

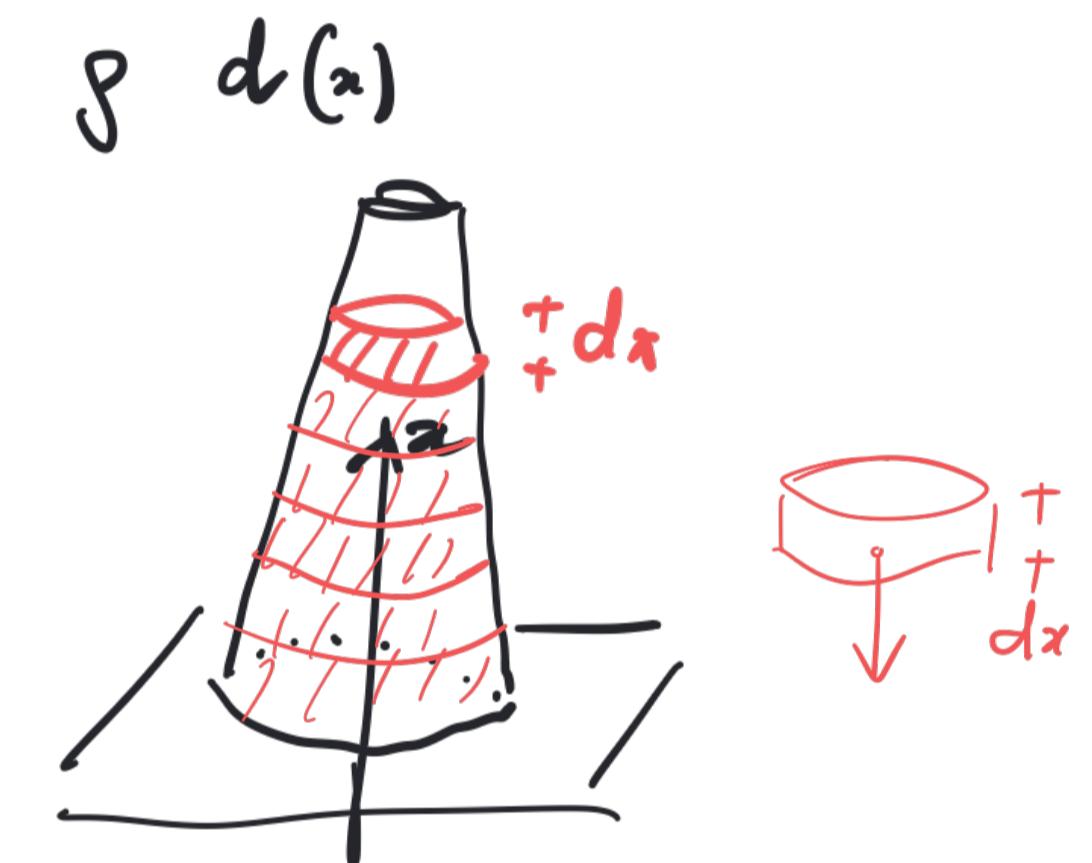
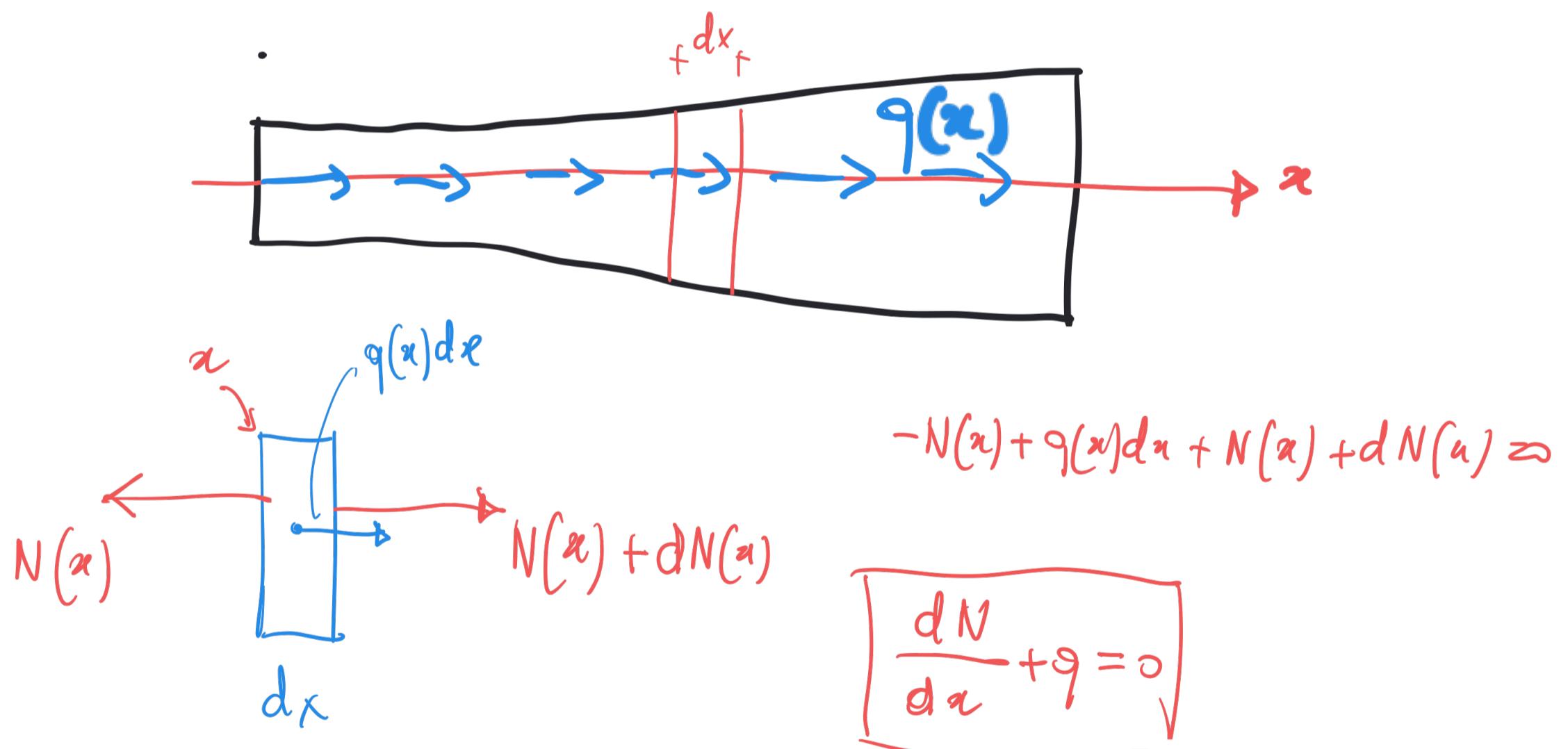
- Equazione d' equilibrio della trave carica  $\vec{q}$  asciumente  

$$\left( \frac{dN}{dx} + q = 0 \right) \quad (\text{rel 8.2})$$
- Equazione d' congruenza  $\frac{df}{dx} = \varepsilon \quad (\text{rel 8.2})$
- Equazione costitutiva  $N = EA\varepsilon$

Capitoli 8 e 9 di Caciu e Vasta

$$\delta = w$$

Equazione differenziale delle trave tesa  
o compressa.



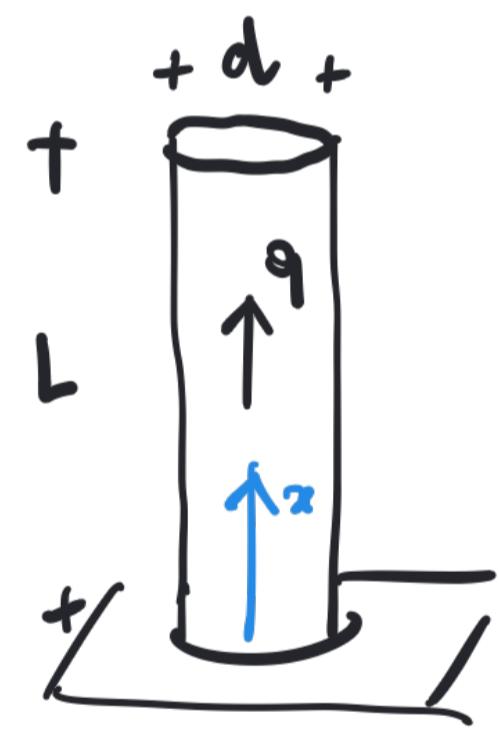
$$dm = g \frac{\pi}{4} d(x)^2 dx$$

$$dF = q dm =$$

$$= q \rho \frac{\pi}{4} d(x)^2 dx$$

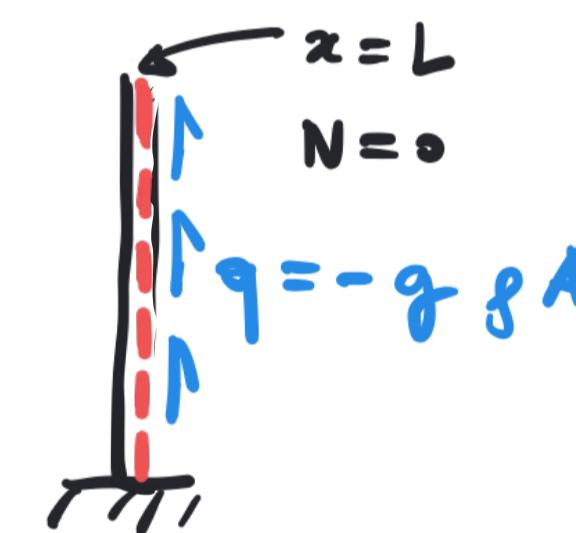
$$\frac{dF}{dx} = q \rho \frac{\pi}{4} d(x)^2$$

$$q = - \frac{dF}{dx}$$



$$\boxed{\frac{dN}{dx} + q = 0}$$

$$q = -g\rho A$$

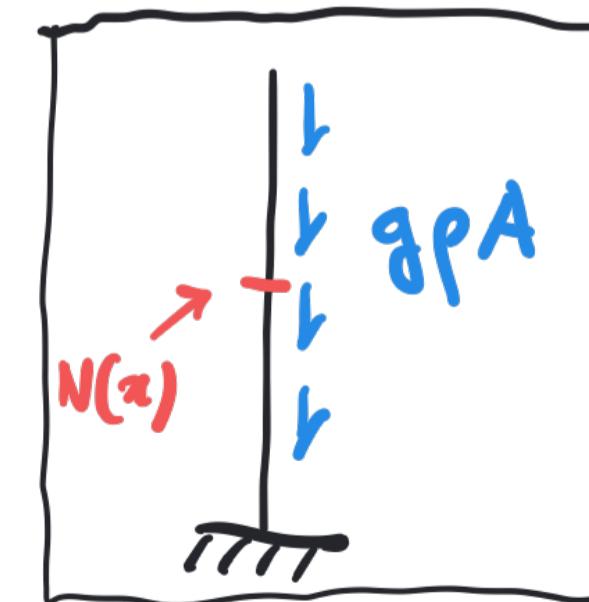


$$N(x) = g\rho A x + c$$

$$N(L) = 0$$

$$c = -g\rho A L$$

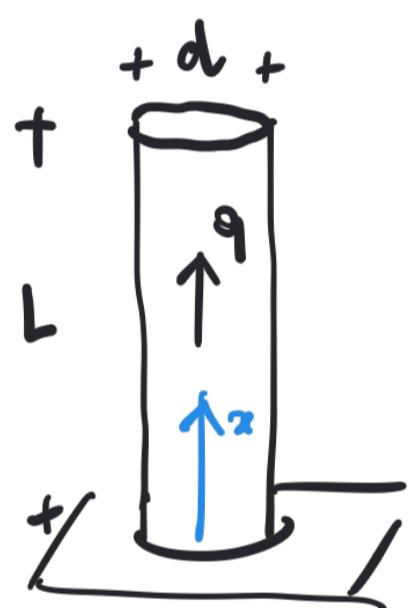
$$N(x) = -g\rho A (L - x) = -g\rho A L (1 - x/L)$$



$$q = \frac{dF}{dx} \quad \boxed{+dr}$$

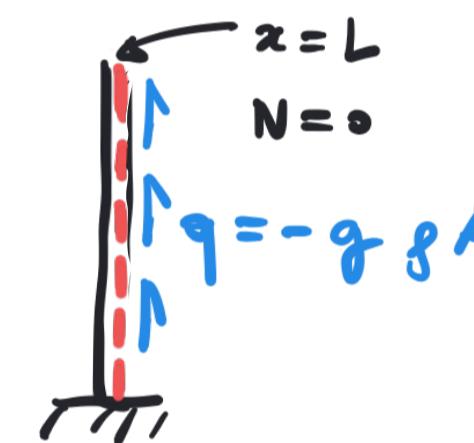
$$dF = -A dx \times \rho$$

$$\frac{dF}{dx} = -g\rho A = q$$



$$\frac{dN}{dx} + q = 0$$

$$q = -g \rho A$$

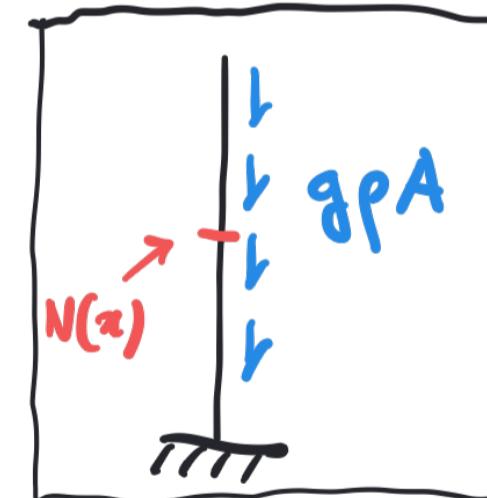


$$N(x) = g \rho A x + c$$

$$N(L) = 0$$

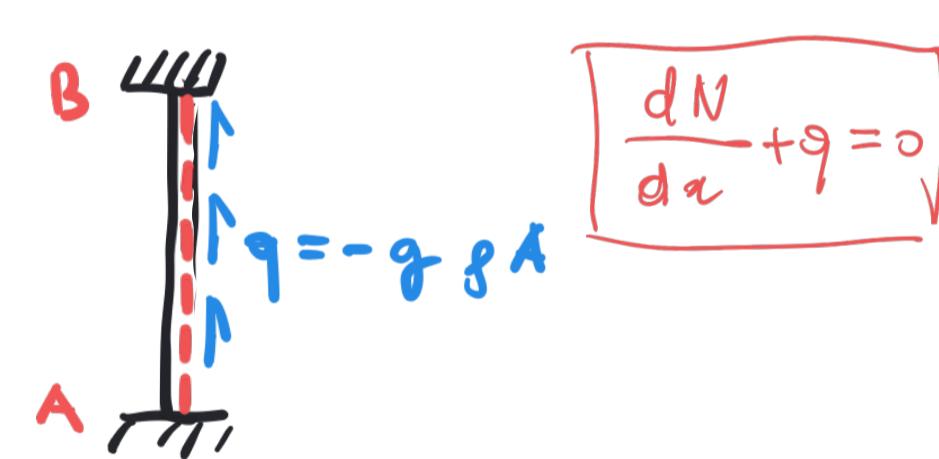
$$c = -g \rho A L$$

$$N(x) = -g \rho A (L - x) = -g \rho A L (1 - x/L)$$

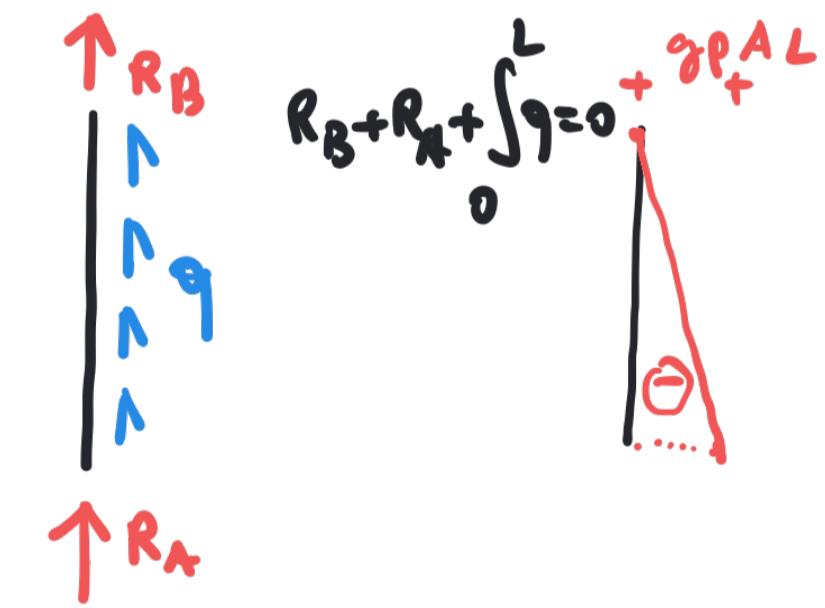


$$\downarrow \sum F_y \Rightarrow N + (L-x)g\rho A = 0$$

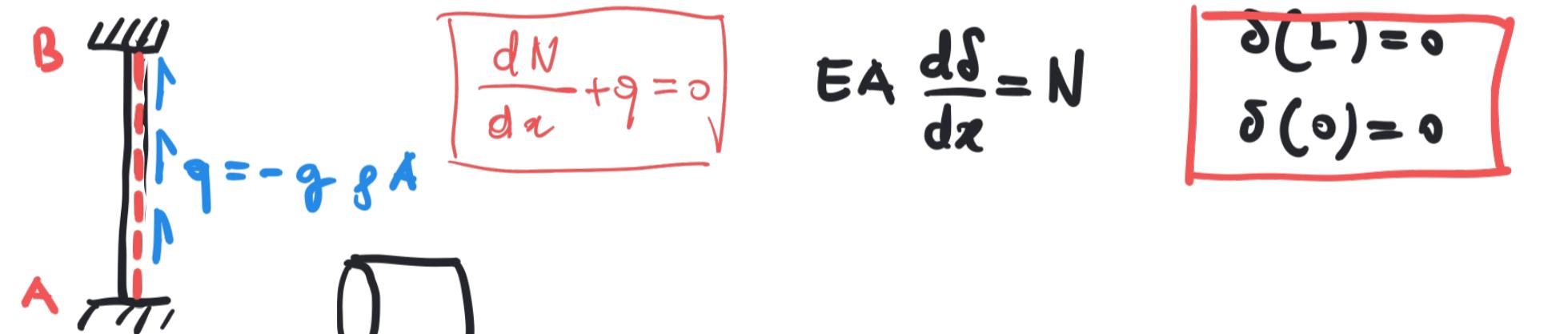
$$\Rightarrow N(x) = -g \rho A (L-x)$$



$$\frac{dN}{dx} + q = 0$$



$$\checkmark N(x) = q \rho A x + c$$



$$EA \frac{d\delta}{dx} = N$$

$$\begin{cases} \delta(0) = 0 \\ \delta(L) = 0 \end{cases}$$

$\delta(x)$  spostamento della sezione  
 $\sigma \in \sigma$   $\rightarrow \sigma = E\varepsilon$   $x$  lungo l'asse

$$\checkmark N(x) = q\rho Ax + c$$

$$\frac{d\delta}{dx} = \frac{N}{EA} = q\rho Ax + c$$

$$\delta(0) = 0 \Rightarrow d = 0$$

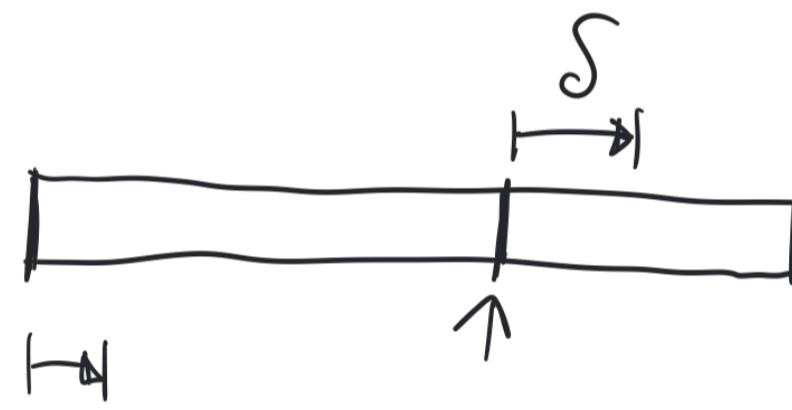
$$\delta(L) = 0 \Rightarrow \frac{1}{2}q\rho AL^2 + CL + d = 0$$

$$C = -\frac{1}{2}q\rho AL$$

$$\delta(x) = \frac{1}{2}q\rho Ax^2 + Cx + d$$

$$N(x) = \frac{1}{2}q\rho Ax^2 - \frac{1}{2}q\rho ALx = \frac{1}{2}q\rho A x(x-L) = \frac{1}{2}q\rho AL^2 \frac{x}{L} \left(\frac{x}{L} - 1\right)$$

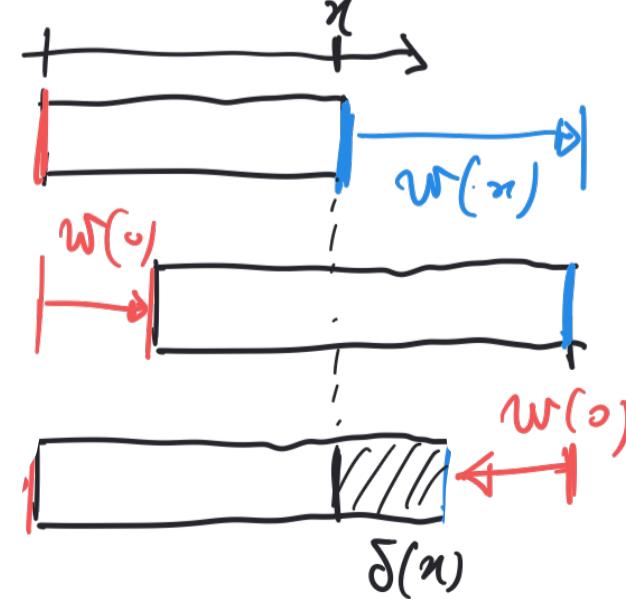
$$N = q\rho Ax - \frac{1}{2}q\rho AL = q\rho A \left(x - \frac{1}{2}\right) = q\rho AL \left(\frac{x}{L} - \frac{1}{2}\right)$$



$\delta(x) = \underline{\text{allungamento del tratto } (0, x)}$  (Hibbeler)

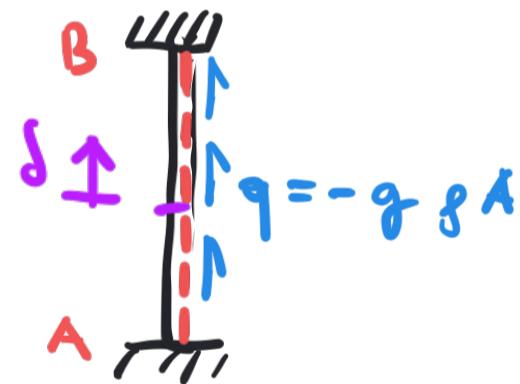
$w(x) = \text{spostamento della posizione } x$  (Casini e Vasta).

$$\delta(x) = w(x) - w(0)$$



$$\varepsilon = \frac{d\delta}{dx} = \frac{dw}{dx}$$

$$\delta(x) = ?$$



(1)	$\frac{dN}{dx} + q = 0$	equilibrio	$\delta(L) = 0$	$\quad  (4)$
(2)	$\epsilon = \frac{d\delta}{dx}$	congruenza	$\delta(0) = 0$	
(3)	$N = EA\epsilon$	constitutiva	incognite $(N, \epsilon, \delta)$	

1) Integrando (1)  $\Rightarrow N(x) = q\rho Ax + c$  }  $\Rightarrow \delta(x) = \frac{1}{2}q\rho Ax^2 + cx + d$

2) (2) + (3)  $\Rightarrow \frac{d\delta}{dx} = \frac{N}{EA} = \frac{q\rho}{E}x + c$  }

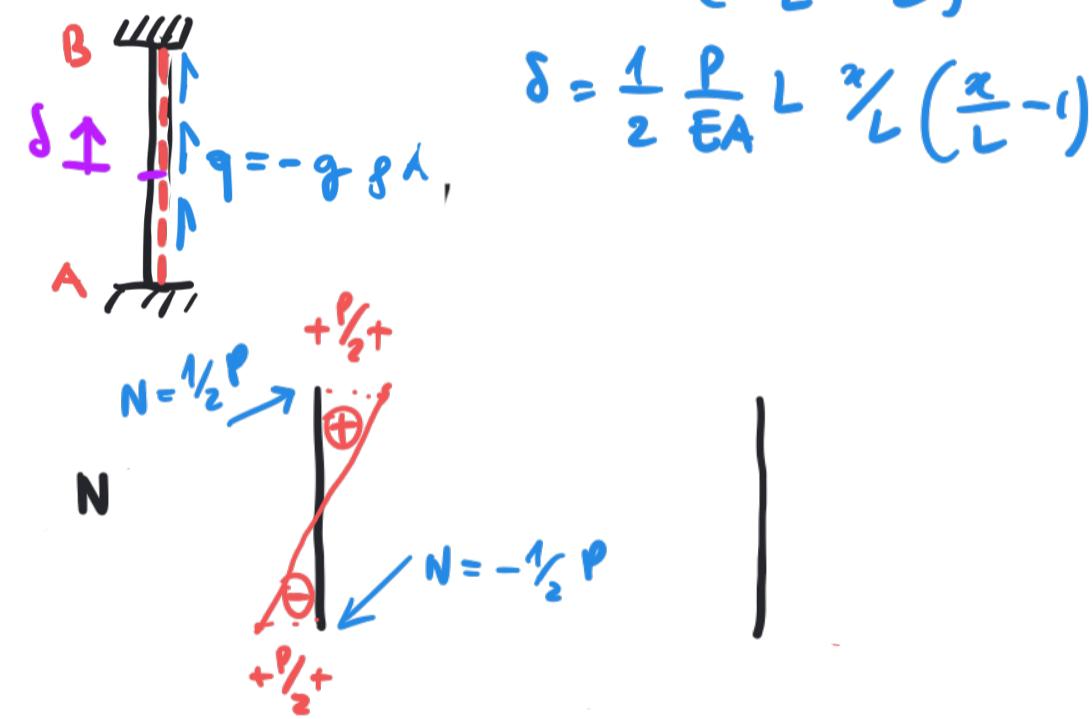
3) (4)  $\Rightarrow \delta(0) = 0 \Rightarrow d = 0$   
 $\delta(L) = 0 \Rightarrow \frac{1}{2}\frac{q\rho}{E}L^2 + CL + d = 0 \Rightarrow c = -\frac{1}{2}\frac{q\rho L^2}{E}$

$\delta(x) = \frac{1}{2}\frac{q\rho}{E}x^2 - \frac{1}{2}\frac{q\rho}{E}Lx = \frac{1}{2}\frac{q\rho}{E}x(x-L) = \frac{1}{2}\frac{q\rho}{E}L^2 \frac{x}{L} \left(\frac{x}{L} - 1\right)$

$\epsilon = \frac{q\rho}{E}x - \frac{1}{2}\frac{q\rho}{E}L = \frac{q\rho}{E}(x - \frac{1}{2}L) = \frac{q\rho}{E}L \left(\frac{x}{L} - \frac{1}{2}\right) \quad N = EA\epsilon$

$$P = g \rho A L$$

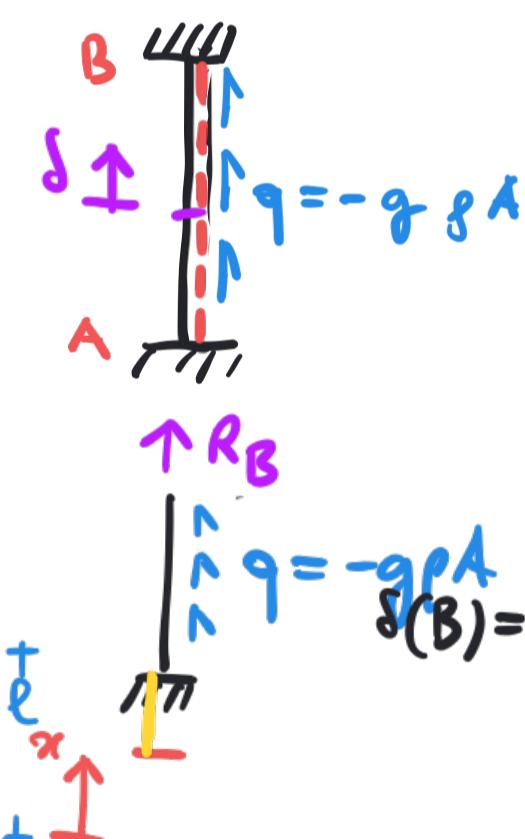
$$N = P \left( \frac{x}{L} - \frac{1}{2} \right)$$



$$\delta = \frac{1}{2} \frac{P}{EA} L \frac{\partial}{\partial z} \left( \frac{x}{L} - 1 \right)$$

$$\boxed{\delta(x) = \frac{1}{2} \frac{q \rho}{E} x^2 - \frac{1}{2} \frac{q \rho}{E} L x = \frac{1}{2} \frac{q \rho}{E} x (x - L) = \frac{1}{2} \frac{q \rho}{E} L^2 \frac{x}{L} \left( \frac{x}{L} - 1 \right)}$$

$$\epsilon = \frac{q \rho}{E} x - \frac{1}{2} \frac{q \rho}{E} L = \frac{q \rho}{E} \left( x - \frac{L}{2} \right) = \frac{q \rho}{E} L \left( \frac{x}{L} - \frac{1}{2} \right) \quad N = EA \epsilon$$



$$(*) N(x) = R_B - g\rho A(l-x)$$

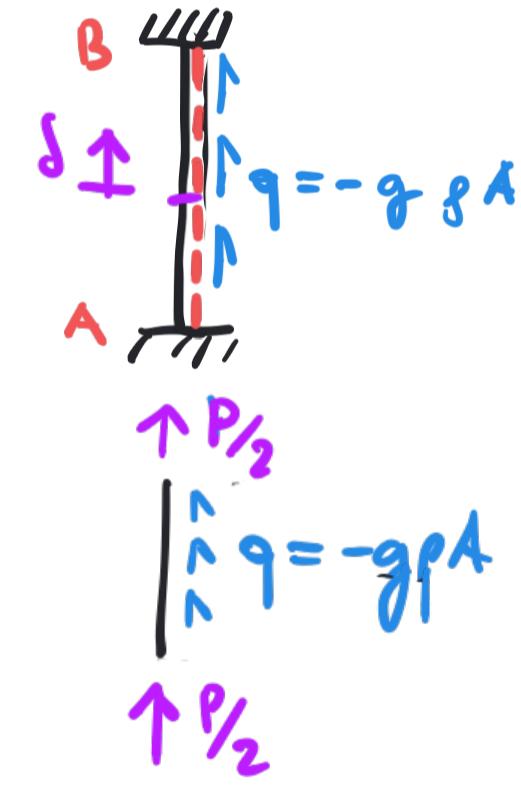
$$\delta(L) = \delta(0) + \int_0^L \frac{d\delta}{dx} dx = \delta(0) + \int_0^L \epsilon dx = \delta(0) + \int_0^L \frac{N}{EA} dx = 0$$

(1)	$\frac{dN}{dx} + q = 0$	equilibrio	$\delta(L) = 0$	(4)
(2)	$\epsilon = \frac{d\delta}{dx}$	congruenza	$\rightarrow \delta(0) = 0$	incogniti $(N, \epsilon, \delta)$
(3)	$N = EA\epsilon$	costitutiva		

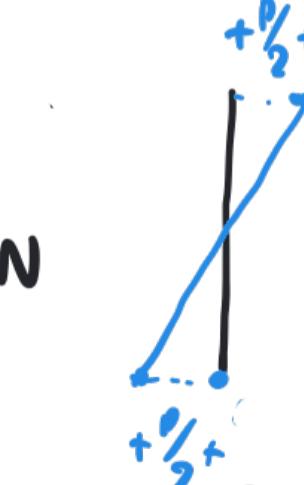
$$\int_0^L \frac{N}{EA} dx = 0$$

(\*) \downarrow EA \text{ cost.}

$$R_B = g\rho A \int_0^L (1 - \frac{x}{l}) dx = \frac{g\rho Al}{2}$$



- |     |                                    |             |                                      |     |
|-----|------------------------------------|-------------|--------------------------------------|-----|
| (1) | $\frac{dN}{dx} + q = 0$            | equilibrio  | $\delta(L) = 0$                      | (4) |
| (2) | $\varepsilon = \frac{d\delta}{dx}$ | congruenza  | $\rightarrow \delta(0) = 0$          |     |
| (3) | $N = EA\varepsilon$                | costitutiva | incogniti $(N, \varepsilon, \delta)$ |     |



$$(*) N(x) = \frac{P}{2} - g p A (l - x) = \frac{P}{2} - P \left(1 - \frac{x}{l}\right) = P \left(\frac{a}{L} - \frac{x}{l}\right)$$

$$P = gpA\ell \quad (\text{peso della colonna}) \quad R_B = \frac{P}{2}$$

# METODO SPOSTAMENTI

$$\left. \begin{array}{l} \frac{dN}{dx} + q = 0 \\ \epsilon = \frac{d\delta}{dx} \\ N = EA\epsilon \end{array} \right\} \Rightarrow \frac{d}{dx} \left( EA \frac{d\delta}{dx} \right) + q = 0$$

$$N = EA\varepsilon$$

$$\left. \begin{aligned} \frac{dN}{dx} + q &= \\ \epsilon &= \frac{d\delta}{dx} \\ N &= EA\epsilon \end{aligned} \right\} \Rightarrow \frac{d}{dx} \left( EA \frac{d\delta}{dx} \right) + q = 0$$

A horizontal line representing a beam is supported by two vertical lines at its ends. Four downward-pointing arrows are placed above the beam, indicating downward loads.

$$\delta(0) = 0 \quad \delta(\infty) \approx$$

The diagram shows a horizontal beam element of length  $L$ . At the left end, there is a vertical spring with stiffness  $EA$  and a fixed support. A vertical force  $F$  is applied downwards at this end. At the right end, there is a roller support. The beam has a small deflection angle  $\delta$  at its center. The beam is discretized into three segments by nodes.

Equations shown:

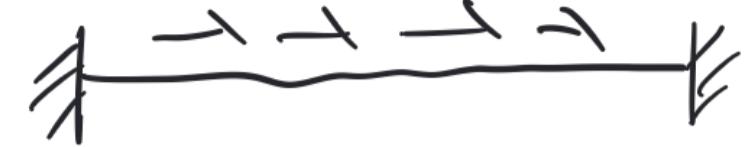
$$EA \frac{d\delta}{dx} = 0$$

$$\delta(0) = 0$$

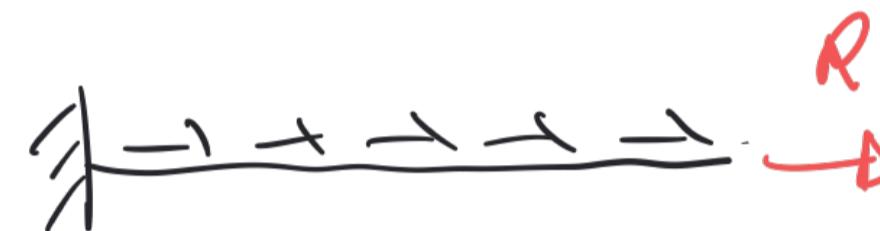
$$N(L) = 0$$

## METODO SPOSTAMENTI

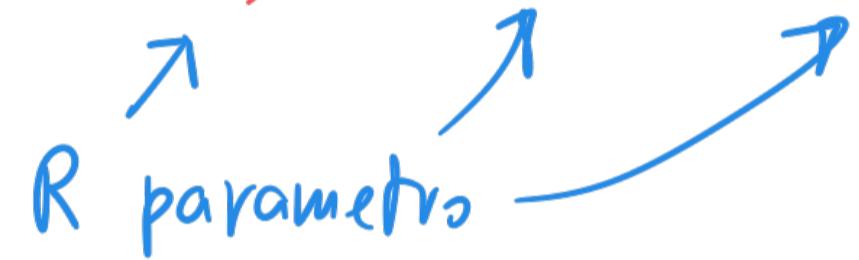
$$\left. \begin{array}{l} \frac{dN}{dx} + q = 0 \\ \epsilon = \frac{d\delta}{dx} \\ N = EA\epsilon \end{array} \right\} \Rightarrow \frac{d}{dx} \left( EA \frac{d\delta}{dx} \right) + q = 0$$



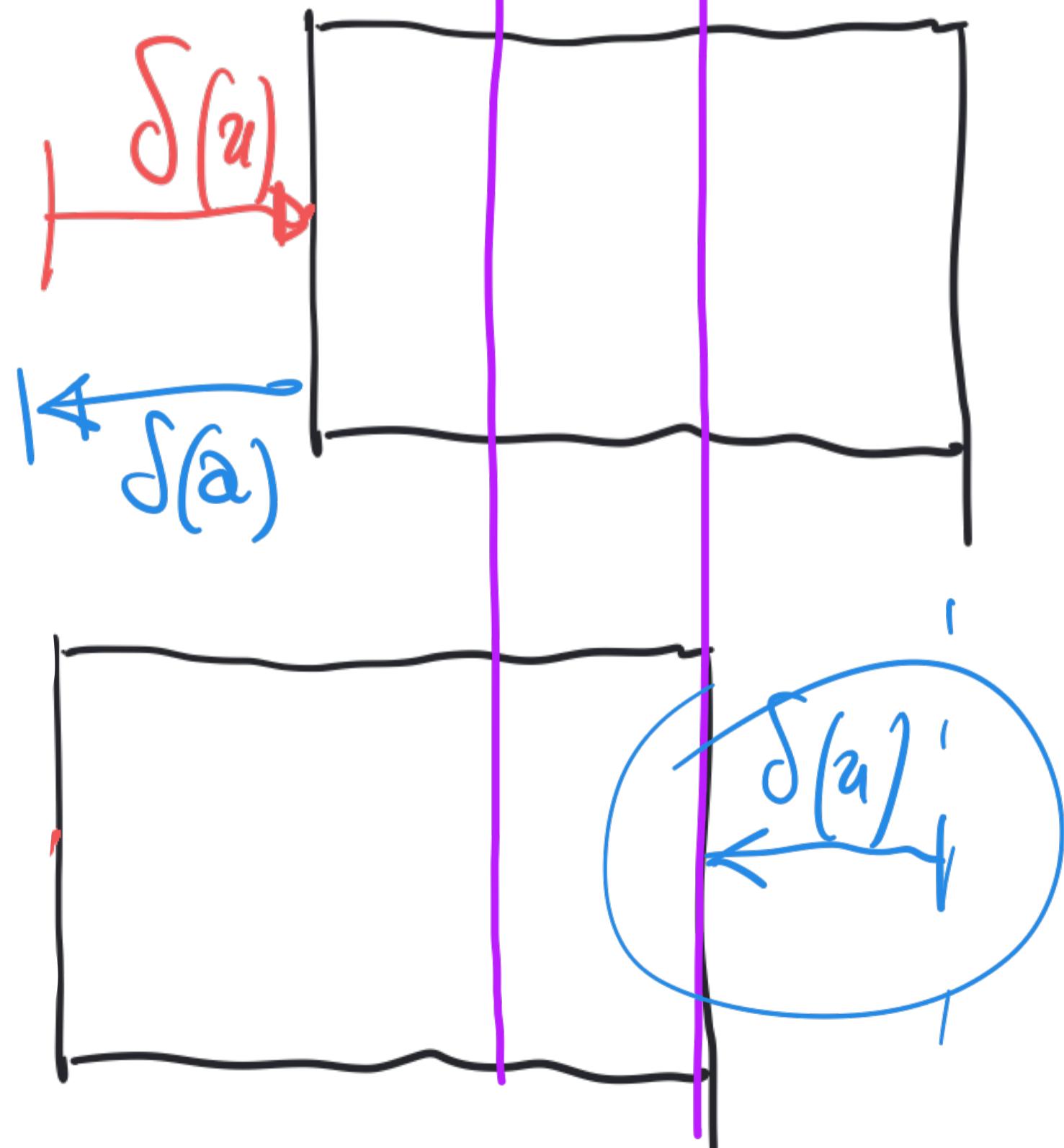
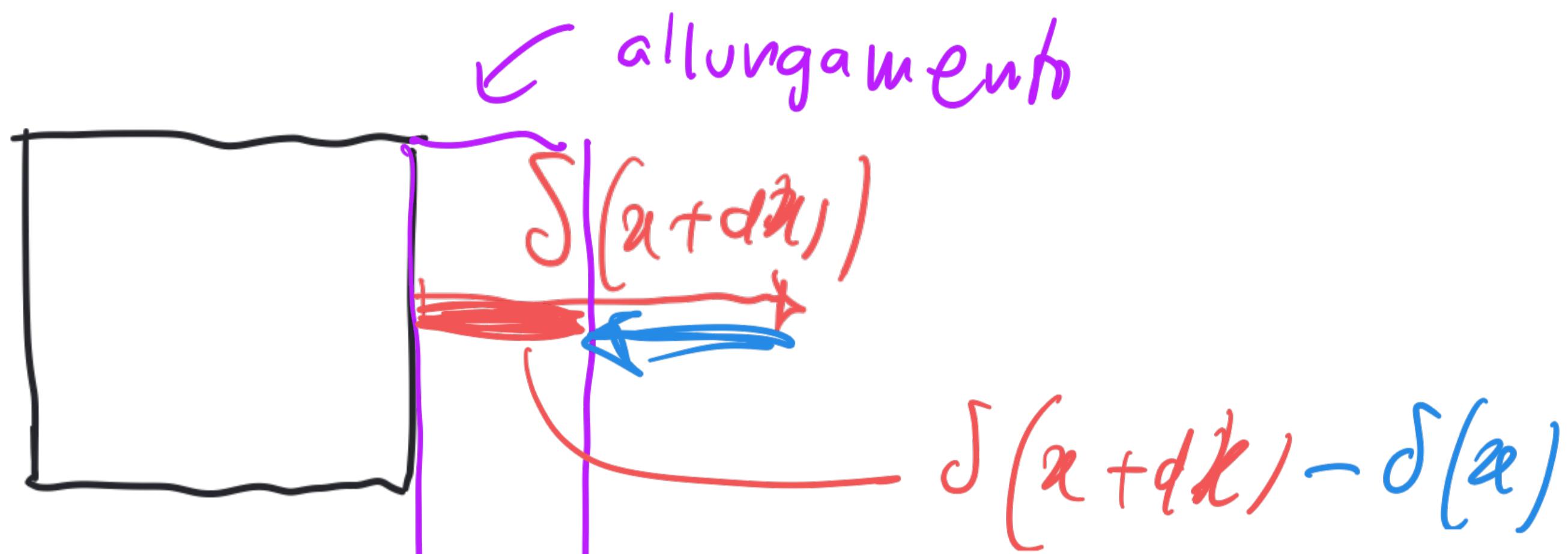
$$\delta(0) = 0 \quad \delta(L) \approx$$

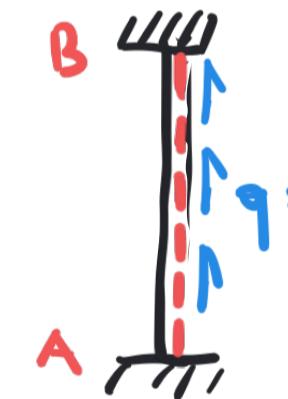


Equilibrio  $\Rightarrow N(x) \Rightarrow \epsilon(x) \Rightarrow \delta(x)$

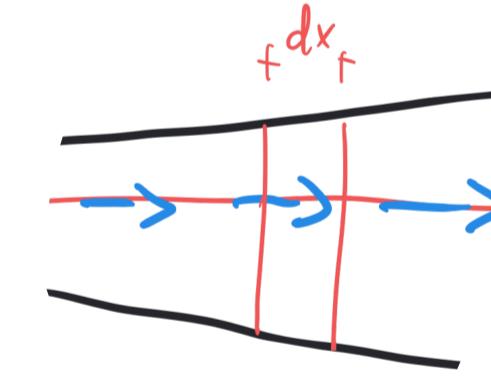


$$\delta(L) = \delta(L|R) = 0 \Rightarrow R$$





$$\boxed{\frac{dN}{dx} + q = 0}$$



✓  $N(x) = q\rho A x + c$

$\delta(x)$  spostamento della sezione

$x$  lungo l'asse

$$\begin{aligned} \delta(x) & \quad \text{displacement of the section} \\ d\alpha & \quad \text{length element} \\ \delta(x+d\alpha) & \quad \text{displacement at position } x+d\alpha \\ d\alpha' & = d\alpha + \delta(x+d\alpha) - \delta(x) \\ \epsilon & = \frac{d\alpha' - d\alpha}{d\alpha} = \frac{d\alpha'}{d\alpha} - 1 \end{aligned}$$

$$\frac{d\alpha'}{d\alpha} = 1 + \frac{\delta(x+d\alpha) - \delta(x)}{d\alpha}$$

$$- \quad \boxed{\epsilon = \frac{\delta(x+d\alpha) - \delta(x)}{d\alpha} = \frac{d\delta}{d\alpha}}$$

