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9–1.

Prove that the sum of the normal stresses $\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'}$ is constant. See Figs. 9–2*a* and 9–2*b*.

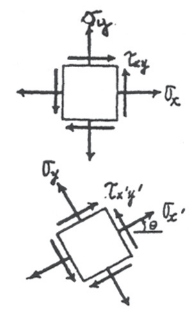
SOLUTION

Stress Transformation Equations: Applying Eqs. 9–1 and 9–3 of the text.

$$\sigma_{x'} + \sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$+ \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y \qquad \textbf{(Q. E. D.)}$$



These solutions represent a preliminary version of the Instructors' Solutions Manual (ISM). It is possible and even likely that at this preliminary stage of preparing the ISM there are some omissions and errors in the draft solutions.

Ans:
N/A

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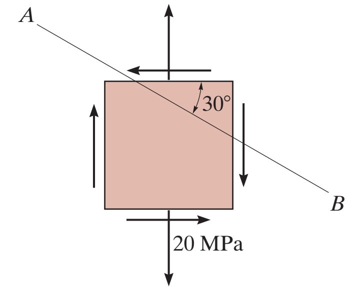
9–2.

Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



SOLUTION

$\swarrow + \Sigma F_{x'} = 0;\quad \sigma_{x'}\Delta A + 20\Delta A \sin 30^\circ \cos 30^\circ + 20\Delta A \cos 30^\circ \cos 60^\circ$

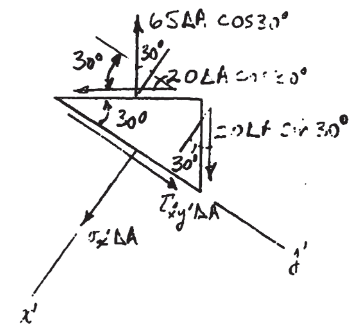
$- 65\Delta A \cos 30^\circ \cos 30^\circ = 0$

$\sigma_{x'} = 31.4 \text{ MPa}$ **Ans.**

$\searrow + \Sigma F_{y'} = 0;\quad \tau_{x'y'}\Delta A + 20\Delta A \sin 30^\circ \sin 30^\circ - 20\Delta A \cos 30^\circ \sin 60^\circ$

$- 65\Delta A \cos 30^\circ \sin 30^\circ = 0$

$\tau_{x'y'} = 38.1 \text{ MPa}$ **Ans.**



Ans:
$\sigma_{x'} = 31.4 \text{ MPa}$,
$\tau_{x'y'} = 38.1 \text{ MPa}$

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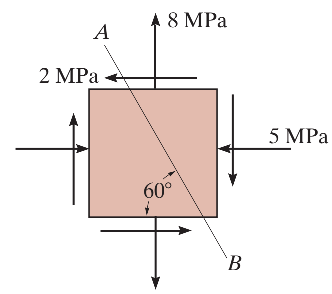
9–3. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



SOLUTION

Referring to Fig *a*, if we assume that the areas of the inclined plane AB is ΔA, then the area of the horizontal and vertical of the triangular element are $\Delta A \cos 60°$ and $\Delta A \sin 60°$ respectively. The forces act acting on these two faces indicated on the FBD of the triangular element, Fig. *b*.

$$+\nearrow \Sigma F_{x'} = 0; \qquad \Delta F_{x'} + 2\Delta A \sin 60° \cos 60° + 5\Delta\ A \sin 60° \sin 60°$$

$$+ 2\Delta A \cos 60° \sin 60° - 8\Delta A \cos 60° \cos 60° = 0$$

$$\Delta F_{x'} = -3.482\ \Delta A$$

$$+\nwarrow \Sigma F_{y'} = 0; \qquad \Delta F_{y'} + 2\Delta A \sin 60° \sin 60° - 5\Delta\ A \sin 60° \cos 60°$$

$$- 8\Delta A \cos 60° \sin 60° - 2\Delta A \cos 60° \cos 60° = 0$$

$$\Delta F_{y'} = 4.629\ \Delta A$$

From the definition,

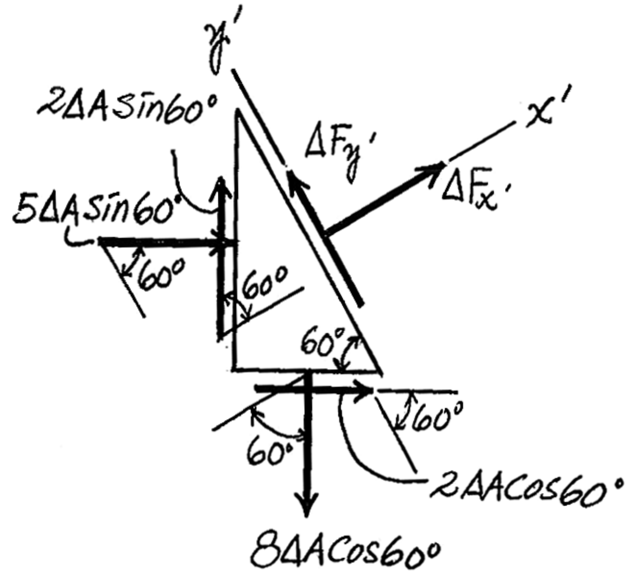
$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -3.48\ \text{MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = 4.63\ \text{MPa} \qquad \textbf{Ans.}$$

The negative sign indicates that $\sigma_{x'}$, is a compressive stress.

(a)

sides



(b)

Ans.

$\sigma_{x'} = -3.48\ \text{MPa}$, $\tau_{x'y'} = 4.63\ \text{MPa}$

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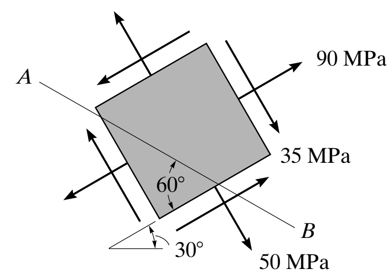
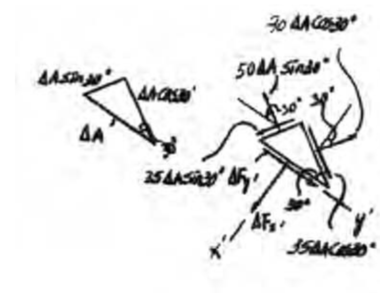
***9–4.** The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



SOLUTION



$\searrow + \Sigma F_{y'} = 0 \qquad \Delta F_{y'} - 50\Delta A \sin 30° \cos 30° - 35\Delta A \sin 30° \cos 60° +$

$$90\Delta A \cos 30° \sin 30° + 35\Delta A \cos 30° \sin 60° = 0$$

$$\Delta F_{y'} = -34.82\Delta A$$

$\swarrow + \Sigma F_{x'} = 0 \qquad \Delta F_{x'} - 50\Delta A \sin 30° \sin 30° + 35\Delta A \sin 30° \sin 60°$

$$-90\Delta A \cos 30° \cos 30° + 35\Delta A \cos 30° \cos 60° = 0$$

$$\Delta F_{x'} = 49.69\ \Delta A$$

$\sigma_{x'} = \lim_{\Delta A \to 0} \dfrac{\Delta F_{x'}}{\Delta A} = 49.7 \text{ MPa}$ **Ans.**

$\tau_{x'y'} = \lim_{\Delta A \to 0} \dfrac{\Delta F_{y'}}{\Delta A} = -34.8 \text{ MPa}$ **Ans.**

The negative signs indicate that the sense of $\sigma_{x'}$, and $\tau_{x'y'}$ are opposite to the shown on FBD.

Ans.
$\sigma_{x'} = 49.7$ MPa, $\tau_{x'y'} = -34.8$ MPa

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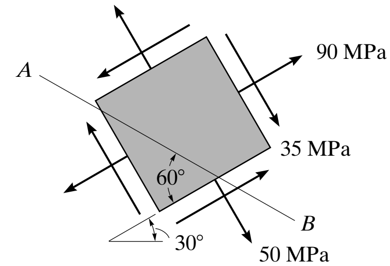
9–5. Solve Prob. 9–6 using the stress-transformation equations developed in Sec. 9.2. Show the result on a sketch.



SOLUTION

$\sigma_x = 90 \text{ MPa} \qquad \sigma_y = 50 \text{ MPa} \qquad \tau_{xy} = -35 \text{ MPa} \qquad \theta = -150^\circ$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{90 + 50}{2} + \frac{90 - 50}{2}\cos(-300^\circ) + (-35)\sin(-300^\circ)$$

$$= 49.7 \text{ MPa}$$ **Ans.**

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{90 - 50}{2}\right)\sin(-300^\circ) + (-35)\cos(-300^\circ) = -34.8 \text{ MPa}$$ **Ans.**

The negative sign indicates $\tau_{x'y'}$ acts in $-y'$ direction.

Ans.
$\sigma_{x'} = 49.7 \text{ MPa}, \tau_{x'y'} = -34.8 \text{ MPa}$

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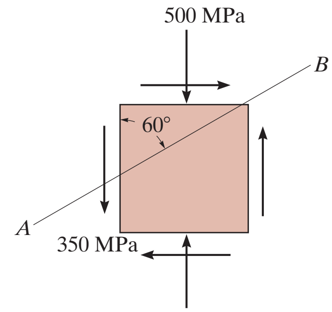
9–6. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



SOLUTION

Referring to Fig. *a*, if we assume that the area of the inclined plane AB is ΔA, then the areas of the horizontal and vertical surfaces of the triangular element are $\Delta A \sin 60°$ and $\Delta A \cos 60°$ respectively. The force acting on these two faces are indicated on the FBD of the triangular element, Fig. *b*

$$+\searrow \Sigma F_{x'} = 0; \quad \Delta F_{x'} + 500\,\Delta A \sin 60° \sin 60° + 350 \Delta A \sin 60° \cos 60°$$

$$+350 \Delta A \cos 60° \sin 60° = 0$$

$$\Delta F_{x'} = -678.11\,\Delta A$$

$$+\nearrow \Sigma F_{y'} = 0; \quad \Delta F_{y'} + 350 \Delta A \sin 60° \sin 60° - 500 \Delta A \sin 60° \cos 60°$$

$$-350 \Delta A \cos 60° \cos 60° = 0$$

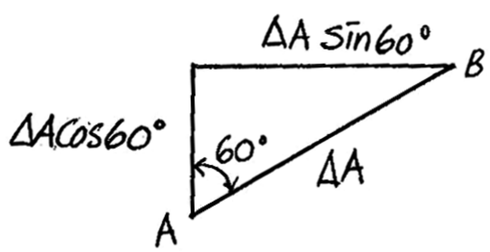
$$\Delta F_{y'} = 41.51\,\Delta A$$

From the definition

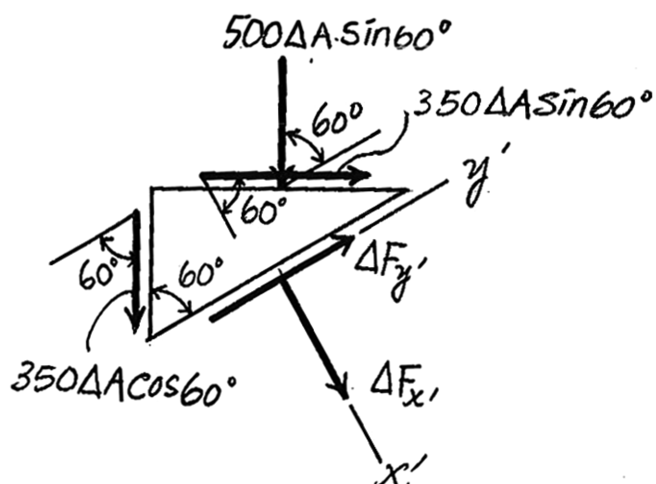
$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -678 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = 41.5 \text{ MPa} \qquad \textbf{Ans.}$$

The negative sign indicates that $\sigma_{x'}$, is a compressive stress.



(a)



(b)

Ans.
$\sigma_{x'} = -678$ MPa, $\tau_{x'y'} = 41.5$ MPa

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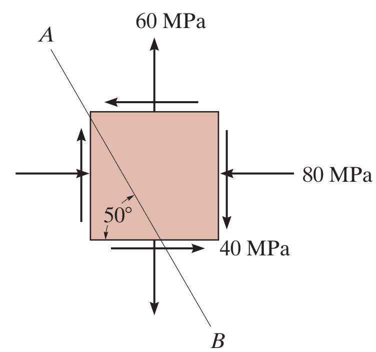
9–7.

Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



SOLUTION

Force Equilibrium: The areas, thus the forces acting on the faces of the lower segment of the sectioned element are shown in Fig. *a*

$$\nwarrow + \Sigma F_{y'} = 0; \quad \Delta F_{y'} + (40\ \Delta A \sin 50°) \sin 50° - (80\ \Delta A \sin 50°) \cos 50°$$
$$- (40\ \Delta A \cos 50°) \cos 50° - (60\ \Delta A \cos 50°) \sin 50° = 0$$
$$\Delta F_{y'} = 61.99\ \Delta A$$

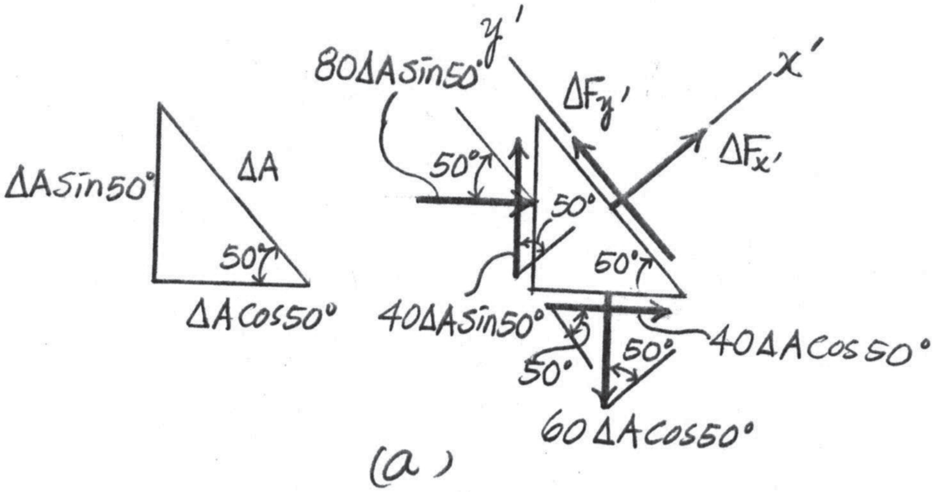
$$+\nearrow \Sigma F_{x'} = 0; \quad \Delta F_{x'} + (40\ \Delta A \sin 50°) \cos 50° + (80\ \Delta A \sin 50°) \sin 50°$$
$$+ (40\ \Delta A \cos 50°) \sin 50° - (60\ \Delta A \cos 50°) \cos 50° = 0$$
$$\Delta F_{x'} = -61.54\ \Delta A$$

Normal And Shear Stresses: For the inclined plane that has an area of ΔA,

$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -61.54 \text{ MPa} = -61.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = 61.99 \text{ MPa} = 62.0 \text{ MPa} \qquad \textbf{Ans.}$$

The negative sign indicates that $\sigma_{x'}$ is a compressive normal stress.



Ans:
$\sigma_{x'} = -61.5$ MPa,
$\tau_{x'y'} = 62.0$ MPa

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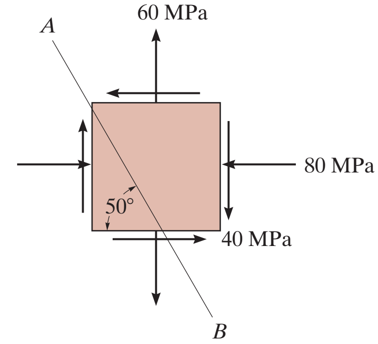
***9–8.**

Solve Prob. 9–7 using the stress transformation equations developed in Sec. 9.2.



SOLUTION

Normal And Shear Stress: In accordance with the established sign conventions,

$\theta = +40°$ (Fig. *a*) $\quad \sigma_x = -80$ MPa $\quad \sigma_y = 60$ MPa $\quad \tau_{xy} = -40$ MPa

Stress Transformation Equations:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-80 + 60}{2} + \frac{-80 - 60}{2}\cos 80° + (-40)\sin 80°$$

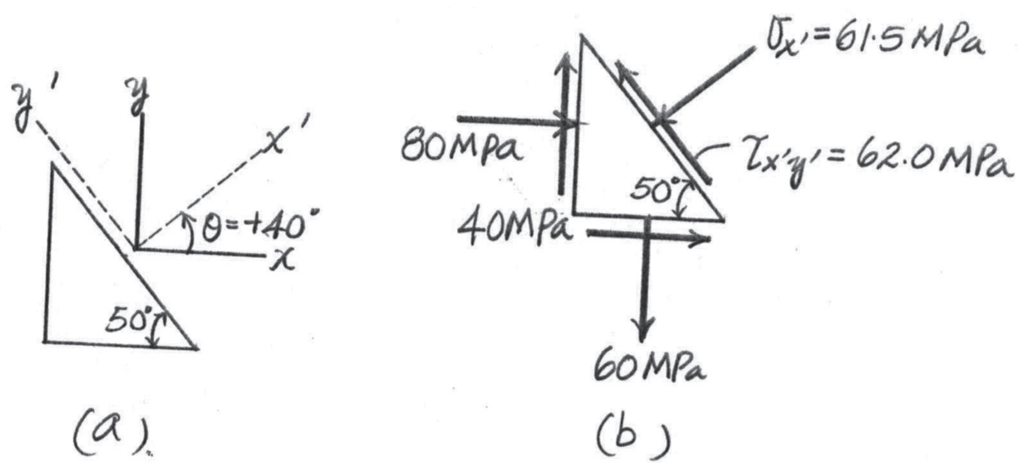
$$= -61.54 \text{ MPa} = -61.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{-80 - 60}{2}\right)\sin 80° + (-40)\cos 80°$$

$$= 61.99 \text{ MPa} = 62.0 \text{ MPa} \qquad \textbf{Ans.}$$

These results are indicated on the sectioned element shown in Fig. *b*



Ans:
$\sigma_{x'} = -61.5$ MPa,
$\tau_{x'y'} = 62.0$ MPa

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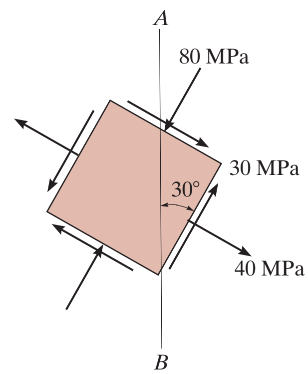
9–9.

Determine the stress components acting on the plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



SOLUTION

Force Equilibrium: The areas, thus the forces acting on the faces of the upper segment of the sectioned element are shown in Fig. *a*

$$+\downarrow \Sigma F_{y'} = 0; \quad \Delta F_{y'} + (80\,\Delta A \sin 30^\circ)\cos 30^\circ + (30\,\Delta A \sin 30^\circ)\sin 30^\circ$$
$$- (30\,\Delta A \cos 30^\circ)\cos 30^\circ + (40\,\Delta A \cos 30^\circ)\sin 30^\circ = 0$$
$$\Delta F_{y'} = -36.96\,\Delta A$$

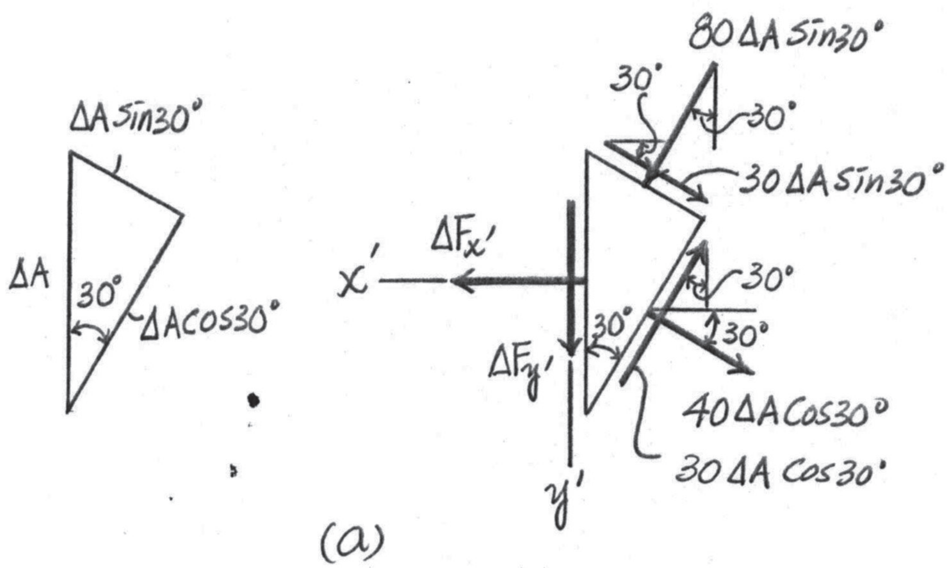
$$\overset{+}{\leftarrow} \Sigma F_{x'} = 0; \quad \Delta F_{x'} - (30\,\Delta A \sin 30^\circ)\cos 30^\circ + (80\,\Delta A \sin 30^\circ)\sin 30^\circ$$
$$- (30\,\Delta A \cos 30^\circ)\sin 30^\circ - (40\,\Delta A \cos 30^\circ)\cos 30^\circ = 0$$
$$\Delta F_{x'} = 35.98\,\Delta A$$

Normal And Shear Stress: For the inclined plane that has an area of ΔA,

$$\sigma_{x'} = \lim_{\Delta A \to 0} = \frac{\Delta F_{x'}}{\Delta A} = 35.98\text{ MPa} = 36.0\text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} = \frac{\Delta F_{y'}}{\Delta A} = -36.96\text{ MPa} = -37.0\text{ MPa} \qquad \textbf{Ans.}$$

The negative sign indicates that $\tau_{x'y'}$ acts in the sense opposite to that shown in the FBD.



Ans:
$\sigma_{x'} = 36.0$ MPa,
$\tau_{x'y'} = -37.0$ MPa

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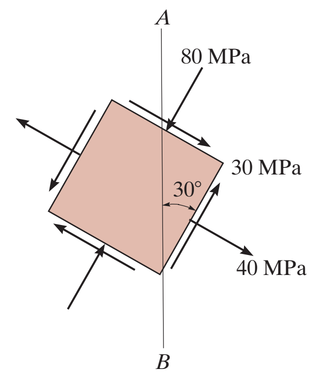
9–10.

Solve Prob. 9–9 using the stress transformation equation developed in Sec. 9.2.



SOLUTION

Normal And Shear Stress: In accordance with the established sign conventions,

$\theta = +120°$ (Fig. a) $\quad \sigma_x = -80$ MPa $\quad \sigma_y = 40$ MPa $\quad \tau_{xy} = -30$ MPa

Stress Transformation Equations:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-80 + 40}{2} + \left(\frac{-80 - 40}{2}\right)\cos 240° + (-30)\sin 240°$$

$$= 35.98 \text{ MPa} = 36.0 \text{ MPa}$$ **Ans.**

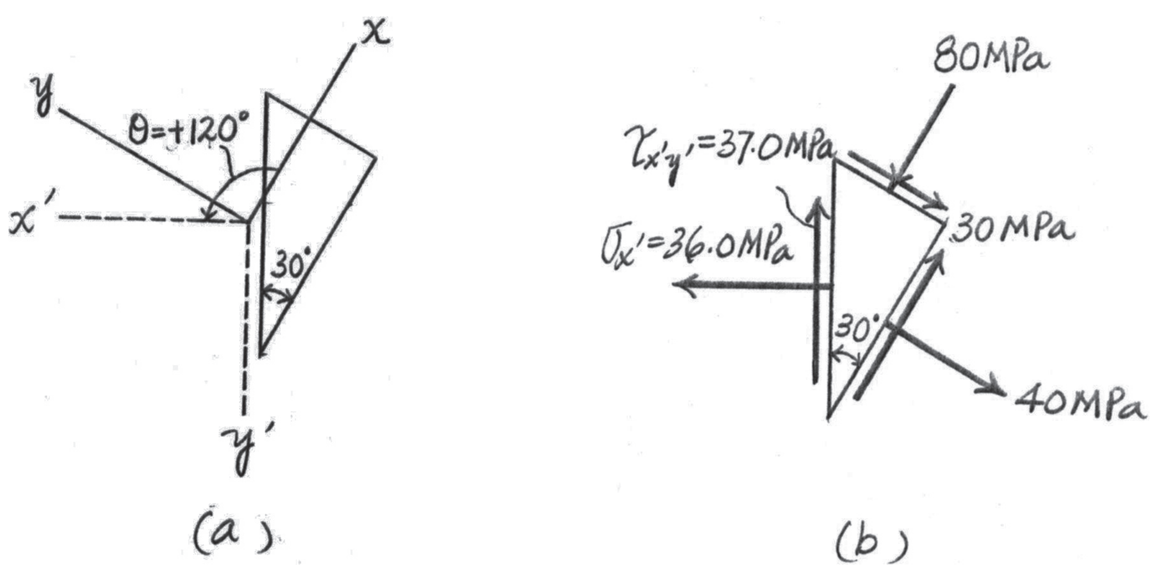
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{-80 - 40}{2}\right)\sin 240° + (-30)\cos 240°$$

$$= -36.96 \text{ MPa} = -37.0 \text{ MPa}$$ **Ans.**

The negative sign indicates that $\tau_{x'y'}$ acts in the negative y' direction.

These results are indicated on the sectioned element shown in Fig. b



Ans:
$\sigma_{x'} = 36.0$ MPa,
$\tau_{x'y'} = -37.0$ MPa

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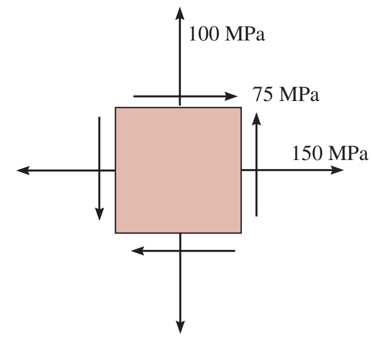
9–11.

Determine the equivalent state of stress on an element at the same point oriented 60° clockwise with respect to the element shown. Sketch the results on the element.



SOLUTION

Stress Transformation Equations:

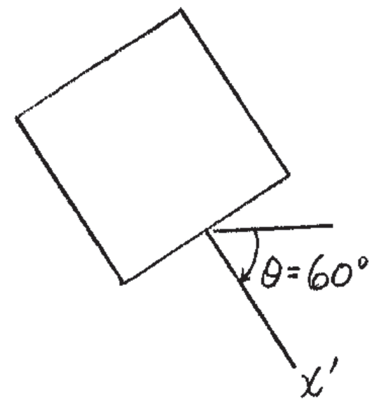
$\theta = -60°$ (Fig. *a*) $\quad \sigma_x = 150\text{ MPa} \quad \sigma_y = 100\text{ MPa} \quad \tau_{xy} = 75\text{ MPa}$

We obtain,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{150 + 100}{2} + \frac{150 - 100}{2}\cos(-120°) + 75\sin(-120°)$$

$$= 47.5\text{ MPa} \qquad \textbf{Ans.}$$



(a)

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{150 + 100}{2} - \frac{150 - 100}{2}\cos(-120°) - 75\sin(-120°)$$

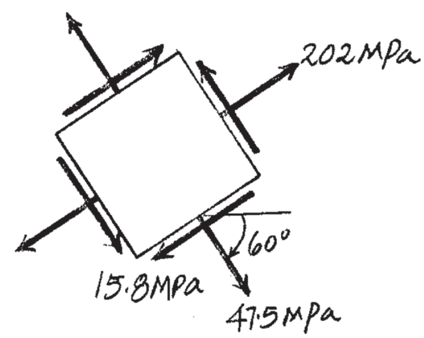
$$= 202\text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{150 - 100}{2}\sin(-120°) + 75\cos(-120°)$$

$$= -15.8\text{ MPa} \qquad \textbf{Ans.}$$

The negative sign indicates that $\tau_{x'y'}$ is directed towards the negative sense of the y' axis. These results are indicated on the element shown in Fig. *b*.



(b)

Ans:
$\sigma_{x'} = 47.5\text{ MPa}, \sigma_{y'} = 202\text{ MPa}$,
$\tau_{x'y'} = -15.8\text{ MPa}$

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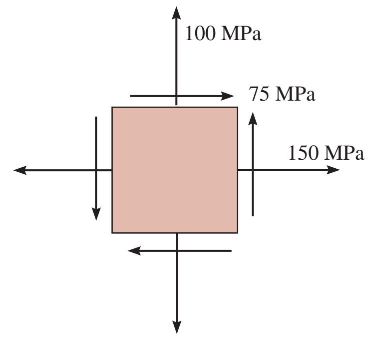
***9–12.**

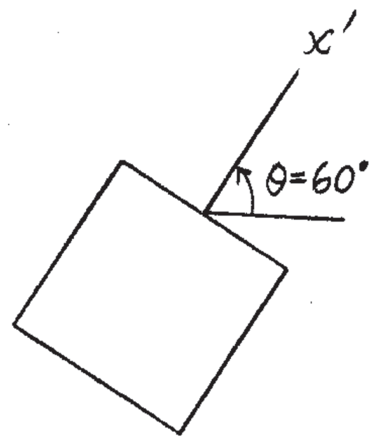
Determine the equivalent state of stress on an element at the same point oriented 60° counterclockwise with respect to the element shown. Sketch the results on the element.



SOLUTION

Stress Transformation Equations:

$\theta = +60°$ (Fig. *a*) $\quad \sigma_x = 150\text{ MPa} \quad \sigma_y = 100\text{ MPa} \quad \tau_{xy} = 75\text{ MPa}$



(a)

We obtain,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{150 + 100}{2} + \frac{150 - 100}{2}\cos 120° + 75\sin 120°$$

$$= 177\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{150 + 100}{2} - \frac{150 - 100}{2}\cos 120° - 75\sin 120°$$

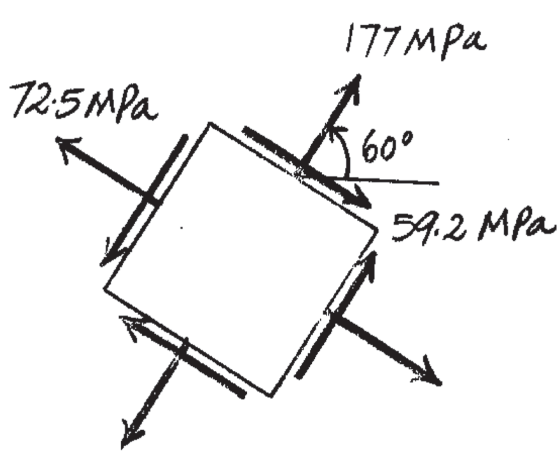
$$= 72.5\text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{150 - 100}{2}\sin 120° + 75\cos 120°$$

$$= -59.2\text{ MPa} \qquad \textbf{Ans.}$$

The negative sign indicates that $\tau_{x'y'}$ is directed towards the negative sense of the y' axis. These results are indicated on the element shown in Fig. *b*.



(b)

Ans:
$\sigma_{x'} = 177\text{ MPa}$, $\sigma_{y'} = 72.5\text{ MPa}$,
$\tau_{x'y'} = -59.2\text{ MPa}$

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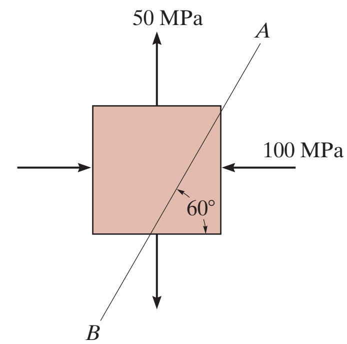
9–13.

Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



SOLUTION

Force Equilibrium: The areas, thus the forces acting on the faces of the lower segment of the sectioned element, are shown in Fig. *a*.

$+\swarrow\Sigma F_{y'} = 0; \qquad \Delta F_{y'} + (100\,\Delta A \sin 60°)\cos 60° + (50\,\Delta A \cos 60°)\sin 60° = 0$

$$\Delta F_{y'} = -64.95\,\Delta A$$

$\nwarrow + \Sigma F_{x'} = 0; \qquad \Delta F_{x'} + (100\,\Delta A \sin 60°)\sin 60° - (50\,\Delta A \cos 60°)\cos 60° = 0$

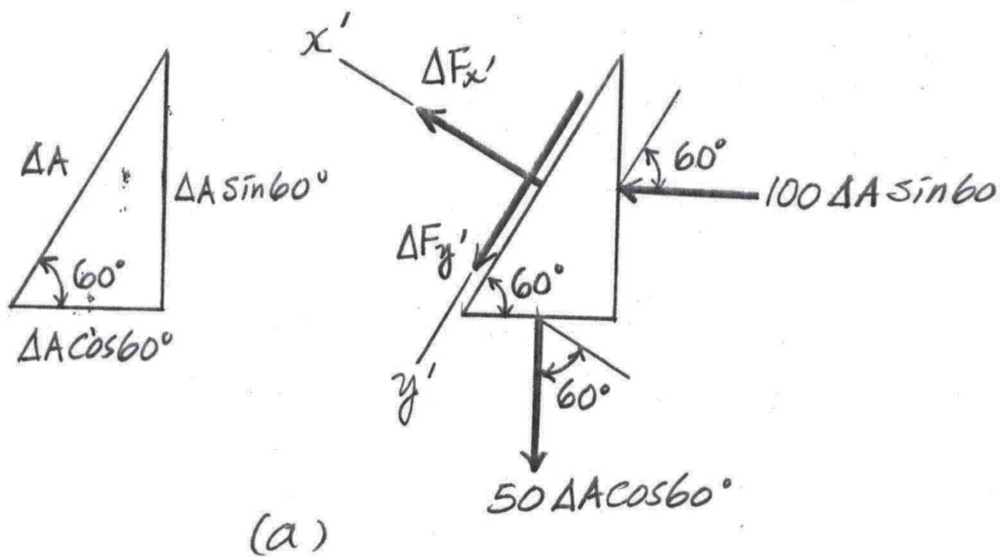
$$\Delta F_{x'} = -62.5\,\Delta A$$

Normal And Shear Stress: For the incline plane that has an area of ΔA,

$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -62.5\text{ MPa} = -62.5\text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = -64.95\text{ MPa} = -65.0\text{ MPa} \qquad \textbf{Ans.}$$

The negative signs indicate that $\sigma_{x'}$ is a compressive normal stress and $\tau_{x'y'}$ acts in the sense opposite to that shown in the FBD.



Ans:
$\sigma_{x'} = -62.5$ MPa,
$\tau_{x'y'} = -65.0$ MPa

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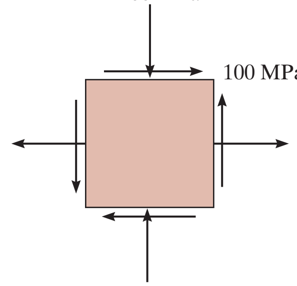
9–14.

Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.



SOLUTION

Normal And Shear Stress: In accordance with the established sign conventions,

$\sigma_x = 300$ MPa $\quad \sigma_y = -200$ MPa $\quad \tau_{xy} = 100$ MPa

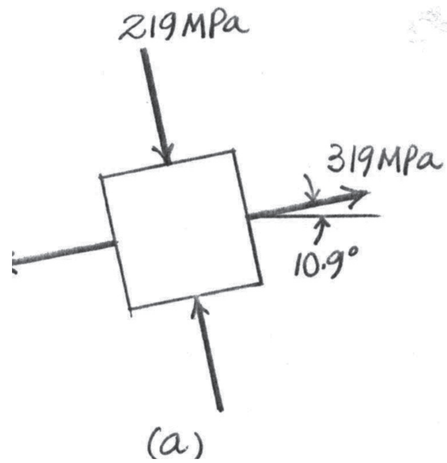
a) In-Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{300 + (-200)}{2} \pm \sqrt{\left[\frac{300 - (-200)}{2}\right]^2 + 100^2}$$

$$= 50 \pm 269.26$$

$\sigma_1 = 319.26$ MPa $= 319$ MPa $\qquad \sigma_2 = -219.26$ MPa $= -219$ MPa **Ans.**



Orientation of Principal Plane:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{100}{[300 - (-200)]/2} = 0.4$$

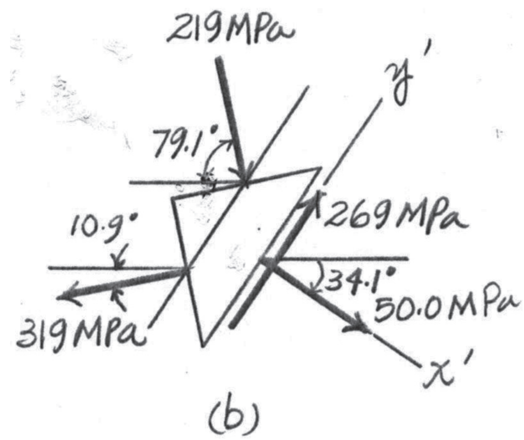
$$\theta_p = 10.90° \text{ and } -79.10°$$

Substitute the result of $\theta = 10.90°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{300 + (-200)}{2} + \frac{300 - (-200)}{2}\cos 21.80° + 100\sin 21.80°$$

$$= 319.26 \text{ MPa} = \sigma_1$$



Hence,

$\theta_{p1} = 10.90° = 10.9° \qquad \theta_{p2} = -79.10° = -79.1°$ **Ans.**

Using these results, the state of in-plane principal stress can be represented by the differential element shown in Fig. *a*.

b) Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \sqrt{\left[\frac{300 - (-200)}{2}\right]^2 + 100^2} = 269.26 \text{ MPa} = 269 \text{ MPa}$$ **Ans.**

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9–14. Continued

Orientation of the Plane for Maximum In-Plane Shear Stress:

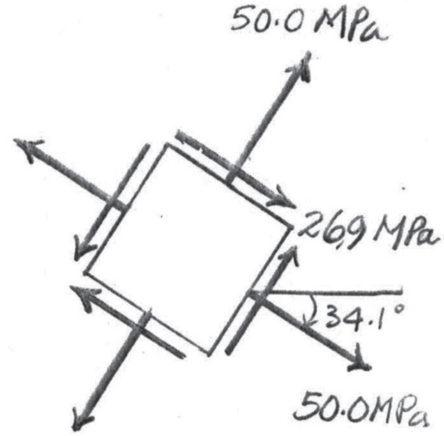
$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-[300 - (-200)]/2}{100} = -2.5$$

$\theta_s = -34.10° = -34.1°$ and $55.90° = 55.9°$ **Ans.**

Average Normal Stress:

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{300 + (-200)}{2} = 50.0 \text{ MPa}$$ **Ans.**

By observing the segment of the element of principal stresses sectioned through the diagonal, Fig. *b*, equilibrium along y' axis requires that $\tau_{\substack{max \\ in\text{-}plane}}$ to act in the direction shown. Thus, the state of maximum in-plane shear stress can be represented by the differential element shown in Fig. *c*.



(c)

Ans:
$\sigma_1 = 319$ MPa,
$\sigma_2 = -219$ MPa,
$\theta_{p1} = 10.9°$,
$\theta_{p2} = -79.1°$,
$\tau_{\substack{max \\ in\text{-}plane}} = 269$ MPa,
$\theta_s = -34.1°$ and $55.9°$,
$\sigma_{avg} = 50.0$ MPa

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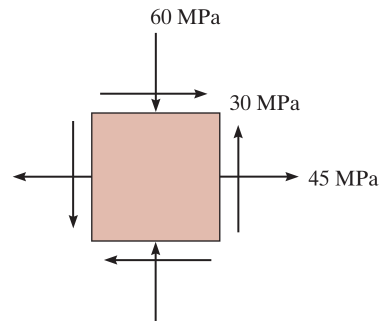
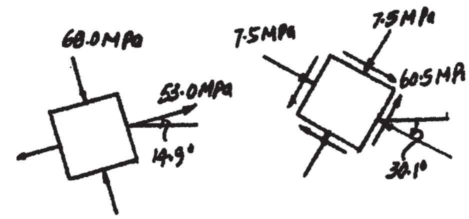
9–15.

The state of stress at a point is shown on the element. Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.



SOLUTION

$\sigma_x = 45\text{ MPa} \quad \sigma_y = -60\text{ MPa} \quad \tau_{xy} = 30\text{ MPa}$



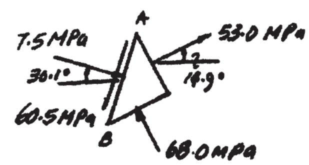
a) $$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\,2}}$$

$$= \frac{45 - 60}{2} \pm \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + (30)^2}$$

$$\sigma_1 = 53.0\text{ MPa} \qquad \sigma_2 = -68.0\text{ MPa}$$ **Ans.**

Orientation of principal stress:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{30}{(45 - (-60))/2} = 0.5714$$



$$\theta_p = 14.87, \quad -75.13$$

Use Eq. 9–1 to determine the principal plane of σ_1 and σ_2:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta, \qquad \text{where } \theta = 14.87°$$

$$= \frac{45 + (-60)}{2} + \frac{45 - (-60)}{2}\cos 29.74° + 30\sin 29.74° = 53.0\text{ MPa}$$

Therefore $\theta_{p1} = 14.9°$ and $\theta_{p2} = -75.1°$ **Ans.**

b)

$$\tau_{\substack{\max\\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\,2}} = \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + 30^2}$$

$$= 60.5\text{ MPa}$$ **Ans.**

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{45 + (-60)}{2} = -7.50\text{ MPa}$$ **Ans.**

Orientation of maximum in-plane shear stress:

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(45 - (-60))/2}{30} = -1.75$$

$$\theta_s = -30.1° \text{ and } \theta_s = 59.9°$$ **Ans.**

By observation, in order to preserve equilibrium along AB, τ_{max} has to act in the direction shown.

Ans:
$\sigma_1 = 53.0\text{ MPa}$,
$\sigma_2 = -68.0\text{ MPa}$,
$\theta_{p1} = 14.9°$,
$\theta_{p2} = -75.1°$,
$\tau_{\substack{\max\\ \text{in-plane}}} = 60.5\text{ MPa}$,
$\sigma_{avg} = -7.50\text{ MPa}$,
$\theta_s = -30.1°$, $\theta_s = 59.9°$

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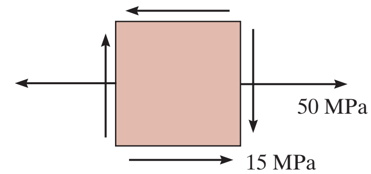
***9–16.**

Determine the equivalent state of stress on an element at the point which represents (a) the principal stresses and (b) the maximum in-plane shear stress and the associated average normal stress. Also, for each case, determine the corresponding orientation of the element with respect to the element shown and sketch the results on the element.



SOLUTION

Normal and Shear Stress:

$$\sigma_x = 50 \text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -15 \text{ MPa}$$

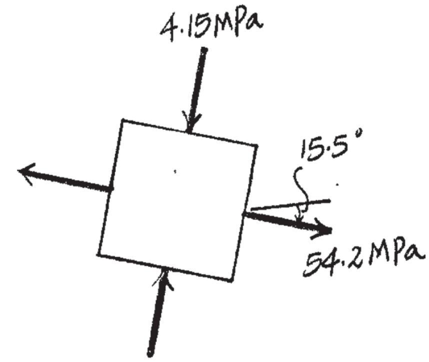
In-Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{50 + 0}{2} \pm \sqrt{\left(\frac{50 - 0}{2}\right)^2 + (-15)^2}$$

$$= 25 \pm \sqrt{850}$$

$$\sigma_1 = 54.2 \text{ MPa} \qquad \sigma_2 = -4.15 \text{ MPa} \qquad \textbf{Ans.}$$



(a)

Orientation of Principal Plane:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-15}{(50 - 0)/2} = -0.6$$

$$\theta_p = -15.48° \text{ and } 74.52°$$

Substitute $\theta = -15.48°$ into

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

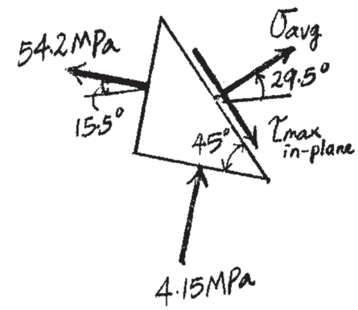
$$= \frac{50 + 0}{2} + \frac{50 - 0}{2} \cos(-30.96°) + (-15) \sin(-30.96°)$$

$$= 54.2 \text{ MPa} = \sigma_1$$

Thus,

$$(\theta_p)_1 = -15.5° \text{ and } (\theta_p)_2 = 74.5° \qquad \textbf{Ans.}$$

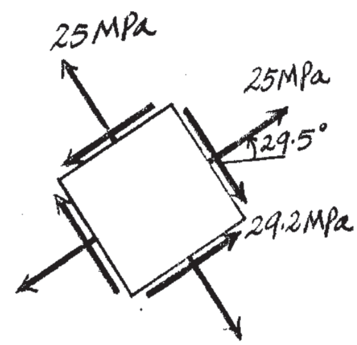
The element that represents the state of principal stress is shown in Fig. *a*.



(b)

Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{50 - 0}{2}\right)^2 + (-15)^2} = 29.2 \text{ MPa} \qquad \textbf{Ans.}$$



(c)

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***9–16. Continued**

Orientation of the Plane of Maximum In-Plane Shear Stress:

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(50 - 0)/2}{-15} = 1.667$$

$$\theta_s = 29.5° \text{ and } 120° \qquad \textbf{Ans.}$$

By inspection, $\tau_{\substack{\max \\ \text{in-plane}}}$ has to act in the same sense shown in Fig. *b* to maintain equilibrium.

Average Normal Stress:

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{50 + 0}{2} = 25 \text{ MPa} \qquad \textbf{Ans.}$$

The element that represents the state of maximum in-plane shear stress is shown in Fig. *c*.

Ans:
$\sigma_1 = 54.2$ MPa,
$\sigma_2 = -4.15$ MPa,
$(\theta_p)_1 = -15.5°$,
$(\theta_p)_2 = 74.5°$,
$\tau_{\substack{\max \\ \text{in-plane}}} = 29.2$ MPa,
$\theta_s = 29.5°$ and $120°$,
$\sigma_{avg} = 25$ MPa

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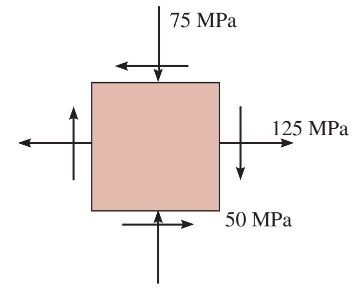
9–17.

Determine the equivalent state of stress on an element at the same point which represents (a) the principal stress, and (b) the maximum in-plane shear stress and the associated average normal stress. Also, for each case, determine the corresponding orientation of the element with respect to the element shown and sketch the results on the element.



SOLUTION

Normal and Shear Stress:

$\sigma_x = 125$ MPa $\qquad \sigma_y = -75$ MPa $\qquad \tau_{xy} = -50$ MPa

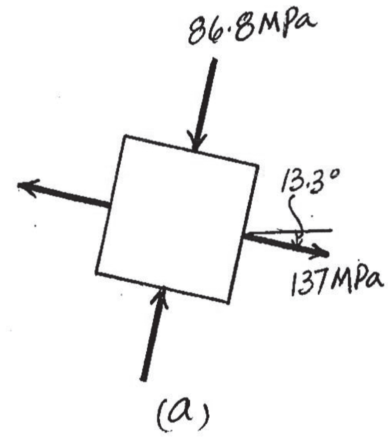
In - Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x - \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{125 + (-75)}{2} \pm \sqrt{\left(\frac{125 - (-75)}{2}\right)^2 + (-50)^2}$$

$$= 25 \pm \sqrt{12500}$$

$\sigma_1 = 137$ MPa $\qquad \sigma_2 = -86.8$ MPa $\qquad$ **Ans.**



Orientation of Principal Plane:

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-50}{(125-(-75))/2} = -0.5$$

$$\theta_p = -13.28° \text{ and } 76.72°$$

Substitute $\theta = -13.28°$ into

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

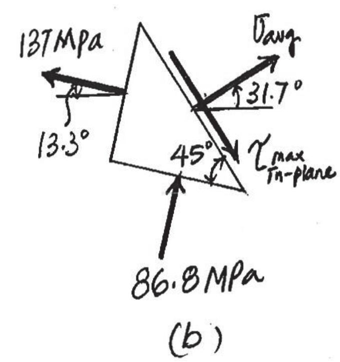
$$= \frac{125 + (-75)}{2} + \frac{125 - (-75)}{2}\cos(-26.57°) + (-50)\sin(-26.57°)$$

$$= 137 \text{ MPa} = \sigma_1$$

Thus,

$$(\theta_p)_1 = -13.3° \text{ and } (\theta_p)_2 = 76.7° \qquad \textbf{Ans.}$$

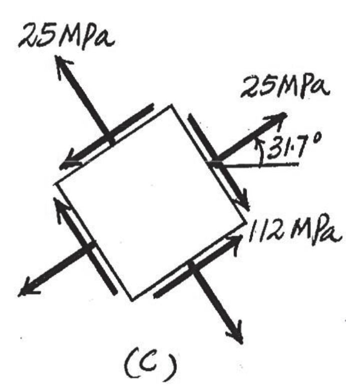
$$125 - (-75)/(-50)$$



The element that represents the state of principal stress is shown in Fig. *a*.

Maximum In - Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{125 - (-75)}{2}\right)^2 + 50^2} = 112 \text{ MPa} \qquad \textbf{Ans.}$$



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9–17. Continued

Orientation of the Plane of Maximum In - Plane Shear Stress:

$$\tan 2\theta_s = -\frac{\left(\sigma_x - \sigma_y\right)/2}{\tau_{xy}} = -\frac{(125 - (-75))/2}{-50} = 2$$

$$\theta_s = 31.7° \text{ and } 122° \qquad \textbf{Ans.}$$

By inspection, $\tau_{\substack{\text{max}\\\text{in-plane}}}$ has to act in the same sense shown in Fig. *b* to maintain equilibrium.

Average Normal Stress:

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{125 + (-75)}{2} = 25 \text{ MPa} \qquad \textbf{Ans.}$$

The element that represents the state of maximum in-plane shear stress is shown in Fig. *c*.

Ans:
$\sigma_1 = 137$ MPa, $\sigma_2 = -86.8$ MPa,
$\theta_{p1} = -13.3°$, $\theta_{p2} = 76.7°$, $\tau_{\substack{\text{max}\\\text{in-plane}}} = 112$ MPa,
$\theta_s = 31.7°$ and $122°$, $\sigma_{\text{avg}} = 25$ MPa

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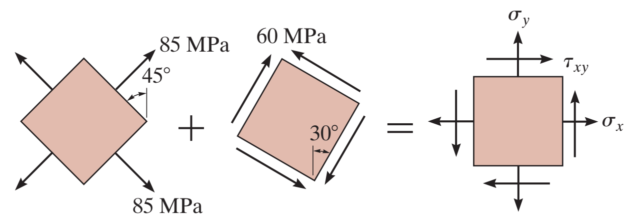
9–18.

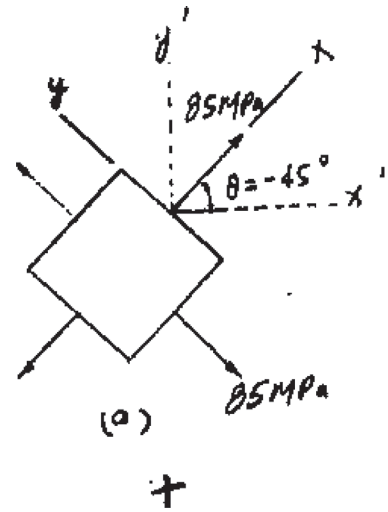
A point on a thin plate is subjected to the two stress components. Determine the resultant state of stress represented on the element oriented as shown on the right.



SOLUTION

For element *a*:

$\sigma_x = \sigma_y = 85 \text{ MPa} \qquad \tau_{xy} = 0 \qquad \theta = -45°$



$$(\sigma_{x'})_a = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{85 + 85}{2} + \frac{85 - 85}{2}\cos(-90°) + 0 = 85 \text{ MPa}$$

$$(\sigma_{y'})_a = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

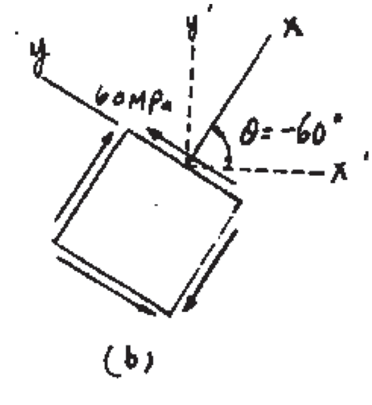
$$= \frac{85 + 85}{2} - \frac{85 - 85}{2}\cos(-90°) - 0 = 85 \text{ MPa}$$

$$(\tau_{x'y'})_a = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{85 - 85}{2}\sin(-90°) + 0 = 0$$

For element *b*:

$\sigma_x = \sigma_y = 0 \qquad \tau_{xy} = 60 \text{ MPa} \qquad \theta = -60°$



$$(\sigma_{x'})_b = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

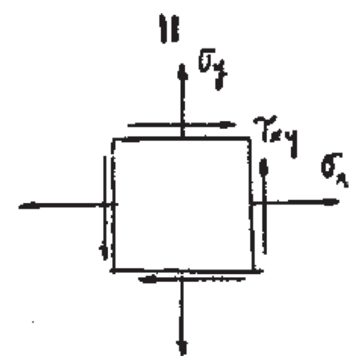
$$= 0 + 0 + 60\sin(-120°) = -51.96 \text{ MPa}$$

$$(\sigma_{y'})_b = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= 0 - 0 - 60\sin(-120°) = 51.96 \text{ MPa}$$

$$(\tau_{x'y'})_b = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$= -\frac{85 - 85}{2}\sin(-120°) + 60\cos(-120°) = -30 \text{ MPa}$$



$\sigma_x = (\sigma_{x'})_a + (\sigma_{x'})_b = 85 + (-51.96) = 33.0 \text{ MPa}$ **Ans.**

$\sigma_y = (\sigma_{y'})_a + (\sigma_{y'})_b = 85 + 51.96 = 137 \text{ MPa}$ **Ans.**

$\tau_{xy} = (\tau_{x'y'})_a + (\tau_{x'y'})_b = 0 + (-30) = -30 \text{ MPa}$ **Ans.**

Ans:
$\sigma_x = 33.0 \text{ MPa}, \sigma_y = 137 \text{ MPa}, \tau_{xy} = -30 \text{ MPa}$

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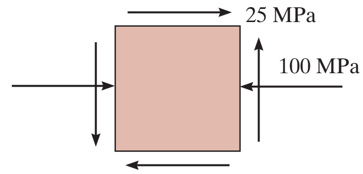
9–19.

Determine the equivalent state of stress on an element at the same point which represents (a) the principal stress, and (b) the maximum in-plane shear stress and the associated average normal stress. Also, for each case, determine the corresponding orientation of the element with respect to the element shown and sketch the results on the element.



SOLUTION

Normal and Shear Stress:

$$\sigma_x = -100 \text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = 25 \text{ MPa}$$

In-Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-100 + 0}{2} \pm \sqrt{\left(\frac{-100 - 0}{2}\right)^2 + 25^2}$$

$$= -50 \pm \sqrt{3125}$$

$$\sigma_1 = 5.90 \text{ MPa} \qquad \sigma_2 = -106 \text{ MPa} \qquad \textbf{Ans.}$$

Orientation of Principal Plane:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{25}{(-100 - 0)/2} = -0.5$$

$$\theta_p = -13.28° \text{ and } 76.72°$$

Substitute $\theta = -13.28°$ into

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-100 + 0}{2} + \frac{-100 - 0}{2}\cos(-26.57°) + 25\sin(-26.57°)$$

$$= -106 \text{ MPa} = \sigma_2$$

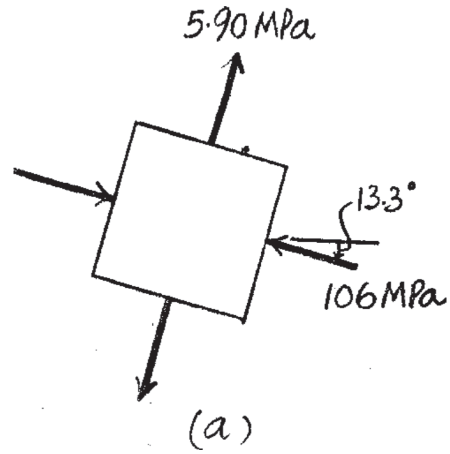
Thus,

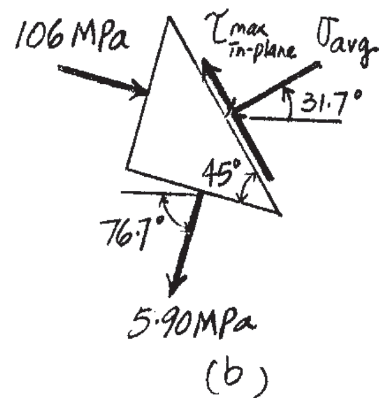
$$(\theta_p)_1 = 76.7° \text{ and } (\theta_p)_2 = -13.3° \qquad \textbf{Ans.}$$

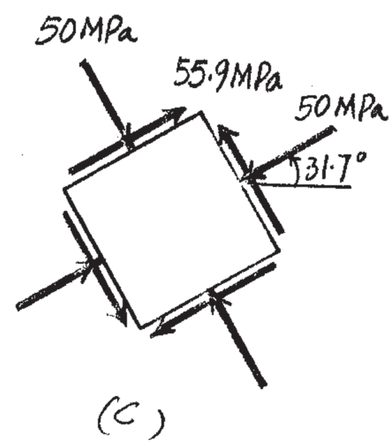
The element that represents the state of principal stress is shown in Fig. *a*.

Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max\\\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-100 - 0}{2}\right)^2 + 25^2} = 55.9 \text{ MPa} \qquad \textbf{Ans.}$$







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9–19. Continued

Orientation of the Plane of Maximum In-Plane Shear Stress:

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = -\frac{(-100 - 0)/2}{25} = 2$$

$$\theta_s = 31.7^\circ \text{ and } 122^\circ \qquad \textbf{Ans.}$$

By inspection, $\tau_{\substack{\max \\ \text{in-plane}}}$ has to act in the same sense shown in Fig. *b* to maintain equilibrium.

Average Normal Stress:

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-100 + 0}{2} = -50 \text{ MPa} \qquad \textbf{Ans.}$$

The element that represents the state of maximum in-plane shear stress is shown in Fig. *c*.

Ans:
$\sigma_1 = 5.90$ MPa, $\sigma_2 = -106$ MPa,
$\theta_{p1} = 76.7^\circ$ and $\theta_{p2} = -13.3^\circ$,
$\tau_{\substack{\max \\ \text{in-plane}}} = 55.9$ MPa, $\sigma_{\text{avg}} = -50$ MPa,
$\theta_s = 31.7^\circ$ and 122°

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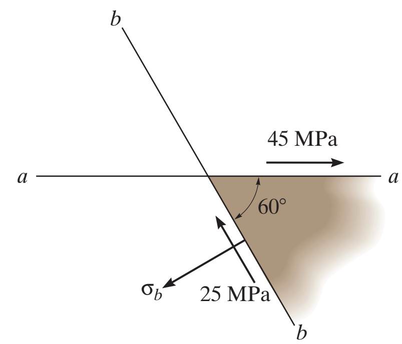
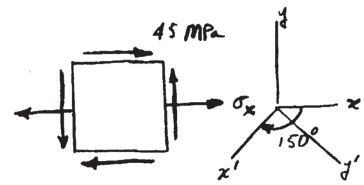
***9–20.**

The stress along two planes at a point is indicated. Determine the normal stresses on plane *b–b* and the principal stresses.



SOLUTION



$$\tau_b = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$-25 = -\frac{(\sigma_x - 0)}{2}\sin(-300^\circ) + 45\cos(-300^\circ)$$

$$\sigma_x = 109.70 \text{ MPa}$$

$$\sigma_b = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{109.70 + 0}{2} + \frac{109.70 - 0}{2}\cos(-300^\circ) + 45\sin(-300^\circ)$$

$$\sigma_b = 121 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{109.70 + 0}{2} \pm \sqrt{\left(\frac{109.70 - 0}{2}\right)^2 + (45)^2}$$

$$\sigma_1 = 126 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = -16.1 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma_b = 121$ MPa,
$\sigma_1 = 126$ MPa,
$\sigma_2 = -16.1$ MPa

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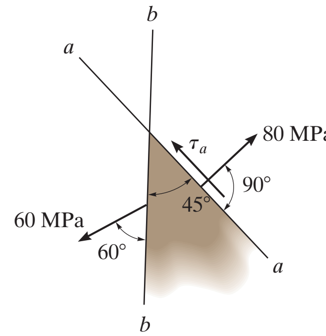
9–21. The stress acting on two planes at a point is indicated. Determine the shear stress on plane *a–a* and the principal stresses at the point.



SOLUTION

$\sigma_x = 60 \sin 60° = 51.962 \text{ MPa}$

$\tau_{xy} = 60 \cos 60° = 30 \text{ MPa}$

$$\sigma_a = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$80 = \frac{51.962 + \sigma_y}{2} + \frac{51.962 - \sigma_y}{2} \cos (90°) + 30 \sin (90°)$$

$\sigma_y = 48.038 \text{ MPa}$

$$\tau_a = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos \theta$$

$$= -\left(\frac{51.962 - 48.038}{2}\right) \sin (90°) + 30 \cos (90°)$$

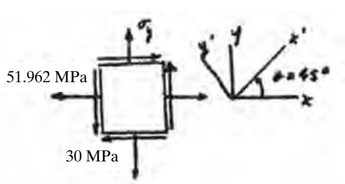
$\tau_a = -1.96 \text{ MPa}$ **Ans.**

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{51.962 + 48.038}{2} \pm \sqrt{\left(\frac{51.962 - 48.038}{2}\right)^2 + (30)^2}$$

$\sigma_1 = 80.1 \text{ MPa}$ **Ans.**

$\sigma_2 = 19.9 \text{ MPa}$ **Ans.**



Ans.
$\sigma_x = 61.962 \text{ MPa}, \tau_{xy} = 30 \text{ MPa}$
$\sigma_1 = 80.1 \text{ MPa}, \sigma_2 = 19.9 \text{ MPa}$

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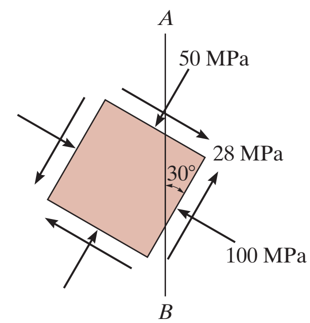
9–22.

The state of stress at a point in a member is shown on the element. Determine the stress components acting on the plane AB.



SOLUTION

Construction of the Circle: In accordance with the sign convention, $\sigma_x = -50$ MPa, $\sigma_y = -100$ MPa, and $\tau_{xy} = -28$ MPa. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-50 + (-100)}{2} = -75.0 \text{ MPa}$$

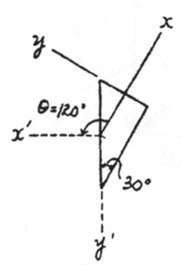
The coordinates for reference points A and C are $A(-50, -28)$ and $C(-75.0, 0)$.

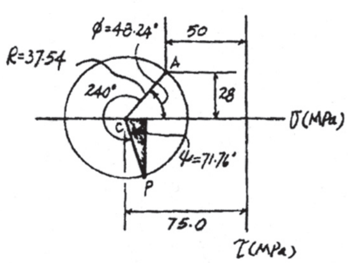
The radius of the circle is $R = \sqrt{(75.0 - 50)^2 + 28^2} = 37.54$ MPa.

Stress on the Rotated Element: The normal and shear stress components $(\sigma_{x'}$ and $\tau_{x'y'})$ are represented by the coordinates of point P on the circle

$$\sigma_{x'} = -75.0 + 37.54 \cos 71.76° = -63.3 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = 37.54 \sin 71.76° = 35.7 \text{ MPa} \qquad \textbf{Ans.}$$





Ans:
$\sigma_{x'} = -63.3$ MPa, $\tau_{x'y'} = 35.7$ MPa

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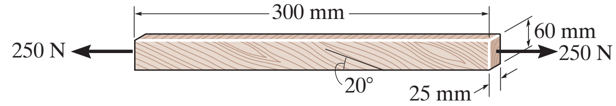
9–23.

The grains of wood in the board make an angle of 20° with the horizontal as shown. Determine the normal and shear stress that act perpendicular and parallel to the grains if the board is subjected to an axial load of 250 N.



SOLUTION

$$\sigma_x = \frac{P}{A} = \frac{250}{(0.06)(0.025)} = 166.67\ \text{kPa}$$

$\sigma_y = 0 \qquad \tau_{xy} = 0$

$\theta = 70°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{166.67 + 0}{2} + \frac{166.67 - 0}{2}\cos 140° + 0 = 19.5\ \text{kPa}$$ **Ans.**

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{166.67 - 0}{2}\right)\sin 140° + 0 = -53.6\ \text{kPa}$$ **Ans.**

Ans:
$\sigma_{x'} = 19.5\ \text{kPa}, \tau_{x'y'} = -53.6\ \text{kPa}$

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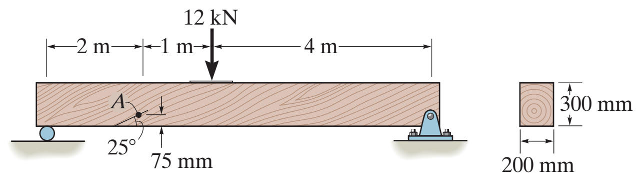
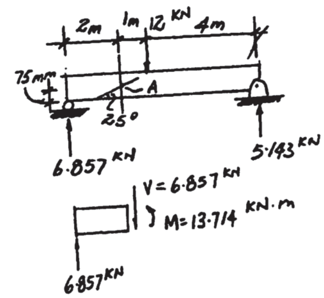
***9–24.**

The wood beam is subjected to a load of 12 kN. If grains of wood in the beam at point A make an angle of 25° with the horizontal as shown, determine the normal and shear stress that act perpendicular to the grains due to the loading.



SOLUTION

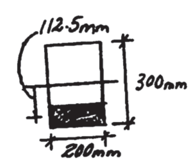


$$I = \frac{1}{12}(0.2)(0.3)^3 = 0.45\left(10^{-3}\right)\text{ m}^4$$

$$Q_A = \bar{y}A' = 0.1125(0.2)(0.075) = 1.6875\left(10^{-3}\right)\text{ m}^3$$

$$\sigma_A = \frac{My_A}{I} = \frac{13.714\left(10^3\right)(0.075)}{0.45\left(10^{-3}\right)} = 2.2857\text{ MPa (T)}$$

$$\tau_A = \frac{VQ_A}{It} = \frac{6.875\left(10^3\right)(1.6875)\left(10^{-3}\right)}{0.45\left(10^{-3}\right)(0.2)} = 0.1286\text{ MPa}$$



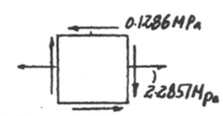
$$\sigma_x = 2.2857\text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -0.1286\text{ MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{2.2857 + 0}{2} \pm \sqrt{\left(\frac{2.2857 - 0}{2}\right)^2 + (-0.1286)^2}$$

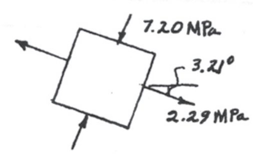
$\sigma_1 = 2.29\text{ MPa}$ **Ans.**

$\sigma_2 = -7.20\text{ kPa}$ **Ans.**



$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-0.1286}{(2.2857 - 0)/2}$$

$\theta_p = -3.21°$ **Ans.**



Check direction of principal stress:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{2.2857 + 0}{2} + \frac{2.2857 - 0}{2}\cos(-6.42°) - 0.1285\sin(-6.42)$$

$$= 2.29\text{ MPa}$$

Ans:
$\sigma_1 = 2.29\text{ MPa}$,
$\sigma_2 = -7.20\text{ kPa}$,
$\theta_p = -3.21°$

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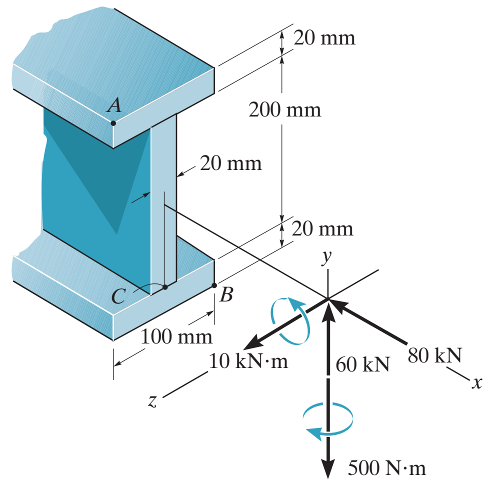
9–25.

The internal loadings at a section of the beam are shown. Determine the in-plane principal stresses at point A. Also compute the maximum in-plane shear stress at this point.



SOLUTION

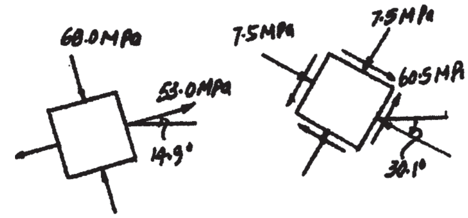
Section Properties: For the wide flange section, Fig. *a*,

$$A = 0.1(0.24) - 0.08(0.2) = 0.008 \text{ m}^2$$

$$I_z = \frac{1}{12}(0.1)(0.24^3) - \frac{1}{12}(0.08)(0.2^3) = 61.8667(10^{-6}) \text{ m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.02)(0.1^3)\right] + \frac{1}{12}(0.2)(0.02^3) = 3.4667(10^{-6}) \text{ m}^4$$

$$(Q_A)_y = 0$$



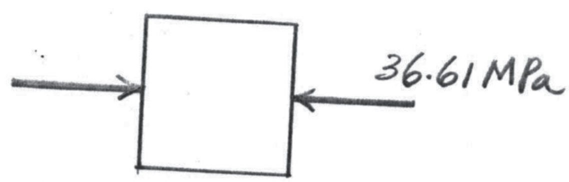
Internal Loadings: Here, $N_x = -80$ kN, $V_y = 60$ kN, $M_y = -0.5$ kN·m and $M_z = 10$ kN·m.

Normal Stress: For the combine loadings,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = \frac{-80(10^3)}{0.008} - \frac{10(10^3)(0.12)}{61.8667(10^{-6})} + \frac{\left[-0.5(10^3)\right](0.05)}{3.4667(10^{-6})}$$

$$= -36.6080(10^6) \text{ Pa} = 36.61 \text{ MPa (C)}$$



(b)

Shear Stress: Since $(Q_A)_y = 0$,

$$\tau_A = 0$$

Thus, the state of stress at point A can be represented by the differential element shown in Fig. *b*

In-Plane Principal Stresses: In accordance to the sign convention $\sigma_x = -36.61$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 0$ for point A, Fig. *a*. Since no shear stress is acting on the element,

$$\sigma_1 = \sigma_y = 0 \qquad \textbf{Ans.}$$

$$\sigma_2 = \sigma_x = -36.61 \text{ MPa} = -36.6 \text{ MPa} \qquad \textbf{Ans.}$$

Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \sqrt{\left(\frac{-36.61 - 0}{2}\right)^2 + 0}$$

$$= 18.30 \text{ MPa} = 18.3 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma_1 = 0$,
$\sigma_2 = -36.6$ MPa,
$\tau_{\substack{\max \\ \text{in-plane}}} = 18.3$ MPa

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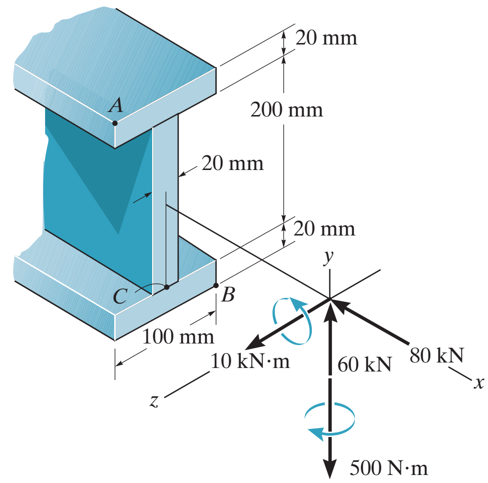
9–26.

Solve Prob. 9–25 for point *B*.



SOLUTION

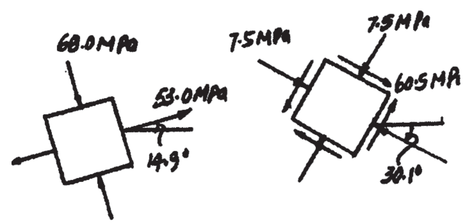
Section Properties: For the wide flange section, Fig. *a*,

$$A = 0.1(0.24) - 0.08(0.2) = 0.008 \text{ m}^2$$

$$I_z = \frac{1}{12}(0.1)(0.24^3) - \frac{1}{12}(0.08)(0.2^3) = 61.8667(10^{-6}) \text{ m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.02)(0.1^3)\right] + \frac{1}{12}(0.2)(0.02^3) = 3.4667(10^{-6}) \text{ m}^4$$

$$(Q_B)_y = 0$$



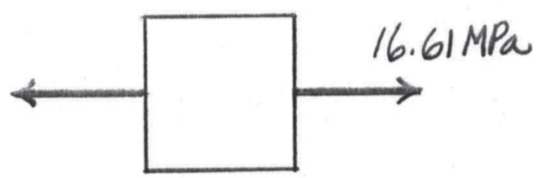
Internal Loadings: Here, $N_x = -80$ kN, $V_y = 60$ kN, $M_y = -0.5$ kN·m and $M_z = 10$ kN·m

Normal Stress: For the combine loadings,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_B = \frac{-80(10^3)}{0.008} - \frac{10(10^3)(-0.12)}{61.8667(10^{-6})} + \frac{[-0.5(10^3)](-0.05)}{3.4667(10^{-6})}$$

$$= 16.6081(10^6) \text{ Pa} = 16.61 \text{ MPa (T)}$$



(b)

Shear Stress: Since $(Q_B)_y = 0$, $\tau_B = 0$.

Thus, the state of stress at point *B* can be represented by the differential element shown in Fig. *b*

In-Plane Principal Stresses: In accordance to the sign convention $\sigma_x = 16.61$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 0$ for point *B*, Fig. *a*. Since no shear stress acting on the element,

$$\sigma_1 = \sigma_x = 16.61 \text{ MPa} = 16.6 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = \sigma_y = 0 \qquad \textbf{Ans.}$$

Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\,2}}$$

$$= \sqrt{\left(\frac{16.61 - 0}{2}\right)^2 + 0}$$

$$= 8.304 \text{ MPa} = 8.30 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma_1 = 16.6$ MPa,
$\sigma_2 = 0$,
$\tau_{\substack{\max \\ \text{in-plane}}} = 8.30$ MPa

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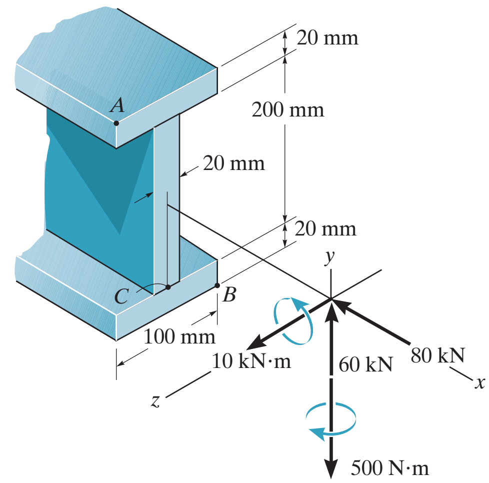
9–27.

Solve Prob. 9–25 for point C.



SOLUTION

Section Properties: For the wide-flange section, Fig. *a*

$$A = 0.1(0.24) - 0.08(0.2) = 0.008 \text{ m}^2$$

$$I_z = \frac{1}{12}(0.1)\left(0.24^3\right) - \frac{1}{12}(0.08)\left(0.2^3\right) = 61.8667\left(10^{-6}\right) \text{ m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.02)\left(0.1^3\right)\right] + \frac{1}{12}(0.2)\left(0.02^3\right) = 3.4667\left(10^{-6}\right) \text{ m}^4$$

For the area shown shaded in Fig. *a*,

$$(Q_C)_y = \bar{y}'A' = 0.11[0.1(0.02)] = 0.22\left(10^{-3}\right) \text{ m}^3$$

Internal Loadings: Here, $N_x = -80 \text{ kN}$, $V_y = 60 \text{ kN}$, $M_y = -0.5 \text{ kN} \cdot \text{m}$ and $M_z = 10 \text{ kN} \cdot \text{m}$.

Normal Stress: For the combine loadings,

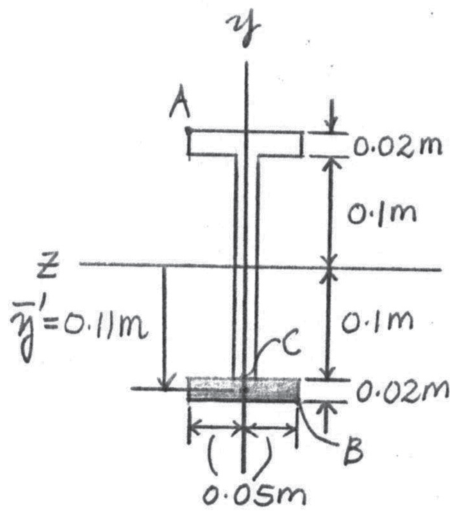
$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_C = \frac{-80\left(10^3\right)}{0.008} - \frac{10\left(10^3\right)(-0.1)}{61.8667\left(10^{-6}\right)} + \frac{\left[-0.5\left(10^3\right)\right](0)}{3.4667\left(10^{-6}\right)}$$

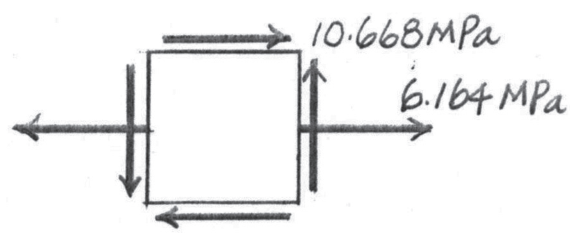
$$= 6.164\left(10^6\right) \text{ Pa} = 6.164 \text{ MPa (T)}$$

Shear Stress: Applying the shear formula,

$$\tau_c = \frac{V_y (Q_C)_y}{I_z t} = \frac{60\left(10^3\right)\left[0.22\left(10^{-3}\right)\right]}{61.8667\left(10^{-6}\right)(0.02)} = 10.668\left(10^6\right) \text{ Pa} = 10.668 \text{ MPa}$$



(a)



(b)

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9–27. Continued

Thus, the state of stress at point C can be represented by the differential element shown in Fig. *b*.

In-Plane Principal Stress: In accordance to the sign convention, $\sigma_x = 6.164$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 10.668$ MPa for point C, Fig. *a*.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{6.164 + 0}{2} \pm \sqrt{\left(\frac{6.164 - 0}{2}\right)^2 + 10.668^2}$$

$$= 3.082 \pm 11.104$$

$\sigma_1 = 14.19 \text{ MPa} = 14.2 \text{ MPa} \qquad \sigma_2 = -8.022 \text{ MPa} = -8.02 \text{ MPa}$ **Ans.**

Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \sqrt{\left(\frac{6.164 - 0}{2}\right)^2 + 10.668^2}$$

$= 11.104 \text{ MPa} = 11.1 \text{ MPa}$ **Ans.**

Ans:
$\sigma_1 = 14.2$ MPa,
$\sigma_2 = -8.02$ MPa,
$\tau_{\substack{\max \\ \text{in-plane}}} = 11.1$ MPa

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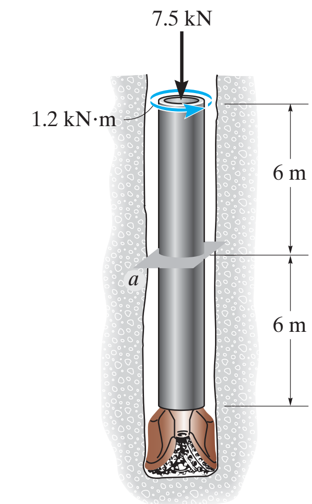
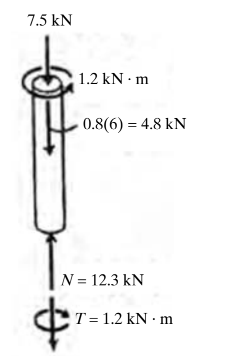
***9–28.** The drill pipe has an outer diameter of 75 mm, a wall thickness of 6 mm, and a weight of 0.8 kN/m. If it is subjected to a torque and axial load as shown, determine (a) the principal stress and (b) the maximum in-plane shear stress at a point on its surface at section *a*.



SOLUTION

Internal Forces and Torque: As shown on FBD(a).



Section Properties:

$$A = \frac{\pi}{4}(0.075^2 - 0.063^2) = 0.414(10^{-3})\pi \text{ m}^2$$

$$J = \frac{\pi}{2}(0.0375^4 - 0.0315^4) = 1.55977(10^{-6}) \text{ m}^4$$

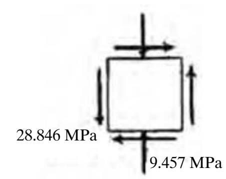
Normal Stress:

$$\sigma = \frac{N}{A} = \frac{-12.3(10^3)}{0.414(10^{-3})\pi} - 9.457(10^6) \text{ N/m}^2 = -9.457 \text{ MPa}$$

Shear Stress: Applying the torsion formula.

$$\tau = \frac{T\,c}{J} = \frac{[1.2(10^3)](0.0375)}{1.55977(10^{-6})} = 28.85(10^6) \text{ N/m}^2 = 28.85 \text{ MPa}$$

a) ***In - Plane Principal Stresses:*** $\sigma_x = 0, \sigma_y = -9.457$ MPa and $\tau_{xy} = 28.846$ MPa for any point on the shaft's surface. Applying Eq. 9-5.



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 + (-9.457)}{2} \pm \sqrt{\left[\frac{0 - (-9.457)}{2}\right]^2 + (28.85^2)}$$

$$= -4.7285 \pm 29.2354$$

$\sigma_1 = 24.5$ MPa **Ans.**

$\sigma_2 = -34.0$ MPa **Ans.**

b) ***Maximum In - Plane Shear Stress:*** Applying Eq. 9-7

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left[\frac{0 - (-9.457)}{2}\right]^2 + (28.85^2)}$$

$$= 29.2$$ **Ans.**

Ans.
$\sigma_1 = 24.5$ MPa, $\sigma_2 = -34.0$ MPa,
$\tau_{\substack{\max \\ \text{in-plane}}} = 29.2$ MPa

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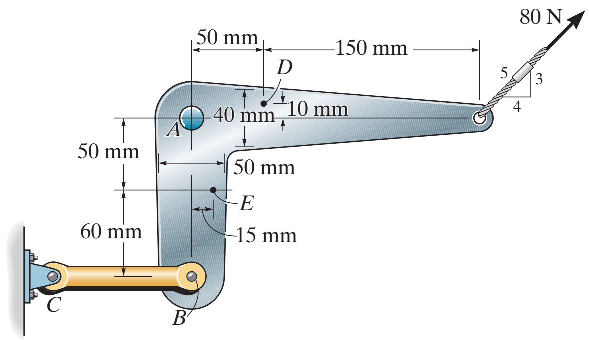
9–29.

The bell crank is pinned at A and supported by a short link BC. If it is subjected to the force of 80 N, determine the principal stresses at (a) point D and (b) point E. The crank is constructed from an aluminum plate having a thickness of 20 mm.



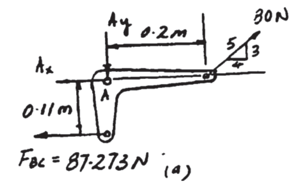
SOLUTION

Point D:

$$A = 0.04(0.02) = 0.8\left(10^{-3}\right)\text{ m}^2$$

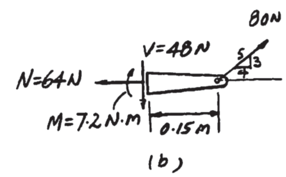
$$I = \frac{1}{12}(0.02)\left(0.04^3\right) = 0.1067\left(10^{-6}\right)\text{ m}^4$$

$$Q_D = \bar{y}'A' = 0.015(0.02)(0.01) = 3\left(10^{-6}\right)\text{ m}^3$$



Normal stress:

$$\sigma_D = \frac{P}{A} + \frac{My}{I} = \frac{64}{0.8\left(10^{-3}\right)} - \frac{7.2(0.01)}{0.1067\left(10^{-6}\right)} = -0.595\text{ MPa}$$



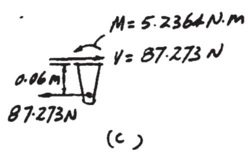
Shear stress:

$$\tau_D = \frac{VQ}{It} = \frac{48(3)\left(10^{-6}\right)}{0.1067\left(10^{-6}\right)(0.02)} = 0.0675\text{ MPa}$$

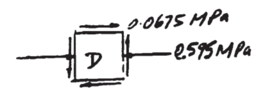
Principal stress: $\sigma_x = -0.595\text{ MPa} \quad \sigma_y = 0 \quad \tau_{xy} = 0.0675\text{ MPa}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-0.595 + 0}{2} \pm \sqrt{\left(\frac{-0.595 - 0}{2}\right)^2 + 0.0675^2}$$

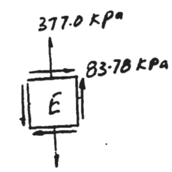


$\sigma_1 = 7.56\text{ kPa}$ **Ans.**
$\sigma_2 = -603\text{ kPa}$ **Ans.**



Point E:

$$I = \frac{1}{12}(0.02)\left(0.05^3\right) = 0.2083\left(10^{-6}\right)\text{ m}^4$$



$$Q_E = \bar{y}'A' = 0.02(0.01)(0.02) = 4.0\left(10^{-6}\right)\text{ m}^3$$

Normal stress:

$$\sigma_E = \frac{My}{I} = \frac{5.2364(0.015)}{0.2083\left(10^{-6}\right)} = 377.0\text{ kPa}$$

Shear stress:

$$\tau_E = \frac{VQ}{It} = \frac{87.273(4.0)\left(10^{-6}\right)}{0.2083\left(10^{-6}\right)(0.02)} = 83.78\text{ kPa}$$

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9–29. Continued

Principal stress: $\sigma_x = 0 \quad \sigma_y = 377.0\text{ kPa} \quad \tau_{xy} = 83.78\text{ kPa}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{0 + 377.0}{2} \pm \sqrt{\left(\frac{0 - 377.0}{2}\right)^2 + 83.78^2}$$

$\sigma_1 = 395\text{ kPa}$ **Ans.**

$\sigma_2 = -17.8\text{ kPa}$ **Ans.**

Ans:
Point D:
$\sigma_1 = 7.56\text{ kPa}$,
$\sigma_2 = -603\text{ kPa}$,
Point E:
$\sigma_1 = 395\text{ kPa}$,
$\sigma_2 = -17.8\text{ kPa}$

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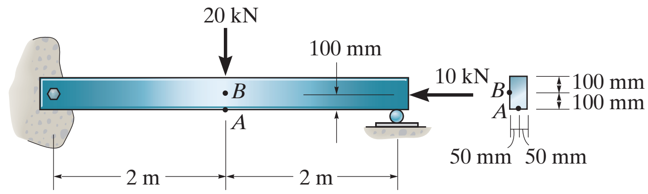
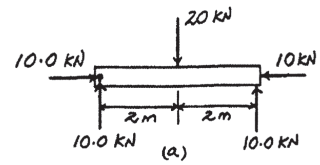
9–30.

The beam has a rectangular cross section and is subjected to the loadings shown. Determine the principal stresses at point A and point B, which are located just to the left of the 20-kN load. Show the results on elements located at these points.



SOLUTION

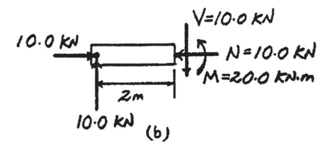
Internal Forces and Moment: As shown on FBD(b).



Section Properties:

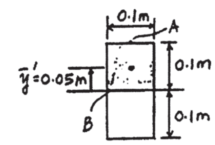
$$A = 0.1\,(0.2) = 0.020 \text{ m}^2$$

$$I = \frac{1}{12}(0.1)\left(0.2^3\right) = 66.667\left(10^{-6}\right) \text{ m}^4$$



$$Q_A = 0$$

$$Q_B = \bar{y}'A' = 0.05(0.1)(0.1) = 0.50\left(10^{-3}\right) \text{ m}^3$$

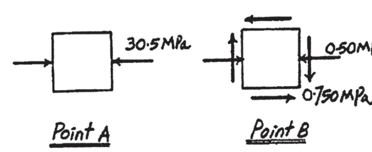


Normal Stresses:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_A = \frac{-10.0\left(10^3\right)}{0.020} - \frac{20.0\left(10^3\right)(0.1)}{66.667\left(10^{-6}\right)} = -30.5 \text{ MPa}$$

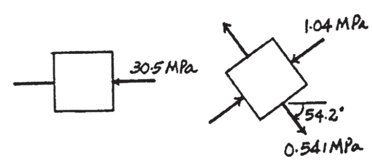
$$\sigma_B = \frac{-10.0\left(10^3\right)}{0.020} - \frac{20.0\left(10^3\right)(0)}{66.667\left(10^{-6}\right)} = -0.500 \text{ MPa}$$



Shear Stress: Applying the shear formula $\tau = \dfrac{VQ}{It}$,

$$\tau_A = 0$$

$$\tau_B = \frac{10.0\left(10^3\right)\left[0.50\left(10^{-3}\right)\right]}{66.667\left(10^{-6}\right)(0.1)} = 0.750 \text{ MPa}$$



In-Plane Principal Stresses: $\sigma_x = -30.5$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element.

$$\sigma_1 = \sigma_y = 0 \qquad \textbf{Ans.}$$

$$\sigma_2 = \sigma_x = -30.5 \text{ MPa} \qquad \textbf{Ans.}$$

$\sigma_x = -0.500$ MPa, $\sigma_y = 0$, and $\tau_{xy} = -0.750$ MPa for point B. Applying Eq. 9–5,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-0.500 + 0}{2} \pm \sqrt{\left(\frac{-0.500 - 0}{2}\right)^2 + (-0.750)^2}$$

$$= -0.250 \pm 0.7906$$

$$\sigma_1 = 0.541 \text{ MPa} \qquad \sigma_2 = -1.04 \text{ MPa} \qquad \textbf{Ans.}$$

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9–30. Continued

Orientation of Principal Plane: Applying Eq. 9–4 for point B.

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-0.750}{(-0.500 - 0)/2} = 3.000$$

$$\theta_P = 35.78^\circ \qquad \text{and} \qquad -54.22^\circ$$

Substituting the results into Eq. 9–1 with $\theta = 35.78^\circ$ yields

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$= \frac{-0.500 + 0}{2} + \frac{-0.500 - 0}{2}\cos 71.56^\circ + (-0.750 \sin 71.56^\circ)$$
$$= -1.04 \text{ MPa} = \sigma_2$$

Hence.

$$\theta_{p1} = -54.2^\circ \qquad \theta_{p2} = 35.8^\circ$$ **Ans.**

Ans:
Point A,
$\sigma_1 = \sigma_y = 0$,
$\sigma_2 = \sigma_x = -30.5$ MPa,
Point B,
$\sigma_1 = 0.541$ MPa,
$\sigma_2 = -1.04$ MPa,
$\theta_{p1} = -54.2^\circ$,
$\theta_{p2} = 35.8^\circ$

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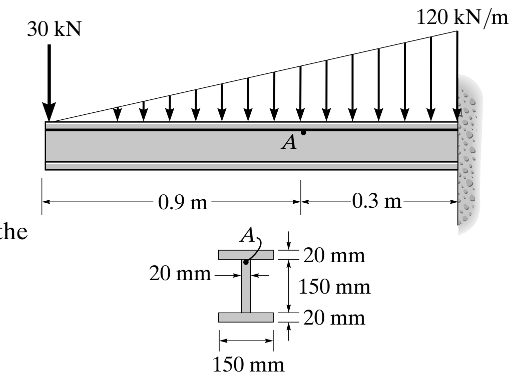
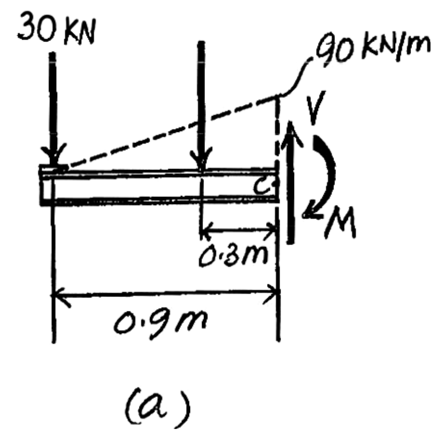
9–31. The wide-flange beam is subjected to the loading shown. Determine the principal stress in the beam at point A, which is located at the top of the web. Although it is not very accurate, use the shear formula to determine the shear stress. Show the result on an element located at this point.



Using the method of sections and consider the FBD of the left cut segment of the bean, Fig. *a*

$$+\uparrow \Sigma F_y = 0; \quad V - \tfrac{1}{2}(90)(0.9) - 30 = 0 \qquad V = 70.5 \text{ kN}$$

$$\circlearrowleft + \Sigma M_C = 0; \quad \tfrac{1}{2}(90)(0.9)(0.3) + 30(0.9) - M = 0 \qquad M = 39.15 \text{ kN}\cdot\text{m}$$



The moment of inertia of the cross - section about the bending axis is

$$I = \frac{1}{12}(0.15)(0.19^3) - \frac{1}{12}(0.13)(0.15^3) = 49.175(10^{-6}) \text{ m}^4$$

Referring to Fig. *b*,

$$Q_A = \bar{y}'A' = 0.085\,(0.02)(0.15) = 0.255\,(10^{-3}) \text{ m}^3$$

The normal stress developed is contributed by bending stress only. For point A, $y = 0.075$ m. Thus,

$$\sigma = \frac{My}{I} = \frac{39.15(10^3)(0.075)}{49.175(10^{-6})} = 59.71(10^6)\text{Pa} = 59.71 \text{ MPa (T)}$$

The shear stress is contributed by the transverse shear stress only. Thus

$$\tau = \frac{VQ_A}{It} = \frac{70.5(10^3)\left[0.255(10^{-3})\right]}{49.175(10^{-6})\,(0.02)} = 18.28(10^6)\text{Pa} = 18.28 \text{ MPa}$$

Here, $\sigma_x = 59.71$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 18.28$ MPa.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}}$$

$$= \frac{59.71 + 0}{2} \pm \sqrt{\left(\frac{59.71 - 0}{2}\right)^2 + 18.28^2}$$

$$= 29.86 \pm 35.01$$

$$\sigma_1 = 64.9 \text{ MPa} \qquad \sigma_2 = -5.15 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{18.28}{(59.71 - 0)/2} = 0.6122$$

$$\theta_P = 15.74^\circ \quad \text{and} \quad -74.26^\circ$$

Substitute $\theta = 15.74^\circ$,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{59.71 + 0}{2} + \frac{59.71 - 0}{2}\cos 31.48^\circ + 18.28 \sin 31.48^\circ$$

$$= 64.9 \text{ MPa} = \sigma_1$$

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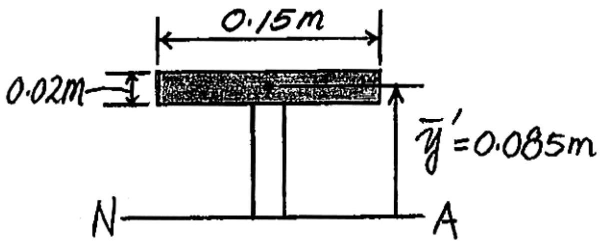
9–31. Continued

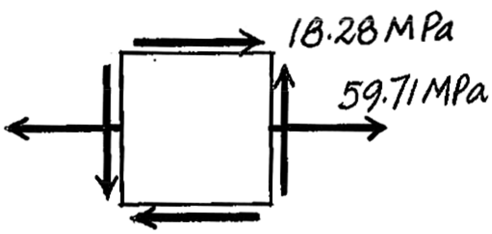
Thus,

$$(\theta_P)_1 = 15.7^\circ \qquad (\theta_P)_2 = -74.3^\circ$$ **Ans.**

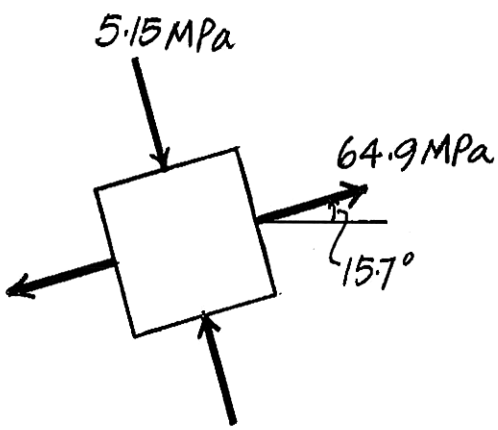
The state of principal stress can be represented by the element shown in Fig. *d*



(b)



(C)



(d)

Ans.
$V = 70.5$ kN, $M = 39.15$ kN·m,
$I = 49.175(10^{-6})$ m^4, $Q_A = 0.255(10^{-3})$ m^3,
$\sigma_1 = 64.9$ MPa, $\sigma_2 = -5.15$ MPa,
$(\theta_p)_1 = 15.7^\circ$, $(\theta_p)_2 = -74.3^\circ$

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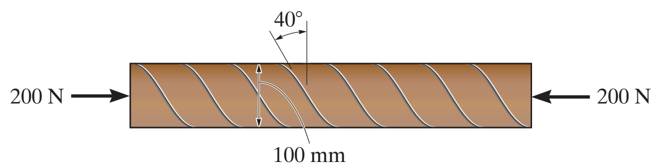
***9–32.**

A paper tube is formed by rolling a cardboard strip in a spiral and then gluing the edges together as shown. Determine the shear stress acting along the seam, which is at 50° from the horizontal, when the tube is subjected to an axial compressive force of 200 N. The paper is 2 mm thick and the tube has an outer diameter of 100 mm.



SOLUTION

Normal And Shear Stresses: The normal stress is caused by the axial force only. Thus

$$\sigma = \frac{N}{A} = \frac{-200}{\frac{\pi}{4}\left(0.1^2 - 0.096^2\right)} = -324.81\left(10^3\right)\text{ Pa} = 324.81\text{ kPa (C)}$$

Since there is no shear force on the cross-section,

$$\tau = 0$$

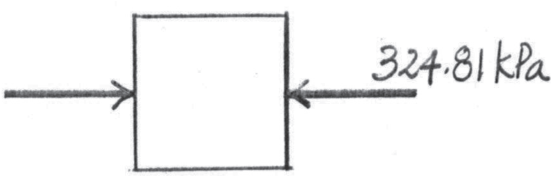
The state of stress of a point on the cross-section can be represented by the element shown in Fig. *a*

Stress Transformation Equations: With $\sigma_x = -324.81$ kPa, $\sigma_y = 0$, $\tau_{xy} = 0$ and $\theta = +40°$ (Fig. *b*),

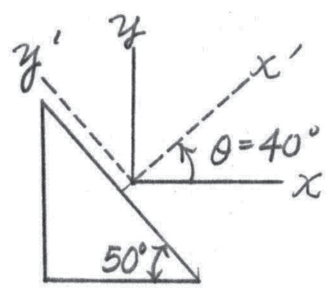
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{(-324.81 - 0)}{2}\sin 80° + 0$$

$$= 159.94\text{ kPa} = 160\text{ kPa}$$ **Ans.**



(a)



(b)

Ans:
$\tau_{x'y'} = 160$ kPa

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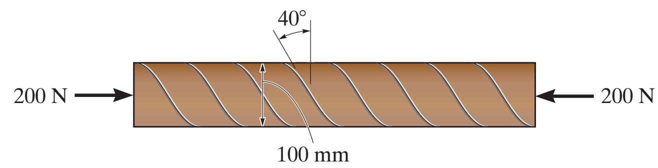
9–33.

Solve Prob. 9–31 for the normal stress acting perpendicular to the seam.



SOLUTION

Normal and Shear Stresses: The normal stress is caused by the axial force only. Thus

$$\sigma = \frac{N}{A} = \frac{-200}{\frac{\pi}{4}\left(0.1^2 - 0.096^2\right)} = -324.81\left(10^3\right)\text{ Pa} = 324.81\text{ kPa (C)}$$

Since there is no shear force on the cross-section,

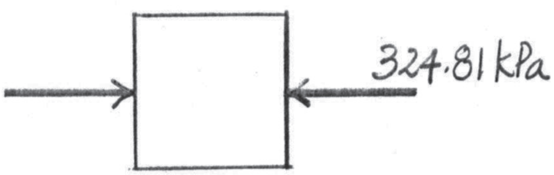
$$\tau = 0$$

The state of stress of a point on the cross-section can be represented by the element shown in Fig. *a*.

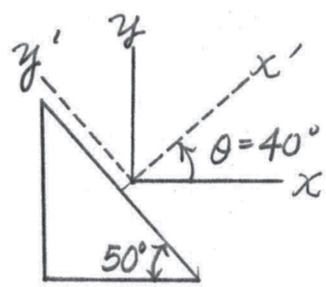
Stress Transformation Equations: With $\sigma_x = -324.81$ kPa, $\sigma_y = 0$, $\tau_{xy} = 0$ and $\theta = +40°$ (Fig. *b*),

$$\begin{aligned}\sigma_{x'} &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \\ &= \frac{-324.81 + 0}{2} + \frac{(-324.81 - 0)}{2}\cos 80° + 0 \\ &= -190.60\text{ kPa} = -191\text{ kPa}\end{aligned}$$

Ans.



(a)



(b)

Ans:
$\sigma_{x'} = -191$ kPa

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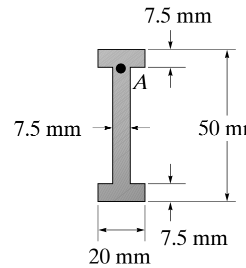
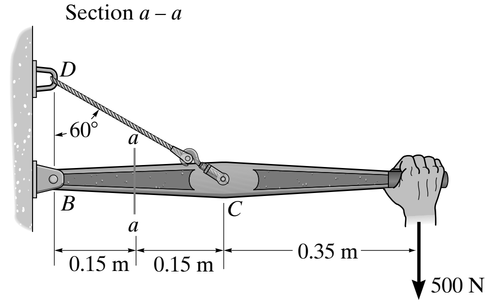
9–34. Determine the principal stress at point A on the cross section of the arm at section a–a. Specify the orientation of this state of stress and indicate the results on an element at the point.



Section $a-a$



Support Reactions: Referring to the free - body diagram of the entire arm shown in Fig. a,

$\Sigma M_B = 0;\ F_{CD} \sin 30°(0.3) - 500(0.65) = 0 \qquad F_{CD} = 2166.67\text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x - 2166.67 \cos 30° = 0 \qquad B_x = 1876.39\text{ N}$

$+\uparrow \Sigma F_y = 0; \qquad 2166.67 \sin 30° - 500 - B_y = 0 \qquad B_y = 583.33\text{ N}$

Internal Loadings: Consider the equilibrium of the free - body diagram of the arm's left segment, Fig. b.

$\xrightarrow{+} \Sigma F_x = 0; \qquad 1876.39 - N = 0 \qquad N = 1876.39\text{ N}$

$+\uparrow \Sigma F_y = 0; \qquad V - 583.33 = 0 \qquad V = 583.33\text{ N}$

$+\Sigma M_O = 0; \qquad 583.33(0.15) - M = 0 \qquad M = 87.5\text{ N}\cdot\text{m}$

Section Properties: The cross - sectional area and the moment of inertia about the z axis of the arm's cross section are

$$A = 0.02(0.05) - 0.0125(0.035) = 0.5625\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.02)\left(0.05^3\right) - \frac{1}{12}(0.0125)\left(0.035^3\right) = 0.16367\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. b,

$$Q_A = \bar{y}'A' = 0.02125(0.0075)(0.02) = 3.1875\left(10^{-6}\right)\text{ m}^3$$

Normal and Shear Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma_A = \frac{N}{A} + \frac{My_A}{I}$$

$$= \frac{-1876.39}{0.5625\left(10^{-3}\right)} + \frac{87.5(0.0175)}{0.16367\left(10^{-6}\right)} = 6.020\text{ MPa}$$

The shear stress is caused by transverse shear stress.

$$\tau_A = \frac{VQ_A}{It} = \frac{583.33\left[3.1875\left(10^{-6}\right)\right]}{0.16367\left(10^{-6}\right)(0.0075)} = 1.515\text{ MPa}$$

The share of stress at point A can be represented on the element shown in Fig. d.

In - Plane Principal Stress: $\sigma_x = 6.020$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 1.515$ MPa. We have

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{6.020 + 0}{2} \pm \sqrt{\left(\frac{6.020 - 0}{2}\right)^2 + 1.515^2}$$

$\sigma_1 = 6.38\text{ MPa} \qquad \sigma_2 = -0.360\text{ MPa}$ **Ans.**

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9–34. Continued

Orientation of the Principal Plane:

$$\tan 2\theta_P = \frac{\tau_{xy}}{\left(\sigma_x - \sigma_y\right)/2} = \frac{1.515}{(6.020 - 0)/2} = 0.5032$$

$$\theta_p = 13.36^\circ \text{ and } 26.71^\circ$$

Substituting $\theta = 13.36^\circ$ into

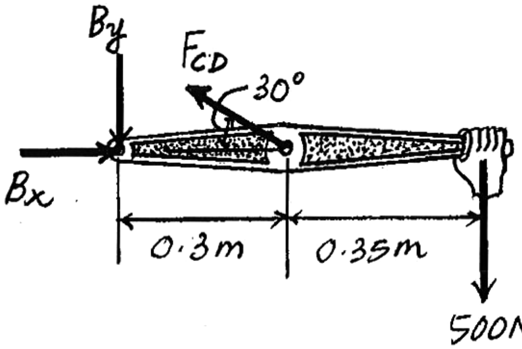
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{6.020 - 0}{2} + \frac{6.020 + 0}{2}\cos 26.71^\circ + 1.515\sin 26.71^\circ$$

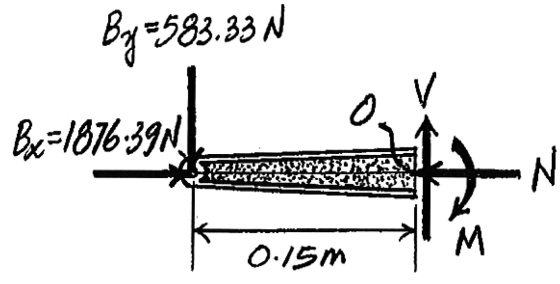
$$= 6.38 \text{ MPa} = \sigma_1$$

Thus, $\left(\theta_P\right)_1 = 13.4$ and $\left(\theta_P\right)_2 = 26.71^\circ$ **Ans.**

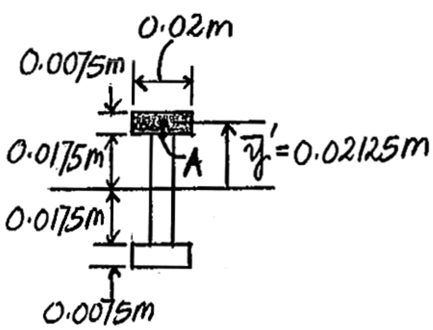
The state of principal stresses is represented by the element shown in Fig. *e*.



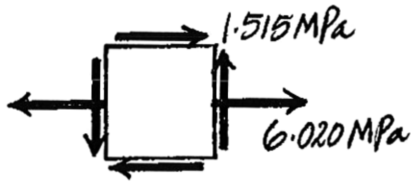
(a)



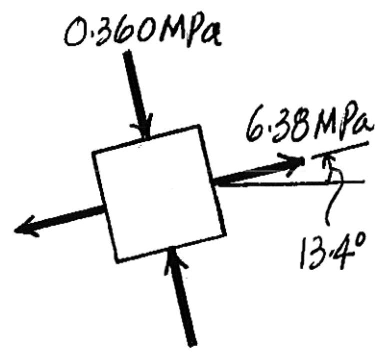
(b)



(C)



(d)



(e)

Ans.
$\sigma_1 = 6.38$ MPa, $\sigma_2 = -0.360$ MPa,
$(\theta_p)_1 = 13.4^\circ$, $(\theta_p)_2 = 103^\circ$

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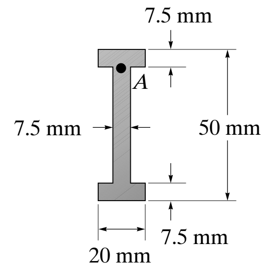
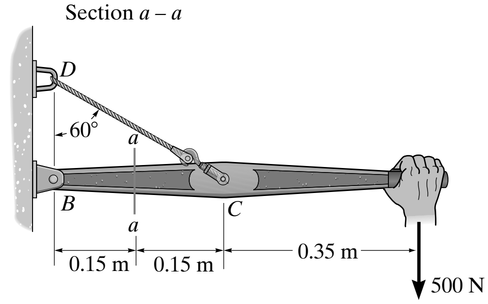
9–35. Determine the maximum in-plane shear stress developed at point A on the cross section of the arm at section a–a. Specify the orientation of this state of stress and indicate the results on an element at the point.



Section $a - a$



SOLUTION

Support Reactions: Referring to the free - body diagram of the entire arm shown in Fig. a,

$\Sigma M_B = 0;\ F_{CD}\sin 30°(0.3) - 500(0.65) = 0 \qquad F_{CD} = 2166.67\text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x - 2166.67\cos 30° = 0 \qquad B_x = 1876.39\text{ N}$

$+\uparrow \Sigma F_y = 0; \qquad 2166.67\sin 30° - 500 - B_y = 0 \qquad B_y = 583.33\text{ N}$

Internal Loadings: Considering the equilibrium of the free - body diagram of the arm's left cut segment, Fig. b,

$\xrightarrow{+} \Sigma F_x = 0; \qquad 1876.39 - N = 0 \qquad N = 1876.39\text{ N}$

$+\uparrow \Sigma F_y = 0; \qquad V - 583.33 = 0 \qquad V = 583.33\text{ N}$

$+\Sigma M_O = 0; \qquad 583.33(0.15) - M = 0 \qquad M = 87.5\text{ N}\cdot\text{m}$

Section Properties: The cross - sectional area and the moment of inertia about the z axis of the arm's cross section are

$$A = 0.02(0.05) - 0.0125(0.035) = 0.5625\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.02)\left(0.05^3\right) - \frac{1}{12}(0.0125)\left(0.035^3\right) = 0.16367\left(10^{-6}\right)\text{ m}^4$$

Referring to Fig. b,

$$Q_A = \bar{y}'A' = 0.02125(0.0075)(0.02) = 3.1875\left(10^{-6}\right)\text{m}^3$$

Normal and Shear Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma_A = \frac{N}{A} + \frac{My_A}{I}$$

$$= \frac{-1876.39}{0.5625\left(10^{-3}\right)} + \frac{87.5(0.0175)}{0.16367\left(10^{-6}\right)} = 6.020\text{ MPa}$$

The shear stress is contributed only by transverse shear stress.

$$\tau_A = \frac{VQ_A}{It} = \frac{583.33\left[3.1875\left(10^{-6}\right)\right]}{0.16367\left(10^{-6}\right)(0.0075)} = 1.515\text{ MPa}$$

Maximum In - Plane Shear Stress: $\sigma_x = 6.020\text{ MPa}, \sigma_y = 0,$ and $\tau_{xy} = 1.515\text{ MPa}.$

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{6.020 - 0}{2}\right)^2 + 1.515^2} = 3.37\text{ MPa} \qquad \textbf{Ans.}$$

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9–35. Continued

Orientation of the Plane of Maximum In - Plane Shear Stress:

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = -\frac{(6.020 - 0)/2}{1.515} = -1.9871$$

$\theta_s = -31.6°$ and $58.4°$ **Ans.**

Substituting $\theta = -31.6°$ into

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{6.020 - 0}{2}\sin(-63.29°) + 1.515\cos(-63.29°)$$

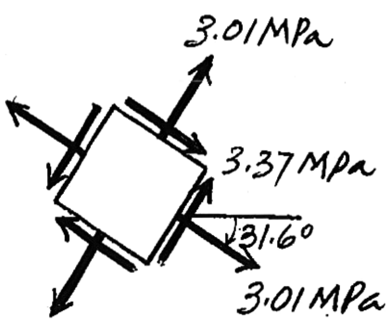
$$= 3.37 \text{ MPa} = \tau_{\substack{\max \\ \text{in-plane}}}$$

This indicates that $\tau_{\substack{\max \\ \text{in-plane}}}$ is directed in the positive sense of the y' axis on the face of the element defined by $\theta_s = -31.6°$.

Average Normal Stress:

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{6.020 + 0}{2} = 3.01 \text{ MPa}$$ **Ans.**

The state of maximum in - plane shear stress is represented on the element shown in Fig. *e*.



(e)

Ans.

$\theta_s = -31.6°$ and $58.4°$, $\sigma_{avg} = 3.01$ MPa, $\tau_{\substack{\max \\ \text{in-plane}}} = 3.37$ MPa

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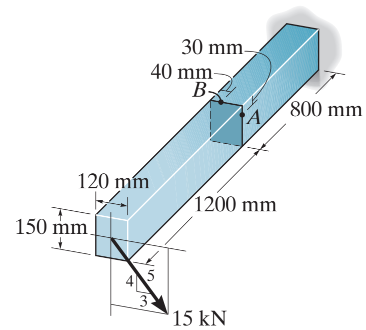
***9–36.**

Determine the principal stresses in the cantilevered beam at points A and B.



SOLUTION

Internal Forces and Moment: As shown on FBD.

Section Properties:

$$I_z = \frac{1}{12}(0.12)(0.15^3) = 33.75(10^{-6})\ \text{m}^4$$

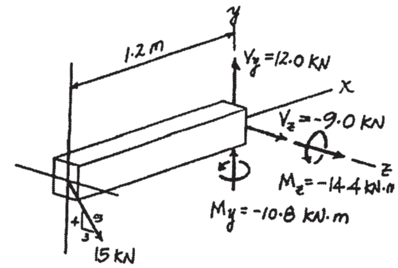
$$I_y = \frac{1}{12}(0.15)(0.12^3) = 21.6(10^{-6})\ \text{m}^4$$

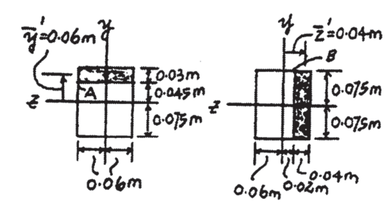
$$(Q_A)_y = \bar{y}'A' = 0.06(0.03)(0.12) = 0.216(10^{-3})\ \text{m}^3$$

$$(Q_A)_z = 0$$

$$(Q_B)_z = \bar{z}'A' = 0.04(0.04)(0.15) = 0.240(10^{-3})\ \text{m}^3$$

$$(Q_B)_y = 0$$





Normal Stress:

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{-14.4(10^3)(0.045)}{33.75(10^{-6})} + \frac{-10.8(10^3)(0.06)}{21.6(10^{-6})}$$

$$= -10.8\ \text{MPa}$$

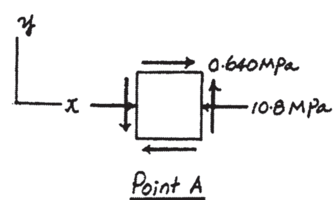
$$\sigma_B = -\frac{-14.4(10^3)(0.075)}{33.75(10^{-6})} + \frac{-10.8(10^3)(-0.02)}{21.6(10^{-6})}$$

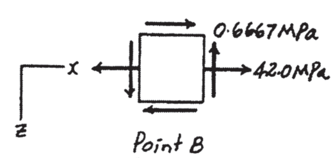
$$= 42.0\ \text{MPa}$$

Shear Stress: Applying the shear formula

$$\tau_A = \frac{V_y (Q_A)_y}{I_z t} = \frac{12.0(10^3)\left[0.216(10^{-3})\right]}{33.75(10^{-6})(0.12)} = 0.640\ \text{MPa}$$

$$\tau_B = \frac{V_z (Q_B)_z}{I_y t} = \frac{-9.00(10^3)\left[0.240(10^{-3})\right]}{21.6(10^{-6})(0.15)} = -0.6667\ \text{MPa}$$





In-Plane Principal Stress: $\sigma_x = -10.8$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 0.640$ MPa for point A.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-10.8 + 0}{2} \pm \sqrt{\left(\frac{-10.8 - 0}{2}\right)^2 + 0.640^2}$$

$$= -5.40 \pm 5.4378$$

$\sigma_1 = 37.8$ kPa $\qquad \sigma_2 = -10.8$ MPa **Ans.**

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***9–36. Continued**

$\sigma_x = 42.0$ MPa, $\sigma_z = 0$, and $\tau_{xz} = 0.6667$ MPa for point *B*.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{42.0 + 0}{2} \pm \sqrt{\left(\frac{42.0 - 0}{2}\right)^2 + 0.6667^2}$$

$$= 21.0 \pm 21.0105$$

$\sigma_1 = 42.0 \text{ MPa} \qquad \sigma_2 = -10.6 \text{ kPa}$ **Ans.**

Ans:
Point *A*,
$\sigma_1 = 37.8$ kPa,
$\sigma_2 = -10.8$ MPa,
Point *B*,
$\sigma_1 = 42.0$ MPa,
$\sigma_2 = -10.6$ kPa

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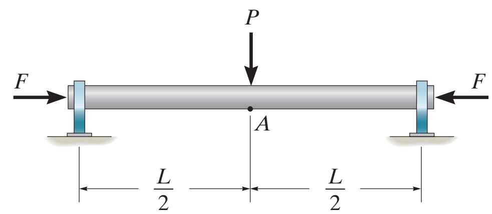
9–37.

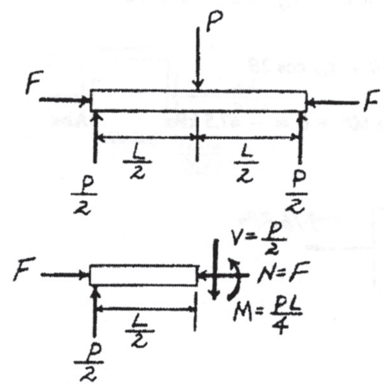
The shaft has a diameter d and is subjected to the loadings shown. Determine the principal stresses and the maximum in-plane shear stress at point A. The bearings only support vertical reactions.



SOLUTION

Support Reactions: As shown on FBD(a).

Internal Forces and Moment: As shown on FBD(b).



Section Properties:

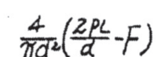
$$A = \frac{\pi}{4} d^2 \quad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64} d^4 \quad Q_A = 0$$

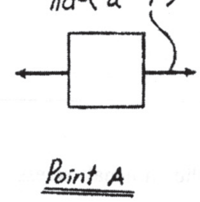
Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

$$= \frac{-F}{\frac{\pi}{4} d^2} \pm \frac{\frac{PL}{4}\left(\frac{d}{2}\right)}{\frac{\pi}{64} d^4}$$

$$\sigma_A = \frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right)$$





Shear Stress: Since $Q_A = 0, \tau_A = 0$

In-Plane Principal Stress: $\sigma_x = \dfrac{4}{\pi d^2}\left(\dfrac{2PL}{d} - F\right)$.

$\sigma_y = 0$ and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_x = \frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right) \qquad \textbf{Ans.}$$

$$\sigma_2 = \sigma_y = 0 \qquad \textbf{Ans.}$$

Maximum In-Plane Shear Stress: Applying Eq. 9–7 for point A,

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \sqrt{\left(\frac{\frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right) - 0}{2}\right)^2 + 0}$$

$$= \frac{2}{\pi d^2}\left(\frac{2PL}{d} - F\right) \qquad \textbf{Ans.}$$

Ans:

$$\sigma_1 = \frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right), \sigma_2 = 0,$$

$$\tau_{\substack{\max \\ \text{in-plane}}} = \frac{2}{\pi d^2}\left(\frac{2PL}{d} - F\right)$$

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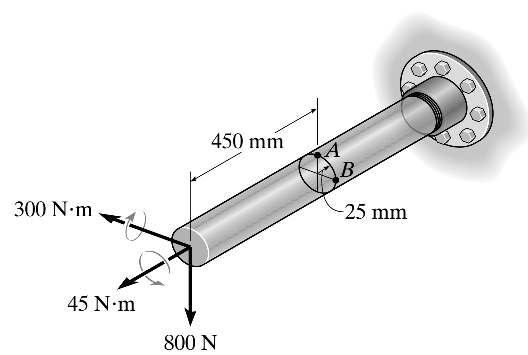
9–38. The solid shaft is subjected to a torque, bending moment, and shear force as shown. Determine the principal stresses acting at point A.

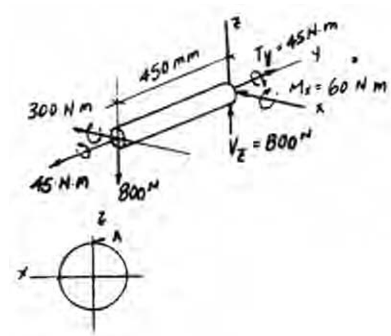


SOLUTION

$$I_x = I_y = \frac{\pi}{4}(0.025)^4 = 0.306796(10^{-6})\ \text{m}^4$$

$$J = \frac{\pi}{2}(0.025)^4 = 0.613592(10^{-6})\ \text{m}^4$$

$$Q_A = 0$$

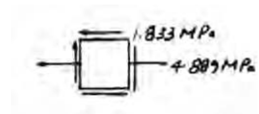


$$\sigma_A = \frac{M_x c}{I} = \frac{60(0.025)}{0.306796(10^{-6})} = 4.889\ \text{MPa}$$

$$\tau_A = \frac{T_y c}{J} = \frac{45(0.025)}{0.613592(10^{-6})} = 1.833\ \text{MPa}$$

$$\sigma_x = 4.889\ \text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -1.833\ \text{MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



$$= \frac{4.889 + 0}{2} \pm \sqrt{\left(\frac{4.889 - 0}{2}\right)^2 + (-1.833)^2}$$

$\sigma_1 = 5.50\ \text{MPa}$ **Ans.**

$\sigma_2 = -0.611\ \text{MPa}$ **Ans.**

Ans.
$\sigma_1 = 5.50\ \text{MPa}, \sigma_2 = -0.611\ \text{MPa}$

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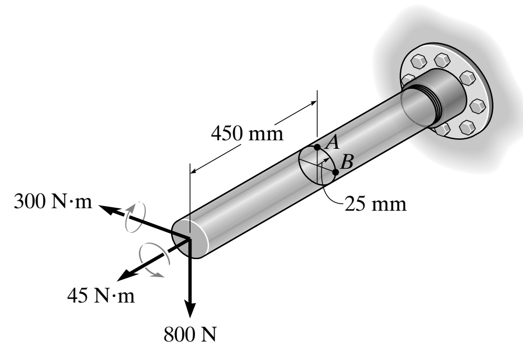
9–39. Solve Prob. 9–47 for point B.

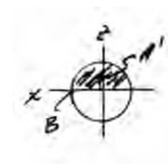


SOLUTION

$$I_x = I_y = \frac{\pi}{4}(0.025)^4 = 0.306796(10^{-6})\ \text{m}^4$$

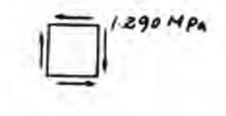
$$J = \frac{\pi}{2}(0.025)^4 = 0.613592(10^{-6})\ \text{m}^4$$

$$Q_B = \bar{y}A' = \frac{4(0.025)}{3\pi}\left(\frac{1}{2}\right)\pi\,(0.025^2) = 10.4167(10^{-6})\ \text{m}^3$$



$$\sigma_B = 0$$

$$\tau_B = \frac{V_z Q_B}{It} - \frac{T_y\,c}{J} = \frac{800(10.4167)(10^{-6})}{0.306796(10^{-6})(0.05)} - \frac{45(0.025)}{0.61359(10^{-6})} = -1.290\ \text{MPa}$$



$$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = -1.290\ \text{MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= 0 \pm \sqrt{(0)^2 + (-1.290)^2}$$

$\sigma_1 = 1.29$ MPa **Ans.**

$\sigma_2 = -1.29$ MPa **Ans.**

Ans.
$\sigma_1 = 1.29$ MPa, $\sigma_2 = -1.29$ MPa

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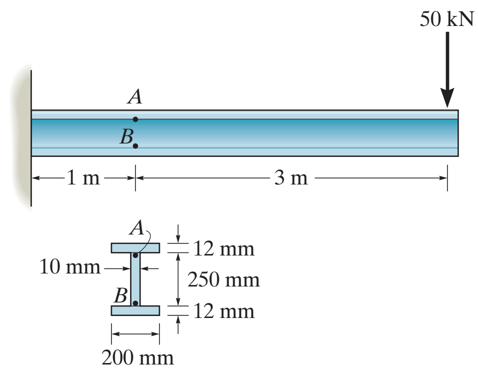
***9–40.**

The wide-flange beam is subjected to the 50-kN force. Determine the principal stresses in the beam at point A located on the *web* at the bottom of the upper flange. Although it is not very accurate, use the shear formula to calculate the shear stress.



SOLUTION

$$I = \frac{1}{12}(0.2)(0.274)^3 - \frac{1}{12}(0.19)(0.25)^3 = 95.451233(10^{-6})\ \text{m}^4$$

$$Q_A = (0.131)(0.012)(0.2) = 0.3144(10^{-3})\ \text{m}^3$$

$$\sigma_A = \frac{My}{I} = \frac{150(10^3)(0.125)}{95.451233(10^{-6})} = 196.43\ \text{MPa}$$

$$\tau_A = \frac{VQ_A}{It} = \frac{50(10^3)(0.3144)(10^{-3})}{95.451233(10^{-6})(0.01)} = 16.47\ \text{MPa}$$

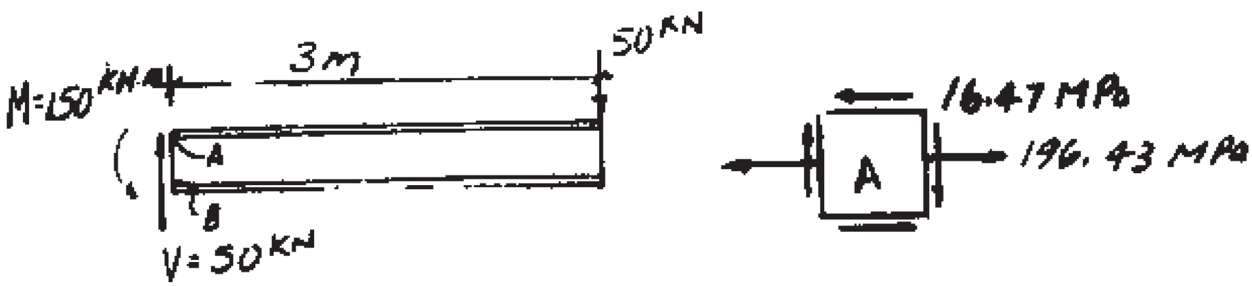
$$\sigma_x = 196.43\ \text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -16.47\ \text{MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{196.43 + 0}{2} \pm \sqrt{\left(\frac{196.43 - 0}{2}\right)^2 + (-16.47)^2}$$

$\sigma_1 = 198\ \text{MPa}$ **Ans.**

$\sigma_2 = -1.37\ \text{MPa}$ **Ans.**



Ans:
$\sigma_1 = 198\ \text{MPa}$,
$\sigma_2 = -1.37\ \text{MPa}$

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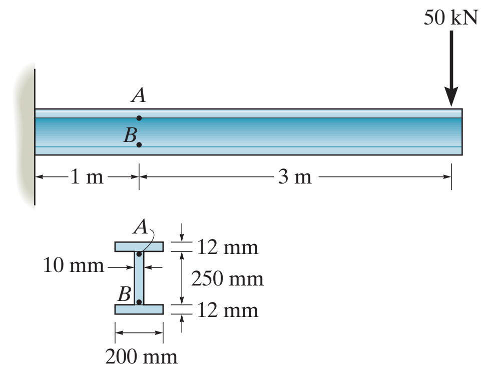
9–41.

Solve Prob. 9–40 for point B located on the *web* at the top of the bottom flange.



SOLUTION

$$I = \frac{1}{12}(0.2)(0.247)^3 - \frac{1}{12}(0.19)(0.25)^3 = 95.451233(10^{-6})\ \text{m}^4$$

$$Q_B = (0.131)(0.012)(0.2) = 0.3144(10^{-3})$$

$$\sigma_B = -\frac{My}{I} = -\frac{150(10^3)(0.125)}{95.451233(10^{-6})} = -196.43\ \text{MPa}$$

$$\tau_B = \frac{VQ_B}{It} = \frac{50(10^3)(0.3144)(10^{-3})}{95.451233(10^{-6})(0.01)} = 16.47\ \text{MPa}$$

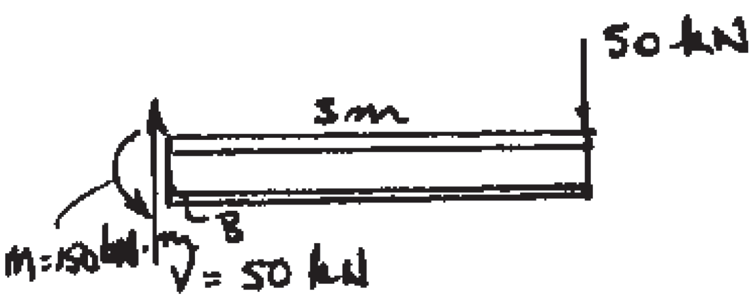
$\sigma_x = -196.43\ \text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -16.47\ \text{MPa}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-196.43 + 0}{2} \pm \sqrt{\left(\frac{-196.43 - 0}{2}\right)^2 + (-16.47)^2}$$

$\sigma_1 = 1.37\ \text{MPa}$ **Ans.**

$\sigma_2 = -198\ \text{MPa}$ **Ans.**



Ans:
$\sigma_1 = 1.37\ \text{MPa}, \sigma_2 = -198\ \text{MPa}$

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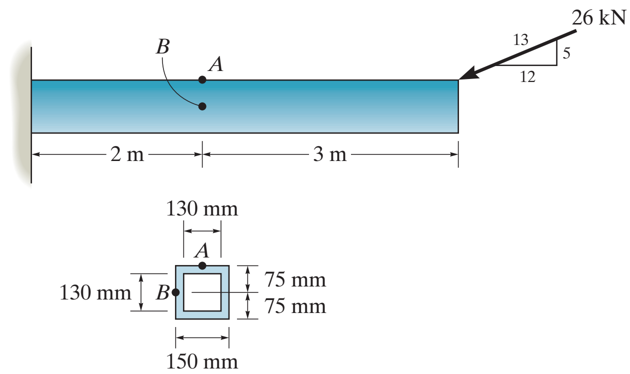
9–42.

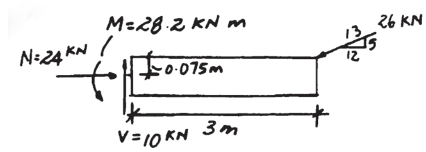
The box beam is subjected to the 26-kN force that is applied at the center of its width, 75 mm from each side. Determine the principal stresses at point A and show the results in an element located at this point. Use the shear formula to calculate the shear stress.



SOLUTION

$$I = \frac{1}{12}(0.15)\left(0.15^3\right) - \frac{1}{12}(0.13)\left(0.13^3\right) = 18.38667\left(10^{-6}\right)\text{ m}^4$$

$$A = 0.15^2 - 0.13^2 = 5.6\left(10^{-3}\right)\text{ m}^2$$



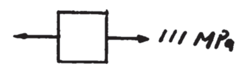
$$Q_A = 0$$

$$\tau_A = 0$$

$$\sigma_A = -\frac{P}{A} + \frac{Mc}{I} = \frac{-24\left(10^3\right)}{5.6\left(10^{-3}\right)} + \frac{28.2\left(10^3\right)(0.075)}{18.38667\left(10^{-6}\right)} = 111\text{ MPa}$$

$\sigma_1 = 111$ MPa **Ans.**

$\sigma_2 = 0$ **Ans.**



Ans:
$\sigma_1 = 111$ MPa,
$\sigma_2 = 0$

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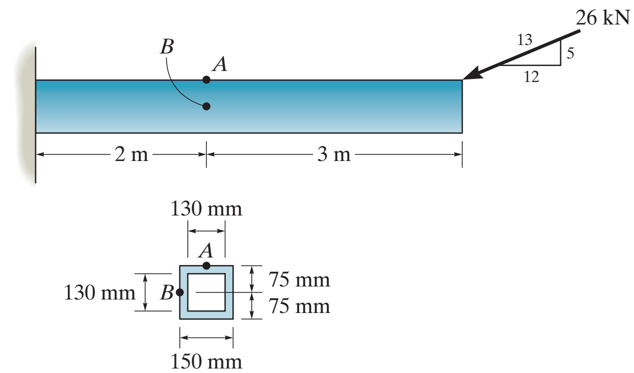
9–43.

Solve Prob. 9–42 for point B.



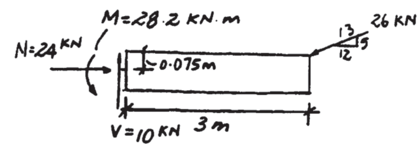
SOLUTION

$$I = \frac{1}{12}(0.15)\left(0.15^3\right) - \frac{1}{12}(0.13)\left(0.13^3\right) = 18.38667\left(10^{-6}\right)\text{ m}^4$$

$$A = 0.15^2 - 0.13^2 = 5.6\left(10^{-3}\right)\text{ m}^2$$

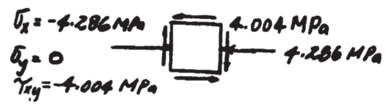
$$Q_B = (0.07)(0.15)(0.01) + 2(0.0325)(0.065)(0.01) = 0.14725\left(10^{-3}\right)\text{ m}^3$$

$$\sigma_B = -\frac{P}{A} = -\frac{24\left(10^3\right)}{5.6\left(10^{-3}\right)} = -4.286\text{ MPa}$$



$$\tau_B = \frac{VQ_B}{It} = \frac{10\left(10^3\right)(0.14725)\left(10^{-3}\right)}{18.38667\left(10^{-6}\right)(0.02)} = 4.004\text{ MPa}$$

$$\sigma_x = -4.286\text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -4.004\text{ MPa}$$

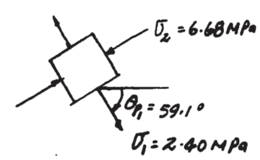


$$\sigma_{1.2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-4.286 + 0}{2} \pm \sqrt{\left(\frac{-4.286 - 0}{2}\right)^2 + \left(-4.004^2\right)}$$

$$\sigma_1 = 2.40\text{ MPa} \qquad \sigma_2 = -6.68\text{ MPa}$$ **Ans.**

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-4.004}{(-4.286 - 0)/2}$$



$$\theta_P = 30.9^\circ \qquad \text{or} \qquad -59.1^\circ$$

Use Eq. 9–1,

$$\theta_{P1} = -59.1^\circ \qquad \theta_{P2} = 30.9^\circ$$

Ans:
$\sigma_1 = 2.40$ MPa,
$\sigma_2 = -6.68$ MPa

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***9–44.**

Solve Prob. 9–2 using Mohr's circle.

SOLUTION

$$\frac{\sigma_x + \sigma_y}{2} = \frac{0 + 65}{2} = 32.5 \text{ MPa}$$

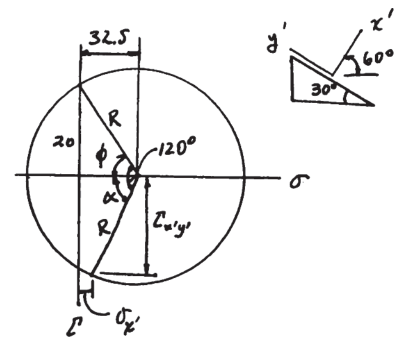
$$R = \sqrt{(32.5)^2 + (20)^2} = 38.1608$$

$$\phi = \tan^{-1}\frac{20}{32.5} = 31.6075°$$

$$\alpha = 120° - 31.6075° = 88.392°$$

$$\sigma_{x'} = 32.5 - 38.1608 \cos 88.392° = 31.4 \text{ MPa}$$ **Ans.**

$$\tau_{x'y'} = 38.1608 \sin 88.392° = 38.1 \text{ MPa}$$ **Ans.**



Ans:
$\sigma_{x'} = 31.4$ MPa,
$\tau_{x'y'} = 38.1$ MPa

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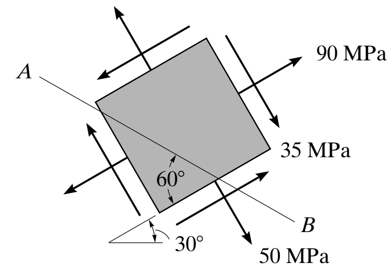
9–45. Solve Prob. 9–4 using Mohr's circle.



SOLUTION

$\sigma_x = 90 \text{ MPa} \qquad \sigma_y = 50 \text{ MPa} \qquad \tau_{xy} = -35 \text{ MPa} \qquad A(90, -35)$

$$\frac{\sigma_x + \sigma_y}{2} = \frac{90 + 50}{2} = 70$$

$$R = \sqrt{(90 - 70)^2 + (35)^2} = 40.311$$

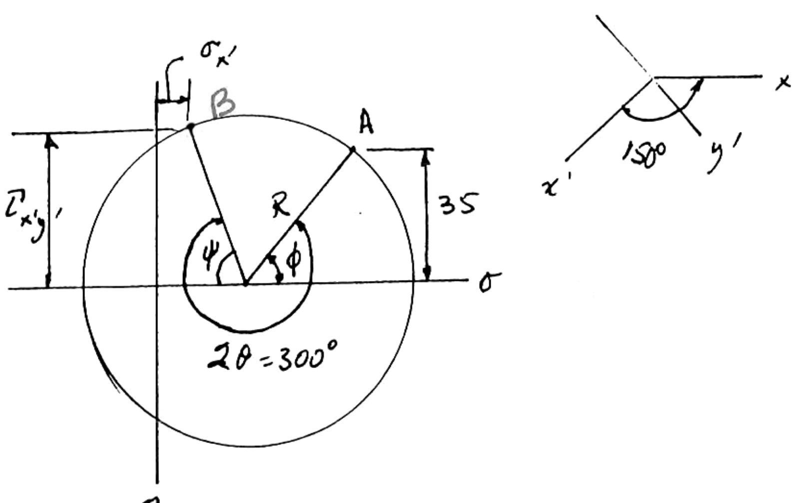
Coordinates of point B:

$$\phi = \tan^{-1}\left(\frac{35}{20}\right) = 60.255°$$

$$\psi = 300° - 180° - 60.255° = 59.745°$$

$\sigma_{x'} = 70 - 40.311 \cos 59.745° = 49.7 \text{ MPa}$ **Ans.**

$\tau_{x'} = -40.311 \sin 59.745° = -34.8 \text{ MPa}$ **Ans.**



Ans.

$\sigma_{x'} = -388$ psi, $\tau_{x'y'} = 455$ psi

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9–46.

SOLUTION

Construction of the Circle: In accordance to the sign convention, $\sigma_x = 0$, $\sigma_y = -500$ MPa and $\tau_{xy} = 350$ MPa. Hence

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-500)}{2} = -250 \text{ MPa}$$

The coordinates for reference point A and center C of the circle are

$$A(0, 350) \qquad C(-250, 0)$$

The radius of the circle is

$$R = \sqrt{[0 - (-250)]^2 + 350^2} = 430.12 \text{ MPa}$$

Using the results, the circle shown in Fig. a can be constructed.

Stress on the Rotated Element: Plane AB can be represented by an element rotating clockwise through an angle $\theta = 60°$, Fig. b. The normal and shear stress components on plane AB $\left(\sigma_{x'} \text{ and } \tau_{x'y'}\right)$ are represented by coordinates of point P on the circle. From the geometry shown on the circle

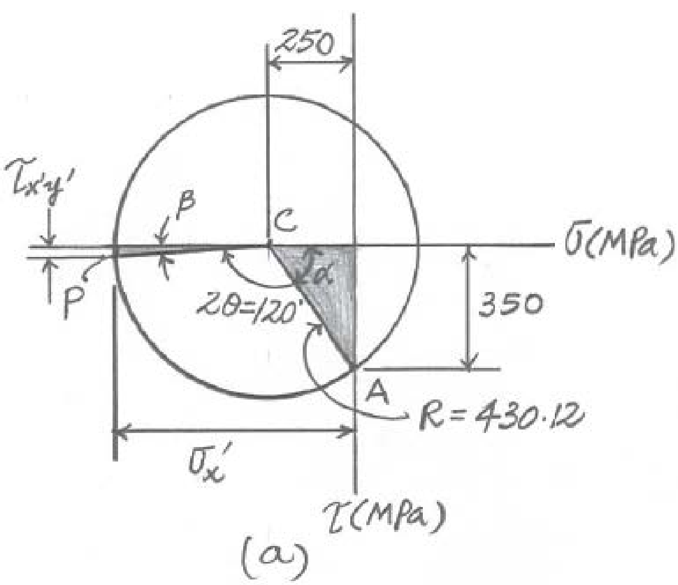
$$\alpha = \tan^{-1}\left(\frac{350}{250}\right) = 54.46°$$

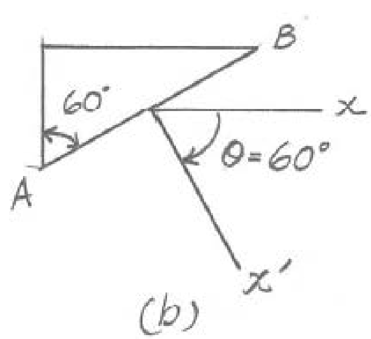
$$\beta = 180° - \alpha - 2\theta = 180° - 54.46° - 120° = 5.538°$$

Then

$\sigma_{x'} = -250 - 430.12\cos 5.538° = -678$ MPa **Ans**

$\tau_{x'y'} = 430.12\sin 5.538° = 41.5$ MPa **Ans**





Ans:
$\sigma_{x'} = -678$ MPa, $\tau_{x'y'} = 41.5$ MPa

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9–47.

Solve Prob. 9–11 using Mohr's circle.

SOLUTION

Construction of the Circle: $\theta = -60°$, $\sigma_x = 150$ MPa, $\sigma_y = 100$ MPa, $\tau_{xy} = 75$ MPa. Thus

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{150 + 100}{2} = 125 \text{ MPa}$$

The coordinates of the reference point A and center C of the circle are

$$A(150, 75) \qquad C(125, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(150 - 125)^2 + (75)^2} = 79.06 \text{ MPa}$$

Normal and Shear Stress on Rotated Element: Here $\theta = 60°$ clockwise. By rotating the radial line CA clockwise $2\theta = 120°$, it coincides with the radial line OP and the coordinates of reference point $P(\sigma_{x'}, \tau_{x'y'})$ represent the normal and shear stresses on the face of the element defined by $\theta = -60°$. $\sigma_{y'}$ can be determined by calculating the coordinates of point Q. From the geometry of the circle, Fig. (a),

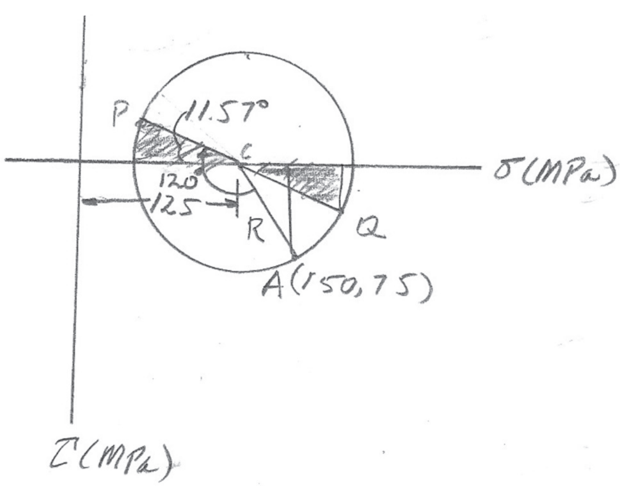
$$\sin\alpha = \frac{75}{79.06}, \ \alpha = 71.57°, \ \beta = 120° + 71.57° - 180° = 11.57°$$

$\sigma_{x'} = 125 - 79.06\cos 11.57° = 47.5$ MPa **Ans.**

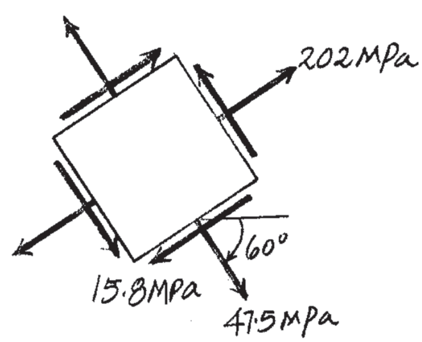
$\tau_{x'y'} = -79.06\sin 11.57° = -15.8$ MPa **Ans.**

$\sigma_{y'} = 125 + 79.06\cos 11.57° = 202$ MPa **Ans.**

The results are shown in Fig. (b).



(a)



(b)

Ans:
$\sigma_{x'} = 47.5$ MPa, $\tau_{x'y'} = -15.8$ MPa,
$\sigma_{y'} = 202$ MPa

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***9–48.**

Solve Prob. 9–15 using Mohr's circle.

SOLUTION

$$\frac{\sigma_x + \sigma_y}{2} = \frac{45 - 60}{2} = -7.5 \text{ MPa}$$

$$R = \sqrt{(45 + 7.5)^2 + (30)^2} = 60.467 \text{ MPa}$$

$\sigma_1 = 60.467 - 7.5 = 53.0$ MPa **Ans.**

$\sigma_2 = -60.467 - 7.5 = -68.0$ MPa **Ans.**

$$2\theta_{p1} = \tan^{-1}\frac{30}{(45 + 7.5)}$$

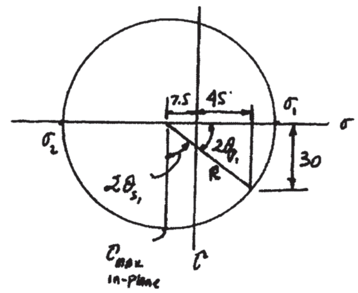
$\theta_{p1} = 14.9°$ counterclockwise **Ans.**

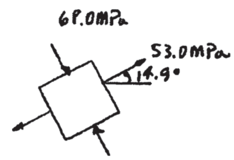
$\tau_{\substack{\max\\\text{in-plane}}} = 60.5$ MPa **Ans.**

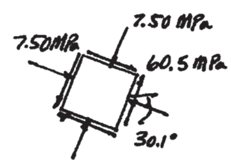
$\sigma_{avg} = -7.50$ MPa **Ans.**

$$2\theta_{s1} = 90° - \tan^{-1}\frac{30}{(45 + 7.5)}$$

$\theta_{s1} = 30.1°$ clockwise **Ans.**







Ans:
$\sigma_1 = 53.0$ MPa,
$\sigma_2 = -68.0$ MPa,
$\theta_{p1} = 14.9°$ counterclockwise,
$\tau_{\substack{\max\\\text{in-plane}}} = 60.5$ MPa,
$\sigma_{avg} = -7.50$ MPa,
$\theta_{s1} = 30.1°$ clockwise

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9–49.

Solve Prob. 9–16 using Mohr's circle.

SOLUTION

Construction of Circle: $\sigma_x = 50$ MPa, $\sigma_y = 0$, $\tau_{xy} = -15$ MPa. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{50 + 0}{2} = 25 \text{ MPa} \qquad \textbf{Ans.}$$

The coordinates of reference point A and center C of the circle are

$$A(50, -15) \qquad C(25, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(50 - 25)^2 + (-15)^2} = \tau_{\substack{\max \\ \text{in-plane}}} = 29.15 \text{ MPa}$$

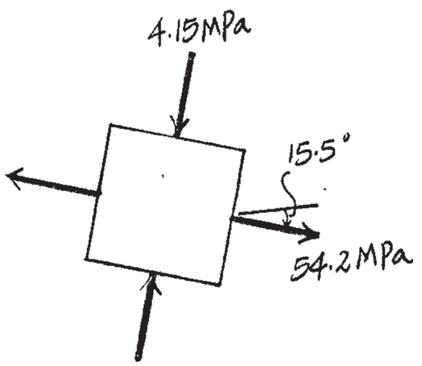
See Fig. (a).

a) **Principal Stress:**

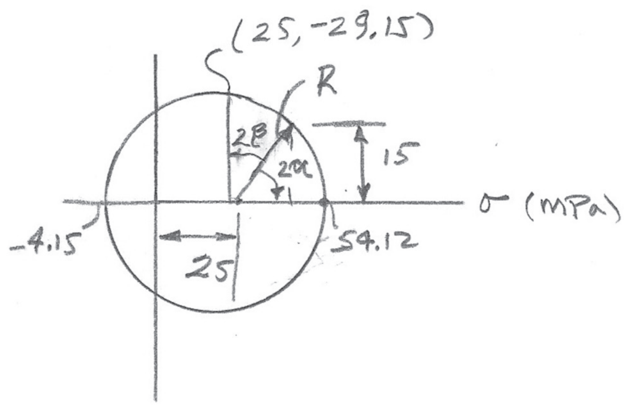
$$\sigma_1 = 54.2 \text{ MPa}, \qquad \sigma_2 = -4.15 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sin 2\alpha = \frac{15}{29.15}, \qquad \alpha = 15.5° \qquad \textbf{Ans.}$$

See Fig. (b).



(a)



(a)

Ans:
$\sigma_1 = 54.2$ MPa, $\sigma_2 = -4.15$ MPa, $\theta_p = -15.5°$,
$\sigma_{avg} = 25$ MPa, $\tau_{\substack{\max \\ \text{in-plane}}} = 29.2$ MPa, $\theta_s = 29.5°$,
$\alpha = 15.5°$

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9–50.

Mohr's circle for the state of stress is shown in Fig. 9–17*a*. Show that finding the coordinates of point $P\,(\sigma_{x'}, \tau_{x'y'})$ on the circle gives the same value as the stress transformation Eqs. 9–1 and 9–2.

SOLUTION

$$A(\sigma_x, \tau_{xy}) \qquad B(\sigma_y, -\tau_{xy}) \qquad C\left(\left(\frac{\sigma_x + \sigma_y}{2}\right), 0\right)$$

$$R = \sqrt{\left[\sigma_x - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 + \tau_{xy}^2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\cos\theta' \qquad \textbf{(1)}$$

$$\theta' = 2\theta_P - 2\theta$$

$$\cos(2\theta_P - 2\theta) = \cos 2\theta_P \cos 2\theta + \sin 2\theta_p \sin 2\theta \qquad \textbf{(2)}$$

From the circle:

$$\cos 2\theta_P = \frac{\sigma_x - \frac{\sigma_x + \sigma_y}{2}}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \qquad \textbf{(3)}$$

$$\sin 2\theta_P = \frac{\tau_{xy}}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \qquad \textbf{(4)}$$

Substitute Eq. (2), (3) and into Eq. (1)

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \textbf{QED}$$

$$\tau_{x'y'} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\,\sin\theta' \qquad \textbf{(5)}$$

$$\sin\theta' = \sin(2\theta_P - 2\theta)$$

$$= \sin 2\theta_P \cos 2\theta - \sin 2\theta \cos 2\theta_P \qquad \textbf{(6)}$$

Substitute Eq. (3), (4), (6) into Eq. (5),

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad \textbf{QED}$$

Ans:
N/A

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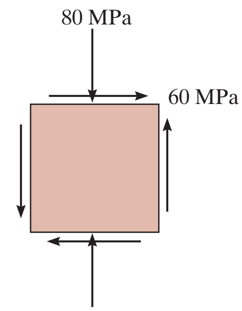
9–51.

Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



SOLUTION

Construction of The Circle: In accordance to the sign convention, $\sigma_x = 0$, $\sigma_y = -80$ MPa and $\tau_{xy} = 60$ MPa. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-80)}{2} = -40.0 \text{ MPa} \qquad \textbf{Ans.}$$

The coordinates for reference point A and center of circle C are

$$A(0, 60) \qquad C(-40.0, 0)$$

Thus, the radius of the circle is

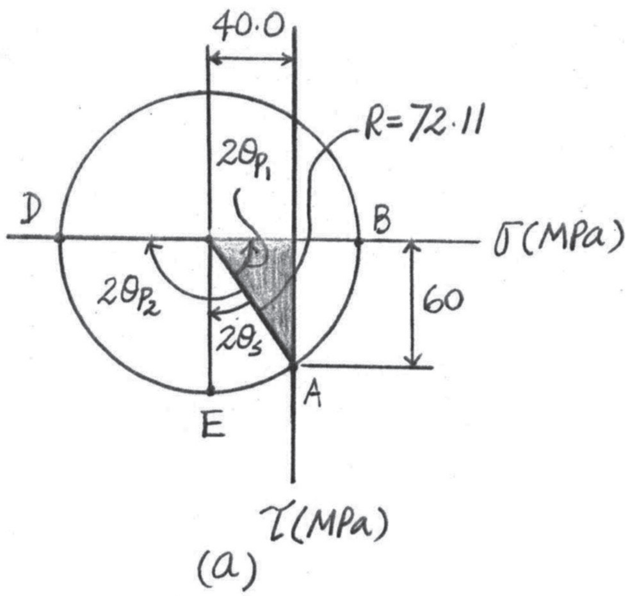
$$R = CA = \sqrt{[0 - (-40.0)]^2 + (60 - 0)^2} = 72.11 \text{ MPa}$$

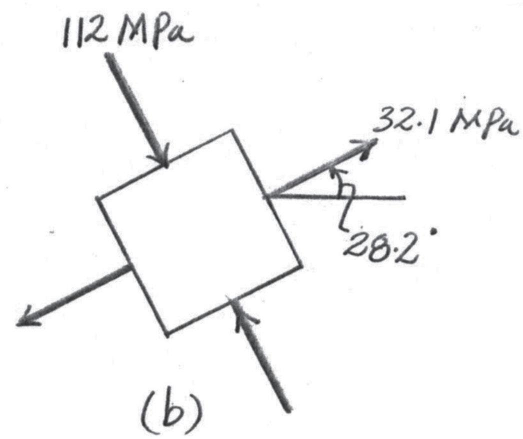
Using these results, the circle shown in Fig. *a* can be constructed.

a) In-plane Principal Stresses: The coordinates of points B and D on the circle represent σ_1 and σ_2, respectively.

$$\sigma_1 = -40.0 + 72.11 = 32.11 \text{ MPa} = 32.1 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = -40.0 - 72.11 = -112.11 \text{ MPa} = -112 \text{ MPa} \qquad \textbf{Ans.}$$





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9–51. Continued

Orientation of Principal Plane: From the shaded triangle on the circle,

$$\tan 2\theta_{p1} = \frac{60}{40.0} = 1.5$$

$$2\theta_{p1} = 56.31°$$

$$\theta_{p1} = 28.15° = 28.2° \textbf{ (counterclockwise)} \qquad \textbf{Ans.}$$

Using these results the state of in-plane principal stresses can be represented by the differential element shown in Fig. *b*.

b) Maximum In-plane Shear Stress: Represented by the coordinates of point E on the circle.

$$\tau_{\substack{\max \\ \text{in-plane}}} = R = 72.11 \text{ MPa} = 72.1 \text{ MPa} \qquad \textbf{Ans.}$$

Orientation of The Plane For Maximum in-plane Shear Stress: From the circle,

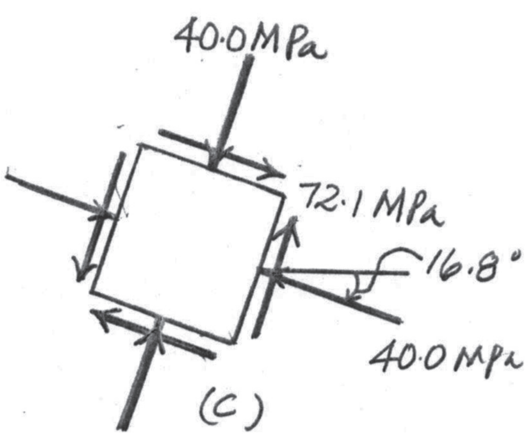
$$\tan 2\theta_s = \frac{40.0}{60} = 0.6667$$

$$2\theta_s = 33.69°$$

$$\theta_s = 16.845° = 16.8° \textbf{ (clockwise)} \qquad \textbf{Ans.}$$

Using these results, the state of maximum in-plane shear stress can be represented by the differential element shown in Fig. *c*.

$$\theta_s = -16.8°$$



Ans:
$\sigma_{avg} = -40.0$ MPa,
$\sigma_1 = 32.1$ MPa,
$\sigma_2 = -112$ MPa,
$\theta_{p1} = 28.2°$,
$\tau_{\substack{\max \\ \text{in-plane}}} = 72.1$ MPa,
$\theta_s = -16.8°$

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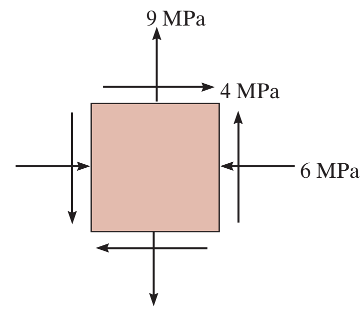
***9–52.** Determine the equivalent state of stress if an element is oriented 30° clockwise from the element shown. Show the result on the element.



In accordance to the established sign convention, $\sigma_x = -6$ MPa, $\sigma_y = 9$ MPa, and $\tau_{xy} = 4$ MPa. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-6 + 9}{2} = 1.50 \text{ MPa}$$

Then, the coordinates of reference point A and C are

$$A(-6, 4) \qquad C(1.5, 0)$$

The radius of the circle is

$$R = CA = \sqrt{(-6 - 1.5)^2 + 4^2} = 8.50 \text{ MPa}$$

Using these results, the circle shown in Fig. *a* can be constructed.

Referring to the geometry of the circle, Fig. *a*,

$$\alpha = \tan^{-1}\left(\frac{4}{6 + 1.5}\right) = 28.07° \qquad \beta = 60° - 28.07° = 31.93°$$

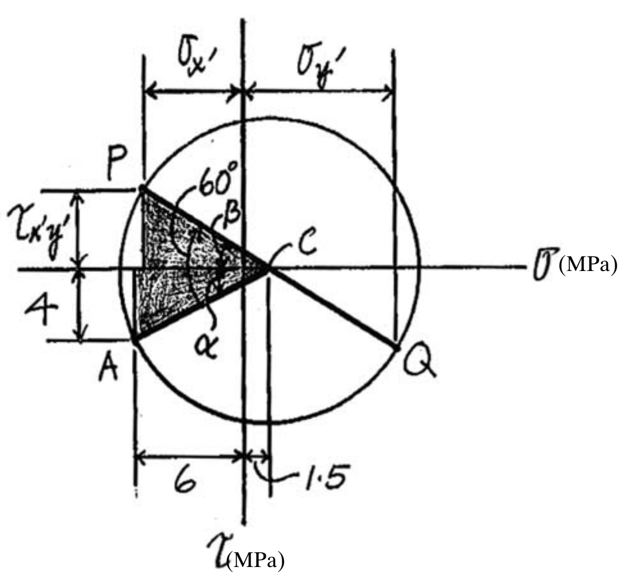
Then,

$$\sigma_{x'} = 1.5 - 8.50 \cos 31.93° = -5.71 \text{ MPa} \qquad \textbf{Ans.}$$

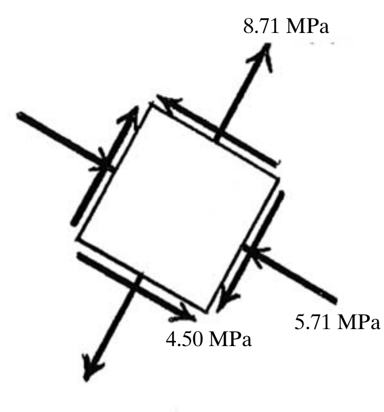
$$\tau_{x'y'} = -8.5 \sin 31.95° = -4.50 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{y'} = 8.71 \text{ MPa} \qquad \textbf{Ans.}$$

The results are shown in Fig. *b*.



(a)



(b)

Ans.
$\sigma_{x'} = -5.71$ MPa, $\tau_{x'y'} = -4.50$ MPa,
$\sigma_{y'} = 8.71$ MPa

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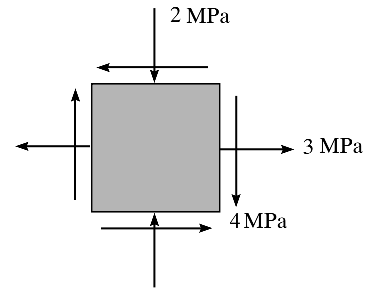
9–53. Determine the equivalent state of stress if an element is oriented 20° clockwise from the element shown.



SOLUTION

Construction of the Circle: In accordance with the sign convention, $\sigma_x = 3$ MPa, $\sigma_y = -2$ MPa, $\tau_{xy} = -4$ MPa. Hence,

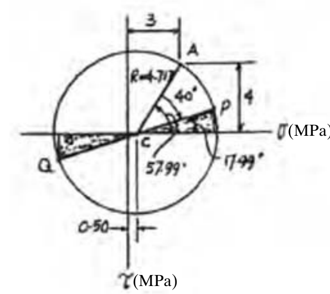
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{3 + (-2)}{2} = 0.500 \text{ MPa}$$

The coordinates for reference points A and C are

$$A(3, -4) \qquad C(0.500, 0)$$

The radius of the circle is

$$R = \sqrt{(3 - 0.500)^2 + 4^2} = 4.717 \text{ MPa}$$

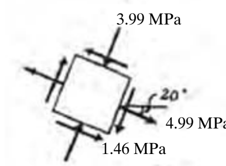


Stress on the Rotated Element: The normal and shear stress components $(\sigma_{x'}$ and $\tau_{x'y'})$ are represented by the coordinate of point P on the circle, $\sigma_{y'}$, can be determined by calculating the coordinates of point Q on the circle.

$$\sigma_{x'} = 0.500 + 4.717 \cos 17.99° = 4.99 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = -4.717 \sin 17.99° = -1.46 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{y'} = 0.500 - 4.717 \cos 17.99° = -3.99 \text{ MPa} \qquad \textbf{Ans.}$$



Ans.
$\sigma_{x'} = 4.99$ MPa, $\tau_{x'y'} = -1.46$ MPa,
$\sigma_{y'} = -3.99$ MPa

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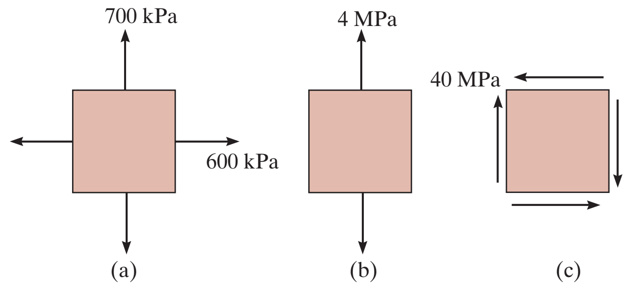
9–54. Draw Mohr's circle that describes each of the following states of stress.



SOLUTION

a) Here, $\sigma_x = 600$ KPa, $\sigma_y = 700$ KPa and $\tau_{xy} = 0$. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{600 + 700}{2} = 650 \text{ KPa}$$

Thus, the coordinate of reference point A and center of circle are

$$A(600, 0) \qquad C(650, 0)$$

Then the radius of the circle is

$$R = CA = 650 - 600 = 50 \text{ KPa}$$

The Mohr's circle represents this state of stress is shown in Fig. *a*.

b) Here, $\sigma_x = 0, \sigma_y = 4$ MPa and $\tau_{xy} = 0$. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + 4}{2} = 2 \text{ MPa}$$

Thus, the coordinate of reference point A and center of circle are

$$A(0, 0) \qquad C(2, 0)$$

Then the radius of the circle is

$$R = CA = 2 - 0 = 2 \text{ MPa}$$

c) Here, $\sigma_x = \sigma_y = 0$ and $\tau_{xy} = -40$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = 0$$

Thus, the coordinate of reference point A and the center of circle are

$$A(0, -40) \qquad C(0, 0)$$

Then, the radius of the circle is

$$R = CA = 40 \text{ MPa}$$

The Mohr's circle represents this state of stress shown in Fig. *c*

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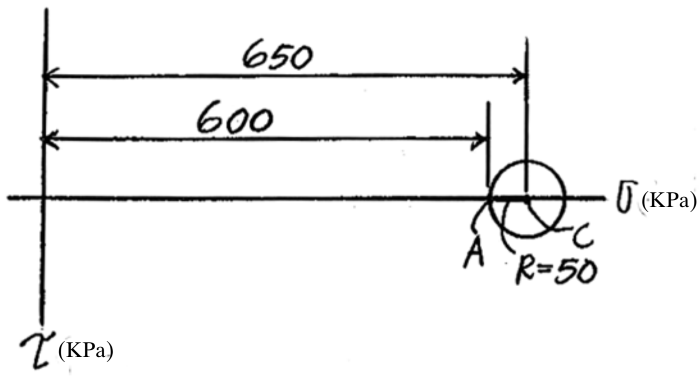
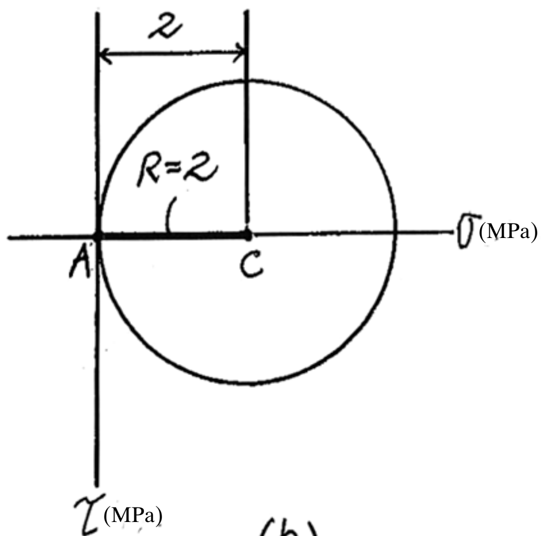
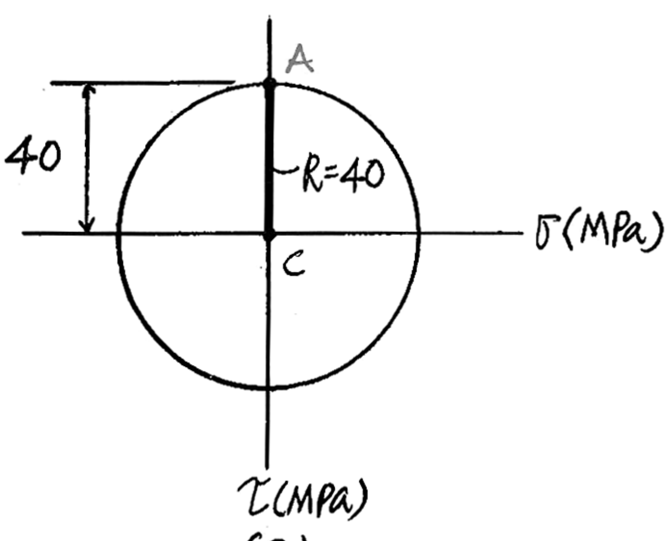
9–54. Continued



(a)



(b)



(c)

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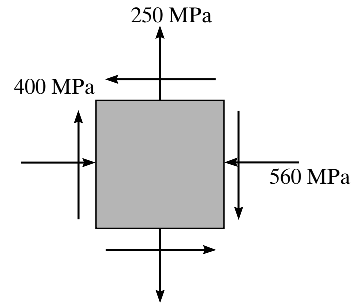
9–55. Determine the equivalent state of stress for an element oriented 60° counterclockwise from the element shown. Show the result on the element.



In accordance to the established sign convention, $\sigma_x = -560$ MPa, $\sigma_y = 250$ MPa and $\tau_{xy} = -400$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-560 + 250}{2} = -155 \text{ MPa}$$

Then, the coordinate of reference points A and C are

$$A(-560, -400) \qquad C(-155, 0)$$

The radius of the circle is

$$R = CA = \sqrt{\left[-560 - (-155)\right]^2 + (-400)^2} = 569.23 \text{ MPa}$$

Using these results, the circle shown in Fig. a can be constructed.

Referring to the geometry of the circle, Fig. a

$$\alpha = \tan^{-1}\left(\frac{400}{560 - 155}\right) = 44.64° \qquad \beta = 120° - 44.64° = 75.36°$$

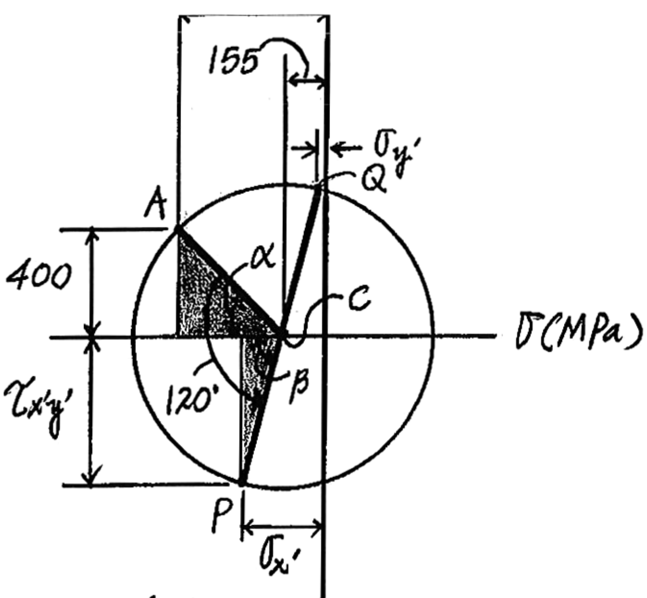
Then,

$$\sigma_{x'} = -155 - 569.23 \cos 75.36° = -299 \text{ MPa} \qquad \textbf{Ans.}$$

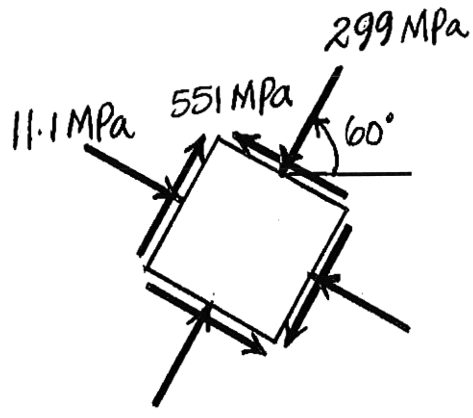
$$\tau_{x'y'} = 569.23 \sin 75.36° = 551 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{y'} = -155 + 569.23 \cos 75.36° = -11.1 \text{ MPa} \qquad \textbf{Ans.}$$

The results are shown in Fig. b.



(a)



(b)

Ans.
$\sigma_{\text{avg}} = -155$ MPa, $R = 569.23$ MPa,
$\sigma_{x'} = -299$ MPa, $\tau_{x'y'} = 551$ MPa,
$\sigma_{y'} = -11.1$ MPa

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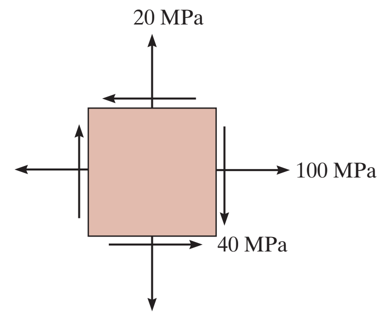
***9–56.**

Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



SOLUTION

Construction of the Circle: In accordance to the sign convention, $\sigma_x = 100$ MPa, $\sigma_y = 20$ MPa, and $\tau_{xy} = -40$ MPa. Hence

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{100 + 20}{2} = 60.0 \text{ MPa} \qquad \textbf{Ans.}$$

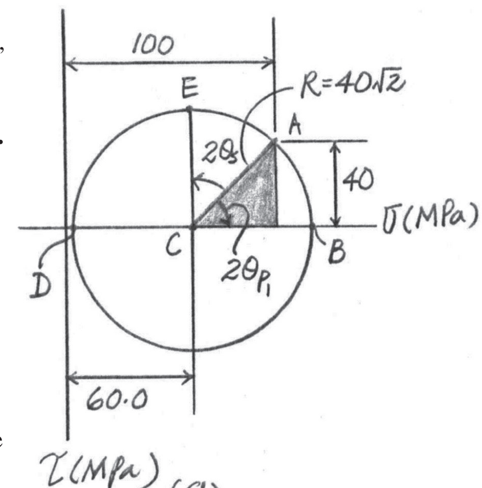
The coordinates for reference point A and center of circle C are

$$A(100, -40) \qquad C(60.0, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(100 - 60.0)^2 + (-40 - 0)^2} = 40\sqrt{2} \text{ MPa}$$

Using these results, the circle shown in Fig. a can be constructed.



(a)

a) In-Plane Principal Stresses: The coordinates of points B and D on the circle represent σ_1 and σ_2 respectively.

$$\sigma_1 = 60.0 + 40\sqrt{2} = 116.57 \text{ MPa} = 117 \text{ MPa} \qquad \textbf{Ans.}$$

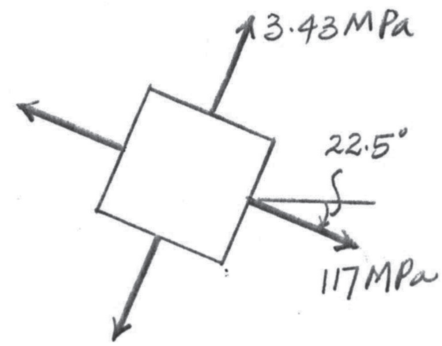
$$\sigma_2 = 60.0 - 40\sqrt{2} = 3.4315 \text{ MPa} = 3.43 \text{ MPa} \qquad \textbf{Ans.}$$

Orientation of Principal Plane: From the shaded triangle on the circle,

$$\tan 2\theta_{p1} = \frac{40}{100 - 60} = 1 \qquad 2\theta_{p1} = 45.0^\circ$$

$$\theta_{p1} = 22.5^\circ \textbf{ (Clockwise)} \qquad \textbf{Ans.}$$

Using these results, the state of in-plane principal stress can be represented by the differential element shown in Fig. b.



(b)

b) Maximum In-Plane Shear Stress: Represented by the coordinates of point E on the circle.

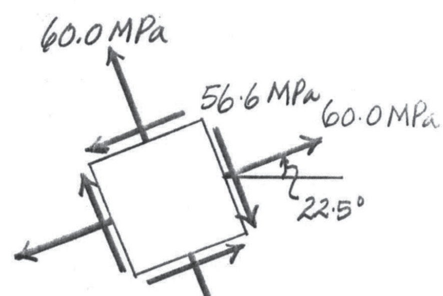
$$\tau_{\substack{\max \\ \text{in-plane}}} = -R = -40\sqrt{2} \text{ MPa} = -56.6 \text{ MPa} \qquad \textbf{Ans.}$$

Orientation of the Plane for Maximum In-Plane Shear Stress: From the circle,

$$\tan 2\theta_s = \frac{100 - 60.0}{40} = 1 \qquad 2\theta_s = 45.0^\circ$$

$$\theta_s = 22.5^\circ \textbf{ (Counterclockwise)} \qquad \textbf{Ans.}$$

Using these results, the state of maximum in-plane shear stress can be represented by the differential element shown in Fig. c.



(c)

Ans:
$\sigma_{avg} = 60.0$ MPa, $\sigma_1 = 117$ MPa, $\sigma_2 = 3.43$ MPa,
$\tau_{\substack{\max \\ \text{in-plane}}} = -56.6$ MPa, $\theta_s = 22.5^\circ$,
$\theta_{p1} = 22.5^\circ$ **(Clockwise)**

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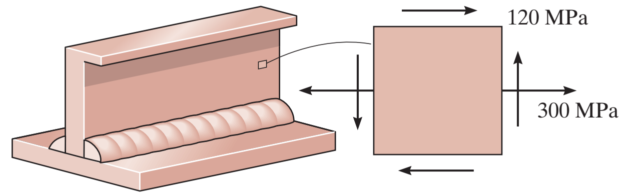
9–57. Determine the principal stress, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.



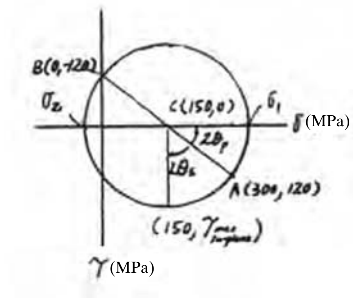
SOLUTION

$A(300, 120) \qquad B(0, -120) \qquad C(150, 0)$

$R = \sqrt{(300 - 150)^2 + 120^2} = 192.094$

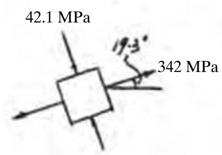
$\sigma_1 = 150 + 192.094 = 342 \text{ MPa}$ **Ans.**

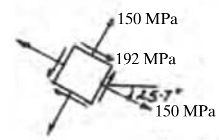
$\sigma_2 = 150 - 192.094 = -42.1 \text{ MPa}$ **Ans.**



$$\tan 2\theta_P = \frac{120}{300 - 150} = 0.8$$

$\theta_{P_1} = 19.3° \quad (Counterclockwise)$ **Ans.**





$\sigma_{avg} = 150 \text{ MPa}$ **Ans.**

$\tau_{\substack{max \\ in\text{-}plane}} = 192 \text{ MPa}$ **Ans.**

$$\tan 2\theta_s = \frac{300 - 150}{120} = 1.25$$

$\theta_s = 25.7° \quad (Clockwise)$

Ans.
$R = 192.094, \sigma_1 = 342 \text{ MPa},$
$\sigma_2 = -42.1 \text{ MPa}, \theta_P = 19.3° \ (Counterclockwise),$
$\sigma_{avg} = 150 \text{ MPa}, \tau_{\substack{max \\ in\text{-}plane}} = 19.2 \text{ MPa},$
$\theta_s = 25.7° \ (Clockwise)$

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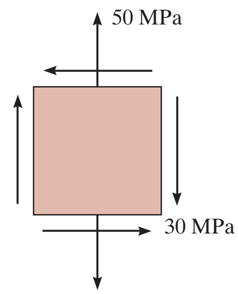
9–58.

Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



SOLUTION

$A(0, -30) \qquad B(50, 30) \qquad C(25, 0)$

$R = CA = \sqrt{(25 - 0)^2 + 30^2} = 39.05$

$\sigma_1 = 25 + 39.05 = 64.1 \text{ MPa}$ **Ans.**

$\sigma_2 = 25 - 39.05 = -14.1 \text{ MPa}$ **Ans.**

$$\tan 2\theta_P = \frac{30}{25 - 0} = 1.2$$

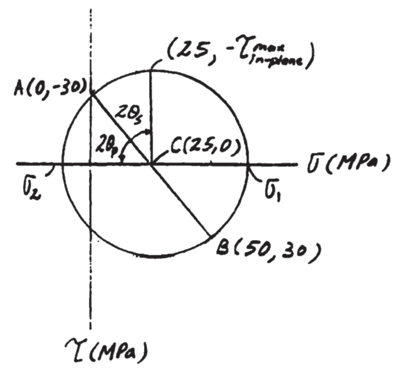
$\theta_P = 25.1^\circ$ **Ans.**

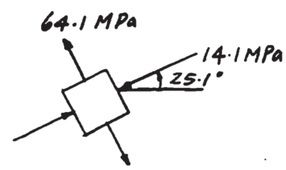
$\sigma_{avg} = 25.0 \text{ MPa}$ **Ans.**

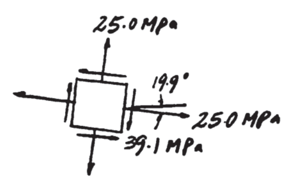
$\tau_{\substack{max \\ in\text{-}plane}} = R = 39.1 \text{ MPa}$ **Ans.**

$$\tan 2\theta_s = \frac{25 - 0}{30} = 0.8333$$

$\theta_s = -19.9^\circ$ **Ans.**







Ans:
$\sigma_1 = 64.1 \text{ MPa}$,
$\sigma_2 = -14.1 \text{ MPa}$,
$\theta_P = 25.1^\circ$,
$\sigma_{avg} = 25.0 \text{ MPa}$,
$\tau_{\substack{max \\ in\text{-}plane}} = 39.1 \text{ MPa}$,
$\theta_s = -19.9^\circ$

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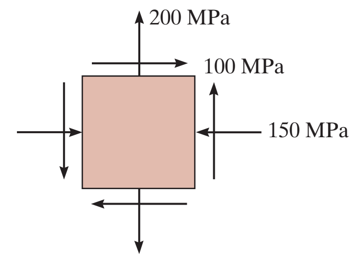
9–59.

Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



SOLUTION

$A(-150, 100) \qquad B(200, -100) \qquad C(25, 0)$

$$R = CA = \sqrt{(150 + 25)^2 + 100^2} = 201.556$$

$$\tan 2\theta_P = \frac{100}{150 + 25} = 0.5714$$

$\theta_P = -14.9°$ **Ans.**

$\sigma_1 = 25 + 201.556 = 227 \text{ MPa}$ **Ans.**

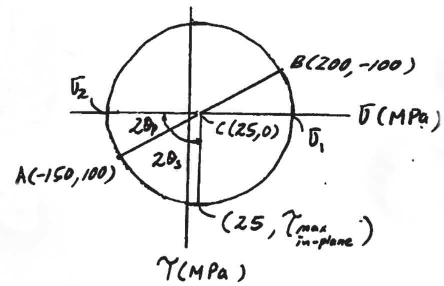
$\sigma_2 = 25 - 201.556 = -177 \text{ MPa}$ **Ans.**

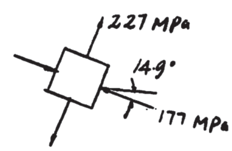
$\tau_{\substack{\max \\ \text{in-plane}}} = R = 202 \text{ MPa}$ **Ans.**

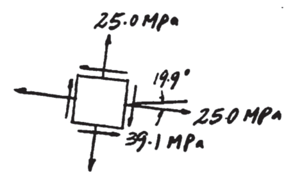
$\sigma_{avg} = 25 \text{ MPa}$ **Ans.**

$$\tan 2\theta_s = \frac{150 + 25}{100} = 1.75$$

$\theta_s = 30.1°$ **Ans.**







Ans:
$\theta_P = -14.9°$,
$\sigma_1 = 227 \text{ MPa}$,
$\sigma_2 = -177 \text{ MPa}$,
$\tau_{\substack{\max \\ \text{in-plane}}} = 202 \text{ MPa}$,
$\sigma_{avg} = 25 \text{ MPa}$,
$\theta_s = 30.1°$

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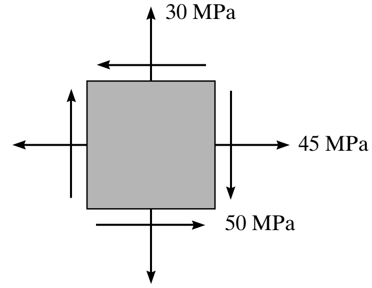
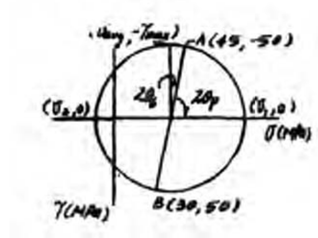
***9–60.** Determine the principal stress, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.



SOLUTION

$A(45, -50) \qquad B(30, 50) \qquad C(37.5, 0)$

$R = CA = CB = \sqrt{7.5^2 + 50^2} = 50.56$

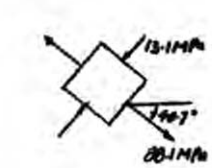


a)

$$\sigma_1 = 37.5 + 50.56 = 88.1 \text{ MPa}$$ **Ans.**

$$\sigma_2 = 37.5 - 50.56 = -13.1 \text{ MPa}$$ **Ans.**

$$\tan 2\theta_P = \frac{50}{7.5} \qquad 2\theta_P = 81.47° \qquad \theta_P = 40.7° \qquad (\textit{Clockwise})$$ **Ans.**



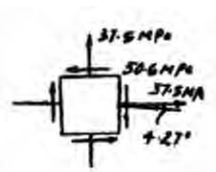
b)

$$\tau_{\substack{\text{max} \\ \text{in-plane}}} = R = 50.6 \text{ MPa}$$ **Ans.**

$$\sigma_{\text{avg}} = 37.5 \text{ MPa}$$ **Ans.**

$$2\theta_s = 90 - 2\theta_P$$

$$\theta_s = 4.27° \qquad (\textit{Counterclockwise})$$ **Ans.**



Ans.
$\sigma_1 = 88.1$ MPa, $\sigma_2 = -13.1$ MPa, $\theta_p = 40.7°$
$\tau_{\substack{\text{max} \\ \text{in-plane}}} = 50.6$ MPa, $\sigma_{\text{avg}} = 37.5$ MPa, $\theta_s = 4.27°$

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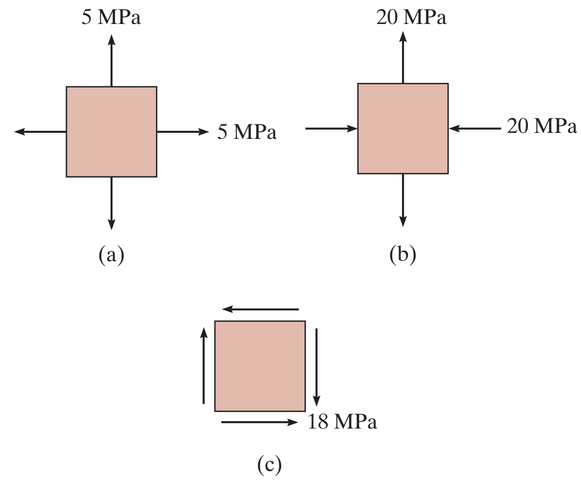
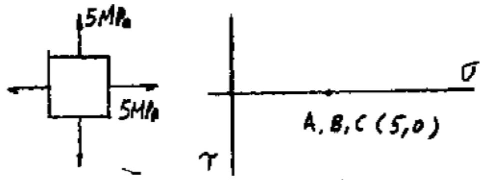
9–61. Draw Mohr's circle that describes each of the following states of stress.

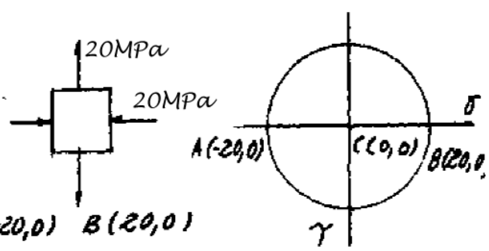


a)



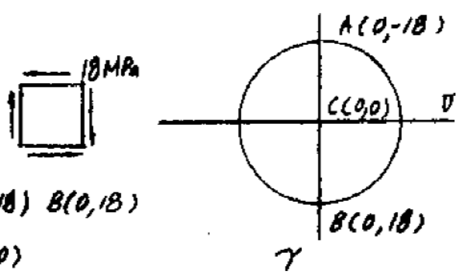
A(5,0) B(5,0) C(5,0)

b)



A(-20,0) B(20,0)
C(0,0)

c)



A(0,-18) B(0,18)
C(0,0)

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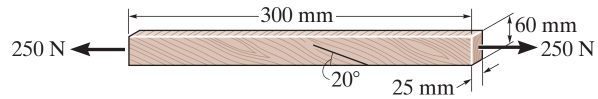
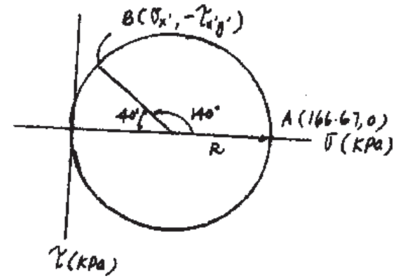
9–62.

The grains of wood in the board make an angle of 20° with the horizontal as shown. Determine the normal and shear stresses that act perpendicular and parallel to the grains if the board is subjected to an axial load of 250 N.



SOLUTION

$$\sigma_x = \frac{P}{A} = \frac{250}{(0.06)(0.025)} = 166.67\text{ kPa}$$



$R = 83.33$

Coordinates of point *B*:

$\sigma_{x'} = 83.33 - 83.33\cos 40°$

$\sigma_{x'} = 19.5\text{ kPa}$ **Ans.**

$\tau_{x'y'} = -83.33\sin 40° = -53.6\text{ kPa}$ **Ans.**

Ans:
$\sigma_{x'} = 19.5\text{ kPa}$, $\tau_{x'y'} = -53.6\text{ kPa}$

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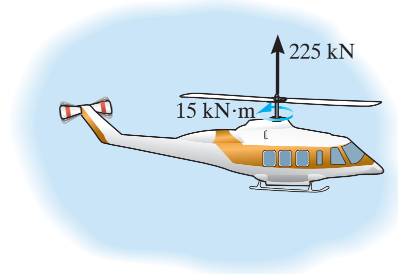
9–63. The rotor shaft of the helicopter is subjected to the tensile force and torque shown when the rotor blades provide the lifting force to suspend the helicopter at midair. If the shaft has a diameter of 150 mm, determine the principal stress and maximum in-plane shear stress at a point located on the surface of the shaft.

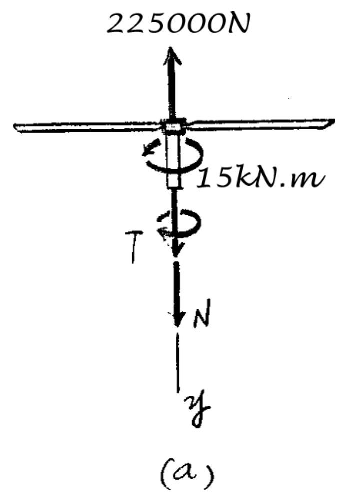


SOLUTION

Internal Loadings: Considering the equilibrium of the free-body diagram of the rotor shaft's upper segment, Fig. *a*,

$\Sigma F_y = 0;\quad N - 225 = 0\quad N = 225\text{ kN}$

$\Sigma M_y = 0;\quad T - 15 = 0\quad T = 15\text{ kN.m}$



Section Properties: The cross-sectional area and the polar moment of inertia of the rotor shaft's cross section are

$$A = \pi(0.075^2) = 5.625(10^{-3})\pi\ \text{m}^2$$

$$J = \frac{\pi}{2}(0.075^4) = 49.7010(10^{-6})\ \text{m}^4$$

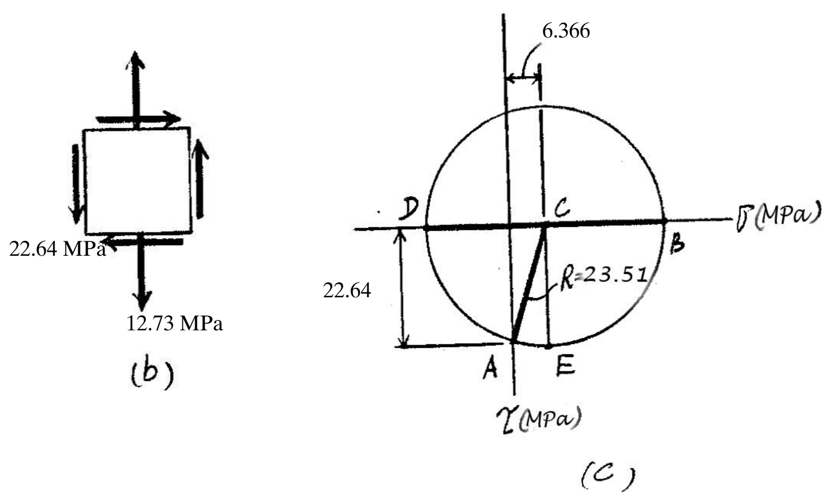
Normal and Shear Stress: The normal stress is contributed by axial stress only.

$$\sigma = \frac{N}{A} = \frac{225(10^3)}{5.625(10^{-3})\pi} = 12.73(10^6)\ \text{N/m}^2 = 12.73\ \text{MPa}$$

The shear stress is contributed by the torsional shear stress only.

$$\tau = \frac{[15(10^3)](0.075)}{49.7010(10^{-6})} = 22.64(10^6)\ \text{N/m}^2 = 22.64\ \text{MPa}$$

The state of stress at point A is represented by the element shown in Fig. *b*.



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9–63. Continued

Construction of the Circle: $\sigma_x = 0$, $\sigma_y = 12.73$ MPa, and $\tau_{xy} = 22.64$ MPa. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + 12.73}{2} = 6.366 \text{ MPa}$$

The coordinates of reference point A and the center C of the circle are

$$A(0, 22.64) \qquad C(6.366, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(0 - 6.366)^2 + 22.64^2} = 23.51 \text{ MPa}$$

Using these results, the circle is shown in Fig. *c*.

In-Plane Principal Stress: The coordinates of reference points B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 6.366 + 23.51 = 29.9 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = 6.366 - 23.51 = -17.1 \text{ MPa} \qquad \textbf{Ans.}$$

Maximum In-Plane Shear Stress: The state of maximum shear stress is represented by the coordinates of point E, Fig. *a*.

$$\tau_{\substack{\max \\ \text{in-plane}}} = R = 23.5 \text{ MPa} \qquad \textbf{Ans.}$$

Ans.

$\tau_{\substack{\max \\ \text{in-plane}}} = 23.5$ MPa, $\sigma_1 = 29.9$ MPa, $\sigma_2 = -17.1$ MPa

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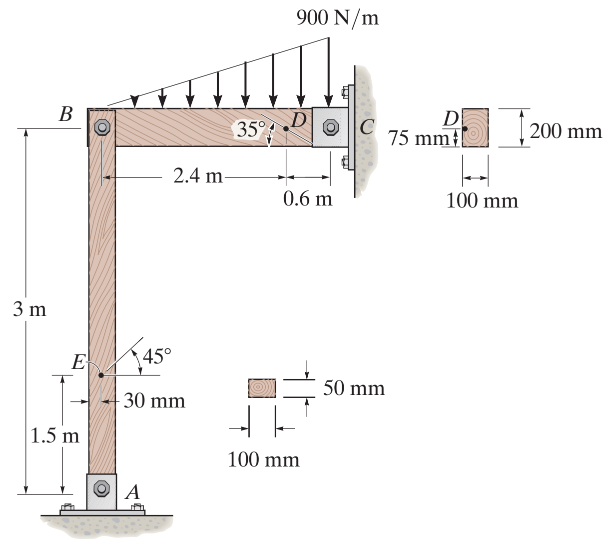
***9–64.**

The frame supports the triangular distributed load shown. Determine the normal and shear stresses at point D that act perpendicular and parallel, respectively, to the grains. The grains at this point make an angle of 35° with the horizontal as shown.



SOLUTION

Support Reactions and Internal Loadings: Referring to the FBD of the entire beam, Fig. *a*,

$$\circlearrowleft + \Sigma M_C = 0; \qquad \left[\frac{1}{2}(900)(3)\right](1) - B_y(3) = 0 \qquad B_y = 450 \text{ N}$$

Referring to the FBD of the left segment of the sectioned beam, Fig. *b*,

$$+\uparrow \Sigma F_y = 0; \qquad 450 - \frac{1}{2}(720)(2.4) - V_D = 0 \qquad V_D = -414 \text{ N}$$

$$\circlearrowleft + \Sigma M_D = 0; \qquad M_D + \left[\frac{1}{2}(720)(2.4)\right](0.8) - 450(2.4) = 0 \qquad M_D = 388.8 \text{ N} \cdot \text{m}$$

Section Properties: For the rectangular cross section, Fig. *c*,

$$I = \frac{1}{12}(0.1)\left(0.2^3\right) = 66.6667\left(10^{-6}\right) \text{ m}^4$$

$$Q_D = \widetilde{y}'A' = 0.0625[0.1(0.075)] = 0.46875\left(10^{-3}\right) \text{ m}^3$$

Normal Stress: The normal stress is contributed by bending stress only. Applying the flexure formula,

$$\sigma_D = \frac{M_D y_D}{I} = \frac{388.8(0.025)}{66.6667\left(10^{-6}\right)} = 145.8\left(10^3\right) \text{ Pa} = 145.8 \text{ kPa (T)}$$

Shear Stress: The shear stress is contributed by the transverse shear stress only. Applying the shear formula,

$$\tau_D = \frac{VQ}{It} = \frac{414\left[0.46875\left(10^{-3}\right)\right]}{66.6667\left(10^{-6}\right)(0.1)} = 29.11\left(10^3\right) \text{ Pa} = 29.11 \text{ kPa}$$

Using these results, the state of stress at point D can be represented by the differential element shown in Fig. *d*.

Construction of The Circle: In accordance to the sign convention, $\sigma_x = 145.8$ kPa, $\sigma_y = 0$ $\tau_{xy} = 29.11$ kPa and $\theta = 55°$ (Counterclockwise, Fig. *e*). Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{145.8 + 0}{2} = 72.9 \text{ kPa}$$

The coordinates of reference point A and center of circle C are

$$A(145.8, 29.11) \qquad C(72.9, 0)$$

The radius of the circle is

$$R = CA = \sqrt{(145.8 - 72.9)^2 + (29.11 - 0)^2} = 78.497 \text{ kPa}$$

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***9–64. Continued**

Using these results, the circle shown in Fig. *f*, can be constructed.

Stress on the Inclined Plane: The normal stress and shear stress on the inclined plane are represented by the coordinates of point $P(\sigma_{x'}, \tau_{x'y'})$ on the circle which can be established by rotating radial line CA $2\theta = 110°$ counterclockwise to coincide with radial line CP, Fig. *f*. Here,

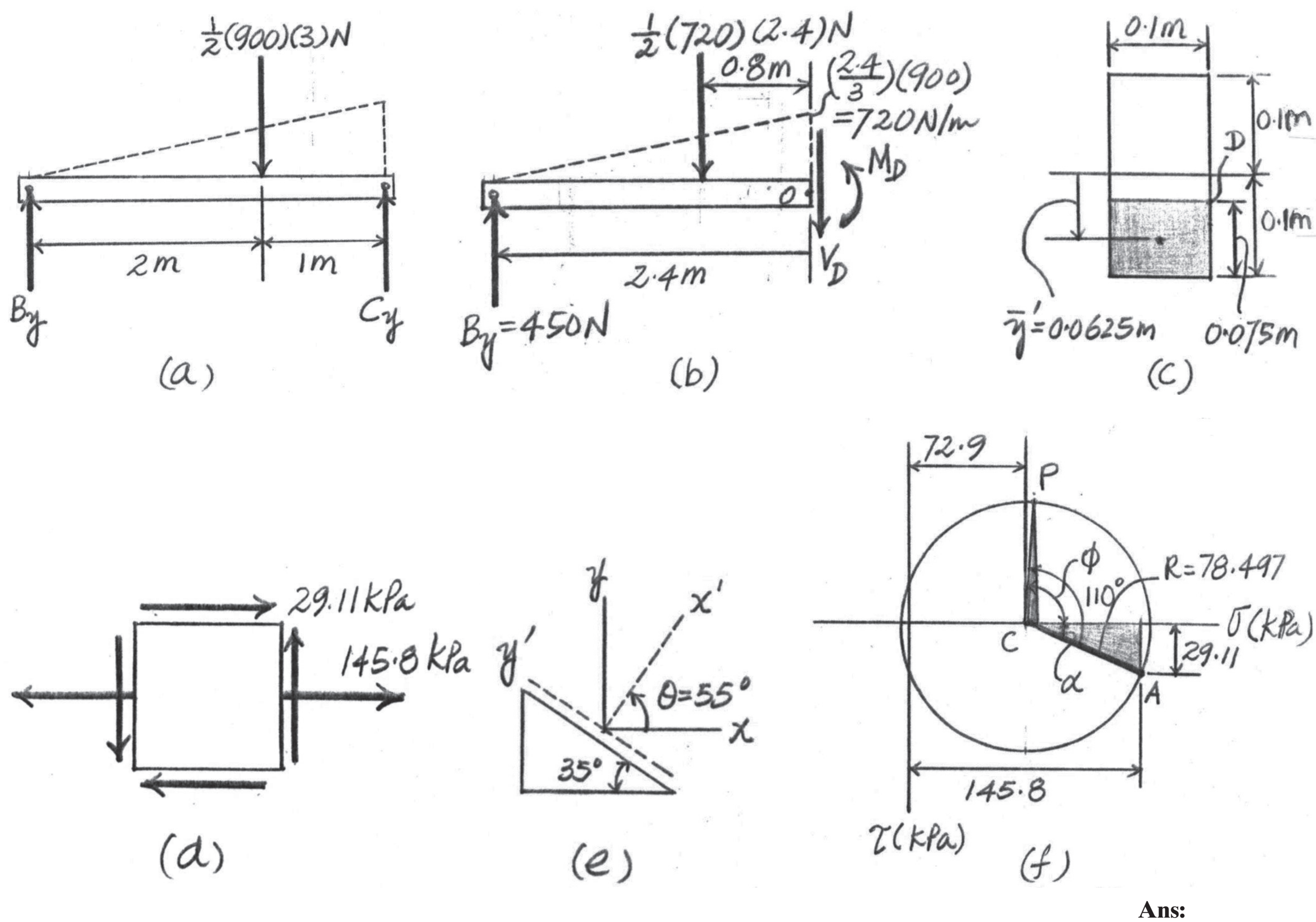
$$\alpha = \tan^{-1}\left(\frac{29.11}{145.8 - 72.9}\right) = 21.77°$$

$$\phi = 110° - 21.77° = 88.23°$$

Then

$$\sigma_{x'} = 72.9 + 78.497 \cos 88.23° = 75.32 \text{ kPa} = 75.3 \text{ kPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = -78.497 \sin 88.23° = -78.46 \text{ kPa} = -78.5 \text{ kPa} \qquad \textbf{Ans.}$$



Ans:
$\sigma_{x'} = 75.3$ kPa,
$\tau_{x'y'} = -78.5$ kPa

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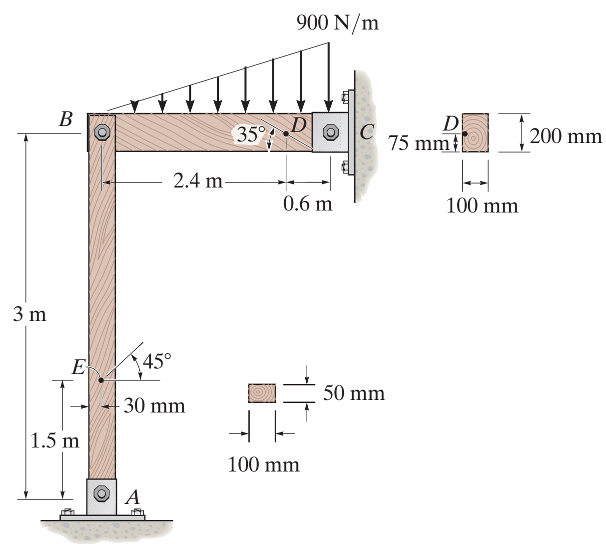
9–65.

The frame supports the triangular distributed load shown. Determine the normal and shear stresses at point E that act perpendicular and parallel, respectively, to the grains. The grains at this point make an angle of 45° with the horizontal as shown.



SOLUTION

Support Reactions and Internal Loadings: Referring to the FBD of the entire beam, Fig. *a*,

$$\circlearrowleft + \Sigma M_C = 0; \qquad \left[\frac{1}{2}(900)(3)\right](1) - B_y(3) = 0 \qquad B_y = 450 \text{ N}$$

Referring to the FBD of the upper segment of the sectioned column, Fig. *b*,

$$+\uparrow \Sigma F_y = 0; \qquad -450 - N_E = 0 \qquad N_E = -450 \text{ N}$$

Section Properties: For the rectangular cross-section,

$$A = 0.05(0.1) = 5.00\left(10^{-3}\right) \text{ m}^2$$

Normal Stress: The normal stress is contributed by axial load only.

$$\sigma_E = \frac{N_E}{A} = \frac{-450}{5.00\left(10^{-3}\right)} = -90.0 \text{ kPa}$$

Using this result, the state of stress at point E can be represented by the differential element shown in Fig. *c*.

Construction of the Circle: In accordance to the sign convention, $\sigma_x = 0$, $\sigma_y = -90.0$ kPa, $\tau_{xy} = 0$ and $\theta = 135°$ (Counterclockwise, Fig. *d*). Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-90.0)}{2} = -45.0 \text{ kPa}$$

The coordinates of reference point A and center of circle C are

$$A(0, 0) \qquad C(-45.0, 0)$$

The radius of the circle is

$$R = CA = 0 - (-45.0) = 45.0 \text{ kPa}$$

Using the results, the circle shown in Fig. *e* can be constructed.

Stress on the Inclined Plane: The normal stress and shear stress on the inclined plane are represented by the coordinates of point $P(\sigma_{x'}, \tau_{x'y'})$ on the circle which can be established by rotating the radial line CA $2\theta = 270°$ counterclockwise to coincide with radial line CP, Fig. *e*. Thus,

$$\sigma_{x'} = -45.0 \text{ kPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = 45.0 \text{ kPa} \qquad \textbf{Ans.}$$

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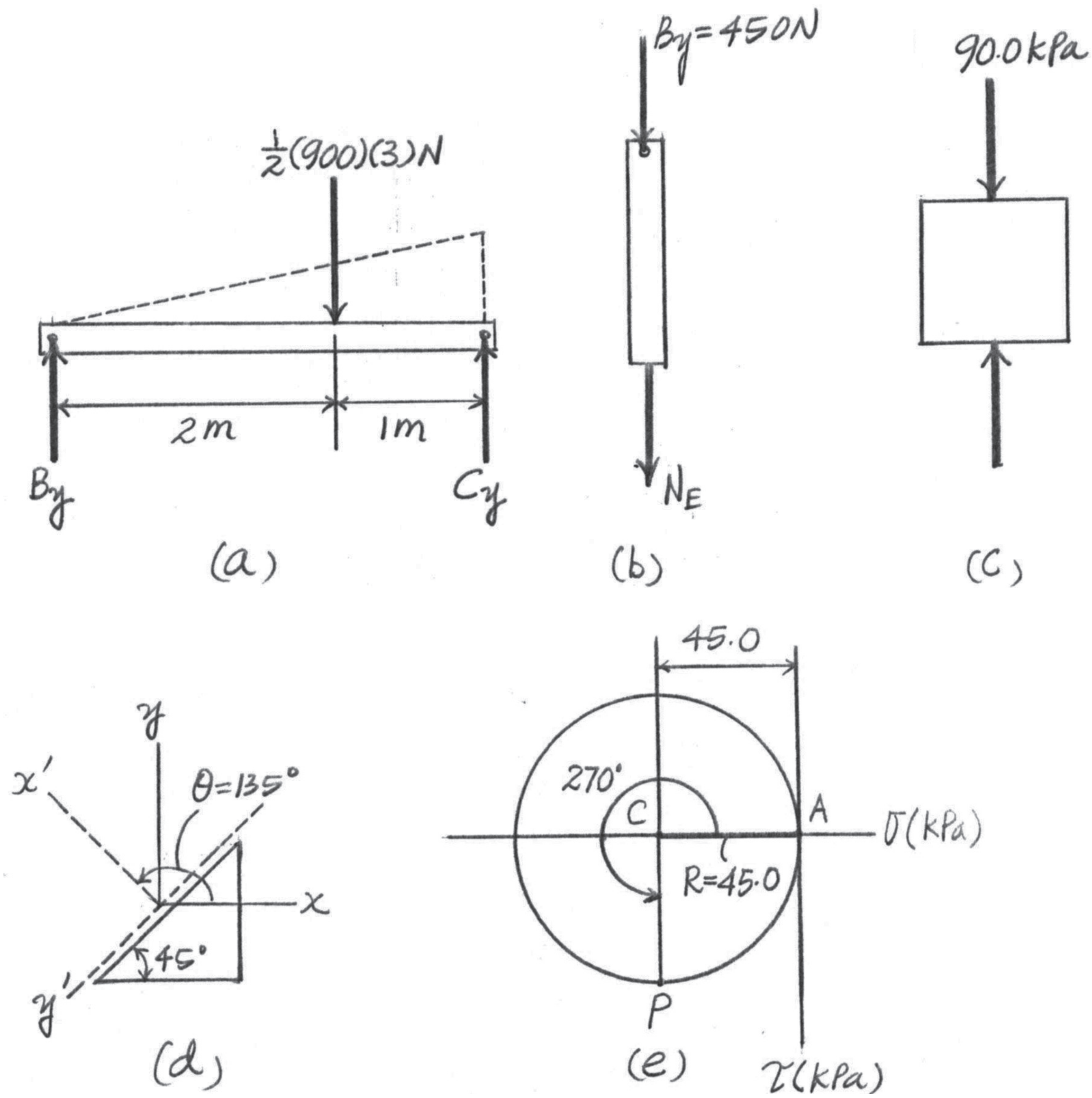
9–65. Continued



Ans:
$\sigma_{x'} = -45.0$ kPa,
$\tau_{x'y'} = 45.0$ kPa

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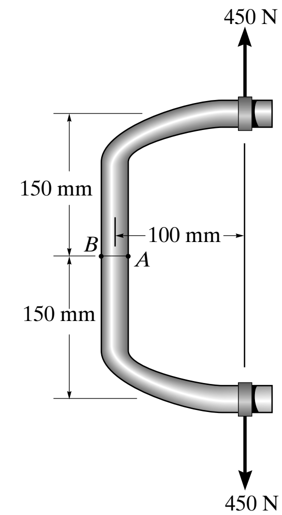
9–66. Determine the principal stresses and the maximum in-plane shear stress that are developed at point A. Show the results on an element located at this point. The rod has a diameter of 40 mm.



SOLUTION

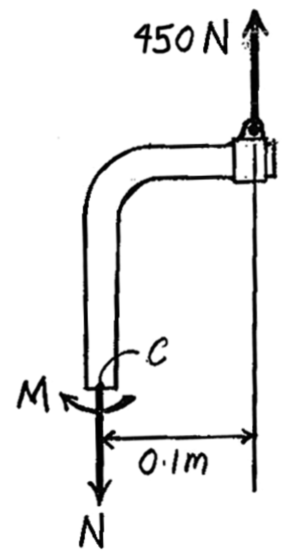
Using the method of sections and consider the FBD of the member's upper cut segment, Fig. *a*,

$+\uparrow \Sigma F_y = 0; \quad 450 - N = 0 \qquad N = 450 \text{ N}$

$\circlearrowleft + \Sigma M_C = 0; \quad 450(0.1) - M = 0 \qquad M = 45 \text{ N} \cdot \text{m}$

$$A = \pi(0.02^2) = 0.4(10^{-3})\pi \text{ m}^2$$

$$I = \frac{\pi}{4}(0.02^4) = 40(10^{-9})\pi \text{ m}^4$$



(a)

The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{My}{I}$$

For point A, $y = C = 0.02$ m.

$$\sigma = \frac{450}{0.4(10^{-3})\pi} + \frac{45\,(0.02)}{40(10^{-9})\pi} = 7.520 \text{ MPa}$$

Since no transverse shear and torque is acting on the cross - section

$$\tau = 0$$

The state of stress at point A can be represented by the element shown in Fig. *b*.

In accordance to the established sign convention $\sigma_x = 0$, $\sigma_y = 7.520$ MPa and $\tau_{xy} = 0$. Thus

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + 7.520}{2} = 3.760 \text{ MPa}$$

Then, the coordinates of reference point A and the center C of the circle are

$$A(0, 0) \qquad C(3.760, 0)$$

Thus, the radius of the circle is

$$R = CA = 3.760 \text{ MPa}$$

Using this results, the circle shown in Fig. *c* can be constructed. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_y = 7.52 \text{ MPa} \qquad \sigma_2 = \sigma_x = 0 \qquad \textbf{Ans.}$$

The state of principal stresses can also be represented by the element shown in Fig. *b*.

The state of maximum in - plane shear stress is represented by point B on the circle, Fig. *c*. Thus.

$$\tau_{\substack{\text{max} \\ \text{in-plane}}} = R = 3.76 \text{ MPa} \qquad \textbf{Ans.}$$

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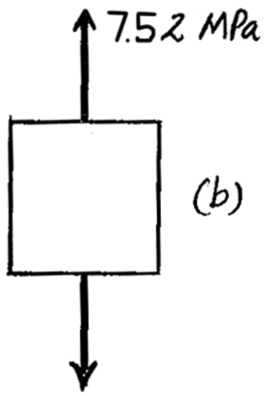
9–66. Continued

From the circle,

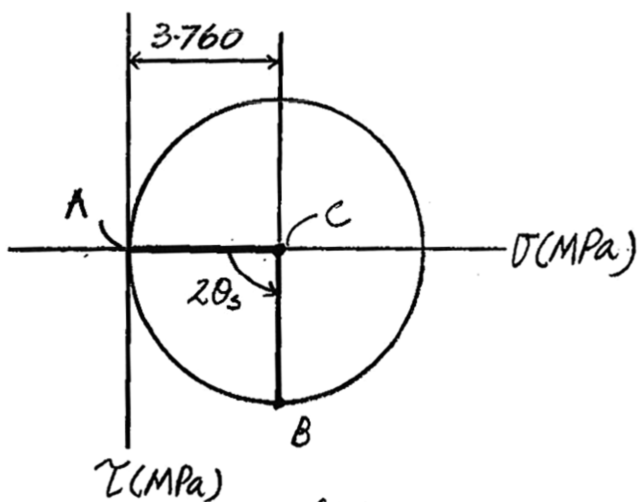
$$2\theta_s = 90^\circ$$

$$\theta_s = 45^\circ \quad (\textit{counter clockwise}) \qquad \textbf{Ans.}$$

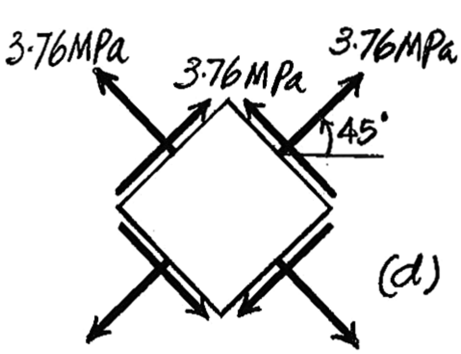
The state of maximum In - Plane shear stress can be represented by the element shown in Fig. *d*.



(b)



(c)



(d)

Ans.

$\sigma_1 = 7.52$ MPa, $\sigma_2 = 0$,
$\tau_{\substack{\max \\ \text{in-plane}}} = 3.76$ MPa,
$\theta_s = 45^\circ$ (*Counterclockwise*)

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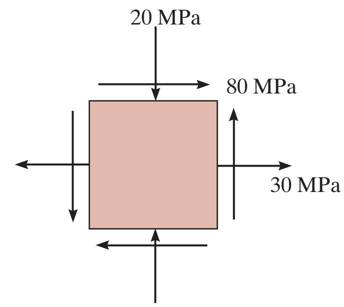
9–67.

Determine the principal stresses, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.



SOLUTION

In accordance to the established sign convention, $\sigma_x = 30$ MPa, $\sigma_y = -20$ MPa and $\tau_{xy} = 80$ MPa. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{30 + (-20)}{2} = 5 \text{ MPa} \qquad \textbf{Ans.}$$

Then, the coordinates of reference point A and the center C of the circle is

$$A(30, 80) \qquad C(5, 0)$$

Thus, the radius of circle is given by

$$R = CA = \sqrt{(30 - 5)^2 + (80 - 0)^2} = 83.815 \text{ MPa}$$

Using these results, the circle shown in Fig. a, can be constructed.

The coordinates of points B and D represent σ_1 and σ_2 respectively. Thus

$$\sigma_1 = 5 + 83.815 = 88.8 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = 5 - 83.815 = -78.8 \text{ MPa} \qquad \textbf{Ans.}$$

Referring to the geometry of the circle, Fig. a

$$\tan 2(\theta_P)_1 = \frac{80}{30 - 5} = 3.20$$

$$\theta_P = 36.3^\circ \ (\textit{Counterclockwise}) \qquad \textbf{Ans.}$$

The state of maximum in-plane shear stress is represented by the coordinate of point E. Thus

$$\tau_{\substack{\max \\ \text{in-plane}}} = R = 83.8 \text{ MPa} \qquad \textbf{Ans.}$$

From the geometry of the circle, Fig. a,

$$\tan 2\theta_s = \frac{30 - 5}{80} = 0.3125$$

$$\theta_s = 8.68^\circ \quad (\textit{Clockwise}) \qquad \textbf{Ans.}$$

The state of maximum in-plane shear stress is represented by the element in Fig. c.

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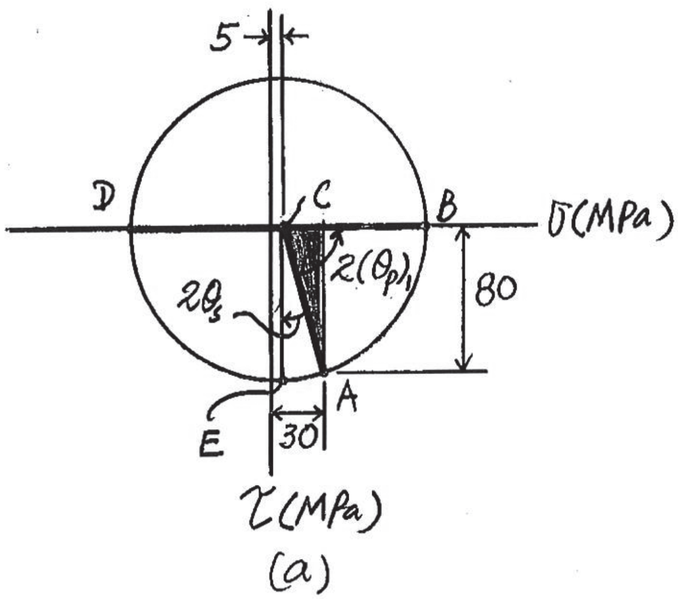
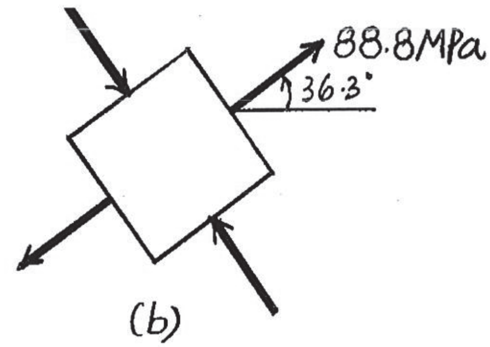
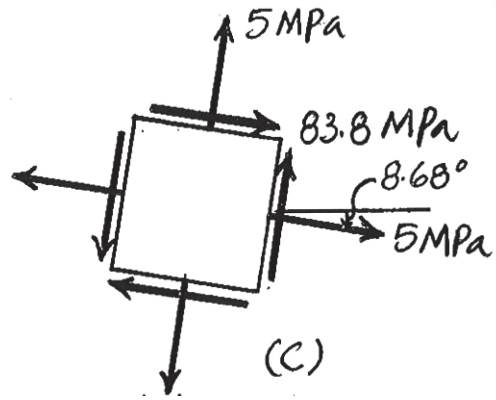
9–67. Continued







Ans:
$\sigma_{avg} = 5$ MPa,
$\sigma_1 = 88.8$ MPa,
$\sigma_2 = -78.8$ MPa,
$\theta_P = 36.3°$ (*Counterclockwise*),
$\tau_{\substack{max \\ in\text{-}plane}} = 83.8$ MPa,
$\theta_s = 8.68°$ (*Clockwise*)

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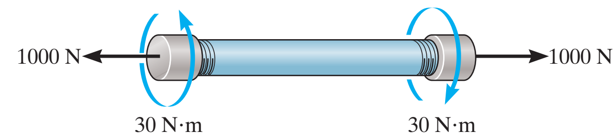
***9–68.** The thin-walled pipe has an inner diameter of 12 mm and a thickness of 0.6 mm. If it is subjected to an internal pressure of 3.5 MPa and the axial tension and torsional loadings shown, determine the principal stress at a point on the surface of the pipe.

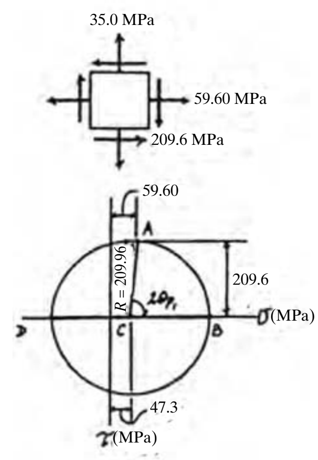


SOLUTION

Section Properties:

$$A = \frac{\pi}{4}(0.0066^2 - 0.006^2) = 7.56(10^{-6})\pi \text{ m}^2$$

$$J = \frac{\pi}{2}(0.0066^4 - 0.006^4) = 0.94479(10^{-9}) \text{ m}^4$$



Normal Stress: Since $\frac{r}{t} = \frac{6}{0.6} = 10$, thin wall analysis is valid.

$$\sigma_{\text{long}} = \frac{N}{A} + \frac{pr}{2t} = \frac{1000}{7.56(10^{-6})\pi} + \frac{[3.5(10^6)](0.006)}{2(0.0006)}$$

$$= 59.60(10^6) \text{ N/m}^2 = 59.60 \text{ MPa}$$

$$\sigma_{\text{hoop}} = \frac{pr}{t} = \frac{[3.5(10^6)](0.006)}{0.0006} = 35.0(10^6) \text{ N/m}^2 = 35.0 \text{ MPa}$$

Shear Stress: Applying the torsion formula,

$$\tau = \frac{Tc}{J} = \frac{30(0.0066)}{0.94479(10^{-9})} = 209.57(10^6) \text{ N/m}^2 = 209.57 \text{ MPa}$$

Construction of the Circle: In accordance with the sign convention $\sigma_x = 59.60$ MPa, $\sigma_y = 35.0$ MPa, and $\tau_{xy} = -209.57$ MPa. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{59.60 + 35.0}{2} = 47.30 \text{ MPa}$$

The coordinates for reference points A and C are

$$A(59.60 - 209.57) \qquad C(47.30, 0)$$

The radius of the circle is

$$R = \sqrt{(59.60 - 47.30)^2 + 209.57^2} = 209.93 \text{ MPa}$$

In - Plane Principal Stress: The coordinates of point B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 47.30 + 209.93 = 257 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = 47.30 - 209.93 = -163 \text{ MPa} \qquad \textbf{Ans.}$$

Ans.
$\sigma_1 = 257$ MPa, $\sigma_2 = -163$ MPa

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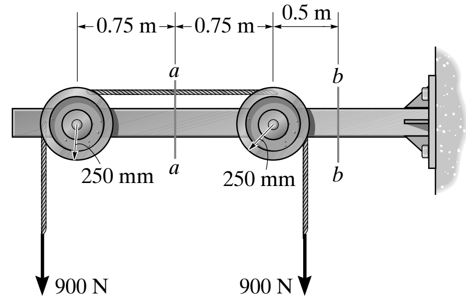
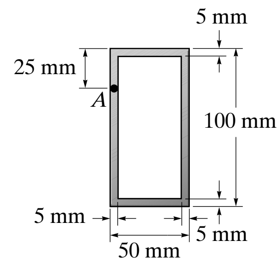
9–69. Determine the principal stress at point A on the cross section of the hanger at section a–a. Specify the orientation of this state of stress and indicate the result on an element at the point.





Sections $a-a$ and $b-b$

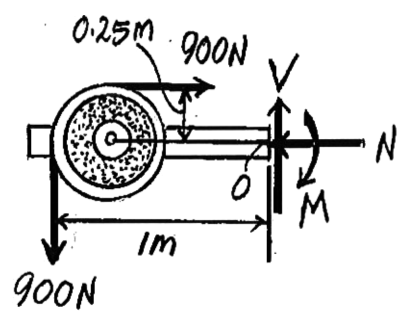
SOLUTION

Internal Loadings: Considering the equilibrium of the free - body diagram of the hanger's left cut segment, Fig. a,

$\xrightarrow{+} \Sigma F_x = 0; \qquad 900 - N = 0 \qquad N = 900\,\text{N}$

$+\uparrow \Sigma F_y = 0; \qquad V - 900 = 0 \qquad V = 900\,\text{N}$

$\circlearrowleft + \Sigma M_O = 0; \qquad 900(1) - 900(0.25) - M = 0 \qquad M = 675\,\text{N}\cdot\text{m}$



(a)

Section Properties: The cross - sectional area and the moment of inertia about the centroidal axis of the hanger's cross section are

$$A = 0.05(0.1) - 0.04(0.09) = 1.4\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.05)\left(0.1^3\right) - \frac{1}{12}(0.04)\left(0.09^3\right) = 1.7367\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. b,

$$Q_A = 2\bar{y}_1'A_1' + \bar{y}_2'A_2' = 2[0.0375(0.025)(0.005)] + 0.0475(0.005)(0.04)$$
$$= 18.875\left(10^{-6}\right)\text{m}^3$$

Normal and Shear Stress: The normal stress is a combination of axial and bending stresses. Thus,

$$\sigma_A = \frac{N}{A} + \frac{My_A}{I} = -\frac{900}{1.4\left(10^{-3}\right)} + \frac{675(0.025)}{1.7367\left(10^{-6}\right)} = 9.074\text{ MPa}$$

The shear stress is caused by the transverse shear stress.

$$\tau_A = \frac{VQ_A}{It} = \frac{900\left[18.875\left(10^{-6}\right)\right]}{1.7367\left(10^{-6}\right)(0.01)} = 0.9782\text{ MPa}$$

The state of stress at point A is represented by the element shown in Fig. c.

Construction of the Circle: $\sigma_x = 9.074$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0.9782$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{9.074 + 0}{2} = 4.537\text{ MPa}$$

The coordinates of reference points A and the center C of the circle are

$$A(9.074, 0.9782) \qquad C(4.537, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(9.074 - 4.537)^2 + 0.9782^2} = 4.641\text{ MPa}$$

Using these results, the circle is shown in Fig. d.

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9–69. Continued

In - Plane Principal Stress: The coordinates of point B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 4.537 + 4.641 = 9.18 \text{ MPa} \qquad \textbf{Ans.}$$

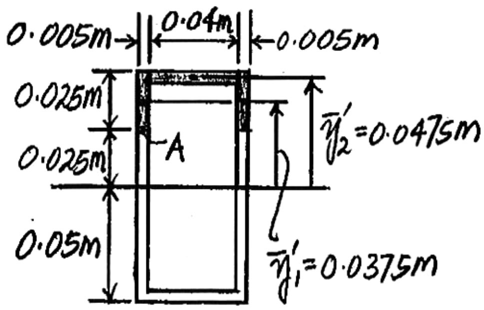
$$\sigma_2 = 4.537 - 4.641 = -0.104 \text{ MPa} \qquad \textbf{Ans.}$$

Orientaion of Principal Plane: Referring to the geometry of the circle, Fig. *d*,

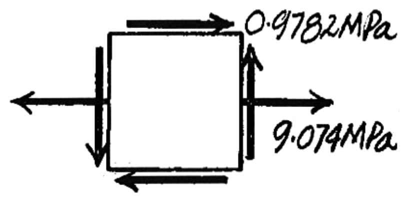
$$\tan 2\left(\theta_P\right)_1 = \frac{0.9782}{9.074 - 4.537} = 0.2156$$

$$\left(\theta_P\right)_1 = 6.08° \text{ (counterclockwise)} \qquad \textbf{Ans.}$$

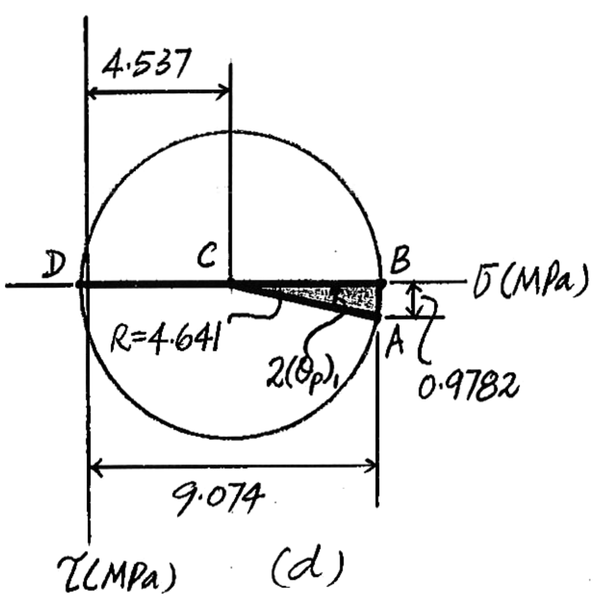
The state of principal stresses is represented on the element shown in Fig. *e*.



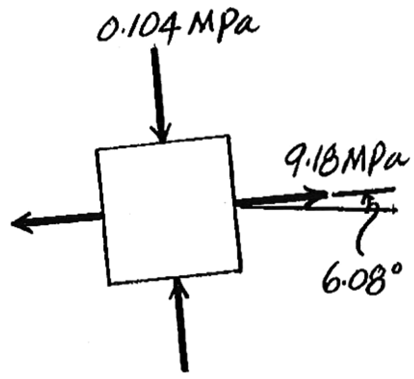
(b)



(c)



(d)



(e)

Ans.
$N = 900$ N, $V = 900$ N, $M = 675$ N·m,
$A = 1.4(10^{-3})$ m^2, $I = 1.7367(10^{-6})$ m^4,
$\sigma_1 = 9.18$ MPa, $\sigma_2 = -0.104$ MPa,
$(\theta_p)_1 = 6.08°$ (*Counterclockwise*)

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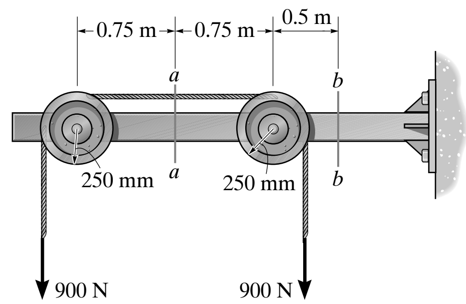
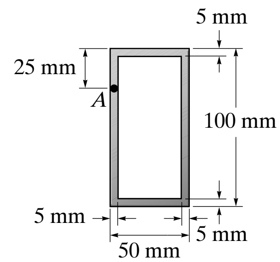
9–70. Determine the principal stress at point A on the cross section of the hanger at section b–b. Specify the orientation of the state of stress and indicate the results on an element at the point.



SOLUTION



Sections a – a and b – b

Internal Loadings: Considering the equilibrium of the free - body diagram of the hanger's left cut segment, Fig. *a*,

$+\uparrow \Sigma F_y = 0; \qquad V - 900 - 900 = 0 \qquad V = 1800\,\text{N}$

$\circlearrowleft + \Sigma M_O = 0; \qquad 900(2.25) + 900(0.25) - M = 0 \qquad M = 2250\,\text{N}\cdot\text{m}$

Section Properties: The cross - sectional area and the moment of inertia about the centroidal axis of the hanger's cross section are

$$A = 0.05(0.1) - 0.04(0.09) = 1.4\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.05)\left(0.1^3\right) - \frac{1}{12}(0.04)\left(0.09^3\right) = 1.7367\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. *b*.

$$Q_A = 2\bar{y}_1'A_1' + \bar{y}_2'A_2' = 2[0.0375(0.025)(0.005)] + 0.0475(0.005)(0.04)$$
$$= 18.875\left(10^{-6}\right)\text{m}^3$$

Normal and Shear Stress: The normal stress is contributed by the bending stress only.

$$\sigma_A = \frac{My_A}{I} = \frac{2250(0.025)}{1.7367\left(10^{-6}\right)} = 32.39\text{ MPa}$$

The shear stress is contributed by the transverse shear stress only.

$$\tau_A = \frac{VQ_A}{It} = \frac{1800\left[18.875\left(10^{-6}\right)\right]}{1.7367\left(10^{-6}\right)(0.01)} = 1.956\text{ MPa}$$

The state stress at point A is represented by the element shown in Fig. *c*.

Construction of the Circle: $\sigma_x = 32.39$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 1.956$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{32.39 + 0}{2} = 16.19\text{ MPa}$$

The coordinates of reference point A and the center C of the circle are

$A(32.39, 1.956) \qquad C(16.19, 0)$

Thus, the radius of the circle is

$$R = CA = \sqrt{(32.39 - 16.19)^2 + 1.956^2} = 16.313\text{ MPa}$$

Using these results, the cricle is shown in Fig. *d*.

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9–70. Continued

In - Plane Principal Stresses: The coordinates of reference point B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 16.19 + 16.313 = 32.5 \text{ MPa} \qquad \textbf{Ans.}$$

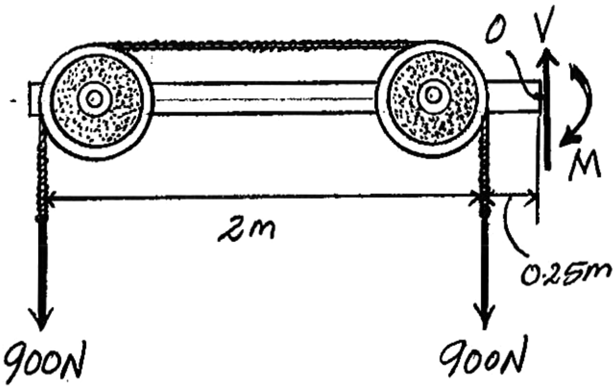
$$\sigma_2 = 16.19 - 16.313 = -0.118 \text{ MPa} \qquad \textbf{Ans.}$$

Orientaion of Principal Plane: Referring to the geometry of the circle, Fig. d,

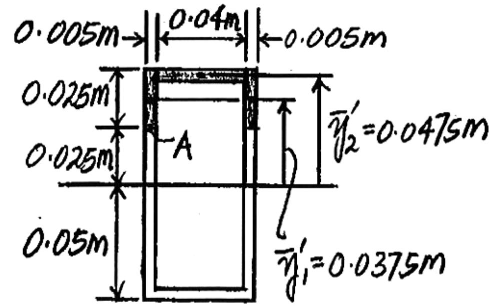
$$\tan 2\left(\theta_P\right)_1 = \frac{1.956}{32.39 - 16.19} = 0.1208$$

$$\left(\theta_P\right)_1 = 3.44° \qquad \text{(counterclockwise)} \qquad \textbf{Ans.}$$

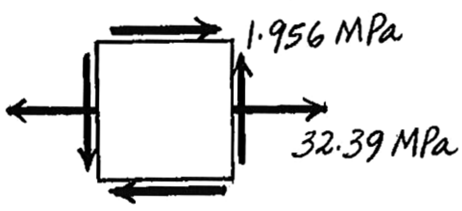
The state of principal stresses is represented on the element shown in Fig. e.



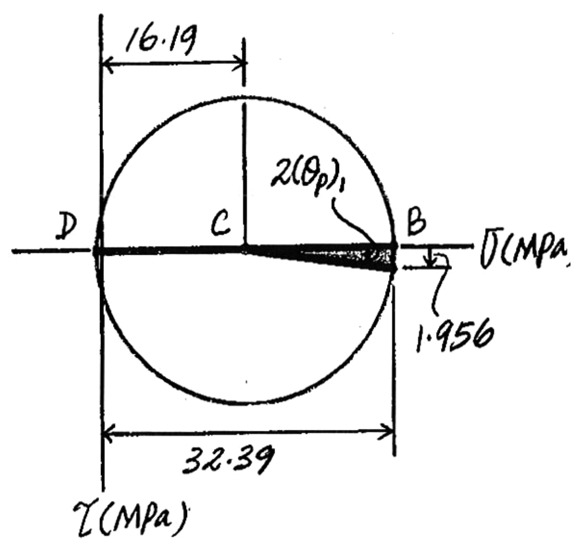
(a)



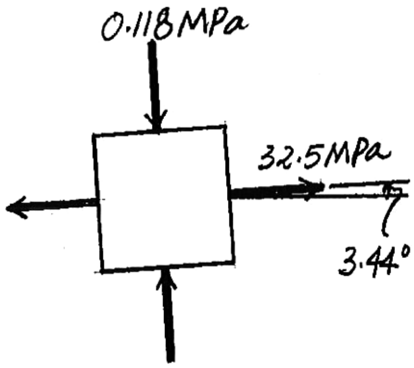
(b)



(C)



(d)



(e)

Ans.
$\sigma_1 = 32.5$ MPa, $\sigma_2 = -0.118$ MPa, $(\theta_p)_1 = 3.44°$ (*Counterclockwise*)

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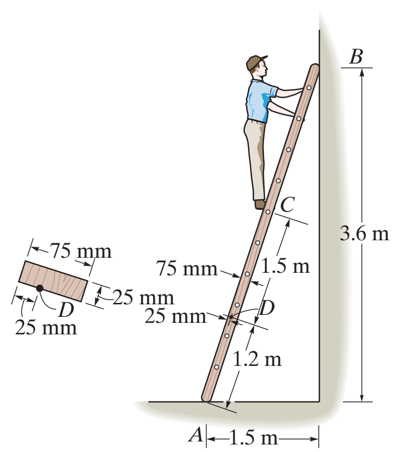
9–71. The ladder is supported on the rough surface at A and by a smooth wall at B. If a man weighing 675 N stands upright at C, determine the principal stresses in one of the legs at point D. Each leg is made from a 25 mm-thick board having a rectangular cross section. Assume that the total weight of the man is exerted vertically on the rung at C and is shared equally by each of the ladder's two legs. Neglect the weight of the ladder and the forces developed by the man's arms.



SOLUTION

$$A = 0.025(0.075) = 1.875(10^{-3})\ \text{m}^2$$

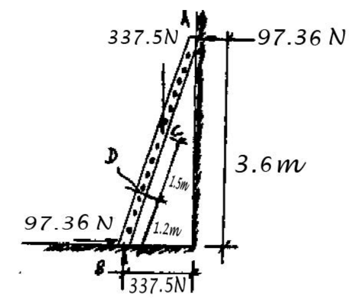
$$I = \frac{1}{12}0.025(0.075^3) = 0.87891(10^{-6})\ \text{m}^4$$

$$Q_D = y'A' = 0.025(0.025)(0.025) = 15.625(10^{-6})\ \text{m}^3$$

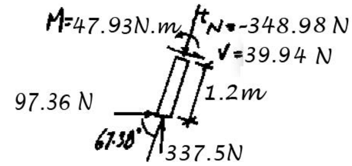
$$\sigma_D = \frac{-N}{A} - \frac{My}{I} = \frac{-348.98}{1.875(10^{-3})} - \frac{(47.93)(0.0125)}{0.87891(10^{-6})}$$

$$= -867.78(10^3)\ \text{N/m}^2 = -867.78\ \text{KPa}$$

$$\tau_D = \frac{VQ_D}{It} = \frac{39.94[15.625(10^{-6})]}{[0.87891(10^{-6})](0.025)} = 28.40(10^3)\ \text{N/m}^2 = 28.40\ \text{KPa}$$



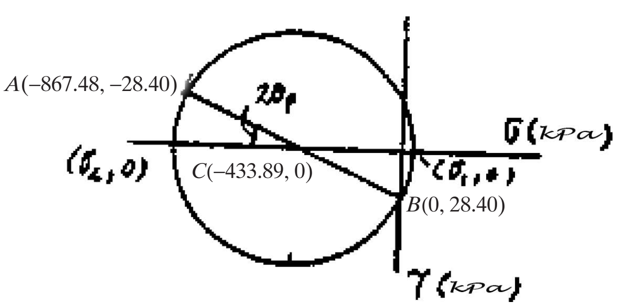
$A(-867.78, -28.40) \qquad B(0, 28.40) \qquad C(-433.89, 0)$

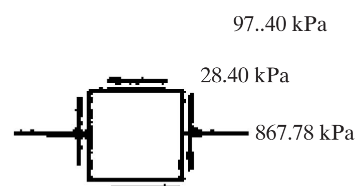


$$R = CA = \sqrt{(-867.78 - 433.89)^2 + 28.40^2} = 434.82\ \text{KPa}$$

$\sigma_1 = -433.89 + 434.82 = 0.929\ \text{KPa}$ **Ans.**

$\sigma_2 = -433.89 - 434.82 = -869\ \text{KPa}$ **Ans.**





Ans.
$\sigma_1 = 0.929\ \text{kPa}, \sigma_2 = -869\ \text{kPa}$

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***9–72.**

A spherical pressure vessel has an inner radius of 1.5 m and a wall thickness of 12 mm. Draw Mohr' Draw Mohr's circle for the state of stress at a point on the vessel and explain the significance of the result. The vessel is subjected to an 0.56 MPa.

SOLUTION

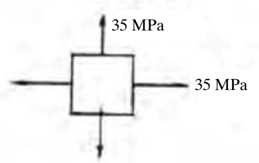
Normal Stress:

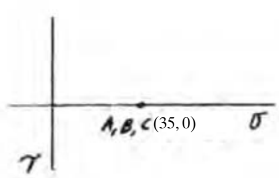
$$\sigma_1 = \sigma_2 = \frac{p\,r}{2\,t} = \frac{0.56(1.5)(1000)}{2(12)} = 35 \text{ MPa}$$

Mohr's circle:

$A(35, 0) \qquad B(35, 0) \qquad C(35, 0)$

Regardless of the orientation of the element, the shear stress is zero and the state of stress is represented by the same two normal stress components.





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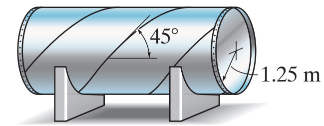
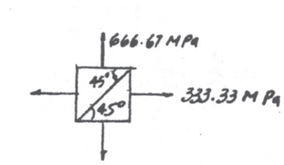
9–73.

The cylindrical pressure vessel has an inner radius of 1.25 m and a wall thickness of 15 mm. It is made from steel plates that are welded along the 45° seam. Determine the normal and shear stress components along this seam if the vessel is subjected to an internal pressure of 8 MPa.



SOLUTION



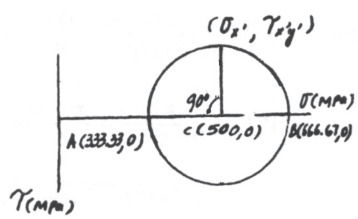
$$\sigma_x = \frac{pr}{2t} = \frac{8(1.25)}{2(0.015)} = 333.33 \text{ MPa}$$

$$\sigma_y = 2\sigma_x = 666.67 \text{ MPa}$$

$A(333.33, 0) \qquad B(666.67, 0) \qquad C(500, 0)$

$$\sigma_{x'} = \frac{333.33 + 666.67}{2} = 500 \text{ MPa}$$ **Ans.**

$$\tau_{x'y'} = -R = 500 - 666.67 = -167 \text{ MPa}$$ **Ans.**



Ans:
$\sigma_{x'} = 500$ MPa, $\tau_{x'y'} = -167$ MPa

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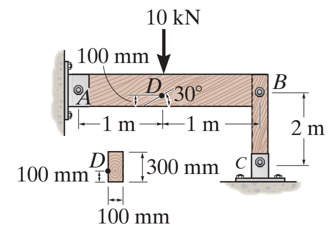
9–74.

Determine the normal and shear stresses at point D that act perpendicular and parallel, respectively, to the grains. The grains at this point make an angle of 30° with the horizontal as shown. Point D is located just to the left of the 10-kN force.



SOLUTION

Using the method of section and consider the FBD of the left cut segment, Fig. a

$+\uparrow \Sigma F_y = 0; \qquad 5 - V = 0 \qquad\qquad V = 5\text{ kN}$

$\circlearrowleft + \Sigma M_C = 0; \qquad M - 5(1) = 0 \qquad M = 5\text{ kN}\cdot\text{m}$

The moment of inertia of the rectangular cross section about the neutral axis is

$$I = \frac{1}{12}(0.1)(0.3^3) = 0.225(10^{-3})\text{ m}^4$$

Referring to Fig. b,

$$Q_D = \bar{y}'A' = 0.1(0.1)(0.1) = 0.001\text{ m}^3$$

The normal stress developed is contributed by bending stress only. For point D, $y = 0.05$ m. Then

$$\sigma = \frac{My}{I} = \frac{5(10^3)(0.05)}{0.225(10^{-3})} = 1.111\text{ MPa (T)}$$

The shear stress is contributed by the transverse shear stress only. Thus,

$$\tau = \frac{VQ_D}{It} = \frac{5(10^3)(0.001)}{0.225(10^{-3})(0.1)} = 0.2222\text{ MPa}$$

The state of stress at point D can be represented by the element shown in Fig. c

In accordance with the established sign convention, $\sigma_x = 1.111$ MPa, $\sigma_y = 0$ and $\tau_{xy} = -0.2222$ MPa. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{1.111 + 0}{2} = 0.5556\text{ MPa}$$

Then, the coordinate of reference point A and the center C of the circle are

$$A(1.111, -0.2222) \qquad C(0.5556, 0)$$

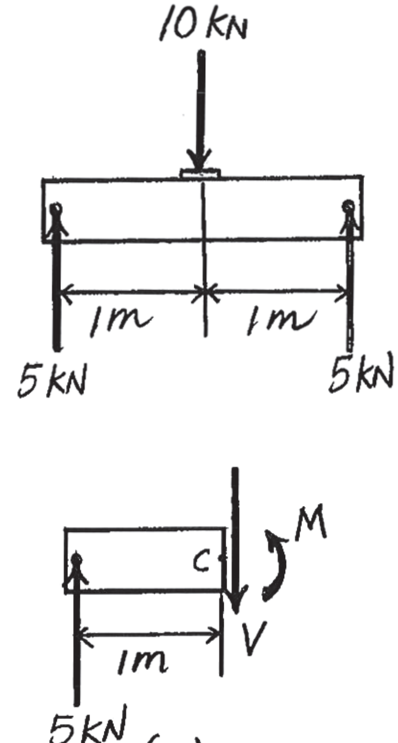
Thus, the radius of the circle is given by

$$R = \sqrt{(1.111 - 0.5556)^2 + (-0.2222)^2} = 0.5984\text{ MPa}$$

Using these results, the circle shown in Fig. d can be constructed.

Referring to the geometry of the circle, Fig. d,

$$\alpha = \tan^{-1}\left(\frac{0.2222}{1.111 - 0.5556}\right) = 21.80° \qquad \beta = 180° - (120° - 21.80°) = 81.80°$$



(a)

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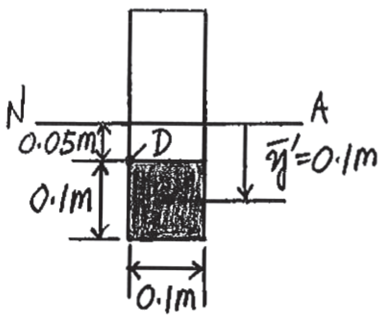
9–74. Continued

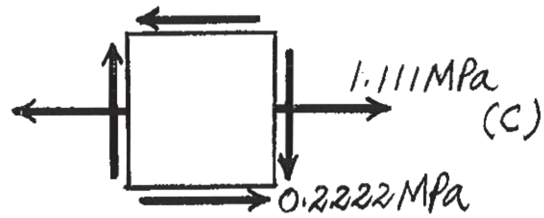
Then

$\sigma_{x'} = 0.5556 - 0.5984 \cos 81.80° = 0.4702 \text{ MPa} = 470 \text{ kPa}$ **Ans.**

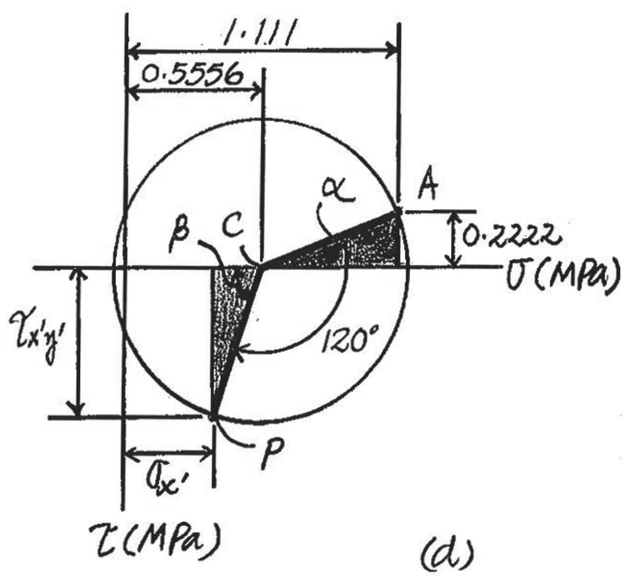
$\tau_{x'y'} = 0.5984 \sin 81.80° = 0.5922 \text{ MPa} = 592 \text{ kPa}$ **Ans.**



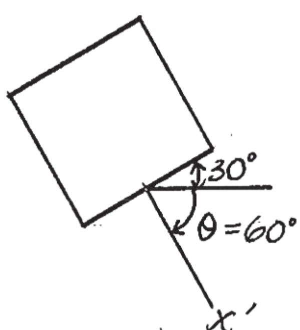
(b)



(c)



(d)



(e)

Ans:
$\sigma_{x'} = 470$ kPa,
$\tau_{x'y'} = 592$ kPa

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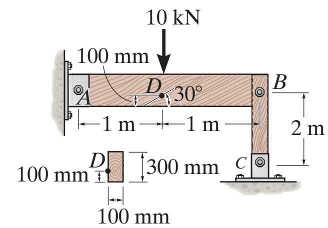
9–75.

Determine the principal stress at point D, which is located just to the left of the 10-kN force.



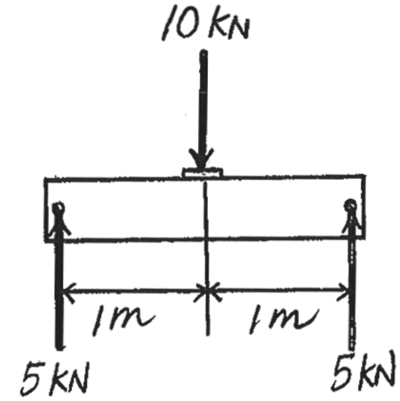
SOLUTION

Using the method of section and consider the FBD of the left cut segment, Fig. *a*,

$+\uparrow\Sigma F_y = 0; \qquad 5 - V = 0 \qquad\qquad V = 5\text{ kN}$

$\circlearrowleft + \Sigma M_C = 0; \qquad M - 5(1) = 0 \qquad M = 5\text{ kN}\cdot\text{m}$

$$I = \frac{1}{12}(0.1)(0.3^3) = 0.225(10^{-3})\text{ m}^4$$

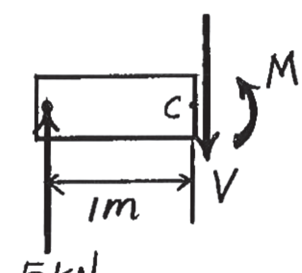


Referring to Fig. *b*,

$$Q_D = \bar{y}'A' = 0.1(0.1)(0.1) = 0.001\text{ m}^3$$

The normal stress developed is contributed by bending stress only. For point D, $y = 0.05$ m

$$\sigma = \frac{My}{I} = \frac{5(10^3)(0.05)}{0.225(10^{-3})} = 1.111\text{ MPa (T)}$$



(a)

The shear stress is contributed by the transverse shear stress only. Thus,

$$\tau = \frac{VQ_D}{It} = \frac{5(10^3)(0.001)}{0.225(10^{-3})(0.1)} = 0.2222\text{ MPa}$$

The state of stress at point D can be represented by the element shown in Fig. *c*.

In accordance with the established sign convention, $\sigma_x = 1.111$ MPa, $\sigma_y = 0$, and $\tau_{xy} = -0.2222$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{1.111 + 0}{2} = 0.5556\text{ MPa}$$

Then, the coordinate of reference point A and center C of the circle are

$$A(1.111, -0.2222) \qquad C(0.5556, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(1.111 - 0.5556)^2 + (-0.2222)^2} = 0.5984\text{ MPa}$$

Using these results, the circle shown in Fig. *d* can be constructed.

In-Plane Principal Stresses: The coordinates of points B and D represent σ_1 and σ_2, respectively. Thus,

$$\sigma_1 = 0.5556 + 0.5984 = 1.15\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = 0.5556 - 0.5984 = -0.0428\text{ MPa} \qquad \textbf{Ans.}$$

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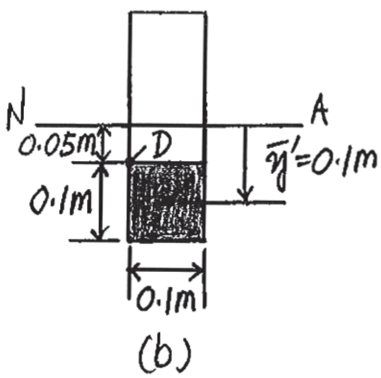
9–75. Continued

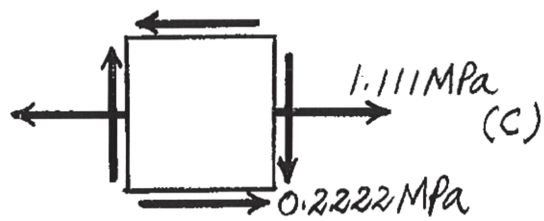
Referring to the geometry of the circle, Fig. *d*,

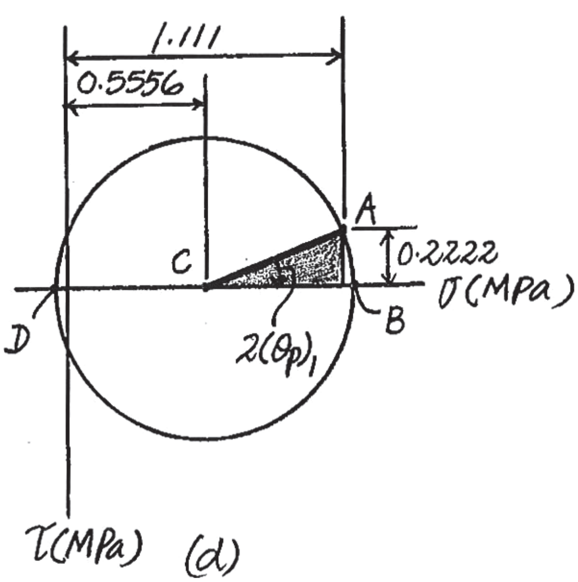
$$\tan (2\theta_P)_1 = \frac{0.2222}{1.111 - 0.5556} = 0.4$$

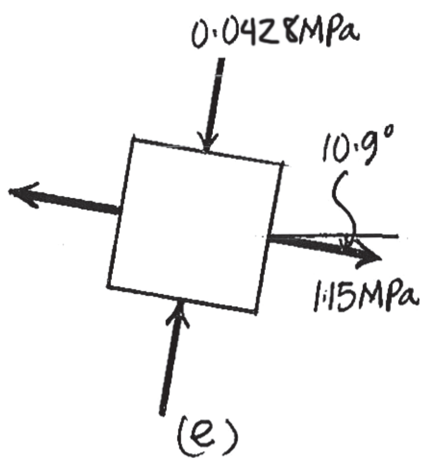
$$(\theta_P)_1 = 10.9° \ (Clockwise) \qquad \textbf{Ans.}$$

The state of principal stresses is represented by the element shown in Fig. *e*.









Ans:
$\sigma_1 = 1.15$ MPa, $\sigma_2 = -0.0428$ MPa

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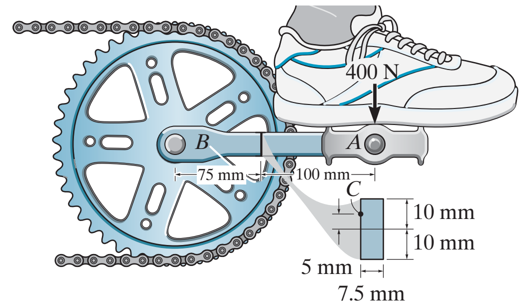
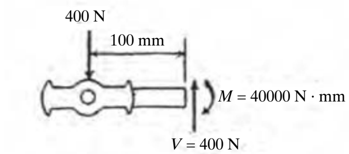
***9–76.** The pedal crank for a bicycle has the cross section shown. If it is fixed to the gear at B and does not rotate while subjected to a force of 400 N, determine the principal stress in the material on the cross section at point C.



SOLUTION

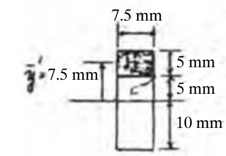
Internal Forces and Moment: As shown on FBD



Section Properties:

$$I = \frac{1}{12}(0.0075)(0.02^3) = 5(10^{-9})\ \text{m}^4$$

$$Q_C = \overline{y}'A' = (0.0075)(0.0075)(0.005) = 0.28125(10^{-6})\ \text{m}^3$$

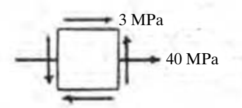


Normal Stress: Applying the flexure formula.

$$\sigma_C = -\frac{My}{I} = -\frac{(-40)(0.005)}{5(10^{-9})} = 40(10^6)\ \text{N/m}^2 = 40\ \text{MPa}$$

Shear Stress: Applying the shear formula.

$$\tau_C = \frac{VQ_C}{It} = \frac{400[0.28125(10^{-6})]}{[5(10^{-9})](0.0075)} = 3(10^6)\ \text{N/m}^2 = 3\ \text{MPa}$$



Construction of the Circle: In accordance with the sign convention, $\sigma_x = 40$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 3$ MPa. Hence,

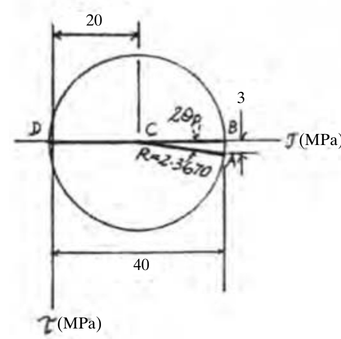
$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{40 + 0}{2} = 20\ \text{MPa}$$

The coordinates for reference points A and C are

$$A(40, 3) \qquad C(20, 0)$$

The radius of the circle is

$$R = \sqrt{(40 - 20)^2 + 3^2} = 20.224\ \text{MPa}$$



In - Plane Principal Stress: The coordinates of point B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 20 + 20.224 = 40.2\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = 20 - 20.224 = -0.224\ \text{MPa} \qquad \textbf{Ans.}$$

Ans.
$\sigma_1 = 40.2$ MPa, $\sigma_2 = -0.224$ MPa

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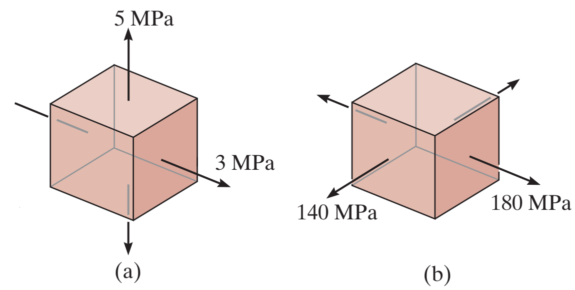
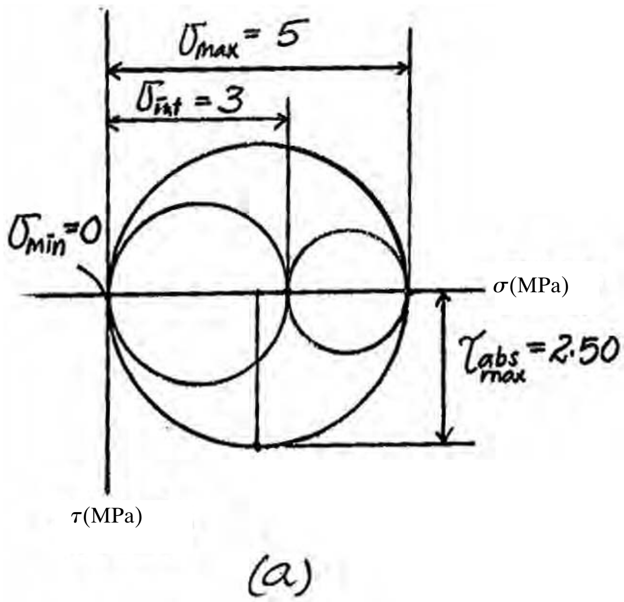
9–77. Draw the three Mohr's circles that describe each of the following states of stress.

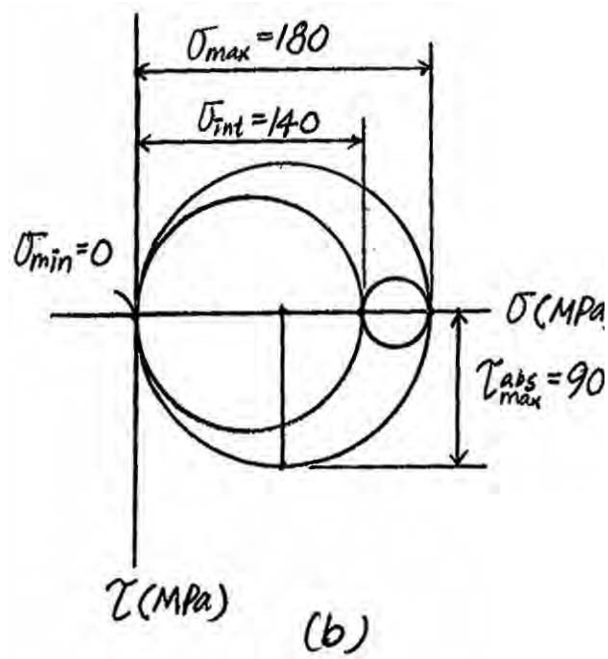


SOLUTION

(a) Here, $\sigma_{min} = 0$, $\sigma_{int} = 3$ MPa and $\sigma_{max} = 5$ MPa. The three Mohr's circle of this state of stress are shown in Fig. *a*

(b) Here, $\sigma_{min} = 0$, $\sigma_{int} = 140$ MPa and $\sigma_{max} = 180$ MPa. The three Mohr's circle of this state of stress are shown in Fig. *b*





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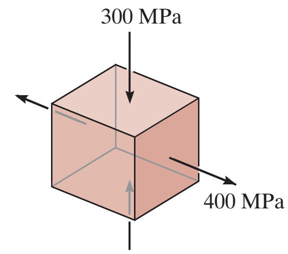
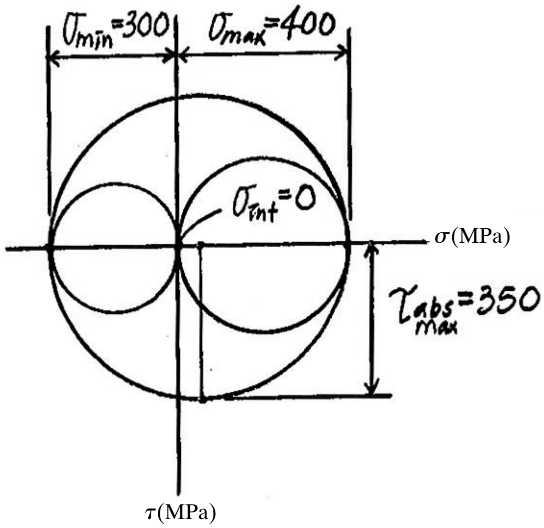
9–78. Draw the three Mohr's circles that describe the following state of stress.



SOLUTION

Here, $\sigma_{\min} = -300$ MPa, $\sigma_{\text{int}} = 0$ and $\sigma_{\max} = 400$ MPa. The three Mohr's circle for this state of stress is shown in Fig. *a*.



(a)

Ans:
$\sigma_{\min} = -300$ MPa, $\sigma_{\text{int}} = 0$, $\sigma_{\max} = 400$ MPa

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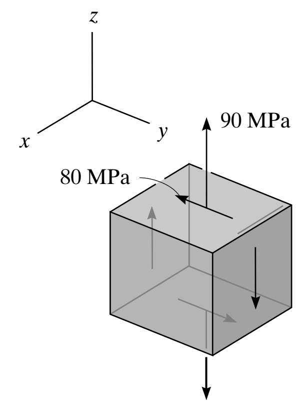
9–79. The stress at a point is shown on the element. Determine the principal stress and the absolute maximum shear stress.



SOLUTION

For $y - z$ plane:

$$A(0, -80) \qquad B(90, 80) \qquad C(45, 0)$$

$$R = \sqrt{45^2 + 80^2} = 91.79$$

$$\sigma_1 = 45 + 91.79 = 136.79 \text{ MPa}$$

$$\sigma_2 = 45 - 91.79 = -46.79 \text{ MPa}$$

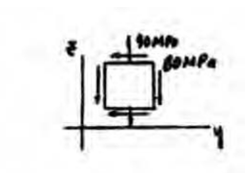
Thus,

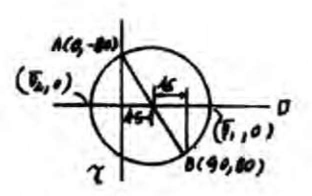
$$\sigma_{\text{Int}} = 0 \qquad \textbf{Ans.}$$

$$\sigma_{\text{Max}} = 137 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{\text{Min}} = -46.8 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} = \frac{136.79 - (-46.79)}{2} = 91.8 \text{ MPa} \qquad \textbf{Ans.}$$





Ans.
$\sigma_{\text{int}} = 0, \sigma_{\text{max}} = 137 \text{ MPa},$
$\sigma_{\text{min}} = -46.8 \text{ MPa},$
$\tau_{\substack{\text{abs}\\\text{max}}} = 91.8 \text{ MPa}$

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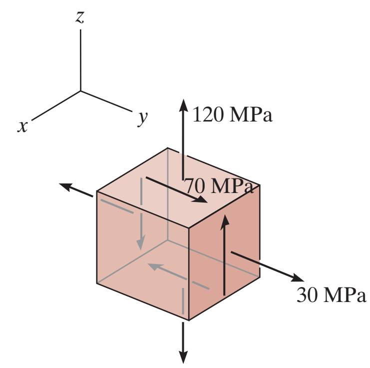
***9–80.** The stress at a point is shown on the element. Determine the principal stress and the absolute maximum shear stress.



SOLUTION

Mohr's circle for the element in y - z plane, Fig. a, will be drawn first. In accordance to the established sign convention, $\sigma_y = 30\,\text{MPa}, \sigma_z = 120\,\text{MPa}$ and $\tau_{yz} = 70\,\text{MPa}$. Thus

$$\sigma_{avg} = \frac{\sigma_y + \sigma_z}{2} = \frac{30 + 120}{2} = 75\text{ MPa}$$

Thus the coordinates of reference point A and the center C of the circle are

$$A(30, 70) \qquad C(75, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(75 - 30)^2 + 70^2} = 83.217\text{ MPa}$$

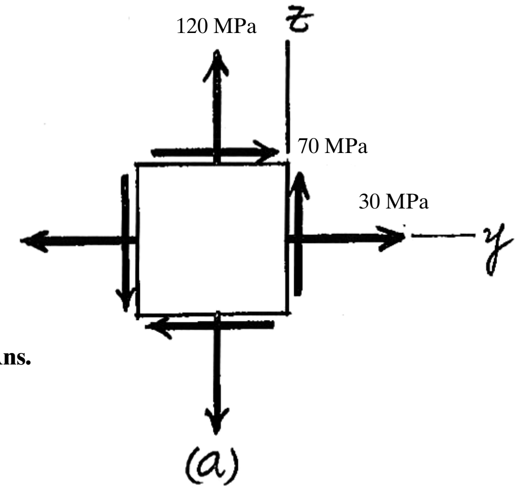
Using these results, the circle shown in Fig. b.

The coordinates of point B and D represent the principal stresses

From the results,

$\sigma_{max} = 158.22\text{ MPa} = 158\text{ MPa} \quad \sigma_{min} = -8.22\text{ MPa} = -8.22\text{ MPa}$ **Ans.**

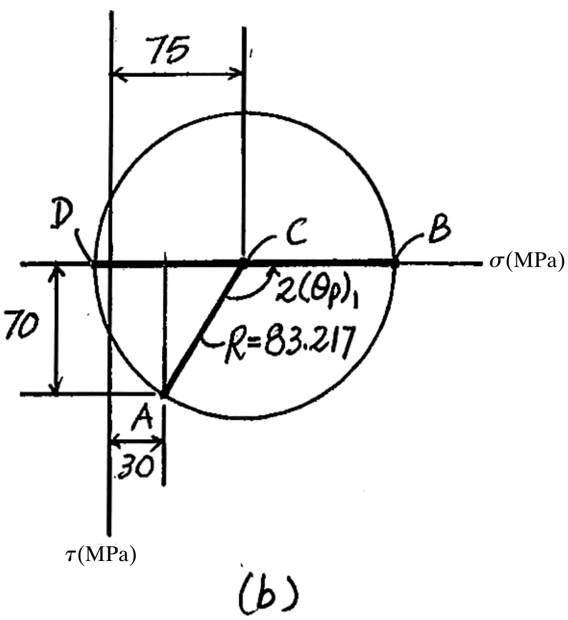
$\sigma_{int} = 0\text{ MPa}$

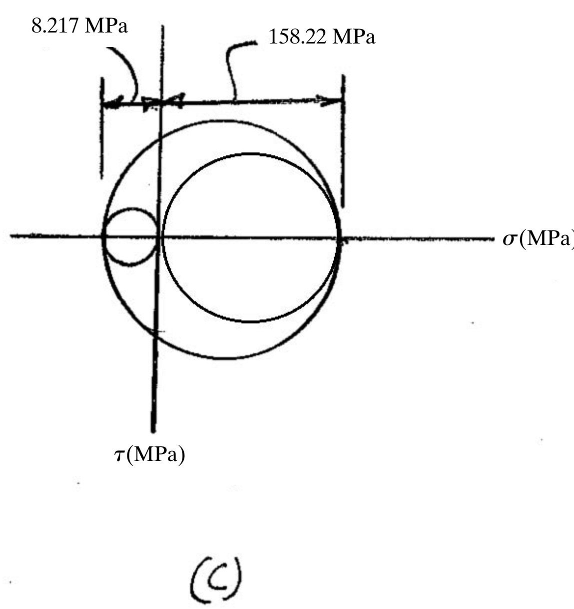


Using these results, the three Mohr's circle are shown in Fig. c,

From the geometry of the three circles,

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{158.22 - (-8.217)}{2} = 83.2\text{ MPa}$$ **Ans.**





Ans.
$\sigma_{int} = 0, \sigma_{max} = 158\text{ MPa},$
$\sigma_{min} = -8.22\text{ MPa},$
$\tau_{\substack{abs\\max}} = 83.2\text{ MPa}$

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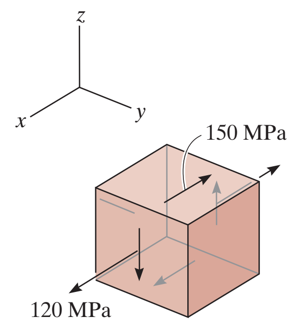
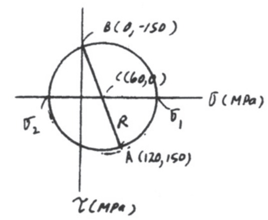
9–81.

Determine the principal stresses and the absolute maximum shear stress.



SOLUTION

For *x* - *z* plane:



$R = CA = \sqrt{(120 - 60)^2 + 150^2} = 161.55$

$\sigma_1 = 60 + 161.55 = 221.55 \text{ MPa}$

$\sigma_2 = 60 - 161.55 = -101.55 \text{ MPa}$

$\sigma_1 = 222 \text{ MPa} \qquad \sigma_2 = -102 \text{ MPa}$ **Ans.**

$\tau_{\substack{\text{abs}\\ \text{max}}} = \dfrac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} = \dfrac{221.55 - (-101.55)}{2} = 162 \text{ MPa}$ **Ans.**

Ans:
$\sigma_1 = 222 \text{ MPa}, \sigma_2 = -102 \text{ MPa},$
$\tau_{\substack{\text{abs}\\ \text{max}}} = 162 \text{ MPa}$

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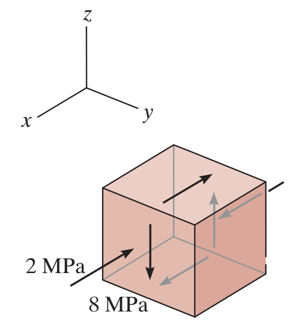
9–82. The stress at a point is shown on the element. Determine the principal stress and the absolute maximum shear stress.



SOLUTION

Mohr's circle for the element in x - z plane, Fig. a, will be drawn first. In accordance to the established sign convention, $\sigma_x = -2$ MPa, $\sigma_z = 0$ and $\tau_{xz} = 8$ MPa. Thus

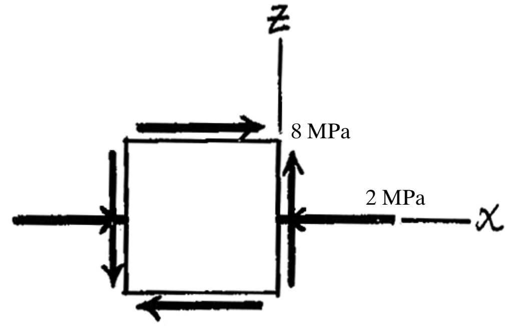
$$\sigma_{avg} = \frac{\sigma_x + \sigma_z}{2} = \frac{-2 + 0}{2} = -1 \text{ MPa}$$

Thus, the coordinates of reference point A and the center C of the circle are

$$A(-2, 8) \qquad C(-1, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{[-2 - (-1)]^2 + 8^2} = \sqrt{65} \text{ MPa}$$



(a)

Using these results, the circle in shown in Fig. b,

The coordinates of points B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = -1 + \sqrt{65} = 7.062 \text{ MPa}$$

$$\sigma_2 = -1 - \sqrt{65} = -9.062 \text{ MPa}$$

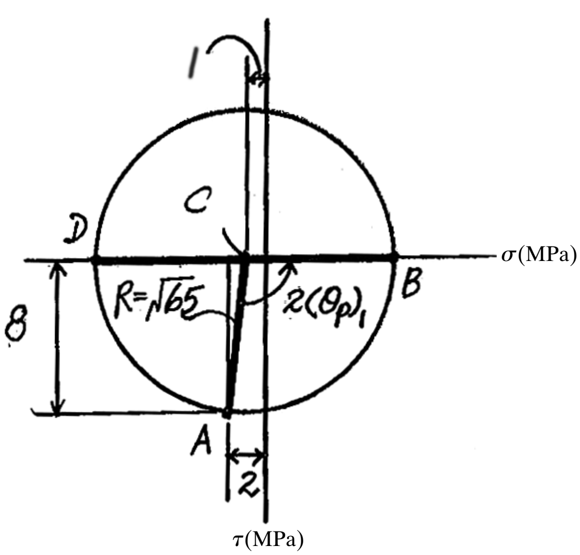
From the results obtained,

$$\sigma_{int} = 0 \text{ MPa} \quad \sigma_{max} = 7.06 \text{ MPa} \quad \sigma_{min} = -9.06 \text{ MPa}$$ **Ans.**

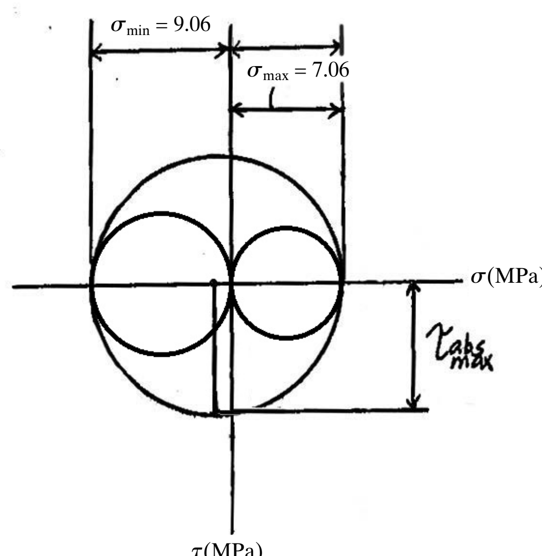
Using these results, the three Mohr's circles are shown in Fig, c.

From the geometry of the cricle,

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{7.062 - (-9.062)}{2} = 8.06 \text{ MPa}$$ **Ans.**



(b)



(c)

Ans.

$\sigma_{int} = 0$ MPa, $\sigma_{max} = 7.06$ MPa,
$\sigma_{min} = -9.06$ MPa, $\tau_{\substack{abs\\max}} = 8.06$ MPa

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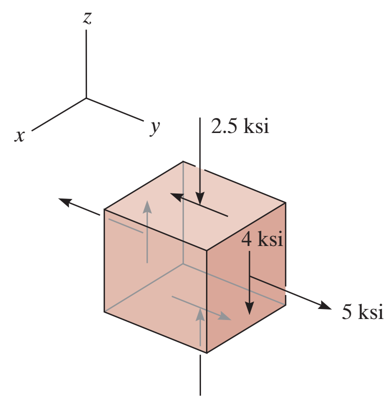
9–83.

Determine the principal stresses and the absolute maximum shear stress.



SOLUTION

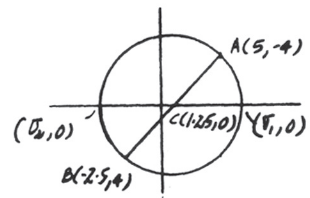
For *y* - *z* plane:

$$A(5, -4) \qquad B(-2.5, 4) \qquad C(1.25, 0)$$

$$R = \sqrt{3.75^2 + 4^2} = 5.483$$

$$\sigma_1 = 1.25 + 5.483 = 6.733 \text{ ksi}$$

$$\sigma_2 = 1.25 - 5.483 = -4.233 \text{ ksi}$$



(a)

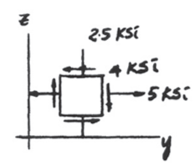
Thus,

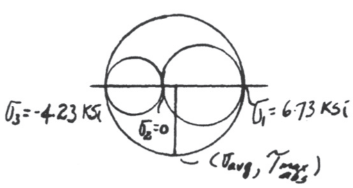
$$\sigma_1 = 6.73 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = -4.23 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_{avg} = \frac{6.73 + (-4.23)}{2} = 1.25 \text{ ksi}$$

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{6.73 - (-4.23)}{2} = 5.48 \text{ ksi} \qquad \textbf{Ans.}$$





Ans:
$\sigma_1 = 6.73$ ksi, $\sigma_2 = -4.23$ ksi,
$\tau_{\substack{abs\\max}} = 5.48$ ksi

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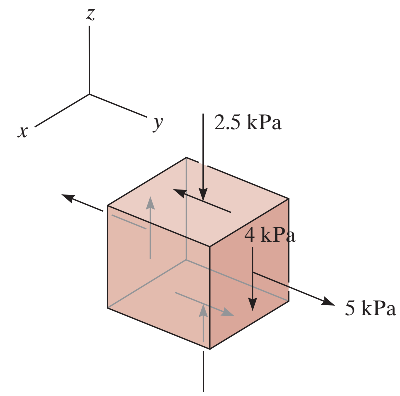
***9–84.** The state of stress at a point is shown on the element. Determine the principal stress and the absolute maximum shear stress.



SOLUTION

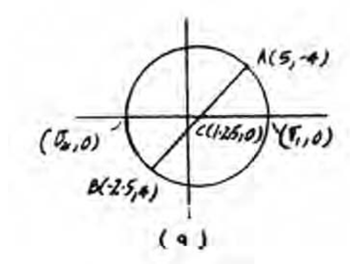
For y - z plane:

$$A(5, -4) \qquad B(-2.5, 4) \qquad C(1.25, 0)$$

$$R = \sqrt{3.75^2 + 4^2} = 5.483$$

$$\sigma_1 = 1.25 + 5.483 = 6.733 \text{ kPa}$$

$$\sigma_2 = 1.25 - 5.483 = -4.233 \text{ kPa}$$

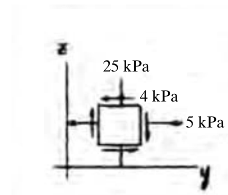


Thus,

$$\sigma_{\text{Max}} = 6.73 \text{ kPa}$$ **Ans.**

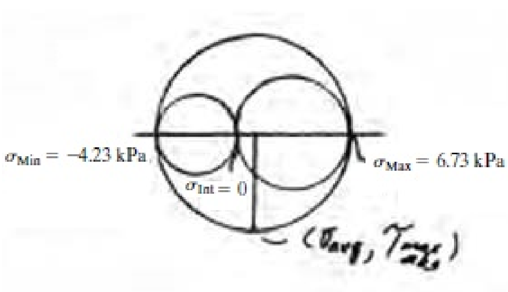
$$\sigma_{\text{Int}} = 0$$ **Ans.**

$$\sigma_{\text{Min}} = -4.23 \text{ kPa}$$ **Ans.**



$$\sigma_{\text{avg}} = \frac{6.733 + (-4.233)}{2} = 1.25 \text{ kPa}$$

$$\tau_{\substack{\text{abs} \\ \text{max}}} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{6.733 - (-4.233)}{2} = 5.48 \text{ kPa}$$ **Ans.**



Ans.
$\sigma_{\text{int}} = 0, \sigma_{\max} = 6.73$ MPa,
$\sigma_{\min} = -4.23$ MPa,
$\tau_{\substack{\text{abs} \\ \text{max}}} = 5.48$ MPa

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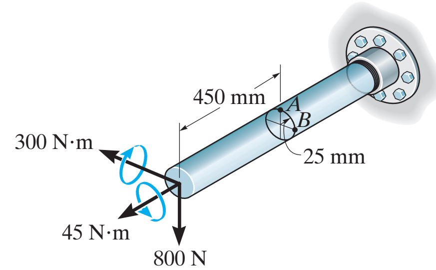
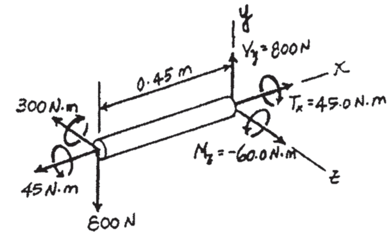
9–85.

The solid shaft is subjected to a torque, bending moment, and shear force. Determine the principal stresses at points A and B and the absolute maximum shear stress.



SOLUTION

Internal Forces and Moment: As shown on FBD.



Section Properties:

$$I_z = \frac{\pi}{4}\left(0.025^4\right) = 0.306796\left(10^{-6}\right)\text{ m}^4$$

$$J = \frac{\pi}{2}\left(0.025^4\right) = 0.613592\left(10^{-6}\right)\text{ m}^4$$

$$\left(Q_A\right)_y = 0$$

$$\left(Q_B\right)_y = \bar{y}'A'$$

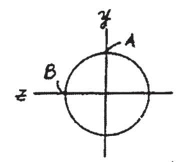
$$= \frac{4(0.025)}{3\pi}\left[\frac{1}{2}(\pi)\left(0.025^2\right)\right] = 10.417\left(10^{-6}\right)\text{ m}^3$$

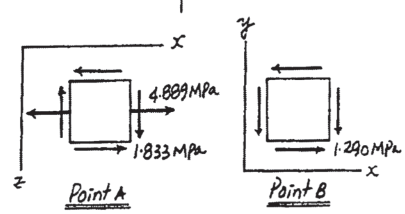
Normal Stress: Applying the flexure formula,

$$\sigma = -\frac{M_z y}{I_z}$$

$$\sigma_A = -\frac{-60.0(0.025)}{0.306796\left(10^{-6}\right)} = 4.889\text{ MPa}$$

$$\sigma_B = -\frac{-60.0(0)}{0.306796\left(10^{-6}\right)} = 0$$





Shear Stress: Applying the torsion formula for point A,

$$\tau_A = \frac{Tc}{J} = \frac{45.0(0.025)}{0.613592\left(10^{-6}\right)} = 1.833\text{ MPa}$$

The transverse shear stress in the y direction and the torsional shear stress can be obtained using shear formula and torsion formula.

$\tau_V = \dfrac{VQ}{It}$ and $\tau_{\text{twist}} = \dfrac{T\rho}{J}$, respectively.

$$\tau_B = \left(\tau_V\right)_y - \tau_{\text{twist}}$$

$$= \frac{800\left[10.417\left(10^{-6}\right)\right]}{0.306796\left(10^{-6}\right)(0.05)} - \frac{45.0(0.025)}{0.613592\left(10^{-6}\right)}$$

$$= -1.290\text{ MPa}$$

Construction of the Circle: $\sigma_x = 4.889$ MPa, $\sigma_z = 0$, and $\tau_{xz} = -1.833$ MPa for point A. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_z}{2} = \frac{4.889 + 0}{2} = 2.445\text{ MPa}$$

The coordinates for reference points A and C are A (4.889, −1.833) and C(2.445, 0).

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9–85. Continued

The radius of the circle is

$$R = \sqrt{(4.889 - 2.445)^2 + 1.833^2} = 3.056 \text{ MPa}$$

$\sigma_x = \sigma_z = 0$ and $\tau_{xy} = -1.290$ MPa for point B. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_z}{2} = 0$$

The coordinates for reference points A and C are $A(0, -1.290)$ and $C(0, 0)$

The radius of the circle is $R = 1.290$ MPa

In-Plane Principal Stresses: The coordinates of points B and D represent σ_1 and σ_2, respectively. For point A

$$\sigma_1 = 2.445 + 3.056 = 5.50 \text{ MPa}$$
$$\sigma_2 = 2.445 - 3.056 = -0.611 \text{ MPa}$$

For point B,

$$\sigma_1 = 0 + 1.290 = 1.29 \text{ MPa}$$
$$\sigma_2 = 0 - 1.290 = -1.290 \text{ MPa}$$

Three Mohr's Circles: From the results obtained above, the principal stresses for point A are

$\sigma_1 = 5.50$ MPa $\quad \sigma_2 = -0.611$ MPa **Ans.**

And for point B,

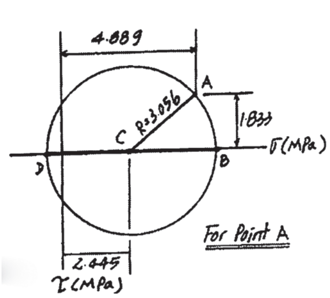
$\sigma_1 = 1.29$ MPa $\quad \sigma_2 = -1.29$ MPa **Ans.**

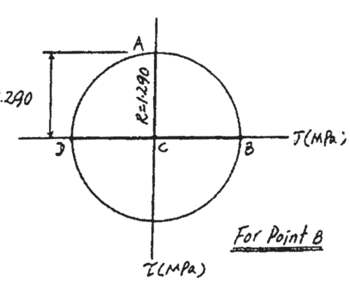
Absolute Maximum Shear Stress: From point A,

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{5.50 - (-0.611)}{2} = 3.06 \text{ MPa}$$ **Ans.**

For point B,

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{1.29 - (-1.29)}{2} = 1.29 \text{ MPa}$$ **Ans.**





Ans:
$\sigma_1 = 5.50$ MPa, $\sigma_2 = -0.611$ MPa,
$\sigma_1 = 1.29$ MPa, $\sigma_2 = -1.29$ MPa,
$\tau_{\substack{abs \\ max}} = 3.06$ MPa,
$\tau_{\substack{abs \\ max}} = 1.29$ MPa

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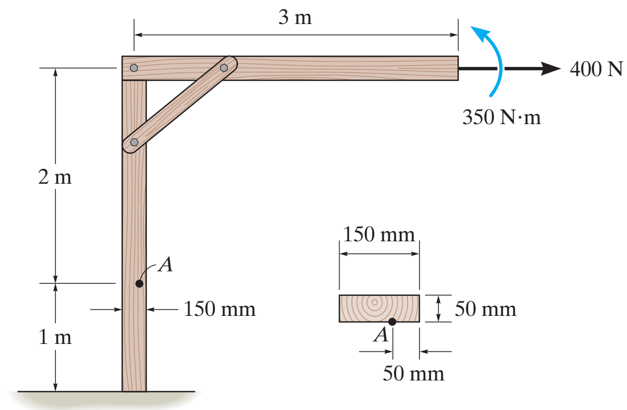
9–86.

The frame is subjected to a horizontal force and couple moment. Determine the principal stresses and the absolute maximum shear stress at point A. The cross-sectional area at this point is shown.



SOLUTION

$$I = \frac{1}{12}(0.05)(0.15^3) = 14.0625(10^{-6})\ \text{m}^4$$

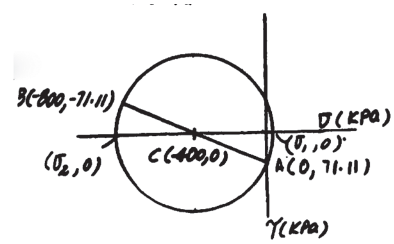
$$Q_A = 0.05(0.05)(0.05) = 0.125(10^{-3})\ \text{m}^3$$

$$\sigma_A = -\frac{Mx}{I} = -\frac{450(0.025)}{14.0625(10^{-6})} = -800\ \text{kPa}$$

$$\tau_A = \frac{VQ_A}{It} = \frac{400(0.125)(10^{-3})}{14.0625(10^{-6})(0.05)} = 71.11\ \text{kPa}$$

$A(0, 71.11) \qquad B(-800, -71.11) \qquad C(-400, 0)$

$$R = \sqrt{400^2 + 71.11^2} = 406.272$$
$$\sigma_1 = -400 + 406.272 = 6.72\ \text{kPa}$$
$$\sigma_2 = -400 - 406.272 = -806\ \text{kPa}$$

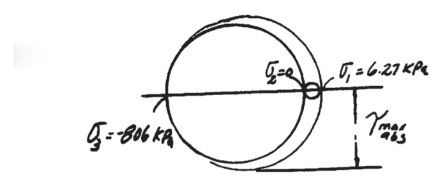


Thus,

$\sigma_1 = 6.27\ \text{kPa}$ **Ans.**

$\sigma_2 = -806\ \text{kPa}$ **Ans.**

$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{6.27 - (-806.27)}{2} = 406\ \text{kPa}$$ **Ans.**



Ans:
$\sigma_1 = 6.27\ \text{kPa}$,
$\sigma_2 = -806\ \text{kPa}$,
$\tau_{\substack{\text{abs}\\ \text{max}}} = 406\ \text{kPa}$

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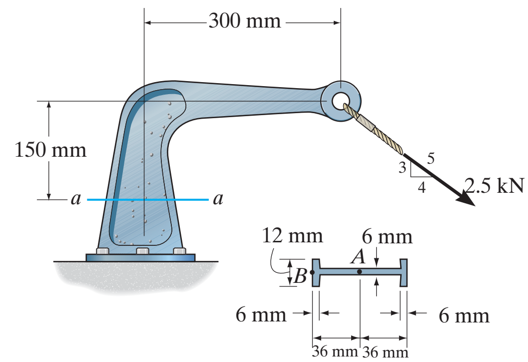
9–87. Determine the principal stress and absolute maximum shear stress developed at point B on the cross section of the bracket at section a–a.



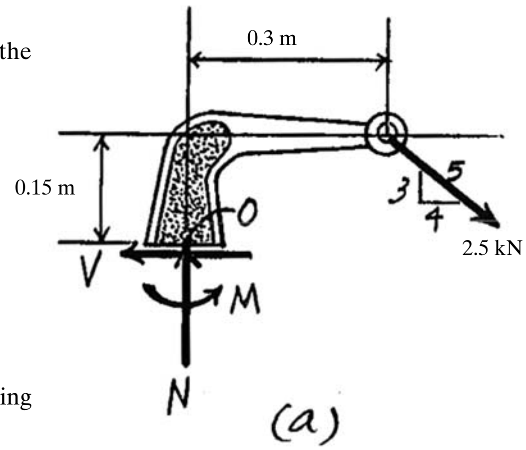
Section $a-a$

Internal Loadings: Considering the equilibrium of the free - body diagram of the bracket's upper cut segment, Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad N - 2.5\left(\frac{3}{5}\right) = 0 \qquad N = 1.5 \text{ kN}$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad V - 2.5\left(\frac{4}{5}\right) = 0 \qquad V = 2.0 \text{ kN}$$

$$\Sigma M_O = 0; \; M - 2.5\left(\frac{3}{5}\right)(0.300) - 2.5\left(\frac{4}{5}\right)(0.150) = 0 \qquad M = 0.75 \text{ kN}\cdot\text{m}$$



Section Properties: The cross - sectional area and the moment of inertia about the centroidal axis of the bracket's cross section are

$$A = 0.012(0.072) - 0.006(0.06) = 0.504(10^{-3})\text{ m}^2$$

$$I = \frac{1}{12}(0.012)(0.072^3) - \frac{1}{12}(0.006)(0.06^3) = 0.265248(10^{-6})\text{ m}^4$$

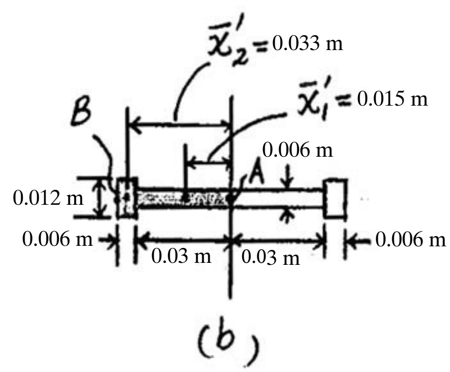
Referring to Fig. b,

$$Q_B = 0$$

Normal and Shear Stress: The normal stress is a combination of axial and bending stress.

$$\sigma_B = \frac{N}{A} + \frac{Mx_B}{I} = \frac{-1.5(10^3)}{0.504(10^{-3})} + \frac{[0.75(10^3)](0.036)}{0.265248(10^{-6})}$$
$$= 98.82(10^6)\text{ N/m}^2 = 98.82 \text{ MPa}$$

Since $Q_B = 0$, $\tau_B = 0$. The state of stress at point B is represented on the element shown in Fig. c.



In - Plane Principal Stresses: Since no shear stress acts on the element,

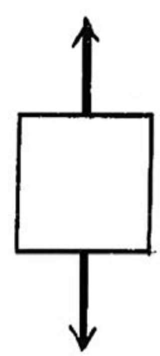
$$\sigma_1 = 98.8 \text{ MPa} \qquad \sigma_2 = 0$$

Three Mohr's Circles: Using these results,

$$\sigma_{max} = 98.8 \text{ MPa} \qquad \sigma_{int} = \sigma_{min} = 0 \qquad \textbf{Ans.}$$

Absolute Maximum Shear Stress:

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{98.82 - 0}{2} = 49.4 \text{ MPa} \qquad \textbf{Ans.}$$



Ans.
$\sigma_{max} = 98.8$ MPa, $\sigma_{int} = \sigma_{min} = 0$
$\tau_{\substack{abs\\max}} = 49.4$ MPa

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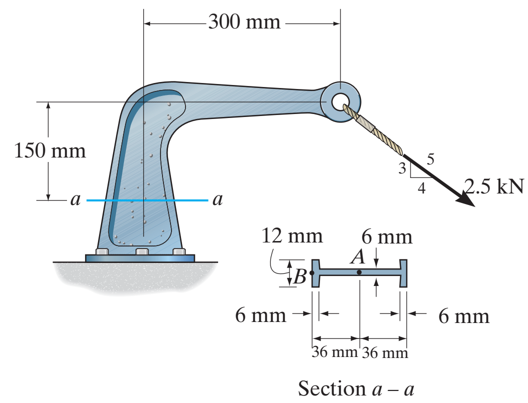
***9–88.** Determine the principal stress and absolute maximum shear stress developed at point A on the cross section of the bracket at section a–a.



SOLUTION

Internal Loadings: Considering the equilibrium of the free - body diagram of the bracket's upper cut segment, Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad N - 2.5\left(\frac{3}{5}\right) = 0 \qquad N = 1.5 \text{ kN}$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad V - 2.5\left(\frac{4}{5}\right) = 0 \qquad V = 2.0 \text{ kN}$$

$$\Sigma M_O = 0; \; M - 2.5\left(\frac{3}{5}\right)(0.300) - 2.5\left(\frac{4}{5}\right)(0.150) = 0 \qquad M = 0.75 \text{ kN} \cdot \text{m}$$

Section Properties: The cross - sectional area and the moment of inertia of the bracket's cross section are

$$A = 0.012(0.072) - 0.006(0.06) = 0.504(10^{-3}) \text{ m}^2$$

$$I = \frac{1}{12}(0.012)(0.072^3) - \frac{1}{12}(0.006)(0.06^3) = 0.265248(10^{-6}) \text{ m}^4$$

Referring to Fig. b.

$$Q_A = \bar{x}_1' A_1' + \bar{x}_2' A_2' = 0.015(0.006)(0.03) + 0.033(0.012)(0.006) = 5.076(10^{-6}) \text{ m}^3$$

Normal and Shear Stress: The normal stress is

$$\sigma_A = \frac{N}{A} = \frac{1.5(10^3)}{0.504(10^{-3})} = -2.976(10^6) \text{ N/m}^2 = -2.976 \text{ MPa}$$

The shear stress is contributed by the transverse shear stress.

$$\tau_A = \frac{VQ_A}{It} = \frac{[2.0(10^3)][5.076(10^{-6})]}{[0.265248(10^{-6})](0.006)} = 6.379(10^6) \text{ N/m}^2 = 6.379 \text{ MPa}$$

The state of stress at point A is represented by the element shown in Fig. c.

Construction of the Circle: $\sigma_x = 0$, $\sigma_y = -2.976$ MPa, and $\tau_{xy} = 6.379$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-2.976)}{2} = -1.488 \text{ MPa}$$

The coordinates of reference point A and the center C of the circle are

$$A(0, -6.379) \qquad C(-1.488, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{[0 - (-1.488)]^2 + 6.379^2} = 6.5502 \text{ MPa}$$

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***9–88. Continued**

Using these results, the cricle is shown in Fig. *d*.

In - Plane Principal Stresses: The coordinates of reference point *B* and *D* represent σ_1 and σ_2, respectively.

$$\sigma_1 = -1.488 + 6.5502 = 5.062 \text{ MPa}$$

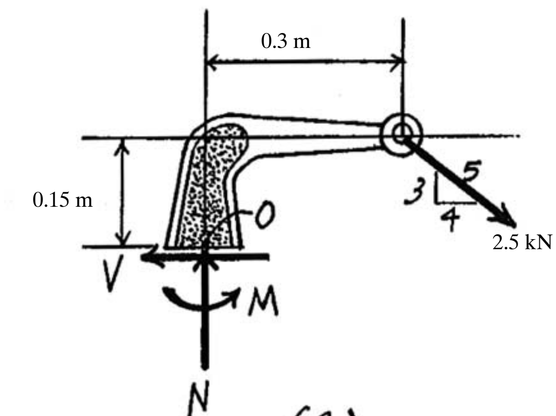
$$\sigma_2 = -1.488 - 6.5502 = -8.038 \text{ MPa}$$

Three Mohr's Circles: Using these results,

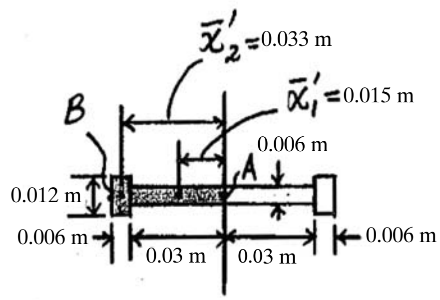
$$\sigma_{max} = 5.06 \text{ MPa} \quad \sigma_{int} = 0 \quad \sigma_{min} = -8.04 \text{ MPa}$$ **Ans.**

Absolute Maximum Shear Stress:

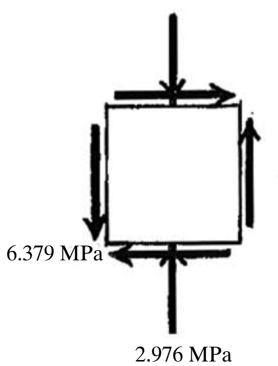
$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{5.062 - (-8.038)}{2} = 6.55 \text{ MPa}$$ **Ans.**



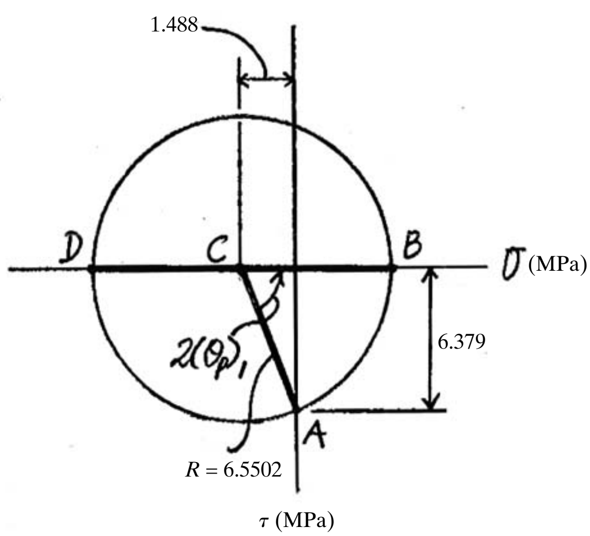
(a)



(b)



(c)



(d)

Ans.
$\sigma_{max} = 98.8$ MPa, $\sigma_{int} = \sigma_{min} = 0$,
$\tau_{\substack{abs\\max}} = 49.4$ MPa

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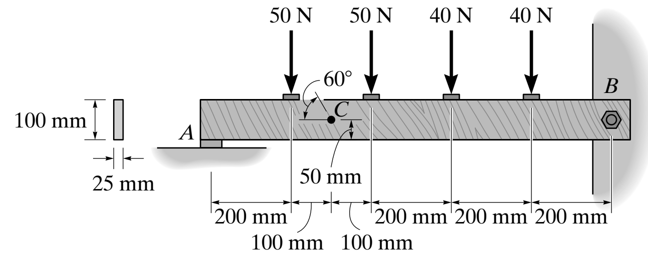
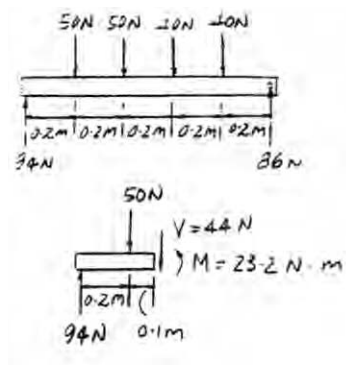
R9–1.

The wooden strut is subjected to the loading shown. Determine the principal stresses that act at point C and specify the orientation of the element at this point. The strut is supported by a bolt (pin) at B and smooth support at A.



SOLUTION



$Q_C = \bar{y}'A' = 0.025(0.05)(0.025) = 31.25(10^{-6})\ \text{m}^3$

$I = \dfrac{1}{12}(0.025)(0.1^3) = 2.0833(10^{-6})\ \text{m}^4$

Normal stress: $\sigma_C = 0$

Shear stress:

$$\tau = \frac{VQ_C}{It} = \frac{44(31.25)(10^{-6})}{2.0833(10^{-6})(0.025)} = 26.4\ \text{kPa}$$

Principal stress:

$\sigma_x = \sigma_y = 0; \qquad \tau_{xy} = -26.4\ \text{kPa}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2{}_{xy}}$$

$$= 0 \pm \sqrt{0 + (26.4)^2}$$

$\sigma_1 = 26.4\ \text{kPa} \qquad ; \qquad \sigma_2 = -26.4\ \text{kPa}$ **Ans.**

Orientation of principal stress:

$$\tan 2\theta_p = \frac{\tau_{xy}}{\dfrac{(\sigma_x - \sigma_y)}{2}} = -\infty$$

$\theta_p = +45°$ and $-45°$

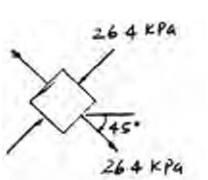
Use Eq. 9-1 to determine the principal plane of σ_1 and σ_2

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$\theta = \theta_p = -45°$

$\sigma_{x'} = 0 + 0 + (-26.4)\sin(-90°) = 26.4\ \text{kPa}$

Therefore, $\theta_{p_1} = -45°; \qquad \theta_{p_2} = 45°$ **Ans.**



Ans.
$Q_C = 31.25(10^{-6})\ \text{m}^3, I = 2.0833(10^{-6})\ \text{m}^4,$
$\tau = 26.4\ \text{kPa}, \sigma_1 = 26.4\ \text{kPa}, \sigma_2 = -26.4\ \text{kPa},$
$\theta_{p_1} = -45°; \quad \theta_{p_2} = 45°$

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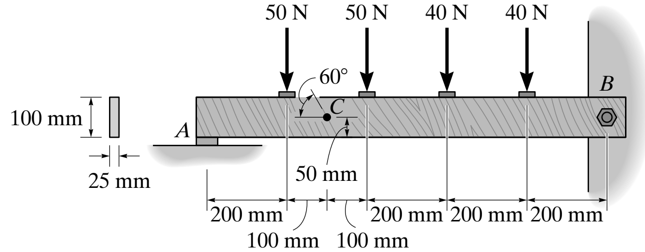
R9–2.

The wooden strut is subjected to the loading shown. If grains of wood in the strut at point C make an angle of 60° with the horizontal as shown, determine the normal and shear stresses that act perpendicular and parallel to the grains, respectively, due to the loading. The strut is supported by a bolt (pin) at B and smooth support at A.



SOLUTION

$$Q_C = y'A' = 0.025(0.05)(0.025) = 31.25(10^{-6})\text{ m}^3$$

$$I = \frac{1}{12}(0.025)(0.1^3) = 2.0833(10^{-6})\text{ m}^4$$

Normal stress: $\sigma_C = 0$

Shear stress:

$$\tau = \frac{VQ_C}{I\,t} = \frac{44(31.25)(10^{-6})}{2.0833(10^{-6})(0.025)} = 26.4\text{ kPa}$$

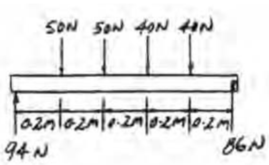
Stress transformation: $\sigma_x = \sigma_y = 0; \qquad \tau_{xy} = -26.4\text{ kPa}; \qquad \theta = 30°$

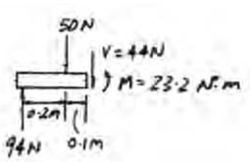
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= 0 + 0 + (-26.4)\sin 60° = -22.9\text{ kPa}$$ **Ans.**

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -0 + (-26.4)\cos 60° = -13.2\text{ kPa}$$ **Ans.**





Ans.
$\sigma_{x'} = -22.9\text{ kPa}$, $\tau_{x'y'} = -13.2\text{ kPa}$

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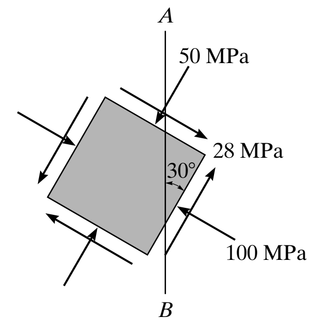
R9–3.

The state of stress at a point in a member is shown on the element. Determine the stress components acting on the plane AB.



SOLUTION

Construction of the Circle: In accordance with the sign convention, $\sigma_x = -50$ MPa, $\sigma_y = -100$ MPa, and $\tau_{xy} = -28$ MPa. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-50 + (-100)}{2} = -75.0 \text{ MPa}$$

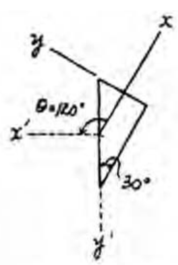
The coordinates for reference points A and C are $A(-50, -28)$ and $C(-75.0, 0)$.

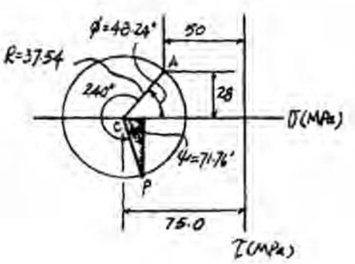
The radius of the circle is $R = \sqrt{(75.0 - 50)^2 + 28^2} = 37.54$ MPa.

Stress on the Rotated Element: The normal and shear stress components $\left(\sigma_{x'} \text{ and } \tau_{x'y'}\right)$ are represented by the coordinates of point P on the circle

$$\sigma_{x'} = -75.0 + 37.54 \cos 71.76° = -63.3 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = 37.54 \sin 71.76° = 35.7 \text{ MPa} \qquad \textbf{Ans.}$$





Ans.
$\sigma_{x'} = -63.3$ MPa, $\tau_{x'y'} = 35.7$ MPa

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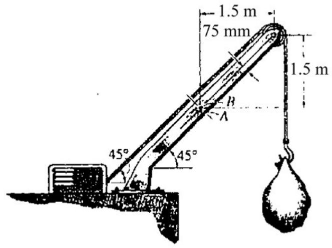
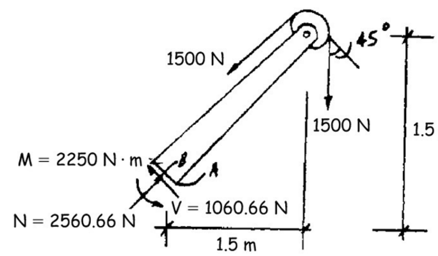
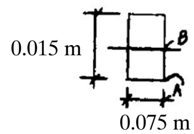
***R9–4.**

The crane is used to support the 1500 N load. Determine the principal stresses acting in the boom at points A and B. The cross section is rectangular and has a width of 150 mm and a thickness of 75 mm. Use Mohr's circle.







SOLUTION

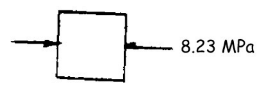
$$A = 0.075(0.15) = 0.01125 \text{ m}^2$$

$$I = \frac{1}{12}(0.075)(0.15^3) = 21.09375(10^{-6}) \text{ m}^4$$

$$Q_B = 0.0375(0.075)(0.075) = 0.2109375(10^{-3}) \text{ m}^3$$

$$Q_A = 0$$

Point A:



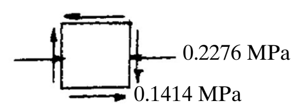
$$\sigma_A = -\frac{P}{A} - \frac{My}{I} = \frac{-2560.66}{0.01125} - \frac{2250(0.075)}{21.09375(10^{-6})}$$

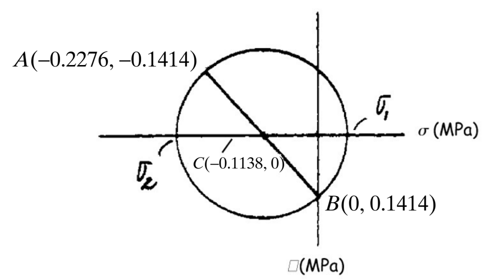
$$= -8.228(10^6) \text{ N/m}^2 = -8.23 \text{ MPa}$$

$$\tau_A = 0$$

$\sigma_1 = 0 \qquad \sigma_2 = -8.23$ MPa **Ans**

Point B:





$$\sigma_B = -\frac{P}{A} = \frac{-2560.66}{0.01125} = -0.2276(10^6) \text{ N/m}^2 = -0.2276 \text{ MPa}$$

$$\tau_B = \frac{VQ_B}{It} = \frac{1060.66[0.2109375(10^{-3})]}{[21.09375(10^{-6})](0.075)} = 0.1414(10^6) \text{ N/m}^2 = 0.1414 \text{ MPa}$$

$A(0.2276, -0.1414) \qquad B(0, 0.1414) \qquad C(-0.1138, 0)$

$$R = \sqrt{0.1138^2 + 0.1414^2} = 0.1815$$

$\sigma_1 = -0.1138 + 0.1815 = 0.06772 \text{ MPa} = 67.72 \text{ KPa}$ **Ans**

$\sigma_2 = -0.1138 - 0.1815 = -0.2953 \text{ MPa} = -295 \text{ KPa}$ **Ans**

Ans:
For point A: $\sigma_1 = 0$, $\sigma_2 = -8.23$ MPa,
For point B: $\sigma_1 = 67.7$ kPa, $\sigma_2 = -295$ kPa

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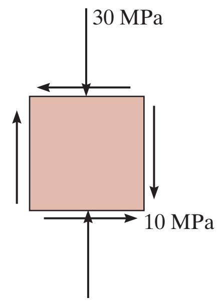
R9–5.

Determine the equivalent state of stress on an element at the same point which represents (a) the principal stresses, and (b) the maximum in-plane shear stress and the associated average normal stress. Also, for each case, determine the corresponding orientation of the element with respect to the element shown and sketch the results on the element.



SOLUTION

Normal and Shear Stress:

$$\sigma_x = 0 \qquad \sigma_y = -30 \text{ MPa} \qquad \tau_{xy} = -10 \text{ MPa}$$

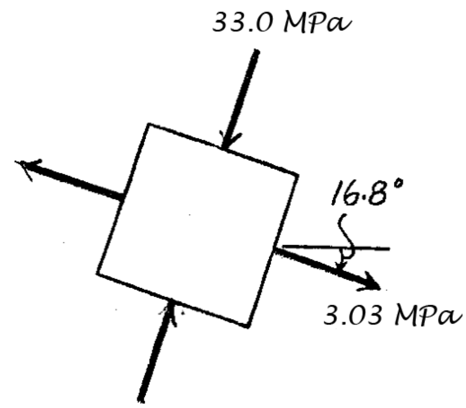
In-Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0+(-30)}{2} \pm \sqrt{\left[\frac{0-(-30)}{2}\right]^2 + (-10)^2}$$

$$= -15 \pm \sqrt{325}$$

$$\sigma_1 = 3.03 \text{ MPa} \qquad \sigma_2 = -33.0 \text{ MPa}$$ **Ans.**



(a)

Orientation of Principal Plane:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-10}{[0 - (-30)]/2} = -0.6667$$

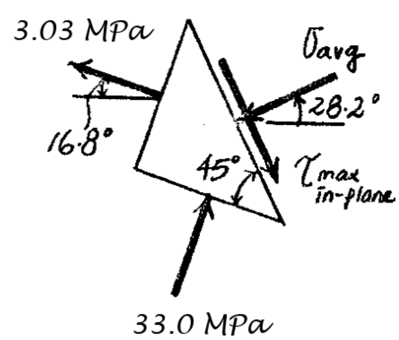
$$\theta_p = -16.845° \text{ and } 73.155°$$

Substituting $\theta = -16.845°$ into

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{0 + (-30)}{2} + \frac{0 - (-30)}{2} \cos(-33.69°) - 10 \sin(-33.69°)$$

$$= 3.03 \text{ MPa} = \sigma_1$$



(b)

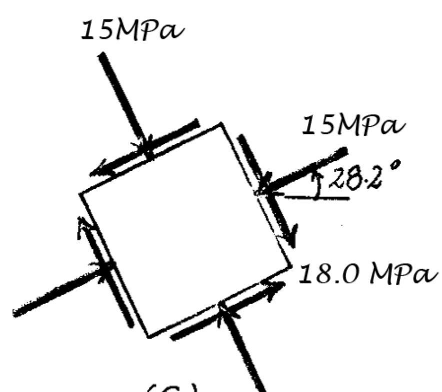
Thus,

$$(\theta_p)_1 = -16.8° \text{ and } (\theta_p)_2 = 73.2°$$ **Ans.**

The element that represents the state of principal stress is shown in Fig. *a*.

Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left[\frac{0-(-30)}{2}\right]^2 + (-10)^2} = 18.0 \text{ MPa}$$ **Ans.**



(C)

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R9–5. Continued

Orientation of the Plane of Maximum In-Plane Shear Stress:

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = -\frac{[0-(-30)]/2}{-10} = 1.5 \qquad \textbf{Ans.}$$

$$\theta_s = 28.2^\circ \text{ and } 118^\circ$$

By inspection, $\tau_{\substack{\max \\ \text{in-plane}}}$ has to act in the same sense shown in Fig. *b* to maintain equilibrium.

Average Normal Stress:

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-30)}{2} = -15 \text{ MPa} \qquad \textbf{Ans.}$$

The element that represents the state of maximum in-plane shear stress is shown in Fig. *c*.

Ans:
$\sigma_1 = 3.03\,\text{MPa}, \sigma_2 = -33.0\,\text{MPa},$
$\theta_{p1} = -16.8^\circ$ and $\theta_{p2} = 73.2^\circ,$
$\tau_{\substack{\max \\ \text{in-plane}}} = 18.0\,\text{MPa}, \sigma_{avg} = -15\,\text{MPa}, \quad \theta_s = 28.2^\circ$

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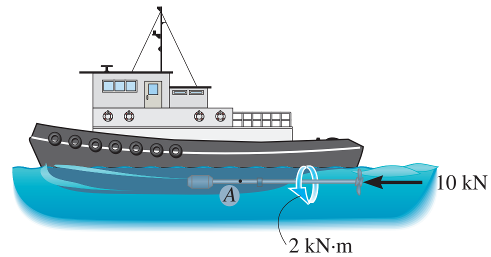
R9–6.

The propeller shaft of the tugboat is subjected to the compressive force and torque shown. If the shaft has an inner diameter of 100 mm and an outer diameter of 150 mm, determine the principal stresses at a point A located on the outer surface.



SOLUTION

Internal Loadings: Considering the equilibrium of the free-body diagram of the propeller shaft's right segment, Fig. *a*,

$\Sigma F_x = 0; \quad 10 - N = 0 \qquad\qquad N = 10 \text{ kN}$

$\Sigma M_x = 0; \quad T - 2 = 0 \qquad\qquad T = 2 \text{ kN}\cdot\text{m}$

Section Properties: The cross-sectional area and the polar moment of inertia of the propeller shaft's cross section are

$$A = \pi\left(0.075^2 - 0.05^2\right) = 3.125\pi\left(10^{-3}\right) \text{ m}^2$$

$$J = \frac{\pi}{2}\left(0.075^4 - 0.05^4\right) = 12.6953125\pi\left(10^{-6}\right) \text{ m}^4$$

Normal and Shear Stress: The normal stress is a contributed by axial stress only.

$$\sigma_A = \frac{N}{A} = -\frac{10\left(10^3\right)}{3.125\pi\left(10^{-3}\right)} = -1.019 \text{ MPa}$$

The shear stress is contributed by the torsional shear stress only.

$$\tau_A = \frac{Tc}{J} = \frac{2\left(10^3\right)(0.075)}{12.6953125\pi\left(10^{-6}\right)} = 3.761 \text{ MPa}$$

The state of stress at point A is represented by the element shown in Fig. *b*.

Construction of the Circle: $\sigma_x = -1.019$ MPa, $\sigma_y = 0$, and $\tau_{xy} = -3.761$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-1.019 + 0}{2} = -0.5093 \text{ MPa}$$

The coordinates of reference point A and the center C of the circle are

$$A(-1.019, -3.761) \qquad C(-0.5093, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{[-1.019 - (-0.5093)]^2 + (-3.761)^2} = 3.795 \text{ MPa}$$

Using these results, the circle is shown in Fig. *c*.

In-Plane Principal Stress: The coordinates of reference points B and D represent σ_1 and σ_2, respectively.

$\sigma_1 = -0.5093 + 3.795 = 3.29 \text{ MPa}$ **Ans.**

$\sigma_2 = -0.5093 - 3.795 = -4.30 \text{ MPa}$ **Ans.**

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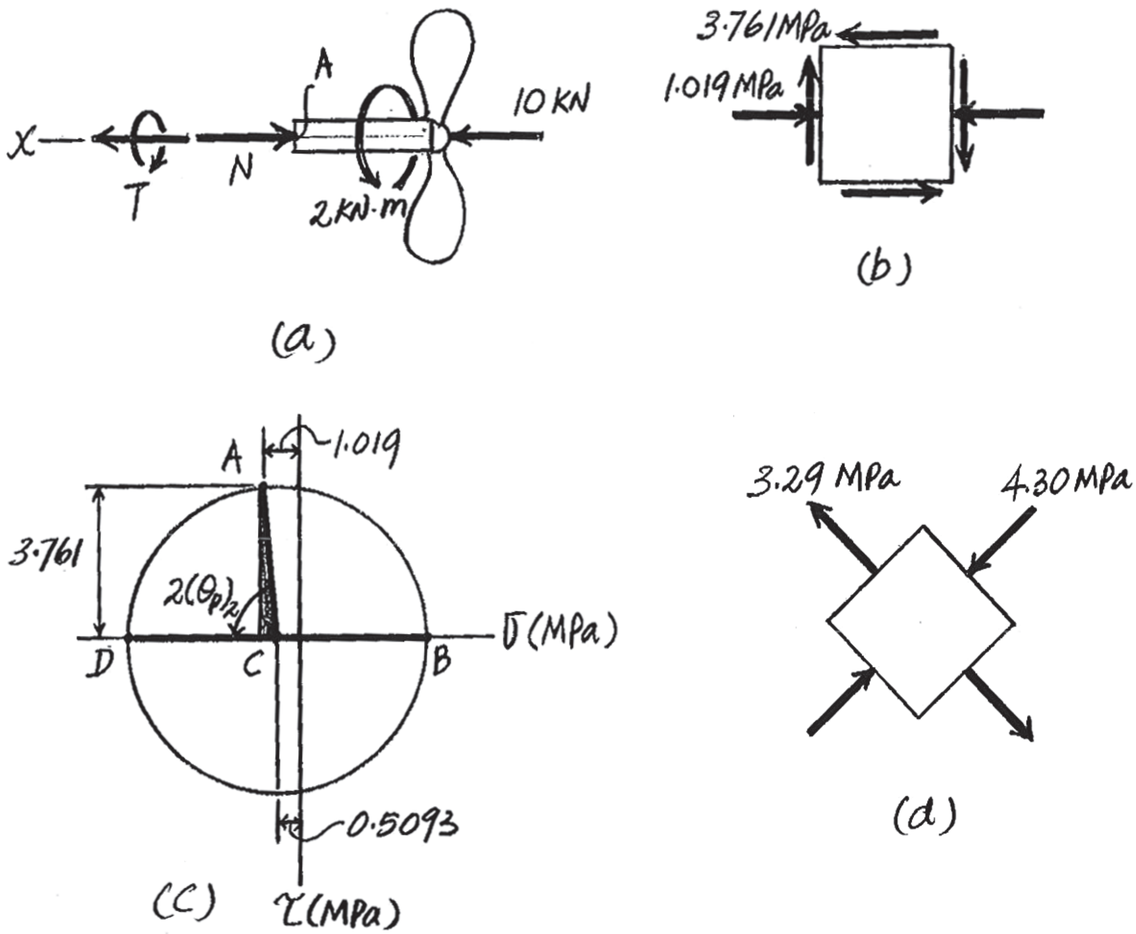
R9–6. Continued



Ans:
$\sigma_1 = 3.29$ MPa,
$\sigma_2 = -4.30$ MPa

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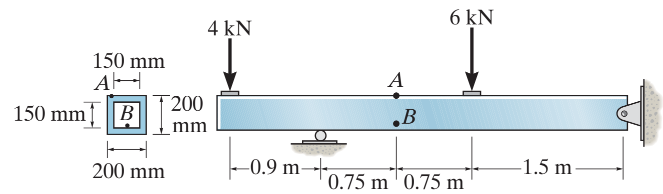
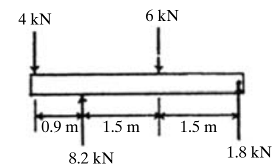
R9–7.

The box beam is subjected to the loading shown. Determine the principal stress in the beam at points A and B.



SOLUTION

Support Reactions: As shown on FBD(a).



Internal Forces and Moment: As shown on FBD(b).

Section Properties:

$$I = \frac{1}{12}(0.2)(0.2^3) - \frac{1}{12}(0.15)(0.15^3) = 91.1458(10^{-6})\ \text{m}^4$$

$$Q_A = Q_B = 0$$

Normal Stress: Applying the flexure formula.

$$\sigma = -\frac{My}{I}$$

$$\sigma_A = -\frac{[-0.45(10^3)](0.1)}{91.1458(10^{-6})} = 0.4937(10^6)\ \text{N/m}^2 = 0.494\ \text{MPa}$$

$$\sigma_B = -\frac{[-0.45(10^3)](-0.075)}{91.1458(10^{-6})} = -0.3703(10^6)\ \text{N/m}^2 = -0.370\ \text{MPa}$$

Shear Stress: Since $Q_A = Q_B = 0$, then $\tau_A = \tau_B = 0$.

In - Plane Principal Stress: $\sigma_x = 0.494$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0$. for point A. Since no shear stress acts on the element,

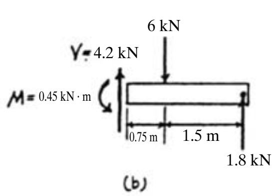
$$\sigma_1 = \sigma_x = 0.494\ \text{MPa} \qquad \textbf{Ans.}$$

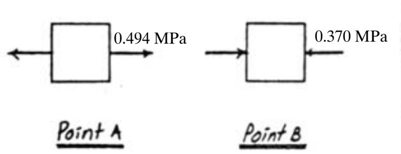
$$\sigma_2 = \sigma_y = 0 \qquad \textbf{Ans.}$$

$\sigma_x = -0.370$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0$ for point B. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_y = 0 \qquad \textbf{Ans.}$$

$$\sigma_2 = \sigma_x = -0.370\ \text{MPa} \qquad \textbf{Ans.}$$





Ans.
$\sigma_1 = \sigma_x = 0.494$ MPa, $\sigma_2 = \sigma_y = 0$;
$\sigma_1 = \sigma_y = 0$, $\sigma_2 = \sigma_x = -0.370$ MPa

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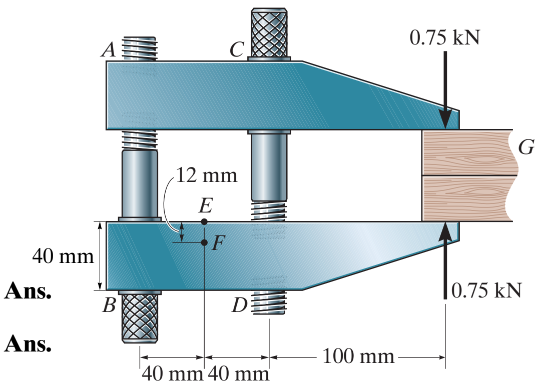
*** R9–8.**

The clamp exerts a force of 0.75 kN on the boards at G. Determine the axial force in each screw, AB and CD, and then compute the principal stresses at points E and F. Show the results on properly oriented elements located at these points. The section through EF is rectangular and is 25 mm wide, 40 mm deep.



Support Reactions: FBD(a).

$\circlearrowleft + \Sigma M_B = 0; \quad F_{CD}(0.080) - 0.75(180) = 0 \quad F_{CD} = 1.6875 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad 1.6875 - 0.75 - F_{AB} = 0 \quad F_{AB} = 0.9375 \text{ kN}$ **Ans.**

Internal Forces and Moment: As shown on FBD(b).

Section Properties:

$$I = \frac{1}{12}(0.025)(0.04^3) = 0.1333(10^{-6}) \text{ m}^4$$

$$Q_E = 0$$

$$Q_F = \overline{y}'A' = 0.014(0.025)(0.012) = 4.2(10^{-6}) \text{ m}^3$$

Normal Stress: Applying the flexure formula $\sigma = -\frac{My}{I}$,

$$\sigma_E = -\frac{(-37.5)(0.02)}{0.1333(10^{-6})} = 5.625(10^6) \text{ N/m}^2 = 5.625 \text{ MPa}$$

$$\sigma_F = -\frac{-37.5(10^3)(8)}{133333} - \frac{(-37.5)(0.008)}{0.1333(10^{-6})} = 2.25(10^6) \text{ N/m}^2 = 2.25 \text{ MPa}$$

Shear Stress: Applying the shear formula $\tau = \frac{VQ}{It}$,

$$\tau_E = \frac{[0.9375(10^{-3})](0)}{[0.1333(10^{-6})](0.025)} = 0$$

$$\tau_F = \frac{[0.9375(10^{-3})][4.2(10^{-6})]}{[0.1333(10^{-6})](0.025)}$$

$$= 1.18125(10^6) \text{ N/m}^2 = 1.18125 \text{ MPa}$$

In - Plane Principal Stress: $\sigma_x = 5.625$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0$. for point E. Since no shear stress acts upon the element.

$$\sigma_1 = \sigma_x = 5.625 \text{ MPa}$$ **Ans.**

$$\sigma_2 = \sigma_y = 0$$ **Ans.**

$\sigma_x = 2.25$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 1.1825$ MPa for point F. Applying Eq. 9-5

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{2.25 + 0}{2} \pm \sqrt{\left(\frac{2.25 - 0}{2}\right)^2 + 1.18125^2}$$

$$= 1.125 \pm 1.63125$$

$$\sigma_1 = 2.76 \text{ MPa} \qquad \sigma_2 = -0.506 \text{ MPa}$$ **Ans.**

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***R9–8. Continued**

Orientation of Principal Plane: Applying Eq. 9-4 for point *F*,

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{1.18125}{(2.25-0)/2} = 1.05$$

$$\theta_p = 23.20° \quad \text{and} \quad -66.80°$$

Substituting the results into Eq. 9-1 with $\theta = 23.20°$ yields

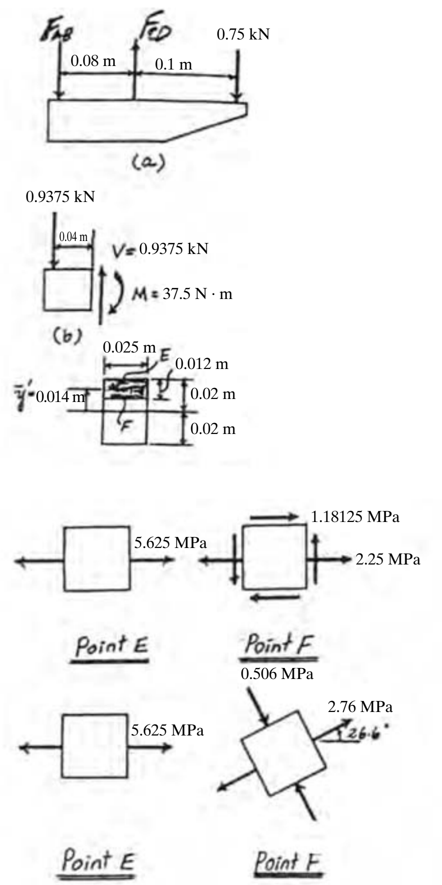
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{2.25+0}{2} + \frac{2.25-0}{2}\cos 46.40° + 1.18125\sin 46.40°$$

$$= 2.76 \text{ MPa} = \sigma_1$$

Hence,

$$\theta_{p1} = 23.2° \qquad \theta_{p2} = -66.8°$$ **Ans.**



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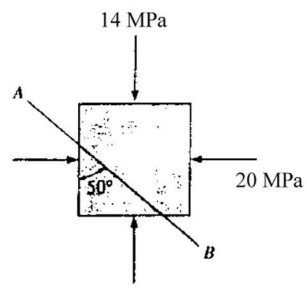
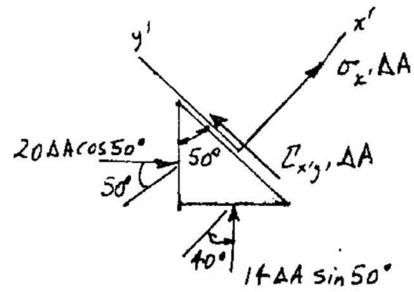
R9–9 The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.





$+\nearrow \Sigma F_{x'} = 0;$ $\quad \sigma_{x'} \Delta A + 14\,\Delta A \sin 50° \cos 40° + 20\,\Delta A \cos 50° \cos 50° = 0$

$\sigma_{x'} = -16.5\text{ MPa}$ **Ans**

$+\nwarrow \Sigma F_{y'} = 0;$ $\quad \tau_{x'y'}\,\Delta A + 14\,\Delta A \sin 50° \sin 40° - 20\,\Delta A \cos 50° \sin 50° = 0$

$\tau_{xy} = 2.95\text{ MPa}$ **Ans**

Ans:
$\sigma_{x'} = -16.5\text{ MPa}, \quad \tau_{x'y'} = 2.95\text{ MPa}$