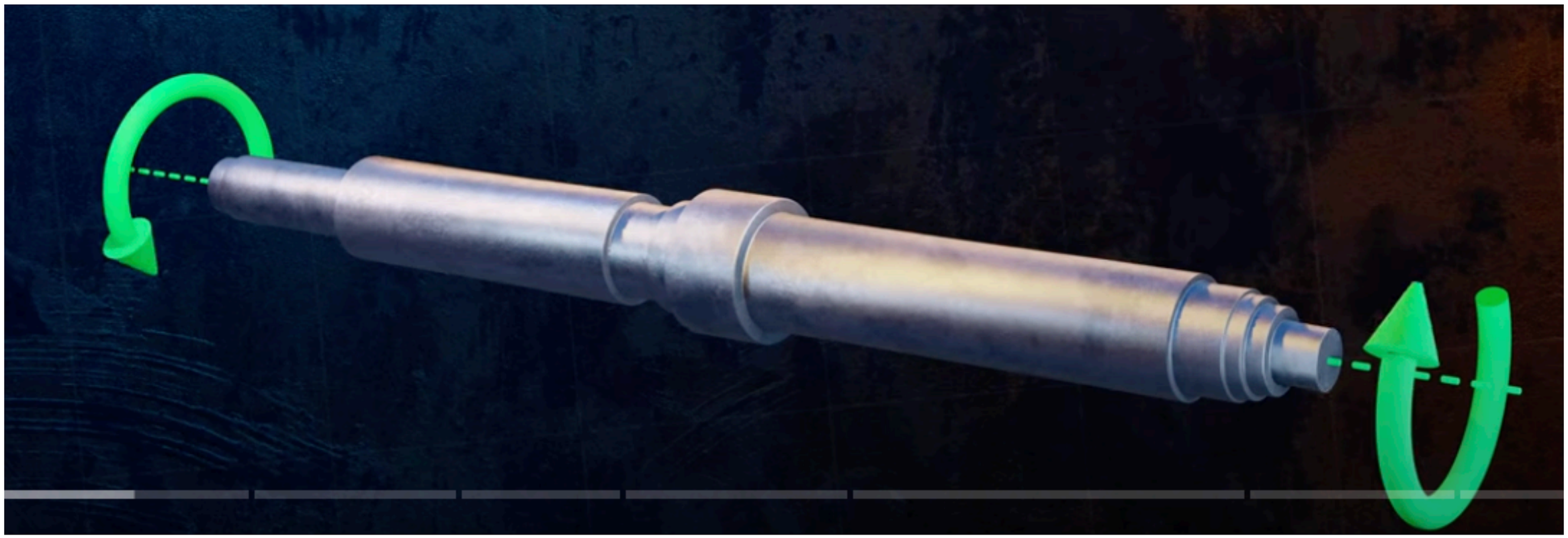
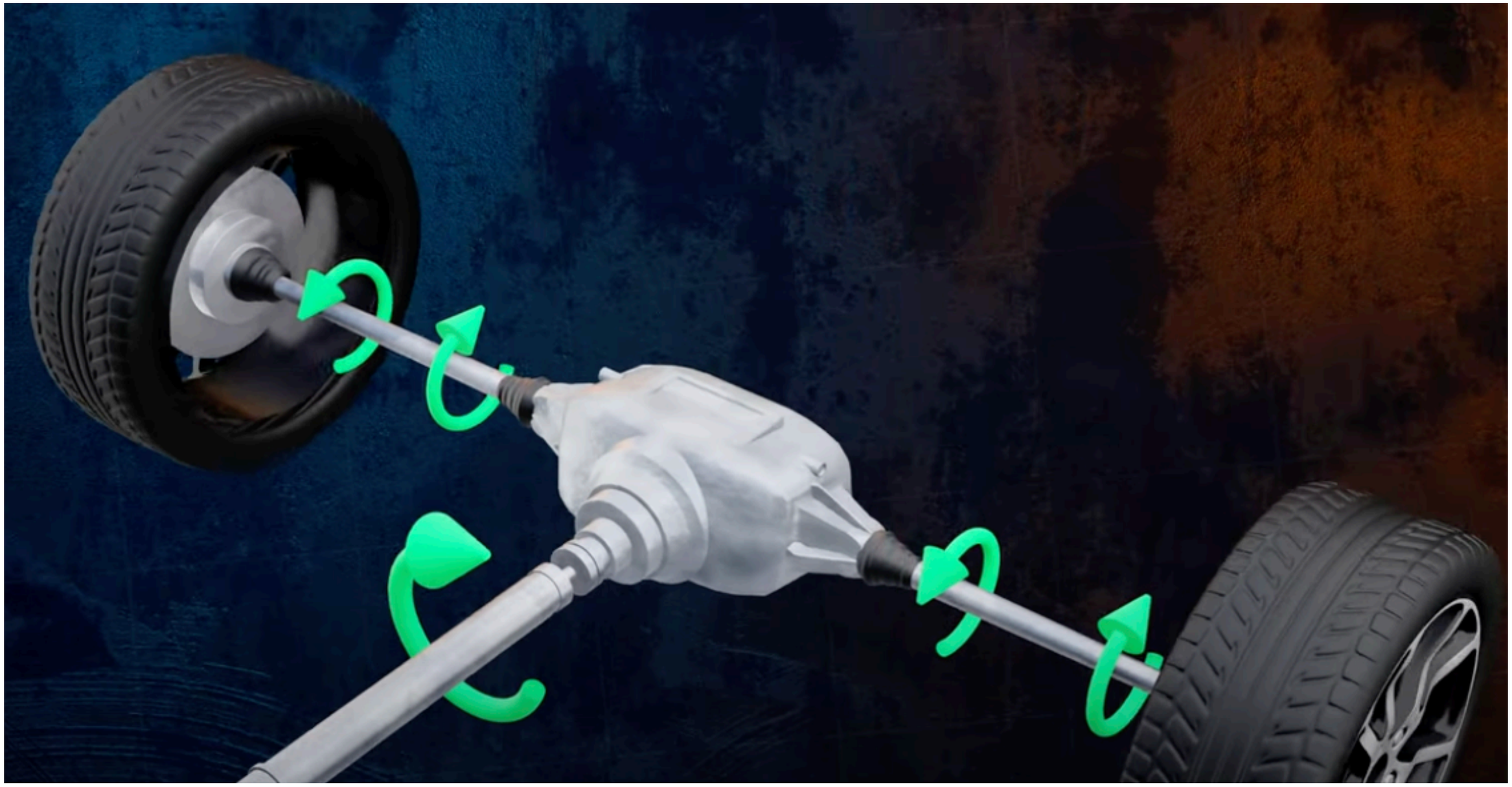
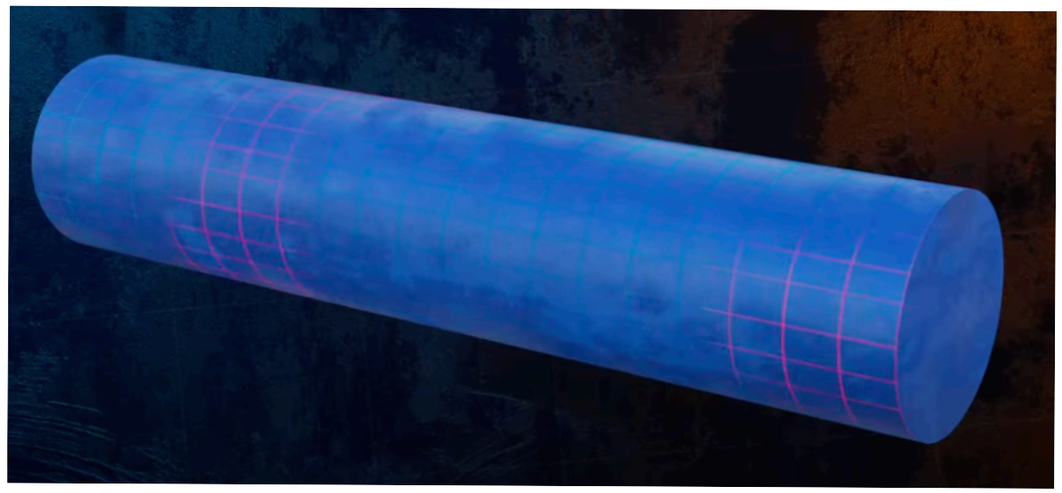
DEFORMATIONAL

TORSIONAL

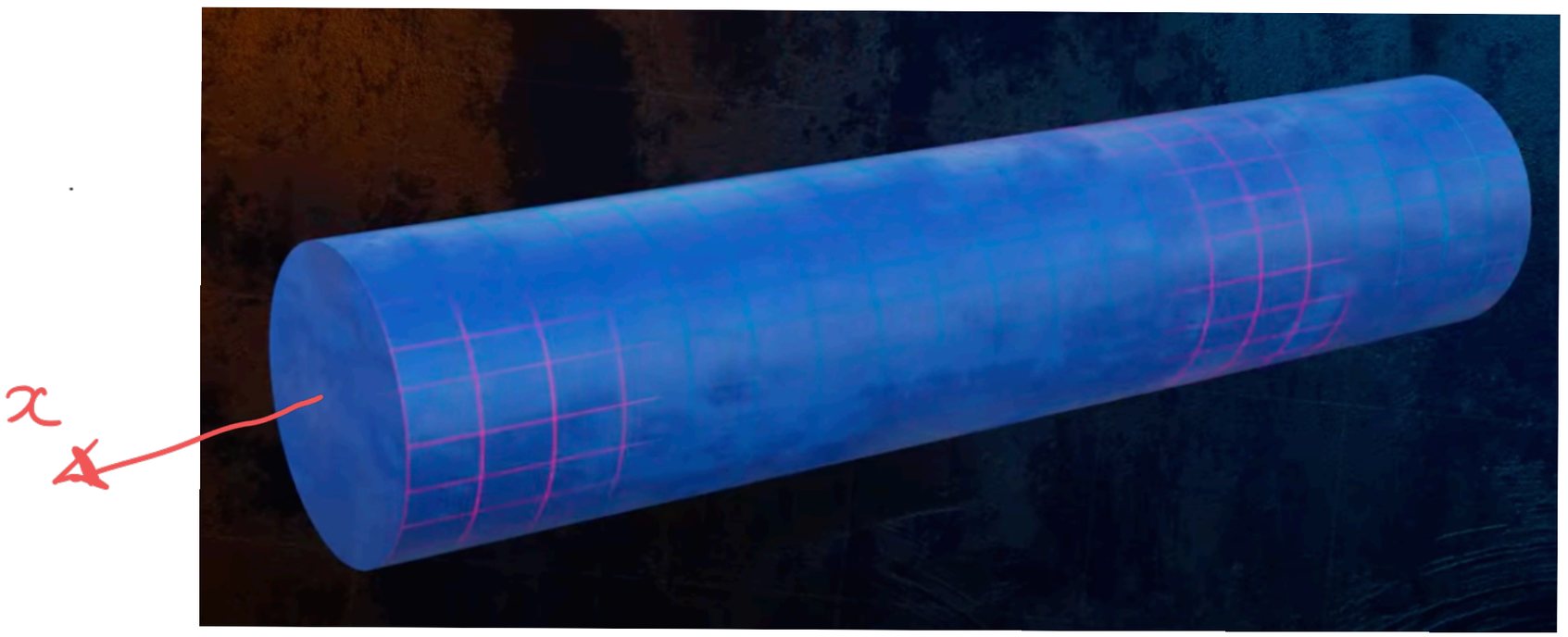
TORSIONE DI UN ALBERO MOTORE

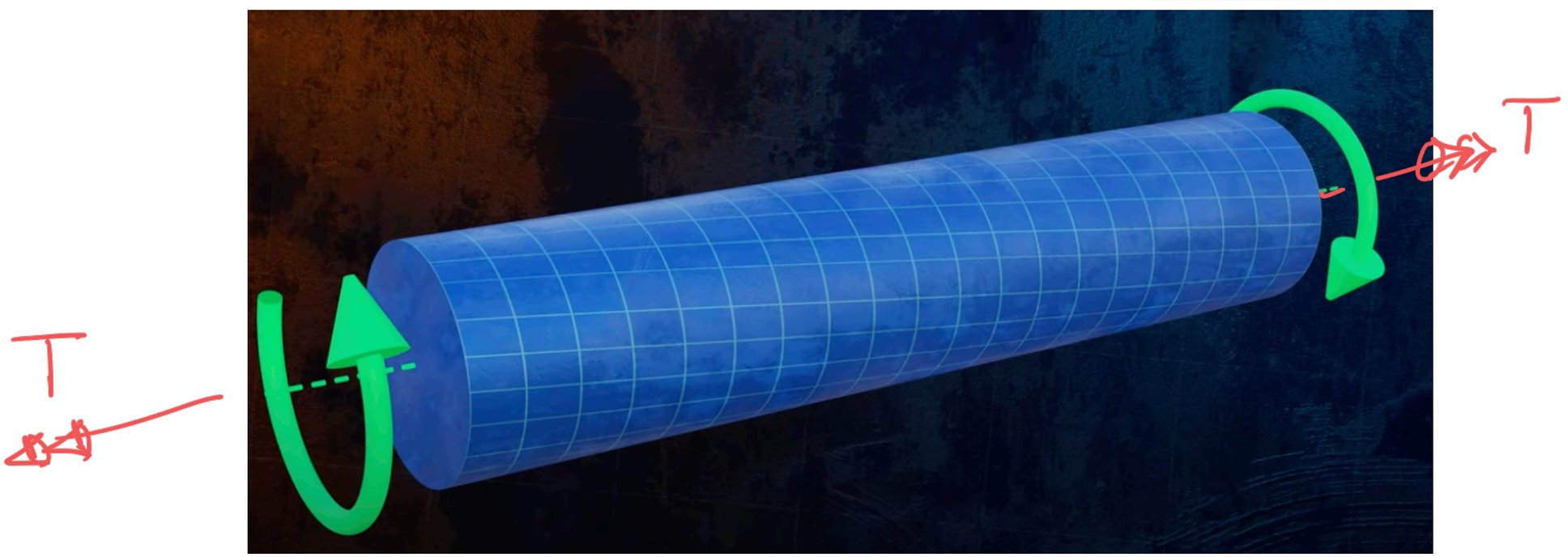


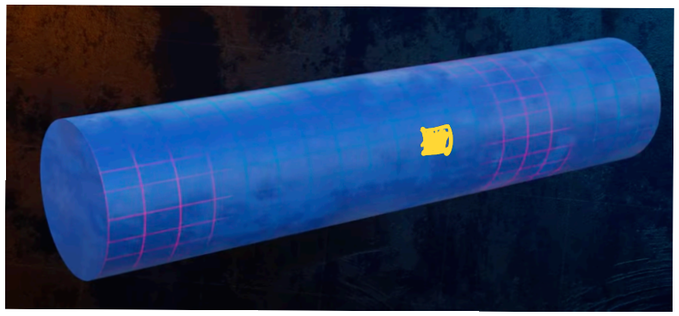


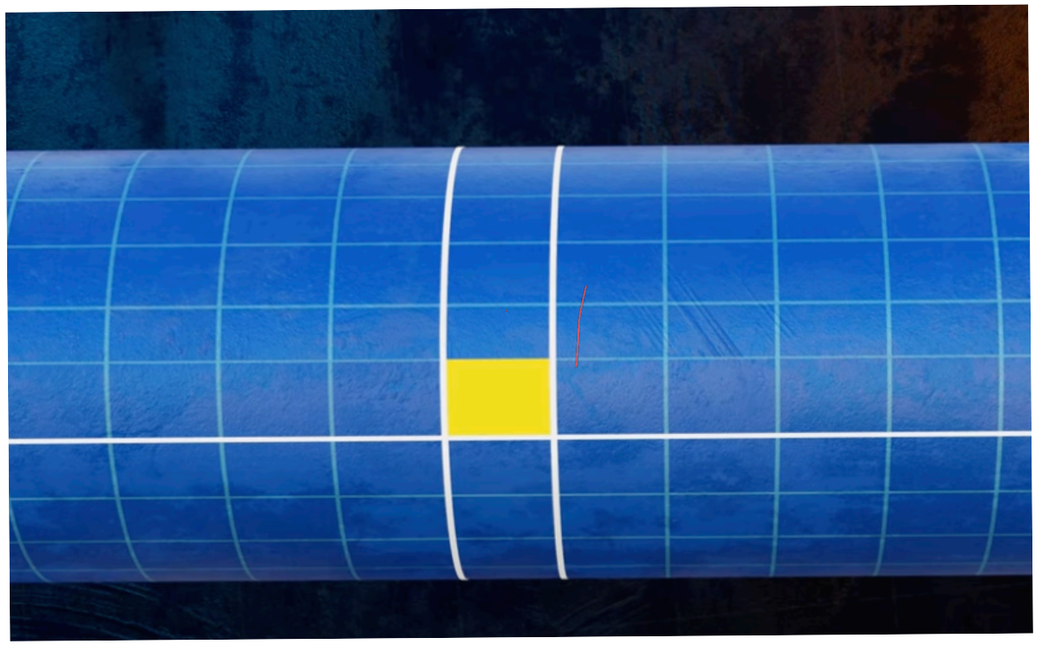


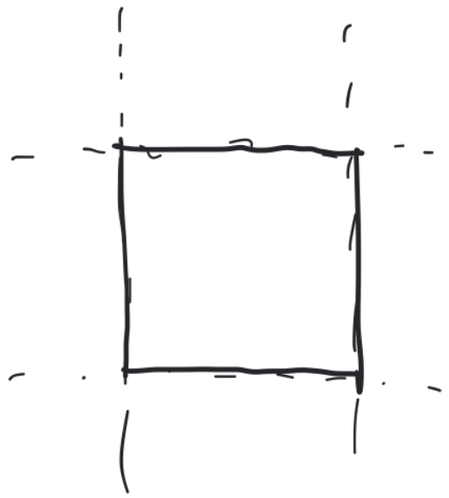
PARTE 1: TORSIONE UNIFORME

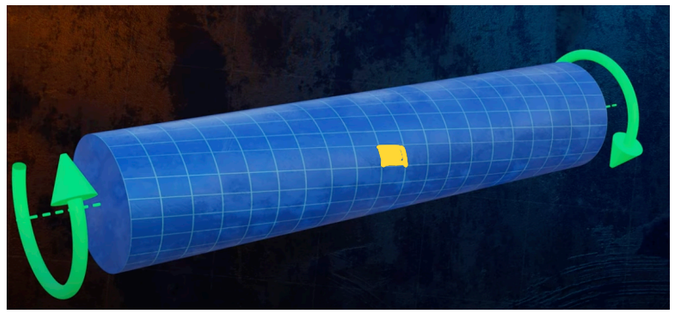


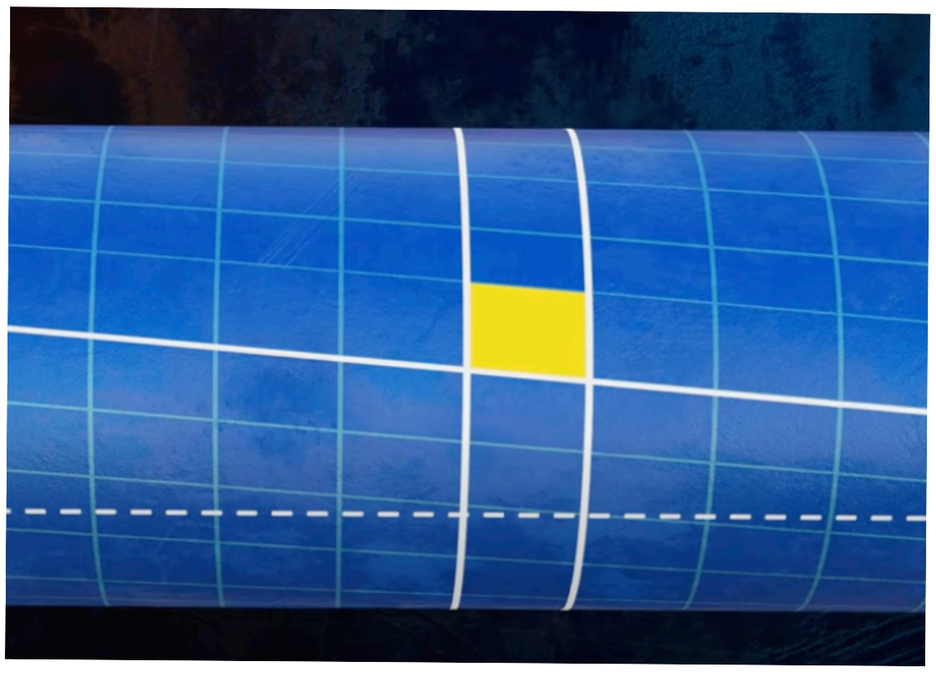


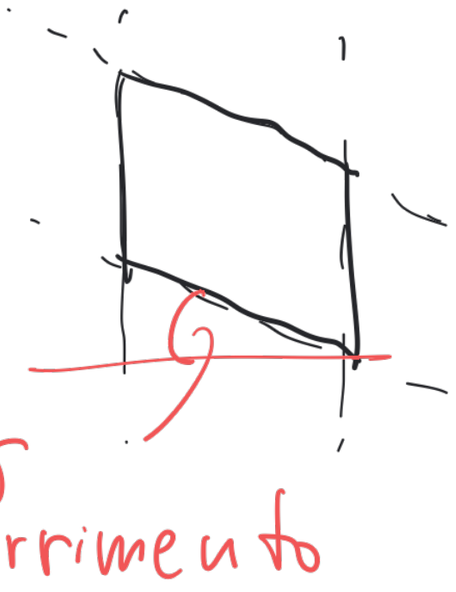








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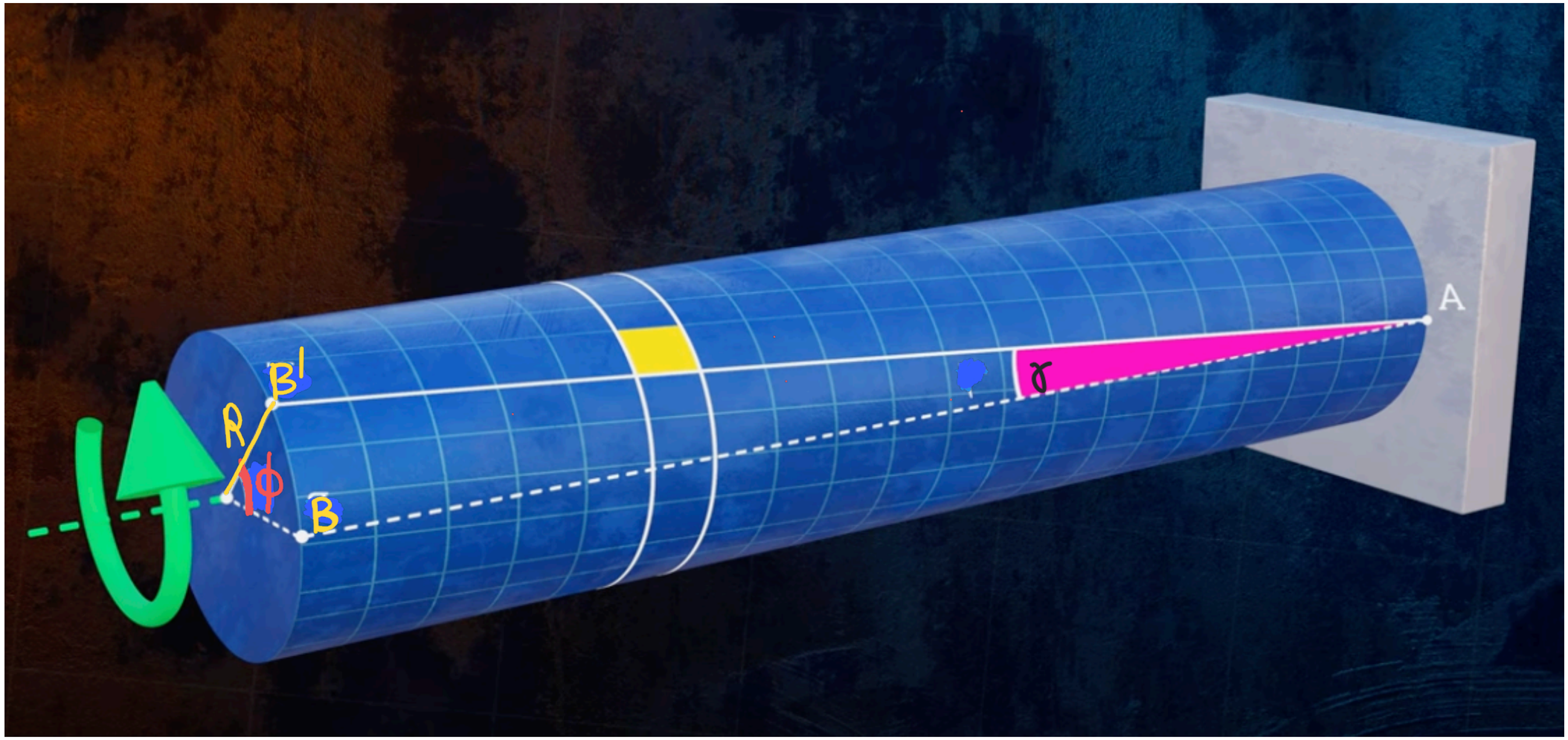
Scorrimento



$|BB'| \simeq AB \tan\gamma \simeq |AB|\gamma \qquad |BB'| = R\phi \Rightarrow \gamma = \frac{R\phi}{|AB|}$

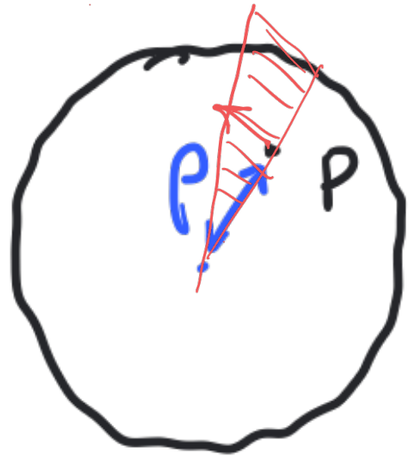
Lo scorrimento sulla superficie del cilindro è $\boxed{\gamma = \frac{R}{L}\phi}$, dove

R è il raggio della sezione, ϕ è l'angolo di torsione.

L è la lunghezza della trave

Punto generico

$$\gamma = \rho \frac{\phi}{L}$$



Tensione tangenziale

$$\tau = G\gamma$$

$$\tau = G\rho \frac{\phi}{L} \qquad \vartheta = \frac{\phi}{L}$$

ϑ: angolo unitario di torsione

$$T = 2\pi \int_0^R \tau(\rho)\, \rho\, d\rho$$

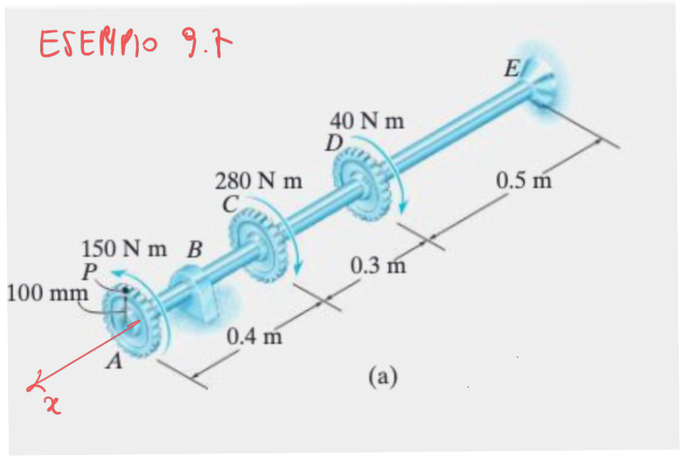
$$T = GJ\vartheta$$

$$\tau = \rho \frac{T}{J}$$

$$T = G J \frac{\phi}{L}$$

$$\tau(\rho) = \frac{\rho T}{J}$$

ESEMPIO 9.1

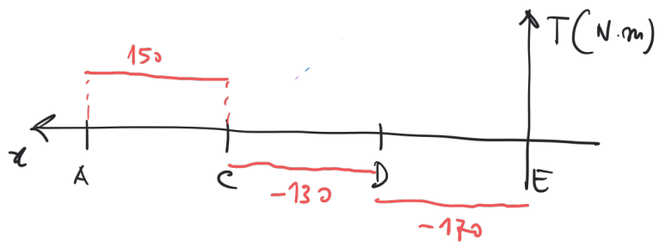


Calcolare lo spostamento del dente P.

$G = 80\,GPa$

$d = 14\,mm$ diametro albero.

Diagramma di T



$$T = GJ\frac{\phi}{L} \qquad \tau(\rho) = \frac{\rho T}{J}$$

Calcolare lo spostamento del dente P.

$G = 80\,GPa$

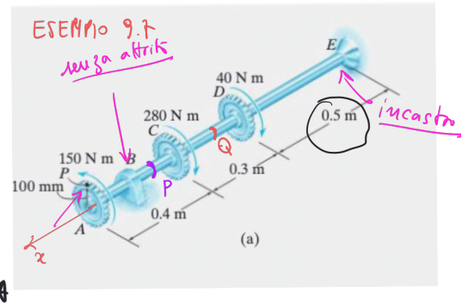
$d = 14\,mm$ diametro albero.

$$\phi_{ED} = \frac{T_{ED} L_{ED}}{GJ}$$

$\phi_{DC} =$ //

$\phi_{CA} =$ //

$$J = \frac{\pi}{2} R^4 = \frac{\pi}{2}(7\,mm)^4 = 3.77 \cdot 10^{-9}\,m^4$$

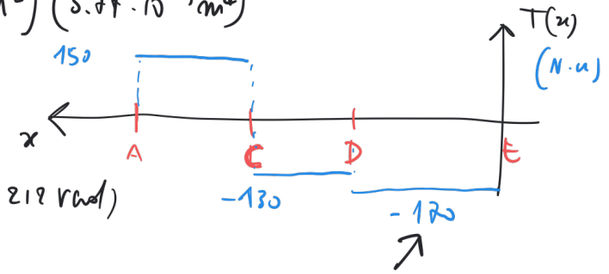


$$\phi_A = \phi_{EA} = \phi_{ED} + \phi_{DC} + \phi_{CA} = \frac{T_{ED}L_{ED} + T_{DC}L_{DC} + T_{CA}L_{CA}}{GJ}$$

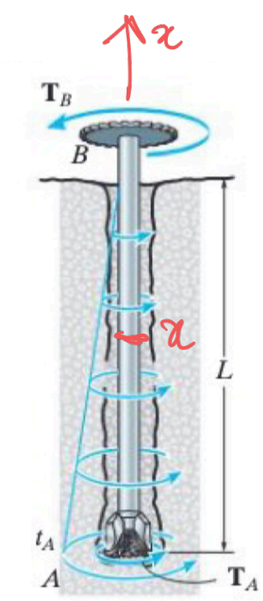
$$= \frac{(-120\,Nm)(0.5\,m) + (-130\,Nm \cdot 0.3\,m) + (150\,Nm)(0.4\,m)}{(80 \cdot 10^9\,N/m^2)(3.77 \cdot 10^{-9}\,m^4)}$$

$$= -0.212\,rad.$$

$$S_p = \phi_A = (100\,mm)(0.212\,rad) = 21.2\,mm$$



Parte 2: torsione non uniforme



Problema 9.12

$$t(x) = t_A\left(1 - x/L\right)$$

$$T(x) = T_A + \int_0^x t_A\left(1 - s/L\right) ds = T_A + t_A\left(x - \tfrac{1}{2}\, x^2/L\right)$$

$\phi(x)$ rotazione relativa tra sezione x e sez. A

$$\phi(0) = 0 \qquad \phi_{AB} = \phi(L) = \int_0^L \frac{d\phi}{dx}\, dx = \int_0^L \frac{1}{GJ}\left(T_A + t_A x\left(1 - \tfrac{1}{2}\, x/L\right)\right)$$

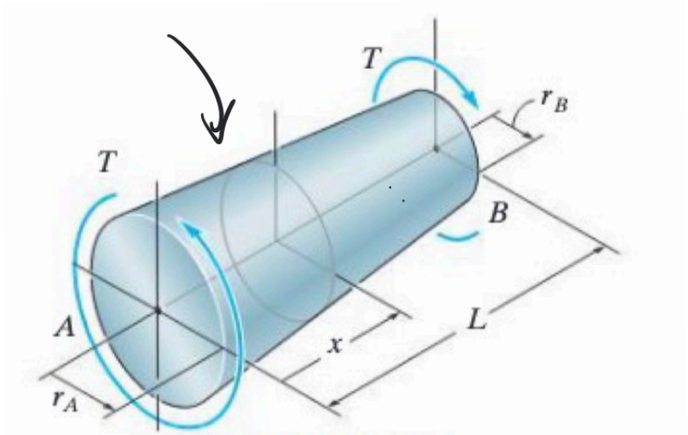
$$= \frac{1}{GJ}\left(T_A L + t_A \frac{L^2}{3}\right)$$

$$\phi = \frac{T}{GJ} L \qquad d\phi = \frac{T}{GJ}\, dx \qquad \frac{d\phi}{dx} = \frac{T}{GJ}$$

$$\phi = \int_0^L \frac{T}{GJ}\,dx = \frac{T}{G}\int_0^L \frac{1}{J(x)}\,dx$$

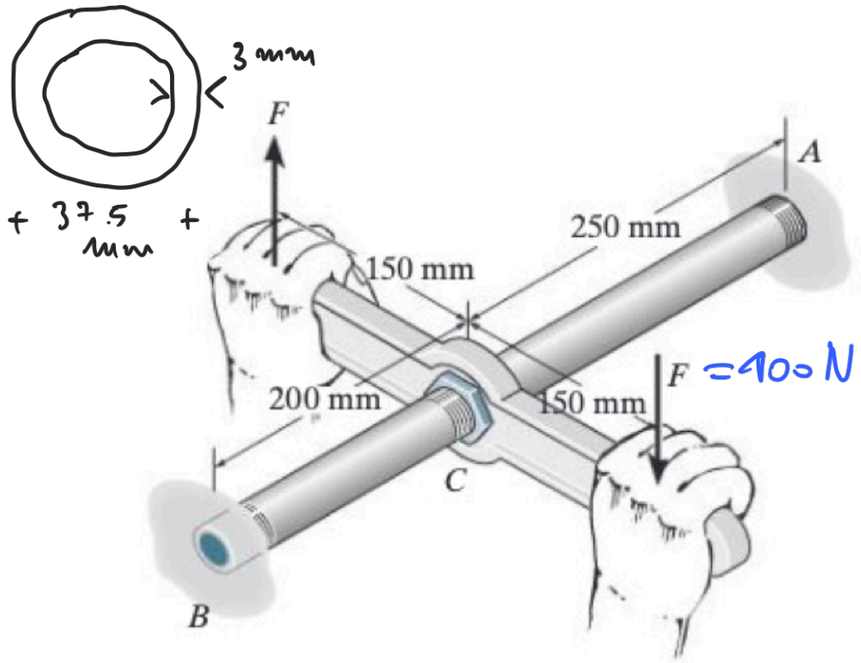
La torsione può essere non uniforme anche a causa di caratteristiche della sezione variabili.

$J = J(x)$



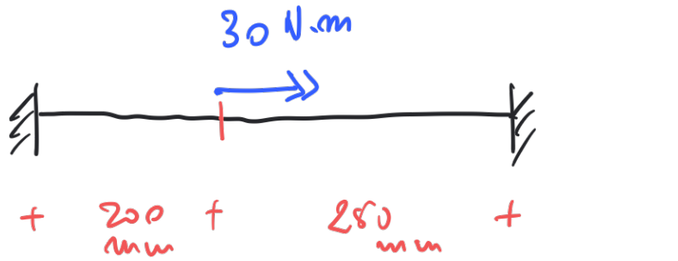
Problema 9.11

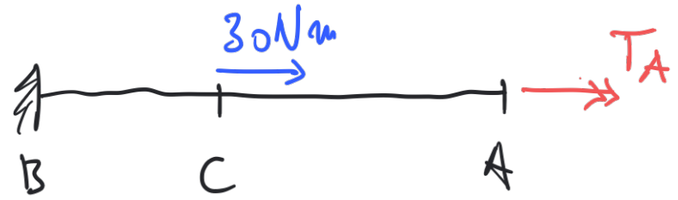
Torsione di travi staticamente indeterminate.



Problema 9.28

Determinare la massima tensione tangenziale





$T_{BC} = T_A + 30\,Nm$

$d\phi = \frac{T}{GJ}\,dx$

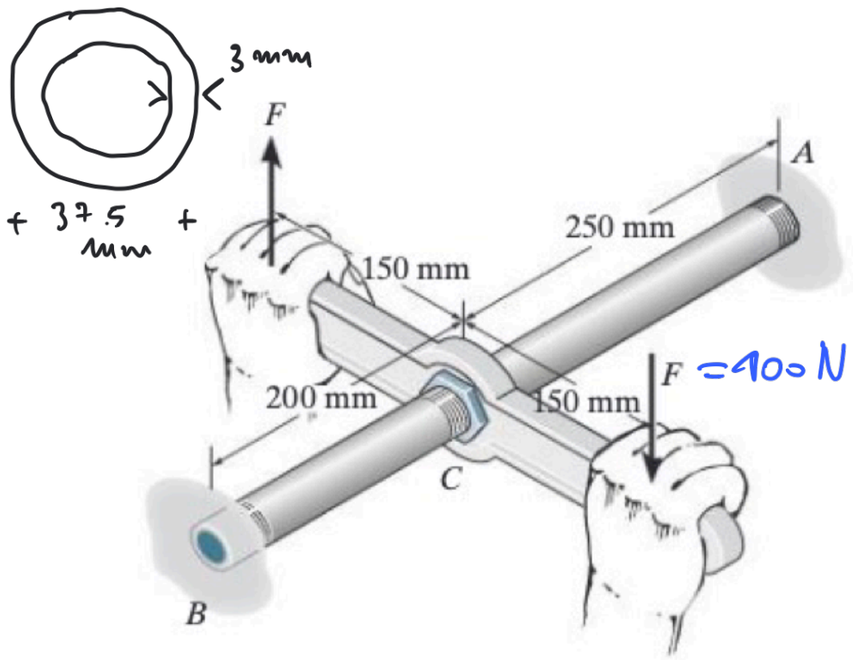
$T_{CA} = T_A$

$$0 = \phi = \int_0^L \frac{T}{GJ}\,dx = \frac{1}{GJ}\left(T_{BC}L_{BC} + T_{CA}L_{CA}\right)$$

$$\Rightarrow T_{BC}L_{BC} + T_{CA}L_{CA} = 0$$

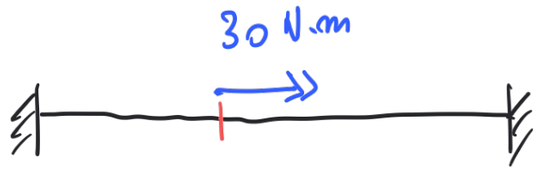
$$T_A \cdot 450\,mm + 30\,Nm \cdot 200\,mm = 0 \Rightarrow T_A = -\frac{200}{450}\,30\,N\cdot m = -13.3\,Nm$$

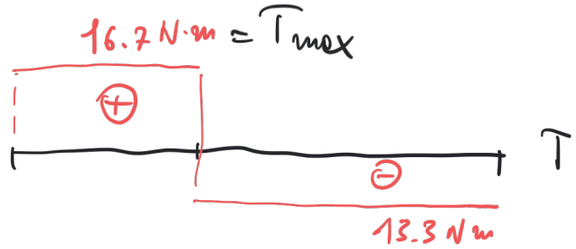
Torsione di travi staticamente indeterminate.

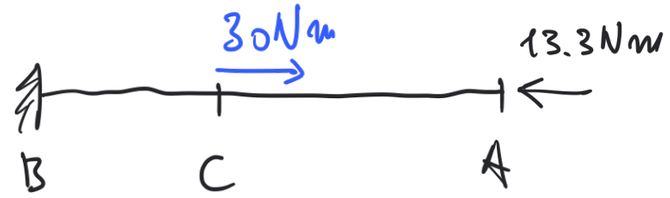


Problema 9.28

Determinare la massima tensione tangenziale







$$J = \frac{\pi}{2}\left(R_e^4 - R_i^4\right)$$

$$\tau_{max} = R_e \frac{T_{max}}{J}$$

$$T_{BC} = T_A + 30\,Nm$$

$$d\phi = \frac{T}{GJ}\,dx$$

$$T_{CA} = T_A$$

$$0 = \phi = \int_0^L \frac{T}{GJ}\,dx = \frac{1}{GJ}\left(T_{BC}L_{BC} + T_{CA}L_{CA}\right)$$

$$\Rightarrow T_{BC}L_{BC} + T_{CA}L_{CA} = 0$$

$$T_A \cdot 450\,mm + 30\,Nm \cdot 200\,mm = 0 \Rightarrow T_A = -\frac{200}{450}\,30\,N\cdot m = -13.3\,Nm$$