10–1.

Prove that the sum of the normal strains in perpendicular directions is constant, i.e., $\epsilon_x + \epsilon_y = \epsilon_{x'} + \epsilon_{y'}$.

SOLUTION

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \qquad \textbf{(1)}$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta \qquad \textbf{(2)}$$

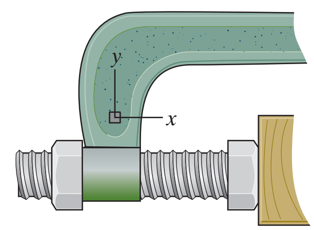
Adding Eq. (1) and Eq. (2) yields:

$$\epsilon_{x'} + \epsilon_{y'} = \epsilon_x + \epsilon_y = \text{constant} \qquad \textbf{(Q.E.D.)}$$

Ans:
N/A

10–2.

The state of strain at the point on the arm has components of $\epsilon_x = 200(10^{-6})$, $\epsilon_y = -300(10^{-6})$, and $\gamma_{xy} = 400(10^{-6})$. Use the strain transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of 30° counterclockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.



SOLUTION

In accordance with the established sign convention,

$$\epsilon_x = 200(10^{-6}), \qquad \epsilon_y = -300(10^{-6}) \qquad \gamma_{xy} = 400(10^{-6}) \qquad \theta = 30^\circ$$

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{200 + (-300)}{2} + \frac{200 - (-300)}{2}\cos 60^\circ + \frac{400}{2}\sin 60^\circ\right](10^{-6})$$

$$= 248\,(10^{-6})$$ **Ans.**

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\epsilon_x - \epsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = \left\{-\left[200 - (-300)\right]\sin 60^\circ + 400\cos 60^\circ\right\}(10^{-6})$$

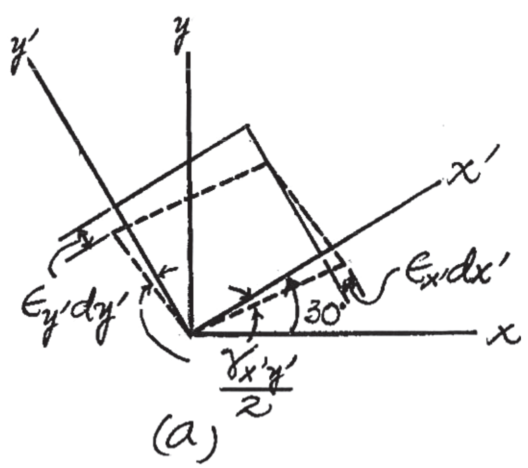
$$= -233(10^{-6})$$ **Ans.**

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{200 + (-300)}{2} - \frac{200 - (-300)}{2}\cos 60^\circ - \frac{400}{2}\sin 60^\circ\right](10^{-6})$$

$$= -348(10^{-6})$$ **Ans.**

The deformed element of this equivalent state of strain is shown in Fig. *a*.

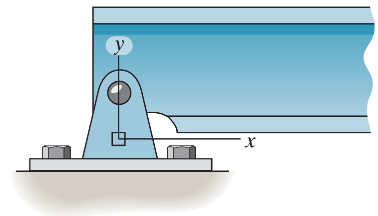


(a)

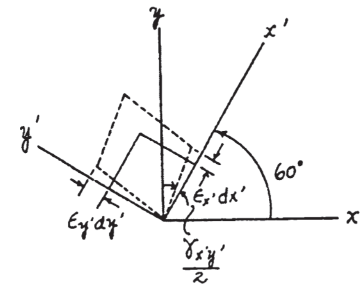
Ans:
$\epsilon_{x'} = 248(10^{-6})$, $\gamma_{x'y'} = -233(10^{-6})$,
$\epsilon_{y'} = -348(10^{-6})$

10–3.

The state of strain at the point on the pin leaf has components of $\epsilon_x = 200(10^{-6})$, $\epsilon_y = 180(10^{-6})$, and $\gamma_{xy} = -300(10^{-6})$. Use the strain transformation equations and determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 60°$ counterclockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.



SOLUTION



Normal Strain and Shear Strain: In accordance with the sign convention,

$$\epsilon_x = 200(10^{-6}) \qquad \epsilon_y = 180(10^{-6}) \qquad \gamma_{xy} = -300(10^{-6})$$

$$\theta = +60°$$

Strain Transformation Equations: Applying Eqs. 10–5, 10–6, and 10–7,

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left(\frac{200 + 180}{2} + \frac{200 - 180}{2}\cos 120° + \frac{-300}{2}\sin 120°\right)(10^{-6})$$

$$= 55.1(10^{-6}) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(200 - 180)\sin 120° + (-300)\cos 120°](10^{-6})$$

$$= 133(10^{-6}) \qquad \textbf{Ans.}$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

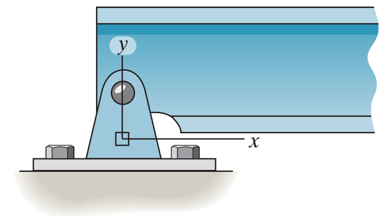
$$= \left(\frac{200 + 180}{2} - \frac{200 - 180}{2}\cos 120° - \frac{-300}{2}\sin 120°\right)(10^{-6})$$

$$= 325(10^{-6}) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_{x'} = 55.1(10^{-6}), \gamma_{x'y'} = 133(10^{-6}),$
$\epsilon_{y'} = 325(10^{-6})$

***10–4.**

Solve Prob. 10–3 for an element oriented $\theta = 30°$ clockwise.

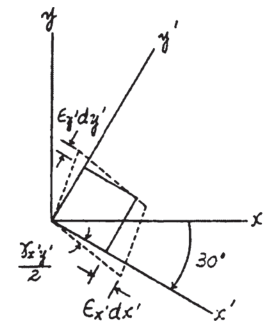


SOLUTION

Normal Strain and Shear Strain: In accordance with the sign convention,

$$\epsilon_x = 200(10^{-6}) \qquad \epsilon_y = 180(10^{-6}) \qquad \gamma_{xy} = -300(10^{-6})$$

$$\theta = -30°$$



Strain Transformation Equations: Applying Eqs. 10–5, 10–6, and 10–7,

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{200 + 180}{2} + \frac{200 - 180}{2}\cos(-60°) + \frac{-300}{2}\sin(-60°)\right](10^{-6})$$

$$= 325(10^{-6}) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(200 - 180)\sin(-60°) + (-300)\cos(-60°)](10^{-6})$$

$$= -133(10^{-6}) \qquad \textbf{Ans.}$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

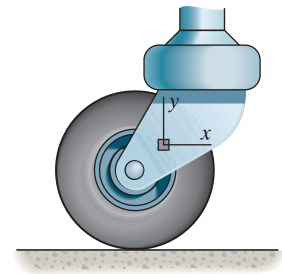
$$= \left(\frac{200 + 180}{2} - \frac{200 - 180}{2}\cos(-60°) - \frac{-300}{2}\sin(-60°)\right)(10^{-6})$$

$$= 55.1(10^{-6}) \qquad \textbf{Ans.}$$

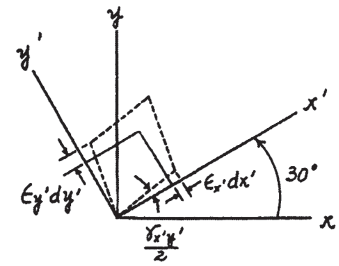
Ans:
$\epsilon_{x'} = 325(10^{-6})$, $\gamma_{x'y'} = -133(10^{-6})$,
$\epsilon_{y'} = 55.1(10^{-6})$

10–5.

The state of strain at the point on the leaf of the caster assembly has components of $\epsilon_x = -400(10^{-6})$, $\epsilon_y = 860(10^{-6})$, and $\gamma_{xy} = 375(10^{-6})$. Use the strain transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 30°$ counterclockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.



SOLUTION



Normal Strain and Shear Strain: In accordance with the sign convention,

$$\epsilon_x = -400(10^{-6}) \qquad \epsilon_y = 860(10^{-6}) \qquad \gamma_{xy} = 375(10^{-6})$$

$$\theta = +30°$$

Strain Transformation Equations: Applying Eqs. 10–5, 10–6, and 10–7,

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left(\frac{-400 + 860}{2} + \frac{-400 - 860}{2}\cos 60° + \frac{375}{2}\sin 60°\right)(10^{-6})$$

$$= 77.4(10^{-6})$$ **Ans.**

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(-400 - 860)\sin 60° + 375\cos 60°](10^{-6})$$

$$= 1279(10^{-6})$$ **Ans.**

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

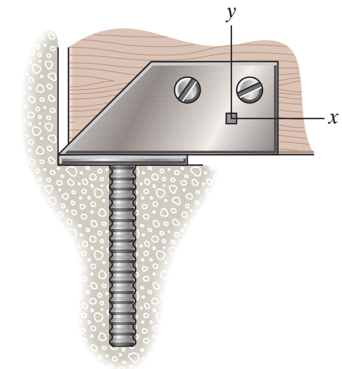
$$= \left(\frac{-400 + 860}{2} - \frac{-400 - 860}{2}\cos 60° - \frac{375}{2}\sin 60°\right)(10^{-6})$$

$$= 383(10^{-6})$$ **Ans.**

Ans:
$\epsilon_{x'} = 77.4(10^{-6})$, $\gamma_{x'y'} = 1279(10^{-6})$,
$\epsilon_{y'} = 383(10^{-6})$

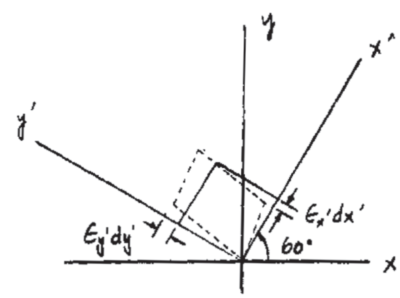
10–6.

The state of strain at a point on the bracket has components of $\epsilon_x = 150(10^{-6})$, $\epsilon_y = 200(10^{-6})$, $\gamma_{xy} = -700(10^{-6})$. Use the strain transformation equations and determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 60^\circ$ counterclockwise from the original position. Sketch the deformed element within the x–y plane due to these strains.



SOLUTION

$\epsilon_x = 150(10^{-6}) \qquad \epsilon_y = 200(10^{-6}) \qquad \gamma_{xy} = -700(10^{-6}) \qquad \theta = 60^\circ$



$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{150 + 200}{2} + \frac{150 - 200}{2}\cos 120^\circ + \left(\frac{-700}{2}\right)\sin 120^\circ\right]10^{-6}$$

$$= -116(10^{-6})$$ **Ans.**

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{150 + 200}{2} - \frac{150 - 200}{2}\cos 120^\circ - \left(\frac{-700}{2}\right)\sin 120^\circ\right]10^{-6}$$

$$= 466(10^{-6})$$ **Ans.**

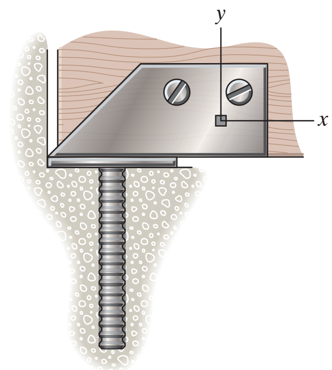
$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = 2\left[-\frac{150 - 200}{2}\sin 120^\circ + \left(\frac{-700}{2}\right)\cos 120^\circ\right]10^{-6} = 393(10^{-6})$$ **Ans.**

Ans:
$\epsilon_{x'} = -116(10^{-6})$, $\epsilon_{y'} = 466(10^{-6})$,
$\gamma_{x'y'} = 393(10^{-6})$

10–7.

Solve Prob. 10–6 for an element oriented $\theta = 30°$ clockwise.



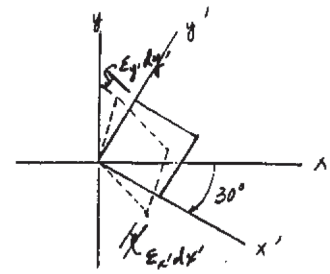
SOLUTION

$\epsilon_x = 150(10^{-6}) \qquad \epsilon_y = 200(10^{-6}) \qquad \gamma_{xy} = -700(10^{-6}) \qquad \theta = -30°$

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{150 + 200}{2} + \frac{150 - 200}{2}\cos(-60°) + \left(\frac{-700}{2}\right)\sin(-60°)\right]10^{-6}$$

$$= 466(10^{-6}) \qquad \textbf{Ans.}$$



$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{150 + 200}{2} - \frac{150 - 200}{2}\cos(-60°) - \left(\frac{-700}{2}\right)\sin(-60°)\right]10^{-6}$$

$$= -116(10^{-6}) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

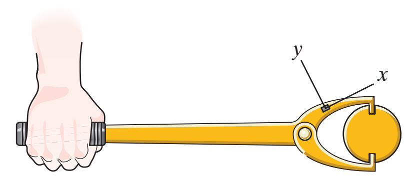
$$\gamma_{x'y'} = 2\left[-\frac{150 - 200}{2}\sin(-60°) + \frac{-700}{2}\cos(-60°)\right]10^{-6}$$

$$= -393(10^{-6}) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_{x'} = 466(10^{-6})$, $\epsilon_{y'} = -116(10^{-6})$,
$\gamma_{x'y'} = -393(10^{-6})$

***10–8.**

The state of strain at the point on the spanner wrench has components of $\epsilon_x = 260(10^{-6})$, $\epsilon_y = 320(10^{-6})$, and $\gamma_{xy} = 180(10^{-6})$. Use the strain transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.



SOLUTION

a)

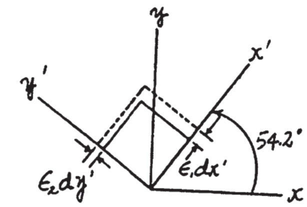
In-Plane Principal Strain: Applying Eq. 10–9,

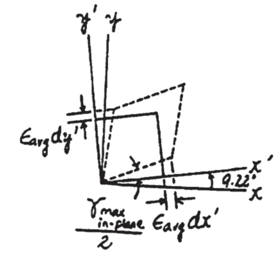
$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{260 + 320}{2} \pm \sqrt{\left(\frac{260 - 320}{2}\right)^2 + \left(\frac{180}{2}\right)^2}\right](10^{-6})$$

$$= 290 \pm 94.87$$

$$\epsilon_1 = 385(10^{-6}) \qquad \epsilon_2 = 195(10^{-6})$$ **Ans.**





Orientation of Principal Strain: Applying Eq. 10–8,

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{180(10^{-6})}{(260 - 320)(10^{-6})} = -3.000$$

$$\theta_p = -35.78° \qquad \text{and} \qquad 54.22°$$

Use Eq. 10–5 to determine which principal strain deforms the element in the x' direction with $\theta = -35.78°$.

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{260 + 320}{2} + \frac{260 - 320}{2}\cos(-71.56°) + \frac{180}{2}\sin(-71.56°)\right](10^{-6})$$

$$= 195(10^{-6}) = \epsilon_2$$

Hence, $\qquad \theta_{p1} = 54.2° \qquad$ and $\qquad \theta_{p1} = -35.8°$ **Ans.**

b)

Maximum In-Plane Shear Strain: Applying Eq. 10–11,

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = 2\left[\sqrt{\left(\frac{260 - 320}{2}\right)^2 + \left(\frac{180}{2}\right)^2}\right](10^{-6})$$

$$= 190(10^{-6})$$ **Ans.**

***10–8. Continued**

Orientation of Maximum In-Plane Shear Strain: Applying Eq. 10–10,

$$\tan 2\theta_s = -\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}} = -\frac{260 - 320}{180} = 0.3333$$

$$\theta_s = 9.22^\circ \qquad \text{and} \qquad -80.8^\circ \qquad \textbf{Ans.}$$

Normal Strain and Shear Strain: In accordance with the sign convention,

$\epsilon_x = 260\left(10^{-6}\right)$ $\qquad \epsilon_y = 320\left(10^{-6}\right)$ $\qquad \gamma_{xy} = 180\left(10^{-6}\right)$

The proper sign of $\gamma_{\substack{\max \\ \text{in-plane}}}$ can be determined by substituting $\theta = 9.22^\circ$ into Eq. 10–6.

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(260 - 320)\sin 18.44^\circ + 180\cos 18.44^\circ]\left(10^{-6}\right)$$

$$= 190\left(10^{-6}\right)$$

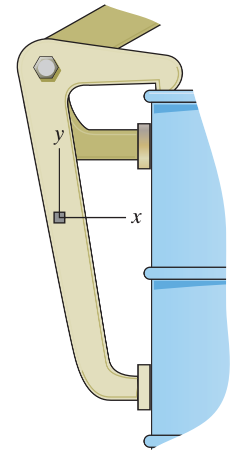
Average Normal Strain: Applying Eq. 10–12,

$$\epsilon_{\text{avg}} = \frac{\epsilon_x + \epsilon_y}{2} = \left[\frac{260 + 320}{2}\right]\left(10^{-6}\right) = 290\left(10^{-6}\right) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_1 = 385\left(10^{-6}\right), \epsilon_2 = 195\left(10^{-6}\right),$
$\theta_{p1} = 54.2^\circ, \theta_{p1} = -35.8^\circ,$
$\gamma_{\substack{\max \\ \text{in-plane}}} = 190\left(10^{-6}\right)$
$\theta_s = 9.22^\circ \quad \text{and} \quad -80.8^\circ,$
$\epsilon_{\text{avg}} = 290\left(10^{-6}\right)$

10–9.

The state of strain at the point on the member has components of $\epsilon_x = 180(10^{-6})$, $\epsilon_y = -120(10^{-6})$, and $\gamma_{xy} = -100(10^{-6})$. Use the strain transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.



SOLUTION

a) In accordance with the established sign convention, $\epsilon_x = 180(10^{-6})$, $\epsilon_y = -120(10^{-6})$ and $\gamma_{xy} = -100(10^{-6})$.

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left\{\frac{180 + (-120)}{2} \pm \sqrt{\left[\frac{180 - (-120)}{2}\right]^2 + \left(\frac{-100}{2}\right)^2}\right\}(10^{-6})$$

$$= (30 \pm 158.11)(10^{-6})$$

$\epsilon_1 = 188(10^{-6})$ $\quad \epsilon_2 = -128(10^{-6})$ **Ans.**

$$\tan 2\theta_P = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-100(10^{-6})}{[180 - (-120)](10^{-6})} = -0.3333$$

$$\theta_P = -9.217^\circ \quad \text{and} \quad 80.78^\circ$$

Substitute $\theta = -9.217^\circ$,

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{180 + (-120)}{2} + \frac{180 - (-120)}{2}\cos(-18.43^\circ) + \frac{-100}{2}\sin(-18.43)\right](10^{-6})$$

$$= 188(10^{-6}) = \epsilon_1$$

Thus,

$(\theta_P)_1 = -9.22^\circ$ $\quad (\theta_P)_2 = 80.8^\circ$ **Ans.**

The deformed element is shown in Fig (a).

b) $\epsilon_{avg} = \dfrac{180(10^{-6}) + (-120)(10^{-6})}{2} = 30(10^{-6})$ **Ans.**

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$\gamma_{\substack{\max \\ \text{in-plane}}} = \left\{2\sqrt{\left[\dfrac{180 - (-120)}{2}\right]^2 + \left(\dfrac{-100}{2}\right)^2}\right\}(10^{-6}) = 316\,(10^{-6})$ **Ans.**

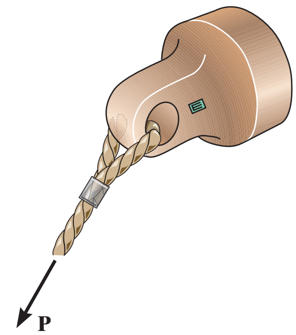
$$\tan 2\theta_s = -\left(\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}}\right) = -\left\{\frac{[180 - (-120)](10^{-6})}{-100(10^{-6})}\right\} = 3$$

$\theta_s = 35.78^\circ = 35.8^\circ$ and $-54.22^\circ = -54.2^\circ$ **Ans.**

Ans:
$\epsilon_1 = 188(10^{-6}), \epsilon_2 = -128(10^{-6})$,
$(\theta_P)_1 = -9.22^\circ, (\theta_P)_2 = 80.8^\circ$
$\epsilon_{avg} = 30(10^{-6})$
$\gamma_{\substack{\max \\ \text{in-plane}}} = 316\,(10^{-6})$,
$\theta_s = 35.8^\circ$ and -54.2°

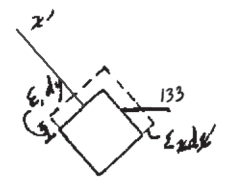
10–10.

The state of strain at the point on the support has components of $\epsilon_x = 350(10^{-6})$, $\epsilon_y = 400(10^{-6})$, $\gamma_{xy} = -675(10^{-6})$. Use the strain-transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.



SOLUTION

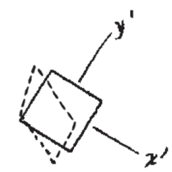
a)



$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \frac{350 + 400}{2} \pm \sqrt{\left(\frac{350 - 400}{2}\right)^2 + \left(\frac{-675}{2}\right)^2}$$

$\epsilon_1 = 713(10^{-6})$ **Ans.** $\epsilon_2 = 36.6(10^{-6})$ **Ans.**



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-675}{(350 - 400)}$$

$\theta_{p1} = 133°$ **Ans.**

b)

$$\frac{(\gamma_{x'y'})_{\max}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\frac{(\gamma_{x'y'})_{\max}}{2} = \sqrt{\left(\frac{350 - 400}{2}\right)^2 + \left(\frac{-675}{2}\right)^2}$$

$(\gamma_{x'y'})_{\max} = 677(10^{-6})$ **Ans.**

$\epsilon_{avg} = \dfrac{\epsilon_x + \epsilon_y}{2} = \dfrac{350 + 400}{2} = 375(10^{-6})$ **Ans.**

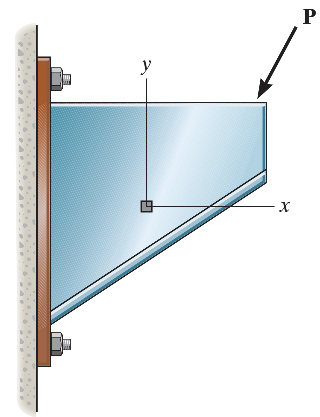
$$\tan 2\theta_s = \frac{-(\epsilon_x - \epsilon_y)}{\gamma_{xy}} = \frac{350 - 400}{675}$$

$\theta_s = -2.12°$ **Ans.**

Ans:
(a) $\epsilon_1 = 713(10^{-6})$, $\epsilon_2 = 36.6(10^{-6})$, $\theta_{p1} = 133°$
(b) $\gamma_{\substack{\max \\ \text{in-plane}}} = 677(10^{-6})$, $\epsilon_{avg} = 375(10^{-6})$,
$\theta_s = -2.12°$

10–11.

Due to the load **P**, the state of strain at the point on the bracket has components of $\epsilon_x = 500(10^{-6})$, $\epsilon_y = 350(10^{-6})$, and $\gamma_{xy} = -430(10^{-6})$. Use the strain transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 30°$ clockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.

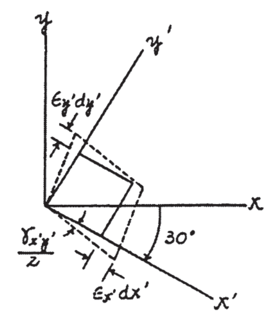


SOLUTION

Normal Strain and Shear Strain: In accordance with the sign convention,

$$\epsilon_x = 500(10^{-6}) \qquad \epsilon_y = 350(10^{-6}) \qquad \gamma_{xy} = -430(10^{-6})$$
$$\theta = -30°$$

Strain Transformation Equations:



$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{500 + 350}{2} + \frac{500 - 350}{2}\cos(-60°) + \frac{-430}{2}\sin(-60°)\right](10^{-6})$$

$$= 649(10^{-6}) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(500 - 350)\sin(-60°) + (-430)\cos(-60°)](10^{-6})$$

$$= -85.1(10^{-6}) \qquad \textbf{Ans.}$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

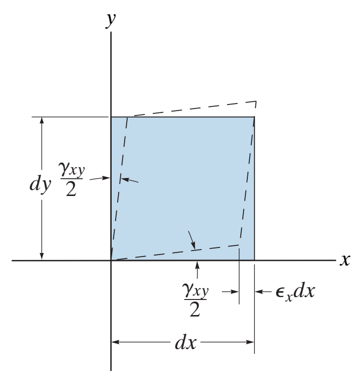
$$= \left(\frac{500 + 350}{2} - \frac{500 - 350}{2}\cos(-60°) - \frac{-430}{2}\sin(-60°)\right)(10^{-6})$$

$$= 201(10^{-6}) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_{x'} = 649(10^{-6})$, $\gamma_{x'y'} = -85.1(10^{-6})$,
$\epsilon_{y'} = 201(10^{-6})$

***10–12.**

The state of strain on an element has components $\epsilon_x = -400(10^{-6})$, $\epsilon_y = 0$, $\gamma_{xy} = 150(10^{-6})$. Determine the equivalent state of strain on an element at the same point oriented 30° clockwise with respect to the original element. Sketch the results on this element.



SOLUTION

Strain Transformation Equations:

$$\epsilon_x = -400(10^{-6}) \qquad \epsilon_y = 0 \qquad \gamma_{xy} = 150(10^{-6}) \qquad \theta = -30°$$

We obtain

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{-400 + 0}{2} + \frac{-400 - 0}{2}\cos(-60°) + \frac{150}{2}\sin(-60°)\right](10^{-6})$$

$$= -365(10^{-6}) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(-400 - 0)\sin(-60°) + 150\cos(-60°)](10^{-6})$$

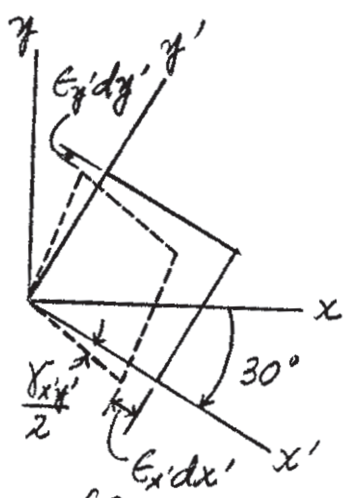
$$= -271(10^{-6}) \qquad \textbf{Ans.}$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{-400 + 0}{2} - \frac{-400 - 0}{2}\cos(-60°) - \frac{150}{2}\sin(-60°)\right](10^{-6})$$

$$= -35.0(10^{-6}) \qquad \textbf{Ans.}$$

The deformed element for this state of strain is shown in Fig. *a*.

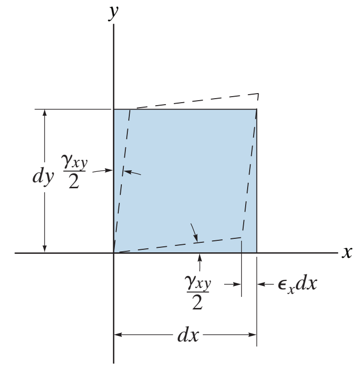


(a)

Ans:
$\epsilon_{x'} = -365(10^{-6})$, $\gamma_{x'y'} = -271(10^{-6})$,
$\epsilon_{y'} = -35.0(10^{-6})$

10–13.

The state of plane strain on the element is $\epsilon_x = -300(10^{-6})$, $\epsilon_y = 0$, and $\gamma_{xy} = 150(10^{-6})$. Determine the equivalent state of strain which represents (a) the principal strains, and (b) the maximum in-plane shear strain and the associated average normal strain. Specify the orientation of the corresponding elements for these states of strain with respect to the original element.



SOLUTION

In-Plane Principal Strains: $\epsilon_x = -300(10^{-6})$, $\epsilon_y = 0$, and $\gamma_{xy} = 150(10^{-6})$. We obtain

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{-300 + 0}{2} \pm \sqrt{\left(\frac{-300 - 0}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\right](10^{-6})$$

$$= (-150 \pm 167.71)(10^{-6})$$

$$\epsilon_1 = 17.7(10^{-6}) \qquad \epsilon_2 = -318(10^{-6})$$ **Ans.**

Orientation of Principal Strain:

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{150(10^{-6})}{(-300 - 0)(10^{-6})} = -0.5$$

$\theta_P = -13.28°$ and $76.72°$

Substituting $\theta = -13.28°$ into Eq. 9–1,

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{-300 + 0}{2} + \frac{-300 - 0}{2}\cos(-26.57°) + \frac{150}{2}\sin(-26.57°)\right](10^{-6})$$

$$= -318(10^{-6}) = \epsilon_2$$

Thus,

$(\theta_P)_1 = 76.7°$ and $(\theta_P)_2 = -13.3°$ **Ans.**

The deformed element of this state of strain is shown in Fig. *a*.

Maximum In-Plane Shear Strain:

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = \left[2\sqrt{\left(\frac{-300 - 0}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\right](10^{-6}) = 335(10^{-6})$$ **Ans.**

Orientation of the Maximum In-Plane Shear Strain:

$$\tan 2\theta_s = -\left(\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}}\right) = -\left[\frac{(-300 - 0)(10^{-6})}{150(10^{-6})}\right] = 2$$

$\theta_s = 31.7°$ and $122°$ **Ans.**

10–13. Continued

The algebraic sign for $\gamma_{\substack{\text{max}\\\text{in-plane}}}$ when $\theta = \theta_s = 31.7°$ can be obtained using

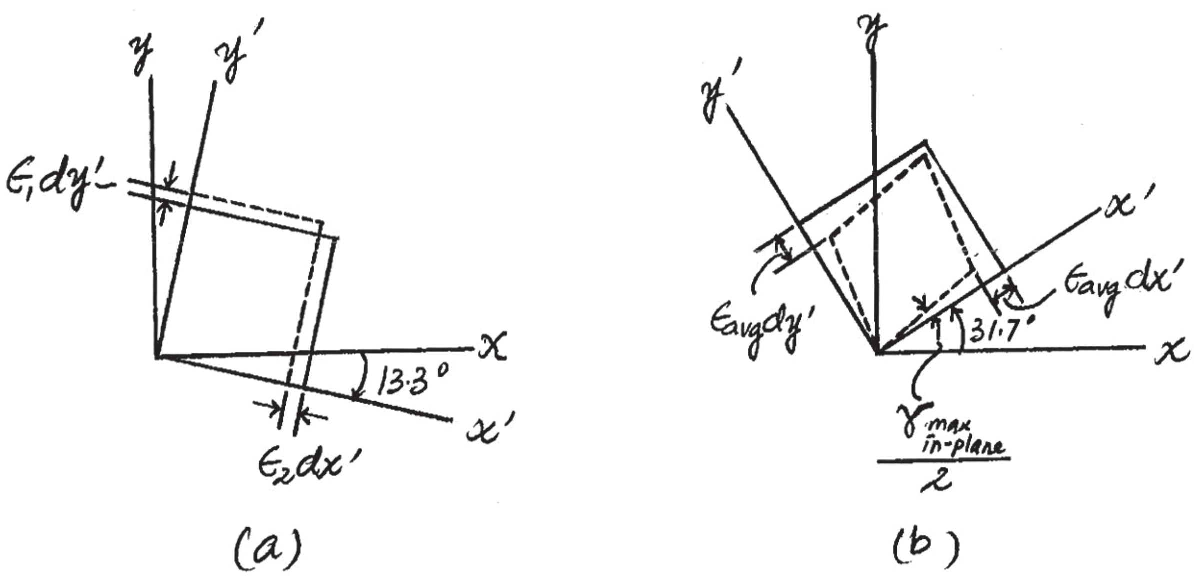
$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\epsilon_x - \epsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(-300 - 0)\sin 63.43° + 150\cos 63.43°]\left(10^{-6}\right)$$
$$= 335\left(10^{-6}\right)$$

Average Normal Strain:

$$\epsilon_{\text{avg}} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{-300 + 0}{2}\right)\left(10^{-6}\right) = -150\left(10^{-6}\right)$$ **Ans.**

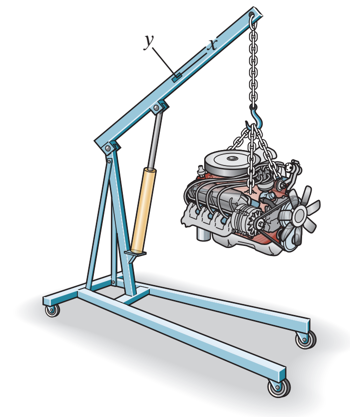
The deformed element for this state of strain is shown in Fig. *b*.



Ans:
$\epsilon_1 = 17.7(10^{-6})$, $\epsilon_2 = -318(10^{-6})$,
$\theta_{p1} = 76.7°$ and $\theta_{p2} = -13.3°$,
$\gamma_{\substack{\text{max}\\\text{in-plane}}} = 335(10^{-6})$, $\theta_s = 31.7°$ and $122°$,
$\epsilon_{\text{avg}} = -150(10^{-6})$

10–14.

The state of strain at the point on a boom of a shop crane has components of $\epsilon_x = 250(0^{-6})$, $\epsilon_y = 300(10^{-6})$, and $\gamma_{xy} = -180(10^{-6})$. Use the strain transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case, specify the orientation of the element and show how the strains deform the element within the x–y plane.



SOLUTION

a)

In-Plane Principal Strain: Applying Eq. 10–9,

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{250 + 300}{2} \pm \sqrt{\left(\frac{250 - 300}{2}\right)^2 + \left(\frac{-180}{2}\right)^2}\right](10^{-6})$$

$$= 275 \pm 93.41$$

$\epsilon_1 = 368(10^{-6})$ $\qquad$ $\epsilon_2 = 182(10^{-6})$ **Ans.**

Orientation of Principal Strain: Applying Eq. 10–8,

$$\tan 2\theta_P = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-180(10^{-6})}{(250 - 300)(10^{-6})} = 3.600$$

$$\theta_P = 37.24° \quad \text{and} \quad -52.76°$$

Use Eq. 10–5 to determine which principal strain deforms the element in the x' direction with $\theta = 37.24°$.

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{250 + 300}{2} + \frac{250 - 300}{2}\cos 74.48° + \frac{-180}{2}\sin 74.48°\right](10^{-6})$$

$$= 182(10^{-6}) = \epsilon_2$$

Hence,

$\theta_{p1} = -52.8°$ and $\theta_{p2} = 37.2°$ **Ans.**

b)

Maximum In-Plane Shear Strain: Applying Eq. 10–11,

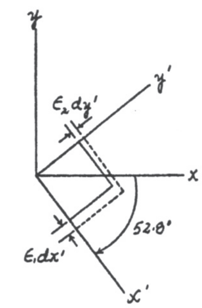
$$\frac{\gamma_{\substack{\max\\\text{in-plane}}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max\\\text{in-plane}}} = 2\left[\sqrt{\left(\frac{250 - 300}{2}\right)^2 + \left(\frac{-180}{2}\right)^2}\right](10^{-6})$$

$$= 187(10^{-6})$$ **Ans.**

10–14. Continued

Orientation of the Maximum In-Plane Shear Strain: Applying Eq. 10–10,



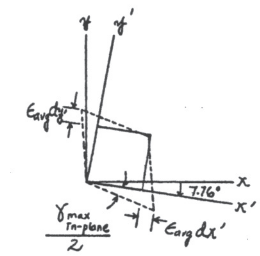
$$\tan 2\theta_s = -\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}} = -\frac{250 - 300}{-180} = -0.2778$$

$\theta_s = -7.76° \quad \text{and} \quad 82.2°$ **Ans.**

The proper sign of $\gamma_{\substack{\max\\ \text{in-plane}}}$ can be determined by substituting $\theta = -7.76°$ into Eq. 10–6.

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = \{-[250 - 300]\sin(-15.52°) + (-180)\cos(-15.52°)\}\left(10^{-6}\right)$$
$$= -187\left(10^{-6}\right)$$



Normal Strain and Shear Strain: In accordance with the sign convention,

$\epsilon_x = 250\left(10^{-6}\right) \quad \epsilon_y = 300\left(10^{-6}\right) \quad \gamma_{xy} = -180\left(10^{-6}\right)$

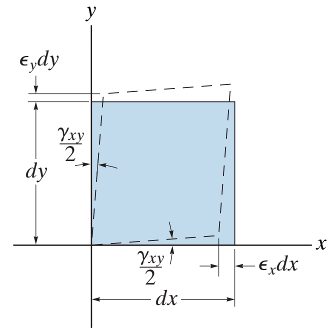
Average Normal Strain: Applying Eq. 10–12,

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} = \left[\frac{250 + 300}{2}\right]\left(10^{-6}\right) = 275\left(10^{-6}\right)$$ **Ans.**

Ans:
$\epsilon_1 = 368(10^{-6})$, $\epsilon_2 = 182(10^{-6})$,
$\theta_{p1} = -52.8°$ and $\theta_{p2} = 37.2°$,
$\gamma_{\substack{\max\\ \text{in-plane}}} = 187(10^{-6})$, $\theta_s = -7.76°$ and $82.2°$,
$\epsilon_{avg} = 275\,(10^{-6})$

***10–16.**

The state of strain on the element has components $\epsilon_x = -300(10^{-6})$, $\epsilon_y = 100(10^{-6})$, $\gamma_{xy} = 150(10^{-6})$. Determine the equivalent state of strain, which represents (a) the principal strains, and (b) the maximum in-plane shear strain and the associated average normal strain. Specify the orientation of the corresponding elements for these states of strain with respect to the original element.



SOLUTION

In-Plane Principal Strains: $\epsilon_x = -300(10^{-6})$, $\epsilon_y = 100(10^{-6})$, and $\gamma_{xy} = 150(10^{-6})$. We obtain

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{-300 + 100}{2} \pm \sqrt{\left(\frac{-300 - 100}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\right](10^{-6})$$

$$= (-100 \pm 213.60)(10^{-6})$$

$$\epsilon_1 = 114(10^{-6}) \qquad \epsilon_2 = -314(10^{-6}) \qquad \textbf{Ans.}$$

Orientation of Principal Strains:

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{150(10^{-6})}{(-300 - 100)(10^{-6})} = -0.375$$

$$\theta_p = -10.28° \text{ and } 79.72°$$

Substituting $\theta = -10.28°$ into

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{-300 + 100}{2} + \frac{-300 - 100}{2}\cos(-20.56°) + \frac{150}{2}\sin(-20.56°)\right](10^{-6})$$

$$= -314(10^{-6}) = \epsilon_2$$

Thus,

$$(\theta_p)_1 = 79.7° \text{ and } (\theta_p)_2 = -10.3° \qquad \textbf{Ans.}$$

The deformed element for the state of principal strain is shown in Fig. *a*.

Maximum In-Plane Shear Strain:

$$\frac{\gamma_{\substack{\max\\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max\\ \text{in-plane}}} = \left[2\sqrt{\left(\frac{-300 - 100}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\right](10^{-6}) = 427(10^{-6}) \qquad \textbf{Ans.}$$

***10–16. Continued**

Orientation of Maximum In-Plane Shear Strain:

$$\tan 2\theta_s = -\left(\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}}\right) = -\left[\frac{(-300 - 100)(10^{-6})}{150(10^{-6})}\right](10^{-6}) = 2.6667$$

$$\theta_s = 34.7° \text{ and } 125° \qquad \textbf{Ans.}$$

The algebraic sign for $\gamma_{\substack{\max \\ \text{in-plane}}}$ when $\theta = \theta_s = 34.7°$ can be determined using

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\epsilon_x - \epsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

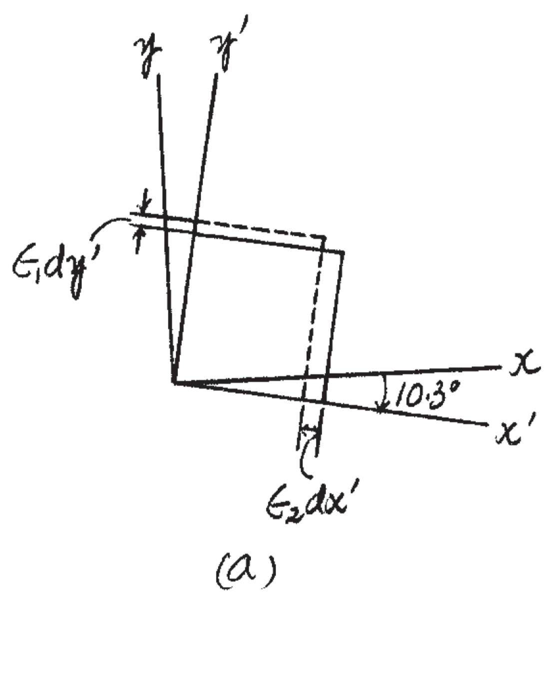
$$\gamma_{x'y'} = [-(-300 - 100)\sin 69.44° + 150\cos 69.44°](10^{-6})$$

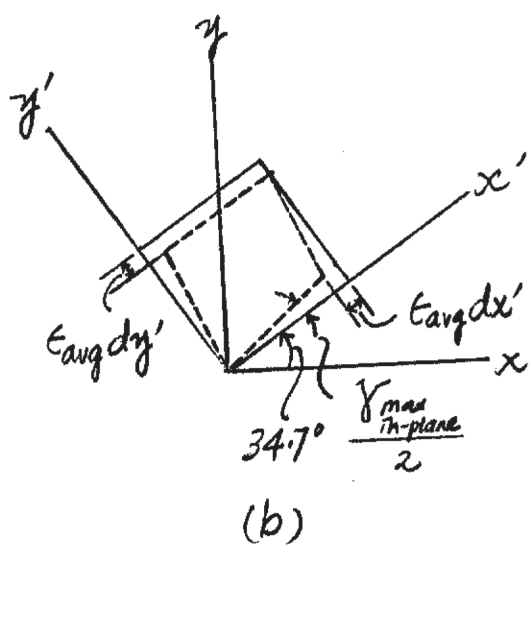
$$= 427(10^{-6})$$

Average Normal Strain:

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{-300 + 100}{2}\right)(10^{-6}) = -100(10^{-6}) \qquad \textbf{Ans.}$$

The deformed element for this state of maximum in-plane shear strain is shown in Fig. *b*.





Ans:
$\epsilon_1 = 114(10^{-6}), \epsilon_2 = -314(10^{-6})$,
$(\theta_p)_1 = 79.7°$ and $(\theta_p)_2 = -10.3°$,
$\gamma_{\substack{\max \\ \text{in-plane}}} = 427(10^{-6}), \theta_s = 34.7°$ and $125°$,
$\epsilon_{avg} = -100(10^{-6})$

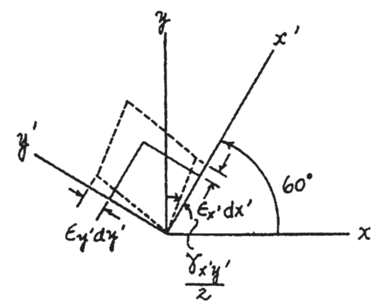
10–17.

Solve Prob. 10–3 using Mohr's circle.

SOLUTION

Construction of the Circle: In accordance with the sign convention, $\varepsilon_x = 200(10^{-6})$, $\varepsilon_y = 180(10^{-6})$, and $\frac{\gamma_{xy}}{2} = -150(10^{-6})$. Hence,

$$\varepsilon_{\text{avg}} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{200 + 180}{2}\right)(10^{-6}) = 190(10^{-6})$$



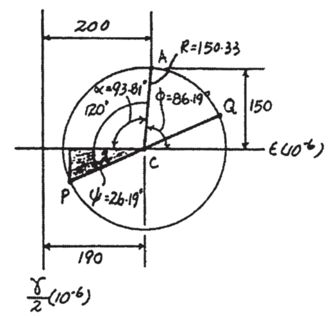
The coordinates for reference points A and C are

$A(200, -150)(10^{-6})$ $\quad$ $C(190, 0)(10^{-6})$

The radius of the circle is

$$R = \left(\sqrt{(200 - 190)^2 + 150^2}\right)(10^{-6}) = 150.33(10^{-6})$$

Strain on The Inclined Element: The normal and shear strain $\left(\varepsilon_{x'} \text{ and } \frac{\gamma_{x'y'}}{2}\right)$ are represented by coordinates of point P on the circle. $\varepsilon_{y'}$ can be determined by calculating the coordinates of point Q on the circle.



$$\varepsilon_{x'} = (190 - 150.33\cos 26.19°)(10^{-6}) = 55.1(10^{-6}) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = (150.33\sin 26.19°)(10^{-6})$$

$$\gamma_{x'y'} = 133(10^{-6}) \qquad \textbf{Ans.}$$

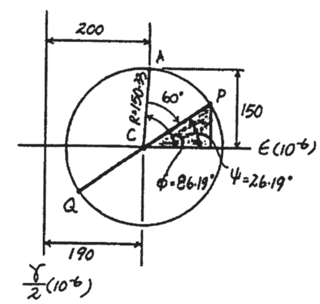
$$\varepsilon_{y'} = (190 + 150.33\cos 26.19°)(10^{-6}) = 325(10^{-6}) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_{x'} = 55.1(10^{-6})$, $\gamma_{x'y'} = 133(10^{-6})$,
$\epsilon_{y'} = 325(10^{-6})$

10–18.

Solve Prob. 10–4 using Mohr's circle.

SOLUTION



Construction of the Circle: In accordance with the sign convention, $\varepsilon_x = 200(10^{-6})$, $\varepsilon_y = 180(10^{-6})$, and $\dfrac{\gamma_{xy}}{2} = -150(10^{-6})$. Hence,

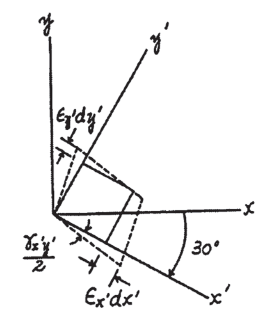
$$\varepsilon_{\text{avg}} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{200 + 180}{2}\right)(10^{-6}) = 190(10^{-6})$$

The coordinates for reference points A and C are

$$A(200, -150)(10^{-6}) \qquad C(190, 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(200 - 190)^2 + 150^2}\right)(10^{-6}) = 150.33(10^{-6})$$



Strain on the Inclined Element: The normal and shear strain $\left(\varepsilon_{x'} \text{ and } \dfrac{\gamma_{x'y'}}{2}\right)$ are represented by coordinates of point P on the circle. $\varepsilon_{y'}$ can be determined by calculating the coordinates of point Q on the circle.

$$\varepsilon_{x'} = (190 + 150.33\cos 26.19°)(10^{-6}) = 325(10^{-6})$$ **Ans.**

$$\frac{\gamma_{x'y'}}{2} = -(150.33\sin 26.19°)(10^{-6})$$

$$\gamma_{x'y'} = -133(10^{-6})$$ **Ans.**

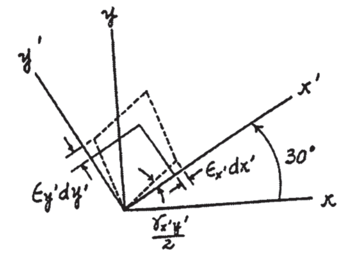
$$\varepsilon_{y'} = (190 - 150.33\cos 26.19°)(10^{-6}) = 55.1(10^{-6})$$ **Ans.**

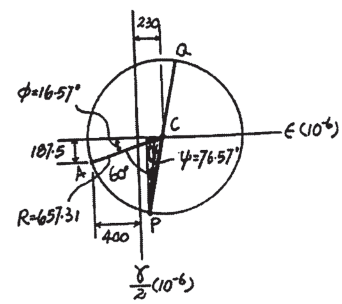
Ans:
$\epsilon_{x'} = 325(10^{-6})$, $\gamma_{x'y'} = -133(10^{-6})$,
$\epsilon_{y'} = 55.1(10^{-6})$

10–19.

Solve Prob. 10–5 using Mohr's circle.

SOLUTION





Construction of the Circle: In accordance with the sign convention $\varepsilon_x = -400(10^{-6})$, $\varepsilon_y = 860(10^{-6})$ and

$\dfrac{\gamma_{xy}}{2} = 187.5(10^{-6})$. Hence,

$$\varepsilon_{\text{avg}} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{-400 + 860}{2}\right)(10^{-6}) = 230(10^{-6})$$

The coordinates for reference points A and C are

$$A(-400, 187.5)(10^{-6}) \qquad C(230, 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(400 + 230)^2 + 187.5^2}\right)(10^{-6}) = 657.31(10^{-6})$$

Strain on the Inclined Element: The normal and shear strain $\left(\varepsilon_{x'} \text{ and } \dfrac{\gamma_{x'y'}}{2}\right)$ are represented by the coordinates of point P on the circle. $\varepsilon_{y'}$ can be determined by calculating the coordinates of point Q on the circle.

$\varepsilon_{x'} = (230 - 657.31\cos 76.57°)(10^{-6}) = 77.4(10^{-6})$ **Ans.**

$$\frac{\gamma_{x'y'}}{2} = (657.31\sin 76.57°)(10^{-6})$$

$\gamma_{x'y'} = 1279(10^{-6})$ **Ans.**

$\varepsilon_{y'} = (230 + 657.31\cos 76.57°)(10^{-6}) = 383(10^{-6})$ **Ans.**

Ans:
$\epsilon_{x'} = 77.4(10^{-6})$, $\gamma_{x'y'} = 1279(10^{-6})$,
$\epsilon_{y'} = 383(10^{-6})$

***10–20.**

Solve Prob. 10–8 using Mohr's circle.

SOLUTION

Construction of the Circle: In accordance with the sign convention $\varepsilon_x = 260(10^{-6})$, $\varepsilon_y = 320(10^{-6})$, and

$\dfrac{\gamma_{xy}}{2} = 90(10^{-6})$. Hence,

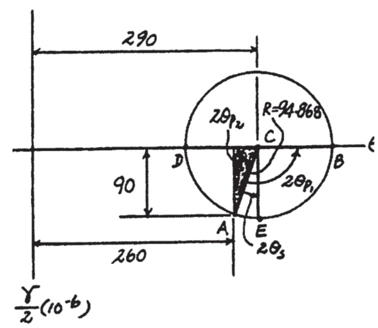
$$\varepsilon_{\text{avg}} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{260 + 320}{2}\right)(10^{-6}) = 290(10^{-6}) \qquad \textbf{Ans.}$$

The coordinates for reference points A and C are

$$A(260, 90)(10^{-6}) \qquad C(290, 0)(10^{-6})$$

The radius of the circle is

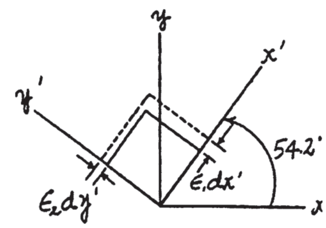
$$R = \left(\sqrt{(290 - 260)^2 + 90^2}\right)(10^{-6}) = 94.868(10^{-6})$$



In-Plane Principal Strain: The coordinates of points B and D represent ε_1 and ε_2, respectively.

$$\varepsilon_1 = (290 + 94.868)(10^{-6}) = 385(10^{-6}) \qquad \textbf{Ans.}$$

$$\varepsilon_2 = (290 - 94.868)(10^{-6}) = 195(10^{-6}) \qquad \textbf{Ans.}$$

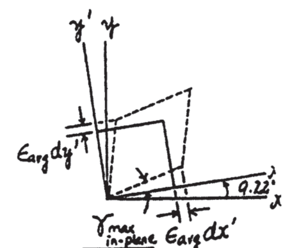


Orientation of Principal Strain: From the circle,

$$\tan 2\theta_{P_2} = \frac{90}{290 - 260} = 3.000 \qquad 2\theta_{P_2} = 71.57^\circ$$

$$2\theta_{P_1} = 180^\circ - 2\theta_{P_2}$$

$$\theta_{P_1} = \frac{180^\circ - 71.57^\circ}{2} = 54.2^\circ \qquad \textbf{(Counterclockwise)} \qquad \textbf{Ans.}$$



Maximum In-Plane Shear Strain: Represented by the coordinates of point E on the circle.

$$\frac{\gamma_{\substack{\max\\ \text{in-plane}}}}{2} = R = 94.868(10^{-6})$$

$$\gamma_{\substack{\max\\ \text{in-plane}}} = 190(10^{-6}) \qquad \textbf{Ans.}$$

Orientation of Maximum In-Plane Shear Strain: From the circle,

$$\tan 2\theta_s = \frac{290 - 260}{90} = 0.3333$$

$$\theta_s = 9.22^\circ \qquad \textbf{(Counterclockwise)} \qquad \textbf{Ans.}$$

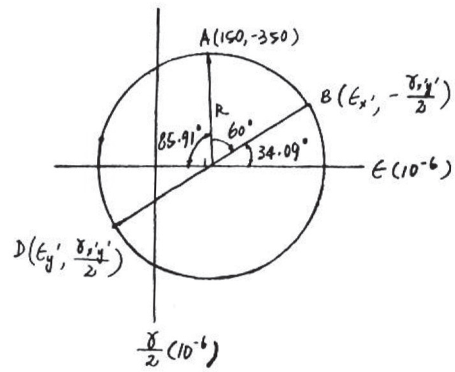
Ans:

$\varepsilon_{\text{avg}} = 290(10^{-6})$,
$\varepsilon_1 = 385(10^{-6})$,
$\varepsilon_2 = 195(10^{-6})$,
$\theta_{P_1} = 54.2^\circ$ **(Counterclockwise)**,
$\gamma_{\substack{\max\\ \text{in-plane}}} = 190(10^{-6})$,
$\theta_s = 9.22^\circ$ **(Counterclockwise)**

10–21.

Solve Prob. 10–7 using Mohr's circle.

SOLUTION



$\epsilon_x = 150(10^{-6}) \qquad \epsilon_y = 200(10^{-6}) \qquad \gamma_{xy} = -700(10^{-6}) \qquad \dfrac{\gamma_{xy}}{2} = -350(10^{-6})$

$\theta = -30^\circ \qquad 2\theta = -60^\circ$

$A(150, -350); \qquad C(175, 0)$

$R = \sqrt{(175 - 150)^2 + (-350)^2} = 350.89$

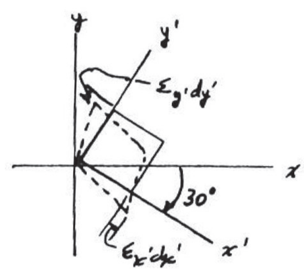
Coordinates of point B:

$\epsilon_{x'} = 350.89 \cos 34.09^\circ + 175$

$= 466(10^{-6})$ **Ans.**

$\dfrac{\gamma_{x'y'}}{2} = -350.89 \sin 34.09^\circ$

$\gamma_{x'y'} = -393(10^{-6})$ **Ans.**



Coordinates of point D:

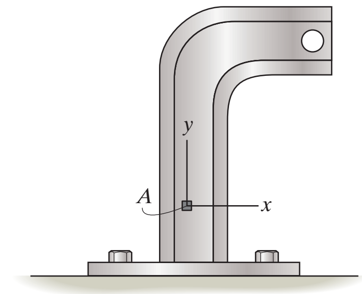
$\epsilon_{y'} = 175 - 350.89 \cos 34.09^\circ$

$= -116(10^{-6})$ **Ans.**

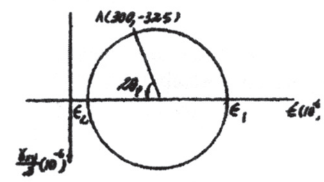
Ans:
$\epsilon_{x'} = 466(10^{-6})$, $\gamma_{x'y'} = -393(10^{-6})$,
$\epsilon_{y'} = -116(10^{-6})$

10–22.

The strain at point A on the bracket has components $\epsilon_x = 300(10^{-6})$, $\epsilon_y = 550(10^{-6})$, $\gamma_{xy} = -650(10^{-6})$, $\epsilon_z = 0$. Determine (a) the principal strains at A in the x–y plane, (b) the maximum shear strain in the x–y plane, and (c) the absolute maximum shear strain.



SOLUTION



$$\epsilon_x = 300(10^{-6}) \quad \epsilon_y = 550(10^{-6}) \quad \gamma_{xy} = -650(10^{-6}) \quad \frac{\gamma_{xy}}{2} = -325(10^{-6})$$

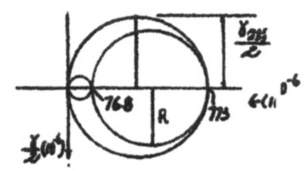
$$A(300, -325)10^{-6} \quad C(425, 0)10^{-6}$$

$$R = \left[\sqrt{(425 - 300)^2 + (-325)^2}\right]10^{-6} = 348.2(10^{-6})$$

a)

$$\epsilon_1 = (425 + 348.2)(10^{-6}) = 773(10^{-6})$$ **Ans.**

$$\epsilon_2 = (425 - 348.2)(10^{-6}) = 76.8(10^{-6})$$ **Ans.**



b)

$$\gamma_{\substack{\max \\ \text{in-plane}}} = 2R = 2(348.2)(10^{-6}) = 696(10^{-6})$$ **Ans.**

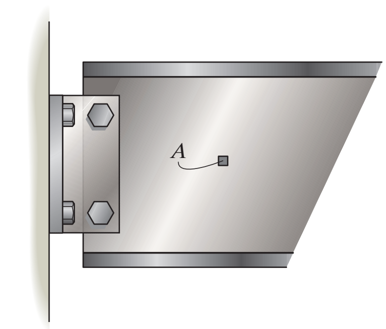
c)

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \frac{773(10^{-6})}{2}; \quad \gamma_{\substack{\text{abs} \\ \max}} = 773(10^{-6})$$ **Ans.**

Ans:
(a) $\epsilon_1 = 773(10^{-6})$, $\epsilon_2 = 76.8(10^{-6})$,
(b) $\gamma_{\substack{\max \\ \text{in-plane}}} = 696(10^{-6})$,
(c) $\gamma_{\substack{\text{abs} \\ \max}} = 773(10^{-6})$

10–23.

The strain at point A on a beam has components $\epsilon_x = 450(10^{-6})$, $\epsilon_y = 825(10^{-6})$, $\gamma_{xy} = 275(10^{-6})$, $\epsilon_z = 0$. Determine (a) the principal strains at A, (b) the maximum shear strain in the x–y plane, and (c) the absolute maximum shear strain.

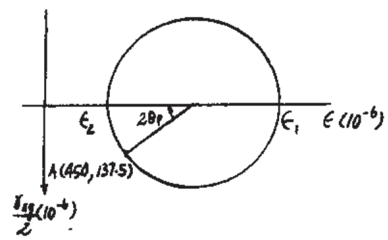


SOLUTION

$\epsilon_x = 450(10^{-6})$ $\quad\epsilon_y = 825(10^{-6})$ $\quad\gamma_{xy} = 275(10^{-6})$ $\quad\dfrac{\gamma_{xy}}{2} = 137.5(10^{-6})$

$A(450, 137.5)10^{-6}$ $\quad C(637.5, 0)10^{-6}$

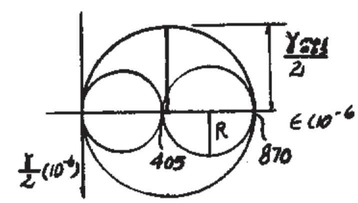
$$R = [\sqrt{(637.5 - 450)^2 + 137.5^2}]10^{-6} = 232.51(10^{-6})$$



a)

$$\epsilon_1 = (637.5 + 232.51)(10^{-6}) = 870(10^{-6})$$ **Ans.**

$$\epsilon_2 = (637.5 - 232.51)(10^{-6}) = 405(10^{-6})$$ **Ans.**



b)

$$\gamma_{\substack{\max \\ \text{in-plane}}} = 2R = 2(232.51)(10^{-6}) = 465(10^{-6})$$ **Ans.**

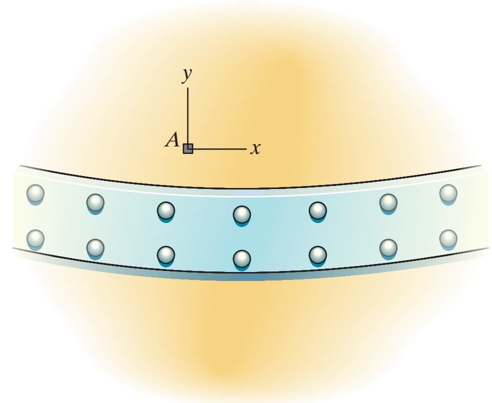
c)

$$\frac{\gamma_{\substack{\text{abs} \\ \max}}}{2} = \frac{870(10^{-6})}{2}; \quad \gamma_{\substack{\text{abs} \\ \max}} = 870(10^{-6})$$ **Ans.**

Ans:
(a) $\epsilon_1 = 870(10^{-6})$, $\epsilon_2 = 405(10^{-6})$,
(b) $\gamma_{\substack{\max \\ \text{in-plane}}} = 465(10^{-6})$,
(c) $\gamma_{\substack{\text{abs} \\ \max}} = 870(10^{-6})$

***10–24.**

The strain at point A on the pressure-vessel wall has components $\epsilon_x = 480(10^{-6})$, $\epsilon_y = 720(10^{-6})$, $\gamma_{xy} = 650(10^{-6})$. Determine (a) the principal strains at A, in the x–y plane, (b) the maximum shear strain in the x–y plane, and (c) the absolute maximum shear strain.



SOLUTION

$\epsilon_x = 480(10^{-6})$ $\quad \epsilon_y = 720(10^{-6})$ $\quad \gamma_{xy} = 650(10^{-6})$ $\quad \dfrac{\gamma_{xy}}{2} = 325(10^{-6})$

$A(480, 325)10^{-6}$ $\quad C(600, 0)10^{-6}$

$$R = \left(\sqrt{(600 - 480)^2 + 325^2}\right)10^{-6} = 346.44(10^{-6})$$

a)

$\epsilon_1 = (600 + 346.44)10^{-6} = 946(10^{-6})$ **Ans.**

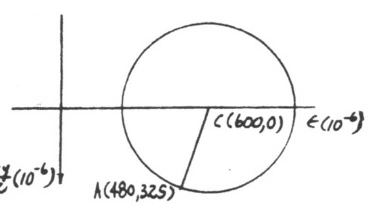
$\epsilon_2 = (600 - 346.44)10^{-6} = 254(10^{-6})$ **Ans.**

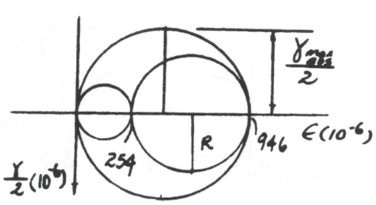
b)

$\gamma_{\substack{\max \\ \text{in-plane}}} = 2R = 2(346.44)10^{-6} = 693(10^{-6})$ **Ans.**

c)

$\dfrac{\gamma_{\substack{\text{abs} \\ \max}}}{2} = \dfrac{946(10^{-6})}{2}; \quad \gamma_{\substack{\text{abs} \\ \max}} = 946(10^{-6})$ **Ans.**

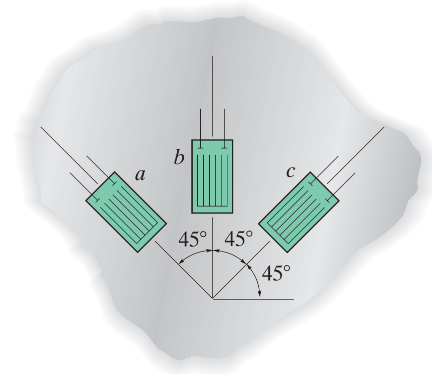




Ans:
$\epsilon_1 = 946(10^{-6})$,
$\epsilon_2 = 254(10^{-6})$,
$\gamma_{\substack{\max \\ \text{in-plane}}} = 693(10^{-6})$,
$\gamma_{\substack{\text{abs} \\ \max}} = 946(10^{-6})$

10–25.

The 45° strain rosette is mounted on the surface of a shell. The following readings are obtained for each gage: $\epsilon_a = -200(10^{-6})$, $\epsilon_b = 300(10^{-6})$, and $\epsilon_c = 250(10^{-6})$. Determine the in-plane principal strains.



SOLUTION

Strain Rosettes (45°): Applying the equation in the text with $\epsilon_a = -200(10^{-6})$, $\epsilon_b = 300(10^{-6})$, $\epsilon_c = 250(10^{-6})$, $\theta_a = 135°$, $\theta_b = 90°$ and $\theta_c = 45°$.

$$300(10^{-6}) = \epsilon_x \cos^2 90° + \epsilon_y \sin^2 90° + \gamma_{xy} \sin 90° \cos 90°$$

$$\epsilon_y = 300(10^{-6})$$

$$-200(10^{-6}) = \epsilon_x \cos^2 135° + 300(10^{-6}) \sin^2 135° + \gamma_{xy} \sin 135° \cos 135°$$

$$\epsilon_x - \gamma_{xy} = -700(10^{-6}) \qquad \textbf{(1)}$$

$$250(10^{-6}) = \epsilon_x \cos^2 45° + 300(10^{-6}) \sin^2 45° + \gamma_{xy} \sin 45° \cos 45°$$

$$\epsilon_x + \gamma_{xy} = 200(10^{-6}) \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$\epsilon_x = -250(10^{-6}) \qquad \gamma_{xy} = 450(10^{-6})$$

Construction of The Circle: With $\epsilon_x = -250(10^{-6})$, $\epsilon_y = 300(10^{-6})$ and $\frac{\gamma_{xy}}{2} = 225(10^{-6})$,

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{-250 + 300}{2}\right)(10^{-6}) = 25.0(10^{-6})$$

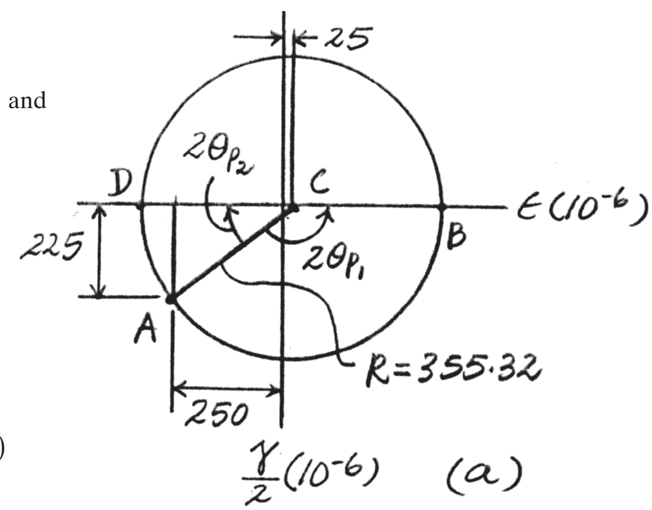
The coordinates of reference point A and center of circle C are

$$A(-250, 225)(10^{-6}) \qquad C(25.0, 0)(10^{-6})$$

The radius of the circle is

$$R = CA = \left[\sqrt{(-250 - 25)^2 + (225 - 0)^2}\right](10^{-6}) = 355.32(10^{-6})$$

Using these results, the circle shown in Fig. a can be constructed.



In-Plane Principal Strain: The coordinates of points B and D represent ϵ_1 and ϵ_2 respectively.

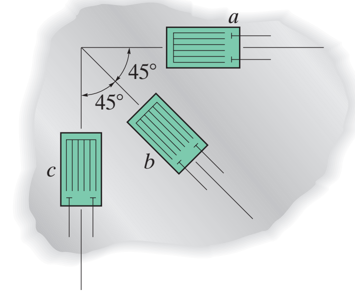
$$\epsilon_1 = (25.0 + 355.32)(10^{-6}) = 380.32(10^{-6}) = 380(10^{-6}) \qquad \textbf{Ans.}$$

$$\epsilon_2 = (25.0 - 355.32)(10^{-6}) = -330.32(10^{-6}) = -330(10^{-6}) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_1 = 380(10^{-6})$,
$\epsilon_2 = -330(10^{-6})$

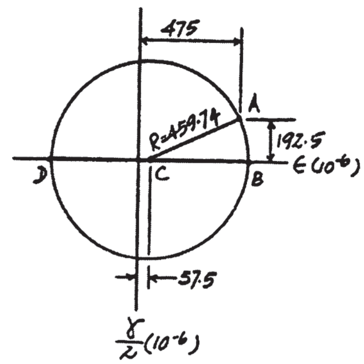
10–26.

The 45° strain rosette is mounted on the surface of a pressure vessel. The following readings are obtained for each gage: $\epsilon_a = 475(10^{-6})$, $\epsilon_b = 250(10^{-6})$, and $\epsilon_c = -360(10^{-6})$. Determine the in-plane principal strains.



SOLUTION

Strain Rosettes (45°): Applying the equations in the text with $\epsilon_a = 475(10^{-6})$, $\epsilon_b = 250(10^{-6})$, $\epsilon_c = -360(10^{-6})$, $\theta_a = 0°$, $\theta_b = -45°$, and $\theta_c = -90°$.



$$475(10^{-6}) = \epsilon_x \cos^2 0° + \epsilon_y \sin^2 0° + \gamma_{xy} \sin 0° \cos 0°$$

$$\epsilon_x = 475(10^{-6})$$

$$250(10^{-6}) = 475(10^{-6}) \cos^2(-45°) + \epsilon_y \sin^2(-45°) + \gamma_{xy} \sin(-45°) \cos(-45°)$$

$$250(10^{-6}) = 237.5(10^{-6}) + 0.5\,\epsilon_y - 0.5\,\gamma_{xy}$$

$$0.5\epsilon_y - 0.5\,\gamma_{xy} = 12.5(10^{-6}) \qquad \textbf{(1)}$$

$$-360(10^{-6}) = 475(10^{-6}) \cos^2(-90°) + \epsilon_y \sin^2(-90°) + \gamma_{xy} \sin(-90°) \cos(-90°)$$

$$\epsilon_y = -360(10^{-6})$$

From Eq. (1), $\gamma_{xy} = -385(10^{-6})$

Therefore, $\epsilon_x = 475(10^{-6})$ $\quad \epsilon_y = -360(10^{-6})$ $\quad \gamma_{xy} = -385(10^{-6})$

Construction of the Circle: With $\dfrac{\gamma_{xy}}{2} = -192.5(10^{-6})$ and

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{475 + (-360)}{2}\right)(10^{-6}) = 57.5(10^{-6})$$

The coordinates for reference points A and C are

$$A(475, -192.5)(10^{-6}) \qquad C(57.5, 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(475 - 57.5)^2 + 192.5^2}\right)(10^{-6}) = 459.74(10^{-6})$$

In-Plane Principal Strain: The coordinates of points B and D represent ϵ_1 and ϵ_2, respectively.

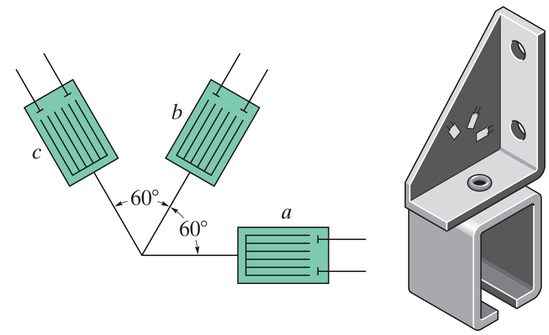
$$\epsilon_1 = (57.5 + 459.74)(10^{-6}) = 517(10^{-6}) \qquad \textbf{Ans.}$$

$$\epsilon_2 = (57.5 - 459.74)(10^{-6}) = -402(10^{-6}) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_1 = 517(10^{-6}), \epsilon_2 = -402(10^{-6})$

10–27.

The 60° strain rosette is mounted on the surface of the bracket. The following readings are obtained for each gage: $\epsilon_a = -780(10^{-6})$, $\epsilon_b = 400(10^{-6})$, and $\epsilon_c = 500(10^{-6})$. Determine (a) the principal strains and (b) the maximum in-plane shear strain and associated average normal strain. In each case show the deformed element due to these strains.



SOLUTION

Strain Rosettes (60°): Applying the equations in the text with $\epsilon_a = -780(10^{-6})$, $\epsilon_b = 400(10^{-6})$, $\epsilon_c = 500(10^{-6})$, $\theta_a = 0°$, $\theta_b = 60°$, and $\theta_c = 120°$,

$$\epsilon_x = \epsilon_a = -780(10^{-6})$$

$$\epsilon_y = \frac{1}{3}(2\epsilon_b + 2\epsilon_c - \epsilon_a)$$

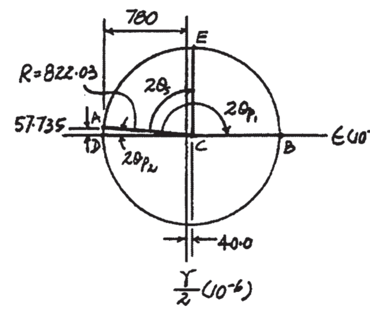
$$= \frac{1}{3}[2(400) + 2(500) - (-780)](10^{-6})$$

$$= 860(10^{-6})$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}(\epsilon_b - \epsilon_c)$$

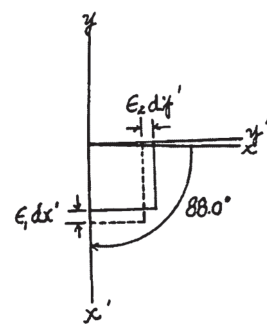
$$= \frac{2}{\sqrt{3}}(400 - 500)(10^{-6})$$

$$= -115.47(10^{-6})$$



Construction of the Circle: With $\epsilon_x = -780(10^{-6})$, $\epsilon_y = 860(10^{-6})$, and $\frac{\gamma_{xy}}{2} = -57.735(10^{-6})$.

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{-780 + 860}{2}\right)(10^{-6}) = 40.0(10^{-6}) \qquad \textbf{Ans.}$$



The coordinates for reference points A and C are

$$A(-780, -57.735)(10^{-6}) \qquad C(40.0, 0)(10^{-6})$$

The radius of the circle is

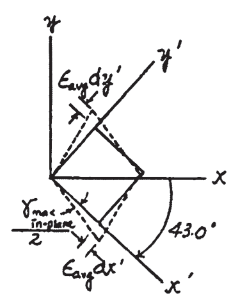
$$R = \left(\sqrt{(780 + 40.0)^2 + 57.735^2}\right)(10^{-6}) = 822.03(10^{-6})$$

a)

In-Plane Principal Strain: The coordinates of point B and D represent ϵ_1 and ϵ_2, respectively.

$$\epsilon_1 = (40.0 + 822.03)(10^{-6}) = 862(10^{-6}) \qquad \textbf{Ans.}$$

$$\epsilon_2 = (40.0 - 822.03)(10^{-6}) = -782(10^{-6}) \qquad \textbf{Ans.}$$



10–27. Continued

Orientation of Principal Strain: From the circle,

$$\tan 2\theta_{p2} = \frac{57.735}{780 + 40} = 0.07041 \quad 2\theta_{p2} = 4.03°$$

$$2\theta_{p1} = 180° - 2\theta_{p2}$$

$$\theta_{p1} = \frac{180° - 4.03°}{2} = 88.0° \quad \textbf{(Clockwise)} \qquad \textbf{Ans.}$$

b)

Maximum In-Plane Shear Strain: Represented by the coordinates of point E on the circle.

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = -R = -822.03(10^{-6})$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = -1644(10^{-6}) \qquad \textbf{Ans.}$$

Orientation of Maximum In-Plane Shear Strain: From the circle,

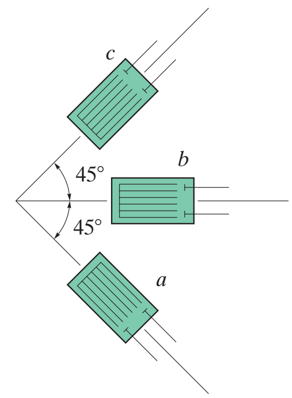
$$\tan 2\theta_s = \frac{780 + 40}{57.735} = 14.2028$$

$$\theta_s = 43.0° \quad \textbf{(Clockwise)} \qquad \textbf{Ans.}$$

Ans:
$\epsilon_{avg} = 40.0(10^{-6})$,
$\epsilon_1 = 862(10^{-6})$,
$\epsilon_2 = -782(10^{-6})$,
$\theta_{p1} = 88.0°$ **(Clockwise)**,
$\gamma_{\substack{\max \\ \text{in-plane}}} = -1644(10^{-6})$,
$\theta_s = 43.0°$ **(Clockwise)**

***10–28.**

The 45° strain rosette is mounted on a steel shaft. The following readings are obtained from each gage: $\epsilon_a = 800(10^{-6})$, $\epsilon_b = 520(10^{-6})$, $\epsilon_c = -450(10^{-6})$. Determine the in-plane principal strains.



SOLUTION

$\epsilon_a = 800(10^{-6})$ $\quad\epsilon_b = 520(10^{-6})$ $\quad\epsilon_c = -450(10^{-6})$

$\theta_a = -45°$ $\quad\theta_b = 0°$ $\quad\theta_c = 45°$

$$\epsilon_b = \epsilon_x \cos^2\theta_b + \epsilon_y \sin^2\theta_b + \gamma_{xy}\sin\theta_b\cos\theta_b$$

$$520(10^{-6}) = \epsilon_x \cos^2 0° + \epsilon_y \sin^2 0° + \gamma_{xy}\sin 0°\cos 0°$$

$$\epsilon_a = 520(10^{-6})$$

$$\epsilon_a = \epsilon_x \cos^2\theta_a + \epsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a\cos\theta_a$$

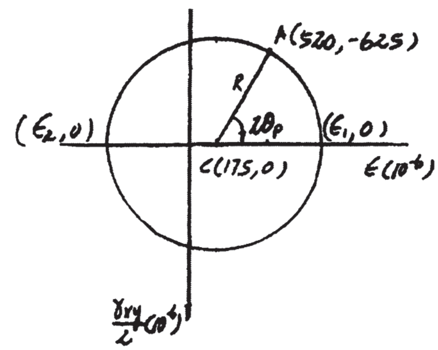
$$800(10^{-6}) = \epsilon_x \cos^2(-45°) + \epsilon_y \sin^2(-45°) + \gamma_{xy}\sin(-45°)\cos(-45°)$$

$$800(10^{-6}) = 0.5\epsilon_x + 0.5\epsilon_y - 0.5\gamma_{xy} \qquad \textbf{(1)}$$

$$\epsilon_c = \epsilon_x \cos^2\theta_c + \epsilon_y \sin^2\theta_c + \gamma_{xy}\sin\theta_c\cos\theta_c$$

$$-450(10^{-6}) = \epsilon_x \cos^2 45° + \epsilon_y \sin^2 45° + \gamma_{xy}\sin 45°\cos 45°$$

$$-450(10^{-6}) = 0.5\epsilon_x + 0.5\epsilon_y + 0.5\gamma_{xy} \qquad \textbf{(2)}$$



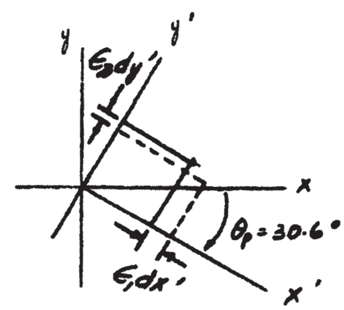
Subtract Eq. (2) from Eq. (1)

$$1250(10^{-6}) = -\gamma_{xy}$$

$$\gamma_{xy} = -1250(10^{-6})$$

$$\epsilon_y = -170(10^{-6})$$

$$\frac{\gamma_{xy}}{2} = -625(10^{-6})$$



$A(520, -625)10^{-6} \qquad C(175, 0)10^{-6}$

$$R = \left[\sqrt{(520 - 175)^2 + 625^2}\right]10^{-6} = 713.90(10^{-6})$$

$$\epsilon_1 = (175 + 713.9)10^{-6} = 889(10^{-6}) \qquad \textbf{Ans.}$$

$$\epsilon_2 = (175 - 713.9)10^{-6} = -539(10^{-6}) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_1 = 889(10^{-6})$,
$\epsilon_2 = -539(10^{-6})$

10–30.

For the case of plane stress, show that Hooke's law can be written as

$$\sigma_x = \frac{E}{(1-\nu^2)}(\epsilon_x + \nu\epsilon_y), \quad \sigma_y = \frac{E}{(1-\nu^2)}(\epsilon_y + \nu\epsilon_x)$$

SOLUTION

Generalized Hooke's Law: For plane stress, $\sigma_z = 0$. Applying Eq. 10–18,

$$\epsilon_x = \frac{1}{E}\left(\sigma_x - \nu\,\sigma_y\right)$$

$$\nu E\epsilon_x = \left(\sigma_x - \nu\,\sigma_y\right)\nu$$

$$\nu E\epsilon_x = \nu\,\sigma_x - \nu^2\,\sigma_y \qquad \textbf{(1)}$$

$$\epsilon_y = \frac{1}{E}(\sigma_y - \nu\,\sigma_x)$$

$$E\,\epsilon_y = -\nu\,\sigma_x + \sigma_y \qquad \textbf{(2)}$$

Adding Eq. (1) and Eq. (2) yields.

$$\nu E\,\epsilon_x + E\,\epsilon_y = \sigma_y - \nu^2\,\sigma_y$$

$$\sigma_y = \frac{E}{1-\nu^2}\left(\nu\epsilon_x + \epsilon_y\right) \qquad \textbf{\textit{(Q.E.D.)}}$$

Substituting σ_y into Eq. (2)

$$E\,\epsilon_y = -\nu\sigma_x + \frac{E}{1-\nu^2}\left(\nu\,\epsilon_x + \epsilon_y\right)$$

$$\sigma_x = \frac{E\left(\nu\,\epsilon_x + \epsilon_y\right)}{\nu\,(1-\nu^2)} - \frac{E\epsilon_y}{\nu}$$

$$= \frac{E\nu\epsilon_x + E\epsilon_y - E\epsilon_y + E\epsilon_y\,\nu^2}{\nu(1-\nu^2)}$$

$$= \frac{E}{1-\nu^2}\,(\epsilon_x + \nu\,\epsilon_y) \qquad \textbf{\textit{(Q.E.D.)}}$$

Ans:
N/A

10–31.

Use Hooke's law, Eq. 10–18, to develop the strain tranformation equations, Eqs. 10–5 and 10–6, from the stress tranformation equations, Eqs. 9–1 and 9–2.

SOLUTION

Stress Transformation Equations:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \qquad \textbf{(1)}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \qquad \textbf{(2)}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta \qquad \textbf{(3)}$$

Hooke's Law:

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu\, \sigma_y}{E} \qquad \textbf{(4)}$$

$$\epsilon_y = \frac{-\nu\, \sigma_x}{E} + \frac{\sigma_y}{E} \qquad \textbf{(5)}$$

$$\tau_{xy} = G\, \gamma_{xy} \qquad \textbf{(6)}$$

$$G = \frac{E}{2(1 + \nu)} \qquad \textbf{(7)}$$

From Eqs. (4) and (5)

$$\epsilon_x + \epsilon_y = \frac{(1 - \nu)(\sigma_x + \sigma_y)}{E} \qquad \textbf{(8)}$$

$$\epsilon_x - \epsilon_y = \frac{(1 + \nu)(\sigma_x - \sigma_y)}{E} \qquad \textbf{(9)}$$

From Eqs. (6) and (7)

$$\tau_{xy} = \frac{E}{2(1 + \nu)} \gamma_{xy} \qquad \textbf{(10)}$$

From Eq. (4)

$$\epsilon_{x'} = \frac{\sigma_{x'}}{E} - \frac{\nu\, \sigma_{y'}}{E} \qquad \textbf{(11)}$$

Substitute Eqs. (1) and (3) into Eq. (11)

$$\epsilon_{x'} = \frac{(1 - \nu)(\sigma_x + \sigma_y)}{2\,E} + \frac{(1 + \nu)(\sigma_x - \sigma_y)}{2\,E} \cos 2\theta + \frac{(1 + \nu)\tau_{xy} \sin 2\theta}{E} \qquad \textbf{(12)}$$

By using Eqs. (8), (9) and (10) and substitute into Eq. (12),

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta \qquad \textbf{(\textit{Q.E.D.})}$$

10–31. Continued

From Eq. (6).

$$\tau_{x'y'} = G\gamma_{x'y'} = \frac{E}{2(1+\nu)}\gamma_{x'y'} \qquad \textbf{(13)}$$

Substitute Eqs. (13), (6) and (9) into Eq. (2),

$$\frac{E}{2(1+\nu)}\gamma_{x'y'} = -\frac{E(\epsilon_x - \epsilon_y)}{2(1+\nu)}\sin 2\theta + \frac{E}{2(1+\nu)}\gamma_{xy}\cos 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{(\epsilon_x - \epsilon_y)}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta \qquad \textit{(Q.E.D.)}$$

Ans:
N/A

***10–32.** A bar of copper alloy is loaded in a tension machine and it is determined that $\epsilon_x = 940(10^{-6})$ and $\sigma_x = 100$ MPa, $\sigma_y = 0$, $\sigma_z = 0$. Determine the modulus of elasticity, E_{cu}, and the dilatation, e_{cu}, of the copper. $\nu_{cu} = 0.35$.

SOLUTION

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$940(10^{-6}) = \frac{1}{E_{cu}}[100(10^6) - 0.35(0+0)]$$

$$E_{cu} = 106.38(10^9)\text{ Pa} = 106\text{ GPa}$$ **Ans.**

$$e_{cu} = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{1-2(0.35)}{106.38(10^9)}[100(10^6) + 0 + 0] = 0.282(10^3)$$ **Ans.**

Ans.
$E_{cu} = 106$ GPa,
$e_{cu} = 0.282(10)^{-3}$

10–33.

A rod has a radius of 10 mm. If it is subjected to an axial load of 15 N such that the axial strain in the rod is $\epsilon_x = 2.75(10^{-6})$, determine the modulus of elasticity E and the change in the rod's diameter. $\nu = 0.23$.

SOLUTION

$$\sigma_x = \frac{15}{\pi(0.01)^2} = 47.746 \text{ kPa}, \qquad \sigma_y = 0, \qquad \sigma_z = 0$$

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$2.75(10^{-6}) = \frac{1}{E}\left[47.746(10^3) - 0.23(0 + 0)\right]$$

$$E = 17.4 \text{ GPa} \qquad \textbf{Ans.}$$

$$\epsilon_y = \epsilon_z = -\nu\epsilon_x = -0.23(2.75)(10^{-6}) = -0.632(10^{-6})$$

$$\Delta d = 20(-0.632(10^{-6})) = -12.6(10^{-6}) \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$E = 17.4$ GPa, $\Delta d = -12.6(10^{-6})$ mm

10–34. The principal strains at a point on the aluminum fuselage of a jet aircraft are $\epsilon_1 = 780(10^{-6})$ and $\epsilon_2 = 400(10^{-6})$. Determine the associated principal stresses at the point in the same plane. $E_{al} = 70$ GPa, $\nu_{al} = 0.33$. *Hint:* See Prob. 10–30.

SOLUTION

Plane stress, $\sigma_3 = 0$

See Prob 10-30,

$$\sigma_1 = \frac{E}{1 - \nu^2}(\varepsilon_1 + \nu\varepsilon_2)$$

$$= \frac{70(10^9)}{1-0.33^2}[780(10^{-6}) + 0.33(400)(10^{-6})] = 71.64(10^6)\text{ Pa} = 71.6\text{ MPa} \qquad \textbf{Ans.}$$

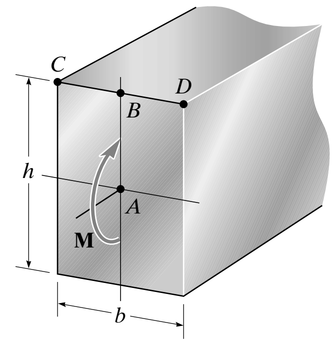
$$\sigma_2 = \frac{E}{1 - \nu^2}(\varepsilon_2 + \nu\varepsilon_1)$$

$$= \frac{70(10^9)}{1-0.33^2}[400(10^{-6}) + 0.33(780)(10^{-6})] = 51.64(10^6)\text{ Pa} = 51.6\text{ MPa} \qquad \textbf{Ans.}$$

Ans:

$\sigma_1 = 71.6$ MPa, $\sigma_2 = 51.6$ MPa

10–35. The cross section of the rectangular beam is subjected to the bending moment **M**. Determine an expression for the increase in length of lines AB and CD. The material has a modulus of elasticity E and Poisson's ratio is ν.



SOLUTION

For line AB,

$$\sigma_z = -\frac{My}{I} = \frac{My}{\frac{1}{12}b\,h^3} = -\frac{12My}{b\,h^3}$$

$$\varepsilon_y = -\frac{\nu\,\sigma_z}{E} = \frac{12\,\nu\,My}{E\,b\,h^3}$$

$$\Delta L_{AB} = \int_0^{\frac{h}{2}} \varepsilon_y\,dy = \frac{12\,\nu\,M}{E\,b\,h^3}\int_0^{\frac{h}{2}} y\,dy$$

$$= \frac{3\,\nu\,M}{2\,E\,b\,h} \qquad \textbf{Ans.}$$

For line CD,

$$\sigma_z = -\frac{Mc}{I} = -\frac{M\,\frac{h}{2}}{\frac{1}{12}b\,h^3} = -\frac{6M}{bh^2}$$

$$\varepsilon_x = -\frac{\nu\,\sigma_z}{E} = \frac{6\,\nu\,M}{E\,b\,h^2}$$

$$\Delta L_{CD} = \varepsilon_x\,L_{CD} = \frac{6\,\nu\,M}{E\,b\,h^2}(b)$$

$$= \frac{6\,\nu\,M}{E\,h^2} \qquad \textbf{Ans.}$$

Ans.

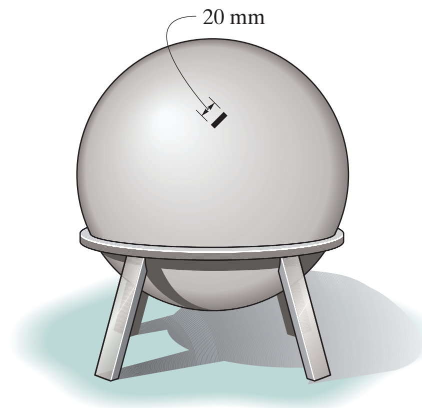
$\sigma_z = -\dfrac{12\,My}{bh^3}$, $\epsilon_y = \dfrac{12\,\nu My}{Ebh^3}$,

$\Delta L_{AB} = \dfrac{3\,\nu M}{2Ebh}$,

$\Delta L_{CD} = \dfrac{6\,\nu M}{Eh^2}$

***10–36.**

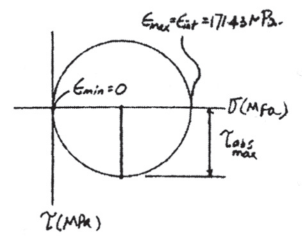
The spherical pressure vessel has an inner diameter of 2 m and a thickness of 10 mm. A strain gage having a length of 20 mm is attached to it, and it is observed to increase in length by 0.012 mm when the vessel is pressurized. Determine the pressure causing this deformation, and find the maximum in-plane shear stress, and the absolute maximum shear stress at a point on the outer surface of the vessel. The material is steel, for which $E_{st} = 200$ GPa and $\nu_{st} = 0.3$.



SOLUTION

Normal Stresses: Since $\frac{r}{t} = \frac{1000}{10} = 100 > 10$, the *thin wall* analysis is valid to determine the normal stress in the wall of the spherical vessel. This is a plane stress problem where $\sigma_{min} = 0$ since there is no load acting on the outer surface of the wall.

$$\sigma_{max} = \sigma_{lat} = \frac{pr}{2t} = \frac{p(1000)}{2(10)} = 50.0p \qquad \textbf{(1)}$$



Normal Strains: Applying the generalized Hooke's Law with

$$\epsilon_{max} = \epsilon_{lat} = \frac{0.012}{20} = 0.600\left(10^{-3}\right) \text{ mm/mm}$$

$$\epsilon_{max} = \frac{1}{E}\left[\sigma_{max} - \nu\left(\sigma_{lat} + \sigma_{min}\right)\right]$$

$$0.600\left(10^{-3}\right) = \frac{1}{200(10^9)}\left[50.0p - 0.3(50.0p + 0)\right]$$

$$p = 3.4286 \text{ MPa} = 3.43 \text{ MPa} \qquad \textbf{Ans.}$$

From Eq. (1) $\sigma_{max} = \sigma_{lat} = 50.0(3.4286) = 171.43$ MPa

Maximum In-Plane Shear (Sphere's Surface): Mohr's circle is simply a dot. As the result, the state of stress is the same, consisting of two normal stresses with zero shear stress regardless of the orientation of the element.

$$\tau_{\substack{max\\ \text{in-plane}}} = 0 \qquad \textbf{Ans.}$$

Absolute Maximum Shear Stress:

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{171.43 - 0}{2} = 85.7 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$p = 3.43$ MPa
$\tau_{\substack{max\\ \text{in-plane}}} = 0$
$\tau_{\substack{abs\\max}} = 85.7$ MPa

10–37. Determine the bulk modulus for each of the following materials: (a) rubber, $E_r = 2.8$ MPa, $\nu_r = 0.48$, and (b) glass, $E_g = 56$ GPa, $\nu_g = 0.24$.

SOLUTION

a) For rubber:

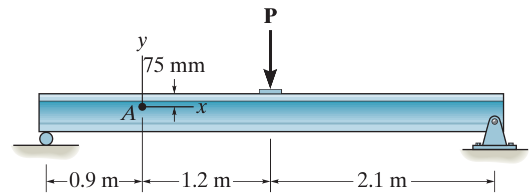
$$K_r = \frac{E_r}{3\,(1 - 2\,\nu_r)} = \frac{2.8}{3[1 - 2(0.48)]} = 23.3 \text{ MPa}$$ **Ans.**

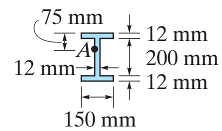
b) For glass:

$$K_g = \frac{E_g}{3\,(1 - 2\,\nu_g)} = \frac{56}{3[1 - 2(0.24)]} = 35.9 \text{ GPa}$$ **Ans.**

Ans.
(a) $K_r = 23.3$ MPa, (b) $K_g = 35.9$ GPa

10–38. The strain in the x direction at point A on the steel beam is measured and found to be $\epsilon_x = -100(10^{-6})$. Determine the applied load P. What is the shear strain γ_{xy} at point A? $E_{st} = 200$ GPa, $\nu_{st} = 0.3$.





SOLUTION

$$I_x = \frac{1}{12}(0.15)(0.224^3) - \frac{1}{12}(0.138)(0.2^3) = 48.4928(10^{-6})\ \text{m}^4$$

$$Q_A = 0.106(0.15)(0.012) + 0.0685(0.012)(0.063) = 0.242586(10^{-3})\ \text{m}^3$$

$$\sigma = E\varepsilon_x = [200(10^9)][100(10^{-6})] = 20(10^6)\ \text{N/m}^2$$

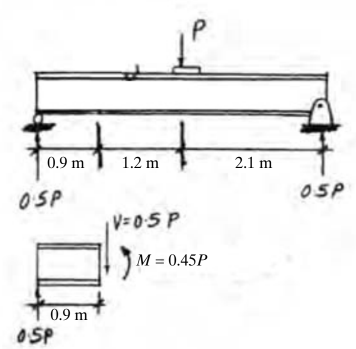
$$\sigma = \frac{My}{I}, \qquad 20(10^6) = \frac{(0.45P)(0.037)}{48.4928(10^{-6})}$$

$$P = 58.24(10^3)\ \text{N} = 58.2\ \text{kN}$$ **Ans.**

$$\tau_A = \frac{VQ}{It} = \frac{0.5[58.24(10^3)][0.242586(10^{-3})]}{[48.4928(10^{-6})](0.012)} = 12.14(10^6)\ \text{Pa}$$

$$G = \frac{E}{2(1+\nu)} = \frac{200}{2(1+0.3)} = 76.92\ \text{GPa}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{12.14(10^6)}{76.92(10^9)} = 0.1578(10^{-3})\ \text{rad} = 0.158(10^{-3})\ \text{rad}$$ **Ans.**



Ans.
$P = 58.2$ kN, $\gamma_{xy} = 0.158(10^{-3})$ rad

10–39. The principal strains in a plane, measured experimentally at a point on the aluminum fuselage of a jet aircraft, are $\epsilon_1 = 630(10^{-6})$ and $\epsilon_2 = 350(10^{-6})$. If this is a case of plane stress, determine the associated principal stresses at the point in the same plane. $E_{al} = 70$ GPa and $\nu_{al} = 0.33$.

SOLUTION

Normal Stresses: For plane stress, $\sigma_3 = 0$.

Normal Strains: Applying the generalized Hooke's Law.

$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - v\,(\sigma_2 + \sigma_3)\right]$$

$$630(10^{-6}) = \frac{1}{70(10^9)}[\sigma_1 - 0.33(\sigma_2 + 0)]$$

$$44.1(10^6) = \sigma_1 - 0.33\sigma_2 \qquad [1]$$

$$\varepsilon_2 = \frac{1}{E}\left[\sigma_2 - v\,(\sigma_1 + \sigma_3)\right]$$

$$350(10^{-6}) = \frac{1}{70(10^9)}[\sigma_2 - 0.33(\sigma_1 + 0)]$$

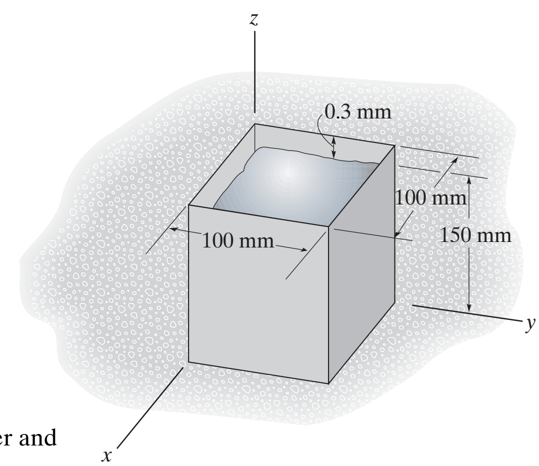
$$24.5(10^6) = \sigma_2 - 0.33\sigma_1 \qquad [2]$$

Solving Eqs.[1] and [2] yields:

$\sigma_1 = 58.56(10^6)$ Pa $= 58.6$ MPa $\qquad \sigma_2 = 43.83(10^6)$ Pa $= 43.8$ MPa **Ans.**

Ans:
$\sigma_1 = 58.6$ MPa, $\sigma_2 = 43.8$ MPa

***10–40.** The smooth rigid-body cavity is filled with liquid 6061-T6 aluminum. When cooled it is 0.3 mm from the top of the cavity. If the top of the cavity is covered and the temperature is increased by 110°C, determine the stress components σ_x, σ_y, and σ_z in the aluminum. *Hint:* Use Eqs. 10–18 with an additional strain term of $\alpha\Delta T$ (Eq. 4–4).



SOLUTION

Normal Strains: Since the aluminum is confined at its sides by a rigid container and allowed to expand in the z direction, $\varepsilon_x = \varepsilon_y = 0$; whereas $\varepsilon_z = \dfrac{0.3}{150} = 0.002$. Applying the generalized Hooke's Law with the additional thermal strain,

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] + \alpha\Delta T$$

$$0 = \frac{1}{68.9(10^9)}[\sigma_x - 0.35(\sigma_y + \sigma_z)] + [24(10^{-6})](110)$$

$$0 = \sigma_x - 0.35\sigma_y - 0.35\sigma_z + 181.896(10^6) \qquad [1]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z) + \alpha\Delta T\right.$$

$$0 = \frac{1}{68.9(10^9)}[\sigma_y - 0.35(\sigma_x + \sigma_z)] + [24(10^{-6})](110)$$

$$0 = \sigma_y - 0.35\sigma_x - 0.35\sigma_z + 181.896(10^6) \qquad [2]$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$

$$0.002 = \frac{1}{68.9(10^9)}[\sigma_z - 0.35(\sigma_x + \sigma_z)] + [24(10^{-6})](110)$$

$$0 = \sigma_z - 0.35\sigma_x - 0.35\sigma_z + 44.096(10^6) \qquad [3]$$

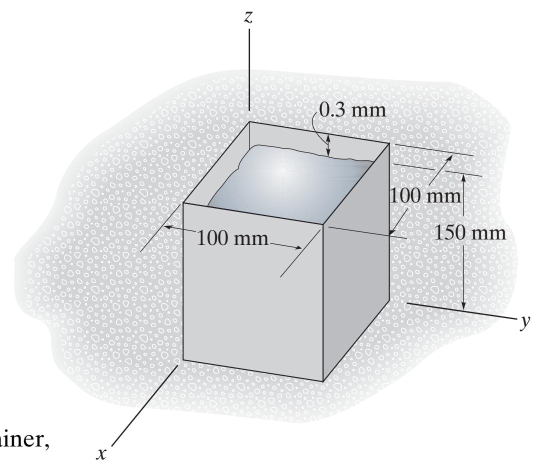
Solving Eqs.[1], [2] and [3] yields:

$$\sigma_x = \sigma_y = -487.23(10^6)\ \text{N/m}^2 = -487\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_z = -385.16(10^6)\ \text{N/m}^2 = -385\ \text{MPa} \qquad \textbf{Ans.}$$

Ans.
$\sigma_x = \sigma_y = -487$ MPa, $\sigma_z = -385$ MPa

10–41. The smooth rigid-body cavity is filled with liquid 6061-T6 aluminum. When cooled it is 0.3 mm from the top of the cavity. If the top of the cavity is not covered and the temperature is increased by 110°C, determine the stress components ϵ_x, ϵ_y, and ϵ_z in the aluminum. *Hint:* Use Eqs. 10–18 with an additional strain term of $\alpha \Delta T$ (Eq. 4–4).



SOLUTION

Normal Strains: Since the aluminum is confined at its sides by a rigid container, then

$$\varepsilon_x = \varepsilon_y = 0 \qquad \textbf{Ans.}$$

and since it is not restrained in *z direction*, $\sigma_z = 0$. Applying the generalized Hooke's Law with the additional thermal strain,

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - v\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$0 = \frac{1}{68.9(10^9)}[\sigma_x - 0.35(\sigma_y + \sigma_z)] + [24(10^{-6})](110)$$

$$0 = \sigma_x - 0.35\sigma_y - 0.35\sigma_z + 181.896(10^6) \qquad [1]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - v(\sigma_x + \sigma_z)\right] + \alpha\Delta T$$

$$0 = \frac{1}{68.9(10^9)}[\sigma_y - 0.35(\sigma_x + \sigma_z)] + [24(10^{-6})](110)$$

$$0 = \sigma_y - 0.35\sigma_x - 0.35\sigma_z + 181.896(10^6) \qquad [2]$$

Solving Eqs. [1] and [2] yields:

$$\sigma_x = \sigma_y = -279.84(10^6)\ \text{N/m}^2$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - v\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$

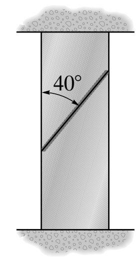
$$= \frac{1}{68.9(10^9)}\left\{0 - 0.35[-279.84(10^6) + (-279.84)(10^6)]\right\} + [24(10^{-6})](110)$$

$$= 5.483(10^{-3}) = 0.00548 \qquad \textbf{Ans.}$$

Ans.
$\epsilon_x = \epsilon_y = 0, \epsilon_z = 5.483(10^{-3})$

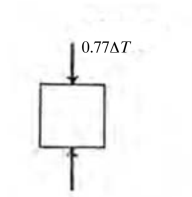
10–42. The block is fitted between the fixed supports. If the glued joint can resist a maximum shear stress of $\tau_{\text{allow}} = 14$ MPa, determine the temperature rise that will cause the joint to fail. Take $E = 70$ GPa, $\nu = 0.2$, and $\alpha = 11(10^{-6})/°\text{C}$.

Hint: Use Eq. 10–18 with an additional strain term of $\alpha\Delta T$ (Eq. 4–4).



SOLUTION

Normal Strain: Since the block is confined along the *y direction* by the rigid frame, then $\varepsilon_y = 0$ and $\sigma_x = \sigma_z = 0$. Applying the generalized Hooke's Law with the additional thermal strain,

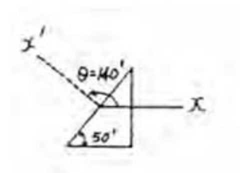


$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] + \alpha\Delta T$$

$$0 = \frac{1}{70(10^9)}[\sigma_y - 0.2(0+0)] + 11(10^{-6})(\Delta T)$$

$$\sigma_y = -770(10^3)\Delta T$$

Construction of the Circle: In accordance with the sign convention. $\sigma_x = 0$, $\sigma_y = -770(10^3)\Delta T$ and $\tau_{xy} = 0$. Hence.



$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + [-770(10^3)\Delta T]}{2} = -385(10^3)\Delta T$$

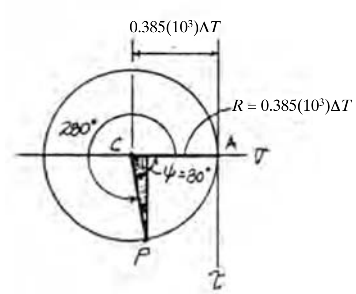
The coordinates for reference points A and C are $A\ (0, 0)$ and $C(-385(10^3)\Delta T, 0)$.

The radius of the circle is $R = \sqrt{\{0 - [-385(10^3)\Delta T]\}^2 + 0} = 385(10^3)\Delta T$

Stress on The inclined plane: The shear stress components $\tau_{x'y'}$, are represented by the coordinates of point P on the circle.

$$\tau_{x'y'} = 385(10^3)\Delta T \sin 80° = 379.15(10^3)\Delta T$$

Allowable Shear Stress:



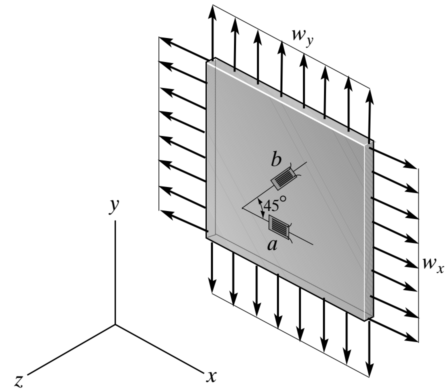
$$\tau_{\text{allow}} = \tau_{x'y'}$$

$$14(10^6) = 379.15(10^3)\Delta T$$

$$\Delta T = 36.92° = 36.9° \qquad \textbf{Ans.}$$

Ans.
$\Delta T = 36.92°\text{C}$

10–43. Two strain gauges a and b are attached to a plate made from a material having a modulus of elasticity of $E = 70$ GPa and Poisson's ratio $\nu = 0.35$. If the gauges give a reading of $\epsilon_a = 450(10^{-6})$ and $\epsilon_b = 100(10^{-6})$, determine the intensities of the uniform distributed load w_x and w_y acting on the plate. The thickness of the plate is 25 mm.



SOLUTION

Normal Strain: Since no shear force acts on the plane along the x and y axes, $\gamma_{xy} = 0$. With $\theta_a = 0$ and $\theta_b = 45°$, we have

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a\cos\theta_a$$

$$450\left(10^{-6}\right) = \varepsilon_x \cos^2 0° + \varepsilon_y \sin^2 0° + 0$$

$$\varepsilon_x = 450\left(10^{-6}\right)$$

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy}\sin\theta_b\cos\theta_b$$

$$100\left(10^{-6}\right) = 450\left(10^{-6}\right)\cos^2 45° + \varepsilon_y \sin^2 45° + 0$$

$$\varepsilon_y = -250\left(10^{-6}\right)$$

Generalized Hooke's Law: This is a case of plane stress. Thus, $\sigma_z = 0$.

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$450\left(10^{-6}\right) = \frac{1}{70\left(10^9\right)}\left[\sigma_y - 0.35\left(\sigma_y + 0\right)\right]$$

$$\sigma_x - 0.35\sigma_y = 31.5\left(10^6\right) \qquad (1)$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right]$$

$$-250\left(10^{-6}\right) = \frac{1}{70\left(10^9\right)}\left[\sigma_y - 0.35\left(\sigma_y + 0\right)\right]$$

$$\sigma_y - 0.35\sigma_x = -17.5\left(10^6\right) \qquad (2)$$

Solving Eqs. (1) and (2),

$$\sigma_y = -7.379\left(10^6\right)\text{N/m}^2 \qquad \sigma_x = 28.917\left(10^6\right)\text{N/m}^2$$

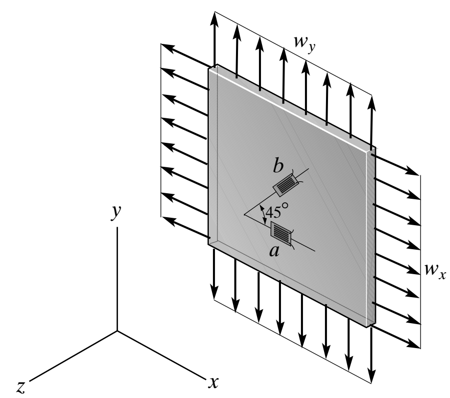
Then,

$$w_y = \sigma_y t = -7.379\left(10^6\right)(0.025) = -184 \text{ N/m} \qquad \textbf{Ans.}$$

$$w_x = \sigma_x t = 28.917\left(10^6\right)(0.025) = 723 \text{ N/m} \qquad \textbf{Ans.}$$

Ans.
$w_y = -184$ kN/m, $w_x = 723$ kN/m

***10–44.** Two strain gauges a and b are attached to the surface of the plate which is subjected to the uniform distributed load $w_x = 700$ kN/m and $w_y = -175$ kN/m. If the gauges give a reading of $\varepsilon_a = 450(10^{-6})$ and $\varepsilon_b = 100(10^{-6})$, determine the modulus of elasticity E, shear modulus G, and Poisson's ratio ν for the material.



Normal Stress and Strain: The normal stresses along the x, y, and z axes are

$$\sigma_x = \frac{700\left(10^3\right)}{0.025} = 28\left(10^6\right)\text{N/m}^2$$

$$\sigma_y = -\frac{175\left(10^3\right)}{0.025} = -7\left(10^6\right)\text{N/m}^2$$

$$\sigma_z = 0 \text{ (plane stress)}$$

Since no shear force acts on the plane along the x and y axes, $\gamma_{xy} = 0$. With $\theta_a = 0°$ and $\theta_b = 45°$, we have

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a \cos\theta_a$$

$$450\left(10^{-6}\right) = \varepsilon_x \cos^2 0° + \varepsilon_y \sin^2 0° + 0$$

$$\varepsilon_x = 450\left(10^{-6}\right)$$

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy}\sin\theta_b \cos\theta_b$$

$$100\left(10^{-6}\right) = 450\left(10^{-6}\right)\cos^2 45° + \varepsilon_y \sin^2 45° + 0$$

$$\varepsilon_y = -250\left(10^{-6}\right)$$

Generalized Hooke's Law:

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$450\left(10^{-6}\right) = \frac{1}{E}\left[28\left(10^6\right) - \nu\left[-7\left(10^6\right) + 0\right]\right]$$

$$450\left(10^{-6}\right)E - 7\left(10^6\right)\nu = 28\left(10^6\right) \qquad (1)$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$-250\left(10^{-6}\right) = \frac{1}{E}\left\{-7\left(10^6\right) - \nu\left[28\left(10^6\right) + 0\right]\right\}$$

$$250\left(10^{-6}\right)E - 28\left(10^6\right)\nu = 7\left(10^6\right) \qquad (2)$$

Solving Eqs. (1) and (2),

$$E = 67.74\left(10^9\right)\text{ N/m}^2 = 67.7\text{ GPa} \qquad \textbf{Ans.}$$

$$\nu = 0.3548 = 0.355 \qquad \textbf{Ans.}$$

Using the above results,

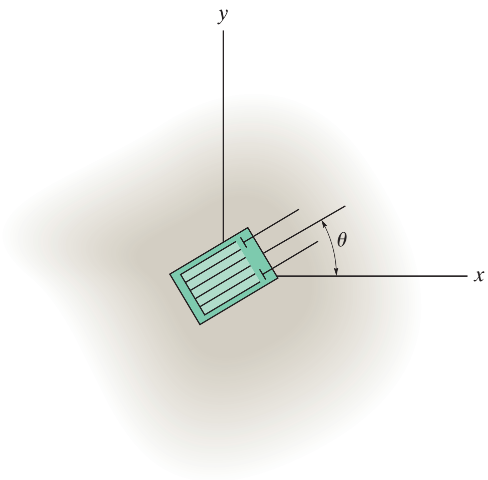
$$G = \frac{E}{2(1+\nu)} = \frac{67.74\left(10^9\right)}{2(1+0.3548)}$$

$$= 25.0\left(10^9\right)\text{N/m}^2 = 25.0\text{ GPa} \qquad \textbf{Ans.}$$

Ans.
$E = 67.7$ GPa, $\nu = 0.355$,
$G = 25.0$ GPa

10–45.

A material is subjected to principal stresses σ_x and σ_y. Determine the orientation θ of the strain gage so that its reading of normal strain responds only to σ_y and not σ_x. The material constants are E and ν.



SOLUTION

$$\sigma_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

Since $\tau_{xy} = 0$,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\sigma_n = \frac{\sigma_x}{2} + \frac{\sigma_y}{2} + (\sigma_x - \sigma_y)\cos^2\theta - \frac{\sigma_x}{2} + \frac{\sigma_y}{2}$$

$$= \sigma_y(1 - \cos^2\theta) + \sigma_x\cos^2\theta$$

$$= \sigma_x\cos^2\theta + \sigma_y\sin^2\theta$$

$$\sigma_{n+90^\circ} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$= \frac{\sigma_x}{2} + \frac{\sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)(2\cos^2\theta - 1)$$

$$= \frac{\sigma_x}{2} + \frac{\sigma_y}{2} - (\sigma_x - \sigma_y)\cos^2\theta + \frac{\sigma_x}{2} - \frac{\sigma_y}{2}$$

$$= \sigma_x(1 - \cos^2\theta) + \sigma_y\cos^2\theta$$

$$= \sigma_x\sin^2\theta + \sigma_y\cos^2\theta$$

$$\epsilon_n = \frac{1}{E}(\sigma_n - \nu\,\sigma_{n+90^\circ})$$

$$= \frac{1}{E}(\sigma_x\cos^2\theta + \sigma_y\sin^2\theta - \nu\,\sigma_x\sin^2\theta - \nu\,\sigma_y\cos^2\theta)$$

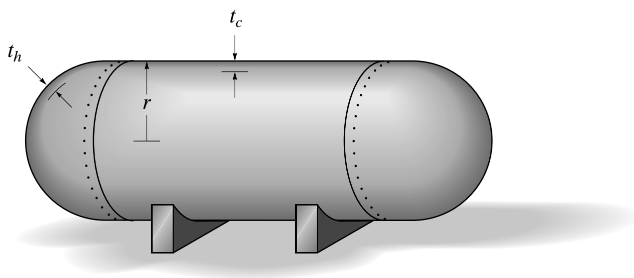
If ϵ_n is to be independent of σ_x, then

$$\cos^2\theta - \nu\sin^2\theta = 0 \qquad \text{or} \qquad \tan^2\theta = 1/\nu$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{\nu}}\right)$$ **Ans.**

Ans:
$\theta = \tan^{-1}\left(\frac{1}{\sqrt{\nu}}\right)$

10–46. The cylindrical pressure vessel is fabricated using hemispherical end caps in order to reduce the bending stress that would occur if flat ends were used. The bending stresses at the seam where the caps are attached can be eliminated by proper choice of the thickness t_h and t_c of the caps and cylinder, respectively. This requires the radial expansion to be the same for both the hemispheres and cylinder. Show that this ratio is $t_c/t_h = (2 - \nu)/(1 - \nu)$. Assume that the vessel is made of the same material and both the cylinder and hemispheres have the same inner radius. If the cylinder is to have a thickness of 12 mm, what is the required of the hemispheres? Take $\nu = 0.3$.



SOLUTION

For cylindrical vessel:

$$\sigma_1 = \frac{p\,r}{t_c}; \qquad \sigma_2 = \frac{p\,r}{2\,t_c}$$

$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu\,(\sigma_2 + \sigma_3)\right] \qquad \sigma_3 = 0$$

$$= \frac{1}{E}\left(\frac{p\,r}{t_c} - \frac{\nu\,p\,r}{2\,t_c}\right) = \frac{p\,r}{E\,t_c}\left(1 - \frac{1}{2}\nu\right)$$

$$d\,r = \varepsilon_1\,r = \frac{p\,r2}{E\,t_c}\left(1 - \frac{1}{2}\nu\right) \qquad (1)$$

For hemispherical end caps:

$$\sigma_1 = \sigma_2 = \frac{p\,r}{2\,t_h}$$

$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu\,(\sigma_2 + \sigma_3)\right]; \qquad \sigma_3 = 0$$

$$= \frac{1}{E}\left(\frac{p\,r}{2\,t_h} - \frac{\nu\,p\,r}{2\,t_h}\right) = \frac{p\,r}{2\,E\,t_h}(1 - \nu)$$

$$d\,r = \varepsilon_1\,r = \frac{p\,r^2}{2\,E\,t_h}(1 - \nu) \qquad (2)$$

Equate Eqs. (1) and (2):

$$\frac{p\,r^2}{E\,t_c}\left(1 - \frac{1}{2}\nu\right) = \frac{p\,r^2}{2\,E\,t_h}(1 - \nu)$$

$$\frac{t_c}{t_h} = \frac{2\,(1 - \frac{1}{2}\nu)}{1 - \nu} = \frac{2 - \nu}{1 - \nu} \qquad \textbf{QED}$$

$$t_h = \frac{(1 - \nu)\,t_c}{2 - \nu} = \frac{(1 - 0.3)\,(12)}{2 - 0.3} = 4.94\text{ mm} \qquad \textbf{Ans.}$$

Ans.
$t_h = 4.94$ mm

10–47. A thin-walled cylindrical pressure vessel has an inner radius r, thickness t, and length L. If it is subjected to an internal pressure p, show that the increase in its inner radius is $dr = r\epsilon_1 = pr^2(1 - \frac{1}{2}\nu)/Et$ and the increase in its length is $\Delta L = pLr(\frac{1}{2} - \nu)/Et$. Using these results, show that the change in internal volume becomes $dV = \pi r^2(1 + \epsilon_1)^2(1 + \epsilon_2)L - \pi r^2 L$. Since ϵ_1 and ϵ_2 are small quantities, show further that the change in volume per unit volume, called *volumetric strain*, can be written as $dV/V = pr(2.5 - 2\nu)/Et$.

SOLUTION

Normal stress:

$$\sigma_1 = \frac{p\,r}{t}; \qquad \sigma_2 = \frac{p\,r}{2\,t}$$

Normal strain: Applying Hooke's law

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - v(\sigma_2 + \sigma_3)], \qquad \sigma_3 = 0$$

$$= \frac{1}{E}\left(\frac{p\,r}{t} - \frac{v\,p\,r}{2\,t}\right) = \frac{p\,r}{E\,t}\left(1 - \frac{1}{2}\,v\right)$$

$$d\,r = \varepsilon_t\,r = \frac{p\,r^2}{E\,t}\left(1 - \frac{1}{2}\,v\right) \qquad \textbf{QED}$$

$$\varepsilon_2 = \frac{1}{E}[\sigma_2 - v(\sigma_1 + \sigma_3)], \qquad \sigma_3 = 0$$

$$= \frac{1}{E}\left(\frac{p\,r}{2\,t} - \frac{v\,p\,r}{t}\right) = \frac{p\,r}{E\,t}\left(\frac{1}{2} - v\right)$$

$$\Delta L = \varepsilon_2\,L = \frac{p\,L\,r}{E\,t}\left(\frac{1}{2} - v\right) \qquad \textbf{QED}$$

$$V' = \pi(r + \varepsilon_1\,r)^2\,(L + \varepsilon_2 L)\,; \qquad V = \pi\,r^2\,L$$

$$dV = V' - V = \pi r^2\,(1 + \varepsilon_1)^2\,(1 + \varepsilon_2)L - \pi r^2\,L \qquad \textbf{QED}$$

$(1 + \varepsilon_1)^2 = 1 + 2\,\varepsilon_1$ neglect ε_1^2 term

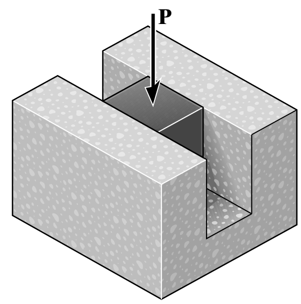
$(1 + \varepsilon_1)^2\,(1 + \varepsilon_2) = (1 + 2\,\varepsilon_1)(1 + \varepsilon_2) = 1 + \varepsilon_2 + 2\,\varepsilon_1$ neglect $\varepsilon_1\varepsilon_2$ term

$$\frac{dV}{V} = 1 + \varepsilon_2 + 2\,\varepsilon_1 - 1 = \varepsilon_2 + 2\,\varepsilon_1$$

$$= \frac{p\,r}{E\,t}\left(\frac{1}{2} - v\right) + \frac{2\,p\,r}{E\,t}\left(1 - \frac{1}{2}\,v\right)$$

$$= \frac{p\,r}{E\,t}\,(2.5 - 2\,v) \qquad \textbf{QED}$$

10–48. The rubber block is confined in the U-shape smooth rigid block. If the rubber has a modulus of elasticity E and Poisson's ratio ν, determine the effective modulus of elasticity of the rubber under the confined condition.



SOLUTION

Generalized Hooke's Law: Under this confined condition, $\varepsilon_x = 0$ and $\sigma_y = 0$. We have

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$0 = \frac{1}{E}(\sigma_x - \nu\sigma_z)$$

$$\sigma_x = \nu\sigma_z \qquad (1)$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + 0)]$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_x) \qquad (2)$$

Substituting Eq. (1) into Eq. (2),

$$\varepsilon_z = \frac{\sigma_z}{E}\left(1 - \nu^2\right)$$

The effective modulus of elasticity of the rubber block under the confined condition can be determined by considering the rubber block as unconfined but rather undergoing the same normal strain of ε_z when it is subjected to the same normal stress σ_z, Thus,

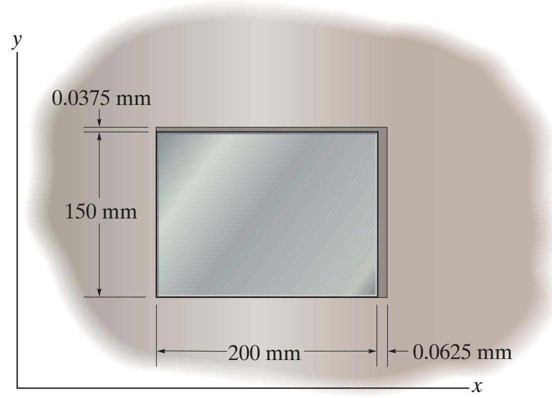
$$\sigma_z = E_{\text{eff}}\,\varepsilon_z$$

$$E_{\text{eff}} = \frac{\sigma_z}{\varepsilon_z} = \frac{\sigma_z}{\frac{\sigma_z}{E}\left(1 - \nu^2\right)} = \frac{E}{1 - \nu^2} \qquad \textbf{Ans.}$$

Ans.

$$E_{\text{eff}} = \frac{E}{1 - \nu^2}$$

10–49. Initially, gaps between the A-36 steel plate and the rigid constraint are as shown. Determine the normal stresses σ_x and σ_y developed in the plate if the temperature is increased by $\Delta T = 55°\text{C}$. To solve, add the thermal strain $\alpha \Delta T$ to the equations for Hooke's Law.



SOLUTION

Generalized Hooke's Law: Since there are gaps between the sides of the plate and the rigid constraint, the plate is allowed to expand before it comes in contact with the constraint. Thus, $\varepsilon_x = \dfrac{\delta_x}{L_x} = \dfrac{0.0625}{200} = 0.3125\left(10^{-3}\right)$ and $\varepsilon_y = \dfrac{\delta_y}{L_y} = \dfrac{0.0375}{150} = 0.25\left(10^{-3}\right)$. However, the plate is allowed to have free expansion along the z direction. Thus, $\sigma_z = 0$. With the additional thermal strain term, we have

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha \Delta T$$

$$0.3125(10^{-3}) = \frac{1}{200(10^9)}[\sigma_x - 0.32(\sigma_y + 0)] + [12(10^{-6})](55)$$

$$\sigma_x - 0.32\sigma_y = -69.5(10^6) \qquad (1)$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha \Delta T$$

$$0.25(10^{-3}) = \frac{1}{200(10^9)}[\sigma_y - 0.32(\sigma_x + 0)] + [12(10^{-6})](55)$$

$$\sigma_y - 0.32\sigma_x = -82(10^6) \qquad (2)$$

Solving Eqs. (1) and (2),

$$\sigma_x = -106.66(10^6)\ \text{N/m}^2 = 107\ \text{MPa (C)} \qquad \textbf{Ans.}$$

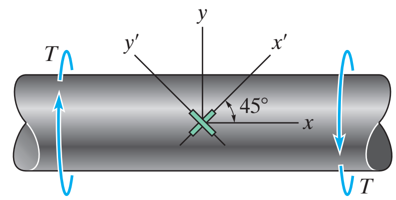
$$\sigma_y = -116.13(10^6)\ \text{N/m}^2 = 116\ \text{MPa (C)} \qquad \textbf{Ans.}$$

Since $\sigma_x < \sigma_Y$ and $\sigma_y < \sigma_Y$, the above results are valid.

Ans.
$\sigma_x = 107$ MPa (C), $\sigma_y = 116.1$ MPa (C)

10–50.

The steel shaft has a radius of 15 mm. Determine the torque T in the shaft if the two strain gages, attached to the surface of the shaft, report strains of $\epsilon_{x'} = -80(10^{-6})$ and $\epsilon_{y'} = 80(10^{-6})$. Also, determine the strains acting in the x and y directions. $E_{st} = 200$ GPa, $\nu_{st} = 0.3$.



SOLUTION

$\epsilon_{x'} = -80(10^{-6}) \qquad \epsilon_{y'} = 80(10^{-6})$

Pure shear $\epsilon_x = \epsilon_y = 0$ **Ans.**

$\epsilon_{x'} = \epsilon_x \cos^2\theta + \epsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$

$\theta = 45°$

$-80(10^{-6}) = 0 + 0 + \gamma_{xy} \sin 45° \cos 45°$

$\gamma_{xy} = -160(10^{-6})$ **Ans.**

Also, $\theta = 135°$

$80(10^{-6}) = 0 + 0 + \gamma \sin 135° \cos 135°$

$\gamma_{xy} = -160(10^{-6})$

$$G = \frac{E}{2(1+\nu)} = \frac{200(10^9)}{2(1+0.3)} = 76.923(10^9)$$

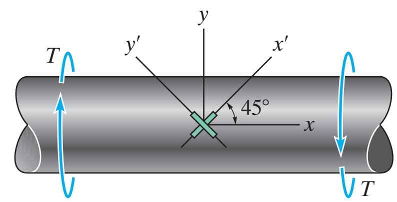
$$\tau = G\gamma = 76.923(10^9)(160)(10^{-6}) = 12.308(10^6)\text{ Pa}$$

$$T = \frac{\tau J}{c} = \frac{12.308(10^6)\left(\frac{\pi}{2}\right)(0.015)^4}{0.015} = 65.2\text{ N}\cdot\text{m}$$ **Ans.**

Ans:
$\epsilon_x = \epsilon_y = 0$, $\gamma_{xy} = -160(10^{-6})$, $T = 65.2\text{ N}\cdot\text{m}$

10–51.

The shaft has a radius of 15 mm and is made of L2 tool steel. Determine the strains in the x' and y' direction if a torque $T = 2\ \text{kN}\cdot\text{m}$ is applied to the shaft.



SOLUTION

$$\tau = \frac{Tc}{J} = \frac{2(10^3)(0.015)}{\frac{\pi}{2}(0.015^4)} = 377.26\ \text{MPa}$$

Stress-Strain Relationship:

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = -\frac{377.26(10^6)}{75.0(10^9)} = -5.030(10^{-3})\ \text{rad}$$

This is a pure shear case, therefore,

$$\epsilon_x = \epsilon_y = 0$$

Applying Eq. 10–15,

$$\epsilon_{x'} = \epsilon_x \cos^2\theta_a + \epsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a\cos\theta_a$$

Here $\theta_a = 45°$

$$\epsilon_{x'} = 0 + 0 - 5.030(10^{-3})\ \sin 45° \cos 45° = -2.52(10^{-3})$$

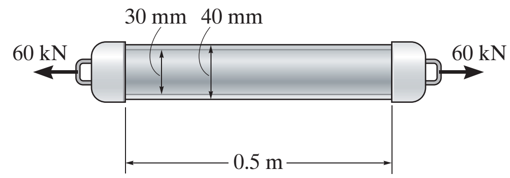
$\epsilon_{x'} = -2.52(10^{-3})$ **Ans.**

$\epsilon_{y'} = 2.52(10^{-3})$ **Ans.**

Ans:
$\varepsilon_{x'} = -2.52(10^{-3}),\ \varepsilon_{y'} = 2.52(10^{-3})$

***10–52.**

The A-36 steel pipe is subjected to the axial loading of 60 kN. Determine the change in volume of the material after the load is applied.



SOLUTION

Normal Stress: The pipe is subjected to uniaxial load. Therefore, $\sigma_v = \sigma_z = 0$ and $\sigma_x = \dfrac{N}{A}$.

Dilatation: Applying Eq. 10–23.

$$\frac{\delta V}{V} = \frac{1-2v}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$\frac{\delta V}{V} = \frac{1-2v}{E}\left(\frac{N}{A}\right)$$

$$\delta V = \frac{1-2v}{E}\left(\frac{N}{A}\right)V \qquad \text{However, } V = AL$$

$$\delta V = \left(\frac{1-2v}{E}\right)NL$$

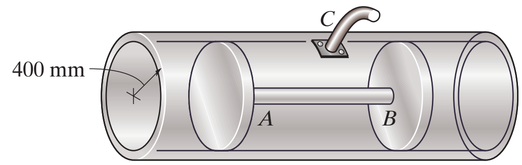
$$= \left[\frac{1-2(0.32)}{200\left(10^9\right)}\right](60)\left(10^3\right)(0.5)$$

$$= 54.0\left(10^{-9}\right)\text{ m}^3 = 54.0\text{ mm}^3 \qquad \textbf{Ans.}$$

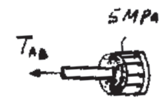
Ans:
$\delta V = 54.0\text{ mm}^3$

10–53.

Air is pumped into the steel thin-walled pressure vessel at C. If the ends of the vessel are closed using two pistons connected by a rod AB, determine the increase in the diameter of the pressure vessel when the internal gage pressure is 5 MPa. Also, what is the tensile stress in rod AB if it has a diameter of 100 mm? The inner radius of the vessel is 400 mm, and its thickness is 10 mm. $E_{st} = 200$ GPa and $\nu_{st} = 0.3$.



SOLUTION



Circumferential Stress:

$$\sigma = \frac{p\,r}{t} = \frac{5(400)}{10} = 200 \text{ MPa}$$

Note: longitudinal and radial stresses are zero.

Circumferential Strain:

$$\epsilon = \frac{\sigma}{E} = \frac{200(10^6)}{200(10^9)} = 1.0(10^{-3})$$

$$\Delta d = \epsilon\, d = 1.0(10^{-3})(800) = 0.800 \text{ mm} \qquad \textbf{Ans.}$$

For rod AB:

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad T_{AB} - 5(10^6)\left(\frac{\pi}{4}\right)(0.8^2 - 0.1^2) = 0$$

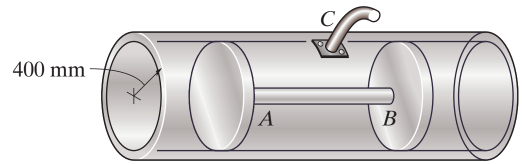
$$T_{AB} = 2474 \text{ kN}$$

$$\sigma_{AB} = \frac{T_{AB}}{A_{AB}} = \frac{2474(10^3)}{\frac{\pi}{4}(0.1^2)} = 315 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\Delta d = 0.800$ mm, $\sigma_{AB} = 315$ MPa

10–54.

Determine the increase in the diameter of the pressure vessel in Prob. 10–53 if the pistons are replaced by walls connected to the ends of the vessel.



SOLUTION

Principal Stress:

$$\sigma_1 = \frac{p\,r}{t} = \frac{5(400)}{10} = 200\text{ MPa}; \qquad \sigma_3 = 0$$

$$\sigma_2 = \frac{1}{2}\,\sigma_1 = 100\text{ MPa}$$

Circumferential Strain:

$$\epsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu(\sigma_2 + \sigma_3)\right] = \frac{1}{200(10^9)}\left[200(10^6) - 0.3\{100(10^6) + 0\}\right]$$

$$= 0.85(10^{-3})$$

$$\Delta d = \epsilon_1\, d = 0.85(10^{-3})(800) = 0.680\text{ mm}$$ **Ans.**

Ans:
$\Delta d = 0.680$ mm

10–55.

A thin-walled spherical pressure vessel having an inner radius r and thickness t is subjected to an internal pressure p. Show that the increase in the volume within the vessel is $\Delta V = (2p\pi r^4/Et)(1 - \nu)$. Use a small-strain analysis.

SOLUTION

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

$$\sigma_3 = 0$$

$$\epsilon_1 = \epsilon_2 = \frac{1}{E}(\sigma_1 - \nu\sigma_2)$$

$$\epsilon_1 = \epsilon_2 = \frac{pr}{2t\,E}(1 - \nu)$$

$$\epsilon_3 = \frac{1}{E}(-\nu(\sigma_1 + \sigma_2))$$

$$\epsilon_3 = -\frac{\nu\,pr}{t\,E}$$

$$V = \frac{4\pi r^3}{3}$$

$$V + \Delta V = \frac{4\pi}{3}(r + \Delta r)^3 = \frac{4\pi r^3}{3}\left(1 + \frac{\Delta r}{r}\right)^3$$

where $\Delta V \ll V, \Delta r \ll r$

$$V + \Delta V = \frac{4\pi\,r^3}{3}\left(1 + 3\frac{\Delta r}{r}\right)$$

$$\epsilon_{\text{Vol}} = \frac{\Delta V}{V} = 3\left(\frac{\Delta r}{r}\right)$$

Since $\epsilon_1 = \epsilon_2 = \dfrac{2\pi(r + \Delta r) - 2\pi\,r}{2\pi\,r} = \dfrac{\Delta r}{r}$

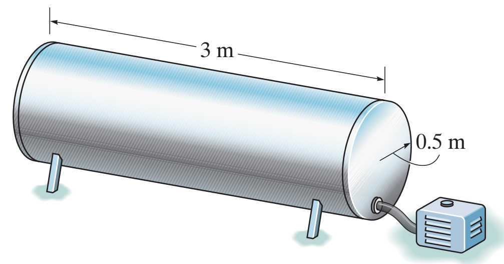
$$\epsilon_{\text{Vol}} = 3\epsilon_1 = \frac{3pr}{2t\,E}(1 - \nu)$$

$$\Delta V = V\epsilon_{\text{Vol}} = \frac{2p\pi\,r^4}{E\,t}(1 - \nu)$$ **(Q.E.D.)**

Ans:
N/A

***10–56.**

The thin-walled cylindrical pressure vessel of inner radius r and thickness t is subjected to an internal pressure p. If the material constants are E and ν, determine the strains in the circumferential and longitudinal directions. Using these results, calculate the increase in both the diameter and the length of a steel pressure vessel filled with air and having an internal gage pressure of 15 MPa. The vessel is 3 m long, and has an inner radius of 0.5 m and a thickness of 10 mm. $E_{st} = 200$ GPa, $\nu_{st} = 0.3$.



SOLUTION

Normal Stress:

$$\sigma_1 = \frac{p\,r}{t} \qquad \sigma_2 = \frac{p\,r}{2\,t} \qquad \sigma_3 = 0$$

Normal Strain:

$$\epsilon_{cir} = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)]$$

$$= \frac{1}{E}\left(\frac{p\,r}{t} - \frac{\nu\,p\,r}{2\,t}\right) = \frac{p\,r}{2\,E\,t}(2 - \nu) \qquad \textbf{Ans.}$$

$$\epsilon_{long} = \frac{1}{E}[\sigma_2 - \nu(\sigma_1 + \sigma_3)]$$

$$= \frac{1}{E}\left(\frac{p\,r}{2\,t} - \frac{\nu\,p\,r}{t}\right) = \frac{p\,r}{2\,E\,t}(1 - 2\nu) \qquad \textbf{Ans.}$$

Numerical Substitution:

$$\epsilon_{cir} = \frac{15(10^6)(0.5)}{2(200)(10^9)(0.01)}(2 - 0.3) = 3.1875\,(10^{-3})$$

$$\Delta d = \epsilon_{cir}\,d = 3.1875\,(10^{-3})(1000) = 3.19 \text{ mm} \qquad \textbf{Ans.}$$

$$\epsilon_{long} = \frac{15(10^6)(0.5)}{2(200)(10^9)(0.01)}(1 - 2(0.3)) = 0.75(10^{-3})$$

$$\Delta L = \epsilon_{long}\,L = 0.75\,(10^{-3})(3000) = 2.25 \text{ mm} \qquad \textbf{Ans.}$$

Ans:

$\epsilon_{cir} = \dfrac{p\,r}{2E\,t}(2 - \nu)$,

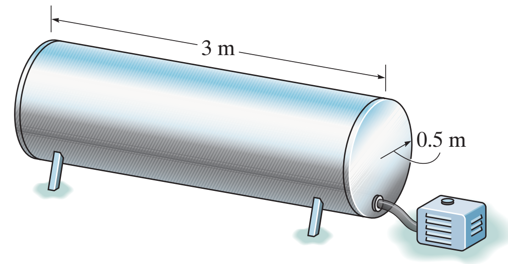
$\epsilon_{long} = \dfrac{p\,r}{2E\,t}(1 - 2\nu)$,

$\Delta d = 3.19$ mm,

$\Delta L = 2.25$ mm

10–57.

Estimate the increase in volume of the pressure vessel in Prob. 10–56.



SOLUTION

By basic principles,

$$\Delta V = \pi(r + \Delta r)^2(L + \Delta L) - \pi r^2 L = \pi(r^2 + \Delta r^2 + 2\, r\, \Delta r)(L + \Delta L) - \pi r^2 L$$
$$= \pi(r^2 L + r^2 \Delta L + \Delta r^2 L + \Delta r^2 \Delta L + 2\, r\, \Delta r\, L + 2\, r\, \Delta r\, \Delta L - r^2 L)$$
$$= \pi(r^2 \Delta L + \Delta r^2 L + \Delta r^2 \Delta L + 2\, r\, \Delta r\, L + 2\, r\, \Delta r\, \Delta L)$$

Neglecting the second order terms,

$$\Delta V = \pi(r^2 \Delta L + 2\, r\, \Delta r\, L)$$

From Prob. 10–56,

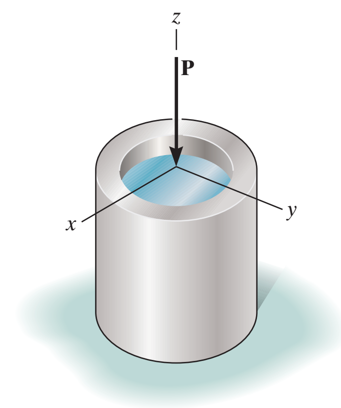
$$\Delta L = 0.00225 \text{ m} \qquad \Delta r = \frac{\Delta d}{2} = 0.00159375 \text{ m}$$

$$\Delta V = \pi[(0.5^2)(0.00225) + 2(0.5)(0.00159375)(3)] = 0.0168 \text{ m}^3$$ **Ans.**

Ans:
$\Delta V = 0.0168 \text{ m}^3$

10–58.

A soft material is placed within the confines of a rigid cylinder which rests on a rigid support. Assuming that $\epsilon_x = 0$ and $\epsilon_y = 0$, determine the factor by which the stiffness of the material, or the apparent modulus of elasticity, will be increased when a load is applied, if $\nu = 0.3$ for the material.



SOLUTION

Normal Strain: Since the material is confined in a rigid cylinder. $\epsilon_x = \epsilon_y = 0$. Applying the generalized Hooke's Law,

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$0 = \sigma_x - \nu(\sigma_y + \sigma_z) \qquad \textbf{(1)}$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$0 = \sigma_y - \nu(\sigma_x + \sigma_z) \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$\sigma_x = \sigma_y = \frac{\nu}{1-\nu}\sigma_z$$

Thus,

$$\begin{aligned}
\epsilon_z &= \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] \\
&= \frac{1}{E}\left[\sigma_z - \nu\left(\frac{\nu}{1-\nu}\sigma_z + \frac{\nu}{1-\nu}\sigma_z\right)\right] \\
&= \frac{\sigma_z}{E}\left[1 - \frac{2\nu^2}{1-\nu}\right] \\
&= \frac{\sigma_z}{E}\left[\frac{1-\nu-2\nu^2}{1-\nu}\right] \\
&= \frac{\sigma_z}{E}\left[\frac{(1+\nu)(1-2\nu)}{1-\nu}\right]
\end{aligned}$$

Thus, when the material is not being confined and undergoes the same normal strain of ϵ_z, then the required modulus of elasticity is

$$E' = \frac{\sigma_z}{\epsilon_z} = \frac{1-\nu}{(1-2\nu)(1+\nu)}E$$

The increase factor is $k = \dfrac{E'}{E} = \dfrac{1-\nu}{(1-2\nu)(1+\nu)}$

$$= \frac{1-0.3}{[1-2(0.3)](1+0.3)}$$

$$= 1.35 \qquad \textbf{Ans.}$$

Ans:
$k = 1.35$

10–59.

A material is subjected to plane stress. Express the distortion energy theory of failure in terms of σ_x, σ_y, and τ_{xy}.

SOLUTION

Maximum distortion energy theory:

$$(\sigma_1^2 - \sigma_1\,\sigma_2 + \sigma_2^2) = \sigma_Y^2 \qquad \textbf{(1)}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Let $a = \dfrac{\sigma_x + \sigma_y}{2}$ and $b = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$$\sigma_1 = a + b; \qquad \sigma_2 = a - b$$

$$\sigma_1^2 = a^2 + b^2 + 2\,a\,b; \qquad \sigma_2^2 = a^2 + b^2 - 2\,a\,b$$

$$\sigma_1\sigma_2 = a^2 - b^2$$

From Eq. (1)

$$(a^2 + b^2 + 2\,a\,b - a^2 + b^2 + a^2 + b^2 - 2\,a\,b) = \sigma_Y^2$$

$$(a^2 + 3\,b^2) = \sigma_Y^2$$

$$\frac{(\sigma_x + \sigma_y)^2}{4} + 3\,\frac{(\sigma_x - \sigma_y)^2}{4} + 3\,\tau_{xy}^2 = \sigma_Y^2$$

$$\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\,\tau_{xy}^2 = \sigma_Y^2 \qquad \textbf{Ans.}$$

Ans:
$\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2 = \sigma_y^2$

***10–60.**

A material is subjected to plane stress. Express the maximum shear stress theory of failure in terms of σ_x, σ_y, and τ_{xy}. Assume that the principal stresses are of different algebraic signs.

SOLUTION

Maximum shear stress theory:

$$|\sigma_1 - \sigma_2| = \sigma_Y \qquad \textbf{(1)}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\left|\sigma_1 - \sigma_2\right| = 2\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

From Eq. (1)

$$4\left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right] = \sigma_Y^2$$

$$(\sigma_x - \sigma_y)^2 + 4\,\tau_{xy}^2 = \sigma_Y^2 \qquad \textbf{Ans.}$$

Ans:
$(\sigma_x - \sigma_y)^2 + 4\,\tau_{xy}^2 = \sigma_Y^2$

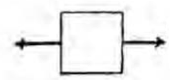
10–61. A bar with a square cross-sectional area is made of a material having a yield stress of $\sigma_Y = 840$ MPa. If the bar is subjected to a bending moment of 10 kN · m, determine the required size of the bar according to the maximum-distortion-energy theory. Use a factor of safety of 1.5 with respect to yielding.

SOLUTION

Normal and Shear Stress: Applying the flexure formula,

$$\sigma = \frac{Mc}{I} = \frac{[10(10^3)](a/2)}{\frac{1}{12}a^4} = \frac{60(10^3)}{a^3}$$

In-Plane Principal Stress: Since no shear stress acts on the element



$$\sigma_1 = \sigma_x = \frac{60(10^3)}{a^3} \qquad \sigma_2 = \sigma_y = 0$$

Maximum Distortion Energy Theory:

$$\sigma_1^2 - \sigma_1\,\sigma_2 + \sigma_2^2 = \sigma_{\text{allow}}^2$$

$$\left[\frac{60(10^3)}{a^3}\right]^2 - 0 + 0 = \left[\frac{840(10^6)}{1.5}\right]^2$$

$$a = 0.047495 \text{ m} = 47.5 \text{ mm}$$ **Ans.**

Ans.

$a = 47.5$ m

10–62. Solve Prob. 10–64 using the maximum-shear-stress theory.

SOLUTION

Normal and Shear Stress: Applying the flexure formula,

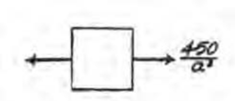
$$\sigma = \frac{Mc}{I} = \frac{[10(10^3)](a/2)}{\frac{1}{12}a^4} = \frac{60(10^3)}{a^3}$$

In-Plane Principal Stress: Since no shear stress acts on the element.

$$\sigma_1 = \sigma_x = \frac{60(10^3)}{a^3} \qquad \sigma_2 = \sigma_x = 0$$

Maximum Shear Stress Theory:

$$|\sigma_2| = 0 < \sigma_{\text{allow}} = \frac{840}{1.5} = 560 \text{ MPa} \qquad \textit{(O.K!)}$$



$$|\sigma_1| = \sigma_{\text{allow}}$$

$$\frac{60(10^3)}{a^3} = \frac{840(10^6)}{1.5}$$

$$a = 0.047495 \text{ m} = 47.5 \text{ mm}$$ **Ans.**

Ans.

$\sigma = \dfrac{60(10^6)}{a^3}, a = 47.50$ mm

10–63. Derive an expression for an equivalent bending moment M_e that, if applied alone to a solid bar with a circular cross section, would cause the same energy of distortion as the combination of an applied bending moment M and torque T.

SOLUTION

Principal stresses:

$$\sigma_1 = \frac{M_e\, c}{I}; \qquad \sigma_2 = 0$$

$$u_d = \frac{1 + v}{3\,E}\,(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)$$

$$(u_d)_1 = \frac{1 + v}{3\,E}\left(\frac{M_e^2 c^2}{I^2}\right) \tag{1}$$

Principal stress:

$$\sigma_{1,2} = \frac{\sigma + 0}{2} \pm \sqrt{\left(\frac{\sigma - 0}{2}\right)^2 + \tau^3}$$

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{4} + \tau^2}; \qquad \sigma_2 = \frac{\sigma}{2} - \sqrt{\frac{\sigma^2}{4} + \tau^2}$$

Distortion Energy:

Let $a = \frac{\sigma}{2}$ $b = \sqrt{\frac{\sigma^2}{4} + \tau^2}$

$$\sigma_1^2 = a^2 + b^2 + 2\,a\,b$$

$$\sigma_1\sigma_2 = a^2 - b^2$$

$$\sigma_2^2 = a^2 + b^2 - 2\,a\,b$$

$$\sigma_2^2 - \sigma_1\sigma_2 + \sigma_2^2 = 3\,b^2 + a^2$$

Apply $\sigma = \frac{M\,c}{I}; \qquad \tau = \frac{T\,c}{J}$

$$(u_d)_2 = \frac{1 + v}{3\,E}\,(3\,b^2 + a^2) = \frac{1 + v}{3\,E}\left(\frac{\sigma^2}{4} + \frac{3\sigma^2}{4} + 3\,\tau^2\right)$$

$$= \frac{1 + v}{3\,E}\,(\sigma^2 + 3\,\tau^2) = \frac{1 + v}{3\,E}\left(\frac{M^2c^2}{I^2} + \frac{3\,T^2c^2}{J^2}\right) \tag{2}$$

10–63. Continued

Equating Eq. (1) and (2) yields:

$$\frac{(1+v)}{3E}\left(\frac{M_e c^2}{I^2}\right) = \frac{1+v}{3E}\left(\frac{M^2 c^2}{I^2} + \frac{3T^2 c^2}{J^2}\right)$$

$$\frac{M_e^2}{I^2} = \frac{M^1}{I^2} + \frac{3T^2}{J^2}$$

$$M_e^2 = M^1 + 3T^2\left(\frac{I}{J}\right)^2$$

For circular shaft

$$\frac{I}{J} = \frac{\frac{\pi}{4}c^4}{\frac{\pi}{2}c^4} = \frac{1}{2}$$

Hence, $M_e^2 = M^2 + 3T^2\left(\frac{1}{2}\right)^2$

$$M_e = \sqrt{M^2 + \frac{3}{4}T^2}$$ **Ans.**

Ans.

$$M_e = \sqrt{M^2 + \frac{3}{4}T^2}$$

***10–64.** Derive an expression for an equivalent bending moment M_e that, if applied alone to a solid bar with a circular cross section, would cause the same maximum shear stress as the combination of an applied moment M and torque T. Assume that the principal stresses are of opposite algebraic signs.

SOLUTION

Bending and Torsion:

$$\sigma = \frac{M\,c}{I} = \frac{M\,c}{\frac{\pi}{4}\,c^4} = \frac{4\,M}{\pi\,c^3}; \qquad \tau = \frac{T\,c}{J} = \frac{T\,c}{\frac{\pi}{2}c^4} = \frac{2\,T}{\pi\,c^3}$$

The principal stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\frac{4\,M}{\pi c^3} + 0}{2} \pm \sqrt{\left(\frac{\frac{4\,M}{\pi\,c^3} - 0}{2}\right)^2 + \left(\frac{2T}{\pi\,c^3}\right)^2}$$

$$= \frac{2\,M}{\pi\,c^3} \pm \frac{2}{\pi\,c^3}\sqrt{M^2 + T^2}$$

$$\tau_{\substack{\text{abs}\\ \text{max}}} = \sigma_1 - \sigma_2 = 2\left[\frac{2}{\pi\,c^3}\sqrt{M^2 + T^2}\right] \qquad (1)$$

Pure bending:

$$\sigma_1 = \frac{M\,c}{I} = \frac{M_e\,c}{\frac{\pi}{4}\,c^4} = \frac{4\,M_e}{\pi\,c^3}; \qquad \sigma_2 = 0$$

$$\tau_{\substack{\text{abs}\\ \text{max}}} = \sigma_1 - \sigma_2 = \frac{4\,M_e}{\pi\,c^3} \qquad (2)$$

Equating Eq. (1) and (2) yields:

$$\frac{4}{\pi\,c^3}\sqrt{M^2 + T^2} = \frac{4\,M_e}{\pi\,c^3}$$

$M_e = \sqrt{M^2 + T^2}$ **Ans.**

Ans.
$M_e = \sqrt{M^2 + T^2}$

10–65.

Derive an expression for an equivalent torque T_e that, if applied alone to a solid bar with a circular cross section, would cause the same energy of distortion as the combination of an applied bending moment M and torque T.

SOLUTION

$$\tau = \frac{T_e c}{J}$$

Principal Stress:

$$\sigma_1 = \tau \quad \sigma_2 = -\tau$$

$$u_d = \frac{1+\nu}{3E}(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)$$

$$(u_d)_1 = \frac{1+\nu}{3E}(3\tau^2) = \frac{1+\nu}{3E}\left(\frac{3T_e^2c^2}{J^2}\right)$$

Bending Moment and Torsion:

$$\sigma = \frac{Mc}{I}; \qquad \tau = \frac{Tc}{J}$$

Principal Stress:

$$\sigma_{1,2} = \frac{\sigma + 0}{2} \pm \sqrt{\left(\frac{\sigma - 0}{2}\right)^2 + \tau^2}$$

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{4} + \tau^2}; \qquad \sigma_2 = \frac{\sigma}{2} - \sqrt{\frac{\sigma^2}{4} + \tau^2}$$

Let $a = \frac{\sigma}{2}$ $\qquad b = \sqrt{\frac{\sigma^2}{4} + \tau^2}$

$$\sigma_1^2 = a^2 + b^2 + 2ab$$

$$\sigma_1\sigma_2 = a^2 - b^2$$

$$\sigma_2^2 = a^2 + b^2 - 2ab$$

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = 3b^2 + a^2$$

$$u_d = \frac{1+\nu}{3E}(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)$$

$$(u_d)_2 = \frac{1+\nu}{3E}(3b^2 + a^2) = \frac{1+\nu}{3E}\left(\frac{3\sigma^2}{4} + 3\tau^2 + \frac{\sigma^2}{4}\right)$$

$$= \frac{1+\nu}{3E}(\sigma^2 + 3\tau^2) = \frac{c^2(1+\nu)}{3E}\left(\frac{M^2}{I^2} + \frac{3T^2}{J^2}\right)$$

$$(u_d)_1 = (u_d)_2$$

$$\frac{c^2(1+\nu)}{3E}\frac{3T_e^2}{J^2} = \frac{c^2(1+\nu)}{3E}\left(\frac{M^2}{I^2} + \frac{3T^2}{J^2}\right)$$

10–65. Continued

For circular shaft

$$\frac{J}{I} = \frac{\frac{\pi}{2}c^4}{\frac{\pi}{4}c^4} = 2$$

$$T_e = \sqrt{\frac{J^2}{I^2}\frac{M^2}{3} + T^2}$$

$$T_e = \sqrt{\frac{4}{3}M^2 + T^2}$$ **Ans.**

Ans:

$$T_e = \sqrt{\frac{4}{3}M^2 + T^2}$$

10–66. An aluminum alloy 6061-T6 is to be used for a solid drive shaft such that it transmits 33 kW at 2400 rev/min. Using a factor of safety of 2 with respect to yielding, determine the smallest-diameter shaft that can be selected based on the maximum-shear-stress theory.

SOLUTION

$$\omega = \left(2400\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{s}}\right) = 80\,\pi\text{ rad/s}$$

$$T = \frac{P}{\omega} = \frac{33(10^3)}{80\pi} = \frac{3300}{(8\pi)}\,\text{N}\cdot\text{m}$$

Applying $\tau = \dfrac{T\,c}{J}$

$$\tau = \frac{\left(\frac{3300}{8\pi}\right)c}{\frac{\pi}{2}\,c^4} = \frac{6600}{8\pi^3 c^3}$$

The principal stresses:

$$\sigma_1 = \tau = \frac{6600}{8\pi^2 c^3}; \qquad \sigma_2 = -\tau = \frac{6600}{8\pi^2 c^3}$$

Maximum shear stress theory: Both principal stresses have opposite sign, hence,

$$\left|\sigma_1 - \sigma_2\right| = \frac{\sigma_Y}{\text{F.S.}}; \qquad 2\left(\frac{6600}{8\pi^2 c^3}\right) = \left|\frac{255(10^6)}{2}\right|$$

$c = 0.010945$ m = 10.945 mm

$d = 21.89$ mm **Ans.**

Ans.

$\omega = 80\,\pi$ rad/s, $T = \dfrac{3300}{8\pi}\,\text{N}\cdot\text{m}$,

$d = 21.89$ mm

10–67. Solve Prob. 10–61 using the maximum-distortion-energy theory.

SOLUTION

$$\omega = \left(2400\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\text{p rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{s}}\right) = 80\,\pi\text{ rad/s}$$

$$T = \frac{P}{\omega} = \frac{33(10^3)}{80\pi} = \frac{3300}{(8\pi)}\text{ N}\cdot\text{m}$$

Applying $\tau = \dfrac{T\,c}{J}$

$$\tau = \frac{\left(\frac{3300}{8\pi}\right)c}{\frac{\pi}{2}c^4} = \frac{6600}{8\pi^3c^3}$$

The principal stresses:

$$\sigma_1 = \tau = \frac{6600}{8\pi^2c^3}; \qquad \sigma_2 = -\tau = -\frac{6600}{8\pi^2c^3}$$

The maximum distortion-energy theory:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \left(\frac{\sigma_Y}{\text{F.S.}}\right)^2$$

$$3\left[\frac{6600}{8\pi^2c^3}\right]^2 = \left(\frac{255(10^6)}{2}\right)^2$$

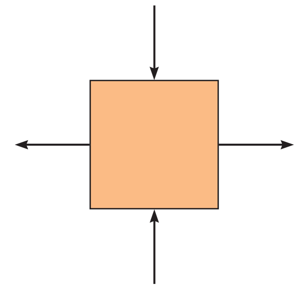
$c = 0.0104328\text{ m} = 10.4328\text{ mm}$

$d = 20.87\text{ mm}$ **Ans.**

Ans.
$d = 20.87\text{ mm}$

***10–68.**

If the material is machine steel having a yield stress of $\sigma_Y = 700$ MPa, determine the factor of safety with respect to yielding if the maximum shear stress theory is considered.



SOLUTION

$\sigma_{max} = 80 \text{ MPa} \qquad \sigma_{min} = -50 \text{ MPa}$

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{80 - (-50)}{2} = 65 \text{ MPa}$$

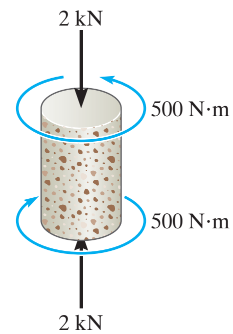
$$\tau_{max} = \frac{\sigma_Y}{2} = \frac{700}{2} = 350 \text{ MPa}$$

$$\text{F.S.} = \frac{\tau_{max}}{\tau_{\substack{abs\\max}}} = \frac{350}{65} = 5.38$$ **Ans.**

Ans:
F.S. = 5.38

10–69.

The short concrete cylinder having a diameter of 50 mm is subjected to a torque of 500 N·m and an axial compressive force of 2 kN. Determine if it fails according to the maximum normal stress theory. The ultimate stress of the concrete is $\sigma_{ult} = 28$ MPa.

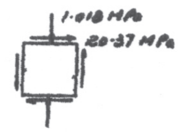


SOLUTION

$$A = \frac{\pi}{4}(0.05)^2 = 1.9635(10^{-3})\ \text{m}^2$$

$$J = \frac{\pi}{2}(0.025)^4 = 0.61359(10^{-4})\ \text{m}^4$$

$$\sigma = \frac{P}{A} = \frac{2(10^3)}{1.9635(10^{-3})} = 1.019\ \text{MPa}$$



$$\tau = \frac{Tc}{J} = \frac{500(0.025)}{0.61359(10^{-6})} = 20.372\ \text{MPa}$$

$$\sigma_x = 0 \qquad \sigma_y = -1.019\ \text{MPa} \qquad \tau_{xy} = 20.372\ \text{MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$\sigma_{1,2} = \frac{0 - 1.018}{2} \pm \sqrt{\left(\frac{0-(-1.019)}{2}\right)^2 + 20.372^2}$$

$$\sigma_1 = 19.87\ \text{MPa} \qquad \sigma_2 = -20.89\ \text{MPa}$$

Failure criteria:

$|\sigma_1| < \sigma_{alt} = 28$ MPa **OK**

$|\sigma_2| < \sigma_{alt} = 28$ MPa **OK**

No. **Ans.**

Ans:
No

10–70. A bar with a circular cross-sectional area is made of SAE 1045 carbon steel having a yield stress of $\sigma_Y = 1000$ MPa. If the bar is subjected to a torque of 3.75 kN · m and a bending moment of 7 kN · m, determine the required diameter of the bar according to the maximum-distortion-energy theory. Use a factor of safety of 2 with respect to yielding.

SOLUTION

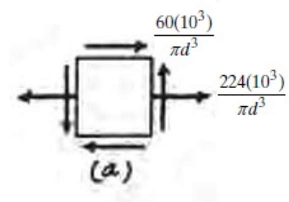
Normal and Shear Stresses: Applying the flexure and torsion formulas.

$$\sigma = \frac{Mc}{I} = \frac{[7(10^3)](d/2)}{\frac{\pi}{4}(d/2)^4} = \frac{224(10^3)}{\pi d^3}$$

$$\tau = \frac{Tc}{J} = \frac{[3.75(10^3)](d/2)}{\frac{\pi}{2}(-d/2)^4} = \frac{60(10^3)}{\pi d^3}$$

The critical state of stress is shown in Fig. (a) or (b), where

$$\sigma_x = \frac{224(10^3)}{\pi d^3} \qquad \sigma_y = 0 \qquad \tau_{xy} = \frac{60(10^3)}{\pi d^3}$$



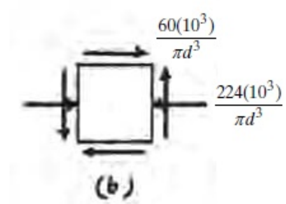
In - Plane Principal Stresses : Applying Eq. 9-5,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \left[\frac{\frac{224}{\pi d^3} + 0}{2} \pm \sqrt{\left(\frac{\frac{224}{\pi d^3} - 0}{2}\right)^2 + \left(\frac{60}{\pi d^3}\right)^2}\right](10^3)$$

$$= \left(\frac{112}{\pi d^3} \pm \frac{127.06}{\pi d^3}\right)(10^3)$$

$$\sigma_1 = \left(\frac{239.06}{\pi d^3}\right)(10^3) \qquad \sigma_2 = \left(-\frac{15.06}{\pi d^3}\right)(10^3)$$



Maximum Distortion Energy Theory :

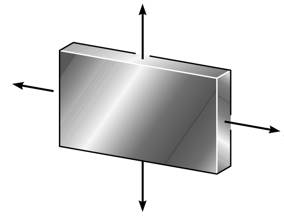
$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{\text{allow}}^2$$

$$\left(\frac{239.06}{\pi d^3}\right)^2 - \left(\frac{239.06}{\pi d^3}\right)\left(-\frac{15.06}{\pi d^3}\right) + \left(-\frac{15.06}{\pi d^3}\right)^2 (10^3)^2 = \left[\frac{1000(10^6)}{2}\right]^2$$

$$d = 0.05397 \text{ m} = 54.0 \text{ mm}$$ **Ans.**

Ans.
$d = 54.0$ m

10–71. The plate is made of hard copper, which yields at $\sigma_Y = 735$ MPa. Using the maximum-shear-stress theory, determine the tensile stress σ_x that can be applied to the plate if a tensile stress $\sigma_y = 0.5\sigma_x$ is also applied.



SOLUTION

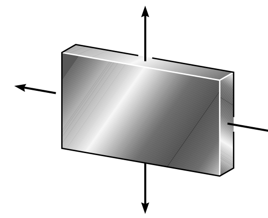
$$\sigma_1 = \sigma_x \qquad \sigma_2 = \frac{1}{2}\sigma_x$$

$$|\sigma_1| = \sigma_Y$$

$$\sigma_x = 735\text{ MPa}$$ **Ans.**

Ans.
$\sigma_x = 735$ MPa

***10–72.** Solve Prob. 10–80 using the maximum-distortion-energy theory.



SOLUTION

$$\sigma_1 = \sigma_x$$

$$\sigma_2 = \frac{\sigma_x}{2}$$

$$\sigma_1^2 - \sigma_1\,\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

$$\sigma_x^2 - \frac{\sigma_x^2}{2} + \frac{\sigma_x^2}{4} = 735^2$$

$\sigma_x = 848.7 \text{ MPa} = 849 \text{ MPa}$ **Ans.**

Ans.
$\sigma_x = 849 \text{ MPa}$

10–73. The state of stress acting at a critical point on the seat frame of an automobile during a crash is shown in the figure. Determine the smallest yield stress for a steel that can be selected for the member, based on the maximum-shear-stress theory.

SOLUTION

Normal and Shear Stress: In accordance with the sign convention.

$$\sigma_x = 560 \text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = 175 \text{ MPa}$$

In - Plane Principal Stress: Applying Eq. 9-5.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{560+0}{2} \pm \sqrt{\left(\frac{560-0}{2}\right)^2 + 175^2}$$

$$= 280 \pm 330.19$$

$$\sigma_1 = 610.19 \text{ MPa} \qquad \sigma_2 = -50.19 \text{ MPa}$$

Maximum Shear Stress Theory: σ_1 and σ_2 have opposite signs so

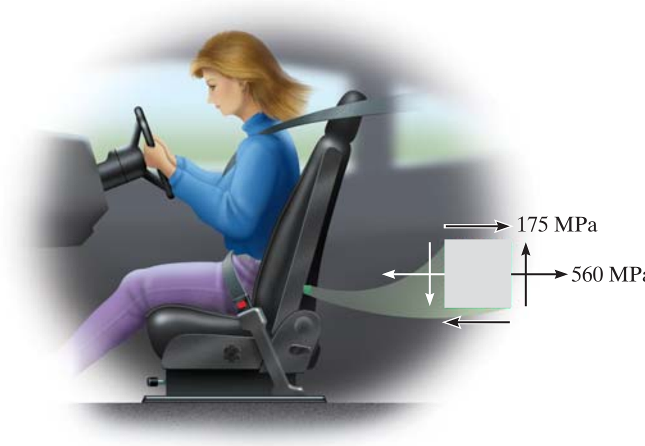
$$|\sigma_1 - \sigma_2| = \sigma_Y$$

$$|610.19 - (-50.19)| = \sigma_Y$$

$$\sigma_Y = 660.38 \text{ MPa} = 660 \text{ MPa}$$ **Ans.**

Ans.
$\sigma_Y = 660$ MPa

10–74. Solve Prob. 10–82 using the maximum-distortion-energy theory.



SOLUTION

Normal and Shear Stress: In accordance with the sign convention.

$$\sigma_x = 560 \text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = 175 \text{ MPa}$$

In - Plane Principal Stress: Applying Eq. 9-5.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{560+0}{2} \pm \sqrt{\left(\frac{560-0}{2}\right)^2 + 175^2}$$

$$= 280 \pm 330.189$$

$$\sigma_1 = 610.19 \text{ MPa} \qquad \sigma_2 = -50.19 \text{ MPa}$$

Maximum Distortion Energy Theory:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

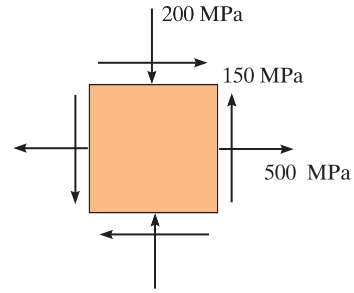
$$610.19^2 - (610.19)(-50.19) + (-50.19)^2 = \sigma_Y^2$$

$$\sigma_Y = 636.77 \text{ MPa} = 637 \text{ MPa}$$ **Ans.**

Ans.
$\sigma_Y = 637$ MPa

10–75.

The components of plane stress at a critical point on a thin steel shell are shown. Determine if failure (yielding) has occurred on the basis of the maximum distortion energy theory. The yield stress for the steel is $\sigma_Y = 700$ MPa.



SOLUTION

Normal And Shear Stresses: In accordance with the sign convention,

$$\sigma_x = 500 \text{ MPa} \qquad \sigma_y = -200 \text{ MPa} \qquad \tau_{xy} = 150 \text{ MPa}$$

In-Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{500 + (-200)}{2} \pm \sqrt{\left[\frac{500 - (-200)}{2}\right]^2 + 150^2}$$

$$= 150 \pm 380.79$$

$$\sigma_1 = 530.79 \text{ MPa} \qquad \sigma_2 = -230.79 \text{ MPa}$$

Maximum Distortion Energy Theory: This theory gives

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2$$

$$= 530.79^2 - 530.79(-230.79) + (-230.79)^2$$

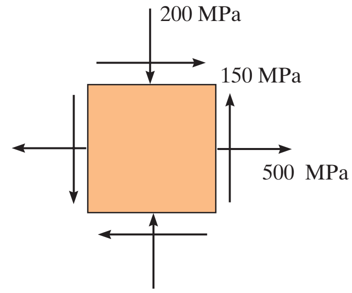
$$= 457500 < \sigma_Y^2 = 490000$$

Based on the results obtained above, **the material will not yield according to the maximum distortion energy theory.** **Ans.**

Ans:
No

***10–76.**

Solve Prob. 10–75 using the maximum shear stress theory.



SOLUTION

Normal And Shear Stresses: In accordance with the sign convention,

$$\sigma_x = 500 \text{ MPa} \qquad \sigma_y = -200 \text{ MPa} \qquad \tau_{xy} = 150 \text{ MPa}$$

In-Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{500 + (-200)}{2} \pm \sqrt{\left[\frac{500 - (-200)}{2}\right]^2 + 150^2}$$

$$= 150 \pm 380.79$$

$$\sigma_1 = 530.79 \text{ MPa} \qquad \sigma_2 = -230.79 \text{ MPa}$$

Maximum Shear-Stress Theory: Here, σ_1 and σ_2 have opposite signs. So

$$|\sigma_1 - \sigma_2|$$

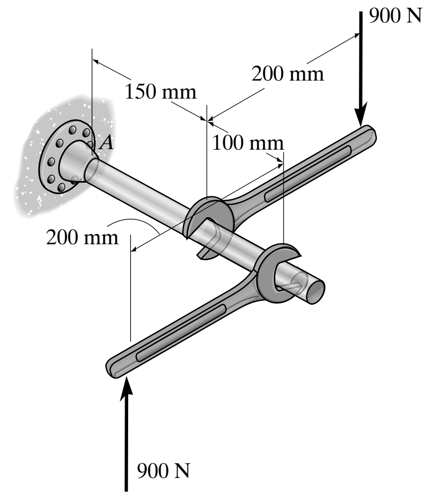
$$= 530.79 - (-230.79)$$

$$= 761.58 \text{ MPa} > \sigma_Y = 700 \text{ MPa}$$

Based on the results obtained above, **the material yield according to the maximum shear-stress theory.** **Ans.**

Ans:
Yes

10–77. If the A-36 steel pipe has outer and inner diameters of 30 mm and 20 mm, respectively, determine the factor of safety against yielding of the material at point A according to the maximum-shear-stress theory.



SOLUTION

Internal Loadings. Considering the equilibrium of the free - body diagram of the post's right cut segment Fig. a,

$$\Sigma F_y = 0; \quad V_y + 900 - 900 = 0 \qquad V_y = 0$$

$$\Sigma M_x = 0; \; T + 900(0.4) = 0 \qquad T = -360\,\text{N}\cdot\text{m}$$

$$\Sigma M_z = 0; \; M_z + 900(0.15) - 900(0.25) = 0 \; M_z = 90\,\text{N}\cdot\text{m}$$

Section Properties. The moment of inertia about the z axis and the polar moment of inertia of the pipe's cross section are

$$I_z = \frac{\pi}{4}\left(0.015^4 - 0.01^4\right) = 10.15625\pi\left(10^{-9}\right)\,\text{m}^4$$

$$J = \frac{\pi}{2}\left(0.015^4 - 0.01^4\right) = 20.3125\pi\left(10^{-9}\right)\,\text{m}^4$$

Normal Stress and Shear Stress. The normal stress is contributed by bending stress. Thus,

$$\sigma_Y = -\frac{My_A}{I_z} = -\frac{90(0.015)}{10.15625\pi\left(10^{-9}\right)} = -42.31\text{MPa}$$

The shear stress is contributed by torsional shear stress.

$$\tau = \frac{Tc}{J} = \frac{360(0.015)}{20.3125\pi\left(10^{-9}\right)} = 84.62\text{ MPa}$$

The state of stress at point A is represented by the two - dimensional element shown in Fig. b.

In - Plane Principal Stress. $\sigma_x = -42.31$ MPa, $\sigma_z = 0$ and $\tau_{xz} = 84.62$ MPa. We have

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}{}^2}$$

$$= \frac{-42.31 + 0}{2} \pm \sqrt{\left(\frac{-42.31 - 0}{2}\right)^2 + 84.62^2}$$

$$= (-21.16 \pm 87.23)\text{ MPa}$$

$$\sigma_1 = 66.07\text{ MPa} \qquad \sigma_2 = -108.38\text{ MPa}$$

10–77. Continued

Maximum Shear Stress Theory. σ_1 and σ_2 have opposite signs. This requires

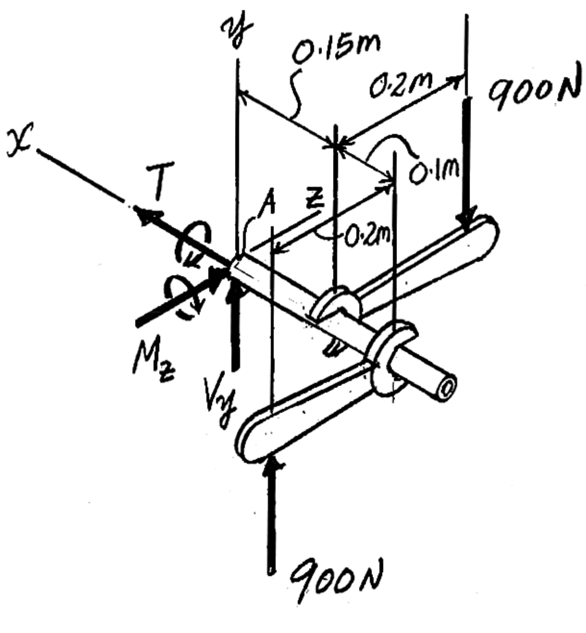
$$|\sigma_1 - \sigma_2| = \sigma_{\text{allow}}$$

$$66.07 - (-108.38) = \sigma_{\text{allow}}$$

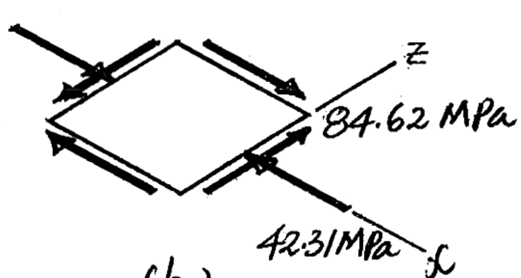
$$\sigma_{\text{allow}} = 174.45 \text{ MPa}$$

The factor of safety is

$$F.S. = \frac{\sigma_Y}{\sigma_{\text{allow}}} = \frac{250}{174.45} = 1.43$$ **Ans.**



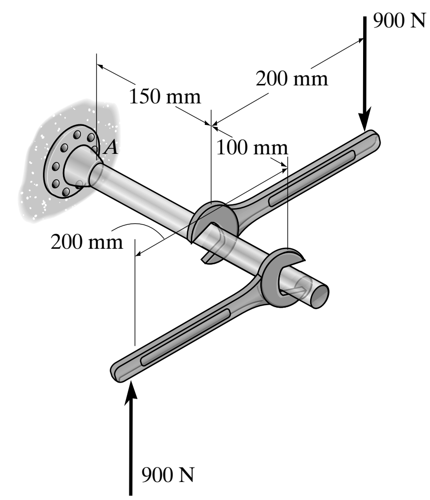
(a)



(b)

Ans.
$F.S. = 1.43$

10–78. If the A-36 steel pipe has an outer and inner diameter of 30 mm and 20 mm, respectively, determine the factor of safety against yielding of the material at point A according to the maximum-distortion-energy theory.



SOLUTION

Internal Loadings: Considering the equilibrium of the free - body diagram of the pipe's right cut segment Fig. *a*,

$\Sigma F_y = 0;\quad V_y + 900 - 900 = 0 \qquad\qquad V_y = 0$

$\Sigma M_x = 0;\; T + 900(0.4) = 0 \qquad\qquad T = -360\,\text{N}\cdot\text{m}$

$\Sigma M_z = 0;\; M_z + 900(0.15) - 900(0.25) = 0 \;\; M_z = 90\,\text{N}\cdot\text{m}$

Section Properties. The moment of inertia about the z axis and the polar moment of inertia of the pipe's cross section are

$$I_z = \frac{\pi}{4}\left(0.015^4 - 0.01^4\right) = 10.15625\pi\left(10^{-9}\right)\text{ m}^4$$

$$J = \frac{\pi}{2}\left(0.015^4 - 0.01^4\right) = 20.3125\pi\left(10^{-9}\right)\text{ m}^4$$

Normal Stress and Shear Stress. The normal stress is caused by bending stress. Thus,

$$\sigma_Y = -\frac{My_A}{I_z} = -\frac{90(0.015)}{10.15625\pi\left(10^{-9}\right)} = -42.31\text{MPa}$$

The shear stress is caused by torsional stress.

$$\tau = \frac{Tc}{J} = \frac{360(0.015)}{20.3125\pi\left(10^{-9}\right)} = 84.62\text{ MPa}$$

The state of stress at point A is represented by the two -dimensional element shown in Fig. *b*.

In - Plane Principal Stress. $\sigma_x = -42.31$ MPa, $\sigma_z = 0$ and $\tau_{xz} = 84.62$ MPa. We have

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}{}^2}$$

$$= \frac{-42.31 + 0}{2} \pm \sqrt{\left(\frac{-42.31 - 0}{2}\right)^2 + 84.62^2}$$

$$= (-21.16 \pm 87.23)\text{ MPa}$$

$\sigma_1 = 66.07$ MPa $\qquad\qquad \sigma_2 = -108.38$ MPa

10–78. Continued

Maximum Distortion Energy Theory.

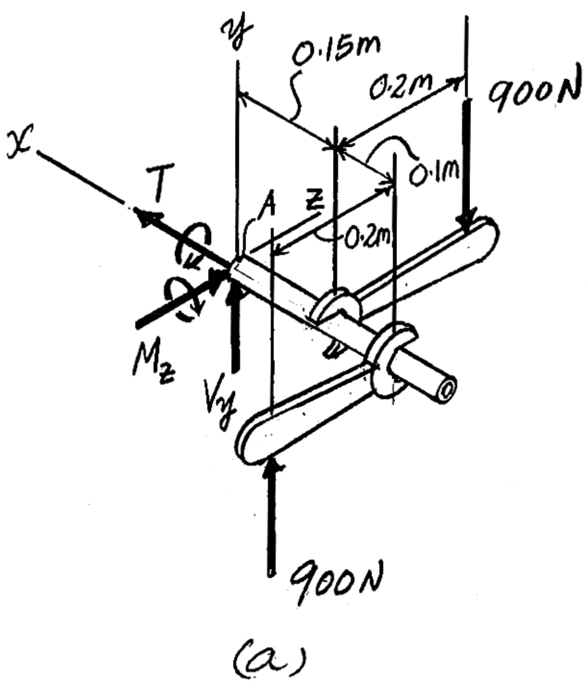
$$\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2{}^2 = \sigma_{\text{allow}}{}^2$$

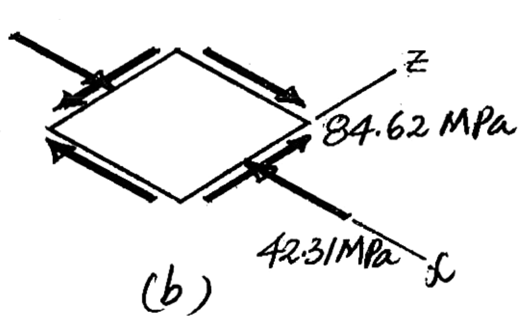
$$66.07^2 - 66.07(-108.38) + (-108.38)^2 = \sigma_{\text{allow}}{}^2$$

$$\sigma_{\text{allow}} = 152.55 \text{ MPa}$$

Thus, the factor of safety is

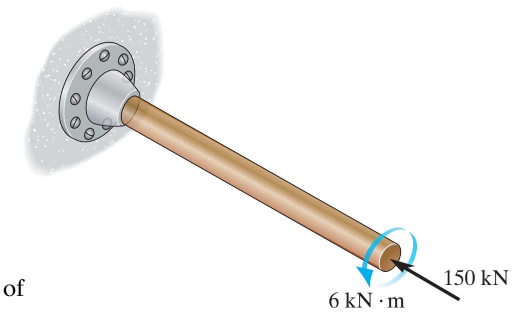
$$F.S. = \frac{\sigma_Y}{\sigma_{\text{allow}}} = \frac{250}{152.55} = 1.64$$ **Ans.**





Ans.
$F.S. = 1.64$

10–79. If the 50-mm diameter shaft is made from brittle material having an ultimate strength of $\sigma_{ult} = 350$ MPa for both tension and compression, determine if the shaft fails according to the maximum-normal-stress theory. Use a factor of safety of 1.5 against rupture.



SOLUTION

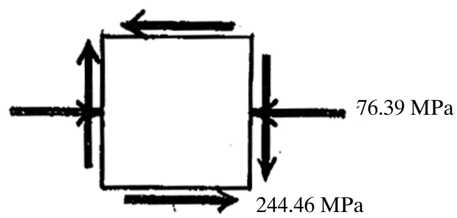
Normal Stress and Shear Stresses. The cross-sectional area and polar moment of inertia of the shaft's cross-section are

$$A = \pi(0.025^2) = 0.625(10^{-3})\pi \text{ m}^2$$

$$J = \frac{\pi}{2}(0.025^4) = 0.1953125(10^{-6})\pi \text{ m}^4$$

The normal stress is caused by axial stress.

$$\sigma = \frac{N}{A} = -\frac{150(10^3)}{0.625(10^{-3})\pi} = -76.39(10^6)\text{ N/m}^2 = -76.39\text{ MPa}$$



(a)

The shear stress is contributed by torsional shear stress.

$$\tau = \frac{Tc}{J} = \frac{[6(10^3)](0.025)}{0.1953125(10^{-6})\pi} = 244.46(10^6)\text{ N/m}^2 = 244.46\text{ MPa}$$

The state of stress at the points on the surface of the shaft is represented on the element shown in Fig. *a*.

In-Plane Principal Stress. $\sigma_x = -76.39$ MPa, $\sigma_y = 0$ and $\tau_{xy} = -244.46$ MPa. We have

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-76.39 + 0}{2} \pm \sqrt{\left(\frac{-76.39 - 0}{2}\right)^2 + (-244.46)^2}$$

$$= -30.20 \pm 247.43$$

$\sigma_1 = 209.23$ MPa $\qquad \sigma_2 = -285.63$ MPa

Maximum Normal-Stress Theory.

$$\sigma_{\text{allow}} = \frac{\sigma_{\text{ult}}}{F.S.} = \frac{350}{1.5} = 233.3\text{ MPa}$$

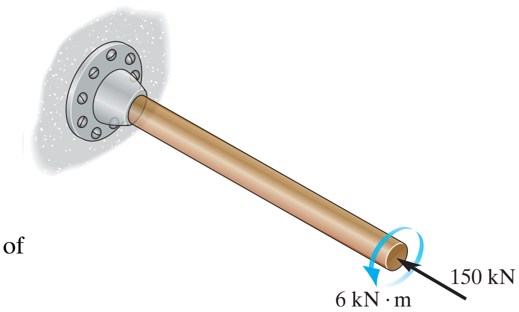
$$|\sigma_1| = 209.23 < \sigma_{\text{allow}} = 233.33\text{ MPa} \qquad \text{(O.K.)}$$

$$|\sigma_2| = 285.63 > \sigma_{\text{allow}} = 233.33\text{ MPa} \qquad \text{(N.G.)}$$

Based on these results, the material *fails* according to the maximum normal-stress theory.

Ans:
Yes

***10–80.** If the 50-mm diameter shaft is made from cast iron having tensile and compressive ultimate strengths of $(\sigma_{ult})_t = 350$ MPa and $(\sigma_{ult})_c = 525$ MPa, respectively, determine if the shaft fails in accordance with Mohr's failure criterion.



SOLUTION

Normal Stress and Shear Stresses. The cross-sectional area and polar moment of inertia of the shaft's cross-section are

$$A = \pi(0.025^2) = 0.625(10^{-3})\pi \text{ m}^2$$

$$J = \frac{\pi}{2}(0.025^4) = 0.1953125(10^{-6})\pi \text{ m}^4$$

The normal stress is caused by axial stress.

$$\sigma = \frac{N}{A} = -\frac{150(10^3)}{0.625(10^{-3})\pi} = -76.39(10^6) \text{ N/m}^2 = -76.39 \text{ MPa}$$

The shear stress is contributed by torsional shear stress.

$$\tau = \frac{Tc}{J} = \frac{[6(10^3)](0.025)}{0.1953125(10^{-6})\pi} = 244.46(10^6) \text{ N/m}^2 = 244.46 \text{ MPa}$$

The state of stress at the points on the surface of the shaft is represented on the element shown in Fig. *a*.

In-Plane Principal Stress. $\sigma_x = -76.39$ MPa, $\sigma_y = 0$ and $\tau_{xy} = -244.46$ MPa. We have

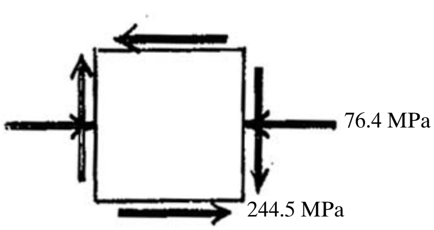
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-76.39 + 0}{2} \pm \sqrt{\left(\frac{-76.39 - 0}{2}\right)^2 + (-244.46)^2}$$

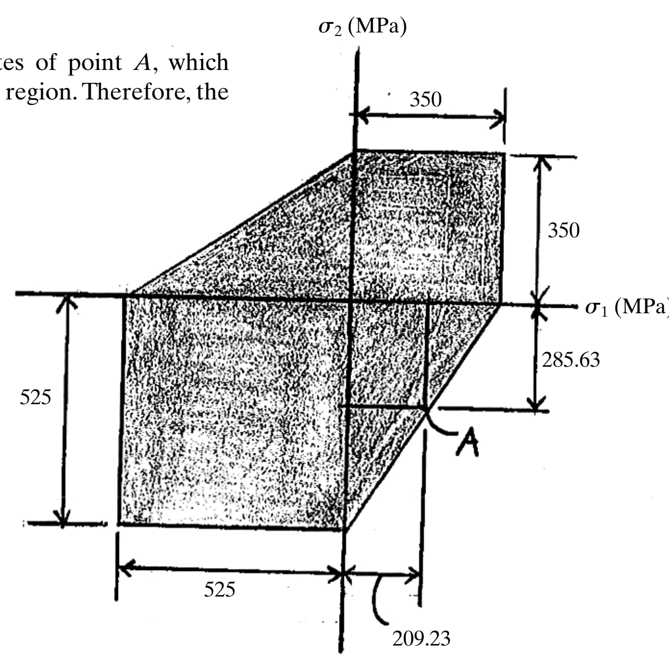
$$= -30.20 \pm 247.43$$

$$\sigma_1 = 209.23 \text{ MPa} \qquad \sigma_2 = -285.63 \text{ MPa}$$

Mohr's Failure Criteria. As shown in Fig. *b*, the coordinates of point *A*, which represent the principal stresses, are located inside the shaded region. Therefore, the material *does not fail* according to Mohr's failure criteria.

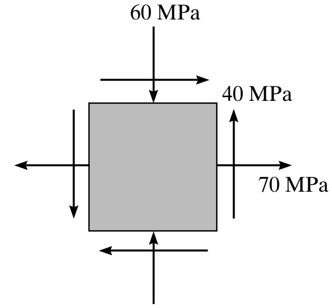


(a)



(b)

10–81. The components of plane stress at a critical point on an A-36 steel shell are shown. Determine if failure (yielding) has occurred on the basis of the maximum-shear-stress theory.



SOLUTION

In accordance to the established sign convention, $\sigma_x = 70$ MPa, $\sigma_y = -60$ MPa and $\tau_{xy} = 40$ MPa.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{70 + (-60)}{2} \pm \sqrt{\left[\frac{70 - (-60)}{2}\right]^2 + 40^2}$$

$$= 5 \pm \sqrt{5825}$$

$$\sigma_1 = 81.32 \text{ MPa} \qquad \sigma_2 = -71.32 \text{ MPa}$$

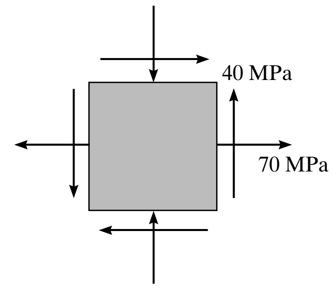
In this case, σ_1 and σ_2 have opposite sign. Thus,

$$|\sigma_1 - \sigma_2| = |81.32 - (-71.32)| = 152.64 \text{ MPa} < \sigma_Y = 250 \text{ MPa}$$

Based on this result, **the steel shell does not yield according to the maximum shear stress theory**.

Ans:
No

10–82. The components of plane stress at a critical point on an A-36 steel shell are shown. Determine if failure (yielding) has occurred on the basis of the maximum-distortion-energy theory.



SOLUTION

In accordance to the established sign convention, $\sigma_x = 70$ MPa, $\sigma_y = -60$ MPa and $\tau_{xy} = 40$ MPa.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{70 + (-60)}{2} \pm \sqrt{\left[\frac{70 - (-60)}{2}\right]^2 + 40^2}$$

$$= 5 \pm \sqrt{5825}$$

$$\sigma_1 = 81.32 \text{ MPa} \qquad \sigma_2 = -71.32 \text{ MPa}$$

$$\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2{}^2 = 81.32^2 - 81.32(-71.32) + (-71.32)^2 = 17{,}500 < \sigma_Y{}^2 = 62500$$

Based on this result, **the steel shell does not yield according to the maximum distortion energy theory**.

Ans:
No

10–83. The yield stress for heat-treated beryllium copper is $\sigma_Y = 900$ MPa. If this material is subjected to plane stress and elastic failure occurs when one principal stress is 1000 MPa, what is the smallest magnitude of the other principal stress? Use the maximum-distortion-energy theory.

SOLUTION

Maximum Distortion Energy Theory : With $\sigma_1 = 1000$ MPa,

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

$$1000^2 - 1000\sigma_2 + \sigma_2^2 = 900^2$$

$$\sigma_2^2 - 1000\sigma_2 + 190000 = 0$$

$$\sigma_2 = \frac{-(-1000) \pm \sqrt{(1000)^2 - 4(1)(190000)}}{2(1)}$$

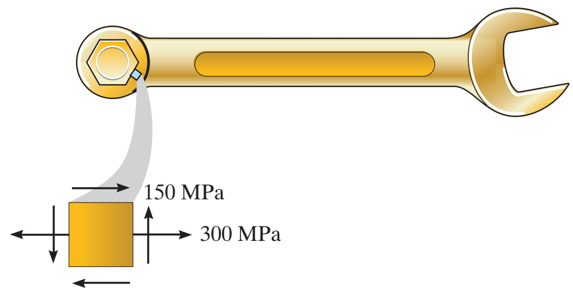
$$= 500 \pm 244.95$$

Choose the smaller root, $\sigma_2 = 255.05$ MPa = 255 MPa **Ans.**

Ans:
$\sigma_2 = 255$ MPa

***10–84.**

The state of stress acting at a critical point on a wrench is shown. Determine the smallest yield stress for steel that might be selected for the part, based on the maximum distortion energy theory.



SOLUTION

Normal And Shear Stress: In accordance with the sign convention,

$$\sigma_x = 300 \text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = 150 \text{ MPa}$$

In-Plane Principal Stress:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{300 + 0}{2} \pm \sqrt{\left(\frac{300 - 0}{2}\right)^2 + 150^2}$$

$$= 150 \pm 150\sqrt{2}$$

$$\sigma_1 = 362.13 \text{ MPa} \qquad \sigma_2 = -62.13 \text{ MPa}$$

Maximum Distortion Energy Theory: The theory gives

$$\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2{}^2 = \sigma_Y{}^2$$

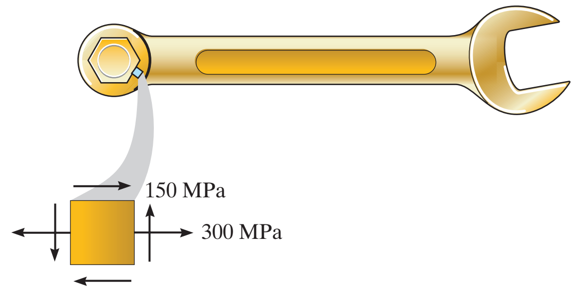
$$362.13^2 - 362.13(-62.13) + (-62.13)^2 - \sigma_Y{}^2$$

$$\sigma_Y = 396.86 \text{ MPa} = 397 \text{ MPa}$$ **Ans.**

Ans:
$\sigma_Y = 397$ MPa

10–85.

The state of stress acting at a critical point on a wrench is shown in the figure. Determine the smallest yield stress for steel that might be selected for the part, based on the maximum shear stress theory.



SOLUTION

Normal And Shear Stresses: In accordance to the sign convention,

$$\sigma_x = 300 \text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = 150 \text{ MPa}$$

In-Plane Principal Stress:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{300 + 0}{2} \pm \sqrt{\left(\frac{300 - 0}{2}\right)^2 + 150^2}$$

$$= 150 \pm 150\sqrt{2}$$

$$\sigma_1 = 362.13 \text{ MPa} \qquad \sigma_2 = -62.13 \text{ MPa}$$

Maximum Shear Stress Theory: Here, σ_1 and σ_2 have opposite signs. So

$$|\sigma_1 - \sigma_2| = \sigma_Y$$

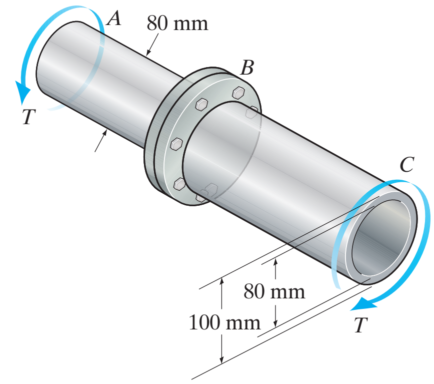
$$|362.13 - (-62.13)| = \sigma_Y$$

$$\sigma_Y = 424.26 \text{ MPa} = 424 \text{ MPa}$$ **Ans.**

Ans:
$\sigma_Y = 424$ MPa

10–86.

The shaft consists of a solid segment AB and a hollow segment BC, which are rigidly joined by the coupling at B. If the shaft is made from A-36 steel, determine the maximum torque T that can be applied according to the maximum shear stress theory. Use a factor of safety of 1.5 against yielding.



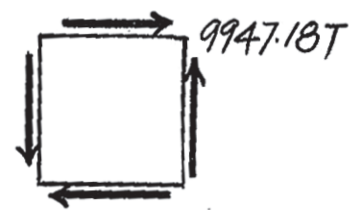
SOLUTION

Shear Stress: This is a case of pure shear, and the shear stress is contributed by torsion. For the hollow segment, $J_h = \frac{\pi}{2}\left(0.05^4 - 0.04^4\right) = 1.845\pi\left(10^{-6}\right)$ m^4. Thus,

$$(\tau_{\max})_h = \frac{Tc_h}{J_h} = \frac{T(0.05)}{1.845\pi\left(10^{-6}\right)} = 8626.28T$$

For the solid segment, $J_s = \frac{\pi}{2}\left(0.04^4\right) = 1.28\pi\left(10^{-6}\right)$ m^4. Thus,

$$(\tau_{\max})_s = \frac{Tc_s}{J_s} = \frac{T(0.04)}{1.28\pi\left(10^{-6}\right)} = 9947.18T$$



(a)

By comparision, the points on the surface of the solid segment are critical and their state of stress is represented on the element shown in Fig. a.

In - Plane Principal Stress. $\sigma_x = \sigma_y = 0$ and $\tau_{xy} = 9947.18T$. We have

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0+0}{2} \pm \sqrt{\left(\frac{0-0}{2}\right)^2 + (9947.18T)^2}$$

$$\sigma_1 = 9947.18T \qquad \sigma_2 = -9947.18T$$

Maximum Shear Stress Theory.

$$\sigma_{\text{allow}} = \frac{\sigma_Y}{F.S.} = \frac{250}{1.5} = 166.67 \text{ MPa}$$

Since σ_1 and σ_2 have opposite sings,

$$|\sigma_1 - \sigma_2| = \sigma_{\text{allow}}$$

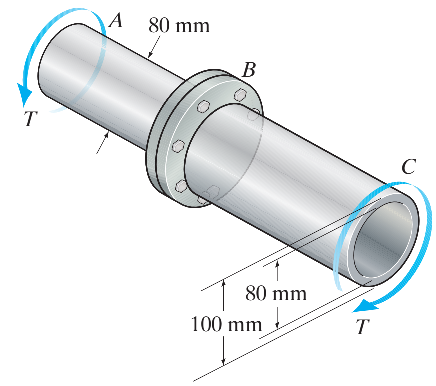
$$9947.18T - (-9947.18T) = 166.67\left(10^6\right)$$

$$T = 8377.58 \text{ N}\cdot\text{m} = 8.38 \text{ kN}\cdot\text{m}$$ **Ans.**

Ans:
$T = 8.38$ kN·m

10–87.

The shaft consists of a solid segment AB and a hollow segment BC, which are rigidly joined by the coupling at B. If the shaft is made from A-36 steel, determine the maximum torque T that can be applied according to the maximum distortion energy theory. Use a factor of safety of 1.5 against yielding.



SOLUTION

Shear Stress. This is a case of pure shear, and the shear stress is contributed by torsion. For the hollow segment, $J_h = \frac{\pi}{2}\left(0.05^4 - 0.04^4\right) = 1.845\pi\left(10^{-6}\right)\text{ m}^4$. Thus,

$$(\tau_{\max})_h = \frac{Tc_h}{J_h} = \frac{T(0.05)}{1.845\pi\left(10^{-6}\right)} = 8626.28T$$

For the solid segment, $J_s = \frac{\pi}{2}\left(0.04^4\right) = 1.28\pi\left(10^{-6}\right)\text{ m}^4$. Thus,

$$(\tau_{\max})_s = \frac{Tc_s}{J_s} = \frac{T(0.04)}{1.28\pi\left(10^{-6}\right)} = 9947.18T$$

By comparision, the points on the surface of the solid segment are critical and their state of stress is represented on the element shown in Fig. *a*.

In - Plane Principal Stress. $\sigma_x = \sigma_y = 0$ and $\tau_{xy} = 9947.18T$. We have

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 + 0}{2} \pm \sqrt{\left(\frac{0 - 0}{2}\right)^2 + (9947.18T)^2}$$

$$\sigma_1 = 9947.18T \qquad \sigma_2 = -9947.18T$$

Maximum Distortion Energy Theory.

$$\sigma_{\text{allow}} = \frac{\sigma_Y}{F.S.} = \frac{250}{1.5} = 166.67\text{ MPa}$$

Then,

$$\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2{}^2 = \sigma_{\text{allow}}{}^2$$

$$(9947.18T)^2 - (9947.18T)(-9947.18T) + (-9947.18T)^2 = \left[166.67\left(10^6\right)\right]^2$$

$$T = 9673.60\text{ N}\cdot\text{m} = 9.67\text{ kN}\cdot\text{m}$$ **Ans.**

Ans:

$T = 9.67\text{ kN}\cdot\text{m}$

***10–88.**

The principal stresses acting at a point on a thin-walled cylindrical pressure vessel are $\sigma_1 = pr/t$, $\sigma_2 = pr/2t$, and $\sigma_3 = 0$. If the yield stress is σ_Y, determine the maximum value of p based on (a) the maximum shear stress theory and (b) the maximum distortion energy theory.

SOLUTION

a) Maximum Shear Stress Theory: σ_1 and σ_2 have the same signs, then

$$|\sigma_2| = \sigma_Y \qquad \left|\frac{pr}{2t}\right| = \sigma_Y \qquad p = \frac{2t}{r}\sigma_Y$$

$$|\sigma_1| = \sigma_Y \qquad \left|\frac{pr}{t}\right| = \sigma_Y \qquad p = \frac{t}{r}\sigma_Y \text{ (Controls!)}$$ **Ans.**

b) Maximum Distortion Energy Theory:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

$$\left(\frac{pr}{t}\right)^2 - \left(\frac{pr}{t}\right)\left(\frac{pr}{2t}\right) + \left(\frac{pr}{2t}\right)^2 = \sigma_Y^2$$

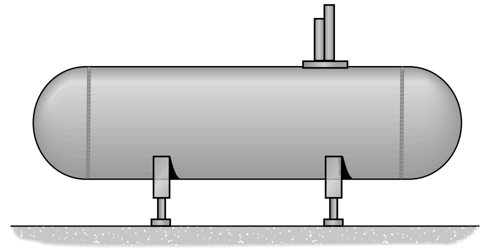
$$p = \frac{2t}{\sqrt{3}r}\sigma_Y$$ **Ans.**

Ans:

(a) $p = \frac{t}{r}\sigma_y$,

(b) $p = \frac{2t}{\sqrt{3}r}\sigma_y$

10–89. The gas tank has an inner diameter of 1.50 m and a wall thickness of 25 mm. If it is made from A-36 steel and the tank is pressured to 5 MPa, determine the factor of safety against yielding using (a) the maximum-shear-stress theory, and (b) the maximum-distortion-energy theory.



SOLUTION

(a) **Normal Stress.** Since $\frac{r}{t} = \frac{0.75}{0.025} = 30 > 10$, thin - wall analysis can be used. We have

$$\sigma_1 = \sigma_h = \frac{pr}{t} = \frac{5(0.75)}{0.025} = 150 \text{ MPa}$$

$$\sigma_2 = \sigma_{\text{long}} = \frac{pr}{2t} = \frac{5(0.75)}{2(0.025)} = 75 \text{ MPa}$$

Maximum Shear Stress Theory. σ_1 and σ_2 have the sign. Thus,

$$|\sigma_1| = \sigma_{\text{allow}}$$

$$\sigma_{\text{allow}} = 150 \text{ MPa}$$

The factor of safety is

$$F.S. = \frac{\sigma_Y}{\sigma_{\text{allow}}} = \frac{250}{150} = 1.67 \qquad \textbf{Ans.}$$

(b) **Maximum Distortion Energy Theory.**

$$\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2{}^2 = \sigma_{\text{allow}}{}^2$$

$$150^2 - 150(75) + 75^2 = \sigma_{\text{allow}}{}^2$$

$$\sigma_{\text{allow}} = 129.90 \text{ MPa}$$

The factor of safety is

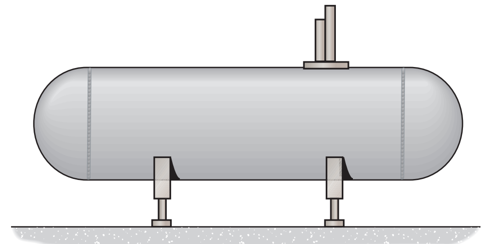
$$F.S. = \frac{\sigma_Y}{\sigma_{\text{allow}}} = \frac{250}{129.90} = 1.92 \qquad \textbf{Ans.}$$

Ans:

$F.S. = 1.67$, $F.S. = 1.92$

10–90.

The gas tank is made from A-36 steel and has an inner diameter of 1.50 m. If the tank is designed to withstand a pressure of 5 MPa, determine the required minimum wall thickness to the nearest millimeter using (a) the maximum shear stress theory, and (b) maximum distortion energy theory. Apply a factor of safety of 1.5 against yielding.



SOLUTION

(a) Normal Stress: Assuming that thin-wall analysis is valid, we have

$$\sigma_1 = \sigma_h = \frac{pr}{t} = \frac{5\left(10^6\right)(0.75)}{t} = \frac{3.75\left(10^6\right)}{t}$$

$$\sigma_2 = \sigma_{\text{long}} = \frac{pr}{2t} = \frac{5\left(10^6\right)(0.75)}{2t} = \frac{1.875\left(10^6\right)}{t}$$

Maximum Shear Stress Theory:

$$\sigma_{\text{allow}} = \frac{\sigma_Y}{FS.} = \frac{250\left(10^6\right)}{1.5} = 166.67\left(10^6\right)\text{Pa}$$

σ_1 and σ_2 have the same sign. Thus,

$$|\sigma_1| = \sigma_{\text{allow}}$$

$$\frac{3.75\left(10^6\right)}{t} = 166.67\left(10^6\right)$$

$$t = 0.0225 \text{ m} = 22.5 \text{ mm}$$ **Ans.**

Since $\frac{r}{t} = \frac{0.75}{0.0225} = 33.3 > 10$, thin-wall analysis is valid.

(b) Maximum Distortion Energy Theory:

$$\sigma_{\text{allow}} = \frac{\sigma_Y}{F.S.} = \frac{250\left(10^6\right)}{1.5} = 166.67\left(10^6\right)\text{Pa}$$

Thus,

$$\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2{}^2 = \sigma_{\text{allow}}{}^2$$

$$\left[\frac{3.75\left(10^6\right)}{t}\right]^2 - \left[\frac{3.75\left(10^6\right)}{t}\right]\left[\frac{1.875\left(10^6\right)}{t}\right] + \left[\frac{1.875\left(10^6\right)}{t}\right]^2 = \left[166.67\left(10^6\right)\right]^2$$

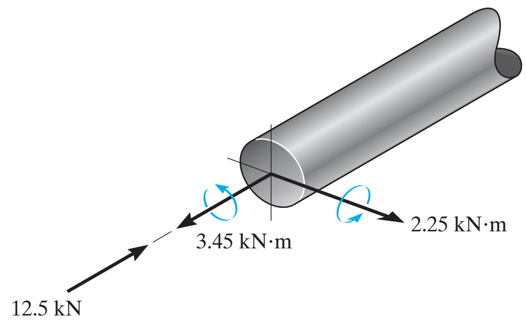
$$\frac{3.2476\left(10^6\right)}{t} = 166.67\left(10^6\right)$$

$$t = 0.01949 \text{ m} = 19.5 \text{ mm}$$ **Ans.**

Since $\frac{r}{t} = \frac{0.75}{0.01949} = 38.5 > 10$, thin-wall analysis is valid.

Ans:
(a) $t = 22.5$ mm,
(b) $t = 19.5$ mm

10–91. The internal loadings at a critical section along the steel drive shaft of a ship are calculated to be a torque of 3.45 KN · m, a bending moment of 2.25 kN · m, and an axial thrust of 12.5 kN. If the yield points for tension and shear are $\sigma_Y = 700$ MPa and $\sigma_Y = 350$ MPa, respectively, determine the required diameter of the shaft using the maximum-shear-stress theory.



SOLUTION

$$A = \pi c^2 \qquad I = \frac{\pi}{4}c^4 \qquad J = \frac{\pi}{2}c^4$$

$$\sigma_A = \frac{P}{A} + \frac{Mc}{I} = -\left[\frac{12.5(10^3)}{\pi c^2} + \frac{2.25(10^3)c}{\frac{\pi}{4}c^4}\right] = -\left[\frac{12.5(10^3)}{\pi c^2} + \frac{9(10^3)}{\frac{\pi}{4}c^3}\right]$$

$$\tau = \frac{Tc}{J} = \frac{3.45(10^3)c}{\frac{\pi}{2}c^4} = \frac{6.9(10^3)}{\pi c^3}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= -\left[\frac{12.5(10^3)c + 9(10^3)}{2\pi c^3}\right] \pm \sqrt{\left[\frac{12.5(10^3)c + 9(10^3)}{2\pi c^3}\right]^2 + \left[\frac{6.9(10^3)}{\pi c^3}\right]^2} \qquad (1)$$

Assume σ_1 and σ_2 have opposite signs:

$$|\sigma_1 - \sigma_2| = \sigma_Y$$

$$2\sqrt{\left[\frac{12.5(10^3)c + 9(10^3)}{2\pi c^3}\right]^2 + \left[\frac{6.9(10^3)}{\pi c^3}\right]^2} = 700(10^6)$$

$$490(10^9)\pi^2 c^6 - 156.25c^2 - 225c - 271.44 = 0$$

By trial and error:

$c = 0.019621$ m

Substitute c into Eq. (1):

$$\sigma_1 = 155.20(10^6)\ \text{N/m}^2 = 155.20\ \text{MPa}$$

$$\sigma_2 = -544.81(10^6)\ \text{N/m}^2 = -544.81\ \text{MPa}$$

σ_1 and σ_2 are of opposite signs **OK**

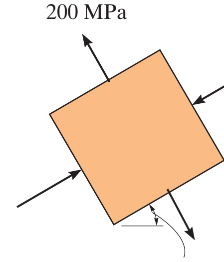
Therefore,

$d = 2(0.019621) = 0.039242$ m $= 39.2$ mm **Ans.**

Ans.
$d = 39.2$ mm

***10–92.**

If the material is machine steel having a yield stress of $\sigma_Y = 750$ MPa, determine the factor of safety with respect to yielding using the maximum distortion energy theory.



SOLUTION

Maximum Distortion Energy Theory: Here, $\sigma_1 = 200$ MPa and $\sigma_2 = -500$ MPa. The theory gives

$$\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2{}^2 = \sigma_{\text{allow}}^2$$

$$200^2 - (200)(-500) + (-500)^2 = \sigma_{\text{allow}}^2$$

$$\sigma_{\text{allow}} = 624.50 \text{ MPa}$$

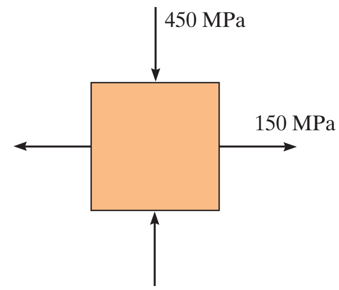
The factor of safety is

$$\text{F.S.} = \frac{\sigma_Y}{\sigma_{\text{allow}}} = \frac{750}{624.50} = 1.20 \qquad \textbf{Ans.}$$

Ans:
F.S. $= 1.20$

10–93.

If the material is machine steel having a yield stress of $\sigma_Y = 750$ MPa, determine the factor of safety with respect to yielding if the maximum shear stress theory is considered.



SOLUTION

Maximum Shear Stress Theory: Here, $\sigma_1 = 150$ MPa and $\sigma_2 = -450$ MPa since σ_1 and σ_2 have opposite signs, then

$$|\sigma_1 - \sigma_2| = \sigma_{allow}$$

$$|150 - (-450)| = \sigma_{allow}$$

$$\sigma_{allow} = 600 \text{ MPa}$$

The factor of safety is

$$\text{F.S.} = \frac{\sigma_Y}{\sigma_{allow}} = \frac{750}{600} = 1.25 \qquad \textbf{Ans.}$$

Ans:
F.S. $= 1.25$

R10–1.

In the case of plane stress, where the in-plane principal strains are given by ϵ_1 and ϵ_2, show that the third principal strain can be obtained from

$$\epsilon_3 = \frac{-\nu(\epsilon_1 + \epsilon_2)}{(1 - \nu)}$$

where ν is Poisson's ratio for the material.

SOLUTION

Generalized Hooke's Law: In the case of plane stress, $\sigma_3 = 0$. Thus,

$$\epsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) \qquad \textbf{(1)}$$

$$\epsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1) \qquad \textbf{(2)}$$

$$\epsilon_3 = -\frac{\nu}{E}(\sigma_1 + \sigma_2) \qquad \textbf{(3)}$$

Solving for σ_1 and σ_2 using Eqs. (1) and (2), we obtain

$$\sigma_1 = \frac{E(\epsilon_1 + \nu\epsilon_2)}{1 - \nu^2} \qquad \sigma_2 = \frac{E(\epsilon_2 + \nu\epsilon_1)}{1 - \nu^2}$$

Substituting these results into Eq. (3),

$$\epsilon_3 = -\frac{\nu}{E}\left[\frac{E(\epsilon_1 + \nu\epsilon_2)}{1 - \nu^2} + \frac{E(\epsilon_2 + \nu\epsilon_1)}{1 - \nu^2}\right]$$

$$\epsilon_3 = -\frac{\nu}{1 - \nu}\left[\frac{(\epsilon_1 + \epsilon_2) + \nu(\epsilon_1 + \epsilon_2)}{1 + \nu}\right]$$

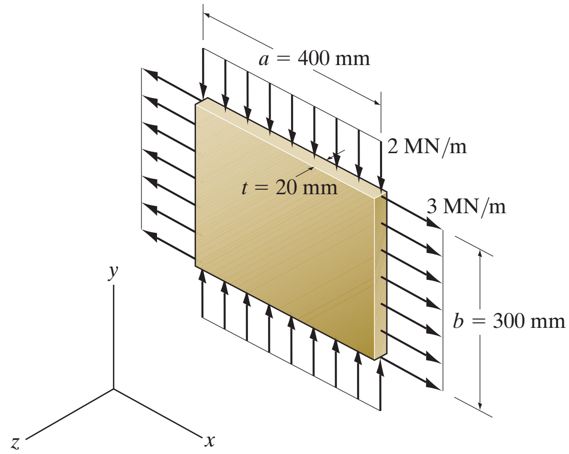
$$\epsilon_3 = -\frac{\nu}{1 - \nu}\left[\frac{(\epsilon_1 + \epsilon_2)(1 + \nu)}{1 + \nu}\right]$$

$$\epsilon_3 = -\frac{\nu}{1 - \nu}(\epsilon_1 + \epsilon_2) \qquad \textbf{(Q.E.D.)} \qquad \textbf{Ans.}$$

Ans:
N/A

R10–2.

The plate is made of material having a modulus of elasticity $E = 200$ GPa and Poisson's ratio $\nu = \frac{1}{3}$. Determine the change in width a, height b, and thickness t when it is subjected to the uniform distributed loading shown.



SOLUTION

Normal Stress: The normal stresses along the x, y, and z axes are

$$\sigma_x = \frac{3(10^6)}{0.02} = 150 \text{ MPa}$$

$$\sigma_y = -\frac{2(10^6)}{0.02} = -100 \text{ MPa}$$

$$\sigma_z = 0$$

Generalized Hooke's Law:

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$= \frac{1}{200(10^9)}\left\{150(10^6) - \frac{1}{3}\left[-100(10^6) + 0\right]\right\}$$

$$= 0.9167(10^{-3})$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$= \frac{1}{200(10^9)}\left\{-100(10^6) - \frac{1}{3}\left[150(10^6) + 0\right]\right\}$$

$$= -0.75(10^{-3})$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$= \frac{1}{200(10^9)}\left\{0 - \frac{1}{3}\left[150(10^6) + (-100)(10^6)\right]\right\}$$

$$= -83.33(10^{-6})$$

Thus, the changes in dimensions of the plate are

$$\delta_a = \epsilon_x a = 0.9167(10^{-3})(400) = 0.367 \text{ mm} \qquad \textbf{Ans.}$$

$$\delta_b = \epsilon_y b = -0.75(10^{-3})(300) = -0.225 \text{ mm} \qquad \textbf{Ans.}$$

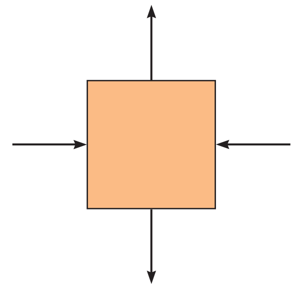
$$\delta_t = \epsilon_z t = -83.33(10^{-6})(20) = -0.00167 \text{ mm} \qquad \textbf{Ans.}$$

The negative signs indicate that b and t contract.

Ans:
$\delta_a = 0.367$ mm, $\delta_b = -0.255$ mm,
$\delta_t = -0.00167$ mm

R10–3.

If the material is machine steel having a yield stress of $\sigma_Y = 500$ MPa, determine the factor of safety with respect to yielding if the maximum shear stress theory is considered.



SOLUTION

Here, the in plane principal stresses are

$$\sigma_1 = \sigma_y = 100 \text{ MPa} \quad \sigma_2 = \sigma_x = -150 \text{ MPa}$$

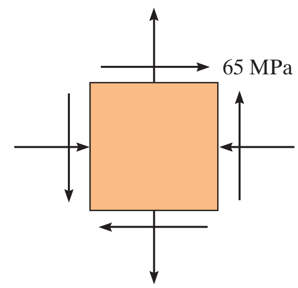
Since σ_1 and σ_2 have the same sign,

$$\text{F.S.} = \frac{\sigma_y}{|\sigma_1 - \sigma_2|} = \frac{500}{|100 - (-150)|} = 2$$ **Ans.**

Ans:
F.S. $= 2$

***R10–4.**

The components of plane stress at a critical point on a thin steel shell are shown. Determine if yielding has occurred on the basis of the maximum distortion energy theory. The yield stress for the steel is $\sigma_Y = 650$ MPa.



SOLUTION

$\sigma_x = -55$ MPa $\quad \sigma_y = 340$ MPa $\quad \tau_{xy} = 65$ MPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-55 + 340}{2} \pm \sqrt{\left(\frac{-55 - 340}{2}\right)^2 + 65^2}$$

$\sigma_1 = 350.42$ MPa $\quad \sigma_2 = -65.42$ MPa

$$(\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2) = [350.42^2 - 350.42(-65.42) + (-65.42)^2]$$

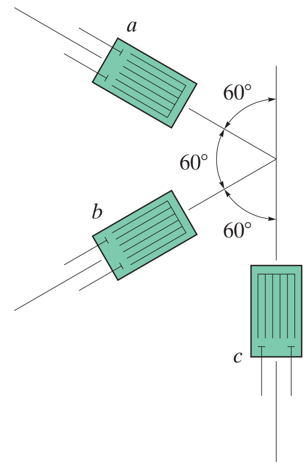
$$= 150\,000 < \sigma_Y^2 = 422\,500 \qquad \textbf{OK}$$

No. **Ans.**

Ans:
No

R10–5.

The 60° strain rosette is mounted on a beam. The following readings are obtained for each gage: $\epsilon_a = 600(10^{-6})$, $\epsilon_b = -700(10^{-6})$, and $\epsilon_c = 350(10^{-6})$. Determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case show the deformed element due to these strains.



SOLUTION

Strain Rosettes (60º): Applying Eq. 10–15 with $\epsilon_x = 600\left(10^{-6}\right)$,

$\epsilon_b = -700\left(10^{-6}\right), \epsilon_c = 350\left(10^{-6}\right), \theta_a = 150°, \theta_b = -150°$ and $\theta_c = -90°$,

$$350\left(10^{-6}\right) = \epsilon_x \cos^2(-90°) + \epsilon_y \sin^2(-90°) + \gamma_{xy} \sin(-90°) \cos(-90°)$$

$$\epsilon_y = 350\left(10^{-6}\right)$$

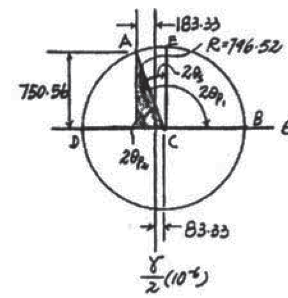
$$600\left(10^{-6}\right) = \epsilon_x \cos^2 150° + 350\left(10^{-6}\right) \sin^2 150° + \gamma_{xy} \sin 150° \cos 150°$$

$$512.5\left(10^{-6}\right) = 0.75\,\epsilon_x - 0.4330\,\gamma_{xy} \quad \textbf{(1)}$$

$$-700\left(10^{-6}\right) = \epsilon_x \cos^2(-150°) + 350\left(10^{-6}\right) \sin^2(-150°) + \gamma_{xy} \sin(-150°) \cos(-150°)$$

$$-787.5\left(10^{-6}\right) = 0.75\epsilon_x + 0.4330\,\gamma_{xy} \quad \textbf{(2)}$$

Solving Eq. (1) and (2) yields $\epsilon_x = -183.33\left(10^{-6}\right) \quad \gamma_{xy} = -1501.11\left(10^{-6}\right)$



Construction of the Circle: With $\epsilon_x = -183.33\left(10^{-6}\right)$, $\epsilon_y = 350\left(10^{-6}\right)$, and $\dfrac{\gamma_{xy}}{2} = -750.56\left(10^{-6}\right)$.

$$\epsilon_{\text{avg}} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{-183.33 + 350}{2}\right)\left(10^{-6}\right) = 83.3\left(10^{-6}\right) \quad \textbf{Ans.}$$

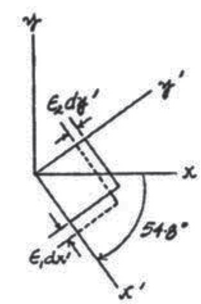
The coordinates for reference points A and C are

$A(-183.33, -750.56)\left(10^{-6}\right) \quad C(83.33, 0)\left(10^{-6}\right)$

The radius of the circle is

$$R = \left(\sqrt{(183.33 + 83.33)^2 + 750.56^2}\right)\left(10^{-6}\right) = 796.52\left(10^{-6}\right)$$

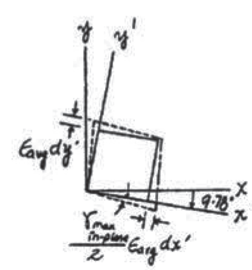
a)



In-Plane Principal Strain: The coordinates of points B and D represent ϵ_1 and ϵ_2, respectively.

$$\epsilon_1 = (83.33 + 796.52)\left(10^{-6}\right) = 880\left(10^{-6}\right) \quad \textbf{Ans.}$$

$$\epsilon_2 = (83.33 - 796.52)\left(10^{-6}\right) = -713\left(10^{-6}\right) \quad \textbf{Ans.}$$



Orientation of Principal Strain: From the circle,

$$\tan 2\theta_{P1} = \frac{750.56}{183.33 + 83.33} = 2.8145 \quad 2\theta_{P2} = 70.44°$$

$$2\theta_{P1} = 180° - 2\theta_{P2}$$

$$\theta_P = \frac{180° - 70.44°}{2} = 54.8° \quad \text{(Clockwise)} \quad \textbf{Ans.}$$

R10–5. Continued

b)

Maximum In-Plane Shear Strain: Represented by the coordinates of point E on the circle.

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = -R = -796.52\left(10^{-6}\right)$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = -1593\left(10^{-6}\right)$$ **Ans.**

Orientation of Maximum In-Plane Shear Strain: From the circle.

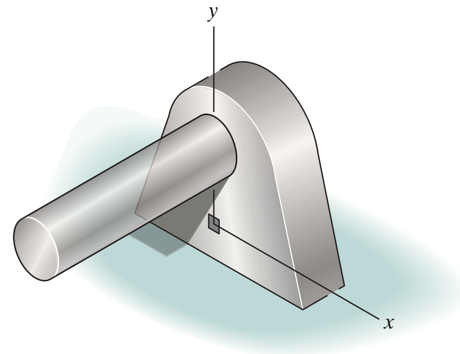
$$\tan 2\theta_s = \frac{183.33 + 83.33}{750.56} = 0.3553$$

$$\theta_s = 9.78^\circ \text{ (Clockwise)}$$ **Ans.**

Ans:
$\epsilon_{avg} = 83.3(10^{-6})$, $\epsilon_1 = 880(10^{-6})$,
$\epsilon_2 = -713(10^{-6})$, $\theta_p = 54.8^\circ$ (clockwise),
$\gamma_{\substack{\max \\ \text{in-plane}}} = -1593(10^{-6})$,
$\theta_s = 9.78^\circ$ (clockwise)

R10–6.

The state of strain at the point on the bracket has components $\epsilon_x = 350(10^{-6})$, $\epsilon_y = -860(10^{-6})$, $\gamma_{xy} = 250(10^{-6})$. Use the strain transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 45°$ clockwise from the original position. Sketch the deformed element within the x–y plane due to these strains.



SOLUTION

$\epsilon_x = 350(10^{-6}) \quad \epsilon_y = -860(10^{-6}) \quad \gamma_{xy} = 250(10^{-6}) \quad \theta = -45°$

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{350 - 860}{2} + \frac{350 - (-860)}{2}\cos(-90°) + \frac{250}{2}\sin(-90°)\right](10^{-6})$$

$$= -380(10^{-6}) \qquad \textbf{Ans.}$$

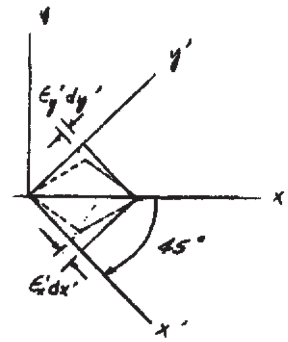
$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{350 - 860}{2} - \frac{350 - (-860)}{2}\cos(-90°) - \frac{250}{2}\sin(-90°)\right](10^{-6})$$

$$= -130(10^{-6}) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma}{2}\cos 2\theta$$

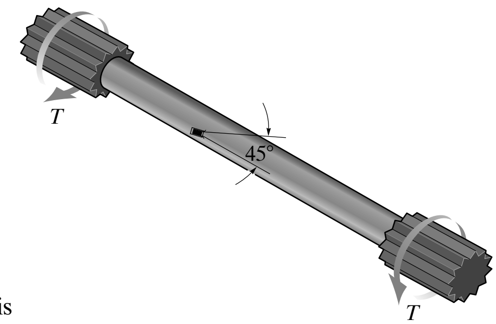
$$\gamma_{x'y'} = 2\left[-\left(\frac{350 - (-860)}{2}\right)\sin(-90°) + \frac{250}{2}\cos(-90°)\right](10^{-6}) = 1.21(10^{-3}) \qquad \textbf{Ans.}$$



Ans:
$\epsilon_{x'} = -380(10^{-6})$, $\epsilon_{y'} = -130(10^{-6})$,
$\gamma_{x'y'} = 1.21(10^{-3})$

R10–7.

A strain gauge forms an angle of 45° with the axis of the 50-mm diameter shaft. If it gives a reading of $P = -200(10^{-6})$ when the torque **T** is applied to the shaft, determine the magnitude of **T**. The shaft is made from A-36 steel.

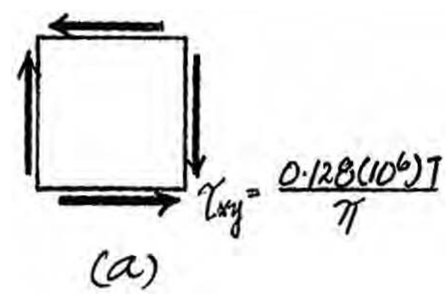


SOLUTION

Shear Stress. This is a case of pure shear, and the shear stress developed is contributed by torsional shear stress. Here, $J = \frac{\pi}{2}\left(0.025^4\right) = 0.1953125\pi\left(10^{-6}\right)\text{m}^4$. Then

$$\tau = \frac{Tc}{J} = \frac{T(0.025)}{0.1953125\pi\left(10^{-6}\right)} = \frac{0.128\left(10^6\right)T}{\pi}$$

The state of stress at points on the surface of the shaft can be represented by the element shown in Fig. *a*.



Shear Strain: For pure shear $\varepsilon_x = \varepsilon_y = 0$. We obtain,

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a\cos\theta_a$$

$$-200\left(10^{-6}\right) = 0 + 0 + \gamma_{xy}\sin 45^\circ \cos 45^\circ$$

$$\gamma_{xy} = -400\left(10^{-6}\right)$$

Shear Stress and Strain Relation: Applying Hooke's Law for shear,

$$\tau_{xy} = G\gamma_{xy}$$

$$-\frac{0.128\left(10^6\right)T}{\pi} = 75\left(10^9\right)\left[-400\left(10^{-6}\right)\right]$$

$$T = 736\ \text{N}\cdot\text{m}$$ **Ans.**

Ans:

$T = 736\ \text{N}\cdot\text{m}$

***R10–8.**

A differential element is subjected to plane strain that has the following components; $\epsilon_x = 950(10^{-6})$, $\epsilon_y = 420(10^{-6})$, $\gamma_{xy} = -325(10^{-6})$. Use the strain-transformation equations and determine (a) the principal strains and (b) the maximum in-plane shear strain and the associated average strain. In each case specify the orientation of the element and show how the strains deform the element.

SOLUTION

$$\epsilon_{1,2} = \frac{\epsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \varepsilon_y}{2}\right)^2 + \gamma_{xy}{}^2}$$

$$= \left[\frac{950 + 420}{2} \pm \sqrt{\left(\frac{950 - 420}{2}\right)^2 + \left(\frac{-325}{2}\right)^2}\right](10^{-6})$$

$$\epsilon_1 = 996(10^{-6}) \qquad \textbf{Ans.}$$

$$\epsilon_2 = 374(10^{-6}) \qquad \textbf{Ans.}$$

Orientation of ϵ_1 and ϵ_2:

$$\tan 2\theta_P = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-325}{950 - 420}$$

$$\theta_P = -15.76^\circ, 74.24^\circ$$

Use Eq. 10–5 to determine the direction of ϵ_1 and ϵ_2.

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\theta = \theta_P = -15.76^\circ$$

$$\epsilon_{x'} = \left\{\frac{950 + 420}{2} + \frac{950 - 420}{2}\cos(-31.52^\circ) + \frac{(-325)}{2}\sin(-31.52^\circ)\right\}(10^{-6}) = 996(10^{-6})$$

$$\theta_{P1} = -15.8^\circ \qquad \textbf{Ans.}$$

$$\theta_{P2} = 74.2^\circ \qquad \textbf{Ans.}$$

b)

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = 2\left[\sqrt{\left(\frac{950 - 420}{2}\right)^2 + \left(\frac{-325}{2}\right)^2}\right](10^{-6}) = 622(10^{-6}) \qquad \textbf{Ans.}$$

$$\epsilon_{\text{avg}} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{950 + 420}{2}\right)(10^{-6}) = 685(10^{-6}) \qquad \textbf{Ans.}$$

***R10–8. Continued**

Orientation of $\gamma_{\max}$:

$$\tan 2\theta_P = \frac{-(\epsilon_x - \epsilon_y)}{\gamma_{xy}} = \frac{-(950 - 420)}{-325}$$

$\theta_P = 29.2^\circ$ and $\theta_P = 119^\circ$ **Ans.**

Use Eq. 10–6 to determine the sign of $\gamma_{\substack{\max \\ \text{in-plane}}}$:

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\theta = \theta_s = 29.2^\circ$$

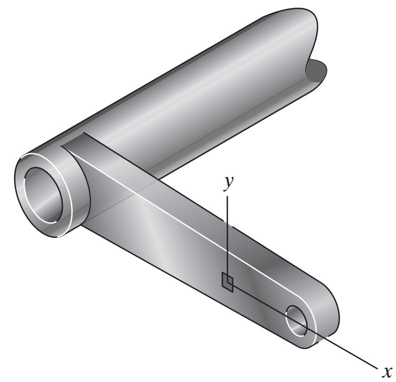
$$\gamma_{x'y'} = 2\left[\frac{-(950 - 420)}{2}\sin(58.4^\circ) + \frac{-325}{2}\cos(58.4^\circ)\right](10^{-6})$$

$$\gamma_{xy} = -622(10^{-6})$$

Ans:
$\epsilon_1 = 996(10^{-6}), \epsilon_2 = 374(10^{-6}), \theta_{p1} = -15.8,$
$\theta_{p2} = 74.2, \gamma_{\substack{\max \\ \text{in-plane}}} = 622(10^{-6}),$
$\epsilon_{avg} = 685(10^{-6}), \theta_s = 29.2^\circ$ and 119°

R10–9.

The state of strain at the point on the bracket has components $\epsilon_x = -130(10^{-6})$, $\epsilon_y = 280(10^{-6})$, $\gamma_{xy} = 75(10^{-6})$. Use the strain transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.



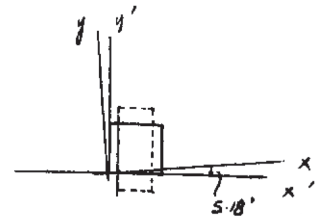
SOLUTION

$\epsilon_x = -130(10^{-6}) \qquad \epsilon_y = 280(10^{-6}) \qquad \gamma_{xy} = 75(10^{-6})$

a)

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{-130 + 280}{2} \pm \sqrt{\left(\frac{-130 - 280}{2}\right)^2 + \left(\frac{75}{2}\right)^2}\right](10^{-6})$$

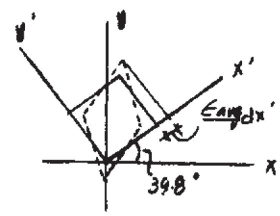


$\epsilon_1 = 283(10^{-6})$ **Ans.** $\qquad \epsilon_2 = -133(10^{-6})$ **Ans.**

Orientation of ϵ_1 and ϵ_2:

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{75}{-130 - 280}$$

$\theta_p = -5.18^\circ$ and 84.82°



Use Eq. 10–5 to determine the direction of ϵ_1 and ϵ_2:

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$\theta = \theta_p = -5.18^\circ$

$$\epsilon_{x'} = \left[\frac{-130 + 280}{2} + \frac{-130 - 280}{2}\cos(-10.37^\circ) + \frac{75}{2}\sin(-10.37^\circ)\right](10^{-6}) = -133(10^{-6})$$

Therefore $\theta_{p1} = 84.8^\circ$ **Ans.** $\qquad \theta_{p2} = -5.18^\circ$ **Ans.**

R10–9. Continued

b)

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = 2\left[\sqrt{\left(\frac{-130 - 280}{2}\right)^2 + \left(\frac{75}{2}\right)^2}\right](10^{-6}) = 417(10^{-6})$$ **Ans.**

$$\epsilon_{\text{avg}} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{-130 + 280}{2}\right)(10^{-6}) = 75.0(10^{-6})$$ **Ans.**

Orientation of $\gamma_{\max}$:

$$\tan 2\theta_s = \frac{-(\epsilon_x - \epsilon_y)}{\gamma_{xy}} = \frac{-(-130 - 280)}{75}$$

$$\theta_s = 39.8° \text{ and } \theta_s = 130°$$ **Ans.**

Use Eq. 10–16 to determine the sign of $\gamma_{\substack{\max \\ \text{in-plane}}}$:

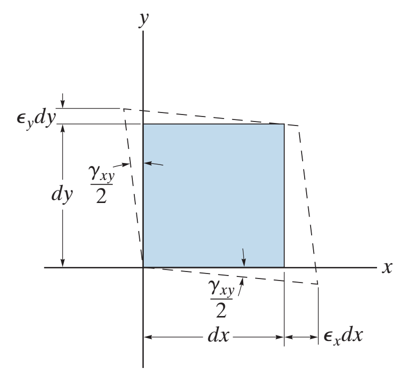
$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta; \qquad \theta = \theta_s = 39.8°$$

$$\gamma_{x'y'} = -(-130 - 280)\sin(79.6°) + (75)\cos(79.6°) = 417(10^{-6})$$

Ans:
$\epsilon_1 = 283(10^{-6})$, $\epsilon_2 = -133(10^{-6})$,
$\theta_{p1} = 84.8°$, $\theta_{p2} = -5.18°$, $\gamma_{\substack{\max \\ \text{in-plane}}} = 417(10^{-6})$,
$\epsilon_{\text{avg}} = 75.0(10^{-6})$, $\theta_s = 39.8°$ and $130°$

R10–10.

The state of plane strain on the element is $\epsilon_x = 400(10^{-6})$, $\epsilon_y = 200(10^{-6})$, and $\gamma_{xy} = -300(10^{-6})$. Determine the equivalent state of strain, which represents (a) the principal strains, and (b) the maximum in-plane shear strain and the associated average normal strain. Specify the orientation of the corresponding element at the point with respect to the original element. Sketch the results on the element.



SOLUTION

Construction of the Circle: $\epsilon_x = 400(10^{-6})$, $\epsilon_y = 200(10^{-6})$, and $\frac{\gamma_{xy}}{2} = -150(10^{-6})$. Thus,

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{400 + 200}{2}\right)(10^{-6}) = 300(10^{-6}) \qquad \textbf{Ans.}$$

The coordinates for reference points A and the center C of the circle are

$$A(400, -150)(10^{-6}) \qquad C(300, 0)(10^{-6})$$

The radius of the circle is

$$R = CA = \sqrt{(400 - 300)^2 + (-150)^2} = 180.28(10^{-6})$$

Using these results, the circle is shown in Fig. *a*.

In-Plane Principal Stresses: The coordinates of points B and D represent ϵ_1 and ϵ_2, respectively. Thus,

$$\epsilon_1 = (300 + 180.28)(10^{-6}) = 480(10^{-6}) \qquad \textbf{Ans.}$$

$$\epsilon_2 = (300 - 180.28)(10^{-6}) = 120(10^{-6}) \qquad \textbf{Ans.}$$

Orientation of Principal Plane: Referring to the geometry of the circle,

$$\tan 2(\theta_p)_1 = \frac{150}{400 - 300} = 1.5$$

$$(\theta_p)_1 = 28.2^\circ \text{ (clockwise)} \qquad \textbf{Ans.}$$

The deformed element for the state of principal strains is shown in Fig. *b*.

Maximum In-Plane Shear Stress: The coordinates of point E represent ϵ_{avg} and $\gamma_{\substack{max \\ in\text{-}plane}}$. Thus

$$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = -R = -180.28(10^{-6})$$

$$\gamma_{\substack{max \\ in\text{-}plane}} = -361(10^{-6}) \qquad \textbf{Ans.}$$

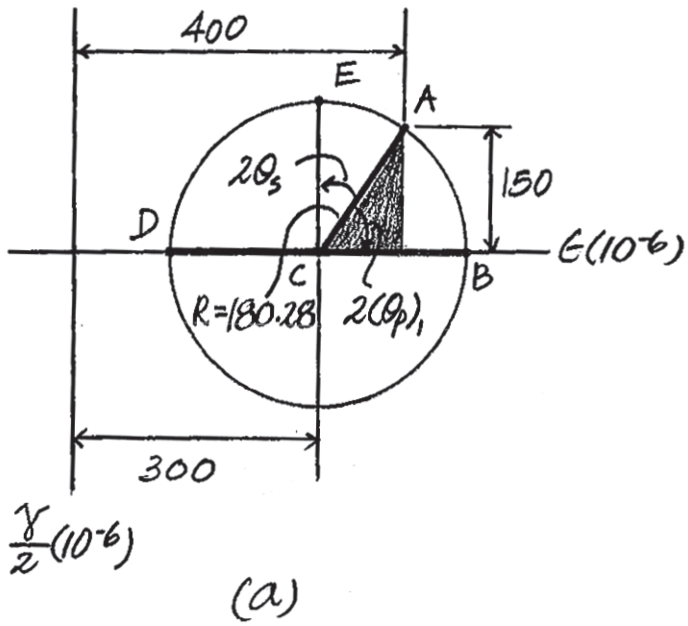
Orientation of the Plane of Maximum In-plane Shear Strain: Referring to the geometry of the circle,

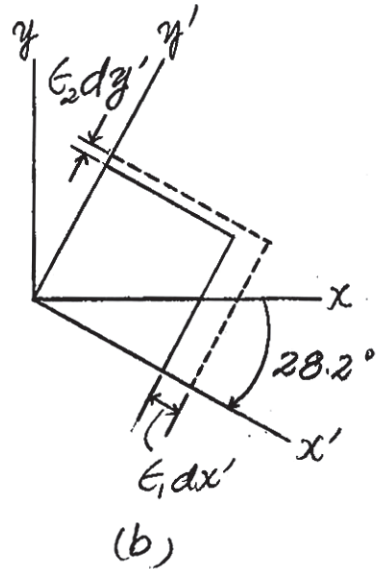
$$\tan 2\theta_s = \frac{400 - 300}{150} = 0.6667$$

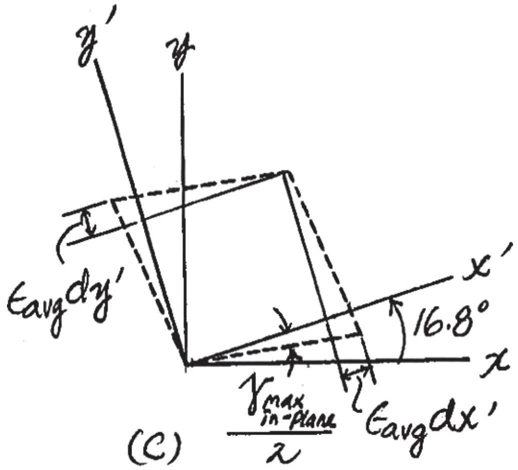
$$\theta_S = 16.8^\circ \text{ (counterclockwise)} \qquad \textbf{Ans.}$$

The deformed element for the state of maximum In-plane shear strain is shown in Fig. *c*.

R10–10. Continued







Ans:
$\epsilon_1 = 480(10^{-6})$, $\epsilon_2 = 120(10^{-6})$,
$\theta_{p1} = 28.2°$ (clockwise),
$\gamma_{\substack{\max \\ \text{in-plane}}} = -361(10^{-6})$,
$\theta_s = 16.8°$ (counterclockwise),
$\epsilon_{avg} = 300(10^{-6})$