2–1.

An air-filled rubber ball has a diameter of 150 mm. If the air pressure within it is increased until the ball's diameter becomes 175 mm, determine the average normal strain in the rubber.

SOLUTION

$d_0 = 150 \text{ mm}$

$d = 175 \text{ mm}$

$$\varepsilon = \frac{\pi d - \pi d_0}{\pi d_0} = \frac{175 - 150}{150} = 0.167 \text{ mm/mm}$$ **Ans.**

Ans:

$\varepsilon = 0.167 \text{ mm/mm}$

2–2.

A thin strip of rubber has an unstretched length of 375 mm. If it is stretched around a pipe having an outer diameter of 125 mm, determine the average normal strain in the strip.

SOLUTION

$L_0 = 375 \text{ mm}$

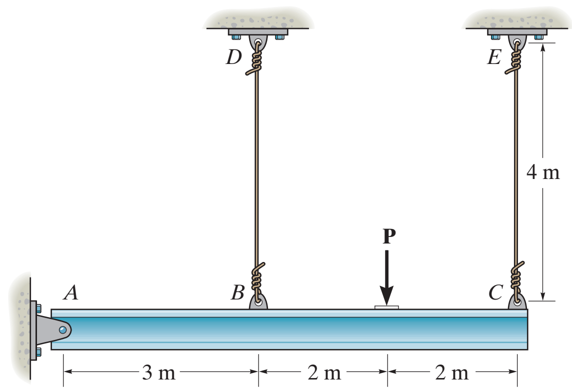
$L = \pi(125 \text{ mm})$

$$\varepsilon = \frac{L - L_0}{L_0} = \frac{125\pi - 375}{375} = 0.0472 \text{ mm/mm}$$ **Ans.**

Ans.
$\varepsilon = 0.0472 \text{ mm/mm}$

2–3.

If the load **P** on the beam causes the end C to be displaced 10 mm downward, determine the normal strain in wires CE and BD.



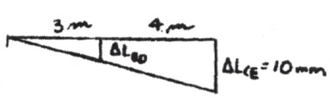
SOLUTION

$$\frac{\Delta L_{BD}}{3} = \frac{\Delta L_{CE}}{7}$$

$$\Delta L_{BD} = \frac{3\,(10)}{7} = 4.286 \text{ mm}$$

$$\epsilon_{CE} = \frac{\Delta L_{CE}}{L} = \frac{10}{4000} = 0.00250 \text{ mm/mm}$$ **Ans.**

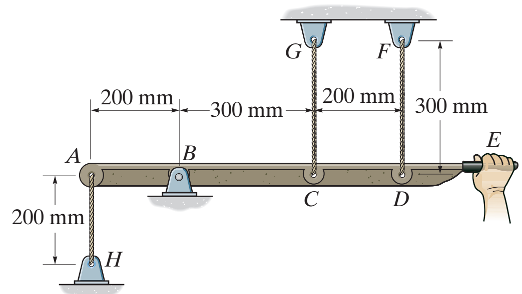
$$\epsilon_{BD} = \frac{\Delta L_{BD}}{L} = \frac{4.286}{4000} = 0.00107 \text{ mm/mm}$$ **Ans.**



Ans:
$\epsilon_{CE} = 0.00250$ mm/mm, $\epsilon_{BD} = 0.00107$ mm/mm

***2–4.**

The force applied at the handle of the rigid lever causes the lever to rotate clockwise about the pin B through an angle of 2°. Determine the average normal strain in each wire. The wires are unstretched when the lever is in the horizontal position.



SOLUTION

Geometry: The lever arm rotates through an angle of $\theta = \left(\frac{2°}{180}\right)\pi \text{ rad} = 0.03491 \text{ rad}$.

Since θ is small, the displacements of points A, C, and D can be approximated by

$$\delta_A = 200(0.03491) = 6.9813 \text{ mm}$$

$$\delta_C = 300(0.03491) = 10.4720 \text{ mm}$$

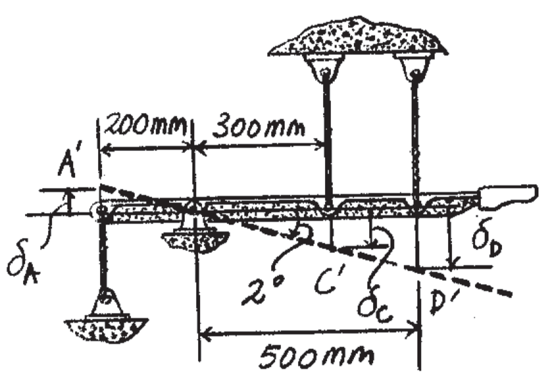
$$\delta_D = 500(0.03491) = 17.4533 \text{ mm}$$

Average Normal Strain: The unstretched length of wires AH, CG, and DF are $L_{AH} = 200$ mm, $L_{CG} = 300$ mm, and $L_{DF} = 300$ mm. We obtain

$$(\epsilon_{\text{avg}})_{AH} = \frac{\delta_A}{L_{AH}} = \frac{6.9813}{200} = 0.0349 \text{ mm/mm} \qquad \textbf{Ans.}$$

$$(\epsilon_{\text{avg}})_{CG} = \frac{\delta_C}{L_{CG}} = \frac{10.4720}{300} = 0.0349 \text{ mm/mm} \qquad \textbf{Ans.}$$

$$(\epsilon_{\text{avg}})_{DF} = \frac{\delta_D}{L_{DF}} = \frac{17.4533}{300} = 0.0582 \text{ mm/mm} \qquad \textbf{Ans.}$$

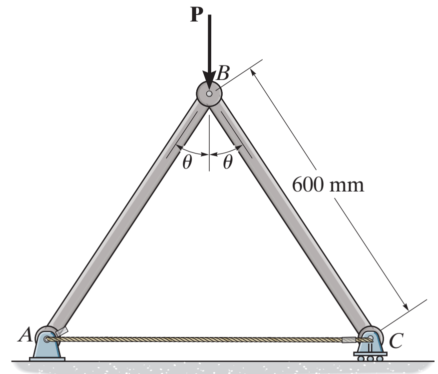


(a)

Ans:
$(\epsilon_{\text{avg}})_{AH} = 0.0349$ mm/mm
$(\epsilon_{\text{avg}})_{CG} = 0.0349$ mm/mm
$(\epsilon_{\text{avg}})_{DF} = 0.0582$ mm/mm

2–5.

The pin-connected rigid rods AB and BC are inclined at $\theta = 30°$ when they are unloaded. When the force **P** is applied θ becomes 30.2°. Determine the average normal strain in wire AC.



SOLUTION

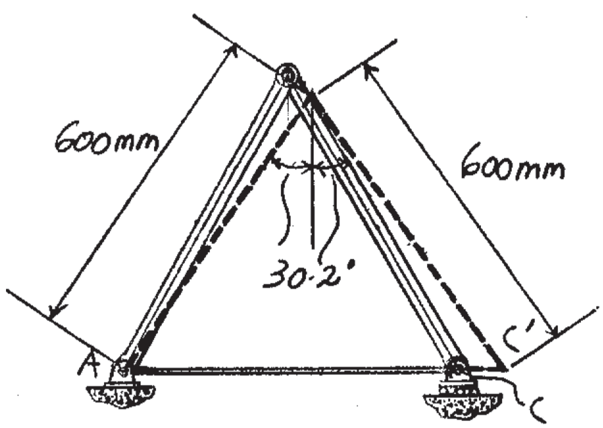
Geometry: Referring to Fig. a, the unstretched and stretched lengths of wire AD are

$$L_{AC} = 2(600 \sin 30°) = 600 \text{ mm}$$

$$L_{AC'} = 2(600 \sin 30.2°) = 603.6239 \text{ mm}$$

Average Normal Strain:

$$(\epsilon_{\text{avg}})_{AC} = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{603.6239 - 600}{600} = 6.04(10^{-3}) \text{ mm/mm} \qquad \textbf{Ans.}$$

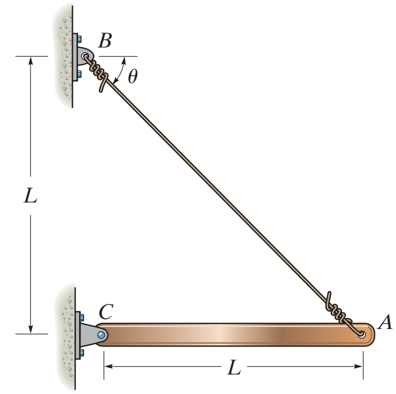


(a)

Ans:
$(\epsilon_{\text{avg}})_{AC} = 6.04(10^{-3}) \text{ mm/mm}$

2–6.

The wire AB is unstretched when $\theta = 45°$. If a load is applied to the bar AC, which causes θ to become 47°, determine the normal strain in the wire.



SOLUTION

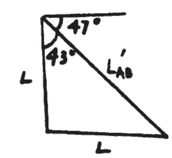
$$L^2 = L^2 + L'^2_{AB} - 2LL'_{AB}\cos 43°$$

$$L'_{AB} = 2L\cos 43°$$

$$\epsilon_{AB} = \frac{L'_{AB} - L_{AB}}{L_{AB}}$$

$$= \frac{2L\cos 43° - \sqrt{2}L}{\sqrt{2}L}$$

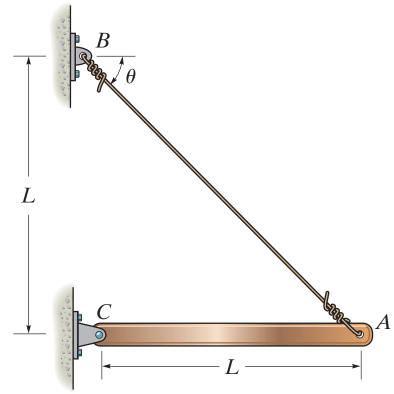
$$= 0.0343 \qquad \textbf{Ans.}$$



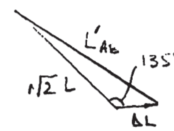
Ans:
$\epsilon_{AB} = 0.0343$

2–7.

If a horizontal load applied to the bar AC causes point A to be displaced to the right by an amount ΔL, determine the normal strain in the wire AB. Originally, $\theta = 45°$.



SOLUTION



$$L'_{AB} = \sqrt{(\sqrt{2}L)^2 + \Delta L^2 - 2(\sqrt{2}L)(\Delta L)\cos 135°}$$

$$= \sqrt{2L^2 + \Delta L^2 + 2L\Delta L}$$

$$\epsilon_{AB} = \frac{L'_{AB} - L_{AB}}{L_{AB}}$$

$$= \frac{\sqrt{2L^2 + \Delta L^2 + 2L\Delta L} - \sqrt{2}L}{\sqrt{2}L}$$

$$= \sqrt{1 + \frac{\Delta L^2}{2L^2} + \frac{\Delta L}{L}} - 1$$

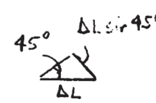
Neglecting the higher-order terms,

$$\epsilon_{AB} = \left(1 + \frac{\Delta L}{L}\right)^{\frac{1}{2}} - 1$$

$$= 1 + \frac{1}{2}\frac{\Delta L}{L} + \ldots\ldots - 1 \qquad \text{(binomial theorem)}$$

$$= \frac{0.5\Delta L}{L} \qquad \textbf{Ans.}$$

Also,



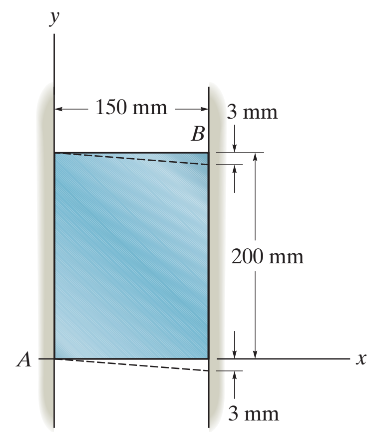
$$\epsilon_{AB} = \frac{\Delta L \sin 45°}{\sqrt{2}L} = \frac{0.5\,\Delta L}{L} \qquad \textbf{Ans.}$$

Ans:

$$\epsilon_{AB} = \frac{0.5\Delta L}{L}$$

***2–8.**

The rectangular plate is subjected to the deformation shown by the dashed line. Determine the average shear strain γ_{xy} in the plate.



SOLUTION

Geometry:

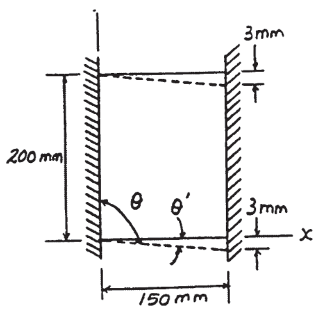
$$\theta' = \tan^{-1}\frac{3}{150} = 0.0200 \text{ rad}$$

$$\theta = \left(\frac{\pi}{2} + 0.0200\right) \text{rad}$$

Shear Strain:

$$\gamma_{xy} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \left(\frac{\pi}{2} + 0.0200\right)$$

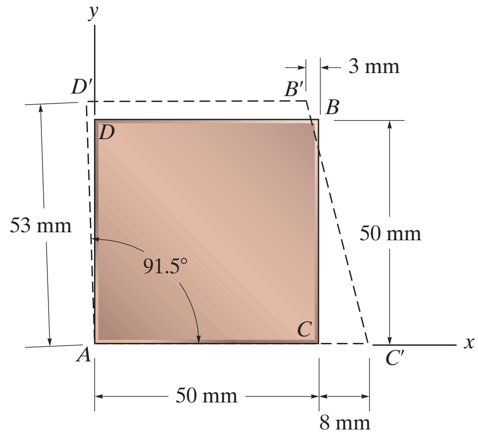
$$= -0.0200 \text{ rad} \qquad \textbf{Ans.}$$



Ans:
$\gamma_{xy} = -0.0200$ rad

2–9.

The square deforms into the position shown by the dashed lines. Determine the shear strain at each of its corners, A, B, C, and D, relative to the x, y axes. Side $D'B'$ remains horizontal.



SOLUTION

Geometry:

$$B'C' = \sqrt{(8 + 3)^2 + (53 \sin 88.5°)^2} = 54.1117 \text{ mm}$$

$$C'D' = \sqrt{53^2 + 58^2 - 2(53)(58) \cos 91.5°}$$

$$= 79.5860 \text{ mm}$$

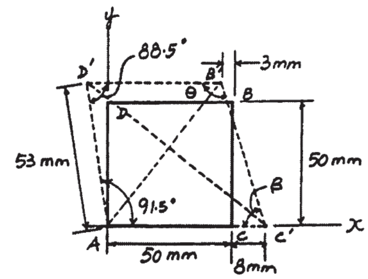
$$B'D' = 50 + 53 \sin 1.5° - 3 = 48.3874 \text{ mm}$$

$$\cos \theta = \frac{(B'D')^2 + (B'C')^2 - (C'D')^2}{2(B'D')(B'C')}$$

$$= \frac{48.3874^2 + 54.1117^2 - 79.5860^2}{2(48.3874)(54.1117)} = -0.20328$$

$$\theta = 101.73°$$

$$\beta = 180° - \theta = 78.27°$$



Shear Strain:

$$(\gamma_A)_{xy} = \frac{\pi}{2} - \pi\left(\frac{91.5°}{180°}\right) = -0.0262 \text{ rad}$$ **Ans.**

$$(\gamma_B)_{xy} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \pi\left(\frac{101.73°}{180°}\right) = -0.205 \text{ rad}$$ **Ans.**

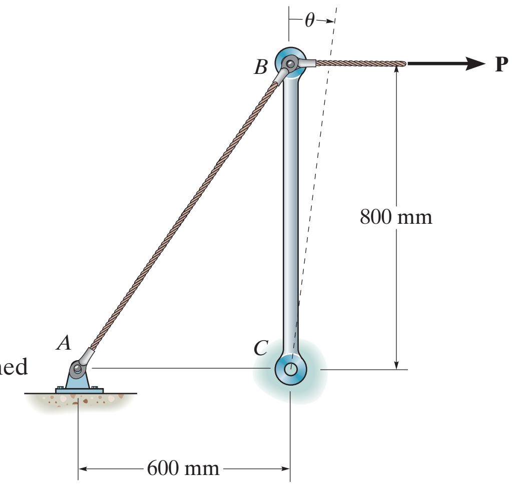
$$(\gamma_C)_{xy} = \beta - \frac{\pi}{2} = \pi\left(\frac{78.27°}{180°}\right) - \frac{\pi}{2} = -0.205 \text{ rad}$$ **Ans.**

$$(\gamma_D)_{xy} = \pi\left(\frac{88.5°}{180°}\right) - \frac{\pi}{2} = -0.0262 \text{ rad}$$ **Ans.**

Ans:
$(\gamma_A)_{xy} = -0.0262$ rad
$(\gamma_B)_{xy} = -0.205$ rad
$(\gamma_C)_{xy} = -0.205$ rad
$(\gamma_D)_{xy} = -0.0262$ rad

2–10.

Part of a control linkage for an airplane consists of a rigid member CB and a flexible cable AB. If a force is applied to the end B of the member and causes it to rotate by $\theta = 0.5°$, determine the normal strain in the cable. Originally the cable is unstretched.



SOLUTION

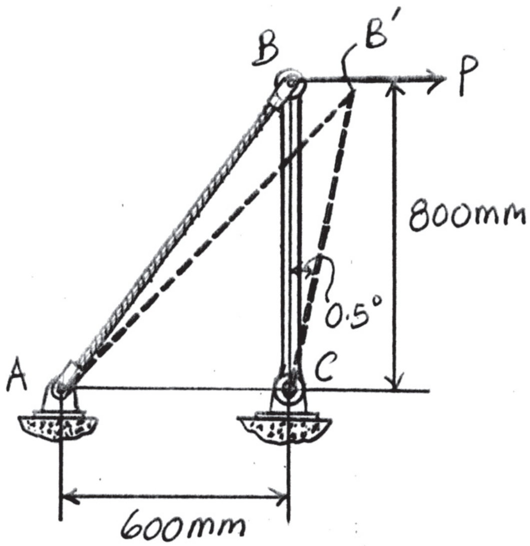
Geometry: Referring to the geometry shown in Fig. a, the unstretched and stretched lengths of cable AB are

$$L_{AB} = \sqrt{600^2 + 800^2} = 1000 \text{ mm}$$

$$L_{AB'} = \sqrt{600^2 + 800^2 - 2(600)(800)\cos 90.5°} = 1004.18 \text{ mm}$$

Average Normal Strain:

$$\epsilon_{AB} = \frac{L_{AB'} - L_{AB}}{L_{AB}} = \frac{1004.18 - 1000}{1000} = 0.00418 \text{ mm/mm}$$ **Ans.**

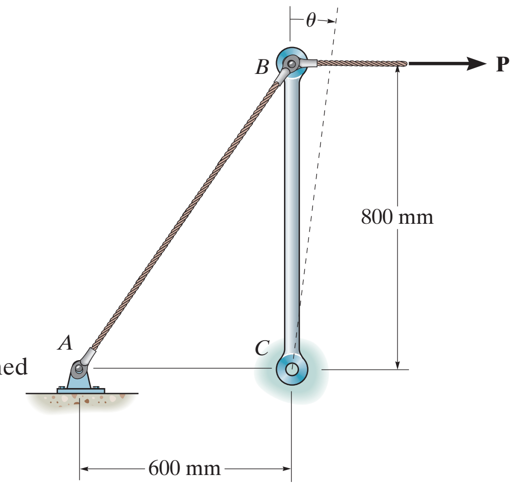


(a)

Ans:
$\epsilon_{AB} = 0.00418$ mm/mm

2–11.

Part of a control linkage for an airplane consists of a rigid member CB and a flexible cable AB. If a force is applied to the end B of the member and causes a normal strain in the cable of 0.004 mm/mm, determine the displacement of point B. Originally the cable is unstretched.



SOLUTION

Geometry: Referring to the geometry shown in Fig. a, the unstretched and stretched lengths of cable AB are

$$L_{AB} = \sqrt{600^2 + 800^2} = 1000 \text{ mm}$$

$$L_{AB'} = \sqrt{600^2 + 800^2 - 2(600)(800)\cos(90^\circ + \theta)}$$

$$L_{AB'} = \sqrt{1(10^6) - 0.960(10^6)\cos(90^\circ + \theta)}$$

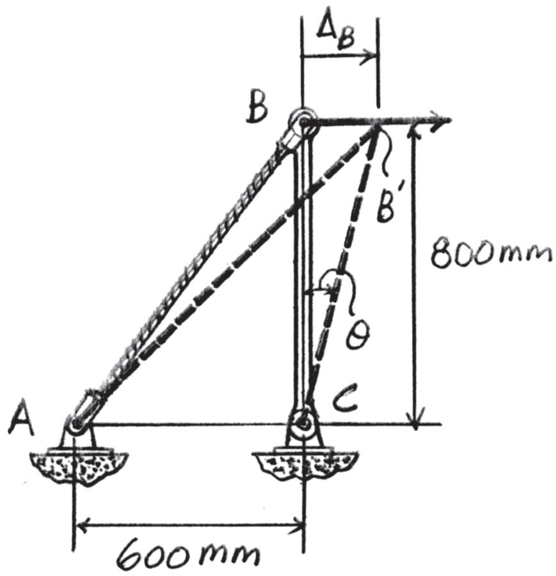
Average Normal Strain:

$$\epsilon_{AB} = \frac{L_{AB'} - L_{AB}}{L_{AB}}; \qquad 0.004 = \frac{\sqrt{1(10^6) - 0.960(10^6)\cos(90^\circ + \theta)} - 1000}{1000}$$

$$\theta = 0.4784^\circ\left(\frac{\pi}{180^\circ}\right) = 0.008350 \text{ rad}$$

Thus,

$$\Delta_B = \theta L_{BC} = 0.008350(800) = 6.68 \text{ mm}$$ **Ans.**

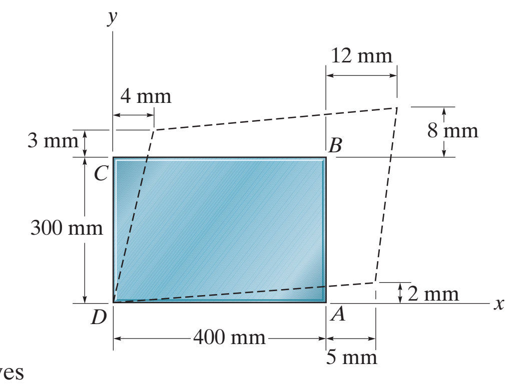


(a)

Ans:
$\Delta_B = 6.68$ mm

***2–12.**

Determine the shear strain γ_{xy} at corners A and B if the plastic distorts as shown by the dashed lines.



SOLUTION

Geometry: Referring to the geometry shown in Fig. *a*, the small-angle analysis gives

$$\alpha = \psi = \frac{7}{306} = 0.022876 \text{ rad}$$

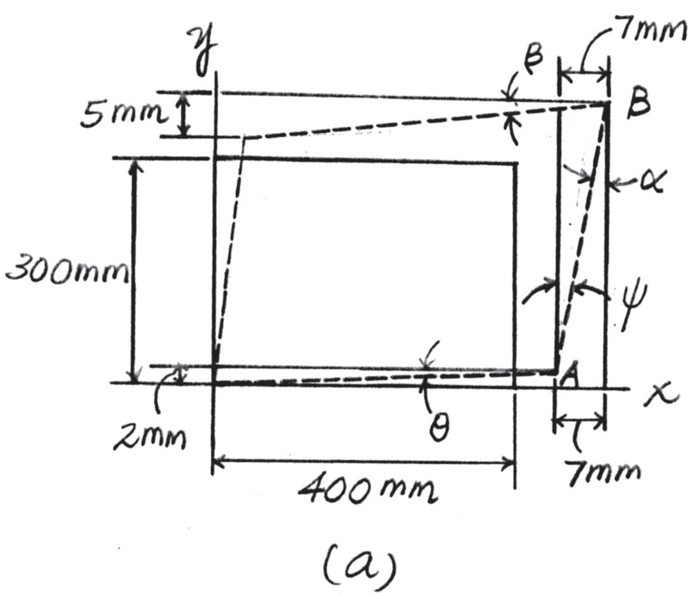
$$\beta = \frac{5}{408} = 0.012255 \text{ rad}$$

$$\theta = \frac{2}{405} = 0.0049383 \text{ rad}$$

Shear Strain: By definition,

$$(\gamma_A)_{xy} = \theta + \psi = 0.02781 \text{ rad} = 27.8(10^{-3}) \text{ rad} \qquad \textbf{Ans.}$$

$$(\gamma_B)_{xy} = \alpha + \beta = 0.03513 \text{ rad} = 35.1(10^{-3}) \text{ rad} \qquad \textbf{Ans.}$$

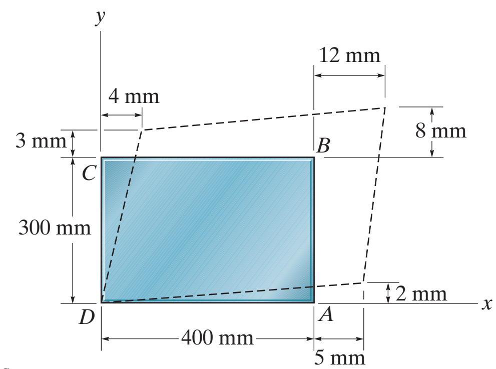


(a)

Ans:
$(\gamma_A)_{xy} = 27.8(10^{-3})$ rad
$(\gamma_B)_{xy} = 35.1(10^{-3})$ rad

2–13.

Determine the shear strain γ_{xy} at corners D and C if the plastic distorts as shown by the dashed lines.



SOLUTION

Geometry: Referring to the geometry shown in Fig. *a*, the small-angle analysis gives

$$\alpha = \psi = \frac{4}{303} = 0.013201 \text{ rad}$$

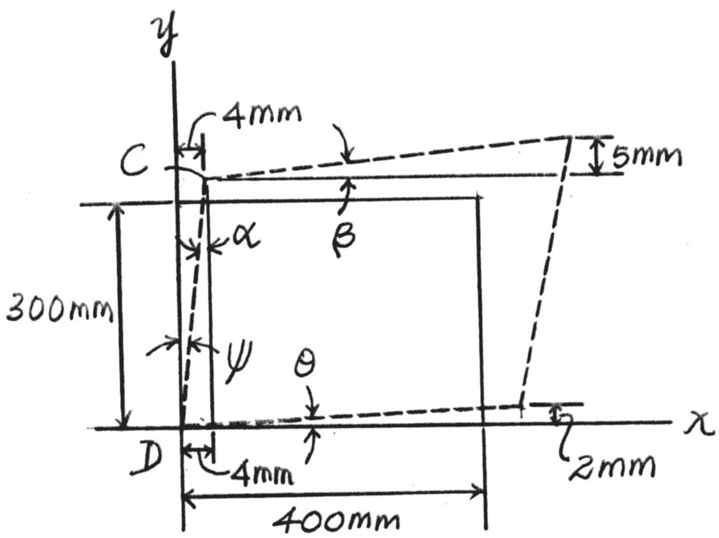
$$\theta = \frac{2}{405} = 0.0049383 \text{ rad}$$

$$\beta = \frac{5}{408} = 0.012255 \text{ rad}$$

Shear Strain: By definition

$$(\gamma_{xy})_C = \alpha + \beta = 0.02546 \text{ rad} = 25.5(10^{-3}) \text{ rad} \qquad \textbf{Ans.}$$

$$(\gamma_{xy})_D = \theta + \psi = 0.01814 \text{ rad} = 18.1(10^{-3}) \text{ rad} \qquad \textbf{Ans.}$$

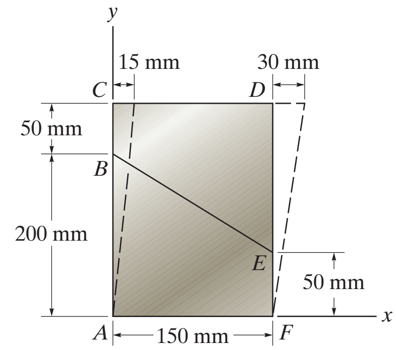


(a)

Ans:
$(\gamma_{xy})_C = 25.5(10^{-3})$ rad
$(\gamma_{xy})_D = 18.1(10^{-3})$ rad

2–14.

The material distorts into the dashed position shown. Determine the average normal strains ϵ_x, ϵ_y and the shear strain γ_{xy} at A, and the average normal strain along line BE.



SOLUTION

Geometry: Referring to the geometry shown in Fig. *a*,

$$\tan\theta = \frac{15}{250}; \qquad \theta = (3.4336°)\left(\frac{\pi}{180°}\text{ rad}\right) = 0.05993 \text{ rad}$$

$$L_{AC'} = \sqrt{15^2 + 150^2} = \sqrt{62725} \text{ mm}$$

$$\frac{BB'}{15} = \frac{200}{250}; \qquad BB' = 12 \text{ mm} \qquad \frac{EE'}{30} = \frac{50}{250}; \qquad EE' = 6 \text{ mm}$$

$$x' = 150 + EE' - BB' = 150 + 6 - 12 = 144 \text{ mm}$$

$$L_{BE} = \sqrt{150^2 + 150^2} = 150\sqrt{2} \text{ mm} \quad L_{B'E'} = \sqrt{144^2 + 150^2} = \sqrt{43236} \text{ mm}$$

Average Normal and Shear Strain: Since no deformation occurs along x axis,

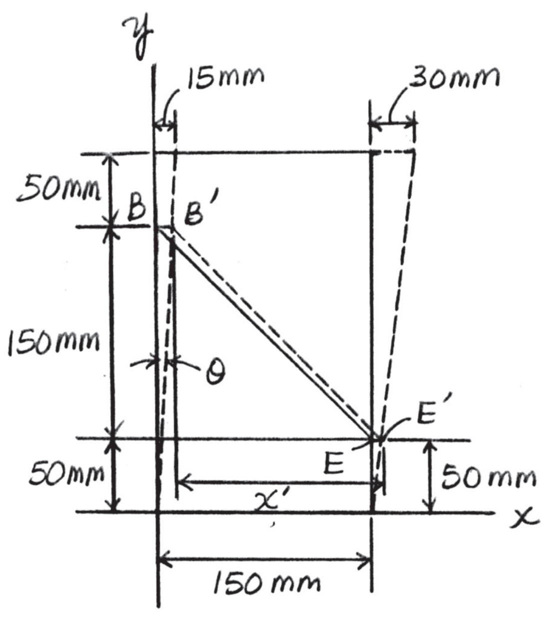
$$(\epsilon_x)_A = 0 \qquad \textbf{Ans.}$$

$$(\epsilon_y)_A = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{\sqrt{62725} - 250}{250} = 1.80(10^{-3}) \text{ mm/mm} \qquad \textbf{Ans.}$$

By definition,

$$(\gamma_{xy})_A = \theta = 0.0599 \text{ rad} \qquad \textbf{Ans.}$$

$$\epsilon_{BE} = \frac{L_{B'E'} - L_{BE}}{L_{BE}} = \frac{\sqrt{43236} - 150\sqrt{2}}{150\sqrt{2}} = -0.0198 \text{ mm/mm} \qquad \textbf{Ans.}$$

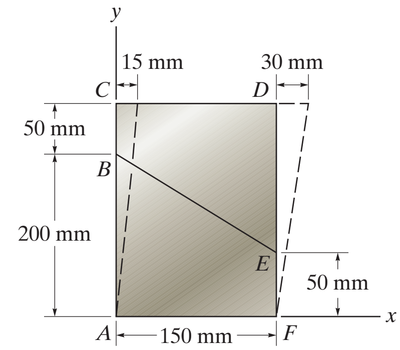


(a)

Ans:
$(\epsilon_x)_A = 0$
$(\epsilon_y)_A = 1.80(10^{-3})$ mm/mm
$(\gamma_{xy})_A = 0.0599$ rad
$\epsilon_{BE} = -0.0198$ mm/mm

2–15.

The material distorts into the dashed position shown. Determine the average normal strains along the diagonals *AD* and *CF*.



SOLUTION

Geometry: Referring to the geometry shown in Fig. *a*,

$$L_{AD} = L_{CF} = \sqrt{150^2 + 250^2} = \sqrt{85000}\text{ mm}$$

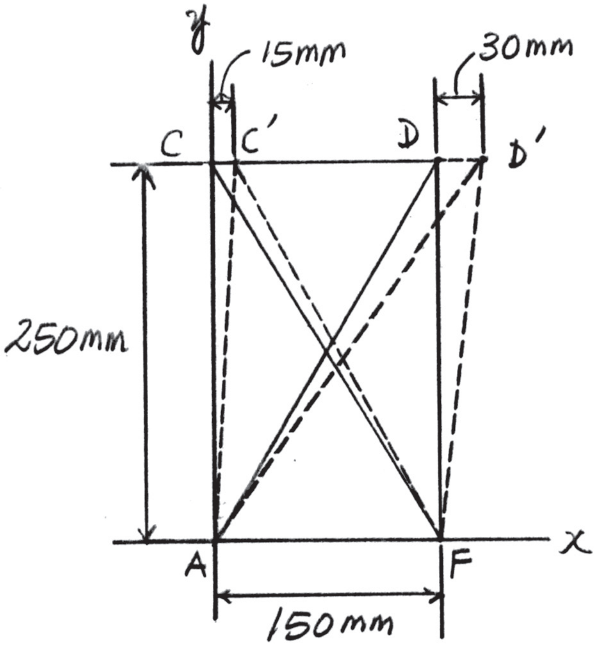
$$L_{AD'} = \sqrt{(150 + 30)^2 + 250^2} = \sqrt{94900}\text{ mm}$$

$$L_{C'F} = \sqrt{(150 - 15)^2 + 250^2} = \sqrt{80725}\text{ mm}$$

Average Normal Strain:

$$\epsilon_{AD} = \frac{L_{AD'} - L_{AD}}{L_{AD}} = \frac{\sqrt{94900} - \sqrt{85000}}{\sqrt{85000}} = 0.0566\text{ mm/mm} \qquad \textbf{Ans.}$$

$$\epsilon_{CF} = \frac{L_{C'F} - L_{CF}}{L_{CF}} = \frac{\sqrt{80725} - \sqrt{85000}}{\sqrt{85000}} = -0.0255\text{ mm/mm} \qquad \textbf{Ans.}$$

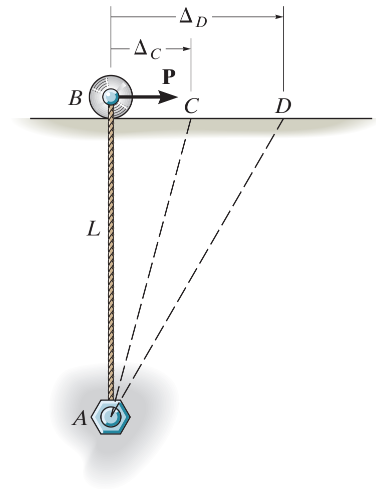


(a)

Ans:
$\epsilon_{AD} = 0.0566\text{ mm/mm}$
$\epsilon_{CF} = -0.0255\text{ mm/mm}$

***2–16.**

The nylon cord has an original length L and is tied to a bolt at A and a roller at B. If a force $\mathbf{P}$ is applied to the roller, determine the normal strain in the cord when the roller is at C, and at D. If the cord is originally unstrained when it is at C, determine the normal strain ϵ'_D when the roller moves to D. Show that if the displacements Δ_C and Δ_D are small, then $\epsilon'_D = \epsilon_D - \epsilon_C$.



SOLUTION

$$L_C = \sqrt{L^2 + \Delta_C^2}$$

$$\epsilon_C = \frac{\sqrt{L^2 + \Delta_C^2} - L}{L}$$

$$= \frac{L\sqrt{1 + \left(\frac{\Delta_C^2}{L^2}\right)} - L}{L} = \sqrt{1 + \left(\frac{\Delta_C^2}{L^2}\right)} - 1$$

For small Δ_C,

$$\epsilon_C = 1 + \frac{1}{2}\left(\frac{\Delta_C^2}{L^2}\right) - 1 = \frac{1}{2}\frac{\Delta_C^2}{L^2}$$ **Ans.**

In the same manner,

$$\epsilon_D = \frac{1}{2}\frac{\Delta_D^2}{L^2}$$ **Ans.**

$$\epsilon_D{}' = \frac{\sqrt{L^2 + \Delta_D^2} - \sqrt{L^2 + \Delta_C^2}}{\sqrt{L^2 + \Delta_C^2}} = \frac{\sqrt{1 + \frac{\Delta_D^2}{L^2}} - \sqrt{1 + \frac{\Delta_C^2}{L^2}}}{\sqrt{1 + \frac{\Delta_C^2}{L^2}}}$$

For small Δ_C and Δ_D,

$$\epsilon_D{}' = \frac{\left(1 + \frac{1}{2}\frac{\Delta_C^2}{L^2}\right) - \left(1 + \frac{1}{2}\frac{\Delta_D^2}{L^2}\right)}{\left(1 + \frac{1}{2}\frac{\Delta_C^2}{L^2}\right)} = \frac{\frac{1}{2L^2}\left(\Delta_C^2 - \Delta_D^2\right)}{\frac{1}{2L^2}\left(2L^2 + \Delta_C^2\right)}$$

$$\epsilon_D{}' = \frac{\Delta_C^2 - \Delta_D^2}{2L^2 - \Delta_C^2} = \frac{1}{2L^2}\left(\Delta_C^2 - \Delta_D^2\right) = \epsilon_C - \epsilon_D$$ **QED**

Also this problem can be solved as follows:

$$A_C = L\sec\theta_C\,; \qquad A_D = L\sec\theta_D$$

$$\epsilon_C = \frac{L\sec\theta_C - L}{L} = \sec\theta_C - 1$$

$$\epsilon_D = \frac{L\sec\theta_D - L}{L} = \sec\theta_D - 1$$

Expanding $\sec\theta$

$$\sec\theta = 1 + \frac{\theta^2}{2\,!} + \frac{5\,\theta^4}{4\,!}\ \ldots\ldots$$

***2–16. Continued**

For small θ neglect the higher order terms

$$\sec\theta = 1 + \frac{\theta^2}{2}$$

Hence,

$$\epsilon_C = 1 + \frac{\theta_C{}^2}{2} - 1 = \frac{\theta_C{}^2}{2}$$

$$\epsilon_D = 1 + \frac{\theta_D{}^2}{2} - 1 = \frac{\theta_D{}^2}{2}$$

$$\epsilon_D{}' = \frac{L\sec\theta_D - L\sec\theta_C}{L\sec\theta_C} = \frac{\sec\theta_D}{\sec\theta_C} - 1 = \sec\theta_D\cos\theta_C - 1$$

Since $\cos\theta = 1 - \frac{\theta^2}{2\,!} + \frac{\theta^4}{4\,!}$ ......

$$\sec\theta_D\cos\theta_C = \left(1 + \frac{\theta_D{}^2}{2}......\right)\left(1 - \frac{\theta_C{}^2}{2}......\right)$$

$$= 1 - \frac{\theta_C^2}{2} + \frac{\theta_D^2}{2} - \frac{\theta_C^2\,\theta_D^2}{4}$$

Neglecting the higher order terms

$$\sec\theta_D\cos\theta_C = 1 + \frac{\theta_D^2}{2} - \frac{\theta_C^2}{2}$$

$$\epsilon_D{}' = \left[1 + \frac{\theta_2{}^2}{2} - \frac{\theta_1{}^2}{2}\right] - 1 = \frac{\theta_D{}^2}{2} - \frac{\theta_C{}^2}{2}$$

$$= \epsilon_D - \epsilon_C$$ **QED**

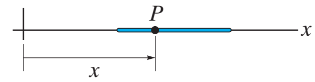
Ans:

$$\epsilon_C = \frac{1}{2}\frac{\Delta_C^2}{L^2}$$

$$\epsilon_D = \frac{1}{2}\frac{\Delta_D^2}{L^2}$$

2–17.

A thin wire, lying along the x axis, is strained such that each point on the wire is displaced $\Delta x = kx^2$ along the x axis. If k is constant, what is the normal strain at any point P along the wire?



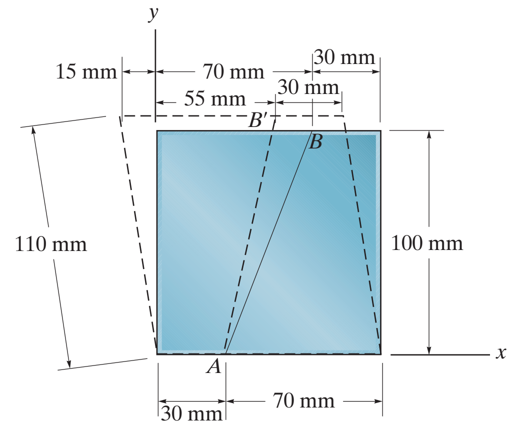
SOLUTION

$$\epsilon = \frac{d(\Delta x)}{dx} = 2\,k\,x$$ **Ans.**

Ans:
$\epsilon = 2kx$

2–18.

The block is deformed into the position shown by the dashed lines. Determine the average normal strain along line AB.



SOLUTION

Geometry:

$$AB = \sqrt{100^2 + (70 - 30)^2} = 107.7033 \text{ mm}$$

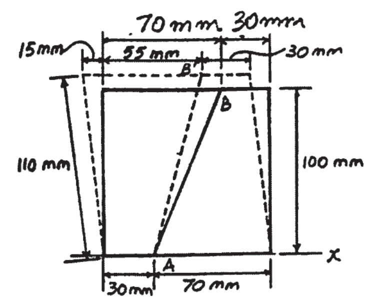
$$AB' = \sqrt{(70 - 30 - 15)^2 + \left(110^2 - 15^2\right)} = 111.8034 \text{ mm}$$

Average Normal Strain:

$$\epsilon_{AB} = \frac{AB' - AB}{AB}$$

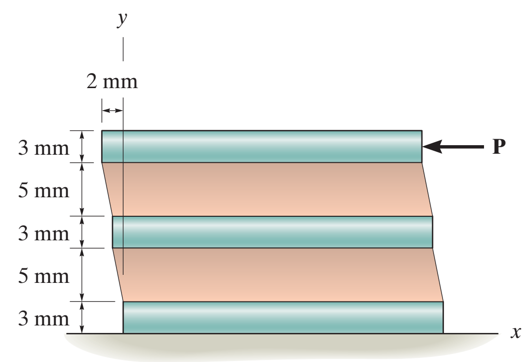
$$= \frac{111.8034 - 107.7033}{107.7033}$$

$$= 0.0381 \text{ mm/mm} = 38.1\left(10^{-3}\right) \text{ mm}$$ **Ans.**



Ans:
$\epsilon_{AB} = 38.1\left(10^{-3}\right)$ mm

2–19. Nylon strips are fused to glass plates. When moderately heated the nylon will become soft while the glass stays approximately rigid. Determine the average shear strain in the nylon due to the load **P** when the assembly deforms as indicated.



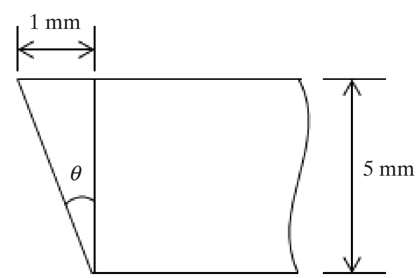
SOLUTION

From the geometry shown in Fig. a

$$\theta = \tan^{-1}\left(\frac{1}{5}\right) = 11.31^\circ = 0.1974 \text{ rad}$$

The shear strain is

$$\gamma = \frac{\pi}{2} - \left(\frac{\pi}{2} + \theta\right) = -0.1974 \text{ rad} = -0.197 \text{ rad}$$ **Ans.**

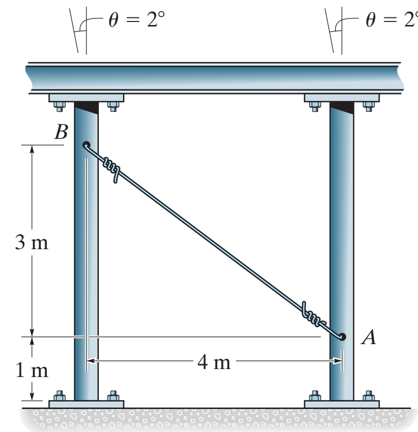


(a)

Ans:
$\gamma = -0.197$ rad

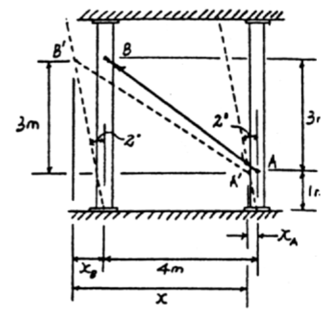
***2–20.**

The guy wire AB of a building frame is originally unstretched. Due to an earthquake, the two columns of the frame tilt $\theta = 2°$. Determine the approximate normal strain in the wire when the frame is in this position. Assume the columns are rigid and rotate about their lower supports.



SOLUTION

Geometry: The vertical displacement is negligible



$$x_A = (1)\left(\frac{2°}{180°}\right)\pi = 0.03491 \text{ m}$$

$$x_B = (4)\left(\frac{2°}{180°}\right)\pi = 0.13963 \text{ m}$$

$$x = 4 + x_B - x_A = 4.10472 \text{ m}$$

$$A'B' = \sqrt{3^2 + 4.10472^2} = 5.08416 \text{ m}$$ **Ans.**

$$AB = \sqrt{3^2 + 4^2} = 5.00 \text{ m}$$

Average Normal Strain:

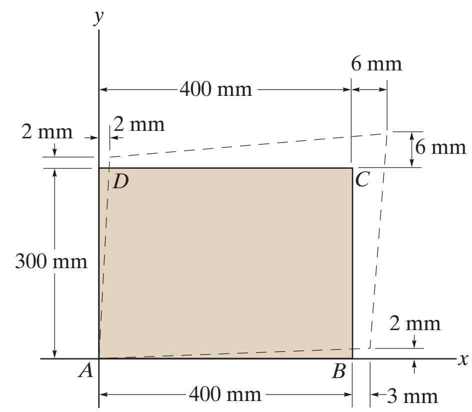
$$\varepsilon_{AB} = \frac{A'B' - AB}{AB}$$

$$= \frac{5.08416 - 5}{5} = 16.8\left(10^{-3}\right) \text{ m/m}$$

Ans:
$A'B' = 5.08416$ m

2–21.

The rectangular plate is deformed into the shape shown by the dashed lines. Determine the average normal strain along diagonal AC, and the average shear strain at corner A relative to the x, y axes.



SOLUTION

Geometry: The unstretched length of diagonal AC is

$$L_{AC} = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

Referring to Fig. a, the stretched length of diagonal AC is

$$L_{AC'} = \sqrt{(400 + 6)^2 + (300 + 6)^2} = 508.4014 \text{ mm}$$

Referring to Fig. a and using small angle analysis,

$$\phi = \frac{2}{300 + 2} = 0.006623 \text{ rad}$$

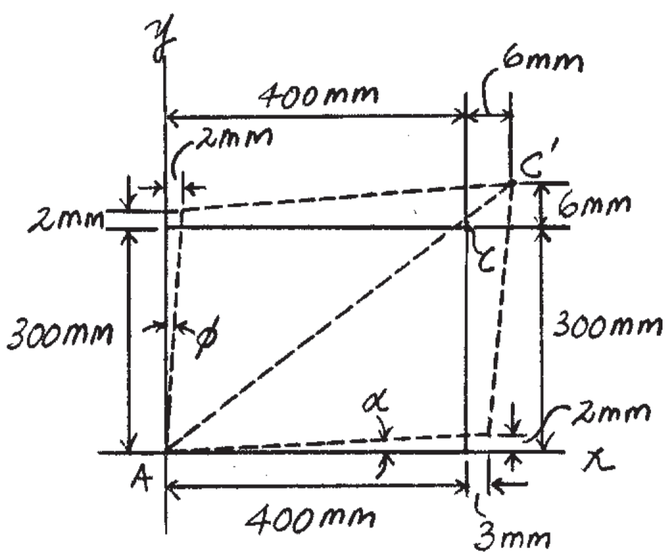
$$\alpha = \frac{2}{400 + 3} = 0.004963 \text{ rad}$$

Average Normal Strain: Applying Eq. 2,

$$(\epsilon_{\text{avg}})_{AC} = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{508.4014 - 500}{500} = 0.0168 \text{ mm/mm} \qquad \textbf{Ans.}$$

Shear Strain: Referring to Fig. a,

$$(\gamma_A)_{xy} = \phi + \alpha = 0.006623 + 0.004963 = 0.0116 \text{ rad} \qquad \textbf{Ans.}$$

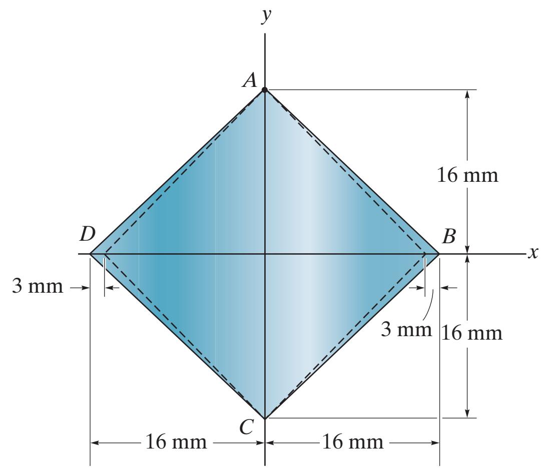


(a)

Ans:
$(\epsilon_{\text{avg}})_{AC} = 0.0168 \text{ mm/mm}$, $(\gamma_A)_{xy} = 0.0116 \text{ rad}$

2–22.

The corners B and D of the square plate are given the displacements indicated. Determine the shear strains at A and B.



SOLUTION

Applying trigonometry to Fig. a

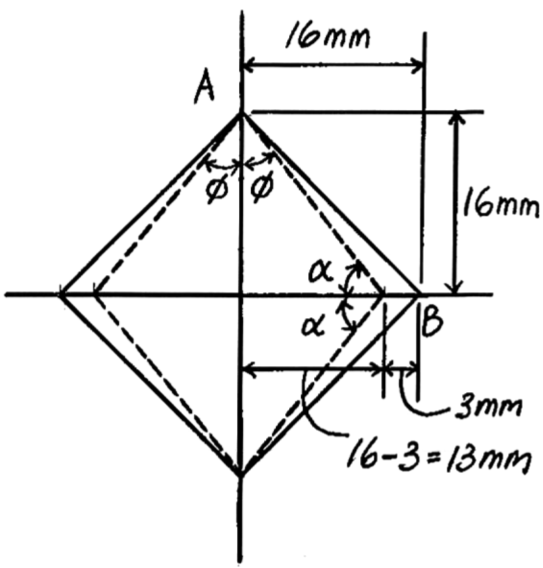
$$\phi = \tan^{-1}\left(\frac{13}{16}\right) = 39.09°\left(\frac{\pi \text{ rad}}{180°}\right) = 0.6823 \text{ rad}$$

$$\alpha = \tan^{-1}\left(\frac{16}{13}\right) = 50.91°\left(\frac{\pi \text{ rad}}{180°}\right) = 0.8885 \text{ rad}$$

By the definition of shear strain,

$$\left(\gamma_{xy}\right)_A = \frac{\pi}{2} - 2\phi = \frac{\pi}{2} - 2(0.6823) = 0.206 \text{ rad}$$ **Ans.**

$$\left(\gamma_{xy}\right)_B = \frac{\pi}{2} - 2\alpha = \frac{\pi}{2} - 2(0.8885) = -0.206 \text{ rad}$$ **Ans.**



(a)

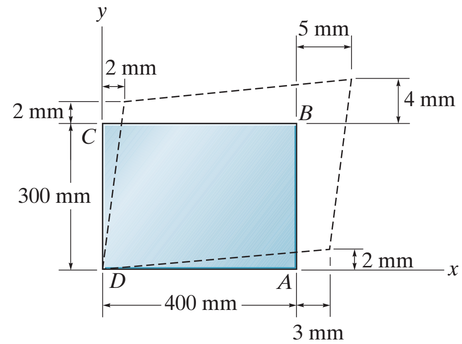
Ans:

$\left(\gamma_{xy}\right)_A = 0.206$ rad

$\left(\gamma_{xy}\right)_B = -0.206$ rad

2–23.

Determine the shear strain γ_{xy} at corners A and B if the plate distorts as shown by the dashed lines.

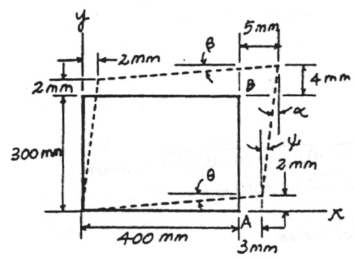


SOLUTION

Geometry: For small angles,

$$\alpha = \psi = \frac{2}{302} = 0.00662252 \text{ rad}$$

$$\beta = \theta = \frac{2}{403} = 0.00496278 \text{ rad}$$



Shear Strain:

$$(\gamma_B)_{xy} = \alpha + \beta$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

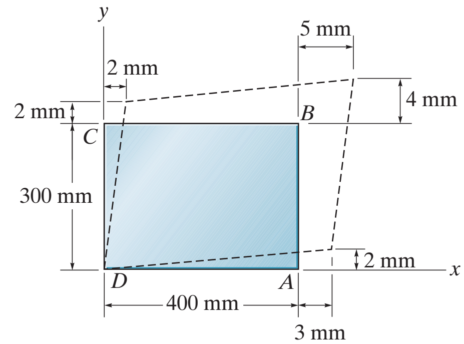
$$(\gamma_A)_{xy} = \theta + \psi$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

Ans:
$(\gamma_B)_{xy} = 11.6(10^{-3})$ rad,
$(\gamma_A)_{xy} = 11.6(10^{-3})$ rad

***2–24.**

Determine the shear strain γ_{xy} at corners D and C if the plate distorts as shown by the dashed lines.

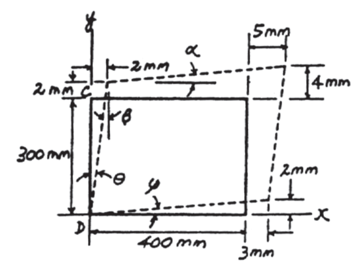


SOLUTION

Geometry: For small angles,

$$\alpha = \psi = \frac{2}{403} = 0.00496278 \text{ rad}$$

$$\beta = \theta = \frac{2}{302} = 0.00662252 \text{ rad}$$



Shear Strain:

$$(\gamma_C)_{xy} = \alpha + \beta$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

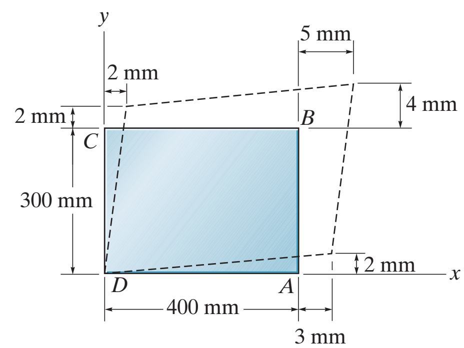
$$(\gamma_D)_{xy} = \theta + \psi$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

Ans:
$(\gamma_C)_{xy} = 11.6(10^{-3})$ rad,
$(\gamma_D)_{xy} = 11.6(10^{-3})$ rad

2–25.

Determine the average normal strain that occurs along the diagonals AC and DB.



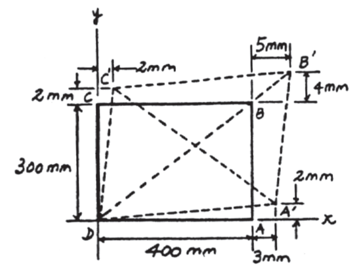
SOLUTION

Geometry:

$$AC = DB = \sqrt{400^2 + 300^2} = 500 \text{ mm}$$

$$DB' = \sqrt{405^2 + 304^2} = 506.4 \text{ mm}$$

$$A'C' = \sqrt{401^2 + 300^2} = 500.8 \text{ mm}$$



Average Normal Strain:

$$\epsilon_{AC} = \frac{A'C' - AC}{AC} = \frac{500.8 - 500}{500}$$

$$= 0.00160 \text{ mm/mm} = 1.60\left(10^{-3}\right) \text{ mm/mm}$$ **Ans.**

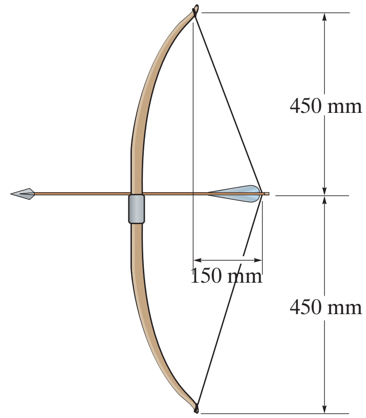
$$\epsilon_{DB} = \frac{DB' - DB}{DB} = \frac{506.4 - 500}{500}$$

$$= 0.0128 \text{ mm/mm} = 12.8\left(10^{-3}\right) \text{ mm/mm}$$ **Ans.**

Ans:
$\epsilon_{AC} = 1.60\left(10^{-3}\right)$ mm/mm
$\epsilon_{DB} = 12.8\left(10^{-3}\right)$ mm/mm

2–26.

If the unstreched length of the bowstring is 887.5 mm, determine the average normal strain in the string when it is streched to the position shown.

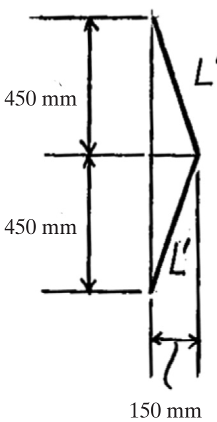


Geometry: Referring to Fig. *a*, the stretched length of the string is

$$L = 2L' = 2\sqrt{450^2 + 150^2} = 948.68 \text{ mm}$$

Average Normal Strain:

$$\varepsilon_{avg} = \frac{L - L_0}{L_0} = \frac{948.68 - 887.5}{887.5} = 0.0689 \text{ mm/mm} \qquad \textbf{Ans.}$$



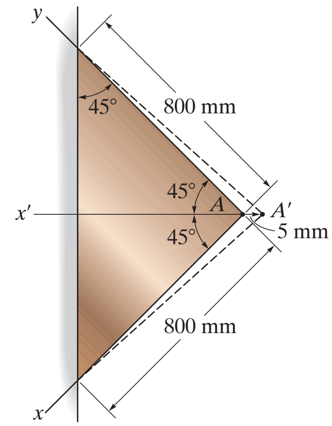
(a)

Ans:

$\varepsilon_{avg} = 0.0689$ mm/mm

2–27.

The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the shear strain, γ_{xy}, at A.



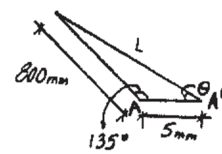
SOLUTION

$$L = \sqrt{800^2 + 5^2 - 2(800)(5)\cos 135^\circ} = 803.54 \text{ mm}$$

$$\frac{\sin 135^\circ}{803.54} = \frac{\sin\theta}{800}; \qquad \theta = 44.75^\circ = 0.7810 \text{ rad}$$

$$\gamma_{xy} = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2(0.7810)$$

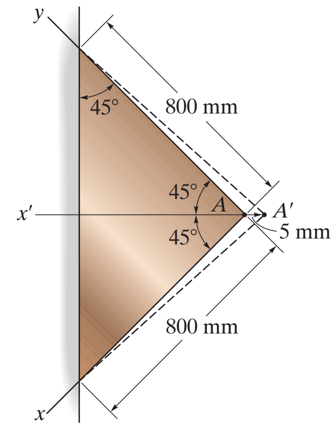
$$= 0.00880 \text{ rad}$$ **Ans.**



Ans:
$\gamma_{xy} = 0.00880$ rad

***2–28.**

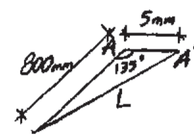
The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the average normal strain ϵ_x along the x axis.



SOLUTION

$$L = \sqrt{800^2 + 5^2 - 2(800)(5)\cos 135^\circ} = 803.54 \text{ mm}$$

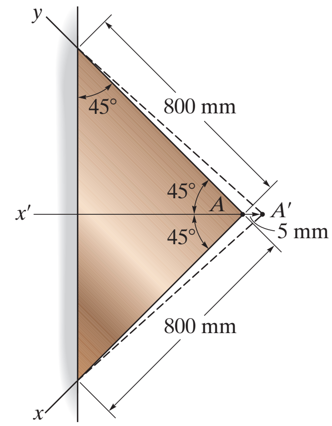
$$\epsilon_x = \frac{803.54 - 800}{800} = 0.00443 \text{ mm/mm}$$ **Ans.**



Ans:
$\epsilon_x = 0.00443$ mm/mm

2–29.

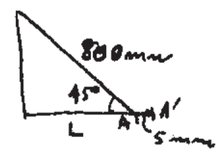
The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the average normal strain $\epsilon_{x'}$ along the x' axis.



SOLUTION

$L = 800 \cos 45° = 565.69 \text{ mm}$

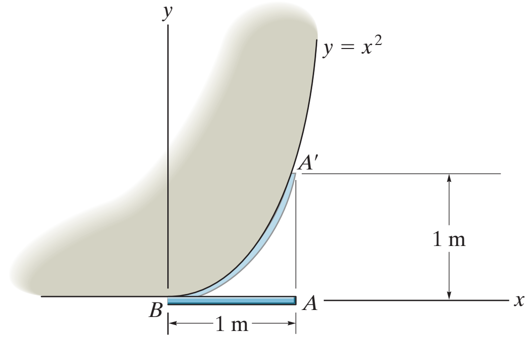
$$\epsilon_{x'} = \frac{5}{565.69} = 0.00884 \text{ mm/mm} \qquad \textbf{Ans.}$$



Ans:
$\epsilon_{x'} = 0.00884 \text{ mm/mm}$

2–30.

The rubber band AB has an unstretched length of 1 m. If it is fixed at B and attached to the surface at point A', determine the average normal strain in the band. The surface is defined by the function $y = (x^2)$ m, where x is in meters.



SOLUTION

Geometry:

$$L = \int_0^{1\text{ m}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

However $y = x^2$ then $\dfrac{dy}{dx} = 2x$

$$L = \int_0^{1\text{ m}} \sqrt{1 + 4x^2}\, dx$$

$$= \frac{1}{4}\left[2x\sqrt{1 + 4x^2} + \ln\left(2x + \sqrt{1 + 4x^2}\right)\right]\Big|_0^{1\text{ m}}$$

$$= 1.47894 \text{ m}$$

Average Normal Strain:

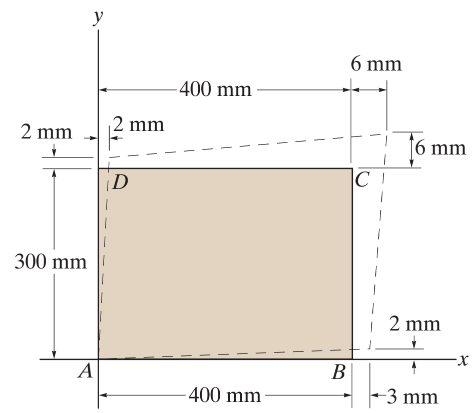
$$\varepsilon_{\text{avg}} = \frac{L - L_0}{L_0} = \frac{1.47894 - 1}{1} = 0.479 \text{ m/m}$$ **Ans.**

Ans:

$\varepsilon_{\text{avg}} = 0.479$ m/m

2–31.

The rectangular plate is deformed into the shape shown by the dashed lines. Determine the average normal strain along diagonal BD, and the average shear strain at corner B relative to the x, y axes.



SOLUTION

Geometry: The unstretched length of diagonal BD is

$$L_{BD} = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

Referring to Fig. a, the stretched length of diagonal BD is

$$L_{B'D'} = \sqrt{(300 + 2 - 2)^2 + (400 + 3 - 2)^2} = 500.8004 \text{ mm}$$

Referring to Fig. a and using small angle analysis,

$$\phi = \frac{2}{403} = 0.004963 \text{ rad}$$

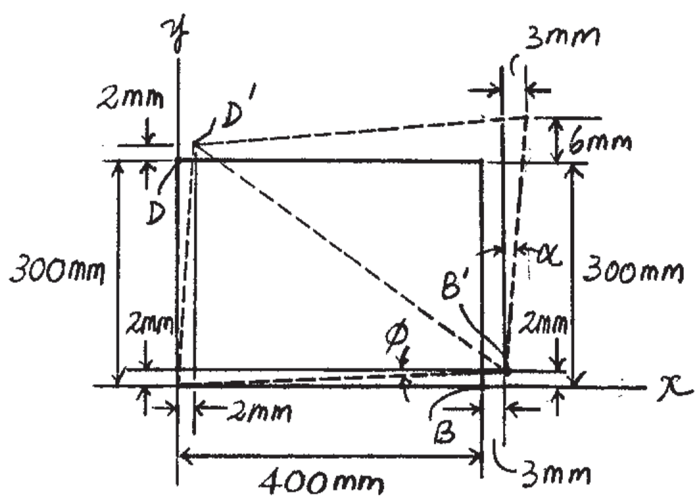
$$\alpha = \frac{3}{300 + 6 - 2} = 0.009868 \text{ rad}$$

Average Normal Strain: Applying Eq. 2,

$$(\epsilon_{\text{avg}})_{BD} = \frac{L_{B'D'} - L_{BD}}{L_{BD}} = \frac{500.8004 - 500}{500} = 1.60(10^{-3}) \text{ mm/mm} \quad \textbf{Ans.}$$

Shear Strain: Referring to Fig. a,

$$(\gamma_B)_{xy} = \phi + \alpha = 0.004963 + 0.009868 = 0.0148 \text{ rad} \quad \textbf{Ans.}$$

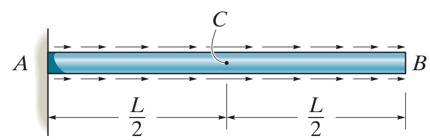


(a)

Ans:
$(\epsilon_{\text{avg}})_{BD} = 1.60(10^{-3})$ mm/mm,
$(\gamma_B)_{xy} = 0.0148$ rad

***2–32.**

The nonuniform loading causes a normal strain in the shaft that can be expressed as $\epsilon_x = k \sin\left(\frac{\pi}{L}x\right)$, where k is a constant. Determine the displacement of the center C and the average normal strain in the entire rod.



SOLUTION

$$\epsilon_x = k \sin\left(\frac{\pi}{L}x\right)$$

$$(\Delta x)_C = \int_0^{L/2} \epsilon_x\, dx = \int_0^{L/2} k \sin\left(\frac{\pi}{L}x\right) dx$$

$$= -k\left(\frac{L}{\pi}\right) \cos\left(\frac{\pi}{L}x\right)\Big|_0^{L/2} = -k\left(\frac{L}{\pi}\right)\left(\cos\frac{\pi}{2} - \cos 0\right)$$

$$= \frac{kL}{\pi}$$ **Ans.**

$$(\Delta x)_B = \int_0^{L} k \sin\left(\frac{\pi}{L}x\right) dx$$

$$= -k\left(\frac{L}{\pi}\right) \cos\left(\frac{\pi}{L}x\right)\Big|_0^{L} = -k\left(\frac{L}{\pi}\right)(\cos\pi - \cos 0) = \frac{2kL}{\pi}$$

$$\epsilon_{avg} = \frac{(\Delta x)_B}{L} = \frac{2k}{\pi}$$ **Ans.**

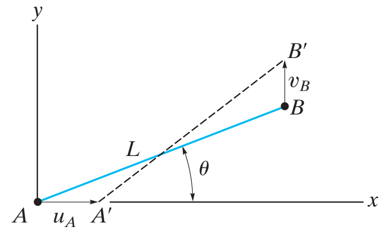
Ans:

$(\Delta x)_C = \frac{kL}{\pi}$

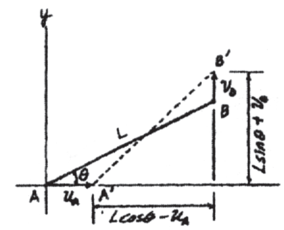
$\epsilon_{avg} = \frac{2k}{\pi}$

2–33.

The fiber AB has a length L and orientation θ. If its ends A and B undergo very small displacements u_A and v_B respectively, determine the normal strain in the fiber when it is in position $A'B'$.



SOLUTION



Geometry:

$$L_{A'B'} = \sqrt{(L\cos\theta - u_A)^2 + (L\sin\theta + v_B)^2}$$

$$= \sqrt{L^2 + u_A^2 + v_B^2 + 2L(v_B\sin\theta - u_A\cos\theta)}$$

Average Normal Strain:

$$\epsilon_{AB} = \frac{L_{A'B'} - L}{L}$$

$$= \sqrt{1 + \frac{u_A^2 + v_B^2}{L^2} + \frac{2(v_B\sin\theta - u_A\cos\theta)}{L}} - 1$$

Neglecting higher terms u_A^2 and v_B^2

$$\epsilon_{AB} = \left[1 + \frac{2(v_B\sin\theta - u_A\cos\theta)}{L}\right]^{\frac{1}{2}} - 1$$

Using the binomial theorem:

$$\epsilon_{AB} = 1 + \frac{1}{2}\left(\frac{2v_B\sin\theta}{L} - \frac{2u_A\cos\theta}{L}\right) + \ \ldots\ -1$$

$$= \frac{v_B\sin\theta}{L} - \frac{u_A\cos\theta}{L}$$ **Ans.**

Ans:

$$\epsilon_{AB} = \frac{v_B\sin\theta}{L} - \frac{u_A\cos\theta}{L}$$

2–34.

If the normal strain is defined in reference to the final length $\Delta s'$, that is,

$$\epsilon' = \lim_{\Delta s' \to 0} \left(\frac{\Delta s' - \Delta s}{\Delta s'} \right)$$

instead of in reference to the original length, Eq. 2–2, show that the difference in these strains is represented as a second-order term, namely, $\epsilon - \epsilon' = \epsilon \epsilon'$.

SOLUTION

$$\epsilon = \frac{\Delta s' - \Delta s}{\Delta s}$$

$$\epsilon - \epsilon' = \frac{\Delta s' - \Delta s}{\Delta s} - \frac{\Delta s' - \Delta s}{\Delta s'}$$

$$= \frac{\Delta s'^2 - \Delta s \Delta s' - \Delta s' \Delta s + \Delta s^2}{\Delta s \Delta s'}$$

$$= \frac{\Delta s'^2 + \Delta s^2 - 2\Delta s' \Delta s}{\Delta s \Delta s'}$$

$$= \frac{(\Delta s' - \Delta s)^2}{\Delta s \Delta s'} = \left(\frac{\Delta s' - \Delta s}{\Delta s} \right)\left(\frac{\Delta s' - \Delta s}{\Delta s'} \right)$$

$$= \epsilon \epsilon'$$ **(Q.E.D)**

Ans:
N/A