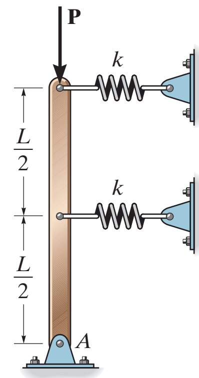
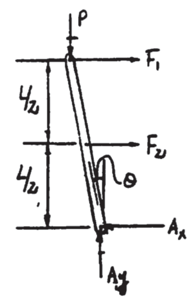
13–1.

Determine the critical buckling load for the column. The material can be assumed rigid.



SOLUTION

$$F_1 = k(L\,\theta); \qquad F_2 = k\left(\frac{L}{2}\,\theta\right)$$



$$\zeta+\Sigma M_A = 0; \qquad P(\theta)(L) - (F_1 L) - F_2\left(\frac{L}{2}\right) = 0$$

$$P(\theta)(L) - kL^2\,\theta - k\left(\frac{L}{2}\right)^2\theta = 0$$

Require:

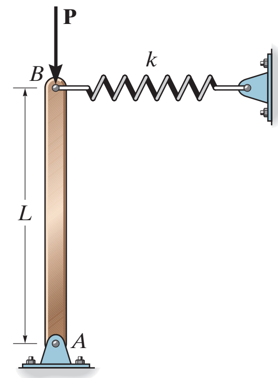
$$P_{cr} = kL + \frac{kL}{4} = \frac{5kL}{4} \qquad \textbf{Ans.}$$

Ans:

$$P_{cr} = \frac{5KL}{4}$$

13–2.

The column consists of a rigid member that is pinned at its bottom and attached to a spring at its top. If the spring is unstretched when the column is in the vertical position, determine the critical load that can be placed on the column.



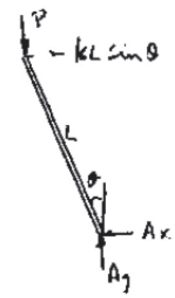
SOLUTION

$\circlearrowleft + \Sigma M_A = 0;\qquad PL\sin\theta - (kL\sin\theta)(L\cos\theta) = 0$

$$P = kL\cos\theta$$

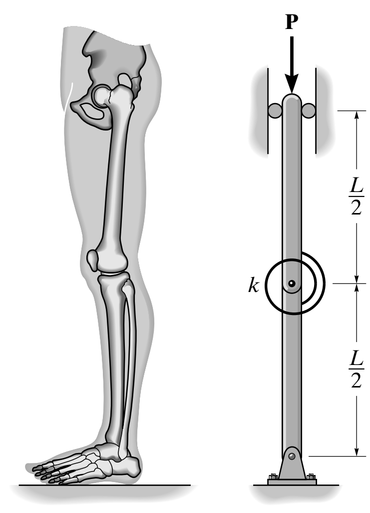
Since θ is small $\qquad \cos\theta = 1$

$$P_{cr} = kL \qquad \textbf{Ans.}$$



Ans:
$P_{cr} = kL$

13–3. The leg in (a) acts as a column and can be modeled (b) by the two pin-connected members that are attached to a torsional spring having a stiffness k (torque/rad). Determine the critical buckling load. Assume the bone material is rigid.



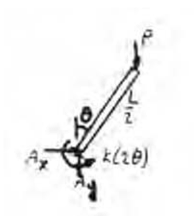
(a) (b)

SOLUTION

$\circlearrowleft + \Sigma M_A = 0; \qquad -P(\theta)\left(\frac{L}{2}\right) + 2\,k\,\theta = 0$

Require:

$$P_{cr} = \frac{4\,k}{L}$$ **Ans.**

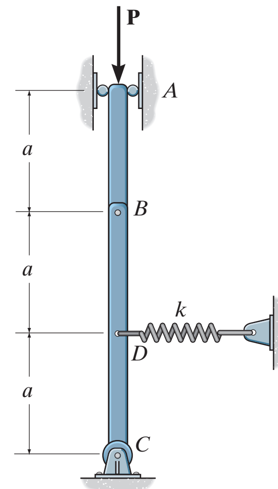


Ans:

$P_{cr} = \frac{4\,k}{L}$

***13–4.**

Rigid bars AB and BC are pin connected at B. If the spring at D has a stiffness k, determine the critical load P_{cr} that can be applied to the bars.



SOLUTION

Equilibrium: The disturbing force F can be related to P by considering the equilibrium of joint A and then the equilibrium of member BC.

Joint A (Fig. b)

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB}\cos\phi - P = 0 \qquad F_{AB} = \frac{P}{\cos\phi}$$

Member BC (Fig. c)

$$\Sigma M_C = 0; F(a\cos\theta) - \frac{P}{\cos\phi}\cos\phi\,(2a\sin\theta) - \frac{P}{\cos\phi}\sin\phi(2a\cos\theta) = 0$$

$$F = 2P(\tan\theta + \tan\phi)$$

Since θ and ϕ are small, $\tan\theta \cong \theta$ and $\tan\phi \cong \phi$. Thus,

$$F = 2P(\theta + \phi) \qquad \textbf{(1)}$$

Also, from the geometry shown in Fig. a,

$$2a\theta = a\phi \qquad \phi = 2\theta$$

Thus Eq. (1) becomes

$$F = 2P(\theta + 2\theta) = 6P\theta$$

Spring Force: The restoring spring force F_{sp} can be determined using the spring formula, $F_{sp} = kx$, where $x = a\theta$, Fig. a. Thus,

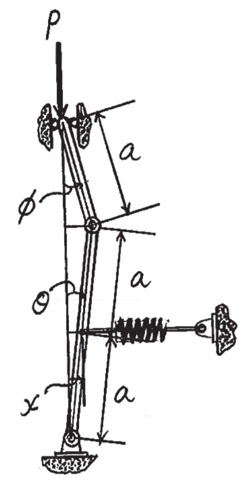
$$F_{sp} = kx = ka\theta$$

Critical Buckling Load: When the mechanism is on the verge of buckling the disturbing force F must be equal to the restoring spring force F_{sp}.

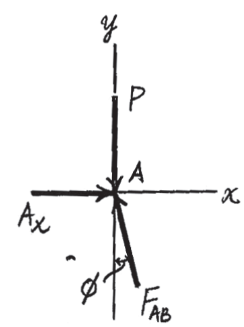
$$F = F_{sp}$$

$$6P_{cr}\theta = ka\theta$$

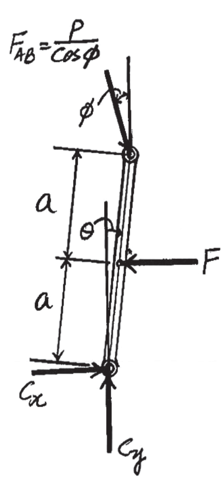
$$P_{cr} = \frac{ka}{6} \qquad \textbf{Ans.}$$



(a)



(b)



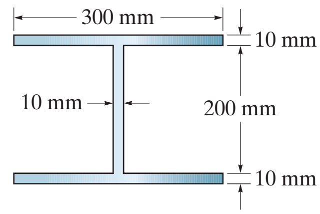
(c)

Ans:

$P_{cr} = \frac{ka}{6}$

13–5.

A 2014-T6 aluminum alloy column has a length of 6 m and is fixed at one end and pinned at the other. If the cross-sectional area has the dimensions shown, determine the critical load. $\sigma_Y = 250$ MPa.



SOLUTION

Section Properties: For the cross-section shown

$$A = 0.3(0.22) - 0.29(0.2) = 8.00\left(10^{-3}\right)\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.3)\left(0.22^3\right) - \frac{1}{12}(0.29)\left(0.2^3\right) = 72.8667\left(10^{-6}\right)\ \text{m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.01)\left(0.3^3\right)\right] + \frac{1}{12}(0.2)\left(0.01^3\right) = 45.0167\left(10^{-6}\right)\ \text{m}^4 \text{ (controls)}$$

Critical Buckling Load: $K = 0.7$ for the column with one end fixed and the other end pinned. For 2014-T6 aluminium alloy, $E = 73.1$ GPa. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2\left[73.1\left(10^9\right)\right]\left[45.0167\left(10^{-6}\right)\right]}{[0.7(6)]^2}$$

$$= 1.8412\left(10^6\right)\ \text{N} = 1.84\ \text{MN} \qquad \textbf{Ans.}$$

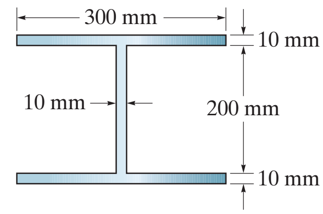
Critical Stress: Euler's formula valid only if $\sigma_{cr} < \sigma_Y$. For 2014-T6 aluminium alloy, $\sigma_Y = 414$ MPa.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1.8412\left(10^6\right)}{8.00\left(10^{-3}\right)} = 230.15\ \text{MPa} < \sigma_Y. \qquad \textbf{(O.K!)}$$

Ans:
$P_{cr} = 1.84$ MN

13–6.

Solve Prob. 13–5 if the column is pinned at its top and bottom.



SOLUTION

Section Properties: For the cross-section shown

$$A = 0.3(0.22) - 0.29(0.2) = 8.00\left(10^{-3}\right)\text{ m}^2$$

$$I_x = \frac{1}{12}(0.3)\left(0.32^3\right) - \frac{1}{12}(0.29)\left(0.2^3\right) = 72.8667\left(10^{-6}\right)\text{ m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.01)\left(0.3^3\right)\right] + \frac{1}{12}(0.2)\left(0.01^3\right) = 45.0167\left(10^{-6}\right)\text{ m}^4 \qquad \text{(controls!)}$$

Critical Buckling Load: $K = 1.0$ for the column pinned at both ends. For 2014-T6 aluminium alloy, $E = 73.1$ GPa. Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2\left[73.1\left(10^9\right)\right]\left[45.0167\left(10^{-6}\right)\right]}{[1.0(6)]^2}$$

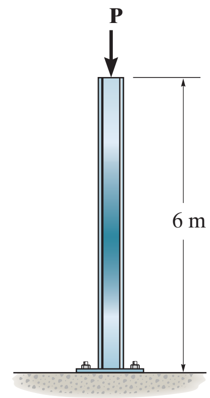
$$= 902.17\left(10^3\right)\text{ N} = 902\text{ kN} \qquad \textbf{Ans.}$$

Critical Stress: Euler's formula valid only if $\sigma_{\text{cr}} < \sigma_Y$. For 2014-T6 aluminium alloy, $\sigma_Y = 414$ MPa.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{902.17\left(10^3\right)}{8.00\left(10^{-3}\right)} = 112.77\text{ MPa} < \sigma_Y \qquad \textbf{(O.K!)}$$

Ans:
$P_{\text{cr}} = 902$ kN

13–7. The W360 × 57 column is made of A-36 steel and is fixed supported at its base. If it is subjected to an axial load of $P = 75$ kN, determine the factor of safety with respect to buckling.



SOLUTION

From the table in appendix, the cross-sectional area and moment of inertia about weak axis (y-axis) for W360 × 57 are

$$A = 7200 \text{ mm}^2 = 7.2(10^{-3}) \text{ m}^2 \qquad I_y = 11.1(10^6) \text{ mm}^4 = 11.1(10^{-6}) \text{ m}^4$$

The column is fixed at its base and free at top, $k = 2$. Here, the column will buckle about the weak axis (y axis). For A36 steel, $E = 200$ GPa and $\alpha_Y = 250$ MPa. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)^2} = \frac{\pi^2[200(10^9)][11.1(10^{-6})]}{[2(6)]^2} = 152.16(10^3) \text{ N} = 152.16 \text{ kN}$$

Thus, the factor of safety with respect to buckling is

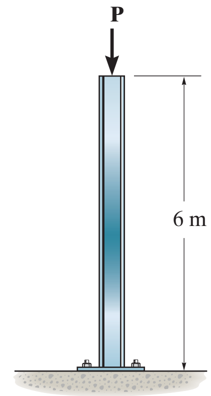
$$F.S = \frac{P_{cr}}{P} = \frac{152.16}{75} = 2.03 \qquad \textbf{Ans.}$$

The Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{152.16(10^3)}{7.2(10^{-3})} = 21.13(10^6) \text{ N/m}^2 = 21.1 \text{ MPa} < \sigma_Y$$
$$= 250 \text{ MPa} \qquad \text{O.K.}$$

Ans:
F.S. = 2.03

***13–8.** The W360 × 57 column is made of A-36 steel. Determine the critical load if its bottom end is fixed supported and its top is free to move about the strong axis and is pinned about the weak axis.



SOLUTION

From the table in appendix, the cross-sectional area and moment of inertia about weak axis (y-axis) for W360 × 57 are

$$A = 7200 \text{ mm}^2 = 7.2(10^{-3}) \text{ m}^2$$

$$I_x = 160(10^6) \text{ mm}^4 = 160(10^{-6}) \text{ m}^4 \qquad I_y = 11.1(10^6) \text{ mm}^4 = 11.1(10^{-6}) \text{ m}^4$$

The column is fixed at its base and free at top about strong axis. Thus, $k_x = 2$. For A36 steel, $E = 200$ GPa and $\sigma_Y = 250$ MPa.

$$P_{cr} = \frac{\pi^2 E I_x}{(K_x L_x)^2} = \frac{\pi^2[200(10^9)][160(10^{-6})]}{[2(6)]^2} = 2.193(10^6) \text{ N} = 2.193 \text{ MN}$$

The column is fixed at its base and pinned at top about weak axis. Thus, $k_y = 0.7$.

$$P_{cr} = \frac{\pi^2 E I_y}{(K_y L_y)^2} = \frac{\pi^2[200(10^9)][11.1(10^{-6})]}{[0.7(6)]^2}$$

$$= 1.242(10^6) \text{ N} = 1.24 \text{ MN} \quad (\textit{Control})$$ **Ans.**

The Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

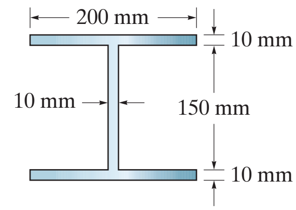
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1.242(10^6)}{7.2(10^{-3})} = 172.51(10^6) \text{ N/m}^2 = 172.51 \text{ MPa} < \sigma_Y$$

$$= 250 \text{ MPa} \qquad \text{O.K.}$$

Ans:
$P_{cr} = 1.24$ MN (*Control*)

13–9.

A steel column has a length of 9 m and is fixed at both ends. If the cross-sectional area has the dimensions shown, determine the critical load. $E_{st} = 200$ GPa, $\sigma_Y = 250$ MPa.



SOLUTION

Section Properties:

$$A = 0.2(0.17) - 0.19(0.15) = 5.50\left(10^{-3}\right)\text{ m}^2$$

$$I_x = \frac{1}{12}(0.2)\left(0.17^3\right) - \frac{1}{12}(0.19)\left(0.15^3\right) = 28.44583\left(10^{-6}\right)\text{ m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.01)\left(0.2^3\right)\right] + \frac{1}{12}(0.15)\left(0.01^3\right) = 13.34583\left(10^{-6}\right)\text{ m}^4 \quad \text{(Controls!)}$$

Critical Buckling Load: $K = 0.5$ for fixed support ends column. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2 (200)\left(10^9\right)(13.34583)\left(10^{-6}\right)}{[0.5(9)]^2}$$

$$= 1300919\text{ N} = 1.30\text{ MN} \qquad \textbf{Ans.}$$

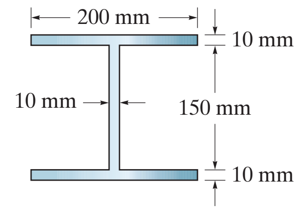
Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1300919}{5.50\left(10^{-3}\right)} = 236.53\text{ MPa} < \sigma_Y = 250\text{ MPa} \qquad \textbf{(O.K!)}$$

Ans:
$P_{cr} = 1.30$ MN

13–10.

A steel column has a length of 9 m and is pinned at its top and bottom. If the cross-sectional area has the dimensions shown, determine the critical load. $E_{st} = 200$ GPa, $\sigma_Y = 250$ MPa.



SOLUTION

Section Properties:

$$A = 0.2(0.17) - 0.19(0.15) = 5.50\left(10^{-3}\right)\text{ m}^2$$

$$I_x = \frac{1}{12}(0.2)\left(0.17^3\right) - \frac{1}{12}(0.19)\left(0.15^3\right) = 28.44583\left(10^{-6}\right)\text{ m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.01)\left(0.2^3\right)\right] + \frac{1}{12}(0.15)\left(0.01^3\right) = 13.34583\left(10^{-6}\right)\text{ m}^4 \text{ (Controls!)}$$

Critical Buckling Load: $K = 1$ for pin supported ends column.
Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2 (200)\left(10^9\right)(13.34583)\left(10^{-6}\right)}{[1(9)]^2}$$

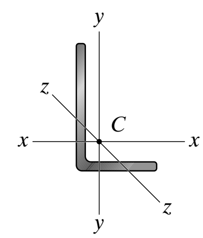
$$= 325229.87\text{ N} = 325\text{ kN}$$ **Ans.**

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{325229.87}{5.50\left(10^{-3}\right)} = 59.13\text{ MPa} < \sigma_Y = 250\text{ MPa}$$ **(O.K!)**

Ans:
$P_{cr} = 325$ kN

13–11. The A-36 steel angle has a cross-sectional area of $A = 1550\ \text{mm}^2$ and a radius of gyration about the x axis of $r_x = 31.5$ mm and about the y axis of $r_y = 21.975$ mm. The smallest radius of gyration occurs about the z axis and is $r_z = 16.1$ mm. If the angle is to be used as a pin-connected 3-m long column, determine the largest axial load that can be applied through its centroid C without causing it to buckle.



SOLUTION

The least radius of gyration:

$\pi_z = 16.1\ \text{mm} = 0.0161\ \text{m}$ controls.

$$\sigma_{\text{cr}} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}; \qquad K = 1.0$$

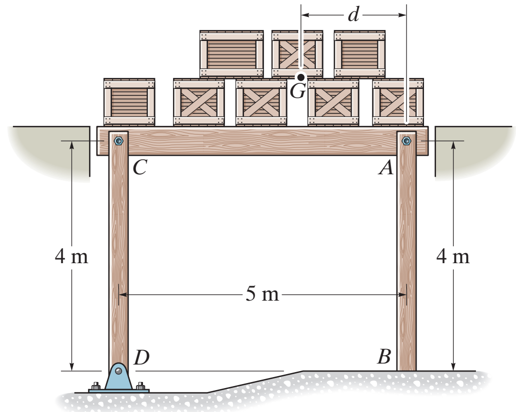
$$= \frac{\pi^2[200(10^9)]}{\left[\frac{1.0(3)}{0.0161}\right]^2} = 56.85(10^6)\ \text{N/m}^2 = 56.85\ \text{MPa} < \sigma_Y = 250\ \text{MPa} \qquad \text{O.K.}$$

$$P_{c\pi} = \sigma_{c\pi} A = [56.85(10^6)][1.55(10^{-3})] = 88.12(10^3)\ \text{N} = 88.1\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$P_{cr} = 88.1$ kN

***13–12.**

The deck is supported by the two 40-mm-square columns. Column AB is pinned at A and fixed at B, whereas CD is pinned at C and D. If the deck is prevented from sidesway, determine the greatest weight of the load that can be applied without causing the deck to collapse. The center of gravity of the load is located at $d = 2$ m. Both columns are made from Douglas Fir.



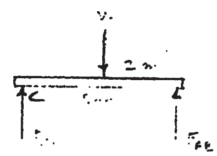
SOLUTION

$$\curvearrowleft+\Sigma M_C = 0; \qquad F_{AB}(5) - W(3) = 0$$

$$F_{AB} = 0.6\,W$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{CD} + 0.6\,W - W = 0$$

$$F_{CD} = 0.4\,W$$



Column CD:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2 (13.1)(10^9)\left(\frac{1}{12}\right)(0.04)^4}{(1(4))^2} = 0.4\,W$$

$$W = 4.31 \text{ kN}$$

Column AB:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2 (13.1)(10^9)\left(\frac{1}{12}\right)(0.04)^4}{(0.7(4))^2} = 0.6\,W$$

$$W = 5.86 \text{ kN}$$

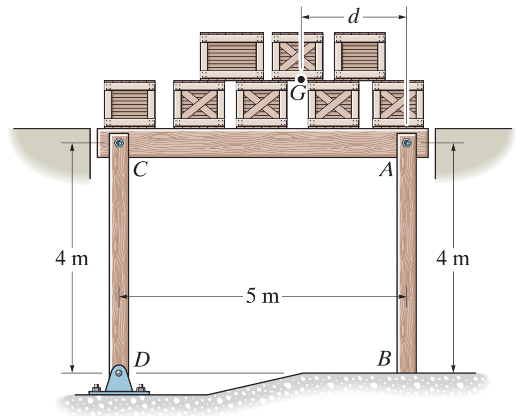
Thus,

$$W = 4.31 \text{ kN}$$ **Ans.**

Ans:
$W = 4.31$ kN

13–13.

The deck is supported by the two 40-mm-square columns. Column AB is pinned at A and fixed at B, whereas CD is pinned at C and D. If the deck is prevented from sidesway, determine the position d of the center of gravity of the load and the load's greatest magnitude without causing the deck to collapse. Both columns are made from Douglas Fir.

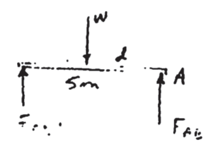


SOLUTION

Column CD:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$F_{CD} = \frac{\pi^2(13.1)(10^9)\left(\frac{1}{12}\right)(0.04)^4}{(1.0(4))^2} = 1.7239(10^3)\text{ N}$$



Column AB:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$F_{AB} = \frac{\pi^2(13.1)(10^9)\left(\frac{1}{12}\right)(0.04)^4}{(0.7(4))^2} = 3.5181(10^3)\text{ N}$$

Thus,

$+\uparrow\Sigma F_y = 0;\qquad 1.7239(10^3) + 3.5181(10^3) - W = 0$

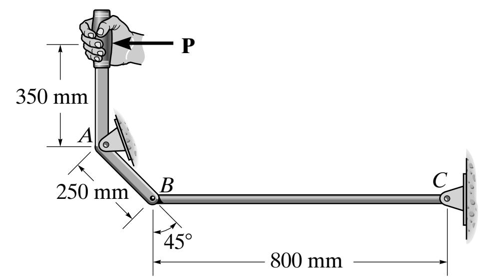
$W = 5.2420(10^3)\text{ N} = 5.24\text{ kN}$ **Ans.**

$\circlearrowleft+\Sigma M_A = 0;\qquad 5.2420(10^3)(d) - 1.7239(10^3)(5) = 0$

$d = 1.64\text{ m}$ **Ans.**

Ans:
$W = 5.24$ kN,
$d = 1.64$ m

13–14. Determine the maximum force P that can be applied to the handle so that the A-36 steel control rod BC does not buckle. The rod has a diameter of 25 mm.



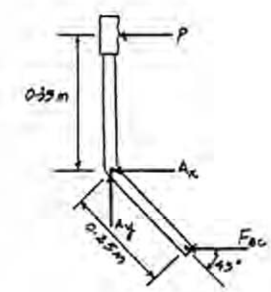
SOLUTION

Support Reactions:

$$\circlearrowleft +\Sigma M_A = 0; \qquad P(0.35) - F_{BC} \sin 45°(0.25) = 0$$

$$F_{BC} = 1.9799P$$

Section Properties:



$$A = \frac{\pi}{4}\left(0.025^2\right) = 0.15625\left(10^{-3}\right)\pi \text{ m}^2$$

$$I = \frac{\pi}{4}\left(0.0125^4\right) = 19.17476\left(10^{-9}\right) \text{ m}^4$$

Critical Buckling Load: $K = 1$ for a column with both ends pinned. Appyling *Euler's* formula,

$$P_{cr} = F_{BC} = \frac{\pi^2 EI}{(KL_{BC})^2}$$

$$1.9799P = \frac{\pi^2(200)(10^9)\left[19.17476(10^{-9})\right]}{[1(0.8)]^2}$$

$$P = 29\,870 \text{ N} = 29.9 \text{ kN} \qquad \textbf{Ans.}$$

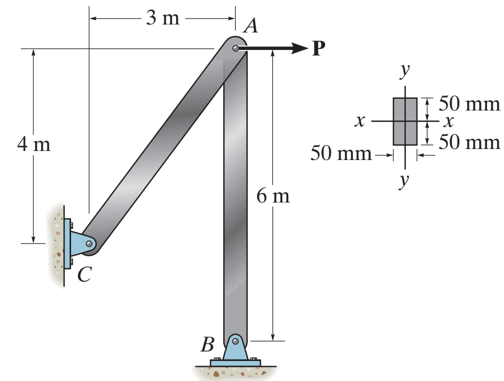
Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1.9799(29\,870)}{0.15625(10^{-3})\pi} = 120.5 \text{ MPa} < \sigma_Y = 250 \text{ MPa} \qquad \text{O.K.}$$

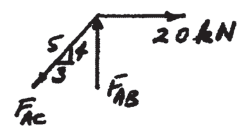
Ans:
$P = 29.9$ kN

13–15.

Determine the maximum load P the frame can support without buckling member AB. Assume that AB is made of steel and is pinned at its ends for y–y axis buckling and fixed at its ends for x–x axis buckling. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION



$\xrightarrow{+} \Sigma F_x = 0; \qquad -F_{AC}\left(\frac{3}{5}\right) + P = 0$

$$F_{AC} = \frac{5}{3}P$$

$+\uparrow \Sigma F_y = 0; \qquad F_{AB} - \frac{5}{3}P\left(\frac{4}{5}\right) = 0$

$$F_{AB} = \frac{4}{3}P$$

$$I_y = \frac{1}{12}(0.10)(0.05)^3 = 1.04167\left(10^{-6}\right)\text{m}^4$$

$$I_x = \frac{1}{12}(0.05)(0.10)^3 = 4.16667\left(10^{-6}\right)\text{m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

x–x axis buckling:

$$P_{cr} = \frac{\pi^2(200)\left(10^9\right)(4.16667)\left(10^{-6}\right)}{(0.5(6))^2} = 914 \text{ kN}$$

y–y axis buckling:

$$P_{cr} = \frac{\pi^2(200)\left(10^9\right)(1.04167)\left(10^{-6}\right)}{(1(6))^2} = 57.12 \text{ kN}$$

y–y axis buckling controls

$$\frac{4}{3}P = 57.12$$

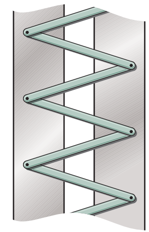
$P = 42.8$ kN **Ans.**

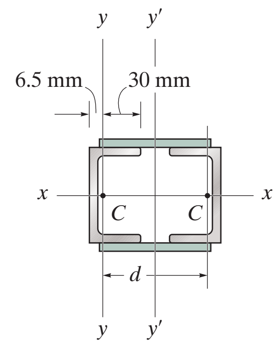
Check:

$\sigma_{cr} = \frac{P}{A} = \frac{57.12\left(10^3\right)}{(0.1)(0.05)} = 11.4 \text{ MPa} < \sigma_Y$ **OK**

Ans:
$P = 42.8$ kN

***13–16.** The two steel channels are to be laced together to form a 9-m long bridge column assumed to be pin connected at its ends. Each channel has a cross-sectional area of $A = 1950\ \text{mm}^2$ and moments of inertia $I_x = 21.60(10^6)\ \text{mm}^4$, $I_y = 0.15(10^6)\ \text{mm}^4$. The centroid C of its area is located in the figure. Determine the proper distance d between the centroids of the channels so that buckling occurs about the x–x and $y'-y'$ axes due to the same load. What is the value of this critical load? Neglect the effect of the lacing. $E_{\text{st}} = 200$ GPa, $\sigma_Y = 350$ MPa.





SOLUTION

$$I_x = 2[21.60(10^{-6})\ \text{m}^4] = 43.2(10^{-6})\ \text{m}^4$$

$$I_y = 2[0.15(10^{-6}) + 1.95(10^{-3})(d/2)^2] = 0.3(10^{-6}) + 0.975(10^{-3})d^2$$

In order for the column to buckle about $x - x$ and $y - y$ at the same time, I_y must be equal to I_x

$$I_y = I_x$$

$$0.3(10^{-6}) + 0.975(10^{-3})d^2 = 43.2(10^{-6})$$

$$d = 0.20976\ \text{m} = 210\ \text{mm}$$ **Ans.**

Check:

$$d > 2(30) = 60\ \text{mm}$$ O.K.

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2[200(10^9)][43.2(10^{-6})]}{[1(9)]^2}$$

$$= 1.053(10^6)\ \text{N} = 1.05\ \text{MN}$$ **Ans.**

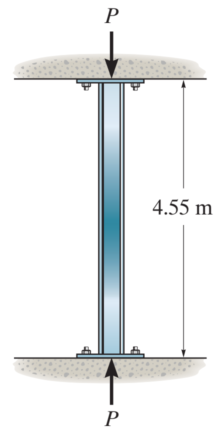
Check stress:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1.053(10^6)}{2[1.95(10^{-3})]} = 269.94(10^6)\ \text{N/m}^2 = 270\ \text{MPa} < \sigma_Y$$

Therefore, Euler's formula is valid.

Ans:
$P_{cr} = 1.05$ MN

13–17. The W250 × 67 is made of A992 steel and is used as a column that has length of 4.55 m. If its ends are assumed pin supported, and it is subjected to an axial load of 500 kN, determine the factor of safety with respect to buckling.



SOLUTION

Critical Buckling Load:

$I_y = 22.2(10^6)\ \text{mm}^4 = 22.2(10^{-6})\ \text{m}^4$ foπ a W250 × 67 wide flange section and $K = 1$ for pin supported ends column. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2[200(10^9)][22.2(10^{-6})]}{[1(4.55)]^2}$$

$$= 2116.70(10^3)\ \text{N} = 2116.70\ \text{kN}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.
$A = 8560\ \text{mm}^2 = 8.56(10^{-3})\ \text{m}^2$ for the W250 × 67 wide-flange section.

$$\sigma_{c\pi} = \frac{P_{c\pi}}{A} = \frac{2116.70(10^3)}{8.56(10^{-3})} = 247.28(10^6)\ \text{N-m}^2$$

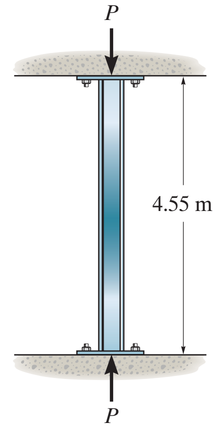
$$= 247.28\ \text{MPa} < \sigma_Y = 345\ \text{MPa} \qquad \text{O.K.}$$

Factor of Safety:

$$\text{F.S.} = \frac{P_{c\pi}}{A} = \frac{2116.70}{500} = 4.23 \qquad \textbf{Ans.}$$

Ans:
F.S. = 4.23

13–18. The W250 × 67 is made of A-36 steel and is used as a column that has length of 4.55 m. If the ends of the column are fixed supported, can the column support the critical load without yielding?



SOLUTION

Critical Buckling Load:

$I_y = 22.2(10^6)\ \text{mm}^4 = 22.2(10^{-6})\ \text{m}^4$ foπ a W250 × 67 wide flange section and $K = 0.5$ for fixed ends support column. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2[200(10^9)][22.2(10^{-6})]}{[0.5(4.55)]^2}$$

$$= 8.4668(10^6)\ \text{N} = 8.47\ \text{MN}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.
$A = 8560\ \text{mm}^2 = 8.56(10^{-3})\ \text{m}^2$ for the W250 × 67 wide-flange section.

$$\sigma_{c\pi} = \frac{P_{c\pi}}{A} = \frac{8.4668(10^6)}{8.56(10^{-3})} = 989.11(10^6)\ \text{N}\cancel{\text{m}}^2$$

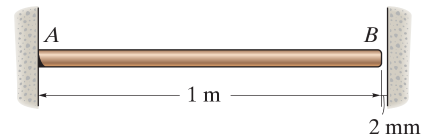
$$= 989\ \text{MPa} < \sigma_Y = 345\ \text{MPa} \quad (No!) \qquad \textbf{Ans.}$$

The column will yield before the axial force achieves the critical load P_{cr} and so *Euler's* formula is not valid.

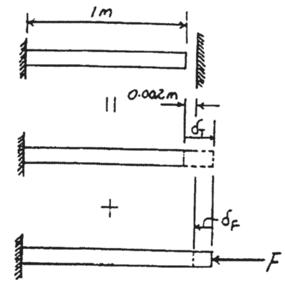
Ans:
$\sigma_{cr} = 345\ \text{MPa}\ (No!)$

13–19.

The 50-mm-diameter C86100 bronze rod is fixed supported at A and has a gap of 2 mm from the wall at B. Determine the increase in temperature ΔT that will cause the rod to buckle. Assume that the contact at B acts as a pin.



SOLUTION



Section Properties:

$$A = \frac{\pi}{4}\left(0.05^2\right) = 0.625\pi\left(10^{-3}\right)\text{ m}^2$$

$$I = \frac{\pi}{4}\left(0.025^4\right) = 97.65625\pi\left(10^{-9}\right)\text{ m}^4$$

Compatibility Condition: This requires

$$(\overset{+}{\rightarrow}) \qquad 0.002 = \delta_T + \delta_F$$

$$0.002 = 17\left(10^{-6}\right)(\Delta T)(1) - \frac{F(1)}{0.625\pi\left(10^{-3}\right)(103)\left(10^9\right)}$$

$$F = 3438.08\Delta T - 404480.05$$

Critical Buckling Load: $K = 0.7$ for a column with one end fixed and the other end pinned. Applying *Euler's* formula,

$$P_{cr} = F = \frac{\pi^2 EI}{(KL)^2}$$

$$3438.08\Delta T - 404\,480.05 = \frac{\pi^2\,(103)\left(10^9\right)\left[97.65625\pi\left(10^{-9}\right)\right]}{[0.7(1)]^2}$$

$$\Delta T = 302.78°\text{C} = 303°\text{C} \qquad \textbf{Ans.}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$P_{cr} = 3438.08(302.78) - 404\,480.05 = 636\,488.86\text{ N}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{636\,488.86}{0.625\pi\left(10^{-3}\right)} = 324.2\text{ MPa} < \sigma_Y = 345\text{ MPa} \qquad \textbf{(O.K!)}$$

Ans:
$\Delta T = 303°\text{C}$

***13–20.**

An A992 steel W200 $\times$ 46 column of length 9 m is fixed at one end and free at its other end. Determine the allowable axial load the column can support if F.S. = 2 against buckling.

SOLUTION

Section Properties: From the table listed in the appendix, the cross-sectional area and moment of inertia about the y axis for a W200 $\times$ 46 are

$$A = 5890\ \text{mm}^2 = 5.89(10^{-3})\ \text{m}^2$$

$$I_y = 15.3(10^6)\ \text{mm}^4 = 15.3(10^{-6})\ \text{m}^4$$

Critical Buckling Load: The column will buckle about the weak (y) axis. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)^2} = \frac{\pi^2[200(10^9)][15.3(10^{-6})]}{[2(9)]^2} = 93.21\ \text{kN}$$

Thus, the allowable centric load is

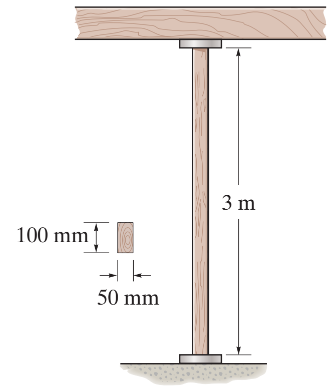
$$P_{allow} = \frac{P_{cr}}{\text{F.S.}} = \frac{93.21}{2} = 46.61\ \text{kN} = 46.6\ \text{kN} \qquad \textbf{Ans.}$$

Critical Stress: Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{93.21(10^3)}{5.89(10^{-3})} = 15.83\ \text{MPa} < \sigma_Y = 345\ \text{MPa} \qquad \textbf{(O.K.)}$$

Ans:
$P_{allow} = 46.6$ kN

13–21. The 3-m wooden rectangular column has the dimension shown. Determine the critical load if the ends are assumed to be pin connected. $E_w = 12$ GPa, $\sigma_Y = 35$ MPa.



SOLUTION

Section Properties:

$$A = 0.1(0.05) = 0.005\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.05)(0.1^3) = 4.1667(10^{-6})\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.1)(0.05^3) = 1.04167(10^{-6})\ \text{m}^4 \quad (\textit{Controls!})$$

Critical Buckling Load: $K = 1$ for pin supported ends column. Applying *Euler's* formula,.

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2[12(10^9)][1.04167(10^{-6})]}{[1(3)]^2}$$

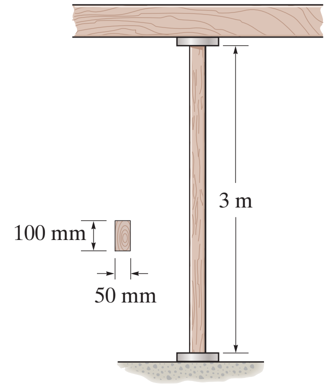
$$= 13.71(10^3)\ \text{N} = 13.7\ \text{kN} \qquad \textbf{Ans.}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{\text{cr}} < \sigma_Y$.

$$\sigma_{c\pi} = \frac{P_{c\pi}}{A} = \frac{13.71(10^3)}{0.005} = 2.742(10^6)\ \text{N/m}^2 = 2.74\ \text{MPa} < \sigma_Y = 35\ \text{MPa} \qquad \text{O.K.}$$

Ans:
$A = 5000\ \text{mm}^2$, $I_x = 4.1667\ (10^6)\ \text{mm}^4$,
$I_y = 1.041667\ (10^6)\ \text{mm}^4$, $P_{\text{cr}} = 13.7\ \text{kN}$

13–22. The 3-m column has the dimensions shown. Determine the critical load if the bottom is fixed and the top is pinned. $E_w = 12$ GPa, $\sigma_Y = 35$ MPa.



SOLUTION

Section Properties:

$A = 0.1(0.05) = 0.005 \text{ m}^2$

$I_x = \frac{1}{12}(0.05)(0.1^3) = 4.1667(10^{-6}) \text{ m}^4$

$I_y = \frac{1}{12}(0.1)(0.05^3) = 1.04167(10^{-6}) \text{ m}^4$ (*Controls!*)

Critical Buckling Load: $K = 0.7$ for column with one end fixed and the other end pinned. Applying *Euler's* formula.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2[12(10^9)][1.04167(10^{-6})]}{[0.7(3)]^2}$$

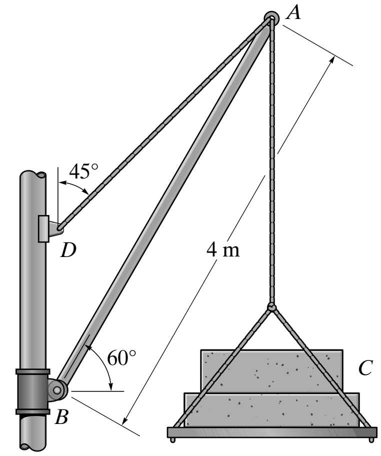
$$= 27.98(10^3) \text{ N} = 28.0 \text{ kN}$$ **Ans.**

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{27.98(10^3)}{0.005} = 5.595(10^6) \text{ N/m}^2 = 5.60 \text{ MPa} < \sigma_Y = 35 \text{ MPa}$$ O.K.

Ans:
$P_{cr} = 28.0$ kN

13–23. If load C has a mass of 500 kg, determine the required minimum diameter of the solid L2-steel rod AB to the nearest mm so that it will not buckle. Use F.S. = 2 against buckling.



SOLUTION

Equilibriun. The compressive force developed in rod AB can be determined by analyzing the equilibrium of joint A, Fig. a.

$\Sigma F_{y'} = 0;\quad F_{AB} \sin 15° - 500(9.81)\cos 45° = 0 \qquad\qquad F_{AB} = 13\,400.71 \text{ N}$

Section Properties. The cross-sectional area and moment of inertia of the solid rod are

$$A = \frac{\pi}{4} d^2 \qquad\qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64} d^4$$

Critical Buckling Load. Since the rod is pinned at both of its ends, $K = 1$. Here, $P_{cr} = F_{AB}\,(\text{F.S.}) = 13400.71(2) = 26801.42$ N. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)^2}$$

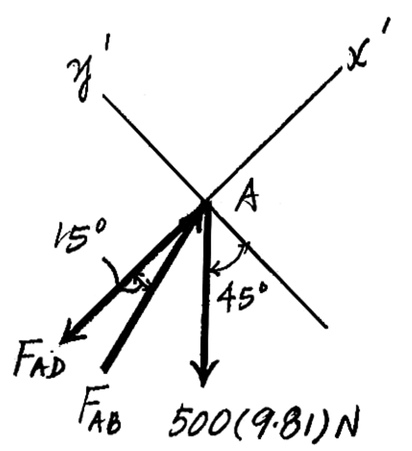
$$26801.42 = \frac{\pi^2\left[200\left(10^9\right)\right]\left[\frac{\pi}{64} d^4\right]}{[1(4)]^2}$$

$$d = 0.04587 \text{ m} = 45.87 \text{ mm}$$

Use $d = 46$ mm **Ans.**

Critical Stress. Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

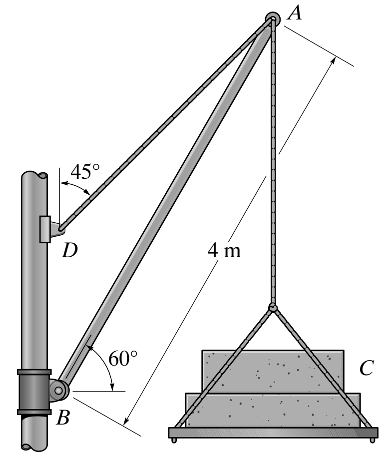
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{26801.42}{\frac{\pi}{4}\left(0.046^2\right)} = 16.13 \text{ MPa} < \sigma_Y = 703 \text{ MPa} \qquad \text{O.K.}$$



(a)

Ans:
Use $d = 46$ mm

***13–24.** If the diameter of the solid L2-steel rod AB is 50 mm, determine the maximum mass C that the rod can support without buckling. Use F.S. = 2 against buckling.



SOLUTION

Equilibrium. The compressive force developed in rod AB can be determined by analyzing the equilibrium of joint A, Fig. a.

$\Sigma F_{y'} = 0;\quad F_{AB} \sin 15° - m(9.81) \cos 45° = 0 \qquad F_{AB} = 26.8014m$

Section Properties. The cross-sectional area and moment of inertia of the rod are

$$A = \frac{\pi}{4}\left(0.05^2\right) = 0.625\left(10^{-3}\right)\pi \text{m}^2$$

$$I = \frac{\pi}{4}\left(0.025^4\right) = 97.65625\left(10^{-9}\right)\pi \text{m}^4$$

Critical Buckling Load. Since the rod is pinned at both of its ends, $K = 1$. Here, $P_{cr} = F_{AB}$ (F.S.) $= 26.8014m(2) = 53.6028m$. Applying Euler's formula,

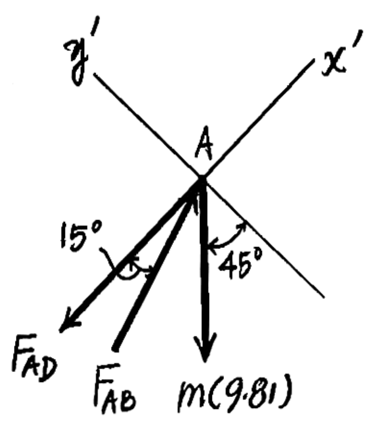
$$P_{cr} = \frac{\pi^2 E I_y}{(KL)^2}$$

$$53.6028m = \frac{\pi^2\left[200\left(10^9\right)\right]\left[97.65625\left(10^{-9}\right)\pi\right]}{[1(4)]^2}$$

$$m = 706.11 \text{ kg} = 7.06 \text{ kg} \qquad \textbf{Ans.}$$

Critical Stress. Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

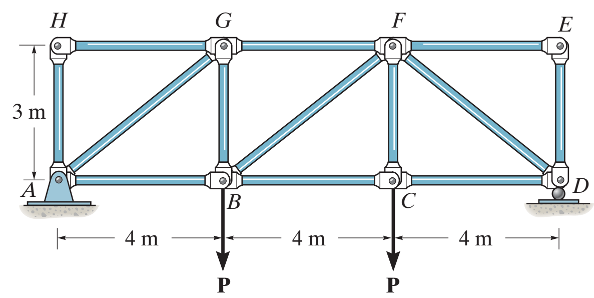
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{53.6028(706.11)}{\pi\, 0.625\left(10^{-3}\right)} = 19.28 \text{ MPa} < \sigma_Y = 703 \text{ MPa} \qquad \text{O.K.}$$



(a)

Ans:
$m = 7.06$ kg

13–25. The members of the truss are assumed to be pin connected. If member GF is an A-36 steel rod having a diameter of 50 mm, determine the greatest magnitude of load **P** that can be supported by the truss without causing this member to buckle.



SOLUTION

Support Reactions: As shown on FBD(a).

Member Forces: Use the method of sections [FBD(b)].

$$+\Sigma M_B = 0; \qquad F_{GF}(3) - P(4) = 0 \qquad F_{GF} = 1.3333P \text{ (C)}$$

Section Properties:

$$A = \frac{\pi}{4}(0.05^2) = 0.625(10^{-3})\pi \text{ m}^2$$

$$I = \frac{\pi}{4}(0.025^4) = 97.65625(10^{-9})\pi \text{ m}^4$$

Critical Buckling Load: $K = 1$ for a column with both ends pinned. Applying *Euler's* formula,

$$P_{cr} = F_{GF} = \frac{\pi^2 EI}{(KL_{GF})^2}$$

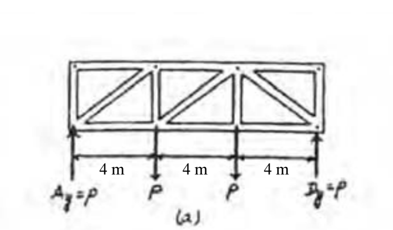
$$1.3333P = \frac{\pi^2[200(10^9)][97.65625(10^{-9})\pi]}{[1(4)]^2}$$

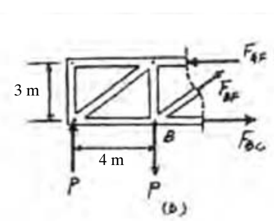
$$P = 28.39(10^3) \text{ N} = 28.4 \text{ kN} \qquad \textbf{Ans.}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1.3333[28.39(10^3)]}{(0.625)(10^{-3})\pi} = 19.28(10^6) \text{ N/m}^2$$

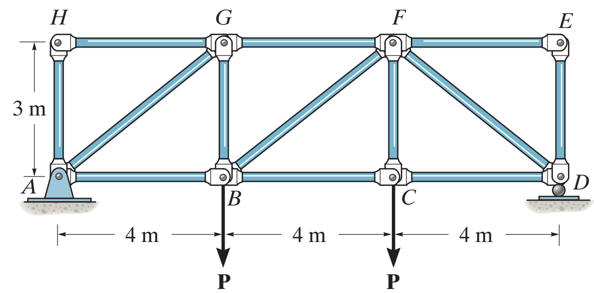
$$= 19.3 \text{ MPa} < \sigma_Y = 250 \text{ MPa} \qquad \text{O.K.}$$





Ans:
$P = 28.4$ kN

13–26. The members of the truss are assumed to be pin connected. If member AG is an A-36 steel rod having a diameter of 50 mm, determine the greatest magnitude of load **P** that can be supported by the truss without causing this member to buckle.



SOLUTION

Support Reactions: As shown on FBD(a).

Member Forces: Use the method of joints [FBD(b)].

$$+\uparrow \Sigma F_y = 0; \qquad P - \frac{3}{5} F_{AG} = 0 \qquad F_{AG} = 1.6667P \text{ (C)}$$

Section Properties:

$$L_{AG} = \sqrt{4^2 + 3^2} = 5 \text{ m}$$

$$A = \frac{\pi}{4}(0.05^2) = 0.625(10^{-3})\pi \text{ m}^2$$

$$I = \frac{\pi}{4}(0.025^4) = 97.65625(10^{-9})\pi \text{ m}^4$$

Critical Buckling Load: $K = 1$ for a column with both ends pinned. Applying *Euler's* formula,

$$P_{cr} = F_{GF} = \frac{\pi^2 EI}{(KL_{GF})^2}$$

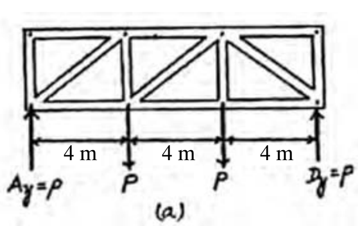
$$1.6667P = \frac{\pi^2[200(10^9)][97.65625(10^{-9})\pi]}{[1(5)]^2}$$

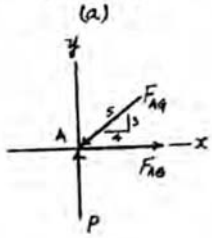
$$P = 14.53(10^3) \text{ N} = 14.5 \text{ kN} \qquad \textbf{Ans.}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} = \sigma_Y$.

$$\sigma_{c\pi} = \frac{P_{c\pi}}{A} = \frac{1.6667[14.53(10^3)]}{(0.625)(10^{-3})\pi} = 12.34(10^6) \text{ N}\cancel{\text{m}}^2$$

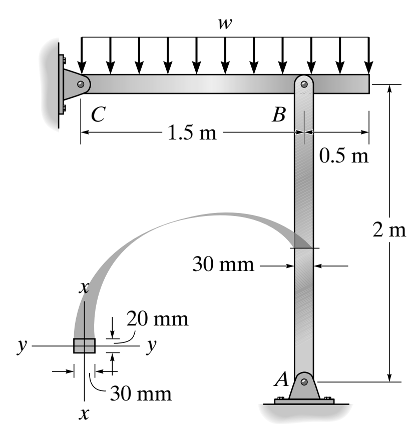
$$= 12.3 \text{ MPa} < \sigma_Y = 250 \text{ MPa} \qquad \text{O.K.}$$





Ans:
$P = 14.5$ kN

13–27. Determine the maximum allowable intensity w of the distributed load that can be applied to member BC without causing member AB to buckle. Assume that AB is made of steel and is pinned at its ends for x–x axis buckling and fixed at its ends for y–y axis buckling. Use a factor of safety with respect to buckling of 3. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION

Moment of inertia:

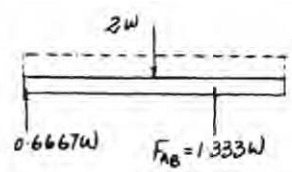
$$I_x = \frac{1}{12}(0.02)(0.03^3) = 45.0(10^{-9})\text{m}^4$$

$$I_y = \frac{1}{12}(0.03)(0.02^3) = 20(10^{-9})\text{ m}^4$$

$x-x$ axis:

$$P_{cr} = F_{AB}\,(\text{F.S.}) = 1.333w(3) = 4.0\,w$$

$$K = 1.0, \qquad L = 2\text{m}$$



$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$4.0w = \frac{\pi^2(200)(10^9)(45.0)(10^{-9})}{[(1.0)(2)]^2}$$

$$w = 5552\text{ N/m} = 5.55\text{ kN/m} \qquad \text{(controls)}$$ **Ans.**

$y-y$ axis

$$K = 0.5, \qquad L = 2\text{m}$$

$$4.0w = \frac{\pi^2\,(200)(10^9)(20)(10^{-9})}{[(0.5)(2)]^2}$$

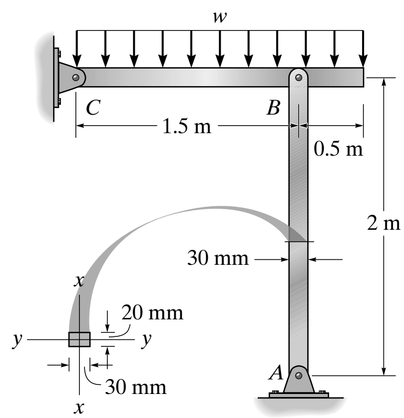
$$w = 9870\text{ N/m} = 9.87\text{ kN/m}$$

Check:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{4(5552)}{(0.02)(0.03)} = 37.0\text{ MPa} < \sigma_\gamma$$ O.K.

Ans:
$w = 5.55$ kN/m

***13–28.** Determine if the frame can support a load of $w = 6$ kN/m if the factor of safety with respect to buckling of member AB is 3. Assume that AB is made of steel and is pinned at its ends for x–x axis buckling and fixed at its ends for y–y axis buckling. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION

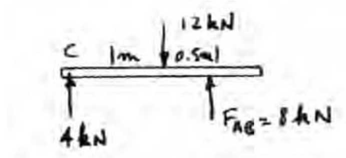
Check $x-x$ axis buckling:

$$I_x = \frac{1}{12}(0.02)(0.03)^3 = 45.0(10^{-9})\ \text{m}^4$$

$K = 1.0 \qquad L = 2$ m

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(45.0)(10^{-9})}{((1.0)(2))^2}$$

$P_{cr} = 22.2$ kN



$\zeta + \Sigma M_C = 0; \qquad F_{AB}(1.5) - 6(2)(1) = 0$

$F_{AB} = 8$ kN

$P_{req'd} = 8(3) = 24\ \text{kN} > 22.2\ \text{kN}$

No, AB will fail. **Ans.**

Ans:
No, AB will fail.

13–29.

A 6061-T6 aluminum alloy solid circular rod of length 4 m is pinned at both of its ends. If it is subjected to an axial load of 15 kN and F.S. = 2 against buckling, determine the minimum required diameter of the rod to the nearest mm.

SOLUTION

Section Properties: The cross-sectional area and moment of inertia of the solid rod are

$$A = \frac{\pi}{4}d^2 \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64}d^4$$

Critical Buckling Load: The critical buckling load is

$$P_{cr} = P_{allow}(\text{F.S.}) = 15(2) = 30\text{ kN}$$

Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI_y}{(KL)^2}$$

$$30(10^3) = \frac{\pi^2\left[68.9(10^9)\right]\left[\frac{\pi}{64}d^4\right]}{[1(4)]^2}$$

$$d = 0.06158\text{ m} = 61.58\text{ mm}$$

Use $d = 62$ mm **Ans.**

Critical Stress: Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{30(10^3)}{\frac{\pi}{4}(0.062^2)} = 9.94\text{ MPa} < \sigma_Y = 255\text{ MPa} \qquad \textbf{(O.K.)}$$

Ans:
Use $d = 62$ mm

13–30.

A 6061-T6 aluminum alloy solid circular rod of length 4 m is pinned at one end while fixed at the other end. If it is subjected to an axial load of 15 kN and F.S. = 2 against buckling, determine the minimum required diameter of the rod to the nearest mm.

SOLUTION

Section Properties: The cross-sectional area and moment of inertia of the solid rod are

$$A = \frac{\pi}{4}d^2 \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64}d^4$$

Critical Buckling Load: The critical buckling load is

$$P_{cr} = P_{allow}(\text{F.S.}) = 15(2) = 30 \text{ kN}$$

Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)^2}$$

$$30(10^3) = \frac{\pi^2\left[68.9(10^9)\right]\left[\frac{\pi}{64}d^4\right]}{[0.7(4)]^2}$$

$$d = 0.05152 \text{ m} = 51.52 \text{ mm}$$

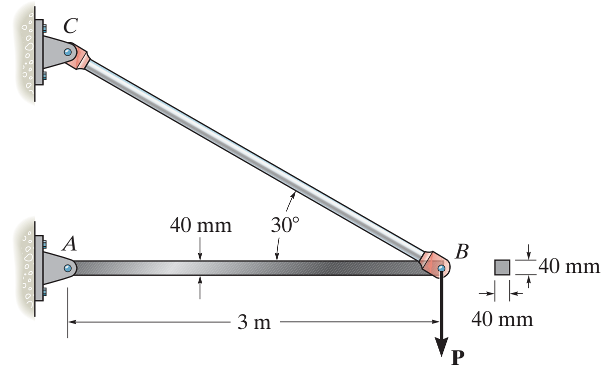
Use $d = 52$ mm **Ans.**

Critical Stress: Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{30(10^3)}{\frac{\pi}{4}(0.052^2)} = 14.13 \text{ MPa} < \sigma_Y = 255 \text{ MPa} \qquad \text{(O.K.)}$$

Ans:
Use $d = 52$ mm

13–31. The A-36 steel bar AB has a square cross section. If it is pin connected at its ends, determine the maximum allowable load P that can be applied to the frame. Use a factor of safety with respect to buckling of 2.



SOLUTION

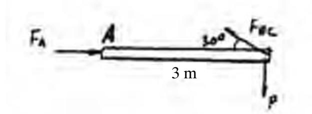
$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BC} \sin 30°(3) - P(3) = 0$

$$F_{BC} = 2\,P$$

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_A - 2P \cos 30° = 0$

$$F_A = 1.732\,P$$

Buckling load:



$$P_{cr} = F_A(\text{F.S.}) = 1.732\,P(2) = 3.464\,P$$

$$L = 3\text{ m}$$

$$I_y = \frac{1}{12}(0.04)(0.04^3) = 0.21333(10^{-6})\text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$0.464P = \frac{\pi^2[200(10^9)][0.21333(10^{-6})]}{[1(3)]^2}$$

$$P = 13.51(10^3)\text{ N} = 13.5\text{ kN} \qquad \textbf{Ans.}$$

$$P_{cr} = F_A(\text{F.S.}) = 1.732(13.51)(2)\text{ N} = 46.79\text{ kN}$$

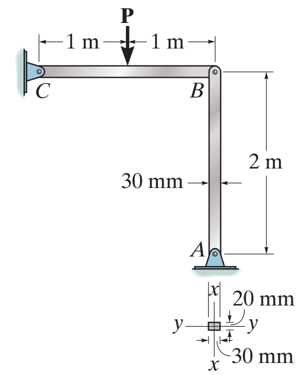
Check:

$$\sigma_{c\pi} = \frac{P_{c\pi}}{A} = \frac{46.79(10^3)}{(0.04)(0.04)} = 29.24(10^6)\text{ N/m}^2 = 29.24\text{ MPa} < \sigma_Y \qquad \text{O.K.}$$

Ans:
$P = 13.5$ kN

***13–32.**

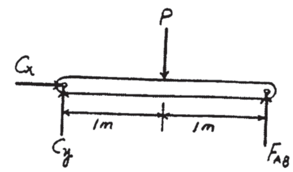
Determine the maximum allowable load P that can be applied to member BC without causing member AB to buckle. Assume that AB is made of steel and is pinned at its ends for x–x axis buckling and fixed at its ends for y–y axis buckling. Use a factor of safety with respect to buckling of F.S. = 3. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION

Support Reactions:

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{AB}(2) - P(1) = 0 \qquad F_{AB} = 0.500P$$



Section Properties:

$$A = 0.02(0.03) = 0.600\left(10^{-3}\right)\text{ m}^2$$

$$I_x = \frac{1}{12}(0.02)\left(0.03^3\right) = 45.0\left(10^{-9}\right)\text{ m}^4$$

$$I_y = \frac{1}{12}(0.03)\left(0.02^2\right) = 20.0\left(10^{-9}\right)\text{ m}^4$$

Critical Buckling Load: With respect to x–x axis, $K = 1$ (column with both ends pinned). Applying *Euler's* formula,

$$P_{cr} = 3F_{AB} = \frac{\pi^2 EI}{(KL)^2}$$

$$3(0.500P) = \frac{\pi^2(200)\left(10^9\right)\left[45.0\left(10^{-9}\right)\right]}{[1(2)]^2}$$

$$P = 14804.4\text{ N} = 14.8\text{ kN (Controls!)}$$ **Ans.**

With respect to y–y axis, $K = 0.5$ (column with both ends fixed).

$$P_{cr} = 3F_{AB} = \frac{\pi^2 EI}{(KL)^2}$$

$$3(0.500P) = \frac{\pi^2(200)\left(10^9\right)\left[20.0\left(10^{-9}\right)\right]}{[0.5(2)]^2}$$

$$P = 26\,318.9\text{ N}$$

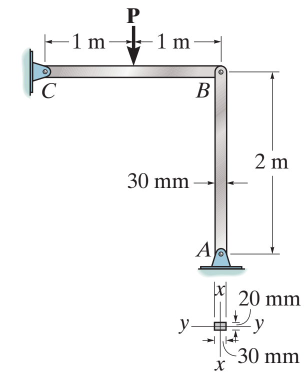
Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{3(0.5)(14\,804.4)}{0.600\left(10^{-3}\right)} = 37.01\text{ MPa} < \sigma_Y = 360\text{ MPa (O.K!)}$$

Ans:
$P = 14.8$ kN

13–33.

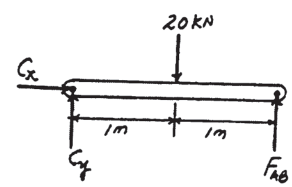
Determine if the frame can support a load of $P = 20$ kN if the factor of safety with respect to buckling of member AB is F.S. = 3. Assume that AB is made of steel and is pinned at its ends for x–x axis buckling and fixed at its ends for y–y axis buckling. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION

Support Reactions:

$$\zeta+ \Sigma M_C = 0; \qquad F_{AB}(2) - 20(1) = 0 \qquad F_{AB} = 10.0 \text{ kN}$$



Section Properties:

$$A = 0.02(0.03) = 0.600\left(10^{-3}\right) \text{ m}^2$$

$$I_x = \frac{1}{12}(0.02)\left(0.03^3\right) = 45.0\left(10^{-9}\right) \text{ m}^4$$

$$I_y = \frac{1}{12}(0.03)\left(0.02^2\right) = 20.0\left(10^{-9}\right) \text{ m}^4$$

Critical Buckling Load: With respect to x–x axis, $K = 1$ (column with both ends pinned). Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$
$$= \frac{\pi^2(200)\left(10^9\right)\left[45.0\left(10^{-9}\right)\right]}{[1(2)]^2}$$
$$= 22\,206.61 = 22.207 \text{ kN (Controls!)}$$

With respect to y–y axis, $K = 0.5$ (column with both ends fixed).

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$
$$= \frac{\pi^2(200)\left(10^9\right)\left[20.0\left(10^{-9}\right)\right]}{[0.5(2)]^2}$$
$$= 39\,478.42 \text{ N}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{32\,127.6}{5.00\left(10^{-3}\right)} = 6.426 \text{ MPa} < \sigma_Y = 360 \text{ MPa} \qquad \textbf{(O.K!)}$$

Factor of Safety: The required factor of safety is 3.

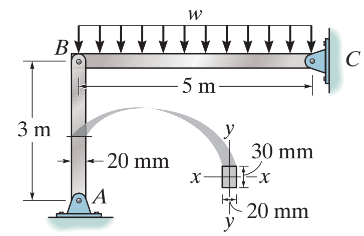
$$\text{F.S} = \frac{P_{cr}}{F_{AB}} = \frac{22.207}{10.0} = 2.22 < 3 \text{ (No Good!)}$$

Hence, the frame cannot support the load with the required F.S. **Ans.**

Ans:
No

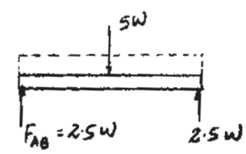
13–34.

The steel bar AB has a rectangular cross section. If it is pin connected at its ends, determine the maximum allowable intensity w of the distributed load that can be applied to BC without causing AB to buckle. Use a factor of safety with respect to buckling of 1.5. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION

Buckling Load:



$$P_{cr} = F_{AB}(\text{F.S.}) = 2.5\,w(1.5) = 3.75\,w$$

$$I = \frac{1}{12}(0.03)(0.02)^3 = 20\,(10^{-9})\text{ m}^4$$

$$K = 1.0$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$3.75\,w = \frac{\pi^2(200)(10^9)(20)(10^{-9})}{[(1.0)(3)]^2}$$

$$w = 1170\text{ N/m} = 1.17\text{ kN/m} \qquad \textbf{Ans.}$$

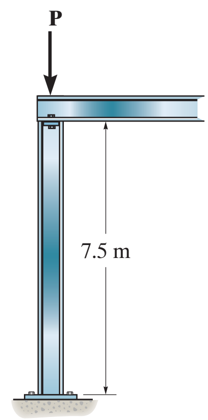
$$P_{cr} = 4.39\text{ kN}$$

Check:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{4.39\,(10^3)}{0.02\,(0.03)} = 7.31\text{ MPa} < \sigma_Y \qquad \textbf{OK}$$

Ans:
$w = 1.17$ kN/m

13–35. The W360 × 45 is used as a structural A-36 steel column that can be assumed pinned at both of its ends. Determine the largest axial force P that can be applied without causing it to buckle.



SOLUTION

From the table in appendix, the cross-sectional area and the moment of inertia about weak axis (y-axis) for W360 × 45 are

$$A = 5710 \text{ mm}^2 = 5.71(10^{-3}) \text{ m}^2 \qquad I_y = 8.16(10^6) \text{ mm}^4 = 8.16(10^{-6}) \text{ m}^4$$

Critical Buckling Load: Since the column is pinned at its base and top, $K = 1$. For A36 steel, $E = 200$ GPa and $\sigma_Y = 250$ MPa. Here, the buckling occurs about the weak axis (y-axis).

$$P = P_{cr} = \frac{\pi^2 E I_y}{(KL)^2} = \frac{\pi^2[200(10^9)][8.16(10^{-6})]}{[1(7.5)]^2}$$

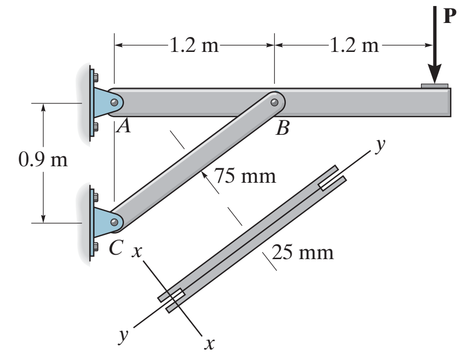
$$= 286.35(10^3) \text{ N} = 286 \text{ kN} \qquad \textbf{Ans.}$$

Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{c\pi} = \frac{P_{c\pi}}{A} = \frac{286.35(10^3)}{5.71(10^{-3})} = 50.14(10^6) \text{ N/m}^2 = 50.1 \text{ MPa} < \sigma_Y = 250 \text{ MPa} \qquad \text{O.K.}$$

Ans:
$A = 5710 \text{ mm}^2, I_y = 8.16\,(10^6) \text{ mm}^4, P = 286 \text{ kN}$

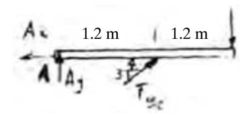
***13–36.** The beam supports the load of $P = 30$ kN. As a result, the A-36 steel member BC is subjected to a compressive load. Due to the forked ends on the member, consider the supports at B and C to act as pins for x–x axis buckling and as fixed supports for y–y axis buckling. Determine the factor of safety with respect to buckling about each of these axes.



SOLUTION

$$\zeta + \Sigma M_A = 0; \qquad F_{BC}\left(\frac{3}{5}\right)(1.2) - 30(2.4) = 0$$

$$F_{BC} = 100 \text{ kN}$$



$x-x$ axis buckling:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2[200(10^9)]\left[\frac{1}{12}(0.025)(0.075^3)\right]}{[1.0(1.5)]^2} = 771.06(10^3)\text{ N} = 771.06\text{ kN}$$

$$\text{F.S.} = \frac{771.06}{100} = 7.71$$ **Ans.**

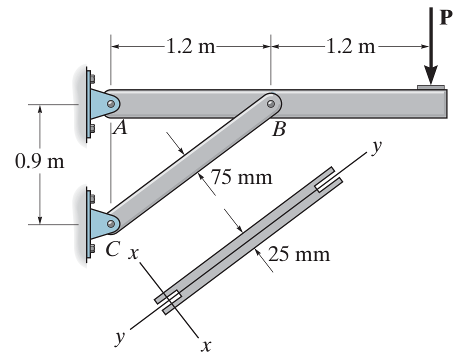
$y-y$ axis buckling:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2[200(10^9)]\left[\frac{1}{12}(0.075)(0.025^3)\right]}{[0.5(1.5)]^2} = 342.69(10^3)\text{ N} = 342.69\text{ kN}$$

$$\text{F.S.} = \frac{342.69}{100} = 3.43$$ **Ans.**

Ans:
x-x axis buckling: F.S. = 7.71,
y-y axis buckling: F.S. = 3.43

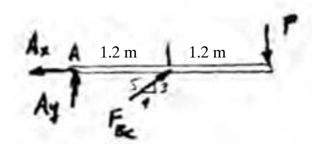
13–37. Determine the greatest load P the frame will support without causing the A-36 steel member BC to buckle. Due to the forked ends on the member, consider the supports at B and C to act as pins for x–x axis buckling and as fixed supports for y–y axis buckling.



SOLUTION

$$\zeta + \Sigma M_A = 0; \quad F_{BC}\left(\frac{3}{5}\right)(1.2) - P(2.4) = 0$$

$$F_{BC} = \left(\frac{10}{3}\right)P$$



$x-x$ axis buckling:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2[200(10^9)]\left[\frac{1}{12}(0.025)(0.075^3)\right]}{[1.0(1.5)]^2} = 771.06(10^3)\text{ N} = 771.06\text{ kN}$$

$y-y$ axis buckling:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2[200(10^9)]\left[\frac{1}{12}(0.075)(0.025^3)\right]}{[0.5(1.5)]^2} = 342.69(10^3)\text{ N} = 342.69\text{ kN}$$

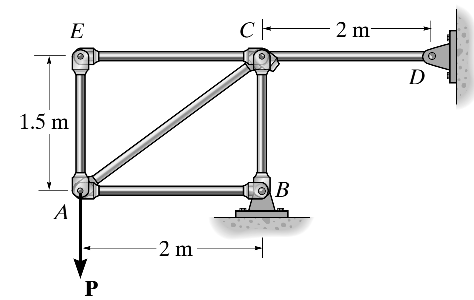
Thus,

$$\left(\frac{10}{3}\right)P = 342.69$$

$P = 102.81\text{ kN} = 103\text{ kN}$ **Ans.**

Ans:
$P = 103\text{ kN}$

13–38. The members of the truss are assumed to be pin connected. If member AB is an A-36 steel rod of 40 mm diameter, determine the maximum force P that can be supported by the truss without causing the member to buckle.



SOLUTION

By inspecting the equilibrium of joint E, $F_{AB} = 0$. Then, the compressive force developed in member AB can be determined by analysing the equilibrium of joint A, Fig. *a*.

$$+\uparrow \Sigma F_y = 0; \quad F_{AC}\left(\frac{3}{5}\right) - P = 0 \qquad F_{AC} = \frac{5}{3}P\ (\text{T})$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad \frac{5}{3}P\left(\frac{4}{5}\right) - F_{AB} = 0 \qquad F_{AB} = \frac{4}{3}P(c)$$

$$A = \pi(0.02^2) = 0.4(10^{-3})\pi \text{ m}^2 \qquad I = \frac{\pi}{4}(0.02^4) = 40(10^{-9})\ \pi \text{ m}^4$$

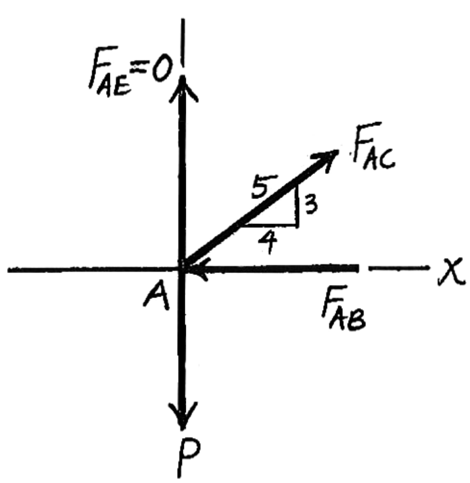
Since both ends of member AB are pinned, $K = 1$. For A36 steel, $E = 200$ GPa and $\sigma_\gamma = 250$ MPa.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}; \qquad \frac{4}{3}P = \frac{\pi^2\left[200(10^9)\right]\left[40(10^{-9})\pi\right]}{\left[1(2)\right]^2}$$

$$P = 46.51(10^3)\text{ N} = 46.5\text{ kN} \qquad \textbf{Ans.}$$

The Euler's formula is valid only if $\sigma_{cr} < \sigma_\gamma$.

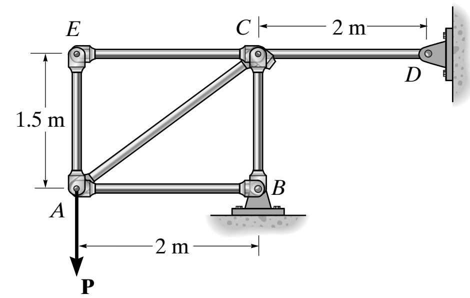
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\frac{4}{3}(46.51)(10^3)}{0.4(10^{-3})\pi} = 49.35(10^6)\text{ Pa} = 49.35\text{ MPa} < \sigma_\gamma = 250\text{ MPa O.K.}$$



(a)

Ans:
$P = 46.5$ kN

13–39. The members of the truss are assumed to be pin connected. If member CB is an A-36 steel rod of 40 mm diameter, determine the maximum load P that can be supported by the truss without causing the member to buckle.



SOLUTION

Section the truss through a–a, the FBD of the left cut segment is shown in Fig. a. The compressive force developed in member CB can be obtained directly by writing the force equation of equilibrium along y axis.

$$+\uparrow \Sigma F_y = 0; \qquad F_{CB} - P = 0 \qquad F_{CB} = P \text{ (C)}$$

$$A = \pi(0.02^2) = 0.4(10^{-3})\pi \text{ m}^2 \qquad I = \frac{\pi}{4}(0.02^4) = 40(10^{-9})\pi \text{ m}^4$$

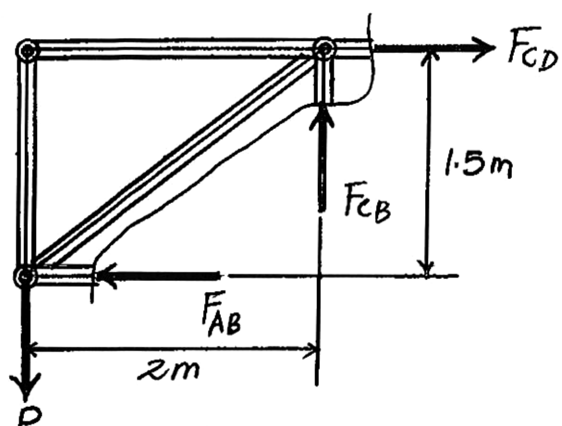
Since both ends of member CB are pinned, $K = 1$. For A36 steel, $E = 200$ GPa and $\sigma_Y = 250$ MPa.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}; \qquad P = \frac{\pi^2\left[200(10^9)\right]\left[40(10^{-9})\pi\right]}{\left[1(1.5)\right]^2}$$

$$= 110.24(10^3) \text{ N} = 110 \text{ kN} \qquad \textbf{Ans.}$$

The Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{110.24(10^3)}{0.4(10^{-3})\pi} = 87.73(10^6) \text{ Pa} = 87.73 \text{ MPa} < \sigma_Y = 250 \text{ MPa} \qquad \text{O.K.}$$

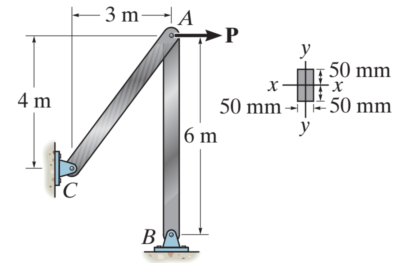


(a)

Ans:
$P = 110$ kN

***13–40.**

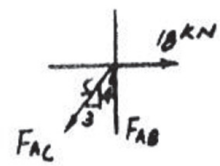
The steel bar AB of the frame is assumed to be pin connected at its ends for y–y axis buckling. If $P = 18$ kN, determine the factor of safety with respect to buckling about the y–y axis. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION

$$I_y = \frac{1}{12}(0.10)(0.05^3) = 1.04167\,(10^{-6})\text{ m}^4$$

Joint A:



$$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad \frac{3}{5}F_{AC} - 18 = 0$$

$$F_{AC} = 30\text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad F_{AB} - \frac{4}{5}(30) = 0$$

$$F_{AB} = 24\text{ kN}$$

$$P_{cr} = \frac{\pi^2 E I}{(KL)^2} = \frac{\pi^2(200)(10^9)(1.04167)(10^{-6})}{[(1.0)(6)]^2} = 57116\text{ N} = 57.12\text{ kN}$$

$$\text{F.S.} = \frac{P_{cr}}{F_{AB}} = \frac{57.12}{24} = 2.38 \qquad \textbf{Ans.}$$

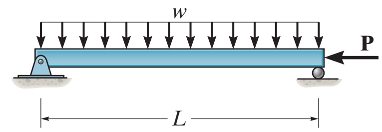
Check:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{57.12\,(10^3)}{0.1\,(0.05)} = 11.4\text{ MPa} < \sigma_Y \qquad \textbf{OK}$$

Ans:
F.S. = 2.38

13–41.

The ideal column has a weight w (force/length) and is subjected to the axial load **P**. Determine the maximum moment in the column at midspan. EI is constant. *Hint:* Establish the differential equation for deflection, Eq. 13–1, with the origin at the midspan. The general solution is $v = C_1 \sin kx + C_2 \cos kx + (w/(2P))x^2 - (wL/(2P))x - (wEI/P^2)$ where $k^2 = P/EI$.

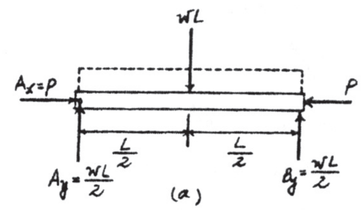


SOLUTION

Moment Functions: FBD(b).

$$\circlearrowleft + \Sigma M_o = 0; \qquad wx\left(\frac{x}{2}\right) - M(x) - \left(\frac{wL}{2}\right)x - Pv = 0$$

$$M(x) = \frac{w}{2}\left(x^2 - Lx\right) - Pv \qquad \textbf{(1)}$$

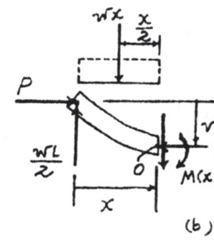


Differential Equation of The Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{w}{2}\left(x^2 - Lx\right) - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = \frac{w}{2EI}\left(x^2 - Lx\right)$$



The solution of the above differential equation is of the form

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{w}{2P}x^2 - \frac{wL}{2P}x - \frac{wEI}{P^2} \qquad \textbf{(2)}$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) + \frac{w}{P}x - \frac{wL}{2P} \qquad \textbf{(3)}$$

The integration constants can be determined from the boundary conditions.

Boundary Condition:

At $x = 0$, $v = 0$. From Eq. (2),

$$0 = C_2 - \frac{wEI}{P^2} \qquad C_2 = \frac{wEI}{P^2}$$

At $x = \frac{L}{2}, \frac{dv}{dx} = 0$. From Eq. (3),

$$0 = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{wEI}{P^2}\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) + \frac{w}{P}\left(\frac{L}{2}\right) - \frac{wL}{2P}$$

$$C_1 = \frac{wEI}{P^2}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)$$

13–41. Continued

SOLUTION

Elastic Curve:

$$v = \frac{w}{P}\left[\frac{EI}{P}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}x\right) + \frac{EI}{P}\cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{x^2}{2} - \frac{L}{2}x - \frac{EI}{P}\right]$$

However, $v = v_{max}$ at $x = \frac{L}{2}$. Then,

$$v_{max} = \frac{w}{P}\left[\frac{EI}{P}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) + \frac{EI}{P}\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L^2}{8} - \frac{EI}{P}\right]$$

$$= \frac{wEI}{P^2}\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{PL^2}{8EI} - 1\right]$$

Maximum Moment: The maximum moment occurs at $x = \frac{L}{2}$. From Eq. (1),

$$M_{max} = \frac{w}{2}\left[\frac{L^2}{4} - L\left(\frac{L}{2}\right)\right] - Pv_{max}$$

$$= -\frac{wL^2}{8} - P\left\{\frac{wEI}{P^2}\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{PL^2}{8EI} - 1\right]\right\}$$

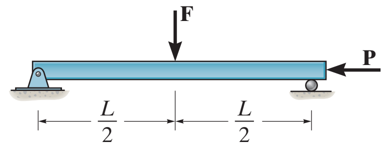
$$= -\frac{wEI}{P}\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - 1\right] \qquad \textbf{Ans.}$$

Ans:

$$M_{max} = -\frac{wEI}{P}\left[\sec\left(\frac{L}{2}\sqrt{\frac{P}{EI}}\right) - 1\right]$$

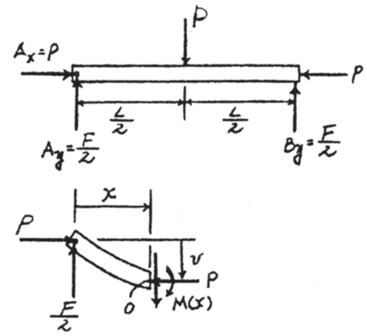
13–42.

The ideal column is subjected to the force **F** at its midpoint and the axial load **P**. Determine the maximum moment in the column at midspan. EI is constant. *Hint:* Establish the differential equation for deflection, Eq. 13–1. The general solution is $v = C_1 \sin kx + C_2 \cos kx - c^2 x/k^2$, where $c^2 = F/2EI, k^2 = P/EI$.



SOLUTION

Moment Functions: FBD(b).



$$\circlearrowleft +\Sigma M_o = 0; \qquad M(x) + \frac{F}{2}x + P(v) = 0$$

$$M(x) = -\frac{F}{2}x - Pv \qquad \textbf{(1)}$$

Differential Equation of The Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = -\frac{F}{2}x - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = -\frac{F}{2EI}x$$

The solution of the above differential equation is of the form

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}\,x\right) - \frac{F}{2P}x \qquad \textbf{(2)}$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{F}{2P} \qquad \textbf{(3)}$$

The integration constants can be determined from the boundary conditions.

Boundary Conditions:

At $x = 0, v = 0$. From Eq. (2), $C_2 = 0$

At $x = \frac{L}{2}, \frac{dv}{dx} = 0$. From Eq. (3),

$$0 = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{F}{2P}$$

$$C_1 = \frac{F}{2P}\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)$$

Elastic Curve:

$$v = \frac{F}{2P}\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{F}{2P}x$$

$$= \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}x\right) - x\right]$$

13–42. Continued

SOLUTION

However, $v = v_{\max}$ at $x = \dfrac{L}{2}$. Then,

$$v_{\max} = \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right]$$

$$= \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right]$$

Maximum Moment: The maximum moment occurs at $x = \dfrac{L}{2}$. From Eq. (1),

$$M_{\max} = -\frac{F}{2}\left(\frac{L}{2}\right) - Pv_{\max}$$

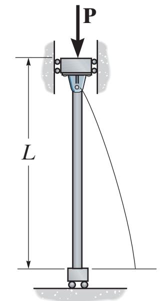
$$= -\frac{FL}{4} - P\left\{\frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right]\right\}$$

$$= -\frac{F}{2}\sqrt{\frac{EI}{P}}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)$$ **Ans.**

Ans:
$M_{\max} = -\dfrac{F}{2}\sqrt{\dfrac{EI}{P}}\tan\left(\dfrac{L}{2}\sqrt{\dfrac{P}{EI}}\right)$

13–43.

The column with constant EI has the end constraints shown. Determine the critical load for the column.



SOLUTION

Moment Function. Referring to the free-body diagram of the upper part of the deflected column, Fig. a,

$\zeta + \Sigma M_O = 0;\qquad M + Pv = 0\qquad\qquad M = -Pv$

Differential Equation of the Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M$$

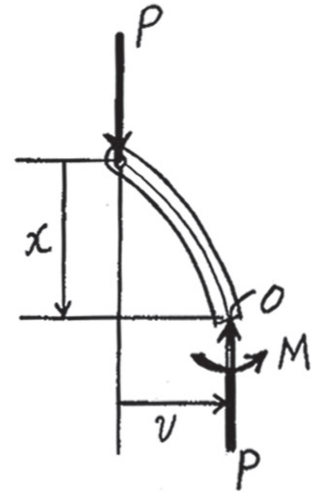
$$EI\frac{d^2v}{dx^2} = -Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = 0$$

The solution is in the form of

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}x\right) \qquad \textbf{(1)}$$

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) \qquad \textbf{(2)}$$



(a)

Boundary Conditions. At $x = 0$, $v = 0$. Then Eq. (1) gives

$$0 = 0 + C_2 \qquad\qquad C_2 = 0$$

At $x = L$, $\dfrac{dv}{dx} = 0$. Then Eq. (2) gives

$$0 = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}L\right)$$

$C_1 = 0$ is the trivial solution, where $v = 0$. This means that the column will remain straight and buckling will not occur regardless of the load P. Another possible solution is

$$\cos\left(\sqrt{\frac{P}{EI}}L\right) = 0$$

$$\sqrt{\frac{P}{EI}}L = \frac{n\pi}{2} \qquad n = 1, 3, 5$$

The smallest critical load occurs when $n = 1$, then

$$\sqrt{\frac{P_{cr}}{EI}}L = \frac{\pi}{2}$$

$$P_{cr} = \frac{\pi^2 EI}{4L^2} \qquad \textbf{Ans.}$$

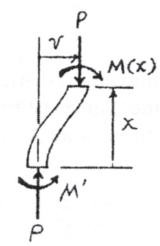
Ans:

$$P_{cr} = \frac{\pi^2 EI}{4L^2}$$

***13–44.**

Consider an ideal column as in Fig. 13–10*c*, having both ends fixed. Show that the critical load on the column is $P_{cr} = 4\pi^2 EI/L^2$. *Hint*: Due to the vertical deflection of the top of the column, a constant moment $\mathbf{M}'$ will be developed at the supports. Show that $d^2v/dx^2 + (P/EI)v = M'/EI$. The solution is of the form $v = C_1 \sin(\sqrt{P/EI}x) + C_2 \cos(\sqrt{P/EI}x) + M'/P$.

SOLUTION



Moment Functions:

$M(x) = M' - Pv$

Differential Equation of The Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = M' - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = \frac{M'}{EI} \qquad \textbf{(Q.E.D.)}$$

The solution of the above differential equation is of the form

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}\,x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}\,x\right) + \frac{M'}{P} \qquad \textbf{(1)}$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\,x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}\,x\right) \qquad \textbf{(2)}$$

The integration constants can be determined from the boundary conditions.

Boundary Conditions:

At $x = 0, v = 0$. From Eq. (1), $C_2 = -\frac{M'}{P}$

At $x = 0, \frac{dv}{dx} = 0$. From Eq. (2), $C_1 = 0$

Elastic Curve:

$$v = \frac{M'}{P}\left[1 - \cos\left(\sqrt{\frac{P}{EI}}\,x\right)\right]$$

and

$$\frac{dv}{dx} = \frac{M'}{P}\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}\,x\right)$$

However, due to symmetry $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$. Then,

$$\sin\left[\sqrt{\frac{P}{EI}}\left(\frac{L}{2}\right)\right] = 0 \qquad \text{or} \qquad \sqrt{\frac{P}{EI}}\left(\frac{L}{2}\right) = n\pi \qquad \text{where } n = 1, 2, 3, \ldots$$

The smallest critical load occurs when $n = 1$.

$$P_{cr} = \frac{4\pi^2 EI}{L^2} \qquad \textbf{(Q.E.D.)}$$

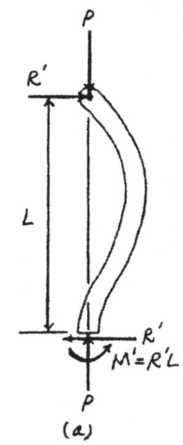
Ans:
N/A

13–45.

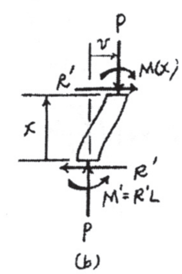
Consider an ideal column as in Fig. 13–10*d*, having one end fixed and the other pinned. Show that the critical load on the column is $P_{cr} = 20.19EI/L^2$. *Hint*: Due to the vertical deflection at the top of the column, a constant moment $\mathbf{M}'$ will be developed at the fixed support and horizontal reactive forces $\mathbf{R}'$ will be developed at both supports. Show that $d^2v/dx^2 + (P/EI)v = (R'/EI)(L - x)$. The solution is of the form $v = C_1 \sin(\sqrt{P/EI}x) + C_2 \cos(\sqrt{P/EI}x) + (R'/P)(L - x)$. After application of the boundary conditions show that $\tan(\sqrt{P/EI}L) = \sqrt{P/EI}\,L$. Solve numerically for the smallest nonzero root.

SOLUTION

Equilibrium. FBD(a).



Moment Functions: FBD(b).



$M(x) = R'(L - x) - Pv$

Differential Equation of The Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = R'(L - x) - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = \frac{R'}{EI}(L - x) \qquad \textbf{(Q.E.D.)}$$

The solution of the above differential equation is of the form

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}\,x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}\,x\right) + \frac{R'}{P}(L - x) \qquad \textbf{(1)}$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\,x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}\,x\right) - \frac{R'}{P} \qquad \textbf{(2)}$$

The integration constants can be determined from the boundary conditions.

Boundary Conditions:

At $x = 0$, $v = 0$. From Eq. (1), $C_2 = -\dfrac{R'L}{P}$

At $x = 0$, $\dfrac{dv}{dx} = 0$. From Eq. (2), $C_1 = \dfrac{R'}{P}\sqrt{\dfrac{EI}{P}}$

Elastic Curve:

$$v = \frac{R'}{P}\sqrt{\frac{EI}{P}}\sin\left(\sqrt{\frac{P}{EI}}\,x\right) - \frac{R'L}{P}\cos\left(\sqrt{\frac{P}{EI}}\,x\right) + \frac{R'}{P}(L - x)$$

$$= \frac{R'}{P}\left[\sqrt{\frac{EI}{P}}\sin\left(\sqrt{\frac{P}{EI}}\,x\right) - L\cos\left(\sqrt{\frac{P}{EI}}\,x\right) + (L - x)\right]$$

13–45. Continued

SOLUTION

However, $v = 0$ at $x = L$. Then,

$$0 = \sqrt{\frac{EI}{P}}\sin\left(\sqrt{\frac{P}{EI}}L\right) - L\cos\left(\sqrt{\frac{P}{EI}}L\right)$$

$$\tan\left(\sqrt{\frac{P}{EI}}L\right) = \sqrt{\frac{P}{EI}}L \qquad \textbf{(Q.E.D.)}$$

By trial and error and choosing the smallest root, we have

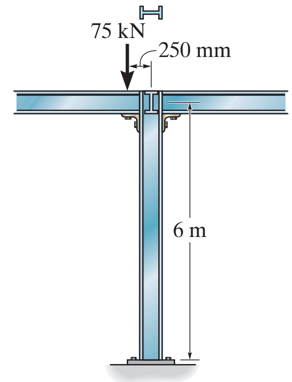
$$\sqrt{\frac{P}{EI}}L = 4.49341$$

Then,

$$P_{\text{cr}} = \frac{20.19EI}{L^2} \qquad \textbf{(Q.E.D.)}$$

Ans:
N/A

13–46. The W360 × 39 structural A-36 steel member is used as 6-m-long column that is assumed to be fixed at its top and fixed at its bottom. If the 75-kN load is applied at an eccentric distance of 250 mm, determine the maximum stress in the column



SOLUTION

Section Properties for W360 × 39

$A = 4960 \text{ mm}^2 = 4.96(10^{-3}) \text{ m}^2$

$d = 353 \text{ mm} = 0.353 \text{ m}$

$\pi_x = 143 \text{ mm} = 0.143 \text{ m}$

Yielding about $x-x$ axis:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{e\,c}{r^2}\sec\left(\frac{KL}{2\,r}\sqrt{\frac{P}{E\,A}}\right)\right]; \qquad K = 0.5$$

$$\frac{P}{A} = \frac{75(10^3)}{4.96(10^{-3})} = 15.12(10^6)\ \text{N-m}^2 = 15.12 \text{ MPa};$$

$$\frac{ec}{\pi^2} = \frac{0.25\left(\frac{0.353}{2}\right)}{0.143^2} = 2.157807$$

$$\frac{KL}{2\pi}\sqrt{\frac{P}{EA}} = \frac{0.5(6)}{2(0.143)}\sqrt{\frac{75(10^3)}{[200(10^9)][4.96(10^{-3})]}}$$

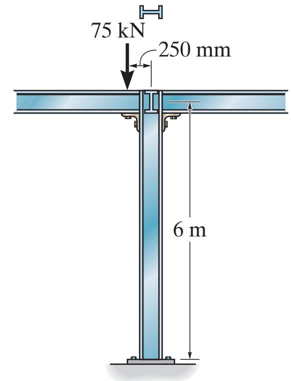
$$\sigma_{\max} = 15.12[1 + 2.157807 \sec(0.091207)]$$

$$= 47.9 \text{ MPa} < \sigma_Y = 250 \text{ MPa} \qquad \text{O.K.}$$

Ans.

Ans:
$\sigma_{\max} = 47.9 \text{ MPa}$

13-47. The W360 × 39 structural A-36 steel member is used as a column that is assumed to be fixed at its top and pinned at its bottom. If the 75-kN load is applied at an eccentric distance of 250 mm, determine the maximum stress in the column



SOLUTION

Section Properties for W360 × 39

$A = 4960 \text{ mm}^2 = 4.96(10^{-3}) \text{ m}^2$

$d = 353 \text{ mm} = 0.353 \text{ m}$

$\pi_x = 143 \text{ mm} = 0.143 \text{ m}$

Yielding about $x-x$ axis:

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{e\,c}{r^2}\sec\left(\frac{KL}{2\,r}\sqrt{\frac{P}{E\,A}}\right)\right]; \qquad K = 0.7$$

$$\frac{P}{A} = \frac{75(10^3)}{4.96(10^{-3})} = 15.12(10^6)\ \text{N}\cancel{\text{m}}^2 = 15.12 \text{ MPa;}$$

$$\frac{ec}{\pi^2} = \frac{0.25\left(\frac{0.353}{2}\right)}{0.143^2} = 2.157807$$

$$\frac{KL}{2\pi}\sqrt{\frac{P}{EA}} = \frac{0.7(6)}{2(0.143)}\sqrt{\frac{75(10^3)}{[200(10^9)][4.96(10^{-3})]}} = 0.127690$$

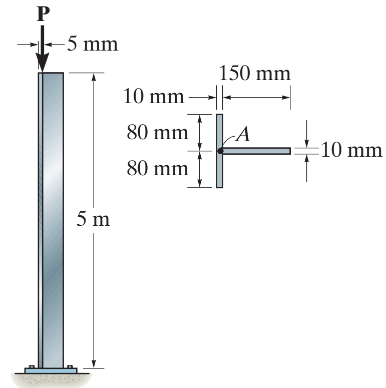
$\sigma_{max} = 15.12[1 + 2.157807 \sec(0.127690)]$

$= 48.0 \text{ MPa} < \sigma_Y = 250 \text{ MPa}$ O.K. **Ans.**

Ans:
$\sigma_{max} = 48.0 \text{ MPa}$

***13–48.**

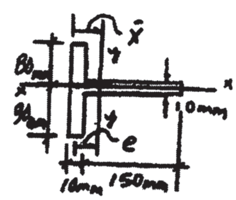
The aluminum column is fixed at the bottom and free at the top. Determine the maximum force P that can be applied at A without causing it to buckle or yield. Use a factor of safety of 3 with respect to buckling and yielding. $E_{al} = 70$ GPa, $\sigma_Y = 95$ MPa.



SOLUTION

$$\bar{x} = \frac{(0.005)(0.16)(0.01) + (0.085)(0.15)(0.01)}{0.16(0.01) + 0.15(0.01)} = 0.04371 \text{ m}$$

$$I_y = \frac{1}{12}(0.16)(0.01)^3 + (0.16)(0.01)(0.04371 - 0.005)^2 + \frac{1}{12}(0.01)(0.15)^3$$
$$+ (0.15)(0.01)(0.085 - 0.04371)^2 = 7.7807\left(10^{-6}\right) \text{m}^4$$



$$I_x = \frac{1}{12}(0.01)\left(0.16^3\right) + \frac{1}{12}(0.15)\left(0.01^3\right) = 3.42583\left(10^{-6}\right) \text{m}^4$$

$$A = (0.16)(0.01) + (0.15)(0.01) = 3.1\left(10^{-3}\right) \text{m}^2$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{7.7807\left(10^{-6}\right)}{3.1\left(10^{-3}\right)}} = 0.0501 \text{ m}$$

Buckling about *x–x* axis:

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(70)\left(10^9\right)(3.42583)\left(10^{-6}\right)}{[(2.0)(5)]^2} = 23668 \text{ N}$$

$P_{allow} = \dfrac{P_{cr}}{3} = 7.89$ kN **(controls)** **Ans.**

Check: $\sigma_{cr} = \dfrac{P_{cr}}{A} = \dfrac{23668 \text{ N}}{3.1\left(10^{-3}\right)} = 7.63 \text{ MPa} < \sigma_Y$ **OK**

Yielding about *y–y* axis:

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}}\right)$$

$$\frac{ec}{r^2} = \frac{(0.03871)(0.04371)}{0.0501^2} = 0.6741$$

$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{2.0(5)}{2(0.0501)}\sqrt{\frac{P}{70\left(10^8\right)(3.1)\left(10^{-3}\right)}} = 6.7749\left(10^{-3}\right)\sqrt{P}$$

$$95\left(10^6\right)(3.1)\left(10^{-3}\right) = P[1 + 0.6741 \sec\left(6.7749\left(10^{-3}\right)\sqrt{P}\right)]$$

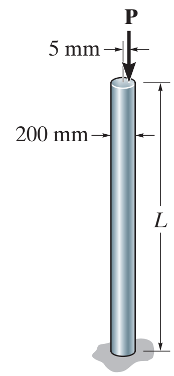
By trial and error:

$P = 45.61$ kN $\qquad P_{allow} = \dfrac{45.61}{3} = 15.2$ kN

Ans:
$P_{allow} = 7.89$ kN

13–49.

The aluminum rod is fixed at its base and free at its top. If the eccentric load $P = 200$ kN is applied, determine the greatest allowable length L of the rod so that it does not buckle or yield. $E_{al} = 72$ GPa, $\sigma_Y = 410$ MPa.



SOLUTION

Section Properties:

$$A = \pi(0.1^2) = 0.031416 \text{ m}^2 \qquad I = \frac{\pi}{4}(0.1^4) = 78.54(10^{-6}) \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{78.54(10^{-6})}{0.031416}} = 0.05 \text{ m}$$

Yielding:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$\frac{P}{A} = \frac{200(10^3)}{0.031416} = 6.3662(10^4) \text{ Pa}$$

$$\frac{ec}{r^2} = \frac{0.005(0.1)}{(0.05)^2} = 0.2$$

$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{2.0(L)}{2(0.05)}\sqrt{\frac{200(10^3)}{72(10^9)(0.031416)}} = 0.188063L$$

$$410(10^4) = 6.3662(10^6)[1 + 0.2\sec(0.188063 \text{ L})]$$

$$L = 8.34 \text{ m} \qquad \text{(controls)}$$ **Ans.**

Buckling about x–x axis:

$$\frac{P}{A} = 6.36 \text{ MPa} < \sigma_Y \qquad \text{Euler formula is valid.}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

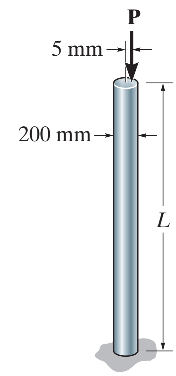
$$200(10^3) = \frac{\pi^2(72)(10^9)(78.54)(10^{-4})}{[(2.0)(L)]^2}$$

$$L = 8.34 \text{ m}$$

Ans:
$L = 8.34$ m

13–50.

The aluminum rod is fixed at its base and free and at its top. If the length of the rod is $L = 2$ m, determine the greatest allowable load P that can be applied so that the rod does not buckle or yield. Also, determine the largest sidesway deflection of the rod due to the loading. $E_{al} = 72$ GPa, $\sigma_Y = 410$ MPa.



SOLUTION

Section Properties:

$$A = \pi(0.1^2) = 0.031416 \text{ m}^2 \qquad I = \frac{\pi}{4}(0.1^4) = 78.54(10^{-6}) \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{78.54(10^{-6})}{0.031416}} = 0.05 \text{ m}$$

Yielding:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$\frac{ec}{r^2} = \frac{(0.005)(0.1)}{0.05^2} = 0.2$$

$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{2(2)}{2(0.05)}\sqrt{\frac{P}{72(10^9)(0.031416)}} = 0.8410\,(10^{-3})\sqrt{P}$$

$$410(10^4)(0.031416) = P[(1 + 0.2\sec(0.8410(10^{-3})\sqrt{P})]$$

By trial and Error:

$P = 3.20$ MN (controls) **Ans.**

Buckling:

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(72)(10^9)(78.54)(10^{-6})}{[(2.0)(2)]^2} = 3488 \text{ kN}$$

Check: $\sigma_{cr} = \dfrac{P_{cr}}{A} = \dfrac{3488(10^3)}{0.031416} = 111 \text{ MPa} < \sigma_Y$ OK

Maximum deflection:

$$v_{\max} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{KL}{2}\right) - 1\right]$$

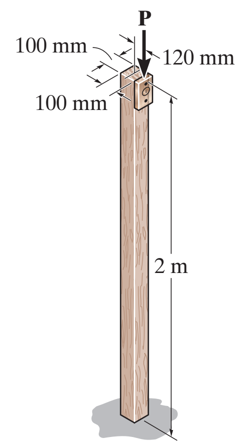
$$\sqrt{\frac{P}{EI}}\frac{KL}{2} = \sqrt{\frac{3.20(10^6)}{72(10^9)(78.54)(10^{-6})}}\left(\frac{2.0(2)}{2}\right) = 1.5045$$

$v_{\max} = 5[\sec(1.5045) - 1] = 70.5$ mm **Ans.**

Ans:
$P = 3.20$ MN,
$v_{\max} = 70.5$ mm

13–51.

The wood column is fixed at its base and free at its top. Determine the load P that can be applied to the edge of the column without causing the column to fail either by buckling or by yielding. $E_w = 12$ GPa, $\sigma_Y = 55$ MPa.



SOLUTION

Section Properties:

$$A = 0.1(0.1) = 0.01\ \text{m}^2 \quad I = \frac{1}{12}(0.1)(0.1)^3 = 8.333(10^{-6})\ \text{m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{8.333(10^{-6})}{0.01}} = 0.02887\ \text{m}$$

Buckling:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(12)(10^9)(8.333)(10^{-6})}{[2.0(2)]^2} = 61.7\ \text{kN}$$

Check: $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{61.7(10^3)}{0.01} = 6.17\ \text{MPa} < \sigma_Y$ **OK**

Yielding:

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$\frac{ec}{r^2} = \frac{0.12(0.05)}{(0.02887)^2} = 7.20$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{2.0(2)}{2(0.02887)}\sqrt{\frac{P}{12(10^9)(0.01)}} = 0.006324\sqrt{P}$$

$$55(10^6)(0.01) = P[1 + 7.20\sec(0.006324\sqrt{P})]$$

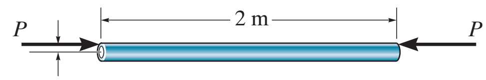
By trial and error:

$P = 31400\ \text{N} = 31.4\ \text{kN}$ controls **Ans.**

Ans:
$P = 31.4$ kN

***13–52.**

The tube is made of copper and has an outer diameter of 35 mm and a wall thickness of 7 mm. Determine the eccentric load P that it can support without failure. The tube is pin supported at its ends. $E_{cu} = 120$ GPa, $\sigma_Y = 750$ MPa.



SOLUTION

Section Properties:

$$A = \frac{\pi}{4}(0.035^2 - 0.021^2) = 0.61575(10^{-3})\ \text{m}^2$$

$$I = \frac{\pi}{4}(0.0175^4 - 0.0105^4) = 64.1152(10^{-9})\ \text{m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{64.1152(10^{-9})}{0.61575(10^{-3})}} = 0.010204\ \text{m}$$

For a column pinned at both ends, $K = 1$. Then $KL = 1(2) = 2$ m.

Buckling: Applying *Euler's* formula,

$$P_{max} = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(120)(10^9)\left[64.1152(10^{-9})\right]}{2^2} = 18983.7\ \text{N} = 18.98\ \text{kN}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{18983.7}{0.61575(10^{-3})} = 30.83\ \text{MPa} < \sigma_Y = 750\ \text{MPa} \qquad \text{O.K.}$$

Yielding: Applying the secant formula,

$$\sigma_{max} = \frac{P_{max}}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{(KL)}{2r}\sqrt{\frac{P_{max}}{EA}}\right)\right]$$

$$750\left(10^6\right) = \frac{P_{max}}{0.61575(10^{-3})}\left[1 + \frac{0.014(0.0175)}{0.010204^2}\sec\left(\frac{2}{2(0.010204)}\sqrt{\frac{P_{max}}{120(10^9)[0.61575(10^{-3})]}}\right)\right]$$

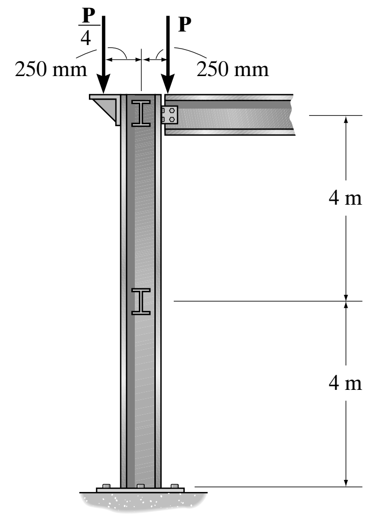
$$750\left(10^6\right) = \frac{P_{max}}{0.61575(10^{-3})}\left(1 + 2.35294\sec 0.0114006\sqrt{P_{max}}\right)$$

Solving by trial and error,

$$P_{max} = 16\,884\ \text{N} = 16.9\ \text{kN} \quad (\textit{Controls!}) \qquad \textbf{Ans.}$$

Ans:
$P_{max} = 16.9$ kN

13–53. The W250 × 45 A-36-steel column is pinned at its top and fixed at its base. Also, the column is braced along its weak axis at mid-height. If $P = 250$ kN, investigate whether the column is adequate to support this loading. Use F.S. = 2 against buckling and F.S. = 1.5 against yielding.



SOLUTION

Section Properties. From the table listed in the appendix, the necessary section properties for a W250 × 45 are

$A = 5700\text{ mm}^2 = 5.70\left(10^{-3}\right)\text{m}^2$ $\qquad r_x = 112\text{ mm} = 0.112\text{ m}$

$I_y = 7.03\left(10^6\right)\text{mm}^4 = 7.03\left(10^{-6}\right)\text{m}^4$ $\qquad c = \dfrac{d}{2} = \dfrac{266}{2} = 133\text{ mm} = 0.133\text{ m}$

The eccentricity of the equivalent force $P' = 250 + \dfrac{250}{4} = 312.5$ kN is

$$e = \frac{250(0.25) - \frac{250}{4}(0.25)}{250 + \frac{250}{4}} = 0.15\text{ m}$$

Buckling About the Weak Axis. The column is braced along the weak axis at midheight and the support provided by the bracing can be considered as a pin. The top portion of the column is critical is since the top is pinned so $K_y = 1$ and $L = 4$ m Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI_y}{(KL)_y{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[7.03\left(10^{-6}\right)\right]}{[1(4)]^2} = 867.29\text{ kN}$$

Euler's equation is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{867.29\left(10^3\right)}{5.70\left(10^{-3}\right)} = 152.16\text{ MPa} < \sigma_Y = 250\text{ MPa} \qquad \text{O.K.}$$

Then,

$$P'_{\text{allow}} = \frac{P_{cr}}{\text{F.S.}} = \frac{867.29}{2} = 433.65\text{ kN}$$

Since $P'_{\text{allow}} > P'$, the column *does not buckle*.

Yielding About Strong Axis. Since the column is fixed at its base and pinned at its top, $K_x = 0.7$ and $L = 8$ m. Applying the secant formula with $P'_{\max} = P'(\text{F.S.}) = 312.5(1.5) = 468.75$ kN

$$\sigma_{\max} = \frac{P'_{\max}}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P'_{\max}}{EA}}\right]\right]$$

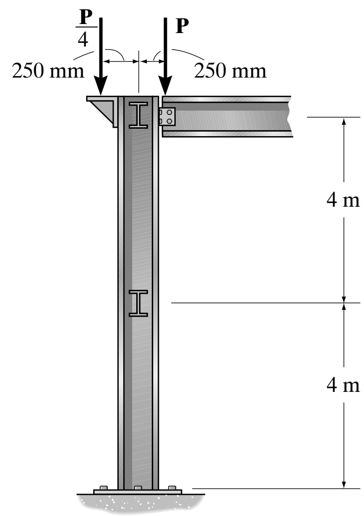
$$= \frac{468.75\left(10^3\right)}{5.70\left(10^{-3}\right)}\left[1 + \frac{0.15(0.133)}{0.112^2}\sec\left[\frac{0.7(8)}{2(0.112)}\sqrt{\frac{468.75\left(10^3\right)}{200\left(10^9\right)\left[5.70\left(10^{-3}\right)\right]}}\right]\right]$$

$$= 231.84\text{ MPa}$$

Since $\sigma_{\max} < \sigma_Y = 250$ MPa, the column *does not yield*.

Ans:
The column is adequate.

***13–54.** The W250 × 45 A-36-steel column is pinned at its top and fixed at its base. Also, the column is braced along its weak axis at mid-height. Determine the allowable force P that the column can support without causing it either to buckle or yield. Use F.S. = 2 against buckling and F.S. = 1.5 against yielding.



Section Properties. From the table listed in the appendix, the necessary section properties for a W250 × 45 are

$A = 5700\text{ mm}^2 = 5.70\left(10^{-3}\right)\text{ m}^2 \qquad r_x = 112\text{ mm} = 0.112\text{ m}$

$I_y = 7.03\left(10^6\right)\text{mm}^4 = 7.03\left(10^{-6}\right)\text{m}^4 \qquad c = \dfrac{d}{2} = \dfrac{266}{2} = 133\text{ mm} = 0.133\text{ m}$

The eccentricity of the equivalent force $P' = P + \dfrac{P}{4} = 1.25P$ is

$$e = \frac{P(0.25) - \dfrac{P}{4}(0.25)}{P + \dfrac{P}{4}} = 0.15\text{ m}$$

Buckling About the Weak Axis. The column is braced along the weak axis at midheight and the support provided by the bracing can be considered as a pin. The top portion of the column is critical is since the top is pinned so $K_y = 1$ and $L = 4$ m. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI_y}{(KL)_y{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[7.03\left(10^{-6}\right)\right]}{[1(4)]^2} = 867.29\text{ kN}$$

Euler's equation is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{867.29\left(10^3\right)}{5.70\left(10^{-3}\right)} = 152.16\text{ MPa} < \sigma_Y = 250\text{ MPa} \qquad \text{O.K.}$$

Then,

$$P'_{\text{allow}} = \frac{P_{cr}}{\text{F.S.}}$$

$$1.25P_{\text{allow}} = \frac{867.29}{2}$$

$$P_{\text{allow}} = 346.92\text{ kN}$$

Yielding About Strong Axis. Since the column is fixed at its base and pinned at its top, $K_x = 0.7$ and $L = 8$ m. Applying the secant formula,

$$\sigma_{\max} = \frac{P'_{\max}}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P'_{\max}}{EA}}\right]\right]$$

$$250\left(10^6\right) = \frac{1.25P_{\max}}{5.70\left(10^{-3}\right)}\left[1 + \frac{0.15(0.133)}{0.112^2}\sec\left[\frac{0.7(8)}{2(0.112)}\sqrt{\frac{1.25P_{\max}}{200\left(10^9\right)\left[5.70\left(10^{-3}\right)\right]}}\right]\right]$$

$$250\left(10^6\right) = \frac{1.25P_{\max}}{5.70\left(10^{-3}\right)}\left(1 + 1.5904\sec(0.00082783)\sqrt{P_{\max}}\right)$$

Solving by trial and error,

$$P_{\max} = 401.75\text{ kN}$$

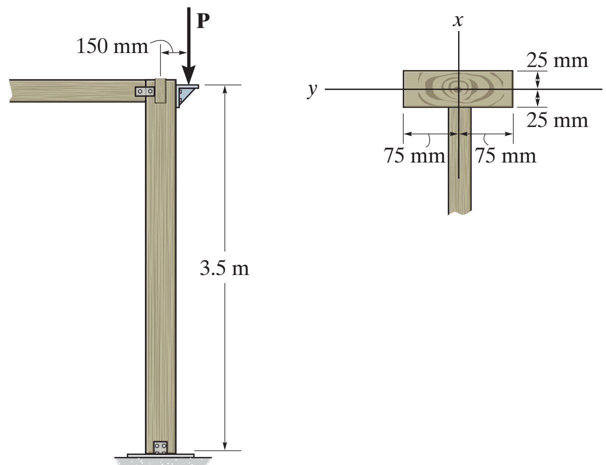
Then,

$$P_{\text{allow}} = \frac{401.75}{1.5} = 267.83\text{ kN} = 268\text{ kN (controls)} \qquad \textbf{Ans.}$$

Ans:
$e = 0.15$ m, $P_{\text{allow}} = 346.92$ kN,
$P_{\text{allow}} = 268$ kN

13–55.

The wood column is pinned at its base and top. If the eccentric force $P = 10$ kN is applied to the column, investigate whether the column is adequate to support this loading without buckling or yielding. Take $E = 10$ GPa and $\sigma_Y = 15$ MPa.



SOLUTION

Section Properties:

$$A = 0.05(0.15) = 7.5\left(10^{-3}\right)\text{ m}^2$$

$$I_x = \frac{1}{12}(0.05)\left(0.15^3\right) = 14.0625\left(10^{-6}\right)\text{ m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{14.0625\left(10^{-6}\right)}{7.5\left(10^{-3}\right)}} = 0.04330\text{ m}$$

$$I_y = \frac{1}{12}(0.15)\left(0.05^3\right) = 1.5625\left(10^{-6}\right)\text{m}^4$$

$e = 0.15$ m $\qquad$ $c = 0.075$ m

For a column that is pinned at both ends, $K = 1$. Then,

$$(KL)_x = (KL)_y = 1(3.5) = 3.5\text{ m}$$

Buckling About the Weak Axis: Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 EI_y}{(KL)_y{}^2} = \frac{\pi^2\left[10\left(10^9\right)\right]\left[1.5625\left(10^{-6}\right)\right]}{3.5^2} = 12.59\text{ kN}$$

Euler's formula is valid if $\sigma_{\text{cr}} < \sigma_Y$.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{12.59\left(10^3\right)}{7.5\left(10^{-3}\right)} = 1.68\text{ MPa} < \sigma_Y = 15\text{ MPa}$$ **O.K.**

Since $P_{\text{cr}} > P = 10$ kN, the column *will not buckle*.

Yielding About Strong Axis: Applying the secant formula,

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right]\right]$$

$$= \frac{10\left(10^3\right)}{7.5\left(10^{-3}\right)}\left[1 + \frac{0.15(0.075)}{0.04330^2}\sec\left[\frac{3.5}{2(0.04330)}\sqrt{\frac{10\left(10^3\right)}{10\left(10^9\right)\left[7.5\left(10^{-3}\right)\right]}}\right]\right]$$

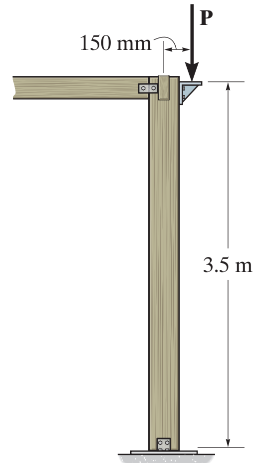
$$= 10.29\text{ MPa}$$

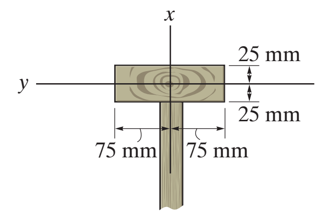
Since $\sigma_{\max} < \sigma_Y = 15$ MPa, the column *will not yield*. **Ans.**

Ans:
Yes

***13–56.**

The wood column is pinned at its base and top. Determine the maximum eccentric force P the column can support without causing it to either buckle or yield. Take $E = 10$ GPa and $\sigma_Y = 15$ MPa.





SOLUTION

Section Properties:

$$A = 0.05(0.15) = 7.5\left(10^{-3}\right)\text{ m}^2$$

$$I_x = \frac{1}{12}(0.05)\left(0.15^3\right) = 14.0625\left(10^{-6}\right)\text{ m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{14.0625\left(10^{-6}\right)}{7.5\left(10^{-3}\right)}} = 0.04330\text{ m}$$

$$I_y = \frac{1}{12}(0.15)\left(0.05^3\right) = 1.5625\left(10^{-6}\right)\text{m}^4$$

$e = 0.15$ m $\qquad c = 0.075$ m

For a column that is pinned at both ends, $K = 1$. Then,

$$(KL)_x = (KL)_y = 1(3.5) = 3.5\text{ m}$$

Buckling About the Weak Axis: Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI_y}{(KL)_y{}^2} = \frac{\pi^2\left[10\left(10^9\right)\right]\left[1.5625\left(10^{-6}\right)\right]}{3.5^2} = 12.59\text{ kN} = 12.6\text{ kN} \qquad \textbf{Ans.}$$

Euler's formula is valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{12.59\left(10^3\right)}{7.5\left(10^{-3}\right)} = 1.68\text{ MPa} < \sigma_Y = 15\text{ MPa} \qquad \textbf{O.K.}$$

Yielding About Strong Axis: Applying the secant formula with $P = P_{cr} = 12.59$ kN,

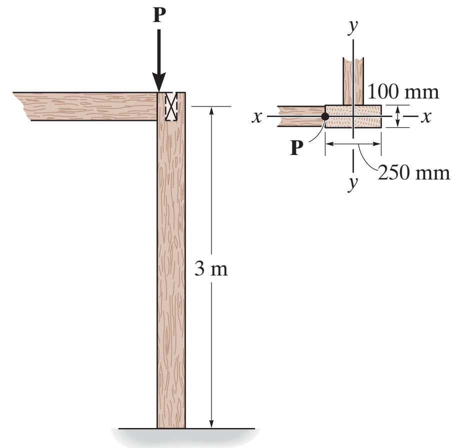
$$\sigma_{max} = \frac{P}{A}\left[\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right]\right]\right]$$

$$= \frac{12.59\left(10^3\right)}{7.5\left(10^{-3}\right)}\left[1 + \frac{0.15(0.075)}{0.04330^2}\sec\left[\frac{3.5}{2(0.04330)}\sqrt{\frac{12.59\left(10^3\right)}{10\left(10^9\right)\left[7.5\left(10^{-3}\right)\right]}}\right]\right]$$

$$= 13.31\text{ MPa} < \sigma_Y = 15\text{ MPa} \qquad \textbf{O.K.}$$

Ans:
$P_{cr} = 12.6$ kN

13–57. The wood column is fixed at its base and can be assumed pin connected at its top. Determine the maximum eccentric load P that can be applied without causing the column to buckle or yield. $E_w = 12$ GPa, $\sigma_Y = 56$ MPa.



SOLUTION

Section Properties:

$$A = 0.25(0.1) = 0.025\ \text{m}^2$$

$$I_y = \frac{1}{12}(0.1)(0.25^3) = 0.13021(10^{-3})\ \text{m}^4$$

$$I_x = \frac{1}{12}(0.25)(0.1^3) = 20.8333(10^{-6})\ \text{m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{0.13021(10^{-3})}{0.025}} = 0.072169\ \text{m}$$

Buckling about $x-x$ axis:

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2[12(10^9)][20.8333(10^{-6})]}{[0.7(3)]^2} = 559.50(10^3)\ \text{N} = 560\ \text{kN}$$

Check: $\sigma_{cr} = \dfrac{P_{cr}}{A} = \dfrac{559.50(10^3)}{0.025} = 22.38(10^6)\ \text{N/m}^2 = 22.4\ \text{MPa} < \sigma_Y$ O.K.

Yielding about $y-y$ axis:

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}}\right]$$

$$\frac{ec}{r^2} = \frac{0.125(0.125)}{0.072169^2} = 3.00$$

$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{0.7(3)}{2(0.072169)}\sqrt{\frac{P}{[12(10^9)](0.125)}} = 0.8400(10^{-3})\sqrt{P}$$

$$56(10^6) = \frac{P}{0.025}\left\{1 + 3.00\sec[0.8400(10^{-3})\sqrt{P}]\right\}$$

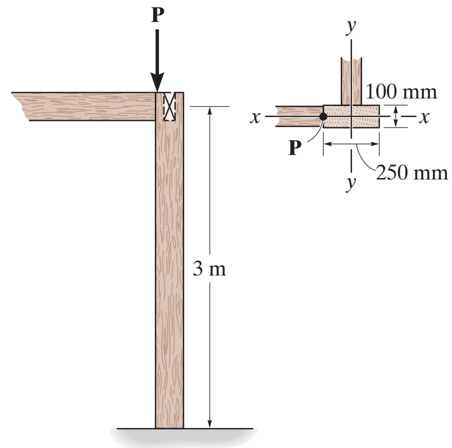
$$1.40(10^6) = P\left\{1 + 3.00\sec[0.8400(10^{-3})\sqrt{P}]\right\}$$

By trial and error:

$P = 320.08(10^3)$ N $= 320$ kN (controls) **Ans.**

Ans:
$P = 320$ kN

13–58. The wood column is fixed at its base and can be assumed fixed connected at its top. Determine the maximum eccentric load P that can be applied without causing the column to buckle or yield. $E_w = 12$ GPa, $\sigma_Y = 56$ MPa.



SOLUTION

Section Properties:

$$A = 0.25(0.1) = 0.025 \text{ m}^2$$

$$I_y = \frac{1}{12}(0.1)(0.25^3) = 0.13021(10^{-3}) \text{ m}^4$$

$$I_x = \frac{1}{12}(0.25)(0.1^3) = 20.8333(10^{-6}) \text{ m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{0.13021(10^{-3})}{0.025}} = 0.072169 \text{ m}$$

Buckling about $x-x$ axis:

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2[12(10^9)][20.8333(10^{-6})]}{[0.5(3)]^2} = 1.097(10^6) \text{ N} = 1.10 \text{ MN}$$

Check: $\sigma_{cr} = \dfrac{P_{cr}}{A} = \dfrac{1.097(10^6)}{0.025} = 43.86(10^6) \text{ N/m}^2 = 43.9 \text{ MPa} < \sigma_Y (= 56 \text{ MPa})$ O.K.

Yielding about $y-y$ axis:

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}}\right]$$

$$\frac{ec}{r^2} = \frac{0.125(0.125)}{0.072169^2} = 3.00$$

$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{0.5(3)}{2(0.072169)}\sqrt{\frac{P}{[12(10^9)](0.025)}} = 0.600(10^{-3})\sqrt{P}$$

$$56(10^6) = \frac{P}{0.025}\left\{1 + 3.00\sec[0.600(10^{-3})\sqrt{P}]\right\}$$

$$1.4(10^6) = P\left\{1 + 3.00\sec[0.600(10^{-3})\sqrt{P}]\right\}$$

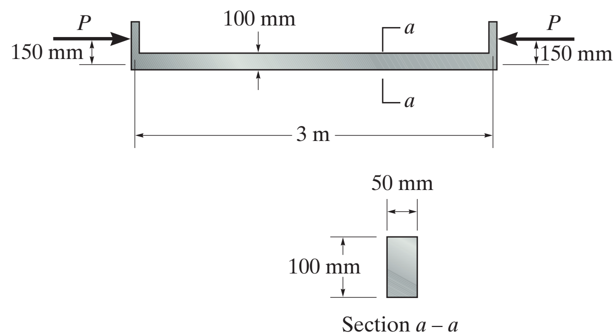
By trial and error:

$P = 334.13(10^3)$ N = 334 kN (controls) **Ans.**

Ans:
$P = 334$ kN

13–59.

Determine the maximum eccentric load P the 2014-T6-aluminum-alloy strut can support without causing it either to buckle or yield. The ends of the strut are pin connected.



SOLUTION

Section Properties: The necessary section properties are

$$A = 0.05(0.1) = 5\left(10^{-3}\right)\text{m}^2$$

$$I_y = \frac{1}{12}(0.1)\left(0.05^3\right) = 1.04167\left(10^{-6}\right)\text{ m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{4.1667\left(10^{-6}\right)}{5\left(10^{-3}\right)}} = 0.02887\text{ m}$$

For a column that is pinned at both of its ends $K = 1$. Thus,

$$(KL)_x = (KL)_y = 1(3) = 3\text{ m}$$

Buckling About the Weak Axis: Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)_y{}^2} = \frac{\pi^2\left[73.1\left(10^9\right)\right]\left[1.04167\left(10^{-6}\right)\right]}{3^2} = 83.50\text{ kN} = 83.5\text{ kN} \qquad \textbf{Ans.}$$

Critical Stress: Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{83.50\left(10^3\right)}{5\left(10^{-3}\right)} = 16.70\text{ MPa} < \sigma_Y = 414\text{ MPa} \qquad \textbf{O.K.}$$

Yielding About Strong Axis: Applying the secant formula,

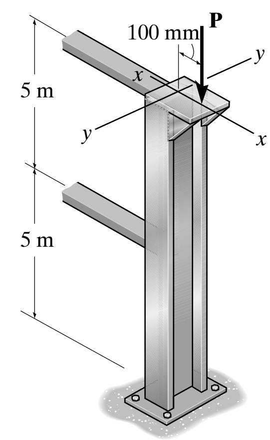
$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right]\right]$$

$$= \frac{83.50\left(10^3\right)}{5\left(10^{-3}\right)}\left[1 + \frac{0.15(0.05)}{0.02887^2}\sec\left[\frac{3}{2(0.02887)}\sqrt{\frac{83.50\left(10^3\right)}{73.1\left(10^9\right)\left[5\left(10^{-3}\right)\right]}}\right]\right]$$

$$= 229.27\text{ MPa} < \sigma_Y = 414\text{ MPa} \qquad \textbf{O.K.}$$

Ans:
$P_{cr} = 83.5$ kN

***13–60.** The W200 × 22 A-36-steel column is fixed at its base. Its top is constrained to rotate about the *y–y* axis and free to move along the *y–y* axis. Also, the column is braced along the *x–x* axis at its mid-height. Determine the allowable eccentric force *P* that can be applied without causing the column either to buckle or yield. Use F.S. = 2 against buckling and F.S. = 1.5 against yielding.



Section Properties. From the table listed in the appendix, the necessary section properties for a W200 × 22 are

$A = 2860\text{ mm}^2 = 2.86\left(10^{-3}\right)\text{ m}^2$ $\qquad r_y = 22.3\text{ mm} = 0.0223\text{ m}$

$I_x = 20.0\left(10^6\right)\text{ mm}^4 = 20.0\left(10^{-6}\right)\text{m}^4$ $\qquad c = \dfrac{b_f}{2} = \dfrac{102}{2} = 51\text{ mm} = 0.051\text{ m}$

$e = 0.1\text{m}$

Buckling About the Strong Axis. Since the column is fixed at the base and free at the top, $K_x = 2$. Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 EI_x}{(KL)_x{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[20.0\left(10^{-6}\right)\right]}{[2(10)]^2} = 98.70\text{kN}$$

Euler's formula is valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{98.70\left(10^3\right)}{2.86\left(10^{-3}\right)} = 34.51\text{ MPa} < \sigma_Y = 250\text{MPa} \qquad \text{O.K.}$$

Then,

$$P_{\text{allow}} = \frac{P_{cr}}{\text{F.S.}} = \frac{98.70}{2} = 49.35\text{ kN}$$

Yielding About Weak Axis. Since the support provided by the bracing can be considered a pin connection, the upper portion of the column is pinned at both of its ends. Then $K_y = 1$ and $L = 5$ m. Applying the secant formula,

$$\sigma_{\max} = \frac{P_{\max}}{A}\left[1 + \frac{ec}{r_y{}^2}\sec\left[\frac{(KL)_y}{2r_y}\sqrt{\frac{P_{\max}}{EA}}\right]\right]$$

$$250\left(10^6\right) = \frac{P_{\max}}{2.86\left(10^{-3}\right)}\left[1 + \frac{0.1(0.051)}{0.0223^2}\sec\left[\frac{1(5)}{2(0.0223)}\sqrt{\frac{P_{\max}}{200\left(10^9\right)\left[2.86\left(10^{-3}\right)\right]}}\right]\right]$$

$$250\left(10^6\right) = \frac{P_{\max}}{2.86\left(10^{-3}\right)}\left[1 + 10.2556\sec 4.6875\left(10^{-3}\right)\sqrt{P_{\max}}\right]$$

Solving by trial and error,

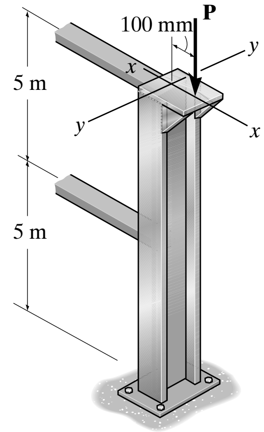
$$P_{\max} = 39.376\text{ kN}$$

Then,

$$P_{\text{allow}} = \frac{P_{\max}}{1.5} = \frac{39.376}{1.5} = 26.3\text{ kN (controls)} \qquad \textbf{Ans.}$$

Ans:
$P_{\text{allow}} = 26.3\text{ kN (controls)}$

13–61. The W200 × 22 A-36-steel column is fixed at its base. Its top is constrained to rotate about the y–y axis and free to move along the y–y axis. Also, the column is braced along the x–x axis at its mid-height. If $P = 25\text{ kN}$, determine the maximum normal stress developed in the column.



SOLUTION

Section Properties. From the table listed in the appendix, necessary section properties for a W200 × 22 are

$$A = 2860\text{ mm}^2 = 2.86\left(10^{-3}\right)\text{ m}^2 \qquad r_y = 22.3\text{ mm} = 0.0223\text{ m}$$

$$I_x = 20.0\left(10^{6}\right)\text{ mm}^4 = 20.0\left(10^{-6}\right)\text{m}^4 \qquad c = \frac{b_f}{2} = \frac{102}{2} = 51\text{ mm} = 0.051\text{ m}$$

$$e = 0.1\text{m}$$

Buckling About the Strong Axis. Since the column is fixed at the base and free at the top, $K_x = 2$. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI_x}{(KL)_x{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[20.0\left(10^{-6}\right)\right]}{[2(10)]^2} = 98.70\text{kN}$$

Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{98.70\left(10^3\right)}{2.86\left(10^{-3}\right)} = 34.51\text{ MPa} < \sigma_Y = 250\text{ MPa} \qquad \text{O.K.}$$

Since $P = 25\text{kN} < P_{cr}$, the column does not buckle.

Yielding About Weak Axis. Since the support provided by the bracing can be considered a pin connection, the upper portion of the column is pinned at both of its ends. Then $K_y = 1$ and $L = 5$ m. Applying the secant formula,

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r_y{}^2}\sec\left[\frac{(KL)}{2r_y}\sqrt{\frac{P}{EA}}\right]\right]$$

$$= \frac{2.5\left(10^3\right)}{2.86\left(10^{-3}\right)}\left[1 + \frac{0.1(0.051)}{0.0223^2}\sec\left[\frac{1(5)}{2(0.0223)}\sqrt{\frac{25\left(10^3\right)}{200\left(10^9\right)\left[2.86\left(10^{-3}\right)\right]}}\right]\right]$$

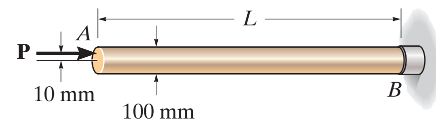
$$= 130.26\text{ MPa} = 130\text{ MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{\max} < \sigma_Y = 250\text{ MPa}$, the column does not yield.

Ans:
$\sigma_{\max} = 130\text{ MPa}$

13–62.

The brass rod is fixed at one end and free at the other end. If the eccentric load $P = 200$ kN is applied, determine the greatest allowable length L of the rod so that it does not buckle or yield. $E_{br} = 101$ GPa, $\sigma_Y = 69$ MPa.



SOLUTION

Section Properties:

$$A = \frac{\pi}{4}(0.1^2) = 2.50(10^{-3})\pi \text{ m}^2$$

$$I = \frac{\pi}{4}(0.05^4) = 1.5625(10^{-6})\pi \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{1.5625(10^{-6})\pi}{2.50(10^{-3})\pi}} = 0.025 \text{ m}$$

For a column fixed at one end and free at the other end, $K = 2$. Then $KL = 2(L) = 2L$.

Buckling: Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$200(10^3) = \frac{\pi^2(101)(10^9)\left[1.5625(10^{-6})\pi\right]}{(2L)^2}$$

$$L = 2.473 \text{ m}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{200(10^3)}{2.50(10^{-3})\pi} = 25.46 \text{ MPa} < \sigma_Y = 69 \text{ MPa} \qquad \textbf{(O.K!)}$$

Yielding: Applying the secant formula,

$$\sigma_Y = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{(KL)}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$69(10^6) = \frac{200(10^3)}{2.50(10^{-3})\pi}\left[1 + \frac{0.01(0.05)}{0.025^2}\sec\left(\frac{2L}{2(0.025)}\sqrt{\frac{200(10^3)}{101(10^9)\left[2.50(10^{-3})\pi\right]}}\right)\right]$$

$$69 = \frac{80}{\pi}(1 + 0.800 \sec 0.635140L)$$

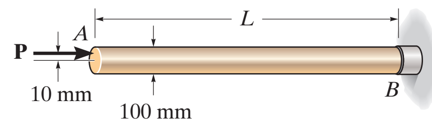
Solving by trial and error,

$$L = 1.7065 \text{ m} = 1.71 \text{ m} \qquad \textbf{(Controls!)} \qquad \textbf{Ans.}$$

Ans:
$L = 1.71$ m

13–63.

The brass rod is fixed at one end and free at the other end. If the length of the rod is $L = 2$ m, determine the greatest allowable load P that can be applied so that the rod does not buckle or yield. Also, determine the largest sidesway deflection of the rod dueto the loading. $E_{br} = 101$ GPa, $\sigma_Y = 69$ MPa.



SOLUTION

Section Properties:

$$A = \frac{\pi}{4}(0.1^2) = 2.50(10^{-3})\pi \text{ m}^2$$

$$I = \frac{\pi}{4}(0.05^4) = 1.5625(10^{-6})\pi \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{1.5625(10^{-6})\pi}{2.50(10^{-3})\pi}} = 0.025 \text{ m}$$

For a column fixed at one end and free at the other end, $K = 2$. Then $KL = 2(2) = 4$ m.

Buckling: Applying *Euler's* formula,

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2(101)(10^9)\left[1.5625(10^{-6})\pi\right]}{4^2}$$

$$= 305823.6 \text{ N} = 305.8 \text{ kN}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{305823.6}{2.50(10^{-3})\pi} = 38.94 \text{ MPa} < \sigma_Y = 69 \text{ MPa} \qquad \textbf{(O.K!)}$$

Yielding: Applying the secant formula,

$$\sigma_Y = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{(KL)}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$69(10^6) = \frac{P}{2.50(10^{-3})\pi}\left[1 + \frac{0.01(0.05)}{0.025^2}\sec\left(\frac{4}{2(0.025)}\sqrt{\frac{P}{101(10^9)\left[2.50(10^{-3})\pi\right]}}\right)\right]$$

$$69(10^6) = \frac{400P}{\pi}\left[1 + 0.800\sec 2.84043(10^{-3})\sqrt{P}\right]$$

Solving by trial and error,

$$P = 173700 \text{ N} = 174 \text{ kN} \qquad \textbf{(Controls!)} \qquad \textbf{Ans.}$$

Maximum Displacement:

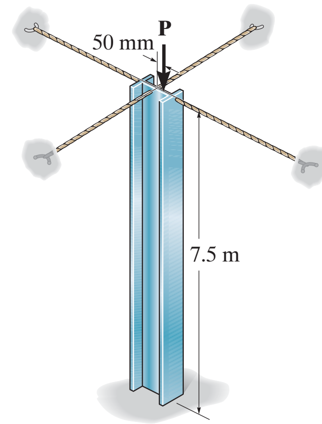
$$v_{max} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{KL}{2}\right) - 1\right]$$

$$= 0.01\left[\sec\left(\sqrt{\frac{173700}{101(10^9)\left[1.5625(10^{-6})\pi\right]}}\left(\frac{4}{2}\right)\right) - 1\right]$$

$$= 0.01650 \text{ m} = 16.5 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$P = 174$ kN
$v_{max} = 16.5$ mm

***13–64.** Determine the load P required to cause the steel W310 × 74 structural A-36 steel column to fail either by buckling or by yielding. The column is fixed at its bottom and the cables at its top act as a pin to hold it.



SOLUTION

Section Properties: For a wide flange section W310 × 74,

$$A = 9480 \text{ mm}^2 = 9.48(10^{-3}) \text{ m}^2$$

$$r_x = 132 \text{ mm} = 0.132 \text{ mm}$$

$$I_y = 23.4(10^6) \text{ mm}^4 = 23.4(10^{-6}) \text{ m}^4$$

$$d = 310 \text{ mm} = 0.310 \text{ mm}$$

For a column fixed at one end and pinned at the other end, $K = 0.7$.

$$(KL)_y = (KL)_x = 0.7(7.5) = 5.25 \text{ m}$$

Buckling About y–y Axis: Applying *Euler's* formula,

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)_y^2}$$

$$= \frac{\pi^2[200(10^9)][23.4(10^{-6})]}{5.25^2}$$

$$= 1.6758(10^6) \text{ N} = 1.68 \text{ MN}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1.6758(10^6)}{9.48(10^{-3})} = 176.77(10^6) \text{ N/m}^2$$

$$= 177 \text{ MPa} < \sigma_Y (= 250 \text{ MPa}) \qquad \text{O. K.}$$

Yielding About x–x Axis: Applying the secant formula,

$$\sigma_{max} = \frac{P}{A}\left\{1 + \frac{ec}{r_x^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right]\right\}$$

$$250(10^6) = \frac{P}{9.48(10^{-3})}\left\{1 + \frac{0.05\left(\frac{0.310}{2}\right)}{0.132^2}\sec\left[\frac{5.25}{2(0.132)}\sqrt{\frac{P}{200(10^9)(9.48)(10^{-3})}}\right]\right\}$$

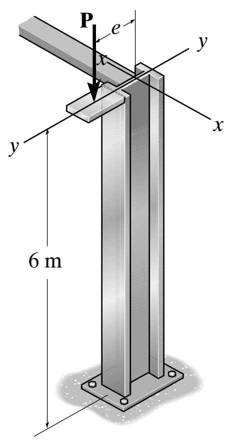
$$2.37(10^6) = P\left\{1 + 0.444789\sec[0.45671(10^{-3})\sqrt{P}]\right\}$$

Solving by trial and error,

$$P_{max} = 1.5511(10^6) \text{ N} = 1.55 \text{ MN } (\textit{Controls!})$$ **Ans.**

Ans:
$P_{max} = 1.55$ MN (*Controls!*)

13–65. The W250 × 28 A-36-steel column is fixed at its base. Its top is constrained to rotate about the y–y axis and free to move along the y–y axis. If $e = 350$ mm, determine the allowable eccentric force P that can be applied without causing the column either to buckle or yield. Use F.S. = 2 against buckling and F.S. = 1.5 against yielding.



SOLUTION

Section Properties. From the table listed in the appendix, necessary section properties for a W250 × 28 are

$$A = 3620\ \text{mm}^2 = 3.62\left(10^{-3}\right)\text{m}^2 \qquad r_x = 105\ \text{mm} = 0.105\ \text{m}$$

$$I_y = 1.78\left(10^6\right)\text{mm}^4 = 1.78\left(10^{-6}\right)\text{m}^4 \qquad c = \frac{d}{2} = \frac{260}{2} = 130\ \text{mm} = 0.13\ \text{m}$$

$$e = 0.35\ \text{m}$$

Buckling About the Strong Axis. Since the column is fixed at the base and pinned at the top, $K_x = 0.7$. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)_y{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[1.78\left(10^{-6}\right)\right]}{[0.7(6)]^2} = 199.18\ \text{kN}$$

Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{199.18\left(10^3\right)}{3.62\left(10^{-3}\right)} = 55.02\ \text{MPa} < \sigma_Y = 250\ \text{MPa} \qquad \text{O.K.}$$

Thus,

$$P_{\text{allow}} = \frac{P_{cr}}{\text{F.S.}} = \frac{199.18}{2} = 99.59\ \text{kN}$$

Yielding About Strong Axis. Since the column is fixed at its base and free at its top, $K_x = 2$. Applying the secant formula,

$$\sigma_{\max} = \frac{P_{\max}}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P_{\max}}{EA}}\right]\right]$$

$$250\left(10^6\right) = \frac{P_{\max}}{3.62\left(10^{-3}\right)}\left[1 + \frac{0.35(0.13)}{0.105^2}\sec\left[\frac{2(6)}{2(0.105)}\sqrt{\frac{P_{\max}}{200\left(10^9\right)\left[3.62\left(10^{-3}\right)\right]}}\right]\right]$$

$$250\left(10^6\right) = \frac{P_{\max}}{3.62\left(10^{-3}\right)}\left(1 + 4.1270\sec\left(0.0021237\right)\sqrt{P_{\max}}\right)$$

Solving by trial and error,

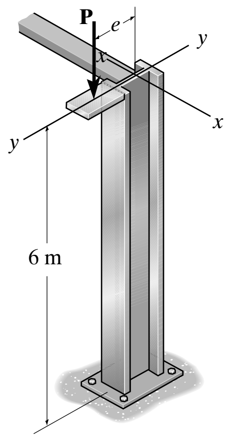
$$P_{\max} = 133.45\ \text{kN}$$

Then,

$$P_{\text{allow}} = \frac{P_{\max}}{1.5} = \frac{133.45}{1.5} = 88.97\ \text{kN} = 89.0\ \text{kN (controls)} \qquad \textbf{Ans.}$$

Ans:
Strong axis yielding controls.
$P_{\text{allow}} = 89.0$ kN

13–66. The W250 × 28 A-36-steel column is fixed at its base. Its top is constrained to rotate about the y–y axis and free to move along the y–y axis. Determine the force **P** and its eccentricity e so that the column will yield and buckle simultaneously.



SOLUTION

Section Properties. From the table listed in the appendix, the necessary section properties for a W250 × 28 are

$A = 3620 \text{ mm}^2 = 3.62\left(10^{-3}\right)\text{m}^2$ $\qquad r_x = 105 \text{ mm} = 0.105 \text{ m}$

$I_y = 1.78\left(10^6\right)\text{mm}^4 = 1.78\left(10^{-6}\right)\text{m}^4$ $\qquad c = \dfrac{d}{2} = \dfrac{260}{2} = 130 \text{ mm} = 0.13 \text{ m}$

Buckling About the Weak Axis. Since the column is fixed at the base and pinned at its top, $K_x = 0.7$. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)_y{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[1.78\left(10^{-6}\right)\right]}{[0.7(6)]^2} = 199.18 \text{ kN} = 199 \text{ kN} \qquad \textbf{Ans.}$$

Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{199.18\left(10^3\right)}{3.62\left(10^{-3}\right)} = 55.02 \text{ MPa} < \sigma_Y = 250 \text{ MPa} \qquad \text{O.K.}$$

Yielding About Strong Axis. Since the column is fixed at its base and free at its top, $K_x = 2$. Applying the secant formula with $P = P_{cr} = 199.18$ kN,

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right]\right]$$

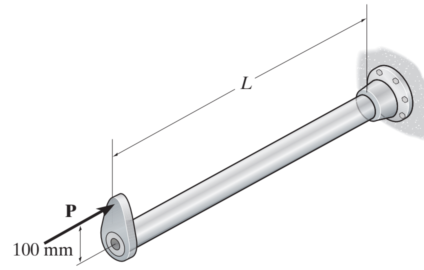
$$250\left(10^6\right) = \frac{199.18\left(10^3\right)}{3.62\left(10^{-3}\right)}\left[1 + \frac{e(0.13)}{0.105^2}\sec\left[\frac{2(6)}{2(0.105)}\sqrt{\frac{199.18\left(10^3\right)}{200\left(10^9\right)\left[3.62\left(10^{-3}\right)\right]}}\right]\right]$$

$e = 0.1753 \text{ m} = 175 \text{ mm}$ **Ans.**

Ans:
$P_{cr} = 199$ kN, $e = 175$ mm

13–67.

The 6061-T6 aluminum alloy solid shaft is fixed at one end but free at the other end. If the shaft has a diameter of 100 mm, determine its maximum allowable length L if it is subjected to the eccentric force $P = 80$ kN.



SOLUTION

Section Properties.

$$A = \pi(0.05^2) = 2.5(10^{-3})\pi \text{ m}^2$$

$$I = \frac{\pi}{4}(0.05^4) = 1.5625(10^{-6})\pi \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{1.5625(10^{-6})\pi}{2.5(10^{-3})\pi}} = 0.025 \text{ m}$$

$$e = 0.1 \text{ m} \qquad c = 0.05 \text{ m}$$

For a column that is fixed at one end and free at the other, $K = 2$. Thus,

$$KL = 2L$$

Buckling. The critical buckling load is $P_{cr} = 80$ kN. Applying Euler's equation,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$80(10^3) = \frac{\pi^2[68.9(10^9)][1.5625(10^{-6})\pi]}{(2L)^2}$$

$$L = 3.230 \text{ m}$$

Euler's equation is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{80(10^3)}{2.5(10^{-3})\pi} = 10.19 \text{ MPa} < \sigma_Y = 255 \text{ MPa} \qquad \text{(O.K.)}$$

Yielding. Applying the secant formula,

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left[\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right]\right]$$

$$255(10^6) = \frac{80(10^3)}{2.5(10^{-3})\pi}\left[1 + \frac{0.1(0.05)}{0.025^2}\sec\left[\frac{2L}{2(0.025)}\sqrt{\frac{80(10^3)}{68.9(10^9)[2.5(10^{-3})\pi]}}\right]\right]$$

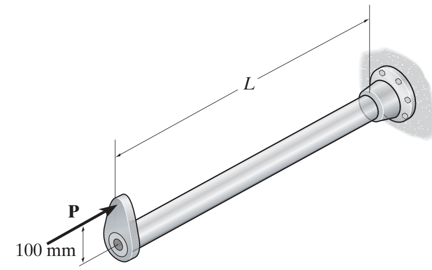
$$\sec 0.4864L = 3.0043$$

$$L = 2.532 \text{ m} = 2.53 \text{ m (controls)} \qquad \textbf{Ans.}$$

Ans:
$L = 2.53$ m

***13–68**

The 6061-T6 aluminum alloy solid shaft is fixed at one end but free at the other end. If the length is $L = 3$ m, determine its minimum required diameter if it is subjected to the eccentric force $P = 60$ kN.



SOLUTION

Section Properties.

$$A = \frac{\pi}{4}d^2 \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64}d^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}d^4}{\frac{\pi}{4}d^2}} = \frac{d}{4}$$

$$e = 0.1 \text{ m} \qquad c = \frac{d}{2}$$

For a column that is fixed at one end and free at the other, $K = 2$. Thus,

$$KL = 2(3) = 6 \text{ m}$$

Buckling. The critical buckling load is $P_{cr} = 60$ kN. Applying Euler's equation,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$60(10^3) = \frac{\pi^2\left[68.9(10^9)\right]\left(\frac{\pi}{64}d^4\right)}{6^2}$$

$$d = 0.08969 \text{ m} = 89.7 \text{ mm}$$

Yielding. Applying the secant formula,

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left[\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right]\right]$$

$$255(10^6) = \frac{60(10^3)}{\frac{\pi}{4}d^2}\left[1 + \frac{0.1\left(\frac{d}{2}\right)}{\left(\frac{d}{4}\right)^2}\sec\left[\frac{6}{2\left(\frac{d}{4}\right)}\sqrt{\frac{60(10^3)}{68.9(10^9)\left(\frac{\pi}{4}d^2\right)}}\right]\right]$$

$$255(10^6) = \frac{240(10^3)}{\pi d^2}\left[1 + \frac{0.8}{d}\sec\left(\frac{0.012636}{d^2}\right)\right]$$

Solving by trial and error,

$$d = 0.09831 \text{ m} = 98.3 \text{ mm (controls)}$$ **Ans.**

Ans:
$d = 98.3$ mm

13–69. A column of intermediate length buckles when the compressive stress is 280 MPa. If the slenderness ratio is 60, determine the tangent modulus.

SOLUTION

$$\sigma_{\text{cr}} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2}; \qquad \left(\frac{KL}{r}\right) = 60$$

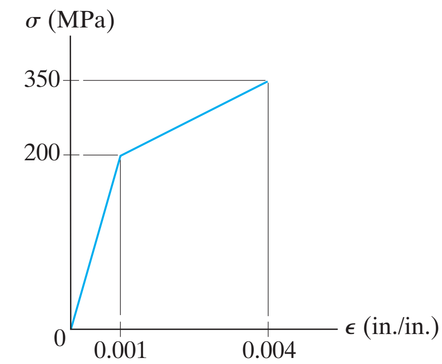
$$280(10^6) = \frac{\pi^2 E_t}{60^2}$$

$$E_t = 102.13(10^9)\ \text{N/m}^2 = 102\ \text{GPa}$$ **Ans.**

Ans:
$E_t = 102.13$ GPa

13–70.

The stress–strain diagram for the material of a column can be approximated as shown. Plot P/A vs. KL/r for the column.



SOLUTION

Tangent Moduli: From the stress-strain diagram,

$$(E_t)_1 = \frac{200(10^6)}{0.001} = 200\,\text{GPa} \qquad 0 \le \sigma < 200\text{ MPa}$$

$$(E_t)_2 = \frac{(350 - 200)(10^6)}{0.004 - 0.001} = 50\,\text{GPa} \qquad 200\text{ MPa} < \sigma \le 350\text{ MPa}$$

Critical Stress: Applying Engesser's equation,

$$\sigma_{cr} = \frac{P}{A} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2} \qquad \textbf{(1)}$$

If $E_t = (E_t)_1 = 200\,\text{GPa}$, Eq. (1) becomes

$$\frac{P}{A} = \frac{\pi^2\left[200(10^9)\right]}{\left(\frac{KL}{r}\right)^2} = \frac{1.974(10^6)}{\left(\frac{KL}{r}\right)^2}\text{ MPa}$$

When $\sigma_{cr} = \frac{P}{A} = \sigma_Y = 200\text{ MPa}$, this equation becomes

$$200(10^6) = \frac{\pi^2\left[200(10^9)\right]}{\left(\frac{KL}{r}\right)^2}$$

$$\frac{KL}{r} = 99.346 = 99.3$$

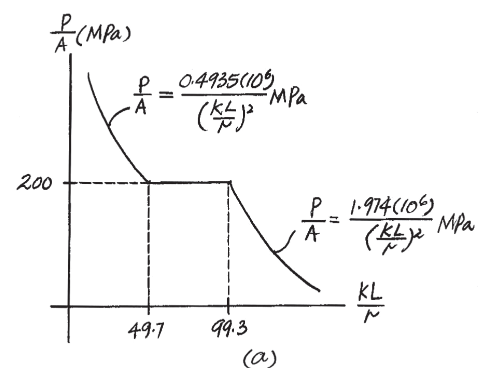
If $E_t = (E_t)_2 = 50\text{ GPa}$, Eq. (1) becomes

$$\frac{P}{A} = \frac{\pi^2\left[50(10^9)\right]}{\left(\frac{KL}{r}\right)^2} = \frac{0.4935(10^6)}{\left(\frac{KL}{r}\right)^2}\text{ MPa}$$

when $\sigma_{cr} = \frac{P}{A} = \sigma_Y = 200\text{ MPa}$, this equation gives

$$200(10^6) = \frac{\pi^2\left[50(10^9)\right]}{\left(\frac{KL}{r}\right)^2}$$

$$\frac{KL}{r} = 49.67 = 49.7$$

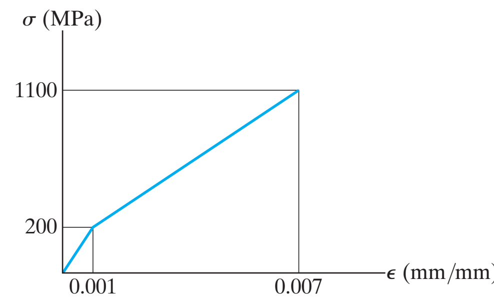


Using these results, the graphs of $\frac{P}{A}$ vs. $\frac{KL}{r}$ as shown in Fig. *a* can be plotted.

Ans:
N/A

13–71.

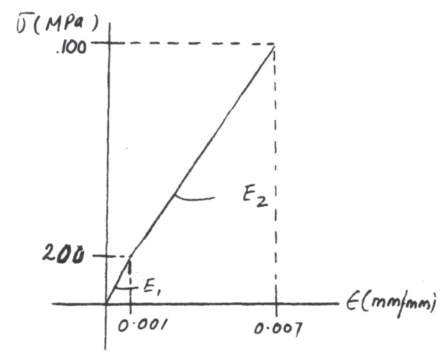
The stress–strain diagram for a material can be approximated by the two line segments shown. If a bar having a diameter of 80 mm and a length of 1.5 m is made from this material, determine the critical load provided the ends are pinned. Assume that the load acts through the axis of the bar. Use Engesser's equation.



SOLUTION

$$E_1 = \frac{200\,(10^6)}{0.001} = 200\text{ GPa}$$

$$E_2 = \frac{1100\,(10^6) - 200\,(10^6)}{0.007 - 0.001} = 150\text{ GPa}$$



Section properties:

$$I = \frac{\pi}{4}c^4; \quad A = \pi c^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}c^4}{\pi c^2}} = \frac{c}{2} = \frac{0.04}{2} = 0.02\text{ m}$$

Engesser's equation:

$$\frac{KL}{r} = \frac{1.0(1.5)}{0.02} = 75$$

$$\sigma_{cr} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 E_t}{(75)^2} = 1.7546(10^{-3})\,E_t$$

Assume $E_t = E_1 = 200$ GPa

$$\sigma_{cr} = 1.7546\,(10^{-3})(200)(10^9) = 351\text{ MPa} > 200\text{ MPa}$$

Therefore, inelastic buckling occurs:

Assume $E_t = E_2 = 150$ GPa

$$\sigma_{cr} = 1.7546\,(10^{-3})(150)(10^9) = 263.2\text{ MPa}$$

$200\text{ MPa} < \sigma_{cr} < 1100\text{ MPa}$ **O.K.**

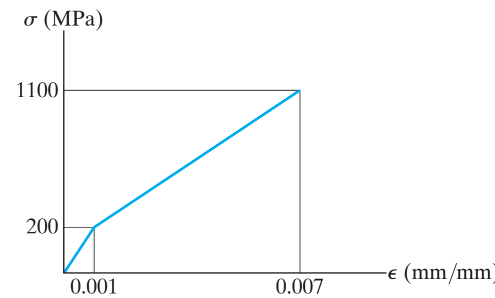
Critical load:

$P_{cr} = \sigma_{cr} A = 263.2\,(10^6)(\pi)(0.04^2) = 1323\text{ kN}$ **Ans.**

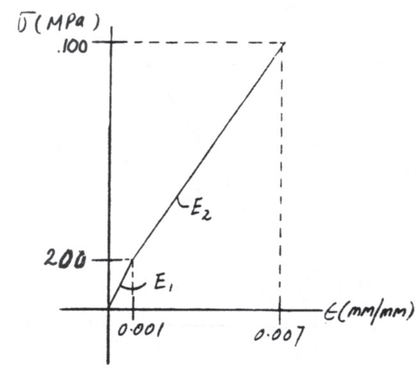
Ans:
$P_{cr} = 1323$ kN

***13–72.**

The stress–strain diagram for a material can be approximated by the two line segments shown. If a bar having a diameter of 80 mm and a length of 1.5 m is made from this material, determine the critical load provided the ends are fixed. Assume that the load acts through the axis of the bar. Use Engesser's equation.



SOLUTION



$$E_1 = \frac{200\,(10^6)}{0.001} = 200 \text{ GPa}$$

$$E_2 = \frac{1100\,(10^6) - 200\,(10^6)}{0.007 - 0.001} = 150 \text{ GPa}$$

Section properties:

$$I = \frac{\pi}{4}c^4; \quad A = \pi c^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}c^4}{\pi c^2}} = \frac{c}{2} = \frac{0.04}{2} = 0.02 \text{ m}$$

Engesser's equation:

$$\frac{KL}{r} = \frac{0.5\,(1.5)}{0.02} = 37.5$$

$$\sigma_{cr} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 E_t}{(37.5)^2} = 7.018385(10^{-3})\,E_t$$

Assume $E_t = E_1 = 200$ GPa

$\sigma_{cr} = 7.018385\,(10^{-3})(200)(10^9) = 1403.7 \text{ MPa} > 200 \text{ MPa}$ NG

Assume $E_t = E_2 = 150$ GPa

$\sigma_{cr} = 7.018385\,(10^{-3})(150)(10^9) = 1052.8 \text{ MPa}$

$200 \text{ MPa} < \sigma_{cr} < 1100 \text{ MPa}$ **O.K.**

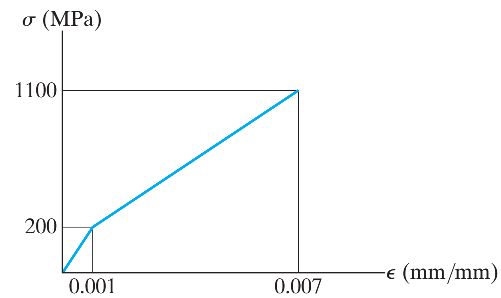
Critical load:

$P_{cr} = \sigma_{cr} A = 1052.8\,(10^6)(\pi)(0.04^2) = 5292 \text{ kN}$ **Ans.**

Ans:
$P_{cr} = 5292$ kN

13–73.

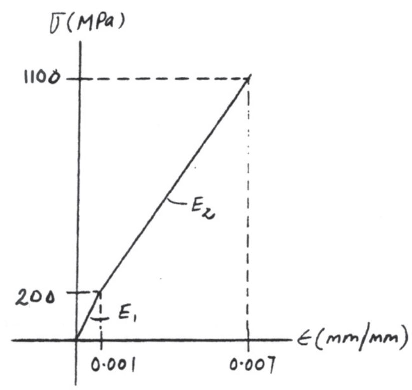
The stress–strain diagram for a material can be approximated by the two line segments shown. If a bar having a diameter of 80 mm and length of 1.5 m is made from this material, determine the critical load provided one end is pinned and the other is fixed. Assume that the load acts through the axis of the bar. Use Engesser's equation.



SOLUTION

$$E_1 = \frac{200\,(10^6)}{0.001} = 200\text{ GPa}$$

$$E_2 = \frac{1100\,(10^6) - 200\,(10^6)}{0.007 - 0.001} = 150\text{ GPa}$$



Section properties:

$$I = \frac{\pi}{4}c^4; \quad A = \pi c^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}c^4}{\pi c^2}} = \frac{c}{2} = \frac{0.04}{2} = 0.02\text{ m}$$

Engesser's equation:

$$\frac{KL}{r} = \frac{0.7\,(1.5)}{0.02} = 52.5$$

$$\sigma_{cr} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 E_t}{(52.5)^2} = 3.58081\,(10^{-3})\,E_t$$

Assume $E_t = E_1 = 200$ GPa

$\sigma_{cr} = 3.58081\,(10^{-3})(200)(10^9) = 716.2\text{ MPa} > 200\text{ MPa}$ NG

Assume $E_t = E_2 = 150$ GPa

$\sigma_{cr} = 3.58081\,(10^{-3})(150)(10^9) = 537.1\text{ MPa}$

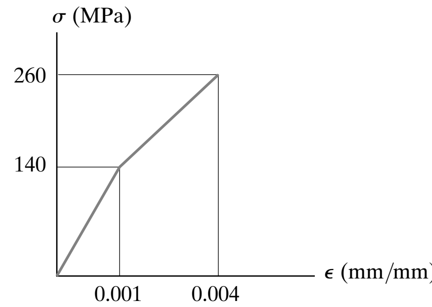
$200\text{ MPa} < \sigma_{cr} < 1100\text{ MPa}$ **O.K.**

Critical load:

$P_{cr} = \sigma_{cr} A = 537.1\,(10^6)(\pi)(0.04^2) = 2700\text{ kN}$ **Ans.**

Ans:
$P_{cr} = 2700$ kN

13–74. Construct the buckling curve, P/A versus L/r, for a column that has a bilinear stress–strain curve in compression as shown. The column is pinned at its ends.



SOLUTION

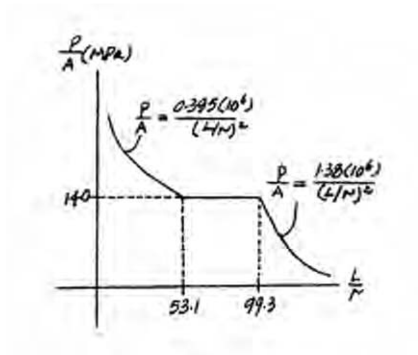
Tangent modulus: From the stress–strain diagram,

$$(E_t)_1 = \frac{140(10^6)}{0.001} = 140 \text{ GPa}$$

$$(E_t)_2 = \frac{(260 - 140)(10^6)}{0.004 - 0.001} = 40 \text{ GPa}$$

Critical Stress: Applying *Engesser's equation*,

$$\sigma_{cr}\frac{P}{A} = \frac{\pi^2 E_t}{\left(\frac{L}{r}\right)^2} \qquad [1]$$



Substituting $(E_t)_1 = 140$ GPa into Eq. [1], we have

$$\frac{P}{A} = \frac{\pi^2\left[140(10^9)\right]}{\left(\frac{L}{r}\right)^2}$$

$$\frac{P}{A} = \frac{1.38(10^6)}{\left(\frac{L}{r}\right)^2} \text{ MPa}$$

When $\dfrac{P}{A} = 140$ MPa, $\dfrac{L}{r} = 99.3$

Substitute $(E_t)_2 = 40$ GPa into Eq. [1], we have

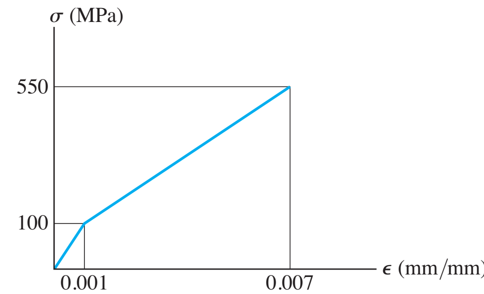
$$\frac{P}{A} = \frac{\pi^2\left[40(10^9)\right]}{\left(\frac{L}{r}\right)^2}$$

$$\frac{P}{A} = \frac{0.395(10^6)}{\left(\frac{L}{r}\right)^2} \text{ MPa}$$

When $\dfrac{P}{A} = 140$ MPa, $\dfrac{L}{r} = 53.1$

13–75.

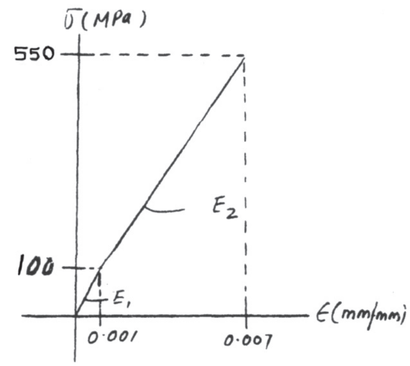
The stress–strain diagram of the material can be approximated by the two line segments. If a bar having a diameter of 80 mm and a length of 1.5 m is made from this material, determine the critical load provided the ends are pinned. Assume that the load acts through the axis of the bar. Use Engesser's equation.



SOLUTION

$$E_1 = \frac{100(10^6)}{0.001} = 100 \text{ GPa}$$

$$E_2 = \frac{550(10^6) - 100(10^6)}{0.007 - 0.001} = 75 \text{ GPa}$$



Section Properties:

$$I = \frac{\pi}{4}c^4; \qquad A = \pi c^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}c^4}{\pi c^2}} = \frac{c}{2} = \frac{0.04}{2} = 0.02 \text{ m}$$

Engesser's Equation:

$$\frac{KL}{r} = \frac{1.0(1.5)}{0.02} = 75$$

$$\sigma_{cr} = \frac{\pi^2 E_r}{(\frac{KL}{r})^2} = \frac{\pi^2 E_r}{(75)^2} = 1.7546(10^{-3})\, E_t$$

Assume $E_t = E_1 = 100$ GPa

$\sigma_{cr} = 1.7546\,(10^{-3})(100)(10^9) = 175 \text{ MPa} > 100 \text{ MPa}$

Therefore, inelastic buckling occurs:

Assume $E_t = E_2 = 75$ GPa

$\sigma_{cr} = 1.7546\,(10^{-3})\,(75)(10^9) = 131.6 \text{ MPa}$

$100 \text{ MPa} < \sigma_{cr} < 550 \text{ MPa}$ OK

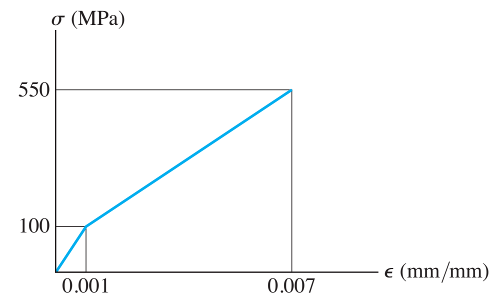
Critical Load:

$P_{cr} = \sigma_{cr} A = 131.6(10^6)(\pi)(0.04^2) = 661 \text{ kN}$ **Ans.**

Ans:
$P_{cr} = 661$ kN

***13–76.**

The stress–strain diagram of the material can be approximated by the two line segments. If a bar having a diameter of 80 mm and a length of 1.5 m is made from this material, determine the critical load provided the ends are fixed. Assume that the load acts through the axis of the bar. Use Engesser's equation.



SOLUTION

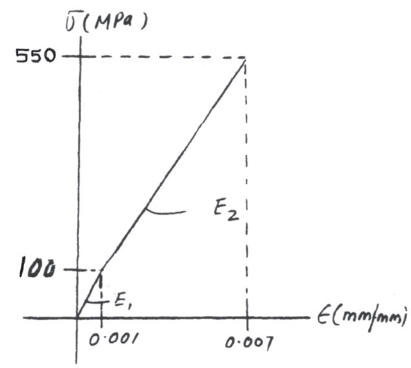
$$E_1 = \frac{100(10^6)}{0.001} = 100 \text{ GPa}$$

$$E_2 = \frac{550(10^6) - 100(10^6)}{0.007 - 0.001} = 75 \text{ GPa}$$

Section Properties:

$$I = \frac{\pi}{4}c^4; \qquad A = \pi c^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}c^4}{\pi c^2}} = \frac{c}{2} = \frac{0.04}{2} = 0.02 \text{ m}$$



Engesser's Equation:

$$\frac{KL}{r} = \frac{0.5(1.5)}{0.02} = 37.5$$

$$\sigma_{cr} = \frac{\pi^2 E_r}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 E_r}{(37.5)^2} = 7.018385(10^{-3})\, E_t$$

Assume $E_t = E_1 = 100$ GPa

$\sigma_{cr} = 7.018385(10^{-3})(100)(10^9) = 701.8 \text{ MPa} > 100 \text{ MPa}$ NG

Assume $E_t = E_2 = 75$ GPa

$\sigma_{cr} = 7.018385(10^{-3})(75)(10^9) = 526.4 \text{ MPa}$

$100 \text{ MPa} < \sigma_{cr} < 550 \text{ MPa}$ **OK**

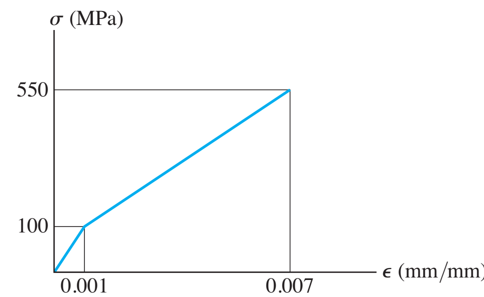
Critical Load:

$P_{cr} = \sigma_{cr} A = 526.4(10^6)(\pi)(0.04^2) = 2.65(10^3) \text{ kN}$ **Ans.**

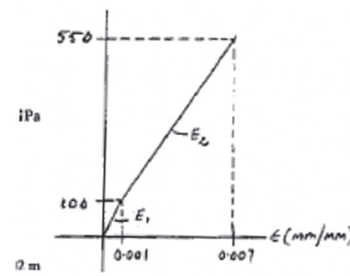
Ans:
$P_{cr} = 2.65(10^3)$ kN

13–77.

The stress–strain diagram of the material can be approximated by the two line segments. If a bar having a diameter of 80 mm and a length of 1.5 m is made from this material, determine the critical load provided one end is pinned and the other is fixed. Assume that the load acts through the axis of the bar. Use Engesser's equation.



SOLUTION



$$E_1 = \frac{100(10^6)}{0.001} = 100 \text{ GPa}$$

$$E_2 = \frac{550(10^6) - 100(10^6)}{0.007 - 0.001} = 75 \text{ GPa}$$

Section Properties:

$$I = \frac{\pi}{4}c^4; \qquad A = \pi c^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}c^4}{\pi c^2}} = \frac{c}{2} = \frac{0.04}{2} = 0.02 \text{ m}$$

Engesser's Equation:

$$\frac{KL}{r} = \frac{0.7(1.5)}{0.02} = 52.5$$

$$\sigma_{cr} = \frac{\pi^2 E_r}{(\frac{KL}{r})^2} = \frac{\pi^2 E_r}{(52.5)^2} = 3.58081(10^{-3})\, E_t$$

Assume $E_t = E_1 = 100$ GPa

$$\sigma_{cr} = 3.58081(10^{-3})(100)(10^9) = 358.1 \text{ MPa} > 100 \text{ MPa} \qquad \text{NG}$$

Assume $E_t = E_2 = 75$ GPa

$$\sigma_{cr} = 3.58081(10^{-3})(75)(10^9) = 268.6 \text{ MPa}$$

$$100 \text{ MPa} < \sigma_{cr} < 550 \text{ MPa} \qquad \text{OK}$$

Critical Load:

$$P_{cr} = \sigma_{cr} A = 268.6(10^6)(\pi)(0.04^2) = 1.35(10^3) \text{ kN}$$ **Ans.**

Ans:
$P_{cr} = 1.35(10^3)$ kN

13–78. Determine the largest length of a structural A-36 steel rod if it is fixed supported and subjected to an axial load of 100 kN. The rod has a diameter of 50 mm. Use the AISC equations.

SOLUTION

Section Properties:

$$A = \pi\left(0.025^2\right) = 0.625\left(10^{-3}\right)\,\pi\ \text{m}^2$$

$$I = \frac{\pi}{4}\left(0.025^4\right) = 97.65625\left(10^{-9}\right)\,\pi\ \text{m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{97.65625(10^{-9})\pi}{0.625(10^{-3})\pi}} = 0.0125\ \text{m}$$

Slenderness Ratio: For a column fixed at both ends, $K = 0.5$. Thus,

$$\frac{KL}{r} = \frac{0.5L}{0.0125} = 40.0L$$

AISC Column Formula: Assume a *long* column.

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23\left(\frac{KL}{r}\right)^2}$$

$$\frac{100(10^3)}{0.625(10^{-3})\pi} = \frac{12\pi^2\left[200(10^9)\right]}{23(40.0L)^3}$$

$$L = 3.555\ \text{m}$$

Here, $\dfrac{KL}{r} = 40.0(3.555) = 142.2$ and for A–36 steel, $\left(\dfrac{KL}{r}\right)_e = \sqrt{\dfrac{2\pi^2 E}{\sigma_\gamma}} = \sqrt{\dfrac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.7$. Since $\left(\dfrac{KL}{r}\right)_e \leq \dfrac{KL}{r} \leq 200$, the assumption is correct.

Thus,

$$L = 3.56\ \text{m}$$ **Ans.**

Ans:
$L = 3.56$ m

13–79. Check if a W250 × 58 column can safely support an axial force of $P = 1150$ kN. The column is 6 m long and is pinned at both ends and braced against its weak axis at mid-height. It is made of steel having $E = 200$ GPa and $\sigma_Y = 350$ MPa. use the AISC column design formulas.

SOLUTION

Section Properties. From the table listed in the appendix, the necessary section properties for a W250 × 58 are

$$A = 7400 \text{ mm}^2 = 7.4(10^{-3}) \text{ m}^2$$

$$r_x = 109 \text{ mm} = 0.109 \text{ m}$$

$$r_y = 50.4 \text{ mm} = 0.0504 \text{ m}$$

Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. Here, $L_x = 6$ m and $L_y = 3$ m. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{1(6)}{0.109} = 55.05$$

$$\left(\frac{KL}{r}\right)_y = \frac{1(3)}{0.0504} = 59.52 \text{ (controls)}$$

AISC Column Formulas. For A-36 steel $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$ $= \sqrt{\frac{2\pi^2[200(10^9)]}{350(10^6)}} = 106.21.$ Since $\left(\frac{KL}{r}\right)_y < \left(\frac{KL}{r}\right)_c$, the column is an intermediate column.

$$\sigma_{\text{allow}} = \frac{\left[1 - \dfrac{(KL/r)^2}{2(KL/r)_c{}^2}\right]\sigma_Y}{\dfrac{5}{3} + \dfrac{3(KL/r)}{8(KL/r)_c} - \dfrac{(KL/r)^3}{8(KL/r)_c{}^3}}$$

$$= \frac{\left[1 - \dfrac{59.62^2}{2(106.21^2)}\right](350)}{\dfrac{5}{3} + \dfrac{3(59.52)}{8(106.21)} - \dfrac{(59.52^3)}{8(106.21^3)}}$$

$$= 159.06 \text{ MPa}$$

Thus, the allowable force is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = [159.06(10^6)][7.4(10^{-3})] = 1177.04(10^3) \text{ N}$$

$$= 1177 \text{ kN} > P = 1150 \text{ kN} \qquad \text{O.K.}$$

Thus, a W250 × 58 column *is adequate.*

Ans:
Yes

***13–80.** A W200 × 36 A-36-steel column of 9-m length is pinned at both ends and braced against its weak axis at mid-height. Determine the allowable axial force P that can be safely supported by the column. Use the AISC column design formulas.

SOLUTION

Section Properties. From the table listed in the appendix, the necessary section properties for a W200 × 36 are

$$A = 4570\text{ mm}^2 = 4.57(10^{-3})\text{ m}^2$$

$$r_x = 86.8\text{ mm} = 0.0868\text{ m} \qquad r_y = 40.9\text{ mm} = 0.0409\text{ m}$$

Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. Here, $L_x = 9$ m and $L_y = 4.5$ m. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{1(9)}{0.0868} = 103.69$$

$$\left(\frac{KL}{r}\right)_y = \frac{1(4.5)}{0.0409} = 110.02 \text{ (controls)}$$

AISC Column Formulas. For A-36 steel $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$ $= \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66$. Since $\left(\frac{KL}{r}\right)_y < \left(\frac{KL}{r}\right)_c$, the column is an intermediate column.

$$\sigma_{\text{allow}} = \frac{\left[1 - \dfrac{(KL/r)^2}{2(KL/r)_c{}^2}\right]\sigma_Y}{\dfrac{5}{3} + \dfrac{3(KL/r)}{8(KL/r)_c} - \dfrac{(KL/r)^3}{8(KL/r)_c{}^3}}$$

$$= \frac{\left[1 - \dfrac{110.02^2}{2(125.66^2)}\right](250)}{\dfrac{5}{3} + \dfrac{3(110.02)}{8(125.66)} - \dfrac{(110.02^3)}{8(125.66^3)}}$$

$$= 80.\,67\text{ MPa}$$

Thus, the allowable force is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = [80.67(10^6)][4.57(10^{-3})] = 368.68(10^3)\text{ N} = 369\text{ kN} \qquad \textbf{Ans.}$$

Ans:

$P_{\text{allow}} = 369$ kN

13–81. Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 9 m long and supports an axial load of 1000 kN. The ends are fixed.

SOLUTION

Tπy W250× 80

$A = 10200\ \text{mm}^2 = 0.0102\ \text{m}^2 \qquad \pi_y = 65.0\ \text{mm} = 0.065\ \text{m}$

$$\left(\frac{KL}{\pi}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66$$

$$\frac{KL}{\pi_y} = \frac{0.5(9)}{0.065} = 69.23$$

$\left(\frac{KL}{r_y}\right) < \left(\frac{KL}{r}\right)_c$ intermediate column.

$$\sigma_{\text{allow}} = \frac{\left\{1 - \frac{1}{2}\left[\frac{\frac{KL}{r}}{\left(\frac{KL}{r}\right)_c}\right]^2\right\}\sigma_Y}{\left\{\frac{5}{3} + \frac{3}{8}\left[\frac{\frac{KL}{r}}{\left(\frac{KL}{r}\right)_c}\right] - \frac{1}{8}\left[\frac{\frac{KL}{r}}{\left(\frac{KL}{r}\right)_c}\right]^3\right\}}$$

$$= \frac{\{1 - \frac{1}{2}[\frac{69.23}{125.66}]^2\}250}{\{\frac{5}{3} + \frac{3}{8}[\frac{69.23}{125.66}] - \frac{1}{8}[\frac{69.23}{125.66}]^3\}} = 114.48\ \text{MPa}$$

$P_{\text{allow}} = \sigma_{\text{allow}} A = [114.48(10^6)](0.0102) = 1167.71(10^3)\ \text{N}$

$= 1168\ \text{kN} > P = 1000\ \text{kN}$ O.K.

Use W250 × 80 **Ans.**

Ans:
Use W250 × 80

13–82. Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 7.2 m long and supports an axial load of 450 kN. The ends are fixed.

SOLUTION

Section Properties: Try a W200 × 36 wide flange section,

$$A = 4570\ \text{mm}^2 = 4.57(10^{-3})\ \text{m}^2 \qquad r_y = 40.9\ \text{mm} = 0.0409\ \text{m}$$

Slenderness Ratio: For a column fixed at both ends, $K = 0.5$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{0.5(7.2)}{0.0409} = 88.02$$

AISC Column Formula: For A–36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66$. Since $\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$, the column is an *intermediate* column. Applying Eq. 13–23,

$$\sigma_{\text{allow}} = \frac{\left[1 - \dfrac{(KL/r)^2}{2(KL/r)_c^2}\right]\sigma_Y}{\dfrac{5}{3} + \dfrac{3(KL/r)}{8(KL/r)_c} - \dfrac{(KL/r)^3}{8(KL/r)_c^3}}$$

$$= \frac{\left[1 - \dfrac{(88.02^2)}{2(125.66^2)}\right](250)}{\dfrac{5}{3} + \dfrac{3(88.02)}{8(125.66)} - \dfrac{(88.02^3)}{8(125.66^3)}}$$

$$= 100.02\ \text{MPa}$$

The allowable load is

$$P_{\text{allow}} = \sigma_{\text{allow}} A$$

$$= [100.02(10^6)][4.57(10^{-3})] = 457.09(10^3)\ \text{N}$$

$$= 457\ \text{kN} > P = 450\ \text{kN} \qquad \text{O.K.}$$

Thus, **Use** W200 × 36 **Ans.**

Ans:
Use W200 × 36

13–83. Determine the largest length of a W250 × 67 A992 structural steel column if it is pin supported and subjected to an axial load of 1450 kN. $E_{st} = 200$ GPa, $\sigma_Y = 350$ MPa. Use the AISC equations.

SOLUTION

Section Properties: For W200 × 46 wide flange section,

$$A = 8560 \text{ mm}^2 = 8.56(10^{-3}) \text{ m}^2 \qquad r_y = 50.9 \text{ mm} = 0.0509 \text{ m}$$

Slenderness Ratio: For a column pinned at both ends, $K = 1$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{1(L)}{0.0509} = 19.6464L$$

AISC Column Formula: Assume a *long* column,

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23\left(\frac{KL}{r}\right)^2}$$

$$\frac{1450(10^3)}{8.56(10^{-3})} = \sqrt{\frac{12\pi^2[200(10^9)]}{23(19.6464L)^2}}$$

$$L = 3.9688 \text{ m}$$

Here, $\frac{KL}{r} = 19.6464(3.9688) = 77.97$ and for A992 steel, $\left(\frac{KL}{r}\right)_c = \frac{2\pi^2 E}{\sigma_Y}$ $= \sqrt{\frac{2\pi^2[200(10^9)]}{345(10^6)}} = 106.97$. Since $\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$, the assumption is not correct.

Thus, the column is an *intermediate* column.

Applying Eq. 13–23,

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{KL/r)^2}{(2(KL/r)_c^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c^3}}$$

$$\frac{1450(10^3)}{8.56(10^{-3})} = \frac{\left[1 - \frac{(19.6464L)^2}{2(106.97^2)}\right][345(10^6)]}{\frac{5}{3} + \frac{3(19.6464L)}{8(106.97)} - \frac{(19.6464L)^3}{8(106.97^3)}}$$

$$-3.80208(10^{-4})L^3 + 0.016865L^2 + 0.0338157L - 0.181679 = 0$$

Solving by trial and error,

$$L = 2.4790 \text{ m} = 2.48 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$L = 2.48$ m

***13–84.** Determine the largest length of a W250 × 18 structural A-36 steel section if it is pin supported and is subjected to an axial load of 140 kN. Use the AISC equations.

SOLUTION

For a W250 × 18, $A = 2280\ \text{mm}^2 = 2.28(10^{-3})\ \text{m}^2$

$r_y = 20.1\ \text{mm} = 0.0201\ \text{m}$

$$\sigma = \frac{P}{A} = \frac{140(10^3)}{2.28(10^{-3})} = 61.40(10^6)\ \text{N/m}^2 = 61.40\ \text{MPa}$$

Assume a long column:

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$\left(\frac{KL}{r}\right) = \sqrt{\frac{12\pi^2 E}{23\sigma_{\text{allow}}}} = \sqrt{\frac{12\pi^2[200(10^9)]}{23[61.40(10^6)]}} = 129.51$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66, \frac{KL}{r} > \left(\frac{KL}{r}\right)_c$$

Long column.

$$\frac{KL}{r} = 129.51$$

$$L = 129.51\left(\frac{r}{K}\right) = 129.51\left(\frac{0.0201}{1}\right) = 2.603\ \text{m} = 2.60\ \text{m}$$ **Ans.**

Ans:
$L = 2.60$ m

13–85. Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 4.2 m long and supports an axial load of 200 kN. The ends are pinned.

SOLUTION

Try, W150 × 22

$A = 2860 \text{ mm}^2 = 2.86(10^{-3}) \text{ m}^2 \qquad r_y = 36.8 \text{ mm} = 0.0368 \text{ m}$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{345(10^6)}} = 106.97$$

$$\left(\frac{KL}{r_y}\right) = \frac{1.0(4.2)}{0.0368} = 114.13, \quad \left(\frac{KL}{r_y}\right) > \left(\frac{KL}{r}\right)_c$$

Long column

$$\sigma_{allow} = \frac{12\,\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2[200(10^9)]}{23(114.13^2)} = 79.06(10^6) \text{ N-m}^2 = 79.06 \text{ MPa}$$

$$P_{allow} = \sigma_{allow} A$$

$$= [79.06(10^6)][2.86(10^{-3})] = 226 \text{ kN} > 200 \text{ kN} \qquad \text{O.K.}$$

Use W150 × 22 **Ans.**

Ans:
Use W150 × 22

13–86. Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 3.6 m long and supports an axial load of 200 kN. The ends are fixed.

SOLUTION

Tπy W150×14

$A = 1730\ \text{mm}^2 = 1.73(10^{-3})\ \text{m}^2 \qquad r_y = 23.0\ \text{mm} = 0.023\ \text{m}$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{345(10^6)}} = 106.97$$

$$\frac{KL}{r_y} = \frac{0.5(3.6)}{0.023} = 78.26$$

$$\frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

Intermediate column

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{1}{2}\left(\frac{KL/r}{(KL/r)_c}\right)^2\right]\sigma_Y}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{(KL/r)_c}\right) - \frac{1}{8}\left(\frac{KL/r}{(KL/r)_c}\right)^3\right]} = \frac{\left[1 - \frac{1}{2}\left(\frac{78.26}{106.97}\right)^2\right](345\ \text{MPa})}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{78.26}{106.97}\right) - \frac{1}{8}\left(\frac{78.26}{106.97}\right)^3\right]} = 133.54\ \text{MPa}$$

$$\begin{aligned} P_{\text{allow}} &= \sigma_{\text{allow}} A \\ &= [133.54(10^6)][1.73(10^{-3})] \\ &= 231.03(10^3)\ \text{N} \\ &= 231\ \text{kN} > 200\ \text{kN} \end{aligned}$$

O.K.

Use W150 × 14 **Ans.**

Ans:
Use W150 × 14

13–87. Check if a W250 × 67 column can safely support an axial force of $P = 1000$ kN. The column is 4.5 m long and is pinned at both of its ends. It is made of steel having $E = 200$ GPa and $\sigma_Y = 350$ MPa. Use the AISC column design formulas.

SOLUTION

Section Properties. Try W250 × 67. From the table listed in the appendix, the necessary section properties are

$$A = 8560 \text{ mm}^2 = 8.56(10^{-3}) \text{ m}^2 \qquad r_y = 50.9 \text{ mm} = 0.0509 \text{ m}$$

Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{1(4.5)}{0.0509} = 88.41$$

AISC Column Formulas. Here, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{350(10^6)}} = 106.21.$

Since $\left(\frac{KL}{r}\right)_y < \left(\frac{KL}{r}\right)_c$, the

column is an intermediate column.

$$\sigma_{\text{allow}} = \frac{\left[1 - \dfrac{(KL/r)^2}{2(KL/r)_c{}^2}\right]\sigma_Y}{\dfrac{5}{3} + \dfrac{3(KL/r)}{8(KL/r)_c} - \dfrac{(KL/r)^3}{8(KL/r)_c{}^3}}$$

$$= \frac{\left[1 - \dfrac{88.41^2}{2(106.21^2)}\right](350)}{\dfrac{5}{3} + \dfrac{3(88.41)}{8(106.21)} - \dfrac{(88.41^3)}{8(106.21^3)}}$$

$$= 199.96 \text{ MPa}$$

Thus, the allowable force is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = [119.96(10^6)][8.56(10^{-3})] = 1026.88(10^3) \text{ N}$$

$$= 1027 \text{ kN} > 1000 \text{ kN} \qquad \text{O.K.}$$

Thus, use

W250× 67 **Ans.**

Ans:
Use W250 × 67

***13–88.** A 1.5-m-long rod is used in a machine to transmit an axial compressive load of 15 kN. Determine its smallest diameter if it is pin connected at its ends and is made of a 2014-T6 aluminum alloy.

SOLUTION

Section properties:

$$A = \frac{\pi}{4}d^2; \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi d^4}{64}}{\frac{\pi}{4}d^2}} = \frac{d}{4}$$

$$\sigma_{\text{allow}} = \frac{P}{A} = \frac{15(10^3)}{\frac{\pi}{4}d^2} = \frac{60(10^3)}{\pi d^2}$$

Assume long column:

$$\frac{KL}{r} = \frac{1.0(1.5)}{d/4} = \frac{6}{d}$$

$$\sigma_{\text{allow}} = \frac{372550 \text{ MPa}}{(KL/r)^2}; \qquad \frac{60(10^3)}{\pi d^2} = \frac{372550(10^6)}{(6/d)^2}$$

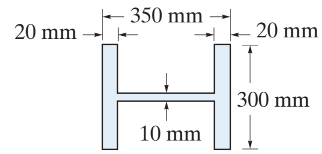
$d = 0.03686 \text{ m} = 36.9 \text{ mm}$ **Ans.**

$$\frac{KL}{r} = \frac{1.0(1.5)}{d/4} = \frac{6}{0.03686} = 162.79 > 55 \text{ (Assumption is correct!)}$$ O.K.

Ans:
$d = 36.9$ mm

13–89.

Using the AISC equations, check if a column having the cross section shown can support an axial force of 1500 kN. The column has a length of 4 m, is made from A992 steel, and its ends are pinned.



SOLUTION

Section Properties:

$$A = 0.3(0.35) - 0.29(0.31) = 0.0151 \text{ m}^2$$

$$I_y = \frac{1}{12}(0.04)\left(0.3^3\right) + \frac{1}{12}(0.31)\left(0.01^3\right) = 90.025833\left(10^{-6}\right) \text{ m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{90.02583(10^{-6})}{0.0151}} = 0.077214 \text{ m}$$

Slenderness Ratio: For a column pinned at both ends, $K = 1$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{1(4)}{0.077214} = 51.80$$

AISC Column Formula: For A992 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{345(10^6)}} = 107$. Since $\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$, the column is an *intermediate* column. Applying Eq. 13–23,

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_c^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c^3}}$$

$$= \frac{\left[1 - \frac{(51.80^2)}{2(107^2)}\right](345)(10^6)}{\frac{5}{3} + \frac{3(51.80)}{8(107)} - \frac{(51.80^3)}{8(107^3)}}$$

$$= 166.1 \text{ MPa}$$

The allowable load is

$$P_{\text{allow}} = \sigma_{\text{allow}} A$$

$$= 166.1\left(10^6\right)(0.0151)$$

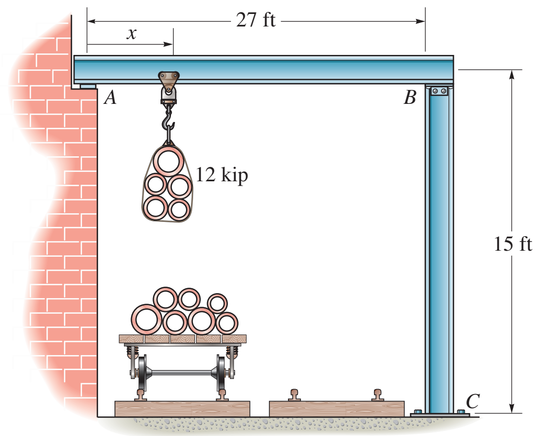
$$= 2508 \text{ kN} > P = 1500 \text{ kN} \qquad \textbf{O.K.}$$

Thus, the column is adequate. **Ans.**

Ans:
Yes

13–90.

The beam and column arrangement is used in a railroad yard for loading and unloading cars. If the maximum anticipated hoist load is 12 kip, determine if the W8 × 31 wide-flange A-36 steel column is adequate for supporting the load. The hoist travels along the bottom flange of the beam, 1 ft ≤ x ≤ 25 ft, and has negligible size. Assume the beam is pinned to the column at B and pin supported at A. The column is also pinned at C and it is braced so it will not buckle out of the plane of the loading.



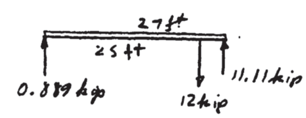
SOLUTION

For W8 × 31, $r_x = 3.47$ in., $A = 9.13$ in^2

Maximum axial load occurs when $x = 25$ ft.

$$\frac{KL}{r} = \frac{(1.0)(15)(12)}{3.47} = 51.87$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1$$



Here $0 < 51.87 < 126.1$

Intermediate Column:

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{1}{2}\left(\frac{KL/r}{(KL/r)c}\right)^2\right]\sigma_Y}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{(KL/r)c}\right) - \frac{1}{8}\left(\frac{KL/r}{(KL/r)c}\right)^3\right]}$$

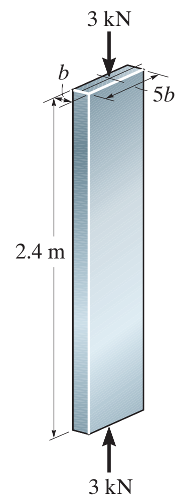
$$= \frac{\left(1 - \frac{1}{2}(51.87/126.1)^2\right)36}{\left\{\frac{5}{3} + \left[\left(\frac{3}{8}\right)(51.87/126.1)\right] - \left[\frac{1}{8}(51.87/126.1)^3\right]\right\}} = 18.2 \text{ ksi}$$

$$\sigma = \frac{P}{A} = \frac{11.11}{9.13} = 1.22 \text{ ksi} < 18.2 \text{ ksi} \qquad \textbf{OK}$$

Yes, the column is adequate. **Ans.**

Ans:
Yes

13–91. The bar is made of a 2014-T6 aluminum alloy. Determine its smallest thickness b if its width is $5b$. Assume that it is pin connected at its ends.



SOLUTION

Section Properties:

$$A = b(5b) = 5b^2$$

$$I_y = \frac{1}{12}(5b)\left(b^3\right) = \frac{5}{12}b^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\frac{5}{12}b^4}{5b^2}} = \frac{\sqrt{3}}{6}b$$

Slenderness Ratio: For a column pinned at both ends, $K = 1$. Thus,

$$\left(\frac{KL}{\pi}\right)_y = \frac{1(2.4)}{\sqrt{\frac{3}{6}}b} = \frac{8.3138}{b}$$

Aluminum (2014 - T6 alloy) Column Formulas: Assume a *long* column and apply Eq. 13–26.

$$\sigma_{\text{allow}} = \frac{372550 \text{ MPa}}{(KL/\pi)^2}$$

$$\frac{3(10^3)}{5b^2} = \frac{372550(10^6)}{(8.3138/b)^2}$$

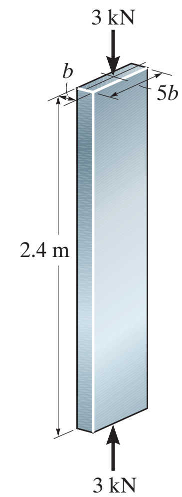
$$b = 0.01827 \text{ m} = 18.3 \text{ mm}$$

Heπe, $\frac{KL}{\pi} = \frac{8.3138}{0.01827} = 455.15$. Since $\frac{KL}{r} > 55$, the assumption is correct. Thus,

$$b = 18.3 \text{ mm}$$ **Ans.**

Ans:
$b = 18.3$ mm

***13–92.** The bar is made of a 2014-T6 aluminum alloy. Determine its smallest thickness b if its width is $5b$. Assume that it is fixed connected at its ends.



SOLUTION

Section Properties:

$$A = b(5b) = 5b^2$$

$$I_y = \frac{1}{12}(5b)\left(b^3\right) = \frac{5}{12}b^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\frac{5}{12}b^4}{5b^2}} = \frac{\sqrt{3}}{6}b$$

Slenderness Ratio: For a column fixed at both ends, $K = 0.5$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{0.5(2.4)}{\sqrt{\frac{3}{6}}b} = \frac{4.1569}{b}$$

Aluminum (2014 - T6 alloy) Column Formulas: Assume a *long* column and apply Eq. 13–26.

$$\sigma_{\text{allow}} = \frac{372550 \text{ MPa}}{(KL/r)^2}$$

$$\frac{3(10^3)}{5b^2} = \frac{372550(10^6)}{(4.1569/b)^2}$$

$$b = 0.01292 \text{ m} = 12.9 \text{ mm}$$

Here, $\frac{KL}{r} = \frac{4.1569}{0.01292} = 321.84$. Since $\frac{KL}{r} > 55$, the assumption is correct.

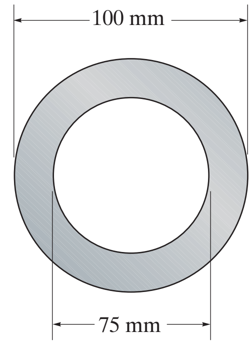
Thus,

$$b = 12.9 \text{ mm}$$ **Ans.**

Ans:

$b = 12.9$ mm

13–93. The 2014-T6 aluminum hollow section has the cross section shown. If the column is 3 m long and is fixed at both ends, determine the allowable axial force P that can be safely supported by the column.



SOLUTION

Section Properties.

$$A = \pi(0.05^2 - 0.0375^2) = 3.4361(10^{-3})\ \text{m}^2$$

$$I = \frac{\pi}{4}(0.05^4 - 0.0375^4) = 3.3556(10^{-6})\ \text{m}^4$$

$$\pi = \sqrt{\frac{I}{A}} = \sqrt{\frac{3.3556(10^{-6})}{3.4361(10^{-3})}} = 0.03125\ \text{m}$$

Slenderness Ratio. For a column fixed at both of its ends, $K = 0.5$. Thus,

$$\frac{KL}{\pi} = \frac{0.5(3)}{0.03125} = 48$$

2014-T6 Aluminum Alloy Column Formulas. Since $12 < \frac{KL}{r} < 55$, the column can be classified as an intermediate column.

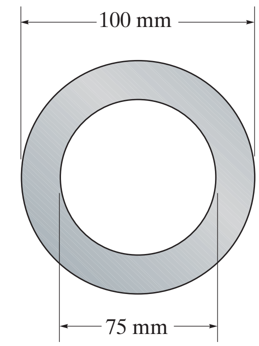
$$\begin{aligned}\sigma_{\text{allow}} &= \left[212 - 1.59\left(\frac{KL}{\pi}\right)\right]\ \text{MPa}\\ &= [212 - 1.59(48)]\ \text{MPa}\\ &= 135.68\ \text{MPa}\end{aligned}$$

Thus, the allowable load is

$$\begin{aligned}P_{\text{allow}} &= \sigma_{\text{allow}} A = [135.68(10^6)][3.4361(10^{-3})]\\ &= 466.21(10^3)\ \text{N} = 466\ \text{kN}\end{aligned}$$ **Ans.**

Ans:
$P_{\text{allow}} = 466$ kN

***13–94.** The 2014-T6 aluminum hollow section has the cross section shown. If the column is fixed at its base and pinned at its top, and is subjected to the axial force $P = 500$ kN, determine the maximum length of the column for it to safely support the load.



SOLUTION

Section Properties.

$$A = \pi(0.05^2 - 0.0375^2) = 3.4361(10^{-3})\ \text{m}^2$$

$$I = \frac{\pi}{4}(0.05^4 - 0.0375^4) = 3.3556(10^{-6})\ \text{m}^4$$

$$\pi = \sqrt{\frac{I}{A}} = \sqrt{\frac{3.3556(10^{-6})}{3.4361(10^{-3})}} = 0.03125\ \text{m}$$

Slenderness Ratio. For a column fixed at its base and pinned at its top, $K = 0.7$. Thus,

$$\frac{KL}{\pi} = \frac{0.7L}{0.03125} = 22.4L$$

2014-T6 Aluminum Alloy Column Formulas. Assuming an intermediate column,

$$\sigma_{\text{allow}} = \left[212 - 1.59\left(\frac{KL}{\pi}\right)\right]\ \text{MPa}$$

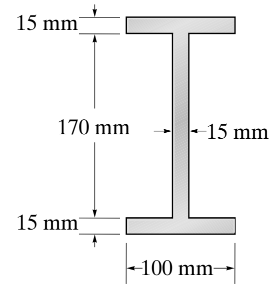
$$\frac{500(10^3)}{3.4361(10^{-3})} = [212 - 1.59(22.4L)](10^6)$$

$$L = 1.8668\ \text{m} = 1.87\ \text{m} \qquad \textbf{Ans.}$$

Heπe, $\dfrac{KL}{\pi} = 22.4(1.8668) = 41.8$. Since $12 < \dfrac{KL}{\pi} < 55$, the assumptions of an intermediate column is correct.

Ans:
$L = 1.87$ m

13–95. The 2014-T6 aluminum column of 3-m length has the cross section shown. If the column is pinned at both ends and braced against the weak axis at its mid-height, determine the allowable axial force P that can be safely supported by the column.



SOLUTION

Section Properties.

$$A = 0.1(0.2) - 0.085(0.17) = 5.55\left(10^{-3}\right)\text{ m}^2$$

$$I_x = \frac{1}{12}(0.1)\left(0.2^3\right) - \frac{1}{12}(0.085)\left(0.17^3\right) = 31.86625\left(10^{-6}\right)\text{ m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.015)\left(0.1^3\right)\right] + \frac{1}{12}(0.17)\left(0.015^3\right) = 2.5478\left(10^{-6}\right)\text{ m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{31.86625\left(10^{-6}\right)}{5.55\left(10^{-3}\right)}} = 0.07577$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{2.5478\left(10^{-6}\right)}{5.55\left(10^{-3}\right)}} = 0.02143\text{ m}$$

Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. Here, $L_x = 3$ m and $L_y = 1.5$ m. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{(1)(3)}{0.07577} = 39.592$$

$$\left(\frac{KL}{r}\right)_y = \frac{(1)(1.5)}{0.02143} = 70.009\text{ (controls)}$$

2014-T6 Alumimum Alloy Column Formulas. Since $\left(\frac{KL}{r}\right)_y > 55$, the column can be classified a long column,

$$\sigma_{\text{allow}} = \left[\frac{372550}{\left(\frac{KL}{r}\right)^2}\right]\text{MPa}$$

$$= \left[\frac{372550}{70.009^2}\right]\text{MPa}$$

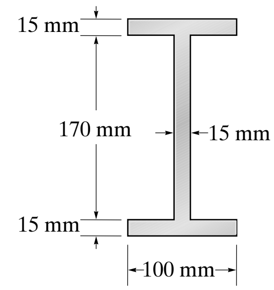
$$= 76.011\text{ MPa}$$

Thus, the allowed force is

$$P_{\text{allow}} = \sigma_{\text{allow}}A = [76.011(10^6)]\,[5.55(10^{-3})] = 421.86(10^3)\text{ N} = 422\text{ kN} \qquad \textbf{Ans.}$$

Ans:
$A = 5.55\,(10^{-3})\text{ m}^2$, $I_x = 31.86625\,(10^{-6})\text{ m}^4$,
$I_y = 2.5478\,(10^{-6})\text{ m}^4$, $P_{\text{allow}} = 422$ kN

***13–96.** The 2014-T6 aluminum column has the cross section shown. If the column is pinned at both ends and subjected to an axial force $P = 100$ kN, determine the maximum length the column can have to safely support the loading.



SOLUTION

Section Properties.

$$A = 0.1(0.2) - 0.085(0.17) = 5.55\left(10^{-3}\right)\text{ m}^2$$

$$I_y = 2\left[\frac{1}{12}(0.015)\left(0.1^3\right)\right] + \frac{1}{12}(0.17)\left(0.015^3\right) = 2.5478\left(10^{-6}\right)\text{ m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{2.5478\left(10^{-6}\right)}{5.55\left(10^{-3}\right)}} = 0.02143\text{ m}$$

Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. Then,

$$\left(\frac{KL}{r}\right)_y = \frac{1(L)}{0.02143} = 46.6727L$$

2014-T6 Alumimum Alloy Column Formulas. Assuming a long column,

$$\sigma_{\text{allow}} = \left[\frac{372550}{\left(\frac{KL}{r}\right)^2}\right]\text{MPa}$$

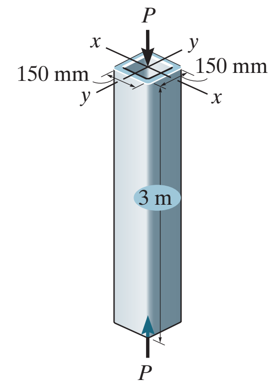
$$\frac{100\left(10^3\right)}{5.55\left(10^{-3}\right)} = \left[\frac{372550}{(46.6727L)^2}\right]\left(10^6\right)\text{ Pa}$$

$$L = 3.0809\text{ m} = 3.08\text{ m}$$ **Ans.**

Since $\left(\frac{KL}{r}\right)_y = 46.6727(3.0809) = 143.79 > 55$, the assumption is correct.

Ans:
$L = 3.08$ m

13–97. The tube is 6 mm thick, is made of a 2014-T6 aluminum alloy, and is fixed at its bottom and pinned at its top. Determine the largest axial load that it can support.



SOLUTION

Section Properties:

$$A = 0.15(0.15) - 0.138(0.138) = 3.456(10^{-3})\ \text{m}^2$$

$$I = \frac{1}{12}(0.15)(0.15^3) - \frac{1}{12}(0.138)(0.138^3) = 11.9647(10^{-6})\ \text{m}^4$$

$$\pi = \sqrt{\frac{I}{A}} = \sqrt{\frac{11.9647(10^{-6})}{3.456(10^{-3})}} = 0.058839\ \text{m}$$

Slenderness Ratio: For a column fixed at one end and pinned at the other end, $K = 0.7$. Thus,

$$\frac{KL}{\pi} = \frac{0.7(3)}{0.058839} = 35.69$$

Aluminium (2014 –∑ T6 alloy) Column Formulas: Since $12 < \frac{KL}{r} < 55$, the column is classified as an *intermediate* column. Applying Eq. 13–25,

$$\sigma_{\text{allow}} = \left[212 - 1.59\left(\frac{KL}{\pi}\right)\right]\ \text{MPa}$$

$$= [212 - 1.59(35.69)]$$

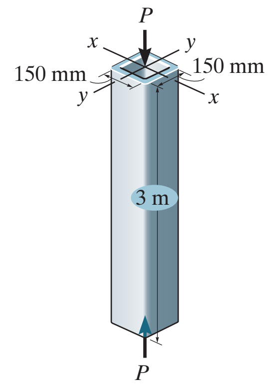
$$= 155.25\ \text{MPa}$$

The allowable load is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = [155.25(10^6)][3.456(10^{-3})] = 536.55(10^3)\ \text{N} = 537\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$P_{\text{allow}} = 537\ \text{kN}$

***13–98.** The tube is 6 mm thick, is made of a 2014-T6 aluminum alloy, and is fixed connected at its ends. Determine the largest axial load that it can support.



SOLUTION

Section Properties:

$$A = 0.15(0.15) - 0.138(0.138) = 3.456(10^{-3})\ \text{m}^2$$

$$I = \frac{1}{12}(0.15)(0.15^3) - \frac{1}{12}(0.138)(0.138^3) = 11.9647(10^{-6})\ \text{m}^4$$

$$\pi = \sqrt{\frac{I}{A}} = \sqrt{\frac{11.9647(10^{-6})}{3.456(10^{-3})}} = 0.058839\ \text{m}$$

Slenderness Ratio: For column fixed at both ends, $K = 0.5$. Thus,

$$\frac{KL}{\pi} = \frac{0.5(3)}{0.058839} = 25.49$$

Aluminium (2014 – T6 alloy) Column Formulas: Since $12 < \frac{KL}{r} < 55$, the column is classified as an *intermediate* column. Applying Eq. 13–25,

$$\sigma_{\text{allow}} = \left[212 - 1.59\left(\frac{KL}{\pi}\right)\right]\ \text{MPa}$$

$$= [212 - 1.59(25.49)]$$

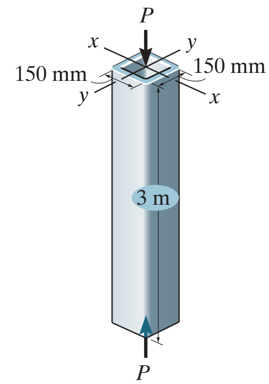
$$= 171.46\ \text{MPa}$$

The allowable load is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = [171.46(10^6)][3.456(10^{-3})] = 592.58(10^3)\ \text{N} = 593\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$P_{\text{allow}} = 593$ kN

13–99. The tube is 6 mm thick, is made of a 2014-T6 aluminum alloy and is pin connected at its ends. Determine the largest axial load it can support.



SOLUTION

Section Properties:

$$A = 0.15(0.15) - 0.138(0.138) = 3.456(10^{-3})\ \text{m}^2$$

$$I = \frac{1}{12}(0.15)(0.15^3) - \frac{1}{12}(0.138)(0.138^3) = 11.9647(10^{-6})\ \text{m}^4$$

$$\pi = \sqrt{\frac{I}{A}} = \sqrt{\frac{11.9647(10^{-6})}{3.456(10^{-3})}} = 0.058839\ \text{m}$$

Slenderness Ratio: For a column pinned as both ends, $K = 1$. Thus,

$$\frac{KL}{\pi} = \frac{1(3)}{0.058839} = 50.99$$

Aluminum (2014 – T6 alloy) Column Formulas: Since $12 < \frac{KL}{r} < 55$, the column is classified as an *intermediate* column. Applying Eq. 13–25,

$$\sigma_{\text{allow}} = \left[212 - 1.59\left(\frac{KL}{\pi}\right)\right]\ \text{MPa}$$

$$= [212 - 1.59(50.99)]$$

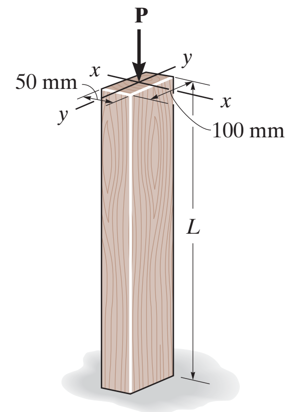
$$= 130.93\ \text{MPa}$$

The allowable load is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = [130.93(10^6)][3.456(10^{-3})] = 452.49(10^3)\ \text{N} = 452\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$P_{\text{allow}} = 452\ \text{kN}$

***13–100.** The column is made of wood. It is fixed at its bottom and free at its top. Use the NFPA formulas to determine its greatest allowable length if it supports an axial load of $P = 10$ KN.



SOLUTION

Slenderness Ratio: For a column fixed at one end and free at the other end, $K = 2$. Thus,

$$\frac{KL}{d} = \frac{2(L)}{0.05} = 40L$$

NFPA Timber Column Formulas: Assume a *long* column. Apply Eq. 13–29,

$$\sigma_{\text{allow}} = \frac{3725}{(KL/d)^2} \text{ MPa}$$

$$\frac{10(10^3)}{0.05(0.1)} = \frac{3725(10^6)}{(40L)^2}$$

$$L = 1.079 \text{ m}$$

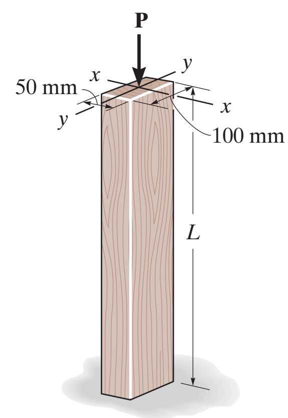
Here, $\frac{KL}{d} = 40(1.079) = 43.16$. Since $26 < \frac{KL}{d} < 50$, the assumption is correct. Thus,

$$L = 1.08 \text{ m} \qquad \textbf{Ans.}$$

Ans:

$L = 1.08$ m

13–101. The column is made of wood. It is fixed at its bottom and free at its top. Use the NFPA formulas to determine the largest allowable axial load P that it can support if it has a length $L = 1.2$ m.



SOLUTION

Slenderness Ratio: For a column fixed at one end and free at the other end, $K = 2$. Thus,

$$\frac{KL}{d} = \frac{2(1.2)}{0.05} = 48.0$$

NFPA Timber Column Formulas: Since $26 < \dfrac{KL}{d} < 50$, it is a *long* column. Apply Eq. 13–29,

$$\sigma_{\text{allow}} = \frac{3725}{(KL/d)^2}\ \text{MPa}$$

$$= \frac{3725}{48.0^2}\ \text{MPa}$$

$$= 1.6168\ \text{MPa}$$

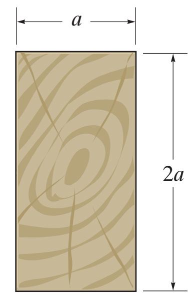
The allowable axial force is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = [1.6168(10^6)][0.05(0.1)] = 8.084(10^3)\ \text{N} = 8.08\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$P_{\text{allow}} = 8.08$ kN

13–102.

A rectangular wooden column has the cross section shown. If $a = 3$ in. and the column is subjected to an axial force of $P = 15$ kip, determine the maximum length the column can have to safely support the load. The column is pinned at its top and fixed at its base.



SOLUTION

Slenderness Ratio: For a column fixed at its base and pinned at its top, $K = 0.7$. Then,

$$\frac{KL}{d} = \frac{0.7L}{3} = 0.2333L$$

NFPA Timber Column Formula: Assuming an intermediate column,

$$\sigma_{\text{allow}} = 1.20\left[1 - \frac{1}{3}\left(\frac{KL/d}{26.0}\right)^2\right] \text{ksi}$$

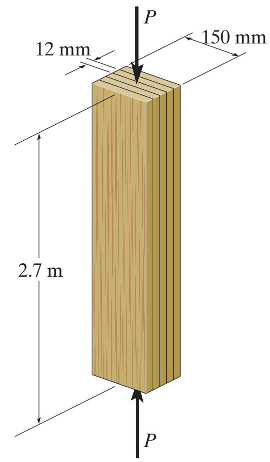
$$\frac{15}{3(6)} = 1.20\left[1 - \frac{1}{3}\left(\frac{0.2333L}{26.0}\right)^2\right]$$

$$L = 106.68 \text{ in.} = 8.89 \text{ ft}$$ **Ans.**

Here, $\frac{KL}{d} = 0.2333(106.68) = 24.89$. Since $11 < \frac{KL}{d} < 26$, the assumption is correct.

Ans:
$L = 8.89$ ft

13–103. The wooden column shown is formed by gluing together the 150 mm × 12 mm boards. If the column is pinned at both ends and is subjected to an axial load $P = 100$ kN, determine the required number of boards needed to form the column in order to safely support the loading.



SOLUTION

Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. If the number of the boards required is n and assuming that $n(0.012) < 0.15$ m. Then, $d = n(0.012)$. Thus,

$$\frac{KL}{d} = \frac{1(2.7)}{n(0.012)} = \frac{225}{n}$$

NFPA Timber Column Formula. Assuming an intermediate column,

$$\sigma_{\text{allow}} = 8.28\left[1 - \frac{1}{3}\left(\frac{KL/d}{26.0}\right)^2\right] \text{ MPa}$$

$$\frac{100(10^3)}{[n(0.012)](0.15)} = 8.28\left[1 - \frac{1}{3}\left(\frac{1.8/d}{26.0}\right)^2\right](10^6)$$

$$n^2 - 6.70961n - 24.963 = 0$$

Solving for the positive root,

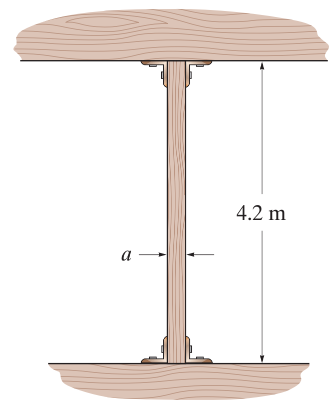
$$n = 9.373$$

Use $n = 10$ **Ans.**

Here, $\dfrac{KL}{d} = \dfrac{225}{10} = 22.5$. Since $n(0.012) = 10(0.012) = 0.12 \text{ m} < 0.15 \text{ m}$ and $11 < \dfrac{KL}{d} < 26$, the assumptions made are correct.

Ans:
Use $n = 10$

***13–104.** The timber column has a square cross section and is assumed to be pin connected at its top and bottom. If it supports an axial load of 250 kN, determine its smallest side dimension a to the nearest multiples of 15 mm. Use the NFPA formulas.



SOLUTION

Section properties:

$$A = a^2 \qquad \sigma_{\text{allow}} = \sigma = \frac{P}{A} = \frac{250(10^3)}{a^2}$$

Assume long column:

$$\sigma_{\text{allow}} = \frac{3725 \text{ MPa}}{(KL/d)^2}$$

$$\frac{250(10^3)}{a^2} = \frac{3725(10^6)}{[1.0(4.2)/a]^2}$$

$$a = 0.1855 \text{ m}$$

$$\frac{KL}{d} = \frac{1.0(4.2)}{0.1855} = 22.6, \quad \frac{KL}{d} < 26 \qquad \text{Assumption NG}$$

Assume intermediate column:

$$\sigma_{\text{allow}} = 8.28\left[1 - \frac{1}{3}\left(\frac{KL/d}{26.0}\right)^2\right] \text{MPa}$$

$$\frac{250(10^3)}{a^2} = 8.28\left\{1 - \frac{1}{3}\left[\frac{1.0(4.2)/a}{26.0}\right]^2\right\}(10^6)$$

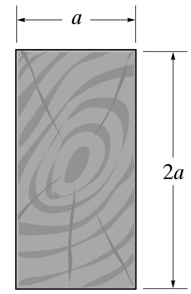
$$a = 0.19721 \text{ m} = 197.21 \text{ mm}$$

$$\frac{KL}{d} = \frac{1(4.2)}{0.19721} = 21.3, \quad 11 < \frac{KL}{d} < 26 \qquad \text{Assumption O.K.}$$

Use $a = 200$ mm **Ans.**

Ans:
Use $a = 200$ mm

13–105. A rectangular wooden column has the cross section shown. If the column is 1.8 m long and subjected to an axial force of $P = 75$ kN, determine the required minimum dimension a of its cross-sectional area to the nearest multiples of 5 mm so that the column can safely support the loading. The column is pinned at both ends.



SOLUTION

Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. Then,

$$\frac{KL}{d} = \frac{1(1.8)}{a} = \frac{1.8}{a}$$

NFPA Timber Column Formula. Assuming an intermediate column,

$$\sigma_{\text{allow}} = 8.28\left[1 - \frac{1}{3}\left(\frac{KL/d}{26.0}\right)^2\right] \text{ MPa}$$

$$\frac{75(10^3)}{2a(a)} = 8.28\left[1 - \frac{1}{3}\left(\frac{1.8/d}{26.0}\right)^2\right](10^6)$$

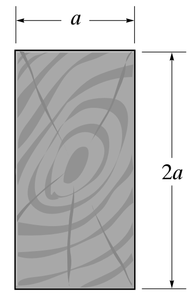
$$a = 0.07827 \text{ m} = 78.27 \text{ mm}$$

Use $a = 80$ mm **Ans.**

$\frac{KL}{d} = \frac{1.8}{0.08} = 22.5$. Since $11 < \frac{KL}{d} < 26$, the assumption is correct.

Ans:
Use $a = 80$ mm

13–106. A rectangular wooden column has the cross section shown. If $a = 75$ mm and the column is 3.6 m long, determine the allowable axial force P that can be safely supported by the column if it is pinned at its top and fixed at its base.



SOLUTION

Slenderness Ratio. For a column fixed at its base and pinned at its top $K = 0.7$. Then,

$$\frac{KL}{d} = \frac{0.7(3.6)}{0.075} = 33.6$$

NFPA Timer Column Formula. Since $26 < \dfrac{KL}{d} < 50$, the column can be classified as a long column.

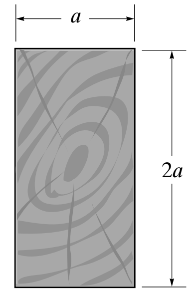
$$\sigma_{\text{allow}} = \frac{3725\ \text{MPa}}{(KL/d)^2} = \frac{3725\ \text{MPa}}{33.6^2} = 3.2995\ \text{MPa}$$

The allowable force is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = [3.2995(10^6)][(0.075)(0.015)] = 37.12(10^3)\ \text{N} = 37.1\ \text{N}$$ **Ans.**

Ans:
$P_{\text{allow}} = 37.1$ kN

13–107. A rectangular wooden column has the cross section shown. If $a = 75$ mm and the column is subjected to an axial force of $P = 75$ kN, determine the maximum length the column can have to safely support the load. The column is pinned at its top and fixed at its base.



SOLUTION

Slenderness Ratio. For a column fixed at its base and pinned at its top, $K = 0.7$. Then,

$$\frac{KL}{d} = \frac{0.7L}{0.075} = 9.3333L$$

NFPA Timber Column Formula. Assuming an intermediate column,

$$\sigma_{\text{allow}} = 8.28\left[1 - \frac{1}{3}\left(\frac{KL/d}{26.0}\right)^2\right] \text{ MPa}$$

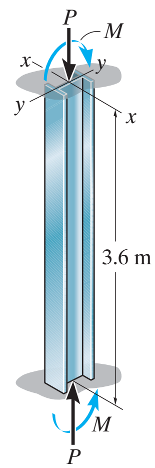
$$\frac{75(10^3)}{0.075(0.15)} = 8.28\left[1 - \frac{1}{3}\left(\frac{9.3333L}{26.0}\right)^2\right](10^6)$$

$$L = 2.1298 \text{ m} = 2.13 \text{ m}$$ **Ans.**

Hence, $\frac{KL}{d} = 9.3333(2.1298) = 19.88$. Since $11 < \frac{KL}{d} < 26$, the assumption is correct.

Ans:
$L = 2.13$ m

***13–108.** The W360 × 33 structural A-36 steel column fixed at its top and bottom. If a horizontal load (not shown) causes it to support end moments of $M = 15$ kN · m, determine the maximum allowable axial force P that can be applied. Bending is about the x–x axis. Use the AISC equations of Sec. 13.6 and Eq. 13–30.



SOLUTION

Section properties for W360 × 33:

$$A = 4190 \text{ mm}^2 = 4.19(10^{-3}) \text{ m}^2 \qquad d = 349 \text{ mm} = 0.349 \text{ m}$$

$$I_x = 82.9(10^6) \text{ mm}^4 = 82.9(10^{-6}) \text{ m}^4 \qquad r_y = 26.4 \text{ mm} = 0.0264 \text{ m}$$

Allowable stress method:

$$\frac{KL}{r_y} = \frac{0.5(3.6)}{0.0264} = 68.182$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66, \quad \frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

Hence,

$$(\sigma_a)_{\text{allow}} = \frac{\left[1 - \frac{1}{2}\left(\frac{\left(\frac{KL}{r}\right)^2}{\left(\frac{KL}{r}\right)_c^2}\right)\right]\sigma_Y}{\left[\frac{5}{3} + \frac{3}{8}\frac{\frac{KL}{r}}{\left(\frac{KL}{r}\right)_c} - \frac{1}{8}\frac{\left(\frac{KL}{r}\right)^3}{\left(\frac{KL}{r}\right)_c^3}\right]} = \frac{[1 - \frac{1}{2}(\frac{68.182}{125.66})^2]250}{[\frac{5}{3} + \frac{3}{8}(\frac{68.182}{125.66}) - \frac{1}{8}(\frac{68.182}{125.66})^3]} = 115.23 \text{ MPa}$$

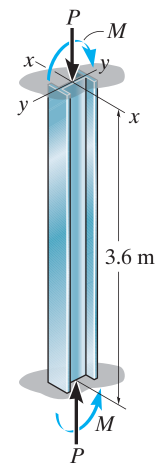
$$\sigma_{\max} = (\sigma_a)_{\text{allow}} = \frac{P}{A} + \frac{\mathrm{M_y}\, c}{\mathrm{I_x}}$$

$$115.23(10^6) = \frac{P}{4.19(10^{-3})} + \frac{[15(10^3)](0.349/2)}{82.9(10^{-6})}$$

$P = 350.53(10^3)$ N = 351 kN **Ans.**

Ans:
$P = 351$ kN

13–109. The W360 × 33 structural A-36 steel column is fixed at its top and bottom. If a horizontal load (not shown) causes it to support end moments of $M = 70$ kN · m, determine the maximum allowable axial force P that can be applied. Bending is about the x–x axis. Use the interaction formula with $(\sigma_b)_{\text{allow}} = 168$ MPa.



SOLUTION

Section Properties for W360 × 33:

$A = 4190 \text{ mm}^2 = 4.19(10^{-3}) \text{ m}^2 \qquad d = 349 \text{ mm} = 0.349 \text{ m}$

$I_x = 82.9(10^6) \text{ mm}^4 = 82.9(10^{-6}) \text{ m}^4 \qquad r_y = 26.4 \text{ mm} = 0.0264 \text{ m}$

Interaction method:

$$\frac{KL}{r_y} = \frac{0.5(3.6)(10^3)}{26.4} = 68.182$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{25(10^6)}} = 125.66, \quad \frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

Hence,

$$(\sigma_a)_{\text{allow}} = \frac{\left[1 - \frac{1}{2}\left(\frac{\left(\frac{KL}{r}\right)^2}{\left(\frac{KL}{r}\right)_c^2}\right)\right]\sigma_Y}{\left[\frac{5}{3} + \frac{3}{8}\frac{\frac{KL}{r}}{\left(\frac{KL}{r}\right)_c} - \frac{1}{8}\frac{\left(\frac{KL}{r}\right)^3}{\left(\frac{KL}{r}\right)_c^3}\right]} = \frac{\left[1 - \frac{1}{2}\left(\frac{68.182}{125.66}\right)^2\right]250}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{68.182}{125.66}\right) - \frac{1}{8}\left(\frac{68.182}{125.66}\right)^3\right]} = 115.2 \text{ MPa}$$

$$\sigma_a = \frac{P}{A} = \frac{P}{4.19(10^{-3})} = 238.66P$$

$$\sigma_b = \frac{M_x c}{I_x} = \frac{[70(10^3)](0.349/2)}{82.9(10^{-6})} = 147.35(10^6) \text{ N/m}^2$$

$$\frac{\sigma_a}{(\sigma_a)_{\text{allow}}} + \frac{\sigma_b}{(\sigma_b)_{\text{allow}}} = 1.0$$

$$\frac{238.66P}{115.23(10^6)} + \frac{147.35(10^6)}{168(10^6)} = 1.0$$

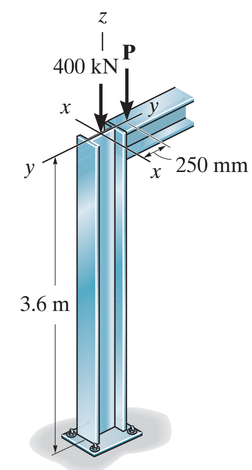
$P = 59.36(10^3) \text{ N} = 59.4 \text{ kN}$ **Ans.**

Since $\dfrac{\sigma_a}{\sigma_{\text{allow}}} = \dfrac{238.66[59.36(10^3)]}{115.23(10^6)} = 0.12 \; < 0.15$

Then the interaction formula is valid.

Ans:
$P = 59.4$ kN

13–110. The W360 × 79 structural A-36 steel column supports an axial load of 400 kN in addition to an eccentric load P. Determine the maximum allowable value of P based on the AISC equations of Sec. 13.6 and Eq. 13–30. Assume the column is fixed at its base, and at its top it is free to sway in the x–z plane while it is pinned in the y–z plane.



SOLUTION

Section Properties: For a W360 × 79 wide flange section.

$$A = 10100 \text{ mm}^2 = 0.0101 \text{ m}^2$$

$$d = 354 \text{ mm} = 0.354 \text{ m}$$

$$I_x = 227(10^6) \text{ mm}^4 = 227(10^{-6}) \text{ m}^4$$

$$r_x = 150 \text{ mm} = 0.15 \text{ m}$$

$$r_x = 48.9 \text{ mm} = 0.0489 \text{ m}$$

Slenderness Ratio: By observation, the largest slenderness ratio is about $y-y$ axis. For a column fixed at one end and free at the other end, $K = 2$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{2(3.6)}{0.0489} = 147.24$$

Allowable Stress: The allowable stress can be determined using *AISC Column Formulas*. For A–36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66$. Since $\left(\frac{KL}{r}\right)_c \leq \frac{KL}{r} \leq 200$, the column is a *long* column. Applying Eq. 13–21,

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$= \frac{12\pi^2[200(10^9)]}{23(147.24^2)}$$

$$= 47.50(10^6) \text{ N/m}^2$$

Maximum Stress: Bending is about $x-x$ axis. Applying we have

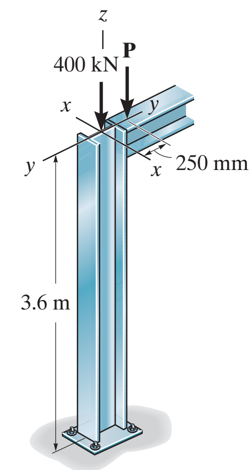
$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$47.50(10^6) = \frac{P + 400(10^3)}{0.0101} + \frac{[P(0.25)](0.354/2)}{227(10^{-6})}$$

$$P = 26.88(10^3) \text{ N} = 26.9 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$P = 26.9$ kN

13–111. The W310 × 67 structural A-36 steel column supports an axial load of 400 kN in addition to an eccentric loads of $P = 30$ kN. Determine if the column fails based on the AISC equations of Sec. 13.6 and Eq. 13–30. Assume that the column is fixed at its base, and at its top it is free to sway in the x–z plane while it is pinned in the y–z plane.



SOLUTION

Section Properties: For a W310 × 67 wide flange section,

$$A = 8530\ \text{mm}^2 = 8.53(10^{-3})\ \text{m}^2$$

$$d = 306\ \text{mm} = 0.306\ \text{m}$$

$$I_x = 145(10^6)\ \text{mm}^4 = 145(10^{-6})\ \text{m}^4$$

$$\pi_x = 130\ \text{mm} = 0.130\ \text{m}$$

$$\pi_x = 49.3\ \text{mm} = 0.0483\ \text{m}$$

Slenderness Ratio: By observation, the largest slenderness ratio is about $y-y$ axis. For a column fixed at one end and free at the other end, $K = 2$. Thus,

$$\left(\frac{KL}{\pi}\right)_y = \frac{2(3.6)}{0.0483} = 146.04$$

Allowable Stress: The allowable stress can be determined using *AISC Column Formulas*. For A–36 steel, $\left(\frac{KL}{\pi}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66$. Since $\left(\frac{KL}{r}\right)_c \leq \frac{KL}{r} \leq 200$, the column is a *long* column. Applying Eq. 13–21,

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$= \frac{12\pi^2[200(10^9)]}{23(146.04^2)}$$

$$= 48.28(10^6)\ \text{N/m}^2 = 48.28\ \text{MPa}$$

Maximum Stress: Bending is about $x-x$ axis. Applying Eq. 1 we have

$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I}$$

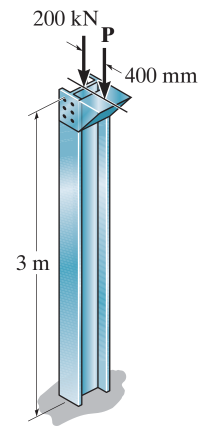
$$= \frac{P + 430(10^3)}{8.53(10^{-3})} + \frac{[30(10^3)](0.306/2)}{145(10^{-6})}$$

$$= 82.07(10^6)\ \text{N/m}^2 = 82.07\ \text{MPa}$$

Since $\sigma_{\max} > \sigma_{\text{allow}}$, the column is not adequate.

Ans:
The column is not adequate.

***13–112.** The W360 × 57 structural A-36 steel column is fixed at its bottom and free at its top. Determine the greatest eccentric load P that can be applied using Eq. 13–30 and the AISC equations of Sec. 13.6.



SOLUTION

Section properties for W360 × 57:

$A = 7200\text{ mm}^2 = 7.2(10^{-3})\text{ m}^2$ $\qquad r_y = 39.3\text{ mm} = 0.0393\text{ m}$

$d = 358\text{ mm} = 0.358\text{ m}$ $\qquad b_f = 172\text{ mm} = 0.172\text{ m}$

$I_x = 11.1(10^6)\text{ mm}^4 = 11.1(10^{-6})\text{ m}^4$

Allowable stress method:

$$\frac{KL}{r_y} = \frac{2(3)}{0.0393} = 152.67$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66, \quad \left(\frac{KL}{r}\right)_c < \frac{KL}{r_y} < 200$$

$$(\sigma_a)_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2[200(10^9)]}{23(152.67^2)} = 44.18(10^6)\text{ N/m}^2$$

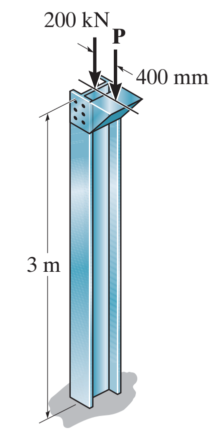
$$\sigma_{\max} = (\sigma_a)_{\text{allow}} = \frac{P}{A} + \frac{M_y c}{I_y}$$

$$44.18(10^6) = \frac{P + 200(10^3)}{7.2(10^{-3})} + \frac{[P(0.4)](0.172/2)}{11.1(10^{-6})}$$

$P = 5.067(10^3)\text{ N} = 5.07\text{ kN}$ **Ans.**

Ans:
$P = 5.07$ kN

13–113. The W250 × 67 structural A-36 steel column is fixed at its bottom and free at its top. If it is subjected to a load of $P = 10$ kN, determine if it is safe based on the AISC equations of Sec. 13.6 and Eq. 13–30.



SOLUTION

Section Properties for W250 × 67:

$A = 8560\ \text{mm}^2 = 8.56(10^{-3})\ \text{m}^2$ $\qquad d = 257\ \text{mm} = 0.257\ \text{m}$

$I_y = 22.2(10^6)\ \text{mm}^4 = 22.2(10^{-6})\ \text{m}^4$ $\qquad r_y = 50.9\ \text{mm} = 0.0509\ \text{m}$

$b_f = 204\ \text{mm} = 0.204\ \text{m}$

Allowable stress method:

$$\frac{KL}{r_y} = \frac{2(3)}{0.0509} = 117.88$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66$$

$$\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$$

$$(\sigma_a)_{\text{allow}} \geq \frac{\left[1 - \frac{1}{2}\frac{(KL/r)^2}{(KL/r)_c^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{KL/r_c}\right) - \frac{1}{8}\frac{(KL/r)^3}{(KL/r_c)^3}} = \frac{\left[1 - \frac{1}{2}\left(\frac{117.88}{125.66}\right)^2\right](250)}{\frac{5}{3} + \frac{3}{8}\left[\frac{117.88}{125.66}\right] - \frac{1}{8}\left[\frac{117.88}{125.66}\right]^3} = 73.10\ \text{MPa}$$

$$(\sigma_a)_{\text{allow}} = \frac{P}{A} + \frac{M_y c}{I_y}$$

$$\sigma_{\max} = \frac{210(10^3)}{8.56(10^{-3})} + \frac{[10(10^3)](0.4)(0.204/2)}{22.2(10^{-3})}$$

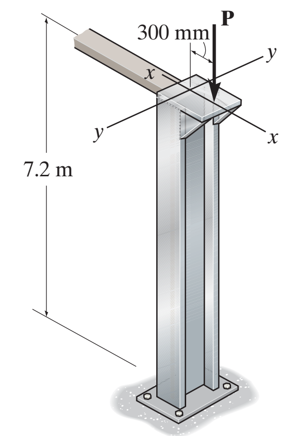
$$= 42.91(10^6)\ \text{N/m}^2 = 42.91\ \text{MPa}$$ O.K.

Since $\sigma_{\text{allow}} > \sigma_{\max}$ the column is safe.

Yes. **Ans.**

Ans:
Yes

13–114. The A-36-steel W250 × 67 column is fixed at its base. Its top is constrained to move along the x–x axis but free to rotate about and move along the y–y axis. Determine the maximum eccentric force P that can be safely supported by the column using an interaction formula. The allowable bending stress is $(\sigma_b)_{\text{allow}} = 100$ MPa.



SOLUTION

Section Properties. From the table listed in the appendix, the section properties for a W250 × 67 are

$A = 8560 \text{ mm}^2 = 8.56(10^{-3}) \text{ m}^2$ $\qquad \pi_x = 110 \text{ mm} = 0.11 \text{ m}$

$I_y = 22.2(10^6) \text{ mm}^4 = 22.2(10^{-6}) \text{ m}^4$ $\qquad \pi_y = 50.9 \text{ mm} = 0.0509 \text{ m}$

$b_f = 204 \text{ mm} = 0.204 \text{ m}$

Slenderness Ratio. Here, $L_x = 7.2$ m and for a column fixed at its base and free at its top, $K_x = 2$. Thus,

$$\left(\frac{KL}{\pi}\right)_x = \frac{2(7.2)}{0.11} = 130.91 \text{ (cont}\pi\text{ols)}$$

Since the column is fixed at its base and pinned at its top, $K_y = 0.7$ and $L_y = 7.2$ m. Then,

$$\left(\frac{KL}{\pi}\right)_y = \frac{0.7(7.2)}{0.0509} = 99.02$$

Allowable Stress. The allowable stress will be determined using the AISC column formulas. For A-36 steel, $\left(\dfrac{KL}{\pi}\right)_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_Y}} = \sqrt{\dfrac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66$. Since $\left(\dfrac{KL}{r}\right)_c < \left(\dfrac{KL}{r}\right)_x < 200$, the column is classified as a long column.

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$= \frac{12\pi^2[200(10^9)]}{23(130.91^2)} = 60.10(10^6) \text{ N/m}^2$$

Interaction Formula. Bending is about the weak axis. Here, $M = P(0.3 \text{ m})$ and $c = \dfrac{b_f}{2} = \dfrac{0.204}{2} = 0.102$ m

$$\frac{P/A}{(\sigma_a)_{\text{allow}}} + \frac{Mc/Ar^2}{(\sigma_b)_{\text{allow}}} = 1$$

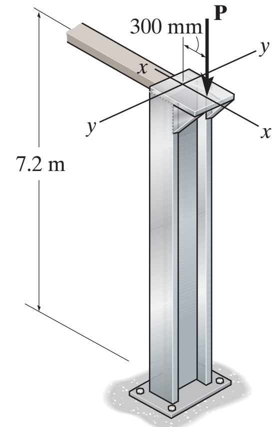
$$\frac{P/8.56(10^{-3})}{60.10(10^6)} + \frac{[P(0.3)](0.102)/[8.56(10^{-3})](0.0509^2)}{100(10^6)}$$

$P = 63.53(10^3) \text{ N} = 63.5 \text{ kN}$ **Ans.**

$$\frac{\sigma_a}{(\sigma_a)_{\text{allow}}} = \frac{63.53(10^3)/8.56(10^{-3})}{60.10(10^6)} = 0.123 < 0.15 \qquad \text{O.K.}$$

Ans:
$P = 63.5$ kN

13–115. The A-36-steel W310 × 74 column is fixed at its base. Its top is constrained to move along the x–x axis but free to rotate about and move along the y–y axis. If the eccentric force $P = 75$ kN is applied to the column, investigate if the column is adequate to support the loading. Use the allowable stress method.



SOLUTION

Section Properties. From the table listed in the appendix, the section properties for a W310 × 74 are

$A = 9480 \text{ mm}^2 = 9.48(10^{-3}) \text{ m}^2 \qquad r_x = 132 \text{ mm} = 0.132 \text{ m}$

$I_y = 23.4(10^6) \text{ mm}^4 = 22.3(10^{-6}) \text{ m}^4 \qquad r_y = 49.7 \text{ mm} = 0.0497 \text{ m}$

$b_f = 205 \text{ mm} = 0.205 \text{ m}$

Slenderness Ratio. Here, $L_x = 7.2$ m and for a column fixed at its base and pinned at its top, $K_x = 2$. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{2(7.2)}{0.132} = 109.09 \text{ (controls)}$$

Since the column is fixed at its base and pinned at its top, $K_y = 0.7$ and $L_y = 7.2$ m. Then,

$$\left(\frac{KL}{r}\right)_y = \frac{0.7(7.2)}{0.0497} = 101.41$$

Allowable Stress. The allowable stress will be determined using the AISC column formulas. For A-36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66$. Since $\left(\frac{KL}{r}\right)_x < \left(\frac{KL}{r}\right)_c$, the column can be classified as an intermediate column.

$$\sigma_{\text{allow}} = \frac{\left[1 - \dfrac{(KL/r)^2}{2(KL/r)_C{}^2}\right]\sigma_Y}{\dfrac{5}{3} + \dfrac{3(KL/r)}{8(KL/r)_C} - \dfrac{(KL/r)^3}{8(KL/r)_C{}^3}}$$

$$= \frac{\left[1 - \left(\dfrac{1}{2}\right)\left(\dfrac{109.09}{125.66}\right)^2\right](250)}{\dfrac{5}{3} + \left(\dfrac{3}{8}\right)\left(\dfrac{109.09}{125.66}\right) - \left(\dfrac{1}{8}\right)\left(\dfrac{109.09}{125.66}\right)^3}$$

$$= 81.55 \text{ MPa}$$

Maximum Stress. Bending is about the weak axis. Since, $M = 75(0.3) = 22.5 \text{ kN} \cdot \text{m}$ and $c = \frac{b_f}{2} = \frac{0.205}{2} = 0.1025$ m

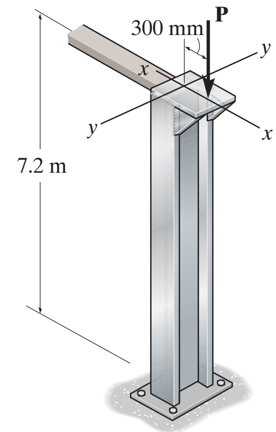
$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I} = \frac{75(10^3)}{9.48(10^{-3})} + \frac{[22.5(10^3)](0.1025)}{23.4(10^{-6})}$$

$$= 106.47(10^6) \text{ N/m}^2 = 106.47 \text{ MPa}$$

Since $\sigma_{\max} > \sigma_{\text{allow}}$, the W310 × 74 column *is inadequate* according to the allowable stress method.

? 1q8
The column is not adequate.

***13–116.** The A-36-steel W250 × 67 column is fixed at its base. Its top is constrained to move along the x–x axis but free to rotate about and move along the y–y axis. Determine the maximum eccentric force P that can be safely supported by the column using the allowable stress method.



SOLUTION

Section Properties. From the table listed in the appendix, the section properties for a W250 × 67 are

$A = 8560\ \text{mm}^2 = 8.56(10^{-3})\ \text{m}^2 \qquad r_x = 110\ \text{mm} = 0.11\ \text{m}$

$I_y = 22.2(10^6)\ \text{mm}^4 = 22.2(10^{-6})\ \text{m}^4 \qquad r_y = 50.9\ \text{mm} = 0.0509\ \text{m}$

$b_f = 204\ \text{mm} = 0.204\ \text{m}$

Slenderness Ratio. Here, $L_x = 7.2$ m and for a column fixed at its base and free at its top, $K_x = 2$. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{2(7.2)}{0.11} = 130.91\ \text{(controls)}$$

Since the column is fixed at its base and pinned at its top, $K_y = 0.7$ and $L_y = 7.2$ m. Then,

$$\left(\frac{KL}{r}\right)_y = \frac{0.7(7.2)}{0.0509} = 99.02$$

Allowable Stress. The allowable stress will be determined using the AISC column formulas. For A-36 steel,

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66.\ \text{Since}\ \left(\frac{KL}{r}\right)_c < \left(\frac{KL}{r}\right)_x < 200,$$

the column is classified as a long column.

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$= \frac{12\pi^2[200(10^9)]}{23(130.91^2)} = 60.10(10^6)\ \text{N/m}^2 = 60.10\ \text{MPa}$$

Maximum Stress. Bending is about the weak axis. Since $M = P(0.3\ \text{m})$ and $c = \frac{b_f}{2} = \frac{0.204}{2} = 0.102\ \text{m}$

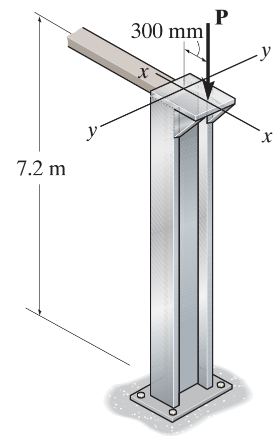
$$\sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$60.10(10^6) = \frac{P}{8.56(10^{-3})} + \frac{[P(0.3)](0.102)}{22.2(10^{-6})}$$

$P = 40.19(10^3)\ \text{N} = 40.2\ \text{kN}$ **Ans.**

Ans:
$P = 40.2$ kN

13–117. The A-36-steel W310 × 74 column is fixed at its base. Its top is constrained to move along the x–x axis but free to rotate about and move along the y–y axis. If the eccentric force $P = 65$ kN is applied to the column, investigate if the column is adequate to support the loading. Use the interaction formula. The allowable bending stress is $(\sigma_b)_{\text{allow}} = 100$ MPa



SOLUTION

Section Properties. From the table listed in the appendix, the section properties for a W310 × 74 are

$A = 9480\text{ mm}^2 = 9.48(10^{-3})\text{ m}^2 \qquad r_x = 132\text{ mm} = 0.132\text{ m}$

$I_y = 23.4(10^6)\text{ mm}^4 = 22.3(10^{-6})\text{ m}^4 \qquad r_y = 49.7\text{ mm} = 0.0497\text{ m}$

$b_f = 205\text{ mm} = 0.205\text{ m}$

Slenderness Ratio. Here, $L_x = 7.2$ m and for a column fixed at its base and pinned at its top, $K_x = 2$. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{2(7.2)}{0.132} = 109.09 \text{ (controls)}$$

Since the column is fixed at its base and pinned at its top, $K_y = 0.7$ and $L_y = 7.2$ m. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{0.7(7.2)}{0.0497} = 101.41$$

Allowable Stress. The allowable stress will be determined using the AISC column formulas. For A-36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66$. Since $\left(\frac{KL}{r}\right)_x < \left(\frac{KL}{r}\right)_c$, the column can be classified as an intermediate column.

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_C{}^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_C} - \frac{(KL/r)^3}{8(KL/r)_C{}^3}}$$

$$= \frac{\left[1 - \left(\frac{1}{2}\right)\left(\frac{109.09}{125.66}\right)^2\right](250)}{\frac{5}{3} + \left(\frac{3}{8}\right)\left(\frac{109.09}{125.66}\right) - \left(\frac{1}{8}\right)\left(\frac{109.09}{125.66}\right)^3}$$

$$= 81.55 \text{ MPa}$$

Interaction Formula. Bending is about the weak axis. Here $M = 65(0.3) = 19.5\text{ kN}\cdot\text{m}$ and $c = \frac{b_f}{2} = \frac{0.205}{2} = 0.1025$ m

$$\frac{P/A}{(\sigma_a)_{\text{allow}}} + \frac{Mc/Ar^2}{(\sigma_b)_{\text{allow}}} = \frac{65(10^3)/9.48(10^{-3})}{81.55(10^6)} + \frac{[19.5(10^3)](0.1025)/[9.48(10^{-3})](0.0497^2)}{100(10^6)}$$

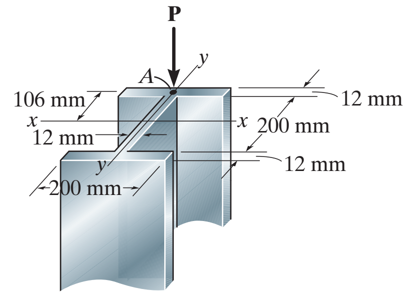
$$= 0.9376 < 1$$

$$\frac{\sigma_a}{(\sigma_a)_{\text{allow}}} = \frac{65(10^3)/9.48(10^{-3})}{81.55(10^6)} = 0.0841 < 0.15 \qquad \text{O.K.}$$

Thus, a W310 × 74 column *is adequate* according to the interaction formula.

Ans:
The column is adequate.

13–118. A 4.8-m-long column is made of aluminum alloy 2014-T6. If it is fixed at its top and bottom, and a compressive load **P** is applied at point A, determine the maximum allowable magnitude of **P** using the equations of Sec. 13.6 and Eq. 13–30.



SOLUTION

Section properties:

$$A = 0.012(0.2) + 2[0.2(0.012)] = 7.2(10^{-3})\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.2)(0.224^3) - \frac{1}{12}(0.188)(0.2^3) = 61.9904(10^{-6})\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.2)(0.012^3) + 2\left[\frac{1}{12}(0.012)(0.2^3)\right] = 16.0288(10^{-6})\ \text{m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{16.0288(10^{-6})}{7.2(10^{-3})}} = 0.04718\ \text{m}$$

Allowable stress method:

$$\frac{KL}{r_y} = \frac{0.5(4.8)}{0.04718} = 50.87,\ 12 < \frac{KL}{r_y} < 55$$

$$\sigma_{\text{allow}} = \left[212 - 1.59\left(\frac{KL}{r}\right)\right]\ \text{MPa}$$

$$= [212 - 1.59(50.87)] = 131.12\ \text{MPa}$$

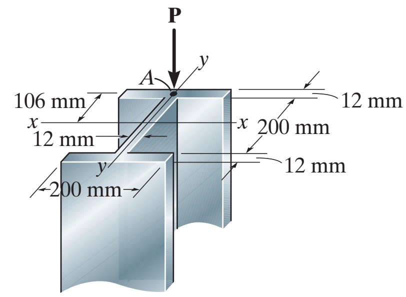
$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{P}{A} + \frac{M_x c}{I_x}$$

$$131.12(10^6) = \frac{P}{7.2(10^{-3})} + \frac{[P(0.106)](0.112)}{61.9904(10^{-6})}$$

$P = 396.86(10^3)\ \text{N} = 397\ \text{kN}$ **Ans.**

Ans:
$P = 397$ kN

13–119. A 4.8-m-long column is made of aluminum alloy 2014-T6. If it is fixed at its top and bottom, and a compressive load **P** is applied at point A, determine the maximum allowable magnitude of **P** using the equations of Sec. 13.6 and the interaction formula with $(\sigma_b)_{\text{allow}} = 140$ MPa.



SOLUTION

Section properties:

$$A = 0.012(0.2) + 2[0.2(0.012)] = 7.2(10^{-3})\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.2)(0.224^3) - \frac{1}{12}(0.188)(0.2^3) = 61.9904(10^{-6})\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.2)(0.012^3) + 2\left[\frac{1}{12}(0.012)(0.2^3)\right] = 16.0288(10^{-6})\ \text{m}^4$$

$$\pi_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{16.0288(10^{-6})}{7.2(10^{-3})}} = 0.04718\ \text{m}$$

Allowable stress method:

$$\frac{KL}{\pi_y} = \frac{0.5(4.8)}{0.04718} = 50.87,\ 12 < \frac{KL}{\pi_y} < 55$$

$$\sigma_{\text{allow}} = \left[212 - 1.59\left(\frac{KL}{\pi}\right)\right]\ \text{MPa}$$

$$= [212 - 1.59(50.87)] = 131.12\ \text{MPa}$$

$$\sigma_a = \frac{P}{A} = \frac{P}{7.2(10^{-3})} = 138.89P$$

$$\sigma_b = \frac{Mc}{I_x} = \frac{[P(0.106)](0.112)}{61.9904(10^{-6})} = 191.51P$$

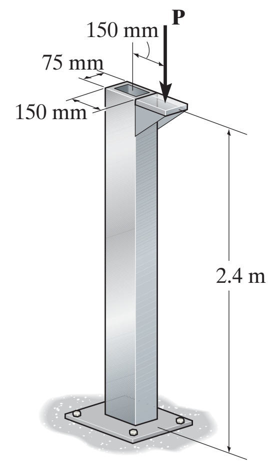
$$\frac{\sigma_a}{(\sigma_a)_{\text{allow}}} + \frac{\sigma_b}{(\sigma_b)_{\text{allow}}} = 1.0$$

$$\frac{138.89P}{131.12(10^6)} + \frac{191.51(10^6)}{140(10^6)} = 1.0$$

$$P = 412.00(10^3)\ \text{N} = 412\ \text{kN}$$ **Ans.**

Ans:
$P = 412$ kN

***13–120.** The 2014-T6 hollow column is fixed at its base and free at its top. Determine the maximum eccentric force P that can be safely supported by the column. Use the allowable stress method. The thickness of the wall for the section is $t = 12$ mm.



SOLUTION

Section Properties.

$$A = 0.15(0.075) - 0.126(0.051) = 4.824(10^{-3})\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.075)(0.15^3) - \frac{1}{12}(0.051)(0.126^3) = 12.592152(10^{-6})\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.15)(0.075^3) - \frac{1}{12}(0.126)(0.051^3) = 3.880602(10^{-6})\ \text{m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{12.592152(10^{-6})}{4.824(10^{-3})}} = 0.05109\ \text{m}$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{3.880602(10^{-6})}{4.824(10^{-3})}} = 0.02836\ \text{m}$$

Slenderness Ratio. For a column fixed at its base and free at its top, $K = 2$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{2(2.4)}{0.02836} = 169.24$$

Allowable Stress. Since $\left(\frac{KL}{r}\right)_y > 55$, the column can be classified as a long column.

$$\sigma_{\text{allow}} = \frac{372550\ \text{MPa}}{(KL/r)^2} = \frac{372550\ \text{MPa}}{169.24^2} = 13.01\ \text{MPa}$$

Maximum Stress. Bending occurs about the strong axis so that $M = P(0.150\ \text{m})$ and $c = \frac{0.15}{2} = 0.075\ \text{m}$

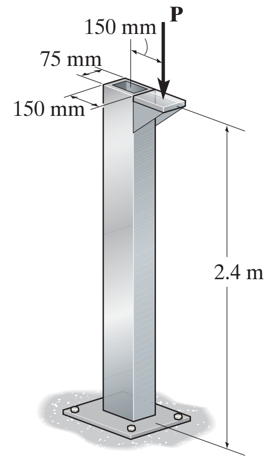
$$\sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$13.01(10^6) = \frac{P}{4.824(10^{-3})} + \frac{[P(0.15)](0.075)}{12.592152(10^{-6})}$$

$$P = 11.82(10^3)\ \text{N} = 11.8\ \text{kN}$$ **Ans.**

Ans:
$P = 11.8$ kN

13–121. The 2014-T6 hollow column is fixed at its base and free at its top. Determine the maximum eccentric force P that can be safely supported by the column. Use the interaction formula. The allowable bending stress is $(\sigma_b)_{\text{allow}} = 200$ MPa. The thickness of the wall for the section is $t = 12$ mm.



SOLUTION

Section Properties.

$$A = 0.15(0.075) - 0.126(0.051) = 4.824(10^{-3})\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.075)(0.15^3) - \frac{1}{12}(0.051)(0.126^3) = 12.592152(10^{-6})\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.15)(0.075^3) - \frac{1}{12}(0.126)(0.051^3) = 3.880602(10^{-6})\ \text{m}^4$$

$$\pi_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{12.592152(10^{-6})}{4.824(10^{-3})}} = 0.05109\ \text{m}$$

$$\pi_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{3.880602(10^{-6})}{4.824(10^{-3})}} = 0.02836\ \text{m}$$

Slenderness Ratio. For a column fixed at its base and free at its top, $K = 2$. Thus,

$$\left(\frac{KL}{\pi}\right)_y = \frac{2(2.4)}{0.02836} = 169.24$$

Allowable Stress. Since $\left(\frac{KL}{r}\right)_y > 55$, the column can be classified as a long column.

$$\sigma_{\text{allow}} = \frac{372550\ \text{MPa}}{(KL/\pi)^2} = \frac{372550\ \text{MPa}}{169.24^2} = 13.01\ \text{MPa}$$

Maximum Stress. Bending occurs about the strong axis so that $M = P(0.150\ \text{m})$ and $c = \frac{0.15}{2} = 0.075$ m

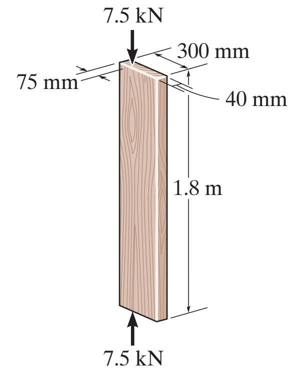
$$\frac{P/A}{(\sigma_a)_{\text{allow}}} + \frac{Mc/Ar^2}{(\sigma_b)_{\text{allow}}} = 1$$

$$\frac{P/4.824(10^{-3})}{13.01(10^6)} + \frac{P(0.15)(0.075)/[4.824(10^{-3})](0.05109^2)}{200(10^6)} = 1$$

$$P = 49.01(10^3)\ \text{N} = 49.0\ \text{kN}$$ **Ans.**

Ans:
$P = 49.0$ kN

13–122. Determine if the column can support the eccentric compressive load of 7.5 kN. Assume that the ends are pin connected. Use the NFPA equations in Sec. 13.6 and Eq. 13–30.



SOLUTION

$A = 0.3(0.04) = 0.012 \text{ m}^2 \qquad I_x = \frac{1}{12}(0.04)(0.3^3) = 90.0(10^{-6}) \text{ m}^4$

$d = 0.04 \text{ m}$

$$\frac{KL}{d} = \frac{1.0(1.8)}{0.04} = 45$$

$$26 < \frac{KL}{d} < 50$$

$$(\sigma_a)_{\text{allow}} = \frac{3725 \text{ MPa}}{(KL/d)^2} = \frac{3725 \text{ MPa}}{45^2} = 1.8395 \text{ MPa}$$

$$\sigma_{\max} = \frac{P}{A} + \frac{M_x c}{I_x}$$

$$= \frac{7.5(10^3)}{0.012} + \frac{[7.5(10^3)](0.075)(0.15)}{90.0(10^{-6})}$$

$$= 1.5625(10^6) \text{ N/m}^2 = 1.5625 \text{ MPa}$$

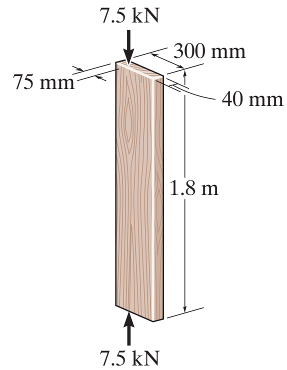
$$(\sigma_a)_{\text{allow}} > \sigma_{\max}$$

The column is adequate.

Yes. **Ans.**

Ans:
Yes.

13–123. Determine if the column can support the eccentric compressive load of 7.5 kN. Assume that the bottom is fixed and the top is pinned. Use the NFPA equations in Sec. 13.6 and Eq. 13–30.



SOLUTION

$A = 0.3(0.04) = 0.012 \text{ m}^2 \qquad I_x = \frac{1}{12}(0.04)(0.3^3) = 90.0(10^{-6}) \text{ m}^4$

$d = 0.04$ m

$$\frac{KL}{d} = \frac{0.7(1.8)}{0.04} = 31.5$$

$$26 < \frac{KL}{d} < 50$$

$$(\sigma_a)_{\text{allow}} = \frac{3725 \text{ MPa}}{(KL/d)^2} = \frac{3725 \text{ MPa}}{31.5^2} = 3.754 \text{ MPa}$$

$$\sigma_{\max} = \frac{P}{A} + \frac{M_x c}{I_x} = \frac{7.5(10^3)}{0.012} + \frac{[7.5(10^3)](0.075)(0.15)}{90.0(10^{-6})}$$

$$= 1.5625(10^6) \text{ N/m}^2 = 1.5625 \text{ MPa}$$

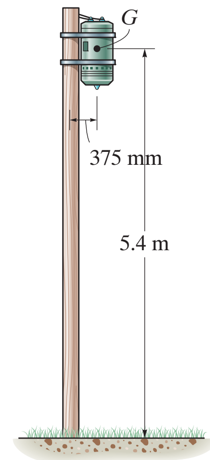
$(\sigma_a)_{\text{allow}} > \sigma_{\max}$

The column is adequate.

Yes. **Ans.**

Ans:
Yes

***13–124.** The 250-mm-diameter utility pole supports the transformer that has a weight of 3 kN and center of gravity at G. If the pole is fixed to the ground and free at its top, determine if it is adequate according to the NFPA equations of Sec. 13.6 and Eq. 13–30.



SOLUTION

$$\frac{KL}{d} = \frac{2(5.4)}{0.25} = 43.2$$

$$26 < \frac{KL}{d} < 50$$

Use Eq. 13–29,

$$\sigma_{\text{allow}} = \frac{3725 \text{ MPa}}{(KL/d)^2} = \frac{3725 \text{ MPa}}{43.2^2} = 1.9960 \text{ MPa} = 1.99 \text{ MPa}$$

$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I}$$

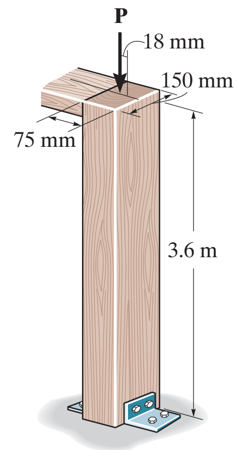
$$\sigma_{\max} = \frac{3(10^3)}{\pi(0.125^2)} + \frac{[3(10^3)(0.375)](0.125)}{\frac{\pi}{4}(0.125^4)}$$

$$\sigma_{\max} = 0.7945(10^6) \text{ N/m}^2 = 0.7945 \text{ MPa} < 1.9960 \text{ MPa}$$ O.K.

Yes. **Ans.**

Ans:
Yes

13–125. Using the NFPA equations of Sec. 13.6 and Eq. 13–30, determine the maximum allowable eccentric load P that can be applied to the wood column. Assume that the column is pinned at both its top and bottom.



SOLUTION

Section Properties:

$$A = 0.15(0.075) = 0.01125 \text{ m}^2$$

$$I_y = \frac{1}{12}(0.15)(0.075^3) = 5.27344(10^{-6}) \text{ m}^4$$

Slenderness Ratio: For a column pinned at both ends, $K = 1.0$. Thus,

$$\left(\frac{KL}{d}\right)_y = \frac{1.0(3.6)}{0.075} = 48.0$$

Allowable Stress: The allowable stress can be determined using *NFPA timber column formulas.* Since $26 < \dfrac{KL}{d} < 50$, it is a *long* column. Applying Eq. 13–29,

$$\sigma_{\text{allow}} = \frac{3725}{(KL/d)^2} \text{ MPa}$$

$$= \frac{3725 \text{ MPa}}{48.0^2} = 1.6168 \text{ MPa}$$

Maximum Stress: Bending is about $y-y$ axis. Applying Eq. 13–30, we have

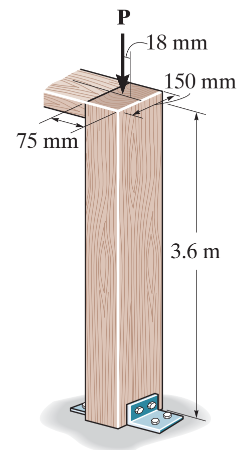
$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$1.6168(10^6) = \frac{P}{0.01125} + \frac{[P(0.018)](0.0375)}{5.27344(10^{-6})}$$

$$P = 7.454(10^3) \text{ N} = 7.45 \text{ kN}$$ **Ans.**

Ans:
$P = 7.45$ kN

13–126. Using the NFPA equations of Sec. 13.6 and Eq. 13–30, determine the maximum allowable eccentric load P that can be applied to the wood column. Assume that the column is pinned at the top and fixed at the bottom.



SOLUTION

Section Properties:

$$A = 0.15(0.075) = 0.01125 \text{ m}^2$$

$$I_y = \frac{1}{12}(0.15)(0.075^3) = 5.27344(10^{-6}) \text{ m}^4$$

Slenderness Ratio: For a column pinned at one end and fixed at the other end, $K = 0.7$. Thus,

$$\left(\frac{KL}{d}\right)_y = \frac{0.7(3.6)}{0.075} = 33.6$$

Allowable Stress: The allowable stress can be determined using *NFPA timber column formulas*. Since $26 < \frac{KL}{d} < 50$, it is a *long* column. Applying Eq. 13–29,

$$\sigma_{\text{allow}} = \frac{3725}{(KL/d)^2} \text{MPa}$$

$$= \frac{3725 \text{ MPa}}{33.6^2} = 3.2995 \text{ MPa}$$

Maximum Stress: Bending is about $y-y$ axis. Applying Eq. 13–30, we have

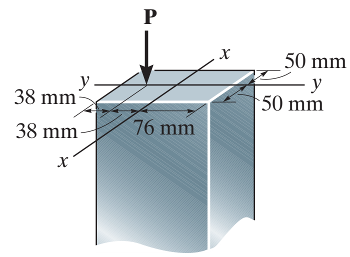
$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$3.2995(10^6) = \frac{P}{0.01125} + \frac{[P(0.018)](0.0375)}{5.27344(10^{-6})}$$

$$P = 15.21(10^3) \text{ N} = 15.2 \text{ kN}$$ **Ans.**

Ans:
$P = 15.2$ kN

13–127. A 3-m-long bar is made of aluminum alloy 2014-T6. If it is fixed at its bottom and pinned at the top, determine the maximum allowable eccentric load **P** that can be applied using the formulas in Sec. 13.6 and Eq. 13–30.



SOLUTION

Section Properties:

$$A = 0.152(0.1) = 0.0152 \text{ m}^2$$

$$I_x = \frac{1}{12}(0.1)(0.152^3) = 29.2651(10^{-6}) \text{ m}^4$$

$$I_y = \frac{1}{12}(0.152)(0.1^3) = 12.6667(10^{-6}) \text{ m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{12.6667(10^{-6})}{0.0152}} = 0.02887 \text{ m}$$

Slenderness Ratio: The largest slenderness ratio is about $y-y$ axis. For a column pinned at one end fixed at the other end, $K = 0.7$. Thus,

$$\left(\frac{KL}{\pi}\right)_y = \frac{0.7(3)}{0.02887} = 72.74$$

Allowable Stress: The allowable stress can be determined using aluminum (2014 –T6 alloy) column formulas. Since $\frac{KL}{r} > 55$, the column is classified as a *long* column. Applying Eq. 13–26,

$$\sigma_{\text{allow}} = \left[\frac{372550}{(KL/\pi)^2}\right]\text{MPa}$$

$$= \frac{372550}{72.74^2}\text{MPa}$$

$$= 70.40 \text{ MPa}$$

Maximum Stress: Bending is about $x-x$ axis. Applying Eq. 13–30, we have

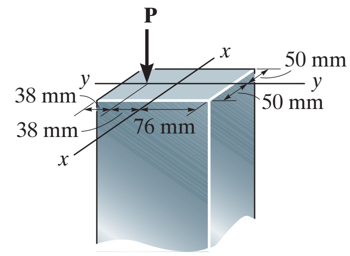
$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$70.40(10^6) = \frac{P}{0.0152} + \frac{[P(0.038)](0.076)}{29.2651(10^{-6})}$$

$$P = 428.02(10^3) \text{ N} = 428 \text{ kN}$$ **Ans.**

Ans:
$P = 428$ kN

***13–128.** The 3-m-long bar is made of aluminum alloy 2014-T6. If it is fixed at its bottom and pinned at the top, determine the maximum allowable eccentric load **P** that can be applied using the equations of Sec. 13.6 and the interaction formula with $(\sigma_b)_{\text{allow}} = 126$ MPa.



SOLUTION

Section Properties:

$$A = 0.152(0.1) = 0.0152 \text{ m}^2$$

$$I_x = \frac{1}{12}(0.1)(0.152^3) = 29.2651(10^{-6}) \text{ m}^4$$

$$I_y = \frac{1}{12}(0.152)(0.1^3) = 12.6667(10^{-6}) \text{ m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{29.2651(10^{-6})}{0.0152}} = 0.04388 \text{ m}$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{12.6667(10^{-6})}{0.0152}} = 0.02887 \text{ m}$$

Slenderness Ratio: The largest slenderness radio is about $y-y$ axis. For a column pinned at one end and fixed at the other end, $K = 0.7$. Thus

$$\left(\frac{KL}{r}\right)_y = \frac{0.7(3)}{0.02887} = 72.74$$

Allowable Stress: The allowable stress can be determined using *aluminum (2014 –T6 alloy) column formulas.* Since $\frac{KL}{r} > 55$, the column is classified as a *long* column. Applying Eq. 13–26,

$$\sigma_{\text{allow}} = \left[\frac{372550}{(KL/r)^2}\right] \text{MPa}$$

$$= \frac{372550}{72.74^2} \text{MPa}$$

$$= 70.40 \text{ MPa}$$

Interaction Formula: Bending is about $x-x$ axis. Applying Eq. 13–31, we have

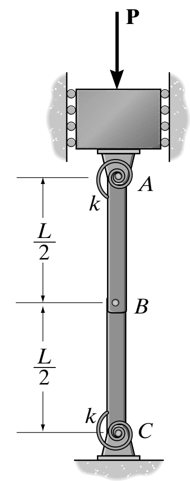
$$\frac{P/A}{(\sigma_a)_{\text{allow}}} + \frac{Mc/Ar^2}{(\sigma_b)_{\text{allow}}} = 1$$

$$\frac{P/0.0152}{70.40(10^6)} + \frac{[P(0.038)](0.076)/0.0152(0.04388^2)}{126(10^6)} = 1$$

$$P = 582.16(10^3) \text{ N} = 582 \text{ kN}$$ **Ans.**

Ans:
$P = 582$ kN

R13–1. If the torsional springs attached to ends A and C of the rigid members AB and BC have a stiffness k, determine the critical load P_{cr}.



SOLUTION

Equilibrium. When the system is given a slight lateral disturbance, the configuration shown in Fig. *a* is formed. The couple moment M can be related to P by considering the equilibrium of members AB and BC.

Member AB

$$+\uparrow\Sigma F_y = 0; \qquad B_y - P = 0 \tag{1}$$

$$\circlearrowleft +\Sigma M_A = 0; \qquad B_y\left(\frac{L}{2}\sin\theta\right) + B_x\left(\frac{L}{2}\cos\theta\right) - M = 0 \tag{2}$$

Member BC

$$\circlearrowleft +\Sigma M_C = 0; \; -B_y\left(\frac{L}{2}\sin\theta\right) + B_x\left(\frac{L}{2}\cos\theta\right) + M = 0 \tag{3}$$

Solving Eqs. (1), (2), and (3), we obtain

$$B_x = 0 \qquad B_y = \frac{2M}{L\sin\theta} \qquad M = \frac{PL}{2}\sin\theta$$

Since θ is very small, the small angle analysis gives $\sin\theta \cong \theta$. Thus,

$$M = \frac{PL}{2}\theta \tag{4}$$

Torslonal Spring Moment. The restoring couple moment M_{sp} can be determined using the torsional spring formula, $M = k\theta$. Thus,

$$M_{sp} = k\theta$$

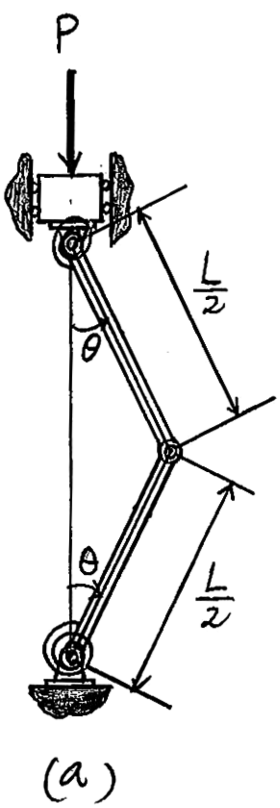
Critical Buckling Load. When the mechanism is on the verge of bucklling M must equal M_{sp}.

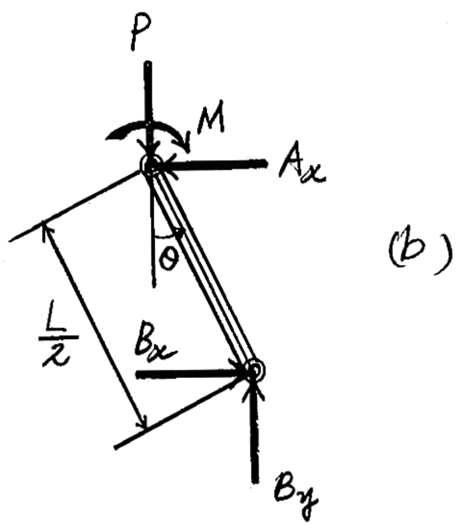
$$M = M_{sp}$$

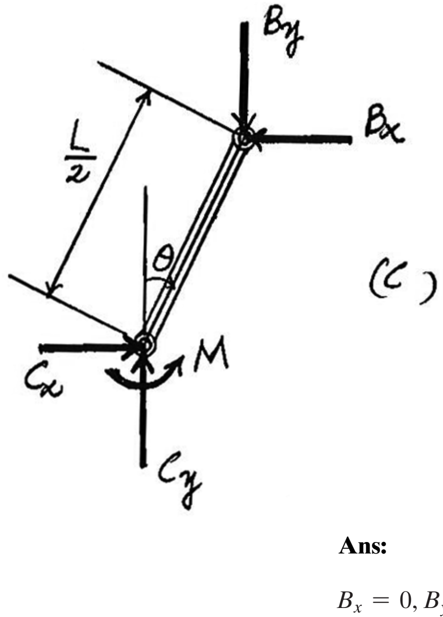
$$\frac{P_{cr}L}{2}\theta = k\theta$$

$$P_{cr} = \frac{2k}{L} \qquad \textbf{Ans.}$$

R13–1. Continued





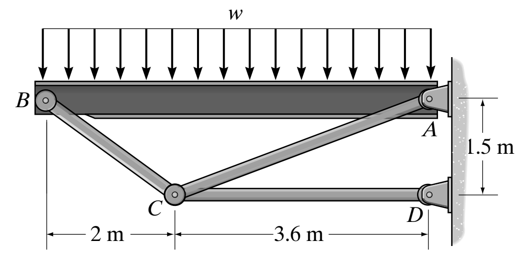


Ans:

$B_x = 0, B_y = \dfrac{2M}{L \sin\theta}, M = \dfrac{PL}{2} \sin\theta,$

$M = \dfrac{PL}{2}\theta, P_{cr} = \dfrac{2k}{L}$

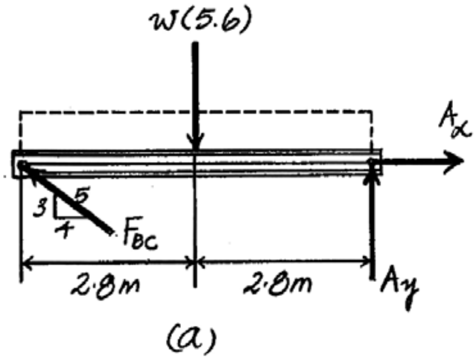
R13–2. Determine the maximum intensity w of the uniform distributed load that can be applied on the beam without causing the compressive members of the supporting truss to buckle. The members of the truss are made from A-36-steel rods having a 60-mm diameter. Use F.S. = 2 against buckling.



SOLUTION

Equilibrium. The force developed in member BC can be determined by considering the equilibrium of the free-body diagram of the beam AB, Fig. a.

$$\zeta + \Sigma M_A = 0; \qquad w(5.6)(2.8) - F_{BC}\left(\frac{3}{5}\right)(5.6) = 0 \quad F_{BC} = 4.6667w$$



(a)

The Force developed in member CD can be obtained by analyzing the equilibrium of joint C, Fig. b,

$$+\uparrow \Sigma F_y = 0; \qquad F_{AC}\left(\frac{5}{13}\right) - 4.6667w\left(\frac{3}{5}\right) = 0 \qquad F_{AC} = 7.28w \text{ (T)}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 4.6667w\left(\frac{4}{5}\right) + 7.28\left(\frac{12}{13}\right)w - F_{CD} = 0 \qquad F_{CD} = 10.4533w \text{ (C)}$$

Section Properties. The cross-sectional area and moment of inertia of the solid circular rod CD are

$$A = \pi\left(0.03^2\right) = 0.9\left(10^{-3}\right)\pi \text{ m}^2 \qquad I = \frac{\pi}{4}\left(0.03^4\right) = 0.2025\left(10^{-6}\right)\pi \text{ m}^4$$

Critical Buckling Load. Since both ends of member CD are pinned, $K = 1$. The critical buckling load is

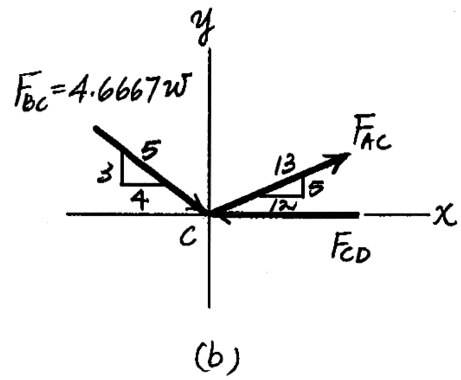
$$P_{cr} = F_{CD}\,(\text{F.S.}) = 10.4533w(2) = 20.9067w$$

Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2}$$

$$20.9067w = \frac{\pi^2\left[200\left(10^9\right)\right]\left[0.2025\left(10^{-6}\right)\pi\right]}{[1(3.6)]^2}$$

$$w = 4634.63 \text{ N/m} = 4.63 \text{ kN/m}$$ **Ans.**



(b)

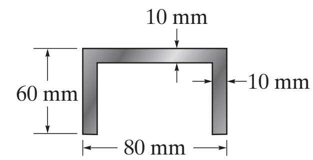
Critical Stress: Euler's formula is valid only if $\sigma_{\text{cr}} < \sigma_Y$.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{20.907(4634.63)}{0.9\left(10^{-3}\right)\pi} = 34.27 \text{ MPa} < \sigma_Y = 250 \text{ MPa}$$ O.K.

Ans:
$w = 4.63$ kN/m

R13–3.

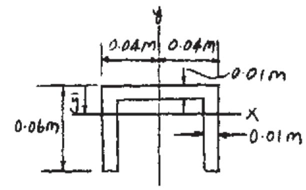
A steel column has a length of 5 m and is free at one end and fixed at the other end. If the cross-sectional area has the dimensions shown, determine the critical load. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION

Section Properties:

$$A = 0.06\,(0.01) + 2\,(0.06)(0.01) = 1.80(10^{-3})\ \text{m}^2$$



$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{0.005\,(0.06)(0.01) + 2[0.03\,(0.06)(0.01)]}{0.06\,(0.01) + 2\,(0.06)(0.01)} = 0.02167\ \text{m}$$

$$I_x = \frac{1}{12}(0.06)(0.01)^3 + 0.06\,(0.01)(0.02167 - 0.005)^2$$

$$+ [\frac{1}{12}(0.01)(0.06)^3 + 0.01\,(0.06)(0.03 - 0.02167)^2] = 0.615\,(10^{-6})\ \text{m}^4 \quad \text{(controls)}$$

$$I_y = \frac{1}{12}\,(0.06)(0.08)^3 - \frac{1}{12}\,(0.05)(0.06)^3 = 1.66\,(10^{-6})\ \text{m}^4$$

Critical Load:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}; \qquad K = 2.0$$

$$= \frac{\pi^2\,(200)(10^9)(0.615)(10^{-6})}{[2.0\,(5)]^2}$$

$$= 12140\ \text{N} = 12.1\ \text{kN}$$ **Ans.**

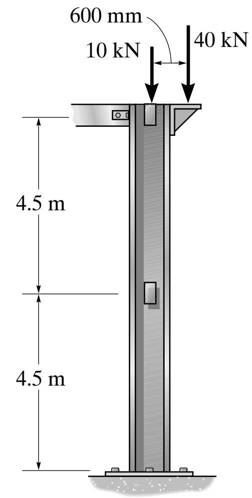
Check Stress:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{12140}{1.80\,(10^{-3})} = 6.74\ \text{MPa} < \sigma_Y = 360\ \text{MPa}$$

Hence, Euler's equation is still valid.

Ans:
$P_{cr} = 12.1$ kN

***R13–4.** The A-36-steel column can be considered pinned at its top and fixed at its base. Also, it is braced at its mid-height along the weak axis. Investigate whether a W250 × 45 section can safely support the loading shown. Use the interaction formula. The allowable bending stress is $(\sigma_b)_{\text{allow}} = 100$ MPa.



SOLUTION

Section Properties. From the table listed in the appendix, the necessary section properties for a W250 × 45 are

$A = 5700 \text{ mm}^2 = 5.70\left(10^{-3}\right) \text{m}^2 \qquad d = 266 \text{ mm} = 0.266 \text{ m}$

$I_x = 71.1\left(10^6\right)\text{mm}^4 = 71.1\left(10^{-6}\right) \text{m}^4 \qquad r_x = 112 \text{ mm} = 0.112 \text{ m} \qquad r_y = 35.1 \text{ mm} = 0.0351 \text{ mm}$

Slenderness Ratio. Here, $L_x = 9$ m and for a column fixed at its base and pinned at its top, $K_x = 0.7$. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{0.7(9)}{0.112} = 56.25$$

Since the bracing provides support equivalent to a pin, $K_y = 1$ and $L_y = 4..5$ m. Then,

$$\left(\frac{KL}{r}\right)_y = \frac{1(4.5)}{0.0351} = 128.21 \text{ (controls)}$$

Allowable Axial Stress. For A–36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$ $= \sqrt{\frac{2\pi^2\left[200\left(10^9\right)\right]}{250\left(10^6\right)}} = 125.66$. Since $\left(\frac{KL}{r}\right)_c < \left(\frac{KL}{r}\right)_y < 200$, the column can be classified as a long column.

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2\left[200\left(10^9\right)\right]}{23(128.21)^2} = 62.657 \text{ MPa}$$

Interaction Formula. Bending is about the strong axis. Here, $P = 10 + 40 = 50$ kN, $M = 40(0.6) = 24 \text{ kN}\cdot\text{m}$ and $c = \frac{d}{2} = \frac{0.266}{2} = 0.133$ m,

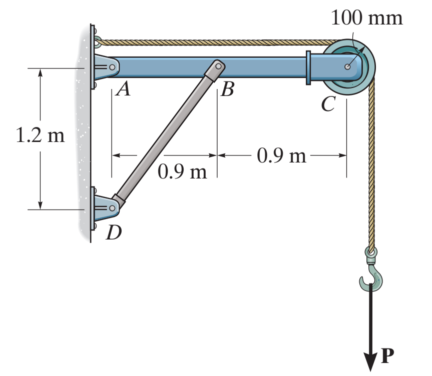
$$\frac{P/A}{(\sigma_a)_{\text{allow}}} + \frac{Mc/Ar^2}{(\sigma_b)_{\text{allow}}} = \frac{50\left(10^3\right)\Big/5.70\left(10^{-3}\right)}{62.657\left(10^6\right)} + \frac{24\left(10^3\right)(0.133)\Big/\left[5.70\left(10^{-3}\right)\left(0.112^2\right)\right]}{100\left(10^6\right)}$$

$$= 0.5864 < 1 \qquad \text{O.K.}$$

$$\frac{\sigma_a}{(\sigma_a)_{\text{allow}}} = \frac{50\left(10^3\right)\Big/5.7\left(10^{-3}\right)}{62.657\left(10^6\right)} = 0.140 < 0.15 \qquad \text{O.K.}$$

Thus, a W250 × 45 column is *adequate* according to the interaction formula.

R13–5. If the A-36 steel solid circular rod BD has a diameter of 50 mm, determine the allowable maximum force **P** that can be supported by the frame without causing the rod to buckle. Use F.S. = 2 against buckling.



SOLUTION

Equilibrium. The compressive force developed in BD can be determined by considering the equilibrium of the free-body diagram of member ABC, Fig. a.

$$\zeta + \Sigma M_A = 0; \qquad F_{BD}\left(\frac{4}{5}\right)(0.9) - P(1.9) + P(0.1) = 0 \qquad F_{BD} = 2.5P$$

Section Properties. The cross-sectional area and moment of inertia of BD are

$$A = \pi(0.025^2) = 0.625(10^{-3})\pi \text{ m}^2 \qquad I = \frac{\pi}{4}(0.025^4) = 97.65625(10^{-9})\pi \text{ m}^4$$

Critical Buckling Load. Since BD is pinned at both of its ends, $K = 1$. The critical buckling load is

$$P_{cr} = F_{BD}(\text{F.S.}) = 2.5P(2) = 5P$$

The length of BD is $L = \sqrt{0.9^2 + 1.2^2} = 1.5$ m. Applying Euler's for,mula

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

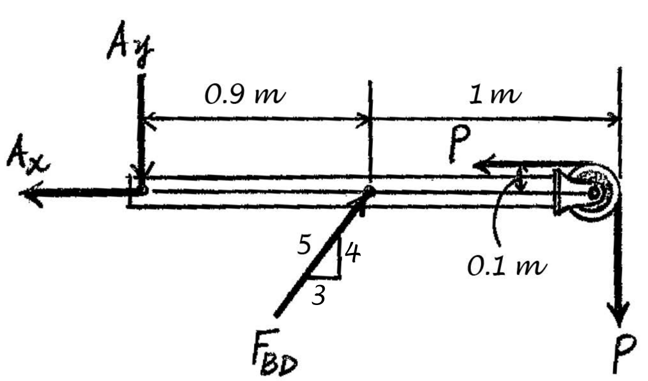
$$5P = \frac{\pi^2[200(10^9)][97.65625(10^{-9})\pi]}{[1.0(1.5)]^2}$$

$$P = 53.83(10^3) \text{ N} = 53.8 \text{ kN}$$ **Ans.**

Critical Stress. Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{5[53.83(10^3)]}{0.625(10^{-3})\pi} = 137.08(10^6) \text{ N}\cancel{\text{m}}^2$$

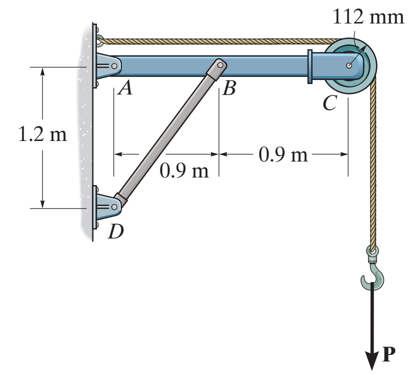
$$= 137.08 \text{ MPa} < \sigma_Y = 250 \text{ MPa} \qquad \text{(O.K.)}$$



(a)

Ans:
$P = 53.8$ kN

R13–6. If $P = 75$ kN, determine the required minimum diameter of the A992 steel solid circular rod BD to the nearest mm. Use F.S. = 2 against buckling.



SOLUTION

Equilibrium. The compressive force developed in BD can be determined by considering the equilibrium of the free-body diagram of member ABC, Fig. a,

$$\zeta + \Sigma M_A = 0; \quad F_{BD}\left(\frac{4}{5}\right)(0.9) - 75(1.9) + 75(0.1) = 0 \qquad F_{BC} = 187.5 \text{ kN}$$

Section Properties. The cross-sectional area and moment of inertia of BD are

$$A = \frac{\pi}{4}d^2 \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64}d^4$$

Critical Buckling Load. Since BD is pinned at both of its ends, $K = 1$. The critical buckling load is

$$P_{cr} = F_{BD}(\text{F.S.}) = 187.5(2) = 375 \text{ kN}$$

The length of BD is $L = \sqrt{0.9^2 + 1.2^2} = 2.5$ m. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$375(10^3) = \frac{\pi^2[200(10^9)]\left(\frac{\pi}{64}d^4\right)}{[1.0(1.5)]^2}$$

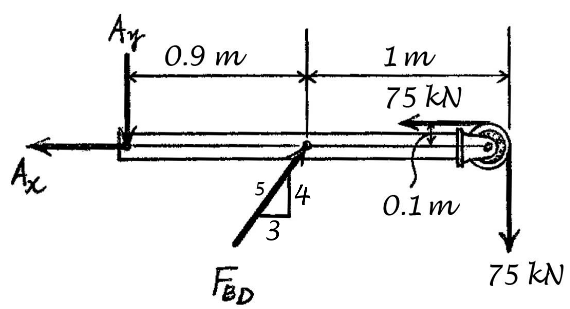
$$d = 0.05432 \text{ m} = 54.32 \text{ mm}$$

Use $d = 55$ mm **Ans.**

Critical Stress. Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{375(10^3)}{\frac{\pi}{4}(0.055^2)} = 157.84(10^6) \text{ N/m}^2$$

$$= 157.85 \text{ MPa} < \sigma_Y = 345 \text{ MPa} \qquad \text{(O.K.)}$$



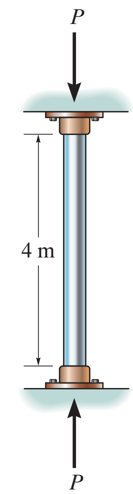
(a)

Ans:

Use $d = 55$ mm

R13–7.

The steel pipe is fixed supported at its ends. If it is 4 m long and has an outer diameter of 50 mm, determine its required thickness so that it can support an axial load of $P = 100$ kN without buckling. $E_{st} = 200$ GPa, $\sigma_Y = 250$ MPa.



SOLUTION

$$I = \frac{\pi}{4}(0.025^4 - r_i^{\,4})$$

Critical Load:

$$P_{cr} = \frac{\pi^2 EI}{(K\,L)^2}; \qquad K = 0.5$$

$$100(10^3) = \frac{\pi^2(200)(10^9)[\frac{\pi}{4}(0.025^4 - r_i^{\,4})]}{[0.5(4)]^2}$$

$$r_i = 0.01908 \text{ m} = 19.1 \text{ mm}$$

$$t = 25 \text{ mm} - 19.1 \text{ mm} = 5.92 \text{ mm}$$ **Ans.**

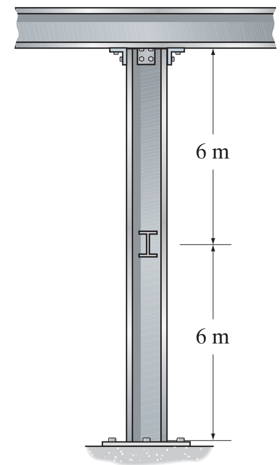
Check Stress:

$$\sigma = \frac{P_{cr}}{A} = \frac{100(10^3)}{\pi(0.025^2 - 0.0191^2)} = 122 \text{ MPa} < \sigma_Y = 345 \text{ MPa}$$ **(OK)**

Ans:
$t = 5.92$ mm

***R13–8.**

The W200 × 46 wide-flange A992-steel column can be considered pinned at its top and fixed at its base. Also, the column is braced at its mid-height against weak axis buckling. Determine the maximum axial load the column can support without causing it to buckle.



SOLUTION

Section Properties: From the table listed in the appendix, the section properties for a W200 × 46 are

$A = 5890\ \text{mm}^2 = 5.89\left(10^{-3}\right)\ \text{m}^2 \qquad I_x = 45.5\left(10^{6}\right)\ \text{mm}^4 = 45.5\left(10^{-6}\right)\ \text{m}^4$

$I_y = 15.3\left(10^{6}\right)\ \text{mm}^4 = 15.3\left(10^{-6}\right)\ \text{m}^4$

Critical Buckling Load: For buckling about the strong axis, $K_x = 0.7$ and $L_x = 12$ m. Since the column is fixed at its base and pinned at its top,

$$P_{cr} = \frac{\pi^2 E I_x}{(KL)_x^{\,2}} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[45.5\left(10^{-6}\right)\right]}{[0.7(12)]^2} = 1.273\left(10^6\right)\text{N} = 1.27\ \text{MN}$$

For buckling about the weak axis, $K_y = 1$ and $L_y = 6$ m since the bracing provides a support equivalent to a pin. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)_y^{\,2}} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[15.3\left(10^{-6}\right)\right]}{[1(6)]^2} = 838.92\ \text{kN} = 839\ \text{kN (controls)}$$

Ans.

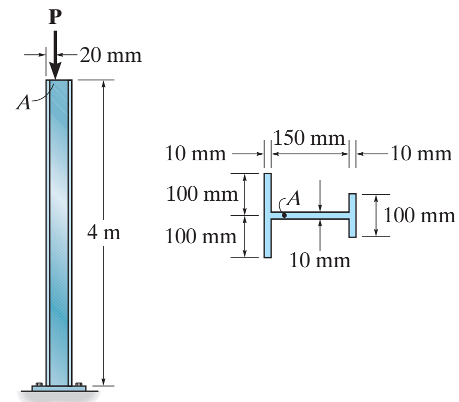
Critical Stress: Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{838.92\left(10^3\right)}{5.89\left(10^{-3}\right)} = 142.43\ \text{MPa} < \sigma_Y = 345\ \text{MPa} \qquad \textbf{(O.K.)}$$

Ans:
$P_{cr} = 839$ kN

R13–9.

The wide-flange A992 steel column has the cross section shown. If it is fixed at the bottom and free at the top, determine the maximum force P that can be applied at A without causing it to buckle or yield. Use a factor of safety of 3 with respect to buckling and yielding.



SOLUTION

Section Properties:

$$\Sigma A = 0.2(0.01) + 0.15\,(0.01) + 0.1(0.01) = 4.5(10^{-3})\ \text{m}^2$$

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{0.005\,(0.2)(0.01) + 0.085\,(0.15)(0.01) + 0.165\,(0.1)(0.01)}{4.5(10^{-3})}$$

$$= 0.06722\ \text{m}$$

$$I_y = \frac{1}{12}(0.2)(0.01^3) + 0.2\,(0.01)(0.06722 - 0.005)^2$$

$$+ \frac{1}{12}(0.01)(0.15^3) + 0.01\,(0.15)(0.085 - 0.06722)^2$$

$$+ \frac{1}{12}(0.1)(0.01^3) + 0.1\,(0.01)(0.165 - 0.06722)^2$$

$$= 20.615278\,(10^{-6})\ \text{m}^4$$

$$I_x = \frac{1}{12}(0.01)(0.2^3) + \frac{1}{12}(0.15)(0.01^3) + \frac{1}{12}(0.01)(0.1^3)$$

$$= 7.5125\,(10^{-6})\ \text{m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{20.615278(10^{-6})}{4.5\,(10^{-3})}} = 0.0676844$$

Buckling about x–x axis:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(7.5125)(10^{-6})}{[2.0(4)]^2}$$

$$= 231.70\ \text{kN} \qquad \text{(controls)}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{231.7\,(10^3)}{4.5\,(10^{-3})} = 51.5\ \text{MPa} < \sigma_y = 345\ \text{MPa}$$

Yielding about y–y axis:

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{e\,c}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]; \qquad e = 0.06722 - 0.02 = 0.04722\ \text{m}$$

$$\frac{e\,c}{r^2} = \frac{0.04722\,(0.06722)}{0.0676844^2} = 0.692919$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{2.0\,(4)}{2(0.0676844)}\sqrt{\frac{P}{200\,(10^9)(4.5)(10^{-3})}} = 1.96992\,(10^{-3})\sqrt{P}$$

$$345(10^6)(4.5)(10^{-3}) = P[1 + 0.692919\sec(1.96992\,(10^{-3})\sqrt{P})]$$

By trial and error:

$$P = 434.342\ \text{kN}$$

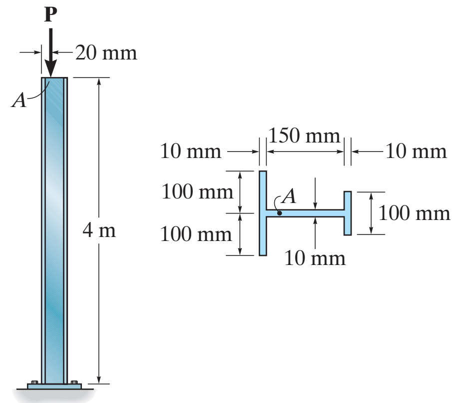
Hence,

$$P_{allow} = \frac{231.70}{3} = 77.2\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$P_{allow} = 77.2$ kN

R13–10.

The wide-flange A992 steel column has the cross section shown. If it is fixed at the bottom and free at the top, determine if the column will buckle or yield when the load $P = 10$ kN is applied at A. Use a factor of safety of 3 with respect to buckling and yielding.



SOLUTION

Section Properties:

$$\Sigma A = 0.2\,(0.01) + 0.15\,(0.01) + 0.1\,(0.01) = 4.5\,(10^{-3})\ \text{m}^2$$

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{0.005\,(0.2)(0.01) + 0.085\,(0.15)(0.01) + 0.165\,(0.1)(0.01)}{4.5\,(10^{-3})} = 0.06722\ \text{m}$$

$$I_y = \frac{1}{12}(0.2)(0.01^3) + 0.2\,(0.01)(0.06722 - 0.005)^2$$

$$+ \frac{1}{12}(0.01)(0.15^3) + 0.01\,(0.15)(0.085 - 0.06722)^2$$

$$+ \frac{1}{12}(0.1)(0.01^3) + 0.1\,(0.01)(0.165 - 0.06722)^2 = 20.615278\,(10^{-6})\ \text{m}^4$$

$$I_x = \frac{1}{12}(0.01)(0.2^3) + \frac{1}{12}(0.15)(0.01^3) + \frac{1}{12}(0.01)(0.1^3) = 7.5125\,(10^{-6})\ \text{m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{20.615278\,(10^{-6})}{4.5\,(10^{-3})}} = 0.0676844\ \text{m}$$

Buckling about *x–x* axis:

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(7.5125)(10^{-6})}{[2.0(4)]^2} = 231.70\ \text{kN}$$

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{231.7\,(10^3)}{4.5\,(10^{-3})} = 51.5\ \text{MPa} < \sigma_\gamma = 345\ \text{MPa} \qquad \textbf{(O.K.)}$$

$$P_{\text{allow}} = \frac{P_{\text{cr}}}{\text{FS}} = \frac{231.7}{3} = 77.2\ \text{kN} > P = 10\ \text{kN}$$

Hence the column does not buckle.

Yielding about *y–y* axis:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{e\,c}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right] \qquad e = 0.06722 - 0.02 = 0.04722\ \text{m}$$

$$P = \frac{10}{3} = 3.333\ \text{kN}$$

$$\frac{P}{A} = \frac{3.333\,(10^3)}{4.5\,(10^{-3})} = 0.7407\ \text{MPa}$$

$$\frac{e\,c}{r^2} = \frac{0.04722\,(0.06722)}{(0.067844)^2} = 0.692919$$

$$\frac{K\,L}{2\,r}\sqrt{\frac{P}{E\,A}} = \frac{2.0\,(4)}{2(0.0676844)}\sqrt{\frac{3.333\,(10^3)}{200\,(10^9)(4.5)(10^{-3})}} = 0.113734$$

$$\sigma_{\max} = 0.7407\,[1 + 0.692919\sec(0.113734)] = 1.26\ \text{MPa} < \sigma_\gamma = 345\ \text{MPa}$$

Hence the column does not yield!

No. **Ans.**

Ans:
It does not buckle or yield