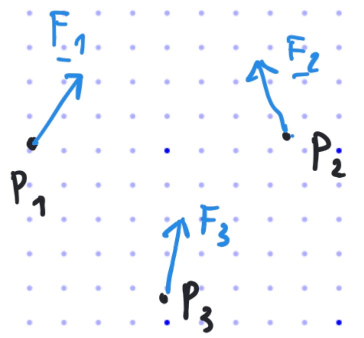
Sistemi di forze (e coppie)



$\Gamma = \{(P_1, \underline{F}_1), \dots, (P_n, \underline{F}_n)\}$

Sistema di forze.

Risultante $\quad \underline{R} = \sum_{i=1}^{n} \underline{F}_i$

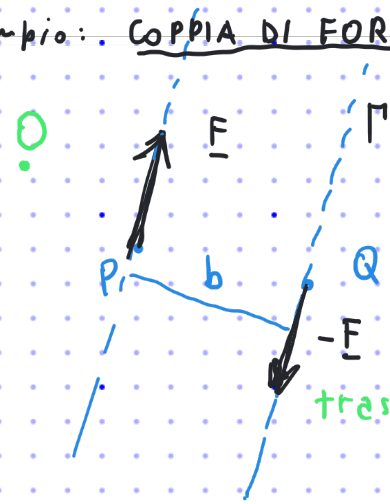
Mom. risult. $\quad \underline{M}_O = \sum_{i=1}^{n} \underbrace{\underline{OP}_i \times \underline{F}_i}_{\underline{M}_O^{(i)}}$ $\quad$ Polo O

Trasporto $\quad \underline{M}_{O'} = \underline{M}_O + \underline{O'O} \times \underline{R}$

Oss: se il sistema e' costituito da una sola forza, allora $\underline{R} = \underline{0} \Rightarrow \underline{M}_O = \underline{0}$.

In generale, $\underline{R} = \underline{0}$ NON IMPLICA $\underline{M}_O = \underline{0}$.

Esempio: COPPIA DI FORZE



$\Gamma = \{(P, \underline{F}), (Q, -\underline{F})\}$

$\underline{R} = \underline{F} + (-\underline{F}) = 0$

$\underline{M}_O \overset{\text{trasporto}}{=} \underline{M}_P = \underline{PQ} \times (-\underline{F})$

$= -bF\,\underline{k}$

mano dx

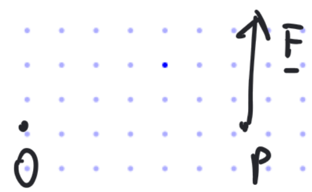
Oss: Γ, Γ' due sistemi

$\Gamma'' = \Gamma' \cup \Gamma''$

$\underline{R}'' = \underline{R} + \underline{R}'$

OSS: $\underline{R} = \underline{0} \Rightarrow \underline{M}_{O'} = \underline{M}_O$

$\underline{M}$ non dipende dal polo.



$\underline{M}_O = \underline{OP} \times \underline{F}$

$\underline{R} = \underline{F}$

$\underline{R} = \underline{0} \Rightarrow \underline{F} = \underline{0} \Rightarrow \underline{M}_O = \underline{0}$

SISTEMA DI FORZE EQUILIBRATO

$\underline{R} = \underline{0}$

$\underline{M}_O = \underline{0}$

Oss: la nozione di S.E. non dipende dalla scelta del polo.

$\underline{M}_Q = \underline{M}_O + \cancel{\underline{QO} \times \underline{R}}$ $\qquad \underline{M}_Q = 0 \Leftrightarrow \underline{M}_O = \underline{0}$

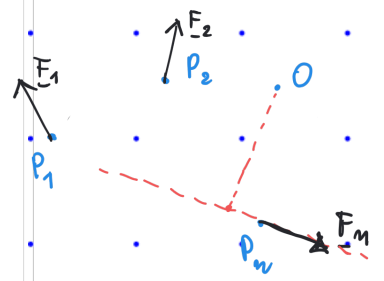
SISTEMI EQUIVALENTI

Γ e Γ' sono equivalenti se

$\underline{R} = \underline{R}'$ e $\underline{M}_O = \underline{M}_O'$

Oss: la def. di S.E. non dipende dal polo.

Operazioni che non alterano l'equivalenza.



1) Trasl. lungo retta di azione

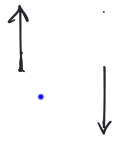
non altera $\underline{R}$. ($\underline{R}$ non dipende dai punti di applicazione)

non cambia $\underline{M}_O$

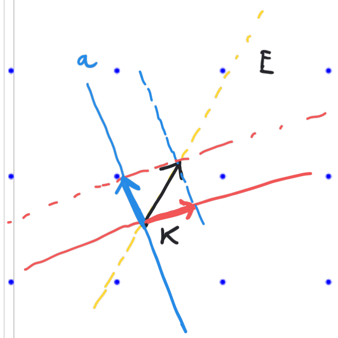
2) Somma di vettori aventi lo stesso punto di applicazione.

procedendo ulteriormente si può pensare di ridurre il sistema a una forza sola.
In realtà ciò non è sempre possibile

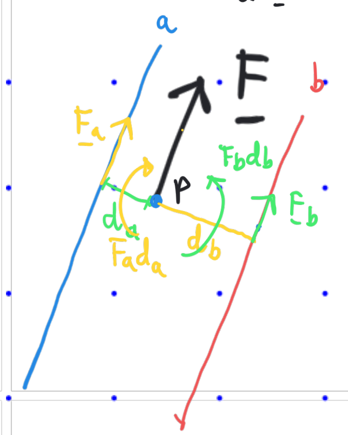
una coppia non è riducibile!



3) Scomposizione di una forza lungo due direzioni assegnate.



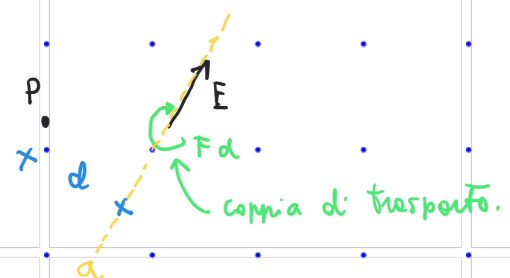
Caso limite: le rette a e b sono parallele a $\underline{F}$.



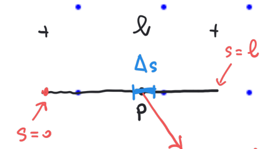
$\underline{M}_P = 0$

$$\boxed{\frac{F_a}{F_b} = \frac{d_b}{d_a}}$$

Trasporto di una forza



FORZE DISTRIBUITE



corpo unidimensionale (fune, asta, trave)

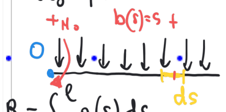
$\underline{b}(P) = \lim_{\Delta s \to 0} \frac{\Delta \underline{F}}{\Delta s}$

densità di forza per unità di lunghezza.

funz. del punto

$\underline{R} = \int_0^\ell \underline{b}(s)\,ds$

Forza peso



$q(s)$ scalare)

$R = \int_0^\ell q(s)\,ds$

$M_0 = \int_0^\ell s\, q(s)\,ds$

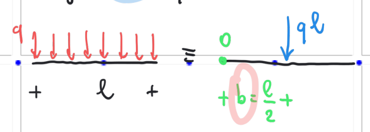
Casi notevoli:

- distr. uniforme $q(s)=q$

$R = q\ell$

$M_0 = q\ell^2/2$ ↺

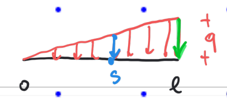
Forza conc. equivalente:



$M_0 = b\,q\ell$
$= \frac{\ell}{2} q\ell$
$= \frac{q\ell^2}{2}$

stessa risultante
stesso mom. risult. } => equiv.

- distrib. triangolare



$R = \frac{q}{\ell}\int_0^\ell s\,ds$

$= \frac{q}{\ell}\,\frac{\ell^2}{2} = \frac{q\ell}{2}$

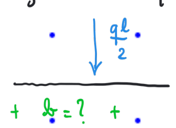
propors.

$q(s) = \frac{s}{\ell} q$

$M_0 = \int_0^\ell s\,\frac{s}{\ell} q\,ds$

$= \frac{q}{\ell}\int_0^\ell s^2 ds = \frac{q\ell^2}{3}$

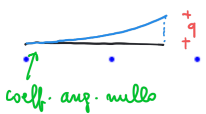
Forza concentr. equiv?



$M_0 = b \cdot \frac{q\ell}{2}$

$b = 2/3\,\ell$

Forza distribuita parabolica



$R = 1/3\, q\ell$

$M_0 = 3/4\, q\ell^2$