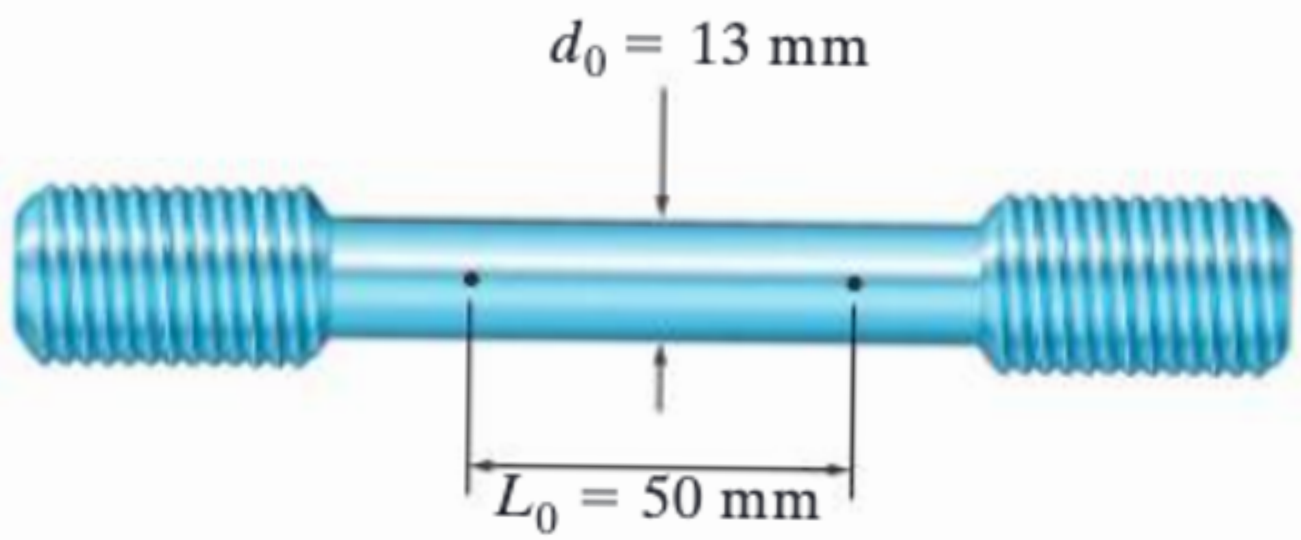
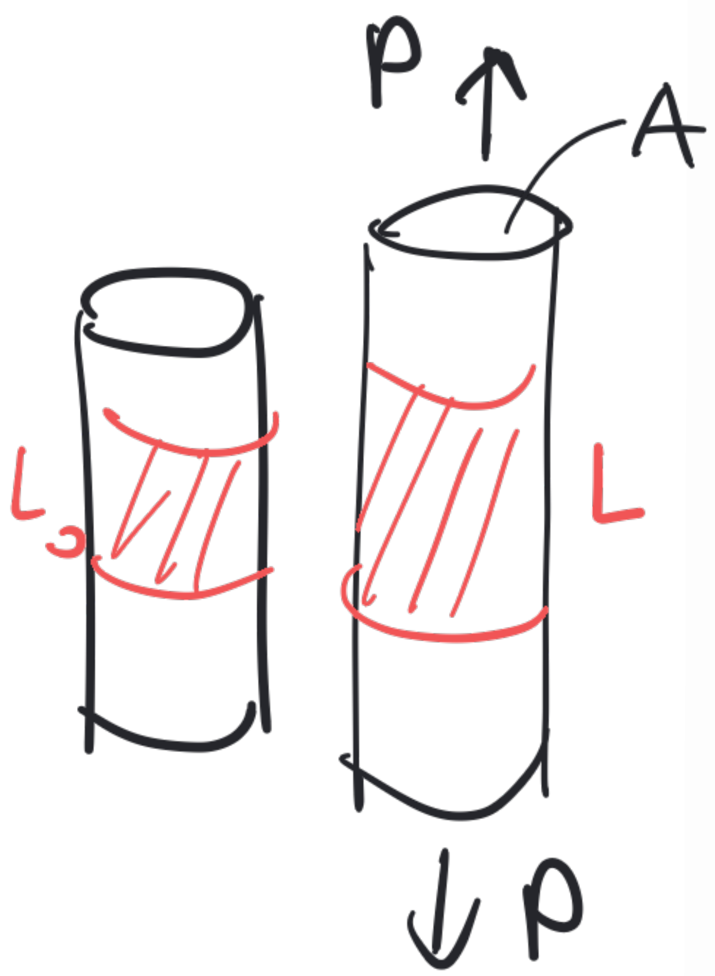
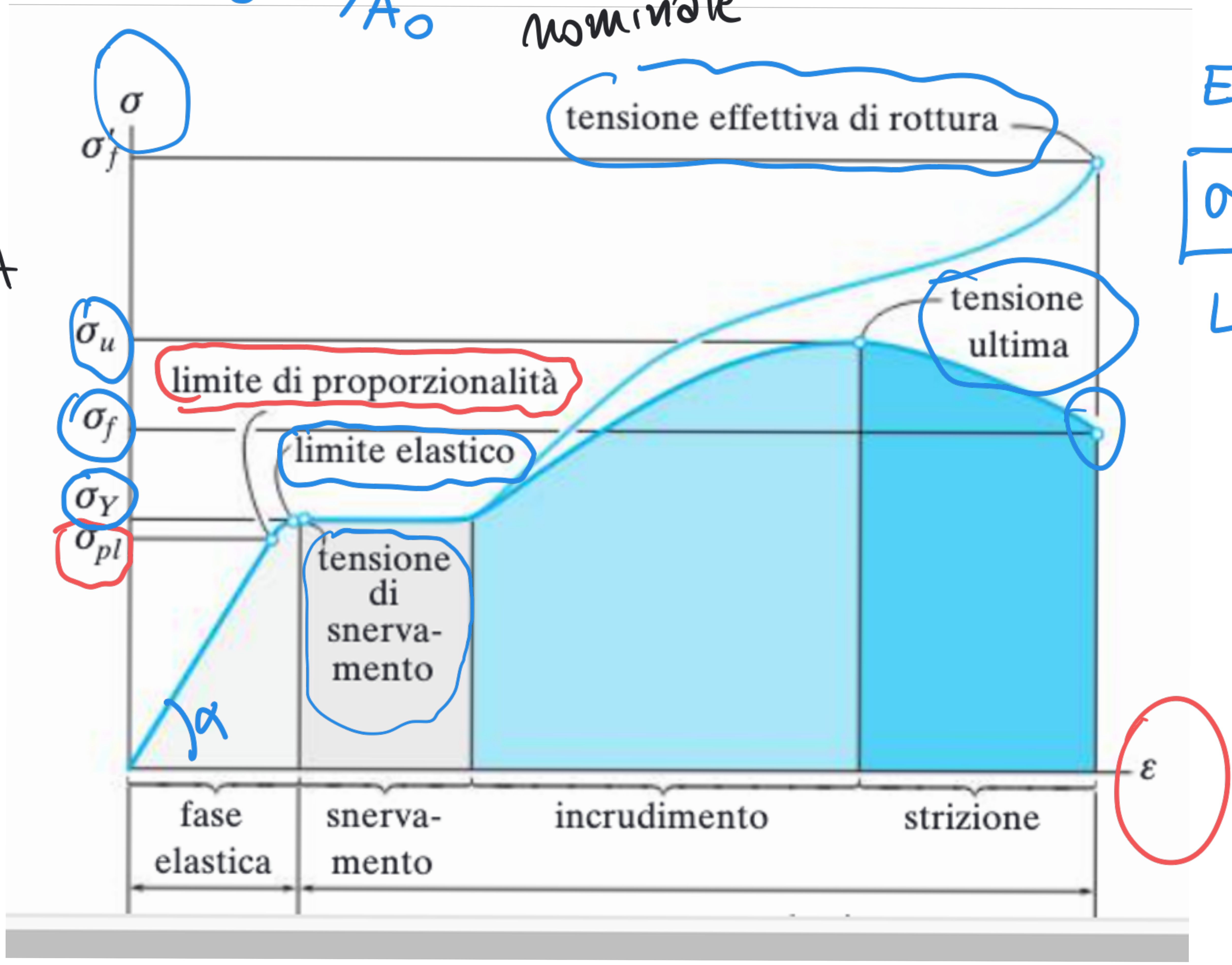


Figura 7.1

▶ Provino di acciaio a sezione circolare sul quale è stato incollato un estensimetro resistivo.

$\sigma = P/A_0$ tensione nominale





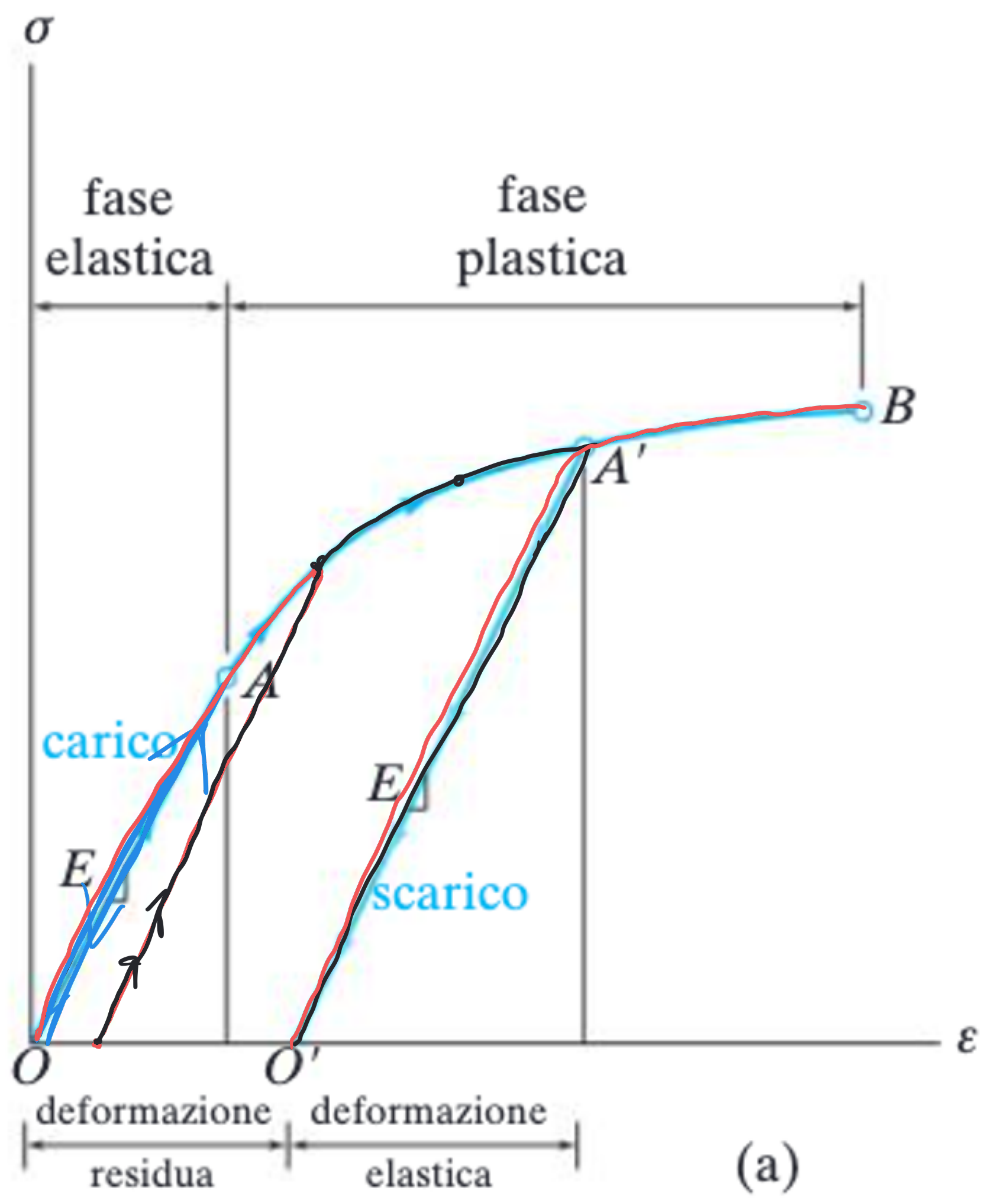
$E = \tan \alpha$

$$\boxed{\sigma = E\varepsilon}$$

Legge di Hooke

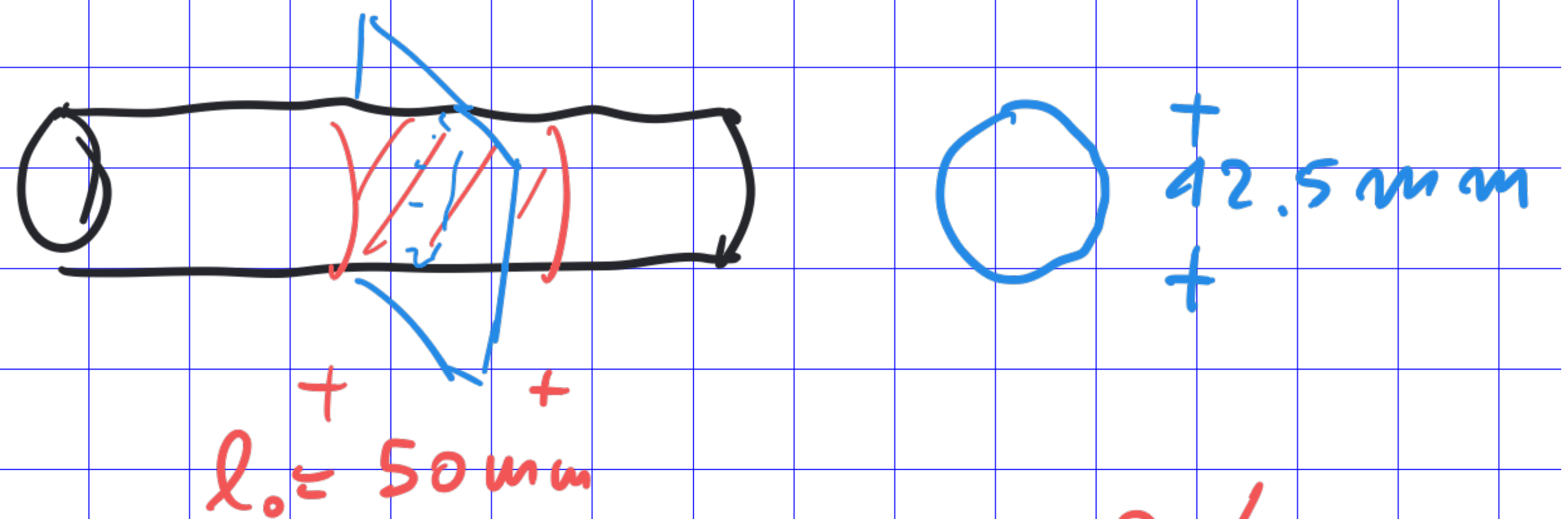
$$\varepsilon = \frac{\delta}{L_0}$$

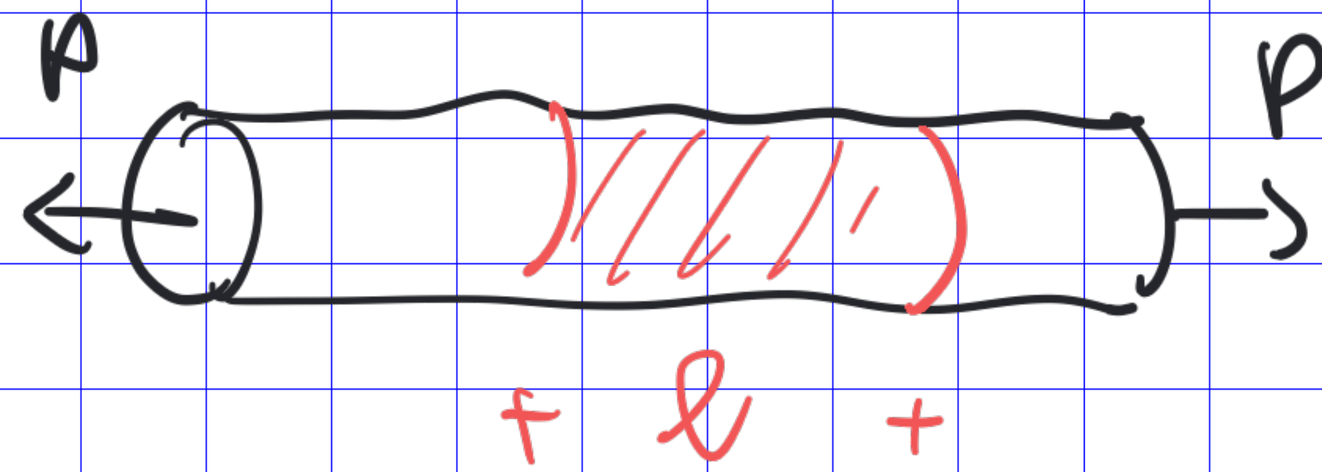
$$\delta = L - L_0$$


σ
fase
elastica
fase
plastica
B
A'
A
carico
E
E
scarico
O
O'
ε
deformazione
residua
deformazione
elastica
(a)

Esempio :

P (kN)	Δℓ (mm)
0	0
7.0	0.0125
21.0	0.0375
36.0	0.0625
50.0	0.0875
53.0	0.125
53.0	0.2
54.0	0.5
75.0	1.0
90.0	2.5
97.0	7.0
87.8	10.0
83.3	11.5





$P \,/\, \Delta\ell$

$\Delta\ell = \ell - \ell_0$

$$A = \frac{\pi}{4} d^2 = 123 \cdot 10^{-6}\ m^2$$

$$\sigma = P/A \qquad \varepsilon = \Delta\ell/\ell_0$$

Esempio:

P (kN)	Δℓ (mm)
0	0
7.0	0.0125
21.0	0.0375
36.0	0.0625
50.0	0.0875
53.0	0.125
53.0	0.2
54.0	0.5
75.0	1.0
90.0	2.5
97.0	7.0
87.8	10.0
83.3	11.5

$$\sigma = \frac{7.0 \cdot 10^3 \, N}{123 \cdot 10^{-6} \, m^2} = 57 \, MPa$$

$$\varepsilon = \frac{0.0125}{50} = 0.00025$$

$\ell_0 = 50 \, mm$

$\varnothing \; 12.5 \, mm$

$$A = \frac{\pi}{4} d^2 = 123 \cdot 10^{-6} \, m^2$$

$$\sigma = P/A \qquad \varepsilon = \Delta\ell / \ell_0$$

σ (MPa)	ε	σ/ε
0	0	?
57	0.00025	228000 MPa

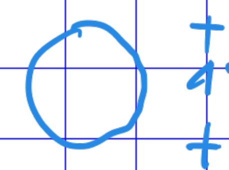
Esempio:

P (kN)	Δl (mm)
0	0
7.0	0.0125
21.0	0.0375
36.0	0.0625
50.0	0.0875
53.0	0.125
53.0	0.2
54.0	0.5
75.0	1.0
90.0	2.5
97.0	7.0
87.8	10.0
83.3	11.5

$$\sigma = \frac{21 \cdot 10^3 \, N}{123 \cdot 10^{-6} \, m^2} = 171 \, MPa$$

$$\varepsilon = \frac{0.0375}{50} = 0.00075$$

$l_0 = 50 \, mm$



12.5 mm

$$A = \frac{\pi}{4} d^2 = 123 \cdot 10^{-6} \, m^2$$

$$\sigma = P/A \qquad \varepsilon = \Delta l / l_0$$

σ (MPa)	ε	σ/ε
0	0	?
57	0.00025	228000 MPa
171	0.00075	228.000 MPa

Esempio:

P (kN)	Δl (mm)
0	0
7.0	0.0125
21.0	0.0375
36.0	0.0625
50.0	0.0875
53.0	0.125
53.0	0.2
54.0	0.5
75.0	1.0
90.0	2.5
97.0	7.0
87.8	10.0
83.3	11.5

$$\sigma = \frac{36 \cdot 10^3\ N}{123 \cdot 10^{-6}\ m^2} = 294\ MPa$$

$$\varepsilon = \frac{0.0625\ mm}{50\ mm} = 0.00125$$

$l_0 = 50\,mm$

12.5 mm

$$A = \frac{\pi}{4} d^2 = 123 \cdot 10^{-6}\ m^2$$

$$\sigma = P/A \qquad \varepsilon = \Delta l / l_0$$

σ (MPa)	ε	σ/ε
0	0	?
57	0.00025	228000 MPa
171	0.00075	228.000 MPa
294	0.00125	234.000 MPa

Esempio:

P (kN)	$\Delta \ell$ (mm)
0	0
7.0	0.0125
21.0	0.0375
36.0	0.0625
50.0	0.0875
53.0	0.125
53.0	0.2
54.0	0.5
75.0	1.0
90.0	2.5
97.0	7.0
87.8	10.0
83.3	11.5

$\ell_0 = 50\,mm$

12.5 mm

$$A = \frac{\pi}{4} d^2 = 123 \cdot 10^{-6}\, m^2$$

$$\sigma = P/A \qquad \varepsilon = \Delta\ell / \ell_0$$

$$\sigma = \frac{50 \cdot 10^3\, N}{123 \cdot 10^{-6}\, m^2} = 408\ MPa$$

$$\varepsilon = \frac{0.0875}{50\ mm} = 0.00175$$

σ (MPa)	ε	σ/ε
0	0	?
57	0.00025	228000 MPa
171	0.00075	228.000 MPa
294	0.00125	234.000 MPa
408	0.00175	233000 MPa

Esemi

$l_0 = 50\,mm$

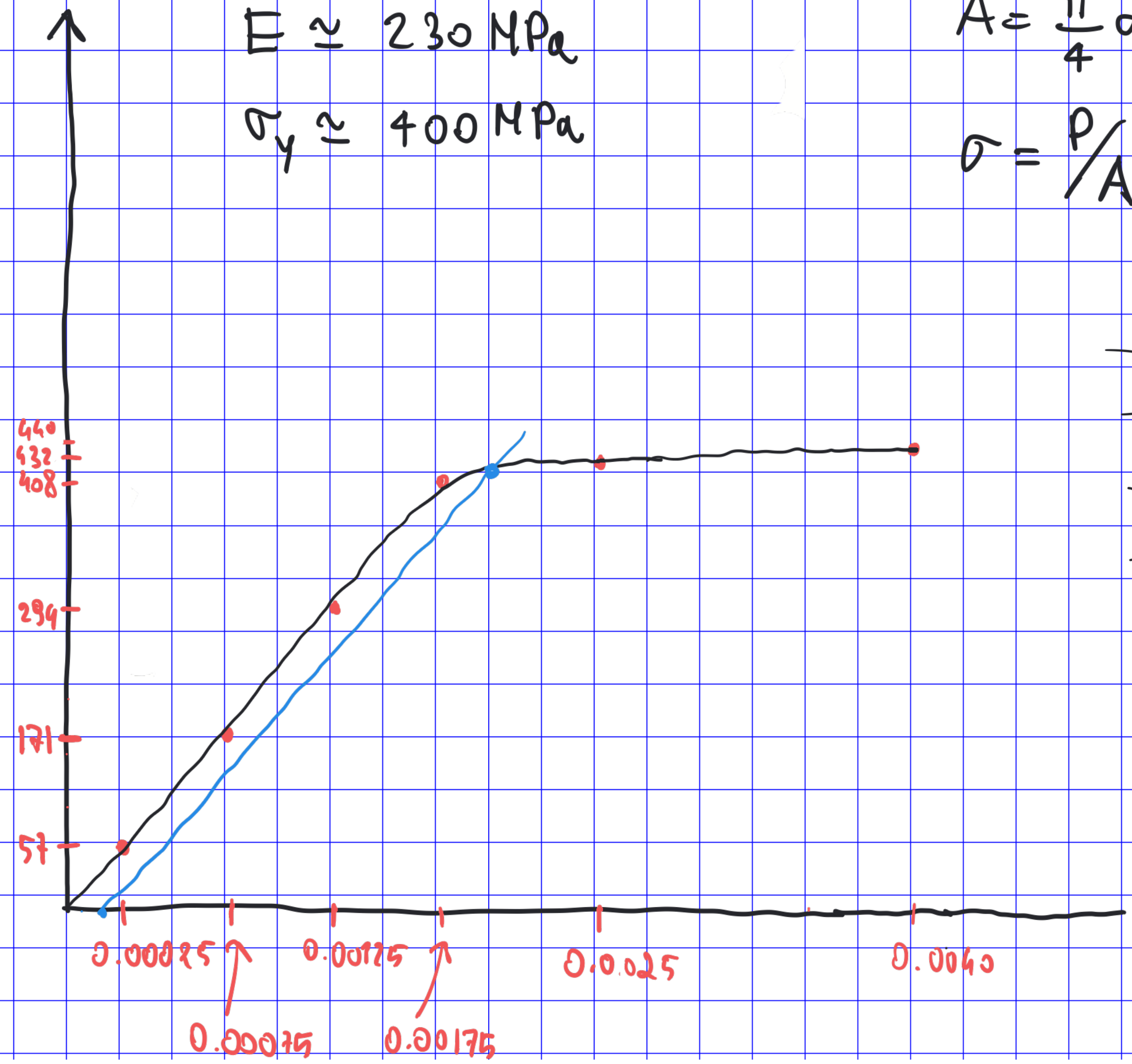
$\varnothing\ 12.5\,mm$

$E \simeq 230\,MPa$

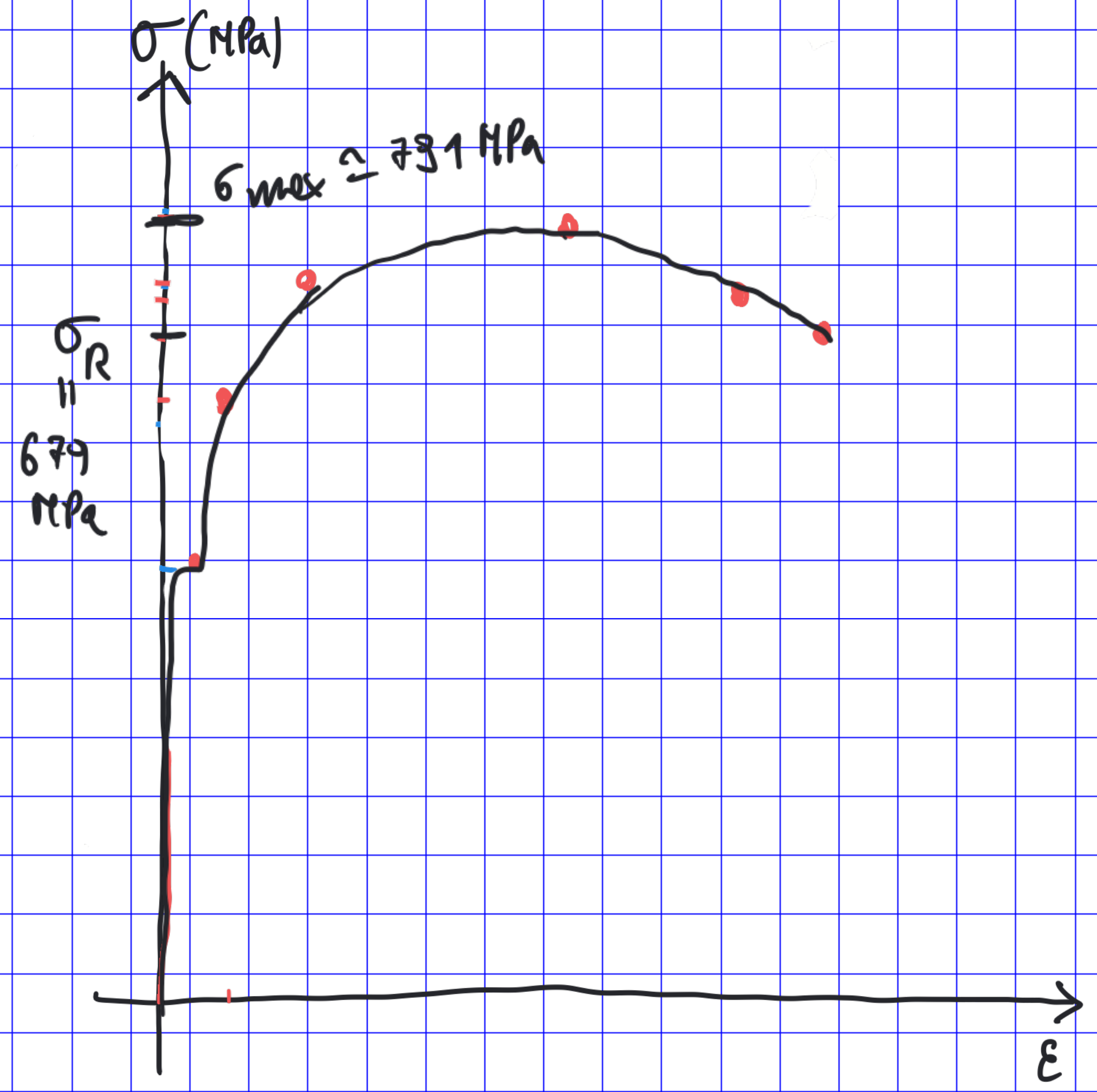
$\sigma_y \simeq 400\,MPa$

$A = \frac{\pi}{4} d^2 = 123 \cdot 10^{-6}\,m^2$

$\sigma = P/A \qquad \varepsilon = \Delta l / l_0$



σ (MPa)	ε	σ/ε
0	0	?
57	0.00025	228000 MPa
171	0.00075	228.000 MPa
294	0.00125	234.000 MPa
408	0.00175	233 000 MPa
432	0.0025	172800 MPa
440	0.0040	

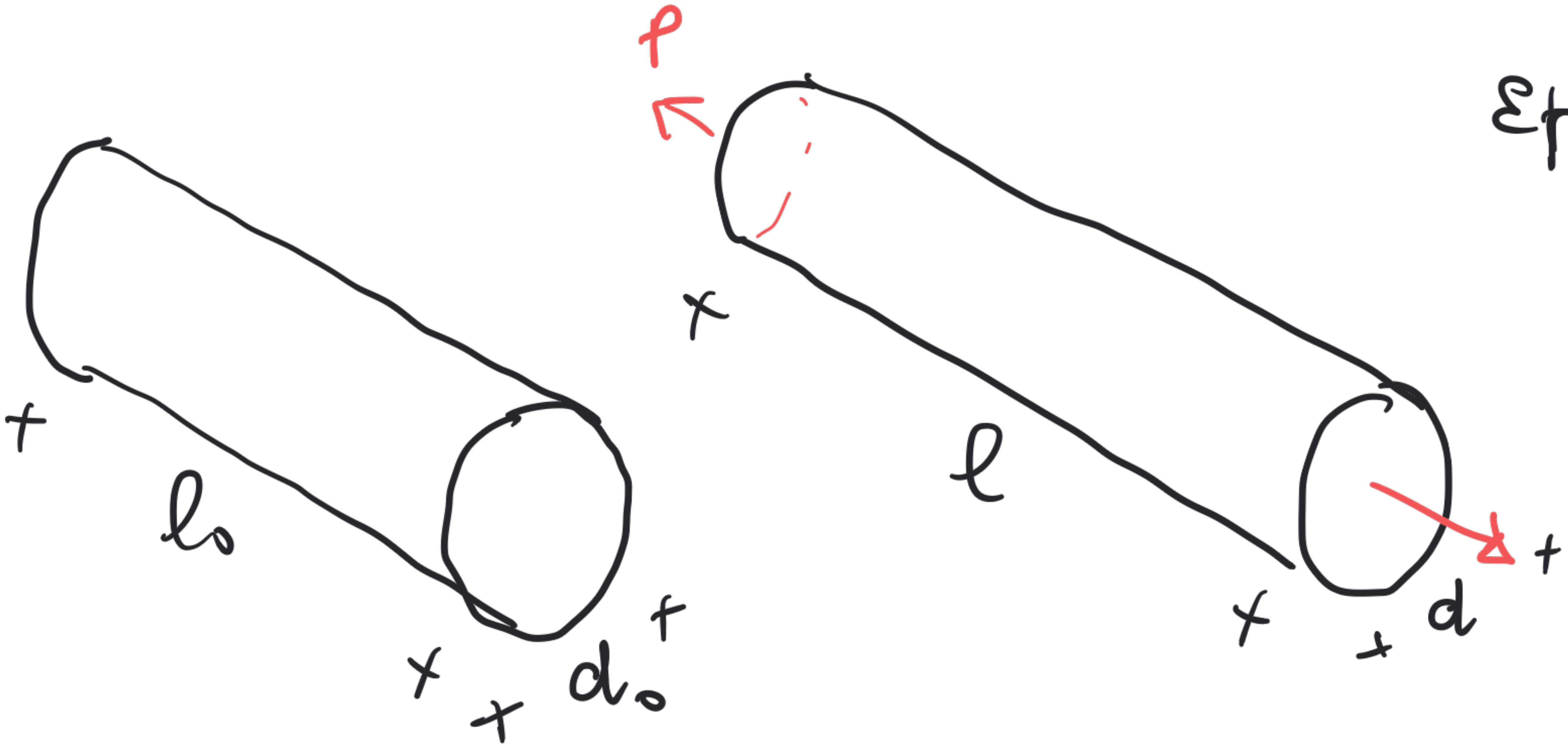


$E \simeq 230$ MPa

$\sigma_y \simeq 400$ MPa

σ(MPa)	ε(mm/mm)
0	0
57.07	0.00025
171.21	0.00075
293.51	0.00125
407.66	0.00175
432.12	0.0025
432.12	0.0040
440.27	0.010
611.49	0.020
733.79	0.050
790.86	0.140
715.85	0.200
679.16	0.230

Coefficiente di Poisson



$$\varepsilon_{long} = \frac{l - l_0}{l_0}$$

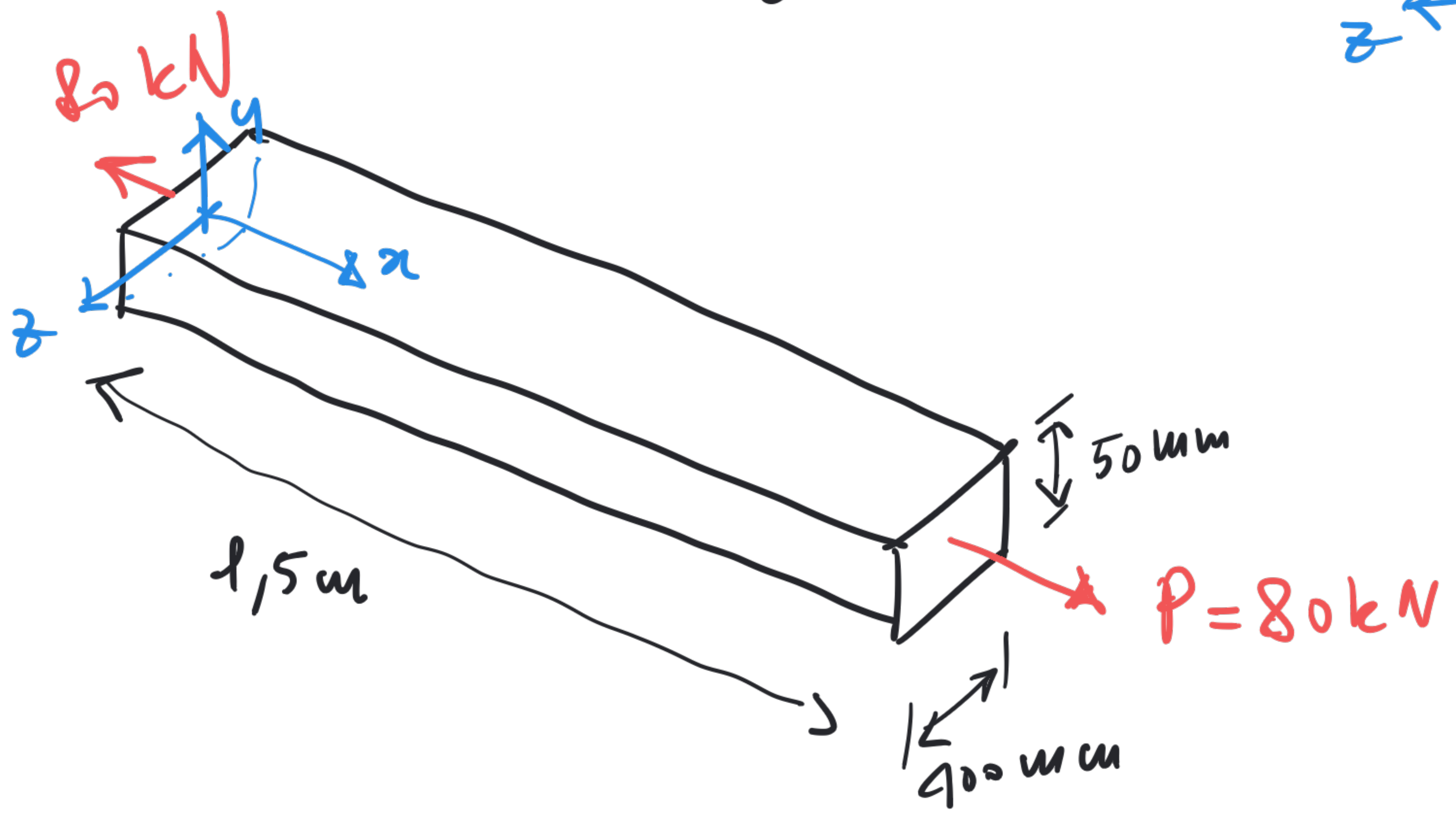
$$\varepsilon_{trasv} = \frac{d - d_0}{d_0}$$

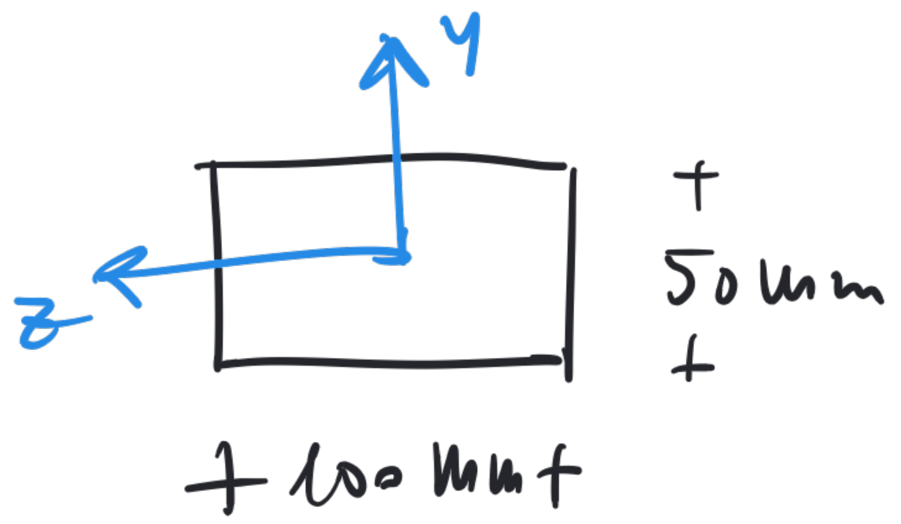
$$-\frac{\varepsilon_{trasv}}{\varepsilon_{long}} = \nu$$

$\nu \leq 0.5$

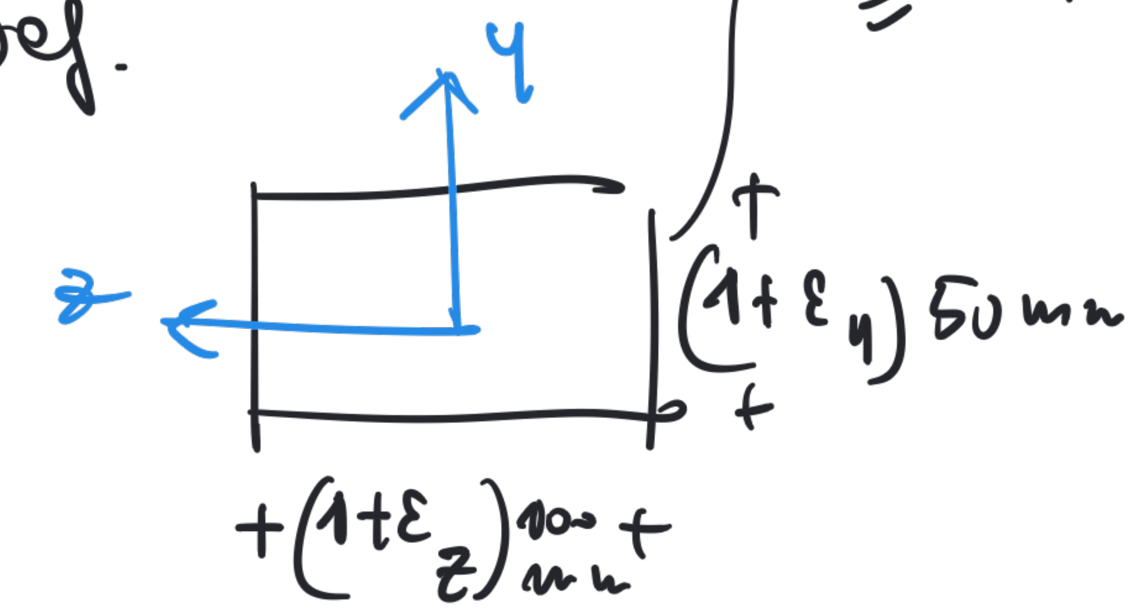
Esempio: $E = 200\ \text{GPa}$

$\nu = 0.32$





Def.



$\delta_y = \varepsilon_y\, 50\ \text{mm}$

$= -1{,}28\ \mu\text{m}$

$\delta_z = (\cancel{1}+\varepsilon_z)\,100\ \text{mm}$

$\cancel{-100\ \text{mm}}$

$= \varepsilon_z \cdot 100\ \text{mm}$

$= -2{,}56\ \mu\text{m}$

Calcolare la variazione di area della regione

$A_0 = 5000\ \text{mm}^2$

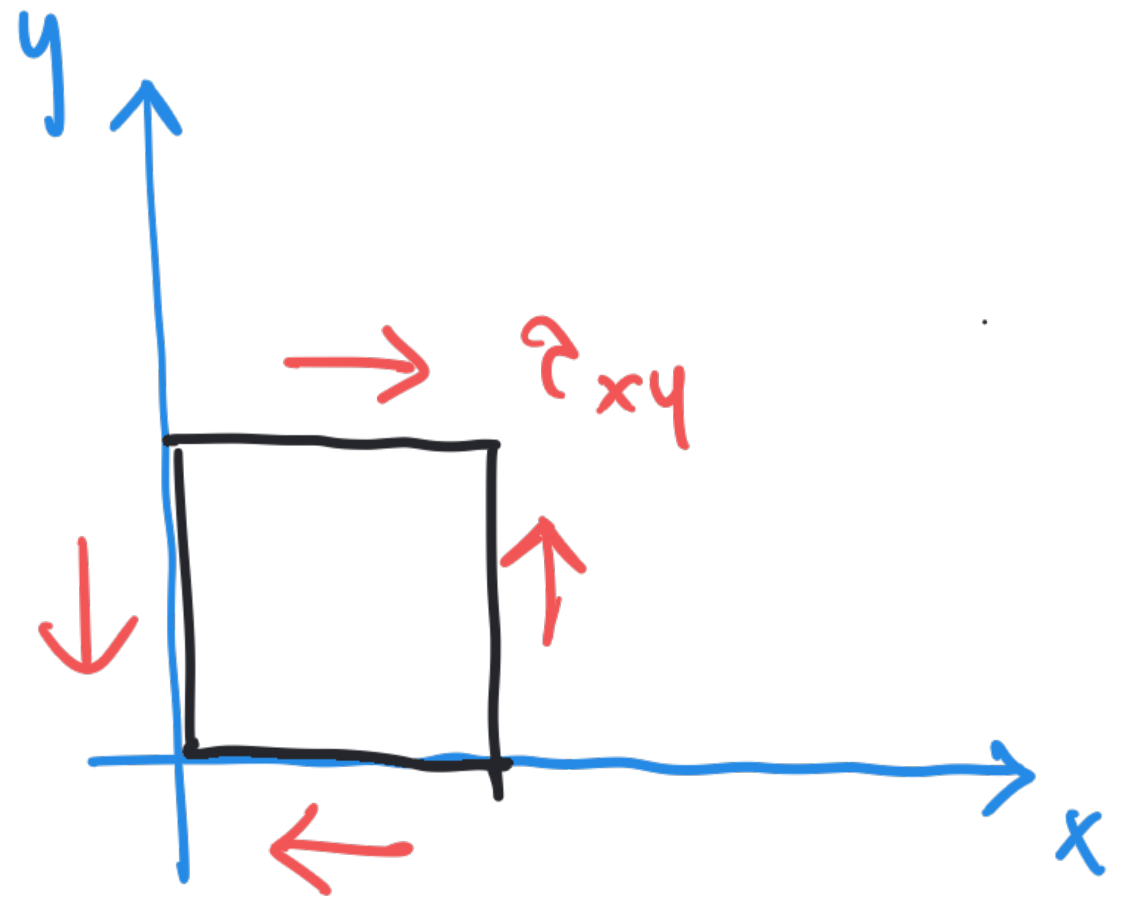
$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{P}{A_0}\,\frac{1}{E} = \frac{80\,(10^3)\ \text{N}}{(0.1)\ \text{m}\,(0.05\ \text{m})} \cdot \frac{1}{200\,(10^9)\ \text{Pa}} = 80\,(10^{-6})$$

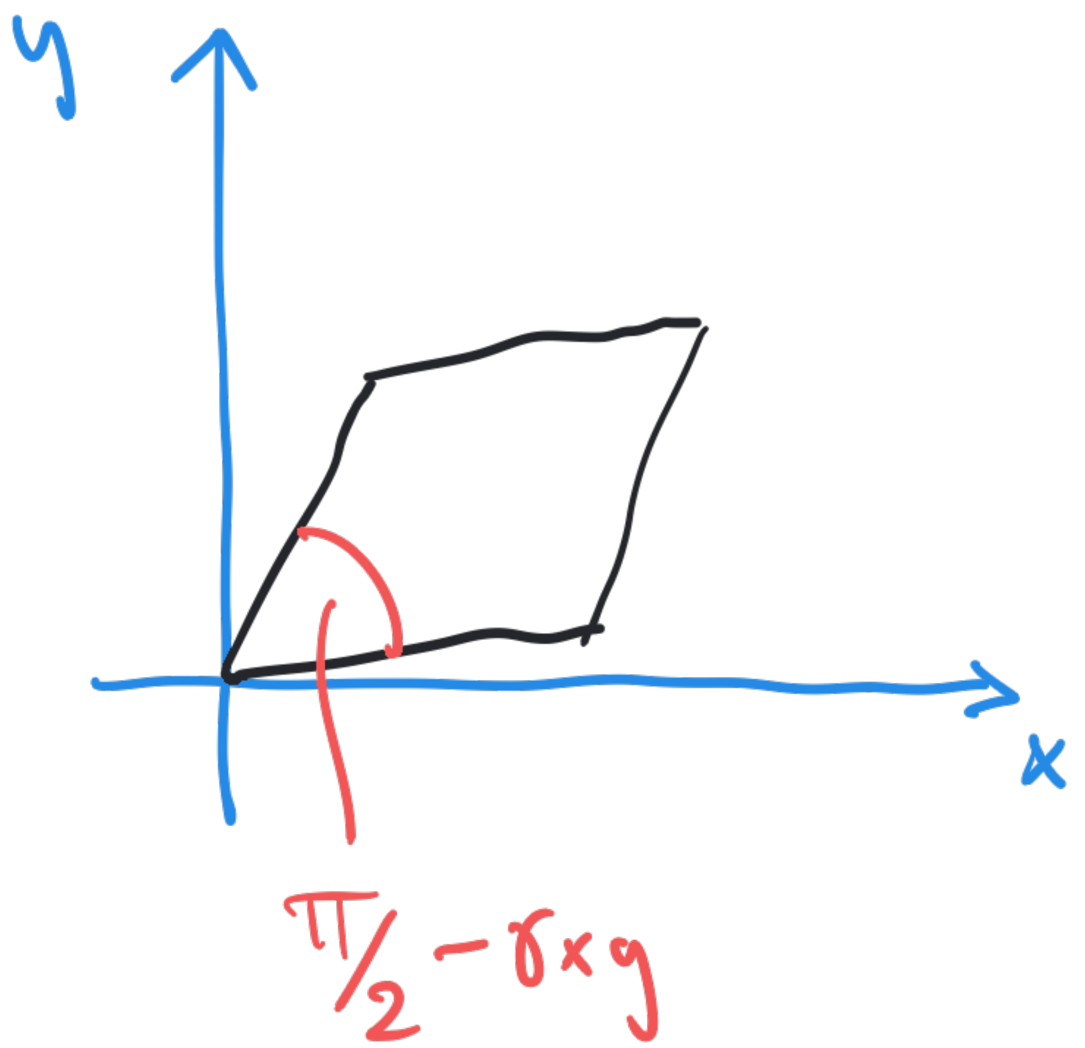
$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} \Rightarrow \varepsilon_y = -\nu\varepsilon_x = -0.32 \cdot 80\,(10^{-6}) = -25.6 \cdot 10^{-6} = -25.6\ \mu\text{m/m}$$

$$\varepsilon_y = \varepsilon_z$$

$$A = (1+\varepsilon_z)(100\ \text{mm}) \times (1+\varepsilon_y)(50\ \text{mm}) = (1+\varepsilon_z)(1+\varepsilon_y)\,A_0 \simeq (1+\varepsilon_z+\varepsilon_y)\,A_0$$

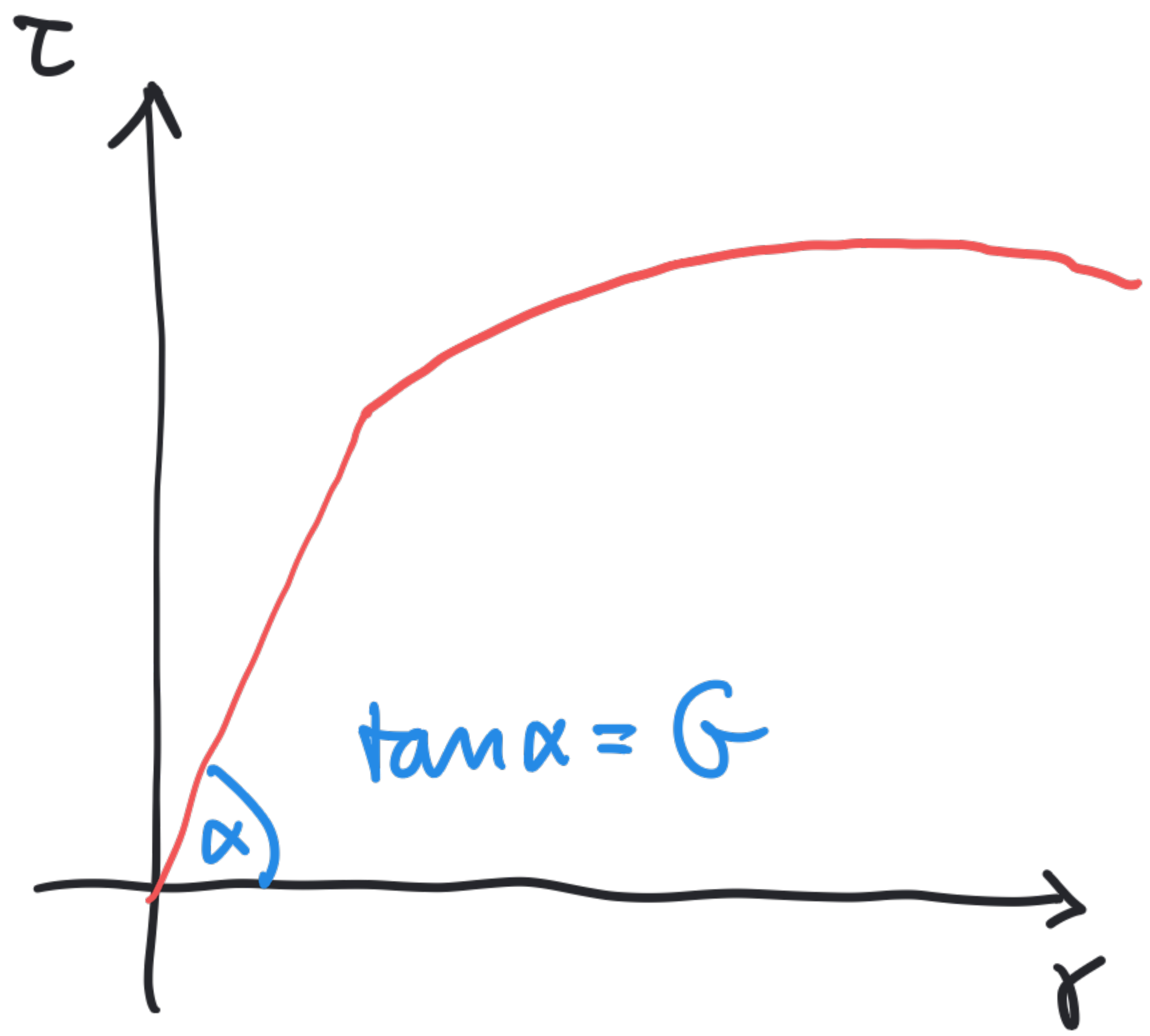
Diagramma tensione tangenziale - scorrimento angolare





$$\tau = G\gamma$$

G modulo di elasticità tangenziale



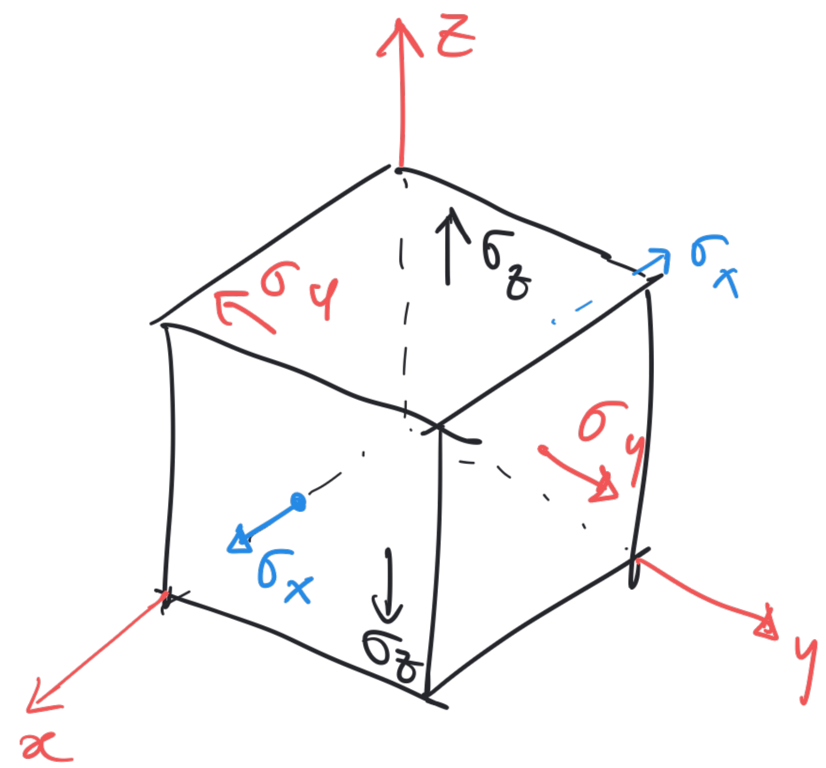
$$G = \frac{E}{2(1+\nu)}$$

Legame costitutivo generalizzato

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E} - \nu \frac{\sigma_x}{E}$$

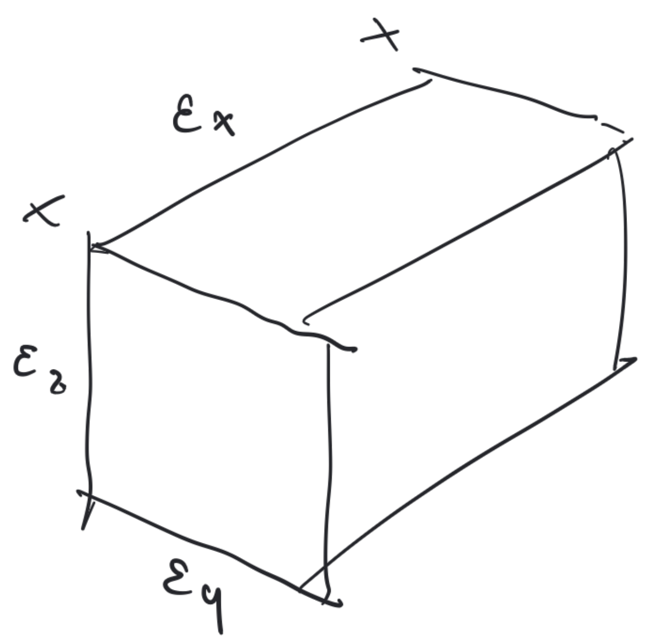
$$\varepsilon_z = -\nu \frac{\sigma_y}{E} - \nu \frac{\sigma_x}{E} + \frac{\sigma_z}{E}$$



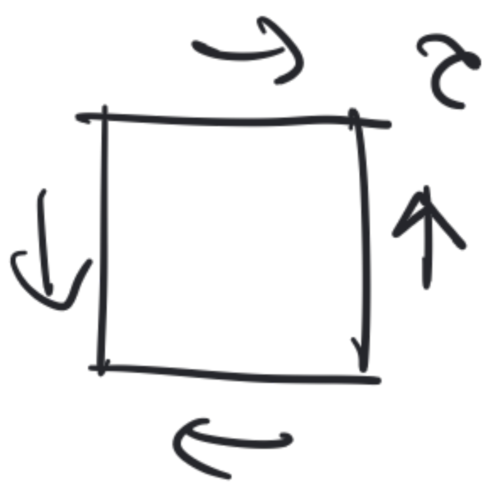
$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right) \qquad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

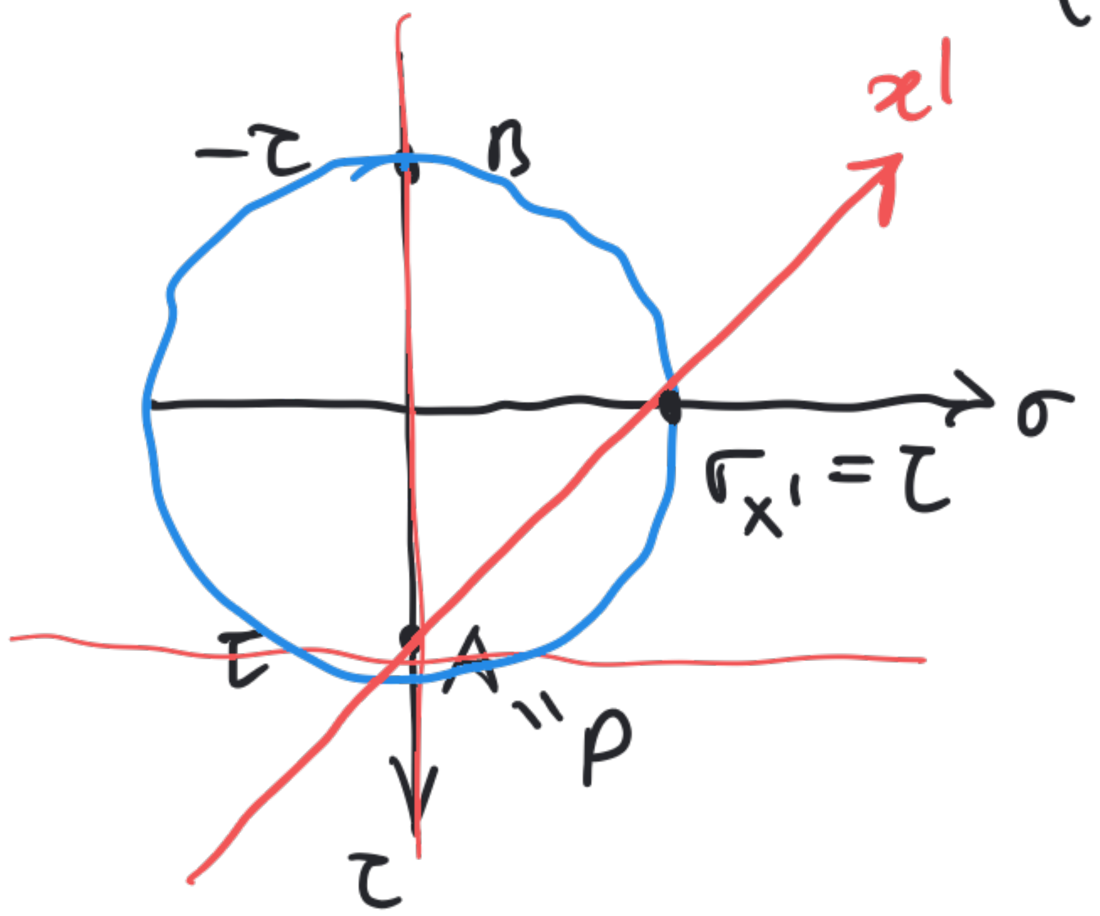
$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right) \qquad \gamma_{yz} = \frac{\tau_{yz}}{G}$$

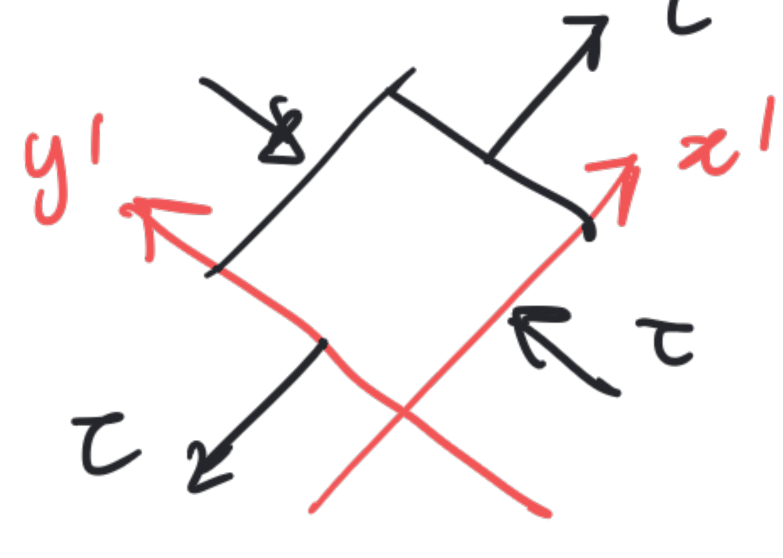
$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right) \qquad \gamma_{zx} = \frac{\tau_{zx}}{G}$$



Dimostrazione della relazione $G = \dfrac{E}{2(1+\nu)}$





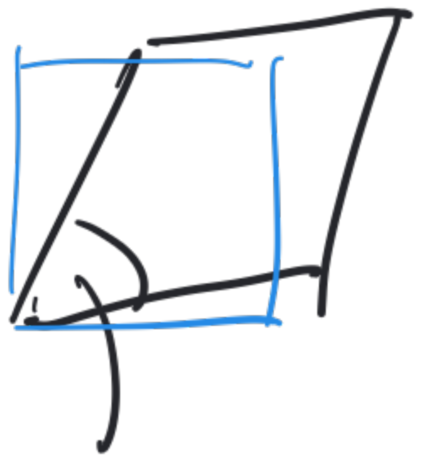


$\sigma_{x'} = \tau$

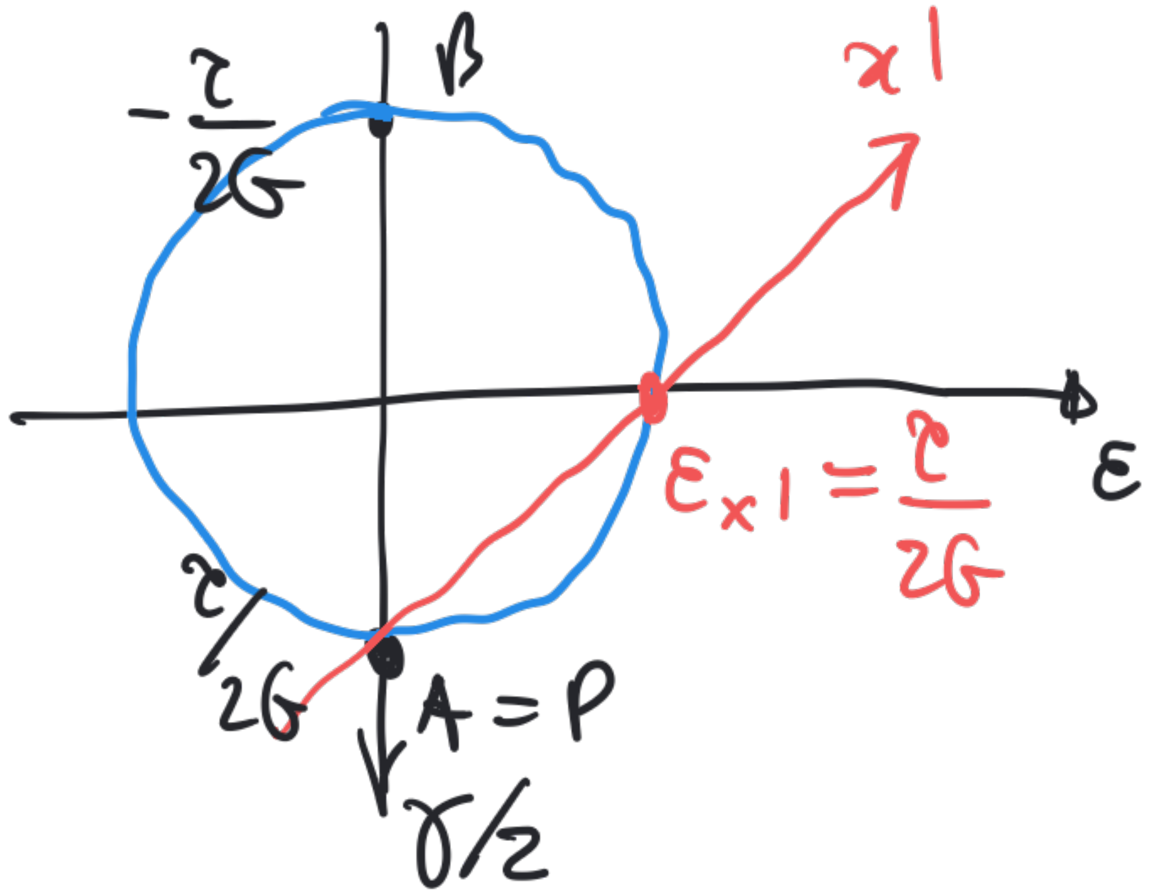
$\sigma_{y'} = -\tau$

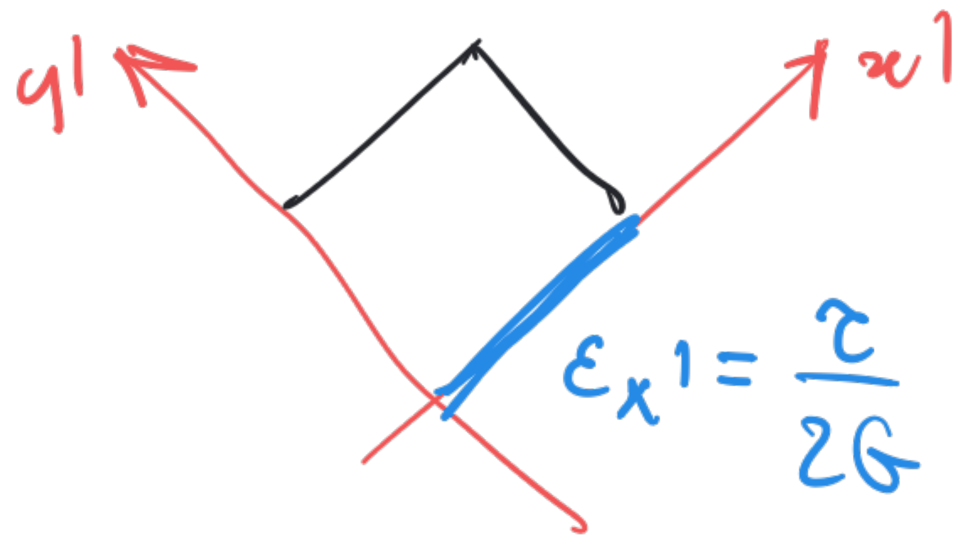
$$\frac{\tau}{2G} = \varepsilon_{x'} = \frac{\sigma_{x'}}{E} - \nu\,\frac{\sigma_{y'}}{E} = \frac{1}{E}(1+\nu)\tau$$

$$\Rightarrow \frac{1}{2G} = \frac{1+\nu}{E} \Rightarrow G = \frac{E}{(1+\nu)2}$$



$\pi/2 - \gamma \qquad \gamma = \dfrac{\tau}{G}$





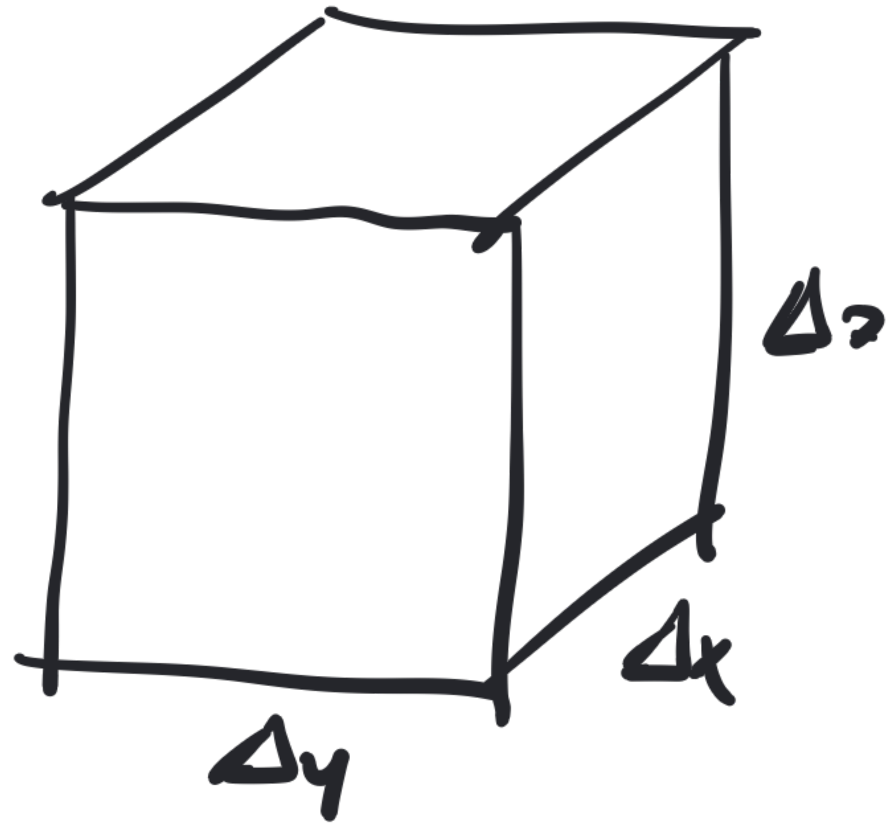
$$e = \frac{\delta V}{V_0} = \frac{V - V_0}{V_0} = \frac{V}{V_0} - 1$$

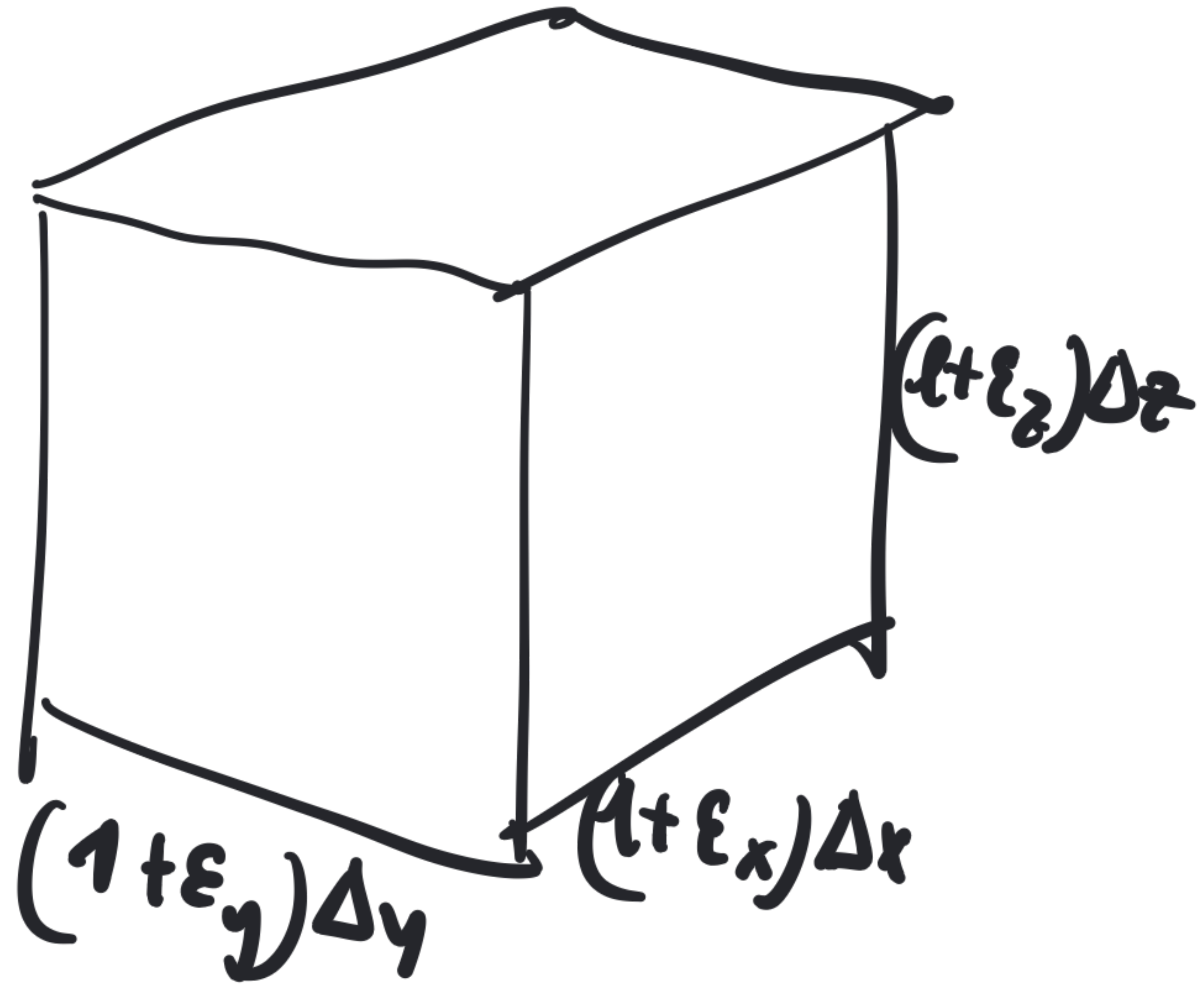
$$= \frac{(1+\varepsilon_x)\Delta x \,(1+\varepsilon_y)\Delta y \,(1+\varepsilon_z)\Delta z}{\Delta x \,\Delta y \,\Delta z} - 1$$

$$= (1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z) - 1$$

$$\simeq \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$= -\frac{1-2\nu}{E} 3p$$





$p = -ke \qquad k = \frac{E}{3(1-2\nu)}$ modulo di elasticità volumetrica