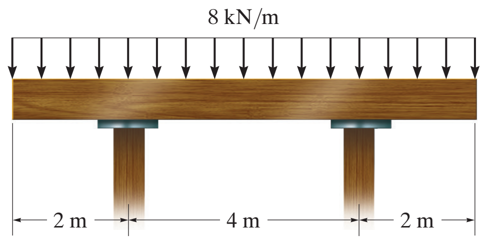
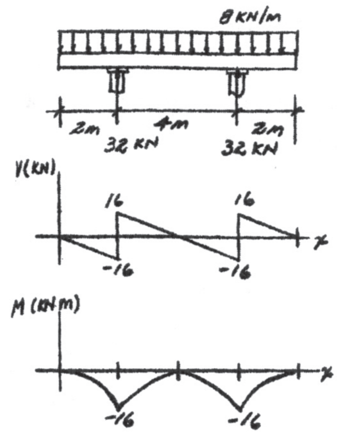
11–1.

The beam is made of timber that has an allowable bending stress of $\sigma_{\text{allow}} = 6.5$ MPa and an allowable shear stress of $\tau_{\text{allow}} = 500$ kPa. Determine its dimensions if it is to be rectangular and have a height-to-width ratio of 1.25. Assume the beam rests on smooth supports.



SOLUTION



$$I_x = \frac{1}{12}(b)(1.25b)^3 = 0.16276b^4$$

$$Q_{\max} = \bar{y}'A' = (0.3125b)(0.625b)(b) = 0.1953125b^3$$

Assume Bending Moment Controls:

$$M_{\max} = 16\ \text{kN}\cdot\text{m}$$

$$\sigma_{\text{allow}} = \frac{M_{\max}\,c}{I}$$

$$6.5(10^6) = \frac{16(10^3)(0.625b)}{0.16276b^4}$$

$$b = 0.21143\ \text{m} = 211\ \text{mm} \qquad \textbf{Ans.}$$

$$h = 1.25b = 264\ \text{mm} \qquad \textbf{Ans.}$$

Check Shear:

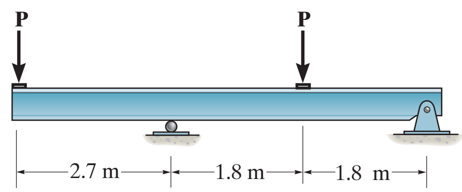
$$Q_{\max} = 1.846159(10^{-3})\ \text{m}^3$$

$$I = 0.325248(10^{-3})\ \text{m}^4$$

$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{16(10^3)(1.846159)(10^{-3})}{0.325248(10^{-3})(0.21143)} = 429\ \text{kPa} < 500\ \text{kPa}, \qquad \textbf{OK}$$

Ans:
$b = 211$ mm, $h = 264$ mm

11–2. Select the lightest-weight W310 steel wide-flang beam from Appendix B that will safely support the loading shown, where $P = 30$ kN. The allowable bending stress is $\sigma_{allow} = 150$ MPa and the allowable shear stress is $\tau_{allow} = 84$ MPa.



SOLUTION

From the Moment Diagram, Fig. *a*, $M_{max} = 81$ kN · m.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{81(10^3)}{150(10^6)}$$

$$= 0.54(10^{-3})\ \text{m}^3 = 540(10^3)\ \text{mm}^3$$

Select W310 × 39 $[S_x = 547(10^3)\ \text{mm}^3, d = 310\ \text{mm}, t_w = 5.84\ \text{mm}]$

From the shear diagram, Fig. *a*, $V_{max} = 37.5$ kN. Provide the shear-stress check for W310 × 39.

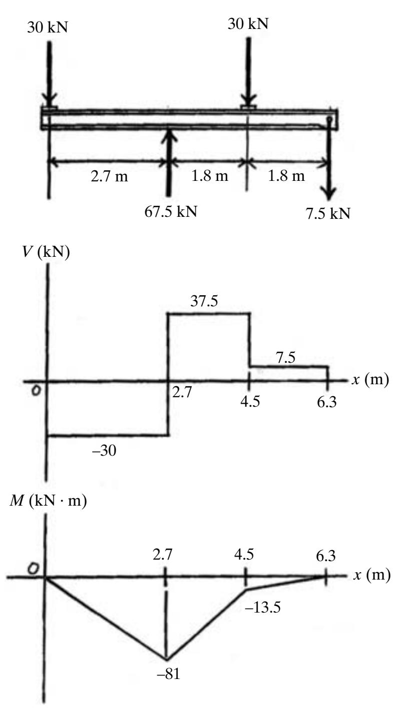
$$\tau_{max} = \frac{V_{max}}{t_w\, d}$$

$$= \frac{37.5(10^3)}{0.00584(0.310)} = 20.71(10^6)\ \text{N/m}^2$$

$$= 20.7\ \text{MPa} < \tau_{allow} = 84\ \text{MPa}\ (O.K!)$$

Hence

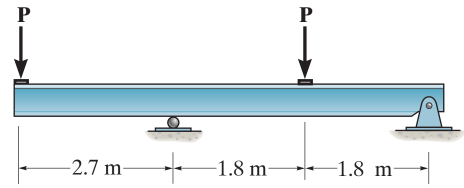
Use W310 × 39 **Ans.**



(a)

Ans:
Use W310 × 39

11–3. Select the lightest-weight W360 steel wide-flang beam from Appendix B that will safely support the loading shown, where $P = 60$ kN. The allowable bending stress is $\sigma_{\text{allow}} = 150$ MPa and the allowable shear stress is $\tau_{\text{allow}} = 84$ MPa.



SOLUTION

$$S_{\text{req'd}} = \frac{M_{\max}}{\sigma_{\text{allow}}}$$

$$= \frac{162(10^3)}{150(10^6)}$$

$$= 1.08(10^{-3})\ \text{m}^3 = 1080(10^3)\ \text{mm}^3$$

Select W360 × 79 $\quad [S_x = 1280(10^3)\ \text{mm}^3, d = 354\ \text{mm}, t_w = 9.4\ \text{mm}]$

From the shear diagram, Fig. a, $V_{max} = 75$ kN. Provide the shear-stress check for W360 × 79,

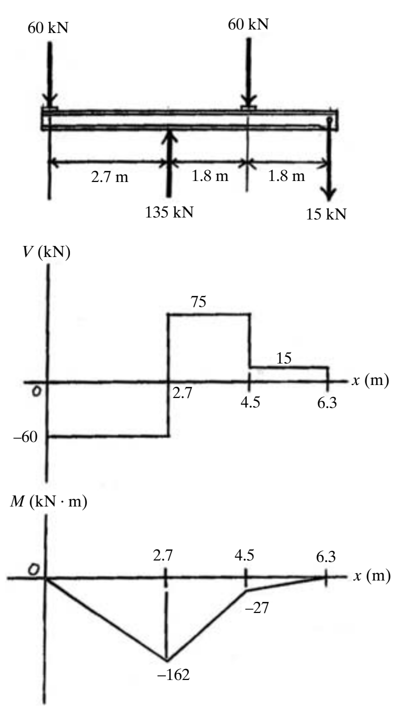
$$\tau_{\max} = \frac{V_{\max}}{t_w\, d}$$

$$= \frac{75(10^3)}{0.0094(0.354)} = 22.54(10^6)\ \text{N/m}^2$$

$$= 22.5\ \text{MPa} < \tau_{\text{allow}} = 84\ \text{MPa}\ (O.K!)$$

Hence,

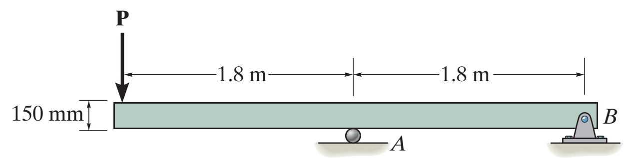
Use W360 × 79 **Ans.**



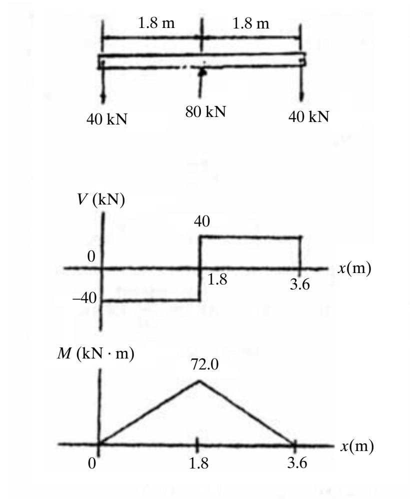
(a)

Ans:
Use W360 × 79

***11–4.** Determine the minimum width of the beam to the nearest multiples of 5 mm that will safely support the loading of $P = 40$ kN. The allowable bending stress is $\sigma_{allow} = 168$ MPa and the allowable shear stress is $\tau_{allow} = 100$ MPa.



SOLUTION



Beam design: Assume moment controls.

$$\sigma_{allow} = \frac{M\,c}{I}; \qquad 168(10^6) = \frac{[72.0(10^3)](0.075)}{\frac{1}{12}b(0.15^3)}$$

$b = 0.1143$ m $= 114.3$ mm

Use $b = 115$ mm **Ans.**

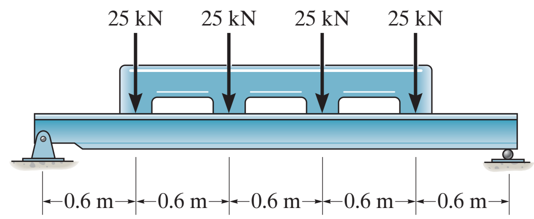
Check shear:

$$\tau_{max} = \frac{V\,Q}{I\,t} = \frac{[40(10^3)][0.0375(0.115)(0.075)]}{\left[\frac{1}{12}(0.115)(0.15^3)\right](0.115)}$$

$$= 3.478(10^6)\ \text{N/m}^2 = 3.48\ \text{MPa} < 100\ \text{MPa OK}$$

Ans:
$b = 114.3$ mm

11–5. Select the lightest-weight steel wide-flange beam from Appendix B that will safely support the machine loading shown. The allowable bending stress is $\sigma_{allow} = 168$ MPa and the allowable shear stress is $\tau_{allow} = 100$ MPa.



SOLUTION

Bending Stress: From the moment diagram, $M_{max} = 45$ kN · m. Assume bending controls the design. Applying the flexure formula.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{45(10^3)}{168(10^6)} = 0.2679(10^{-3})\ \text{m}^3 = 267.9(10^3)\ \text{mm}^3$$

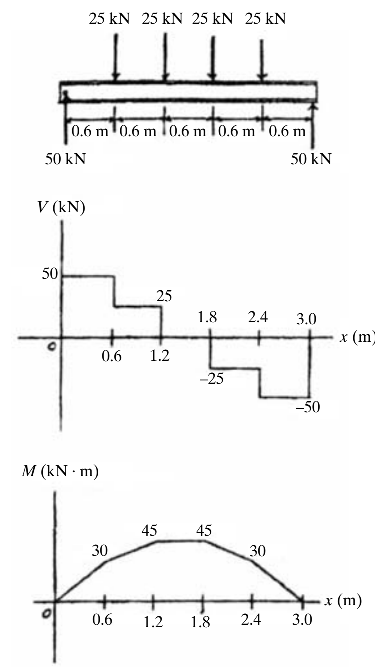
Select W310 × 24 $(S_x = 281(10^3)\ \text{mm}^3, d = 305\ \text{mm}, t_w = 5.59\ \text{mm})$

Shear Stress: Provide a shear stress check using $\tau = \dfrac{V}{t_w d}$ for the W310 × 24 wide - flange section. From the shear diagram, $V_{max} = 50$ kN

$$\tau_{max} = \frac{V_{max}}{t_w d}$$

$$= \frac{50(10^3)}{0.00559(0.305)} = 29.33(10^6)\ \text{N/m}^2$$

$$= 27.8\ \text{MPa} < \tau_{allow} = 100\ \text{MPa (O.}K!)$$

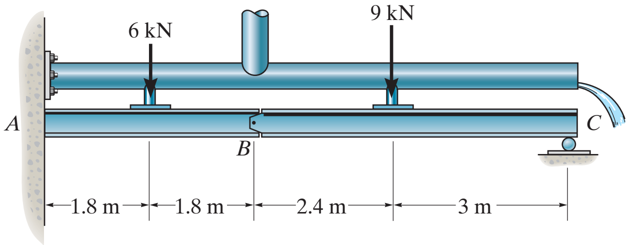


Hence,

Use W310 × 24 **Ans.**

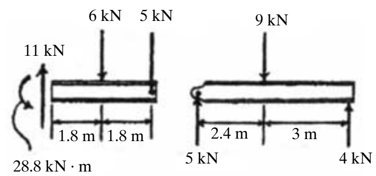
Ans:
Use W310 × 24

11–6. The compound beam is made from two sections, which are pinned together at B. Use Appendix B and select the lightest-weight wide-flange beam that would be safe for each section if the allowable bending stress is $\sigma_{allow} = 168$ MPa and the allowable shear stress is $\tau_{allow} = 100$ MPa. The beam supports a pipe loading of 6 kN and 9 kN as shown.



SOLUTION

Bending Stress: From the moment diagram, $M_{max} = 28.8$ kN·m for member AB. Assuming bending controls the design, applying the flexure formula.



$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{28.8(10^3)}{168(10^6)} = 0.1714(10^{-3})\ \text{m}^3 = 171.4(10^3)\ \text{mm}^3$$

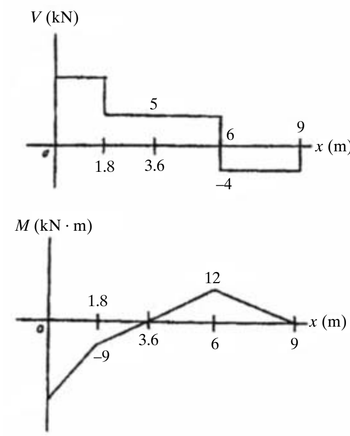
Select W250 × 18 $(S_x = 179(10^3)\ \text{mm}^3, d = 251\ \text{mm}, t_w = 4.83\ \text{mm})$

For member BC, $M_{max} = 12$ kN·m.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{12(10^3)}{168(10^6)} = 71.43(10^{-6})\ \text{m}^3 = 71.43(10^3)\ \text{mm}^3$$

Select W150 × 14 $(S_x = 91.2(10^3)\ \text{mm}^3, d = 150\ \text{mm}, t_w = 4.32\ \text{mm})$



Shear Stress: Provide a shear stress check using $\tau = \dfrac{V}{t_w d}$ for the W250 × 18 wide - flange section for member AB. From the shear diagram, $V_{max} = 11$ kN.

$$\tau_{max} = \frac{V_{max}}{t_w d}$$

$$= \frac{11(10^3)}{0.00483(0.251)} = 9.073(10^6)\ \text{N/m}^2 = 9.07\ \text{MPa}$$

$$= 9.07\ \text{MPa} < \tau_{allow} = 100\ \text{MPa}\ (O.K!)$$

Use W250 × 18 **Ans.**

For member BC (W150 × 14), $V_{max} = 5$ kN

$$\tau_{max} = \frac{V_{max}}{t_w d}$$

$$= \frac{5(10^3)}{0.00432(0.15)} = 7.716(10^6)\ \text{N/m}^2 = 7.72\ \text{MPa}$$

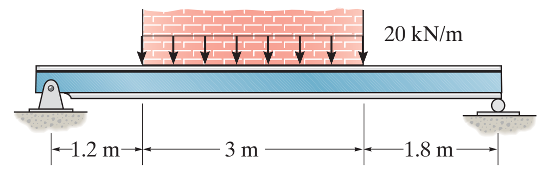
$$= 7.72\ \text{MPa} < \tau_{allow} = 100\ \text{MPa}\ (O.K!)$$

Hence,

Use W150 × 14 **Ans.**

Ans:
Use W250 × 18, Use W150 × 14

11–7. The brick wall exerts a uniform distributed load of 20 kN/m on the beam. If the allowable bending stress is $\sigma_{allow} = 154$ MPa and the allowable shear stress is $\tau_{allow} = 84$ MPa, select the lightest wide-flange section from Appendix B that will safely support the load.



SOLUTION

Bending Stress: From the moment diagram, $M_{max} = 66.825\ \text{kN}\cdot\text{m}$. Assuming bending controls the design and applying the flexure formula.

$$S_{req\,d} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{66.825(10^3)}{154(10^6)} = 0.4339(10^{-3})\ \text{m}^3 = 433.9(10^3)\ \text{mm}^3$$

Select W360 × 33 $(S_x = 475(10^3)\ \text{mm}^3,\ d = 349\ \text{mm},\ t_w = 5.84\ \text{mm})$

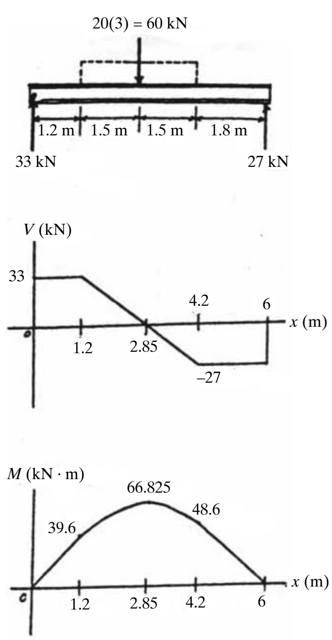
Shear Stress: Provide a shear stress check using $\tau = \dfrac{V}{t_w d}$ for the W360 × 33 wide - flange section. From the shear diagram, $V_{max} = 33$ kN.

$$\tau_{max} = \frac{V_{max}}{t_w d}$$

$$= \frac{33(10^3)}{0.00584(0.349)} = 16.19(10^6)\ \text{N/m}^2 = 16.2\ \text{MPa}$$

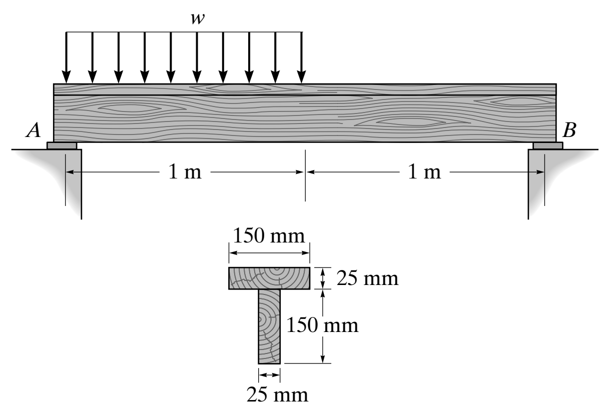
$$= 16.2\ \text{MPa} < \tau_{allow} = 84\ \text{MPa}\ (O.K!)$$

Hence, **Use** W360 × 33 **Ans.**



Ans:
Use W360 × 33

***11–8.** If the bearing pads at A and B support only vertical forces, determine the greatest magnitude of the uniform distributed loading w that can be applied to the beam. $\sigma_{\text{allow}} = 15$ MPa, $\tau_{\text{allow}} = 1.5$ MPa.



SOLUTION

The location of c, Fig. b, is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.1625(0.025)(0.15) + 0.075(0.15)(0.025)}{0.025(0.15) + 0.15(0.025)}$$

$$= 0.11875 \text{ m}$$

$$I = \frac{1}{12}(0.025)(0.15^3) + (0.025)(0.15)(0.04375)^2$$

$$+ \frac{1}{12}(0.15)(0.025^3) + 0.15(0.025)(0.04375)^2$$

$$= 21.5820(10^{-6}) \text{ m}^4$$

Referring to Fig. b,

$$Q_{\max} = \bar{y}'A' = 0.059375\,(0.11875)(0.025)$$

$$= 0.17627(10^{-3}) \text{ m}^3$$

Referring to the moment diagram, $M_{\max} = 0.28125\,w$. Applying the Flexure formula with $C = \bar{y} = 0.11875$ m,

$$\sigma_{\text{allow}} = \frac{M_{\max}\,c}{I}; \qquad 15(10^6) = \frac{0.28125w(0.11875)}{21.5820(10^{-6})}$$

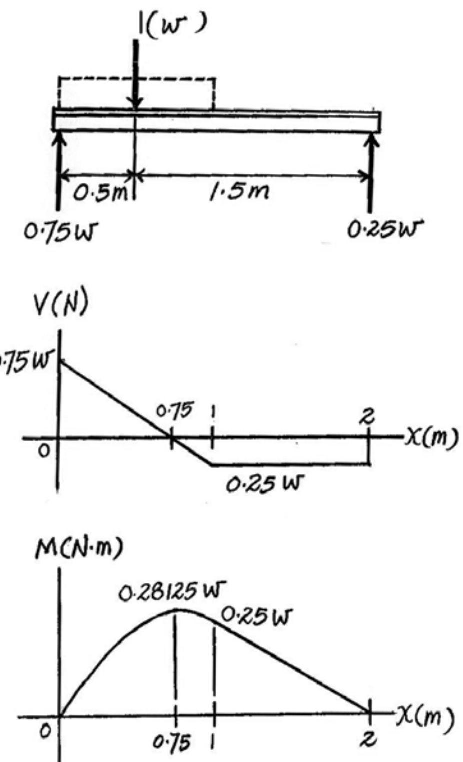
$$W = 9.693(10^3) \text{ N/m}$$

Referring to shear diagram, Fig. a, $V_{\max} = 0.75\,w$.

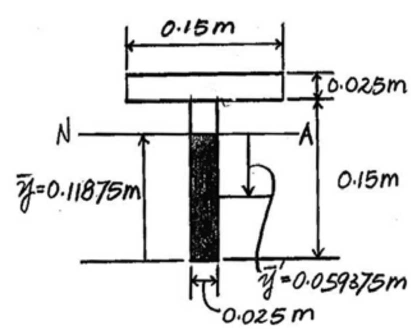
$$\tau_{\text{allow}} = \frac{V_{\text{allow}}\,Q_{\max}}{It}; \qquad 1.5(10^6) = \frac{0.75w\left[0.17627(10^{-3})\right]}{21.5820(10^{-6})(0.025)}$$

$$W = 6.122(10^3) \text{ N/m}$$

$$= 6.12 \text{ kN/m } (\textit{Control!})$$ **Ans.**



(a)



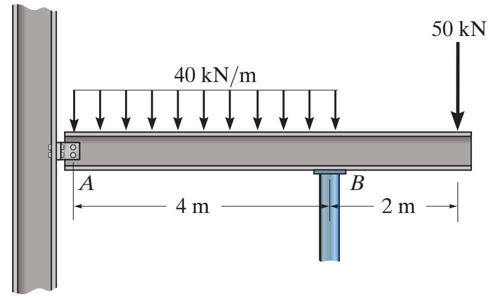
(b)

Ans:

$W = 6.12$ kN/m (*Control!*)

11–9.

Select the lightest W360 wide-flange beam from Appendix B that can safely support the loading. The beam has an allowable normal stress of $\sigma_{allow} = 150$ MPa and an allowable shear stress of $\tau_{allow} = 80$ MPa. Assume there is a pin at A and a roller support at B.



SOLUTION

Shear and Moment Diagram: As shown in Fig. *a*.

Bending Stress: Referring to the moment diagram, Fig. *a*, $M_{max} = 100\ \text{kN}\cdot\text{m}$. Applying the flexure formula,

$$S_{required} = \frac{M_{max}}{\sigma_{allow}} = \frac{100(10^3)}{150(10^6)}$$

$$= 0.6667(10^{-3})\ \text{m}^3 = 666.67(10^3)\ \text{mm}^3$$

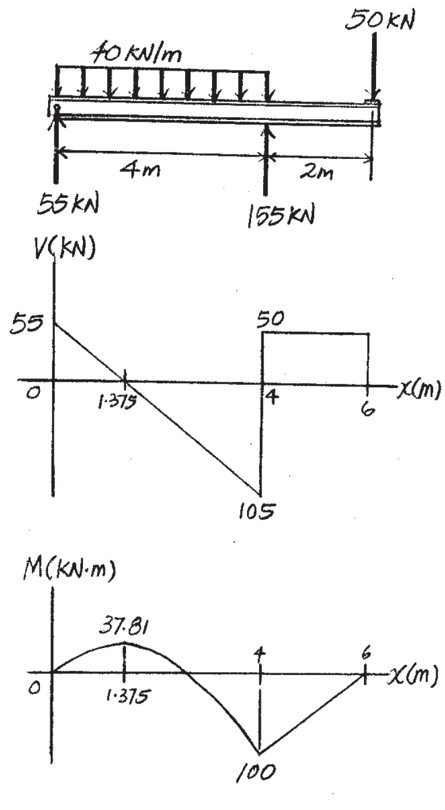
Select W360 × 45($S_x = 688(10^3)\ \text{mm}^3$, $d = 352$ mm and $t_w = 6.86$ mm)

Shear Stress: Referring to the shear diagram, Fig. *a*, $V_{max} = 105$ kN. We have

$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{105(10^3)}{6.86(10^{-3})(0.352)}$$

$$= 43.48\ \text{MPa} < \tau_{allow} = 80\ \text{MPa} \qquad \textbf{(OK)}$$

Hence, use W360 × 45 **Ans.**

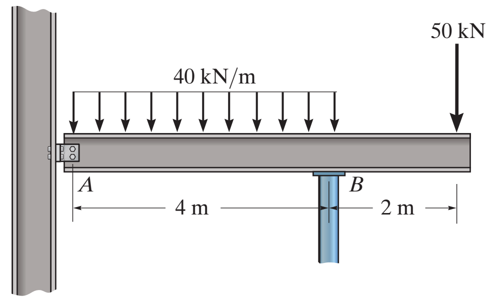


(a)

Ans:
Use W360 × 45

11–10.

Investigate if the W250 × 58 beam can safely support the loading. The beam has an allowable normal stress of $\sigma_{\text{allow}} = 150$ MPa and an allowable shear stress of $\tau_{\text{allow}} = 80$ MPa. Assume there is a pin at A and a roller support at B.



SOLUTION

Shear and Moment Diagram: As shown in Fig. *a*.

Bending Stress: Referring to the moment diagram, Fig. *a*, $M_{\max} = 100\text{ kN}\cdot\text{m}$. For a W250 × 58 section, $S_x = 693(10^3)\text{ mm}^3 = 0.693(10^{-3})\text{ m}^4$. Applying the flexure formula,

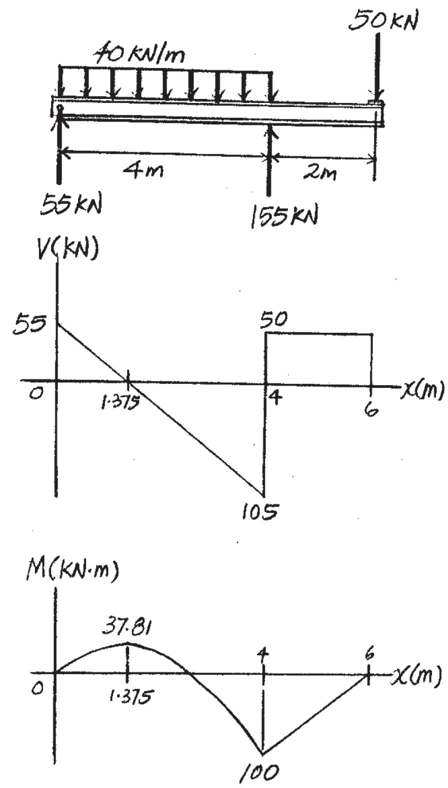
$$\sigma_{\max} = \frac{M_{\max}}{S_x} = \frac{100(10^3)}{0.693(10^{-3})} = 144.30\text{ MPa} < \sigma_{\text{allow}} = 150\text{ MPa} \qquad \textbf{(OK)}$$

Shear Stress: Referring to the shear diagram, Fig. *a*, $V_{\max} = 105$ kN. For a W250 × 58 section, $d = 252$ mm and $t_w = 8.00$ mm. We have

$$\tau_{\max} = \frac{V_{\max}}{t_w d} = \frac{105(10^3)}{8.00(10^{-3})(0.252)}$$

$$= 52.08\text{ MPa} < \tau_{\text{allow}} = 80\text{ MPa} \qquad \textbf{(OK)}$$

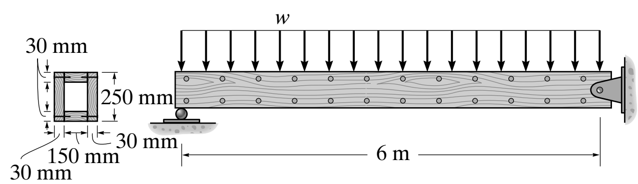
The W250 × 58 can safely support the loading. **Ans.**

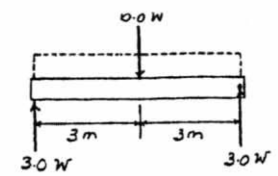


(a)

Ans:
Yes, it can

11–11. The box beam has an allowable bending stress of $\sigma_{allow} = 10$ MPa and an allowable shear stress of $\tau_{allow} = 775$ kPa. Determine the maximum intensity w of the distributed loading that it can safely support. Also, determine the maximum safe nail spacing for each third of the length of the beam. Each nail can resist a shear force of 200 N.





SOLUTION

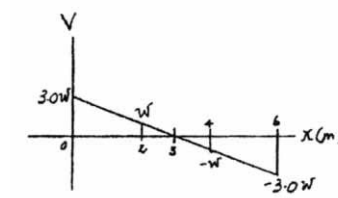
Section Properties:

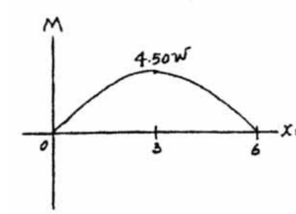
$$I = \frac{1}{12}(0.21)\left(0.25^3\right) - \frac{1}{12}(0.15)\left(0.19^3\right) = 0.1877\left(10^{-3}\right)\text{ m}^4$$

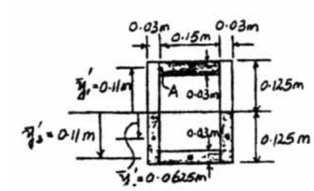
$$Q_A = \bar{y}_1{}'A' = 0.11(0.03)(0.15) = 0.495\left(10^{-3}\right)\text{ m}^3$$

$$Q_{max} = \Sigma\bar{y}'A' = 0.11(0.03)(0.15) + 0.0625(0.125)(0.06)$$

$$= 0.96375\left(10^{-3}\right)\text{ m}^3$$







Bending Stress: From the moment diagram, $M_{max} = 4.50w$. Assume bending controls the design. Applying the flexure formula.

$$\sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$10\left(10^6\right) = \frac{4.50w\,(0.125)}{0.1877\,(10^{-3})}$$

$$w = 3336.9\text{ N/m}$$

Shear Stress: Provide a shear stress check using the shear formula. From the shear diagram, $V_{max} = 3.00w = 10.01$ kN.

$$\tau_{max} = \frac{V_{max}\,Q_{max}}{It}$$

$$= \frac{10.01(10^3)\left[0.96375(10^{-3})\right]}{0.1877(10^{-3})(0.06)}$$

$$= 857\text{ kPa} > \tau_{allow} = 775\text{ kPa } (\textit{No Good!})$$

Hence, shear stress controls.

$$\tau_{allow} = \frac{V_{max}\,Q_{max}}{It}$$

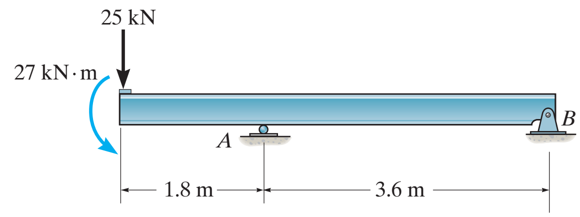
$$775\left(10^3\right) = \frac{3.00w\left[0.96375(10^{-3})\right]}{0.1877(10^{-3})(0.06)}$$

$$w = 3018.8\text{ N/m} = 3.02\text{ kN/m}$$ **Ans.**

Shear Flow: Since there are two rows of nails, the allowable shear flow is $q = \frac{2(200)}{s} = \frac{400}{s}$.

Ans:
$w = 3.02$ kN/m, $s = 16.7$ mm, $s = 50.2$ mm

***11–12.** Select the lightest-weight steel wide-flange beam from Appendix B that will safely support the loading shown. The allowable bending stress is $\sigma_{allow} = 150$ MPa and the allowable shear stress is $\tau_{allow} = 84$ MPa.



SOLUTION

From the moment diagram, Fig. a, $M_{max} = 72$ kN·m.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{72(10^3)}{150(10^6)} = 0.48(10^{-3})\text{ m}^3 = 480(10^3)\text{ mm}^3$$

$$= 480(10^3)\text{ mm}^3$$

The choices are

W310 × 39 W360 × 39 W410 × 39

The section that has shortest depth is preferable, so

Select W310 × 39 $[S_x = 547(10^3)\text{ mm}^3, d = 310\text{ mm and } t_w = 5.84\text{ mm}]$

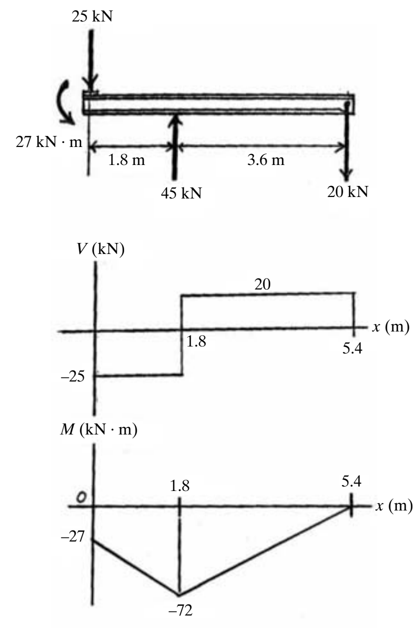
From the shear diagram, Fig. a, $V_{max} = 25$ kN. Provide the shear stress check for W 310 × 39,

$$\tau_{max} = \frac{V_{max}}{t_w d}$$

$$= \frac{25(10^3)}{0.00584(0.3101)} = 13.81(10^6)\text{ N/m}^2 = 13.8\text{ MPa}$$

$$= 13.8\text{ MPa} < \tau_{allow} = 84\text{ MPa } (O.K!)$$

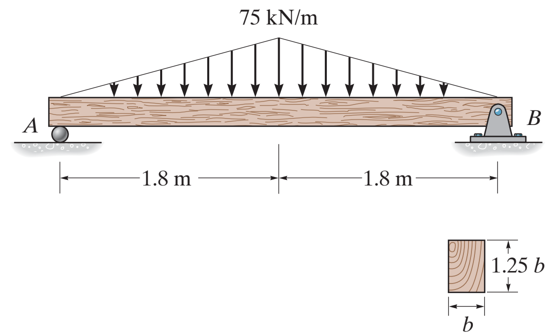
Use W310 × 39 **Ans.**



(a)

Ans:
Use W310 × 39

11–13. The simply supported beam is made of timber that has an allowable bending stress of $\sigma_{allow} = 6.72$ MPa and an allowable shear stress of $\tau_{allow} = 0.525$ MPa. Determine the dimension b if it is to be rectangular and have a height-to-width ratio of 1.25.



SOLUTION

$$I = \frac{1}{12}(b)(1.25b)^3 = 0.16276b^4$$

$$S_{req'd} = \frac{I}{c} = \frac{0.16276b^4}{0.625b} = 0.26042b^3$$

Assume bending moment controls:

$$M_{max} = 81 \text{ kN} \cdot \text{m}$$

$$\sigma_{allow} = \frac{M_{max}}{S_{req'd}}$$

$$6.72(10^6) = \frac{81(10^3)}{0.26042b^3}$$

$$b = 0.35904 \text{ m} = 359.04 \text{ mm}$$

Check shear:

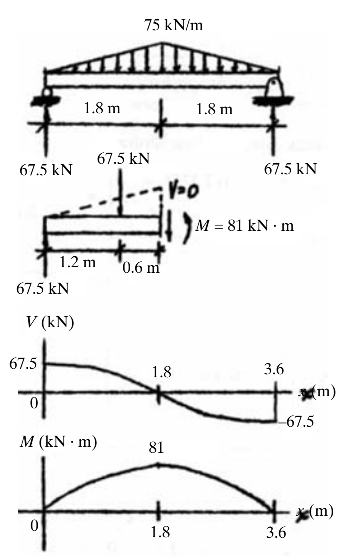
$$\tau_{max} = \frac{1.5V}{A} = \frac{1.5[67.5(10^3)]}{(0.35904)[1.25(0.35904)]} = 0.6283(10^6) \text{ N/m}^2$$

$$= 0.628 \text{ MPa} > 0.525 \text{ MPa (NO Good!)}$$

Shear controls:

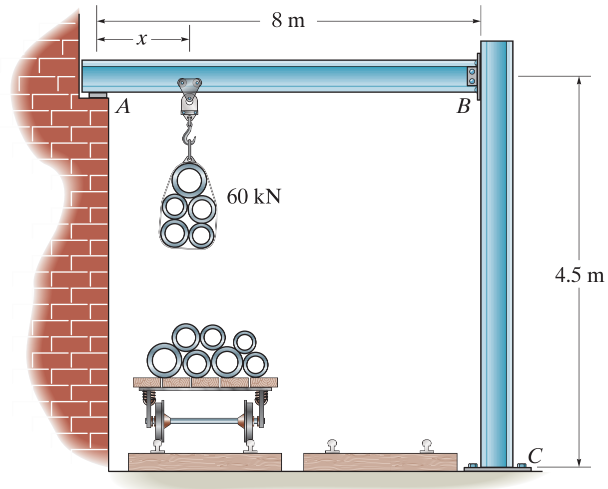
$$\tau_{allow} = \frac{1.5V}{A}; \quad 0.525(10^6) = \frac{1.5[67.5(10^3)]}{b(1.25b)}$$

$$b = 0.3928 \text{ m} = 393 \text{ mm}$$ **Ans.**



Ans:
$b = 393$ mm

11–14. The beam is used in a railroad yard for loading and unloading cars. If the maximum anticipated hoist load is 60 kN, select the lightest-weight steel wide-flange section from Appendix B that will safely support the loading. The hoist travels along the bottom flange of the beam, $0.3 \text{ m} \le x \le 7.5 \text{ m}$, and has negligible size. Assume the beam is pinned to the column at B and roller supported at A. The allowable bending stress is $\sigma_{allow} = 168$ MPa and the allowable shear stress is $\tau_{allow} = 84$ MPa.



SOLUTION

Maximum moment occurs when load is in the center of beam.

$$M_{max} = (30 \text{ kN})(4 \text{ m}) = 120 \text{ kN} \cdot \text{m}$$

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

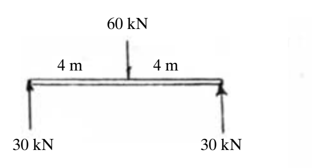
$$= \frac{120(10^3)}{168(10^6)} = 0.7143(10^{-3}) \text{ m}^3 = 714.3(10^3) \text{ mm}^3$$

Select a W410 × 46 $\qquad S_x = 774(10^3) \text{ mm}^3, d = 403 \text{ mm}, t_w = 6.99 \text{ mm}$

At $x = 0.3$ m, $V = 57.75$ kN

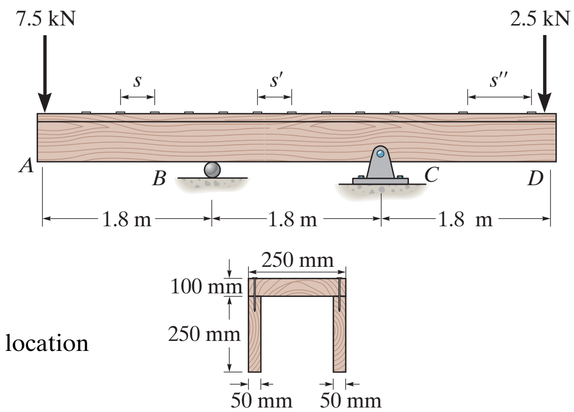
$$\tau = \frac{V}{A_{web}} = \frac{57.75(10^3)}{0.00699(0.403)} = 20.50(10^6) \text{ N/m}^2 = 20.50 \text{ MPa} < 84 \text{ MPa}$$

Use W410 × 46 **Ans.**



Ans:
Use W410 × 46

11–15. The beam is constructed from three boards as shown. If each nail can support a shear force of 1.5 kN, determine the maximum allowable spacing of the nails, s, s', s'', for regions AB, BC, and CD to the nearest 5 mm respectively. Also, if the allowable bending stress is $\sigma_{\text{allow}} = 10$ MPa and the allowable shear stress is $\tau_{\text{allow}} = 1$ MPa, determine if it can safely support the load.



SOLUTION

The neutral axis passes through centroid c of the beam's cross-section. The location of c, Fig. b, is

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.3(0.25)(0.1) + 2[0.125(0.05)(0.25)]}{0.25(0.1) + 2(0.05)(0.25)}$$

$$= 0.2125 \text{ m}$$

The moment of inertia of the beam's cross-section about the neutral axis is

$$I = \left[\frac{1}{12}(0.05)(0.25^3) + 0.05(0.25)(0.2125 - 0.125)^2\right]$$

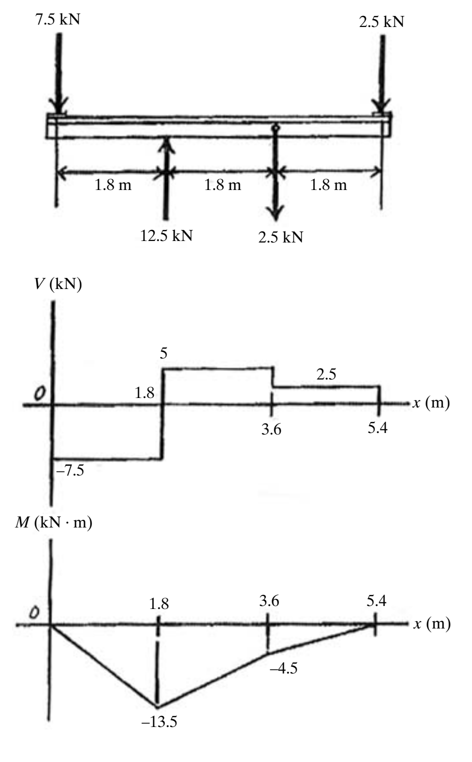
$$+ \frac{1}{12}(0.25)(0.1^3) + 0.25(0.1)(0.3 - 0.2125)^2$$

$$= 0.53385(10^{-3}) \text{ m}^4$$

Referring to Fig. b,

$$Q_{\max} = 2\bar{y}_2' A_2' = 2[(0.10625)(0.05)(0.2125)] = 2.2578(10^{-3}) \text{ m}^3$$

$$Q_A = \bar{y}_1' A_1' = 0.0875(0.25)(0.1) = 2.1875(10^{-3}) \text{ m}^3$$



(a)

Referring to the moment diagram, Fig. a, $M_{\max} = 13.5$ kN · m. Applying flexure formula with $c = \bar{y} = 0.2125$ mm,

$$\sigma_{\max} = \frac{M_{\max}\, c}{I}$$

$$= \frac{[13.5(10^3)](0.2125)}{0.53385(10^{-3})} = 5.374(10^6) \text{ N/m}^2$$

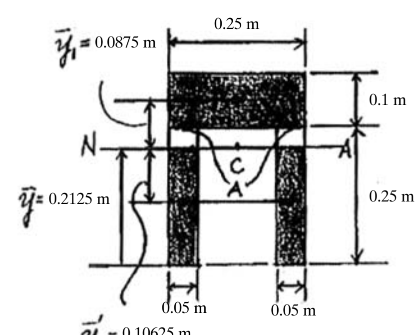
$$= 5.37 \text{ MPa} < \sigma_{\text{allow}} = 10 \text{ MPa } (O.K!)$$

Referring to shear diagram, Fig. a, $V_{\max} = 7.5$ kN.

$$\tau_{\max} = \frac{V_{\max} Q_{\max}}{It}$$

$$= \frac{[7.5(10^3)][2.2578(10^{-3})]}{[0.53385(10^{-3})](0.1)} = 0.3172(10^6) \text{ N/m}^2$$

$$= 0.317 \text{ MPa} < \tau_{\text{allow}} = 1 \text{ MPa } (O.K!)$$



(b)

Yes, it can support the load. **Ans.**

11–15. Continued

Since there are two rows of nails, the allowable shear flow is
$q_{\text{allow}} = \dfrac{2F}{S} = \dfrac{2(1.5)(10^3)}{S} = \dfrac{3000}{S}$. For region AB, $V = 7.5$ kN. Thus

$$q_{\text{allow}} = \frac{VQ_A}{I}; \qquad \frac{3000}{S} = \frac{[7.5(10^3)][2.1875(10^{-3})]}{0.53385(10^{-3})} \qquad S = 0.09762 \text{ m} = 97.62 \text{ mm}$$

Use $S = 95$ mm **Ans.**

For region BC, $V = 5$ kN. Thus

$$q_{\text{allow}} = \frac{VQ_A}{I}; \qquad \frac{3000}{S'} = \frac{[5(10^3)][2.1875(10^{-3})]}{0.53385(10^{-3})} \qquad S' = 0.14643 \text{ m} = 146.43 \text{ mm}$$

Use $S' = 145$ mm **Ans.**

For region CD, $V = 2.5$ kN. Thus

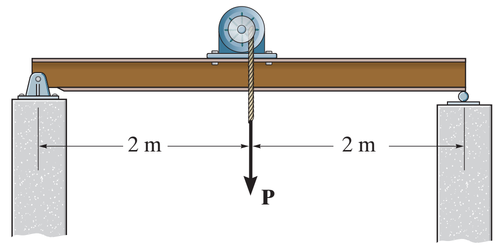
$$q_{\text{allow}} = \frac{VQ_A}{I}; \qquad \frac{3000}{S''} = \frac{[2.5(10^3)][2.1875(10^{-3})]}{0.53385(10^{-3})} \qquad S'' = 0.29286 \text{ m} = 292.86 \text{ mm}$$

Use $S'' = 290$ mm **Ans.**

Ans:
Yes, it can support the load, Use $s = 95$ mm, Use $s' = 145$ mm, Use $s'' = 290$ mm

***11–16.**

If the cable is subjected to a maximum force of $P = 50$ kN, select the lightest W310 wide-flange beam that can safely support the load. The beam has an allowable normal stress of $\sigma_{\text{allow}} = 150$ MPa and an allowable shear stress of $\tau_{\text{allow}} = 85$ MPa.



SOLUTION

Shear and Moment Diagram: As shown in Fig. *a*.

Bending Stress: From the moment diagram, Fig. *a*, $M_{\max} = 50$ kN·m. Applying the flexure formula,

$$S_{\text{required}} = \frac{M_{\max}}{\sigma_{\text{allow}}} = \frac{50(10^3)}{150(10^6)}$$

$$= 0.3333(10^{-3})\ \text{m}^3 = 333.33(10^3)\ \text{mm}^3$$

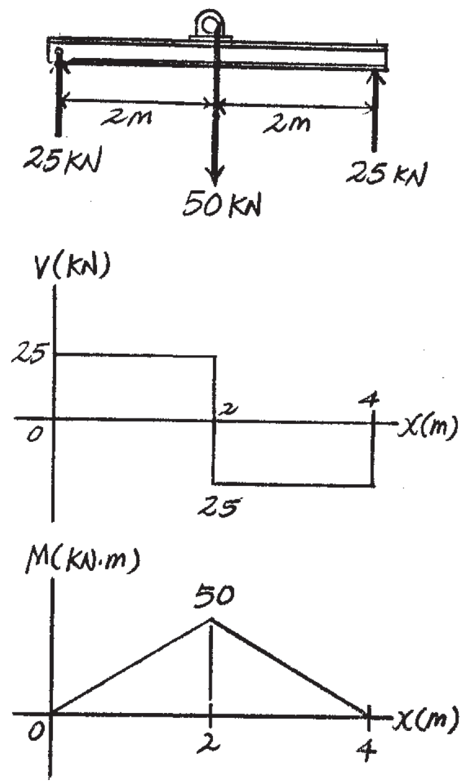
Select a W310 × 33[$S_x = 415(10^3)$ mm^3, $d = 313$ mm, and $t_w = 6.60$ mm]

Shear Stress: From the shear diagram, Fig. *a*, $V_{\max} = 25$ kN. We have

$$\tau_{\max} = \frac{V_{\max}}{t_w\, d} = \frac{25(10^3)}{6.60(10^{-3})(0.313)}$$

$$= 12.10\ \text{MPa} < \tau_{\text{allow}} = 85\ \text{MPa} \qquad \textbf{OK}$$

Hence, Use W310 × 33. **Ans.**

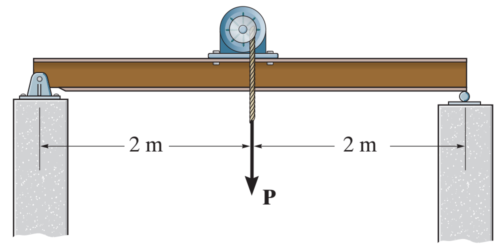


(a)

Ans:
Use W310 × 33

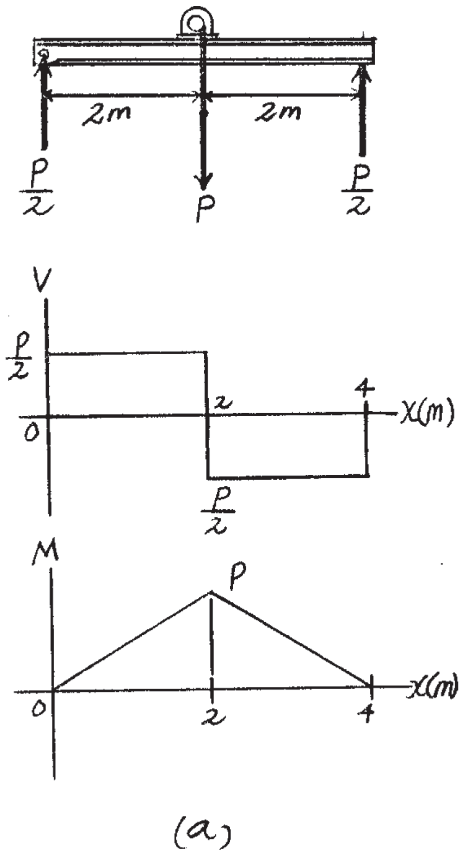
11–17.

If the W360 × 45 wide-flange beam has an allowable normal stress of $\sigma_{\text{allow}} = 150$ MPa and an allowable shear stress of $\tau_{\text{allow}} = 85$ MPa, determine the maximum cable force **P** that can safely be supported by the beam.



SOLUTION

Shear and Moment Diagram: As shown in Fig. *a*.



Bending Stress: From the moment diagram, Fig. *a*, $M_{\text{max}} = P$. For W360 × 45 section, $S_x = 688(10^3)\ \text{mm}^3 = 0.688(10^{-3})\ \text{m}^3$.

Applying the flexure formula,

$$M_{\text{max}} = S_x \sigma_{\text{allow}}$$

$$P = 0.688(10^{-3})[150(10^6)]$$

$$P = 103\,200\ \text{N} = 103\ \text{kN} \qquad \textbf{Ans.}$$

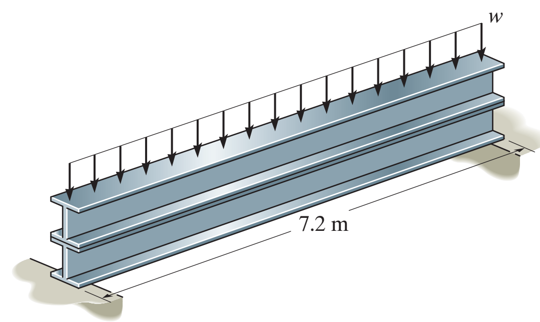
Shear Stress: From the shear diagram, Fig. *a*, $V_{\text{max}} = \dfrac{P}{2} = \dfrac{103\,200}{2} = 51\,600$ N. For W360 × 45 section, $d = 352$ mm and $t_w = 6.86$ mm. We have

$$\tau_{\text{max}} = \frac{V_{\text{max}}}{t_w d} = \frac{51\,600}{6.86(10^{-3})(0.352)}$$

$$= 21.37\ \text{MPa} < \tau_{\text{allow}} = 85\ \text{MPa} \qquad \textbf{(OK!)}$$

Ans:
$P = 103$ kN

11–18. The simply supported beam is composed of two W310 $\times$ 33 sections built up as shown. Determine the maximum uniform loading w the beam will support if the allowable bending stress of $\sigma_{allow} = 150$ MPa and the allowable shear stress is $\tau_{allow} = 100$ MPa.



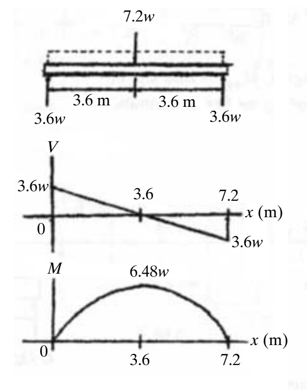
SOLUTION

Section properties:

For W310 $\times$ 33 ($d = 313$ mm $I_x = 65(10^6)$ mm^4 $t_w = 6.60$ mm $A = 4180$ mm^2)

$$I = 2\left[65(10^6) + 4180\left(\frac{313}{2}\right)^2\right] = 334.76(10^6)\ \text{mm}^4$$

$$S = \frac{I}{c} = \frac{334.76(10^6)}{313} = 1.0695(10^6)\ \text{mm}^3 = 1.0695(10^{-3})\ \text{m}^3$$



Maximum Loading: Assume moment controls.

$M = \sigma_{allow} S;$

$6.48w = [150(10^6)][1.0695(10^{-3})]$

$w = 24.76(10^3)\ \text{N/m} = 24.8\ \text{kN/m}$ **Ans.**

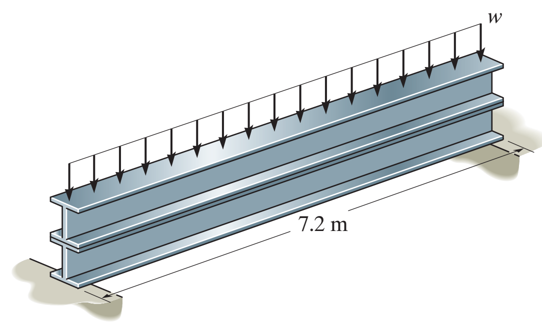
Check Shear: (Neglect area of flanges.)

$$\tau_{max} = \frac{V_{max}}{A_w} = \frac{3.6(24.76)(10^3)}{2(0.0066)(0.313)} = 21.57(10^6)\ \text{N/m}^2$$

$$= 21.6\ \text{MPa} < \tau_{allow} = 100\ \text{MPa}\ (O.K!)$$

Ans:
$w = 24.8$ kN/m

11–19. The simply supported beam is composed of two W310 × 33 sections built up as shown. Determine if the beam will safely support a loading of $w = 30$ kN/m. The allowable bending stress is $\sigma_{allow} = 150$ MPa and the allowable shear stress is $\tau_{allow} = 100$ MPa.



SOLUTION

Section properties:

For W310 × 33 $\quad (d = 313 \text{ mm} \quad I_x = 65(10^6) \text{ mm}^4 \quad t_w = 6.60 \text{ mm} \quad A = 4180 \text{ mm}^2)$

$$I = 2\left[65(10^6) + 4180\left(\frac{313}{2}\right)^2\right] = 334.76(10^6)\text{ mm}^4$$

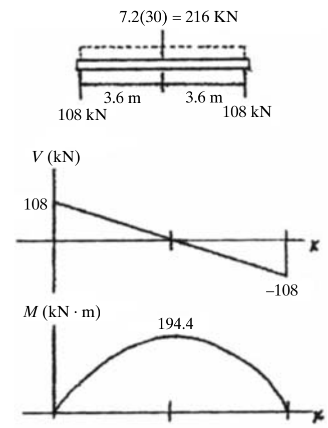
$$S = \frac{I}{c} = \frac{334.76(10^6)}{313} = 1.0695(10^6)\text{ mm}^3 = 1.0695(10^{-3})\text{ m}^3$$

Bending stress:

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{194.4(10^3)}{1.0695(10^{-3})} = 181.77(10^6)\text{ N/m}^2$$

$$= 181.8 \text{ MPa} > \sigma_{allow} = 150 \text{ MPa}$$

No, the beam fails due to bending stress criteria. **Ans.**



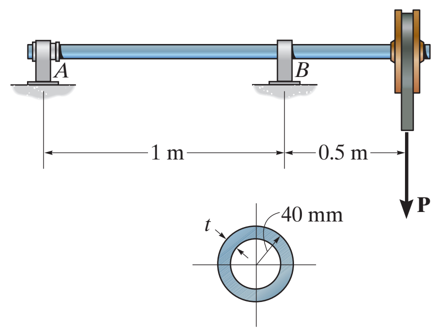
Check shear: (Neglect area of flanges.)

$$\tau_{max} = \frac{V_{max}}{A_w} = \frac{108(10^3)}{2(0.0066)(0.313)} = 26.14(10^6)\text{ N/m}^2$$

$$= 26.1 \text{ MPa} < \tau_{allow} = 100 \text{ MPa} \quad \text{OK}$$

Ans:
The beam fails.

***11–20.** The shaft is supported by a smooth thrust bearing at A and a smooth journal bearing at B. If $P = 5$ kN and the shaft is made from steel having an allowable normal stress of $\sigma_{\text{allow}} = 150$ MPa and an allowable shear stress of $\tau_{\text{allow}} = 85$ MPa, determine the required minimum wall thickness t of the shaft to the nearest millimeter to safely support the load.



SOLUTION

Shear and Moment Diagram: As shown in Fig. a,

Bending Stress: From the moment diagram, Fig. a, $M_{\text{max}} = 2.5$ kN · m. The moment of inertia of the shaft about the bending axis is $I = \frac{\pi}{4}(0.04^4 - r_i^{\,4})$. Applying the flexure formula,

$$\sigma_{\text{allow}} = \frac{M_{\text{max}}\,c}{I}$$

$$150(10^6) = \frac{2.5(10^3)(0.04)}{\frac{\pi}{4}(0.04^4 - r_i^{\,4})}$$

$$r_i = 0.03617 \text{ m} = 36.17 \text{ mm}$$

Thus,

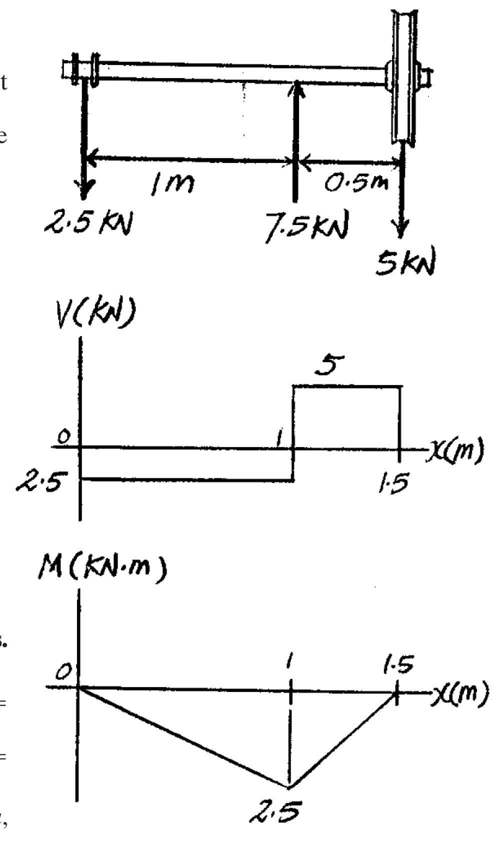
$$t = r_0 - r_i = 40 - 36.17 = 3.83 \text{ mm}$$

Use $t = 4$ mm **Ans.**

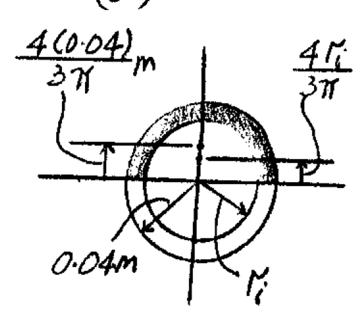
Shear Stress: Using this result, $r_i = 0.04 - 0.004 = 0.036$ m. Then $Q_{\text{max}} = \frac{4(0.04)}{3\pi}\left[\frac{\pi}{2}(0.04^2)\right] - \frac{4(0.036)}{3\pi}\left[\frac{\pi}{2}(0.036^2)\right] = 11.5627(10^{-6})\text{ m}^3$, Fig. b, and $I = \frac{\pi}{4}(0.04^4 - 0.036^4) = 0.69145(10^{-6})\text{ m}^4$. Referring to the shear diagram, Fig. a, $V_{\text{max}} = 5$ kN.

$$\tau_{\text{max}} = \frac{V_{\text{max}}Q_{\text{max}}}{It} = \frac{5(10^3)[11.5627(10^{-6})]}{0.69145(10^{-6})(0.008)}$$

$$= 10.45 \text{ MPa} < \tau_{\text{allow}} = 85 \text{ MPa (OK)}$$



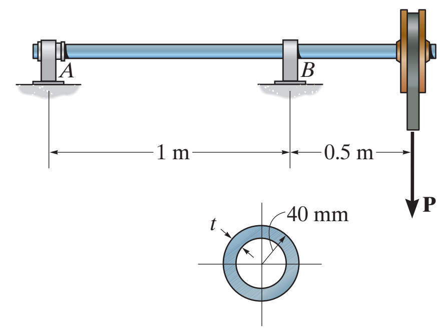
(a)



(b)

Ans:
Use $t = 4$ mm

11–21. The shaft is supported by a smooth thrust bearing at A and a smooth journal bearing at B. If the shaft is made from steel having an allowable normal stress of $\sigma_{allow} = 150$ MPa and allowable shear stress of $\tau_{allow} = 85$ MPa, determine the maximum allowable force $\mathbf{P}$ that can be applied to the shaft. The thickness of the shaft's wall is $t = 5$ mm.



SOLUTION

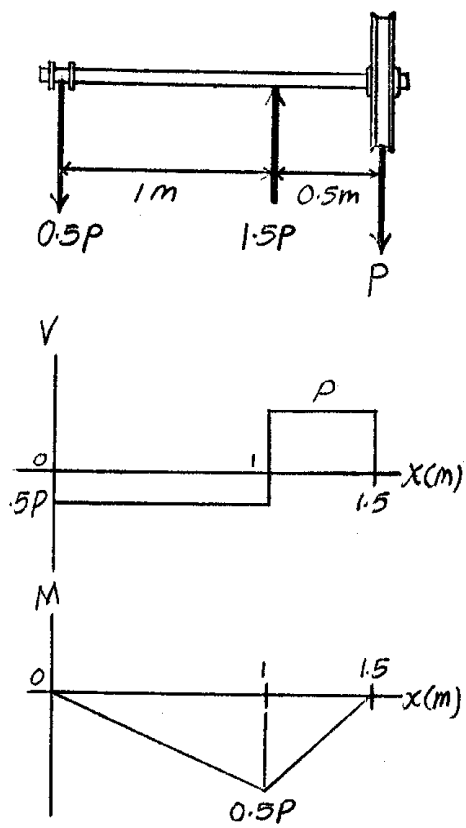
Shear and Moment Diagram: As shown in Fig. *a*,

Bending Stress: The moment of inertia of the shaft about the bending axis is $I = \frac{\pi}{4}(0.04^4 - 0.035^4) = 0.8320(10^{-6})\ \text{m}^4$. Referring to the moment diagram, Fig. *a*, $M_{max} = 0.5P$. Applying the flexure formula,

$$\sigma_{allow} = \frac{M_{max}\, c}{I}$$

$$150(10^6) = \frac{0.5P(0.04)}{0.8320(10^{-6})}$$

$$P = 6240.23\text{N} = 6.24\ \text{kN} \qquad \textbf{Ans.}$$



(a)

Shear Stress:

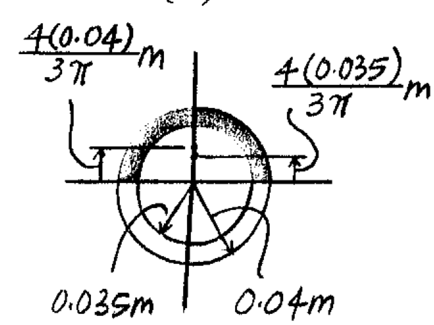
$$Q_{max} = \frac{4(0.04)}{3\pi}\left[\frac{\pi}{2}(0.04^2)\right] - \frac{4(0.035)}{3\pi}\left[\frac{\pi}{2}(0.035^2)\right]$$

$$= 14.0833(10^{-6})\ \text{m}^3$$

Referring to the shear diagram, Fig. *a*, $V_{max} = P = 6240.23$ N. Applying the shear formula,

$$\tau_{max} = \frac{V_{max}\, Q_{max}}{It} = \frac{6240.23[14.0833(10^{-6})]}{0.8320(10^{-6})(0.01)}$$

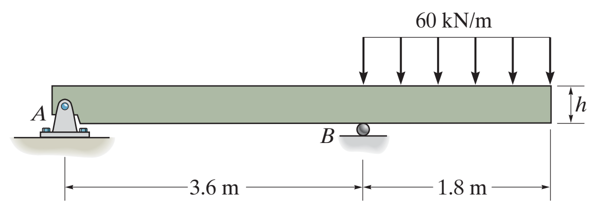
$$= 10.56\ \text{MPa} < \tau_{allow} = 85\ \text{MPa (OK!)}$$



(b)

Ans:
$P = 6.24$ kN

11–22. Determine the minimum depth h of the beam to the nearest multiples of 5 mm that will safely support the loading shown. The allowable bending stress is $\sigma_{\text{allow}} = 147$ MPa and the allowable shear stress is $\tau_{\text{allow}} = 70$ MPa. The beam has a uniform thickness of 75 mm.



SOLUTION

The section modulus of the rectangular cross-section is

$$S = \frac{I}{C} = \frac{\frac{1}{12}(0.075)(h^3)}{h/2} = 0.0125h^2$$

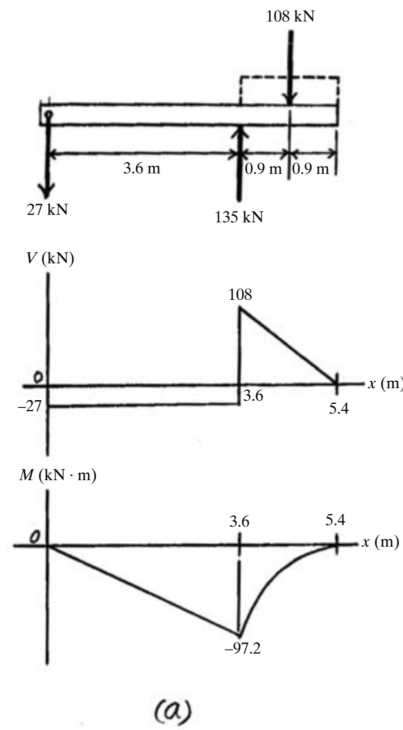
From the moment diagram, $M_{\text{max}} = 97.2 \text{ kN} \cdot \text{m}$.

$$S_{\text{req'd}} = \frac{M_{\text{max}}}{\sigma_{\text{allow}}}$$

$$0.0125h^2 = \frac{97.2(10^3)}{147(10^6)}$$

$$h = 0.229995 \text{ m} = 230 \text{ mm}$$

Use $h = 230$ mm **Ans.**



From the shear diagram, Fig. a, $V_{\text{max}} = 108$ kN. Referring to Fig. b,

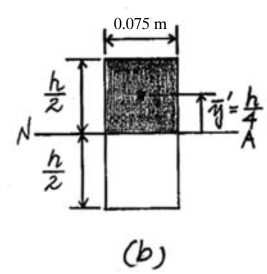
$$Q_{\text{max}} = \bar{y}' A' = \left(\frac{0.23}{4}\right)(0.075)\left(\frac{0.23}{2}\right) = 0.4959375(10^{-3}) \text{ m}^3 \text{ and}$$

$I = \frac{1}{12}(0.075)(0.23^3) = 76.04375(10^{-6}) \text{ m}^4$. Provide the shear stress check by providing shear formula,

$$\tau_{\text{max}} = \frac{V_{\text{max}} Q_{\text{max}}}{It}$$

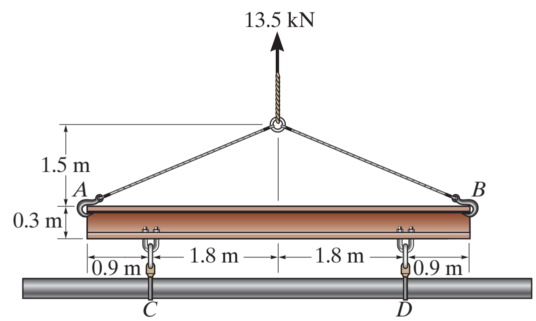
$$= \frac{[108(10^3)][0.4959375(10^{-3})]}{[76.04375(10^{-6})](0.075)}$$

$$= 9.391(10^6) \text{ N/m}^2 = 9.39 \text{ MPa} < \tau_{\text{allow}} = 70 \text{ MPa } (O.K!)$$



Ans:
Use $h = 230$ mm

11–23. The spreader beam AB is used to slowly lift the 13.5 kN pipe that is centrally located on the straps at C and D. If the beam is a W310×67, determine if it can safely support the load. The allowable bending stress is $\sigma_{\text{allow}} = 154$ MPa and the allowable shear stress is $\tau_{\text{allow}} = 84$ MPa.



SOLUTION

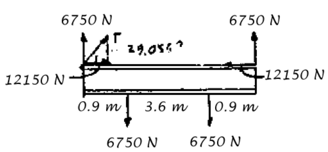
$$F_h = \frac{6750}{\tan 29.055°} = 12150 \text{ N}$$

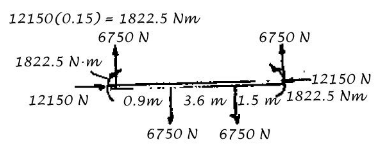
For W310 × 67 $\quad [S = 948(10^3)\text{ mm}^3, d = 306 \text{ mm}, t_w = 8.51 \text{ mm}]$

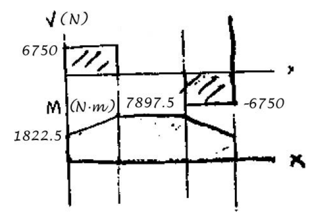
$$\sigma_{\max} = \frac{M_{\max}}{S}; \quad \sigma_{\max} = \frac{7897.5}{0.948(10^{-3})} = 8.331(10^6)\text{ N/m}^2 = 8.33 \text{ MPa} < 154 \text{ MPa} \quad \text{OK}$$

$$\tau_{\max} = \frac{V_{\max}}{A_{web}}; \quad \tau = \frac{6750}{(0.00851)(0.306)} = 2.592(10^6)\text{ N/m}^2 = 2.59 \text{ MPa} < 84 \text{ MPa} \quad \text{OK}$$

Yes. **Ans.**

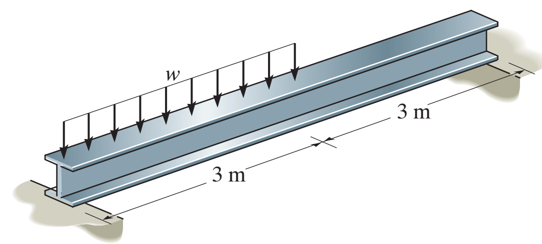






Ans:
Yes.

***11–24.** Determine the maximum uniform loading w the W310 × 21 beam will support if the allowable bending stress is $\sigma_{\text{allow}} = 150$ MPa and the allowable shear stress is $\tau_{\text{allow}} = 84$ MPa.



SOLUTION

From the moment diagram, Fig. a, $M_{\max} = 2.53125\,w$. For W310 × 21, $S_x = 244(10^3)\text{ mm}^3$, $d = 303$ mm and $t_w = 5.08$ mm.

$$\sigma_{\text{allow}} = \frac{M_{\max}}{S}$$

$$150(10^6) = \frac{2.53125w}{0.244(10^{-3})}$$

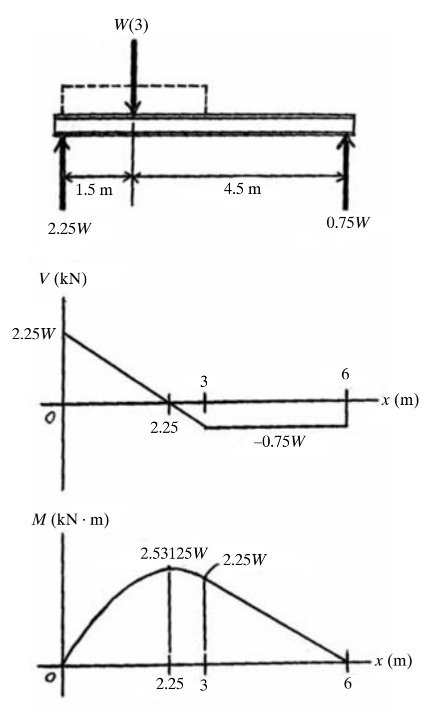
$$w = 14.46(10^3)\text{ N/m} = 14.5\text{ kN/m}$$ **Ans.**

From the shear diagram, Fig. a, $V_{\max} = 2.25(14.46) = 32.53$ kN. Provide a shear stress check on W310 × 21,

$$\tau_{\max} = \frac{V_{\max}}{t_w\, d}$$

$$= \frac{32.53(10^3)}{0.00508(0.303)} = 21.14(10^6)\text{ N/m}^2$$

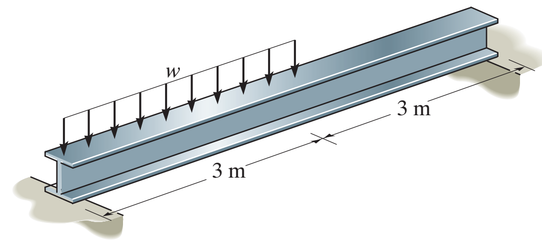
$$= 21.1\text{ MPa} < \tau_{\text{allow}} = 84\text{ MPa }(O.K)$$



(a)

Ans:
$w = 14.5$ kN/m

11–25. Determine if the W360 × 33 beam will safely support a loading of $w = 25$ kN/m. The allowable bending stress is $\sigma_{\text{allow}} = 150$ MPa and the allowable shear stress is $\tau_{\text{allow}} = 84$ MPa.



SOLUTION

For W360 × 33 $\quad S_x = 475(10^3)\ \text{mm}^3 \quad d = 349$ mm and $t_w = 5.84$ mm. From the moment diagram, Fig. *a*, $M_{\max} = 63.28125$ kN · m.

$$\sigma_{\max} = \frac{M_{\max}}{S}$$

$$= \frac{63.28125(10^3)}{0.475(10^{-3})} = 133.22(10^6)\ \text{N/m}^2$$

$$= 133\ \text{MPa} < \sigma_{\text{allow}} = 150\ \text{MPa}\ (O.K!)$$

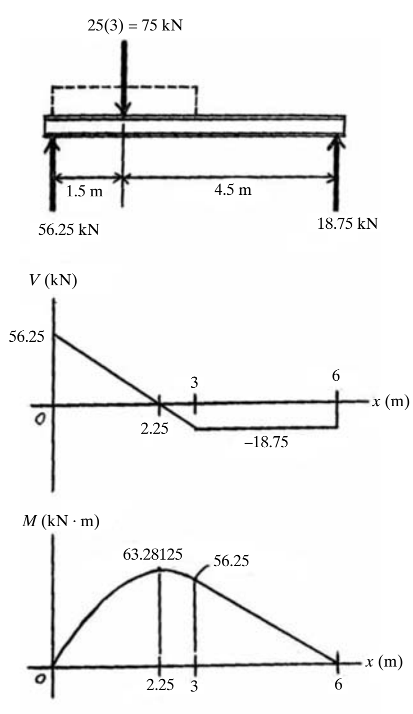
From the shear diagram, Fig. *a*, $V_{\max} = 56.25$ kN.

$$\tau_{\max} = \frac{V_{\max}}{t_w\, d}$$

$$= \frac{56.25(10^3)}{0.00584(0.349)} = 27.60(10^6)\ \text{N/m}^2$$

$$= 27.6\ \text{MPa} < \tau_{\text{allow}} = 84\ \text{MPa (O.K!)}$$

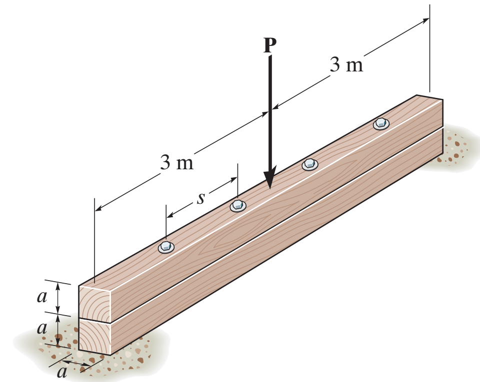
Based on the investigated results, we conclude that W360 × 33 can safely support the loading.



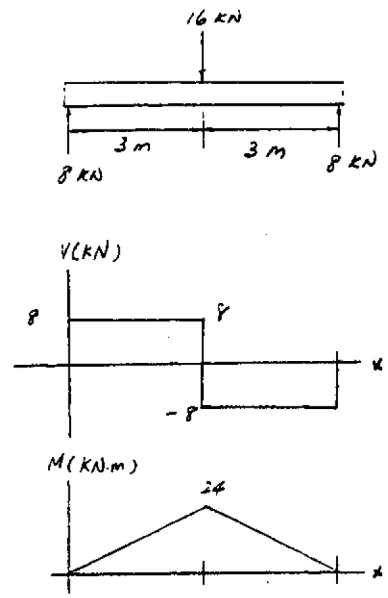
(a)

Ans:
Yes

11–26. The simply supported beam supports a load of $P = 16$ kN. Determine the smallest dimension a of each timber if the allowable bending stress for the wood is $\sigma_{allow} = 30$ MPa and the allowable shear stress is $\tau_{allow} = 800$ kPa. Also, if each bolt can sustain a shear of 2.5 kN, determine the spacing s of the bolts at the calculated dimension a.



SOLUTION



Section Properties:

$$I = \frac{1}{12}(a)(2\,a)^3 = 0.66667\,a^4$$

$$Q_{max} = \bar{y}'A' = \frac{a}{2}(a)(a) = 0.5\,a^3$$

Assume bending controls.

$$\sigma_{allow} = \frac{M_{max}c}{I}; \qquad 30(10^6) = \frac{24(10^3)a}{0.66667\,a^4}$$

$$a = 0.106266\text{ m} = 106\text{ mm}$$ **Ans.**

Check shear:

$$\tau_{max} = \frac{VQ}{It} = \frac{8(10^3)(0.106266/2)(0.106266)^2}{0.66667(0.106266^4)(0.106266)} = 531\text{ kPa} < \tau_{allow} = 800\text{ kPa} \quad \text{OK}$$

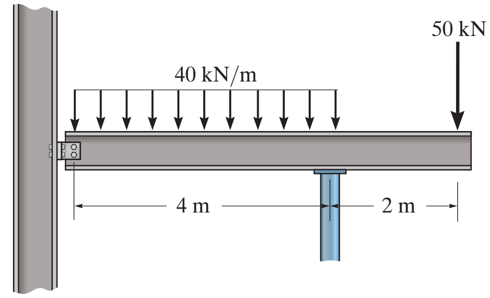
Bolt spacing:

$$q = \frac{VQ}{I} = \frac{8(10^3)(0.106266/2)(0.106266^2)}{0.66667(0.106266^4)} = 56462.16\text{ N/m}$$

$$s = \frac{2.5(10^3)}{56462.16} = 0.04427\text{ m} = 44.3\text{ mm}$$ **Ans.**

Ans:
$a = 106$ mm, $s = 44.3$ mm

11–27. Select the lightest W360 shape section from Appendix B that can safely support the loading acting on the overhanging beam. The beam is made from steel having an allowable normal stress of $\sigma_{allow} = 150$ MPa and an allowable shear stress of $\tau_{allow} = 80$ MPa.



SOLUTION

Shear and Moment Diagram: As shown in Fig. *a*.

Bending Stress: Referring to the moment diagram, Fig. *a*, $M_{max} = 100 \text{ kN}\cdot\text{m}$. Applying the flexure formula,

$$S_{required} = \frac{M_{max}}{\sigma_{allow}} = \frac{100(10^3)}{150(10^6)}$$

$$= 0.6667(10^{-3}) \text{ m}^3 = 666.67(10^3) \text{ mm}^3$$

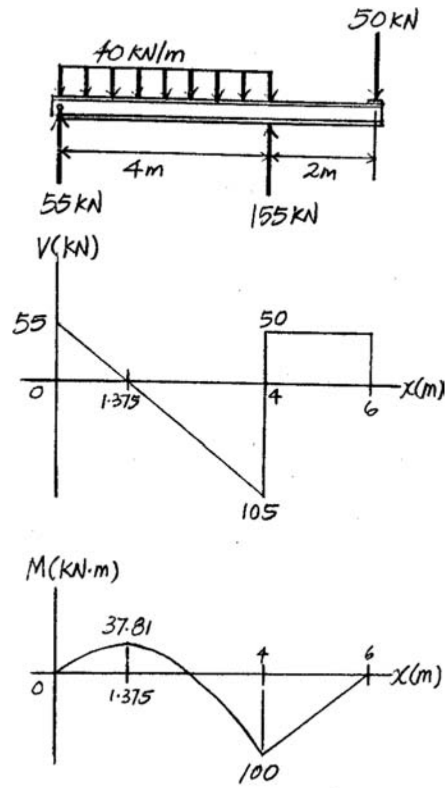
Select W360 × 45($S_x = 688(10^3)$ mm^3, $d = 352$ mm and $t_w = 6.86$ mm)

Shear Stress: Referring to the shear diagram, Fig. *a*, $V_{max} = 105$ kN. We have

$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{105(10^3)}{6.86(10^{-3})(0.352)}$$

$$= 43.48 \text{ MPa} < \tau_{allow} = 80 \text{ MPa} \quad \text{(OK)}$$

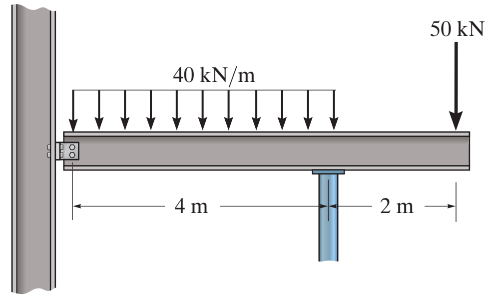
Hence, use W360 × 45 **Ans.**



(a)

Ans:
Use W360 × 45

***11–28.** Investigate if a W250 × 58 shape section can safely support the loading acting on the overhanging beam. The beam is made from steel having an allowable normal stress of $\sigma_{\text{allow}} = 150$ MPa and an allowable shear stress of $\tau_{\text{allow}} = 80$ MPa.



SOLUTION

Shear and Moment Diagram: As shown in Fig. *a*,

Bending Stress: Referring to the moment diagram, Fig. *a*, $M_{\text{max}} = 100\ \text{kN}\cdot\text{m}$. For a W250 × 58 section, $S_x = 693(10^3)\ \text{mm}^3 = 0.693(10^{-3})\ \text{m}^4$. Applying the flexure formula,

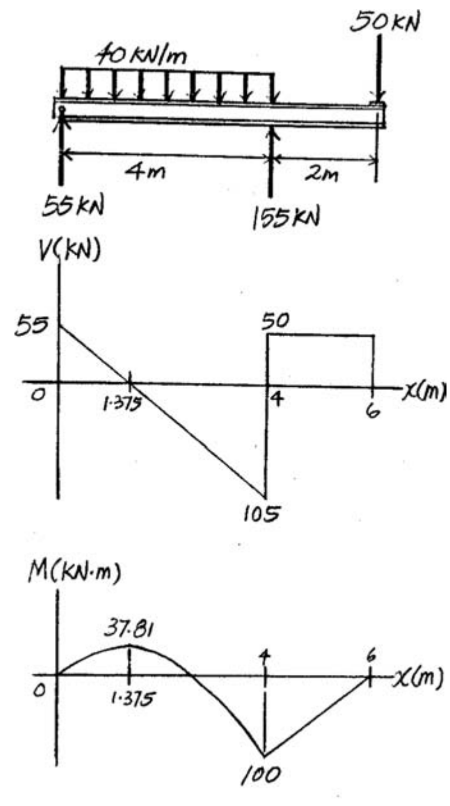
$$\sigma_{\text{max}} = \frac{M_{\text{max}}}{S_x} = \frac{100(10^3)}{0.693(10^{-3})} = 144.30\ \text{MPa} < \sigma_{\text{allow}} = 150\ \text{MPa} \quad \text{(OK)}$$

Shear Stress: Referring to the shear diagram, Fig. *a*, $V_{\text{max}} = 105$ kN. For a W250 × 58 section, $d = 252$ mm and $t_w = 8.00$ mm. We have

$$\tau_{\text{max}} = \frac{V_{\text{max}}}{t_w d} = \frac{105(10^3)}{8.00(10^{-3})(0.252)}$$

$$= 52.08\ \text{MPa} < \tau_{\text{allow}} = 80\ \text{MPa} \quad \text{(OK)}$$

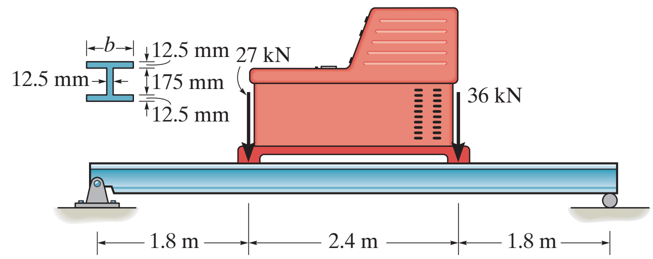
The W250 × 58 can safely support the loading. **Ans.**



(a)

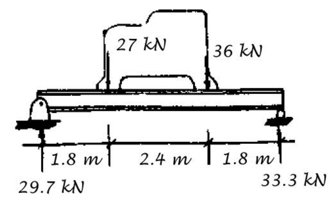
Ans:
Yes.

11–29. The beam is to be used to support the machine, which exerts the forces of 27 kN and 36 kN as shown. If the maximum bending stress is not to exceed $\sigma_{allow} = 154$ MPa, determine the required width b of the flanges.



SOLUTION

Section Properties:

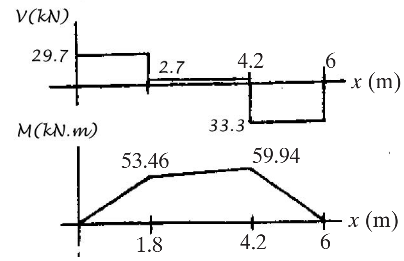


$$I = \frac{1}{12}(b)(0.2^3) - \frac{1}{12}(b - 0.0125)(0.175^3)$$

$$= 0.220052(10^{-3})b + 5.58268(10^{-6})$$

$$S = \frac{I}{c} = \frac{0.220052(10^{-3})b + 5.58268(10^{-6})}{0.1}$$

$$= 2.20052(10^{-3})b + 55.8268(10^{-6})$$



$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

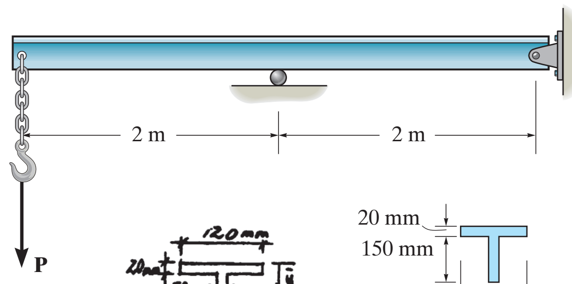
$$2.20052(10^{-3})b + 55.8268(10^{-6}) = \frac{59.94(10^3)}{154(10^6)}$$

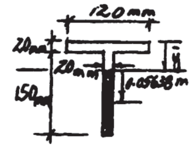
$b = 0.15151 \text{ m} = 152 \text{ mm}$ **Ans.**

Ans:
$b = 151.5$ mm.

11–30.

The steel beam has an allowable bending stress $\sigma_{\text{allow}} = 140$ MPa and an allowable shear stress of $\tau_{\text{allow}} = 90$ MPa. Determine the maximum load that can safely be supported.





SOLUTION

Section Properties:

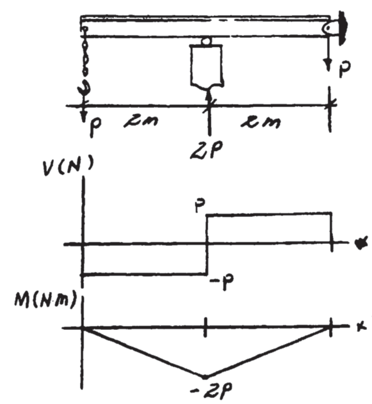
$$\bar{y} = \frac{(10)(120)(20) + (95)(150)(20)}{120(20) + 150(20)} = 57.22 \text{ mm}$$

$$Q_{\max} = \bar{y}'A' = (0.05638)(0.02)(0.170 - 0.05722) = 0.127168\left(10^{-3}\right) \text{ m}^3$$

$$I = \frac{1}{12}(0.12)\left(0.02^3\right) + 0.12(0.02)(0.05722 - 0.01)^2 +$$

$$\frac{1}{12}(0.02)\left(0.15^3\right) + 0.15(0.02)(0.095 - 0.05722)^2 = 15.3383\left(10^{-6}\right) \text{ m}^4$$

$$S = \frac{I}{c} = \frac{15.3383\left(10^{-6}\right)}{(0.170 - 0.05722)} = 0.136005\left(10^{-3}\right) \text{ m}^3$$



For Moment:

$$M = \sigma_{\text{allow}} S$$

$$2P = 140\left(10^6\right)(0.136005)\left(10^{-3}\right)$$

$$P = 9520 \text{ N} = 9.52 \text{ kN} \qquad \textbf{(Controls)} \qquad \textbf{Ans.}$$

For Shear:

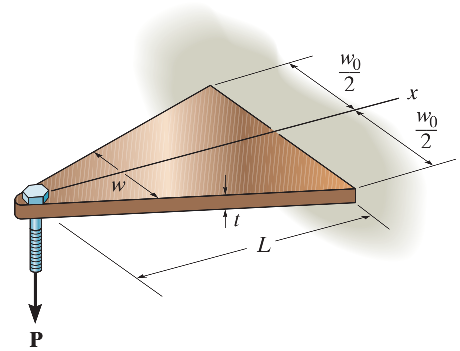
$$V \stackrel{?}{=} \tau_{\text{allow}}\left(\frac{It}{Q_{\max}}\right)$$

$$P = 90\left(10^6\right)\left(\frac{15.3383\left(10^{-6}\right)(0.02)}{0.127168\left(10^{-3}\right)}\right) = 217106 = 217 \text{ kN}$$

Ans:
$P = 9.52$ kN

11–31.

Determine the variation in the width w as a function of x for the cantilevered beam that supports a concentrated force **P** at its end so that it has a maximum bending stress σ_{allow} throughout its length. The beam has a constant thickness t.

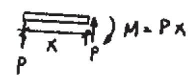


SOLUTION

Section Properties:

$$I = \frac{1}{12}(w)(t^3) \qquad S = \frac{I}{c} = \frac{\frac{1}{12}(w)(t^3)}{t/2} = \frac{wt^2}{6}$$

$$\sigma_{\text{allow}} = \frac{M}{S} = \frac{Px}{wt^2/6} \qquad \textbf{(1)}$$



At $x = L$,

$$\sigma_{\text{allow}} = \frac{PL}{w_0\, t^2/6} \qquad \textbf{(2)}$$

Equate Eqs. (1) and (2),

$$\frac{Px}{wt^2/6} = \frac{PL}{w_0\, t^2/6}$$

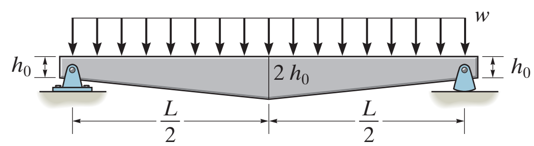
$$w = \frac{w_0}{L}x \qquad \textbf{Ans.}$$

Ans:

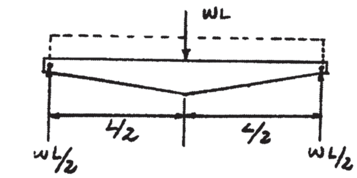
$w = \frac{w_0}{L}x$

***11–32.**

The tapered beam supports a uniform distributed load w. If it is made from a plate and has a constant width b, determine the absolute maximum bending stress in the beam.



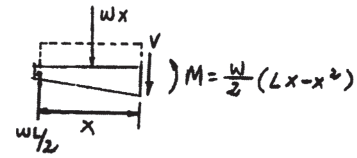
SOLUTION



Section Properties:

$$\frac{h - h_0}{x} = \frac{h_0}{\frac{L}{2}}; \qquad h = h_0\left(\frac{2}{L}x + 1\right)$$

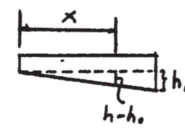
$$I = \frac{1}{12}bh_0^3\left(\frac{2}{L}x + 1\right)^3$$



$$S = \frac{I}{c} = \frac{\frac{1}{12}bh_0^3\left(\frac{2}{L}x + 1\right)^3}{\frac{h_0}{2}\left(\frac{2}{L}x + 1\right)} = \frac{1}{6}bh_0^2\left(\frac{2}{L}x + 1\right)^2$$

Bending Stress:

$$\sigma = \frac{M}{S} = \frac{\frac{w}{2}(Lx - x^2)}{\frac{1}{6}bh_0^2\left(\frac{2}{L}x + 1\right)^2} = \frac{3w}{bh_0^2}\left[\frac{Lx - x^2}{\left(\frac{2}{L}x + 1\right)^2}\right] \qquad \textbf{(1)}$$



$$\frac{d\sigma}{dx} = \frac{3w}{bh_0^2}\left[\frac{\left(\frac{2}{L}x + 1\right)^2(L - 2x) - (Lx - x^2)(2)\left(\frac{2}{L}x + 1\right)\left(\frac{2}{L}\right)}{\left(\frac{2}{L}x + 1\right)^4}\right] = 0$$

$$\left(\frac{2}{L}x + 1\right)(L - 2x) - \frac{4}{L}(Lx - x^2) = 0; \qquad x = \frac{L}{4}$$

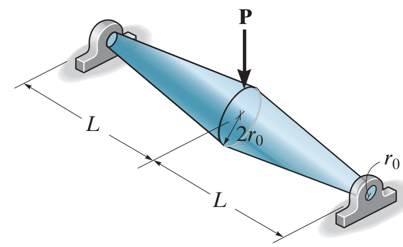
Hence, from Eq. (1),

$$\sigma_{\max} = \frac{3w}{bh_0^2}\left[\frac{L\left(\frac{L}{4}\right) - \left(\frac{L}{4}\right)^2}{\left(\frac{2}{L}\left(\frac{L}{4}\right) + 1\right)^2}\right] = \frac{wL^2}{4bh_0^2} \qquad \textbf{Ans.}$$

Ans:
$\sigma_{\max} = \dfrac{WL^2}{4bh_0^2}$

11–33.

The tapered beam supports the concentrated force **P** at its center. Determine the absolute maximum bending stress in the beam. The reactions at the supports are vertical.



SOLUTION

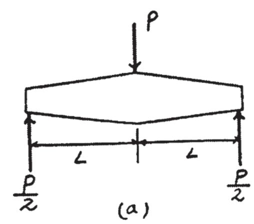
Moment Function: As shown on FBD(b).

Section Properties:

$$\frac{r - r_0}{x} = \frac{r_0}{L} \qquad r = \frac{r_0}{L}(x + L)$$

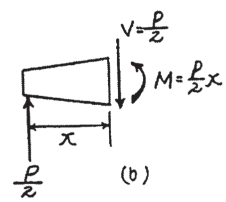
$$I = \frac{\pi}{4}\left[\frac{r_0}{L}(x + L)\right]^4 = \frac{\pi r_0^4}{4L^4}(x + L)^4$$

$$S = \frac{I}{c} = \frac{\frac{\pi r_0^4}{4L^4}(x + L)^4}{\frac{r_0}{L}(x + L)} = \frac{\pi r_0^3}{4L^3}(x + L)^3$$



Bending Stress: Applying the flexure formula,

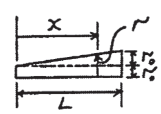
$$\sigma = \frac{M}{S} = \frac{\frac{P}{2}x}{\frac{\pi r_0^3}{4L^3}(x + L)^3} = \frac{2PL^3x}{\pi r_0^3(x + L)^3} \qquad \textbf{(1)}$$



In order to have the absolute maximum bending stress, $\frac{d\sigma}{dx} = 0$.

$$\frac{d\sigma}{dx} = \frac{2PL^3}{\pi r_0^3}\left[\frac{(x + L)^3(1) - x(3)(x + L)^2(1)}{(x + L)^6}\right] = 0$$

$$x = \frac{L}{2}$$



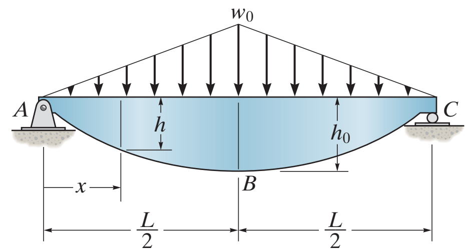
Substituting $x = \frac{L}{2}$ into Eq. (1) yields

$$\sigma_{\max} = \frac{8PL}{27\pi r_0^3} \qquad \textbf{Ans.}$$

Ans:
$\sigma_{\max} = \frac{8PL}{27\pi r_0^3}$

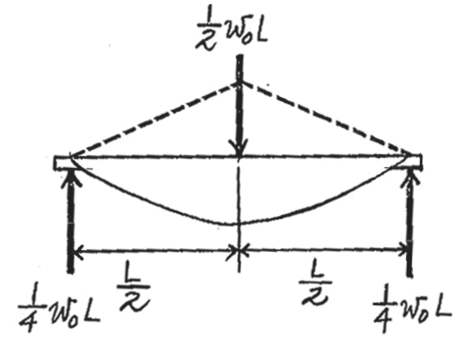
11–34.

The beam is made from a plate that has a constant thickness b. If it is simply supported and carries the distributed loading shown, determine the variation of its depth h as a function of x so that it maintains a constant maximum bending stress σ_{allow} throughout its length.



SOLUTION

Support Reactions: As shown on the free-body diagram of the entire beam, Fig. *a*.



(a)

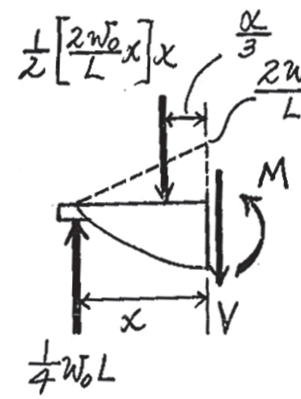
Moment Function: The distributed load as a function of x is

$$\frac{w}{x} = \frac{w_0}{L/2} \qquad w = \frac{2w_0}{L}x$$

The free-body diagram of the beam's left cut segment is shown in Fig. *b*. Considering the moment equilibrium of this free-body diagram,

$$\circlearrowleft + \Sigma M_O = 0; \qquad M + \frac{1}{2}\left[\frac{2w_0}{L}x\right]x\left(\frac{x}{3}\right) - \frac{1}{4}w_0Lx = 0$$

$$M = \frac{w_0}{12L}\left(3L^2x - 4x^3\right)$$



(b)

Section Properties: At position x, the height of the beam's cross section is h. Thus

$$I = \frac{1}{12}bh^3$$

Then

$$S = \frac{I}{c} = \frac{\frac{1}{12}bh^3}{h/2} = \frac{1}{6}bh^2$$

Bending Stress: The maximum bending stress σ_{max} as a function of x can be obtained by applying the flexure formula.

$$\sigma_{max} = \frac{M}{S} = \frac{\frac{w_0}{12L}\left(3L^2x - 4x^3\right)}{\frac{1}{6}bh^2} = \frac{w_0}{2bh^2L}\left(3L^2x - 4x^3\right), \qquad \textbf{(1)}$$

At $x = \frac{L}{2}$, $h = h_0$. From Eq. (1),

$$\sigma_{max} = \frac{w_0L^2}{2bh_0^{\,2}} \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2),

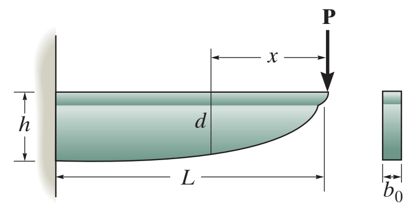
$$\frac{w_0}{2bh^2L}\left(3L^2x - 4x^3\right) = \frac{w_0L^2}{2bh_0^{\,2}}$$

$$h = \frac{h_0}{L^{3/2}}\left(3L^2x - 4x^3\right)^{1/2} \qquad \textbf{Ans.}$$

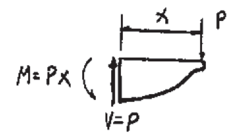
Ans:
$h = \frac{h_0}{L^{3/2}}(3L^2x - 4x^3)^{1/2}$

11–35.

Determine the variation in the depth d of a cantilevered beam that supports a concentrated force **P** at its end so that it has a constant maximum bending stress σ_{allow} throughout its length. The beam has a constant width b_0.



SOLUTION



Section Properties:

$$I = \frac{1}{12} b_0 d^3; \qquad S = \frac{I}{c} = \frac{\frac{1}{12} b_0 d^3}{\frac{d}{2}} = \frac{b_0 d^2}{6}$$

Maximum Bending Stress:

$$\sigma_{\text{allow}} = \frac{M}{S} = \frac{Px}{b_0 \frac{d^2}{6}} = \frac{6Px}{b_0 d^2} \qquad \textbf{(1)}$$

At $x = L, \quad d = h$

$$\sigma_{\text{allow}} = \frac{6PL}{b_0 h^2} \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2),

$$\frac{6Px}{b_0 d^2} = \frac{6PL}{b_0 h^2}$$

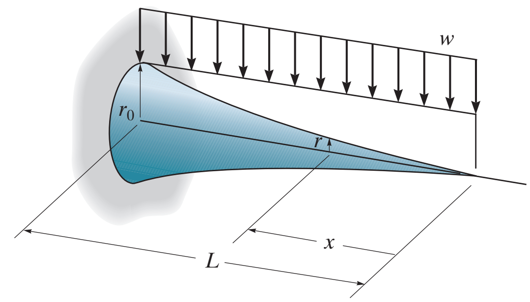
$$d = h\sqrt{\frac{x}{L}} \qquad \textbf{Ans.}$$

Ans:

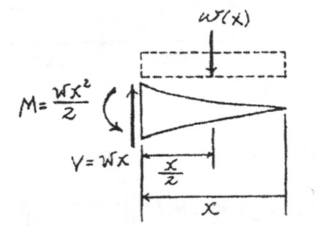
$d = h\sqrt{\frac{x}{L}}$

***11–36.**

Determine the variation of the radius r of the cantilevered beam that supports the uniform distributed load so that it has a constant maximum bending stress σ_{max} throughout its length.



SOLUTION



Moment Function: As shown on FBD.

Section Properties:

$$I = \frac{\pi}{4} r^4 \qquad S = \frac{I}{c} = \frac{\frac{\pi}{4} r^4}{r} = \frac{\pi}{4} r^3$$

Bending Stress: Applying the flexure formula,

$$\sigma_{max} = \frac{M}{S} = \frac{\frac{wx^2}{2}}{\frac{\pi}{4} r^3}$$

$$\sigma_{max} = \frac{2wx^2}{\pi r^3} \qquad \textbf{(1)}$$

At $x = L$, $r = r_0$. From Eq. (1),

$$\sigma_{max} = \frac{2wL^2}{\pi r_0^3} \qquad \textbf{(2)}$$

Equating Eq. (1) and (2) yields

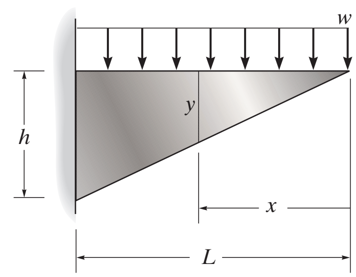
$$r^3 = \frac{r_0^3}{L^2} x^2 \qquad \textbf{Ans.}$$

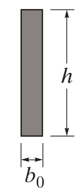
Ans:

$$r^3 = \frac{r_0^3}{L^2} x^2$$

11–37.

The tapered beam supports a uniform distributed load w. If it is made from a plate that has a constant width b_0, determine the absolute maximum bending stress in the beam.





SOLUTION

Section Properties:

$$\frac{y}{h} = \frac{x}{L}; \qquad y = \frac{h}{L}x$$

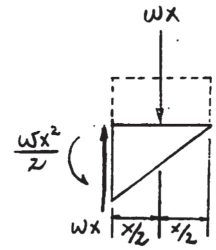
$$I = \frac{1}{12}(b_0)\left(\frac{h}{L}x\right)^3 = \frac{b_0\,h^3}{12L^3}x^3$$

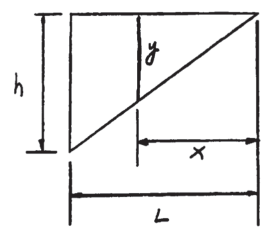
$$S = \frac{I}{c} = \frac{\frac{b_0\,h^3}{12L^3}x^3}{\frac{h}{2L}x} = \frac{b_0\,h^2}{6L^2}x^2$$

Bending Stress:

$$\sigma_{\max} = \frac{M}{S} = \frac{\frac{w}{2}x^2}{\frac{b_0\,h^2}{6L^2}x^2} = \frac{3wL^2}{b_0h^2} \qquad \textbf{Ans.}$$

The bending stress is independent of x. Therefore, the stress is constant throughout the span.



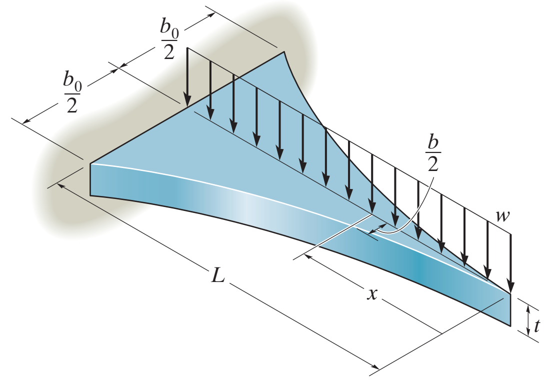


Ans:

$\sigma_{\max} = \dfrac{3wL^2}{b_0h^2}$

11–38.

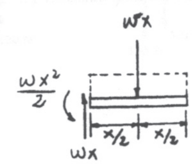
Determine the variation in the width b as a function of x for the cantilevered beam that supports a uniform distributed load along its centerline so that it has the same maximum bending stress σ_{allow} throughout its length. The beam has a constant depth t.



SOLUTION

Section Properties:

$$I = \frac{1}{12} b\, t^3 \qquad S = \frac{I}{c} = \frac{\frac{1}{12} b\, t^3}{\frac{t}{2}} = \frac{t^2}{6} b$$



Bending Stress:

$$\sigma_{\text{allow}} = \frac{M}{S} = \frac{\frac{wx^2}{2}}{\frac{t^2}{6}b} = \frac{3wx^2}{t^2 b} \qquad \textbf{(1)}$$

At $x = L, b = b_0$

$$\sigma_{\text{allow}} = \frac{3wL^2}{t^2 b_0} \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2) yields:

$$\frac{3wx^2}{t^2 b} = \frac{3wL^2}{t^2 b_0}$$

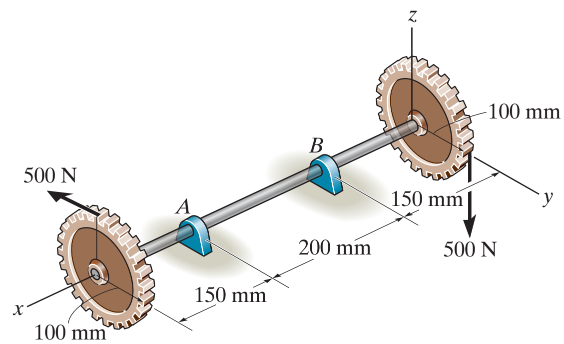
$$b = \frac{b_0}{L^2} x^2 \qquad \textbf{Ans.}$$

Ans:

$b = \frac{b_0}{L^2} x^2$

11–39.

The tubular shaft has an inner diameter of 15 mm. Determine to the nearest millimeter its minimum outer diameter if it is subjected to the gear loading. The bearings at A and B exert force components only in the y and z directions on the shaft. Use an allowable shear stress of $\tau_{\text{allow}} = 70$ MPa, and base the design on the maximum shear stress theory of failure.



SOLUTION

$$I = \frac{\pi}{4}(c^4 - 0.0075^4) \text{ and } J = \frac{\pi}{2}(c^4 - 0.0075^4)$$

$$\tau_{\text{allow}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\text{allow}} = \sqrt{\left(\frac{Mc}{2I}\right)^2 + \left(\frac{Tc}{J}\right)^2}$$

$$\tau_{\text{allow}}^2 = \frac{M^2c^2}{4I^2} + \frac{T^2c^2}{J^2}$$

$$\tau_{\text{allow}}^2\left(\frac{c^4 - 0.0075^4}{c}\right)^2 = \frac{4M^2}{\pi^2} + \frac{4T^2}{\pi^2}$$

$$\frac{c^4 - 0.0075^4}{c} = \frac{2}{\pi\,\tau_{\text{allow}}}\sqrt{M^2 + T^2}$$

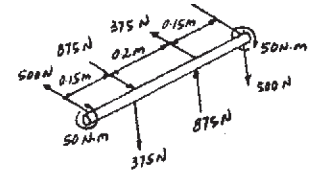
$$\frac{c^4 - 0.0075^4}{c} = \frac{2}{\pi(70)(10^6)}\sqrt{75^2 + 50^2}$$

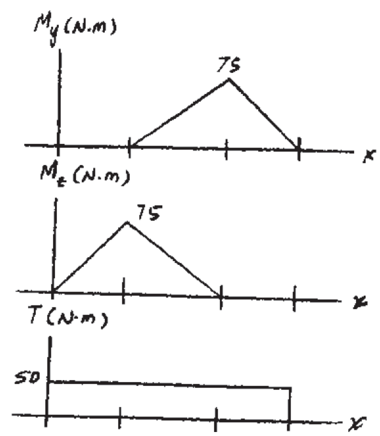
$$c^4 - 0.0075^4 = 0.8198(10^{-6})c$$

Solving, $c = 0.0103976$ m

$d = 2c = 0.0207952 \text{ m} = 20.8 \text{ mm}$

Use $d = 21$ mm **Ans.**

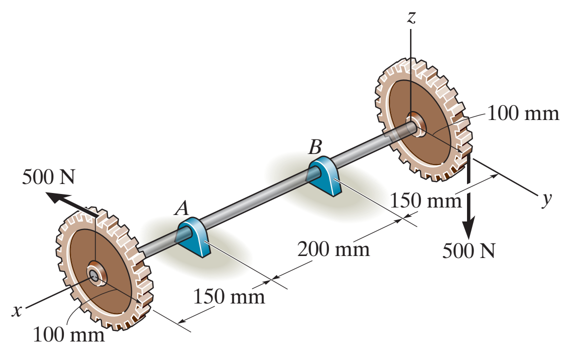




Ans:
Use $d = 21$ mm

***11–40.**

Determine to the nearest millimeter the minimum diameter of the solid shaft if it is subjected to the gear loading. The bearings at A and B exert force components only in the y and z directions on the shaft. Base the design on the maximum distortion energy theory of failure with $\sigma_{\text{allow}} = 150$ MPa.

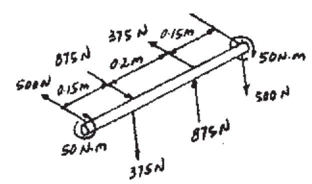


SOLUTION

$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

Let $a = \dfrac{\sigma_x}{2}$, $b = \sqrt{\dfrac{\sigma_x^2}{4} + \tau_{xy}^2}$

$$\sigma_1 = a + b, \quad \sigma_2 = a - b$$

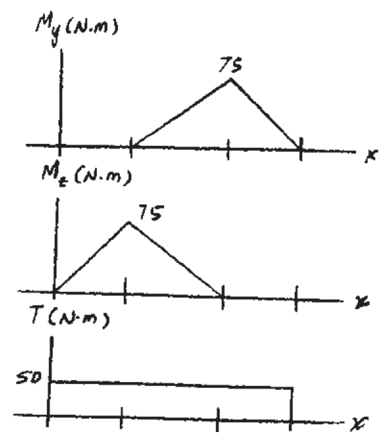


Require

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{\text{allow}}^2$$

$$a^2 + 2ab + b^2 - [a^2 - b^2] + a^2 - 2ab + b^2 = \sigma_{\text{allow}}$$

$$a^2 + 3b^2 = \sigma_{\text{allow}}^2$$



$$\frac{\sigma_x^2}{4} + 3\left(\frac{\sigma_x^2}{4} + \tau_{xy}^2\right) = \sigma_{\text{allow}}^2$$

$$\sigma_x^2 + 3\tau_{xy}^2 = \sigma_{\text{allow}}^2$$

$$\left(\frac{Mc}{\frac{\pi}{4}c^4}\right)^2 + 3\left(\frac{Tc}{\frac{\pi}{2}c^4}\right)^2 = \sigma_{\text{allow}}^2$$

$$\frac{1}{c^6}\left[\left(\frac{4M}{\pi}\right)^2 + 3\left(\frac{2T}{\pi}\right)^2\right] = \sigma_{\text{allow}}^2$$

$$c^6 = \frac{16}{\sigma_{\text{allow}}^2\pi^2}M^2 + \frac{12T^2}{\sigma_{\text{allow}}^2\pi^2}$$

$$c = \left(\frac{4}{\sigma_{\text{allow}}^2\pi^2}\left(4M^2 + 3T^2\right)\right)^{\frac{1}{6}}$$

$$= \left[\frac{4}{(150(10^6))^2(\pi)^2}(4(75)^2 + 3(50)^2)\right]^{\frac{1}{6}} \infty \ 0.009025 \text{ m}$$

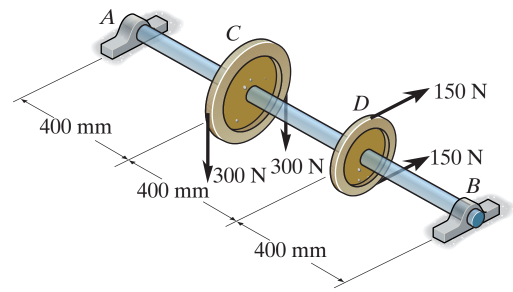
$$d = 2c = 0.0181 \text{ m}$$

Use $d = 19$ mm. **Ans.**

Ans:
Use $d = 19$ mm

11–41.

The 50-mm-diameter shaft is supported by journal bearings at A and B. If the pulleys C and D are subjected to the loadings shown, determine the absolute maximum bending stress in the shaft.



SOLUTION

Internal Moment Components: The shaft is subjected to two bending moment components $\mathbf{M}_z$ and $\mathbf{M}_y$.

Bending Stress: Since all the axes through the centroid of the circular cross section of the shaft are principal axes, then the resultant moment $M = \sqrt{M_y{}^2 + M_z{}^2}$ can be used to determine the maximum bending stress. The maximum bending moment occurs at C ($x = 0.4$ m). Then, $M_{\max} = \sqrt{40^2 + 160^2} = 164.92\ \mathrm{N \cdot m}$.

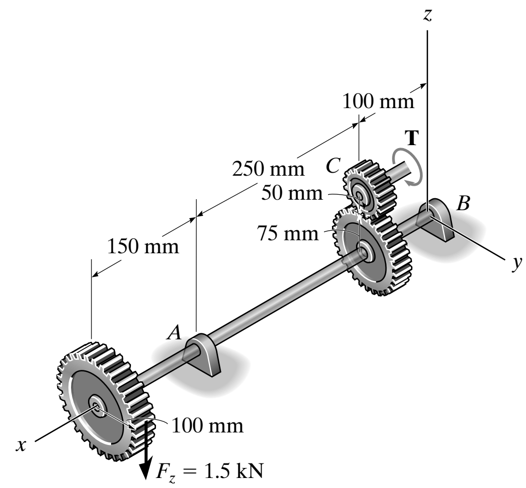
$$\sigma_{\max} = \frac{M_{\max}\, c}{I}$$

$$= \frac{164.92(0.025)}{\frac{\pi}{4}(0.025^4)}$$

$$= 13.439\ \mathrm{MPa} = 13.4\ \mathrm{MPa}$$ **Ans.**

Ans:
$\sigma_{\max} = 13.4\ \mathrm{MPa}$

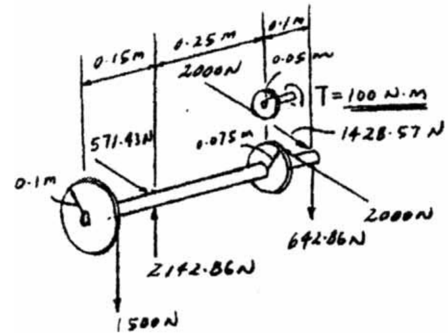
11–42. The end gear connected to the shaft is subjected to the loading shown. If the bearings at A and B exert only y and z components of force on the shaft, determine the equilibrium torque T at gear C and then determine the smallest diameter of the shaft to the nearest millimeter that will support the loading. Use the maximum-shear-stress theory of failure with $\tau_{\text{allow}} = 60$ MPa.



SOLUTION

From the free - body diagrams:

$T = 100\ \text{N}\cdot\text{m}$ **Ans.**



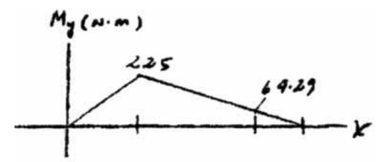
Critical section is at support A.

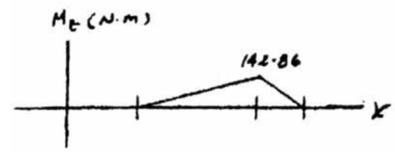
$$c = \left[\frac{2}{\pi\,\tau_{\text{allow}}}\sqrt{M^2 + T^2}\right]^{\frac{1}{3}} = \left[\frac{2}{\pi(60)(10^6)}\sqrt{225^2 + 150^2}\right]^{\frac{1}{3}}$$

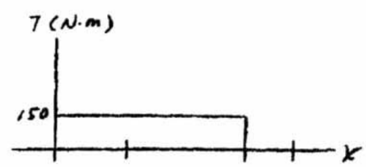
$$= 0.01421\ \text{m}$$

$d = 2c = 0.0284\ \text{m} = 28.4\ \text{mm}$

Use $d = 29$ mm **Ans.**



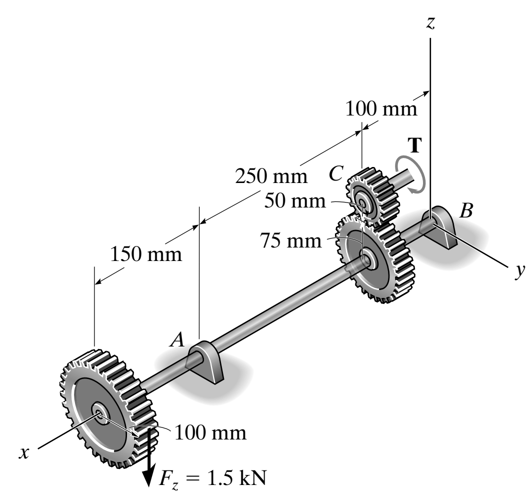




Ans:

$T = 100\ \text{N}\cdot\text{m}$, Use $d = 29$ mm

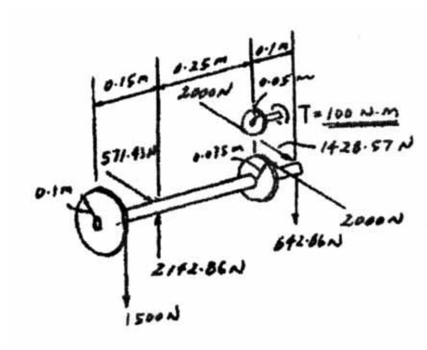
11–43. The end gear connected to the shaft is subjected to the loading shown. If the bearings at A and B exert only y and z components of force on the shaft, determine the equilibrium torque T at gear C and then determine the smallest diameter of the shaft to the nearest millimeter that will support the loading. Use the maximum-distortion-energy theory of failure with $\sigma_{\text{allow}} = 80$ MPa.



SOLUTION

From the free-body diagrams:

$T = 100\ \text{N}\cdot\text{m}$ **Ans.**



Critical section is at support A.

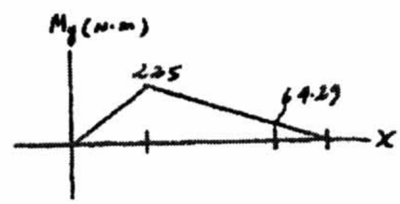
$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

Let $a = \dfrac{\sigma_x}{2}, b = \sqrt{\dfrac{\sigma_x^2}{4} + \tau_{xy}^2}$

$\sigma_1 = a + b, \sigma_2 = a - b$

Require,

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{\text{allow}}^2 a^2 + 2ab + b^2 - [a^2 - b^2] + a^2 - 2ab + b^2 = \sigma_{\text{allow}}^2$$

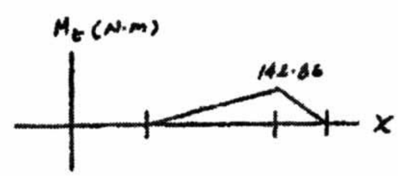


$$a^2 + 3b^2 = \sigma_{\text{allow}}^2$$

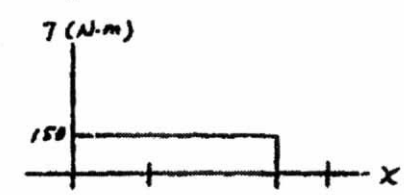
$$\frac{\sigma_x^2}{4} + 3\left(\frac{\sigma_x^2}{4} + \tau_{xy}^2\right) = \sigma_{\text{allow}}^2$$

$$\sigma_x^2 + 3\tau_{xy}^2 = \sigma_{\text{allow}}^2$$

$$\left(\frac{Mt}{\frac{\pi}{4}c^4}\right)^2 + 3\left(\frac{Tc}{\frac{\pi}{2}c^4}\right)^2 = \sigma_{\text{allow}}^2$$



$$\frac{1}{c^4}\left[\left(\frac{4M}{\pi}\right)^2 + 3\left(\frac{2T}{\pi}\right)^2\right] = \sigma_{\text{allow}}^2$$



$$c^4 = \frac{16}{\sigma_{\text{allow}}^2\,\pi^2}M^2 + \frac{12T^2}{\sigma_{\text{allow}}^2\,\pi^2}$$

$$c = \left(\frac{4}{\sigma_{\text{allow}}^2\,\pi^2}(4M^2 + 3T^2)\right)^{\frac{1}{2}}$$

$$= \left[\frac{4}{(80(10^6))^2(\pi)^2}(4(225)^2 + 3(150)^2)\right]^{\frac{1}{2}}$$

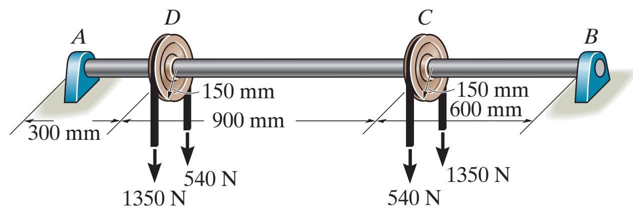
$$= 0.01605 \text{ m}$$

$d = 2c = 0.0321 \text{ m} = 32.1 \text{ mm}$

Use $d = 33$ mm **Ans.**

Ans:
$T = 100\ \text{N}\cdot\text{m}$, Use $d = 33$ mm

***11–44.** The two pulleys attached to the shaft are loaded as shown. If the bearings at A and B exert only vertical forces on the shaft, determine the required diameter of the shaft to the nearest mm using the maximum-distortion energy theory. $\sigma_{allow} = 469$ MPa.



SOLUTION

Section just to the left of point C is the most critical. Both states of stress will yield the same result.

$$\sigma_{a,b} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

Let $\frac{\sigma}{2} = A$ and $\sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = B$

$$\sigma_a^2 = (A + B)^2 \qquad \sigma_b^2 = (A - B)^2$$

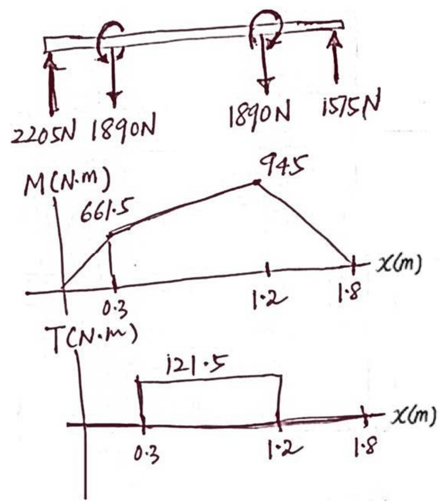
$$\sigma_a \sigma_b = (A + B)(A - B)$$

$$\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2 = A^2 + B^2 + 2AB - A^2 + B^2 + A^2 + B^2 - 2AB$$

$$= A^2 + 3B^2$$

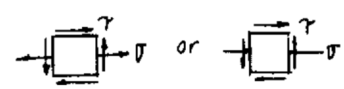
$$= \frac{\sigma^2}{4} + 3\left(\frac{\sigma^2}{4} + \tau^2\right)$$

$$= \sigma^2 + 3\tau^2$$



$$\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2 = \sigma_{allow}^2$$

$$\sigma^2 + 3\tau^2 = \sigma_{allow}^2 \qquad (1)$$



$$\sigma = \frac{Mc}{I} = \frac{Mc}{\frac{\pi}{4}c^4} = \frac{4M}{\pi c^3}$$

$$\tau = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} = \frac{2T}{\pi c^3}$$

From Eq (1)

$$\frac{16M^2}{\pi^2 c^6} + \frac{12T^2}{\pi^2 c^6} = \sigma_{allow}^2$$

$$c = \left(\frac{16M^2 + 12T^2}{\pi^2 \sigma_{allow}^2}\right)^{1/6} = \left\{\frac{16(945^2) + 12(121.5^2)}{\pi^2[469(10^6)]^2}\right\}^{1/6}$$

$c = 0.013718$ m

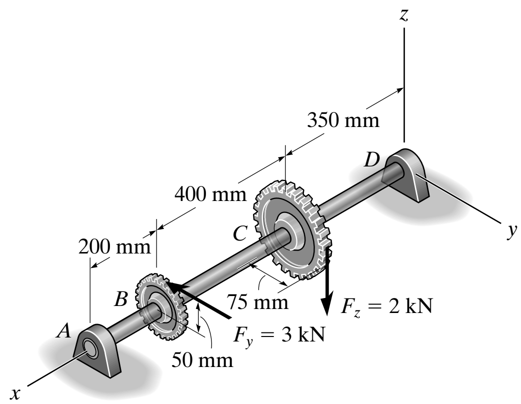
$d = 2c = 2(0.013718) = 0.027435$ m $= 27.44$ mm

Use $d = 28$ mm **Ans.**

Ans:

Use $d = 28$ mm

11–45. The bearings at A and D exert only y and z components of force on the shaft. If $\tau_{\text{allow}} = 60$ MPa, determine to the nearest millimeter the smallest-diameter shaft that will support the loading. Use the maximum-shear-stress theory of failure.



SOLUTION

Critical moment is at point B:

$$M = \sqrt{(473.7)^2 + (147.4)^2} = 496.1 \text{ N} \cdot \text{m}$$

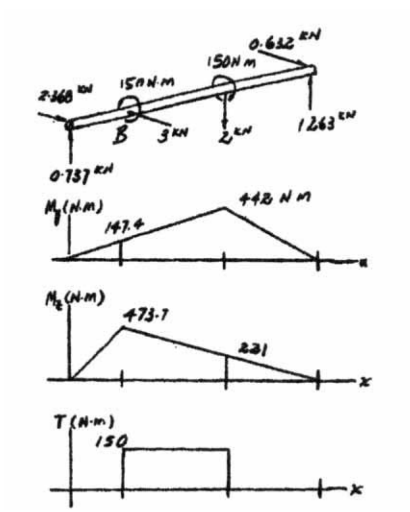
$$T = 150 \text{ N} \cdot \text{m}$$

$$c = \left(\frac{2}{\pi\,\tau_{\text{allow}}}\sqrt{M^2 + T^2}\right)^{1/3} = \left(\frac{2}{\pi(60)(10^6)}\sqrt{496.1^2 + 150^2}\right)^{1/3} = 0.0176 \text{ m}$$

$$c = 0.0176 \text{ m} = 17.6 \text{ mm}$$

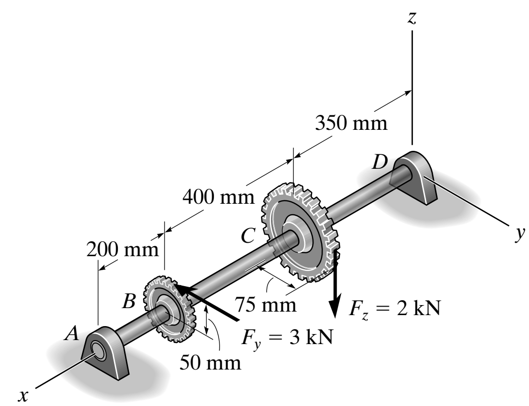
$$d = 2c = 35.3 \text{ mm}$$

Use $d = 36$ mm **Ans.**



Ans:
Use $d = 36$ mm

11–46. The bearings at A and D exert only y and z components of force on the shaft. If $\tau_{\text{allow}} = 60$ MPa, determine to the nearest millimeter the smallest-diameter shaft that will support the loading. Use the maximum-distortion-energy theory of failure. $\sigma_{\text{allow}} = 130$ MPa.



SOLUTION

The critical moment is at B.

$$M = \sqrt{(473.7)^2 + (147.4)^2} = 496.1 \text{ N}\cdot\text{m}$$

$$T = 150 \text{ N}\cdot\text{m}$$

Since,

$$\sigma_{a,b} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

Let $\dfrac{\sigma}{2} = A$ and $\sqrt{\left(\dfrac{\sigma}{2}\right)^2 + \tau^2} = B$

$$\sigma_a^2 = (A + B)^2 \qquad \sigma_b^2 = (A - B)^2$$

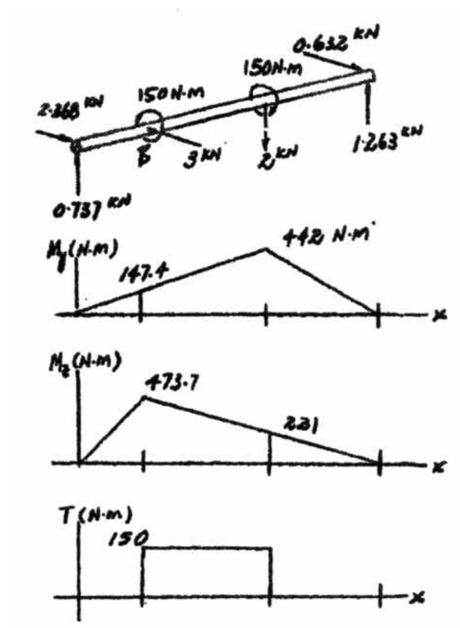
$$\sigma_a \sigma_b = (A + B)(A - B)$$

$$\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2 = A^2 + B^2 + 2AB - A^2 + B^2 + A^2 + B^2 - 2AB$$

$$= A^2 + 3B^2$$

$$= \frac{\sigma^2}{4} + 3\left(\frac{\sigma^2}{4} + \tau^2\right)$$

$$= \sigma^2 + 3\tau^2$$



$$\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2 = \sigma_{\text{allow}}^2$$

$$\sigma^2 + 3\tau^2 = \sigma_{\text{allow}}^2 \qquad \textbf{(1)}$$

$$\sigma = \frac{Mc}{I} = \frac{Mc}{\frac{\pi}{4}c^4} = \frac{4M}{\pi c^3}$$

$$\tau = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} = \frac{2T}{\pi c^3}$$

From Eq (1)

$$\frac{16M^2}{\pi^2 c^4} + \frac{12T^2}{\pi^2 c^4} = \sigma_{\text{allow}}^2$$

$$c = \left(\frac{16M^2 + 12T^2}{\pi^2 \sigma_{\text{allow}}^2}\right)^{1/6}$$

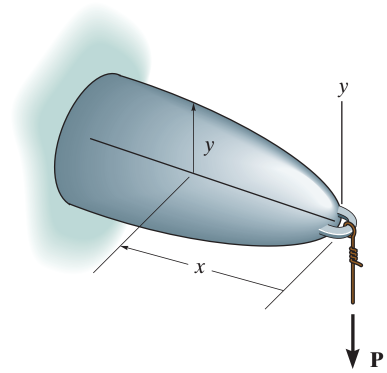
$$= \left(\frac{16(496.1)^2 + 12(150)^2}{\pi^2((130)(10^4))^2}\right)^{1/4} = 0.01712 \text{ m}$$

$$d = 2c = 34.3 \text{ mm}$$ **Ans.**

Ans:
Use $d = 34.3$ mm

R11–1.

The cantilevered beam has a circular cross section. If it supports a force **P** at its end, determine its radius y as a function of x so that it is subjected to a constant maximum bending stress σ_{allow} throughout its length.



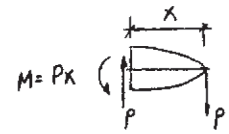
SOLUTION

Section Properties:

$$I = \frac{\pi}{4} y^4$$

$$S = \frac{I}{c} = \frac{\frac{\pi}{4} y^4}{y} = \frac{\pi}{4} y^3$$

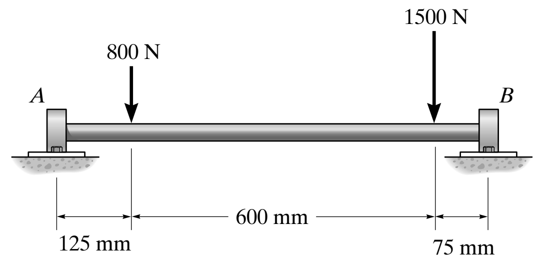
$$\sigma_{\text{allow}} = \frac{M}{S} = \frac{Px}{\frac{\pi}{4} y^3}$$



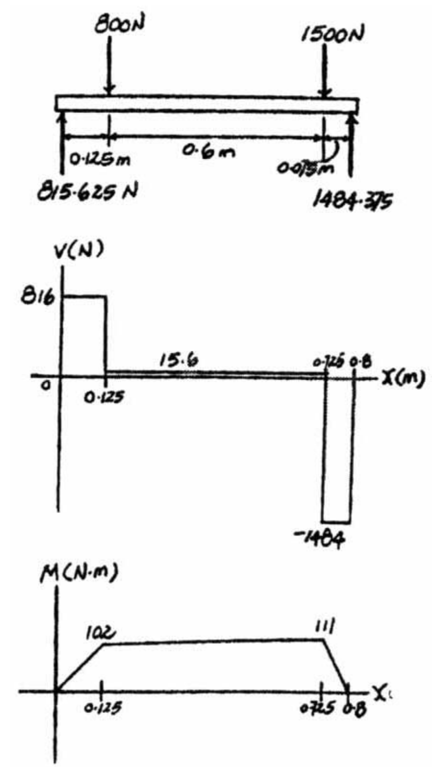
$$y = \left[\frac{4P}{\pi\,\sigma_{\text{allow}}} x\right]^{\frac{1}{3}}$$ **Ans.**

Ans:
$$y = \left[\frac{4P}{\pi\,\sigma_{\text{allow}}} x\right]^{1/3}$$

R11–2. Draw the shear and moment diagrams for the shaft, and then determine its required diameter to the nearest millimeter if $\sigma_{allow} = 140$ MPa and $\tau_{allow} = 80$ MPa. The bearings at A and B exert only vertical reactions on the shaft.



SOLUTION



Bending Stress: From the moment diagram, $M_{max} = 111$ N·m. Assume bending controls the design. Applying the flexure formula.

$$\sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$140\left(10^6\right) = \frac{111\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

$$d = 0.02008 \text{ m} = 20.1 \text{ mm}$$

Use $d = 21$ mm **Ans.**

Shear Stress: Provide a shear stress check using the shear formula with

$$I = \frac{\pi}{4}\left(0.0105^4\right) = 9.5466\left(10^{-9}\right) \text{ m}^4$$

$$Q_{max} = \frac{4(0.0105)}{3\pi}\left[\frac{1}{2}(\pi)(0.0105)^2\right] = 0.77175\left(10^{-6}\right) \text{ m}^3$$

From the shear diagram, $V_{max} = 1484$ N.

$$\tau_{max} = \frac{V_{max}\,Q_{max}}{It}$$

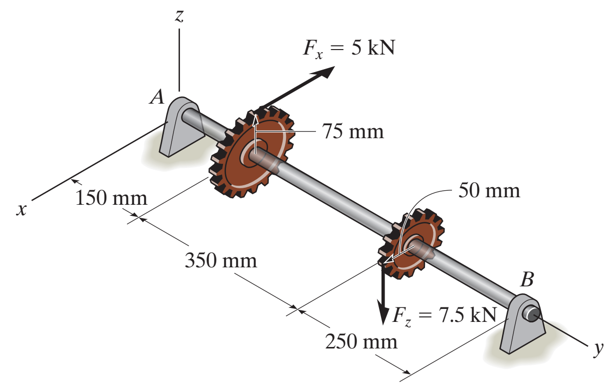
$$= \frac{1484\left[0.77175(10^{-6})\right]}{9.5466(10^{-9})(0.021)}$$

$$= 5.71 \text{ MPa} < \tau_{allow} = 80 \text{ MPa } (O.K!)$$

Ans:
Use $d = 21$ mm

R11–3.

The journal bearings at A and B exert only x and z components of force on the shaft. Determine the shaft's diameter to the nearest millimeter so that it can resist the loadings without exceeding an allowable shear stress of $\tau_{allow} = 80$ MPa. Use the maximum shear stress theory of failure.



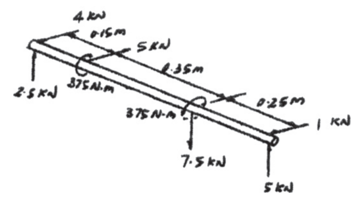
SOLUTION

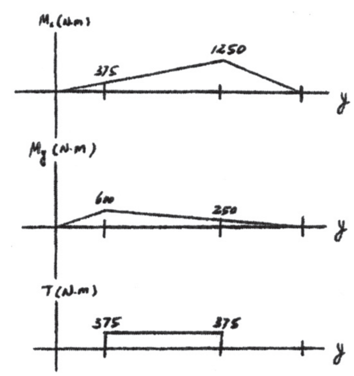
Maximum resultant moment $M = \sqrt{1250^2 + 250^2} = 1274.75\ \text{N}\cdot\text{m}$

$$c = \left[\frac{2}{\pi\,\tau_{allow}}\sqrt{M^2 + T^2}\right]^{\frac{1}{3}} = \left[\frac{2}{\pi(80)(10^6)}\sqrt{1274.75^2 + 375^2}\right]^{\frac{1}{3}} = 0.0219\ \text{m}$$

$d = 2c = 0.0439\ \text{m} = 43.9\ \text{mm}$

Use $d = 44$ mm. **Ans.**

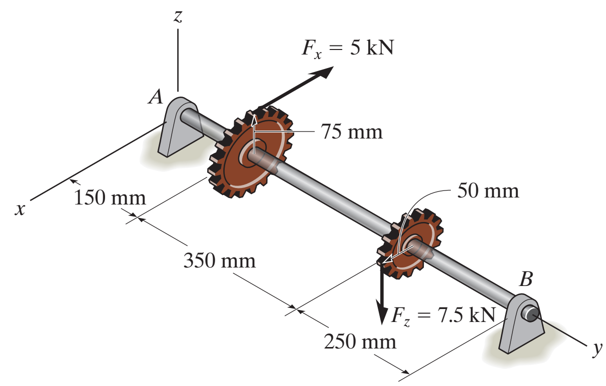




Ans:
Use $d = 44$ mm

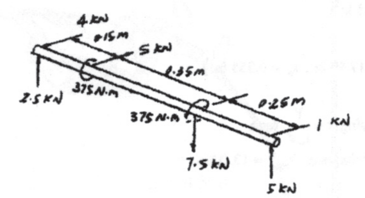
***R11–4.**

The journal bearings at A and B exert only x and z components of force on the shaft. Determine the shaft's diameter to the nearest millimeter so that it can resist the loadings. Use the maximum distortion energy theory of failure with $\sigma_{\text{allow}} = 200$ MPa.



SOLUTION

Maximum resultant moment $M = \sqrt{1250^2 + 250^2} = 1274.75\ \text{N}\cdot\text{m}$



$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

Let $a = \dfrac{\sigma_x}{2},\ b = \sqrt{\dfrac{\sigma_x^2}{4} + \tau_{xy}^2}$

$$\sigma_1 = a + b, \qquad \sigma_2 = a - b$$

Require,

$$\sigma_1^2 - \sigma_1\,\sigma_2 + \sigma_2^2 = \sigma_{\text{allow}}^2$$

$$a^2 + 2ab + b^2 - [a^2 - b^2] + a^2 - 2ab + b^2 = \sigma_{\text{allow}}^2$$

$$a^2 + 3b^2 = \sigma_{\text{allow}}^2$$

$$\frac{\sigma_x^2}{4} + 3\left(\frac{\sigma_x^2}{4} + \tau_{xy}^2\right) = \sigma_{\text{allow}}^2$$

$$\sigma_x^2 + 3\tau_{xy}^2 = \sigma_{\text{allow}}^2$$

$$\left(\frac{Mc}{\frac{\pi}{4}c^4}\right)^2 + 3\left(\frac{Tc}{\frac{\pi}{2}c^4}\right)^2 = \sigma_{\text{allow}}^2$$

$$\frac{1}{c^6}\left[\left(\frac{4M}{\pi}\right)^2 + 3\left(\frac{2T}{\pi}\right)^2\right] = \sigma_{\text{allow}}^2$$

$$c^6 = \frac{16}{\sigma_{\text{allow}}^2\,\pi^2}M^2 + \frac{12T^2}{\sigma_{\text{allow}}^2\,\pi^2}$$

$$c = \left[\frac{4}{\sigma_{\text{allow}}^2\,\pi^2}(4M^2 + 3\,T^2)\right]^{\frac{1}{6}}$$

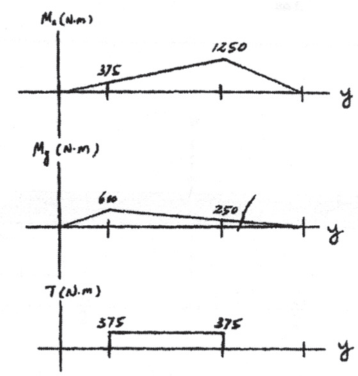
$$= \left[\frac{4}{(200(10^6))^2(\pi)^2}(4(1274.75)^2 + 3(375)^2)\right]^{\frac{1}{6}}$$

$$= 0.0203\ \text{m} = 20.3\ \text{mm}$$

$$d = 40.6\ \text{mm}$$

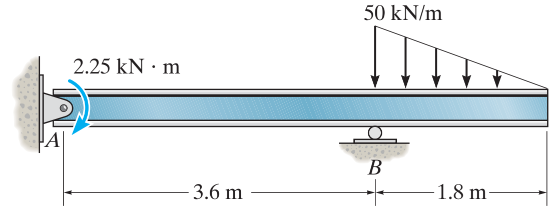
Use

$d = 41$ mm **Ans.**



Ans:
Use $d = 41$ mm

R11–5. Draw the shear and moment diagrams for the beam. Then select the lightest-weight steel wide-flange beam from Appendix B that will safely support the loading. Take $\sigma_{allow} = 150$ MPa and $\tau_{allow} = 84$ MPa.



SOLUTION

Bending Stress: From the moment diagram, $M_{max} = 27$ kN · m. Assume bending controls the design. Applying the flexure formula.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{27(10^3)}{150(10^6)} = 0.18(10^{-3})\ \text{m}^3 = 180(10^3)\ \text{mm}^3$$

Select W310 × 21 ($S_x = 244(10^3)$ mm^3, $d = 303$ mm and $t_w = 5.08$ mm)

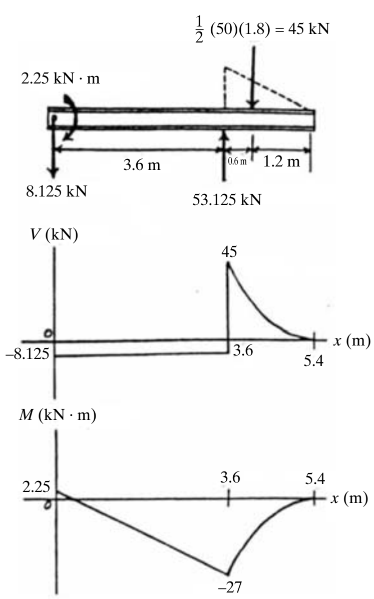
Shear Stress: Provide a shear stress check using $\tau = \dfrac{V}{t_w d}$ for the W310 × 21 wide - flange section. From the shear diagram, $V_{max} = 45$ kN

$$\tau_{max} = \frac{V_{max}}{t_w d}$$

$$= \frac{45(10^3)}{0.00508(0.303)} = 29.24(10^6)\ \text{N/m}^2$$

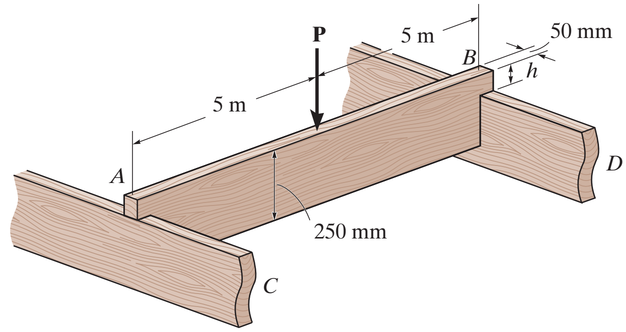
$$= 29.2\ \text{MPa} < \tau_{allow} = 84\ \text{MPa}\ (O.K!)$$

Hence, **Use** W310 × 21 **Ans.**



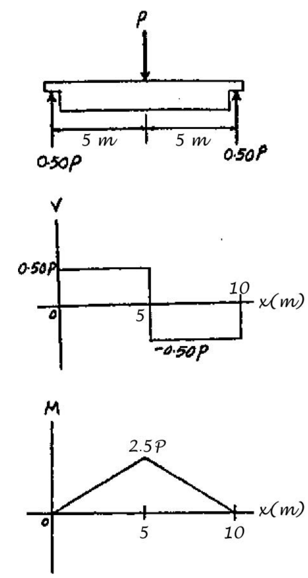
Ans:
Use W310 × 21

R11–6. The simply supported joist is used in the construction of a floor for a building. In order to keep the floor low with respect to the sill beams C and D, the ends of the joists are notched as shown. If the allowable shear stress for the wood is $\tau_{\text{allow}} = 2.5$ MPa and the allowable bending stress is $\sigma_{\text{allow}} = 10.5$ MPa determine the height h that will cause the beam to reach both allowable stresses at the same time. Also, what load P causes this to happen? Neglect the stress concentration at the notch.



SOLUTION

Bending Stress: From the moment diagram, $M_{\text{max}} = 2.5P$. Applying the flexure formula.



$$\sigma_{\text{allow}} = \frac{M_{\text{max}}c}{I}$$

$$10.5(10^6) = \frac{(2.5P)(0.125)}{\frac{1}{12}(0.05)(0.25^3)}$$

$$P = 2187.5 \text{ N} = 2.19 \text{ kN} \qquad \textbf{Ans.}$$

Shear Stress: From the shear diagram, $V_{\text{max}} = 0.500P = 1093.75$ N. The notch is the critical section. Using the shear formula for a rectangular section,

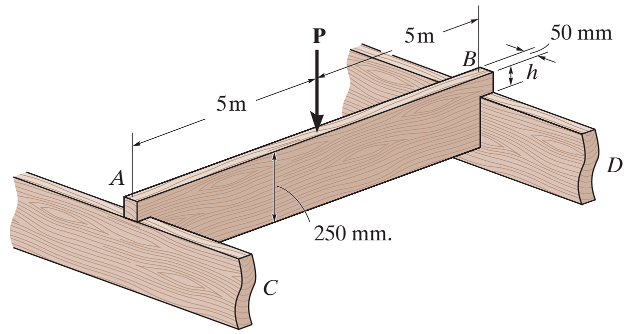
$$\tau_{\text{allow}} = \frac{3V_{\text{max}}}{2A}$$

$$2.5(10^6) = \frac{3(1093.75)}{2[0.05(h)]}$$

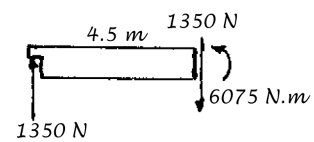
$$h = 0.013125 \text{ m} = 13.1 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$P = 2.19$ kN, $h = 13.1$ mm

R11–7. The simply supported joist is used in the construction of a floor for a building. In order to keep the floor low with respect to the sill beams C and D, the ends of the joists are notched as shown. If the allowable shear stress for the wood is $\tau_{allow} = 2.5$ MPa and the allowable bending stress is $\sigma_{allow} = 12$ MPa, determine the smallest height h so that the beam will support a load of $P = 2$ kN. Also, will the entire joist safely support the load? Neglect the stress concentration at the notch.



SOLUTION



The reaction at the support is $\dfrac{2(10^3)}{2} = 1000$ N

$$\tau_{allow} = \frac{1.5V}{A}; \qquad 2.5(10^6) = \frac{1.5(1000)}{0.05h}$$

$h = 0.012$ m $= 12.0$ mm **Ans.**

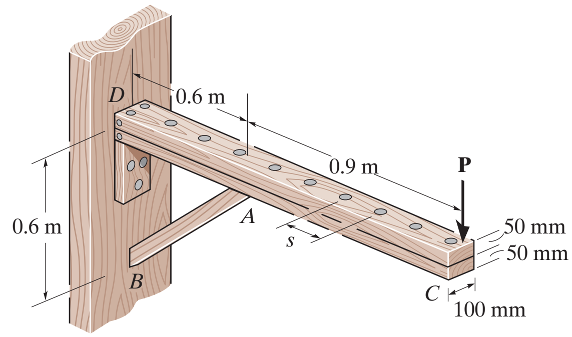
$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{5000(0.125)}{\frac{1}{12}(0.05)(0.25^3)}$$

$$= 9.60(10^6)\ \text{N/m}^2 = 9.60\ \text{MPa} < 9.60\ \text{MPa} \quad \text{OK}$$

Yes, the joist will safely support the load. **Ans.**

Ans:
$h = 16.2$ mm. Yes, the joist will support the load.

***R11–8** The overhang beam is constructed using two 50-mm by 100-mm pieces of wood braced as shown. If the allowable bending stress of σ_{allow} = 4.2 MPa, determine the largest load P that can be applied. Also, determine the associated maximum spacing of nails, s, along the beam section AC if each nail can resist a shear force of 4 kN. Assume the beam is pin-connected at A, B, and D. Neglect the axial force developed in the beam along DA.



SOLUTION

$M_A = M_{max} = 0.9P$

Section properties:

$$I = \frac{1}{12}(0.1)(0.1^3) = 8.3333(10^{-6})\text{ m}^4$$

$$S = \frac{I}{c} = \frac{8.3333(10^{-6})}{0.05} = 0.16667(10^{-3})\text{ m}^3$$

$$M_{max} = \sigma_{allow}\, S$$

$$0.9P = [4.2(10^{-6})][0.16667(10^{-3})]$$

$$P = 777.78\text{ N} = 778\text{ N}$$ **Ans.**

Nail Spacing:

$$V = P = 777.78\text{ N}$$

$$Q = 0.025(0.1)(0.05) = 0.125(10^{-3})\text{ m}^3$$

$$q_{allow} = \frac{4(10^3)}{S}$$

Then

$$q_{allow} = \frac{VQ}{I}; \qquad \frac{4(10^3)}{S} = \frac{777.78[0.125(10^{-3})]}{8.3333(10^{-6})}$$

$$S = 0.34286\text{ m} = 343\text{ mm}$$

Use $S = 340$ mm **Ans.**

Ans.
Use $S = 340$ mm