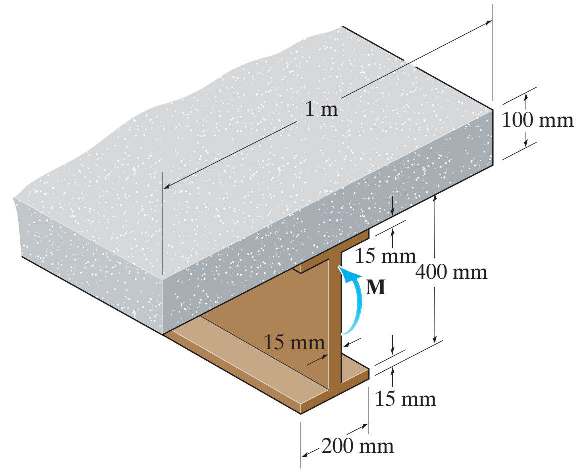
Assume steel fails:

$$(\sigma_{st})_{allow} = n\left(\frac{My}{I}\right); \qquad 280(10^6) = \frac{M(0.4+0.0034112)}{1.36811(10^{-3})}$$

$$M = 127.98(10^3)\ \text{N}\cdot\text{m} = 128\ \text{kN}\cdot\text{m (Controls)} \qquad \textbf{Ans.}$$

Ans:
$M = 128$ kN · m (Controls)

6–125. The low strength concrete floor slab is integrated with a wide-flange A-36 steel beam using shear studs (not shown) to form the composite beam. If the allowable bending stress for the concrete is $(\sigma_{\text{allow}})_{\text{con}} = 10$ MPa, and allowable bending stress for steel is $(\sigma_{\text{allow}})_{\text{st}} = 165$ MPa, determine the maximum allowable internal moment **M** that can be applied to the beam.



Section Properties: The beam cross section will be transformed into that of steel. Here, $n = \dfrac{E_{con}}{E_{st}} = \dfrac{22.1}{200} = 0.1105$. Thus, $b_{st} = nb_{con} = 0.1105(1) = 0.1105$ m. The location of the transformed section is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{0.0075(0.015)(0.2) + 0.2(0.37)(0.015) + 0.3925(0.015)(0.2) + 0.45(0.1)(0.1105)}{0.015(0.2) + 0.37(0.015) + 0.015(0.2) + 0.1(0.1105)}$$

$$= 0.3222 \text{ m}$$

The moment of inertia of the transformed section about the neutral axis is

$$\begin{aligned} I = \Sigma \bar{I} + Ad^2 = {} & \frac{1}{12}(0.2)\left(0.015^3\right) \\ & + 0.2(0.015)(0.3222 - 0.0075)^2 \\ & + \frac{1}{12}(0.015)\left(0.37^3\right) + 0.015(0.37)(0.3222 - 0.2)^2 \\ & + \frac{1}{12}(0.2)\left(0.015^3\right) + 0.2(0.015)(0.3925 - 0.3222)^2 \\ & + \frac{1}{12}(0.1105)\left(0.1^3\right) + 0.1105(0.1)(0.45 - 0.3222)^2 \\ & = 647.93\left(10^{-6}\right) \text{ m}^4 \end{aligned}$$

Bending Stress: Assuming failure of steel,

$$(\sigma_{\text{allow}})_{st} = \frac{Mc_{st}}{I}; \quad 165\left(10^6\right) = \frac{M(0.3222)}{647.93\left(10^{-6}\right)}$$

$$M = 331\,770.52 \text{ N} \cdot \text{m} = 332 \text{ kN} \cdot \text{m}$$

Assuming failure of concrete,

$$(\sigma_{\text{allow}})_{con} = n\frac{Mc_{con}}{I}; \qquad 10\left(10^6\right) = 0.1105\left[\frac{M(0.5 - 0.3222)}{647.93\left(10^{-6}\right)}\right]$$

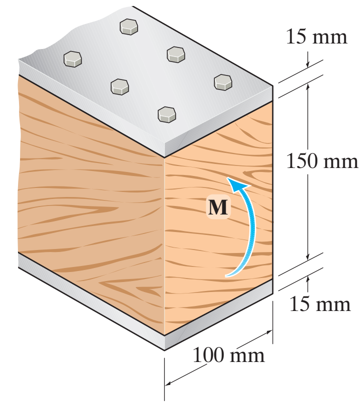
$$M = 329\,849.77 \text{ N} \cdot \text{m} = 330 \text{ kN} \cdot \text{m (controls)} \quad \textbf{Ans.}$$

Ans:

$M = 330$ kN · m

6–126.

The wooden section of the beam is reinforced with two steel plates as shown. Determine the maximum moment M that the beam can support if the allowable stresses for the wood and steel are $(\sigma_{\text{allow}})_{\text{w}} = 6$ MPa, and $(\sigma_{\text{allow}})_{\text{st}} = 150$ MPa, respectively. Take $E_{\text{w}} = 10$ GPa and $E_{\text{st}} = 200$ GPa.



SOLUTION

Section Properties: The cross section will be transformed into that of steel as shown in Fig. *a*. Here, $n = \dfrac{E_{\text{w}}}{E_{\text{st}}} = \dfrac{10}{200} = 0.05$. Thus, $b_{\text{st}} = nb_{\text{w}} = 0.05(0.1) = 0.005$ m. The moment of inertia of the transformed section about the neutral axis is

$$I = \frac{1}{12}(0.1)(0.18^3) - \frac{1}{12}(0.095)(0.15^3) = 21.88125(10^{-6})\ \text{m}^4$$

Bending Stress: Assuming failure of the steel,

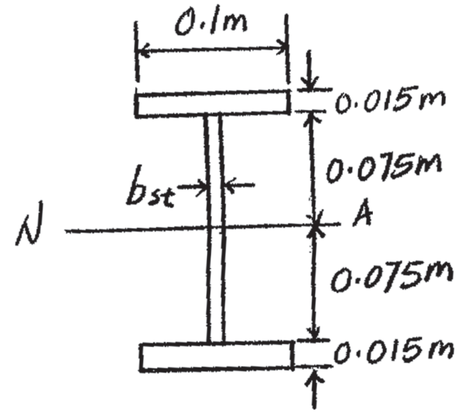
$$(\sigma_{\text{allow}})_{\text{st}} = \frac{Mc_{\text{st}}}{I}; \qquad 150(10^6) = \frac{M(0.09)}{21.88125(10^{-6})}$$

$$M = 36\,468.75\ \text{N}\cdot\text{m} = 36.5\ \text{kN}\cdot\text{m}$$

Assuming failure of wood,

$$(\sigma_{\text{allow}})_{\text{w}} = n\frac{Mc_{\text{w}}}{I}; \qquad 6(10^6) = 0.05\left[\frac{M(0.075)}{21.88125(10^{-6})}\right]$$

$$M = 35\,010\ \text{N}\cdot\text{m} = 35.0\ \text{kN}\cdot\text{m}\ \text{(controls)} \qquad \textbf{Ans.}$$

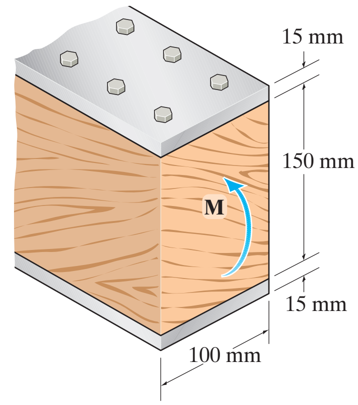


(a)

Ans:
$M = 35.0\ \text{kN}\cdot\text{m}$

6–127.

The wooden section of the beam is reinforced with two steel plates as shown. If the beam is subjected to a moment of $M = 30$ kN·m, determine the maximum bending stresses in the steel and wood. Sketch the stress distribution over the cross section. Take $E_w = 10$ GPa and $E_{st} = 200$ GPa.



SOLUTION

Section Properties: The cross section will be transformed into that of steel as shown in Fig. *a*. Here, $n = \frac{E_w}{E_{st}} = \frac{10}{200} = 0.05$. Thus, $b_{st} = nb_w = 0.05(0.1) = 0.005\,\text{m}$. The moment of inertia of the transformed section about the neutral axis is

$$I = \frac{1}{12}(0.1)(0.18^3) - \frac{1}{12}(0.095)(0.15^3) = 21.88125(10^{-6})\ \text{m}^4$$

Maximum Bending Stress: For the steel,

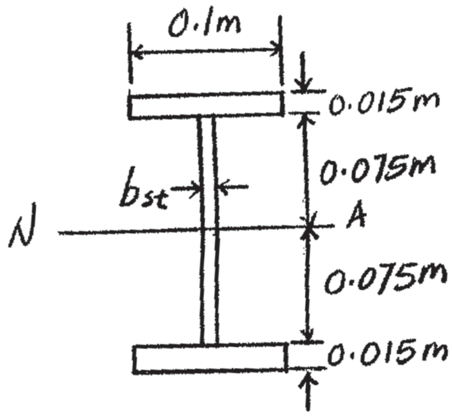
$$(\sigma_{\max})_{st} = \frac{Mc_{st}}{I} = \frac{30(10^3)(0.09)}{21.88125(10^{-6})} = 123\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_{st}\big|_{y=0.075\ \text{m}} = \frac{My}{I} = \frac{30(10^3)(0.075)}{21.88125(10^{-6})} = 103\ \text{MPa}$$

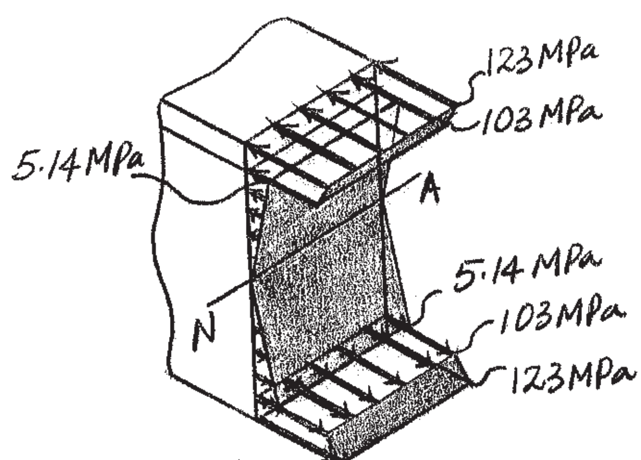
For the wood,

$$(\sigma_{\max})_w = n\frac{Mc_w}{I} = 0.05\left[\frac{30(10^3)(0.075)}{21.88125(10^{-6})}\right] = 5.14\ \text{MPa} \qquad \textbf{Ans.}$$

The bending stress distribution across the cross section is shown in Fig. *b*.



(a)

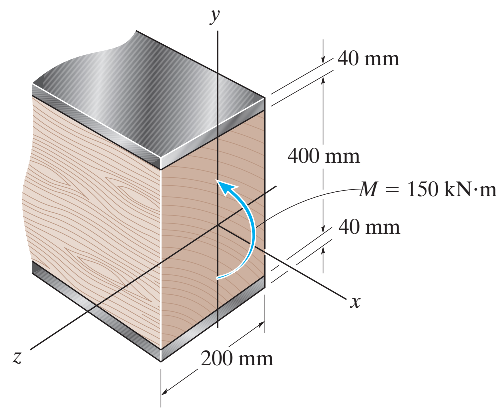


(b)

Ans:
$(\sigma_{\max})_{st} = 123$ MPa, $(\sigma_{\max})_w = 5.14$ MPa

***6–128.**

A wood beam is reinforced with steel straps at its top and bottom as shown. Determine the maximum bending stress developed in the wood and steel if the beam is subjected to a moment of $M = 150$ kN · m. Sketch the stress distribution acting over the cross section. Take $E_w = 10$ GPa, $E_{st} = 200$ GPa.



SOLUTION

Section Properties: Here, $n = \dfrac{E_w}{E_{st}} = \dfrac{10 \text{ GPa}}{200 \text{ GPa}} = 0.05$. Then $b_{st} = nb_w = 0.05(0.2) = 0.01$ m. For the transformed section shown in Fig. *a*,

$$I_{NA} = \frac{1}{12}(0.2)\left(0.48^3\right) - \frac{1}{12}(0.19)\left(0.4^3\right) = 0.82987\left(10^{-3}\right) \text{ m}^4$$

Maximum Bending Stress: Applying the flexure formula,

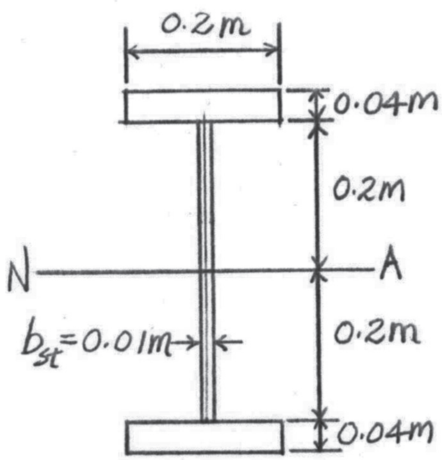
$$(\sigma_{st})_{max} = \frac{Mc_{st}}{I} = \frac{150\left(10^3\right)(0.24)}{0.82987\left(10^{-3}\right)} = 43.38\left(10^6\right) \text{ Pa} = 43.4 \text{ MPa} \qquad \textbf{Ans.}$$

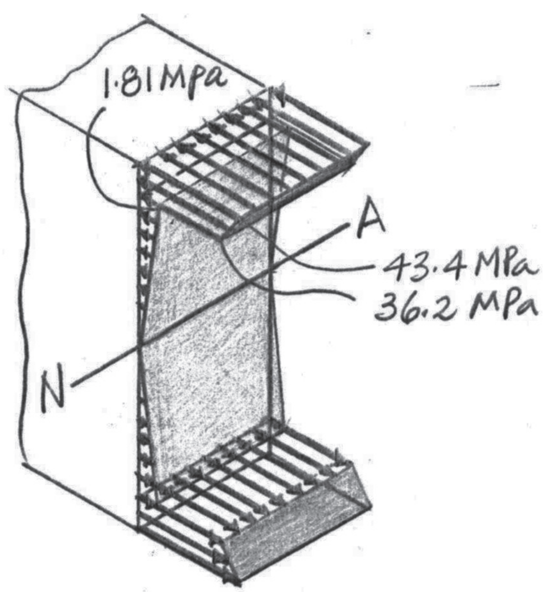
$$(\sigma_w)_{max} = n\frac{Mc_w}{I} = 0.05\left[\frac{150\left(10^3\right)(0.2)}{0.82987\left(10^{-3}\right)}\right] = 1.8075\left(10^6\right) \text{ Pa} = 1.81 \text{ MPa} \qquad \textbf{Ans.}$$

The bending stress on the steel at $y = 0.2$ m is

$$\sigma_{st} = \frac{My}{I} = \frac{150\left(10^3\right)(0.2)}{0.82987\left(10^{-3}\right)} = 36.15\left(10^6\right) \text{ Pa} = 36.2 \text{ MPa}$$

Using these results, the stress distribution on the beam's cross-section shown in Fig. *b* can be sketched.

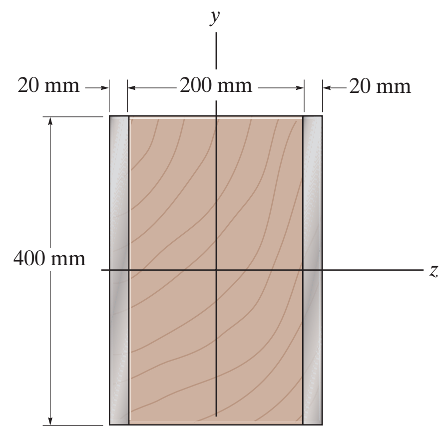




Ans:
$(\sigma_{st})_{max} = 43.4$ MPa,
$(\sigma_w)_{max} = 1.81$ MPa

6–129.

The Douglas Fir beam is reinforced with A-992 steel straps at its sides. Determine the maximum stress in the wood and steel if the beam is subjected to a moment of $M_z = 80\ \text{kN}\cdot\text{m}$. Sketch the stress distribution acting over the cross section.



SOLUTION

Section Properties: Here, $n = \dfrac{E_w}{E_{st}} = \dfrac{13.1\ \text{GPa}}{200\ \text{GPa}} = 0.0655$. Then $b_{st} = nb_w = 0.0655(0.2) = 0.0131$ m. For the transformed section shown in Fig. *a*,

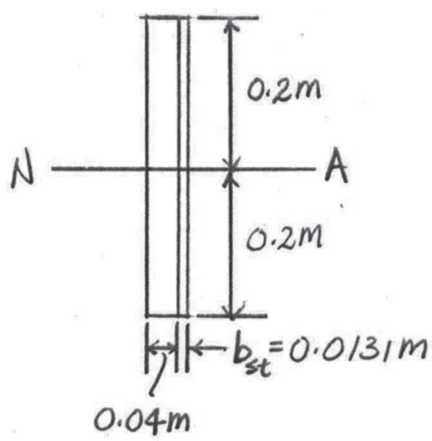
$$I_{NA} = \frac{1}{12}(0.04 + 0.0131)(0.4^3) = 0.2832\,(10^{-3})\ \text{m}^4$$

Maximum Bending Stress: Applying the flexure formula,

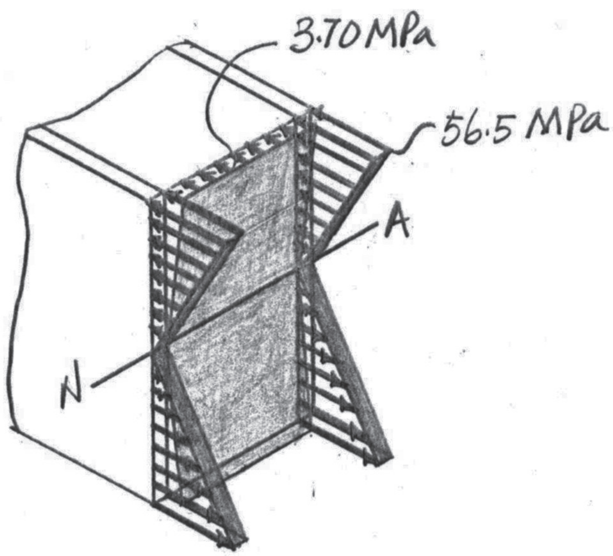
$$(\sigma_{st})_{max} = \frac{Mc}{I} = \frac{80(10^3)(0.2)}{0.2832(10^{-3})} = 56.497\,(10^6)\ \text{Pa} = 56.5\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\sigma_w)_{max} = n\frac{Mc}{I} = 0.0655\left[\frac{80(10^3)(0.2)}{0.2832(10^{-3})}\right] = 3.7006(10^6)\ \text{Pa} = 3.70\ \text{MPa} \quad \textbf{Ans.}$$

Using these results, the stress distribution on the beam's cross-section shown in Fig. *b* can be sketched.



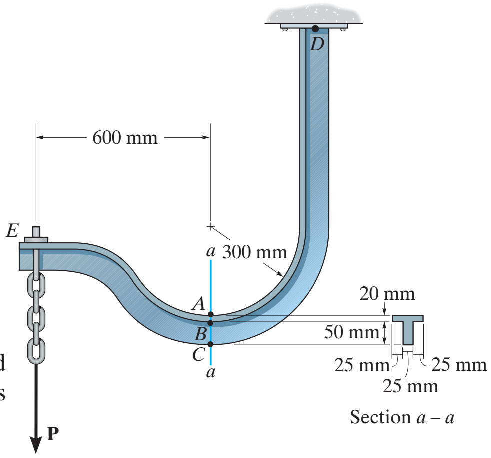
(a)



Ans:
$(\sigma_{st})_{max} = 56.5$ MPa,
$(\sigma_w)_{max} = 3.70$ MPa

6–130.

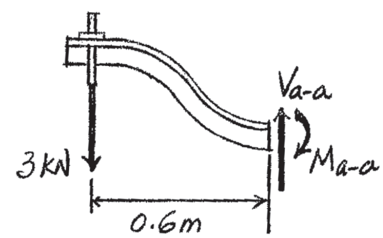
If $P = 3$ kN, determine the bending stress at points A, B, and C of the cross section at section a–a. Using these results, sketch the stress distribution on section a–a.



SOLUTION

Internal Moment: The internal moment developed at section a–a can be determined by writing the moment equation of equilibrium about the neutral axis of the cross section at a–a. Using the free-body diagram shown in Fig. a,

$$\circlearrowleft + \Sigma M_{NA=0;} \qquad 3(0.6) - M_{a-a} = 0 \qquad M_{a-a} = 1.8\ \text{kN} \cdot \text{m}$$



(a)

Here, $\mathbf{M}_{a-a}$ is considered negative since it tends to reduce the curvature of the curved segment of the beam.

Section Properties: Referring to Fig. b, the location of the centroid of the cross section from the center of the beam's curvature is

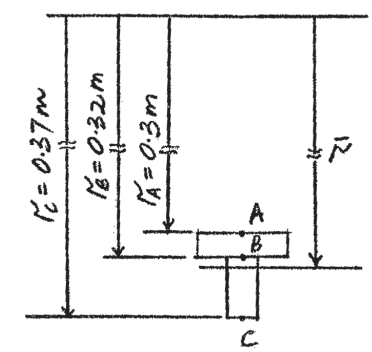
$$\bar{r} = \frac{\Sigma \bar{r}A}{\Sigma A} = \frac{0.31(0.02)(0.075) + 0.345(0.05)(0.025)}{0.02(0.075) + 0.05(0.025)} = 0.325909 \text{ m}$$

The location of the neutral surface from the center of the beam's curvature can be determined from

$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}}$$

where $A = 0.02(0.075) + 0.05(0.025) = 2.75(10^{-3})\ \text{m}^2$

$$\Sigma \int_A \frac{dA}{r} = 0.075 \ln \frac{0.32}{0.3} + 0.025 \ln \frac{0.37}{0.32} = 8.46994(10^{-3})\ \text{m}$$



(b)

Thus,

$$R = \frac{2.75(10^{-3})}{8.46994(10^{-3})} = 0.324678 \text{ m}$$

then

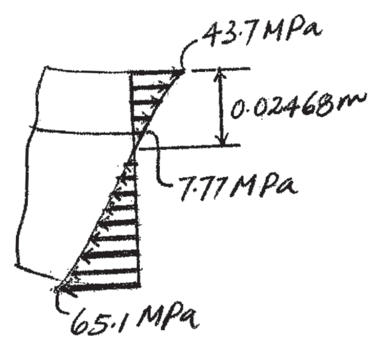
$$e = \bar{r} - R = 0.325909 - 0.324678 = 1.23144(10^{-3}) \text{ m}$$

Normal Stress:

$$\sigma_A = \frac{M(R - r_A)}{Aer_A} = \frac{1.8(10^3)(0.324678 - 0.3)}{2.75(10^{-3})(1.23144)(10^{-3})(0.3)} = 43.7 \text{ MPa (T)} \qquad \textbf{Ans.}$$

$$\sigma_B = \frac{M(R - r_B)}{Aer_B} = \frac{1.8(10^3)(0.324678 - 0.32)}{2.75(10^{-3})(1.23144)(10^{-3})(0.32)} = 7.77 \text{ MPa (T)} \qquad \textbf{Ans.}$$

$$\sigma_C = \frac{M(R - r_C)}{Aer_C} = \frac{1.8(10^3)(0.324678 - 0.37)}{2.75(10^{-3})(1.23144)(10^{-3})(0.37)} = -65.1 \text{ MPa (C)} \qquad \textbf{Ans.}$$

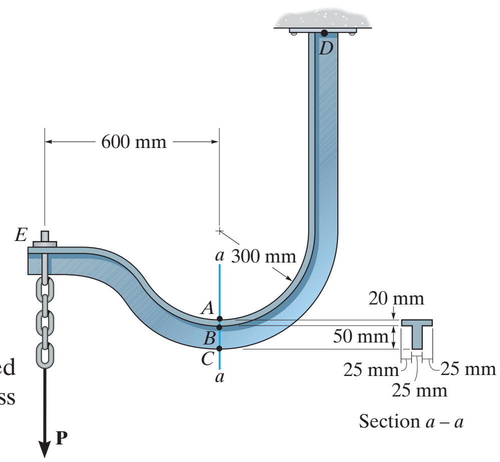


(c)

Ans:
$\sigma_A = 43.7$ MPa (T), $\sigma_B = 7.77$ MPa (T),
$\sigma_C = -65.1$ MPa (C)

6–131.

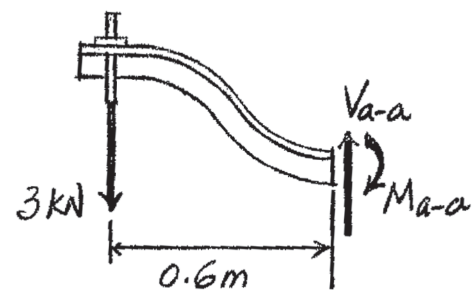
If the maximum bending stress at section a–a is not allowed to exceed $\sigma_{allow} = 150$ MPa, determine the maximum allowable force **P** that can be applied to the end E.



SOLUTION

Internal Moment: The internal Moment developed at section a–a can be determined by writing the moment equation of equilibrium about the neutral axis of the cross section at a–a.

$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad P(0.6) - M_{a-a} = 0 \qquad M_{a-a} = 0.6P$$



(a)

Here, $\mathbf{M}_{a-a}$ is considered positive since it tends to decrease the curvature of the curved segment of the beam.

Section Properties: Referring to Fig. b, the location of the centroid of the cross section from the center of the beam's curvature is

$$\bar{r} = \frac{\Sigma \tilde{r} A}{\Sigma A} = \frac{0.31(0.02)(0.075) + 0.345(0.05)(0.025)}{0.02(0.075) + 0.05(0.025)} = 0.325909 \text{ m}$$

The location of the neutral surface from the center of the beam's curvature can be determined from

$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}}$$

where $A = 0.02(0.075) + 0.05(0.025) = 2.75(10^{-3})\ \text{m}^2$

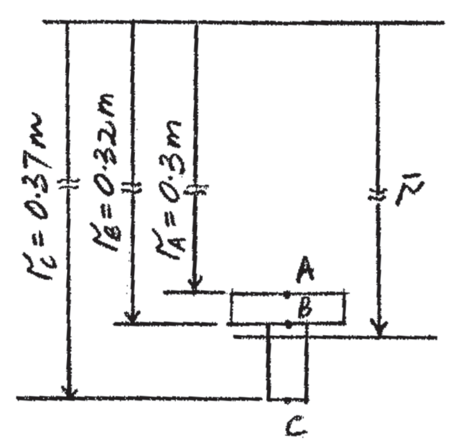
$$\Sigma \int_A \frac{dA}{r} = 0.075 \ln \frac{0.32}{0.3} + 0.025 \ln \frac{0.37}{0.32} = 8.46994(10^{-3}) \text{ m}$$

Thus,

$$R = \frac{2.75(10^{-3})}{8.46994(10^{-3})} = 0.324678 \text{ m}$$

then

$$e = \bar{r} - R = 0.325909 - 0.324678 = 1.23144(10^{-3}) \text{ m}$$



(b)

Allowable Normal Stress: The maximum normal stress occurs at either points A or C. For point A, which is in tension,

$$\sigma_{allow} = \frac{M(R - r_A)}{Aer_A}; \qquad 150(10^6) = \frac{0.6P(0.324678 - 0.3)}{2.75(10^{-3})(1.23144)(10^{-3})(0.3)}$$

$$P = 10\,292.09 \text{ N} = 10.3 \text{ kN}$$

For point C, which is in compression,

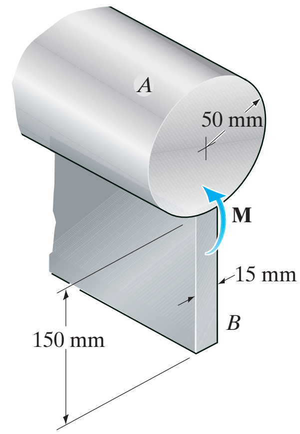
$$\sigma_{allow} = \frac{M(R - r_C)}{Aer_C}; \qquad -150(10^6) = \frac{0.6P(0.324678 - 0.37)}{2.75(10^{-3})(1.23144)(10^{-3})(0.37)}$$

$$P = 6911.55 \text{ N} = 6.91 \text{ kN (controls)}$$ **Ans.**

Ans:
$P = 6.91$ kN

***6–132.**

If the beam is subjected to a moment of $M = 45\ \text{kN}\cdot\text{m}$, determine the maximum bending stress in the A-36 steel section A and the 2014-T6 aluminum alloy section B.



SOLUTION

Section Properties: The cross section will be transformed into that of steel as shown in Fig. a. Here, $n = \dfrac{E_{al}}{E_{st}} = \dfrac{73.1(10^9)}{200(10^9)} = 0.3655$. Thus, $b_{st} = nb_{al} = 0.3655(0.015) = 0.0054825$ m. The location of the transformed section is

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{0.075(0.15)(0.0054825) + 0.2\left[\pi\left(0.05^2\right)\right]}{0.15(0.0054825) + \pi\left(0.05^2\right)}$$

$$= 0.1882 \text{ m}$$

The moment of inertia of the transformed section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2 = \frac{1}{12}(0.0054825)\left(0.15^3\right) + 0.0054825(0.15)(0.1882 - 0.075)^2$$

$$+ \frac{1}{4}\pi\left(0.05^4\right) + \pi\left(0.05^2\right)(0.2 - 0.1882)^2$$

$$= 18.08\left(10^{-6}\right) \text{ m}^4$$

Maximum Bending Stress: For the steel,

$$(\sigma_{max})_{st} = \frac{Mc_{st}}{I} = \frac{45\left(10^3\right)(0.06185)}{18.08\left(10^{-6}\right)} = 154 \text{ MPa} \qquad \textbf{Ans.}$$

For the aluminum alloy,

$$(\sigma_{max})_{al} = n\frac{Mc_{al}}{I} = 0.3655\left[\frac{45\left(10^3\right)(0.1882)}{18.08\left(10^{-6}\right)}\right] = 171 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$(\sigma_{max})_{st} = 154$ MPa, $(\sigma_{max})_{al} = 171$ MPa

6–133.

For the curved beam in Fig. 6–40*a*, show that when the radius of curvature approaches infinity, the curved-beam formula, Eq. 6–24, reduces to the flexure formula, Eq. 6–13.

SOLUTION

Normal Stress: Curved-beam formula

$$\sigma = \frac{M(R - r)}{Ar(\bar{r} - R)} \qquad \text{where } A' = \int_A \frac{dA}{r} \quad \text{and } R = \frac{A}{\int_A \frac{dA}{r}} = \frac{A}{A'}$$

$$\sigma = \frac{M(A - rA')}{Ar(\bar{r}A' - A)} \tag{1}$$

$$r = \bar{r} + y \tag{2}$$

$$\bar{r}A' = \bar{r}\int_A \frac{dA}{r} = \int_A \left(\frac{\bar{r}}{\bar{r} + y} - 1 + 1\right)dA$$

$$= \int_A \left(\frac{\bar{r} - \bar{r} - y}{\bar{r} + y} + 1\right) dA$$

$$= A - \int_A \frac{y}{\bar{r} + y}\, dA \tag{3}$$

Denominator of Eq. (1) becomes,

$$Ar(\bar{r}A' - A) = Ar\left(A - \int_A \frac{y}{\bar{r} + y}\, dA - A\right) = -Ar\int_A \frac{y}{\bar{r} + y}\, dA$$

Using Eq. (2),

$$Ar(\bar{r}A' - A) = -A\int_A \left(\frac{\bar{r}y}{\bar{r} + y} + y - y\right)dA - Ay\int_A \frac{y}{\bar{r} + y}\, dA$$

$$= A\int_A \frac{y^2}{\bar{r} + y}\, dA - A\int_A y\, dA - Ay\int_A \frac{y}{\bar{r} + y}\, dA$$

$$= \frac{A}{\bar{r}}\int_A \left(\frac{y^2}{1 + \frac{y}{\bar{r}}}\right)dA - A\int_A y\, dA - \frac{Ay}{\bar{r}}\int_A \left(\frac{y}{1 + \frac{y}{\bar{r}}}\right) dA$$

But, $\qquad \int_A y\, dA = 0, \qquad \text{as } \frac{y}{\bar{r}} \to 0$

Then, $\qquad Ar(\bar{r}A' - A) \to \frac{A}{\bar{r}}I$

Eq. (1) becomes $\qquad \sigma = \frac{M\bar{r}}{AI}(A - rA')$

Using Eq. (2), $\qquad \sigma = \frac{M\bar{r}}{AI}(A - \bar{r}A' - yA')$

Using Eq. (3),

$$\sigma = \frac{M\bar{r}}{AI}\left[A - \left(A - \int_A \frac{y}{\bar{r} + y}\, dA\right) - y\int_A \frac{dA}{\bar{r} + y}\right]$$

$$= \frac{M\bar{r}}{AI}\left[\int_A \frac{y}{\bar{r} + y}\, dA - y\int_A \frac{dA}{\bar{r} + y}\right]$$

6–133. (Continued)

$$= \frac{M\bar{r}}{AI}\left[\int_A \left(\frac{\frac{y}{\bar{r}}}{1+\frac{y}{\bar{r}}}\right)dA - \frac{y}{\bar{r}}\int_A \left(\frac{dA}{1+\frac{y}{\bar{r}}}\right)\right]$$

As $\dfrac{y}{\bar{r}} \rightarrow 0$

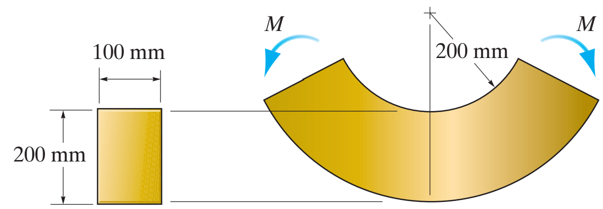
$$\int_A \left(\frac{\frac{y}{\bar{r}}}{1+\frac{y}{\bar{r}}}\right) dA = 0 \quad \text{and} \quad \frac{y}{\bar{r}}\int_A \left(\frac{dA}{1+\frac{y}{\bar{r}}}\right) = \frac{y}{\bar{r}}\int_A dA = \frac{yA}{\bar{r}}$$

Therefore, $\sigma = \dfrac{M\bar{r}}{AI}\left(-\dfrac{yA}{\bar{r}}\right) = -\dfrac{My}{I}$ **(Q.E.D)**

Ans:
N/A

6–134.

The curved member is subjected to the moment of $M = 50\ \text{kN}\cdot\text{m}$. Determine the percentage error introduced in the calculation of maximum bending stress using the flexure formula for straight members.



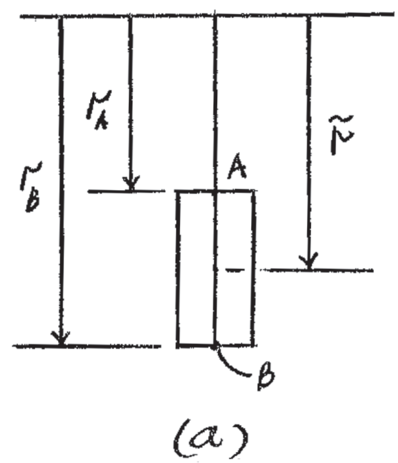
SOLUTION

Straight Member: The maximum bending stress developed in the straight member

$$\sigma_{\max} = \frac{Mc}{I} = \frac{50(10^3)(0.1)}{\frac{1}{12}(0.1)(0.2^3)} = 75\ \text{MPa}$$

Curved Member: When $r = 0.2$ m, $\bar{r} = 0.3$ m, $r_A = 0.2$ m and $r_B = 0.4$ m, Fig. *a*. The location of the neutral surface from the center of curvature of the curve member is

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.1(0.2)}{0.1\ln\frac{0.4}{0.2}} = 0.288539\ \text{m}$$



Then

$$e = \bar{r} - R = 0.011461\ \text{m}$$

Here, $M = 50\ \text{kN}\cdot\text{m}$. Since it tends to decrease the curvature of the curved member,

$$\sigma_B = \frac{M(R - r_B)}{Aer_B} = \frac{50(10^3)(0.288539 - 0.4)}{0.1(0.2)(0.011461)(0.4)}$$

$$= -60.78\ \text{MPa} = 60.78\ \text{MPa (C)}$$

$$\sigma_A = \frac{M(R - r_A)}{Aer_A} = \frac{50(10^3)(0.288539 - 0.2)}{0.1(0.2)(0.011461)(0.2)}$$

$$= 96.57\ \text{MPa (T) (Max.)}$$

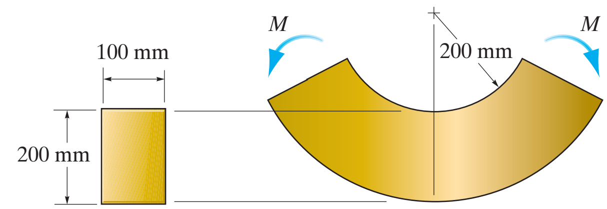
Thus,

$$\%\ \text{of error} = \left(\frac{96.57 - 75}{96.57}\right)100 = 22.3\%$$ **Ans.**

Ans:
% of error $= 22.3\%$

6–135.

The curved member is made from material having an allowable bending stress of $\sigma_{\text{allow}} = 100$ MPa. Determine the maximum allowable moment M that can be applied to the member.



SOLUTION

Internal Moment: M is negative since it tends to decrease the curvature of the curved member.

Section Properties: Referring to Fig. *a*, the location of the neutral surface from the center of curvature of the curve beam is

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.1(0.2)}{0.1\ln\frac{0.4}{0.2}} = 0.288539 \text{ m}$$

Then

$$e = \bar{r} - R = 0.3 - 0.288539 = 0.011461 \text{ m}$$

Allowable Bending Stress: The maximum stress occurs at either point A or B. For point A, which is in tension,

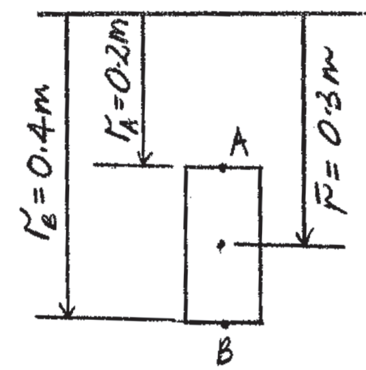
$$\sigma_{\text{allow}} = \frac{M(R - r_A)}{Aer_A}; \qquad 100(10^6) = \frac{M(0.288539 - 0.2)}{0.1(0.2)(0.011461)(0.2)}$$

$$M = 51\,778.27 \text{ N}\cdot\text{m} = 51.8 \text{ kN}\cdot\text{m (controls)} \qquad \textbf{Ans.}$$

For point B, which is in compression,

$$\sigma_{\text{allow}} = \frac{M(R - r_B)}{Aer_B}; \qquad -100(10^6) = \frac{M(0.288539 - 0.4)}{0.1(0.2)(0.011461)(0.4)}$$

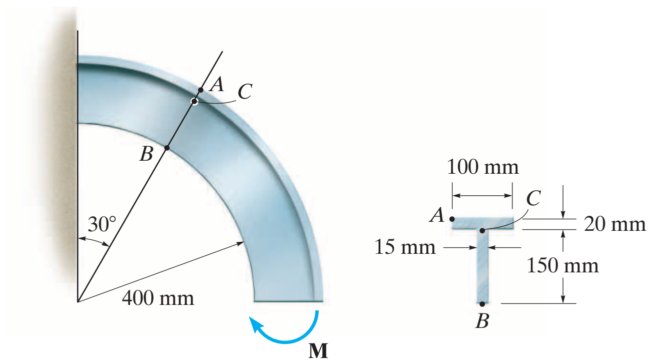
$$M = 82\,260.10 \text{ N}\cdot\text{m} = 82.3 \text{ kN}\cdot\text{m}$$



(a)

Ans:
$M = 51.8 \text{ kN}\cdot\text{m}$

***6–136.** The curved beam is subjected to a bending moment of $M = 900\ \text{N}\cdot\text{m}$ as shown. Determine the stress at points A and B, and show the stress on a volume element located at each of these points.



Internal Moment: $M = -900\ \text{N}\cdot\text{m}$ is negative since it tends to decrease the beam's radius curvature.

Section Properties:

$$\Sigma A = 0.15(0.015) + 0.1(0.02) = 0.00425\ \text{m}^2$$

$$\Sigma \bar{r}A = 0.475(0.15)(0.015) + 0.56(0.1)(0.02) = 2.18875(10^{-3})\ \text{m}^3$$

$$\bar{r} = \frac{\Sigma \bar{r}A}{\Sigma A} = \frac{2.18875\ (10^{-3})}{0.00425} = 0.5150\ \text{m}$$

$$\Sigma \int_A \frac{dA}{r} = 0.015 \ln \frac{0.55}{0.4} + 0.1 \ln \frac{0.57}{0.55} = 8.348614(10^{-3})\ \text{m}$$

$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}} = \frac{0.00425}{8.348614(10^{-3})} = 0.509067\ \text{m}$$

$$\bar{r} - R = 0.515 - 0.509067 = 5.933479(10^{-3})\ \text{m}$$

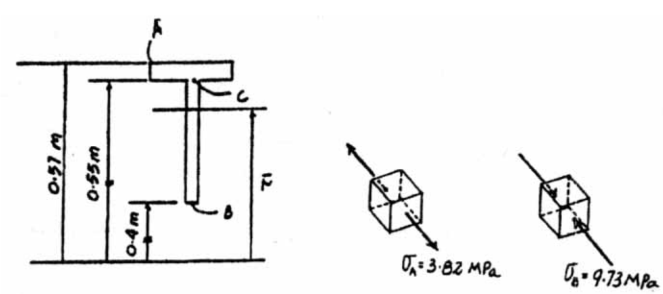
Normal Stress: Applying the curved-beam formula

$$\sigma_A = \frac{M(R - r_A)}{Ar_A\,(\bar{r} - R)} = \frac{-900(0.509067 - 0.57)}{0.00425(0.57)(5.933479)(10^{-3})}$$

$$= 3.82\ \text{MPa (T)} \qquad \textbf{Ans.}$$

$$\sigma_B = \frac{M(R - r_B)}{Ar_B\,(\bar{r} - R)} = \frac{-900(0.509067 - 0.4)}{0.00425(0.4)(5.933479)(10^{-3})}$$

$$= -9.73\ \text{MPa} = 9.73\ \text{MPa (C)} \qquad \textbf{Ans.}$$

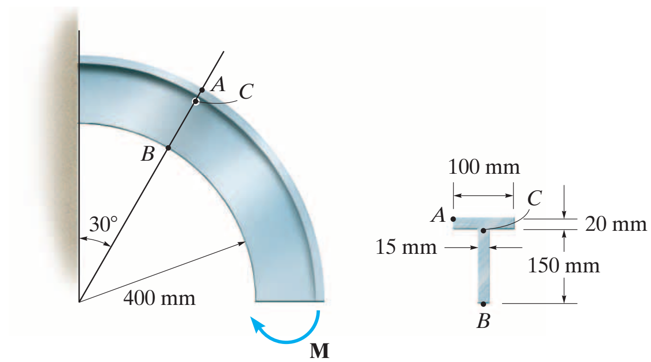


Ans:

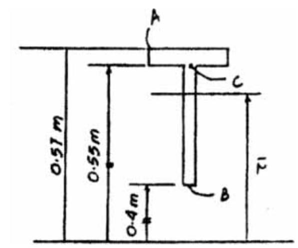
σ_A = 1,60 KN_ &R'*

σ_B = 9.73 MPa (C)

6–137. The curved beam is subjected to a bending moment of $M = 900\ \text{N}\cdot\text{m}$. Determine the stress at point C.



Internal Moment: $M = -900\ \text{N}\cdot\text{m}$ is negative since it tends to decrease the beam's radius of curvature.



Section Properties:

$$\Sigma A = 0.15(0.015) + 0.1(0.02) = 0.00425\ \text{m}^2$$

$$\Sigma \bar{r} A = 0.475(0.15)(0.015) + 0.56(0.1)(0.02) = 2.18875(10^{-3})\ \text{m}$$

$$\bar{r} = \frac{\Sigma \bar{r} A}{\Sigma A} = \frac{2.18875\ (10^{-3})}{0.00425} = 0.5150\ \text{m}$$

$$\Sigma \int_A \frac{dA}{r} = 0.015 \ln \frac{0.55}{0.4} + 0.1 \ln \frac{0.57}{0.55} = 8.348614(10^{-3})\ \text{m}$$

$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}} = \frac{0.00425}{8.348614(10^{-3})} = 0.509067\ \text{m}$$

$$\bar{r} - R = 0.515 - 0.509067 = 5.933479(10^{-3})\ \text{m}$$

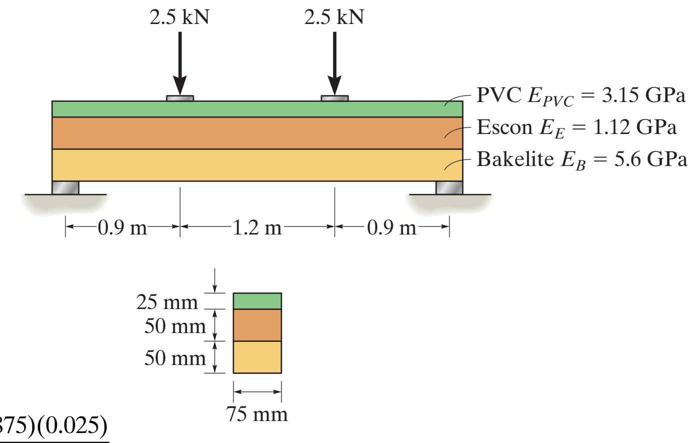
Normal Stress: Applying the curved-beam formula

$$\sigma_C = \frac{M(R - r_C)}{A r_C(\bar{r} - R)} = \frac{-900(0.509067 - 0.55)}{0.00425(0.55)(5.933479)(10^{-3})}$$

$$= 2.66\ \text{MPa (T)}$$ **Ans.**

Ans:
$\Sigma A = 0.00425\ \text{m}^2$, $\bar{r} = 0.5150$ m,
$\Sigma \int_A \frac{dA}{r} = 8.348614(10^{-3})$ m,
$\sigma_C = 2.66$ MPa (T)
$(\sigma_{\max})_{\text{pvc}} = 12.3$ MPa

6–138. The beam is made from three types of plastic that are identified and have the moduli of elasticity shown in the figure. Determine the maximum bending stress in the PVC.

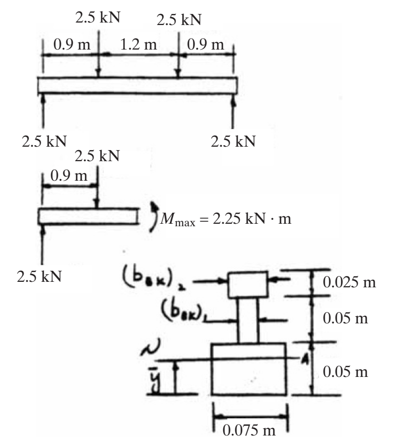


$$(b_{bk})_1 = n_1 b_{Es} = \left(\frac{1.12}{5.6}\right)(0.075) = 0.015 \text{ m}$$

$$(b_{bk})_2 = n_2 b_{pvc} = \left(\frac{3.15}{5.6}\right)(0.075) = 0.0421875 \text{ m}$$

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{0.025(0.075)(0.05) + 0.075(0.015)(0.05) + 0.1125(0.0421875)(0.025)}{0.075(0.05) + 0.015(0.05) + 0.0421875(0.025)}$$

$$= 0.048365 \text{ m}$$



$$I = 2\left[\frac{1}{12}(0.075)(0.05^3) + 0.075(0.05)(0.048365 - 0.025)^2\right]$$

$$+ \frac{1}{12}(0.015)(0.05^3) + 0.015(0.05)(0.075 - 0.048365)^2$$

$$+ \frac{1}{12}(0.0421875)(0.025^3) + 0.0421875(0.025)(0.1125 - 0.048365)^2$$

$$= 7.90996(10^{-6}) \text{ m}^4$$

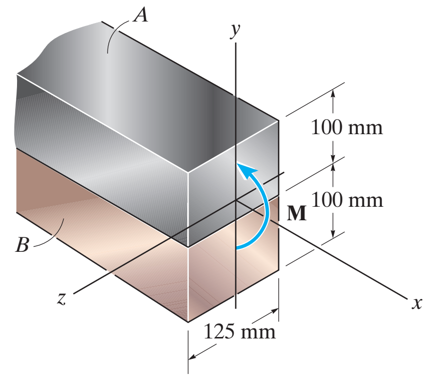
$$(\sigma_{\max})_{pvc} = n_2 \frac{Mc}{I} = \left(\frac{3.15}{5.6}\right)\left\{\frac{[2.25(10^3)](0.1125 - 0.048365)}{7.90996(10^{-6})}\right\}$$

$$= 12.26(10^6) \text{ N/m}^2 = 12.3 \text{ MPa}$$ **Ans.**

Ans:
$(\sigma_{\max})_{\text{pvc}} = 12.3$ MPa

6–139.

The composite beam is made of A-36 steel (A) bonded to C83400 red brass (B) and has the cross section shown. If it is subjected to a moment of $M = 6.5\ \text{kN}\cdot\text{m}$, determine the maximum stress in the brass and steel. Also, what is the stress in each material at the seam where they are bonded together?

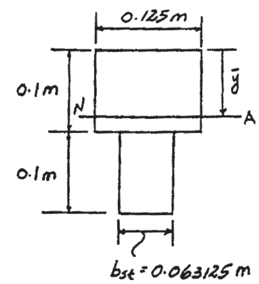


SOLUTION

Section Properties: For the transformed section.

$$n = \frac{E_{br}}{E_{st}} = \frac{101(10^9)}{200(10^9)} = 0.505$$

$$b_{st} = nb_{br} = 0.505(0.125) = 0.063125\ \text{m}$$



$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A}$$

$$= \frac{0.05(0.125)(0.1) + 0.15(0.1)(0.063125)}{0.125(0.1) + 0.1(0.063125)}$$

$$= 0.08355\ \text{m}$$

$$I_{NA} = \frac{1}{12}(0.125)(0.1^3) + 0.125(0.1)(0.08355 - 0.05)^2$$

$$+ \frac{1}{12}(0.063125)(0.1^3) + 0.063125(0.1)(0.15 - 0.08355)^2$$

$$= 57.62060(10^{-6})\ \text{m}^4$$

Maximum Bending Stress: Applying the flexure formula

$$(\sigma_{max})_{st} = \frac{My}{I} = \frac{6.5(10^3)(0.08355)}{57.62060(10^{-6})}$$

$$= 9.42\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\sigma_{max})_{br} = n\frac{Mc}{I} = 0.505\left[\frac{6.5(10^3)(0.2 - 0.08355)}{57.62060(10^{-6})}\right]$$

$$= 6.63\ \text{MPa} \qquad \textbf{Ans.}$$

Bending Stress: At seam

$$\sigma_{st} = \frac{My}{I}$$

$$= \frac{6.5(10^3)(0.1 - 0.08355)}{57.62060(10^{-6})}$$

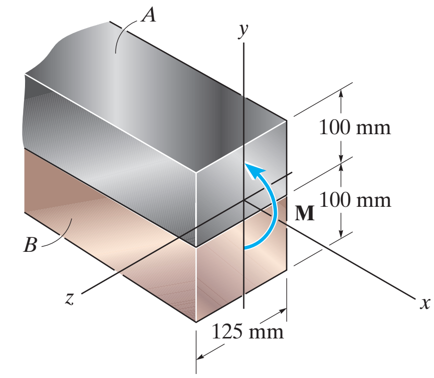
$$= 1.855\ \text{MPa} = 1.86\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_{br} = n\frac{My}{I} = 0.505(1.855) = 0.937\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$(\sigma_{max})_{st} = 9.42$ MPa,
$(\sigma_{max})_{br} = 6.63$ MPa,
$\sigma_{st} = 1.86$ MPa,
$\sigma_{br} = 0.937$ MPa

***6–140.**

The composite beam is made of A-36 steel (A) bonded to C83400 red brass (B) and has the cross section shown. If the allowable bending stress for the steel is $(\sigma_{\text{allow}})_{\text{st}} = 180$ MPa and for the brass $(\sigma_{\text{allow}})_{\text{br}} = 60$ MPa, determine the maximum moment M that can be applied to the beam.

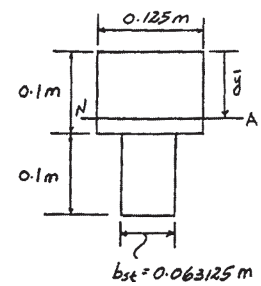


SOLUTION

Section Properties: For the transformed section.

$$n = \frac{E_{\text{br}}}{E_{\text{st}}} = \frac{101(10^9)}{200(10^9)} = 0.505$$

$$b_{\text{st}} = nb_{\text{br}} = 0.505(0.125) = 0.063125 \text{ m}$$



$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A}$$

$$= \frac{0.05(0.125)(0.1) + 0.15(0.1)(0.063125)}{0.125(0.1) + 0.1(0.063125)}$$

$$= 0.08355 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.125)(0.1^3) + 0.125(0.1)(0.08355 - 0.05)^2$$

$$+ \frac{1}{12}(0.063125)(0.1^3) + 0.063125(0.1)(0.15 - 0.08355)^2$$

$$= 57.62060(10^{-6}) \text{ m}^4$$

Allowable Bending Stress: Applying the flexure formula

Assume failure of steel

$$(\sigma_{\max})_{\text{st}} = (\sigma_{\text{allow}})_{\text{st}} = \frac{My}{I}$$

$$180(10^6) = \frac{M(0.08355)}{57.62060(10^{-6})}$$

$$M = 124130 \text{ N}\cdot\text{m} = 124 \text{ kN}\cdot\text{m}$$

Assume failure of brass

$$(\sigma_{\max})_{\text{br}} = (\sigma_{\text{allow}})_{\text{br}} = n\frac{Mc}{I}$$

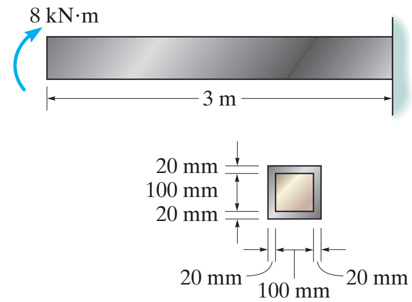
$$60(10^6) = 0.505\left[\frac{M(0.2 - 0.08355)}{57.62060(10^{-6})}\right]$$

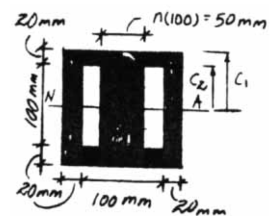
$$M = 58792 \text{ N}\cdot\text{m}$$

$$= 58.8 \text{ kN}\cdot\text{m} \quad \textbf{\textit{(Controls!)}}$$ **Ans.**

Ans:
$M = 58.8 \text{ kN}\cdot\text{m}$

6–141. The member has a brass core bonded to a steel casing. If a couple moment of 8 kN # m is applied at its end, determine the maximum bending stress in the member. $E_{br} = 100$ GPa, $E_{st} = 200$ GPa.





$$n = \frac{E_{br}}{E_{st}} = \frac{100}{200} = 0.5$$

$$I = \frac{1}{12}(0.14)(0.14)^3 - \frac{1}{12}(0.05)(0.1)^3 = 27.84667(10^{-6})\text{m}^4$$

Maximum stress in steel:

$$(\sigma_{st})_{\max} = \frac{Mc_1}{I} = \frac{8(10^3)(0.07)}{27.84667(10^{-6})} = 20.1 \text{ MPa} \qquad \text{(max)}$$ **Ans.**

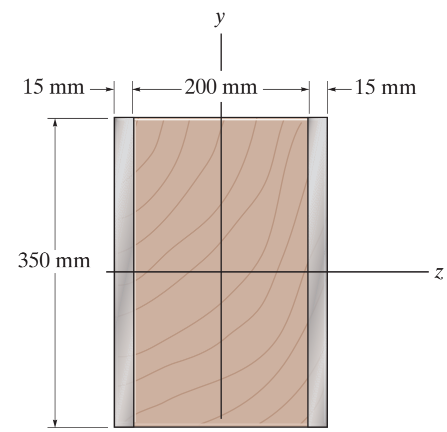
Maximum stress in brass:

$$(\sigma_{br})_{\max} = \frac{nMc_2}{I} = \frac{0.5(8)(10^3)(0.05)}{27.84667(10^{-6})} = 7.18 \text{ MPa}$$

Ans:
$(\sigma_{st})_{\max} = 20.1$ MPa

6–142.

The Douglas Fir beam is reinforced with A-36 steel straps at its sides. Determine the maximum stress in the wood and steel if the beam is subjected to a bending moment of $M_z = 4\ \text{kN}\cdot\text{m}$. Sketch the stress distribution acting over the cross section.



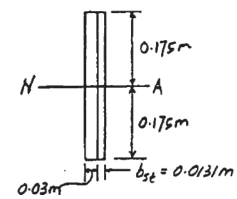
SOLUTION

Section Properties: For the transformed section,

$$n = \frac{E_w}{E_{st}} = \frac{13.1(10^9)}{200(10^9)} = 0.0655$$

$$b_{st} = nb_w = 0.0655(0.2) = 0.0131\ \text{m}$$

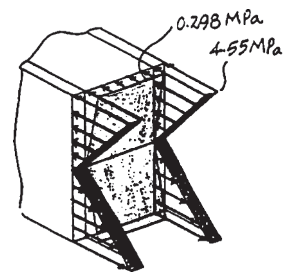
$$I_{NA} = \frac{1}{12}(0.03 + 0.0131)(0.35^3) = 153.99(10^{-6})\ \text{m}^4$$



Maximum Bending Stress: Applying the flexure formula

$$(\sigma_{max})_{st} = \frac{Mc}{I} = \frac{4(10^3)(0.175)}{153.99(10^{-6})} = 4.55\ \text{MPa}$$ **Ans.**

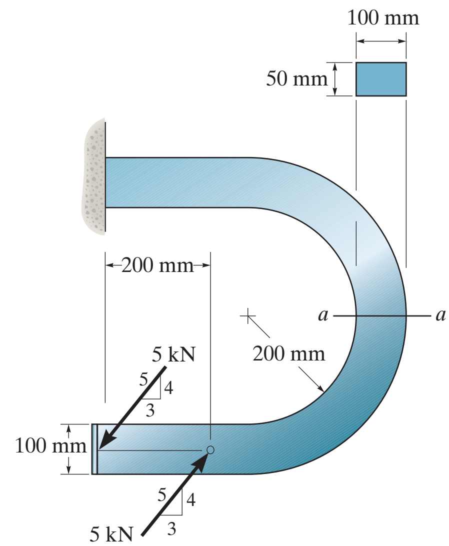
$$(\sigma_{max})_w = n\frac{Mc}{I} = 0.0655\left[\frac{4(10^3)(0.175)}{153.99(10^{-6})}\right] = 0.298\ \text{MPa}$$ **Ans.**



Ans:
$(\sigma_{max})_{st} = 4.55\ \text{MPa},$
$(\sigma_{max})_w = 0.298\ \text{MPa}$

6–143.

The curved bar used on a machine has a rectangular cross section. If the bar is subjected to a couple as shown, determine the maximum tensile and compressive stress acting at section *a–a*. Sketch the stress distribution on the section in three dimensions.



SOLUTION

Internal Moment: Referring to the FBD of the lower segment of the curve beam sectioned through *a–a*, Fig. *a*,

$$\circlearrowleft + \Sigma M_o = 0; \qquad 5\left(\frac{4}{5}\right)(0.2) - M = 0 \qquad M = 0.8\ \text{kN}\cdot\text{m} = 800\ \text{N}\cdot\text{m}$$

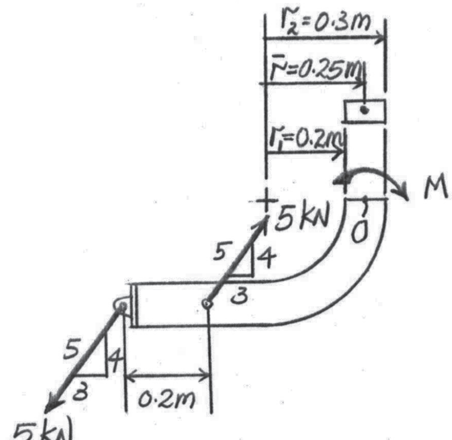
Section Properties: Referring to Fig. *a*,

$$\bar{r} = \frac{0.2 + 0.3}{2} = 0.25\ \text{m} \qquad A = (0.05)(0.1) = 0.005\ \text{m}^2$$

$$\int_A \frac{dA}{r} = b \ln \frac{r_2}{r_1} = 0.05 \ln\left(\frac{0.3}{0.2}\right) = 0.020273255\ \text{m}$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.005}{0.020273255} = 0.2466303\ \text{m}$$

$$\bar{r} - R = 0.25 - 0.2466303 = 0.003369654\ \text{m}$$



(a)

Normal Stress: Here $M = 800\ \text{N}\cdot\text{m}$ is positive, since it tends to increase the beam's radius of curvature. Applying the curved beam formula,

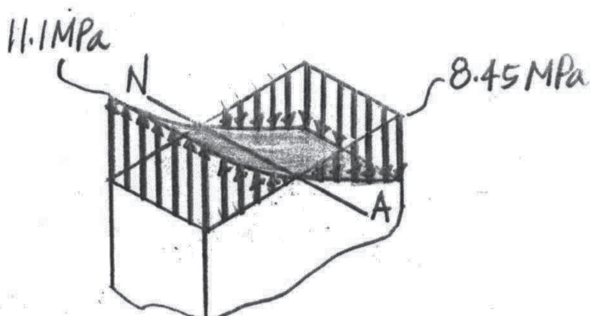
$$(\sigma_T)_{\max} = \frac{M(R - r_1)}{Ar_1(\bar{r} - R)}$$

$$= \frac{800(0.2466303 - 0.2)}{0.005(0.2)(0.003369654)} = 11.07\left(10^6\right)\ \text{Pa} = 11.1\ \text{MPa (T)} \qquad \textbf{Ans.}$$

$$(\sigma_C)_{\max} = \frac{M(R - r_2)}{Ar_2(\bar{r} - R)}$$

$$= \frac{800(0.2466303 - 0.3)}{0.005(0.3)(0.003369654)} = -8.447\left(10^6\right)\ \text{Pa} = 8.45\ \text{MPa (C)} \qquad \textbf{Ans.}$$

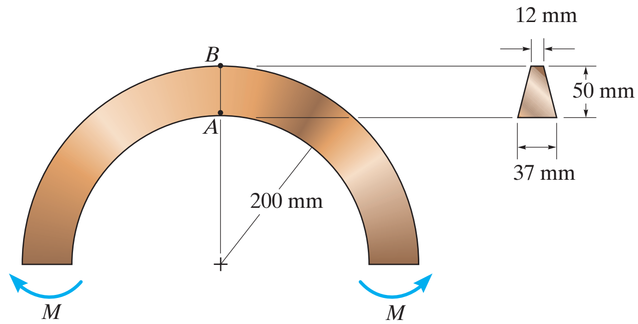
Using these results, the normal stress distribution on the beam's cross-section shown in Fig. *b* can be sketched.



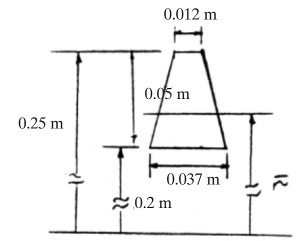
(b)

Ans:
$(\sigma_T)_{\max} = 11.1\ \text{MPa (T)}$,
$(\sigma_C)_{\max} = 8.45\ \text{MPa (C)}$

***6–144.** The curved member is symmetric and is subjected to a moment of $M = 900\ \text{N}\cdot\text{m}$. Determine the bending stress in the member at points A and B. Show the stress acting on volume elements located at these points.



$$A = 0.012(0.05) + \frac{1}{2}(0.025)(0.05) = 0.001225\ \text{m}^2$$



$$\bar{r} = \frac{\Sigma \bar{r} A}{\Sigma A} = \frac{0.225(0.012)(0.05) + 0.21667\left[\frac{1}{2}(0.025)(0.05)\right]}{0.001225} = 0.220748\ \text{m}$$

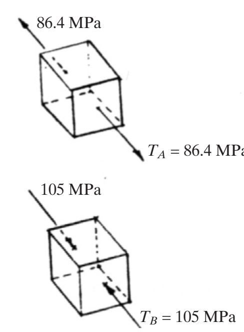
$$\int_A \frac{dA}{r} = 0.012 \ln\left(\frac{0.25}{0.2}\right) + \left[\frac{0.025(0.25)}{0.25-0.2}\right]\ln\left(\frac{0.25}{0.2}\right) - 0.025 = 0.005570667\ \text{m}$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.001225}{0.005570667} = 0.2199018$$

$$\bar{r} - R = 0.220748 - 0.2199018 = 0.846427(10^{-3})\ \text{m}$$

$$\sigma = \frac{M(R-r)}{Ar(\bar{r}-R)}$$

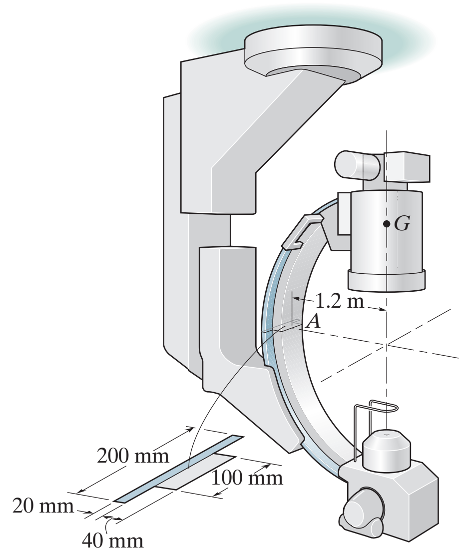
$$\sigma_A = \frac{900(0.2199018 - 0.2)}{0.001225(0.2)[0.846427(10^{-3})]} = 86.37(10^6)\ \text{N/m}^2 = 86.4\ \text{MPa (T)}$$ **Ans.**



$$\sigma_B = \frac{900(0.2199018 - 0.25)}{0.001225(0.25)[0.846427(10^{-3})]} = -104.50(10^6)\ \text{N/m}^2 = 105\ \text{MPa (C)}$$ **Ans.**

Ans:
$\sigma_A = 86.4$ MPa (T), $\sigma_B = 105$ MPa (C)

6–145. The ceiling-suspended C-arm is used to support the X-ray camera used in medical diagnoses. If the camera has a mass of 150 kg, with center of mass at G, determine the maximum bending stress at section A.



Section Properties:

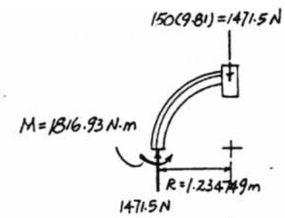
$$\bar{r} = \frac{\Sigma \bar{r} A}{\Sigma A} = \frac{1.22(0.1)(0.04) + 1.25(0.2)(0.02)}{0.1(0.04) + 0.2(0.02)} = 1.235 \text{ m}$$

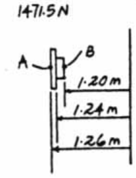
$$\Sigma \int_A \frac{dA}{r} = 0.1 \ln \frac{1.24}{1.20} + 0.2 \ln \frac{1.26}{1.24} = 6.479051\left(10^{-3}\right)\text{m}$$

$$A = 0.1(0.04) + 0.2(0.02) = 0.008 \text{ m}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.008}{6.479051\ (10^{-3})} = 1.234749 \text{ m}$$

$$\bar{r} - R = 1.235 - 1.234749 = 0.251183\left(10^{-3}\right) \text{ m}$$





Internal Moment: The internal moment must be computed about the neutral axis as shown on FBD. $M = -1816.93 \text{ N} \cdot \text{m}$ is negative since it tends to decrease the beam's radius of curvature.

Maximum Normal Stress: Applying the curved-beam formula

$$\sigma_A = \frac{M(R - r_A)}{A r_A (\bar{r} - R)}$$

$$= \frac{-1816.93(1.234749 - 1.26)}{0.008(1.26)(0.251183)(10^{-3})}$$

$$= 18.1 \text{ MPa (T)}$$

$$\sigma_B = \frac{M(R - r_B)}{A r_B (\bar{r} - R)}$$

$$= \frac{-1816.93(1.234749 - 1.20)}{0.008(1.20)(0.251183)(10^{-3})}$$

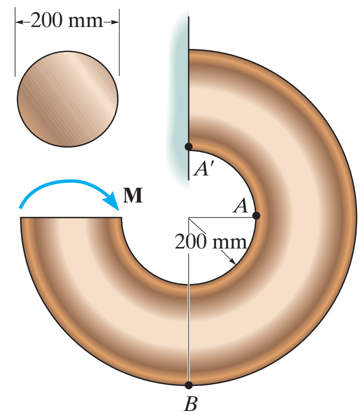
$$= -26.2 \text{ MPa} = 26.2 \text{ MPa (C)} \qquad \textbf{(Max)} \qquad \textbf{Ans.}$$

Ans:

$\bar{r} = 1.235$ m, $\Sigma \int_A \frac{dA}{r} = 6.479051(10^{-3})$ m,
$A = 0.008 \text{ m}^2, \sigma_B = 26.2$ MPa (C)

6–146

The member has a circular cross section. If it is subjected to a moment of $M = 5\ \text{kN}\cdot\text{m}$, determine the stress at points A and B. Is the stress at point A', which is located on the member near the wall, the same as that at A? Explain.



SOLUTION

Section Properties: Referring to Fig. *a*

$$\bar{r} = \frac{0.2 + 0.4}{2} = 0.3\text{ m} \qquad A = \pi C^2 = \pi\left(0.1^2\right) = 0.01\pi\text{ m}^2$$

$$\int_A \frac{dA}{r} = 2\pi\left(r - \sqrt{\bar{r}^2 - C^2}\right)$$

$$= 2\pi\left(0.3 - \sqrt{0.3^2 - 0.1^2}\right) = 0.1078024\text{ m}$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.012r}{0.1078024} = 0.291421\text{ m}$$

$$\bar{r} - R = 0.3 - 0.291421 = 0.00857864\text{ m}$$

Bending Stress: Here, $M = 5\ \text{kN}\cdot\text{m}$ is negative since it tends to decrease the beam's radius of curvature. Applying the curved beam formula,

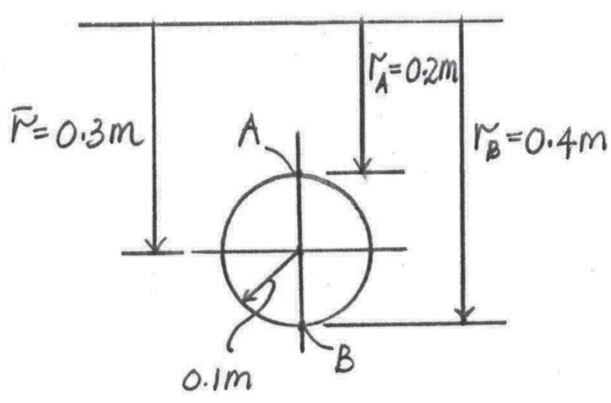
$$\sigma_A = \frac{M(R - r_A)}{Ar_A(\bar{r} - R)} = \frac{-5\left(10^3\right)(0.291421 - 0.2)}{(0.01\pi)(0.2)(0.00857864)} = -8.4805\left(10^6\right)\text{ Pa} = 8.48\text{ MPa (C)}$$

Ans.

$$\sigma_B = \frac{M(R - r_B)}{Ar_B(\bar{r} - R)} = \frac{-5\left(10^3\right)(0.291421 - 0.4)}{(0.01\pi)(0.4)(0.00857864)} = 5.0360\left(10^6\right)\text{ Pa} = 5.04\text{ MPa (T)}$$

Ans.

No! Point A' is at the fixed support where stress concentration occurs. The application of the curved beam formula is only valid for sections sufficiently removed from the support in accordance to Saint-Venant's principle.

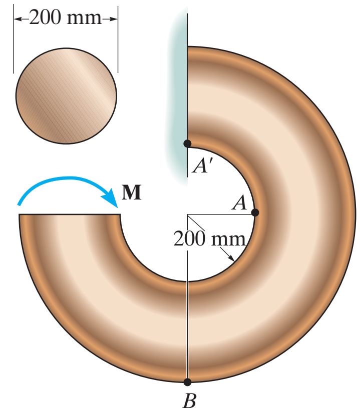


(a)

Ans:
$\sigma_A = 8.48$ MPa (C),
$\sigma_B = 5.04$ MPa (T)
No, it is not the same.

6–147.

The member has a circular cross section. If the allowable bending stress is $\sigma_{allow} = 100$ MPa, determine the maximum moment M that can be applied to the member.



SOLUTION

Section Properties: Referring to Fig. *a*,

$$\bar{r} = \frac{0.2 + 0.4}{2} = 0.3 \text{ m} \qquad A = \pi C^2 = \pi\left(0.1^2\right) = 0.01\pi \text{ m}^2$$

$$\int_A \frac{dA}{r} = 2\pi\left(r - \sqrt{\bar{r}^2 - C^2}\right)$$

$$= 2\pi\left(0.3 - \sqrt{0.3^2 - 0.1^2}\right) = 0.1078024 \text{ m}$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.01\pi}{0.1078024} = 0.291421 \text{ m}$$

$$\bar{r} - R = 0.3 - 0.291421 = 0.00857864 \text{ m}$$

Bending Stress: Here, M is negative since it tends to decrease the beam's radius of curvature. Applying the curve beam's formula by assuming tension failure,

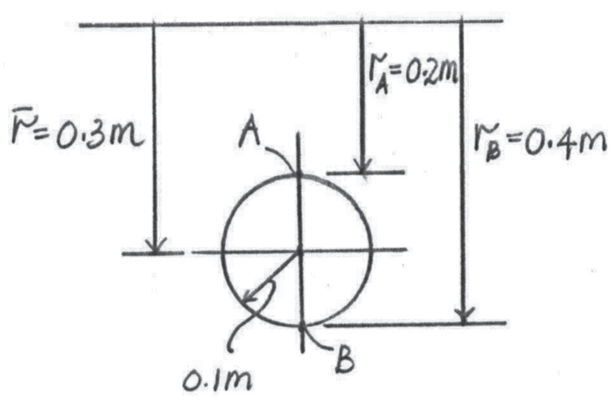
$$\sigma_B = \sigma_{allow} = \frac{M(R - r_B)}{Ar_B(\bar{r} - R)}; \quad 100\left(10^6\right) = \frac{-M(0.291421 - 0.4)}{(0.01\pi)(0.4)(0.00857864)}$$

$$M = 99.29\left(10^3\right) \text{ N}\cdot\text{m} = 99.3 \text{ kN}\cdot\text{m}$$

Assuming compression failure,

$$\sigma_A = \sigma_{allow} = \frac{M(R - r_A)}{Ar_A(\bar{r} - R)}; \quad -100\left(10^6\right) = \frac{-M(0.291421 - 0.2)}{(0.01\pi)(0.2)(0.00857864)}$$

$$M = 58.96\left(10^3\right) \text{ N}\cdot\text{m} = 59.0 \text{ kN}\cdot\text{m (control!)} \quad \textbf{Ans.}$$

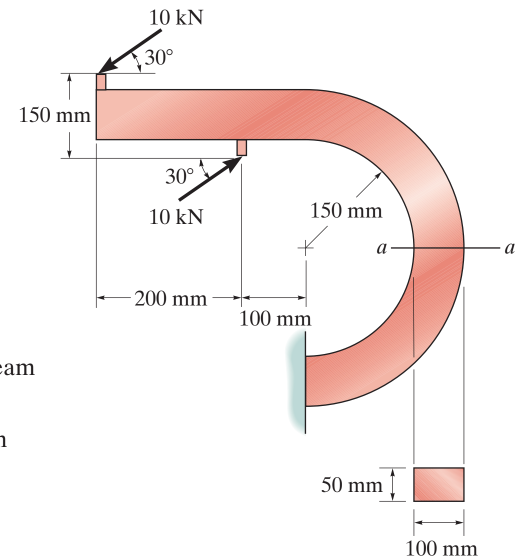


(a)

Ans:
$M = 59.0 \text{ kN}\cdot\text{m}$

***6–148.**

The curved bar used on a machine has a rectangular cross section. If the bar is subjected to a couple as shown, determine the maximum tensile and compressive stresses acting at section *a–a*. Sketch the stress distribution on the section in three dimensions.



SOLUTION

Internal Moment: Referring to the FBD of the upper segment of the curved beam sectioned through *a–a*, Fig. *a*

$\circlearrowleft + \Sigma M_0 = 0 \quad (10 \sin 30°)(0.2) + 10 \cos 30°(0.15) - M = 0 \quad M = 2.2990 \text{ kN} \cdot \text{m}$

Section Properties: Referring to Fig. *a*

$$\bar{r} = \frac{0.15 + 0.25}{2} = 0.2 \text{ m} \qquad A = 0.05(0.1) = 0.005 \text{ m}^2$$

$$\int_A \frac{dA}{r} = b/n \frac{r_2}{r_1} = 0.05 \text{ m}\left(\frac{0.25}{0.15}\right) = 0.025541281 \text{ m}$$

$$R = \frac{A}{\rho_A \frac{dA}{r}} = \frac{0.005}{0.025541281} = 0.1957615 \text{ m}$$

$$\bar{r} - R = 0.2 - 0.1957615 = 0.00423848 \text{ m}$$

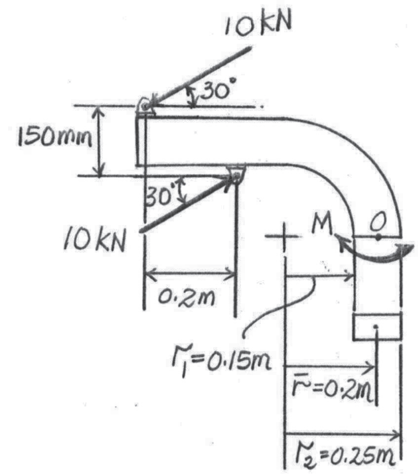
Normal Stress: Here, $M = -2.2990 \text{ kN} \cdot \text{m}$ is negative since it tends to decrease the beam's radius of curvature. Applying curved-beam formula,

$$(\sigma_C)_{\max} = \frac{M(R - r_1)}{Ar_1(\bar{r} - R)} = \frac{-2.2990(10^3)(0.1957615 - 0.15)}{0.005(0.15)(0.00423848)} = -33.10(10^6) \text{ Pa}$$

$$= -33.10\,(10^6) \text{ Pa} = 33.1 \text{ MPa (C)} \qquad \textbf{Ans.}$$

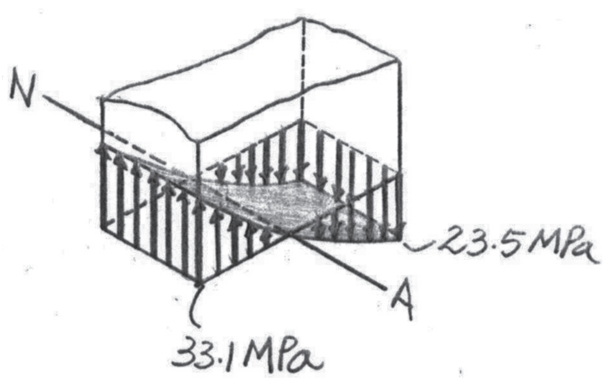
$$(\sigma_T)_{\max} = \frac{M(R - r_2)}{Ar_2(\bar{r} - R)} = \frac{-2.2990(10^3)(0.1957615 - 0.25)}{0.005(0.25)(0.00423848)} =$$

$$= 23.54\,(10^6) \text{ Pa} = 23.5 \text{ MPa (T)} \qquad \textbf{Ans.}$$



(a)

Using these results, the normal stress distribution on the beam's cross-section shown in Fig. *b* can be sketched.

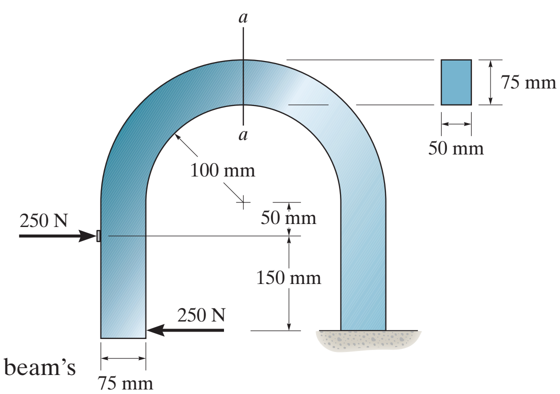


(b)

Ans:
$(\sigma_C)_{\max} = 33.1$ MPa (C),
$(\sigma_T)_{\max} = 23.5$ MPa (T)

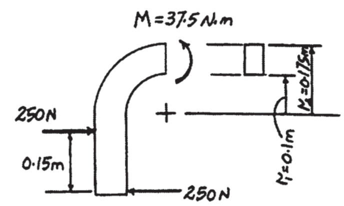
6–149.

The curved bar used on a machine has a rectangular cross section. If the bar is subjected to a couple as shown, determine the maximum tensile and compressive stresses acting at section *a–a*. Sketch the stress distribution on the section in three dimensions.



SOLUTION

Internal Moment: $M = 37.5\ \text{N}\cdot\text{m}$ is positive since it tends to increase the beam's radius of curvature.



Section Properties:

$$\bar{r} = \frac{0.1 + 0.175}{2} = 0.1375\ \text{m}$$

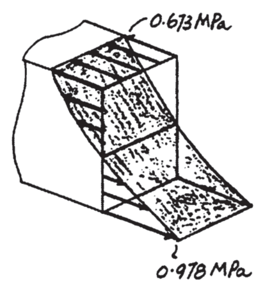
$$A = 0.075(0.05) = 0.00375\ \text{m}^2$$

$$\int_A \frac{dA}{r} = 0.05 \ln \frac{0.175}{0.1} = 0.027981\ \text{m}$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.00375}{0.027981} = 0.134021\ \text{m}$$

$$\bar{r} - R = 0.1375 - 0.134021 = 3.479478\left(10^{-3}\right)\ \text{m}$$

Normal Stress: Applying the curved-beam formula



$$(\sigma_{\max})_t = \frac{M(R - r_1)}{Ar_1(\bar{r} - R)}$$

$$= \frac{37.5(0.134021 - 0.1)}{0.00375(0.1)(3.479478)\left(10^{-3}\right)}$$

$$= 0.978\ \text{MPa (T)}$$ **Ans.**

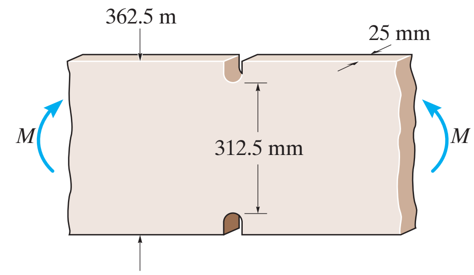
$$(\sigma_{\max})_c = \frac{M(R - r_2)}{Ar_2(\bar{r} - R)}$$

$$= \frac{37.5(0.134021 - 0.175)}{0.00375(0.175)(3.479478)\left(10^{-3}\right)}$$

$$= -0.673\ \text{MPa} = 0.673\ \text{MPa (C)}$$ **Ans.**

Ans:
$(\sigma_{\max})_t = 0.978$ MPa (T),
$(\sigma_{\max})_c = 0.673$ MPa (C)

6–150. If the radius of each notch on the plate is $r = 12.5$ mm, determine the largest moment that can be applied. The allowable bending stress for the material is stress of $\sigma_{\text{allow}} = 125$ MPa.



$$b = \frac{362.5 - 312.5}{2} = 25 \text{ mm}$$

$$\frac{b}{r} = \frac{25}{12.5} = 2.0 \qquad \frac{r}{h} = \frac{12.5}{312.5} = 0.04$$

From Fig. 6-44:

$$K = 2.60$$

$$\sigma_{\max} = K\frac{Mc}{I}$$

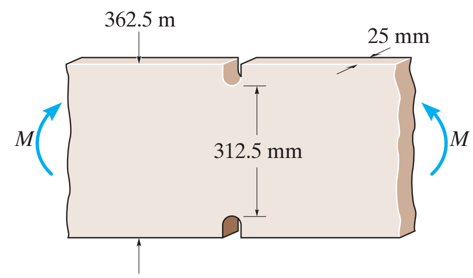
$$125(10^6) = 2.60\left[\frac{M(0.15625)}{\frac{1}{12}(0.025)(0.3125^3)}\right]$$

$$M = 19.56(10^3)\text{ N}\cdot\text{m} = 19.6\text{ kN}\cdot\text{m}$$ **Ans.**

Ans:

$K = 2.60$, $M = 19.6\text{ kN}\cdot\text{m}$

6–151. The symmetric notched plate is subjected to bending. If the radius of each notch is $r = 12.5$ mm and the applied moment is $M = 15\ \text{kN} \cdot \text{m}$, determine the maximum bending stress in the plate.



$$b = \frac{362.5 - 312.5}{2} = 25 \text{ mm}$$

$$\frac{b}{r} = \frac{25}{12.5} = 2.0 \qquad \frac{r}{h} = \frac{12.5}{312.5} = 0.04$$

From Fig. 6-44:

$$K = 2.60$$

$$\sigma_{\max} = K\frac{Mc}{I} = 2.60\left[\frac{[15(10^3)](0.15625)}{\frac{1}{12}(0.025)(0.3125^3)}\right]$$

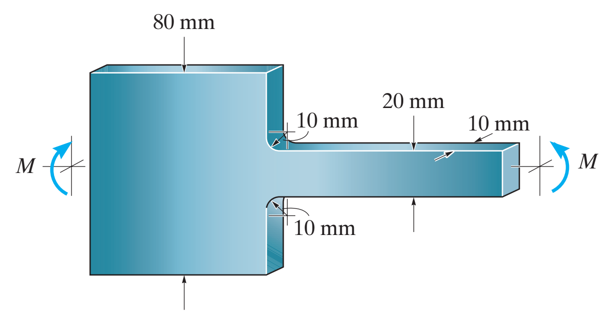
$$= 36.864(10^6)\ \text{N/m}^2 = 36.9 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:

$\sigma_{\max} = 36.9$ MPa

***6–152.**

The bar is subjected to a moment of $M = 100\ \text{N} \cdot \text{m}$. Determine the maximum bending stress in the bar and sketch, approximately, how the stress varies over the critical section.



SOLUTION

Stress Concentration Factor: Entering $\dfrac{w}{h} = \dfrac{80}{20} = 4.0$ and $\dfrac{r}{h} = \dfrac{10}{20} = 0.5$ into Fig. 6–43, we obtain $K = 1.25$.

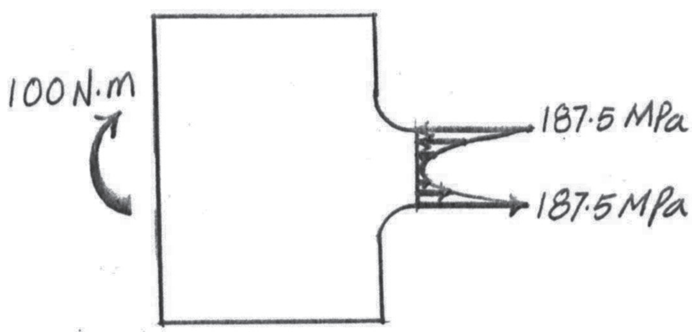
Maximum Bending Stress:

$$\sigma_{\max} = K\frac{Mc}{I}$$

$$= 1.25\left[\frac{100(0.01)}{\frac{1}{12}(0.01)(0.02^3)}\right]$$

$$= 187.5(10^6)\ \text{Pa} = 187.5\ \text{MPa} \qquad \textbf{Ans.}$$

The bending stress distribution across the critical section is shown in Fig. *a*.

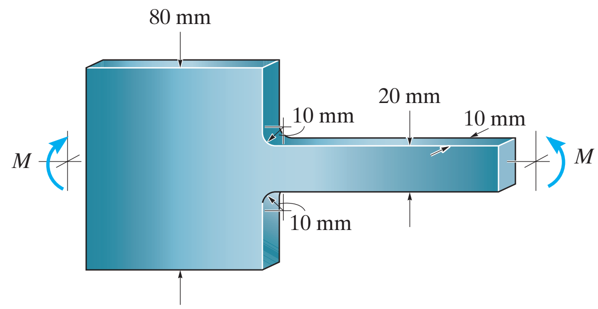


(a)

Ans:
$\sigma_{\max} = 187.5\ \text{MPa}$

6–153.

The allowable bending stress for the bar is $\sigma_{allow} = 200$ MPa. Determine the maximum moment M that can be applied to the bar.



SOLUTION

Stress Concentration Factor: Entering $\frac{w}{h} = \frac{80}{20} = 4.0$ and $\frac{r}{h} = \frac{10}{20} = 0.5$ into Fig. 6–43, we obtain $K = 1.25$.

Maximum Bending Stress:

$$\sigma_{max} = \sigma_{allow} = K\frac{Mc}{I}$$

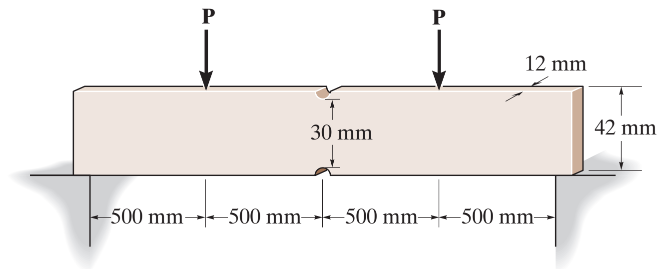
$$200(10^6) = 1.25\left[\frac{M(0.01)}{\frac{1}{12}(0.01)(0.02^3)}\right]$$

$$M = 106.67\ \text{N}\cdot\text{m}$$

$$= 107\ \text{N}\cdot\text{m}$$ **Ans.**

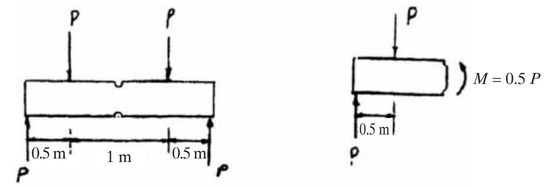
Ans:
$M = 107\ \text{N}\cdot\text{m}$

6–154. The simply supported notched bar is subjected to two forces **P**. Determine the largest magnitude of **P** that can be applied without causing the material to yield. The material is A-36 steel. Each notch has a radius of $r = 3$ mm.



$$b = \frac{42 - 30}{2} = 6 \text{ mm}$$

$$\frac{b}{r} = \frac{6}{3} = 2; \qquad \frac{r}{h} = \frac{3}{30} = 0.1$$



From Fig. 6-44. $K = 1.92$

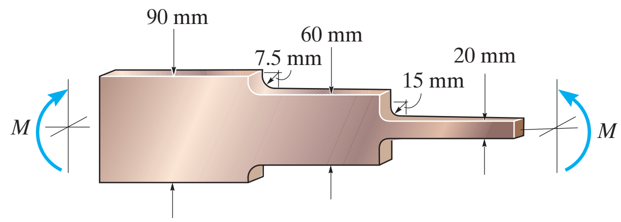
$$\sigma_Y = K\frac{Mc}{I}; \qquad 250(10^6) = 1.92\left[\frac{0.5P(0.015)}{\frac{1}{12}(0.012)(0.3^3)}\right]$$

$P = 468.75 \text{ N} = 469 \text{ N}$ **Ans.**

Ans:
$P = 469$ N

6–155.

The stepped bar has a thickness of 10 mm. Determine the maximum moment that can be applied to its ends if the allowable bending stress is $\sigma_{\text{allow}} = 150$ MPa.



SOLUTION

Stress Concentration Factor: For the smaller section, entering $\frac{w}{h} = \frac{60}{20} = 3.0$ and $\frac{r}{h} = \frac{15}{20} = 0.75$ into Fig. 6–43, we obtain $K = 1.15$. For the larger section, $\frac{w}{h} = \frac{90}{60} = 1.5$ and $\frac{r}{h} = \frac{7.5}{60} = 0.125$ gives $K = 1.65$.

Maximum Bending Stress: For the smaller section,

$$\sigma_{\max} = \sigma_{\text{allow}} = K\frac{Mc}{I}$$

$$150(10^6) = 1.15\left[\frac{M(0.01)}{\frac{1}{12}(0.01)(0.02^3)}\right]$$

$$M = 86.96\ \text{N}\cdot\text{m} = 87.0\ \text{N}\cdot\text{m (controls!)}$$ **Ans.**

For the larger section,

$$\sigma_{\max} = \sigma_{\text{allow}} = K\frac{Mc}{I}$$

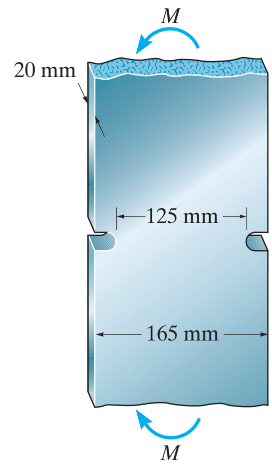
$$150(10^6) = 1.65\left[\frac{M(0.03)}{\frac{1}{12}(0.01)(0.06^3)}\right]$$

$$M = 545.45\ \text{N}\cdot\text{m}$$

Ans:
$M = 87.0\ \text{N}\cdot\text{m}$

***6–156.**

If the radius of each notch on the plate is $r = 10$ mm, determine the largest moment M that can be applied. The allowable bending stress is $\sigma_{\text{allow}} = 180$ MPa.



SOLUTION

Stress Concentration Factor: From the graph in the text with $\dfrac{b}{r} = \dfrac{20}{10} = 2$ and $\dfrac{r}{h} = \dfrac{10}{125} = 0.08$, then $K = 2.1$.

Allowable Bending Stress:

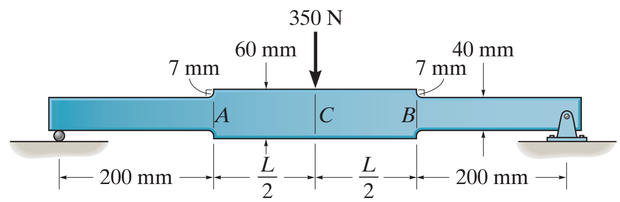
$$\sigma_{\max} = \sigma_{\text{allow}} = K\frac{Mc}{I}$$

$$180\left(10^6\right) = 2.1\left[\frac{M(0.0625)}{\frac{1}{12}(0.02)\left(0.125^3\right)}\right]$$

$$M = 4464\ \text{N}\cdot\text{m} = 4.46\ \text{kN}\cdot\text{m}$$ **Ans.**

Ans:
$M = 4.46\ \text{kN}\cdot\text{m}$

6–157. Determine the length L of the center portion of the bar so that the maximum bending stress at A, B, and C is the same. The bar has a thickness of 10 mm.



$$\frac{w}{h} = \frac{60}{40} = 1.5 \qquad \frac{r}{h} = \frac{7}{40} = 0.175$$

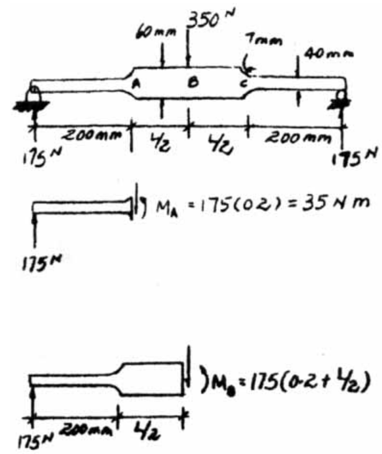
From Fig. 6-43, $K = 1.5$

$$(\sigma_A)_{\max} = K\frac{M_A c}{I} = 1.5\left[\frac{(35)(0.02)}{\frac{1}{12}(0.01)(0.04^3)}\right] = 19.6875 \text{ MPa}$$

$$(\sigma_B)_{\max} = (\sigma_A)_{\max} = \frac{M_B c}{I}$$

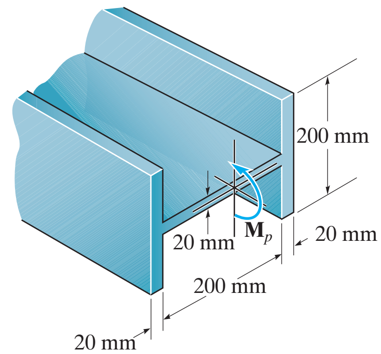
$$19.6875(10^6) = \frac{175(0.2 + \frac{L}{2})(0.03)}{\frac{1}{12}(0.01)(0.06^3)}$$

$$L = 0.95 \text{ m} = 950 \text{ mm} \qquad \textbf{Ans.}$$

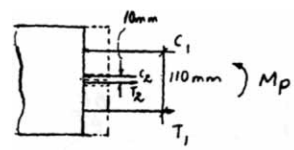


Ans:
$M = 87.0 \text{ kN} \cdot \text{m}$

6–158. Determine the shape factor for the cross section of the H-beam.



$$I_x = \frac{1}{12}(0.2)(0.02^3) + 2\left(\frac{1}{12}\right)(0.02)(0.2^3) = 26.8(10^{-6})\text{m}^4$$



$$C_1 = T_1 = \sigma_Y(2)(0.09)(0.02) = 0.0036\sigma_y$$

$$C_2 = T_2 = \sigma_Y(0.01)(0.24) = 0.0024\sigma_y$$

$$M_p = 0.0036\sigma_Y(0.11) + 0.0024\sigma_Y(0.01) = 0.00042\sigma_Y$$

$$\sigma_Y = \frac{M_Y c}{I}$$

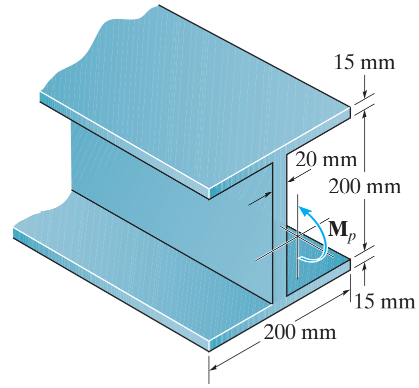
$$M_Y = \frac{\sigma_Y(26.8)(10^{-6})}{0.1} = 0.000268\sigma_Y$$

$$k = \frac{M_p}{M_Y} = \frac{0.00042\sigma_Y}{0.000268\sigma_Y} = 1.57$$ **Ans.**

Ans:
$I_x = 26.8\,(10^{-6})\text{ m}^4$, $M_p = 0.00042\sigma_Y$,
$M_Y = 0.000268\sigma_Y$, $k = 1.57$

6–159.

Determine the shape factor for the wide-flange beam.



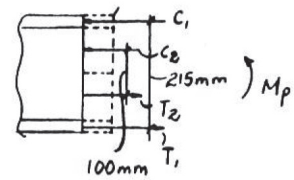
SOLUTION

$$I_x = \frac{1}{12}(0.2)(0.23)^3 - \frac{1}{12}(0.18)(0.2)^3 = 82.78333(10^{-6})\text{ m}^4$$

$$C_1 = T_1 = \sigma_Y(0.2)(0.015) = 0.003\,\sigma_Y$$

$$C_2 = T_2 = \sigma_Y(0.1)(0.02) = 0.002\,\sigma_Y$$

$$M_p = 0.003\,\sigma_Y(0.215) + 0.002\,\sigma_Y(0.1) = 0.000845\,\sigma_Y$$



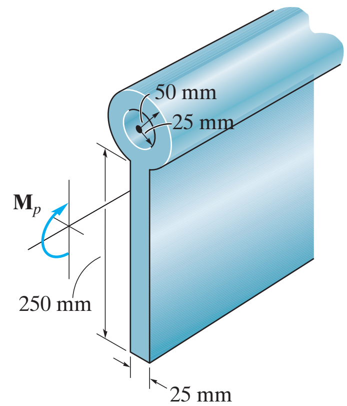
$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{\sigma_Y(82.78333)(10^{-6})}{0.115} = 0.000719855\,\sigma_Y$$

$$k = \frac{M_p}{M_Y} = \frac{0.000845\,\sigma_Y}{0.000719855\,\sigma_Y} = 1.17$$ **Ans.**

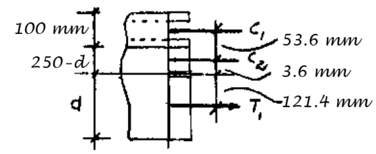
Ans:
$k = 1.17$

***6–160.** Determine the plastic moment $\mathbf{M}_p$ that can be supported by a beam having the cross section shown. $\sigma_Y = 210$ MPa.



$\int \sigma \, dA = 0$

$C_1 + C_2 - T_1 = 0$



$$[\pi(0.05^2 - 0.025^2)][210(10^6)] + [(0.25 - d)(0.025)][210(10^6)] - [d(0.025)][210(10^6)] = 0$$

$d = 0.24281 \text{ m} < 0.25 \text{ m}$ (o.k.!)

$$M_p = [\pi(0.05^2 - 0.025^2)][210(10^6)](0.0571903)$$

$$+ [0.0071903(0.025)][210(10^6)](0.0035951)$$

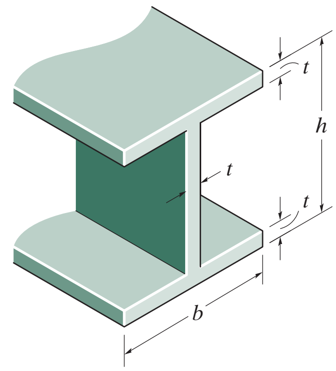
$$+ [0.242810(0.025)][210(10^6)](0.121405)$$

$$= 225.64(10^3) \text{ N} \cdot \text{m} = 226 \text{ kN} \cdot \text{m}$$ **Ans.**

Ans:
$M_p = 226 \text{ kN} \cdot \text{m}.$

6–161.

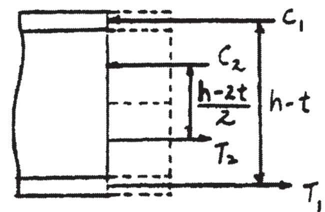
The beam is made of an elastic perfectly plastic material for which $\sigma_Y = 250$ MPa. Determine the residual stress in the beam at its top and bottom after the plastic moment $\mathbf{M}_p$ is applied and then released.



SOLUTION

Plastic analysis:

$$T_1 = C_1 = \sigma_Y bt; \qquad T_2 = C_2 = \sigma_Y\left(\frac{h-2t}{2}\right)t$$



$$M_p = \sigma_Y bt(h-t) + \sigma_Y\left(\frac{h-2t}{2}\right)(t)\left(\frac{h-2t}{2}\right)$$

$$= \sigma_Y\left[bt(h-t) + \frac{t}{4}(h-2t)^2\right]$$

Elastic analysis:

$$I = \frac{1}{12}bh^3 - \frac{1}{12}(b-t)(h-2t)^3$$

$$= \frac{1}{12}\left[bh^3 - (b-t)(h-2t)^3\right]$$

$$M_Y = \frac{\sigma_y I}{c} = \frac{\sigma_Y\left(\frac{1}{12}\right)[bh^3 - (b-t)(h-2t)^3]}{\frac{h}{2}}$$

$$= \frac{bh^3 - (b-t)(h-2t)^3}{6h}\sigma_Y$$

Shape factor:

$$k = \frac{M_P}{M_Y} = \frac{[bt(h-t) + \frac{t}{4}(h-2t)^2]\sigma_Y}{\frac{bh^3 - (b-t)(h-2t)^3}{6h}\sigma_Y}$$

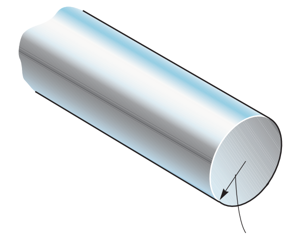
$$= \frac{3h}{2}\left[\frac{4bt(h-t) + t(h-2t)^2}{bh^3 - (b-t)(h-2t)^3}\right] \qquad \textbf{Ans.}$$

Ans:

$$k = \frac{3h}{2}\left[\frac{4bt(h-t) + t(h-2t)^2}{bh^3 - (b-t)(h-2t)^3}\right]$$

6–162.

The rod has a circular cross section. If it is made of an elastic perfectly plastic material, determine the shape factor.

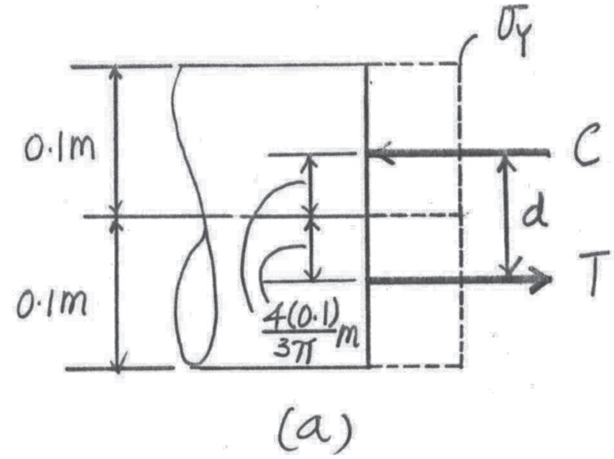


SOLUTION

Maximum Elastic Moment: Applying the flexure formula with $\sigma = \sigma_Y$ and $I = \frac{\pi}{4} r^4 = \frac{\pi}{4}(0.1^4) = 25(10^{-6})\pi \text{ m}^4$,

$$\sigma = \frac{Mc}{I}; \qquad \sigma_Y = \frac{M_Y(0.1)}{25(10^{-6})\pi}$$

$$M_Y = 0.25\,(10^{-3})\pi\sigma_Y$$



Plastic Moment: Referring to Fig. *a*,

$$C = T = \sigma_Y\left(\frac{\pi r^2}{2}\right) = \sigma_Y\left[\frac{\pi(0.1^2)}{2}\right] = 5(10^{-3})\pi\sigma_Y$$

$$d = 2\left(\frac{4r}{3\pi}\right) = 2\left[\frac{4(0.1)}{3\pi}\right] = \frac{0.26667}{\pi} = \text{m}$$

Thus,

$$M_p = Cd = (5(10^{-3})\pi\,\sigma_Y)\left(\frac{0.26667}{\pi}\right) = 1.3333(10^{-3})\sigma_Y$$

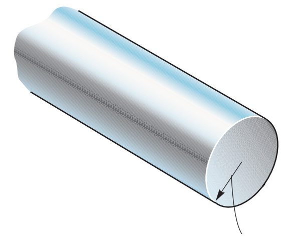
Shape Factor:

$$k = \frac{M_p}{M_Y} = \frac{1.3333(10^{-3})\sigma_Y}{0.25(10^{-3})\pi\sigma_Y} = 1.6976 = 1.70$$ **Ans.**

Ans:
$k = 1.70$

6–163.

The rod has a circular cross section. If it is made of an elastic perfectly plastic material where $\sigma_Y = 345$ MPa, determine the maximum elastic moment and plastic moment that can be applied to the cross section.



SOLUTION

Maximum Elastic Moment: Applying the flexure formula with $\sigma = \sigma_y = 345$ MPa and $I = \frac{\pi}{4}r^4 = \frac{\pi}{4}\left(0.1^4\right) = 25\left(10^{-6}\right)\pi \text{ m}^4$,

$$\sigma = \frac{Mc}{I}; \qquad 345\left(10^6\right) = \frac{M_Y(0.1)}{25\left(10^{-6}\right)\pi}$$

$$M_Y = 270.96\left(10^3\right) \text{ N}\cdot\text{m} = 271 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Plastic Moment: Referring to Fig. *a*,

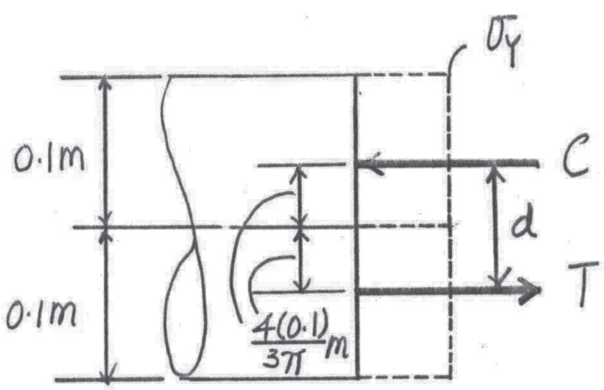
$$C = T = \sigma_y\left(\frac{\pi r^2}{2}\right) = 345\left(10^6\right)\left[\frac{\pi\left(0.1^2\right)}{2}\right] = 1.725\left(10^6\right)\pi \text{ N}$$

$$d = 2\left(\frac{4r}{3\pi}\right) = 2\left[\frac{4(0.1)}{3\pi}\right] = \frac{0.26667}{\pi} \text{ m}$$

Thus,

$$M_P = Cd = \left[1.725\left(10^6\right)\pi\right]\left[\frac{0.26667}{\pi}\right]$$

$$= 460\left(10^3\right) \text{ N}\cdot\text{m} = 460 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

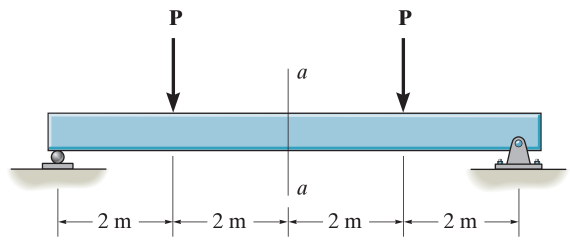


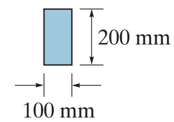
(a)

Ans:
$M_Y = 271 \text{ kN}\cdot\text{m}$,
$M_P = 460 \text{ kN}\cdot\text{m}$

***6–164.**

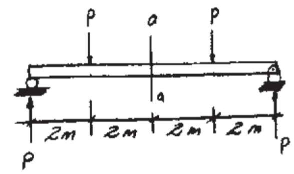
The beam is made of an elastic perfectly plastic material for which $\sigma_Y = 200$ MPa. If the largest moment in the beam occurs within the center section a–a, determine the magnitude of each force **P** that causes this moment to be (a) the largest elastic moment and (b) the largest plastic moment.





SOLUTION

$M = 2P$ **(1)**

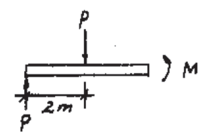


a) Elastic moment

$$I = \frac{1}{12}(0.1)(0.2^3) = 66.667(10^{-6})\text{ m}^4$$

$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{200(10^6)(66.667)(10^{-6})}{0.1}$$



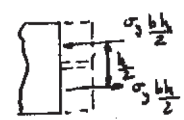
$$= 133.33\text{ kN}\cdot\text{m}$$

From Eq. (1)

$$133.33 = 2\,P$$

$P = 66.7$ kN **Ans.**

b) Plastic moment



$$M_p = \frac{b\,h^2}{4}\sigma_Y$$

$$= \frac{0.1(0.2^2)}{4}(200)(10^6)$$

$$= 200\text{ kN}\cdot\text{m}$$

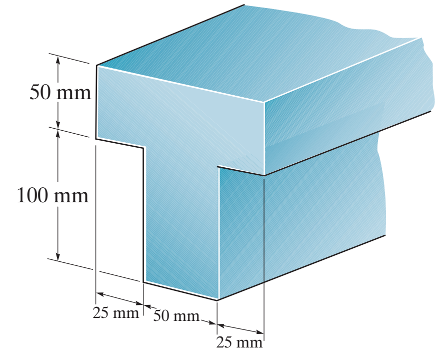
From Eq. (1)

$$200 = 2\,P$$

$P = 100$ kN **Ans.**

Ans:
Elastic: $P = 66.7$ kN,
Plastic: $P = 100$ kN

6–165. Determine the shape factor of the beam's cross section.



Referring to Fig. *a*, the location of centroid of the cross-section is

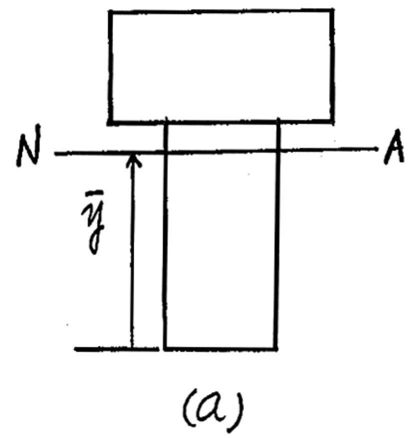
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.125(0.1)(0.05) + 0.05(0.05)(0.1)}{0.1(0.05) + 0.05(0.1)} = 0.0875 \text{ m}$$

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.05)(0.1^3) + 0.05(0.1)(0.0875 - 0.05)^2$$

$$+ \frac{1}{12}(0.1)(0.05^3) + 0.1(0.05)(0.125 - 0.0875)^2$$

$$= 19.27083(10^{-6}) \text{ m}^4$$



Here $\sigma_{\max} = \sigma_Y$ and $c = \bar{y} = 0.0875$ m. Thus

$$\sigma_{\max} = \frac{Mc}{I}; \qquad \sigma_Y = \frac{M_Y(0.0875)}{19.27083(10^{-6})}$$

$$M_Y = 0.22024(10^{-3})\sigma_Y$$

Referring to the stress block shown in Fig. *b*,

$$\int_A \sigma dA = 0; \qquad T - C_1 - C_2 = 0$$

$$[d(0.05)]\sigma_Y - [(0.1 - d)(0.05)]\sigma_Y - [0.05(0.1)]\sigma_Y = 0$$

$$d = 0.1 \text{ m}$$

Since $d = 0.1$ m, $c_1 = 0$, Fig. c. Here

$$T = C = [0.05(0.1)]\sigma_Y = 0.005\sigma_Y$$

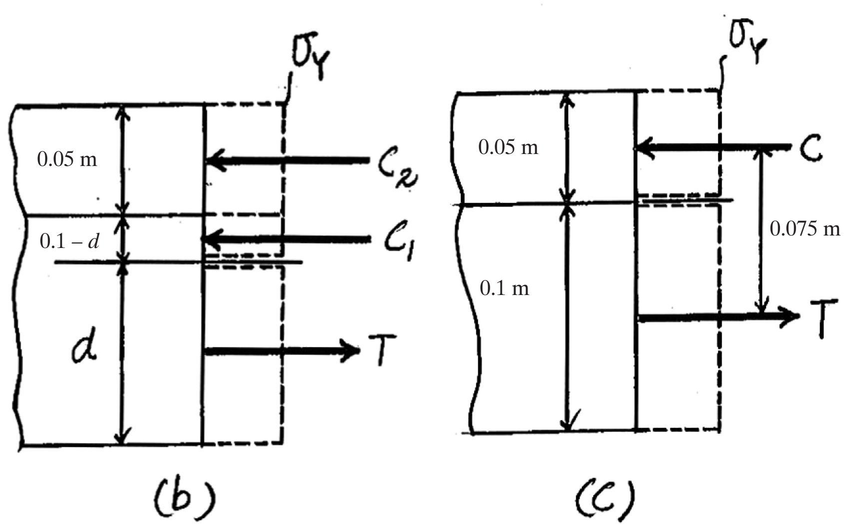
Thus,

$$M_p = T(0.075) = 0.005\sigma_Y(0.075) = 0.375(10^{-3})\sigma_Y$$

Thus,

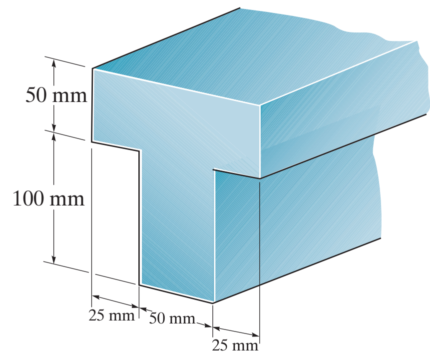
Ans.

$$k = \frac{M_p}{M_Y} = \frac{0.375(10^{-3})\sigma_Y}{0.22024(10^{-3})\sigma_Y} = 1.7027 = 1.70$$



Ans:
$k = 1.70$

6–166. The beam is made of elastic-perfectly plastic material. Determine the maximum elastic moment and the plastic moment that can be applied to the cross section. Take $\sigma_Y = 250$ MPa.



Referring to Fig. *a*, the location of centroid of the cross-section is

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.125(0.1)(0.05) + 0.05(0.05)(0.1)}{0.1(0.05) + 0.05(0.1)} = 0.0875 \text{ m}$$

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.05)(0.1^3) + 0.05(0.1)(0.0875 - 0.05)^2$$

$$+ \frac{1}{12}(0.1)(0.05^3) + 0.1(0.05)(0.125 - 0.0875)^2$$

$$= 19.27083(10^{-6}) \text{ m}^4$$

Here $\sigma_{\max} = \sigma_Y = 250$ MPa and $c = \bar{y} = 0.0875$ m. Then

$$\sigma_{\max} = \frac{Mc}{I}; \qquad 250(10^6) = \frac{M_Y(0.0875)}{19.27083(10^{-6})}$$

$$M_Y = 55.06(10^3) \text{ N}\cdot\text{m} = 55.1 \text{ kN}\cdot\text{m}$$ **Ans.**

Referring to the stress block shown in Fig. *b*,

$$\int_A \sigma dA = 0; \qquad T - C_1 - C_2 = 0$$

$$[d(0.05)][250(10^6)] - [(0.1 - d)(0.05)][250(10^6)] - [0.05(0.1)][250(10^6)] = 0$$

$$d = 0.1 \text{ m}$$

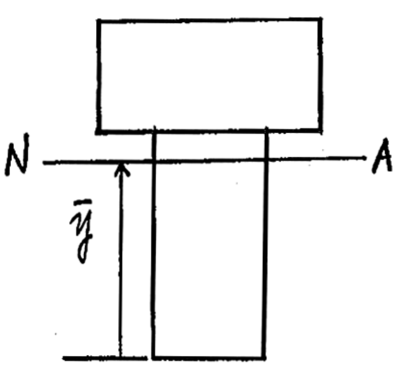
Since $d = 0.1$ m, $c_1 = 0$,

Here,

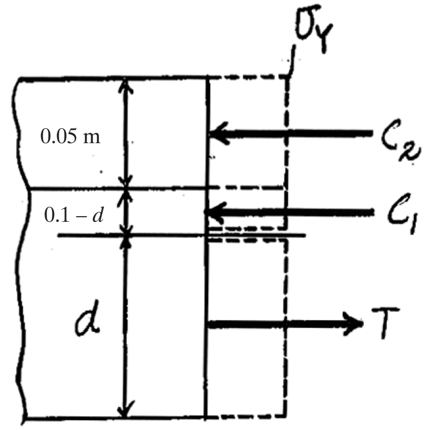
$$T = C = [0.05(0.1)][250(10^6)] = 1250(10^3) \text{ N} = 1250 \text{ kN}$$

Thus,

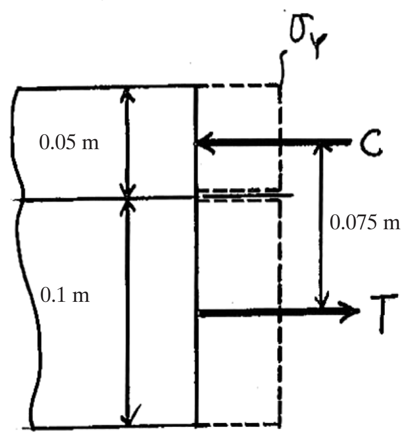
$$M_p = T(0.075) = (1250)(0.075) = 93.75 \text{ kN}$$ **Ans.**



(a)



(b)

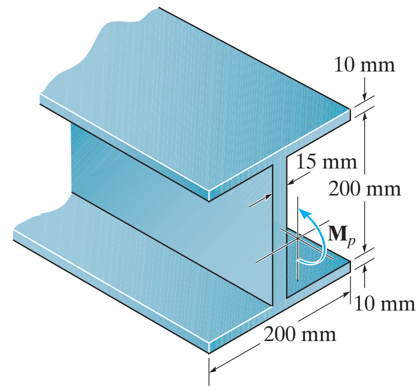


(c)

Ans:
$M_Y = 55.1$ kN·m, $M_P = 93.75$ kN·m

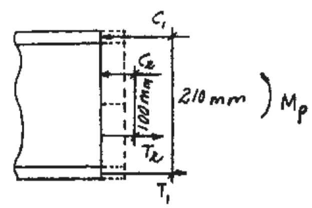
6–167.

Determine the shape factor for the beam.



SOLUTION

$$I = \frac{1}{12}(0.2)(0.22)^3 - \frac{1}{12}(0.185)(0.2)^3 = 54.133(10^{-6})\ \text{m}^4$$



$$C_1 = \sigma_Y(0.01)(0.2) = (0.002)\,\sigma_Y$$

$$C_2 = \sigma_Y(0.1)(0.015) = (0.0015)\,\sigma_Y$$

$$M_p = 0.002\,\sigma_Y(0.21) + 0.0015\,\sigma_Y(0.1) = 0.0005\,\sigma_Y$$

$$\sigma_Y = \frac{M_Y c}{I}$$

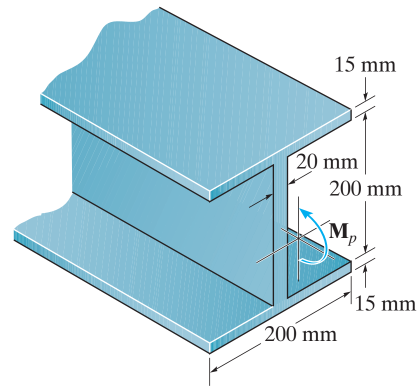
$$M_Y = \frac{\sigma_Y(54.133)(10^{-6})}{0.11} = 0.000492\,\sigma_Y$$

$$k = \frac{M_p}{M_Y} = \frac{0.00057\,\sigma_Y}{0.000492\,\sigma_Y} = 1.16$$ **Ans.**

Ans:
$k = 1.16$

***6–168.**

The beam is made of an elastic perfectly plastic material for which $\sigma_Y = 250$ MPa. Determine the residual stress in the beam at its top and bottom after the plastic moment $\mathbf{M}_p$ is applied and then released.



SOLUTION

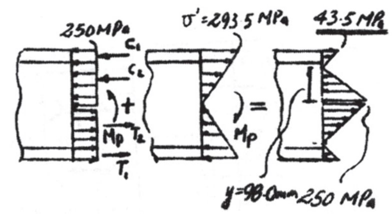
$$I_x = \frac{1}{12}(0.2)(0.23)^3 - \frac{1}{12}(0.18)(0.2)^3 = 82.78333(10^{-6})\text{ m}^4$$

$$C_1 = T_1 = \sigma_Y(0.2)(0.015) = 0.003\,\sigma_Y$$

$$C_2 = T_2 = \sigma_Y(0.1)(0.02) = 0.002\,\sigma_Y$$

$$M_p = 0.003\,\sigma_Y(0.215) + 0.002\,\sigma_Y(0.1) = 0.000845\,\sigma_Y$$

$$= 0.000845(250)(10^6) = 211.25\text{ kN}\cdot\text{m}$$



$$\sigma' = \frac{M_p c}{I} = \frac{211.25(10^3)(0.115)}{82.78333(10^{-6})} = 293.5\text{ MPa}$$

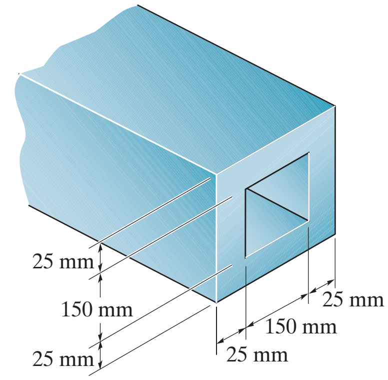
$$\frac{y}{250} = \frac{0.115}{293.5}; \qquad y = 0.09796\text{ m} = 98.0\text{ mm}$$

$$\sigma_{top} = \sigma_{bottom} = 293.5 - 250 = 43.5\text{ MPa}$$ **Ans.**

Ans:
$\sigma_{top} = \sigma_{bottom} = 43.5$ MPa

6–169.

The box beam is made of an elastic perfectly plastic material for which $\sigma_Y = 250$ MPa. Determine the residual stress in the top and bottom of the beam after the plastic moment $\mathbf{M}_p$ is applied and then released.

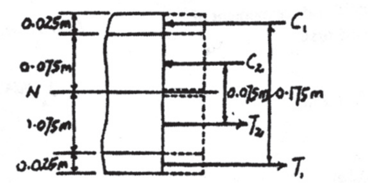


SOLUTION

Plastic Moment:

$$M_P = 250\left(10^6\right)\,(0.2)(0.025)(0.175) + 250\left(10^6\right)\,(0.075)(0.05)(0.075)$$

$$= 289062.5 \text{ N} \cdot \text{m}$$

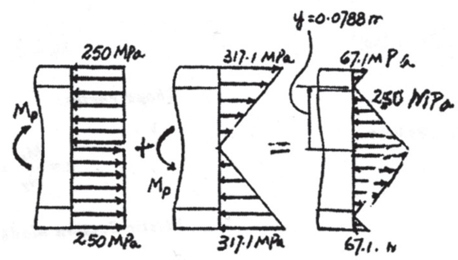


Modulus of Rupture: The modulus of rupture σ_r can be determined using the flexure formula with the application of reverse, plastic moment $M_P = 289062.5 \text{ N} \cdot \text{m}$.

$$I = \frac{1}{12}(0.2)\left(0.2^3\right) - \frac{1}{12}(0.15)\left(0.15^3\right)$$

$$= 91.14583\left(10^{-6}\right) \text{ m}^4$$

$$\sigma_r = \frac{M_P\, c}{I} = \frac{289062.5\ (0.1)}{91.14583\left(10^{-6}\right)} = 317.41 \text{ MPa}$$



Residual Bending Stress: As shown on the diagram.

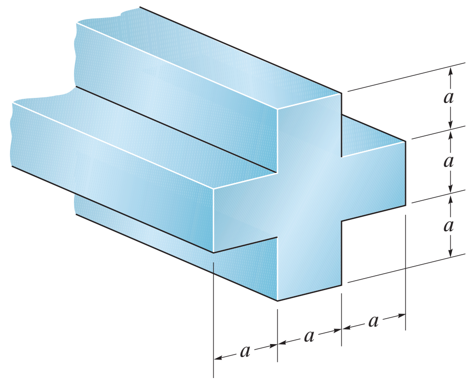
$$\sigma'_{\text{top}} = \sigma'_{\text{bot}} = \sigma_r - \sigma_Y$$

$$= 317.14 - 250 = 67.1 \text{ MPa}$$ **Ans.**

Ans:
$\sigma_{\text{top}} = \sigma_{\text{bottom}} = 67.1$ MPa

6–170.

Determine the shape factor of the cross section.



SOLUTION

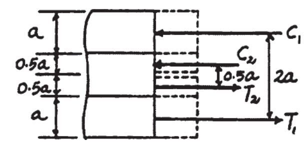
Maximum Elastic Moment: The moment of inertia about neutral axis must be determined first.

$$I_{NA} = \frac{1}{12}(a)(3a)^3 + \frac{1}{12}(2a)\left(a^3\right) = 2.41667\,a^4$$

Applying the flexure formula with $\sigma = \sigma_Y$, we have

$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y\left(2.41667a^4\right)}{1.5a} = 1.6111a^3\sigma_Y$$



Plastic Moment:

$$M_p = \sigma_Y(a)(a)(2a) + \sigma_Y(0.5a)(3a)(0.5a)$$
$$= 2.75a^3\sigma_Y$$

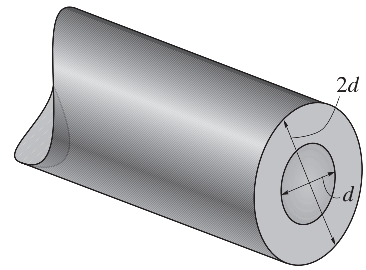
Shape Factor:

$$k = \frac{M_p}{M_Y} = \frac{2.75a^3\sigma_Y}{1.6111a^3\sigma_Y} = 1.71$$ **Ans.**

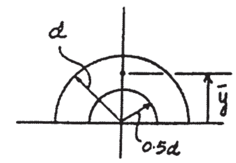
Ans:
$k = 1.71$

6–171.

Determine the shape factor for the member having the tubular cross section.



SOLUTION



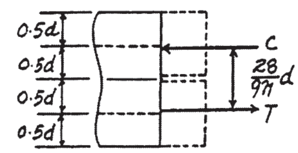
Maximum Elastic Moment: The moment of inertia about neutral axis must be determined first.

$$I_{NA} = \frac{\pi}{4}d^4 - \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{15\pi}{64}d^4$$

Applying the flexure formula with $\sigma = \sigma_Y$, we have

$$\sigma_Y = \frac{M_Y\, c}{I}$$

$$M_Y = \frac{\sigma_Y\, I}{c} = \frac{\sigma_Y\left(\frac{15\pi}{64}d^4\right)}{d} = \frac{15\pi}{64}d^3\,\sigma_Y$$



Plastic Moment:

$$\bar{y} = \frac{\Sigma\bar{y}A}{\Sigma A} = \frac{\frac{4d}{3\pi}\left(\frac{\pi d^2}{2}\right) - \frac{4\left(\frac{d}{2}\right)}{3\pi}\left(\frac{\frac{\pi}{4}d^2}{2}\right)}{\frac{\pi d^2}{2} - \frac{\frac{\pi}{4}d^2}{2}} = \frac{14}{9\pi}d$$

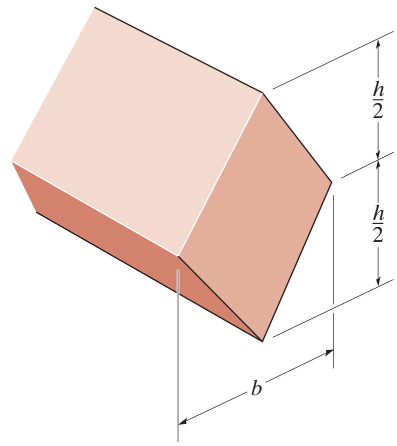
$$M_P = \sigma_Y\left(\frac{\pi d^2}{2} - \frac{\frac{\pi}{4}d^2}{2}\right)\frac{28}{9\pi}d = \frac{7}{6}d^3\,\sigma_Y$$

Shape Factor:

$$k = \frac{M_P}{M_Y} = \frac{\frac{7}{6}d^3\,\sigma_Y}{\frac{15\pi}{64}d^3\,\sigma_Y} = 1.58$$ **Ans.**

Ans:
$k = 1.58$

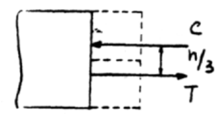
***6–172.** Determine the shape factor for the member.



Plastic analysis:

$$T = C = \frac{1}{2}(b)\left(\frac{h}{2}\right)\sigma_Y = \frac{b\,h}{4}\sigma_Y$$

$$M_P = \frac{b\,h}{4}\sigma_Y\left(\frac{h}{3}\right) = \frac{b\,h^2}{12}\sigma_Y$$



Elastic analysis:

$$I = 2\left[\frac{1}{12}(b)\left(\frac{h}{2}\right)^3\right] = \frac{b\,h^3}{48}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y\left(\frac{bh^3}{48}\right)}{\frac{h}{2}} = \frac{b\,h^2}{24}\sigma_Y$$

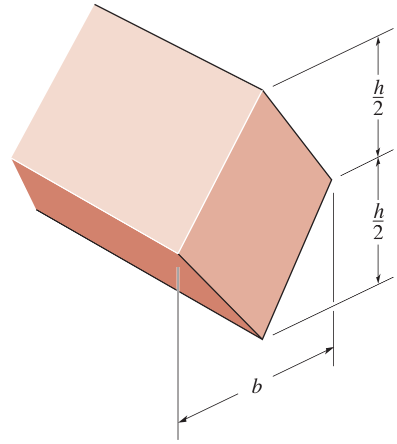
Shape factor:

$$k = \frac{M_p}{M_Y} = \frac{\frac{bh^2}{12}\sigma_Y}{\frac{bh^2}{24}\sigma_Y} = 2$$ **Ans.**

Ans:

$$k = \frac{M_p}{M_Y} = \frac{\frac{bh^2}{12}\sigma_Y}{\frac{bh^2}{24}\sigma_Y} = 2$$

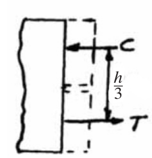
6–173. The member is made from an elastic-plastic material. Determine the maximum elastic moment and the plastic moment that can be applied to the cross section. Take $b = 100$ mm, $h = 150$ mm, $\sigma_Y = 250$ MPa.



Elastic analysis:

$$I = 2\left[\frac{1}{12}(0.1)(0.075^3)\right] = 7.03125(10^{-6})\ \text{m}^4$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{[250(10^6)][7.03125(10^{-6})]}{0.075} = 23.4375(10^3)\ \text{N}\cdot\text{m} = 23.4\ \text{kN}\cdot\text{m}$$ **Ans.**



Plastic analysis:

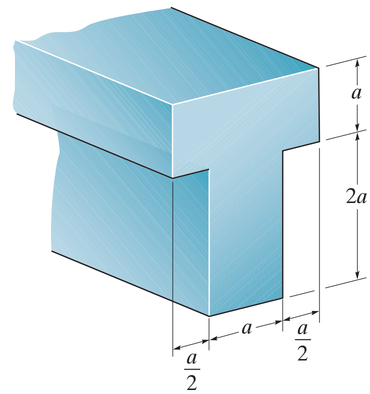
$$T = C = \left[\frac{1}{2}(0.1)(0.075)\right][250(10^6)] = 937.5(10^3)\ \text{N}$$

$$M_p = T\left(\frac{h}{3}\right) = [937.5(10^3)]\left(\frac{0.15}{3}\right) = 46.875(10^3)\ \text{N}\cdot\text{m} = 46.9\ \text{kN}\cdot\text{m}$$ **Ans.**

Ans:
$M_Y = 23.4$ kN·m, $M_P = 46.9$ kN·m

6–174.

Determine the shape factor of the cross section.



SOLUTION

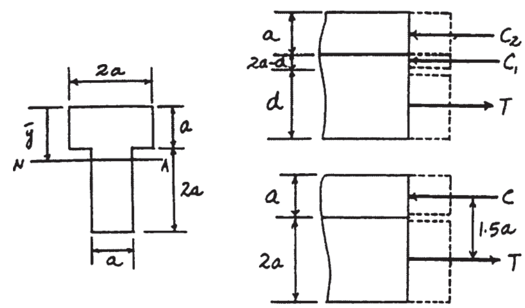
Maximum Elastic Moment: The centroid and the moment of inertia about neutral axis must be determined first.

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.5a(a)(2a) + 2a(2a)(a)}{a(2a) + 2a(a)} = 1.25a$$

$$I_{NA} = \frac{1}{12}(2a)\left(a^3\right) + 2a(a)(1.25a - 0.5a)^2$$

$$+ \frac{1}{12}(a)(2a)^3 + a(2a)(2a - 1.25a)^2$$

$$= 3.0833a^4$$



Applying the flexure formula with $\sigma = \sigma_Y$, we have

$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y\left(3.0833a^4\right)}{(3a - 1.25a)}$$

$$= 1.7619a^3\,\sigma_Y$$

Plastic Moment:

$$\int_A \sigma dA = 0; \qquad T - C_1 - C_2 = 0$$

$$\sigma_Y(d)(a) - \sigma_Y(2a - d)(a) - \sigma_Y(a)(2a) = 0$$

$$d = 2a$$

$$M_P = \sigma_Y(2a)(a)(1.5a) = 3.00a^3\,\sigma_Y$$

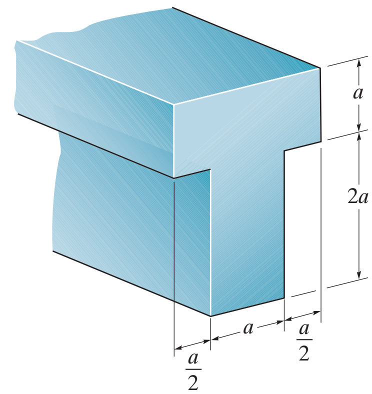
Shape Factor:

$$k = \frac{M_P}{M_Y} = \frac{3.00a^3\,\sigma_Y}{1.7619a^3\,\sigma_Y} = 1.70$$ **Ans.**

Ans:
$k = 1.70$

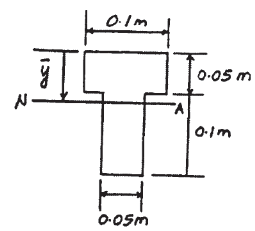
6–175.

The beam is made of elastic perfectly plastic material. Determine the maximum elastic moment and the plastic moment that can be applied to the cross section. Take $a = 50$ mm and $\sigma_Y = 230$ MPa.



SOLUTION

Maximum Elastic Moment: The centroid and the moment of inertia about neutral axis must be determined first.



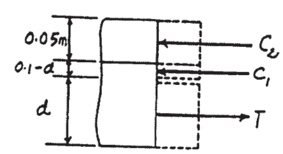
$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.025(0.05)\,(0.1) + 0.1\,(0.1)\,(0.05)}{0.05(0.1) + 0.1(0.05)} = 0.0625 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.1)\left(0.05^3\right) + 0.1(0.05)\,(0.0625 - 0.025)^2$$

$$+ \frac{1}{12}(0.05)\left(0.1^3\right) + 0.05(0.1)\,(0.1 - 0.0625)^2$$

$$= 19.2709\left(10^{-6}\right) \text{ m}^4$$

Applying the flexure formula with $\sigma = \sigma_Y$, we have

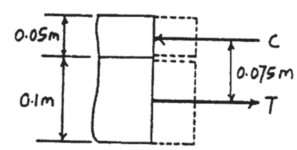


$$\sigma_Y = \frac{M_Y\,c}{I}$$

$$M_Y = \frac{\sigma_Y\,I}{c} = \frac{230\left(10^6\right)(19.2709)\left(10^{-6}\right)}{(0.15 - 0.0625)}$$

$$= 50654.8 \text{ N}\cdot\text{m} = 50.7 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Plastic Moment:



$$\int_A \sigma dA = 0; \qquad T - C_1 - C_2 = 0$$

$$\sigma_Y\,(d)\,(0.05) - \sigma_Y\,(0.1 - d)(0.05) - \sigma_Y\,(0.05)(0.1) = 0$$

$$d = 0.100 \text{ m}$$

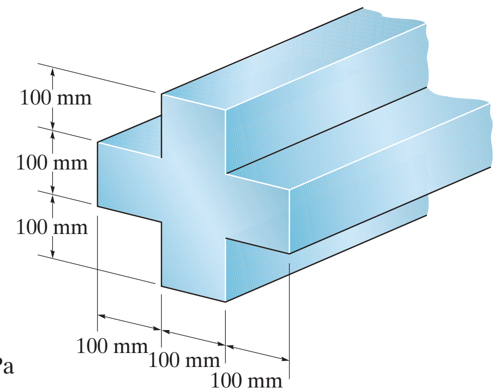
$$M_P = 230\left(10^6\right)\,(0.100)\,(0.05)\,(0.075)$$

$$= 86250 \text{ N}\cdot\text{m} = 86.25 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$M_Y = 50.7$ kN·m,
$M_P = 86.25$ kN·m

***6–176.**

The beam is made of elastic perfectly plastic material for which $\sigma_Y = 345$ MPa. Determine the maximum elastic moment and the plastic moment that can be applied to the cross section.



SOLUTION

Maximum Elastic Moment: Applying the flexure formula with $\sigma = \sigma_y = 345$ MPa and $I = \frac{1}{12}(0.1)(0.3^3) + \frac{1}{12}(0.2)(0.1^3) = 0.24167(10^{-3})\text{ m}^4$,

$$\sigma = \frac{Mc}{I}; \qquad 345(10^6) = \frac{M_Y(0.15)}{0.24167(10^{-3})}$$

$$M_Y = 555.83(10^3)\text{ N}\cdot\text{m} = 556\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Plastic Moment: Referring to Fig. *a*,

$$C_1 = T_1 = \sigma_y(0.1)(0.1) = 0.01\sigma_y = 0.01\left[345(10^6)\right] = 3.45(10^6)\text{ N}$$

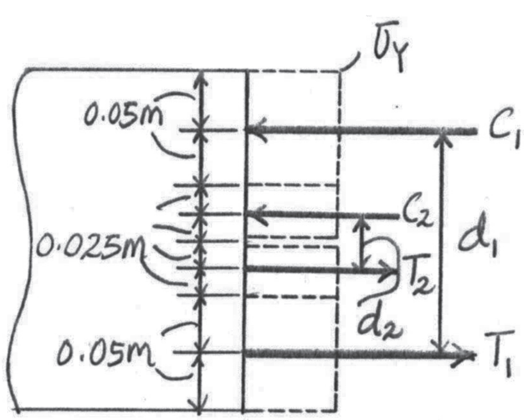
$$C_2 = T_2 = \sigma_y(0.3)(0.05) = 0.015\sigma_y = 0.015\left[345(10^6)\right] = 5.175(10^6)\text{ N}$$

$$d_1 = 4(0.05) = 0.2\text{ m} \qquad d_2 = 2(0.025) = 0.05\text{ m}$$

Thus,

$$M_P = C_1d_1 + C_2d_2 = \left[3.45(10^6)\right](0.2) + \left[5.175(10^6)\right](0.05)$$

$$= 948.75(10^3)\text{ N}\cdot\text{m} = 949\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

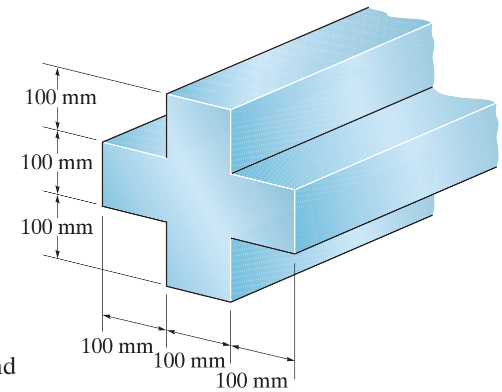


(a)

Ans:
$M_Y = 556\text{ kN}\cdot\text{m}$,
$M_P = 949\text{ kN}\cdot\text{m}$

6–177.

Determine the shape factor of the cross section.



SOLUTION

Maximum Elastic Moment: Applying the flexure formula with $\sigma = \sigma_y$ and $I = \frac{1}{12}(0.1)(0.3^3) + \frac{1}{12}(0.2)(0.1^3) = 0.24167(10^{-3})\text{ m}^4$,

$$\sigma = \frac{Mc}{I}; \qquad \sigma_y = \frac{M_y(0.15)}{0.24167(10^{-3})} \qquad M_y = 1.6111(10^{-3})\sigma_y$$

Plastic Moment: Referring to Fig. *a*,

$$C_1 = T_1 = \sigma_y(0.1)(0.1) = 0.01\ \sigma_y$$

$$C_2 = T_2 = \sigma_y(0.3)(0.05) = 0.015\ \sigma_y$$

$$d_1 = 4(0.05) = 0.2\text{ m}$$

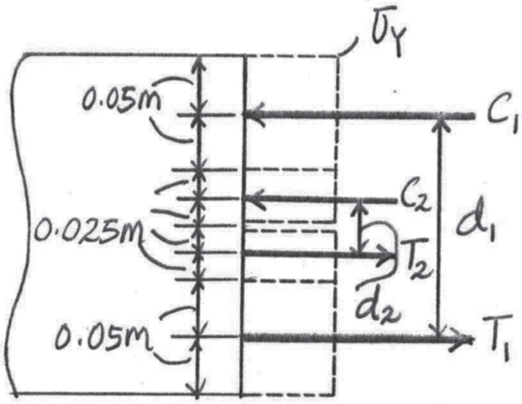
$$d_2 = 2(0.025) = 0.05\text{ m}$$

Thus,

$$M_p = C_1 d_1 + C_2 d_2 = (0.01\ \sigma_y)(0.2) + (0.015\ \sigma_y)(0.05)$$
$$= 2.75\ (10^{-3})\sigma_y$$

Shape Factor:

$$k = \frac{M_p}{M_y} = \frac{2.75(10^{-3})\sigma_y}{1.6111(10^{-3})\sigma_y} = 1.7069 = 1.71 \qquad \textbf{Ans.}$$

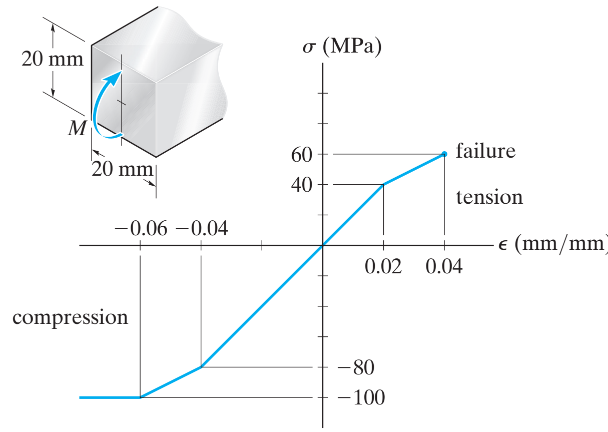


(a)

Ans:
$k = 1.71$

6–178.

The plexiglass bar has a stress–strain curve that can be approximated by the straight-line segments shown. Determine the largest moment M that can be applied to the bar before it fails.

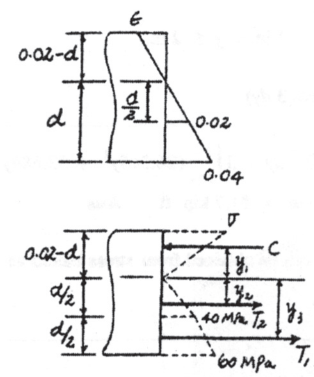


SOLUTION

Ultimate Moment:

$$\int_A \sigma\, dA = 0; \quad C - T_2 - T_1 = 0$$

$$\sigma\left[\frac{1}{2}(0.02 - d)(0.02)\right] - 40(10^6)\left[\frac{1}{2}\left(\frac{d}{2}\right)(0.02)\right] - \frac{1}{2}(60 + 40)(10^6)\left[(0.02)\frac{d}{2}\right] = 0$$



Since $\dfrac{0.04}{d} = \dfrac{\epsilon}{0.02-d}$, then $0.02 - d = 25\epsilon d$.

And since $\dfrac{\sigma}{\epsilon} = \dfrac{40(10^6)}{0.02} = 2(10^9)$, then $\epsilon = \dfrac{\sigma}{2(10^9)}$.

So then $0.02 - d = \dfrac{23\sigma d}{2(10^9)} = 1.25(10^{-8})\sigma d$.

Substituting for $0.02 - d$, then solving for σ, yields $\sigma = 74.833$ MPa. Then $\epsilon = 0.037417$ mm/mm and $d = 0.010334$ m.

Therefore,

$$C = 74.833\,(10^6)\left[\frac{1}{2}(0.02 - 0.010334)(0.02)\right] = 7233.59 \text{ N}$$

$$T_1 = \frac{1}{2}(60 + 40)\,(10^6)\left[(0.02)\left(\frac{0.010334}{2}\right)\right] = 5166.85 \text{ N}$$

$$T_2 = 40(10^6)\left[\frac{1}{2}(0.02)\left(\frac{0.010334}{2}\right)\right] = 2066.74 \text{ N}$$

$$y_1 = \frac{2}{3}(0.02 - 0.010334) = 0.0064442 \text{ m}$$

$$y_2 = \frac{2}{3}\left(\frac{0.010334}{2}\right) = 0.0034445 \text{ m}$$

$$y_3 = \frac{0.010334}{2} + \left[1 - \frac{1}{3}\left(\frac{2(40) + 60}{40 + 60}\right)\right]\left(\frac{0.010334}{2}\right) = 0.0079225 \text{ m}$$

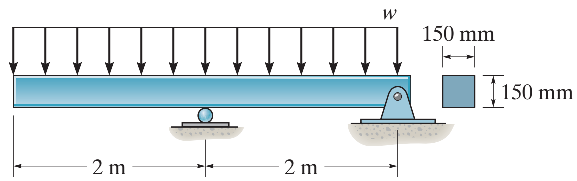
$$M = 7233.59(0.0064442) + 2066.74(0.0034445) + 5166.85(0.0079255)$$

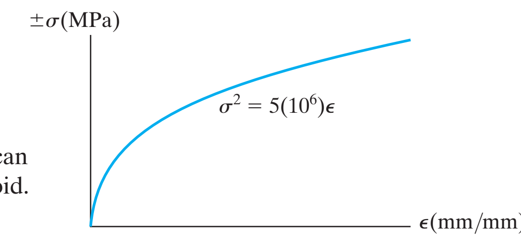
$$= 94.7 \text{ N}\cdot\text{m}$$ **Ans.**

Ans:
$M = 94.7\ \text{N}\cdot\text{m}$

6–179.

The beam is made of phenolic, a structural plastic, that has the stress–strain curve shown. If a portion of the curve can be represented by the equation $\sigma = (5(10^6)\epsilon)^{1/2}$ MPa, determine the magnitude w of the distributed load that can be applied to the beam without causing the maximum strain in its fibers at the critical section to exceed $\epsilon_{max} = 0.005$ mm/mm.





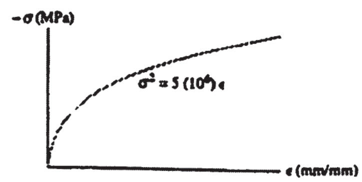
SOLUTION

Resultant Internal Forces: The resultant internal forces T and C can be evaluated from the volume of the stress block which is a paraboloid. When $\epsilon = 0.005$ mm/mm, then

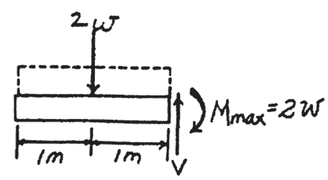
$$\sigma = \sqrt{5(10^6)(0.005)} = 158.11 \text{ MPa}$$

$$T = C = \frac{2}{3}\left[158.11(10^6)(0.075)\right](0.150) = 1.1859 \text{ MN}$$

$$d = 2\left[\frac{3}{5}(0.075)\right] = 0.090 \text{ m}$$



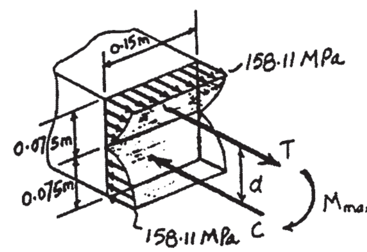
Maximum Internal Moment: The maximum internal moment $M = 2w$ occurs at the overhang support as shown on FBD.



$$M_{max} = Td$$

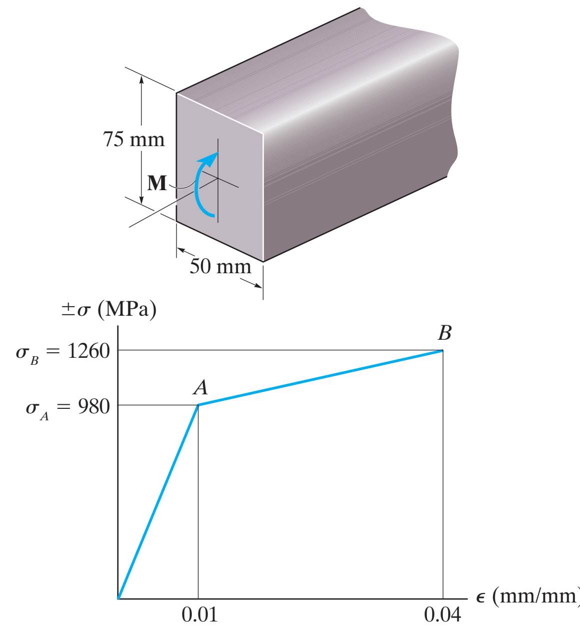
$$2w = 1.1859(10^6)(0.090)$$

$$w = 53363 \text{ N/m} = 53.4 \text{ kN/m} \qquad \textbf{Ans.}$$



Ans:
$w = 53.4$ kN/m

***6–180.** The stress–strain diagram for a titanium alloy can be approximated by the two straight lines. If a strut made of this material is subjected to bending, determine the moment resisted by the strut if the maximum stress reaches a value of (a) σ_A and (b) σ_B.



a) ***Maximum Elastic Moment :*** Since the stress is linearly related to strain up to point A, the flexure formula can be applied.

$$\sigma_A = \frac{Mc}{I}$$

$$M = \frac{\sigma_A I}{c}$$

$$= \frac{[980(10^6)]\left[\frac{1}{12}(0.05)(0.075^3)\right]}{0.0375}$$

$$= 45.9375(10^3)\ \text{N}\cdot\text{m} = 45.9\ \text{kN}\cdot\text{m}$$ **Ans.**

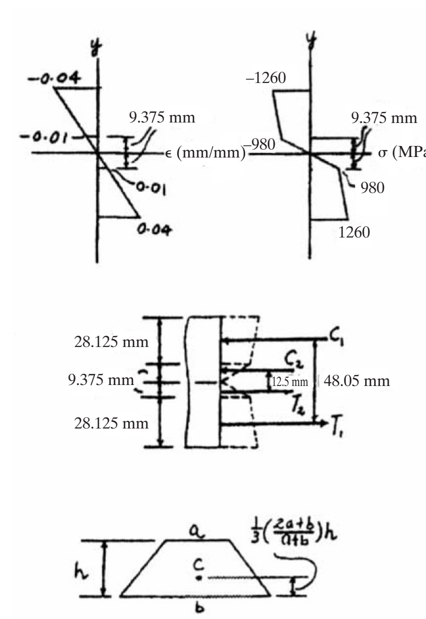
b) ***The Ultimate Moment :***

$$C_1 = T_1 = \frac{1}{2}[(1260 + 980)(10^6)](0.028125)(0.05) = 1575(10^3)\ \text{N}$$

$$C_2 = T_2 = \frac{1}{2}[980(10^6)](0.009375)(0.05) = 229.6875(10^3)\ \text{N}$$

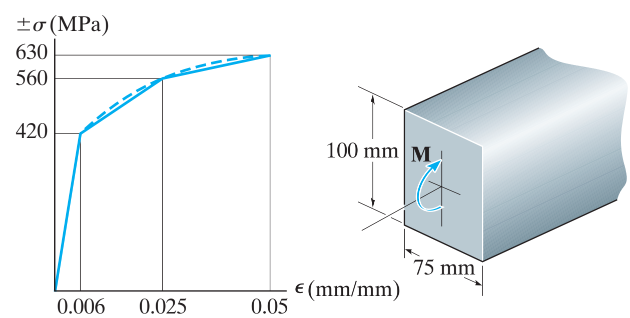
$$M = [1575(10^3)](0.04805) + [229.6875(10^3)](0.0125)$$

$$= 78.54(10^3)\ \text{N}\cdot\text{m} = 78.5\ \text{kN}\cdot\text{m}$$ **Ans.**

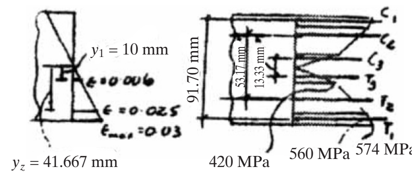


Note: The centroid of a trapezodial area was used in calculation of moment.

6–181. The bar is made of an aluminum alloy having a stress–strain diagram that can be approximated by the straight line segments shown. Assuming that this diagram is the same for both tension and compression, determine the moment the bar will support if the maximum strain at the top and bottom fibers of the beam is $\epsilon_{max} = 0.03$.



$$\frac{\sigma - 560}{0.03 - 0.025} = \frac{630 - 560}{0.05 - 0.025}; \qquad \sigma = 574 \text{ MPa}$$



$$C_1 = T_1 = \frac{1}{2}[(560 + 574)(10^6)](0.008333)(0.075) = 354.375(10^3) \text{ N}$$

$$C_2 = T_2 = \frac{1}{2}[(560 + 420)(10^6)](0.031667)(0.075) = 1163.75(10^3) \text{ N}$$

$$C_3 = T_3 = \frac{1}{2}[420(10^6)](0.01)(0.075) = 157.5(10^3) \text{ N}$$

$$M = [354.375(10^3)](0.09170) + [1163.75(10^3)](0.05317) + [157.5(10^3)](0.01333)$$

$$= 96.48(10^3) \text{ N}\cdot\text{m} = 96.5 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

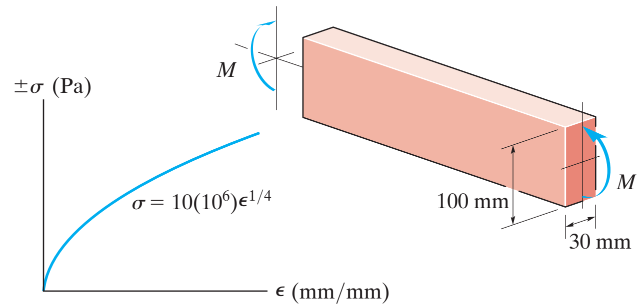
Note: The centroid of a trapezodial area was used in calculation of moment areas.

Ans:

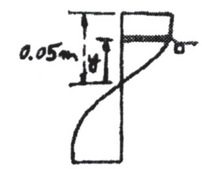
$\sigma = 574$ MPa, $M = 96.5$ kN·m

6–182.

A beam is made from polypropylene plastic and has a stress–strain diagram that can be approximated by the curve shown. If the beam is subjected to a maximum tensile and compressive strain of $\epsilon = 0.02$ mm/mm, determine the moment M.



SOLUTION

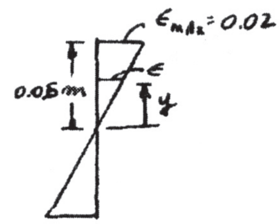


$\epsilon_{\max} = 0.02$

$\sigma_{\max} = 10\left(10^6\right)(0.02)^{1/4} = 3.761 \text{ MPa}$

$$\frac{0.02}{0.05} = \frac{\epsilon}{y}$$

$\epsilon = 0.4\,y$



$\sigma = 10\left(10^6\right)(0.4)^{1/4}y^{1/4}$

$$M = \int_A y\,\sigma\,dA = 2\int_0^{0.05} y(7.9527)\left(10^6\right)y^{1/4}(0.03)dy$$

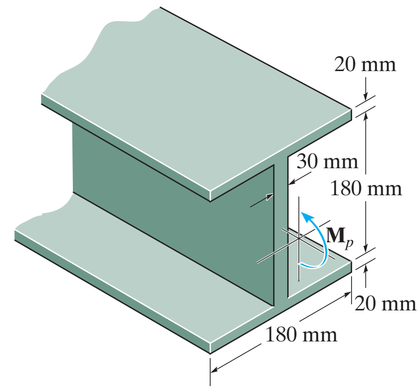
$$M = 0.47716\left(10^6\right)\int_0^{0.05} y^{5/4}dy = 0.47716\left(10^6\right)\left(\frac{4}{9}\right)(0.05)^{9/4}$$

$M = 251 \text{ N}\cdot\text{m}$ **Ans.**

Ans:
$M = 251 \text{ N}\cdot\text{m}$

R6–1.

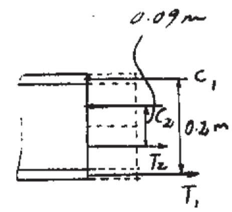
Determine the shape factor for the wide-flange beam.



SOLUTION

$$I = \frac{1}{12}(0.18)(0.22^3) - \frac{1}{12}(0.15)(0.18^3)$$

$$= 86.82(10^{-6})\text{ m}^4$$



Plastic moment:

$$M_p = \sigma_Y(0.18)(0.02)(0.2) + \sigma_Y(0.09)(0.03)(0.09)$$
$$= 0.963(10^{-3})\sigma_Y$$

Shape Factor:

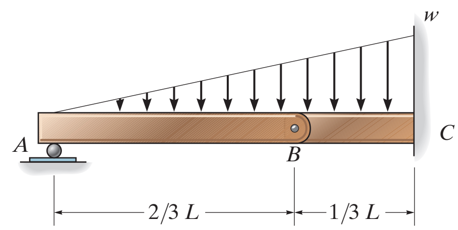
$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y(86.82)(10^{-6})}{0.11} = 0.789273(10^{-3})\sigma_Y$$

$$k = \frac{M_p}{M_Y} = \frac{0.963(10^{-3})\sigma_Y}{0.789273(10^{-3})\sigma_Y} = 1.22$$ **Ans.**

Ans:
$k = 1.22$

R6–2.

The compound beam consists of two segments that are pinned together at B. Draw the shear and moment diagrams if it supports the distributed loading shown.



SOLUTION

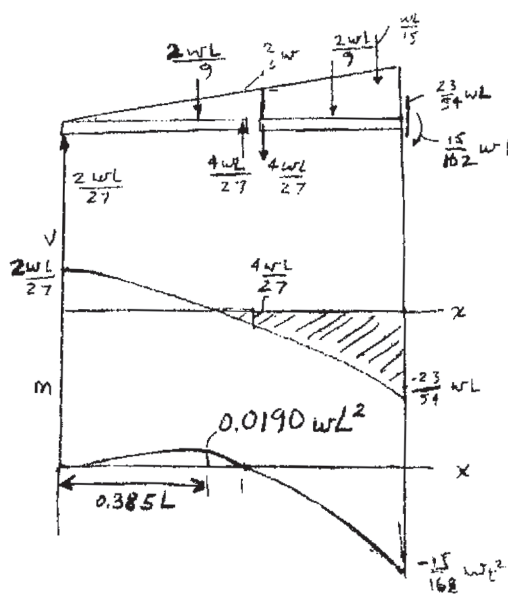
$+\uparrow\Sigma F_y = 0;\qquad \dfrac{2wL}{27} - \dfrac{1}{2}\dfrac{w}{L}x^2 = 0$

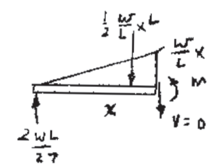
$$x = \sqrt{\frac{4}{27}}L = 0.385\ L$$

$\circlearrowleft + \Sigma M = 0;\qquad M + \dfrac{1}{2}\dfrac{w}{L}(0.385L)^2\left(\dfrac{1}{3}\right)(0.385L) - \dfrac{2wL}{27}(0.385L) = 0$

$$M = 0.0190\ wL^2$$

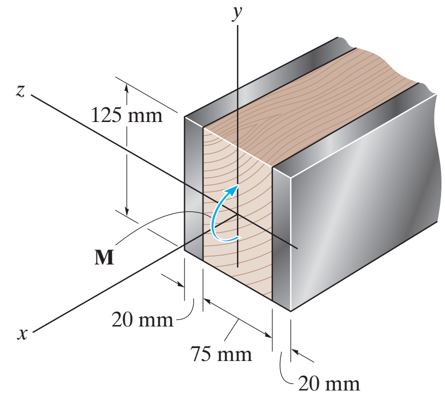
$M = 0.0190\ wL^2$





R6–3.

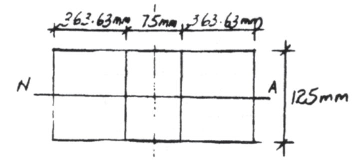
The composite beam consists of a wood core and two plates of steel. If the allowable bending stress for the wood is $(\sigma_{allow})_w = 20$ MPa, and for the steel $(\sigma_{allow})_{st} = 130$ MPa, determine the maximum moment that can be applied to the beam. $E_w = 11$ GPa, $E_{st} = 200$ GPa.



SOLUTION

$$n = \frac{E_{st}}{E_w} = \frac{200(10^9)}{11(10^9)} = 18.182$$

$$I = \frac{1}{12}(0.80227)(0.125^3) = 0.130578(10^{-3})\text{m}^4$$



Failure of wood:

$$(\sigma_w)_{max} = \frac{Mc}{I}$$

$$20(10^6) = \frac{M(0.0625)}{0.130578(10^{-3})}; \qquad M = 41.8 \text{ kN}\cdot\text{m}$$

Failure of steel:

$$(\sigma_{st})_{max} = \frac{nMc}{I}$$

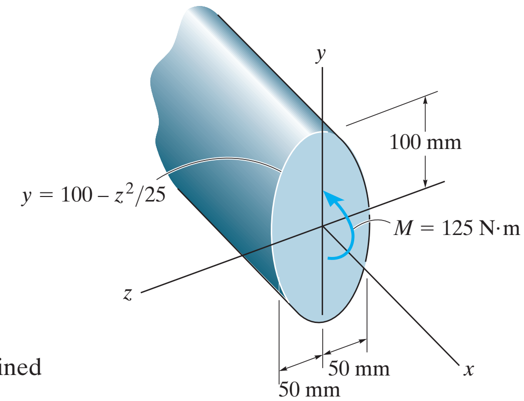
$$130(10^6) = \frac{18.182(M)(0.0625)}{0.130578(10^{-3})}$$

$$M = 14.9 \text{ kN}\cdot\text{m} \quad \text{(controls)}$$ **Ans.**

Ans:
$M = 14.9$ kN·m

***R6–4.**

A shaft is made of a polymer having a parabolic upper and lower cross section. If it resists a moment of $M = 125\ \text{N}\cdot\text{m}$, determine the maximum bending stress in the material (a) using the flexure formula and (b) using integration. Sketch a three-dimensional view of the stress distribution acting over the cross-sectional area. *Hint:* The moment of inertia is determined using Eq. A–3 of Appendix A.



SOLUTION

Maximum Bending Stress: The moment of inertia about y axis must be determined first in order to use flexure formula

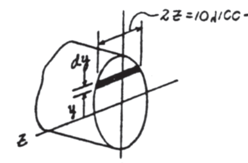
$$I = \int_A y^2\,dA$$

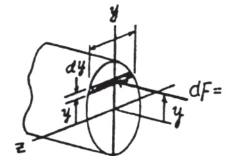
$$= 2\int_0^{100\text{ mm}} y^2\,(2z)\,dy$$

$$= 20\int_0^{100\text{ mm}} y^2\sqrt{100-y}\,dy$$

$$= 20\left[-\frac{3}{2}y^2(100-y)^{\frac{3}{2}} - \frac{8}{15}y(100-y)^{\frac{5}{2}} - \frac{16}{105}(100-y)^{\frac{7}{2}}\right]\Bigg|_0^{100}\text{ mm}$$

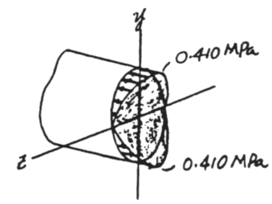
$$= 30.4762\left(10^6\right)\text{ mm}^4 = 30.4762\left(10^{-6}\right)\text{ m}^4$$





Thus,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{125(0.1)}{30.4762(10^{-6})} = 0.410\text{ MPa}$$ **Ans.**



Maximum Bending Stress: Using integration

$$dM = 2[y(\sigma\,dA)] = 2\left\{y\left[\left(\frac{\sigma_{\max}}{100}\right)y\right](2z\,dy)\right\}$$

$$M = \frac{\sigma_{\max}}{5}\int_0^{100\text{ mm}} y^2\sqrt{100-y}\,dy$$

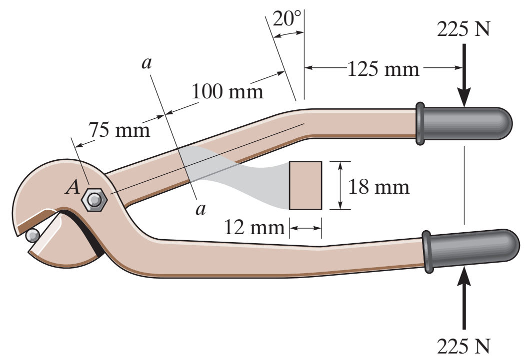
$$125\left(10^3\right) = \frac{\sigma_{\max}}{5}\left[-\frac{3}{2}y^2(100-y)^{\frac{3}{2}} - \frac{8}{15}y(100-y)^{\frac{5}{2}} - \frac{16}{105}(100-y)^{\frac{7}{2}}\right]\Bigg|_0^{100}\text{ mm}$$

$$125\left(10^3\right) = \frac{\sigma_{\max}}{5}(1.5238)\left(10^6\right)$$

$$\sigma_{\max} = 0.410\text{ N/mm}^2 = 0.410\text{ MPa}$$ **Ans.**

Ans:
$\sigma_{\max} = 0.410\text{ MPa}$

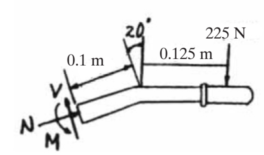
R6–5. Determine the maximum bending stress in the handle of the cable cutter at section *a–a*. A force of 225 N is applied to the handles. The cross-sectional area is shown in the figure.



$\circlearrowleft + \Sigma M = 0;$ $\quad M - 225(0.125 + 0.1\cos 20°) = 0$

$$M = 49.27 \text{ N} \cdot \text{m}$$

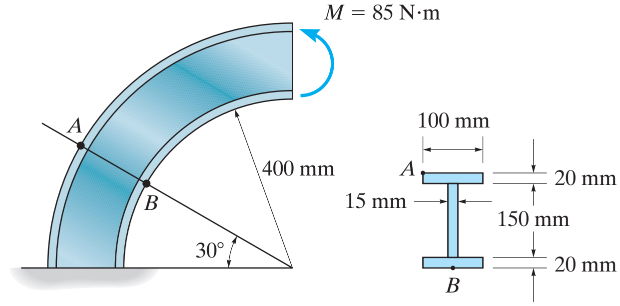
$$\sigma_{\max} = \frac{Mc}{I} = \frac{49.27(0.009)}{\frac{1}{12}(0.012)(0.018^3)} = 76.03(10^6) \text{ N/m}^2 = 76.0 \text{ MPa}$$ **Ans.**



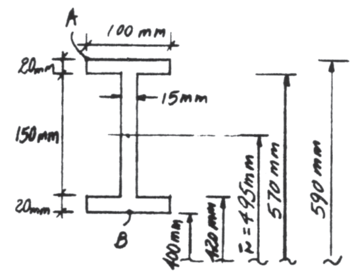
Ans:
$\sigma_{\max} = 76.0$ MPa

R6–6.

The curved beam is subjected to a bending moment of $M = 85\ \text{N}\cdot\text{m}$ as shown. Determine the stress at points A and B and show the stress on a volume element located at these points.



SOLUTION



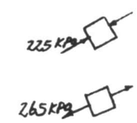
$$\int_A \frac{dA}{r} = b \ln \frac{r_2}{r_1} = 0.1 \ln \frac{0.42}{0.40} + 0.015 \ln \frac{0.57}{0.42} + 0.1 \ln \frac{0.59}{0.57}$$

$$= 0.012908358\ \text{m}$$

$$A = 2(0.1)(0.02) + (0.15)(0.015) = 6.25(10^{-3})\ \text{m}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{6.25(10^{-3})}{0.012908358} = 0.484182418\ \text{m}$$

$$\bar{r} - R = 0.495 - 0.484182418 = 0.010817581\ \text{m}$$



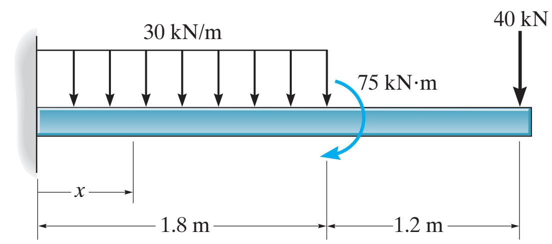
$$\sigma_A = \frac{M(R - r_A)}{Ar_A(\bar{r} - R)} = \frac{85(0.484182418 - 0.59)}{6.25(10^{-3})(0.59)(0.010817581)} = -225.48\ \text{kPa}$$

$\sigma_A = 225\ \text{kPa (C)}$ **Ans.**

$$\sigma_B = \frac{M(R - r_B)}{Ar_B(\bar{r} - R)} = \frac{85(0.484182418 - 0.40)}{6.25(10^{-3})(0.40)(0.010817581)} = 265\ \text{kPa (T)}$$ **Ans.**

Ans:
$\sigma_A = 225\ \text{kPa (C)}, \sigma_B = 265\ \text{kPa (T)}$

R6–7. Draw the shear and moment diagrams for the beam and determine the shear and moment in the beam as functions of x, where $0 \leq x < 1.8$ m.

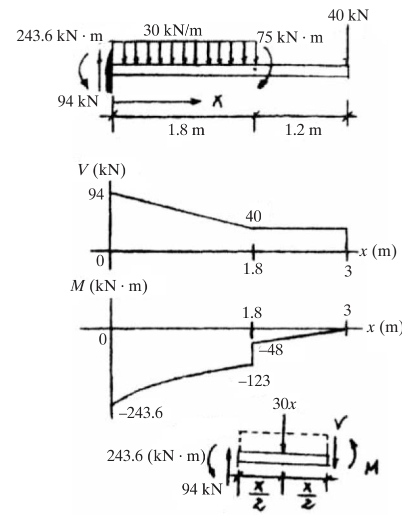


$+\uparrow\ \Sigma F_y = 0;\quad 94 - 30x - V = 0$

$V = \{94 - 30x\}$ kN **Ans.**

$\circlearrowleft + \Sigma M_{NA} = 0;\quad 94x - 243.6 - 30x\left(\dfrac{x}{2}\right) - M = 0$

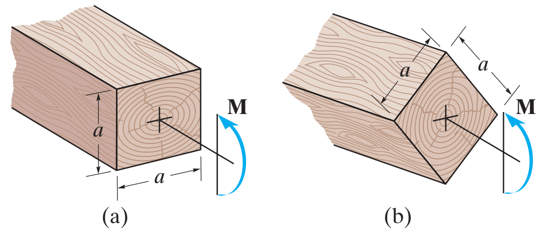
$M = \{-15x^2 + 94x - 243.6\}$ kN·m **Ans.**



Ans:
$V = 94 - 30x$, $M = -15x^2 + 94x - 243.6$

***R6–8.**

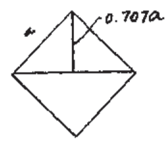
A wooden beam has a square cross section as shown. Determine which orientation of the beam provides the greatest strength at resisting the moment **M**. What is the difference in the resulting maximum stress in both cases?



SOLUTION

Case (a):

$$\sigma_{\max} = \frac{Mc}{I} = \frac{M(a/2)}{\frac{1}{12}(a)^4} = \frac{6M}{a^3}$$



Case (b):

$$I = 2\left[\frac{1}{36}\left(\frac{2}{\sqrt{2}}a\right)\left(\frac{1}{\sqrt{2}}a\right)^3 + \frac{1}{2}\left(\frac{2}{\sqrt{2}}a\right)\left(\frac{1}{\sqrt{2}}a\right)\left[\left(\frac{1}{\sqrt{2}}a\right)\left(\frac{1}{3}\right)\right]^2\right] = 0.08333\,a^4$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{M\left(\frac{1}{\sqrt{2}}a\right)}{0.08333\,a^4} = \frac{8.4853\,M}{a^3}$$

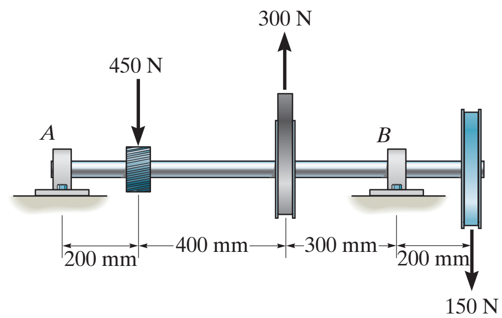
Case (a) provides higher strength, since the resulting maximum stress is less for a given M and a.

$$\Delta\sigma_{\max} = \frac{8.4853\,M}{a^3} - \frac{6M}{a^3} = 2.49\left(\frac{M}{a^3}\right) \qquad \textbf{Ans.}$$

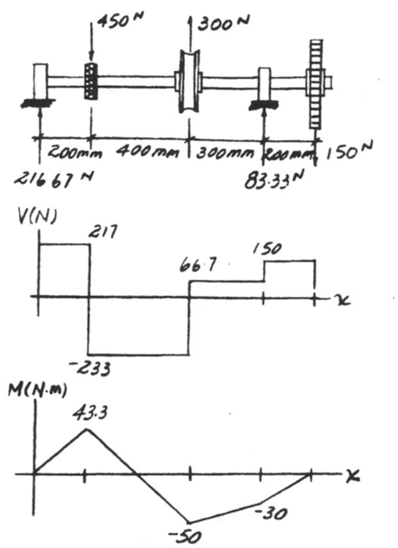
Ans:
Case (a),
$\Delta\sigma_{\max} = 2.49\left(\frac{M}{a^3}\right)$

R6–9.

Draw the shear and moment diagrams for the shaft if it is subjected to the vertical loadings. The bearings at A and B exert only vertical reactions on the shaft.

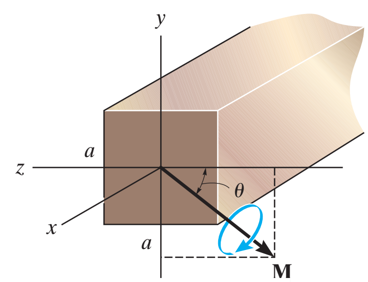


SOLUTION

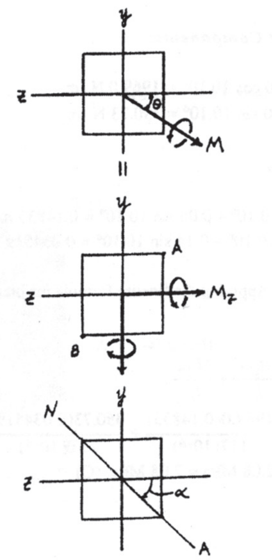


R6–10.

The strut has a square cross section a by a and is subjected to the bending moment **M** applied at an angle θ as shown. Determine the maximum bending stress in terms of a, M, and θ. What angle θ will give the largest bending stress in the strut? Specify the orientation of the neutral axis for this case.



SOLUTION



Internal Moment Components:

$$M_z = -M\cos\theta \qquad M_y = -M\sin\theta$$

Section Property:

$$I_y = I_z = \frac{1}{12}a^4$$

Maximum Bending Stress: By inspection, maximum bending stress occurs at A and B. Applying the flexure formula for biaxial bending at point A

$$\sigma_{\max} = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$= -\frac{-M\cos\theta\left(\frac{a}{2}\right)}{\frac{1}{12}a^4} + \frac{-M\sin\theta\left(-\frac{a}{2}\right)}{\frac{1}{12}a^4}$$

$$= \frac{6M}{a^3}(\cos\theta + \sin\theta) \qquad \textbf{Ans.}$$

$$\frac{d\sigma}{d\theta} = \frac{6M}{a^3}(-\sin\theta + \cos\theta) = 0$$

$$\cos\theta - \sin\theta = 0$$

$$\theta = 45^\circ \qquad \textbf{Ans.}$$

Orientation of Neutral Axis:

$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$

$$\tan\alpha = (1)\tan(45^\circ)$$

$$\alpha = 45^\circ \qquad \textbf{Ans.}$$

Ans:

$\sigma_{\max} = \dfrac{6M}{a^3}(\cos\theta + \sin\theta)$,

$\theta = 45^\circ$, $\alpha = 45^\circ$