- Equazione di equilibrio della trave caricata assialmente
  $\left(\frac{dN}{dx} + q = 0\right)$ (rel 8.2)

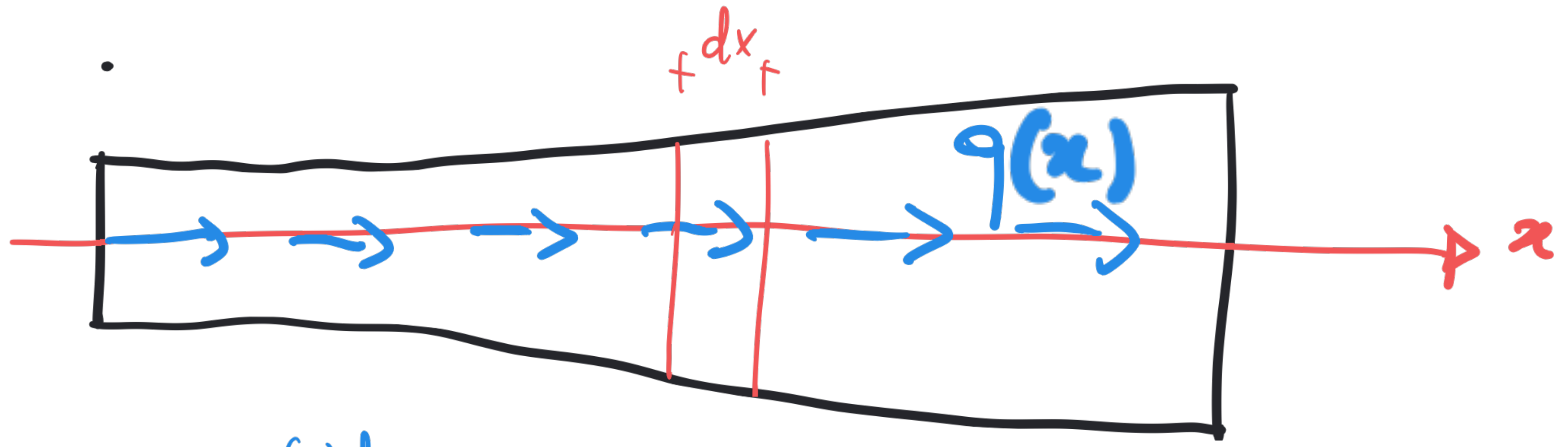
- Equazione di congruenza $\frac{d\delta}{dx} = \varepsilon$ (rez. 8.2)

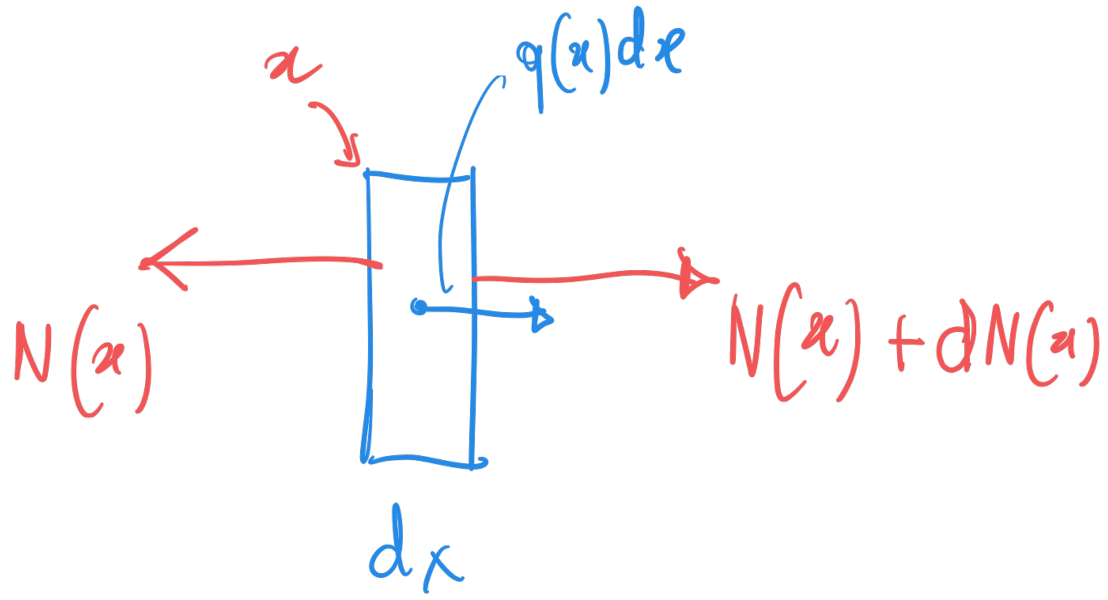
- Equazione costitutiva $N = EA\varepsilon$

Capitolo 8 e 9 di Carini e Vasta

$$\delta = w$$

Equazione differenziale della trave tesa o compressa.

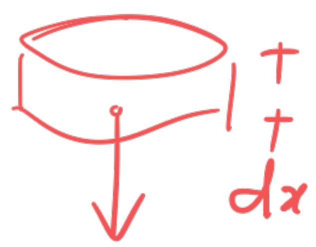




$$-N(x) + q(x)dx + N(x) + dN(x) = 0$$

$$\boxed{\frac{dN}{dx} + q = 0}$$

$\rho \quad d(x)$



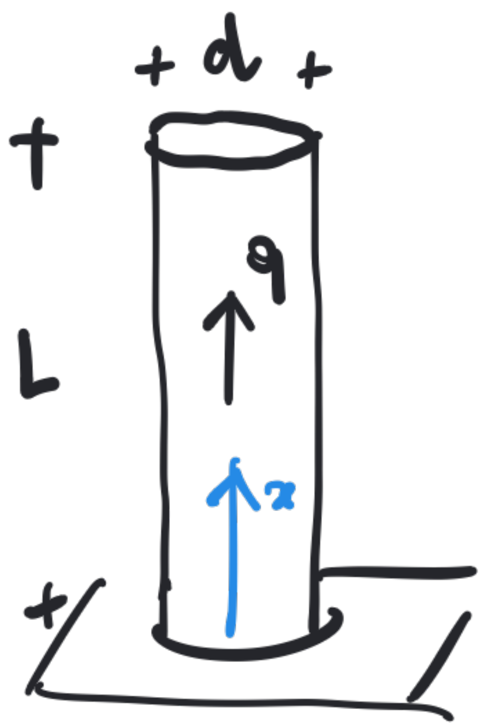
$$dm = \rho \frac{\pi}{4} d(x)^2 dx$$

$$dF = g\,dm =$$

$$= g\rho \frac{\pi}{4} d(x)^2 dx$$

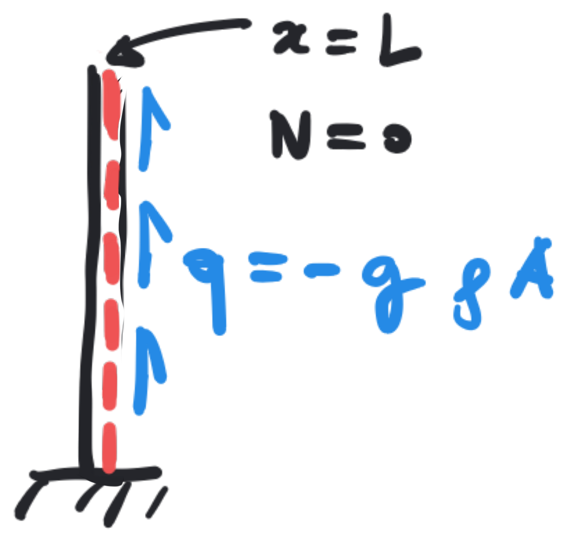
$$\frac{dF}{dx} = g\rho \frac{\pi}{4} d(x)^2$$

$$q = -\frac{dF}{dx}$$



$$\boxed{\frac{dN}{dx}+q=0}$$

$$q=-g\rho A$$

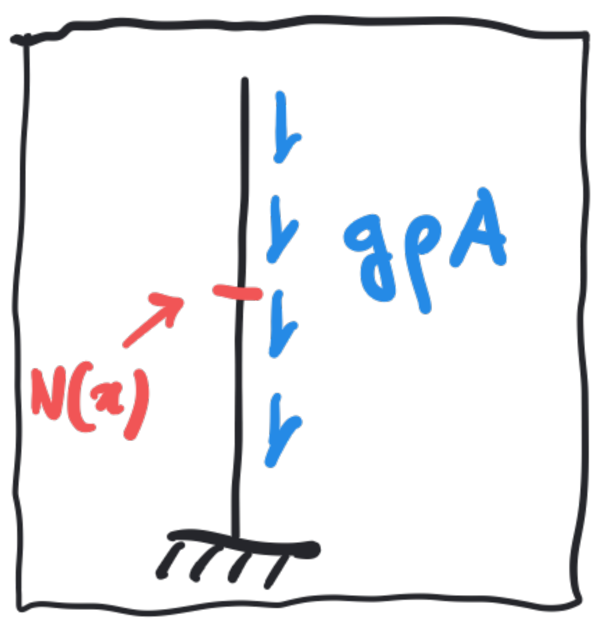


$$N(x)=g\rho A x+c$$

$$N(L)=0$$

$$c=-g\rho AL$$

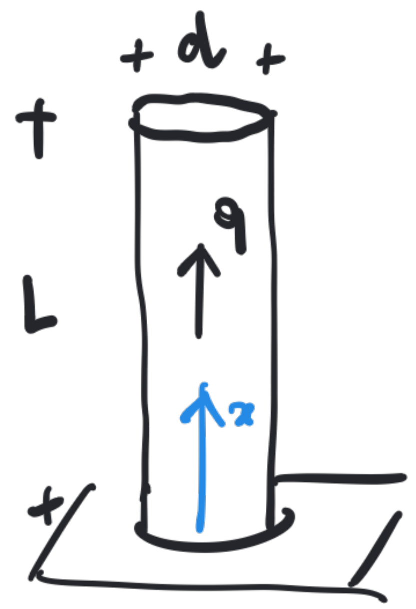
$$N(x)=-g\rho A(L-x)=-g\rho AL(1-x/L)$$



$$q=\frac{dF}{dx}$$

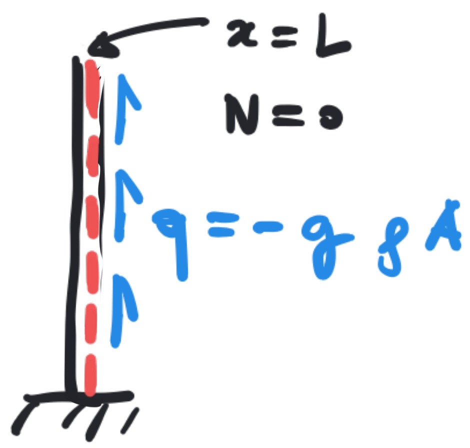
$$dF=-A\,dx\,\rho$$

$$\frac{dF}{dx}=-g\rho A=q$$



$$\boxed{\frac{dN}{dx} + q = 0}$$

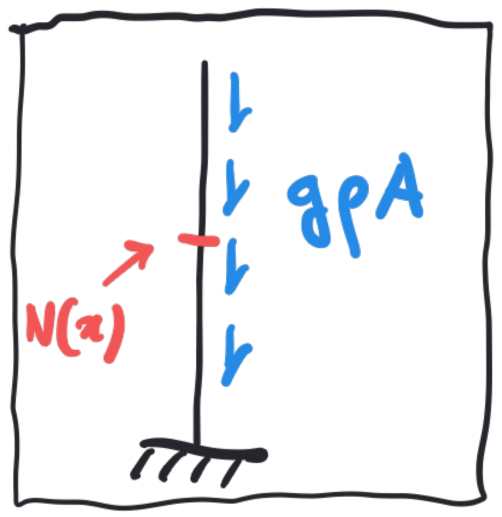
$$q = -g\rho A$$

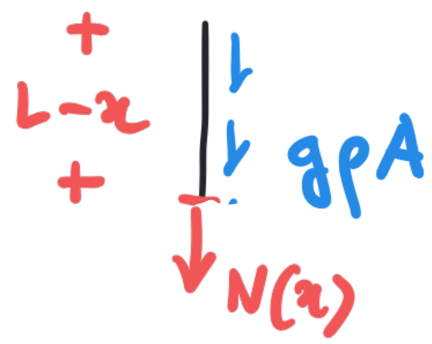


$$N(x) = g\rho A x + c$$

$$N(L) = 0$$

$$c = -g\rho A L$$

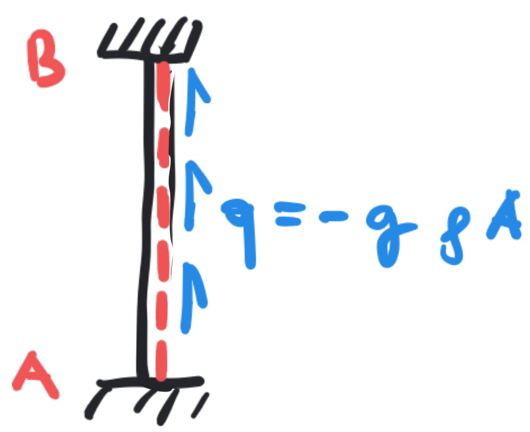




$$\downarrow \sum F_y \Rightarrow N + (L-x)g\rho A = 0$$

$$\boxed{N(x) = -g\rho A(L-x)} = -g\rho A L\left(1 - x/L\right)$$

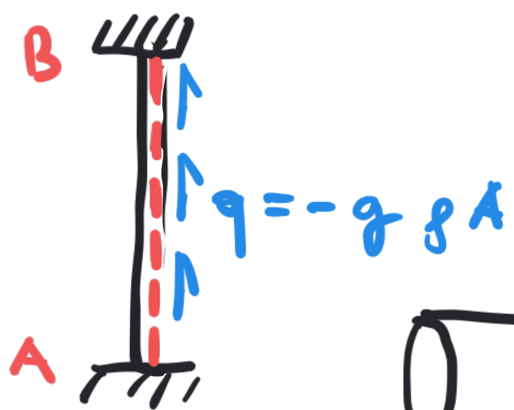
$$\Rightarrow \boxed{N(x) = -g\rho A(L-x)}$$



$$\boxed{\frac{dN}{dx} + q = 0}$$

$$R_B + R_A + \int_0^L q = 0$$

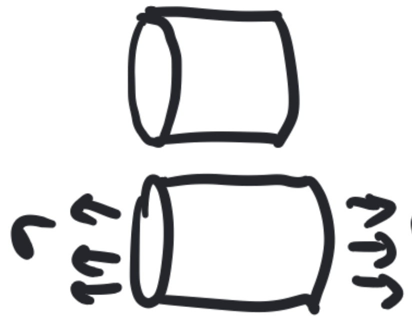
$$N(x) = g\rho A x + c$$



$$\boxed{\frac{dN}{dx} + q = 0}$$

$$EA\frac{d\delta}{dx} = N$$

$$\boxed{\begin{array}{l}\delta(L) = 0\\ \delta(0) = 0\end{array}}$$



$\sigma = E\varepsilon$

$\delta(x)$ spostamento della sezione x lungo l'asse

$N(x) = g\rho A x + c$

Hooke

$\sigma = E\varepsilon = E\dfrac{d\delta}{dx}$

$N = A\sigma = EA\dfrac{d\delta}{dx}$

$$\frac{d\delta}{dx} = \frac{N}{EA} = g\rho A x + c$$

$$\delta(x) = \tfrac{1}{2} g\rho A x^2 + cx + d$$

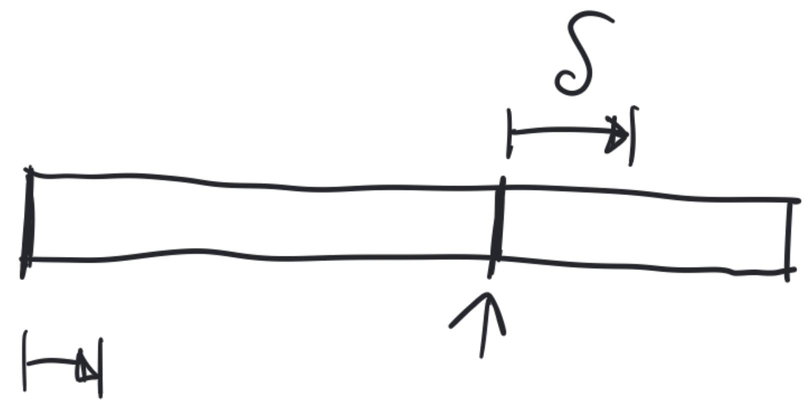
$\delta(0) = 0 \Rightarrow d = 0$

$\delta(L) = 0 \Rightarrow \frac{1}{2} g\rho A L^2 + cL + d = 0$

$c = -\frac{1}{2} gAL$

$$\delta(x) = \tfrac{1}{2} g\rho A x^2 - \tfrac{1}{2} gALx = \tfrac{1}{2} g\rho A x(x-L) = \tfrac{1}{2} g\rho A L^2 \frac{x}{L}\left(\frac{x}{L} - 1\right)$$

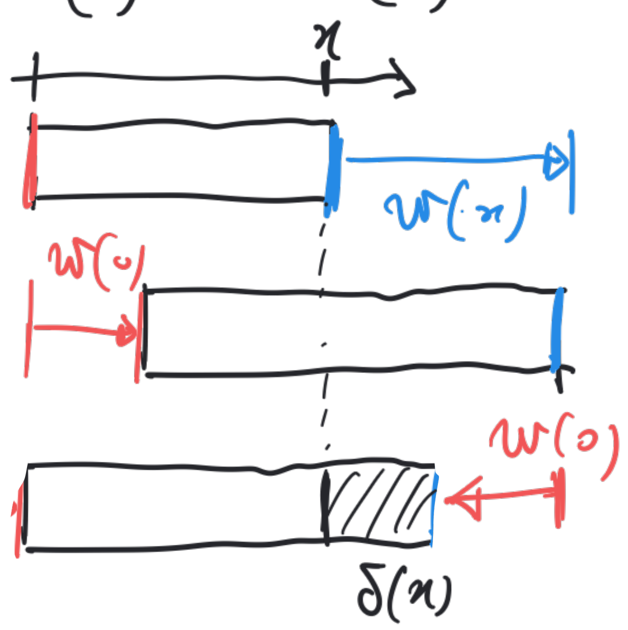
$$N = g\rho A x - \tfrac{1}{2} gAL = g\rho A\left(x - \tfrac{L}{2}\right) = g\rho A L\left(\tfrac{x}{L} - \tfrac{1}{2}\right)$$



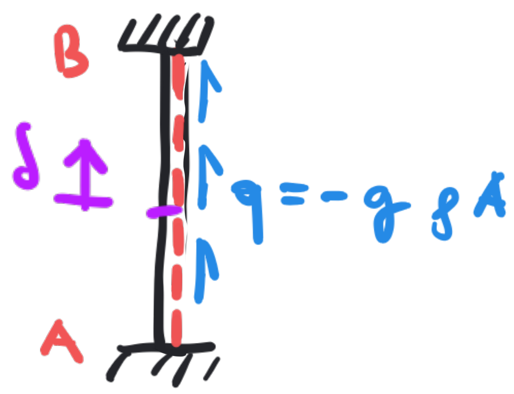
$\delta(x)$ = $\underbrace{\text{allungamento del tratto}}$ $(0,x)$ (Hibbeler)

$w(x)$ = spostamento della sezione x (Casini e Vasta).

$$\delta(x) = w(x) - w(0) \qquad \varepsilon = \frac{d\delta}{dx} = \frac{dw}{dx}$$



$\delta(x) = ?$



(1) $\frac{dN}{dx} + q = 0$ equilibrio

(2) $\varepsilon = \frac{d\delta}{dx}$ congruenza

(3) $N = EA\varepsilon$ costitutiva

$\left.\begin{array}{l}\delta(L)=0\\ \delta(0)=0\end{array}\right\}(4)$

incognite (N, ε, δ)

1) Integrando (1) $\Rightarrow N(x) = g\rho A x + c$

2) (2)+(3) $\Rightarrow \frac{d\delta}{dx} = \frac{N}{EA} = \frac{g\rho}{E} x + c$ $\Bigg\} \Rightarrow \delta(x) = \frac{1}{2} g\rho A x^2 + cx + d$

3) (4) $\Rightarrow$ $\left.\begin{array}{l}\delta(0)=0 \Rightarrow d=0\\ \delta(L)=0 \Rightarrow \frac{1}{2}\frac{g\rho}{E}L^2 + cL + d = 0\end{array}\right\} \Rightarrow \begin{array}{l} d=0\\ c=-\frac{1}{2}\frac{gL}{E}\rho\end{array}$

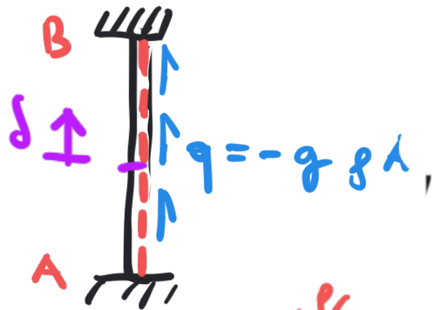
$$\boxed{\delta(x) = \frac{1}{2}\frac{g\rho}{E}x^2 - \frac{1}{2}\frac{g\rho}{E}Lx = \frac{1}{2}\frac{g\rho}{E}x(x-L) = \frac{1}{2}\frac{g\rho}{E}L^2\frac{x}{L}\left(\frac{x}{L}-1\right)}$$

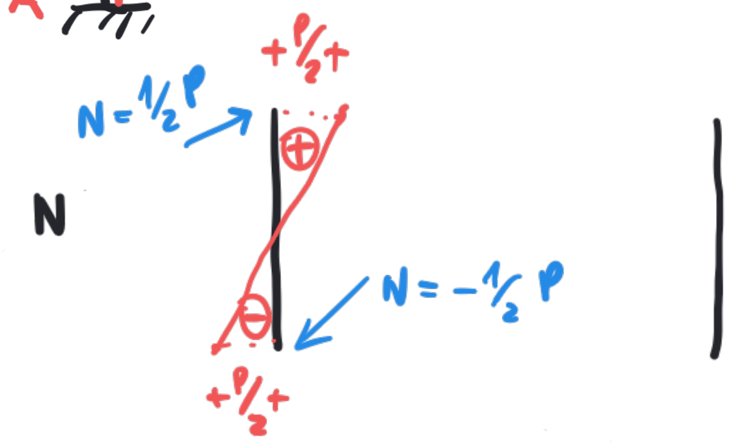
$\varepsilon = \frac{g\rho}{E}x - \frac{1}{2}\frac{g\rho}{E}L = \frac{g\rho}{E}\left(x - \frac{L}{2}\right) = \frac{g\rho}{E}L\left(\frac{x}{L} - \frac{1}{2}\right)$ $\quad N = EA\varepsilon$

$$P = g\rho A L$$

$$N = P\left(\frac{x}{L} - \frac{1}{2}\right)$$

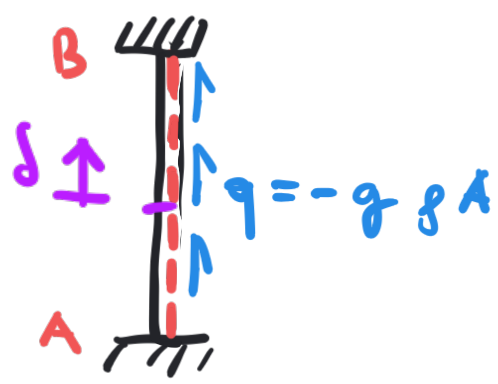
$$\delta = \frac{1}{2}\frac{P}{EA} L \frac{x}{L}\left(\frac{x}{L} - 1\right)$$





$$\boxed{\delta(x) = \frac{1}{2}\frac{g\rho}{E}x^2 - \frac{1}{2}\frac{g\rho}{E}Lx = \frac{1}{2}\frac{g\rho}{E}x(x-L) = \frac{1}{2}\frac{g\rho}{E}L^2\frac{x}{L}\left(\frac{x}{L}-1\right)}$$

$$\varepsilon = \frac{g\rho}{E}x - \frac{1}{2}\frac{g\rho}{E}L = \frac{g\rho}{E}\left(x - \frac{L}{2}\right) = \frac{g\rho}{E}L\left(\frac{x}{L} - \frac{1}{2}\right) \qquad N = EA\varepsilon$$



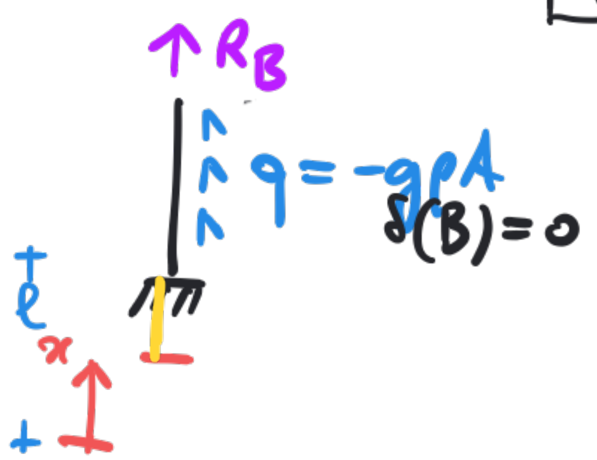
(1) $\frac{dN}{dx}+q=0$, equilibrio

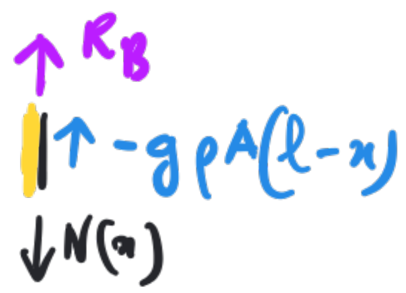
(2) $\varepsilon=\frac{d\delta}{dx}$ congruenza

(3) $N=EA\varepsilon$ costitutiva

$\left.\begin{array}{l}\delta(L)=0\\ \delta(0)=0\end{array}\right\}(4)$

→ incognite (N,ε,δ)





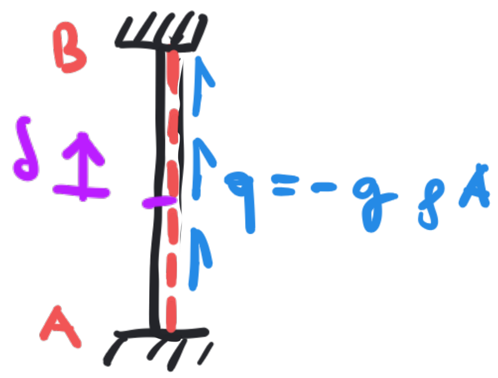
$$\int_0^\ell \frac{N}{\cancel{EA}}dx=0 \quad EA\ \text{cost.}$$

(*) $\Downarrow$

$$R_B = g\rho A\int_0^\ell \underbrace{\left(1-\frac{x}{\ell}\right)dx}_{\ell/2} = \frac{g\rho A\ell}{2}$$

(*) $N(x)=R_B-g\rho A(\ell-x)$

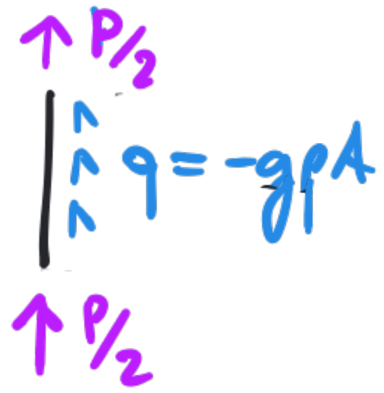
$$\delta(L)=\delta(0)+\int_0^\ell \frac{d\delta}{dx}dx = \delta(0)+\int_0^\ell \varepsilon\, dx = \overset{0}{\overset{\|}{\delta(0)}}+\int_0^\ell \frac{N}{EA}dx=0$$

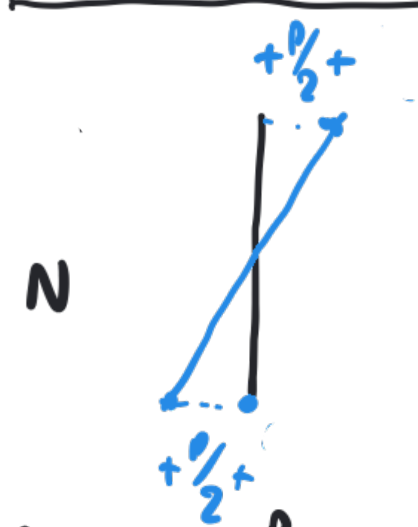


(1)	$\frac{dN}{dx} + q = 0$	equilibrio
(2)	$\varepsilon = \frac{d\delta}{dx}$	congruenza
(3)	$N = EA\varepsilon$	costitutiva

$$\left.\begin{aligned}\delta(L) &= 0\\ \rightarrow \delta(0) &= 0\end{aligned}\right\} (4)$$

incognite (N, ε, δ)



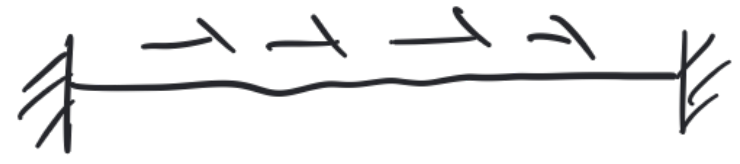


$$(*)\; N(x) = \frac{P}{2} - g\rho A(\ell - x) = P/2 - P(1 - x/\ell) = P(x/L - 1/2)$$

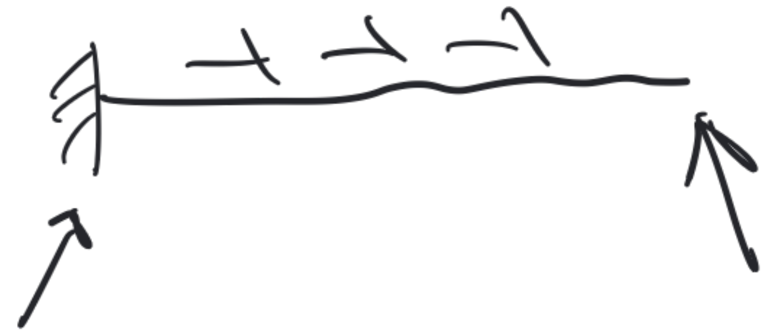
$P = g\rho A\ell$ (peso della colonna) $\quad R_B = P/2$

METODO SPOSTAMENTI

$$\left.\begin{aligned} &\frac{dN}{dx}+q=0 \\ &\varepsilon=\frac{d\delta}{dx} \\ &N=EA\varepsilon \end{aligned}\right\} \Rightarrow \frac{d}{dx}\left(EA\frac{d\delta}{dx}\right)+q=0$$



$\delta(0)=0 \quad \delta(L)=0$



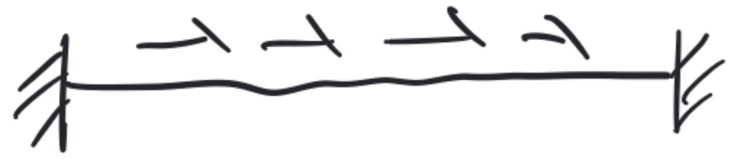
$$EA\frac{d\delta}{dx}=0$$

$\delta(0)$

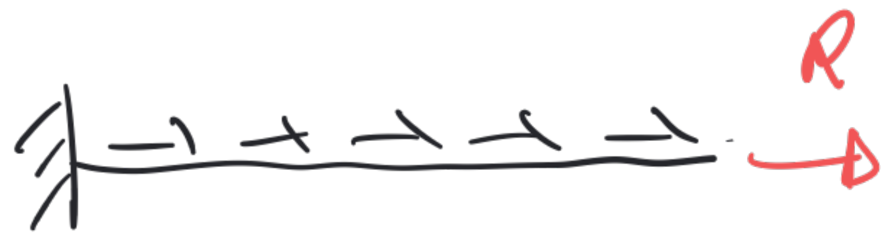
$N(L)=0$

METODO SPOSTAMENTI

$$\left.\begin{array}{l} \dfrac{dN}{dx} + q = 0 \\ \varepsilon = \dfrac{d\delta}{dx} \\ N = EA\varepsilon \end{array}\right\} \Rightarrow \frac{d}{dx}\left(EA\frac{d\delta}{dx}\right) + q = 0$$



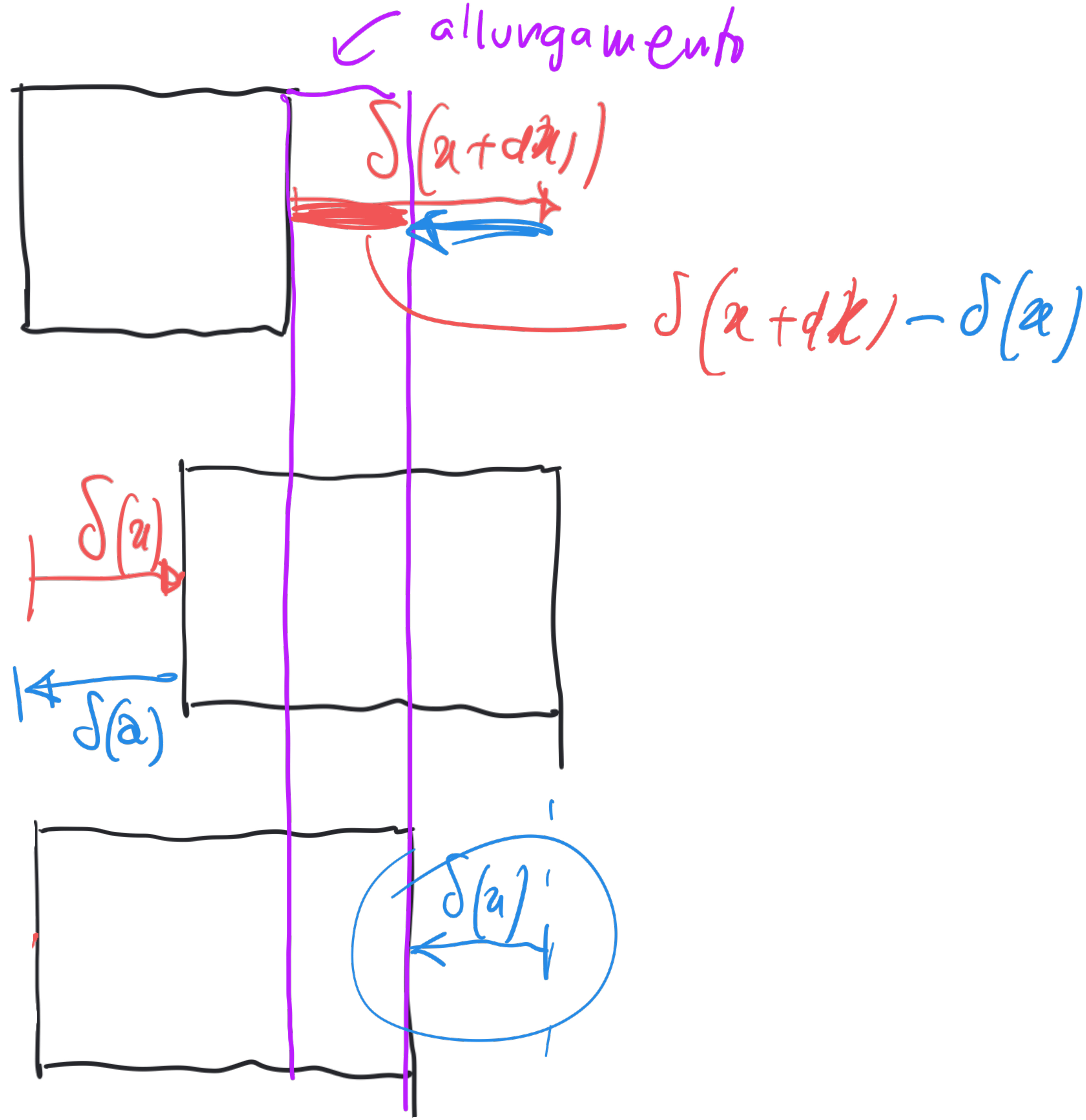
$\delta(0) = 0 \quad \delta(L) = 0$

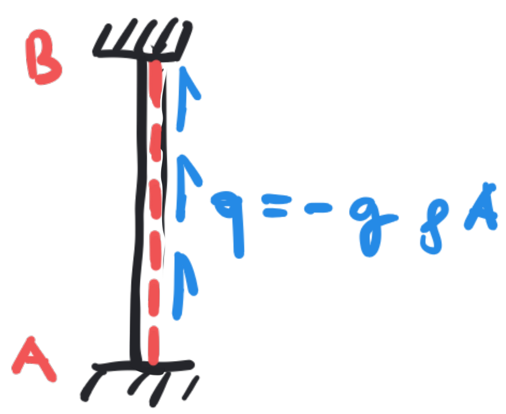


Equilibrio $\Rightarrow N(x) \Rightarrow \varepsilon(x) \Rightarrow \delta(x)$

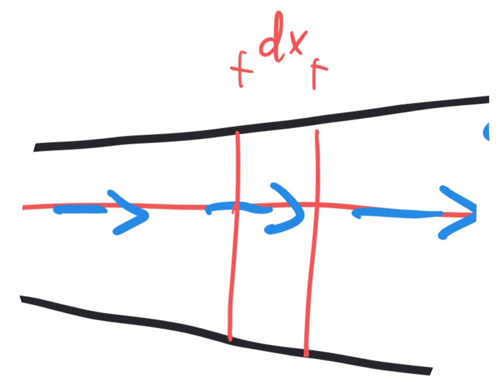
R parametro

$\delta(L) = \delta(L, R) = 0 \Rightarrow R$


allungamento
δ(x+dx)
δ(x+dx) − δ(x)
δ(x)
δ(x)
δ(x)

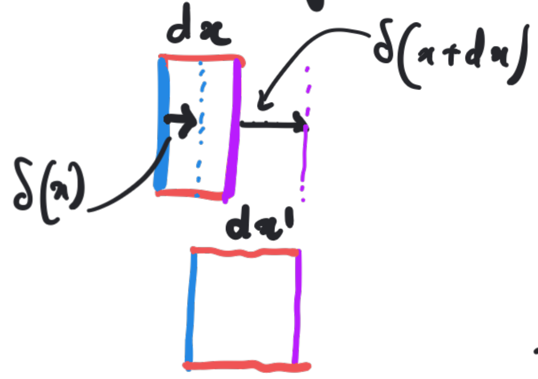


$$\boxed{\frac{dN}{dx} + q = 0}$$



$\delta(x)$ spostamento della sezione x lungo l'asse

✓ $N(x) = g\rho A x + c$



$$dx' = dx + \delta(x+dx) - \delta(x)$$

$$\varepsilon = \frac{dx' - dx}{dx} = \frac{dx'}{dx} - 1$$

$$\frac{dx'}{dx} = 1 + \frac{\delta(x+dx) - \delta(x)}{dx}$$

$$\boxed{\varepsilon = \frac{\delta(x+dx) - \delta(x)}{dx} = \frac{d\delta}{dx}}$$