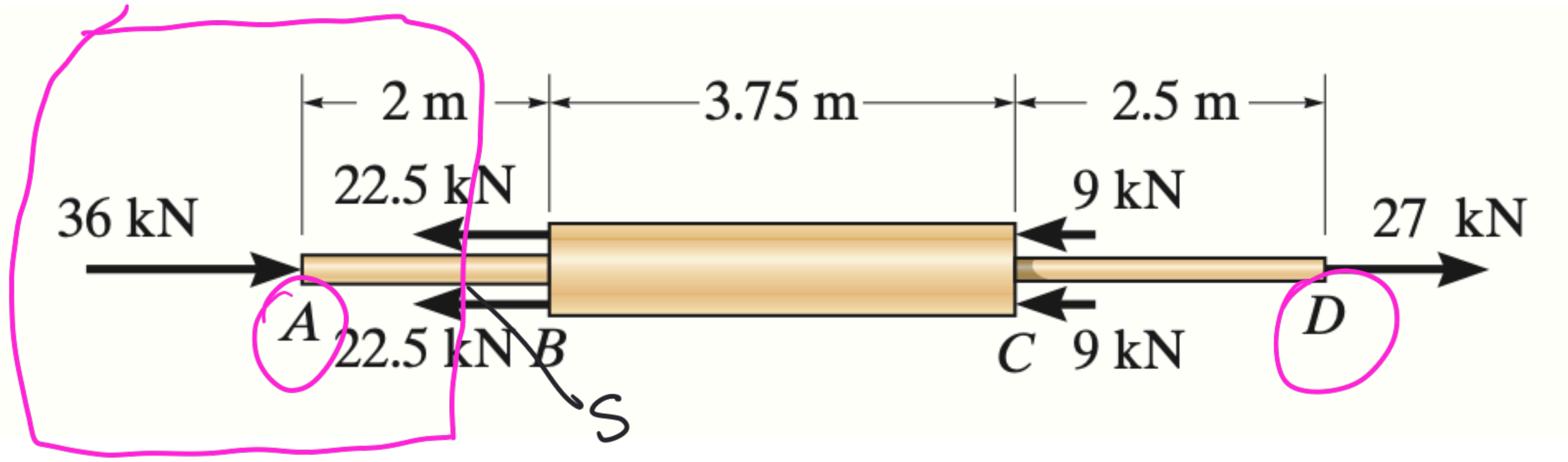
Riferimenti: Cap. 8 Hibbeler

Parzialmente: Capp. 8 e 9 Casini & Vasta.

- ESEMPI SU DEF. ESTENSIONALI
  SOVRAPPOSIZIONE
- TRAVI CARICATE ASSIALMENTE STATICAMENTE INDETERMINATE (METODO DELLE FORZE)
- TENSIONI DOVUTE A VARIAZIONI DI TEMPERATURA



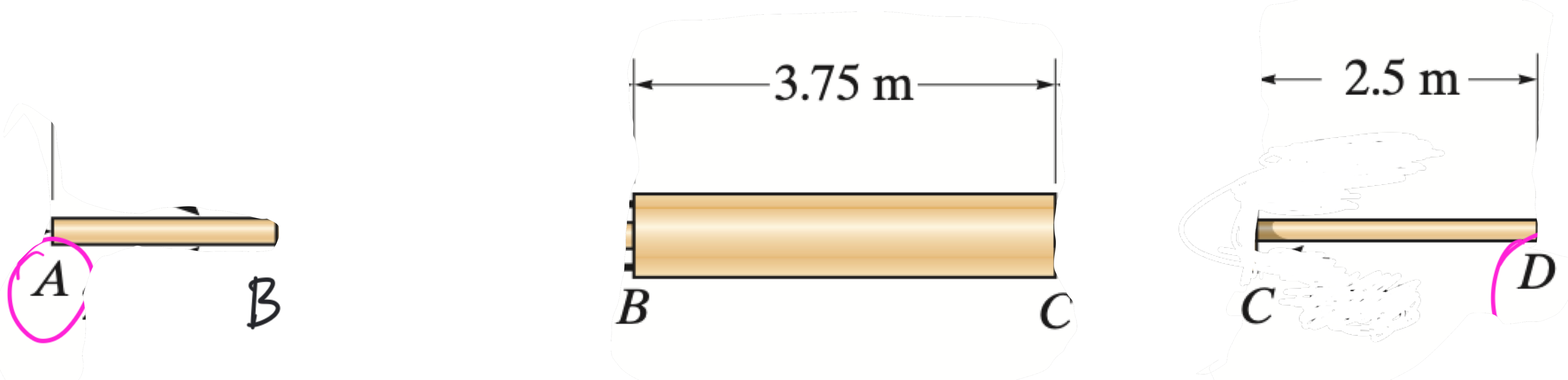
$d_{AB} = 20\,mm$

$d_{BC} = 25\,mm$

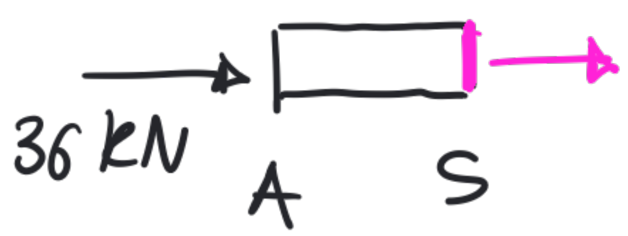
$d_{DC} = 12\,mm$

Determinare l'allungamento δ_{AD} della barra

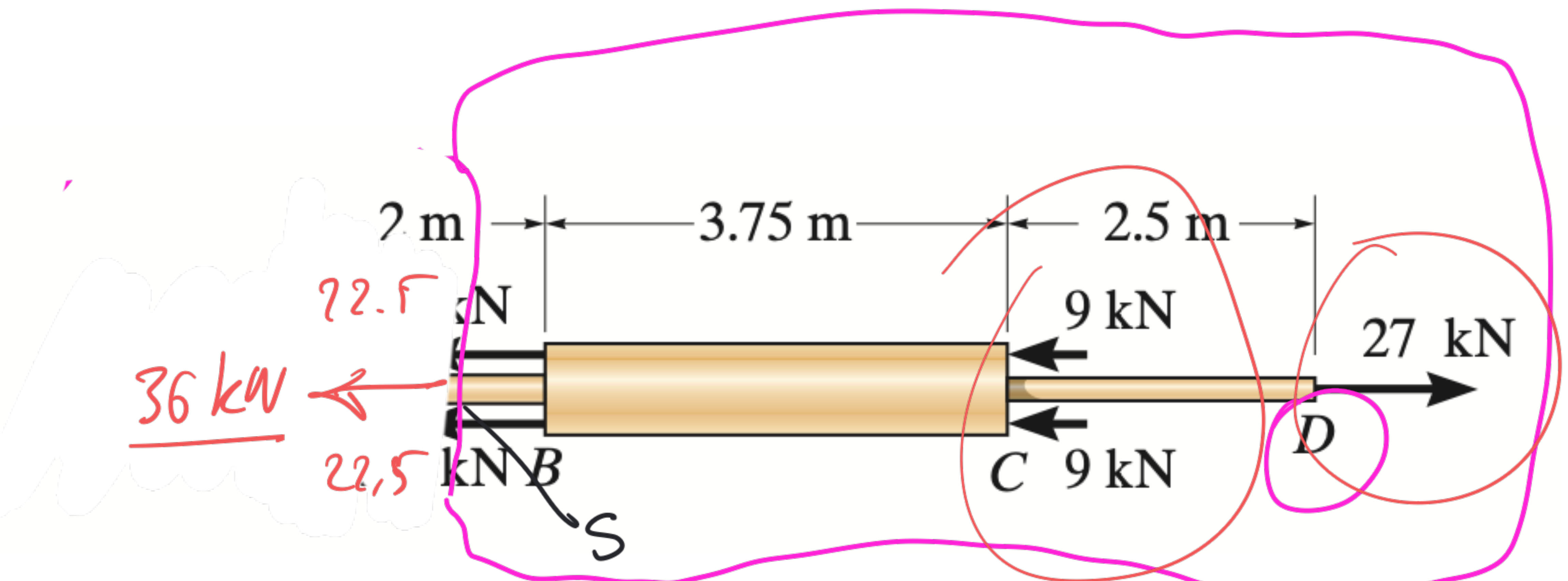
$E = 126\,GPa$



$N_{AB} \leftarrow$ metodo delle sezioni



$N_{AB} = -36\,kN$ (indipendente da S, purché S sia compreso tra A e B)



$d_{AB} = 20\,mm$

$d_{BC} = 25\,mm$

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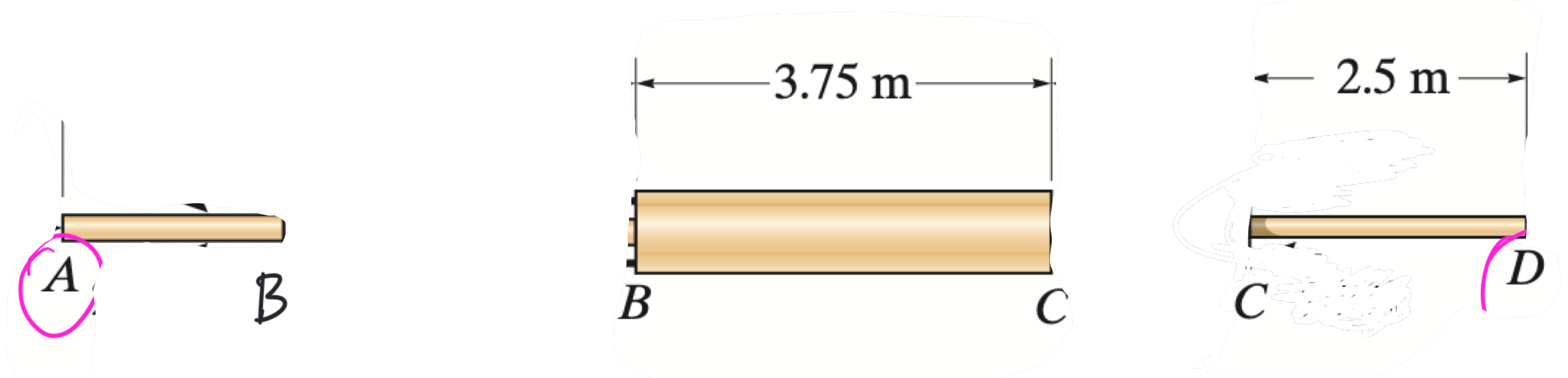
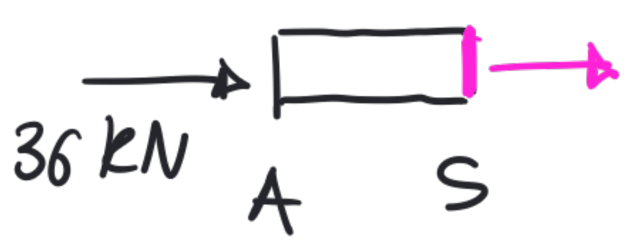
Determinare l'allungamento δ_{AD} della barra

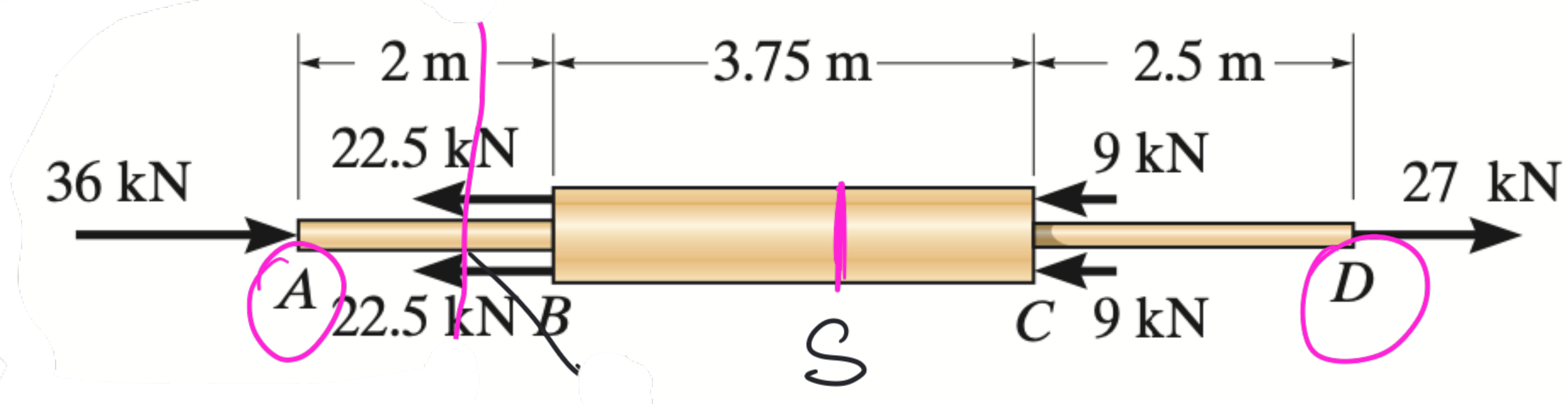
$E = 126\,GPa$



N_{AB} ← metodo delle sezioni



$N_{AB} = -36\,kN$ (indipendente da S, purché S sia compreso tra A e B)



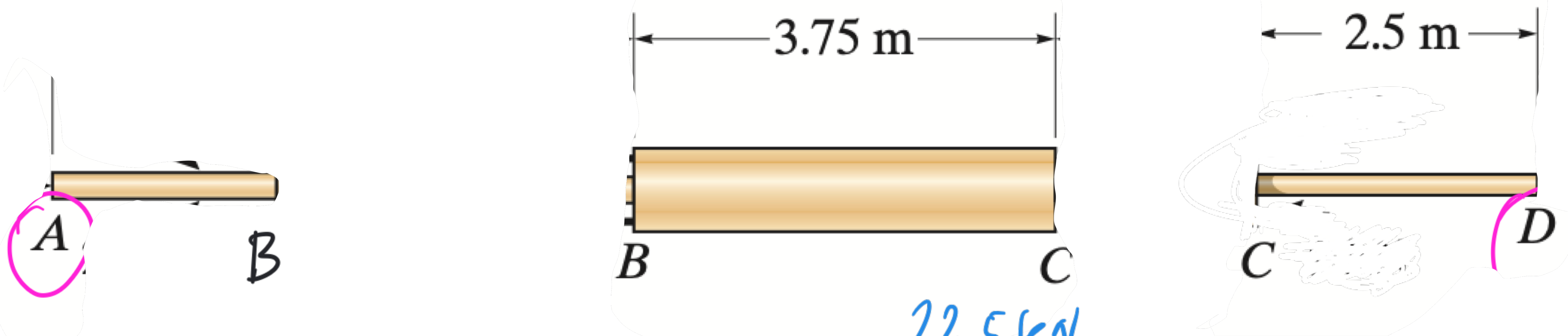
$d_{AB} = 20\ mm$

$d_{BC} = 25\ mm$

$d_{DC} = 12\ mm$

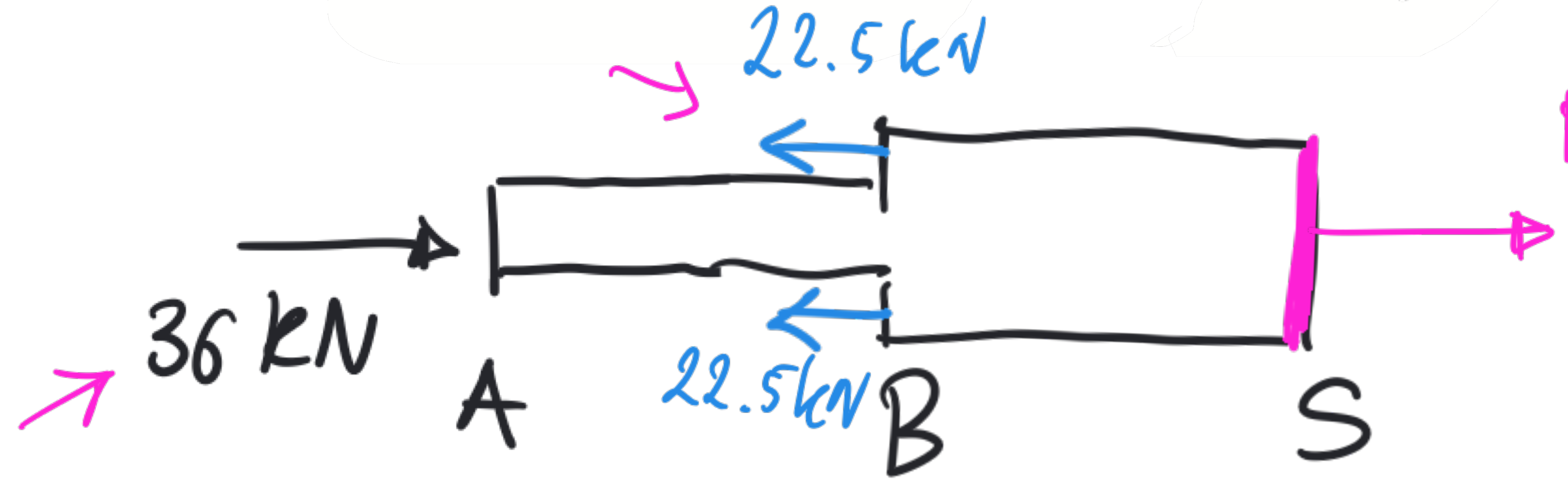
Determinare l'allungamento δ_{AD} della barra

$E = 126\ GPa$

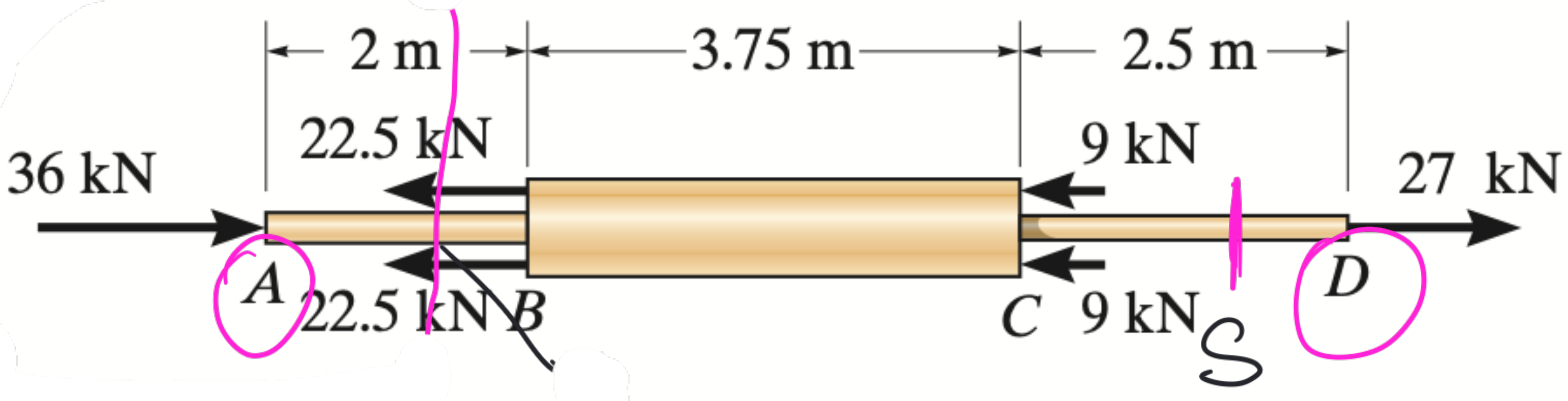


$N_{AB} = -36\ kN$

$N_{BC} = 9\ kN$



$N = (-36 + 45)\ kN$
$= 9\ kN$



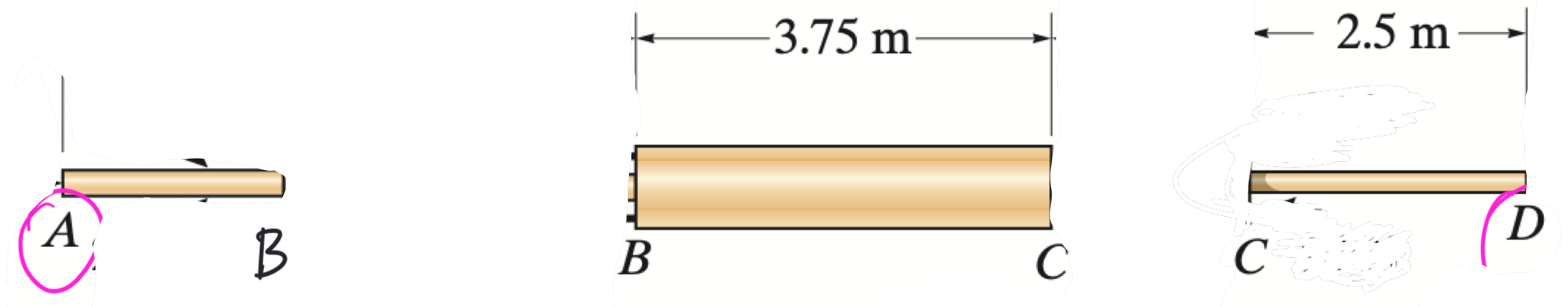
$d_{AB} = 20\,mm$

$d_{BC} = 25\,mm$

$d_{DC} = 12\,mm$

Determinare l'allungamento δ_{AD} della barra

$E = 126\ GPa$



$N_{AB} = -36\ kN$

$N_{BC} = 9\ kN$

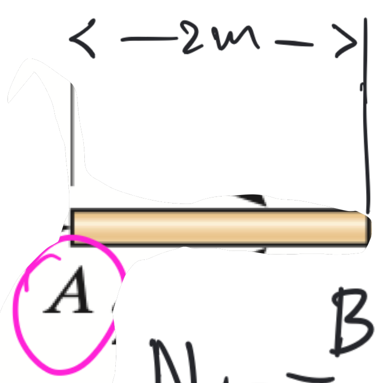
$N_{CD} = 27\ kN$

$d_{BC} = \ldots$

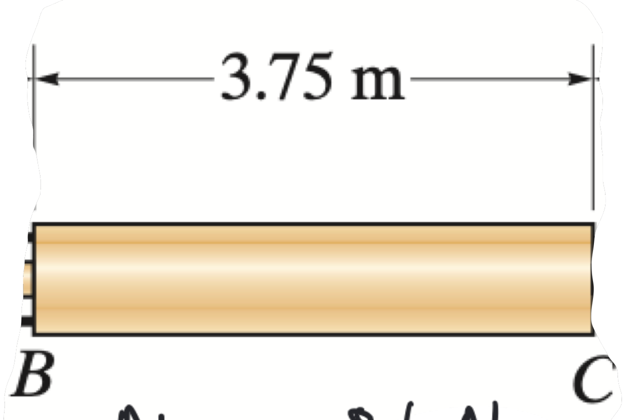
$d_{DC} = 12\ mm$

Determinare l'allungamento δ_{AD} della barra

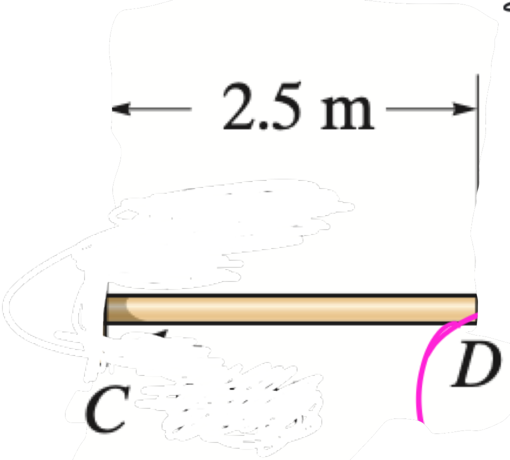
$E = 126\ GPa$



$N_{AB} = -36\ kN$



$N_{BC} = 8\ kN$



$N_{CD} = 27\ kN$

$$\varepsilon_{AB} = \frac{l_{AB} - 2m}{2m} = \frac{\delta_{AB}}{2m}$$

Legge di Hooke $\sigma_{AB} = E\varepsilon_{AB}$ -

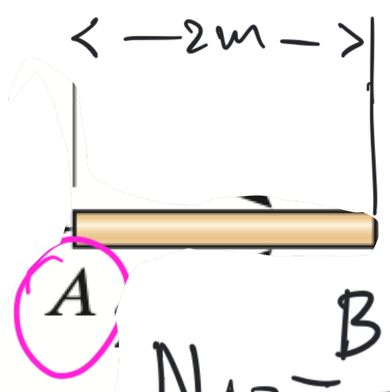
$$\sigma_{AB} = \frac{N_{AB}}{A_{AB}} \Rightarrow N_{AB} = EA\,\varepsilon_{AB}$$

$d_{BC} = 25$ mm

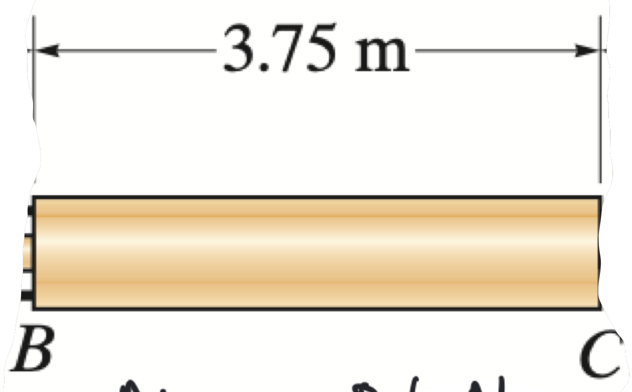
$d_{DC} = 12$ mm

Determinare l'allungamento δ_{AD} della barra

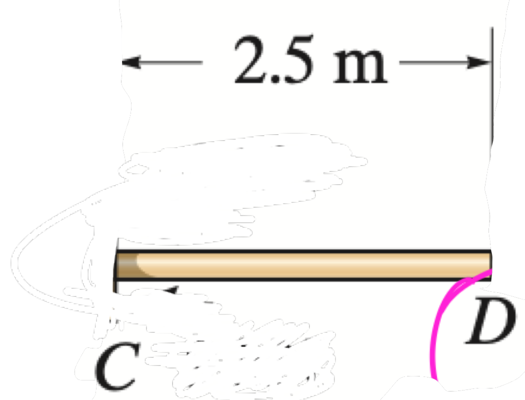
$E = 126$ GPa



$N_{AB} = -36$ kN

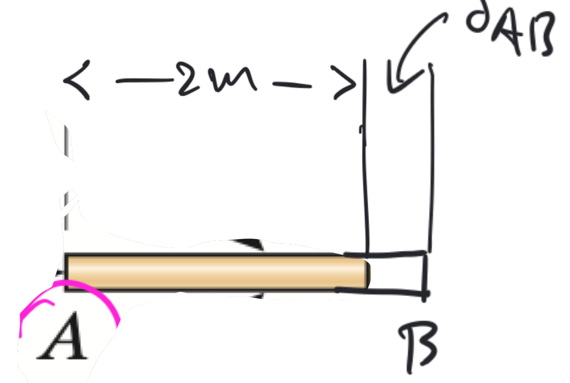


$N_{BC} = 9$ kN

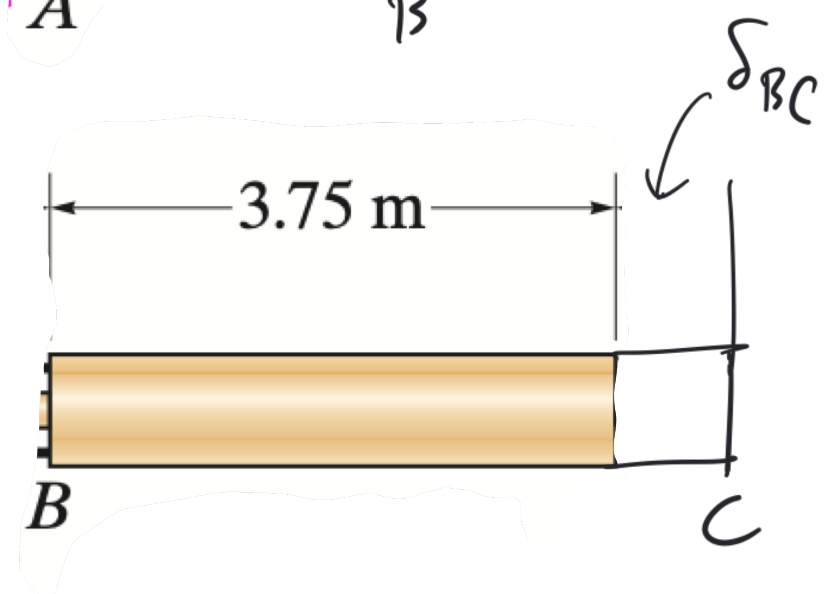


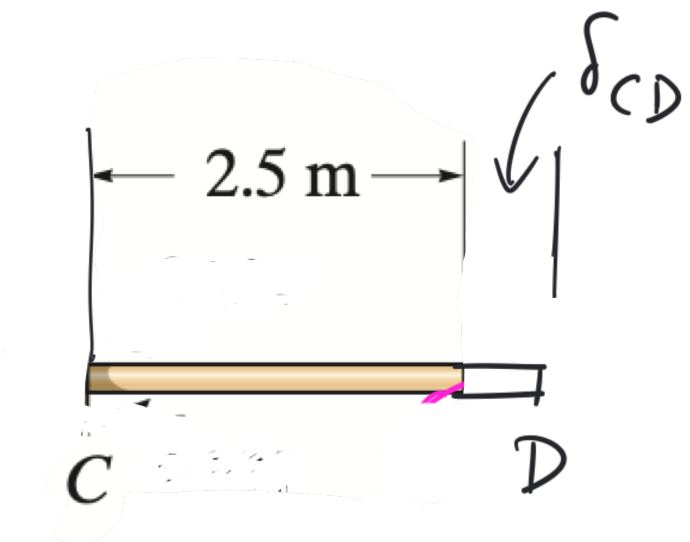
$N_{CD} = 27$ kN

$$\delta_{AB} = L_{AB} \frac{N_{AB}}{EA_{AB}}$$



$$\delta_{BC} = L_{BC} \frac{N_{BC}}{EA_{BC}}$$





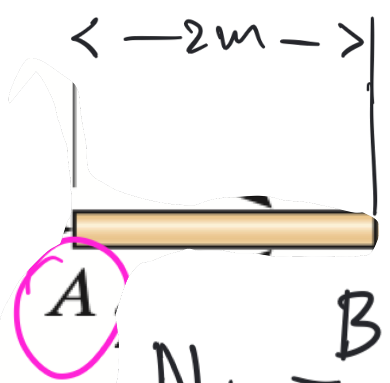
$$\delta_{CD} = L_{CD} \frac{N_{CD}}{EA_{CD}}$$

$d_{BC} = 25\ mm$

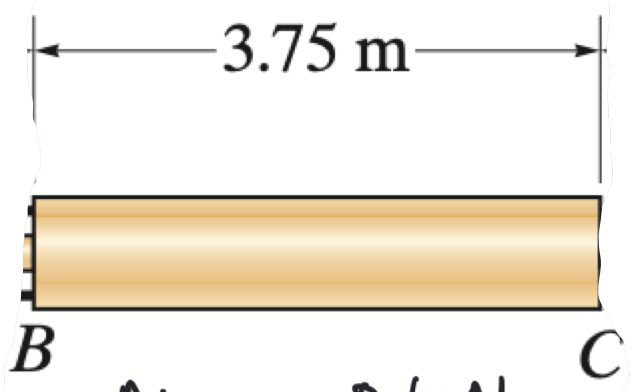
$d_{DC} = 12\ mm$

Determinare l'allungamento δ_{AD} della barra

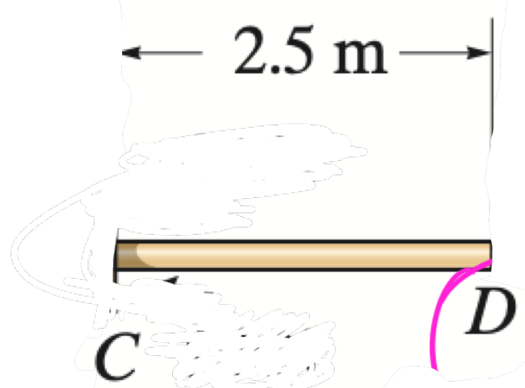
$E = 126\ GPa$



$N_{AB} = -36\ kN$



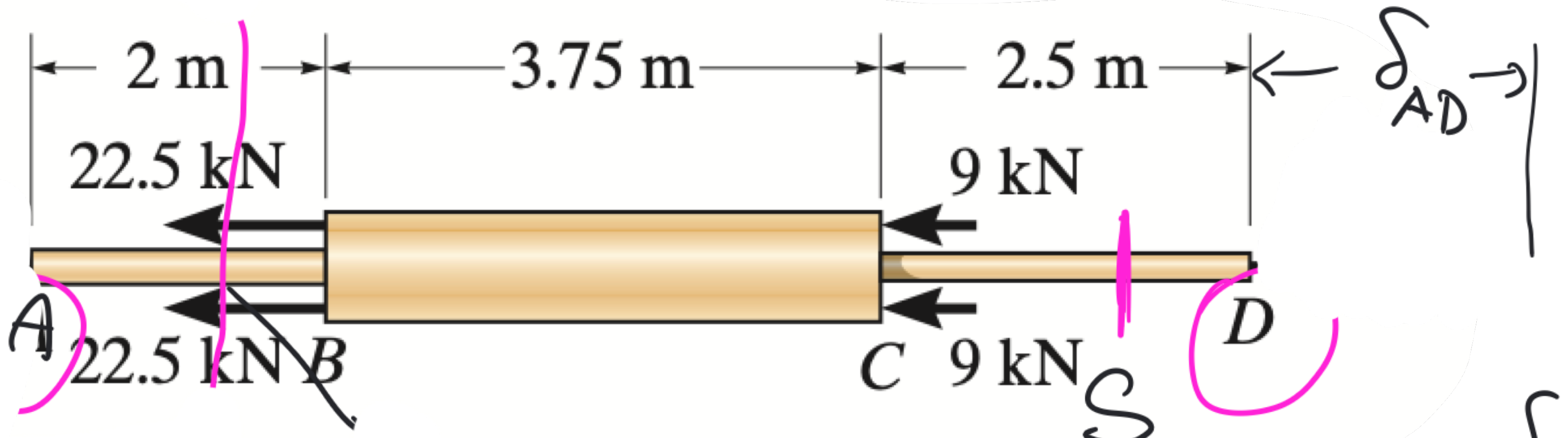
$N_{BC} = 9\ kN$



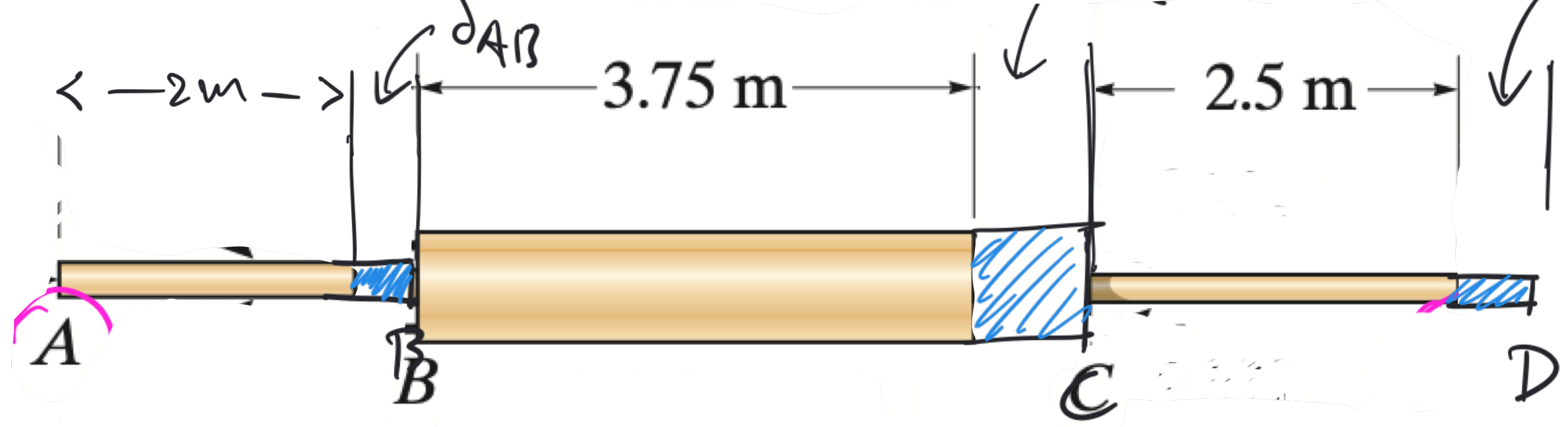
$\delta_{AB} + \delta_{BC} + \delta_{CD}$

$\parallel$

δ_{AD}



$$\boxed{\delta_{AD} = L_{AB}\frac{N_{AB}}{EA_{AB}} + L_{BC}\frac{N_{BC}}{EA_{BC}} + L_{CD}\frac{N_{CD}}{EA_{CD}}}$$



In sintesi:

1) calcoliamo N_i su ciascun tratto. (equilibrio)

2) Legge di Hooke $\sigma_i = E_i \varepsilon_i$

3) Forza normale $N_i = A_i \sigma_i$

$\Rightarrow N_i = E_i A_i \varepsilon_i$

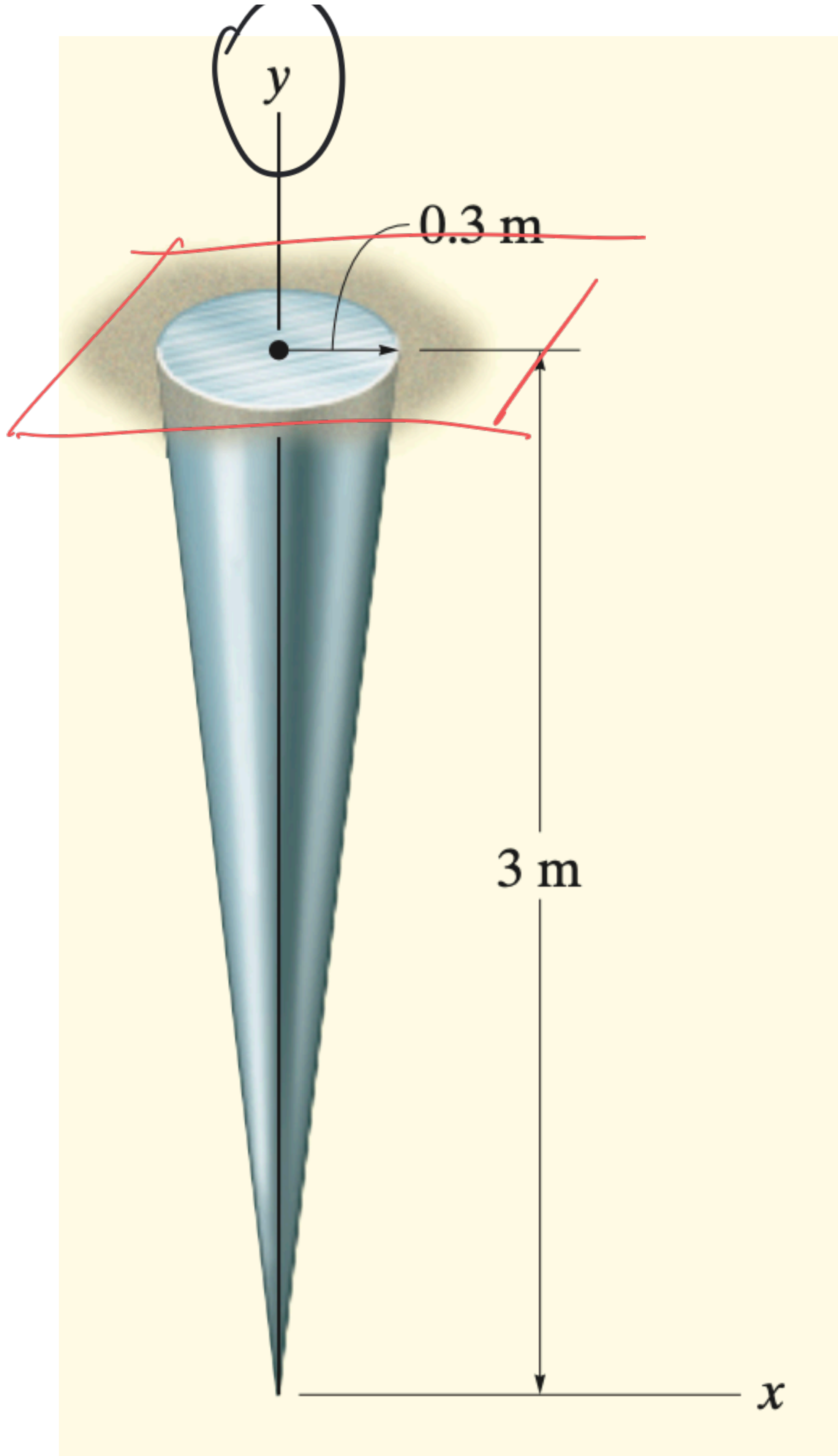
equazione costitutiva

4) Definizione di ε $\quad \varepsilon_i = \delta_i / L_{0i}$

$$\delta_i = \frac{N_i}{E_i A_i} L_{0i}$$

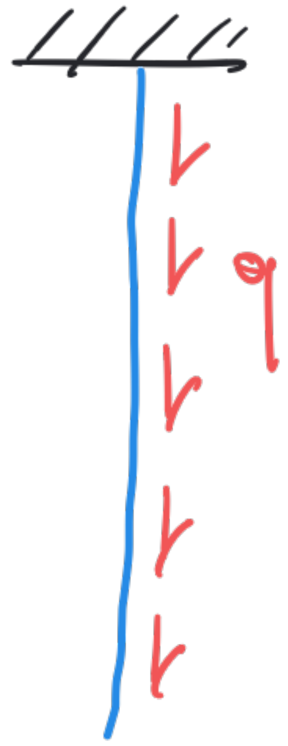
$$\delta = \sum_{i=1}^{N} \delta_i = \sum_{i=1}^{N} \frac{N_i L_{0i}}{E_i A_i}$$

N = numero di tratti (con E, A, N costanti)



Determinare δ

Schema strutturale



peso specifico $\gamma = 6\ kN/m$

$E = 9\ GPa$

(piccolo!)

Piano:

1) calcolare N/σ

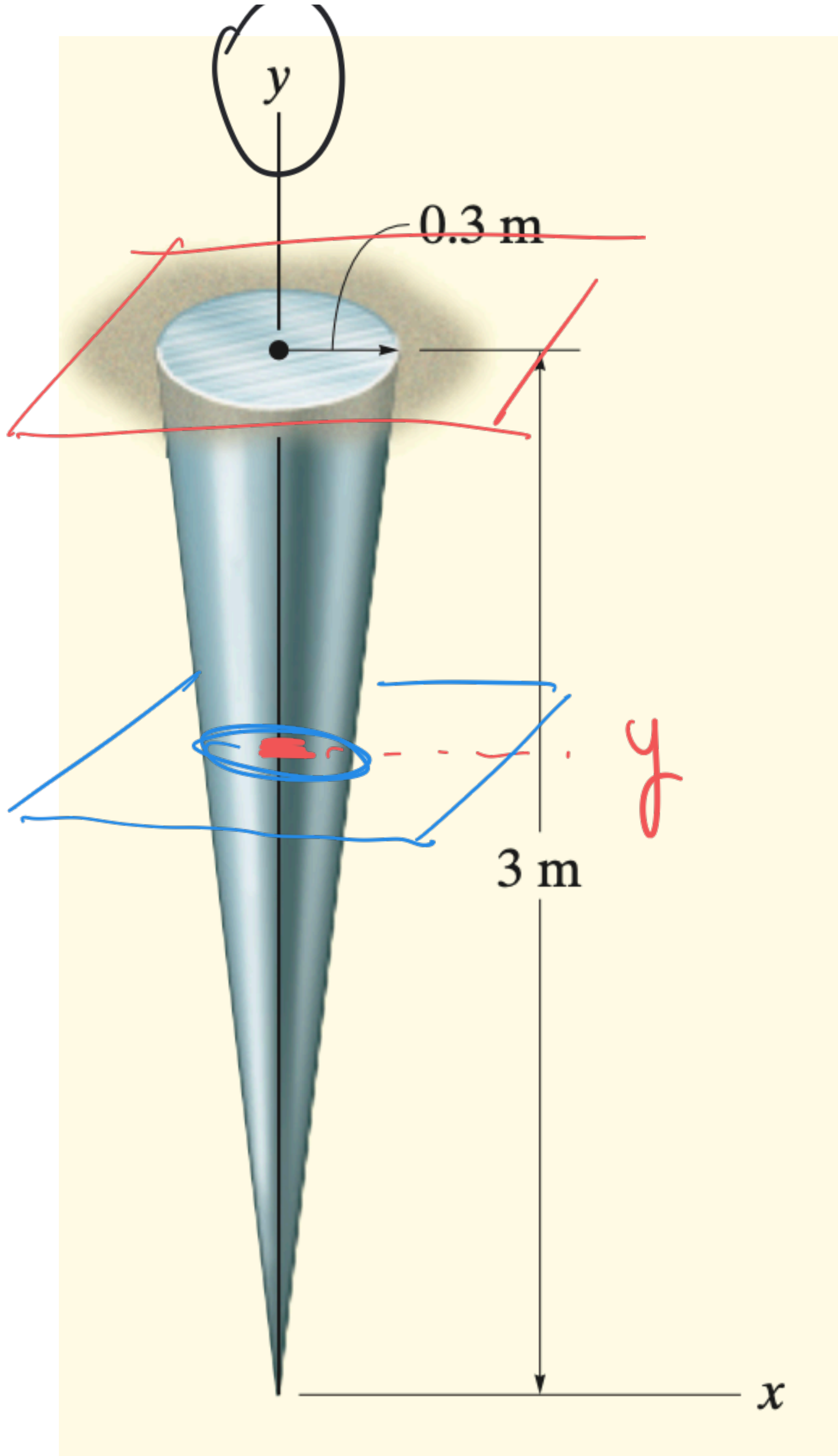
$N = A\sigma$

ipotesi $\rightarrow$ $\sigma \simeq$ costante su ogni sezione

2) Hooke $\sigma = E\varepsilon$

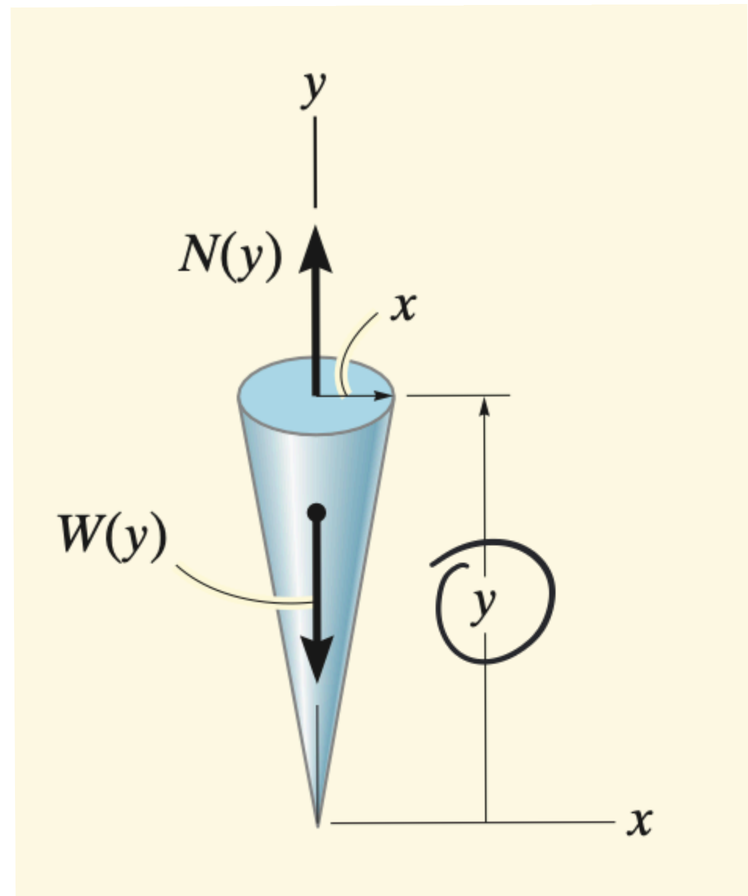
$\Rightarrow \varepsilon$ (dilatazione lungo y)

3) $\varepsilon \rightarrow \delta$



Determinare δ

Metodo delle sezioni



$$W(y) = \gamma \frac{\pi}{3} y x^2 = \gamma \frac{\pi}{3} (0.01) y^3$$

$$x/y = \frac{0.3\,m}{3\,m} \Rightarrow x = 0.1\, y$$

peso specifico $\gamma = 6\ kN/m$

$E = 9\ GPa$

(piccolo!)

Piano:

1) $\boxed{\text{calcolare } N/\sigma}$

$N = A\sigma$

ipotesi $\rightarrow$ $\sigma \simeq$ costante su ogni sezione

2) Hooke $\sigma = E\varepsilon$

$\Rightarrow \varepsilon$ (dilatazione lungo y)

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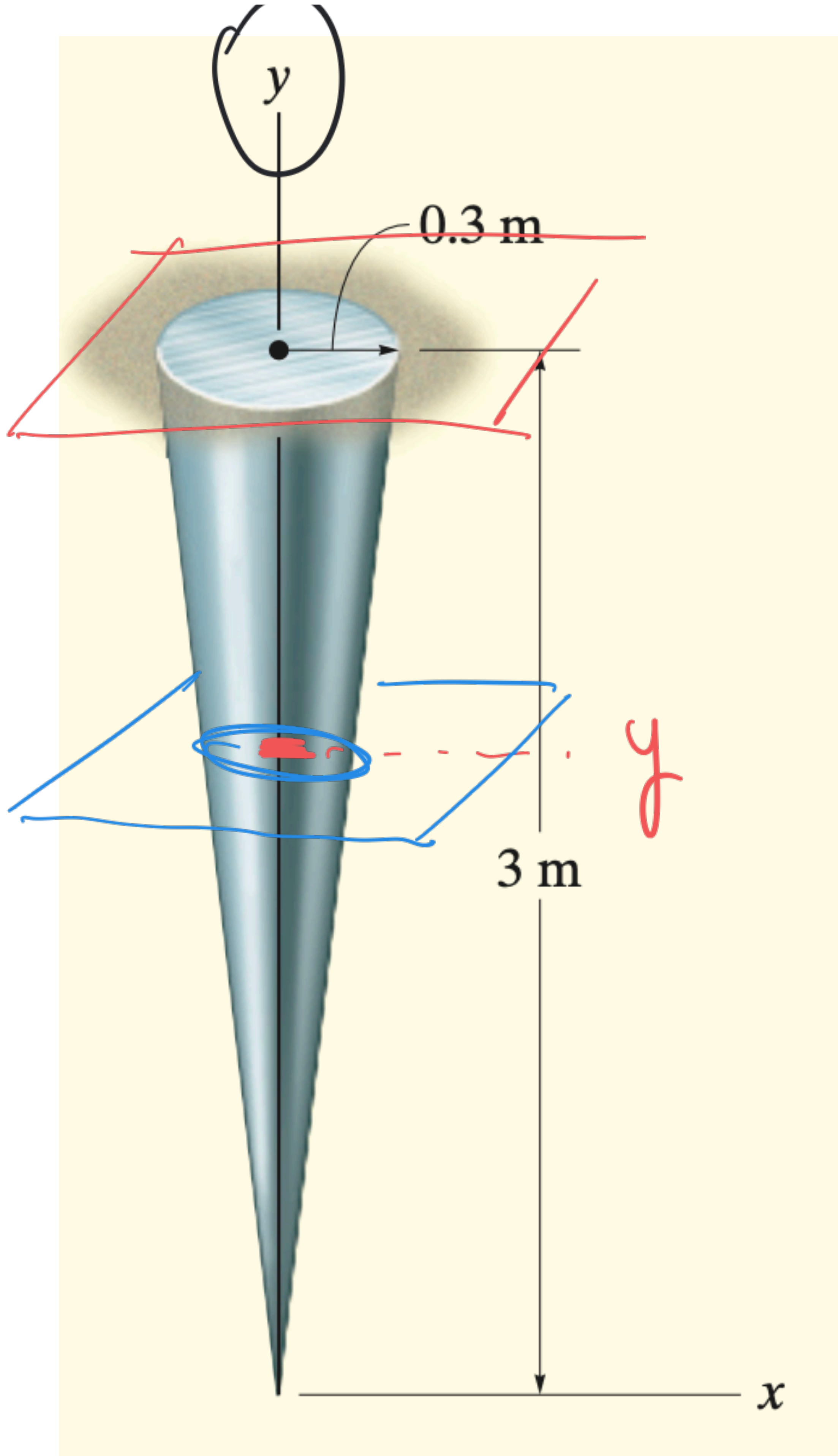


Determinare δ

Metodo delle sezioni

$$N(y) = \gamma \frac{\pi}{3}(0.01)\, y^3$$

$$\varepsilon(y) = \frac{N(y)}{A(y)\, E}$$

$\delta = ??$

$$\delta = \sum_{i=1}^{N} \delta_i = \sum_{i=1}^{N} \varepsilon_i L_i$$

$$\varepsilon = \delta / L$$

peso specifico $\gamma = 6\ kN/m$

$E = 9\ GPa$

(piccolo!)

Piano:

1) $\boxed{\text{calcolare } N/\sigma}$

$N = A\sigma$

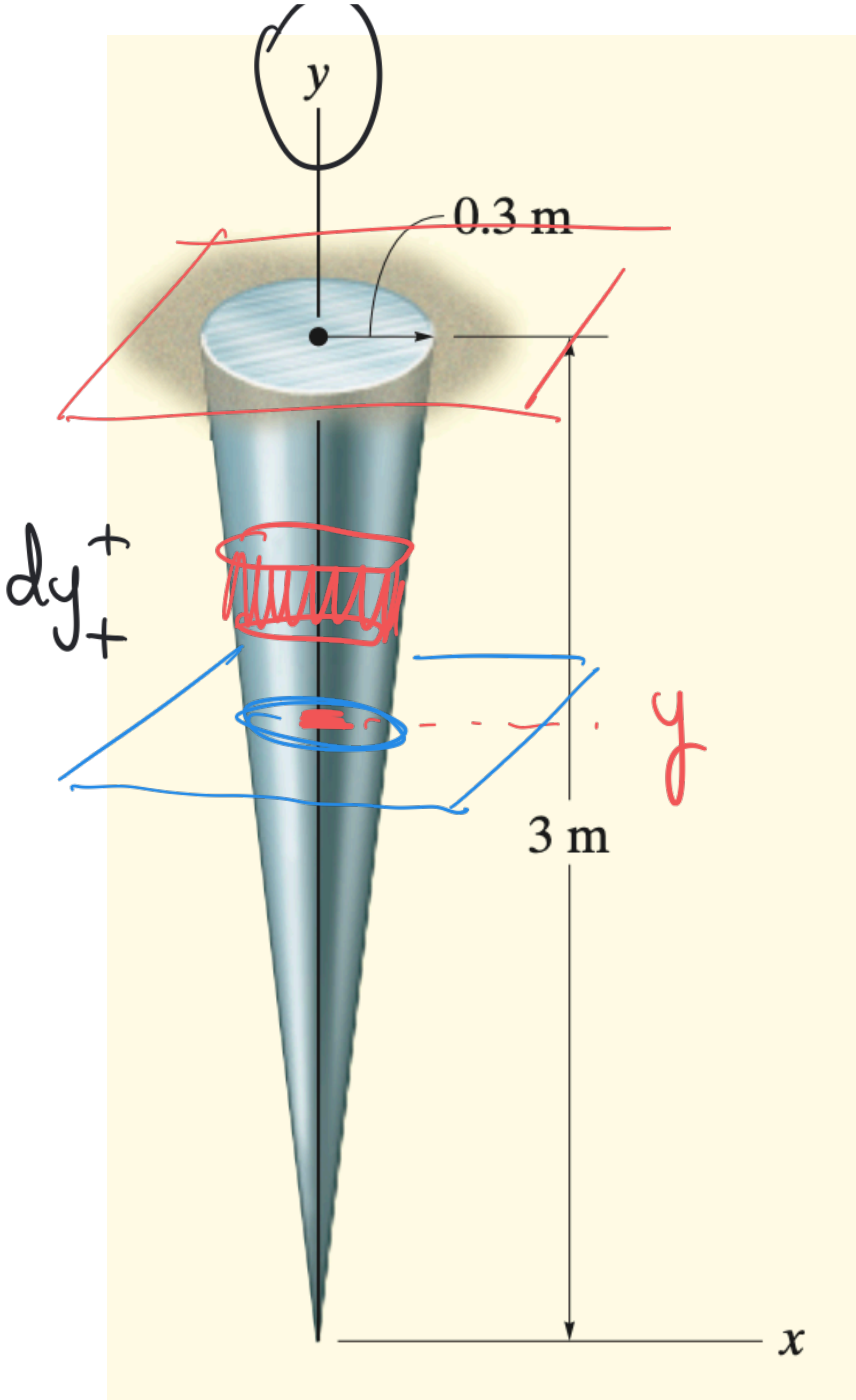
ipotesi $\rightarrow$ $\sigma \simeq$ costante su ogni sezione

2) Hooke $\sigma = E\varepsilon$

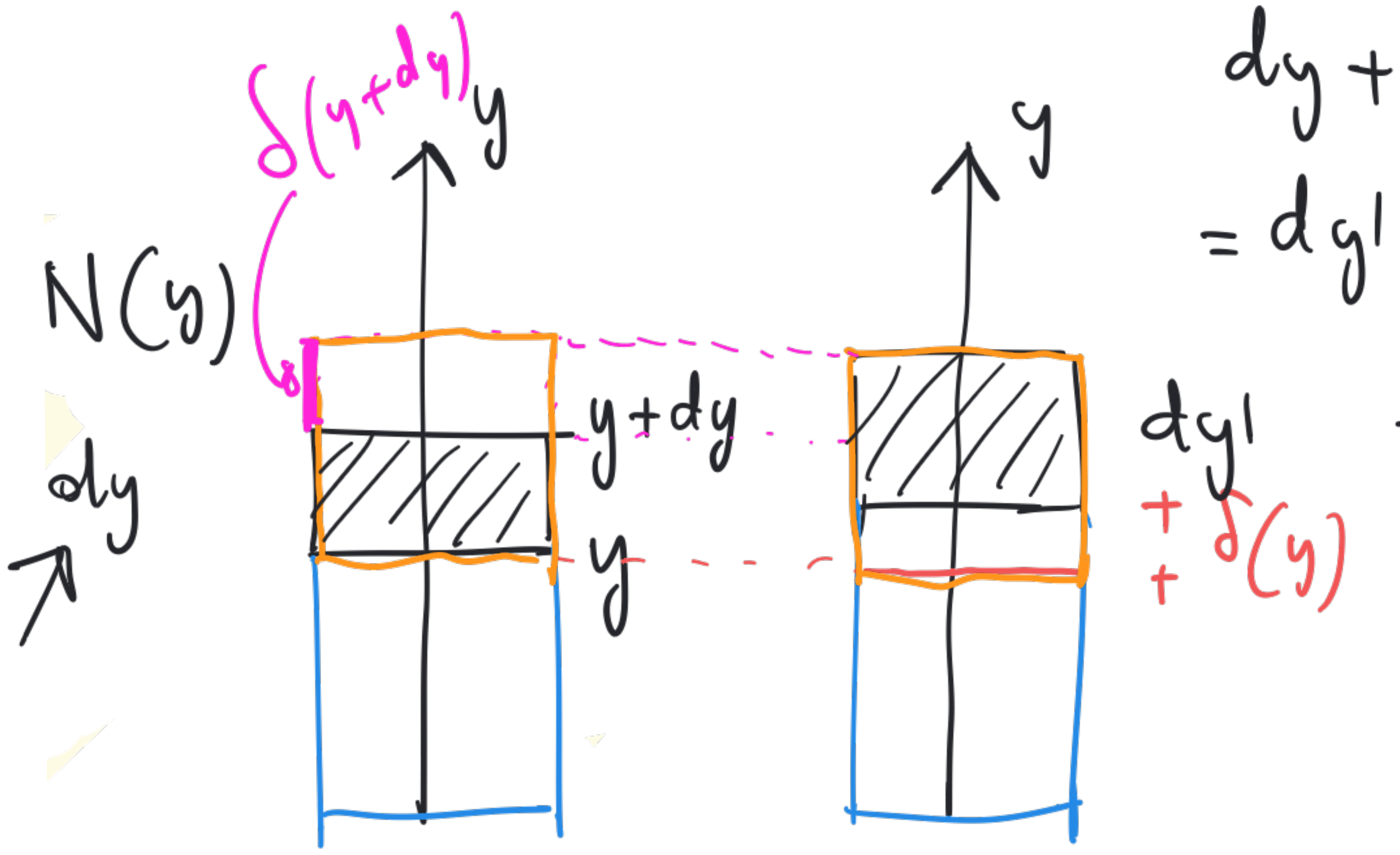
$\Rightarrow$ ε (dilatazione lungo y)

3) $\varepsilon \rightarrow \delta$

$$\delta = \int_0^h \varepsilon(y)\, dy$$



Determinare δ



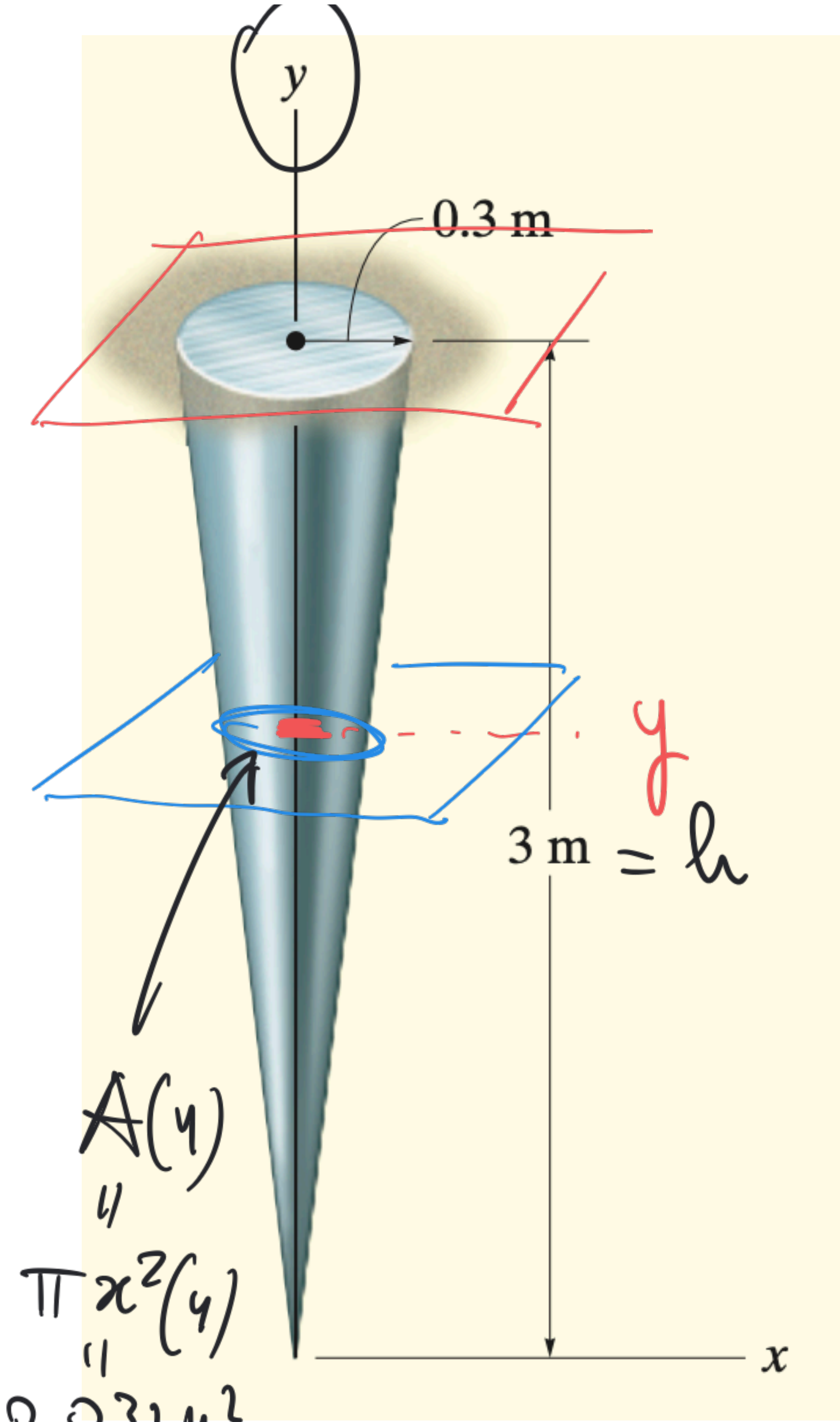
$$dy + \delta(y+dy) = dy' + \delta(y)$$

$$\Rightarrow dy' - dy = \delta(y+dy) - \delta(y)$$

$\delta(y)$ = allungamento del tratto $(0, y)$

$\delta(y+dy) - \delta(y)$ = allungamento del tratto dy

$$\varepsilon(y) = \frac{\delta(y+dy) - \delta(y)}{dy} = \frac{d\delta}{dy} \qquad \delta(0) = 0 \qquad \delta(y) = \int_0^y \varepsilon(\bar{y})\, d\bar{y}$$



$A(y)$
$\parallel$
$\pi x^2(y)$
$\parallel$
$0.031\, y^2$

Determinare δ

Metodo delle sezioni

$$N(y) = \gamma \frac{\pi}{3}(0.01)\, y^3$$

$$\varepsilon(y) = \frac{N(y)}{A(y)E}$$

$$\delta(y) = \int_0^y \frac{N(\bar{y})}{A(\bar{y})E}\, d\bar{y}$$

$$\delta = \int_0^h \frac{N(\bar{y})}{A(\bar{y})E}\, d\bar{y}$$

$$= \frac{\gamma\, \pi/3\, (0.01)}{(0.031)\ 9\,\text{GPa}} \int_0^h \frac{\cancel{\bar{y}^3}}{\cancel{\bar{y}^2}}\, d\bar{y}$$

$$= 1\,\mu m.$$

Piano:

1) $\boxed{\text{calcolare } N / \sigma}$

$N = A\sigma$

ipotesi $\rightarrow$ $\sigma \simeq$ costante su ogni sezione

2) Hooke $\sigma = E\varepsilon$

$\Rightarrow \varepsilon$ (dilatazione lungo y)

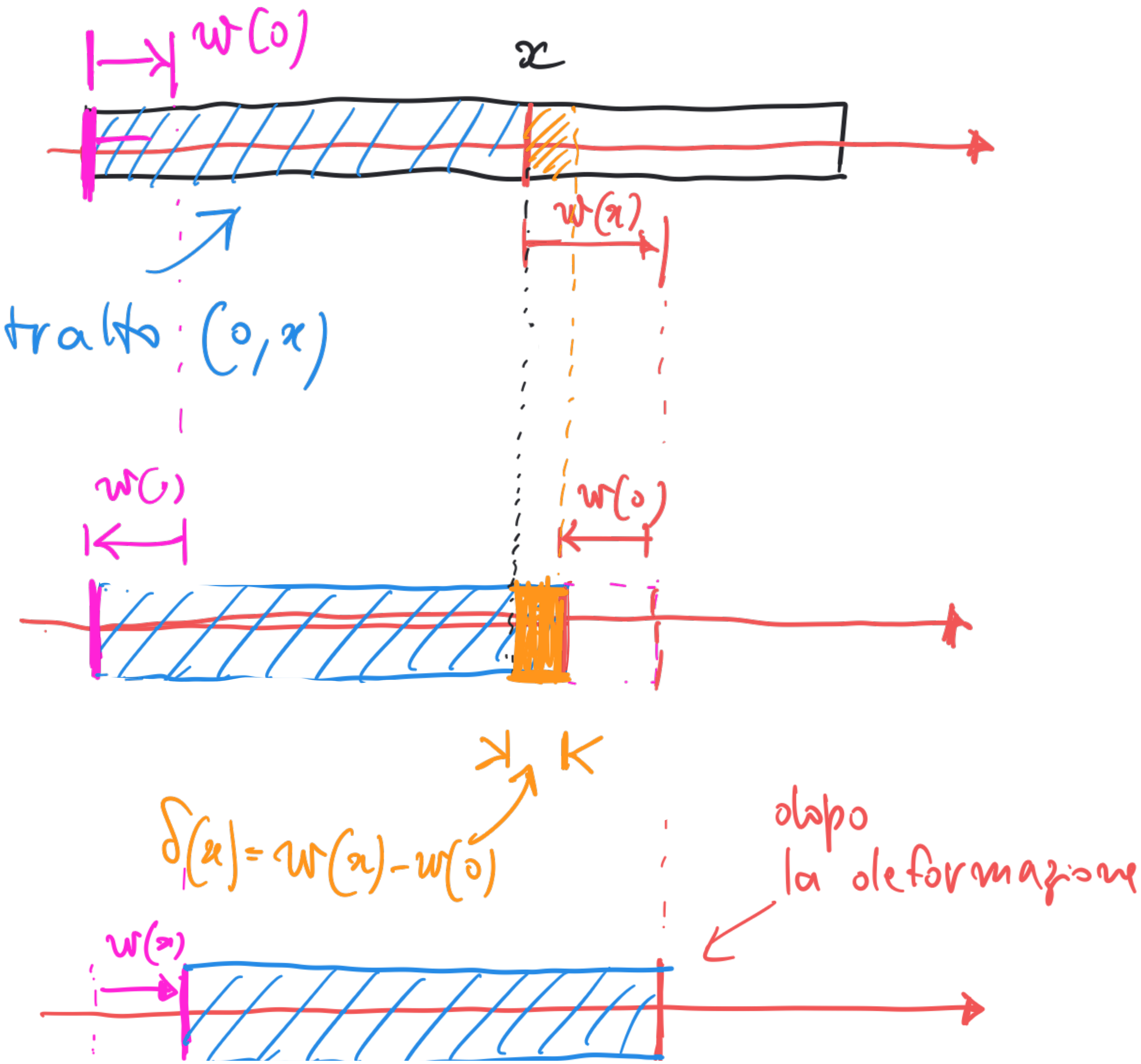
3) $\varepsilon \rightarrow \delta$

peso specifico $\gamma = 6\, kN/m$

$E = 9\, GPa$

(piccolo!)

Equazione di congruenza



$w(x)$ Casini e Vasta
spostamento lungo x della sezione x.

$\delta(x)$ allungamento del tratto $(0,x)$

Vale la seguente relazione:

$$\boxed{\delta(x) = w(x) - w(0)}$$