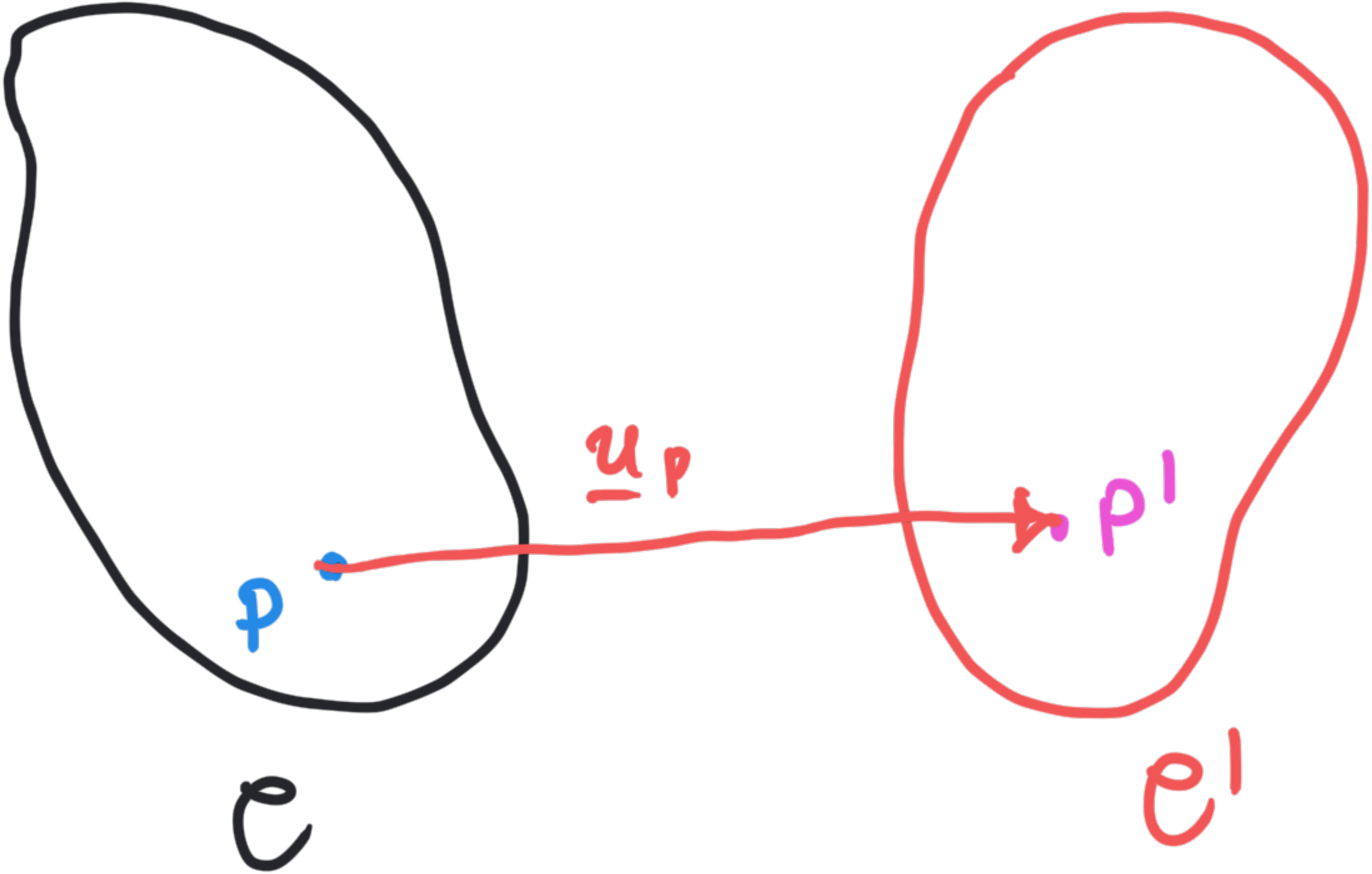
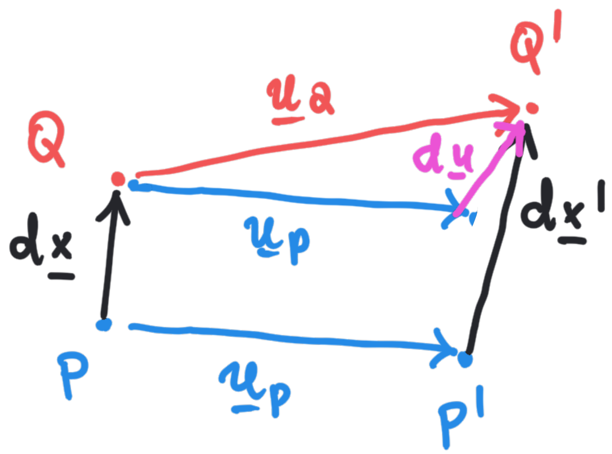


conf. riferimento o iniziale

conf. corrente o deformata

Descrizione locale della deformazione



$$[d\underline{x}] = \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix}$$

$$[\underline{u}] = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \qquad [d\underline{u}] = \begin{bmatrix} du \\ dv \\ dw \end{bmatrix}$$

$$\underline{u}_Q = \underline{u}_P + d\underline{u}$$

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz$$

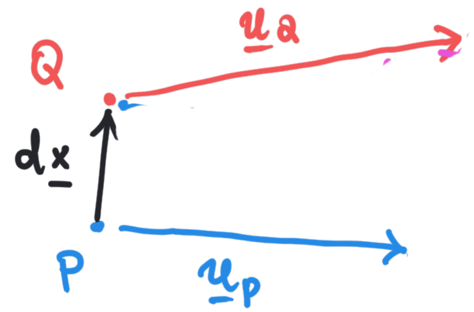
$$dw = \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy + \frac{\partial w}{\partial z}dz$$

$$d\underline{u} = \underline{F}\, d\underline{x}$$

gradienti dello spostamento.

$$[\underline{F}] = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix}$$

Descrizione locale della deformazione



$$[d\underline{x}] = \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix}$$

$$[\underline{u}] = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \qquad [du] = \begin{bmatrix} du \\ dv \\ dw \end{bmatrix}$$

$$\underline{u}_Q = \underline{u}_P + \underline{F}\, d\underline{x}$$

gradienti dello spostamento.

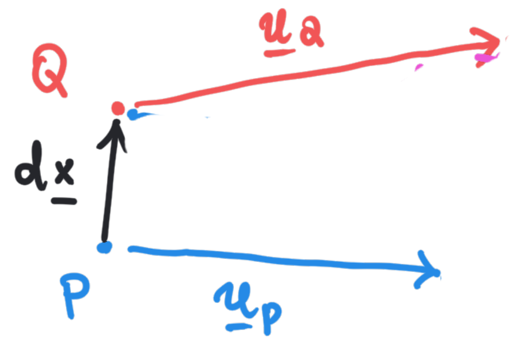
$$[\underline{F}] = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix}$$

$$\underline{F} = \underline{E} + \underline{\Omega}$$

$$\underline{E} := \frac{1}{2}(\underline{F} + \underline{F}^T) \Rightarrow \underline{E} = \underline{E}^T$$

$$\underline{\Omega} := \frac{1}{2}(\underline{F} - \underline{F}^T) \Rightarrow \underline{\Omega} = -\underline{\Omega}^T$$

$$\underline{u}_Q = \underbrace{\underline{u}_P + \underline{\Omega}\, d\underline{x}}_{\text{rigido}} + \underline{E}\, d\underline{x}$$



$$\underline{u}_Q = \underline{u}_p + \underline{F}\, d\underline{x}$$

$$\underline{E} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & * \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & \frac{\partial v}{\partial y} & * \\ \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) & \frac{\partial w}{\partial z} \end{bmatrix}$$

$$\underline{F} = \underline{E} + \underline{\Omega}$$

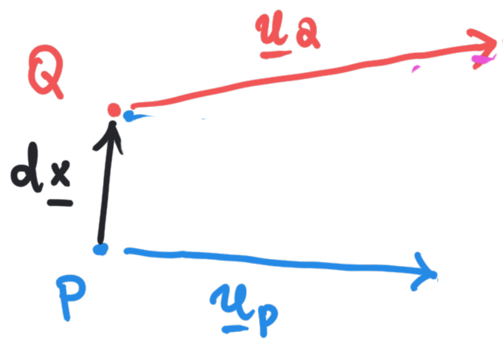
$$\underline{E} := \frac{1}{2}\left(\underline{F} + \underline{F}^T\right) \Rightarrow \underline{E} = \underline{E}^T$$

$$\underline{\Omega} := \frac{1}{2}\left(\underline{F} - \underline{F}^T\right) \Rightarrow \underline{\Omega} = -\underline{\Omega}^T$$

$$\underline{u}_Q = \underbrace{\underline{u}_p + \underline{\Omega}\, d\underline{x}}_{\text{rigido}} + \underline{E}\, d\underline{x}$$

gradienti dello spostamento.

$$[\underline{F}] = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix}$$



$$\underline{u}_Q = \underline{u}_P + \underline{\Omega}\, d\underline{x} + \underline{E}\, d\underline{x}$$

(13.9)

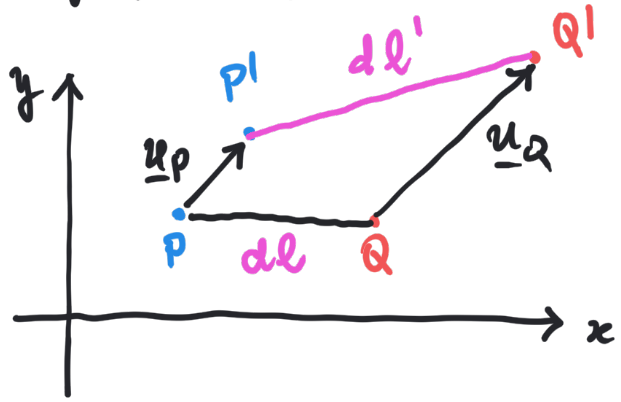
$$[\underline{E}] = \begin{bmatrix} \dfrac{\partial u}{\partial x} & \frac{1}{2}\left(\dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y}\right) & * \\ \frac{1}{2}\left(\dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y}\right) & \dfrac{\partial v}{\partial y} & * \\ \frac{1}{2}\left(\dfrac{\partial w}{\partial x} + \dfrac{\partial u}{\partial z}\right) & \frac{1}{2}\left(\dfrac{\partial w}{\partial y} + \dfrac{\partial v}{\partial z}\right) & \dfrac{\partial w}{\partial z} \end{bmatrix}$$

$$[\underline{E}] = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{yx} & \frac{1}{2}\gamma_{zx} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & \frac{1}{2}\gamma_{zy} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_z \end{bmatrix}$$

$$\varepsilon_x = \frac{\partial u}{\partial x}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial z} = \gamma_{yx}$$

Significato geometrico delle componenti di $\underline{\underline{E}}$.



$$\text{dilat} = \frac{dl' - dl}{dl}$$

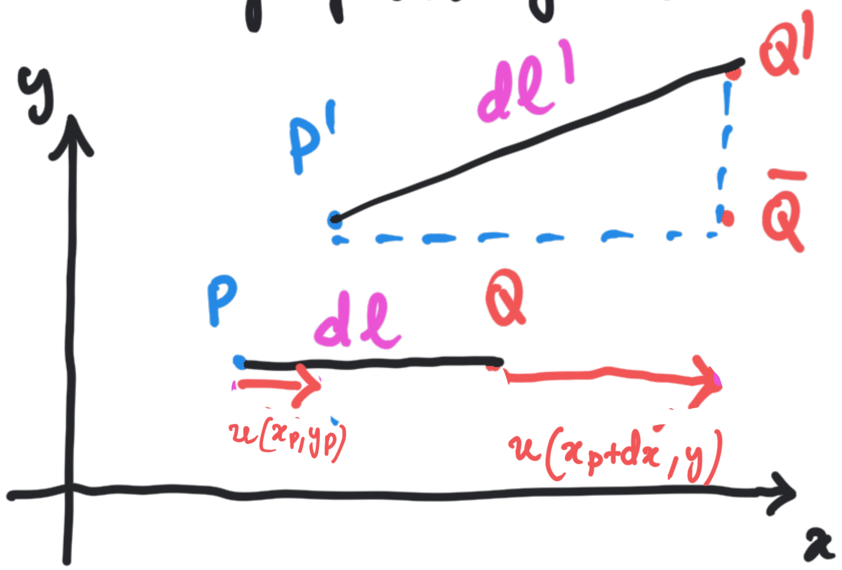
$$[\underline{u}_P] = \begin{bmatrix} u \\ v \end{bmatrix}$$

$$[\underline{\underline{E}}] = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{yx} & \frac{1}{2}\gamma_{zx} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & \frac{1}{2}\gamma_{zy} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_z \end{bmatrix}$$

$$\varepsilon_x = \frac{\partial u}{\partial x}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial z} = \gamma_{yx}$$

Significato geometrico delle componenti di $\underline{\underline{E}}$.



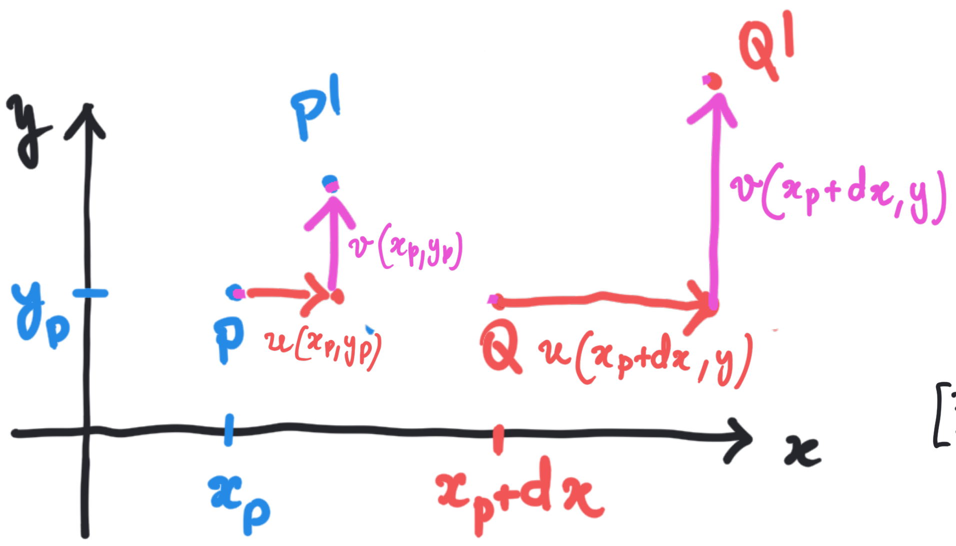
$$\text{dilat} = \frac{dl' - dl}{dl} = \frac{\partial u}{\partial x} = \varepsilon_x$$

$$|P'\bar{Q}| = |PQ| + u(x_p+dx, y_p) - u(x_p, y_p)$$

$$= |PQ| + \frac{\partial u}{\partial x}(x_p, y_p)\,dx$$

$$dl' - dl = \frac{\partial u}{\partial x}\,dx$$

$$dl = dx$$



$$[E] = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{yx} & \frac{1}{2}\gamma_{zx} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & \frac{1}{2}\gamma_{zy} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_z \end{bmatrix}$$

$$\varepsilon_x = \frac{\partial u}{\partial x}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial z} = \gamma_{yx}$$