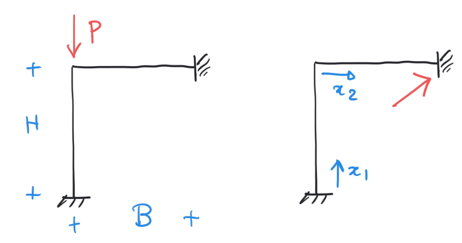
CARICO CRITICO IN UN TELAIO



Problema incrementale:

$EI v_1^{IV} + P v_1'' = 0$ in $(0, H)$

$EI v_2^{IV} = 0$ in $(0, B)$

Soluz. generale

$$v_1(x_1) = A_1 \cos\left(\sqrt{\frac{P}{EI}}\, x_1\right) + A_2 \sin\left(\sqrt{\frac{P}{EI}}\, x_1\right) + A_3 + A_4 x_1$$

$$v_2(x_2) = B_1 x_2^3 + B_2 x_2^2 + B_3 x_2 + B_4$$

Condiz. contorno/raccordo

$v_1(0) = 0 \qquad v_1'(0) = 0$

$v_1(H) = 0 \qquad v_1'(H) = v_2'(0)$

$v_1''(H) = v_2''(0)$

$v_2(0) = 0$

$v_2(B) = 0 \qquad v_2'(B) = 0$

Sostituendo la soluzione generale nelle condiz. di raccordo si trova:

$A_1 + A_3 = 0 \qquad A_4 + A_2 \sqrt{\frac{P}{EI}} = 0$

$A_3 + A_4 H + A_1 \cos\left(H\sqrt{\frac{P}{EI}}\right) + A_2 \sin\left(H\sqrt{\frac{P}{EI}}\right) = 0$

$A_4 + \sqrt{\frac{P}{EI}}\left(A_2 \cos\left(H\sqrt{\frac{P}{EI}}\right) - A_1 \sin\left(H\sqrt{\frac{P}{EI}}\right)\right) = B_3$

$-\dfrac{P\left(A_1 \cos\left(H\sqrt{\frac{P}{EI}}\right) + A_2 \sin\left(H\sqrt{\frac{P}{EI}}\right)\right)}{EI} = 2B_2$

$B_4 = 0$

$B^2(B_1 + B_2) + B B_3 + B_4 = 0$

$3B^2 B_1 + 2B B_2 + B_3 = 0$

Matrice del sistema:

$$M = \begin{pmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & \sqrt{\frac{P}{EI}} & 0 & 1 & 0 & 0 & 0 & 0 \\
\cos\left[H\sqrt{\frac{P}{EI}}\right] & \sin\left[H\sqrt{\frac{P}{EI}}\right] & 1 & H & 0 & 0 & 0 & 0 \\
-\sqrt{\frac{P}{EI}}\sin\left[H\sqrt{\frac{P}{EI}}\right] & \sqrt{\frac{P}{EI}}\cos\left[H\sqrt{\frac{P}{EI}}\right] & 0 & 1 & 0 & 0 & -1 & 0 \\
-\frac{P\cos\left[H\sqrt{\frac{P}{EI}}\right]}{EI} & -\frac{P\sin\left[H\sqrt{\frac{P}{EI}}\right]}{EI} & 0 & 0 & 0 & -2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & B^3 & B^2 & B & 1 \\
0 & 0 & 0 & 0 & 3B^2 & 2B & 1 & 0
\end{pmatrix}$$

$$\det M = \frac{B^3\left(-8EI\sqrt{\frac{P}{EI}} + \sqrt{\frac{P}{EI}}\,(8EI + BHP)\cos\left[H\sqrt{\frac{P}{EI}}\right] - (B - 4H)\,P\sin\left[H\sqrt{\frac{P}{EI}}\right]\right)}{EI}$$

$\sqrt{\frac{P}{EI}}\, H = \alpha$

$$\boxed{-8 + 8\cos\alpha + \frac{B}{H}\alpha^2\cos\alpha + 4\alpha\sin\alpha - \frac{B}{H}\alpha\sin\alpha = 0}$$

$\left(8 + \frac{B}{H}\alpha^2\right)\cos\alpha + \alpha\left(4 - B/H\right)\sin\alpha = 8$

Si noti che per $B \to \infty$ si recupera il risultato noto:

$B \to \infty \qquad \alpha\cos\alpha = \sin\alpha \qquad \alpha = \tan\alpha \qquad \alpha \simeq 4.5$

$P_c = \dfrac{\alpha^2 EI}{H^2} = \dfrac{\pi^2 EI}{H^2}\dfrac{\alpha^2}{\pi^2} \simeq \dfrac{\pi^2 EI}{(0.7H)^2}$