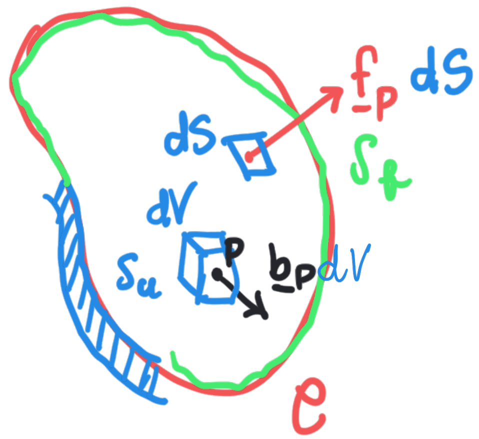
Consideriamo un corpo continuo tridimensionale

Supponiamo che una parte della sua superficie S_u sia vincolata a terra e che la porzione rimanente sia soggetta a forze note f

forze esterne di volume descritte da campo vettoriale $\underline{b}$ tale che la forza che agisce su un elemento dV è pari a $\underline{b}\,dV$.

FORZE ESTERNE



superficie

$\uparrow$

$P \in S_f \quad \underline{f}_P$

$P \in \mathcal{C} \quad \underline{b}_P$

$\downarrow$

volume

$$\underline{R}(S') = \int_{S'} \underline{f}_p \, dS \qquad \underline{R}(C') = \int_{C'} \underline{b}_p \, dV$$

superficie

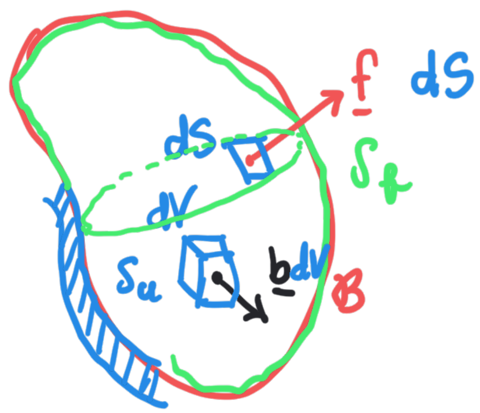
$P \in S_f \quad \underline{f}_p$

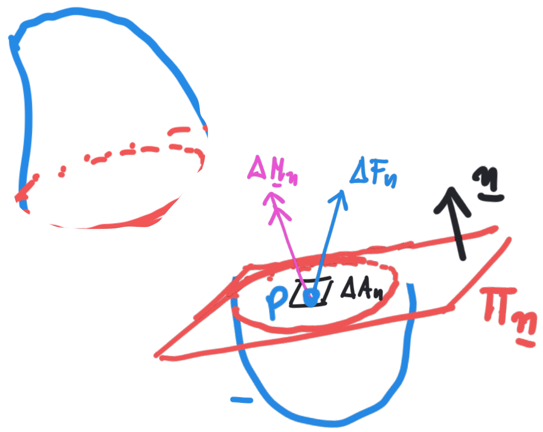
$P \in \mathcal{B} \quad \underline{b}_p$

volume

Per effetto delle forze esterne, si instaura nel corpo delle tensioni interne. Per caratterizzarle consideriamo un piano π

$$\underline{R}(S') = \int_{S'} f_p \, dS \qquad \underline{R}(S) = \int_{C'} b_p \, dV$$

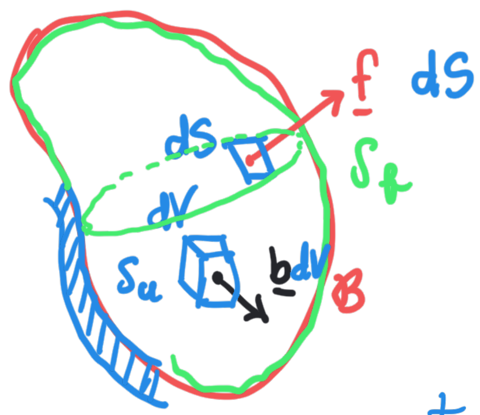


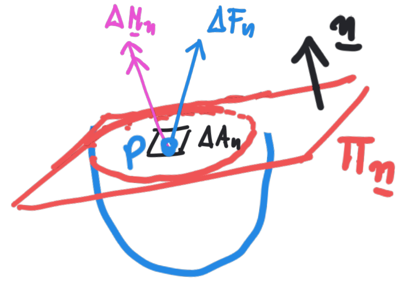


$$\underline{t}_n := \lim_{\Delta A_n \to 0} \frac{\Delta \underline{F}_n}{\Delta A_n} \qquad \underline{m}_n := \lim_{\Delta A_n \to 0} \frac{\Delta \underline{M}_n}{\Delta A_n}$$

Continuo di Cauchy : $\underline{m}_n = \underline{0}$

- $\underline{t}_n = \underline{t}_n(P)$ tensione di Cauchy





$$\underline{t}_n := \lim_{\Delta A_n \to 0} \frac{\Delta \underline{F}_n}{\Delta A_n} \qquad \underline{m}_n := \lim_{\Delta A_n \to 0} \frac{\Delta \underline{M}_n}{\Delta A_n}$$

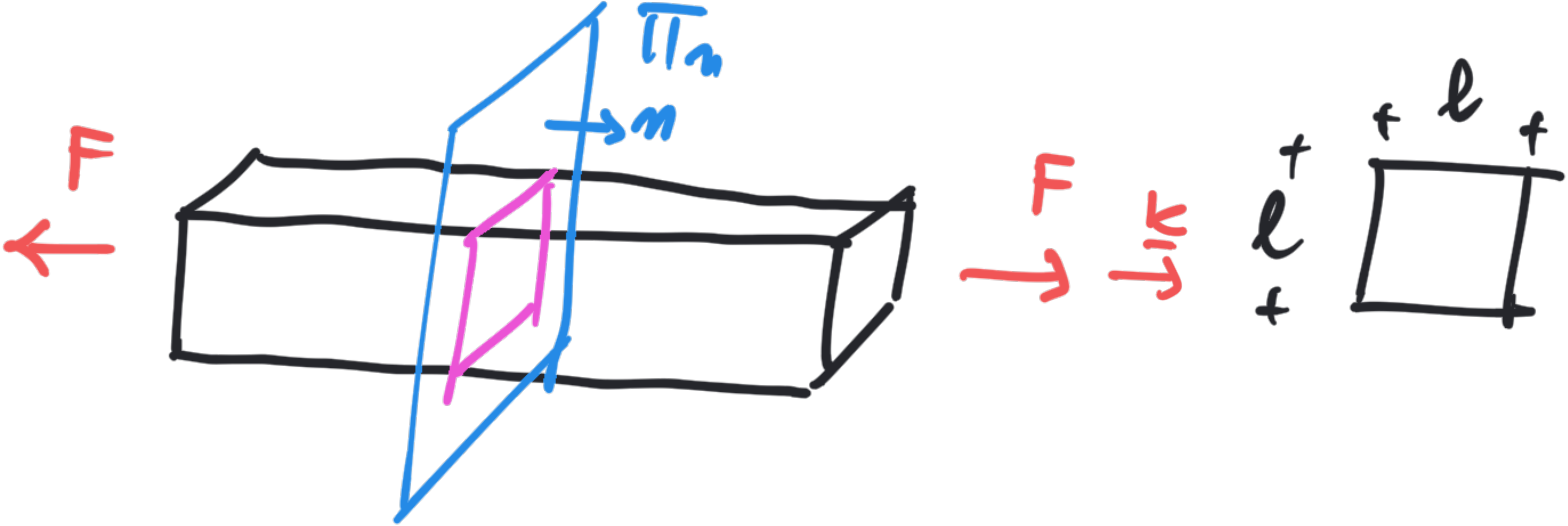
Continuo di Cauchy : $\underline{m}_n = \underline{0}$

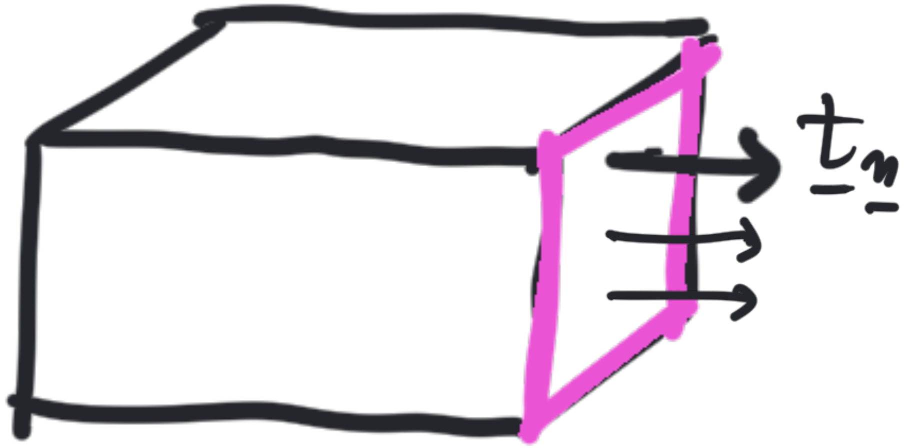
$\underline{t}_n = \underline{t}_n(P)$ "tensione di Cauchy in P agente secondo la giacitura di normale n"

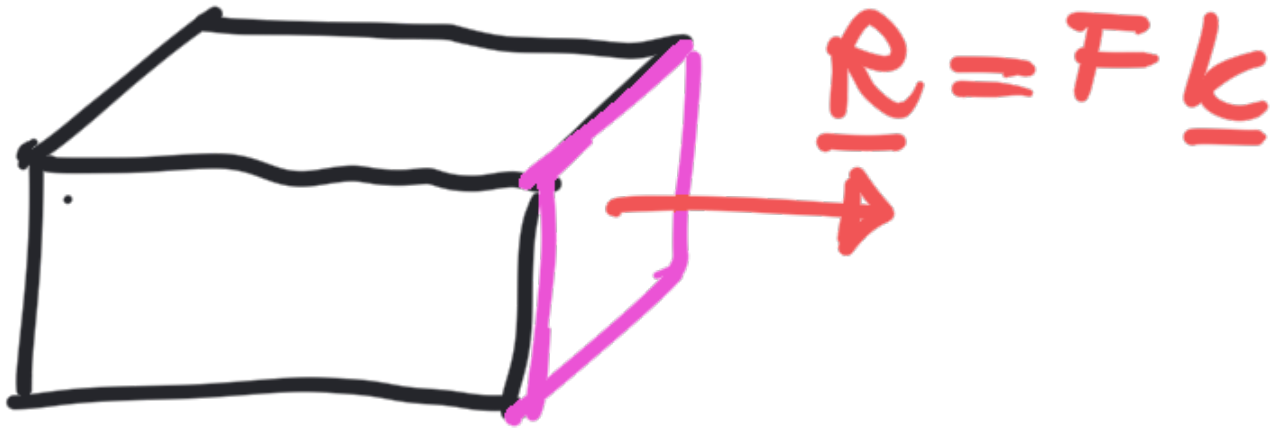
$[t_n] = F L^{-2}$ (Pa) pascal

minuscolo

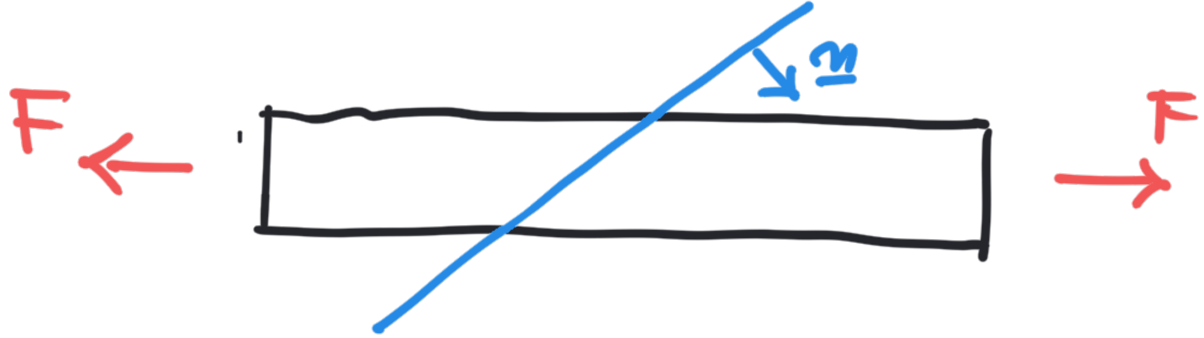
$\underline{m} \neq 0$ continuo "polari". (fratelli Cosserat)

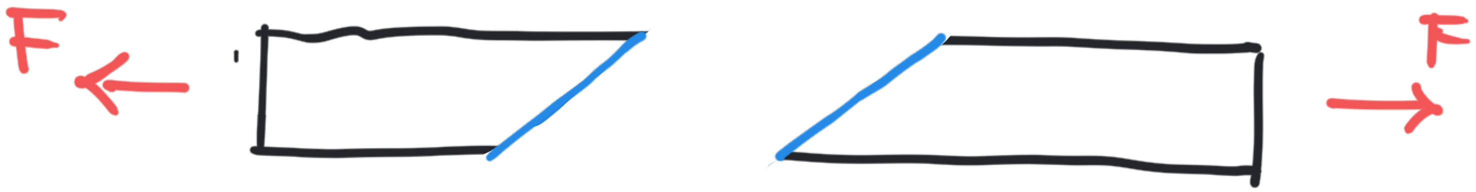


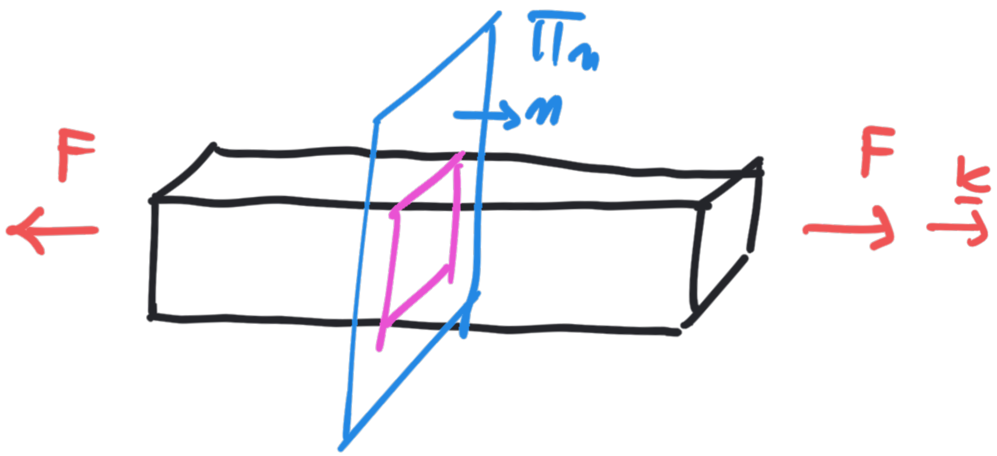


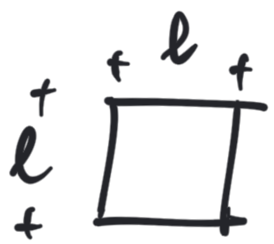


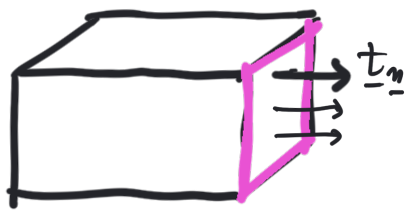
$$|\underline{t}_n| = \frac{F}{\ell^2}$$

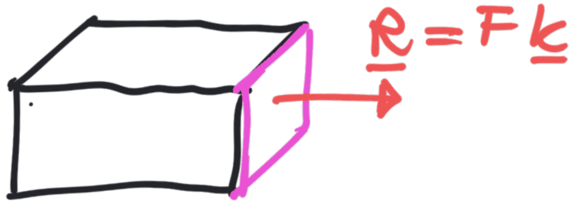






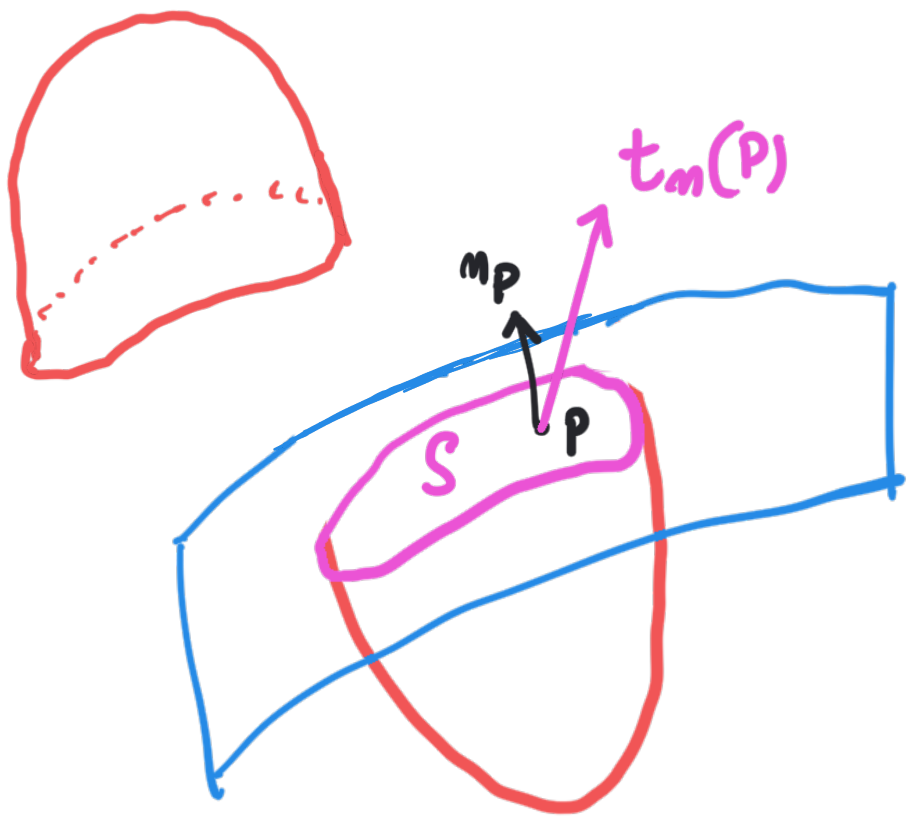




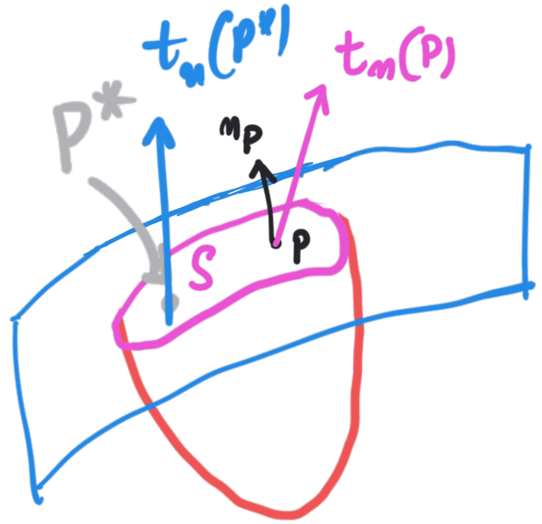


$$|\underline{t}_n| = \frac{F}{\ell^2}$$

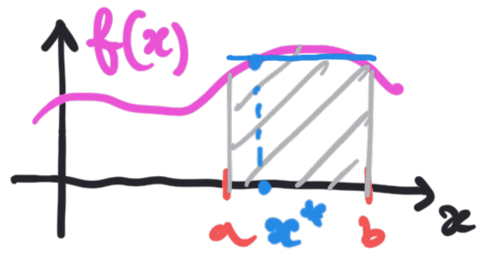
Consideriamo un corpo continuo. Supponiamo di tagliarlo con una superficie di forma arbitraria



$$\underline{R} = \int_S \underline{t}_n(P)\, dA$$ forza risult. agente su S



$$\underline{R} = \int_S \underline{t}_n(P)\, dA$$ forza risult. agente su S



Teor. valor medio

$$\underline{R} = \text{area}(S) \cdot \underline{t}_n(P^*)$$

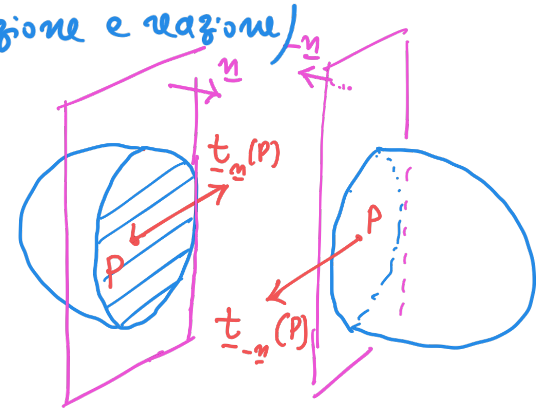
$P^* \in S$

tensione media

$$\int_a^b f(x)\, dx = (b-a)\, f(x^*)$$

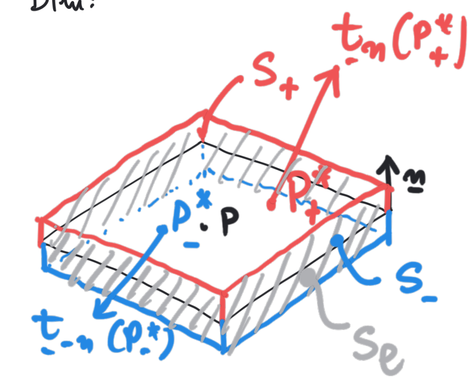
$x^* \in (a,b)$

Lemma di Cauchy: (teorema di azione e reazione)



$$\underline{t}_n(P) = -\,\underline{t}_{-n}(P)$$

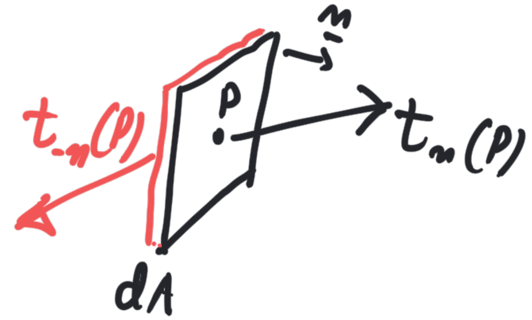
Dim:



$\mathcal{P}$

$P_+^* \in S_+$ $\quad$ $P_-^* \in S_-$

$P^* \in \mathcal{P}$



$$\underline{t}_n(P_+^*) + \underline{t}_{-n}(P_-^*) + \frac{\underline{f}_e}{\varepsilon^2} + \varepsilon^2\,\underline{b}(P^*) = \underline{0}$$

$\varepsilon \to$ $\quad$ $P_+^*, P_-^* \to P$

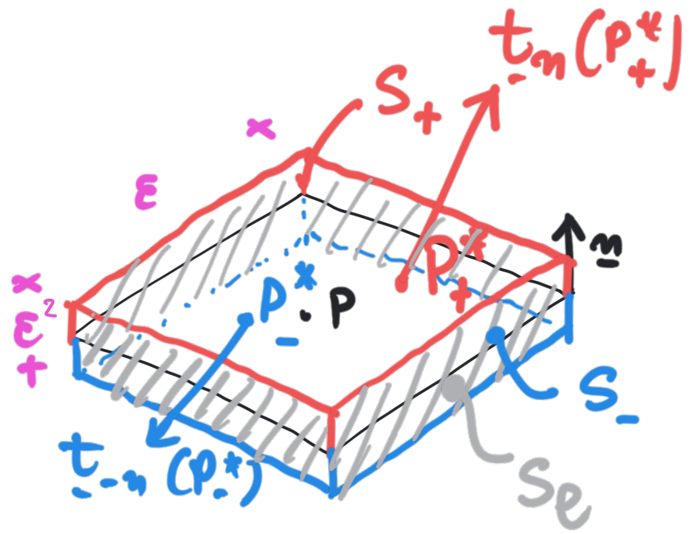
$\text{area}(S_e) = 4\varepsilon^3$

$$\underline{t}_n(P) + \underline{t}_{-n}(P) = 0$$

Scomposizione del vettore tensione di Cauchy

$$\underline{t}_n = t_{nx}\,\underline{i} + t_{ny}\,\underline{j} + t_{nz}\,\underline{k} \qquad [\underline{t}_n] = \begin{bmatrix} t_{nx} \\ t_{ny} \\ t_{nz} \end{bmatrix} \qquad [\underline{n}] = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

$$\alpha = \underline{n}\cdot\underline{i} \qquad \beta = \underline{n}\cdot\underline{j} \qquad \gamma = \underline{n}\cdot\underline{k}$$



$P_+^* \in S_+$ $\qquad$ $P_-^* \in S_-$

$P^* \in P$

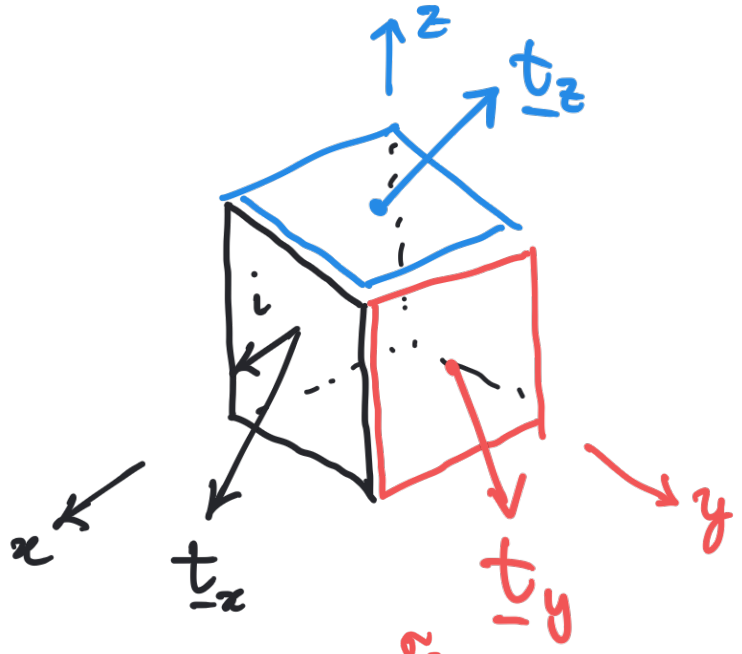
$$\underline{t}_n(P_+^*) + \underline{t}_{-n}(P_-^*) + \underbrace{\frac{\underline{f}_\varepsilon}{\varepsilon^2}}_{\to 0} + \underbrace{\varepsilon\,\underline{b}(P^*)}_{\to 0} = \underline{0}$$

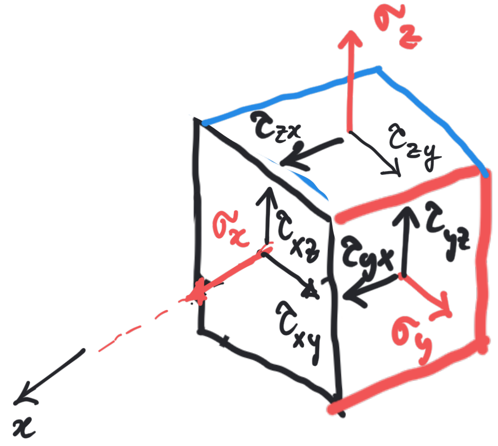
$\varepsilon \to$ $\qquad$ $P_+^*,\ P_-^* \to P$

$$\underline{t}_n(P) + \underline{t}_{-n}(P) = 0$$

$\text{area}(S_\varepsilon) = 4\varepsilon^3$

Componenti speciali della tensione





notazione

$\underline{t}_x \equiv \underline{t}_i \qquad \underline{t}_y = \underline{t}_j$

$\underline{t}_z \equiv \underline{t}_k \qquad \underline{t}_{-x} \equiv \underline{t}_{-i}$

$$\qquad\qquad\qquad\; \| $$

$$-\underline{t}_x = -\underline{t}_i$$

tens. normale relativa alla giacitura di normale $\underline{i}$

$$[\underline{t}_x] = \begin{bmatrix} \sigma_x \\ \tau_{xy} \\ \tau_{xz} \end{bmatrix}$$

$$[\underline{t}_y] = \begin{bmatrix} \tau_{yx} \\ \sigma_y \\ \tau_{yz} \end{bmatrix}$$

$$[\underline{t}_z] = \begin{bmatrix} \tau_{zx} \\ \tau_{zy} \\ \sigma_z \end{bmatrix}$$

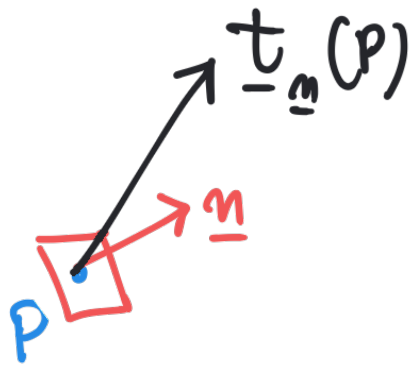
componenti speciali della tens

$$[\underline{T}(P)] \quad \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

tensore degli sforzi

matrice rappresentativa del tensore $\underline{T}$

FORMULA DI CAUCHY



$$[\underline{t}_{\underline{n}}(P)] = [\underline{T}(P)]\,[\underline{n}]$$

notazione

$\underline{t}_x \equiv \underline{t}_i$ $\quad \underline{t}_y = \underline{t}_j$

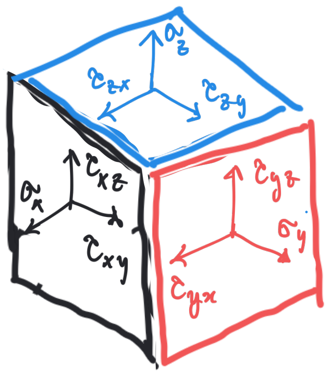
$\underline{t}_z \equiv \underline{t}_k$ $\quad \underline{t}_{-x} \equiv \underline{t}_{-i} = -\underline{t}_x = -\underline{t}_i$

$$[\underline{n}] = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

Es: $\underline{n} = \underline{i}$ $\quad [\underline{n}] = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\quad [T(P)][n] = \begin{bmatrix} \sigma_x \\ \tau_{xy} \\ \tau_{xz} \end{bmatrix}$

$$[\underline{t}_x] = \begin{bmatrix} \sigma_x \\ \tau_{xy} \\ \tau_{xz} \end{bmatrix}$$

$$[\underline{t}_y] = \begin{bmatrix} \tau_{yx} \\ \sigma_y \\ \tau_{yz} \end{bmatrix}$$

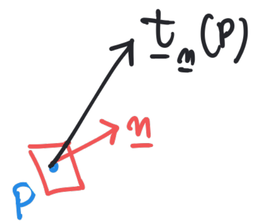


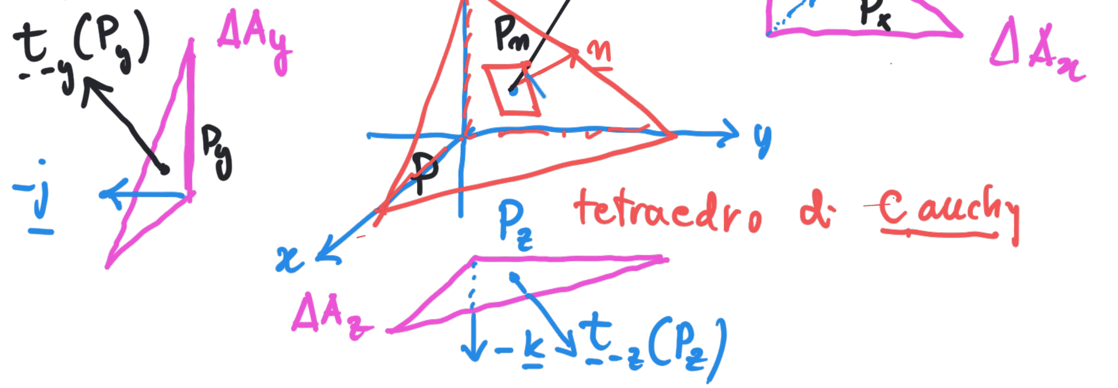
$$[\underline{T}(P)] = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

$$[\underline{t}_z] = \begin{bmatrix} \tau_{zx} \\ \tau_{zy} \\ \sigma_z \end{bmatrix}$$

FORMULA DI CAUCHY

$$[\underline{t}_{\underline{n}}(P)] = [\underline{\underline{T}}(P)]\,[\underline{n}]$$





$$\underline{n} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

$$\underline{t}_{n}(P_n)\cancel{\Delta A_n} + \underline{t}_{-x}(P_x)\frac{\Delta A_x}{\Delta A_n} + \underline{t}_{-y}(P_y)\frac{\Delta A_y}{\Delta A_n} + \underline{t}_{-z}(P_z)\frac{\Delta A_z}{\Delta A_n} + b(P)\frac{\Delta V}{\Delta A_n} = 0$$

$\downarrow$

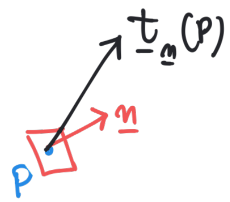
$$\underline{t}_{\underline{n}}(P) + \underline{t}_{-x}(P)\,\alpha + \underline{t}_{-y}(P)\,\beta + \underline{t}_{-z}(P)\,\gamma + 0 = 0$$

$\Delta A_n \to 0$

$P_n,\ P_x, P_y, P_z \to P$

$$\boxed{\underline{t}_{\underline{n}}(P) = \underline{t}_{x}(P)\,\alpha + \underline{t}_{y}(P)\,\beta + \underline{t}_{z}(P)\,\gamma}$$

FORMULA DI CAUCHY



$$[\underline{t}_n(P)] = [T(P)]\,[\underline{n}]$$

matrice

$$[\underline{t}_x] = \begin{bmatrix} \sigma_x \\ \tau_{xy} \\ \tau_{xz} \end{bmatrix} \qquad [\underline{t}_y] = \begin{bmatrix} \tau_{yx} \\ \sigma_y \\ \tau_{yz} \end{bmatrix} \qquad [\underline{t}_z] = \begin{bmatrix} \tau_{zx} \\ \tau_{zy} \\ \sigma_z \end{bmatrix}$$

$$[\underline{t}_n(P)] = \begin{bmatrix} t_{nx} \\ t_{ny} \\ t_{nz} \end{bmatrix} = \begin{bmatrix} \sigma_x \\ \tau_{xy} \\ \tau_{xz} \end{bmatrix} \alpha + \begin{bmatrix} \tau_{yx} \\ \sigma_y \\ \tau_{yz} \end{bmatrix} \beta + \begin{bmatrix} \tau_{zx} \\ \tau_{zy} \\ \sigma_z \end{bmatrix} \gamma$$

$$= \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = [T(P)]\,[\underline{n}]$$

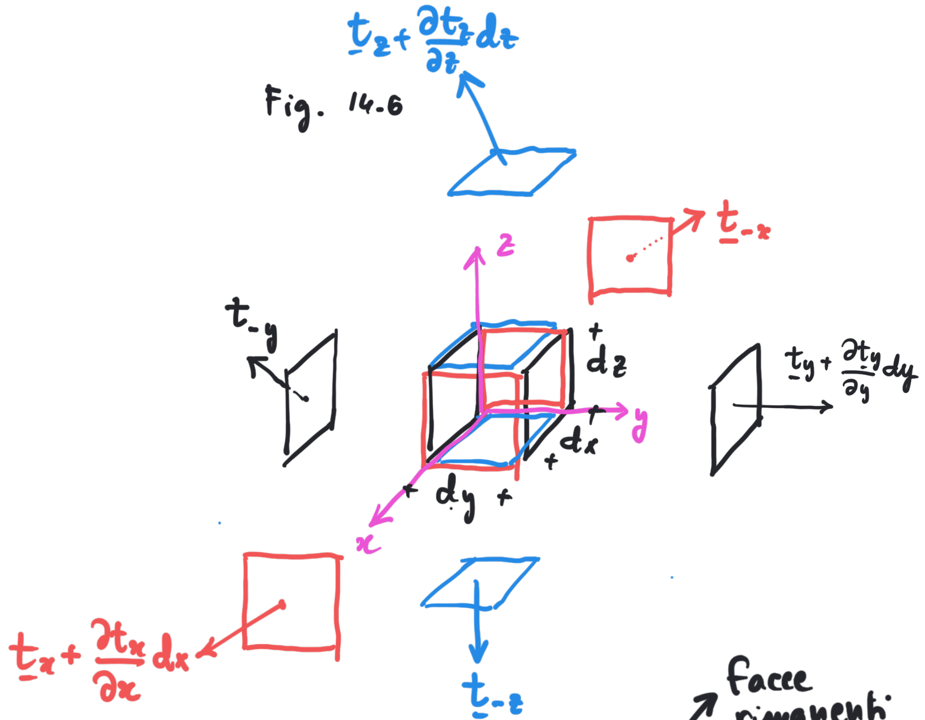
$$\underline{t}_n(P) = \underline{T}(P)\,\underline{n}$$

tensore (appl. lin.)

$$[\underline{T}(P)] = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

$$\boxed{\underline{t}_n(P) = \underline{t}_x(P)\,\alpha + \underline{t}_y(P)\,\beta + \underline{t}_z(P)\,\gamma} \quad (*)$$

Fig. 14.6



facce rimanenti

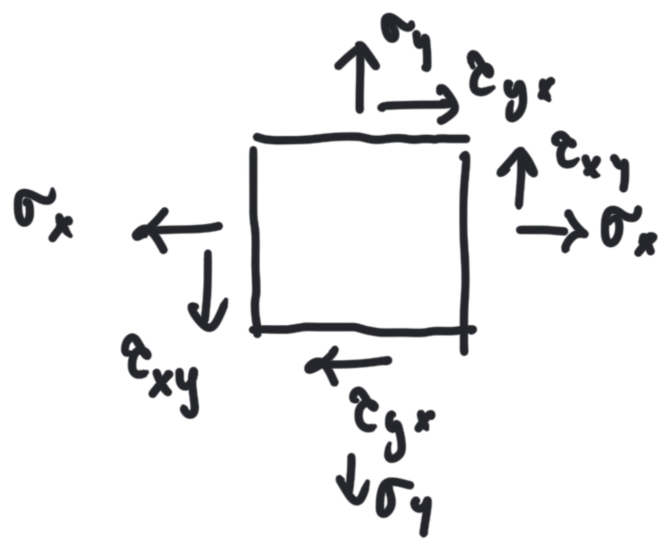
$$\left(\cancel{\underline{t}_x} + \frac{\partial \underline{t}_x}{\partial x} dx\right) dy\,dz - \cancel{\underline{t}_x}\, dy\,dz + \cdots + \underline{b}\, dx\,dy\,dz = \underline{0}$$

(14.15) $$\underbrace{\boxed{\frac{\partial \underline{t}_x}{\partial x} + \frac{\partial \underline{t}_y}{\partial y} + \frac{\partial \underline{t}_z}{\partial z}}}_{\mathrm{div}\,\underline{T}} + \underline{b} = \underline{0}$$ ← eq.ni indefinite di equilibrio

(14.16)
$$\frac{\partial}{\partial x}\sigma_x + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx} + b_x = 0$$
$$\frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\sigma_y + \frac{\partial}{\partial z}\tau_{zy} + b_y = 0$$
$$\frac{\partial}{\partial x}\tau_{xz} + \frac{\partial}{\partial y}\tau_{yz} + \frac{\partial}{\partial z}\sigma_z + b_z = 0$$

$$\mathrm{div}\,\underline{T} + \underline{b} = \underline{0}$$

Eq. momento



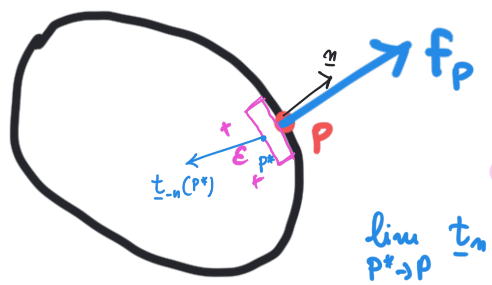
$$\begin{bmatrix} \sigma_x & \tau_{yx} \\ \tau_{xy} & \sigma_y \end{bmatrix}$$

$$\tau_{xy} = \tau_{yx}$$

$$\underline{T} = \underline{T}^T \quad \text{(simmetria)}$$

$$\operatorname{div} \underline{T} + \underline{b} = 0$$

CONDIZ. CONTORNO



$$t_{-n}(P^*) + f_p + O(\varepsilon) = 0$$

$$t_n(P^*) = f_p + O(\varepsilon)$$

$$\lim_{P^* \to P} t_n(P^*) = f_p$$

$$\| $$

$$T(P^*)\, n$$

$$\lim_{P^* \to P} T(P^*)\, n_p = f_p$$

$$\|$$

$$T(P)$$

$$T(P)\, n_p = f_p$$

$$T = T^T \quad (\text{simmetria})$$

$$\operatorname{div} T + b = 0$$

PROBLEMA STATICO

$$\left.\begin{array}{l} \mathrm{div}\,\underline{T} + \underline{b} = 0 \\ \\ \underline{T} = \underline{T}^T \quad (\text{simmetria}) \end{array}\right\} \text{ in } \mathcal{C}$$

$$\underline{T}\,\underline{n} = \underline{f} \quad \text{su } \mathcal{S}_f$$

NB: 3 eq.ni diff. li

6 incognite

PROBLEMA STATICO

$$\left.\begin{array}{l} \operatorname{div} \underline{T} + \underline{b} = \underline{0} \\ \\ \underline{T} = \underline{T}^T \quad (\text{simmetria}) \end{array}\right\} \text{ in } \mathcal{C}$$

NB: 3 eq.ni diff. li

6 incognite

$$\underline{T}\,\underline{n} = \underline{f} \quad \text{su } \mathcal{S}_f$$