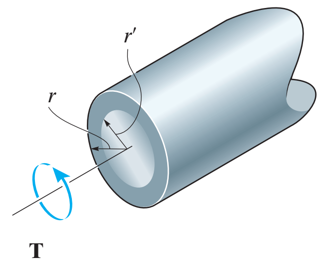
5–1.

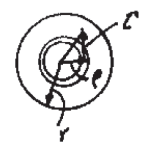
The solid shaft of radius r is subjected to a torque **T**. Determine the radius r' of the inner core of the shaft that resists one-half of the applied torque $(T/2)$. Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.



SOLUTION

$$\text{a) } \tau_{\max} = \frac{Tc}{J} = \frac{Tr}{\frac{\pi}{2} r^4} = \frac{2T}{\pi r^3}$$

$$\tau = \frac{\left(\frac{T}{2}\right) r'}{\frac{\pi}{2}(r')^4} = \frac{T}{\pi (r')^3}$$



$$\text{Since } \tau = \frac{r'}{r}\tau_{\max}; \qquad \frac{T}{\pi (r')^3} = \frac{r'}{r}\left(\frac{2T}{\pi r^3}\right)$$

$$r' = \frac{r}{2^{\frac{1}{4}}} = 0.841r$$ **Ans.**

$$\text{b) } \int_0^{\frac{r}{2}} dT = 2\pi \int_0^{r'} \tau \rho^2 \, d\rho$$

$$\int_0^{\frac{r}{2}} dT = 2\pi \int_0^{r'} \frac{\rho}{r}\tau_{\max} \rho^2 \, d\rho$$

$$\int_0^{\frac{r}{2}} dT = 2\pi \int_0^{r'} \frac{\rho}{r}\left(\frac{2T}{\pi r^3}\right) \rho^2 \, d\rho$$

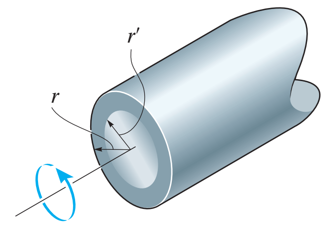
$$\frac{T}{2} = \frac{4T}{r^4} \int_0^{r'} \rho^3 \, d\rho$$

$$r' = \frac{r'}{2^{\frac{1}{4}}} = 0.841r$$ **Ans.**

Ans:
$r' = 0.841r$

5–2.

The solid shaft of radius r is subjected to a torque **T**. Determine the radius r' of the inner core of the shaft that resists one-quarter of the applied torque $(T/4)$. Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.

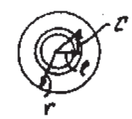


SOLUTION

a) $\tau_{max} = \dfrac{Tc}{J} = \dfrac{T(r)}{\frac{\pi}{2}(r^4)} = \dfrac{2T}{\pi r^3}$

Since $\tau = \dfrac{r'}{r}\tau_{max} = \dfrac{2Tr'}{\pi r^4}$

$\tau' = \dfrac{T'c'}{J'}; \quad \dfrac{2Tr'}{\pi r^4} = \dfrac{(\frac{T}{4})r'}{\frac{\pi}{2}(r')^4}$



$r' = \dfrac{r}{4^{\frac{1}{4}}} = 0.707\, r$ **Ans.**

b) $\tau = \dfrac{\rho}{c}\tau_{max} = \dfrac{\rho}{r}\left(\dfrac{2T}{\pi r^3}\right) = \dfrac{2T}{\pi r^4}\rho; \qquad dA = 2\pi\rho\, d\rho$

$dT = \rho\tau\, dA = \rho\left[\dfrac{2T}{\pi r^4}\rho\right](2\pi\rho\, d\rho) = \dfrac{4T}{r^4}\rho^3 d\rho$

$\displaystyle\int_0^{\frac{T}{4}} dT = \frac{4T}{r^4}\int^{r'} \rho^3 d\rho$

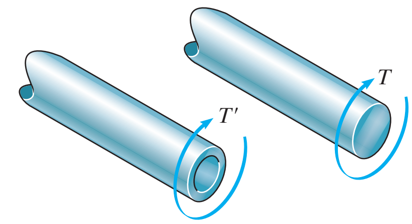
$\dfrac{T}{4} = \dfrac{4T}{r^4}\dfrac{\rho^4}{4}\Big|_0^{r'}; \qquad \dfrac{1}{4} = \dfrac{(r')^4}{r^4}$

$r' = 0.707\, r$ **Ans.**

Ans:
$r' = 0.707\, r$

5–3.

A shaft is made of an aluminum alloy having an allowable shear stress of $\tau_{allow} = 100$ MPa. If the diameter of the shaft is 100 mm, determine the maximum torque **T** that can be transmitted. What would be the maximum torque **T**′ if a 75-mm-diameter hole were bored through the shaft? Sketch the shear-stress distribution along a radial line in each case.



SOLUTION

Allowable Shear Stress: Torsion formula can be applied. For the solid shaft,

$$\tau_{max} = \tau_{allow} = \frac{Tc}{J}; \; 100\left(10^6\right) = \frac{T(0.05)}{\frac{\pi}{2}\left(0.05^4\right)}$$

$$T = 19.63\left(10^3\right)\text{ N}\cdot\text{m} = 19.6\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

For the hollow shaft,

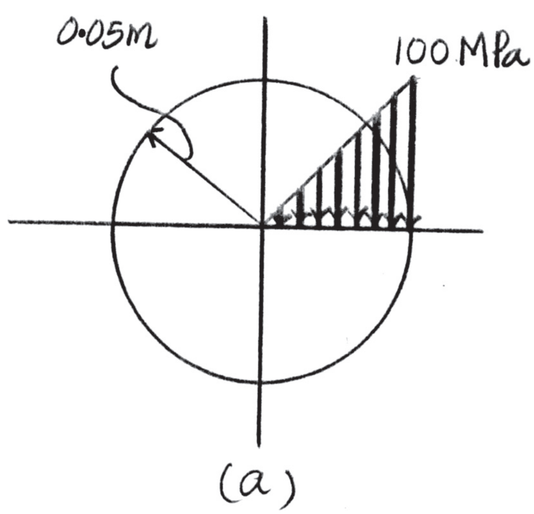
$$\tau_{max} = \tau_{allow} = \frac{Tc}{J}; \; 100\left(10^6\right) = \frac{T'(0.05)}{\frac{\pi}{2}\left(0.05^4 - 0.0375^4\right)}$$

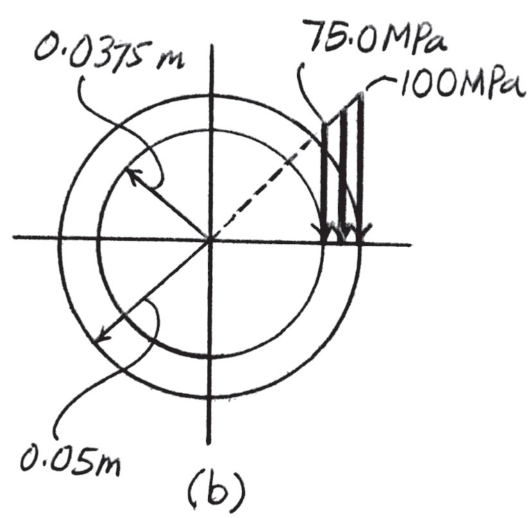
$$T' = 13.42\left(10^3\right)\text{ N}\cdot\text{m} = 13.4\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

The shear stress at the inner surface where $\rho = 0.0375$ m is

$$\tau_\rho = 0.0375\text{ m} = \frac{T'\rho}{\text{J}} = \frac{13.42\left(10^3\right)(0.0375)}{\frac{\pi}{2}\left(0.05^4 - 0.0375^4\right)} = 75.0\left(10^6\right)\text{ Pa} = 75.0\text{ MPa}$$

The shear stress distribution along the radius of the cross-section of the solid and hollow shafts are shown in Figs. *a* and *b* respectively.

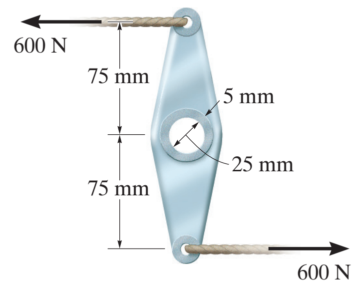




Ans:
$T = 19.6$ kN · m,
$T' = 13.4$ kN · m

***5–4.**

The link acts as part of the elevator control for a small airplane. If the attached aluminum tube has an inner diameter of 25 mm and a wall thickness of 5 mm, determine the maximum shear stress in the tube when the cable force of 600 N is applied to the cables. Also, sketch the shear-stress distribution over the cross section.

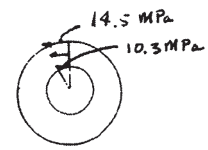


SOLUTION

$T = 600(0.15) = 90\ \text{N}\cdot\text{m}$

$$\tau_{max} = \frac{Tc}{J} = \frac{90(0.0175)}{\frac{\pi}{2}\left[(0.0175)^4 - (0.0125)^4\right]} = 14.5\ \text{MPa}$$ **Ans.**

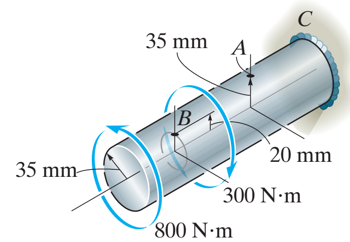
$$\tau_i = \frac{T\rho}{J} = \frac{90(0.0125)}{\frac{\pi}{2}\left[(0.0175)^4 - (0.0125)^4\right]} = 10.3\ \text{MPa}$$



Ans:
$\tau_{max} = 14.5\ \text{MPa}$

5–5.

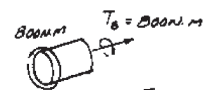
The solid shaft is fixed to the support at C and subjected to the torsional loadings. Determine the shear stress at points A and B on the surface, and sketch the shear stress on volume elements located at these points.

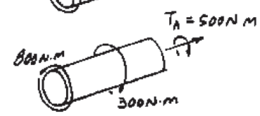


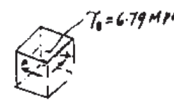
SOLUTION

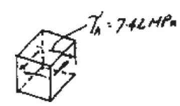
$$\tau_B = \frac{T_B\,\rho}{J} = \frac{800(0.02)}{\frac{\pi}{2}(0.035^4)} = 6.79 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_A = \frac{T_A\,c}{J} = \frac{500(0.035)}{\frac{\pi}{2}(0.035^4)} = 7.42 \text{ MPa} \qquad \textbf{Ans.}$$





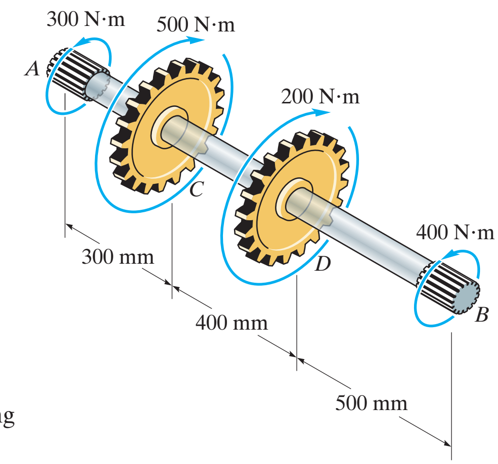




Ans:
$\tau_B = 6.79$ MPa, $\tau_A = 7.42$ MPa

5–6.

The solid 30-mm-diameter shaft is used to transmit the torques applied to the gears. Determine the absolute maximum shear stress in the shaft.



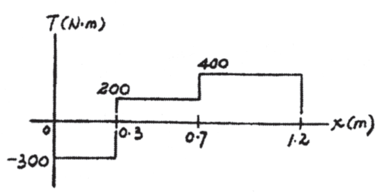
SOLUTION

Internal Torque: As shown on torque diagram.

Maximum Shear Stress: From the torque diagram $T_{max} = 400\ \text{N} \cdot \text{m}$. Then, applying torsion formula,

$$\tau_{\substack{abs \\ max}} = \frac{T_{max}\, c}{J}$$

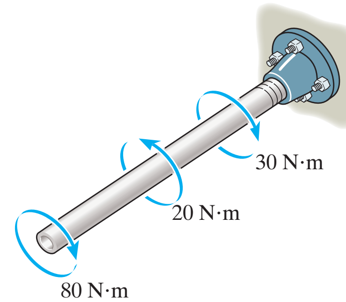
$$= \frac{400(0.015)}{\frac{\pi}{2}(0.015^4)} = 75.5\ \text{MPa} \qquad \textbf{Ans.}$$



Ans:
$\tau_{\substack{abs \\ max}} = 75.5\ \text{MPa}$

5–7.

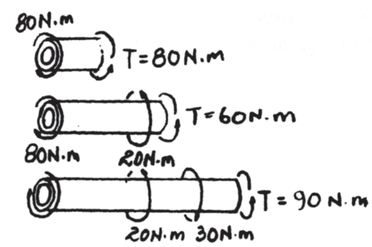
The copper pipe has an outer diameter of 40 mm and an inner diameter of 37 mm. If it is tightly secured to the wall and three torques are applied to it, determine the absolute maximum shear stress developed in the pipe.



SOLUTION

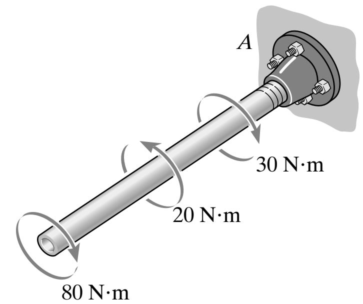
$$\tau_{\max} = \frac{T_{\max}\,c}{J} = \frac{90(0.02)}{\frac{\pi}{2}(0.02^4 - 0.0185^4)}$$

$$= 26.7 \text{ MPa}$$ **Ans.**



Ans:
$\tau_{\max} = 26.7$ MPa

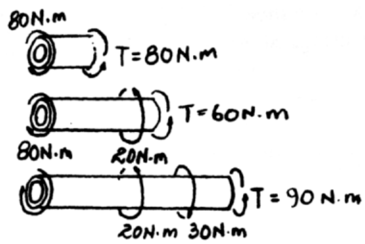
***5–8.** The copper pipe has an outer diameter of 40 mm and an inner diameter of 37 mm. If it is tightly secured to the wall at A and three torques are applied to it as shown, determine the absolute maximum shear stress developed in the pipe.



SOLUTION

$$\tau_{\max} = \frac{T_{\max}\, c}{J} = \frac{90(0.02)}{\frac{\pi}{2}(0.02^4 - 0.0185^4)}$$

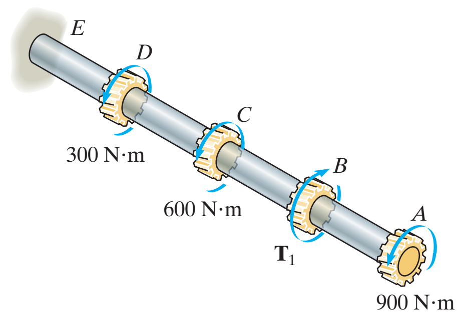
$$= 26.7 \text{ MPa}$$ **Ans..**



Ans:
$\tau_{\max} = 26.7$ MPa

5–9.

The solid aluminum shaft has a diameter of 50 mm and an allowable shear stress of $\tau_{allow} = 60$ MPa. Determine the largest torque T_1 that can be applied to the shaft if it is also subjected to the other torsional loadings. It is required that $\mathbf{T}_1$ act in the direction shown. Also, determine the maximum shear stress within regions CD and DE.



SOLUTION

Internal Torque: Assuming that failure occurs at region BC of the shaft, where the torque will be greatest. Referring to the FBD of the right segment of the shaft sectioned through region BC, Fig. *a*

$$\Sigma M_x = 0; \qquad T_1 - 900 - T_{BC} = 0 \qquad T_{BC} = T_1 - 900$$

Maximum Shear Stress: Applying the torsion formula,

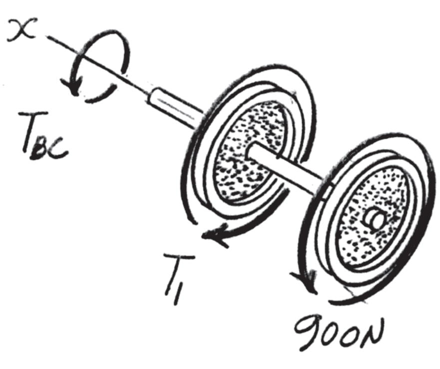
$$\tau_{max} = \tau_{allow} = \frac{T_{BC}\,c}{J}; \qquad 60(10^6) = \frac{(T_1 - 900)(0.025)}{\frac{\pi}{2}(0.025^4)}$$

$$(T_1)_{max} = T_1 = 2372.62 \text{ N}\cdot\text{m} = 2.37 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

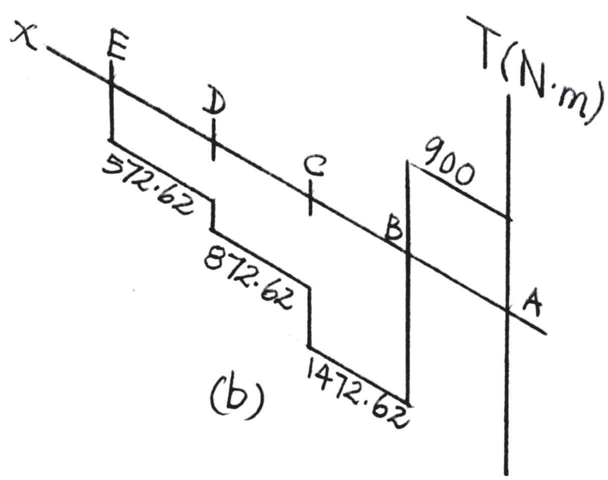
using this result the torque diagram shown in Fig. *b* can be plotted. This indicates that region BC indeed is subjected to maximum internal torque, thus, the critical region. From the torque diagram, the internal torques in regions CD and DE are $T_{CD} = 872.62$ N · m and $T_{DE} = 572.62$ N · m respectively.

$$(\tau_{max})_{CD} = \frac{T_{CD}\,C}{J} = \frac{872.62\,(0.025)}{\frac{\pi}{2}(0.025^4)} = 35.55\,(10^6)\text{ Pa} = 35.6\text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{max})_{DE} = \frac{T_{DC}\,C}{J} = \frac{572.62\,(0.025)}{\frac{\pi}{2}(0.025^4)} = 23.33\,(10^6)\text{ Pa} = 23.3\text{ MPa} \qquad \textbf{Ans.}$$



(a)

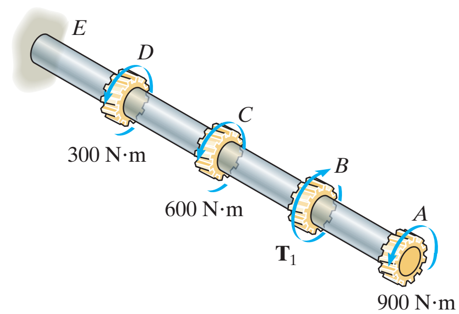


(b)

Ans:
$(T_1)_{max} = 2.37$ kN · m,
$(\tau_{max})_{CD} = 35.6$ MPa,
$(\tau_{max})_{DE} = 23.3$ MPa

5–10.

The solid aluminum shaft has a diameter of 50 mm. Determine the absolute maximum shear stress in the shaft and sketch the shear-stress distribution along a radial line of the shaft where the shear stress is maximum. Set $T_1 = 2000\ \text{N} \cdot \text{m}$.

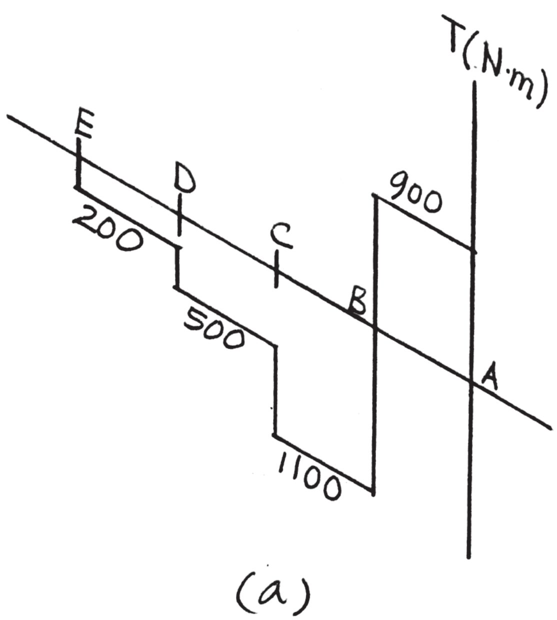


SOLUTION

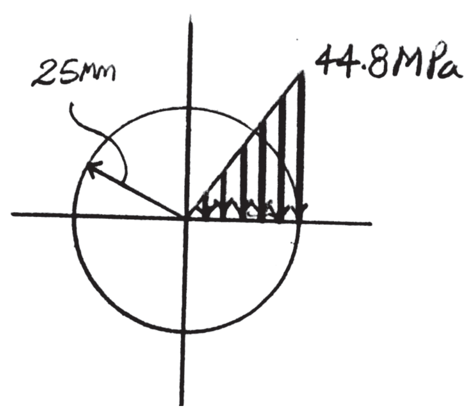
Internal Torque: The torque diagram plotted in Fig. *a* indicates that region *BC* of the shaft is subjected to the greatest internal torque; thus, the critical region where the absolute maximum shear stress occurs. Here $T_{BC} = 1100\ \text{N} \cdot \text{m}$. Applying the torsion formula,

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{T_{BC}\,c}{J} = \frac{1100\,(0.025)}{\frac{\pi}{2}\left(0.025^4\right)} = 44.82\left(10^6\right)\ \text{Pa} = 44.8\ \text{MPa} \qquad \textbf{Ans.}$$

The shear stress distribution along the radius of the cross-section of the shaft is shown in Fig. *b*.



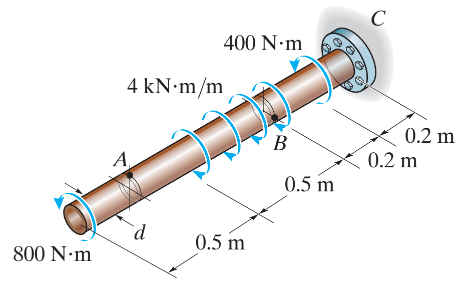
(a)



Ans:
$\tau_{\substack{\text{abs}\\\text{max}}} = 44.8\ \text{MPa}$

5–11.

The 60-mm-diameter solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the absolute maximum and minimum shear stresses on the shaft's surface and specify their locations, measured from the free end.



SOLUTION

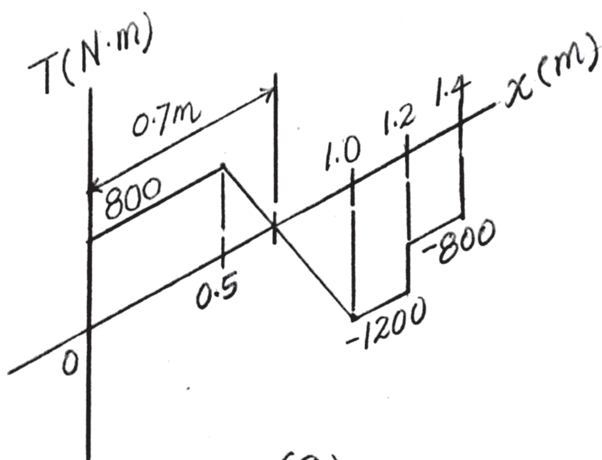
Internal Torque: The torque diagram plotted in Fig. *a* indicates that region $1.0\ \text{m} < x < 1.2\ \text{m}$ of the shaft is subjected to the greatest internal torque where $T_{\max} = 1200\ \text{N} \cdot \text{m}$, whereas the minimum internal torque, $T_{\min} = 0$, occurs at $x = 0.7$ m.

Shear Stress: Applying the torsion formula,

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{T_{\max}\,C}{J} = \frac{1200\,(0.03)}{\frac{\pi}{2}\left(0.03^4\right)} = 28.29\left(10^6\right)\ \text{Pa} = 28.3\ \text{MPa} \qquad \textbf{Ans.}$$

occurs within the region $1.0\ \text{m} < x < 1.2\ \text{m}$ **Ans.**

$$\tau_{\substack{\text{abs}\\\text{min}}} = \frac{T_{\min}\,C}{J} \qquad \text{occurs at } x = 0.700\ \text{m} \qquad \textbf{Ans.}$$

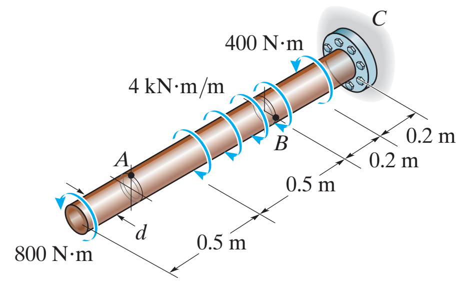


(a)

Ans:
$\tau_{\substack{\text{abs}\\\text{max}}} = 28.3$ MPa,
for $1.0\ \text{m} < x < 1.2\ \text{m}$,
$\tau_{\substack{\text{abs}\\\text{min}}} = 0$,
at $x = 0.700$ m

***5–12.**

The solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the required diameter d of the shaft if the allowable shear stress for the material is $\tau_{allow} = 60$ MPa.



SOLUTION

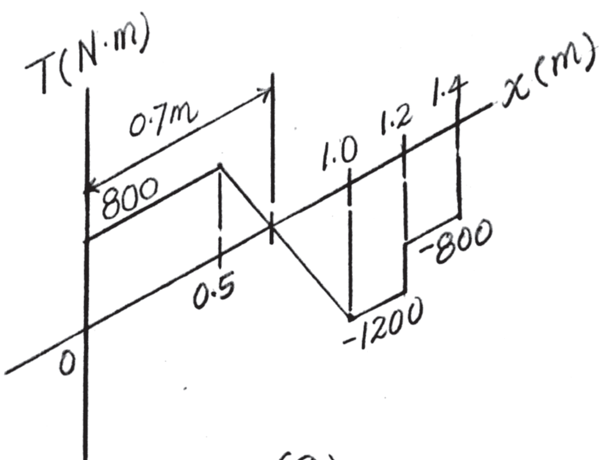
Internal Torque: The torque diagram plotted in Fig. *a* indicates that region $1.0\text{ m} < x < 1.2\text{ m}$ of the shaft is subjected to the greatest internal torque $T_{max} = 1200\text{ N}\cdot\text{m}$; thus, that is the critical region where the absolute maximum shear stress occurs.

Allowable Shear Stress: Applying the torsion formula,

$$\tau_{\substack{abs\\max}} = \tau_{allow} = \frac{T_{max}\,C}{J}$$

$$60\left(10^6\right) = \frac{1200\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

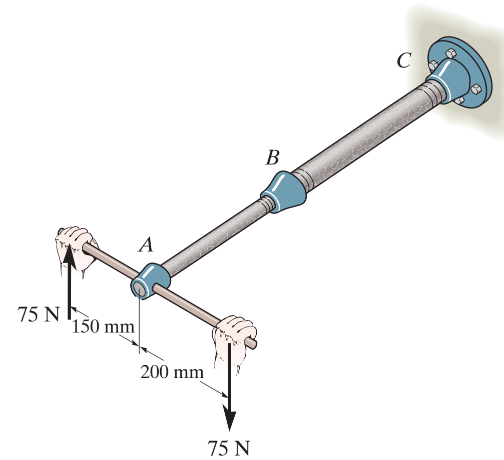
$$d = 0.04670\text{ m} = 46.7\text{ mm}$$ **Ans.**



(a)

Ans:
$d = 46.7$ mm

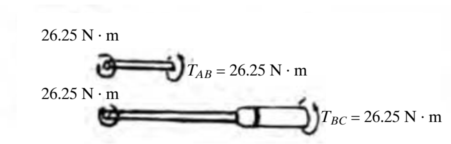
5–13. The assembly consists of two sections of galvanized steel pipe connected together using a reducing coupling at B. The smaller pipe has an outer diameter of 18.75 mm and an inner diameter of 17 mm, whereas the larger pipe has an outer diameter of 25 mm and an inner diameter of 21.5 mm. If the pipe is tightly secured to the wall at C, determine the maximum shear stress developed in each section of the pipe when the couple shown is applied to the handles of the wrench.



SOLUTION

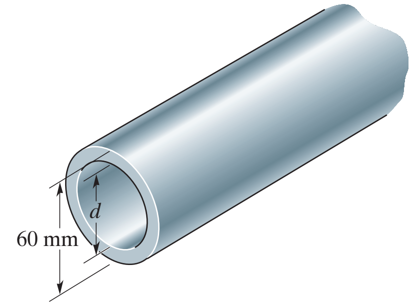
$$\tau_{AB} = \frac{Tc}{J} = \frac{26.25(0.009375)}{\frac{\pi}{2}(0.009375^4 - 0.0085^4)} = 62.54(10^6)\ \text{N/m}^2 = 62.5\ \text{MPa}$$ **Ans.**

$$\tau_{BC} = \frac{Tc}{J} = \frac{26.25(0.0125)}{\frac{\pi}{2}(0.0125^4 - 0.01075^4)} = 18.89(10^6)\ \text{N/m}^2 = 18.9\ \text{MPa}$$ **Ans.**



Ans:
$\tau_{AB} = 62.5$ MPa, $\tau_{BC} = 18.9$ MPa

5–14. A steel tube having an outer diameter of 60 mm is used to transmit 6.75 kW when turning at 27 rev/min. Determine the inner diameter d of the tube to the nearest mm if the allowable shear stress is $\tau_{allow} = 70$ MPa.



SOLUTION

$$\omega = \left(27\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 0.9\pi\text{ rad/s}$$

$$P = T\omega$$

$$6.75(10^3) = T(0.9\pi)$$

$$T = 2.387(10^3)\text{ N}\cdot\text{m}$$

$$\tau_{max} = \tau_{allow} = \frac{Tc}{J}$$

$$70(10^6) = \frac{2.387(10^3)(0.03)}{\frac{\pi}{2}\left(0.03^4 - c_i^4\right)}$$

$$c_i = 0.01996\text{ m}$$

$$d = 2c_i = 2(0.01996) = 0.03992\text{ m} = 39.92\text{ mm}$$

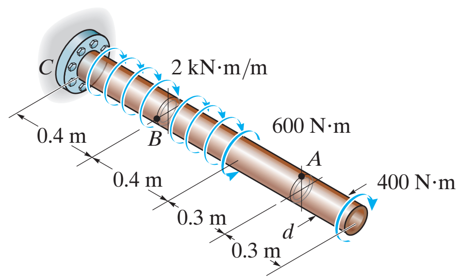
Use $d = 40$ mm **Ans.**

Ans:

Use $d = 40$ mm

5–15.

The 60-mm-diameter solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the shear stress at points A and B, and sketch the shear stress on volume elements located at these points.



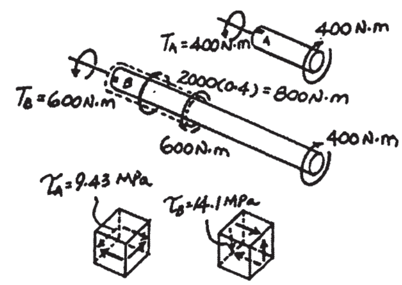
SOLUTION

Internal Torque: As shown on FBD.

Maximum Shear Stress: Applying the torsion formula

$$\tau_A = \frac{T_A\, c}{J} = \frac{400(0.03)}{\frac{\pi}{2}(0.03^4)} = 9.43 \text{ MPa}$$ **Ans.**

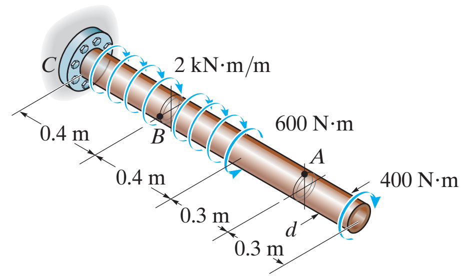
$$\tau_B = \frac{T_B\, c}{J} = \frac{600(0.03)}{\frac{\pi}{2}(0.03^4)} = 14.1 \text{ MPa}$$ **Ans.**



Ans:
$\tau_A = 9.43$ MPa,
$\tau_B = 14.1$ MPa

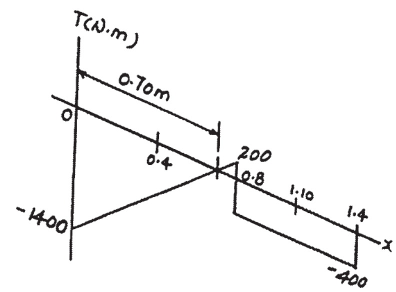
***5–16.**

The 60-mm-diameter solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the absolute maximum and minimum shear stresses on the shaft's surface, and specify their locations, measured from the fixed end C.



SOLUTION

Internal Torque: From the torque diagram, the maximum torque $T_{max} = 1400\ \text{N}\cdot\text{m}$ occurs at the fixed support and the minimum torque $T_{min} = 0$ occurs at $x = 0.700$ m.



Shear Stress: Applying the torsion formula

$$\tau_{\substack{abs\\min}} = \frac{T_{min}\,c}{J} = 0 \qquad \text{occurs at} \qquad x = 0.700 \text{ m}$$ **Ans.**

$$\tau_{\substack{abs\\max}} = \frac{T_{max}\,c}{J} = \frac{1400(0.03)}{\frac{\pi}{2}(0.03^4)} = 33.0 \text{ MPa}$$ **Ans.**

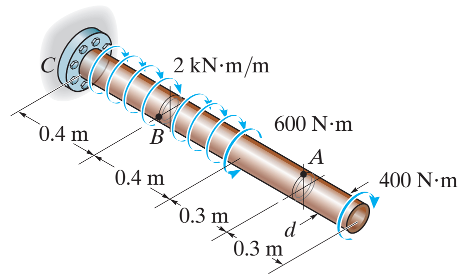
$$\text{occurs at} \qquad x = 0$$ **Ans.**

According to Saint-Venant's principle, application of the torsion formula should be at points sufficiently removed from the supports or points of concentrated loading. Therefore, $\tau_{\substack{abs\\max}}$ obtained *is not valid*.

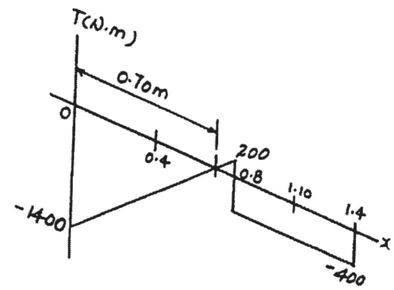
Ans:
$\tau_{\substack{abs\\max}} = 0$ occurs at $x = 0.700$ m,
$\tau_{\substack{abs\\min}} = 33.0$ MPa occurs at $x = 0$

5–17.

The solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the required diameter d of the shaft if the allowable shear stress for the material is $\tau_{allow} = 1.6$ MPa.



SOLUTION



Internal Torque: From the torque diagram, the maximum torque $T_{max} = 1400\ \text{N} \cdot \text{m}$ occurs at the fixed support.

Allowable Shear Stress: Applying the torsion formula

$$\tau_{\substack{abs\\max}} = \tau_{allow} = \frac{T_{max}\, c}{J}$$

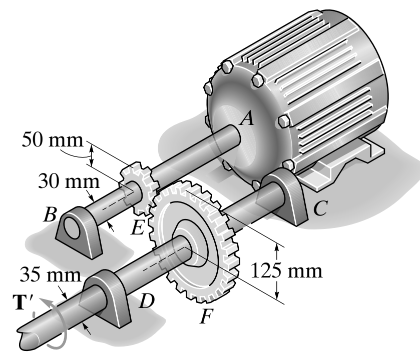
$$175(10^6) = \frac{1400\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

$$d = 0.03441\ \text{m} = 34.4\ \text{mm}$$ **Ans.**

According to Saint-Venant's principle, application of the torsion formula should be at points sufficiently removed from the supports or points of concentrated loading. Therefore, the above analysis *is not valid*.

Ans:
$d = 34.4$ mm

5–18. The motor delivers a torque of $50\ \text{N}\cdot\text{m}$ to the shaft AB. This torque is transmitted to shaft CD using the gears at E and F. Determine the equilibrium torque $\mathbf{T}'$ on shaft CD and the maximum shear stress in each shaft. The bearings B, C, and D allow free rotation of the shafts.



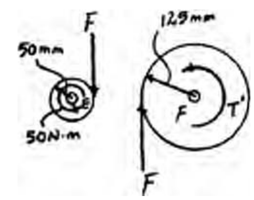
SOLUTION

Equilibrium:

$\zeta + \Sigma M_E = 0; \qquad 50 - F(0.05) = 0 \qquad F = 1000\ \text{N}$

$\zeta + \Sigma M_F = 0; \qquad T' - 1000(0.125) = 0$

$$T' = 125\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

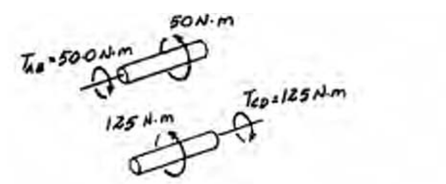


Internal Torque: As shown on FBD.

Maximum Shear Stress: Applying torsion Formula.

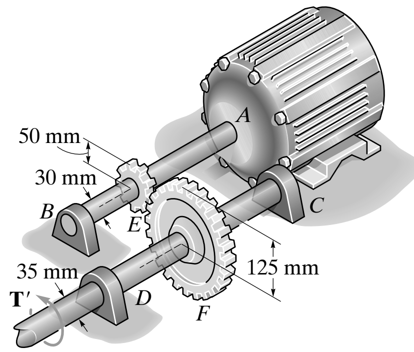
$$(\tau_{AB})_{\max} = \frac{T_{AB}\,c}{J} = \frac{50.0(0.015)}{\frac{\pi}{2}(0.015^4)} = 9.43\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{CD})_{\max} = \frac{T_{CD}c}{J} = \frac{125(0.0175)}{\frac{\pi}{2}(0.0175^4)} = 14.8\ \text{MPa} \qquad \textbf{Ans.}$$



Ans.
$(\tau_{AB})_{\max} = 9.43\ \text{MPa}$
$(\tau_{CD})_{\max} = 14.8\ \text{MPa}$

5–19. If the applied torque on shaft CD is $T' = 75\ \text{N}\cdot\text{m}$, determine the absolute maximum shear stress in each shaft. The bearings B, C, and D allow free rotation of the shafts, and the motor holds the shafts fixed from rotating.



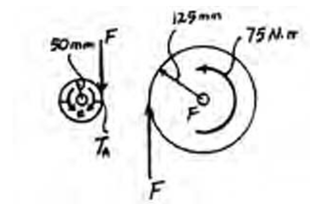
SOLUTION

Equilibrium:

$\circlearrowleft + \Sigma M_F = 0; \qquad 75 - F(0.125) = 0; \qquad F = 600\ \text{N}$

$\circlearrowleft + \Sigma M_E = 0; \qquad 600(0.05) - T_A = 0$

$T_A = 30.0\ \text{N}\cdot\text{m}$

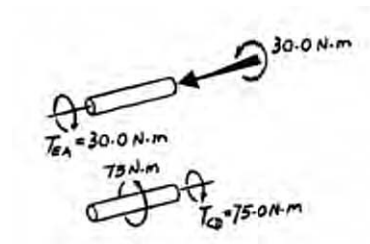


Internal Torque: As shown on FBD.

Maximum Shear Stress: Applying the torsion formula

$$(\tau_{EA})_{\max} = \frac{T_{EA}\,c}{J} = \frac{30.0(0.015)}{\frac{\pi}{2}(0.015^4)} = 5.66\ \text{MPa} \qquad \textbf{Ans.}$$

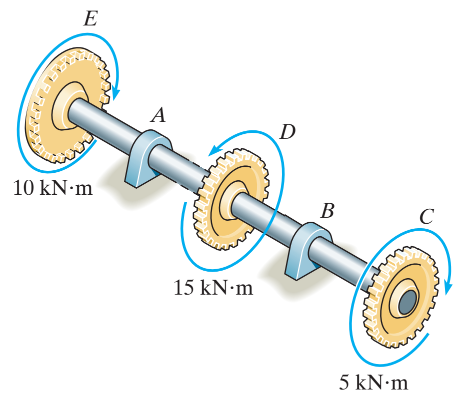
$$(\tau_{CD})_{\max} = \frac{T_{CD}c}{J} = \frac{75.0(0.0175)}{\frac{\pi}{2}(0.0175^4)} = 8.91\ \text{MPa} \qquad \textbf{Ans.}$$



Ans.
$(\tau_{EA})_{\max} = 5.66\ \text{MPa}$
$(\tau_{CD})_{\max} = 8.91\ \text{MPa}$

***5–20.**

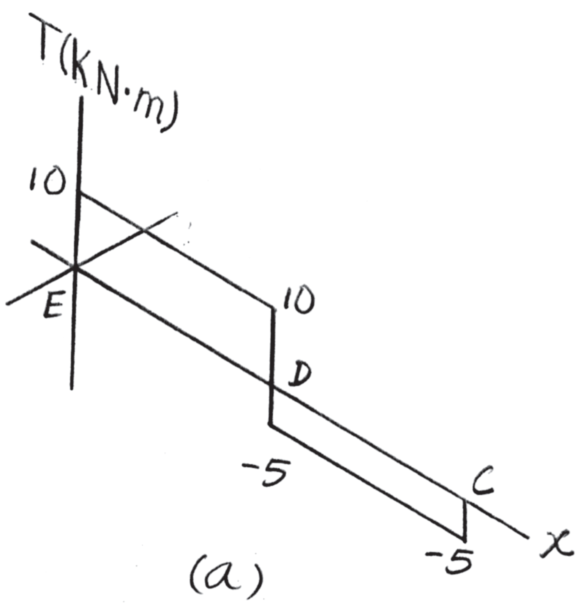
The shaft has an outer diameter of 100 mm and an inner diameter of 80 mm. If it is subjected to the three torques, determine the absolute maximum shear stress in the shaft. The smooth bearings A and B do not resist torque.



SOLUTION

Maximum Shear Stress: Referring to the torque diagram shown in Fig. *a*, region ED is subjected to the largest internal torque, which is $T_{max} = 10\ \text{kN}\cdot\text{m}$. Thus, the absolute maximum shear stress occurs in this region. Applying the torsion formula,

$$\tau_{\substack{abs\\max}} = \frac{T_{max}\,C}{J} = \frac{10\left(10^3\right)(0.05)}{\frac{\pi}{2}\left(0.05^4 - 0.04^4\right)} = 86.26\left(10^6\right)\text{ Pa} = 86.3\text{ MPa} \qquad \textbf{Ans.}$$

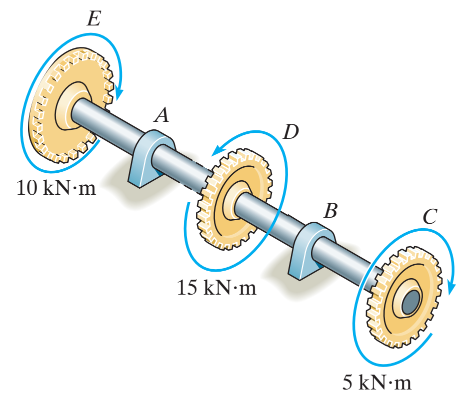


(a)

Ans:
$\tau_{\substack{abs\\max}} = 86.3\text{ MPa}$

5–21.

The shaft has an outer diameter of 100 mm and an inner diameter of 80 mm. If it is subjected to the three torques, plot the shear stress distribution along a radial line for the cross section within region *CD* of the shaft. The smooth bearings at *A* and *B* do not resist torque.



SOLUTION

Shear Stress: Referring to the torque diagram shown in Fig. *a*, Region *CD* of the shaft is subjected to an internal torque of $T_{CD} = 5\ \text{kN}\cdot\text{m}$. The torsion formula will be applied. The maximum shear stress is

$$(\tau_{\max})_{CD} = \frac{T_{CD}\,C}{J} = \frac{5(10^3)(0.05)}{\frac{\pi}{2}\left(0.05^4 - 0.04^4\right)} = 43.13\left(10^6\right)\text{ Pa} = 43.1\text{ MPa}$$

The shear stress at the inner surface of the hollow shaft is

$$(\tau_p = 0.04\text{ m})_{CD} = \frac{T_{CD}\rho}{J} = \frac{5(10^3)(0.04)}{\frac{\pi}{2}\left(0.05^4 - 0.04^4\right)} = 34.51\left(10^6\right)\text{ Pa} = 34.5\text{ MPa}$$

Also,

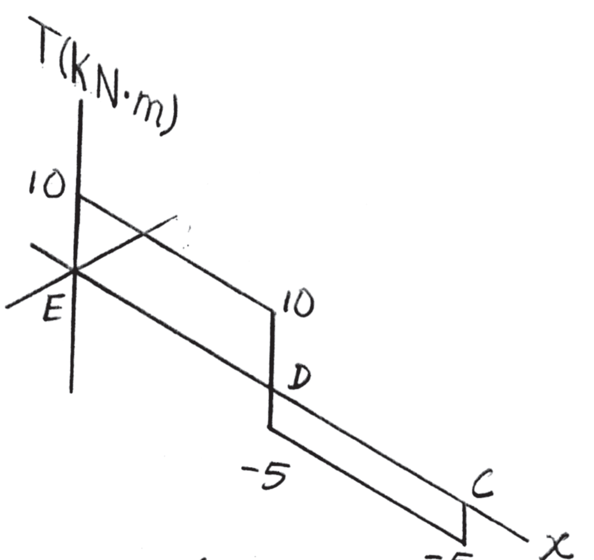
$$\frac{\tau_{\max}}{C} = \frac{\tau_p}{\rho}; \quad (\tau_{p=0.04\text{ m}})_{CD} = \left(\frac{\rho}{C}\right)\tau_{\max}$$

$$= \left(\frac{0.04}{0.05}\right)(43\cdot 13)$$

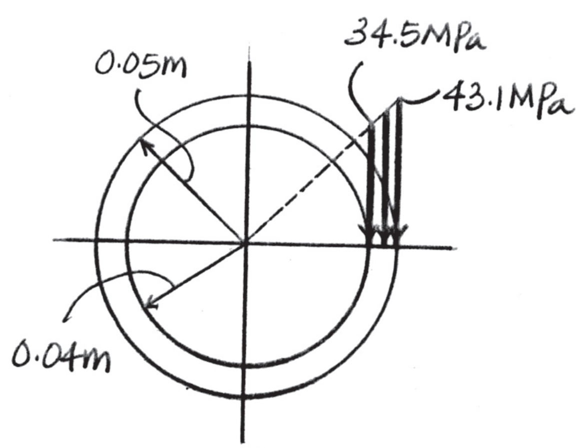
$$= 34.51\text{ MPa}$$

$$= 34.5\text{ MPa}$$

The shear stress distribution along the radius of the cross-section of the shaft is shown in Fig. *b*.



(a)

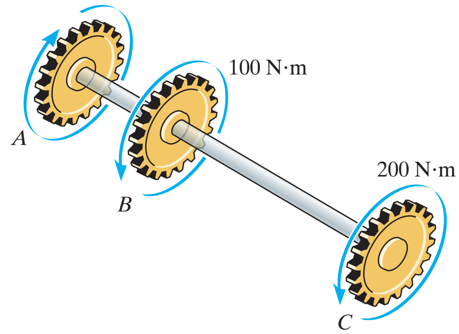


(b)

Ans:
N/A

5–22.

If the gears are subjected to the torques shown, determine the maximum shear stress in the segments AB and BC of the A-36 steel shaft. The shaft has a diameter of 40 mm.



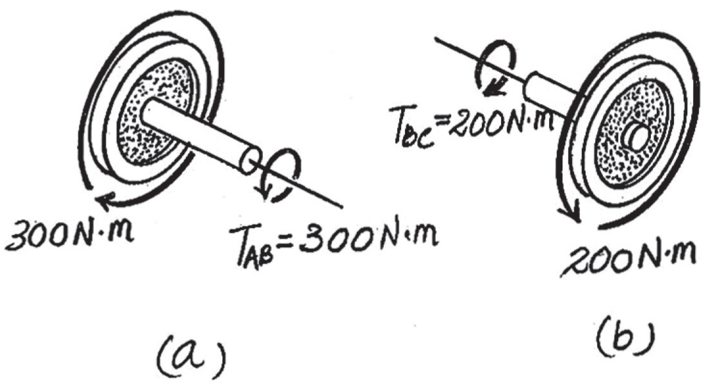
SOLUTION

The internal torque developed in segments AB and BC are shown in their respective FBDs, Figs. a and b.

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.02^4) = 80(10^{-9})\pi \text{ m}^4$. Thus,

$$(\tau_{AB})_{max} = \frac{T_{AB}\,c}{J} = \frac{300(0.02)}{80(10^{-9})\pi} = 23.87(10^6)\text{Pa} = 23.9 \text{ MPa} \qquad \textbf{Ans.}$$

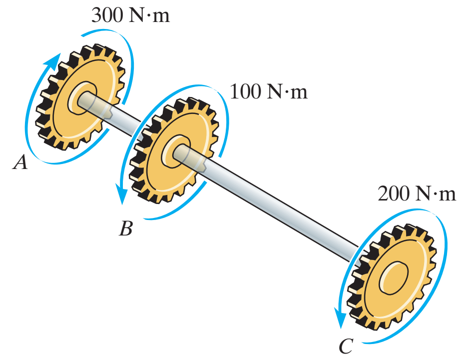
$$(\tau_{BC})_{max} = \frac{T_{BC}\,c}{J} = \frac{200(0.02)}{80(10^{-9})\pi} = 15.92(10^6)\text{Pa} = 15.9 \text{ MPa} \qquad \textbf{Ans.}$$



Ans:
$(\tau_{AB})_{max} = 23.9$ MPa,
$(\tau_{BC})_{max} = 15.9$ MPa

5–23.

If the gears are subjected to the torques shown, determine the required diameter of the A-36 steel shaft to the nearest mm if $\tau_{\text{allow}} = 60$ MPa.



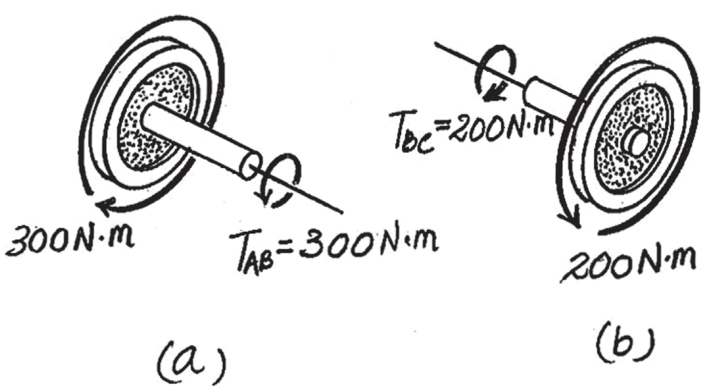
SOLUTION

The internal torque developed in segments AB and BC are shown in their respective FBDs, Fig. a and b.

Here, segment AB is critical since its internal torque is the greatest. The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$. Thus,

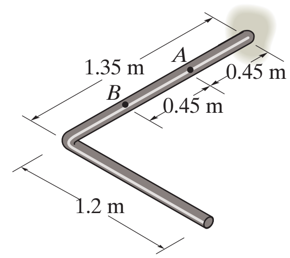
$$\tau_{\text{allow}} = \frac{Tc}{J}; \quad 60\left(10^6\right) = \frac{300\left(d/2\right)}{\pi d^4/32}$$

$$d = 0.02942 \text{ m} = 30 \text{ mm} \qquad \textbf{Ans.}$$



Ans:
$d = 30$ mm

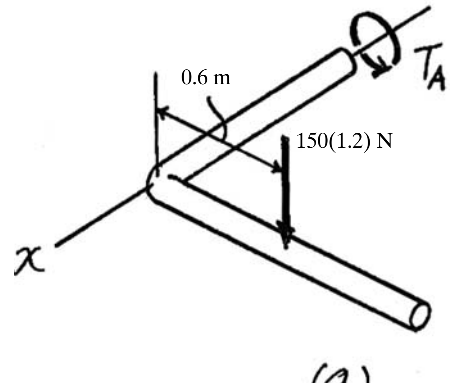
***5–24.** The rod has a diameter of 25 mm and a weight of 150 N/m. Determine the maximum torsional stress in the rod at a section located at A due to the rod's weight.



SOLUTION

Here, we are only interested in the internal torque. Thus, other components of the internal loading are not indicated in the FBD of the cut segment of the rod, Fig. a.

$$\Sigma M_x = 0; \qquad T_A - 150(1.2)(0.6) = 0 \qquad T_A = 108 \text{ N} \cdot \text{m}$$



(a)

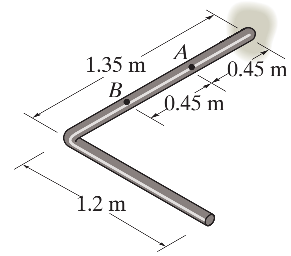
The polar moment of inertia of the cross section at A is

$J = \frac{\pi}{2}(0.0125^4) = 38.35(10^{-9})\text{ m}^4$. Thus

$$\tau_{max} = \frac{T_A c}{J} = \frac{108(0.0125)}{38.35(10^{-9})} = 35.20(10^6)\text{ N/m}^2 = 35.2 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\tau_{max} = 35.2$ MPa

5–25. The rod has a diameter of 25 mm and a weight of 225 N/m. Determine the maximum torsional stress in the rod at a section located at B due to the rod's weight.



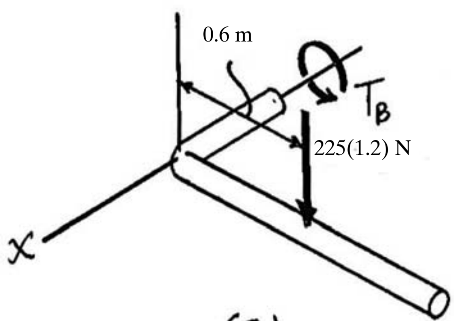
SOLUTION

Here, we are only interested in the internal torque. Thus, other components of the internal loading are not indicated in the FBD of the cut segment of the rod, Fig. a.

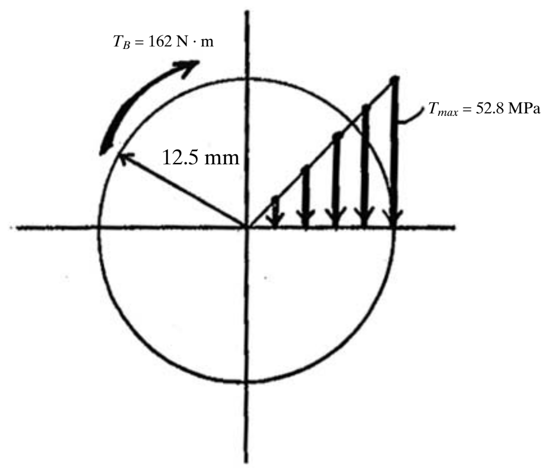
$$\Sigma M_x = 0; \qquad T_B - 225(1.2)(0.6) = 0 \qquad T_B = 162 \text{ N} \cdot \text{m}$$

The polar moment of inertia of the cross-section at B is $J = \frac{\pi}{2}(0.0125^4) = 38.35(10^{-9}) \text{ m}^4$. Thus,

$$\tau_{\max} = \frac{T_B\, c}{J} = \frac{162(0.0125)}{38.35(10^{-9})} = 52.80(10^6) \text{ N/m}^2 = 52.8 \text{ MPa} \qquad \textbf{Ans.}$$



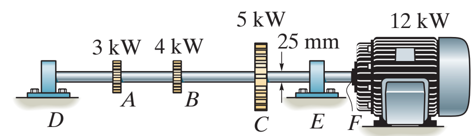
(a)



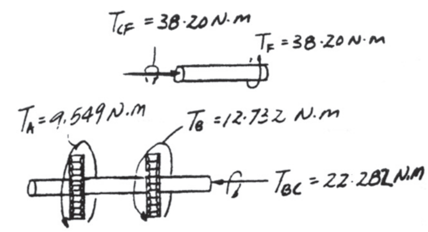
Ans:
$\tau_{\max} = 52.8$ MPa

5–26.

The solid steel shaft DF has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at F, which delivers 12 kW of power to the shaft while it is turning at 50 rev/s. If gears A, B, and C remove 3 kW, 4 kW, and 5 kW respectively, determine the maximum shear stress in the shaft within regions CF and BC. The shaft is free to turn in its support bearings D and E.



SOLUTION



$$\omega = 50\,\frac{\text{rev}}{\text{s}}\left[\frac{2\pi\text{ rad}}{\text{rev}}\right] = 100\,\pi\text{ rad/s}$$

$$T_F = \frac{P}{\omega} = \frac{12(10^3)}{100\,\pi} = 38.20\text{ N}\cdot\text{m}$$

$$T_A = \frac{P}{\omega} = \frac{3(10^3)}{100\,\pi} = 9.549\text{ N}\cdot\text{m}$$

$$T_B = \frac{P}{\omega} = \frac{4(10^3)}{100\,\pi} = 12.73\text{ N}\cdot\text{m}$$

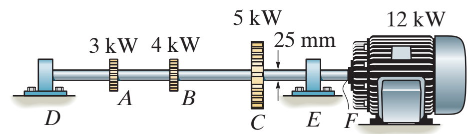
$$(\tau_{\max})_{CF} = \frac{T_{CF}\,c}{J} = \frac{38.20(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 12.5\text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{\max})_{BC} = \frac{T_{BC}\,c}{J} = \frac{22.282(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 7.26\text{ MPa} \qquad \textbf{Ans.}$$

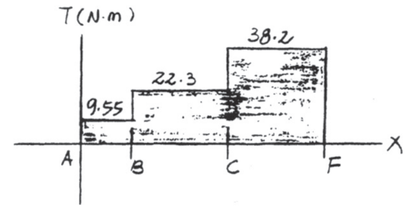
Ans:
$(\tau_{\max})_{CF} = 12.5\text{ MPa}$, $(\tau_{\max})_{BC} = 7.26\text{ MPa}$

5–27.

The solid steel shaft DF has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at F, which delivers 12 kW of power to the shaft while it is turning at 50 rev/s. If gears A, B, and C remove 3 kW, 4 kW, and 5 kW respectively, determine the absolute maximum shear stress in the shaft.



SOLUTION



$$\omega = 50\,\frac{\text{rev}}{\text{s}}\left[\frac{2\pi\ \text{rad}}{\text{rev}}\right] = 100\,\pi\ \text{rad/s}$$

$$T_F = \frac{P}{\omega} = \frac{12(10^3)}{100\pi} = 38.20\ \text{N}\cdot\text{m}$$

$$T_A = \frac{P}{\omega} = \frac{3(10^3)}{100\pi} = 9.549\ \text{N}\cdot\text{m}$$

$$T_B = \frac{P}{\omega} = \frac{4(10^3)}{100\pi} = 12.73\ \text{N}\cdot\text{m}$$

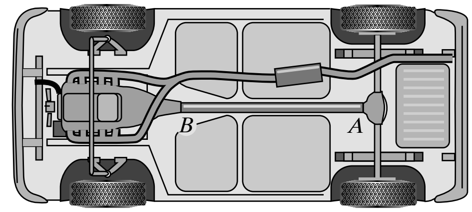
From the torque diagram,

$$T_{\max} = 38.2\ \text{N}\cdot\text{m}$$

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{Tc}{J} = \frac{38.2(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 12.5\ \text{MPa}$$ **Ans.**

Ans:
$\tau_{\substack{\text{abs}\\\text{max}}} = 12.5\ \text{MPa}$

***5–28.** The drive shaft AB of an automobile is made of a steel having an allowable shear stress of $\tau_{allow} = 56$ MPa. If the outer diameter of the shaft is 62.5 mm and the engine delivers 165 kW to the shaft when it is turning at 1140 rev/min, determine the minimum required thickness of the shaft's wall.



SOLUTION

$$\omega = \left(1140\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 38\pi\ \text{rad/s}$$

$$P = T\omega$$

$$165(10^3) = T(38\pi)$$

$$T = 1.382(10^3)\ \text{N}\cdot\text{m}$$

$$\tau_{allow} = \frac{Tc}{J}$$

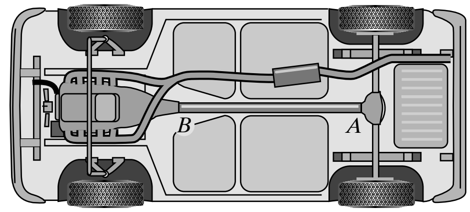
$$56(10^6) = \frac{[1.382(10^3)](0.03125)}{\frac{\pi}{2}(0.03125^4 - r_i^4)}$$

$$r_i = 0.02608\ \text{m}$$

$$t = r_0 - r_i = 0.03125 - 0.02608 = 0.005170\ \text{m} = 5.17\ \text{mm}$$ **Ans.**

Ans.
$t = 5.17$ mm

5–29. The drive shaft AB of an automobile is to be designed as a thin-walled tube. The engine delivers 125 kW when the shaft is turning at 1500 rev/min. Determine the minimum thickness of the shaft's wall if the shaft's outer diameter is 62.5 mm. The material has an allowable shear stress of $\tau_{allow} = 50$ MPa.



SOLUTION

$$\omega = \left(1500\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 50\pi\text{ rad/s}$$

$$P = T\omega$$

$$125(10^3) = T(50\pi)$$

$$T = 795.77\text{ N}\cdot\text{m}$$

$$\tau_{allow} = \frac{Tc}{J}$$

$$50(10^6) = \frac{795.77(0.03125)}{\frac{\pi}{2}(0.03125^4 - r_i^4)}$$

$$r_i = 0.02825\text{ m}$$

$$t = r_0 - r_i = 0.03125 - 0.02825 = 0.002998\text{ m} = 3.00\text{ mm}$$ **Ans.**

Ans.
$t = 3.00$ mm

5–30. A ship has a propeller drive shaft that is turning at 1500 rev/min while developing 1500 kW. If it is 2.4 m long and has a diameter of 100 mm, determine the maximum shear stress in the shaft caused by torsion.

SOLUTION

Internal Torque:

$$\omega = \left(1500\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 50.0\pi\text{ rad/s}$$

$$P = 1500\text{ kW} = 1500(10^3)\text{ N}\cdot\text{m/s}$$

$$T = \frac{P}{\omega} = \frac{1500(10^3)}{50.0\pi} = 9549.30\text{ N}\cdot\text{m}$$

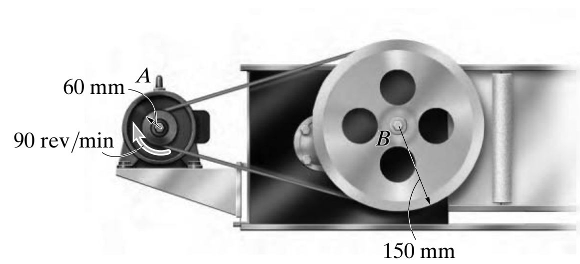
Maximum Shear Stress: Applying torsion formula

$$\tau_{\max} = \frac{Tc}{J} = \frac{9549.30(0.05)}{\frac{\pi}{2}(0.05^4)} = 48.63(10^3)\text{ N/m}^2 = 48.6\text{ MPa}$$ **Ans.**

Ans.

$\tau_{\max} = 48.6\text{ MPa}$

5–31. The motor A develops a power of 300 W and turns its connected pulley at 90 rev/min. Determine the required diameters of the steel shafts on the pulleys at A and B if the allowable shear stress is $\tau_{\text{allow}} = 85$ MPa.



SOLUTION

Internal Torque: For shafts A and B

$$\omega_A = 90\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\frac{1\ \text{min}}{60\ \text{s}} = 3.00\pi\ \text{rad/s}$$

$$P = 300\ \text{W} = 300\ \text{N}\cdot\text{m/s}$$

$$T_A = \frac{P}{\omega_A} = \frac{300}{3.00\pi} = 31.83\ \text{N}\cdot\text{m}$$

$$\omega_B = \omega_A\left(\frac{r_A}{r_B}\right) = 3.00\pi\left(\frac{0.06}{0.15}\right) = 1.20\pi\ \text{rad/s}$$

$$P = 300\ \text{W} = 300\ \text{N}\cdot\text{m/s}$$

$$T_B = \frac{P}{\omega_B} = \frac{300}{1.20\pi} = 79.58\ \text{N}\cdot\text{m}$$

Allowable Shear Stress: For shaft A

$$\tau_{\max} = \tau_{\text{allow}} = \frac{T_A\, c}{J}$$

$$85\left(10^6\right) = \frac{31.83\left(\frac{d_A}{2}\right)}{\frac{\pi}{2}\left(\frac{d_A}{2}\right)^4}$$

$$d_A = 0.01240\ \text{m} = 12.4\ \text{mm}$$ **Ans.**

For shaft B

$$\tau_{\max} = \tau_{\text{allow}} = \frac{T_B\, c}{J}$$

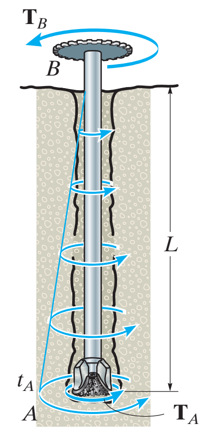
$$85\left(10^6\right) = \frac{79.58\left(\frac{d_B}{2}\right)}{\frac{\pi}{2}\left(\frac{d_B}{2}\right)^4}$$

$$d_B = 0.01683\ \text{m} = 16.8\ \text{mm}$$ **Ans.**

Ans.
$d_A = 12.4$ mm, $d_B = 16.8$ mm

***5–32.**

When drilling a well at constant angular velocity, the bottom end of the drill pipe encounters a torsional resistance T_A. Also, soil along the sides of the pipe creates a distributed frictional torque along its length, varying uniformly from zero at the surface B to t_A at A. Determine the minimum torque T_B that must be supplied by the drive unit to overcome the resisting torques, and calculate the maximum shear stress in the pipe. The pipe has an outer radius r_o and an inner radius r_i.



SOLUTION

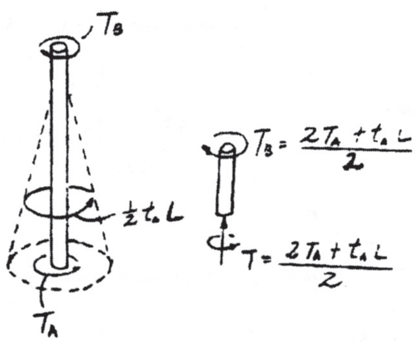
$$T_A + \frac{1}{2} t_A L - T_B = 0$$

$$T_B = \frac{2T_A + t_A L}{2}$$ **Ans.**

Maximum shear stress: The maximum torque is within the region above the distributed torque.

$$\tau_{\max} = \frac{Tc}{J}$$

$$\tau_{\max} = \frac{\left[\frac{(2T_A + t_A L)}{2}\right](r_o)}{\frac{\pi}{2}(r_o^4 - r_i^4)} = \frac{(2T_A + t_A L)r_o}{\pi(r_o^4 - r_i^4)}$$ **Ans.**



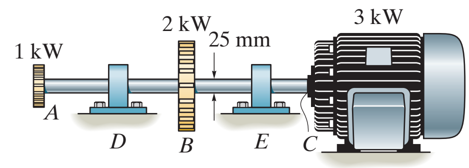
Ans:

$T_B = \dfrac{2T_A + t_A L}{2}$,

$\tau_{\max} = \dfrac{(2T_A + t_A L)r_o}{\pi(r_o^4 - r_i^4)}$

5–33.

The solid steel shaft AC has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at C, which delivers 3 kW of power to the shaft while it is turning at 50 rev/s. If gears A and B remove 1 kW and 2 kW, respectively, determine the maximum shear stress in the shaft within regions AB and BC. The shaft is free to turn in its support bearings D and E.



SOLUTION

$$T_C = \frac{P}{\omega} = \frac{3(10^3)}{50(2\pi)} = 9.549 \text{ N}\cdot\text{m}$$

$$T_A = \frac{1}{3}T_C = 3.183 \text{ N}\cdot\text{m}$$

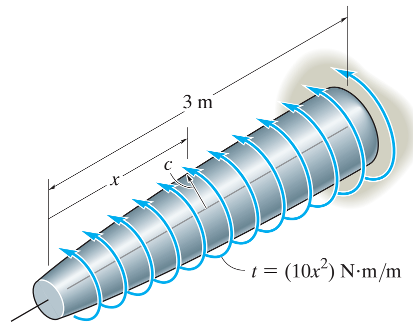
$$(\tau_{AB})_{\max} = \frac{T_C}{J} = \frac{3.183\,(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 1.04 \text{ MPa}$$ **Ans.**

$$(\tau_{BC})_{\max} = \frac{T_C}{J} = \frac{9.549\,(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 3.11 \text{ MPa}$$ **Ans.**

Ans:
$(\tau_{AB})_{\max} = 1.04$ MPa, $(\tau_{BC})_{\max} = 3.11$ MPa

5–34.

The shaft is subjected to a distributed torque along its length of $t = (10x^2)\ \mathrm{N \cdot m/m}$, where x is in meters. If the maximum stress in the shaft is to remain constant at 80 MPa, determine the required variation of the radius c of the shaft for $0 \le x \le 3$ m.



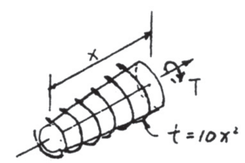
SOLUTION

$$T = \int t\,dx = \int_0^x 10\,x^2 dx = \frac{10}{3}x^3$$

$$\tau = \frac{Tc}{J};\quad 80(10^6) = \frac{(\frac{10}{3})x^3\,c}{\frac{\pi}{2}\,c^4}$$

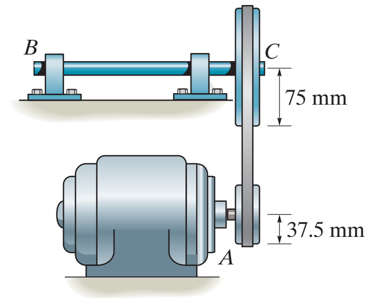
$$c^3 = 26.526(10^{-9})\,x^3$$

$$c = (2.98\,x)\ \mathrm{mm}$$ **Ans.**



Ans:
$c = (2.98\,x)$ mm

5–35. The motor delivers 12 kW to the pulley A while turning at a constant rate of 1800 rpm. Determine to the nearest multiples of 5 mm the smallest diameter of shaft BC if the allowable shear stress is $\tau_{\text{allow}} = 84$ MPa. The belt does not slip on the pulley.



SOLUTION

The angular velocity of shaft BC can be determined using the pulley ratio that is

$$\omega_{BC} = \left(\frac{r_A}{r_C}\right)\omega_A = \left(\frac{0.0375}{0.075}\right)\left(1800\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 30\pi\text{ rad/s}$$

The power is

$$P = 12\text{ kW} = 12(10^3)\text{ N}\cdot\text{m/s}$$

Thus,

$$T = \frac{P}{\omega} = \frac{12(10^3)}{30\pi} = 127.32\text{ N}\cdot\text{m}$$

The polar moment of inertia of the shaft is $J = \dfrac{\pi}{2}\left(\dfrac{d}{2}\right)^4 = \dfrac{\pi d^4}{32}$. Thus,

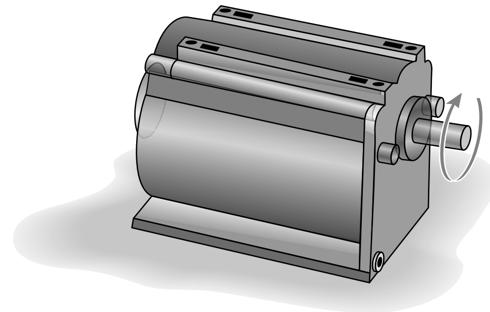
$$\tau_{\text{allow}} = \frac{Tc}{J}; \qquad 84(10^6) = \frac{127.32(d/2)}{\pi d^4/32}$$

$$d = 0.01976\text{ m} = 19.76\text{ mm}$$

Use $d = 20$ mm **Ans.**

Ans.
$d = 20$ mm

***5–36.** The gear motor can develop 1.6 kW when it turns at 450 rev/min. If the shaft has a diameter of 25 mm, determine the maximum shear stress developed in the shaft.



SOLUTION

The angular velocity of the shaft is

$$\omega = \left(450\,\frac{\cancel{\text{rev}}}{\cancel{\text{min}}}\right)\left(\frac{2\pi\text{ rad}}{1\,\cancel{\text{rev}}}\right)\left(\frac{1\,\cancel{\text{min}}}{60\text{ s}}\right) = 15\pi\text{ rad/s}$$

and the power is

$$P = 1.6\text{ kW} = 1.6(10^3)\text{ N}\cdot\text{m/s}$$

Then

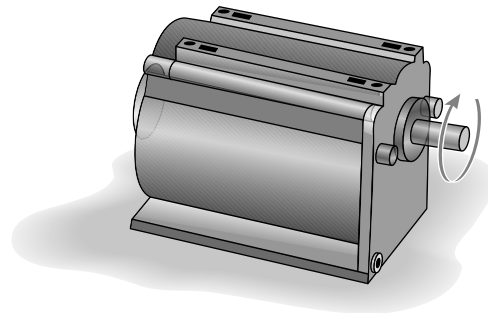
$$T = \frac{P}{\omega} = \frac{1.6(10^3)}{15\pi} = 33.95\text{ N}\cdot\text{m}$$

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.0125^4) = 38.35(10^{-9})\text{ m}^4$. Thus,

$$\tau_{\max} = \frac{T\,c}{J} = \frac{33.95(0.0125)}{38.35(10^{-9})} = 11.07(10^6)\text{ N/m}^2 = 11.1\text{ MPa} \qquad \textbf{Ans.}$$

Ans.
$\tau_{\max} = 11.1$ MPa

5–37. The gear motor can develop 2.4 kW when it turns at 150 rev/min. If the allowable shear stress for the shaft is $\tau_{\text{allow}} = 84$ MPa, determine the smallest diameter of the shaft to the nearest multiples of 5 mm that can be used.



SOLUTION

The angular velocity of the shaft is

$$\omega = \left(150\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 5\pi\ \text{rad/s}$$

and the power is

$$P = 2.4\ \text{kW} = 2.4(10^3)\ \text{N}\cdot\text{m/s}$$

Then

$$T = \frac{P}{\omega} = \frac{2.4(10^3)}{5\pi} = 152.79\ \text{N}\cdot\text{m}$$

The polar moment of inertia of the shaft is $J = \dfrac{\pi}{2}\left(\dfrac{d}{2}\right)^4 = \dfrac{\pi d^4}{32}$. Thus,

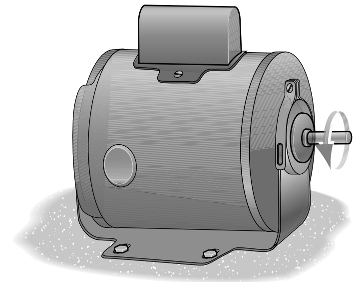
$$\tau_{\text{allow}} = \frac{T\,c}{J}; \qquad 84(10^6) = \frac{152.79(d/2)}{\pi d^4/32}$$

$$d = 0.02100\ \text{m} = 21.00\ \text{mm} \qquad \textbf{Ans.}$$

use $d = 25$ mm

Ans.
$d = 21.00$ mm

5–38. The 25-mm-diameter shaft on the motor is made of a material having an allowable shear stress of $\tau_{\text{allow}} = 75\text{ MPa}$. If the motor is operating at its maximum power of 5 kW, determine the minimum allowable rotation of the shaft.



SOLUTION

Allowable Shear Stress: The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(0.0125^4\right) = 38.3495(10^{-9})\text{ m}^4$.

$$\tau_{\text{allow}} = \frac{Tc}{J}; \qquad 75(10^6) = \frac{T(0.0125)}{38.3495(10^{-9})}$$

$$T = 230.10\text{ N}\cdot\text{m}$$

Internal Loading:

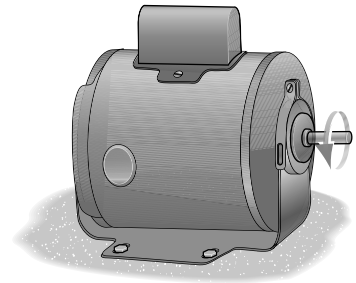
$$T = \frac{P}{\omega}; \qquad 230.10 = \frac{5(10^3)}{\omega}$$

$$\omega = 21.7\text{ rad/s} \qquad \textbf{Ans.}$$

Ans.

$\omega = 21.7\text{ rad/s}$

5–39. The drive shaft of the motor is made of a material having an allowable shear stress of $\tau_{\text{allow}} = 75$ MPa. If the outer diameter of the tubular shaft is 20 mm and the wall thickness is 2.5 mm, determine the maximum allowable power that can be supplied to the motor when the shaft is operating at an angular velocity of 1500 rev/min.



SOLUTION

Internal Loading: The angular velocity of the shaft is

$$\omega = \left(1500\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 50\pi\text{ rad/s}$$

We have

$$T = \frac{P}{\omega} = \frac{P}{50\pi}$$

Allowable Shear Stress: The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(0.01^4 - 0.0075^4\right) = 10.7379(10^{-9})\text{ m}^4$.

$$\tau_{\text{allow}} = \frac{Tc}{J}; \qquad 75(10^6) = \frac{\left(\frac{P}{50\pi}\right)(0.01)}{10.7379(10^{-9})}$$

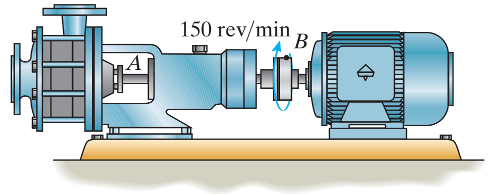
$$P = 12\,650.25\text{ W} = 12.7\text{ kW} \qquad \textbf{Ans.}$$

Ans.

$P = 12.7$ kW

***5–40.**

The pump operates using the motor that has a power of 85 W. If the impeller at B is turning at 150 rev/min, determine the maximum shear stress in the 20-mm-diameter transmission shaft at A.



SOLUTION

Internal Torque:

$$\omega = 150 \frac{\text{rev}}{\text{min}}\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\frac{1 \text{ min}}{60 \text{ s}} = 5.00\pi \text{ rad/s}$$

$$P = 85 \text{ W} = 85 \text{ N}\cdot\text{m/s}$$

$$T = \frac{P}{\omega} = \frac{85}{5.00\pi} = 5.411 \text{ N}\cdot\text{m}$$

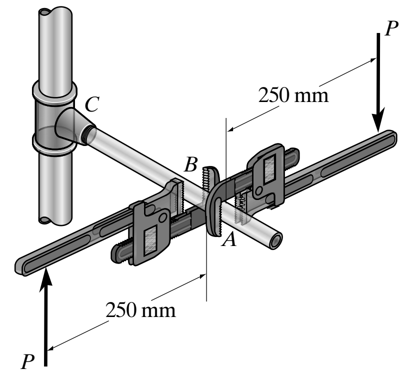
Maximum Shear Stress: Applying torsion formula

$$\tau_{\max} = \frac{T\,c}{J}$$

$$= \frac{5.411\,(0.01)}{\frac{\pi}{2}(0.01^4)} = 3.44 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\tau_{\max} = 3.44$ MPa

5–41. Two wrenches are used to tighten the pipe. If $P = 300$ N is applied to each wrench, determine the maximum torsional shear stress developed within regions AB and BC. The pipe has an outer diameter of 25 mm and inner diameter of 20 mm. Sketch the shear stress distribution for both cases.



SOLUTION

Internal Loadings: The internal torque developed in segments AB and BC of the pipe can be determined by writing the moment equation of equilibrium about the x axis by referring to their respective free - body diagrams shown in Figs. *a* and *b*.

$\Sigma M_x = 0;\ T_{AB} - 300(0.25) = 0 \qquad T_{AB} = 75\,\text{N}\cdot\text{m}$

And

$\Sigma M_x = 0;\ T_{BC} - 300(0.25) - 300(0.25) = 0 \qquad T_{BC} = 150\,\text{N}\cdot\text{m}$

Allowable Shear Stress: The polar moment of inertia of the pipe is $J = \frac{\pi}{2}\left(0.0125^4 - 0.01^4\right) = 22.642(10^{-9})\text{m}^4$.

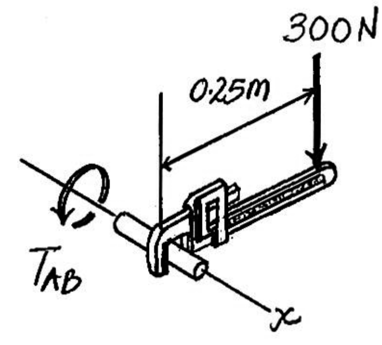
$$\left(\tau_{\max}\right)_{AB} = \frac{T_{AB}\,c}{J} = \frac{75(0.0125)}{22.642(10^{-9})} = 41.4\text{ MPa} \qquad \textbf{Ans.}$$

$$\left(\tau_{AB}\right)_{\rho=0.01\text{ m}} = \frac{T_{AB}\,\rho}{J} = \frac{75(0.01)}{22.642(10^{-9})} = 33.1\text{ MPa}$$

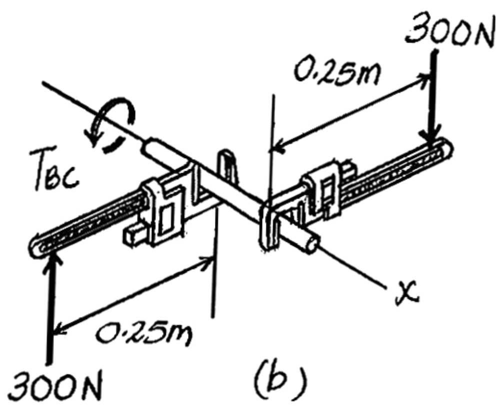
$$\left(\tau_{\max}\right)_{BC} = \frac{T_{BC}\,c}{J} = \frac{150(0.0125)}{22.642(10^{-9})} = 82.8\text{ MPa} \qquad \textbf{Ans.}$$

$$\left(\tau_{BC}\right)_{\rho=0.01\text{ m}} = \frac{T_{BC}\,\rho}{J} = \frac{150(0.01)}{22.642(10^{-9})} = 66.2\text{ MPa}$$

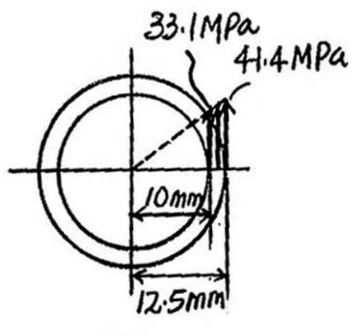
The shear stress distribution along the radial line of segments AB and BC of the pipe is shown in Figs. *c* and *d*, respectively.



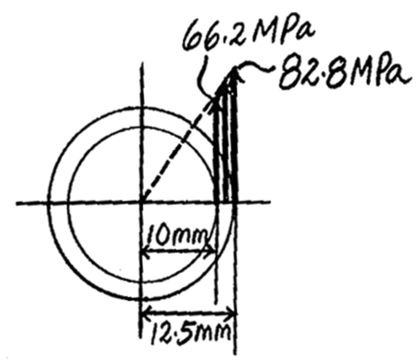
(a)



(b)



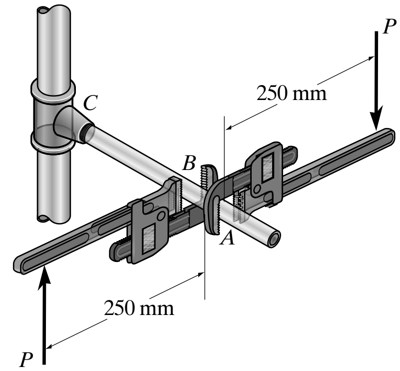
(c)



(d)

Ans:
$(\tau_{\max})_{AB} = 41.4$ MPa, $(\tau_{\max})_{BC} = 82.8$ MPa

5–42. Two wrenches are used to tighten the pipe. If the pipe is made from a material having an allowable shear stress of $\tau_{allow} = 85$ MPa, determine the allowable maximum force **P** that can be applied to each wrench. The pipe has an outer diameter of 25 mm and inner diameter of 20 mm.



SOLUTION

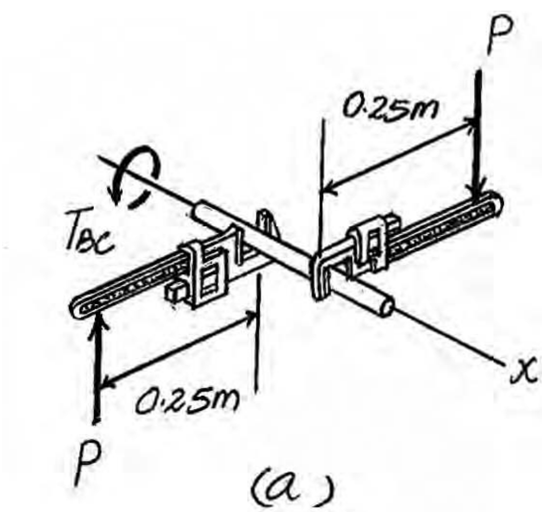
Internal Loading: By observation, segment BC of the pipe is critical since it is subjected to a greater internal torque than segment AB. Writing the moment equation of equilibrium about the x axis by referring to the free-body diagram shown in Fig. a, we have

$$\Sigma M_x = 0;\ T_{BC} - P(0.25) - P(0.25) = 0 \qquad T_{BC} = 0.5P$$

Allowable Shear Stress: The polar moment of inertia of the pipe is $J = \frac{\pi}{2}\left(0.0125^4 - 0.01^4\right) = 22.642(10^{-9})\text{m}^4$

$$\tau_{allow} = \frac{T_{BC}\,c}{J}; \qquad 85(10^6) = \frac{0.5P(0.0125)}{22.642(10^{-9})}$$

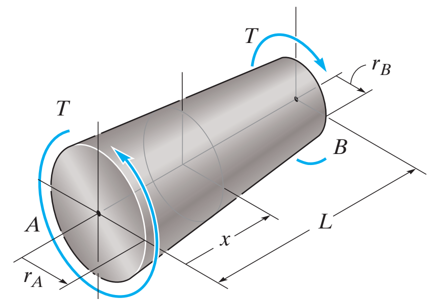
$$P = 307.93\text{N} = 308\text{N}$$ **Ans.**



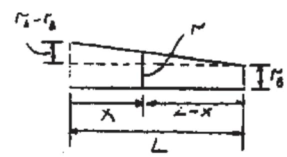
Ans.
$P = 308\text{N}$

5–43.

The solid shaft has a linear taper from r_A at one end to r_B at the other. Derive an equation that gives the maximum shear stress in the shaft at a location x along the shaft's axis.



SOLUTION



$$r = r_B + \frac{r_A - r_B}{L}(L - x) = \frac{r_B L + (r_A - r_B)(L - x)}{L}$$

$$= \frac{r_A(L - x) + r_B x}{L}$$

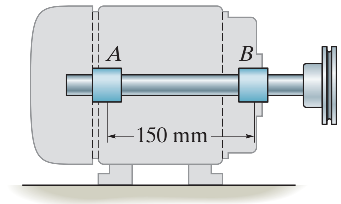
$$\tau_{max} = \frac{Tc}{J} = \frac{Tr}{\frac{\pi}{2} r^4} = \frac{2T}{\pi r^3}$$

$$= \frac{2T}{\pi\left[\frac{r_A(L - x) + r_B x}{L}\right]^3} = \frac{2TL^3}{\pi[r_A(L - x) + r_B x]^3}$$ **Ans.**

Ans:

$$\tau_{max} = \frac{2TL^3}{\pi[r_A(L - x) + r_B x]^3}$$

***5–44.** A motor delivers 375 kW to the shaft, which is tubular and has an outer diameter of 50 mm. If it is rotating at 200 rad/s, determine its largest inner diameter to the nearest mm if the allowable shear stress for the material is $\tau_{\text{allow}} = 175$ MPa.



SOLUTION

$$P = 375(10^3)\ \text{N}\cdot\text{m/s}$$

$$T = \frac{P}{\omega} = \frac{375(10^3)}{200} = 1.875(10^3)\ \text{N}\cdot\text{m}$$

$$\tau_{\max} = \tau_{\text{allow}} = \frac{Tc}{J}$$

$$175(10^6) = \frac{1.875(10^3)(0.025)}{\frac{\pi}{2}\left(0.025^4 - c_i^4\right)}$$

$$c_i = 0.02166\ \text{m}$$

$$d = 2c_i = 2(0.02166) = 0.04332\ \text{m} = 43.32\ \text{mm}$$

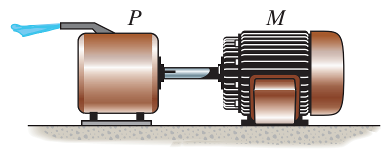
Use $d_i = 43$ mm **Ans.**

Ans:

Use $d_i = 43$ mm

5–45.

The A-36 steel tubular shaft is 2 m long and has an outer diameter of 50 mm. When it is rotating at 40 rad/s, it transmits 25 kW of power from the motor M to the pump P. Determine the smallest thickness of the tube if the allowable shear stress is $\tau_{\text{allow}} = 80$ MPa.



SOLUTION

The internal torque in the shaft is

$$T = \frac{P}{\omega} = \frac{25(10^3)}{40} = 625 \text{ N} \cdot \text{m}$$

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.025^4 - c_i^{\,4})$. Thus,

$$\tau_{\text{allow}} = \frac{Tc}{J}; \qquad 80(10^6) = \frac{625(0.025)}{\frac{\pi}{2}(0.025^4 - c_i^{\,4})}$$

$$c_i = 0.02272 \text{ m}$$

So that

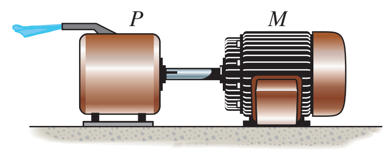
$$t = 0.025 - 0.02272$$

$$= 0.002284 \text{ m} = 2.284 \text{ mm} = 2.28 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$t = 2.28$ mm

5–46.

The A-36 solid steel shaft is 2 m long and has a diameter of 60 mm. It is required to transmit 60 kW of power from the moor M to the pump P. Determine the smallest angular velocity the shaft if the allowable shear stress is $\tau_{\text{allow}} = 80$ MPa.



SOLUTION

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\pi\ \text{m}^4$. Thus,

$$\tau_{\text{allow}} = \frac{Tc}{J}; \qquad 80(10^6) = \frac{T(0.03)}{0.405(10^{-6})\pi}$$

$$T = 3392.92\ \text{N} \cdot \text{m}$$

$$P = T\omega; \qquad 60(10^3) = 3392.92\,\omega$$

$$\omega = 17.68\ \text{rad/s} = 17.7\ \text{rad/s}$$ **Ans.**

Ans:
$\omega = 17.7\ \text{rad/s}$

5–47.

The propellers of a ship are connected to an A-36 steel shaft that is 60 m long and has an outer diameter of 340 mm and inner diameter of 260 mm. If the power output is 4.5 MW when the shaft rotates at 20 rad/s, determine the maximum torsional stress in the shaft and its angle of twist.

SOLUTION

$$T = \frac{P}{\omega} = \frac{4.5(10^6)}{20} = 225(10^3)\ \text{N}\cdot\text{m}$$

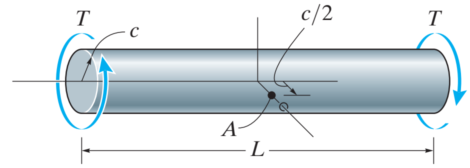
$$\tau_{\max} = \frac{Tc}{J} = \frac{225(10^3)(0.170)}{\frac{\pi}{2}[(0.170)^4 - (0.130)^4]} = 44.3\ \text{MPa}$$ **Ans.**

$$\phi = \frac{TL}{JG} = \frac{225\left(10^3\right)(60)}{\frac{\pi}{2}[(0.170)^4 - (0.130)^4)75(10^9)} = 0.2085\ \text{rad} = 11.9^\circ$$ **Ans.**

Ans:
$\tau_{\max} = 44.3\ \text{MPa}, \phi = 11.9^\circ$

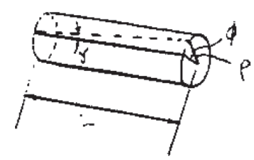
***5–48.**

The solid shaft of radius c is subjected to a torque $\mathbf{T}$ at its ends. Show that the maximum shear strain in the shaft is $\gamma_{\max} = Tc/JG$. What is the shear strain on an element located at point A, $c/2$ from the center of the shaft? Sketch the shear strain distortion of this element.



SOLUTION

From the geometry:



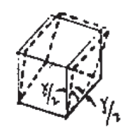
$$\gamma L = \rho\,\phi; \qquad \gamma = \frac{\rho\,\phi}{L}$$

Since $\phi = \dfrac{TL}{JG}$, then

$$\gamma = \frac{T\rho}{JG} \qquad \textbf{(1)}$$

However the maximum shear strain occurs when $\rho = c$

$$\gamma_{\max} = \frac{Tc}{JG} \qquad \textbf{QED}$$



Shear strain when $\rho = \dfrac{c}{2}$ is from Eq. (1),

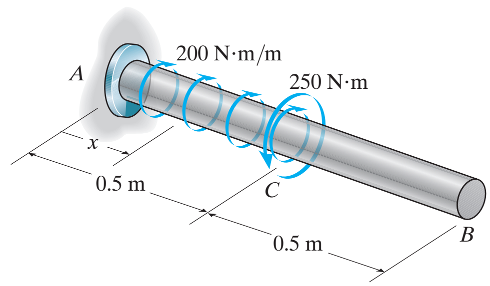
$$\gamma = \frac{T(c/2)}{JG} = \frac{Tc}{2\,JG} \qquad \textbf{Ans.}$$

Ans:

$\gamma = \dfrac{Tc}{2JG}$

5–49.

The A-36 steel shaft has a diameter of 50 mm and is subjected to the distributed and concentrated loadings shown. Determine the absolute maximum shear stress in the shaft and plot a graph of the angle of twist of the shaft in radians versus x.



SOLUTION

Internal Torque: As shown on FBD.

Maximum Shear Stress: The maximum torque occurs at $x = 0.5$ m where $T_{max} = 150 + 200(0.5) = 250\ \text{N}\cdot\text{m}$.

$$\tau_{\substack{max \\ ABS}} = \frac{T_{max}\,c}{J} = \frac{250(0.025)}{\frac{\pi}{2}(0.025^4)} = 10.2\ \text{MPa} \qquad \textbf{Ans.}$$

Angle of Twist:

For $0 \le x < 0.5$ m

$$\phi(x) = \int_0^L \frac{T(x)\,dx}{JG}$$

$$= \int_0^x \frac{(150 + 200x)\,dx}{JG}$$

$$= \frac{150x + 100x^2}{JG}$$

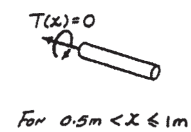
$$= \frac{150x + 100x^2}{\frac{\pi}{2}\left(0.025^4\right)75.0\left(10^9\right)}$$

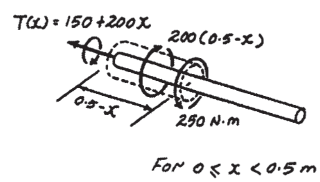
$$= \left[3.26x + 2.17x^2\right](10^{-3})\ \text{rad}$$

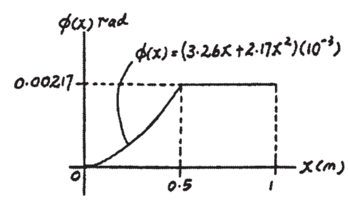
At $x = 0.5$ m, $\qquad \phi = \phi_C = 0.00217$ rad

For $0.5\ \text{m} < x < 1$ m $\qquad$ Since $T(x) = 0$, then

$$\phi(x) = \phi_C = 0.00217\ \text{rad}$$





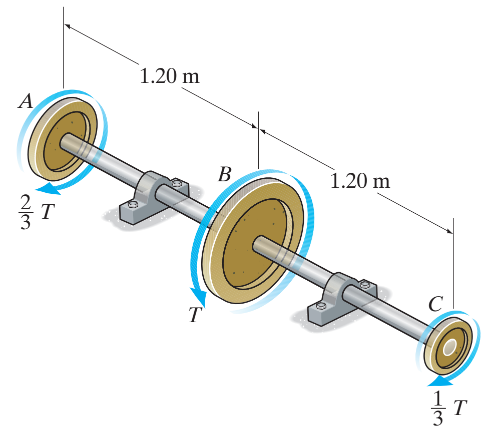


Ans:

$\tau_{\substack{max \\ ABS}} = 10.2$ MPa

5–50.

The 60-mm-diameter shaft is made of 6061-T6 aluminum having an allowable shear stress of $\tau_{allow} = 80$ MPa. Determine the maximum allowable torque $\mathbf{T}$. Also, find the corresponding angle of twist of disk A relative to disk C.



SOLUTION

Internal Loading: The internal torques developed in segments AB and BC of the shaft are shown in Figs. *a* and *b*, respectively.

Allowable Shear Stress: Segment AB is critical since it is subjected to a greater internal torque. The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.03^4) = 0.405\left(10^{-6}\right)\pi\ \text{m}^4$. We have

$$\tau_{allow} = \frac{T_{AB}\,c}{J}; \qquad 80(10^6) = \frac{(\frac{2}{3}T)(0.03)}{0.405(10^{-6})\pi}$$

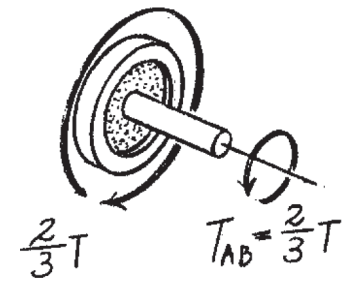
$$T = 5089.38\ \text{N}\cdot\text{m} = 5.09\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Angle of Twist: The internal torques developed in segments AB and BC of the shaft are $T_{AB} = \frac{2}{3}(5089.38) = 3392.92\ \text{N}\cdot\text{m}$ and $T_{BC} = -\frac{1}{3}(5089.38) = -1696.46\ \text{N}\cdot\text{m}$. We have

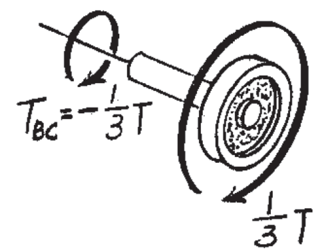
$$\phi_{A/C} = \Sigma\frac{T_iL_i}{J_iG_i} = \frac{T_{AB}L_{AB}}{JG_{al}} + \frac{T_{BC}L_{BC}}{JG_{al}}$$

$$\phi_{A/C} = \frac{3392.92(1.20)}{0.405(10^{-6})\pi(26)(10^9)} + \frac{-1696.46(1.20)}{0.405(10^{-6})\pi(26)(10^9)}$$

$$= 0.06154\ \text{rad} = 3.53^\circ \qquad \textbf{Ans.}$$



(a)

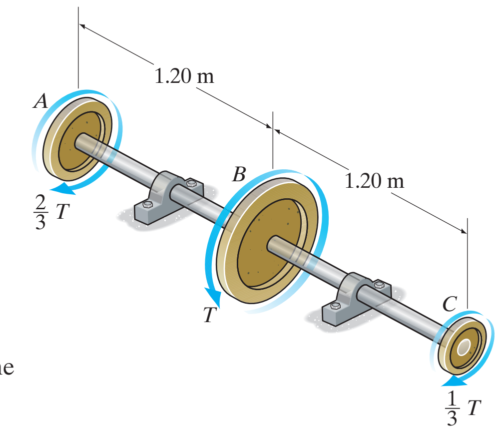


(b)

Ans:
$T = 5.09\ \text{kN}\cdot\text{m}$, $\phi_{A/C} = 3.53^\circ$

5–51.

The 60-mm-diameter shaft is made of 6061-T6 aluminum. If the allowable shear stress is $\tau_{allow} = 80$ MPa, and the angle of twist of disk A relative to disk C is limited so that it does not exceed 0.06 rad, determine the maximum allowable torque **T**.



SOLUTION

Internal Loading: The internal torques developed in segments AB and BC of the shaft are shown in Figs. a and b, respectively.

Allowable Shear Stress: Segment AB is critical since it is subjected to a greater internal torque. The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\pi\ \text{m}^4$. We have

$$\tau_{allow} = \frac{T_{AB}\,c}{J}; \qquad 80(10^3) = \frac{(\frac{2}{3}T)(0.03)}{0.405(10^{-6})\pi}$$

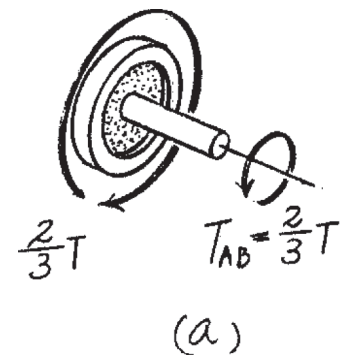
$$T = 5089.38\ \text{N}\cdot\text{m} = 5.089\ \text{kN}\cdot\text{m}$$

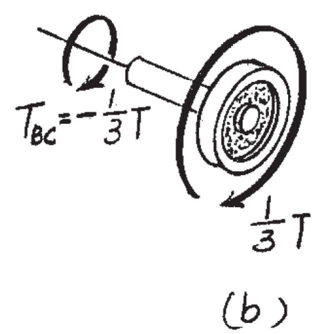
Angle of Twist: It is required that $\phi_{A/C} = 0.06$ rad. We have

$$\phi_{A/C} = \Sigma\frac{T_iL_i}{J_iG_i} = \frac{T_{AB}L_{AB}}{JG_{al}} + \frac{T_{BC}L_{BC}}{JG_{al}}$$

$$0.06 = \frac{(\frac{2}{3}T)(1.2)}{0.405(10^{-6})\pi(26)(10^9)} + \frac{(-\frac{1}{3}T)(1.2)}{0.405(10^{-6})\pi(26)(10^9)}$$

$$T = 4962.14\ \text{N}\cdot\text{m} = 4.96\ \text{kN}\cdot\text{m (controls)}$$ **Ans.**

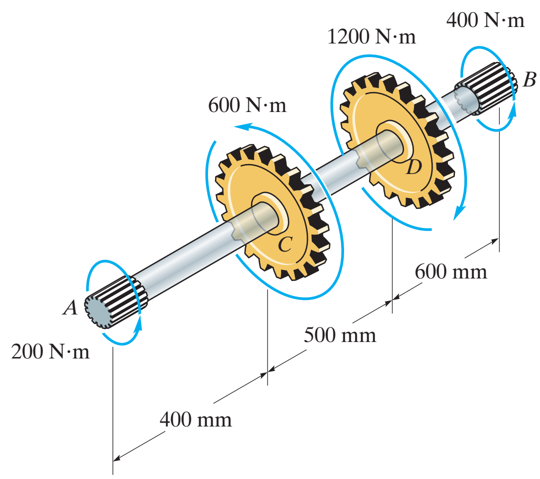




Ans:
$T = 4.96\ \text{kN}\cdot\text{m}$

***5–52.**

The splined ends and gears attached to the A992 steel shaft are subjected to the torques shown. Determine the angle of twist of end B with respect to end A. The shaft has a diameter of 40 mm.



SOLUTION

Internal Torque: The torque diagram shown in Fig. *a* can be plotted. From this diagram, $T_{AC} = 200\ \text{N}\cdot\text{m}$, $T_{CD} = 800\ \text{N}\cdot\text{m}$ and $T_{DB} = 400\ \text{N}\cdot\text{m}$.

Angle of Twist:

$$\phi_{B/A} = \Sigma \frac{TL}{JG}$$

$$= \frac{1}{JG}\left(T_{AC}L_{AC} + T_{CD}L_{CD} + T_{DB}L_{DB}\right)$$

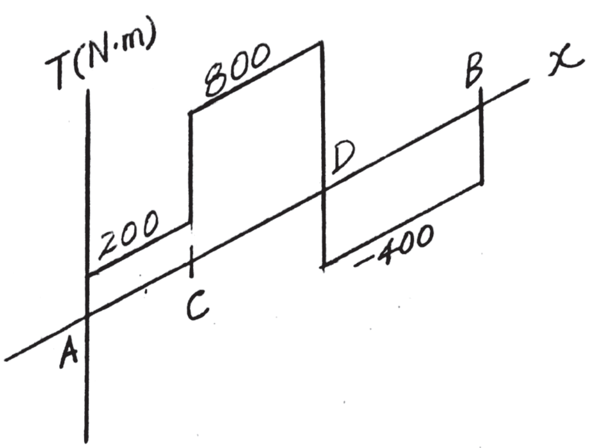
$$= \frac{1}{JG}\left[200(0.4) + 800(0.5) + (-400)(0.6)\right]$$

$$= \frac{240\ \text{N}\cdot\text{m}^2}{JG}$$

For A992 steel, $G = 75$ GPa. Then

$$\phi_{B/A} = \frac{240}{\frac{\pi}{2}\left(0.02^4\right)\left[75\left(10^9\right)\right]}$$

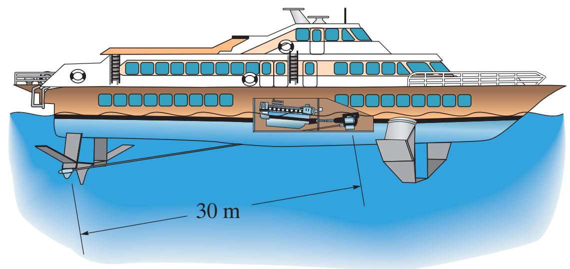
$$= (0.01273\ \text{rad})\left(\frac{180^\circ}{\pi\ \text{rad}}\right) = 0.730^\circ$$ **Ans.**



(a)

Ans:
$\phi_{B/A} = 0.730^\circ$

5–53. The hydrofoil boat has an A-36 steel propeller shaft that is 30 m long. It is connected to an in-line diesel engine that delivers a maximum power of 2000 kW and causes the shaft to rotate at 1700 rpm. If the outer diameter of the shaft is 200 mm and the wall thickness is 10 mm, determine the maximum shear stress developed in the shaft. Also, what is the "wind up," or angle of twist in the shaft at full power?



SOLUTION

Internal Torque:

$$\omega = \left(1700\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 56.667\pi\text{ rad/s}$$

$$P = 2000\text{ kW} = 2000(10^3)\text{ N}\cdot\text{m/s}$$

$$T = \frac{P}{\omega} = \frac{2000(10^3)}{56.667\pi} = 11.23(10^3)\text{ N}\cdot\text{m}$$

Maximum Shear Stress: Applying torsion Formula.

$$\tau_{\max} = \frac{T\,c}{J}$$

$$= \frac{11.23(10^3)(0.1)}{\frac{\pi}{2}(0.1^4 - 0.09^4)} = 20.80(10^6)\text{ N/m}^2 = 20.8\text{ MPa} \qquad \textbf{Ans.}$$

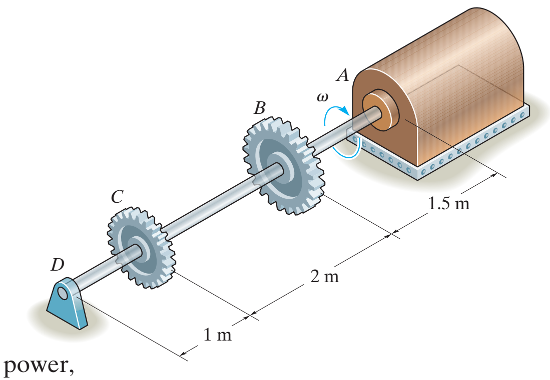
Angle of Twist:

$$\phi = \frac{TL}{JG} = \frac{11.23(10^3)(30)}{[\frac{\pi}{2}(0.1^4 - 0.09^4)][75(10^9)]}$$

$$= 0.08319\text{ rad} = 4.77^\circ \qquad \textbf{Ans.}$$

5–54.

The turbine develops 300 kW of power, which is transmitted to the gears such that both B and C receive an equal amount. If the rotation of the 100-mm-diameter A992 steel shaft is $\omega = 600$ rev/min., determine the absolute maximum shear stress in the shaft and the rotation of end D of the shaft relative to A. The journal bearing at D allows the shaft to turn freely about its axis.



SOLUTION

External Applied Torque: Gears B and C withdraw equal amount of power, $P_B = P_C = P = \dfrac{300}{2} = 150$ kw. Here, the angular velocity of the shaft is

$$\omega = \left(600\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 20\pi\text{ rad/s}$$

Then, the torque exerted on gears B and C can be determined from

$$T_B = T_C\,\frac{p}{\omega} = \frac{150(10^3)}{20\pi} = \frac{7500}{\pi}\text{ N}\cdot\text{m}$$

Using this result, the torque diagram shown in Fig. *a* can be plotted.

Maximum Shear Stress: From the torque diagram, we notice that the maximum torque $T_{\max} = \dfrac{15000}{\pi}$ N · m occurs at region BA. Thus it is the critical region where the absolute maximim shear stress occurs. Applying the torsion formula,

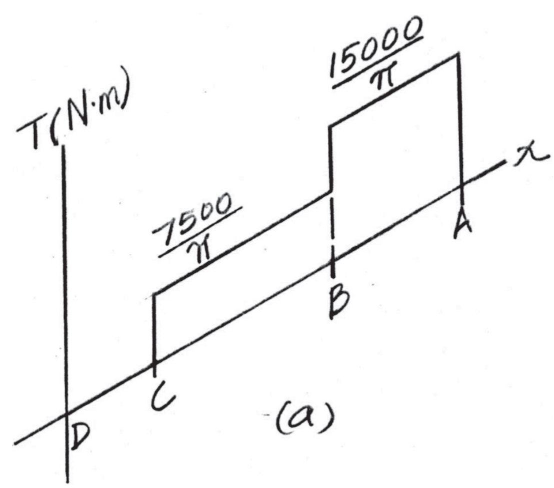
$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{T_{\max}C}{J} = \frac{\frac{15000}{\pi}(0.05)}{\frac{\pi}{2}(0.05^4)} = 24.32\,(10^6)\text{ Pa} = 24.3\text{ MPa} \qquad \textbf{Ans.}$$

Angle of twist: From the torque diagram, $T_{DC} = 0$, $T_{CB} = \dfrac{7500}{\pi}$ N · m and $T_{BA} = \dfrac{15000}{\pi}$ N · m

$$\phi_{D/A} = \Sigma\frac{TL}{JG} = \frac{1}{JG}(T_{DC}L_{DC} + T_{CB}L_{CB} + T_{BA}L_{BA})$$

$$= \frac{1}{JG}\left[0 + \left(\frac{7500}{\pi}\right)(2) + \left(\frac{15000}{\pi}\right)(1.5)\right]$$

$$= \frac{37500\text{ N}\cdot\text{m}^2}{\pi JG}$$



For A992 steel, $G = 75$ GPa. Thus

$$\phi_{D/A} = \frac{37500}{\pi\left[\frac{\pi}{2}(0.05^4)\right][75(10^9)]}$$

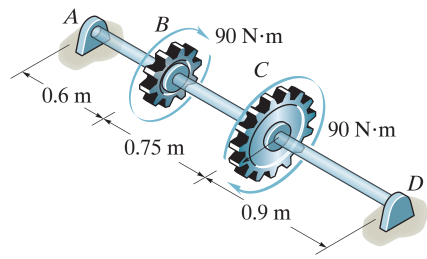
$$= (0.016211\text{ rad})\left(\frac{180°}{\pi\text{ rad}}\right)$$

$$= 0.9288°$$

$$= 0.929° \qquad \textbf{Ans.}$$

Ans:
$\tau_{\substack{\text{abs}\\\text{max}}} = 24.3$ MPa
$\phi_{D/A} = 0.929°$

5–55. The shaft is made of A992 steel. It has a diameter of 25 mm and is supported by bearings at A and D, which allow free rotation. Determine the angle of twist of B with respect to D.



SOLUTION

The internal torques developed in segments BC and CD are shown in Figs. a and b.

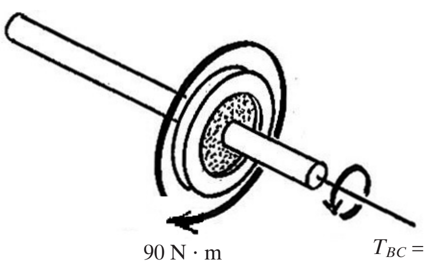
The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.0125^4) = 38.35(10^{-9})\ \text{m}^4$. Thus,

$$\phi_{B/D} = \sum \frac{T_i L_i}{J_i G_i} = \frac{T_{BC} L_{BC}}{J\, G_{st}} + \frac{T_{CD} L_{CD}}{J\, G_{st}}$$

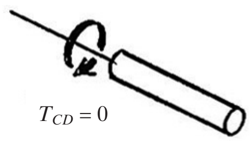
$$= \frac{90(0.75)}{[38.35(10^{-9})]75(10^6)} + 0$$

$$= 0.02347\ \text{rad} = 1.34°$$

Ans.



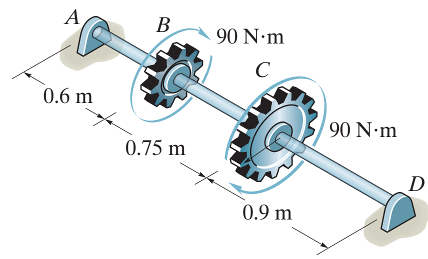
(a)



(b)

Ans:
$\phi_{B/D} = 1.34°$

5–56. The shaft is made of A-36 steel. It has a diameter of 25 mm and is supported by bearings at A and D, which allow free rotation. Determine the angle of twist of gear C with respect to B.



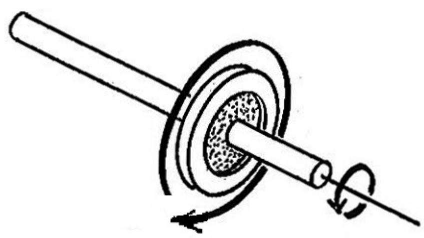
SOLUTION

The internal torque developed in segment BC is shown in Fig. a

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.0125^4) = 38.35(10^{-9})\ \text{m}^4$. Thus,

$$\phi_{C/B} = \frac{T_{BC}\,L_{BC}}{J\,G_{st}} = \frac{90(0.75)}{[38.35(10^{-9})]75(10^6)}$$

$$= 0.02347\ \text{rad} = 1.34° \qquad \textbf{Ans.}$$



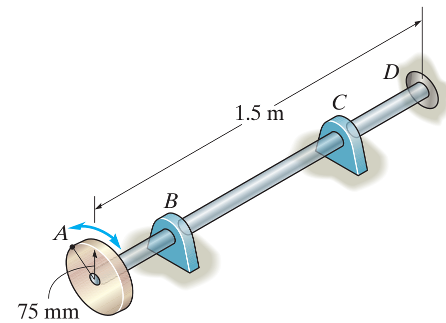
(a)

Ans:

$\phi_{C/B} = 1.34°$

5–57.

The rotating flywheel-and-shaft, when brought to a sudden stop at D, begins to oscillate clockwise-counter-clockwise such that a point A on the outer edge of the fly-wheel is displaced through a 6-mm arc. Determine the maximum shear stress developed in the tubular A-36 steel shaft due to this oscillation. The shaft has an inner diameter of 24 mm and an outer diameter of 32 mm. The bearings at B and C allow the shaft to rotate freely, whereas the support at D holds the shaft fixed.



SOLUTION

$s = r\,\theta$

$6 = 75\,\phi \qquad \phi = 0.08\text{ rad}$

$$\phi = \frac{TL}{JG}$$

$$0.08 = \frac{T(1.5)}{J(75)\left(10^9\right)}$$

$$T = 4\left(10^9\right)J$$

$$\tau_{\max} = \frac{Tc}{J}$$

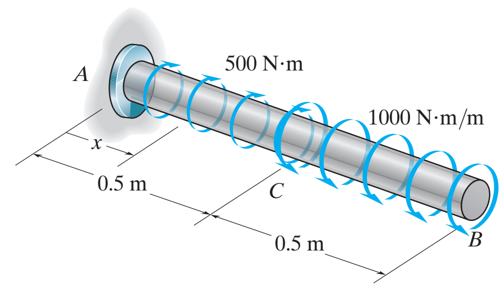
$$= \frac{4(10^9)(J)(0.016)}{J}$$

$$= 64.0\text{ MPa}$$ **Ans.**

Ans:
$\tau_{\max} = 64.0\text{ MPa}$

5–58.

The A992 steel shaft has a diameter of 50 mm and is subjected to the distributed loadings shown. Determine the absolute maximum shear stress in the shaft and plot a graph of the angle of twist of the shaft in radians versus x.



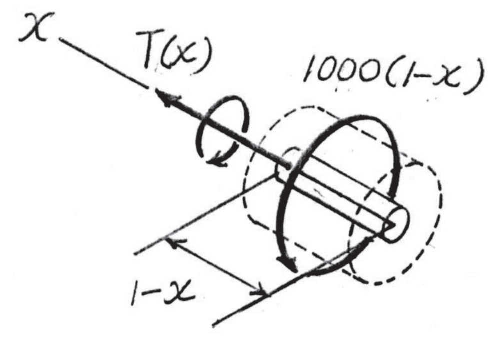
SOLUTION

Internal Torque: Referring to the FBD of the right segment of the shaft shown in Fig. *a* ($0.5\text{ m} < x \leq 1\text{ m}$)

$$\Sigma M_x = 0; \quad T(x) - 1000\,(1-x) = 0 \qquad T(x) = \{1000 - 1000x\}\text{ N}\cdot\text{m}$$

And Fig. *b* ($0 \leq x < 0.5$ m)

$$\Sigma M_x = 0; \quad T(x) + 500\,(0.5-x) - 1000\,(0.5) = 0 \quad T_{Ac} = \{500x + 250\}\text{ N}\cdot\text{m}$$



(a)

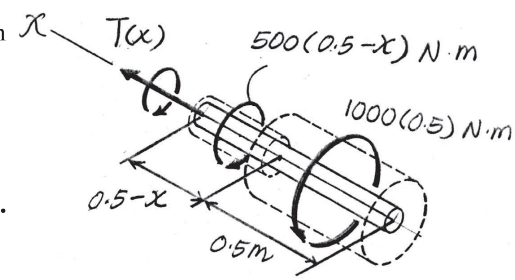
Maximum Shear Stress: The maximum torque at $x = 0.5$ m, where $T_{max} = 1000 - 1000(0.5) = 500\text{ N}\cdot\text{m}$. Applying the torsion formula,

$$\tau_{\substack{abs\\max}} = \frac{T_{max}C}{J} = \frac{500\,(0.025)}{\frac{\pi}{2}(0.025^4)} = 20.37\left(10^6\right) P_a = 20.4\text{ MPa} \qquad \textbf{Ans.}$$

Angle of Twist: For A992 steel $G = 75$ GPa. For region $0 \leq x < 0.5$ m (between B and C).

$$\phi(x) = \int \frac{T(x)dx}{JG} = \frac{1}{\left[\frac{\pi}{2}(0.025^4)\right]\left[75\left(10^9\right)\right]}\int_o^x (500\,x + 250)\,dx$$

$$= \left\{0.005432\left(x^2 + x\right)\right\}\text{rad} \qquad \textbf{Ans.}$$



(b)

At $x = 0.5$ m, $\phi = \phi_C = 0.004074$ rad. For region $0.5\text{ m} < x \leq 1$ m

$$\phi\,(x) = \phi_C + \int \frac{T(x)\,dx}{JG}$$

$$= 0.004074 + \frac{1}{\left[\frac{\pi}{2}(0.025^4)\left[75\left(10^9\right)\right]\right]}\int_{0.5\text{ m}}^x (1000 - 1000x)\,dx$$

$$= \{-0.01086x^2 + 0.02173\,x - 0.004074\}\text{ rad} \qquad \textbf{Ans.}$$

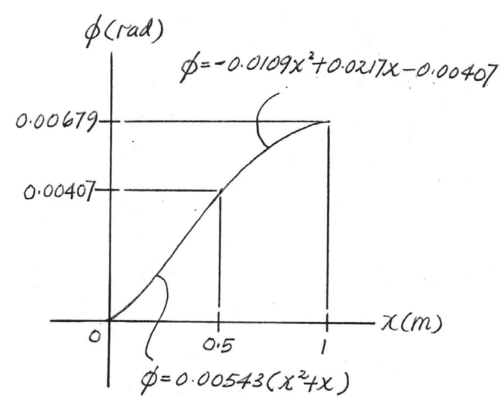
The maximum ϕ occurs at where $\dfrac{d\phi}{dx} = 0.$

$$\frac{d\phi}{dx} = -0.02173x + 0.02173 = 0$$

$$x = 1\text{ m}$$

Thus, $\phi_{max} = \phi_B = -0.01086\left(1^2\right) + 0.02173(1) - 0.004074 = 0.006791$ rad

The plot of ϕ vs x is as shown in Fig. *c*.



(c)

Ans:

$\tau_{\substack{abs\\max}} = 20.4$ MPa

For $0 \leq x < 0.5$ m,

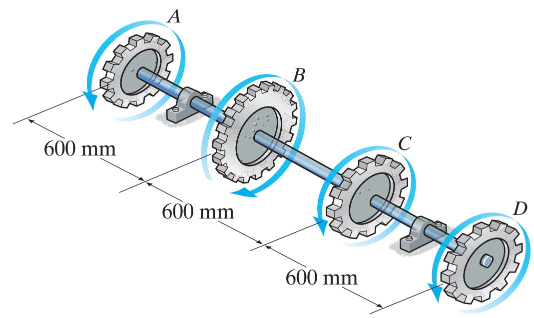
$\phi(x) = \left\{0.005432\left(x^2 + x\right)\right\}$ rad

For $0.5\text{ m} < x \leq 1$ m,

$\phi(x) = \{-0.01086x^2 + 0.02173\,x - 0.004074\}$ rad

5–59.

The shaft is made of A992 steel with the allowable shear stress of $\tau_{allow} = 75$ MPa. If gear B supplies 15 kW of power, while gears A, C and D withdraw 6 kW, 4 kW and 5 kW, respectively, determine the required minimum diameter d of the shaft to the nearest millimeter. Also, find the corresponding angle of twist of gear A relative to gear D. The shaft is rotating at 600 rpm.



SOLUTION

Internal Loading: The angular velocity of the shaft is

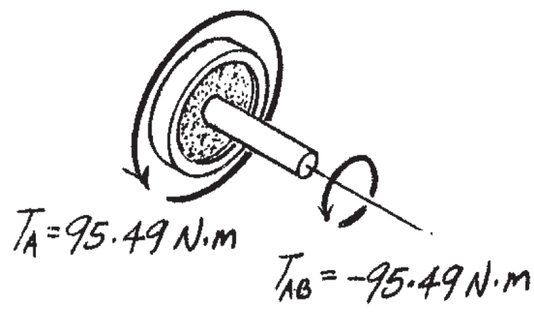
$$\omega = \left(600\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 20\pi\text{ rad/s}$$

Thus, the torque exerted on gears A, C, and D are

$$T_A = \frac{P_A}{\omega} = \frac{6(10^3)}{20\pi} = 95.49\text{ N}\cdot\text{m}$$

$$T_C = \frac{P_C}{\omega} = \frac{4(10^3)}{20\pi} = 63.66\text{ N}\cdot\text{m}$$

$$T_D = \frac{P_D}{\omega} = \frac{5(10^3)}{20\pi} = 79.58\text{ N}\cdot\text{m}$$



(a)

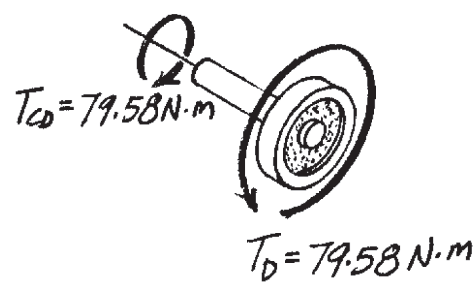
The internal torque developed in segments AB, CD, and BC of the shaft are shown in Figs. a, b, and c, respectively.

Allowable Shear Stress: Segment BC of the shaft is critical since it is subjected to a greater internal torque.

$$\tau_{allow} = \frac{T_{BC}\,c}{J}; \qquad 75(10^6) = \frac{143.24\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

$$d = 0.02135\text{ m} = 21.35\text{ mm}$$

Use $\qquad d = 22$ mm **Ans.**



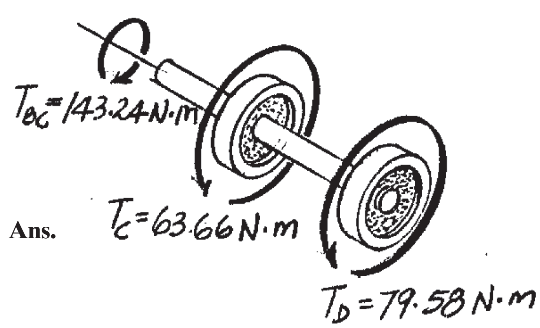
(b)

Angle of Twist: The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.011^4) = 7.3205(10^{-9})\pi\text{ m}^4$. We have

$$\phi_{A/D} = \Sigma\frac{T_iL_i}{J_iG_i} = \frac{T_{AB}L_{AB}}{JG_{st}} + \frac{T_{BC}L_{BC}}{JG_{st}} + \frac{T_{CD}L_{CD}}{JG_{st}}$$

$$\phi_{A/D} = \frac{0.6}{7.3205(10^{-9})\pi(75)(10^9)}(-95.49 + 143.24 + 79.58)$$

$$= 0.04429\text{ rad} = 2.54°$$ **Ans.**

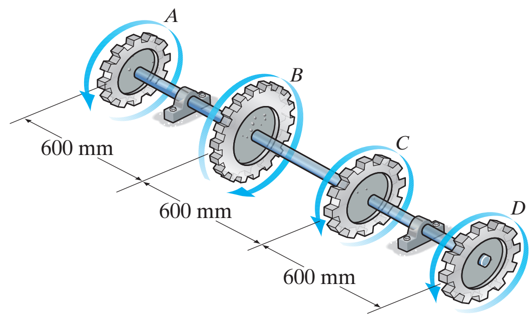


(c)

Ans:
Use $d = 22$ mm, $\phi_{A/D} = 2.54°$

***5–60.**

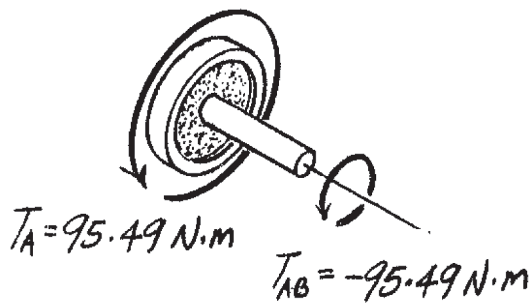
Gear B supplies 15 kW of power, while gears A, C and D withdraw 6 kW, 4 kW and 5 kW, respectively. If the shaft is made of steel with the allowable shear stress of $\tau_{allow} = 75$ MPa, and the relative angle of twist between any two gears cannot exceed 0.05 rad, determine the required minimum diameter d of the shaft to the nearest millimeter. The shaft is rotating at 600 rpm.



SOLUTION

Internal Loading: The angular velocity of the shaft is

$$\omega = \left(600\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\,\text{s}}\right)\left(\frac{2\pi\,\text{rad}}{1\text{ rev}}\right) = 20\pi\,\text{rad/s}$$



(a)

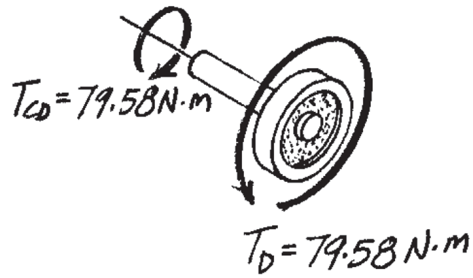
Thus, the torque exerted on gears A, C, and D are

$$T_A = \frac{P_A}{\omega} = \frac{6(10^3)}{20\pi} = 95.49\,\text{N}\cdot\text{m}$$

$$T_C = \frac{P_C}{\omega} = \frac{4(10^3)}{20\pi} = 63.66\,\text{N}\cdot\text{m}$$

$$T_D = \frac{P_D}{\omega} = \frac{5(10^3)}{20\pi} = 79.58\,\text{N}\cdot\text{m}$$

The internal torque developed in segments AB, CD, and BC of the shaft are shown in Figs. a, b, and c, respectively.



(b)

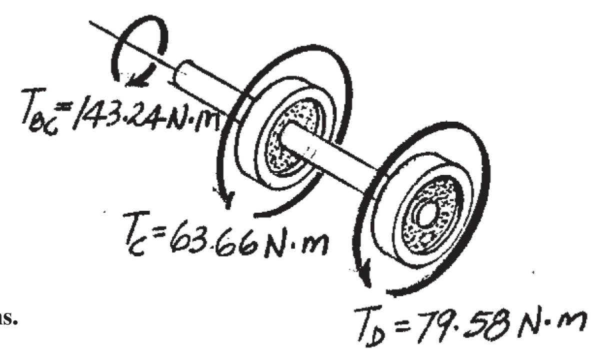
Allowable Shear Stress: Segment BC of the shaft is critical since it is subjected to a greater internal torque.

$$\tau_{allow} = \frac{T_{BC}\,c}{J}; \qquad 75(10^6) = \frac{143.24\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

$$d = 0.02135\text{ m} = 21.35\text{ mm}$$

Angle of Twist: By observation, the relative angle of twist of gear D with respect to gear B is the greatest.

Thus, the requirement is $\phi_{D/B} = 0.05$ rad.



(c)

$$\phi_{D/B} = \Sigma\frac{T_i L_i}{J_i G_i} = \frac{T_{BC}L_{BC}}{JG_{st}} + \frac{T_{CD}L_{CD}}{JG_{st}} = 0.05$$

$$\frac{0.6}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4(75)(10^9)}(143.24 + 79.58) = 0.05$$

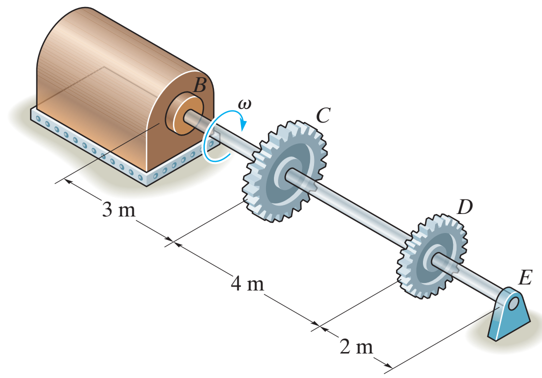
$$d = 0.02455\text{ m} = 24.55\text{ mm} = 25\text{ mm (controls!)}$$

Use $\quad d = 25$ mm **Ans.**

Ans:
Use $d = 25$ mm

5–61.

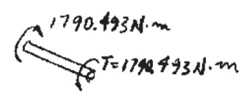
The turbine develops 150 kW of power, which is transmitted to the gears such that C receives 70% and D receives 30%. If the rotation of the 100-mm-diameter A-36 steel shaft is $\omega = 800$ rev/min., determine the absolute maximum shear stress in the shaft and the angle of twist of end E of the shaft relative to B. The journal bearing at E allows the shaft to turn freely about its axis.



SOLUTION

$$P = T\omega; \qquad 150(10^3)\text{ W} = T\left(800\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ sec}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)$$

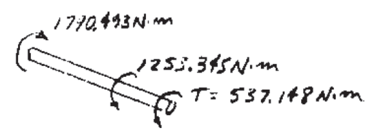
$$T = 1790.493\text{ N}\cdot\text{m}$$



$$T_C = 1790.493(0.7) = 1253.345\text{ N}\cdot\text{m}$$

$$T_D = 1790.493(0.3) = 537.148\text{ N}\cdot\text{m}$$

Maximum torque is in region BC.



$$\tau_{\max} = \frac{T_C}{J} = \frac{1790.493(0.05)}{\frac{\pi}{2}(0.05)^4} = 9.12\text{ MPa} \qquad \textbf{Ans.}$$

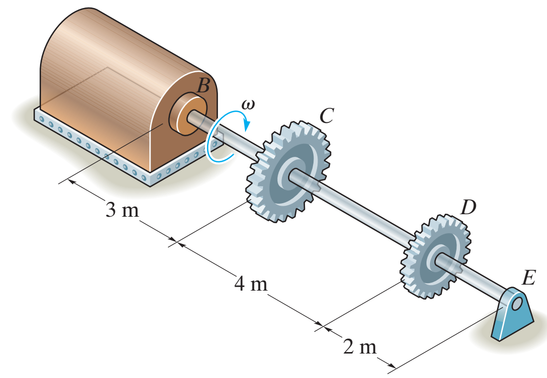
$$\phi_{E/B} = \Sigma\left(\frac{TL}{JG}\right) = \frac{1}{JG}[1790.493(3) + 537.148(4) + 0]$$

$$= \frac{7520.171}{\frac{\pi}{2}(0.05)^4(75)(10^9)} = 0.0102\text{ rad} = 0.585° \qquad \textbf{Ans.}$$

Ans:
$\tau_{\max} = 9.12$ MPa, $\phi_{E/B} = 0.585°$

5–62.

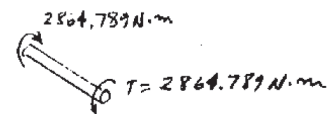
The turbine develops 150 kW of power, which is transmitted to the gears such that both C and D receive an equal amount. If the rotation of the 100-mm-diameter A-36 steel shaft is $\omega = 500$ rev/min., determine the absolute maximum shear stress in the shaft and the rotation of end B of the shaft relative to E. The journal bearing at E allows the shaft to turn freely about its axis.



SOLUTION

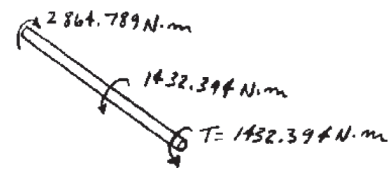
$$P = T\omega; \qquad 150(10^3)\ \text{W} = T\left(500\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{sec}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)$$

$$T = 2864.789\ \text{N}\cdot\text{m}$$



$$T_C = T_D = \frac{T}{2} = 1432.394\ \text{N}\cdot\text{m}$$

Maximum torque is in region BC.



$$\tau_{\max} = \frac{T_C}{J} = \frac{2864.789(0.05)}{\frac{\pi}{2}(0.05)^4} = 14.6\ \text{MPa} \qquad \textbf{Ans.}$$

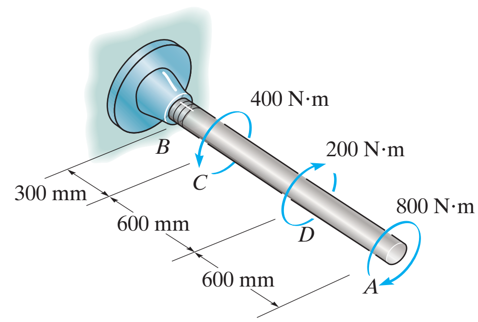
$$\phi_{B/E} = \Sigma\left(\frac{TL}{JG}\right) = \frac{1}{JG}[2864.789(3) + 1432.394(4) + 0]$$

$$= \frac{14323.945}{\frac{\pi}{2}(0.05)^4(75)(10^9)} = 0.0195\ \text{rad} = 1.11^\circ \qquad \textbf{Ans.}$$

Ans:
$\tau_{\max} = 14.6\ \text{MPa}$, $\phi_{B/E} = 1.11^\circ$

5–63.

The 50-mm-diameter A992 steel shaft is subjected to the torques shown. Determine the angle of twist of the end A.



SOLUTION

Internal Torque: The torque diagram shown in Fig. *a* can be plotted. From this diagram, $T_{AD} = -800\ \text{N}\cdot\text{m}$, $T_{DC} = -1000\ \text{N}\cdot\text{m}$ and $T_{CB} = -600\ \text{N}\cdot\text{m}$.

Angle of Twist:

$$\phi_{B/A} = \Sigma \frac{TL}{JG}$$

$$= \frac{1}{JG}\left(T_{AD}\,L_{AD} + T_{DC}\,L_{DC} + T_{CB}\,L_{CB}\right)$$

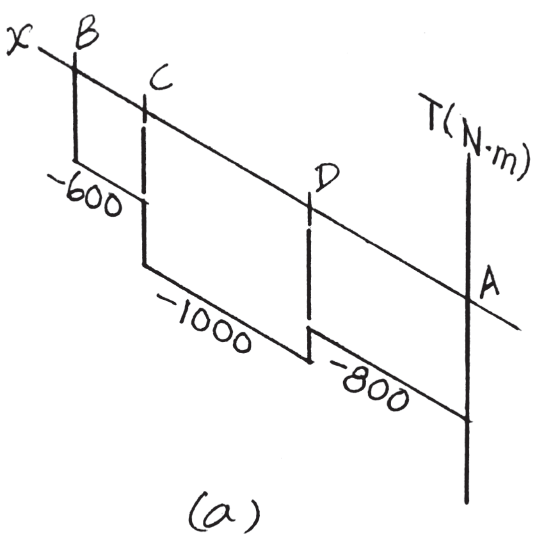
$$= \frac{1}{JG}[(-800)(0.6) + (-1000)(0.6) + (-600)(0.3)]$$

$$= -\frac{1260\ \text{N}\cdot\text{m}^2}{JG}$$

For A992 steel, $G = 75$ GPa. Then

$$\phi_A = \frac{1260}{\frac{\pi}{2}(0.025^4)\left[75(10^9)\right]}$$

$$= (-0.02738\ \text{rad})\left(\frac{180^\circ}{\pi\ \text{rad}}\right) = -1.569^\circ = 1.57^\circ$$ **Ans.**

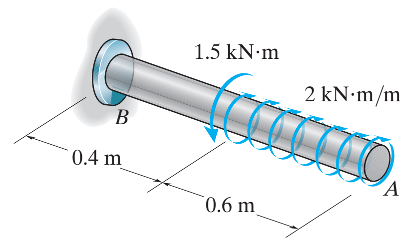


(a)

Ans:
$\phi_A = 1.57^\circ$

***5–64.**

The 60-mm-diameter solid shaft is made of 2014-T6 aluminum and is subjected to the distributed and concentrated torsional loadings shown. Determine the angle of twist at the free end A of the shaft.



SOLUTION

Internal Torque: Referring to the FBD of the right segments of the shaft shown in Fig. *a* and *b*

$\Sigma M_x = 0;\quad T_{AC} + 2000x = 0\qquad T_{AC} = (-2000x)\ \text{N}\cdot\text{m}$

And

$\Sigma M_x = 0;\quad T_{CB} + 2000\,(0.6) - 1500 = 0\qquad T_{CB} = 300\ \text{N}\cdot\text{m}$

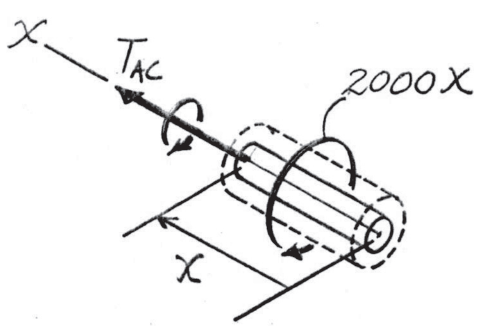
Angle of Twist: For 2014 - T6 Aluminum, $G = 27$ GPa.

$$\phi_A = \Sigma\frac{TL}{JG} = \frac{1}{JG}\left[\int_0^x T_{AC}\,dx + T_{CB}\,L_{CB}\right]$$

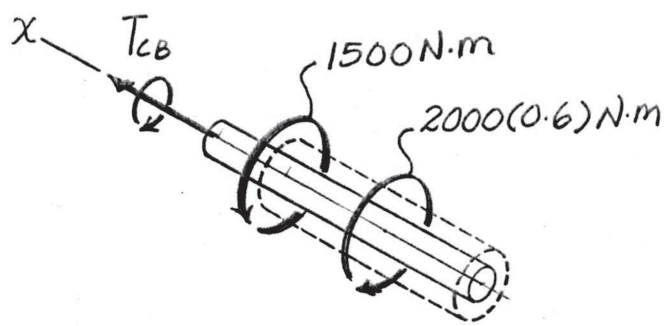
$$= \frac{1}{\left[\frac{\pi}{2}(0.03)^4\right]\left[27(10^9)\right]}\left[\int_0^{0.6\text{ m}}(-2000\,x)dx + 300\,(0.4)\right]$$

$$= (-0.006986\text{ rad})\left(\frac{180^\circ}{\pi\text{ rad}}\right)$$

$$= -0.400^\circ = 0.400^\circ$$ **Ans.**



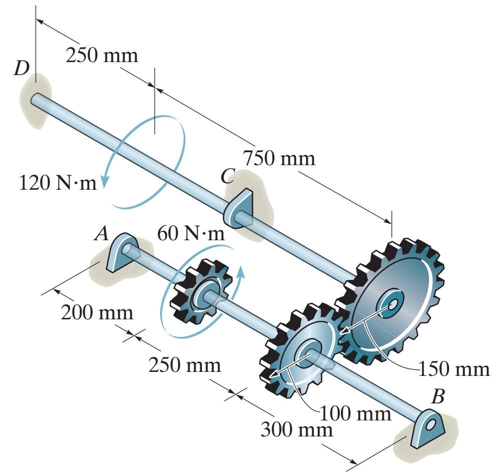
(a)



(b)

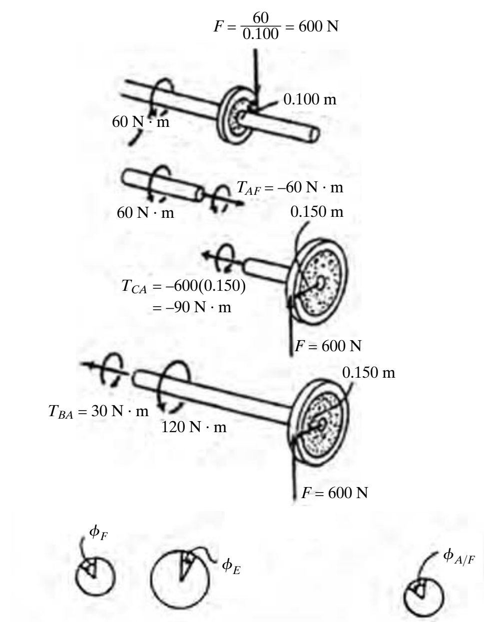
Ans:
$\phi_A = 0.400^\circ$

5–65. The two shafts are made of A-36 steel. Each has a diameter of 25 mm, and they are supported by bearings at A, B, and C, which allow free rotation. If the support at D is fixed, determine the angle of twist of end A when the torques are applied to the assembly as shown.



SOLUTION

Internal Torque: As shown on FBD.



Angle of Twist:

$$\phi_E = \sum \frac{TL}{JG}$$

$$= \frac{1}{[\frac{\pi}{2}(0.0125^4)][75(10^9)]}\{-90(0.75) + 30(0.25)\}$$

$$= -0.020861 = 0.020861 \text{ rad}$$

$$\phi_F = \left(\frac{150}{100}\right)\phi_E = \left(\frac{150}{100}\right)(0.020861) = 0.031291 \text{ rad}$$

$$\phi_{A/F} = \frac{T_{GF}\,L_{GF}}{JG}$$

$$= \frac{-60(0.25)}{[\frac{\pi}{2}(0.0125^4)][75(10^9)]}$$

$$= -0.0052152 \text{ rad} = 0.0052152 \text{ rad}$$

$$\phi_A = \phi_F + \phi_{A/F}$$

$$= 0.031291 + 0.0052152$$

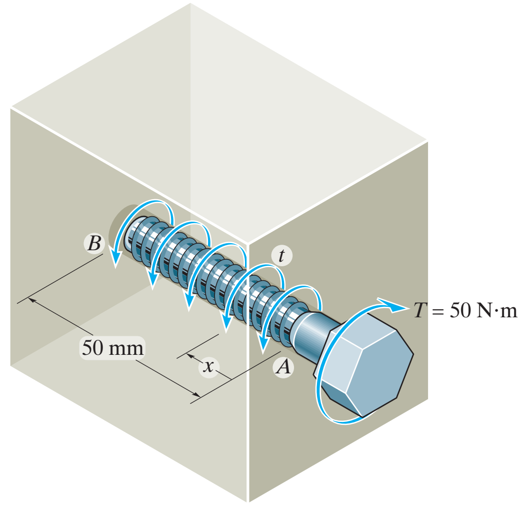
$$= 0.036506 \text{ rad} = 2.09°$$

Ans.

Ans.
$\phi_A = 2.09°$

5–66.

The A-36 steel bolt is tightened within a hole so that the reactive torque on the shank AB can be expressed by the equation $t = (kx^2)\ \mathrm{N\cdot m/m}$, where x is in meters. If a torque of $T = 50\ \mathrm{N\cdot m}$ is applied to the bolt head, determine the constant k and the amount of twist in the 50-mm length of the shank. Assume the shank has a constant radius of 4 mm.



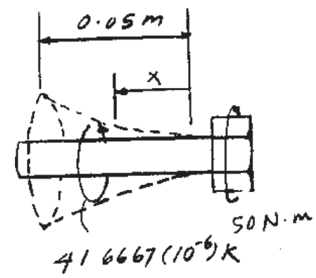
SOLUTION

$dT = t\,dx$

$$T = \int_0^{0.05\text{ m}} kx^2 dx = k\frac{x^3}{3}\Bigg|_0^{0.05} = 41.667(10^{-6})k$$

$$50 - 41.6667(10^{-6})\,k = 0$$

$$k = 1.20(10^6)\ \mathrm{N/m^2}$$ **Ans.**



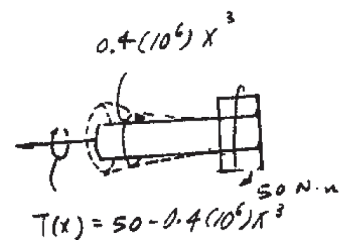
In the general position, $T = \displaystyle\int_0^x 1.20(10^6)x^2 dx = 0.4(10^6)x^3$

$$\phi = \int \frac{T(x)dx}{JG} = \frac{1}{JG}\int_0^{0.05\text{ m}} [50 - 0.4(10^6)x^3]dx$$

$$= \frac{1}{JG}\left[50x - \frac{0.4(10^6)x^4}{4}\right]\Bigg|_0^{0.05\text{ m}}$$

$$= \frac{1.875}{JG} = \frac{1.875}{\frac{\pi}{2}(0.004^4)(75)(10^9)}$$

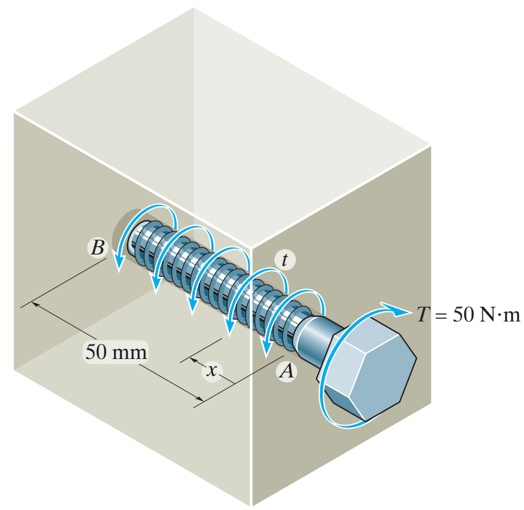
$$= 0.06217\text{ rad} = 3.56°$$ **Ans.**



Ans:
$k = 1.20(10^6)\ \mathrm{N/m^2}$, $\phi = 3.56°$

5–67.

The A-36 steel bolt is tightened within a hole so that the reactive torque on the shank AB can be expressed by the equation $t = (kx^{2/3})\ \text{N}\cdot\text{m/m}$, where x is in meters. If a torque of $T = 50\ \text{N}\cdot\text{m}$ is applied to the bolt head, determine the constant k and the amount of twist in the 50-mm length of the shank. Assume the shank has a constant radius of 4 mm.



SOLUTION

$dT = t\,dx$

$$T = \int_0^{0.05} kx^{\frac{2}{3}}\,dx = k\frac{3}{5}x^{\frac{5}{3}}\Big|_0^{0.05} = (4.0716)(10^{-3})\,k$$

$$50 - 4.0716\,(10^{-3})\,k = 0$$

$$k = 12.3(10^3)\ \text{N/m}^{\left(\frac{2}{3}\right)}$$ **Ans.**

In the general position,

$$T = \int_0^x 12.28(10^3)x^{\frac{2}{3}}\,dx = 7.368(10^3)\,x^{\frac{5}{3}}$$

Angle of twist:

$$\phi = \int \frac{T(x)\,dx}{JG} = \frac{1}{JG}\int_0^{0.05\ \text{m}} [50 - 7.3681(10^3)x^{\frac{8}{3}}]dx$$

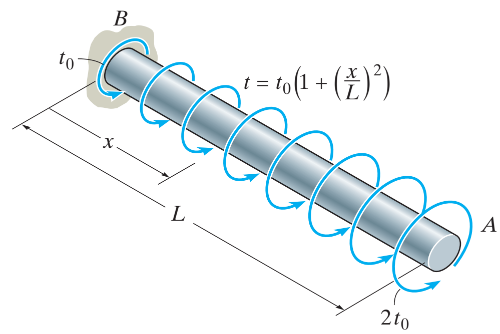
$$= \frac{1}{JG}\left[50x - 7.3681(10^3)\left(\frac{3}{8}\right)x^{\frac{8}{3}}\right]\Big|_0^{0.05\ \text{m}}$$

$$= \frac{1.5625}{\frac{\pi}{2}(0.004^4)(75)(10^9)} = 0.0518\ \text{rad} = 2.97°$$ **Ans.**

Ans:
$k = 12.3(10^3)\ \text{N/m}^{2/3}$, $\phi = 2.97°$

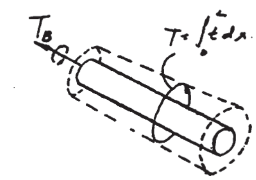
***5–68.**

The shaft of radius c is subjected to a distributed torque t, measured as torque/length of shaft. Determine the angle of twist at end A. The shear modulus is G.



SOLUTION

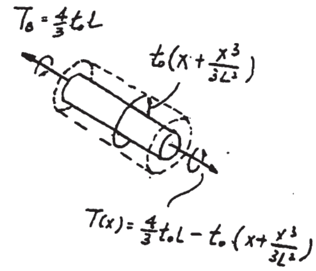
$$T_B - \int t\,dx = 0$$



$$T_B = \int t\,dx = t_0 \int \left(1 + \frac{x^2}{L^2}\right) dx$$

$$= t_0 \left[x + \frac{x^3}{3L^2}\right]\Bigg|_0^L = t_0\left(L + \frac{L}{3}\right) = \frac{4}{3} t_0 L$$

$$\phi = \int \frac{T(x)\,dx}{JG}$$



$$= \frac{1}{JG}\int_0^L \left[\frac{4}{3} t_0\,L - t_0\left(x + \frac{x^3}{3L^2}\right)\right] dx$$

$$= \frac{t_0}{JG}\left[\frac{4}{3} Lx - \left(\frac{x^2}{2} + \frac{x^4}{12L^2}\right)\right]\Bigg|_0^L = \frac{7\,t_0\,L^2}{12\,JG}$$

However $J = \frac{\pi}{2} c^4$,

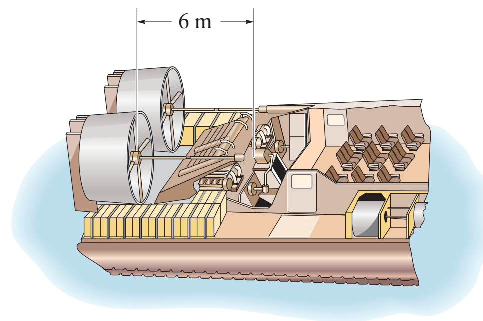
$$\phi = \frac{7\,t_0\,L^2}{6\,\pi\,c^4 G}$$ **Ans.**

Ans:

$$\phi = \frac{7\,t_0\,L^2}{6\,\pi\,c^4 G}$$

5–69.

The tubular drive shaft for the propeller of a hovercraft is 6 m long. If the motor delivers 4 MW of power to the shaft when the propellers rotate at 25 rad/s, determine the required inner diameter of the shaft if the outer diameter is 250 mm. What is the angle of twist of the shaft when it is operating? Take $\tau_{allow} = 90$ MPa and $G = 75$ GPa.



SOLUTION

Internal Torque:

$$P = 4\left(10^6\right)\text{ W} = 4\left(10^6\right)\text{ N}\cdot\text{m/s}$$

$$T = \frac{P}{\omega} = \frac{4\left(10^6\right)}{25} = 160(10^3)\text{ N}\cdot\text{m}$$

Maximum Shear Stress: Applying torsion formula.

$$\tau_{\max} = \tau_{\text{allow}} = \frac{Tc}{J}$$

$$90\left(10^6\right) = \frac{160\left(10^3\right)\left(0.125\right)}{\frac{\pi}{2}\left[0.125^4 - \left(\frac{d_t}{2}\right)^4\right]}$$

$$d_t = 0.2013\text{ m} = 201\text{ mm} \qquad \textbf{Ans.}$$

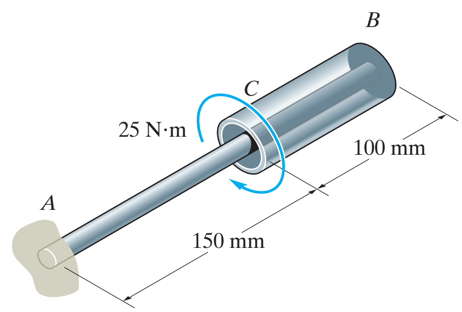
Angle of Twist:

$$\phi = \frac{TL}{JG} = \frac{160\left(10^3\right)\left(6\right)}{\frac{\pi}{2}\left(0.125^4 - 0.10065^4\right)75\left(10^9\right)}$$

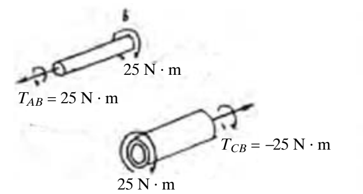
$$= 0.0576\text{ rad} = 3.30° \qquad \textbf{Ans.}$$

Ans:
$d_t = 201$ mm,
$\phi = 3.30°$

5–70. The A-36 steel assembly consists of a tube having an outer radius of 25 mm and a wall thickness of 3 mm. Using a rigid plate at B, it is connected to the solid 25-mm-diameter shaft AB. Determine the rotation of the tube's end C if a torque of 25 N · m is applied to the tube at this end. The end A of the shaft is fixed supported.



SOLUTION



$$\phi_B = \frac{T_{AB}L}{JG} = \frac{25(0.25)}{[\frac{\pi}{2}(0.0125^4)][75(10^9)]} = 0.0021730 \text{ rad}$$

$$\phi_{C/B} = \frac{T_{CB}L}{JG} = \frac{-25(0.1)}{[\frac{\pi}{2}(0.025^4 - 0.022^4)][75(10^9)]}$$

$$= -0.0001357 \text{ rad} = 0.0001357 \text{ rad}$$

$(+)\quad \phi_C = \phi_B + \phi_{C/B}$

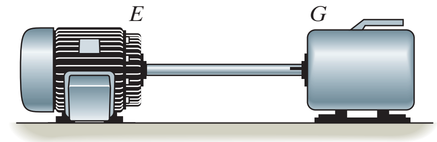
$= 0.0021730 + 0.0001357$

$= 0.0023087 \text{ rad} = 0.132°$ **Ans.**

Ans.
$\phi_C = 0.113°$

5–71.

The A-36 hollow steel shaft is 2 m long and has an outer diameter of 40 mm. When it is rotating at 80 rad/s, it transmits 32 kW of power from the engine E to the generator G. Determine the smallest thickness of the shaft if the allowable shear stress is $\tau_{\text{allow}} = 140$ MPa and the shaft is restricted not to twist more than 0.05 rad.



SOLUTION

$P = T\omega$

$$32(10^3) = T(80)$$

$$T = 400\ \text{N}\cdot\text{m}$$

Shear stress failure

$$\tau = \frac{Tc}{J}$$

$$\tau_{\text{allow}} = 140(10^6) = \frac{400(0.02)}{\frac{\pi}{2}(0.02^4 - r_i^{\,4})}$$

$$r_i = 0.01875\ \text{m}$$

Angle of twist limitation:

$$\phi = \frac{TL}{JG}$$

$$0.05 = \frac{400(2)}{\frac{\pi}{2}(0.02^4 - r_i^{\,4})(75)(10^9)}$$

$$r_i = 0.01247\ \text{m} \qquad \text{(controls)}$$

$$t = r_o - r_i = 0.02 - 0.01247$$

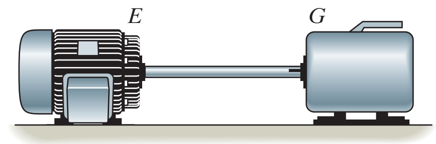
$$= 0.00753\ \text{m}$$

$$= 7.53\ \text{mm}$$ **Ans.**

Ans:
$t = 7.53$ mm

***5–72.**

The A-36 solid steel shaft is 3 m long and has a diameter of 50 mm. It is required to transmit 35 kW of power from the engine E to the generator G. Determine the smallest angular velocity of the shaft if it is restricted not to twist more than 1°.



SOLUTION

$$\phi = \frac{TL}{JG}$$

$$\frac{1^\circ(\pi)}{180^\circ} = \frac{T(3)}{\frac{\pi}{2}(0.025^4)(75)(10^9)}$$

$$T = 267.73\ \text{N}\cdot\text{m}$$

$$P = T\omega$$

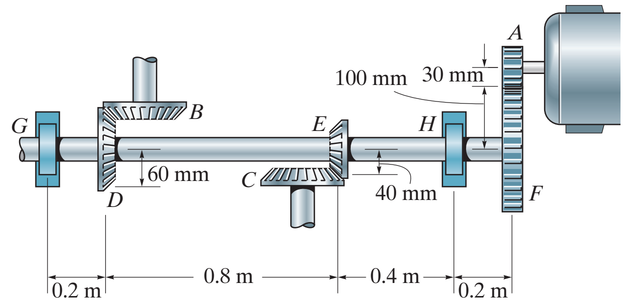
$$35(10^3) = 267.73(\omega)$$

$$\omega = 131\ \text{rad/s}$$ **Ans.**

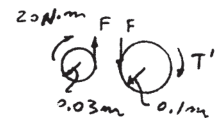
Ans:
$\omega = 131\ \text{rad/s}$

5–73.

The motor produces a torque of $T = 20\ \mathrm{N \cdot m}$ on gear A. If gear C is suddenly locked so it does not turn, yet B can freely turn, determine the angle of twist of F with respect to E and F with respect to D of the L2-steel shaft, which has an inner diameter of 30 mm and an outer diameter of 50 mm. Also, calculate the absolute maximum shear stress in the shaft. The shaft is supported on journal bearings at G at H.



SOLUTION



$$F(0.03) = 20$$

$$F = 666.67\ \mathrm{N}$$

$$T' = (666.67)(0.1) = 66.67\ \mathrm{N \cdot m}$$

Since shaft is held fixed at C, the torque is only in region EF of the shaft.

$$\phi_{F/E} = \frac{TL}{JG} = \frac{66.67(0.6)}{\frac{\pi}{2}\left[(0.025)^4 - (0.015)^4\right]75\left(10^9\right)} = 0.999\left(10\right)^{-3}\ \mathrm{rad} \qquad \textbf{Ans.}$$

Since the torque in region ED is zero,

$$\phi_{F/D} = 0.999(10)^{-3}\ \mathrm{rad} \qquad \textbf{Ans.}$$

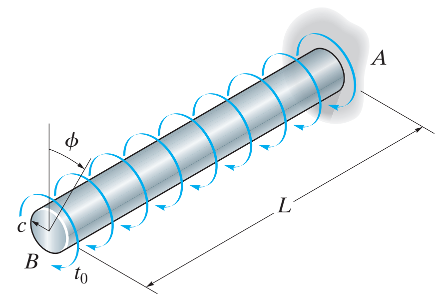
$$\tau_{\max} = \frac{Tc}{J} = \frac{66.67(0.025)}{\frac{\pi}{2}\left((0.025)^4 - (0.015)^4\right)}$$

$$= 3.12\ \mathrm{MPa} \qquad \textbf{Ans.}$$

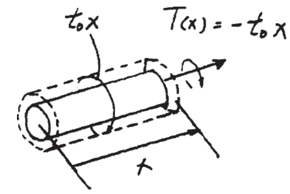
Ans:
$\phi_{F/E} = 0.999(10)^{-3}$ rad,
$\phi_{F/D} = 0.999(10)^{-3}$ rad,
$\tau_{\max} = 3.12$ MPa

5–74.

The shaft has a radius c and is subjected to a torque per unit length of t_0, which is distributed uniformly over the shaft's entire length L. If it is fixed at its far end A, determine the angle of twist ϕ of end B. The shear modulus is G.



SOLUTION



$$\phi = \int \frac{T(x)\,dx}{JG} = \frac{-t_0}{JG}\int_0^L x\,dx$$

$$= \frac{-t_0}{JG}\left[\frac{x^2}{2}\right]\Bigg|_0^L = \frac{-t_0}{JG}\,\frac{L^2}{2}$$

$$= \frac{-t_0 L^2}{2JG}$$

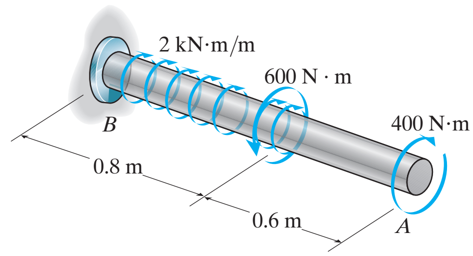
However, $J = \dfrac{\pi}{2}c^4$

$$\phi = \frac{-t_0 L^2}{\pi c^4 G} = \frac{t_0 L^2}{\pi c^4 G}$$ **Ans.**

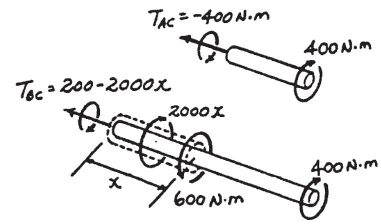
Ans:
$\phi = \dfrac{t_0 L^2}{\pi c^4 G}$

5–75.

The 60-mm-diameter solid shaft is made of A-36 steel and is subjected to the distributed and concentrated torsional loadings shown. Determine the angle of twist at the free end A of the shaft due to these loadings.



SOLUTION



Internal Torque: As shown on FBD.

Angle of Twist:

$$\phi_A = \Sigma \frac{TL}{JG}$$

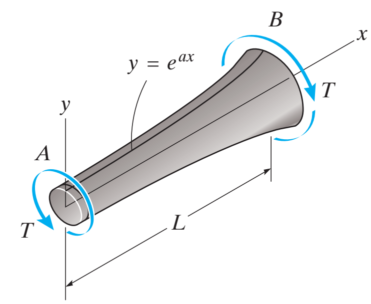
$$= \frac{-400(0.6)}{\frac{\pi}{2}(0.03^4)\,75.0(10^9)} + \int_0^{0.8\,\text{m}} \frac{(200 - 2000x)dx}{\frac{\pi}{2}(0.03^4)\,75.0(10^9)}$$

$$= -0.007545 \text{ rad} = 0.432°$$ **Ans.**

Ans:
$\phi_A = 0.432°$

***5–76.**

The contour of the surface of the shaft is defined by the equation $y = e^{ax}$, where a is a constant. If the shaft is subjected to a torque T at its ends, determine the angle of twist of end A with respect to end B. The shear modulus is G.



SOLUTION

$$\phi = \int \frac{T\,dx}{J(x)G} \qquad \text{where, } J(x) = \frac{\pi}{2}(e^{ax})^4$$

$$= \frac{2T}{\pi G}\int_0^L \frac{dx}{e^{4ax}} = \frac{2T}{\pi G}\left(-\frac{1}{4\,a\,e^{4ax}}\right)\Bigg|_0^L$$

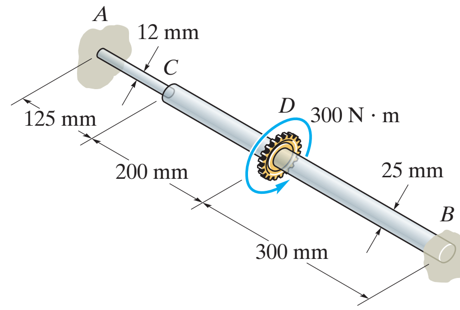
$$= \frac{2T}{\pi G}\left(-\frac{1}{4\,a\,e^{4aL}} + \frac{1}{4a}\right) = \frac{T}{2\,a\,\pi\,G}\left(\frac{e^{4aL} - 1}{e^{4aL}}\right)$$

$$= \frac{T}{2\,a\,\pi\,G}(1 - e^{-4aL}) \qquad \textbf{Ans.}$$

Ans:

$$\phi = \frac{T}{2a\pi G}(1 - e^{-4aL})$$

5–77. The steel shaft is made from two segments: AC has a diameter of 12 mm, and CB has a diameter of 25 mm. If it is fixed at its ends A and B and subjected to a torque of 300 N · m, determine the maximum shear stress in the shaft. $G_{st} = 75$ GPa.

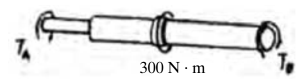


SOLUTION

Equilibrium:

$$T_A + T_B - 300 = 0 \qquad \textbf{(1)}$$

Compatibility condition:



$$\phi_{D/A} = \phi_{D/B}$$

$$\frac{T_A(0.125)}{[\frac{\pi}{2}(0.006^4)]G} + \frac{T_A(0.2)}{[\frac{\pi}{2}(0.0125^4)]G} = \frac{T_B(0.3)}{[\frac{\pi}{2}(0.0125^4)]G}$$

$$T_A = 0.11743T_B \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$T_A = 31.53 \text{ N}\cdot\text{m} \qquad T_B = 268.47 \text{ N}\cdot\text{m}$$

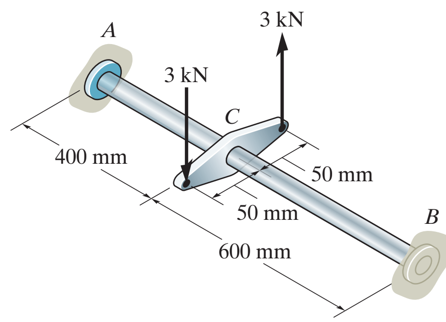
$$(\tau_{AC})_{\max} = \frac{Tc}{J} = \frac{31.53(0.06)}{\frac{\pi}{2}(0.006^4)} = 92.92(10^6) \text{ N/m}^2 = 92.9 \text{ MPa (Max)} \qquad \textbf{Ans.}$$

$$(\tau_{DB})_{\max} = \frac{Tc}{J} = \frac{268.47(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 87.51(10^6) \text{ N/m}^2 = 87.5 \text{ MPa}$$

Ans:
$(\tau_{AC})_{\max} = 92.9$ MPa

5–78.

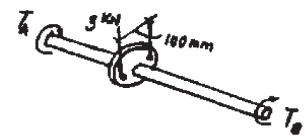
The steel shaft has a diameter of 40 mm and is fixed at its ends A and B. If it is subjected to the couple, determine the maximum shear stress in regions AC and CB of the shaft. $G_{st} = 75$ GPa.



SOLUTION

Equilibrium:

$$T_A + T_B - 3000(0.1) = 0 \qquad \textbf{(1)}$$



Compatibility condition:

$$\phi_{C/A} = \phi_{C/B}$$

$$\frac{T_A\,(400)}{JG} = \frac{T_B(600)}{JG}$$

$$T_A = 1.5\,T_B \qquad \textbf{(2)}$$

Solving Eqs (1) and (2) yields:

$$T_B = 120\ \text{N}\cdot\text{m}$$

$$T_A = 180\ \text{N}\cdot\text{m}$$

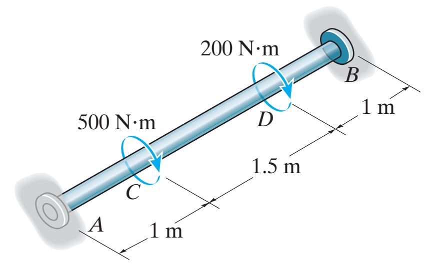
$$(\tau_{AC})_{\max} = \frac{Tc}{J} = \frac{180(0.02)}{\frac{\pi}{2}(0.02^4)} = 14.3\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{CB})_{\max} = \frac{Tc}{J} = \frac{120(0.02)}{\frac{\pi}{2}(0.02^4)} = 9.55\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$(\tau_{AC})_{\max} = 14.3$ MPa, $(\tau_{CB})_{\max} = 9.55$ MPa

5–79.

The A992 steel shaft has a diameter of 60 mm and is fixed at its ends A and B. If it is subjected to the torques shown, determine the absolute maximum shear stress in the shaft.



SOLUTION

Referring to the FBD of the shaft shown in Fig. *a*,

$$\Sigma M_x = 0; \quad T_A + T_B - 500 - 200 = 0 \qquad \textbf{(1)}$$

Using the method of superposition, Fig. *b*

$$\phi_A = (\phi_A)_{T_A} - (\phi_A)_T$$

$$0 = \frac{T_A\,(3.5)}{JG} - \left[\frac{500\,(1.5)}{JG} + \frac{700\,(1)}{JG}\right]$$

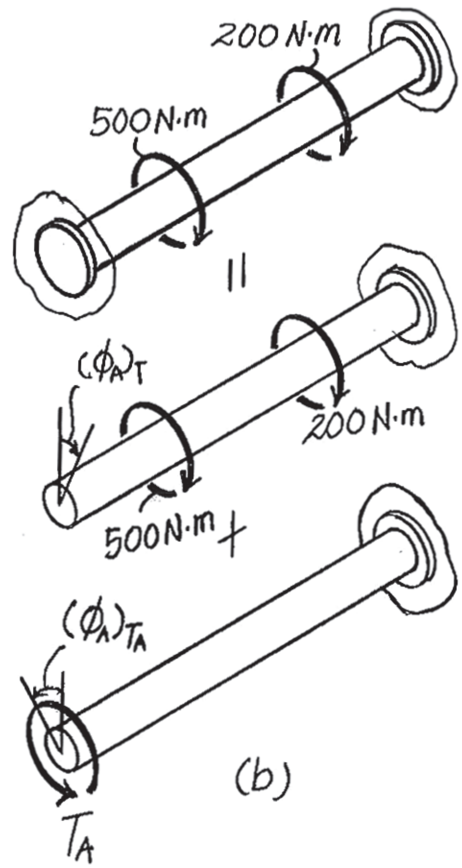
$$T_A = 414.29\ \text{N}\cdot\text{m}$$

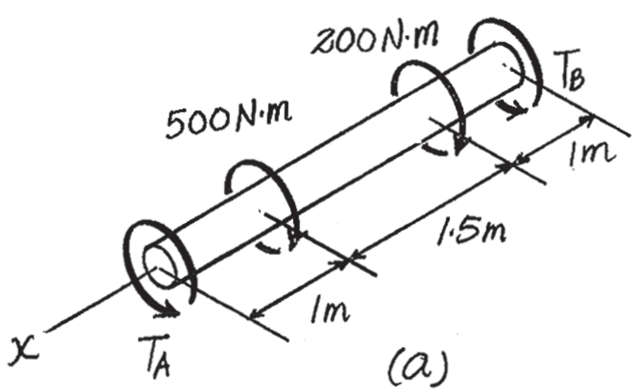
Substitute this result into Eq (1),

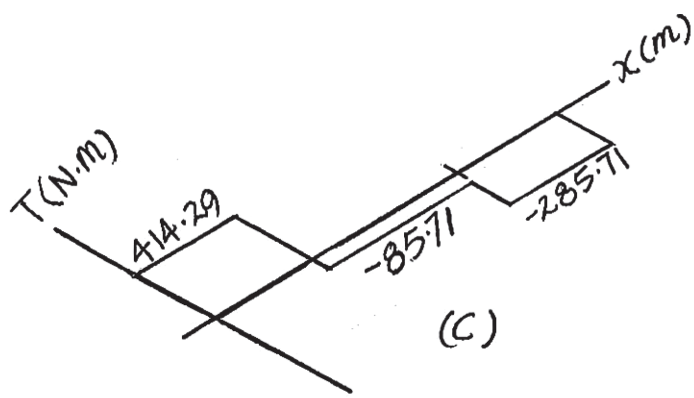
$$T_B = 285.71\ \text{N}\cdot\text{m}$$

Referring to the torque diagram shown in Fig. *c*, segment AC is subjected to maximum internal torque. Thus, the absolute maximum shear stress occurs here.

$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{T_{AC}\,c}{J} = \frac{414.29\,(0.03)}{\frac{\pi}{2}(0.03)^4} = 9.77\ \text{MPa} \qquad \textbf{Ans.}$$





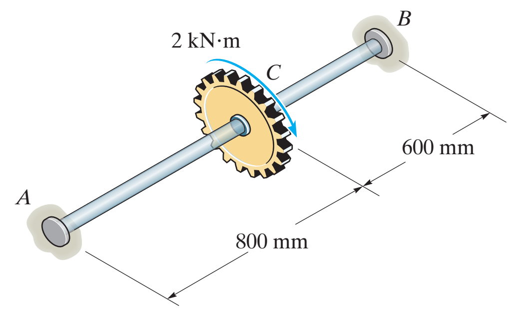


Ans:

$\tau_{\substack{\text{abs}\\ \text{max}}} = 9.77\ \text{MPa}$

***5–80.**

The shaft is made of L2 tool steel, has a diameter of 40 mm, and is fixed at its ends A and B. If it is subjected to the torque, determine the maximum shear stress in regions AC and CB.



SOLUTION

Equilibrium: Referring to the *FBD* of the shaft, Fig. *a*,

$$\Sigma M_x = 0; \qquad T_A + T_B - 2000 = 0 \qquad \textbf{(1)}$$

Compatibility: It is required that

$$\phi_{CA} = \phi_{CB}$$

$$\frac{T_A L_{CA}}{JG} = \frac{T_B L_{CB}}{JG}$$

$$\frac{T_A (0.8)}{JG} = \frac{T_B (0.6)}{JG}$$

$$T_A = 0.75\, T_B \qquad \textbf{(2)}$$

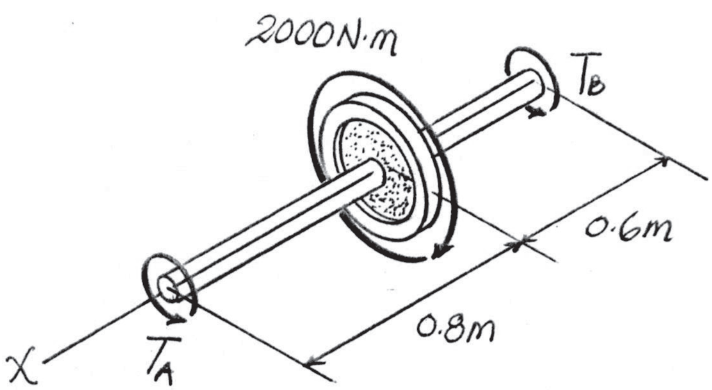
Solving Eqs (1) and (2)

$$T_B = 1142.86 \text{ N}\cdot\text{m} \qquad T_A = 857.14 \text{ N}\cdot\text{m}$$

Maximum Shear Stress: Applying the torsion formula,

$$(\tau_{\max})_{AC} = \frac{T_A c}{J} = \frac{857.14(0.02)}{\frac{\pi}{2}(0.02^4)} = 68.21(10^6) \text{ Pa} = 68.2 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{\max})_{BC} = \frac{T_B c}{J} = \frac{1142.86(0.02)}{\frac{\pi}{2}(0.02^4)} = 90.946(10^6) \text{ Pa} = 90.9 \text{ MPa} \qquad \textbf{Ans.}$$

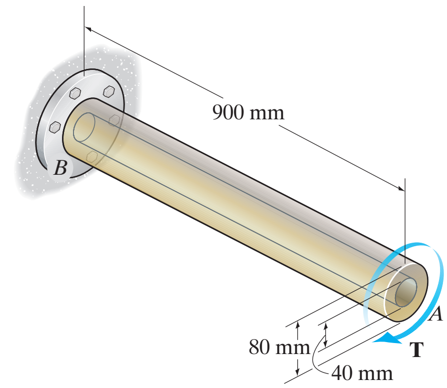


(a)

Ans:
$(\tau_{\max})_{AC} = 68.2$ MPa,
$(\tau_{\max})_{BC} = 90.9$ MPa

5–81.

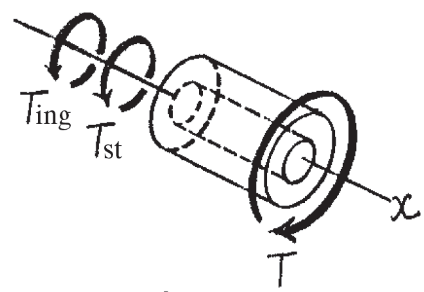
The Am1004-T61 magnesium tube is bonded to the A-36 steel rod. If the allowable shear stresses for the magnesium and steel are $(\tau_{allow})_{mg} = 45$ MPa and $(\tau_{allow})_{st} = 75$ MPa, respectively, determine the maximum allowable torque that can be applied at A. Also, find the corresponding angle of twist of end A.



SOLUTION

Equilibrium: Referring to the free-body diagram of the cut part of the assembly shown in Fig. *a*,

$$\Sigma M_x = 0; \quad T_{mg} + T_{st} - T = 0 \qquad \textbf{(1)}$$



(a)

Compatibility Equation: Since the steel rod is bonded firmly to the magnesium tube, the angle of twist of the rod and the tube must be the same. Thus,

$$(\phi_{st})_A = (\phi_{mg})_A$$

$$\frac{T_{st}L}{\frac{\pi}{2}(0.02^4)(75)(10^9)} = \frac{T_{mg}L}{\frac{\pi}{2}(0.04^4 - 0.02^4)(18)(10^9)}$$

$$T_{st} = 0.2778T_{mg} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$T_{mg} = 0.7826T \qquad T_{st} = 0.2174T$$

Allowable Shear Stress:

$$(\tau_{allow})_{mg} = \frac{T_{mg}\,c}{J}; \quad 45(10^6) = \frac{0.7826T(0.04)}{\frac{\pi}{2}(0.04^4 - 0.02^4)}$$

$$T = 5419.25 \text{ N}\cdot\text{m}$$

$$(\tau_{allow})_{st} = \frac{T_{st}c}{J}; \quad 75(10^6) = \frac{0.2174T(0.02)}{\frac{\pi}{2}(0.02^4)}$$

$$T = 4335.40 \text{ N}\cdot\text{m} = 4.34 \text{ kN}\cdot\text{m (control!)} \qquad \textbf{Ans.}$$

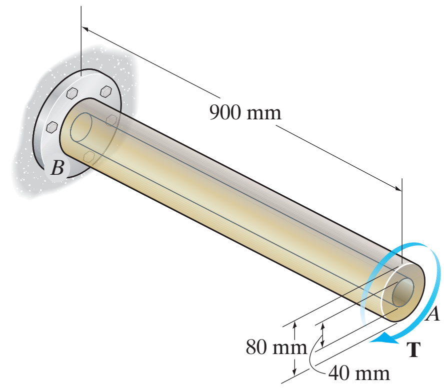
Angle of Twist: Using the result of T, $T_{st} = 942.48$ N·m. We have

$$\phi_A = \frac{T_{st}L}{J_{st}G_{st}} = \frac{942.48(0.9)}{\frac{\pi}{2}(0.02^4)(75)(10^9)} = 0.045 \text{ rad} = 2.58° \qquad \textbf{Ans.}$$

Ans:
$T = 4.34$ kN·m,
$\phi_A = 2.58°$

5–82.

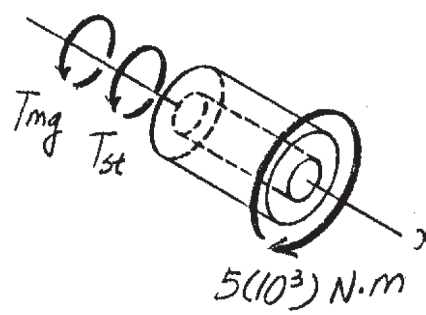
The Am1004-T61 magnesium tube is bonded to the A-36 steel rod. If a torque of $T = 5\ \text{kN}\cdot\text{m}$ is applied to end A, determine the maximum shear stress in each material. Sketch the shear stress distribution.



SOLUTION

Equilibrium: Referring to the free-body diagram of the cut part of the assembly shown in Fig. *a*,

$$\Sigma M_x = 0; \quad T_{mg} + T_{st} - 5(10^3) = 0 \tag{1}$$



(a)

Compatibility Equation: Since the steel rod is bonded firmly to the magnesium tube, the angle of twist of the rod and the tube must be the same. Thus,

$$(\phi_{st})_A = (\phi_{mg})_A$$

$$\frac{T_{st}L}{\frac{\pi}{2}(0.02^4)(75)(10^9)} = \frac{T_{mg}L}{\frac{\pi}{2}(0.04^4 - 0.02^4)(18)(10^9)}$$

$$T_{st} = 0.2778T_{mg} \tag{2}$$

Solving Eqs. (1) and (2),

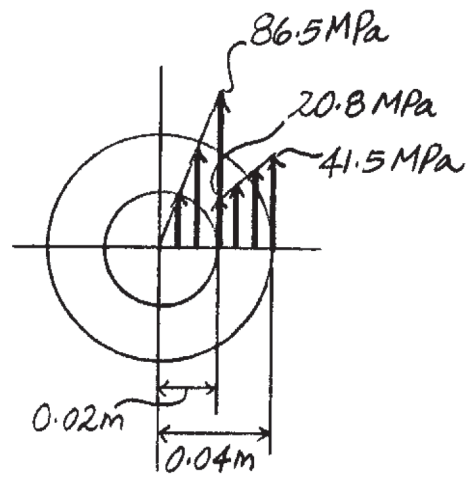
$$T_{mg} = 3913.04\ \text{N}\cdot\text{m} \qquad T_{st} = 1086.96\ \text{N}\cdot\text{m}$$

Maximum Shear Stress:

$$(\tau_{st})_{max} = \frac{T_{st}c_{st}}{J_{st}} = \frac{1086.96(0.02)}{\frac{\pi}{2}(0.02^4)} = 86.5\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{mg})_{max} = \frac{T_{mg}c_{mg}}{J_{mg}} = \frac{3913.04(0.04)}{\frac{\pi}{2}(0.04^4 - 0.02^4)} = 41.5\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{mg})\big|_{\rho=0.02\ \text{m}} = \frac{T_{mg}\,\rho}{J_{mg}} = \frac{3913.04(0.02)}{\frac{\pi}{2}(0.04^4 - 0.02^4)} = 20.8\ \text{MPa} \qquad \textbf{Ans.}$$

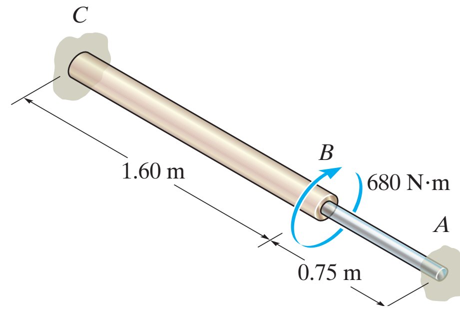


(b)

Ans:
$(\tau_{st})_{max} = 86.5$ MPa, $(\tau_{mg})_{max} = 41.5$ MPa,
$(\tau_{mg})\big|_{\rho=0.02\text{m}} = 20.8$ MPa

5–83.

A rod is made from two segments: AB is steel and BC is brass. It is fixed at its ends and subjected to a torque of $T = 680\ \text{N}\cdot\text{m}$. If the steel portion has a diameter of 30 mm, determine the required diameter of the brass portion so the reactions at the walls will be the same. $G_{st} = 75\ \text{GPa}$, $G_{br} = 39\ \text{GPa}$.

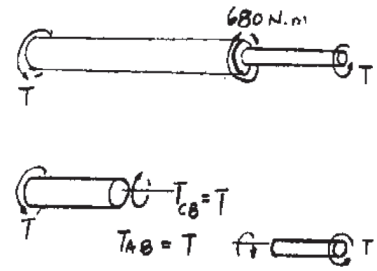


SOLUTION

Compatibility Condition:

$$\phi_{B/C} = \phi_{B/A}$$

$$\frac{T(1.60)}{\frac{\pi}{2}(c^4)(39)(10^9)} = \frac{T(0.75)}{\frac{\pi}{2}(0.015^4)(75)(10^9)}$$



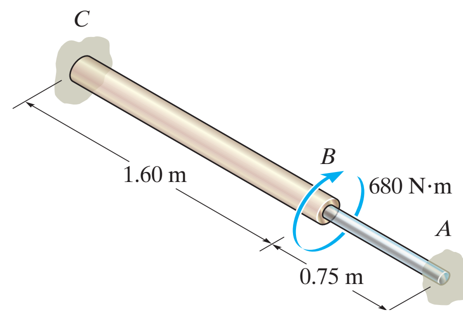
$c = 0.02134$ m

$d = 2c = 0.04269\ \text{m} = 42.7\ \text{mm}$ **Ans.**

Ans:
$d = 42.7$ mm

***5–84.**

Determine the absolute maximum shear stress in the shaft of Prob. 5–88.

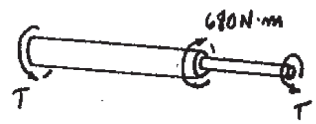


SOLUTION

Equilibrium,

$$2T = 680$$

$$T = 340\ \text{N} \cdot \text{m}$$



$\tau_{\underset{\text{max}}{\text{abs}}}$ occurs in the steel. See solution to Prob. 5–88.

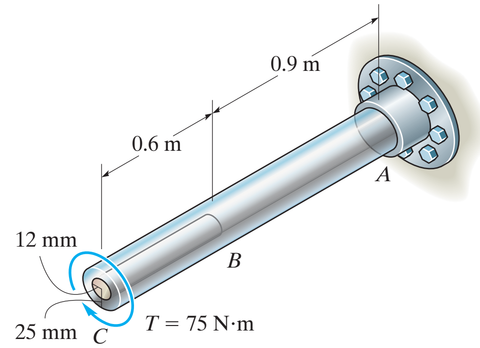
$$\tau_{\underset{\text{max}}{\text{abs}}} = \frac{Tc}{J} = \frac{340(0.015)}{\frac{\pi}{2}(0.015)^4}$$

$$= 64.1\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:

$\tau_{\underset{\text{max}}{\text{abs}}} = 64.1\ \text{MPa}$

5–85. The shaft is made from a solid steel section AB and a tubular portion made of steel and having a brass core. If it is fixed to a rigid support at A, and a torque of $T = 75\ \text{N}\cdot\text{m}$ is applied to it at C, determine the angle of twist that occurs at C and compute the maximum shear stress and maximum shear strain in the brass and steel. Take $G_{st} = 75$ GPa, $G_{br} = 38$ GPa.

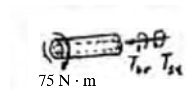


SOLUTION

Equilibrium:

$$T_{br} + T_{st} - 75 = 0 \qquad \textbf{(1)}$$

Both the steel tube and brass core undergo the same angle of twist $\phi_{C/B}$



$$\phi_{C/B} = \frac{TL}{JG} = \frac{T_{br}(0.6)}{[\frac{\pi}{2}(0.0125^4)][38(10^9)]} = \frac{T_{st}(0.6)}{[\frac{\pi}{2}(0.025^4 - 0.0125^4)][75(10^9)]}$$

$$T_{br} = 0.033778\ T_{st} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$T_{st} = 72.549\ \text{N}\cdot\text{m}; \qquad T_{br} = 2.451\ \text{N}\cdot\text{m}$$

$$\phi_C = \Sigma\frac{TL}{JG} = \frac{2.451(0.6)}{[\frac{\pi}{2}(0.0125^4)][38(10^9)]} + \frac{75(0.9)}{[\frac{\pi}{2}(0.025^4)][75(10^9)]}$$

$$= 0.002476\ \text{rad} = 0.142° \qquad \textbf{Ans.}$$

$$(\tau_{st})_{\max AB} = \frac{T_{AB}c}{J} = \frac{75(0.025)}{\frac{\pi}{2}(0.025^4)} = 3.056(10^6)\ \text{N/m}^2 = 3.056\ \text{MPa}$$

$$(\tau_{st})_{\max BC} = \frac{T_{st}\,c}{J} = \frac{72.549(0.025)}{[\frac{\pi}{2}(0.025^4 - 0.0125^4)]}$$

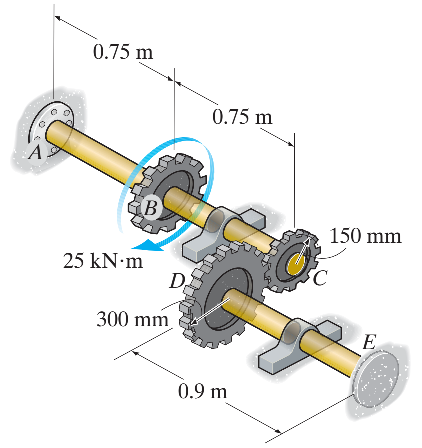
$$= 3.153(10^6)\ \text{N/m}^2 = 3.15\ \text{MPa (Max)} \qquad \textbf{Ans.}$$

$$(\gamma_{st})_{\max} = \frac{(\tau_{st})_{\max}}{G} = \frac{3.153(10^6)}{75(10^9)} = 42.0(10^{-6})\ \text{rad} \qquad \textbf{Ans.}$$

$$(\tau_{br})_{\max} = \frac{T_{br}\,c}{J} = \frac{2.451(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 0.7988(10^6)\ \text{N/m}^2 = 0.799\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\gamma_{br})_{\max} = \frac{(\tau_{br})_{\max}}{G} = \frac{0.7988(10^6)}{38(10^9)} = 21.0(10^{-6})\ \text{rad} \qquad \textbf{Ans.}$$

5–86. The shafts are made of A-36 steel and have the same diameter of 100 mm. If a torque of 25 kN · m is applied to gear B, determine the absolute maximum shear stress developed in the shaft.



SOLUTION

Equilibrium: Referring to the free - body diagrams of shafts ABC and DE shown in Figs. a and b, respectively, we have

$$\Sigma M_x = 0; \quad T_A + F(0.15) - 25 = 0 \qquad \textbf{(1)}$$

and

$$\Sigma M_x = 0; \quad F(0.3) - T_E = 0 \qquad \textbf{(2)}$$

Internal Loadings: The internal torques developed in segments AB and BC of shaft ABC and shaft DE are shown in Figs. c, d, and e, respectively.

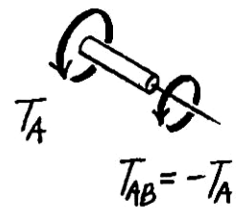
Compatibility Equation:

$$\phi_C r_C = \phi_D r_D$$

$$\left(\frac{T_{AB}L_{AB}}{JG_{st}} + \frac{T_{BC}L_{BC}}{JG_{st}}\right)r_C = \left(\frac{T_{DE}\,L_{DE}}{JG_{st}}\right)r_D$$

$$[-T_A(0.75) + F(0.15)(0.75)](0.15) = -T_E(0.9)(0.3)$$

$$T_A - 0.15F = 2.4T_E \qquad \textbf{(3)}$$



(c)

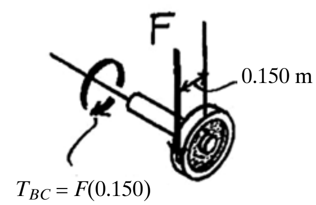
Solving Eqs. (1), (2), and (3), we have

$F = 24.51$ kN $\qquad T_E = 7.353$ kN · m $\qquad T_A = 21.32$ kN · m

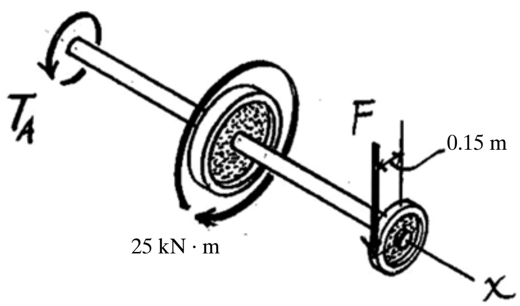
Maximum Shear Stress: By inspection, segment AB of shaft ABC is subjected to the greater torque.

$$\left(\tau_{\max}\right)_{\text{abs}} = \frac{T_{AB}c}{J_{st}} = \frac{[21.32(10^3)](0.05)}{\frac{\pi}{2}(0.05^4)}$$

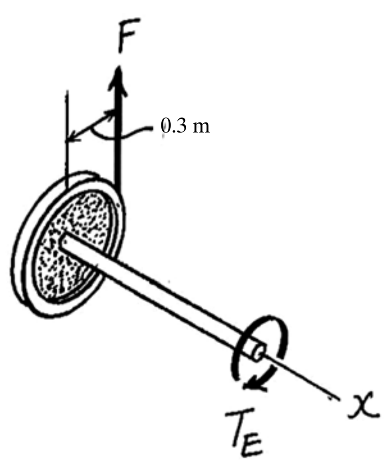
$$= 108.60(10^6)\ \text{N/m}^2 = 109\ \text{MPa} \qquad \textbf{Ans.}$$



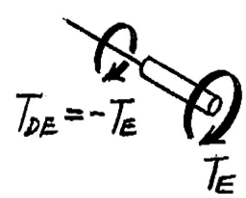
(d)



(a)



(b)

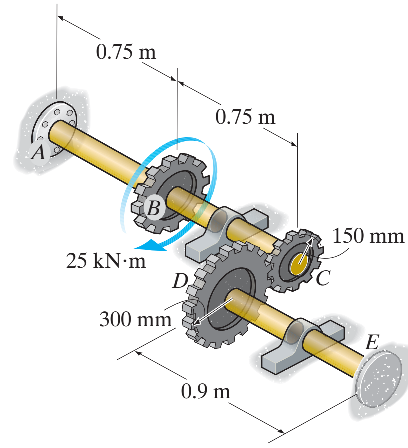


(e)

Ans.

$\left(\tau_{\max}\right)_{\text{abs}} = 109$ MPa

5–87. The shafts are made of A-36 steel and have the same diameter of 100 mm. If a torque of 25 kN · m is applied to gear B, determine the angle of twist of gear B.



SOLUTION

Equilibrium: Referring to the free - body diagrams of shafts ABC and DE shown in Figs. a and b, respectively,

$$\Sigma M_x = 0; \quad T_A + F(0.15) - 25 = 0 \qquad \textbf{(1)}$$

and

$$\Sigma M_x = 0; \quad F(0.3) - T_E = 0 \qquad \textbf{(2)}$$

Internal Loadings: The internal torques developed in segments AB and BC of shaft ABC and shaft DE are shown in Figs. c, d, and e, respectively.

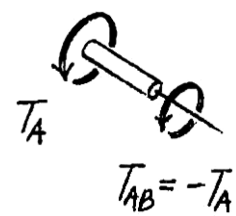
Compatibility Equation: It is required that

$$\phi_C r_C = \phi_D r_D$$

$$\left(\frac{T_{AB}\,L_{AB}}{JG_{st}} + \frac{T_{BC}\,L_{BC}}{JG_{st}}\right) r_C = \left(\frac{T_{DE}\,L_{DE}}{JG_{st}}\right) r_D$$

$$[-T_A(0.75) + F(0.15)(0.75)](0.15) = -T_E(0.9)(0.3)$$

$$T_A - 0.15F = 2.4T_E \qquad \textbf{(3)}$$



(c)

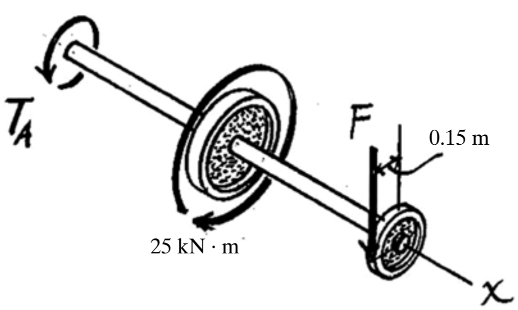
Solving Eqs. (1), (2), and (3),

$F = 24.51$ kN $\qquad T_E = 7.353$ kN · m $\qquad T_A = 21.32$ kN · m

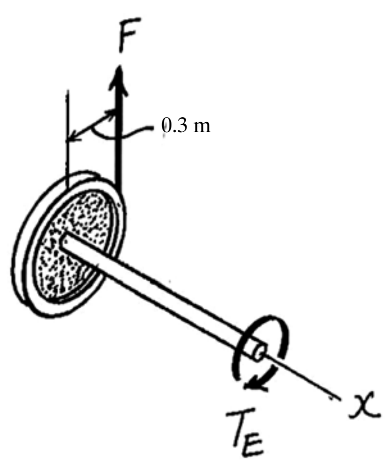
Angle of Twist: Here, $T_{AB} = -T_A = -21.32$ kN · m

$$\phi_B = \frac{T_{AB}\,L_{AB}}{JG_{st}} = \frac{[-21.32(10^3)](0.75)}{[\frac{\pi}{2}(0.05^4)][75(10^9)]}$$

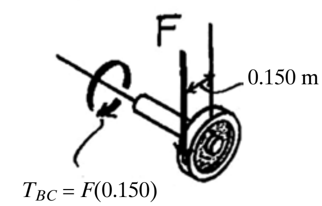
$$= -0.02172 \text{ rad} = 1.24^\circ \qquad \textbf{Ans.}$$



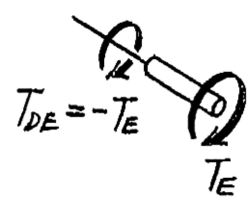
(a)



(b)



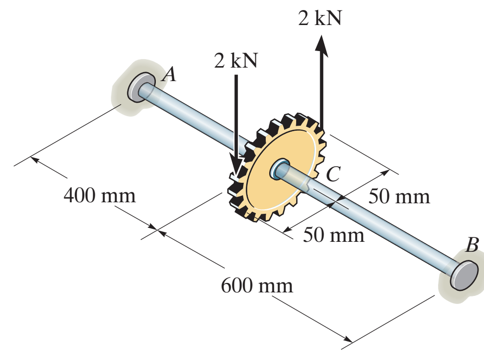
(d)



(e)

***5–88.**

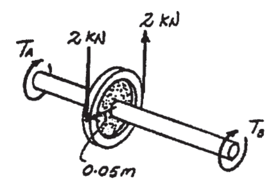
The shaft is made of L2 tool steel, has a diameter of 40 mm, and is fixed at its ends A and B. If it is subjected to the couple, determine the maximum shear stress in regions AC and CB.



SOLUTION

Equilibrium:

$$T_A + T_B - 2(0.1) = 0 \qquad \textbf{(1)}$$



Compatibility:

$$\phi_{C/A} = \phi_{C/B}$$

$$\frac{T_A(0.4)}{JG} = \frac{T_B(0.6)}{JG}$$

$$T_A = 1.50 T_B \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$T_B = 0.080 \text{ kN} \cdot \text{m} \qquad T_A = 0.120 \text{ kN} \cdot \text{m}$$

Maximum Shear Stress:

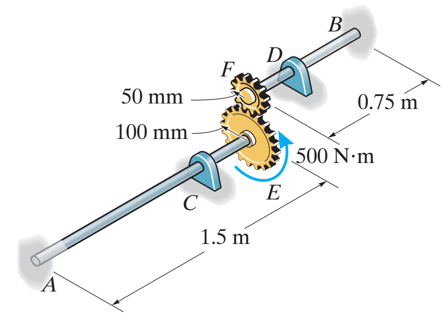
$$(\tau_{AC})_{\max} = \frac{T_A\, c}{J} = \frac{0.12(10^3)(0.02)}{\frac{\pi}{2}(0.02^4)} = 9.55 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{CB})_{\max} = \frac{T_B\, c}{J} = \frac{0.08(10^3)(0.02)}{\frac{\pi}{2}(0.02^4)} = 6.37 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$(\tau_{AC})_{\max} = 9.55$ MPa,
$(\tau_{CB})_{\max} = 6.37$ MPa

5–89.

The two shafts are made of A-36 steel. Each has a diameter of 25 mm and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at *A* and *B*. They are also supported by journal bearings at *C* and *D*, which allow free rotation of the shafts along their axes. If a torque of 500 N·m is applied to the gear at *E*, determine the reactions at *A* and *B*.

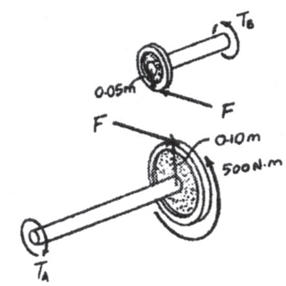


SOLUTION

Equilibrium:

$$T_A + F(0.1) - 500 = 0 \qquad \textbf{(1)}$$

$$T_B - F(0.05) = 0 \qquad \textbf{(2)}$$



From Eqs. (1) and (2)

$$T_A + 2T_B - 500 = 0 \qquad \textbf{(3)}$$

Compatibility:

$$0.1\phi_E = 0.05\phi_F$$

$$\phi_E = 0.5\phi_F$$

$$\frac{T_A(1.5)}{JG} = 0.5\left[\frac{T_B(0.75)}{JG}\right]$$

$$T_A = 0.250T_B \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields:

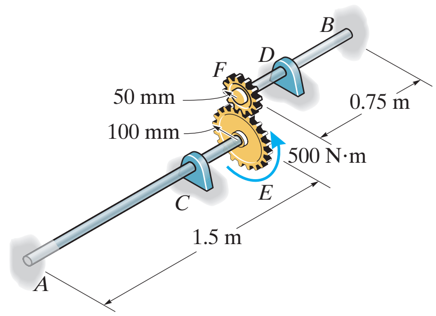
$$T_B = 222 \text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

$$T_A = 55.6 \text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$T_B = 22.2 \text{ N}\cdot\text{m}$, $T_A = 55.6 \text{ N}\cdot\text{m}$

5–90.

The two shafts are made of A-36 steel. Each has a diameter of 25 mm and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at A and B. They are also supported by journal bearings at C and D, which allow free rotation of the shafts along their axes. If a torque of $500\ \text{N}\cdot\text{m}$ is applied to the gear at E, determine the rotation of this gear.

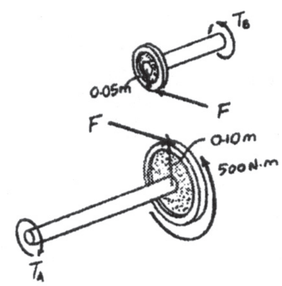


SOLUTION

Equilibrium:

$$T_A + F(0.1) - 500 = 0 \qquad \textbf{(1)}$$

$$T_B - F(0.05) = 0 \qquad \textbf{(2)}$$



From Eqs. (1) and (2)

$$T_A + 2T_B - 500 = 0 \qquad \textbf{(3)}$$

Compatibility:

$$0.1\phi_E = 0.05\phi_F$$

$$\phi_E = 0.5\phi_F$$

$$\frac{T_A(1.5)}{JG} = 0.5\left[\frac{T_B(0.75)}{JG}\right]$$

$$T_A = 0.250T_B \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields:

$$T_B = 222.22\ \text{N}\cdot\text{m} \qquad T_A = 55.56\ \text{N}\cdot\text{m}$$

Angle of Twist:

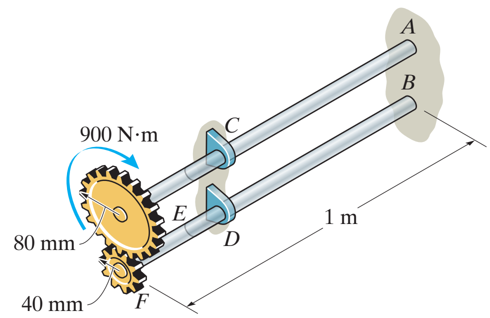
$$\phi_E = \frac{T_A L}{JG} = \frac{55.56(1.5)}{\frac{\pi}{2}(0.0125^4)(75.0)(10^9)}$$

$$= 0.02897\ \text{rad} = 1.66° \qquad \textbf{Ans.}$$

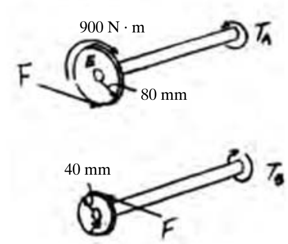
Ans:

$\phi_E = 1.66°$

5–91. The two 1-m-long shafts are made of 2014-T6 aluminum. Each has a diameter of 30 mm and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at A and B. They are also supported by bearings at C and D, which allow free rotation of the shafts along their axes. If a torque of 900 N · m is applied to the top gear as shown, determine the maximum shear stress in each shaft.



SOLUTION



$$T_A + F(0.08) - 900 = 0 \qquad \textbf{(1)}$$

$$T_B - F(0.04) = 0 \qquad \textbf{(2)}$$

From Eqs. (1) and (2)

$$T_A + 2T_B - 900 = 0 \qquad \textbf{(3)}$$

$$80(\phi_E) = 40(\phi_F); \qquad \phi_E = 0.5\phi_F$$

$$\frac{T_A L}{JG} = 0.5\left(\frac{T_B L}{JG}\right); \qquad T_A = 0.5T_B \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields:

$$T_B = 360 \text{ N} \cdot \text{m}; \qquad T_A = 180 \text{ N} \cdot \text{m}$$

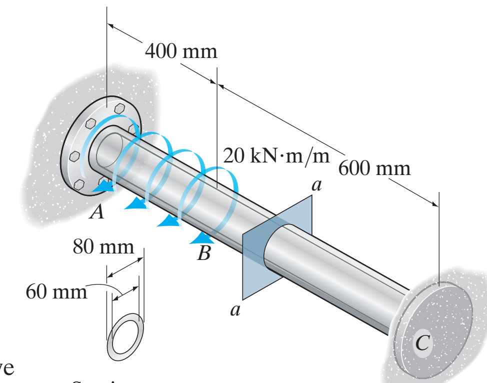
$$(\tau_{BD})_{\max} = \frac{T_B\, c}{J} = \frac{360(0.015)}{\frac{\pi}{2}(0.015^4)} = 67.91(10^6) \text{ N/m}^2 = 67.9 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{AC})_{\max} = \frac{T_A\, c}{J} = \frac{180(0.015)}{\frac{\pi}{2}(0.015^4)} = 33.95(10^6) \text{ N/m}^2 = 34.0 \text{ MPa} \qquad \textbf{Ans.}$$

Ans.
$(\tau_{BD})_{\max} = 67.9$ MPa, $(\tau_{AC})_{\max} = 34.0$ MPa

***5–92.**

If the shaft is subjected to a uniform distributed torque of $t = 20\ \text{kN} \cdot \text{m/m}$, determine the maximum shear stress developed in the shaft. The shaft is made of 2014-T6 aluminum alloy and is fixed at A and C.



Section a–a

SOLUTION

Equilibrium: Referring to the free-body diagram of the shaft shown in Fig. a, we have

$$\Sigma M_x = 0; \quad T_A + T_C - 20(10^3)(0.4) = 0 \qquad \textbf{(1)}$$

Compatibility Equation: The resultant torque of the distributed torque within the region x of the shaft is $T_R = 20(10^3)x\ \text{N} \cdot \text{m}$. Thus, the internal torque developed in the shaft as a function of x when end C is free is $T(x) = 20(10^3)x\ \text{N} \cdot \text{m}$, Fig. b. Using the method of superposition, Fig. c,

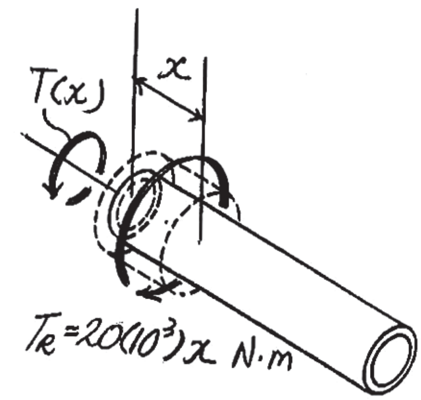
$$\phi_C = (\phi_C)_t - (\phi_C)_{T_C}$$

$$0 = \int_0^{0.4\text{ m}} \frac{T(x)dx}{JG} - \frac{T_C L}{JG}$$

$$0 = \int_0^{0.4\text{ m}} \frac{20(10^3)x dx}{JG} - \frac{T_C(1)}{JG}$$

$$0 = 20(10^3)\left(\frac{x^2}{2}\right)\Bigg|_0^{0.4\text{ m}} - T_C$$

$$T_C = 1600\ \text{N} \cdot \text{m}$$



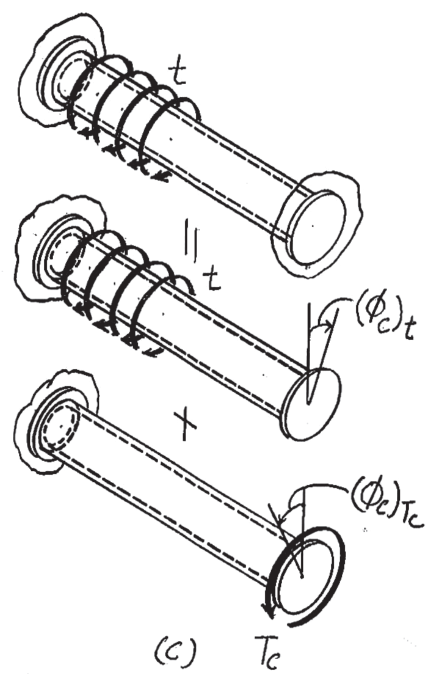
(b)

Substituting this result into Eq. (1),

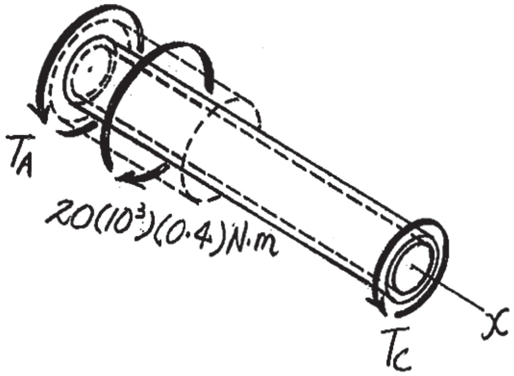
$$T_A = 6400\ \text{N} \cdot \text{m}$$

Maximum Shear Stress: By inspection, the maximum internal torque occurs at support A. Thus,

$$\left(\tau_{\max}\right)_{\text{abs}} = \frac{T_A\, c}{J} = \frac{6400(0.04)}{\dfrac{\pi}{2}\left(0.04^4 - 0.03^4\right)} = 93.1\ \text{MPa} \qquad \textbf{Ans.}$$



(c)

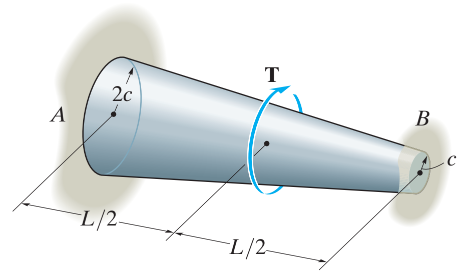


(a)

Ans:
$\tau_{\substack{\text{abs}\\\text{max}}} = 93.1\ \text{MPa}$

5–93.

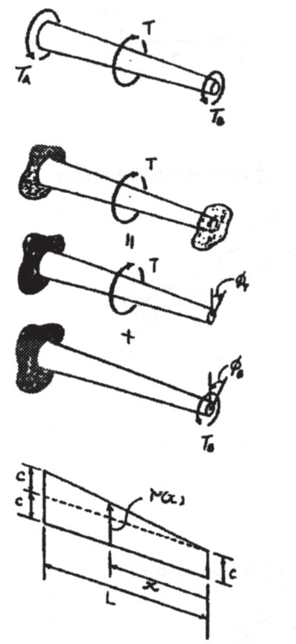
The tapered shaft is confined by the fixed supports at A and B. If a torque $\mathbf{T}$ is applied at its mid-point, determine the reactions at the supports.



SOLUTION

Equilibrium:

$$T_A + T_B - T = 0 \qquad \textbf{(1)}$$



Section Properties:

$$r(x) = c + \frac{c}{L}x = \frac{c}{L}(L + x)$$

$$J(x) = \frac{\pi}{2}\left[\frac{c}{L}(L + x)\right]^4 = \frac{\pi c^4}{2L^4}(L + x)^4$$

Angle of Twist:

$$\phi_T = \int \frac{Tdx}{J(x)G} = \int_{\frac{L}{2}}^{L} \frac{Tdx}{\frac{\pi c^4}{2L^4}(L + x)^4 G}$$

$$= \frac{2TL^4}{\pi c^4 G}\int_{\frac{L}{2}}^{L} \frac{dx}{(L + x)^4}$$

$$= -\frac{2TL^4}{3\pi c^4 G}\left[\frac{1}{(L + x)^3}\right]\Bigg|_{\frac{L}{2}}^{L}$$

$$= \frac{37TL}{324\,\pi c^4 G}$$

$$\phi_B = \int \frac{Tdx}{J(x)G} = \int_0^L \frac{T_B dx}{\frac{\pi c^4}{2L^4}(L + x)^4 G}$$

$$= \frac{2T_B L^4}{\pi c^4 G}\int_0^L \frac{dx}{(L + x)^4}$$

$$= -\frac{2T_B L^4}{3\pi c^4 G}\left[\frac{1}{(L + x)^3}\right]\Bigg|_0^L$$

$$= \frac{7T_B L}{12\pi c^4 G}$$

Compatibility:

$$0 = \phi_T - \phi_B$$

$$0 = \frac{37TL}{324\pi c^4 G} - \frac{7T_B L}{12\pi c^4 G}$$

$$T_B = \frac{37}{189}T \qquad \textbf{Ans.}$$

Substituting the result into Eq. (1) yields:

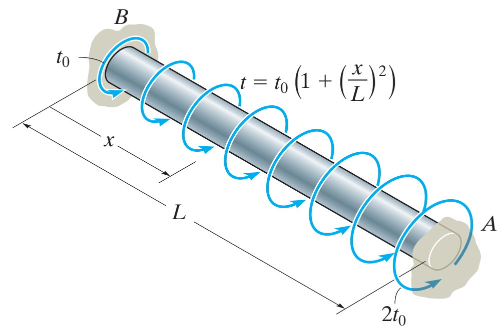
$$T_A = \frac{152}{189}T \qquad \textbf{Ans.}$$

Ans:

$T_B = \dfrac{37}{189}T,\ T_A = \dfrac{152}{189}T$

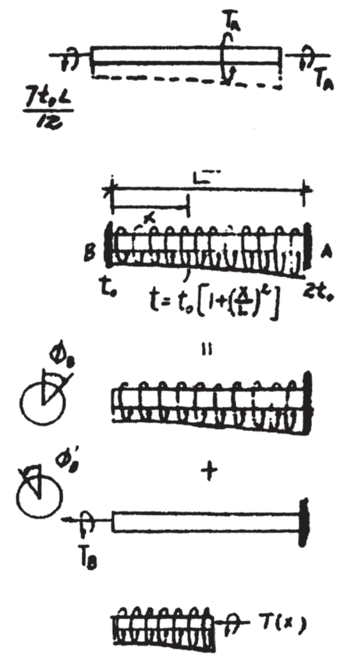
5–94.

The shaft of radius c is subjected to a distributed torque t, measured as torque/length of shaft. Determine the reactions at the fixed supports A and B.



SOLUTION

$$T(x) = \int_0^x t_0\left(1 + \frac{x^2}{L^2}\right)dx = t_0\left(x + \frac{x^3}{3L^2}\right) \qquad \textbf{(1)}$$



By superposition:

$$0 = \phi - \phi_B$$

$$0 = \int_0^L \frac{t_0\left(x + \frac{x^3}{3L^2}\right)dx}{JG} - \frac{T_B(L)}{JG} = \frac{7t_0L^2}{12} - T_B(L)$$

$$T_B = \frac{7t_0L}{12} \qquad \textbf{Ans.}$$

From Eq. (1),

$$T = t_0\left(L + \frac{L^3}{3L^2}\right) = \frac{4t_0L}{3}$$

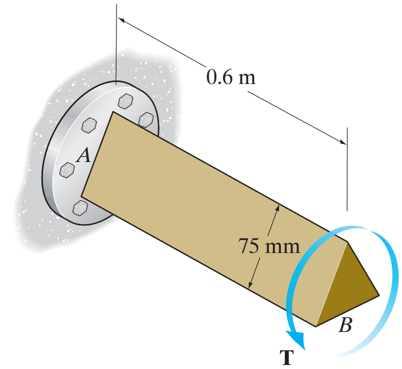
$$T_A + \frac{7t_0L}{12} - \frac{4t_0L}{3} = 0$$

$$T_A = \frac{3t_0L}{4} \qquad \textbf{Ans.}$$

Ans:

$T_B = \dfrac{7t_0L}{12}$, $T_A = \dfrac{3t_0L}{4}$

5–95. If end B of the shaft, which has an equilateral triangle cross section, is subjected to a torque of $T = 1200\ \text{N}\cdot\text{m}$, determine the maximum shear stress developed in the shaft. Also, find the angle of twist of end B. The shaft is made from 6061-T1 aluminum.



SOLUTION

Maximum Shear Stress:

$$\tau_{\max} = \frac{20T}{a^3} = \frac{20(1200)}{0.075^3} = 56.89(10^6)\ \text{N/m}^2 = 56.9\ \text{MPa} \qquad \textbf{Ans.}$$

Angle of Twist:

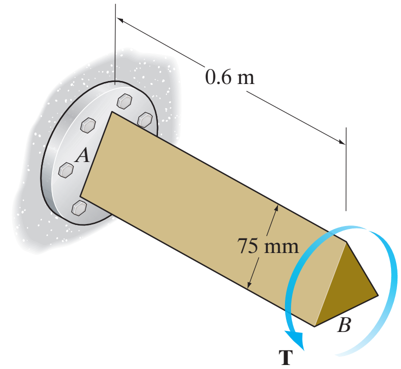
$$\phi = \frac{46TL}{a^4G}$$

$$= \frac{46(1200)(0.6)}{(0.075^4)[26(10^9)]}$$

$$= 0.04026\ \pi\text{ad} = 2.31° \qquad \textbf{Ans.}$$

Ans.
$\tau_{\max} = 56.9$ MPa
$\phi = 2.31°$

***5–96.** If the shaft has an equilateral triangle cross section and is made from 606 |-T| aluminum alloy that has an allowable shear stress of $\tau_{allow} = 84$ MPa, determine the maximum allowable torque **T** that can be applied to end *B*. Also, find the corresponding angle of twist of end *B*.



SOLUTION

Allowable Shear Stress:

$$\tau_{allow} = \frac{20T}{a^3}; \qquad 84(10^6) = \frac{20T}{0.075^3}$$

$$T = 1.772(10^3)\ \text{N}\cdot\text{m} = 1.77\ \text{kN}\cdot\text{m}$$ **Ans.**

Angle of Twist:

$$\phi = \frac{46TL}{a^4G}$$

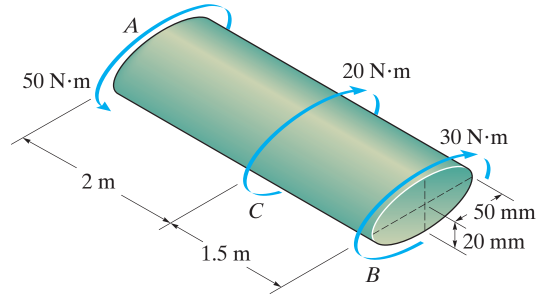
$$= \frac{46[1.772(10^3)](0.6)}{(0.075^4)[26(10^9)]}$$

$$= 0.05945\ \text{rad} = 3.41°$$ **Ans.**

Ans:
$T = 1.77$ kN $\cdot$ m, $\phi = 3.41°$

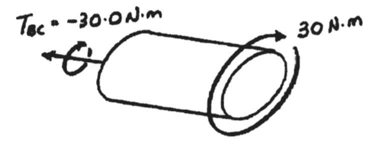
5–97.

The shaft is made of red brass C83400 and has an elliptical cross section. If it is subjected to the torsional loading, determine the maximum shear stress within regions AC and BC, and the angle of twist ϕ of end B relative to end A.



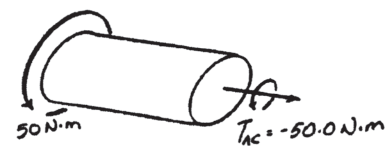
SOLUTION

Maximum Shear Stress:



$$(\tau_{BC})_{\max} = \frac{2T_{BC}}{\pi\, a\, b^2} = \frac{2(30.0)}{\pi(0.05)(0.02^2)}$$

$$= 0.955 \text{ MPa} \qquad \textbf{Ans.}$$



$$(\tau_{AC})_{\max} = \frac{2T_{AC}}{\pi\, a\, b^2} = \frac{2(50.0)}{\pi(0.05)(0.02^2)}$$

$$= 1.59 \text{ MPa} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi_{B/A} = \sum \frac{(a^2 + b^2)T\,L}{\pi\, a^3 b^3\, G}$$

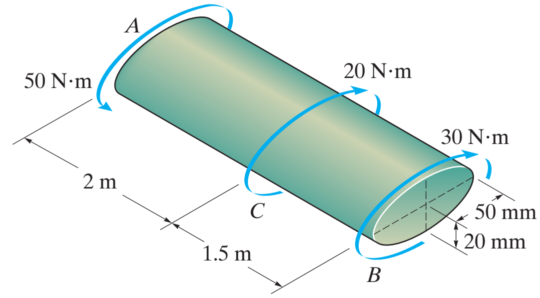
$$= \frac{(0.05^2 + 0.02^2)}{\pi(0.05^3)(0.02^3)(37.0)(10^9)}[(-30.0)(1.5) + (-50.0)(2)]$$

$$= -0.003618 \text{ rad} = 0.207° \qquad \textbf{Ans.}$$

Ans:
$(\tau_{BC})_{\max} = 0.955$ MPa, $(\tau_{AC})_{\max} = 1.59$ MPa,
$\phi_{B/A} = 0.207°$

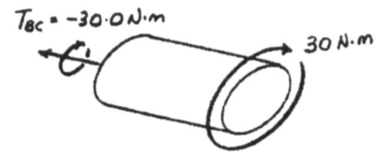
5–98.

Solve Prob. 5–98 for the maximum shear stress within regions AC and BC, and the angle of twist ϕ of end B relative to C.



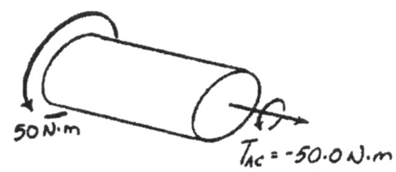
SOLUTION

Maximum Shear Stress:



$$(\tau_{BC})_{\max} = \frac{2T_{BC}}{\pi\, a\, b^2} = \frac{2(30.0)}{\pi(0.05)(0.02^2)}$$

$$= 0.955\text{ MPa} \qquad \textbf{Ans.}$$



$$(\tau_{AC})_{\max} = \frac{2T_{AC}}{\pi\, a\, b^2} = \frac{2(50.0)}{\pi(0.05)(0.02^2)}$$

$$= 1.59\text{ MPa} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi_{B/C} = \frac{(a^2 + b^2)\, T_{BC}\, L}{\pi\, a^3\, b^3\, G}$$

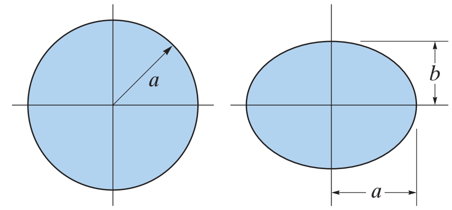
$$= \frac{(0.05^2 + 0.02^2)(-30.0)(1.5)}{\pi(0.05^3)(0.02^3)(37.0)(10^9)}$$

$$= -0.001123\text{ rad} = 0.0643° \qquad \textbf{Ans.}$$

Ans:
$(\tau_{BC})_{\max} = 0.955$ MPa, $(\tau_{AC})_{\max} = 1.59$ MPa,
$\phi_{B/C} = 0.0643°$

5–99.

If $a = 25$ mm and $b = 15$ mm, determine the maximum shear stress in the circular and elliptical shafts when the applied torque is $T = 80$ N·m. By what percentage is the shaft of circular cross section more efficient at withstanding the torque than the shaft of elliptical cross section?



SOLUTION

For the circular shaft:

$$(\tau_{\max})_c = \frac{T\,c}{J} = \frac{80(0.025)}{\frac{\pi}{2}(0.025^4)} = 3.26 \text{ MPa} \qquad \textbf{Ans.}$$

For the elliptical shaft:

$$(\tau_{\max})_e = \frac{2T}{\pi\,a\,b^2} = \frac{2(80)}{\pi(0.025)(0.015^2)} = 9.05 \text{ MPa} \qquad \textbf{Ans.}$$

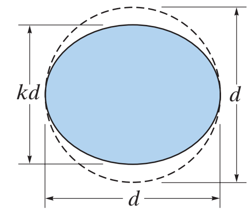
$$\%\text{ more efficient} = \frac{(\tau_{\max})_e - (\tau_{\max})_c}{(\tau_{\max})_c}(100\%)$$

$$= \frac{9.05 - 3.26}{3.26}(100\%) = 178\% \qquad \textbf{Ans.}$$

Ans:
$(\tau_{\max})_c = 3.26$ MPa,
$(\tau_{\max})_e = 9.05$ MPa,
% more efficient $= 178\%$

***5–100.**

It is intended to manufacture a circular bar to resist torque; however, the bar is made elliptical in the process of manufacturing, with one dimension smaller than the other by a factor k as shown. Determine the factor by which the maximum shear stress is increased.



SOLUTION

For the circular shaft:

$$(\tau_{\max})_c = \frac{Tc}{J} = \frac{T\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4} = \frac{16T}{\pi\, d^3}$$

For the elliptical shaft:

$$(\tau_{\max})_e = \frac{2T}{\pi\, a\, b^2} = \frac{2T}{\pi\left(\frac{d}{2}\right)\left(\frac{kd}{2}\right)^2} = \frac{16T}{\pi\, k^2\, d^3}$$

$$\text{Factor of increase in maximum shear stress} = \frac{(\tau_{\max})_e}{(\tau_{\max})_c} = \frac{\frac{16T}{\pi\, k^2 d^3}}{\frac{16T}{\pi\, d^3}}$$

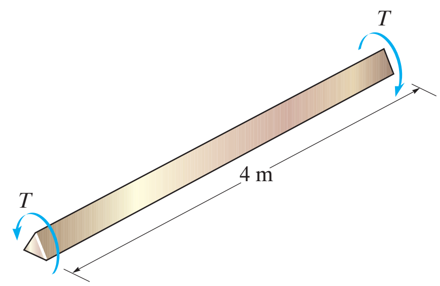
$$= \frac{1}{k^2}$$ **Ans.**

Ans:

Factor of increase $= \dfrac{1}{k^2}$

5–101.

The brass wire has a triangular cross section, 2 mm on a side. If the yield stress for brass is $\tau_Y = 205$ MPa, determine the maximum torque T to which it can be subjected so that the wire will not yield. If this torque is applied to the 4-m-long segment, determine the greatest angle of twist of one end of the wire relative to the other end that will not cause permanent damage to the wire. $G_{br} = 37$ GPa.



SOLUTION

Allowable Shear Stress:

$$\tau_{max} = \tau_Y = \frac{20T}{a^3}$$

$$205(10^6) = \frac{20T}{0.002^3}$$

$$T = 0.0820\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

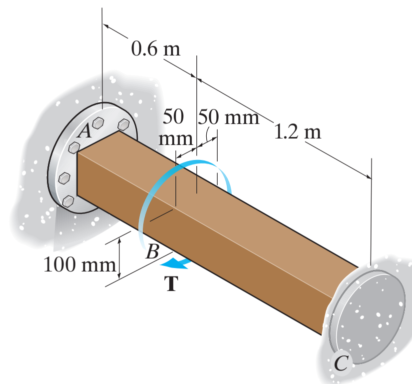
Angle of Twist:

$$\phi = \frac{46TL}{a^4G} = \frac{46(0.0820)(4)}{(0.002^4)(37)(10^9)}$$

$$= 25.5\ \text{rad} \qquad \textbf{Ans.}$$

Ans:
$T = 0.0820\ \text{N}\cdot\text{m}$,
$\phi = 25.5$ rad

5–102. If the solid shaft is made from red brass C83400 copper having an allowable shear stress of $\tau_{allow} = 28$ MPa, determine the maximum allowable torque **T** that can be applied at B.



SOLUTION

Equilibrium: Referring to the free-body diagram of the square bar shown in Fig. *a*, we have

$$\Sigma M_x = 0; \quad T_A + T_C - T = 0 \qquad (1)$$

Compatibility Equation: Here, it is required that

$$\phi_{B/A} = \phi_{B/C}$$

$$\frac{7.10T_A(0.6)}{a^4G} = \frac{7.10T_C(1.2)}{a^4G}$$

$$T_A = 2T_C \qquad (2)$$

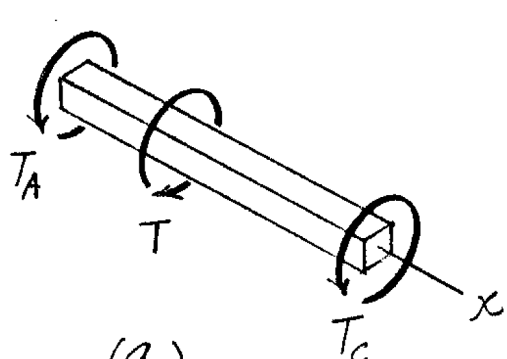
Solving Eqs. (1) and (2),

$$T_C = \frac{1}{3}T \qquad T_A = \frac{2}{3}T$$

Allowable Shear Stress: Segment AB is critical since it is subjected to the greater internal torque.

$$\tau_{allow} = \frac{4.81T_A}{a^3}; \qquad 28(10^6) = \frac{4.81\left(\frac{2}{3}T\right)}{0.1^3}$$

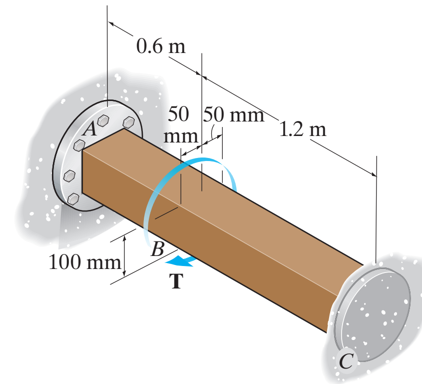
$$T = 8.732(10^3)\ \text{N}\cdot\text{m} = 8.73\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

Ans:
$T = 8.73$ kN · m

5–103. If the solid shaft is made from red brass C83400 copper and it is subjected to a torque $T = 8$ kN · m at B, determine the maximum shear stress developed in segments AB and BC.



SOLUTION

Equilibrium: Referring to the free-body diagram of the square bar shown in Fig. *a*, we have

$$\Sigma M_x = 0; \quad T_A + T_C - 8 = 0 \tag{1}$$

Compatibility Equation: Here, it is required that

$$\phi_{B/A} = \phi_{B/C}$$

$$\frac{7.10T_A(0.6)}{a^4G} = \frac{7.10T_C(1.2)}{a^4G}$$

$$T_A = 2T_C \tag{2}$$

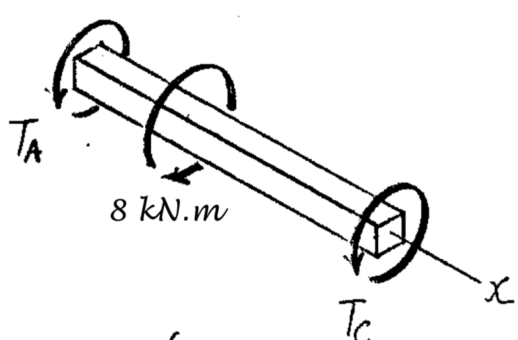
Solving Eqs. (1) and (2),

$$T_C = 2.667 \text{ kN}\cdot\text{m} \qquad T_A = 5.333 \text{ kN}\cdot\text{m}$$

Maximum Shear Stress:

$$(\tau_{\max})_{AB} = \frac{4.81T_A}{a^3} = \frac{4.81[5.333(10^3)]}{0.1^3} = 25.65(10^6)\text{ N/m}^2 = 25.7\text{ MPa} \quad \textbf{Ans.}$$

$$(\tau_{\max})_{BC} = \frac{4.81T_C}{a^3} = \frac{4.81[2.667(10^3)]}{0.1^3} = 12.83(10^6)\text{ N/m}^2 = 12.8\text{ MPa} \quad \textbf{Ans.}$$

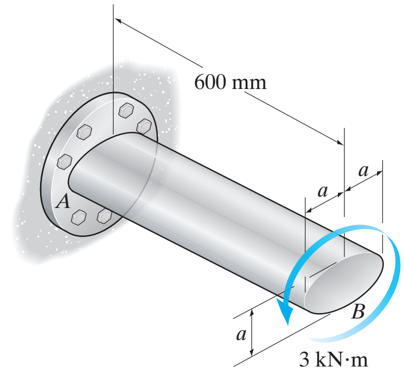


(a)

Ans.
$(\tau_{\max})_{AB} = 25.7$ MPa, $(\tau_{\max})_{BC} = 12.8$ MPa

***5–104.**

If the shaft is subjected to the torque of 3 kN · m, determine the maximum shear stress developed in the shaft. Also, find the angle of twist of end B. The shaft is made from A-36 steel. Set $a = 50$ mm.



SOLUTION

Maximum Shear Stress:

$$\tau_{\max} = \frac{2T}{\pi a b^2} = \frac{2(3)(10^3)}{\pi(0.05)(0.025^2)} = 61.1 \text{ MPa} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi = \frac{(a^2 + b^2)TL}{\pi a^3 b^3 G}$$

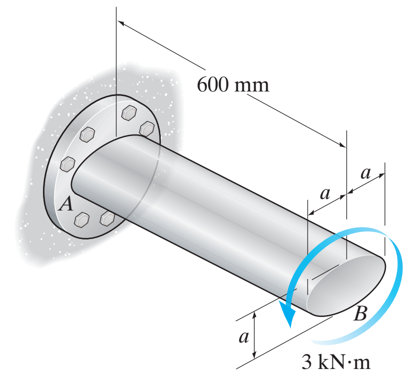
$$= \frac{(0.05^2 + 0.025^2)(3)(10^3)(0.6)}{\pi(0.05^3)(0.025^3)(75)(10^9)}$$

$$= 0.01222 \text{ rad} = 0.700° \qquad \textbf{Ans.}$$

Ans:
$\tau_{\max} = 61.1$ MPa, $\phi_B = 0.700°$

5–105.

If the shaft is made from A-36 steel having an allowable shear stress of $\tau_{allow} = 75$ MPa, determine the minimum dimension a for the cross section to the nearest millimeter. Also, find the corresponding angle of twist at end B.



SOLUTION

Allowable Shear Stress:

$$\tau_{allow} = \frac{2T}{\pi a b^2}; \qquad 75(10^6) = \frac{2(3)(10^3)}{\pi(a)\left(\frac{a}{2}\right)^2}$$

$$a = 0.04670 \text{ m}$$

Use $\quad a = 47$ mm **Ans.**

Angle of Twist:

$$\phi = \frac{(a^2 + b^2)TL}{\pi a^3 b^3 G}$$

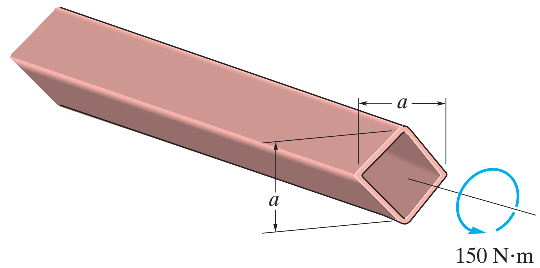
$$= \frac{\left[0.047^2 + \left(\frac{0.047}{2}\right)^2\right](3)(10^3)(0.6)}{\pi(0.047^3)\left(\frac{0.047}{2}\right)^3(75)(10^9)}$$

$$= 0.01566 \text{ rad} = 0.897°$$ **Ans.**

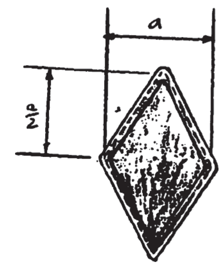
Ans:
Use $a = 47$ mm, $\phi_B = 0.897°$

5–106.

The plastic tube is subjected to a torque of 150 N · m. Determine the mean dimension a of its sides if the allowable shear stress is $\tau_{\text{allow}} = 60$ MPa. Each side has a thickness of $t = 3$ mm.



SOLUTION



$$A_m = 4\left[\frac{1}{2}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)\right] = \frac{a^2}{2}$$

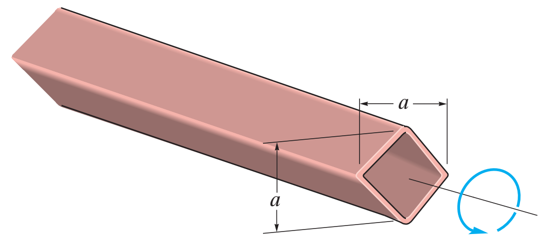
$$\tau_{\text{avg}} = \frac{T}{2\,tA_m}; \qquad 60(10^6) = \frac{150}{2(0.003)\,\frac{1}{2}\,a^2}$$

$$a = 0.0289 \text{ m} = 28.9 \text{ mm}$$ **Ans.**

Ans:
$a = 28.9$ mm

5–107.

The plastic tube is subjected to a torque of 150 N · m. Determine the average shear stress in the tube if the mean dimension $a = 200$ mm. Each side has a thickness of $t = 3$ mm.



SOLUTION

$$A_m = 4\left[\frac{1}{2}\left(\frac{0.2}{2}\right)\left(\frac{0.2}{2}\right)\right] = 0.02 \text{ m}^2$$

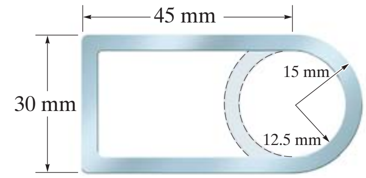
$$\tau_{avg} = \frac{T}{2\,t\,A_m} = \frac{150}{2(0.003)(0.02)}$$

$$= 1.25 \text{ MPa}$$

Ans.

Ans:
$\tau_{avg} = 1.25$ MPa

***5–108.** For a given maximum shear stress, determine the factor by which the torque carrying capacity is increased if the half-circular section is reversed from the dashed-line position to the section shown. The tube is 2.5 mm thick.



SOLUTION

$$A_m = (0.0275)(0.04375) - \frac{\pi(0.01375^2)}{2} = 0.9061(10^{-3})\ \text{m}^2$$

$$A_m' = (0.0275)(0.04375) + \frac{\pi(0.01375^2)}{2} = 1.5001(10^{-3})\ \text{m}^2$$

$$\tau_{\max} = \frac{T}{2t\ A_m}$$

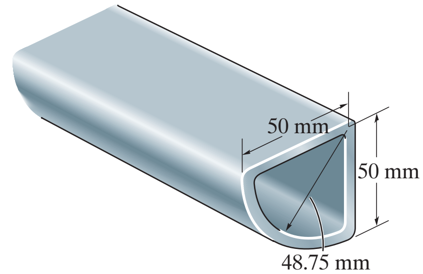
$$T = 2\ t\ A_m\ \tau_{\max}$$

$$\text{Factor} = \frac{2t\ A_m'\ \tau_{\max}}{2t\ A_m\ \tau_{\max}}$$

$$= \frac{A_m'}{A_m} = \frac{1.5001(10^{-3})}{0.9061(10^{-3})} = 1.655 = 1.66$$ **Ans.**

Ans:
Factor = 1.66

5–109. A torque of 200 N · m is applied to the tube. If the wall thickness is 2.5 mm, determine the average shear stress in the tube.



SOLUTION

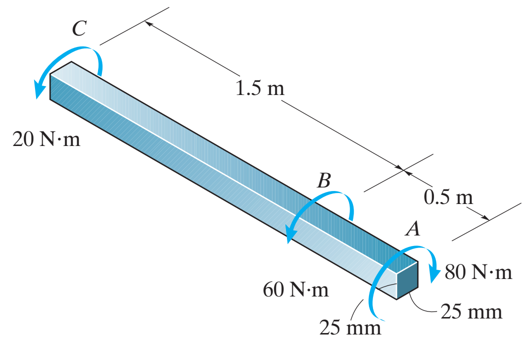
$$A_m = \frac{\pi(0.04875^2)}{4} = 1.8665(10^{-3})\ \text{m}^2$$

$$\tau_{\text{avg}} = \frac{T}{2t\ A_m} = \frac{200}{2(0.0025)[1.8665(10^{-3})]} = 21.43(10^6)\ \text{N/m}^2 = 21.4\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$\tau_{\text{avg}} = 21.44$ MPa

5–110.

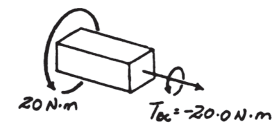
The 6061-T6 aluminum bar has a square cross section of 25 mm by 25 mm. If it is 2 m long, determine the maximum shear stress in the bar and the rotation of one end relative to the other end.



SOLUTION

Maximum Shear Stress:

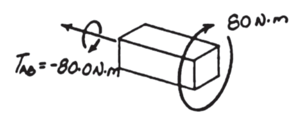
$$\tau_{\max} = \frac{4.81 T_{\max}}{a^3} = \frac{4.81(80.0)}{(0.025^3)} = 24.6 \text{ MPa} \qquad \textbf{Ans.}$$



Angle of Twist:

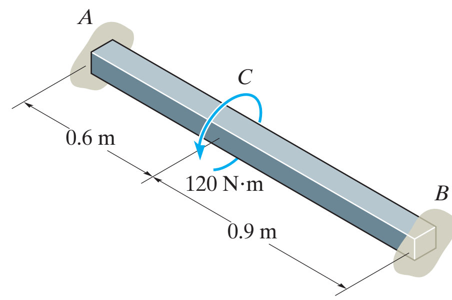
$$\phi_{A/C} = \sum \frac{7.10 TL}{a^4 G} = \frac{7.10(-20.0)(1.5)}{(0.025^4)(26.0)(10^9)} + \frac{7.10(-80.0)(0.5)}{(0.025^4)(26.0)(10^9)}$$

$$= -0.04894 \text{ rad} = 2.80° \qquad \textbf{Ans.}$$



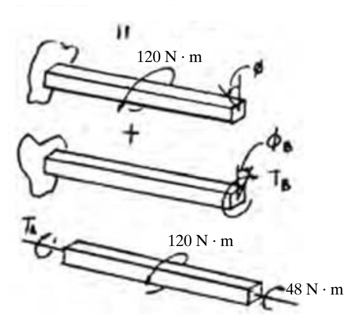
Ans:
$\tau_{\max} = 24.6$ MPa,
$\phi_{A/C} = 2.80°$

5–111. The aluminum strut is fixed between the two walls at A and B. If it has a 50 mm by 50 mm square cross section, and it is subjected to the torque of 120 N · m at C, determine the reactions at the fixed supports. Also, what is the angle of twist at C? $G_{al} = 27$ GPa.



SOLUTION

By superposition:



$$0 = \phi - \phi_B$$

$$0 = \frac{7.10(120)(0.6)}{a^4 G} - \frac{7.10(T_B)(1.5)}{a^4 G}$$

$$T_B = 48 \text{ N} \cdot \text{m}$$ **Ans.**

$$T_A + 48 - 120 = 0$$

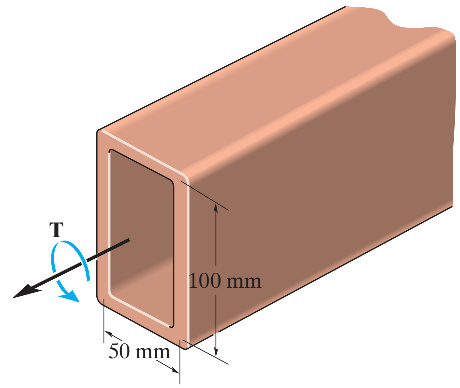
$$T_A = 72 \text{ N} \cdot \text{m}$$ **Ans.**

$$\phi_C = \frac{7.10(48)(0.9)}{(0.05^4)[27(10^9)]} = 0.0018176 \text{ rad} = 0.104°$$ **Ans.**

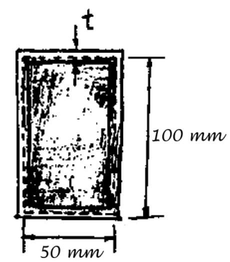
Ans:

$T_B = 48$ N · m,
$T_A = 72$ N · m,
$\phi_C = 0.104°$

***5–112.** Determine the constant thickness of the rectangular tube if average stress is not to exceed 84 MPa when a torque of $T = 2$ kN · m is applied to the tube. Neglect stress concentrations at the corners. The mean dimensions of the tube are shown.



SOLUTION



$$A_m = (0.05)(0.1) = 0.005 \text{ m}^2$$

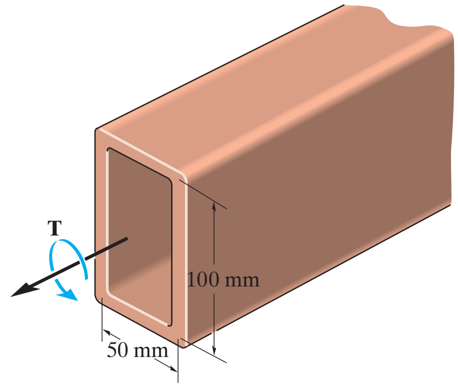
$$\tau_{avg} = \frac{T}{2t\, A_m}$$

$$84(10^6) = \frac{2(10^3)}{2t(0.005)}$$

$t = 0.002381 \text{ m} = 2.38 \text{ mm}$ **Ans.**

Ans:
$t = 2.38$ mm

5–113. Determine the torque T that can be applied to the rectangular tube if the average shear stress is not to exceed 84 MPa. Neglect stress concentrations at the corners. The mean dimensions of the tube are shown and the tube has a thickness of 3 mm.



SOLUTION

$A_m = (0.05)(0.1) = 0.005 \text{ m}^2$

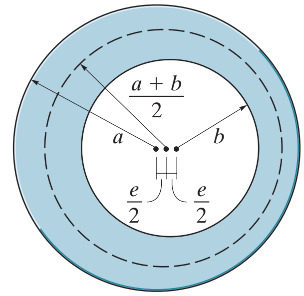
$\tau_{\text{avg}} = \dfrac{T}{2t\,A_m}; \qquad 84(10^6) = \dfrac{2(10^3)}{2(0.003)(0.005)}$

$t = 2.52(10^3) \text{ N} \cdot \text{m} = 2.52 \text{ kN} \cdot \text{m}$ **Ans.**

Ans:
$T = 2.52 \text{ kN} \cdot \text{m}$

5–114.

Due to a fabrication error the inner circle of the tube is eccentric with respect to the outer circle. By what percentage is the torsional strength reduced when the eccentricity e is one-fourth of the difference in the radii?



SOLUTION

Average Shear Stress:

For the aligned tube

$$\tau_{avg} = \frac{T}{2\,t\,A_m} = \frac{T}{2(a-b)(\pi)\left(\frac{a+b}{2}\right)^2}$$

$$T = \tau_{avg}\,(2)(a-b)(\pi)\left(\frac{a+b}{2}\right)^2$$

For the eccentric tube

$$\tau_{avg} = \frac{T'}{2\,t\,A_m}$$

$$t = a - \frac{e}{2} - \left(\frac{e}{2} + b\right) = a - e - b$$

$$= a - \frac{1}{4}(a-b) - b = \frac{3}{4}(a-b)$$

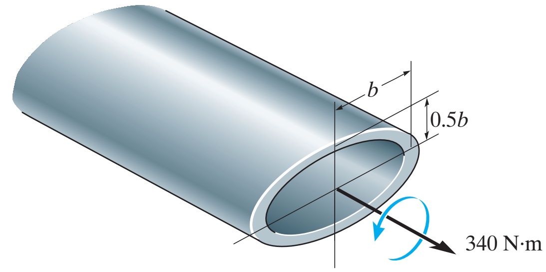
$$T' = \tau_{avg}\,(2)\left[\frac{3}{4}(a-b)\right](\pi)\left(\frac{a+b}{2}\right)^2$$

$$\text{Factor} = \frac{T'}{T} = \frac{\tau_{avg}\,(2)\left[\frac{3}{4}(a-b)\right](\pi)\left(\frac{a+b}{2}\right)^2}{\tau_{avg}\,(2)(a-b)(\pi)\left(\frac{a+b}{2}\right)^2} = \frac{3}{4}$$

$$\text{Percent reduction in strength} = \left(1 - \frac{3}{4}\right) \times 100\% = 25\%$$ **Ans.**

Ans:
Percent reduction in strength $= 25\%$

5–115. The steel tube has an elliptical cross section of mean dimensions shown and a constant thickness of $t = 5$ mm. If the allowable shear stress is $\tau_{allow} = 56$ MPa, and the tube is to resist a torque of $T = 340$ N · m, determine the necessary dimension b. The mean area for the ellipse is $A_m = \pi b(0.5b)$.



SOLUTION

$$\tau_{avg} = \tau_{allow} = \frac{T}{2tA_m}$$

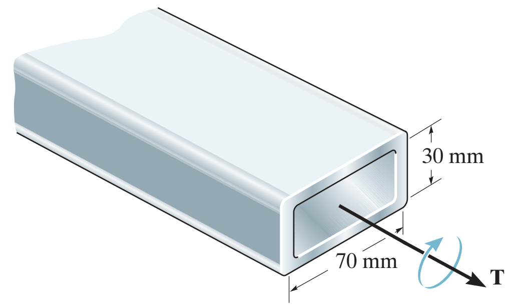
$$56(10^6) = \frac{340}{2(0.005)[\pi b(0.5b)]}$$

$b = 0.01966 \text{ m} = 19.7 \text{ mm}$ **Ans.**

Ans:
$b = 19.7$ mm

***5–116.**

The 304 stainless steel tube has a thickness of 10 mm. If the allowable shear stress is $\tau_{allow} = 80$ MPa, determine the maximum torque T that it can transmit. Also, what is the angle of twist of one end of the tube with respect to the other if the tube is 4 m long? The mean dimensions are shown.

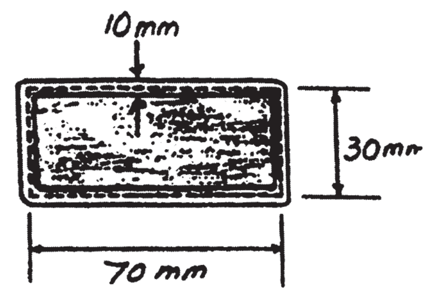


SOLUTION

Section Properties:

$$A_m = 0.07(0.03) = 0.00210 \text{ m}^2$$

$$\int ds = 2(0.07) + 2(0.03) = 0.200 \text{ m}$$



Allowable Average Shear Stress:

$$\tau_{avg} = \tau_{allow} = \frac{T}{2\,t\,A_m}$$

$$80\left(10^6\right) = \frac{T}{2(0.01)(0.00210)}$$

$$T = 3360 \text{ N}\cdot\text{m} = 3.36 \text{ kN}\cdot\text{m}$$ **Ans.**

Angle of Twist:

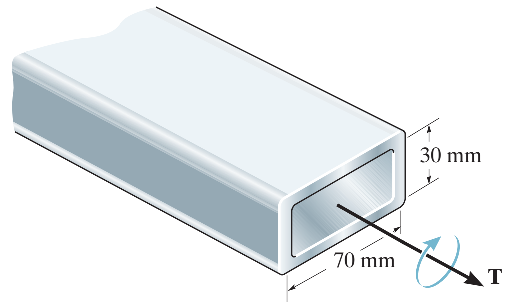
$$\phi = \frac{TL}{4\,A_m^2 G}\int\frac{ds}{t} = \frac{3360(4)(0.200)}{4\left(0.00210^2\right)(75.0)\left(10^9\right)(0.01)}$$

$$= 0.2032 \text{ rad} = 11.6^\circ$$ **Ans.**

Ans:
$T = 3.36$ kN·m,
$\phi = 11.6^\circ$

5–117.

The 304 stainless steel tube has a thickness of 10 mm. If the applied torque is $T = 50\ \mathrm{N \cdot m}$, determine the average shear stress in the tube. The mean dimensions are shown.

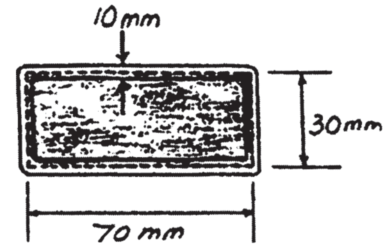


SOLUTION

Section Properties:

$$A_m = 0.07(0.03) = 0.00210\ \mathrm{m}^2$$

Average Shear Stress:



$$\tau_{\text{avg}} = \frac{T}{2\,t\,A_m}$$

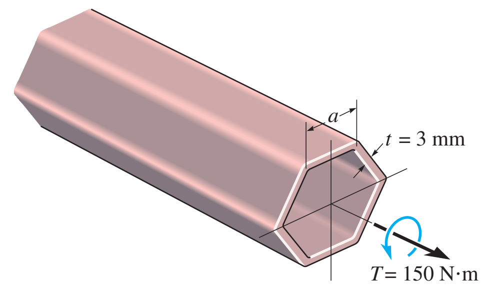
$$= \frac{50}{2(0.01)(0.00210)}$$

$$= 1.19\ \mathrm{MPa} \qquad \textbf{Ans.}$$

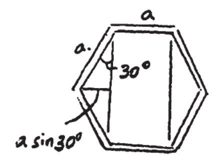
Ans:
$\tau_{\text{avg}} = 1.19\ \mathrm{MPa}$

5–118.

The plastic hexagonal tube is subjected to a torque of 150 N · m. Determine the mean dimension a of its sides if the allowable shear stress is $\tau_{allow} = 60$ MPa. Each side has a thickness of $t = 3$ mm.



SOLUTION



$$A_m = 4\left[\frac{1}{2}(a\cos 30°)(a\sin 30°)\right] + (a)(2a)\cos 30° = 2.5981\,a^2$$

$$\tau_{avg} = \tau_{allow} = \frac{T}{2\,t\,A_m}$$

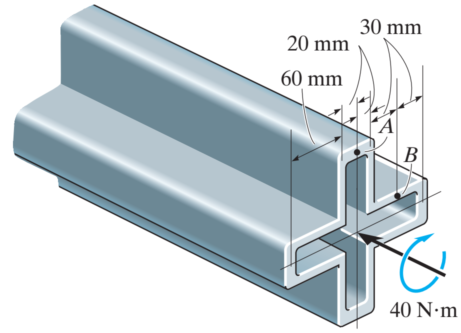
$$60\left(10^6\right) = \frac{150}{(2)(0.003)\left(2.5981\,a^2\right)}$$

$$a = 0.01266 \text{ m} = 12.7 \text{ mm}$$ **Ans.**

Ans:
$a = 12.7$ mm

5–119.

The symmetric tube is made from a high-strength steel, having the mean dimensions shown and a thickness of 5 mm. If it is subjected to a torque of $T = 40\ \text{N}\cdot\text{m}$, determine the average shear stress developed at points A and B. Indicate the shear stress on volume elements located at these points.



SOLUTION

$$A_m = 4(0.04)(0.06) + (0.04)^2 = 0.0112\ \text{m}^2$$

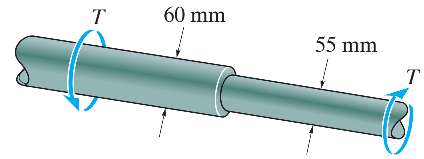
$$\tau_{\text{avg}} = \frac{T}{2\,t\,A_m}$$

$$(\tau_{\text{avg}})_A = (\tau_{\text{avg}})_B = \frac{40}{2(0.005)(0.0112)} = 357\ \text{kPa}$$ **Ans.**

Ans:
$(\tau_{\text{avg}})_A = (\tau_{\text{avg}})_B = 357$ kPa

***5–120.**

The stepped shaft is subjected to a torque **T** that produces yielding on the surface of the larger diameter segment. Determine the radius of the elastic core produced in the smaller diameter segment. Neglect the stress concentration at the fillet.



SOLUTION

Maximum Elastic Torque: For the larger diameter segment

$$\tau_Y = \frac{T_Y\, c}{J}$$

$$T_Y = \frac{\tau_Y\, J}{c}$$

$$= \frac{\tau_Y \left(\frac{\pi}{2}\right)\left(0.03^4\right)}{0.03}$$

$$= 13.5\left(10^{-6}\right)\,\pi\tau_Y$$

Elastic - Plastic Torque: For the smaller diameter segment
$T_P = \frac{2\pi}{3}\,\tau_Y\, c^3 = \frac{2\pi}{3}\,\tau_Y\left(0.0275^3\right) = 13.86\left(10^{-6}\right)\pi\tau_Y > 13.5\left(10^{-6}\right)\pi\tau_Y.$
Applying Eq. 5–26 of the text, we have

$$T = \frac{\pi\,\tau_Y}{6}\left(4\,c^3 - \rho_Y^3\right)$$

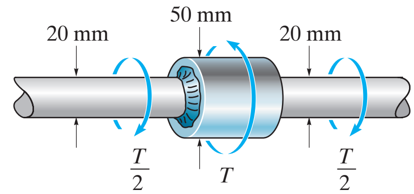
$$13.5\left(10^{-6}\right)\pi\tau_Y = \frac{\pi\,\tau_Y}{6}\left[4\left(0.0275^3\right) - \rho_Y^3\right]$$

$$\rho_Y = 0.01298\text{ m} = 13.0\text{ mm}$$ **Ans.**

Ans:
$\rho_Y = 13.0$ mm

5–121.

The steel step shaft has an allowable shear stress of $\tau_{\text{allow}} = 8$ MPa. If the transition between the cross sections has a radius $r = 4$ mm, determine the maximum torque T that can be applied.



SOLUTION

Allowable Shear Stress:

$$\frac{D}{d} = \frac{50}{20} = 2.5 \qquad \text{and} \qquad \frac{r}{d} = \frac{4}{20} = 0.20$$

From the text, $K = 1.25$

$$\tau_{\max} = \tau_{\text{allow}} = K\frac{Tc}{J}$$

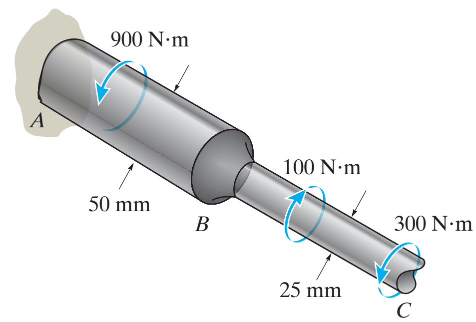
$$8(10^6) = 1.25\left[\frac{\frac{T}{2}(0.01)}{\frac{\pi}{2}(0.01^4)}\right]$$

$$T = 20.1\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$T = 20.1\ \text{N}\cdot\text{m}$

5–122.

The shaft is fixed to the wall at A and is subjected to the torques shown. Determine the maximum shear stress in the shaft. A fillet weld having a radius of 2.75 mm is used to connect the shafts at B.



SOLUTION

Internal Torque: The internal torque in regions CD, BD and AE are indicated in their respective FBD, Fig. a,

Maximum Shear Stress: For region CD,

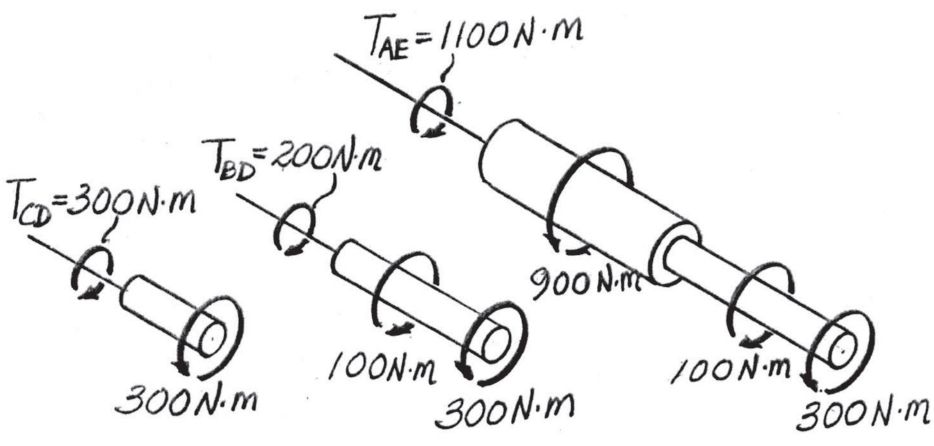
$$(\tau_{\max})_{CD} = \frac{T_{CD}C}{J} = \frac{300(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 97.78\left(10^6\right)\text{ Pa} = 97.8\text{ MPa } (\textbf{Max.}) \qquad \textbf{Ans.}$$

For Region AE,

$$(\tau_{\max})_{AE} = \frac{T_{AE}C}{J} = \frac{1100(0.025)}{\frac{\pi}{2}(0.025^4)} = 44.82\left(10^6\right)\text{ Pa} = 44.8\text{ MPa}$$

For the fillet, enter $\frac{D}{d} = \frac{50}{25} = 2$ and $\frac{r}{d} = \frac{2.75}{25} = 0.11$ into Fig. 5-32, we get $K = 1.375$.

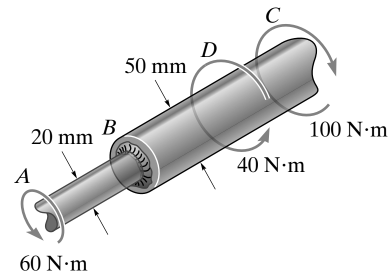
$$(\tau_{\max})_f = K\left(\frac{T_{BD}C}{J}\right) = 1.375\left[\frac{200(0.0125)}{\frac{\pi}{2}(0.0125^4)}\right] = 89.64\left(10^6\right)\text{Pa} = 89.6\text{ MPa}$$



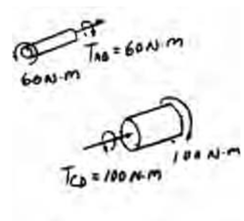
(a)

Ans:
$(\tau_{\max}) = 97.8$ MPa

5–123. The steel shaft is made from two segments: AB and BC, which are connected using a fillet weld having a radius of 2.8 mm. Determine the maximum shear stress developed in the shaft.



SOLUTION



$$(\tau_{\max})_{CD} = \frac{T_{CD}c}{J} = \frac{100(0.025)}{\frac{\pi}{2}(0.025^4)}$$

$$= 4.07 \text{ MPa}$$

For the fillet:

$$\frac{D}{d} = \frac{50}{20} = 2.5; \qquad \frac{r}{d} = \frac{2.8}{20} = 0.14$$

From Fig. 5-32, $K = 1.325$

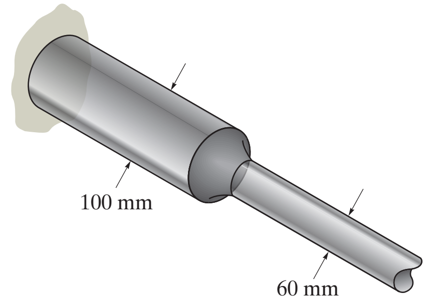
$$(\tau_{\max})_f = K\frac{T_{AB}c}{J} = 1.325\left[\frac{60(0.01)}{\frac{\pi}{2}(0.01^4)}\right]$$

$$= 50.6 \text{ MPa (max)}$$ **Ans.**

Ans.
$(\tau_{\max})_f = 50.6$ MPa (max)

***5–124.**

The built-up shaft is to be designed to rotate at 450 rpm while transmitting 230 kW of power. Is this possible? The allowable shear stress is $\tau_{allow} = 150$ MPa.



SOLUTION

Internal Torque: Here,

$$\omega = \left(450\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 15\pi\ \text{rad/s}$$

Then,

$$T = \frac{P}{\omega} = \frac{230\left(10^3\right)}{15\,\pi} = 4880.75\ \text{N}\cdot\text{m}$$

Maximum Shear Stress: At the fillet,

$$\tau_{\max} = \tau_{allow} = K\left(\frac{Tc}{J}\right)$$

$$150\left(10^6\right) = K\left[\frac{4880.75\,(0.03)}{\frac{\pi}{2}\left(0.03^4\right)}\right]$$

$$K = 1.30$$

Enter this value and $\frac{D}{d} = \frac{100}{60} = 1.67$ into Fig. 5-32, we get $\frac{r}{d} = 0.14$. Thus,

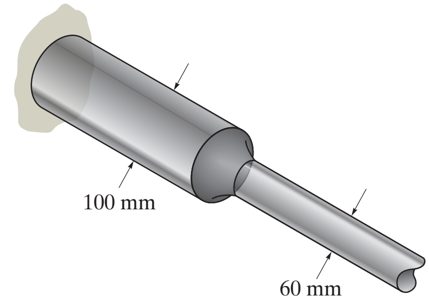
$$\frac{r}{60} = 0.14\,; \qquad r = 8.4\ \text{mm}$$

Since $\frac{D-d}{2} = \frac{100-60}{2} = 20\ \text{mm} > r = 8.4\ \text{mm}$, **it is possible to construct the transition.** **Ans.**

Ans:
It is possible.

5–125.

The built-up shaft is designed to rotate at 450 rpm. If the radius of the fillet weld connecting the shafts is $r = 13.2$ mm, and the allowable shear stress for the material is $\tau_{\text{allow}} = 150$ MPa, determine the maximum power the shaft can transmit.



SOLUTION

Internal Torque: Here,

$$\omega = \left(450\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 15\pi\text{ rad/s}$$

Then,

$$T = \frac{P}{\omega} = \frac{P}{15\,\pi}$$

Maximum Shear Stress: Enter $\frac{D}{d} = \frac{100}{60} = 1.67$ and $\frac{r}{d} = 0.22$ into Fig 5-32, we get $K = 1.2$. Then

$$\tau_{\text{max}} = \tau_{\text{allow}} = K\left(\frac{Tc}{J}\right)$$

$$150\left(10^6\right) = 1.2\left[\frac{\frac{P}{15\,\pi}(0.03)}{\frac{\pi}{2}\left(0.03^4\right)}\right]$$

$$P = 249.82\left(10^3\right)\text{W} = 250\text{ kW}$$ **Ans.**

Ans:
$P = 250$ kW

5–126.

A solid shaft has a diameter of 40 mm and length of 1 m. It is made from an elastic-plastic material having a yield stress of $\tau_Y = 100$ MPa. Determine the maximum elastic torque T_Y and the corresponding angle of twist. What is the angle of twist if the torque is increased to $T = 1.2T_Y$? $G = 80$ GPa.

SOLUTION

Maximum elastic torque T_Y,

$$\tau_Y = \frac{T_Y\, c}{J}$$

$$T_Y = \frac{\tau_Y J}{c} = \frac{100\left(10^6\right)\left(\frac{\pi}{2}\right)\left(0.02^4\right)}{0.02} = 1256.64\ \text{N}\cdot\text{m} = 1.26\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Angle of twist:

$$\gamma_Y = \frac{\tau_Y}{G} = \frac{100\left(10^6\right)}{80\left(10^9\right)} = 0.00125\ \text{rad}$$

$$\phi = \frac{\gamma_Y}{\rho_Y} L = \frac{0.00125}{0.02}(1) = 0.0625\ \text{rad} = 3.58^\circ \qquad \textbf{Ans.}$$

Also,

$$\phi = \frac{T_Y L}{JG} = \frac{1256.64(1)}{\frac{\pi}{2}\left(0.02^4\right)(80)\left(10^9\right)} = 0.0625\ \text{rad} = 3.58^\circ$$

From Eq. 5–26 of the text,

$$T = \frac{\pi \tau_Y}{6}\left(4\, c^3 - \rho_Y^3\right); \qquad 1.2(1256.64) = \frac{\pi(100)\left(10^6\right)}{6}\left[4\left(0.02^3\right) - \rho_Y^3\right]$$

$$\rho_Y = 0.01474\ \text{m}$$

$$\phi' = \frac{\gamma_Y}{\rho_Y} L = \frac{0.00125}{0.01474}(1) = 0.0848\ \text{rad} = 4.86^\circ \qquad \textbf{Ans.}$$

Ans:
$T_Y = 1.26\ \text{kN}\cdot\text{m}$,
$\phi = 3.58^\circ$,
$\phi' = 4.86^\circ$

5–127.

Determine the torque needed to twist a short 2-mm-diameter steel wire through several revolutions if it is made from steel assumed to be elastic perfectly plastic and having a yield stress of $\tau_Y = 50$ MPa. Assume that the material becomes fully plastic.

SOLUTION

Fully plastic torque is applied.

$$T_P = \frac{2\pi}{3}\tau_Y c^3 = \frac{2\pi}{3}(50)(10^6)(0.001^3) = 0.105\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$T_P = 0.105\ \text{N}\cdot\text{m}$

5–128. A solid shaft is subjected to the torque T, which causes the material to yield. If the material is elastic plastic, show that the torque can be expressed in terms of the angle of twist ϕ of the shaft as $T = \frac{4}{3}T_Y(1 - \phi^3{}_Y/4\phi^3)$, where T_Y and ϕ_Y are the torque and angle of twist when the material begins to yield.

SOLUTION

$$\phi = \frac{\gamma L}{\rho} = \frac{\gamma_Y}{\rho_Y}L$$

$$\rho_Y = \frac{\gamma_Y L}{\phi} \qquad \textbf{(1)}$$

When $\rho_Y = c, \phi = \phi_Y$

From Eq. (1),

$$c = \frac{\gamma_Y L}{\phi_Y} \qquad \textbf{(2)}$$

Dividing Eq. (1) by Eq. (2) yields:

$$\frac{\rho_Y}{c} = \frac{\phi_Y}{\phi} \qquad \textbf{(3)}$$

Use Eq. 5-26 from the text.

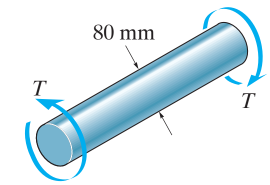
$$T = \frac{\pi\ \tau_Y}{6}(4\,c^3 - \rho_Y^3) = \frac{2\pi\ \tau_Y c^3}{3}\left(1 - \frac{\rho_Y^3}{4\,c^3})\right)$$

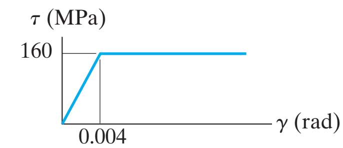
Use Eq. 5-24, $T_Y = \frac{\pi}{2}\tau_Y c^3$ from the text and Eq. (3)

$$T = \frac{4}{3}T_Y\left(1 - \frac{\phi_Y^3}{4\,\phi^3}\right) \qquad \textbf{QED}$$

5–129.

The solid shaft is made of an elastic perfectly plastic material. Determine the torque T needed to form an elastic core in the shaft having a radius of $\rho_Y = 20$ mm. If the shaft is 3 m long, through what angle does one end of the shaft twist with respect to the other end? When the torque is removed, determine the residual stress distribution in the shaft and the permanent angle of twist.





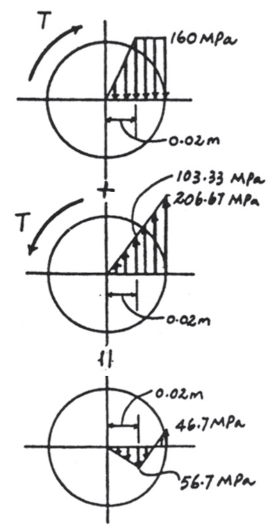
SOLUTION

Elastic-Plastic Torque:

$$T = \frac{\pi\,\tau_Y}{6}\left(4c^3 - \rho_Y^3\right)$$

$$= \frac{\pi(160)(10^6)}{6}\left[4\left(0.04^3\right) - 0.02^3\right]$$

$$= 20776.40\ \text{N}\cdot\text{m} = 20.8\ \text{kN}\cdot\text{m}$$ **Ans.**



Angle of Twist:

$$\phi = \frac{\gamma_Y}{\rho_Y}L = \left(\frac{0.004}{0.02}\right)(3) = 0.600\ \text{rad} = 34.4°$$ **Ans.**

When the reverse $T = 20776.4\ \text{N}\cdot\text{m}$ is applied,

$$G = \frac{160(10^6)}{0.004} = 40\ \text{GPa}$$

$$\phi' = \frac{TL}{JG} = \frac{20776.4(3)}{\frac{\pi}{2}(0.04^4)(40)(10^9)} = 0.3875\ \text{rad}$$

The permanent angle of twist is,

$$\phi_r = \phi - \phi'$$

$$= 0.600 - 0.3875 = 0.2125\ \text{rad} = 12.2°$$ **Ans.**

Residual Shear Stress:

$$(\tau')_{\rho=c} = \frac{Tc}{J} = \frac{20776.4(0.04)}{\frac{\pi}{2}(0.04^4)} = 206.67\ \text{MPa}$$

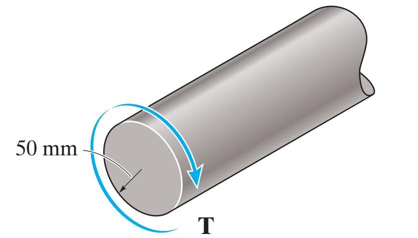
$$(\tau')_{\rho=0.02\ \text{m}} = \frac{Tc}{J} = \frac{20776.4(0.02)}{\frac{\pi}{2}(0.04^4)} = 103.33\ \text{MPa}$$

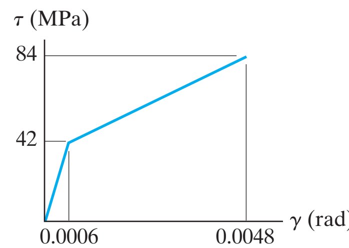
$$(\tau_r)_{\rho=c} = -160 + 206.67 = 46.7\ \text{MPa}$$

$$(\tau_r)_{\rho=0.02\ \text{m}} = -160 + 103.33 = -56.7\ \text{MPa}$$ **Ans.**

Ans:
$T = 20.8\ \text{kN}\cdot\text{m}$, $\phi = 34.4°$, $(\tau_r)_{\max} = 56.7\ \text{MPa}$, $\phi_r = 12.2°$

5–130. The shaft is subjected to a maximum shear strain of 0.0048 rad. Determine the torque applied to the shaft if the material has strain hardening as shown by the shear stress–strain diagram.





SOLUTION

From the shear - strain diagram,

$$\frac{\rho_Y}{0.0006} = \frac{0.05}{0.0048}; \qquad \rho_Y = 0.00625 \text{ m}$$

From the shear stress–strain diagram,

$$\tau_1 = \left[\frac{42(10^6)}{0.00625}\right]\rho = 6.72(10^9)\rho$$

$$\frac{\tau_2 - 42(10^6)}{\rho - 0.00625} = \frac{84(10^6) - 42(10^6)}{0.05 - 0.00625}$$

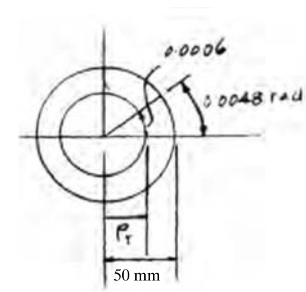
$$\tau_2 = 0.96(10^9)\rho + 36(10^6)$$

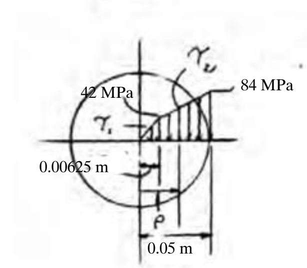
$$T = 2\pi\int_0^c \tau\rho^2 d\rho$$

$$= 2\pi\int_0^{0.00625\text{ m}}[6.72(10^9)\rho]\rho^2 d\rho + 2\pi\int_{0.00625\text{ m}}^{0.05\text{ m}}[0.96(10^9)\rho + 36(10^6)]\rho^2 d\rho$$

$$= 2\pi[1.68(10^9)\rho^4]\Big|_0^{0.00625\text{ m}} + 2\pi[0.24(10^9)\rho^4 + 12(10^6)\rho^3]\Big|_{0.00625\text{ m}}^{0.05\text{ m}}$$

$$= 18.84(10^3)\text{ N}\cdot\text{m} = 18.8\text{ kN}\cdot\text{m}$$ **Ans.**





Ans:

$T = 18.8$ kN · m

5–131. A solid shaft having a diameter of 50 mm is made of elastic-plastic material having a yield stress of $\tau_Y = 112$ MPa and shear modulus of $G = 84$ GPa. Determine the torque required to developo an elastic core in the shaft having a diameter of 25 mm. Also, what is the plasstic torque?

SOLUTION

Use Eq. 5-26 from the text:

$$T = \frac{\pi \tau_Y}{6}(4c^3 - \rho_Y{}^3) = \frac{\pi[112(10^6)]}{6}[4(0.025^3) - 0.0125^3]$$

$$= 3.551(10^3)\ \text{N}\cdot\text{m} = 3.55\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Use Eq. 5-27 from the text:

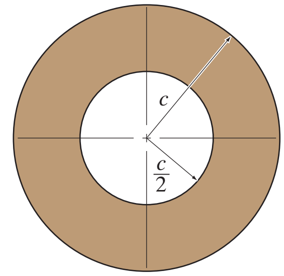
$$T_P = \frac{2\pi}{3}\tau_Y c^3 = \frac{2\pi}{3}[112(10^6)](0.025^3)$$

$$= 3.665(10^3)\ \text{N}\cdot\text{m} = 3.67\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans.
$T = 3.55$ kN · m, $T_P = 3.67$ kN · m

***5–132.**

The hollow shaft has the cross section shown and is made of an elastic perfectly plastic material having a yield shear stress of τ_Y. Determine the ratio of the plastic torque T_p to the maximum elastic torque T_Y.



SOLUTION

Maximum Elastic Torque: In this case, the torsion formula is still applicable.

$$\tau_Y = \frac{T_Y c}{J}$$

$$T_Y = \frac{J}{c}\tau_Y$$

$$= \frac{\frac{\pi}{2}\left[c^4 - \left(\frac{c}{2}\right)^4\right]\tau_Y}{c}$$

$$= \frac{15}{32}\pi c^3 \tau_Y$$

Plastic Torque: Using the general equation, with $\tau = \tau_Y$,

$$T_P = 2\pi\tau_Y \int_{c/2}^{c} \rho^2 d\rho$$

$$= 2\pi\tau_Y\left(\frac{\rho^3}{3}\right)\Bigg|_{c/2}^{c}$$

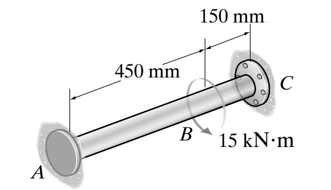
$$= \frac{7}{12}\pi c^3 \tau_Y$$

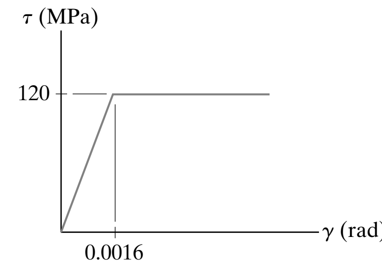
The ratio is

$$\frac{T_P}{T_Y} = \frac{\frac{7}{12}\pi c^3 \tau_Y}{\frac{15}{32}\pi c^3 \tau_Y} = 1.24$$ **Ans.**

Ans:
$\frac{T_P}{T_Y} = 1.24$

5–133. The hollow shaft has inner and outer diameters of 60 mm and 80 mm, respectively. If it is made of an elastic-perfectly plastic material, which has the $\tau-\gamma$ diagram shown, determine the reactions at the fixed supports A and C.





SOLUTION

Equation of Equilibrium. Refering to the free - body diagram of the shaft shown in Fig. *a*,

$$\Sigma M_x = 0;\ T_A + T_C - 15\left(10^3\right) = 0 \qquad \textbf{(1)}$$

Elastic Analysis. It is required that $\phi_{B/A} = \phi_{B/C}$. Thus, the compatibility equation is

$$\phi_{B/A} = \phi_{B/C}$$

$$\frac{T_A L_{AB}}{JG} = \frac{T_C L_{BC}}{JG}$$

$$T_A(0.45) = T_C(0.15)$$

$$T_C = 3T_A \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$T_A = 3750\ \text{N}\cdot\text{m} \qquad T_C = 11\,250\text{N}\cdot\text{m}$$

The maximum elastic torque and plastic torque in the shaft can be determined from

$$T_Y = \frac{J}{c}\tau_Y = \left[\frac{\frac{\pi}{2}\left(0.04^4 - 0.03^4\right)}{0.04}\right](120)\left(10^6\right) = 8246.68\,\text{N}\cdot\text{m}$$

$$T_P = 2\pi\tau_Y\int_{c_i}^{c_o}\rho^2 d\rho$$

$$= 2\pi(120)\left(10^6\right)\left(\frac{\rho^3}{3}\right)\Bigg|_{0.03\text{ m}}^{0.04\text{ m}} = 9299.11\,\text{N}\cdot\text{m}$$

Since $T_C > T_Y$, the results obtained using the elastic analysis are not valid.

Plastic Analysis. Assuming that segment BC is fully plastic,

$$T_C = T_P = 9299.11\ \text{N}\cdot\text{m} = 9.3\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$T_A = 5700\ \text{N}\cdot\text{m} = 5.70\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

5–133. Continued

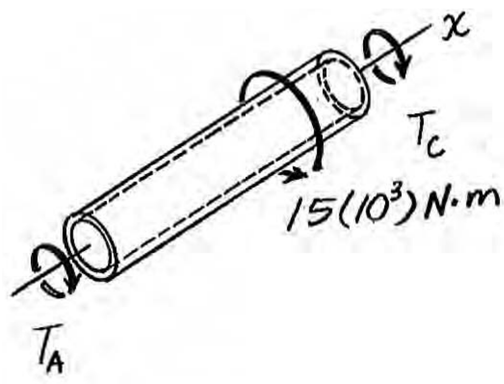
Since $T_A < T_Y$, segment AB of the shaft is still linearly elastic. Here, $G = \dfrac{120(10^6)}{0.0016} = 75\text{GPa}$.

$$\phi_{B/C} = \phi_{B/A} = \frac{T_A L_{AB}}{JG} = \frac{5700.89(0.45)}{\frac{\pi}{2}(0.04^4 - 0.03^4)(75)(10^9)} = 0.01244 \text{ rad}$$

$$\phi_{B/C} = \frac{\gamma_i}{c_i} L_{BC}; \qquad 0.01244 = \frac{\gamma_i}{0.03}(0.15)$$

$$\gamma_i = 0.002489 \text{ rad}$$

Since $\gamma_i > \gamma_Y$, segment BC of the shaft is indeed fully plastic.



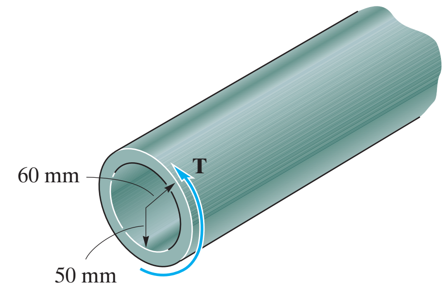
(a)

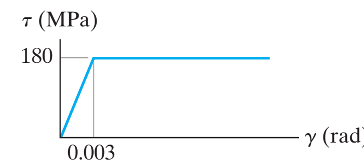
Ans.

$T_A = 5.70 \text{ kN} \cdot \text{m}, \ T_C = 9.3 \text{ kN} \cdot \text{m}$

5–134.

The 2-m-long tube is made of an elastic perfectly plastic material as shown. Determine the applied torque T, which subjects the material at the tube's outer edge to a shear strain of $\gamma_{\max} = 0.006$ rad. What would be the permanent angle of twist of the tube when this torque is removed? Sketch the residual stress distribution in the tube.





SOLUTION

Plastic Torque: The tube will be fully plastic if $\gamma_i \geq \gamma_Y = 0.003$ rad. From Fig. a,

$$\frac{\gamma_i}{0.05} = \frac{0.006}{0.06}; \qquad \gamma_i = 0.005 \text{ rad} > \gamma_Y$$

The tube indeed is fully plastic. Then

$$T_p = 2\pi \int_{c_i}^{c_0} \tau_Y \rho^2 dp$$

$$= \frac{2}{3}\pi\tau_Y \left(C_0^3 - C_i^3\right)$$

$$= \frac{2}{3}\pi\left[180\left(10^6\right)\right]\left(0.06^3 - 0.05^3\right)$$

$$= 34.306\left(10^3\right)\text{N}\cdot\text{m} = 34.3 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi_P = \frac{\gamma_{\max}}{C_0}L = \left(\frac{0.006}{0.06}\right)(2) = 0.2 \text{ rad}$$

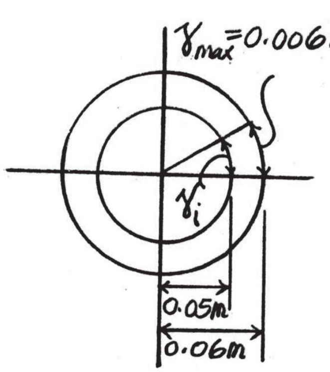
Here, $G = \dfrac{\tau_Y}{\gamma_Y} = \dfrac{180\left(10^6\right)}{0.003} = 60\left(10^9\right)$Pa. When a reverse torque of

$T'_P = 34.306\left(10^3\right)$ N $\cdot$ m is applied

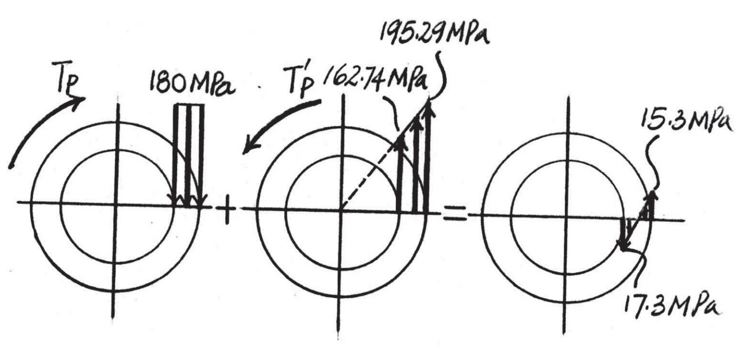
$$\phi'_P = \frac{T_P'L}{JG} = \frac{34.306\left(10^3\right)(2)}{\frac{\pi}{2}\left(0.06^4 - 0.05^4\right)\left[60\left(10^9\right)\right]} = 0.1085 \text{ rad}$$

Thus, the permanent angle of twist is

$$\phi_r = \phi_p - \phi'_p = 0.2 - 0.1085 = (0.09151 \text{ rad})\left(\frac{180°}{\pi \text{ rad}}\right) = 5.24° \quad \textbf{Ans.}$$



(a)



(b)

5–134. Continued

Residual Shear Stress:

$$\left(\tau'_P\right)_o = \frac{T'_P C}{J} = \frac{34.306\left(10^3\right)(0.06)}{\frac{\pi}{2}\left(0.06^4 - 0.05^4\right)} = 195.29\left(10^6\right)\text{Pa} = 195.29\text{ MPa}$$

$$\left(\tau'_P\right)_o = \frac{T'_P \rho}{J} = \frac{34.306\left(10^3\right)(0.05)}{\frac{\pi}{2}\left(0.06^4 - 0.05^4\right)} = 162.74\left(10^6\right)\text{Pa} = 162.74\text{ MPa}$$

Thus

$$(\tau_r)_o = -\tau_Y + \left(\tau'_p\right)_o = -180 + 195.29 = 15.3\text{ MPa} \qquad \textbf{Ans.}$$

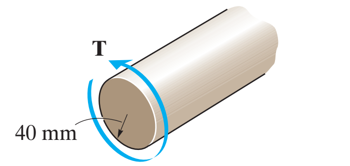
$$(\tau_r)_i = -\tau_Y + \left(\tau'_P\right)_i = -180 + 162.74 = -17.3\text{ MPa} \qquad \textbf{Ans.}$$

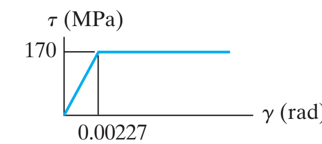
The sketch of the residual shear stress distribution is shown in Fig. *b*.

Ans:
$T_P = 34.3\text{ kN}\cdot\text{m}$,
$\phi_r = 5.24°$,
$(\tau_r)_o = 15.3\text{ MPa}$,
$(\tau_r)_i = -17.3\text{ MPa}$

5–135.

The shaft is made of an elastic perfectly plastic material as shown. Plot the shear-stress distribution acting along a radial line if it is subjected to a torque of $T = 20$ kN · m. What is the residual stress distribution in the shaft when the torque is removed?





SOLUTION

Elastic - Plastic Torque: The maximum elastic torque is

$$T_Y = \frac{\pi}{2}\tau_Y C^3 = \frac{\pi}{2}\left[170(10^6)\right](0.04^3) = 17.09\,(10^3)\text{N}\cdot\text{m}$$

And the plastic torque is

$$T_p = \frac{2}{3}\pi\,\tau_Y C^3 = \frac{2}{3}\pi\,\left[170(10^6)\right](0.04^3) = 22.79\,(10^3)\text{N}\cdot\text{m}$$

Since $T_Y < T < T_P$, the shaft exhibits elastic-plastic behavior.

$$T = \frac{\pi\tau_Y}{6}(4C^3 - \rho_Y^3)$$

$$20(10^3) = \frac{\pi\left[170(10^6)\right]}{6}\left[4(0.04^3) - \rho_Y^3\right]$$

$$\rho_Y = 0.03152\text{ m} = 31.5\text{ mm}$$

Residual shear stress: When the reverse torque $T' = 20$ kN · m is applied,

$$\tau'_C = \frac{T'C}{J} = \frac{20(10^3)(0.04)}{\frac{\pi}{2}(0.04^4)} = 198.94(10^6)\text{Pa} = 198.94\text{ MPa}$$

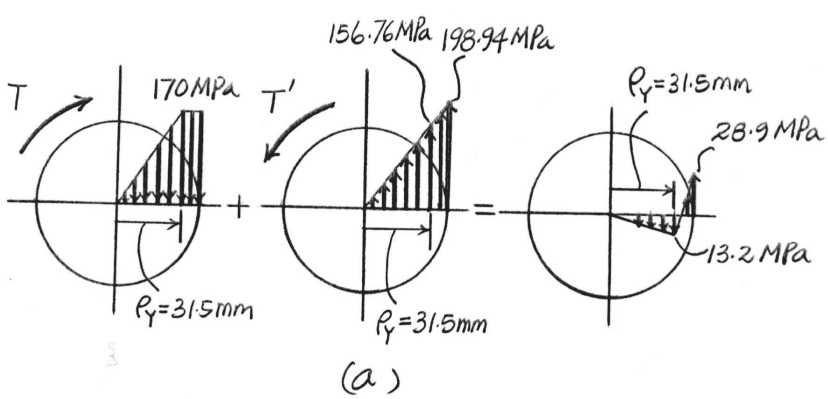
$$\tau'_{\rho Y} = \frac{T'\rho_Y}{J} = \frac{20(10^3)(0.03152)}{\frac{\pi}{2}(0.04^4)} = 156.76(10^6)\text{Pa} = 156.76\text{ MPa}$$

Thus,

$$(\tau_r)_c = -\tau_Y + \tau'_C = -170 + 198.94 = 28.9\text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_r)_{\rho_Y} = -\tau_Y + \tau'_{\rho_Y} = -170 + 156.76 = -13.2\text{ MPa} \qquad \textbf{Ans.}$$

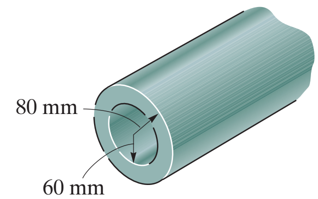
The sketch of the residual shear stress distribution is shown in Fig. *b*.

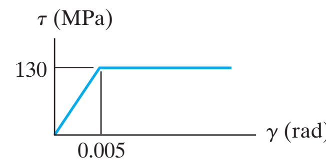


Ans:
$(\tau_r)_c = 28.9$ MPa,
$(\tau_r)\rho_Y = -13.2$ MPa

***5–136.**

The tube has a length of 2 m and is made of an elastic perfectly plastic material as shown. Determine the torque needed to just cause the material to become fully plastic. What is the permanent angle of twist of the tube when this torque is removed?





SOLUTION

Plastic Torque:

$$T_p = 2\pi \int_{c_i}^{c_o} \tau_Y \rho^2 dp$$

$$= \frac{2}{3}\pi\tau_Y \left(C_0^3 - C_i^3\right)$$

$$= \frac{2}{3}\pi\left[130\left(10^6\right)\right]\left(0.08^3 - 0.06^3\right)$$

$$= 80.592\left(10^3\right)\text{N}\cdot\text{m} = 80.6\ \text{KN}\cdot\text{m}$$ **Ans.**

Angle of Twist: Here, $\rho_Y = C_i = 0.06$ m.

$$\phi_P = \frac{\gamma_Y}{\rho_Y}L = \left(\frac{0.005}{0.06}\right)(2) = 0.16667 \text{ rad}$$

Here, $G = \dfrac{\tau_Y}{\gamma_Y} = \dfrac{130\left(10^6\right)}{0.005} = 26\left(10^9\right)$ Pa. When a reverse torque of $T'_P = 80.592$ $\left(10^3\right)$ N · m is applied,

$$\phi'_P = \frac{T'_P L}{JG} = \frac{80.592\left(10^3\right)(2)}{\frac{\pi}{2}\left(0.08^4 - 0.06^4\right)\left[26\left(10^9\right)\right]} = 0.14095 \text{ rad}$$

Thus, the permanent angle of twist is

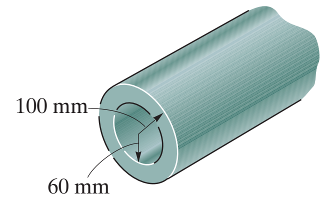
$$\phi_r = \phi_p - \phi'_p = 0.16667 - 0.14095$$

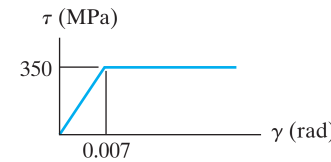
$$= (0.02571 \text{ rad})\left(\frac{180°}{\pi \text{ rad}}\right) = 1.47°$$ **Ans.**

Ans:
$T_P = 80.6$ kN · m,
$\phi_r = 1.47°$

5–137.

The tube has a length of 2 m and is made of an elastic perfectly plastic material as shown. Determine the torque needed to just cause the material to become fully plastic. What is the permanent angle of twist of the tube when this torque is removed?





SOLUTION

Plastic Torque:

$$T_P = 2\pi \int_{c_i}^{c_o} \tau_Y \rho^2 \, d\rho$$

$$= \frac{2\pi\, \tau_Y}{3}\left(c_o^3 - c_i^3\right)$$

$$= \frac{2\pi(350)(10^6)}{3}\left(0.05^3 - 0.03^3\right)$$

$$= 71837.75 \text{ N}\cdot\text{m} = 71.8 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Angle of twist:

$$\phi_\rho = \frac{\gamma_Y}{\rho_Y} L \qquad \text{Where } \rho_Y = c_i = 0.03 \text{ m}$$

$$= \left(\frac{0.007}{0.03}\right)(2) = 0.4667 \text{ rad}$$

When a reverse $T_P = 71837.75 \text{ N}\cdot\text{m}$ is applied.

$$G = \frac{350(10^6)}{0.007} = 50 \text{ GPa}$$

$$\phi_P' = \frac{T_P L}{JG} = \frac{71837.75(2)}{\frac{\pi}{2}\left(0.05^4 - 0.03^4\right)50\left(10^9\right)} = 0.3363 \text{ rad}$$

Permanent angle of twist:

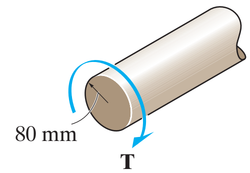
$$\phi_r = \phi_P - \phi_P' = 0.4667 - 0.3363$$

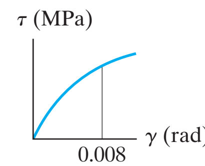
$$= 0.1304 \text{ rad} = 7.47^\circ \qquad \textbf{Ans.}$$

Ans:
$T_P = 71.8 \text{ kN}\cdot\text{m}$,
$\phi_r = 7.47^\circ$

5–138.

A torque is applied to the shaft having a radius of 80 mm. If the material obeys a shear stress–strain relation of $\tau = 500\ \gamma^{1/4}$ MPa, determine the torque that must be applied to the shaft so that the maximum shear strain becomes 0.008 rad.





SOLUTION

τ-ρ Function: First, relate γ to ρ.

$$\gamma = \frac{\rho}{c}\gamma_{\max} = \left(\frac{\rho}{0.08}\right)(0.008) = 0.1\rho$$

Then

$$\tau = 500\left(10^6\right)(0.1\rho)^{1/4} = 281.17\left(10^6\right)\rho^{1/4}$$

Torque:

$$T = 2\pi\int_o^c \tau\rho^2\,dp$$

$$= 2\pi\int_o^{0.08m} 281.17\left(10^6\right)\rho^{\frac{9}{4}}\,dp$$

$$= 1.7666\left(10^9\right)\left(\frac{4}{13}\rho^{13/4}\right)\Big|_0^{0.08\text{ m}}$$

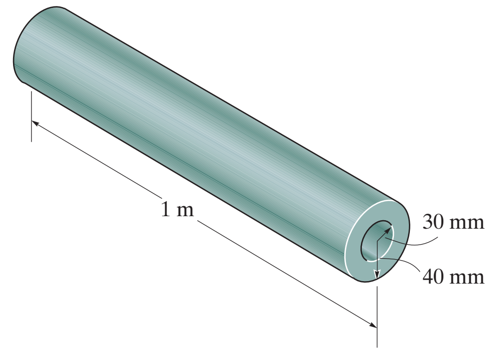
$$= 148.02\left(10^3\right)\text{ N}\cdot\text{m}$$

$$= 148\text{ kN}\cdot\text{m}$$ **Ans.**

Ans:
$T = -148\text{ kN}\cdot\text{m}$

5–139.

A tubular shaft has an inner diameter of 60 mm, an outer diameter of 80 mm, and a length of 1 m. It is made of an elastic perfectly plastic material having a yield stress of $\tau_Y = 150$ MPa. Determine the maximum torque it can transmit. What is the angle of twist of one end with respect to the other end if the inner surface of the tube is about to yield? $G = 75$ GPa.



SOLUTION

Plastic Torque: The plastic torque is the maximum torque a circular shaft can transmit.

$$T_p = 2\pi \int_{C_i}^{C_o} \tau_Y \, p^2 dp$$

$$= 2\pi \, \tau_Y \left(\frac{p^3}{3}\right)\Bigg|_{C_i}^{C_o}$$

$$= \frac{2}{3}\pi \, \tau_Y \left(C_0^3 - C_i^3\right)$$

$$= \frac{2}{3}\pi \left[150\left(10^6\right)\right]\left(0.04^3 - 0.03^3\right)$$

$$= 11.624\left(10^3\right) \text{ N} \cdot \text{m}$$

$$= 11.6 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$

Angle of Twist: The yield strain at the inner surface of the tube can be determined from

$$\tau_Y = G\,\gamma_Y; \qquad 150\left(10^6\right) = 75\left(10^9\right)\gamma_Y$$

$$\gamma_Y = 0.002 \text{ rad}$$

Then, the angle of twist is

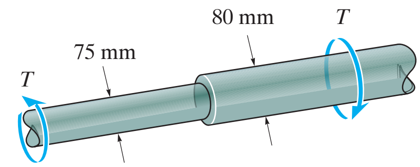
$$\phi = \frac{\gamma_Y}{\rho_Y} L = \left(\frac{0.002}{0.03}\right)(1)$$

$$= (0.06667 \text{ rad})\left(\frac{180^\circ}{\pi \text{ rad}}\right) = 3.82^\circ \qquad \textbf{Ans.}$$

Ans:
$T_P = 11.6 \text{ kN} \cdot \text{m}$,
$\phi = 3.82^\circ$

***5–140.**

The stepped shaft is subjected to a torque **T** that produces yielding on the surface of the larger diameter segment. Determine the radius of the elastic core produced in the smaller diameter segment. Neglect the stress concentration at the fillet.



SOLUTION

Maximum Elastic Torque: For the larger diameter segment, the torsion formula can be applied.

$$\tau_Y = \frac{T_Y c}{J}$$

$$T = \frac{\tau_Y J}{C} = \frac{\tau_Y\left[\frac{\pi}{2}\left(0.04^4\right)\right]}{0.04} = 32.0\left(10^{-6}\right)\pi\,\tau_Y$$

Elastic - Plastic Torque: For the smaller diameter segment, the plastic torque can be determined from

$$T_p = 2\pi\int_0^c \tau_Y\,\rho^2 dp$$

$$= \frac{2\pi}{3}\tau_Y c^3$$

$$= \frac{2\pi}{3}\tau_Y\left(0.0375^3\right)$$

$$= 35.16\left(10^{-6}\right)\pi\,\tau_Y$$

Since $T < T_P$, the smaller segment is not fully plastic

$$T = \frac{\pi\,\tau_Y}{6}\left(4c^3 - \rho_Y^3\right)$$

$$32.0\left(10^{-6}\right)\pi\,\tau_Y = \frac{\pi\,\tau_Y}{6}\left[4\left(0.0375^3\right) - \rho_Y^3\right]$$

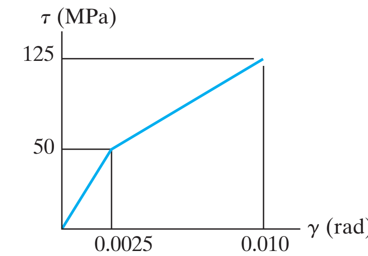
$$\rho_Y = 0.02665 \text{ m}$$

$$= 26.7 \text{ mm}$$ **Ans.**

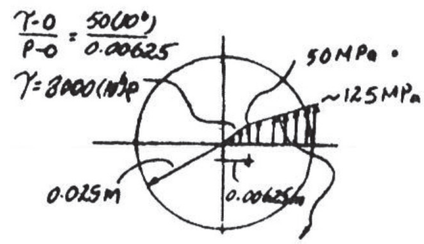
Ans:
$\rho_Y = 26.7$ mm

5–141.

The shear stress–strain diagram for a solid 50-mm-diameter shaft can be approximated as shown in the figure. Determine the torque required to cause a maximum shear stress in the shaft of 125 MPa. If the shaft is 3 m long, what is the corresponding angle of twist?



SOLUTION



$$\gamma = \frac{\rho}{c}\gamma_{\max}$$

$$\gamma_{\max} = 0.01$$

When $\gamma = 0.0025$

$$\rho = \frac{c\gamma}{\gamma_{\max}} = \frac{0.025(0.0025)}{0.010} = 0.00625$$

$$\frac{\tau - 0}{\rho - 0} = \frac{50(10^6)}{0.00625}$$

$$\tau = 8000(10^6)(\rho)$$

$$\frac{\tau - 50(10^6)}{\rho - 0.00625} = \frac{125(10^6) - 50(10^6)}{0.025 - 0.00625}$$

$$\tau = 4000(10^6)(\rho) + 25(10^6)$$

$$T = 2\pi\int_0^c \tau\rho^2\,d\rho$$

$$= 2\pi\int_0^{0.00625} 8000(10^6)\rho^3\,d\rho + 2\pi\int_{0.00625}^{0.025}[4000(10^6)\rho + 25(10^6)]\rho^2\,d\rho$$

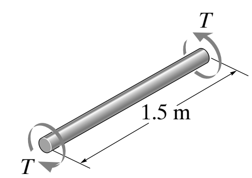
$$T = 3269\text{ N}\cdot\text{m} = 3.27\text{ kN}\cdot\text{m}$$ **Ans.**

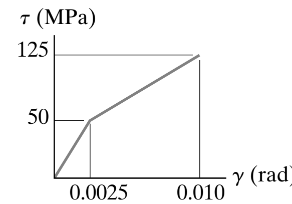
$$\phi = \frac{\gamma_{\max}}{c}L = \frac{0.01}{0.025}(3)$$

$$= 1.20\text{ rad} = 68.8°$$ **Ans.**

Ans:
$T = 3.27\text{ kN}\cdot\text{m}$, $\phi = 68.8°$

5–142. The shear stress–strain diagram for a solid 50-mm-diameter shaft can be approximated as shown in the figure. Determine the torque T required to cause a maximum shear stress in the shaft of 125 MPa. If the shaft is 1.5 m long, what is the corresponding angle of twist?





SOLUTION

Strain Diagram:

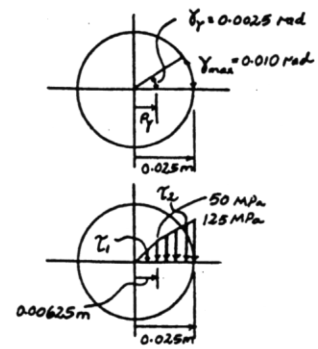
$$\frac{\rho_\gamma}{0.0025} = \frac{0.025}{0.01}; \qquad \rho_\gamma = 0.00625 \text{ m}$$

Stress Diagram:

$$\tau_1 = \frac{50(10^6)}{0.00625}\,\rho = 8(10^9)\,\rho$$

$$\frac{\tau_2 - 50(10^6)}{\rho - 0.00625} = \frac{125(10^6) - 50(10^6)}{0.025 - 0.00625}$$

$$\tau_2 = 4\left(10^9\right)\,\rho + 25\left(10^6\right)$$



The Ultimate Torque:

$$T = 2\pi \int_0^c \tau\,\rho^2 d\rho$$

$$= 2\pi \int_0^{0.00625\text{ m}} 8\left(10^9\right)\rho^3\,d\rho$$

$$+ 2\pi \int_{0.00625\text{ m}}^{0.025\text{ m}} \left[4\left(10^9\right)\rho + 25\left(10^6\right)\right]\rho^2 d\rho$$

$$= 2\pi\left[2\left(10^9\right)\rho^4\right]\Big|_0^{0.00625\text{ m}}$$

$$+ 2\pi\left[1\left(10^9\right)\rho^4 + \frac{25(10^6)\rho^3}{3}\right]\Bigg|_{0.00625\text{ m}}^{0.025\text{ m}}$$

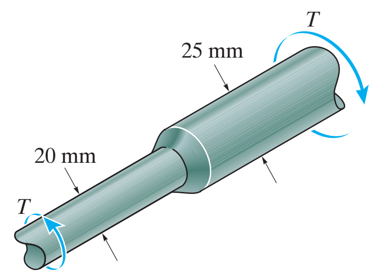
$$= 3269.30 \text{ N}\cdot\text{m} = 3.27 \text{ kN}\cdot\text{m}$$ **Ans.**

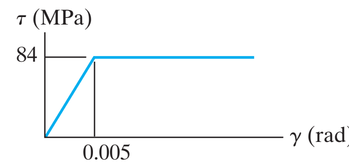
Angle of Twist:

$$\phi = \frac{\gamma_{\max}}{c}L = \left(\frac{0.01}{0.025}\right)(1.5) = 0.60 \text{ rad} = 34.4^\circ$$ **Ans.**

Ans.
$T = 3.27 \text{ kN}\cdot\text{m}, \ \phi = 34.4^\circ$

5–143. The shaft consists of two sections that are rigidly connected. If the material is elastic plastic as shown, determine the largest torque T that can be applied to the shaft. Also, draw the shear-stress distribution over a radial line for each section. Neglect the effect of stress concentration.





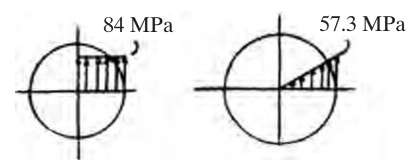
SOLUTION

20-mm diameter segment will be fully plastic. From Eq. 5-27 of the text:

$$T = T_p = \frac{2\pi}{3}\tau_Y c^3$$

$$= \frac{2\pi}{3}[84(10^6)](0.01^3)$$

$$= 175.93\ \text{N}\cdot\text{m} = 176\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$



For 25-mm diameter segment:

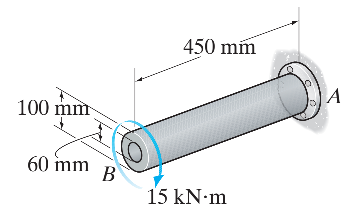
$$\tau_{\max} = \frac{Tc}{J} = \frac{175.93(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 57.34(10^6)\ \text{N/m}^2$$

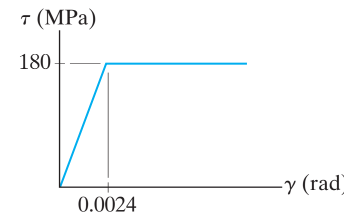
$$= 57.3\ \text{MPa} < \tau_Y$$

Ans.
$T = T_p = 176\ \text{N}\cdot\text{m}$

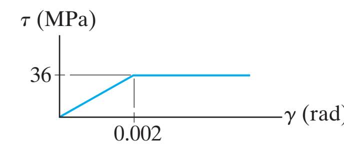
***5–144.**

A steel alloy core is bonded firmly to the copper alloy tube to form the shaft shown. If the materials have the $\tau-\gamma$ diagrams shown, determine the torque resisted by the core and the tube.





Steel Alloy



Copper Alloy

SOLUTION

Equation of Equilibrium. Referring to the free-body diagram of the cut part of the assembly shown in Fig. *a*,

$$\Sigma M_x = 0; \quad T_c + T_t - 15\left(10^3\right) = 0 \qquad \textbf{(1)}$$

Elastic Analysis. The shear modulus of steel and copper are $G_{st} = \dfrac{180\left(10^6\right)}{0.0024} = 75\text{ GPa}$ and $G_{\infty} = \dfrac{36\left(10^6\right)}{0.002} = 18\text{ GPa}$. Compatibility requires that

$$\phi_C = \phi_t$$

$$\frac{T_c L}{J_c G_{st}} = \frac{T_t L}{J_t G_{\infty}}$$

$$\frac{T_c}{\frac{\pi}{2}\left(0.03^4\right)(75)\left(10^9\right)} = \frac{T_t}{\frac{\pi}{2}\left(0.05^4 - 0.03^4\right)(18)\left(10^9\right)}$$

$$T_c = 0.6204 T_t \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$T_t = 9256.95\text{ N}\cdot\text{m} \qquad T_c = 5743.05\text{ N}\cdot\text{m}$$

The maximum elastic torque and plastic torque of the core and the tube are

$$(T_Y)_c = \frac{1}{2}\pi c^3 (\tau_Y)_{st} = \frac{1}{2}\pi\left(0.03^3\right)(180)\left(10^6\right) = 7634.07\text{ N}\cdot\text{m}$$

$$(T_P)_c = \frac{2}{3}\pi c^3 (\tau_Y)_{st} = \frac{2}{3}\pi\left(0.03^3\right)(180)\left(10^6\right) = 10\,178.76\text{ N}\cdot\text{m}$$

and

$$(T_Y)_t = \frac{J}{c}\tau_Y = \left[\frac{\frac{\pi}{2}\left(0.05^4 - 0.03^4\right)}{0.05}\right]\left[(36)\left(10^6\right)\right] = 6152.49\text{ N}\cdot\text{m}$$

$$(T_P)_t = 2\pi(\tau_Y)_{\infty}\int_{c_i}^{c_o}\rho^2 d\rho = 2\pi(36)\left(10^6\right)\left(\frac{\rho^3}{3}\right)\Bigg|_{0.03\text{ m}}^{0.05\text{ m}} = 7389.03\text{ N}\cdot\text{m}$$

Since $T_t > (T_Y)_t$, the results obtained using the elastic analysis are not valid.

5–144. Continued

Plastic Analysis. Assuming that the tube is fully plastic,

$$T_t = (T_P)_t = 7389.03 \text{ N} \cdot \text{m} = 7.39 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$T_c = 7610.97 \text{ N} \cdot \text{m} = 7.61 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$

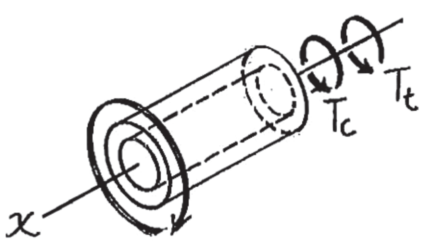
Since $T_c < (T_Y)_c$, the core is still linearly elastic. Thus,

$$\phi_t = \phi_{tc} = \frac{T_c L}{J_c G_{st}} = \frac{7610.97(0.45)}{\frac{\pi}{2}(0.03^4)(75)(10^9)} = 0.03589 \text{ rad}$$

$$\phi_t = \frac{\gamma_i}{c_i} L; \qquad 0.3589 = \frac{\gamma_i}{0.03}(0.45)$$

$$\gamma_i = 0.002393 \text{ rad}$$

Since $\gamma_i > (\gamma_Y)_\infty = 0.002$ rad, the tube is indeed fully plastic.

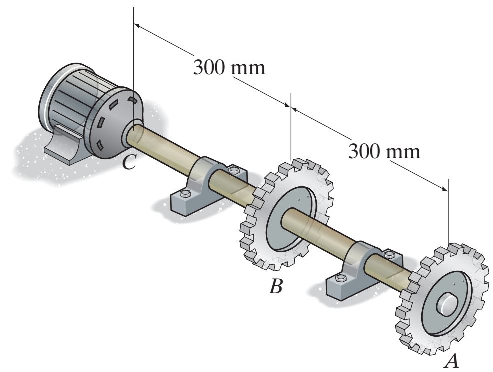


(a)

Ans:
$T_t = 7.39 \text{ kN} \cdot \text{m}$, $T_c = 7.61 \text{ kN} \cdot \text{m}$

R5–1.

The shaft is made of A992 steel and has an allowable shear stress of $\tau_{\text{allow}} = 75$ MPa. When the shaft is rotating at 300 rpm, the motor supplies 8 kW of power, while gears A and B withdraw 5 kW and 3 kW, respectively. Determine the required minimum diameter of the shaft to the nearest millimeter. Also, find the rotation of gear A relative to C.



SOLUTION

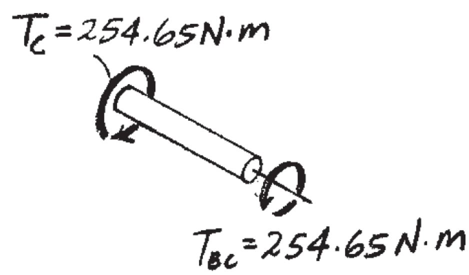
Applied Torque: The angular velocity of the shaft is

$$\omega = \left(300\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{rad}}{1\text{ rev}}\right) = 10\pi\text{ rad/s}$$

Thus, the torque at C and gear A are

$$T_C = \frac{P_C}{\omega} = \frac{8(10^3)}{10\pi} = 254.65\text{ N}\cdot\text{m}$$

$$T_A = \frac{P_A}{\omega} = \frac{5(10^3)}{10\pi} = 159.15\text{ N}\cdot\text{m}$$



(a)

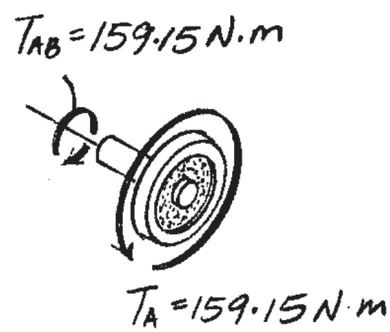
Internal Loading: The internal torque developed in segment BC and AB of the shaft are shown in Figs. a and b, respectively.

Allowable Shear Stress: By inspection, segment BC is critical.

$$\tau_{\text{allow}} = \frac{T_{BC}\,c}{J};\qquad 75(10^6) = \frac{254.65(\frac{d}{2})}{\frac{\pi}{2}(\frac{d}{2})^4}$$

$$d = 0.02586\text{ m}$$

Use $\quad d = 26$ mm **Ans.**



(b)

Angle of Twist: Using $d = 26$ mm,

$$\phi_{A/C} = \Sigma\frac{T_iL_i}{J_iG_i} = \frac{T_{AB}L_{AB}}{JG} + \frac{T_{BC}L_{BC}}{JG}$$

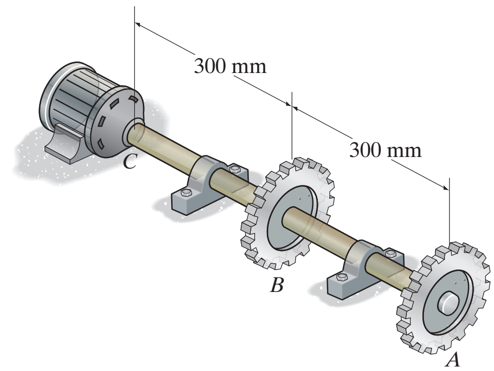
$$= \frac{0.3}{\frac{\pi}{2}(0.013^4)(75)(10^9)}(159.15 + 254.65)$$

$$= 0.03689\text{ rad} = 2.11°$$ **Ans.**

Ans:
Use $d = 26$ mm, $\phi_{A/C} = 2.11°$

R5–2.

The shaft is made of A992 steel and has an allowable shear stress of $\tau_{\text{allow}} = 75$ MPa. When the shaft is rotating at 300 rpm, the motor supplies 8 kW of power, while gears A and B withdraw 5 kW and 3 kW, respectively. If the angle of twist of gear A relative to C is not allowed to exceed 0.03 rad, determine the required minimum diameter of the shaft to the nearest millimeter.



SOLUTION

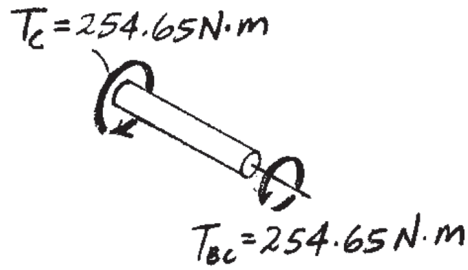
Applied Torque: The angular velocity of the shaft is

$$\omega = \left(300\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{rad}}{1\text{ rev}}\right) = 10\pi\text{ rad/s}$$

Thus, the torque at C and gear A are

$$T_C = \frac{P_C}{\omega} = \frac{8(10^3)}{10\pi} = 254.65\text{ N}\cdot\text{m}$$

$$T_A = \frac{P_A}{\omega} = \frac{5(10^3)}{10\pi} = 159.15\text{ N}\cdot\text{m}$$



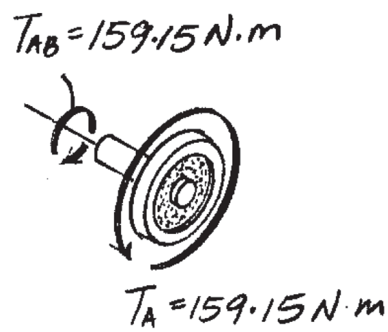
(a)

Internal Loading: The internal torque developed in segment BC and AB of the shaft are shown in Figs. a and b, respectively.

Allowable Shear Stress: By inspection, segment BC is critical.

$$\tau_{\text{allow}} = \frac{T_{BC}\,c}{J}; \qquad 75(10^3) = \frac{254.65\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

$$d = 0.02586$$



(b)

Angle of Twist:

$$\phi_{A/C} = \Sigma\frac{T_iL_i}{J_iG_i} = \frac{T_{AB}L_{AB}}{JG} + \frac{T_{BC}L_{BC}}{JG}$$

$$0.03 = \frac{0.3}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4(75)(10^9)}(159.15 + 254.65)$$

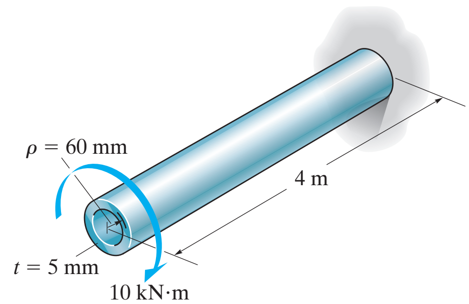
$$d = 0.02738\text{ m(controls)}$$

Use $d = 28$ mm **Ans.**

Ans:
Use $d = 28$ mm

R5–3.

The A-36 steel circular tube is subjected to a torque of 10 kN·m. Determine the shear stress at the mean radius $\rho = 60$ mm and calculate the angle of twist of the tube if it is 4 m long and fixed at its far end. Solve the problem using Eqs. 5–7 and 5–15 and by using Eqs. 5–18 and 5–20.



SOLUTION

We show that two different methods give similar results:

Shear Stress:

Applying Eq. 5-7,

$$r_o = 0.06 + \frac{0.005}{2} = 0.0625 \text{ m} \qquad r_i = 0.06 - \frac{0.005}{2} = 0.0575 \text{ m}$$

$$\tau_{\rho=0.06\text{ m}} = \frac{T\rho}{J} = \frac{10(10^3)(0.06)}{\frac{\pi}{2}(0.0625^4 - 0.0575^4)} = 88.27 \text{ MPa}$$

Applying Eq. 5-18,

$$\tau_{\text{avg}} = \frac{T}{2\,t\,A_m} = \frac{10(10^3)}{2(0.005)(\pi)(0.06^2)} = 88.42 \text{ MPa}$$

Angle of Twist:

Applying Eq. 5-15,

$$\phi = \frac{TL}{JG}$$

$$= \frac{10(10^3)(4)}{\frac{\pi}{2}(0.0625^4 - 0.0575^4)(75.0)(10^9)}$$

$$= 0.0785 \text{ rad} = 4.495^\circ$$

Applying Eq. 5-20,

$$\phi = \frac{TL}{4A_m^2 G}\int \frac{ds}{t}$$

$$= \frac{TL}{4A_m^2 G\,t}\int ds \qquad \text{Where} \qquad \int ds = 2\pi\rho$$

$$= \frac{2\pi TL\rho}{4A_m^2 G\,t}$$

$$= \frac{2\pi(10)(10^3)(4)(0.06)}{4[(\pi)(0.06^2)]^2(75.0)(10^9)(0.005)}$$

$$= 0.0786 \text{ rad} = 4.503^\circ$$

Rounding to three significant figures, we find

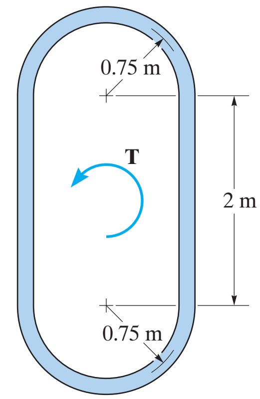
$\tau = 88.3$ MPa **Ans.**

$\phi = 4.50^\circ$ **Ans.**

Ans:
$\tau = 88.3$ MPa, $\phi = 4.50^\circ$

***R5–4.**

A portion of an airplane fuselage can be approximated by the cross section shown. If the thickness of its 2014-T6-aluminum skin is 10 mm, determine the maximum wing torque **T** that can be applied if $\tau_{allow} = 4$ MPa. Also, in a 4-m-long section, determine the angle of twist.



SOLUTION

$$\tau_{avg} = \frac{T}{2tA_m}$$

$$4(10^6) = \frac{T}{2(0.01)[(\pi)(0.75)^2 + 2(1.5)]}$$

$$T = 381.37(10^3) = 381 \text{ kN} \cdot \text{m}$$ **Ans.**

$$\phi = \frac{TL}{4A_m^2 G} \int \frac{ds}{t}$$

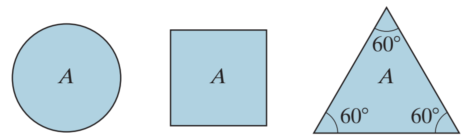
$$\phi = \frac{381.37(10^3)(4)}{4[(\pi(0.75)^2 + 2(1.5))^2 27(10^9)]} \left[\frac{4 + 2\pi(0.75)}{0.010}\right]$$

$$\phi = 0.542(10^{-3}) \text{ rad} = 0.0310^\circ$$ **Ans.**

Ans:
$T = 381$ kN·m, $\phi = 0.0310°$

R5–5.

The material of which each of three shafts is made has a yield stress of τ_Y and a shear modulus of G. Determine which shaft geometry will resist the largest torque without yielding. What percentage of this torque can be carried by the other two shafts? Assume that each shaft is made from the same amount of material and that it has the same cross-sectional area A.



SOLUTION

For circular shaft:

$$A = \pi c^2; \qquad c = \left(\frac{A}{\pi}\right)^{\frac{1}{2}}$$

$$\tau_{\max} = \frac{T c}{J}, \qquad \tau_Y = \;= \frac{T c}{\frac{a}{2} c^4}$$

$$T = \frac{\pi c^3}{2}\tau_Y = \frac{\pi\left(\frac{A}{\pi}\right)^{\frac{3}{2}}}{2}\tau_Y$$

$$T_{cir} = 0.2821\, A^{\frac{1}{2}}\,\tau_Y$$

For the square shaft:

$$A = a^2; \qquad a = A^{\frac{1}{2}}$$

$$\tau_{\max} = \frac{4.81\ T}{a^3}; \qquad \tau_Y = \frac{4.81\ T}{A^{\frac{3}{2}}}$$

$$T = 0.2079\ A^{\frac{3}{2}}\tau_Y$$

For the triangular shaft:

$$A = \frac{1}{2}(a)(a \sin 60°); \qquad a = 1.5197A^{\frac{1}{2}}$$

$$\tau_{\max} = \frac{20\ T}{a^3}; \qquad \tau_Y = \frac{20\ T}{(1.5197)^3 A^{\frac{3}{2}}}$$

$$T = 0.1755A^{\frac{3}{2}}\,\tau_Y$$

The circular shaft will resist the largest torque. **Ans.**

For the square shaft:

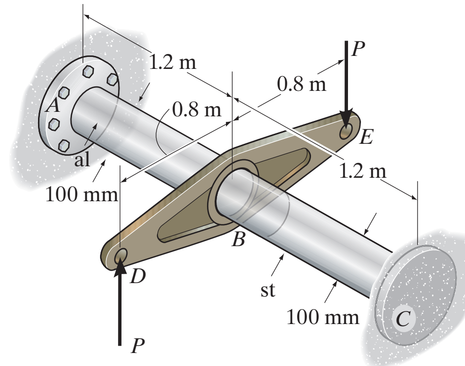
$$\% = \frac{0.2079}{0.2821}(100\%) = 73.7\%$$ **Ans.**

For the triangular shaft:

$$\% = \frac{0.1755}{0.2821}(100\%) = 62.2\ \%$$ **Ans.**

Ans:
The circular shaft will resist the largest torque.
For the square shaft: 73.7%,
For the triangular shaft: 62.2%

R5–6. Segments AB and BC of the assembly are made from 6061-T6 aluminum and A992 steel, respectively. If couple forces $P = 15$ kN are applied to the lever arm, determine the maximum shear stress developed in each segment. The assembly is fixed at A and C.



SOLUTION

Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. a,

$$\Sigma M_x = 0; \qquad T_A + T_C - 15(1.6) = 0 \qquad (1)$$

Compatibility Equation: It is required that

$$\phi_{B/A} = \phi_{B/C}$$

$$\frac{T_A L_{AB}}{JG_{al}} = \frac{T_C L_{BC}}{JG_{st}}$$

$$\frac{T_A L}{J[26(10^9)]} = \frac{T_C L}{J[75(10^9)]}$$

$$T_A = 0.3467\, T_C \qquad (2)$$

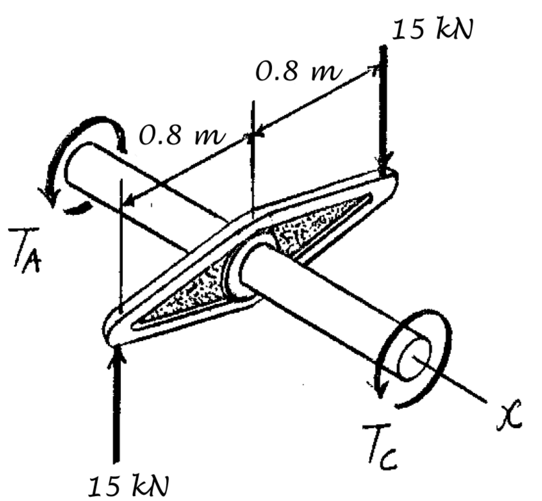
Solving Eqs. (1) and (2),

$$T_C = 17.82 \text{ kN}\cdot\text{m} \qquad T_A = 6.178 \text{ kN}\cdot\text{m}$$

Maximum Shear Stress:

$$(\tau_{\max})_{AB} = \frac{T_A c}{J} = \frac{[6.178(10^3)](0.05)}{\frac{\pi}{2}(0.05^4)} = 31.47(10^6) \text{ N/m}^2 = 31.5 \text{ MPa} \qquad \textbf{Ans.}$$

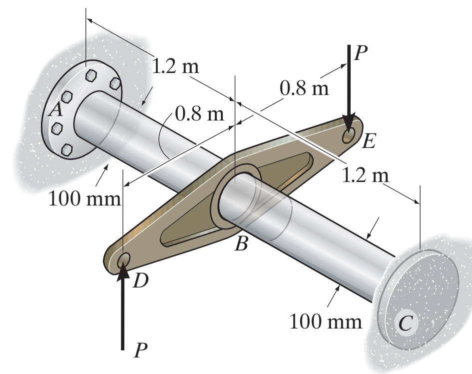
$$(\tau_{\max})_{BC} = \frac{T_C c}{J} = \frac{[17.82(10^3)](0.05)}{\frac{\pi}{2}(0.05^4)} = 90.77(10^6) \text{ N/m}^2 = 90.8 \text{ MPa} \qquad \textbf{Ans.}$$



(a)

Ans.
$(\tau_{\max})_{AB} = 31.5$ MPa
$(\tau_{\max})_{BC} = 90.8$ MPa

R5–7. Segments AB and BC of the assembly are made from 6061-T6 aluminum and A992 steel, respectively. If the allowable shear stress for the aluminum is $(\tau_{\text{allow}})_{al} =$ 90 MPa and for the steel $(\tau_{\text{allow}})_{st} =$ 120 MPa, determine the maximum allowable couple forces P that can be applied to the lever arm. The assembly is fixed at A and C.



SOLUTION

Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. a,

$$\Sigma M_x = 0; \qquad T_A + T_C - P(1.6) = 0 \tag{1}$$

Compatibility Equation: It is required that

$$\phi_{B/A} = \phi_{B/C}$$

$$\frac{T_A L_{AB}}{JG_{\text{al}}} = \frac{T_C L_{BC}}{JG_{\text{st}}}$$

$$\frac{T_A L}{J[26(10^9)]} = \frac{T_C L}{J[75(10^9)]}$$

$$T_A = 0.3467\, T_C \tag{2}$$

Solving Eqs. (1) and (2),

$$T_C = 1.1881P \qquad\qquad T_A = 0.4119P$$

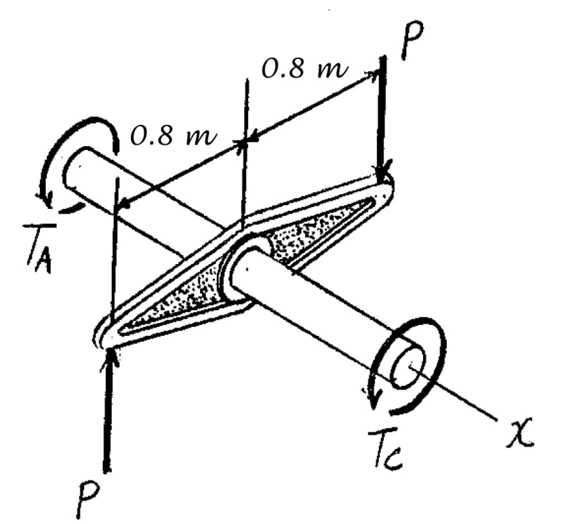
Allowable Shear Stress:

$$(\tau_{\text{allow}})_{al} = \frac{T_A c}{J}; \qquad 90(10^6) = \frac{(0.4119P)(0.05)}{\frac{\pi}{2}(0.05^4)}$$

$$P = 42.90(10^3)\text{ N} = 42.9\text{ kN}$$

$$(\tau_{\text{allow}})_{st} = \frac{T_C c}{J}; \qquad 120(10^6) = \frac{(1.1881P)(0.05)}{\frac{\pi}{2}(0.05^4)}$$

$$P = 19.83(10^3)\text{ N} = 19.8\text{ kN (controls)}$$ **Ans.**

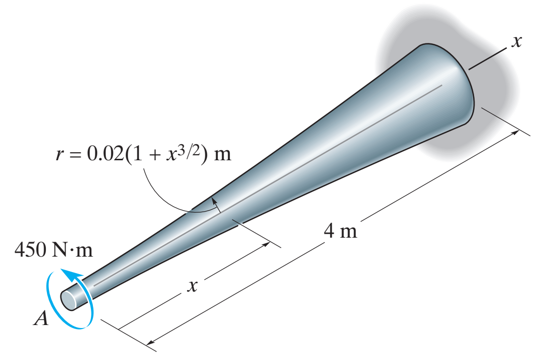


(a)

Ans:
$P = 19.8$ kN

***R5–8.**

The tapered shaft is made from 2014-T6 aluminum alloy, and has a radius which can be described by the equation $r = 0.02(1 + x^{3/2})$ m, where x is in meters. Determine the angle of twist of its end A if it is subjected to a torque of 450 N · m.



SOLUTION

$T = 450 \text{ N} \cdot \text{m}$

$$\phi_A = \int \frac{T dx}{JG} = \int_0^4 \frac{450\, dx}{\frac{\pi}{2}(0.02)^4(1 + x^{\frac{3}{2}})^4(27)(10^9)} = 0.066315 \int_0^4 \frac{dx}{(1 + x^{\frac{3}{2}})^4}$$

Evaluating the integral numerically, we have

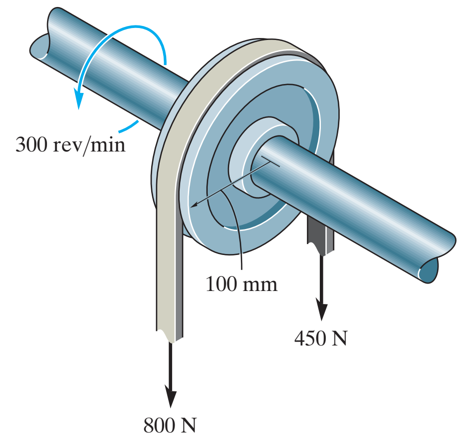
$\phi_A = 0.066315\,[0.4179]$ rad

$= 0.0277 \text{ rad} = 1.59°$ **Ans.**

Ans:
$\phi_A = 1.59°$

R5–9.

The 60-mm-diameter shaft rotates at 300 rev/min. This motion is caused by the unequal belt tensions on the pulley of 800 N and 450 N. Determine the power transmitted and the maximum shear stress developed in the shaft.



SOLUTION

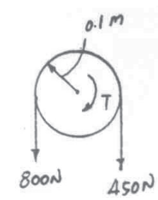
$$\omega = 300\frac{\text{rev}}{\text{min}}\left[\frac{2\pi\text{ rad}}{1\text{ rev}}\right]\frac{1\text{ min}}{60\text{ s}} = 10\,\pi\text{ rad/s}$$

$$T + 450(0.1) - 800(0.1) = 0$$

$$T = 35.0\text{ N}\cdot\text{m}$$

$$P = T\omega = 35.0(10\pi) = 1100\text{ W} = 1.10\text{ kW}$$ **Ans.**

$$\tau_{\max} = \frac{Tc}{J} = \frac{35.0(0.03)}{\frac{\pi}{2}(0.03^4)} = 825\text{ kPa}$$ **Ans.**



Ans:
$P = 1.10$ kW,
$\tau_{\max} = 825$ kPa