3–1. A tension test was performed on a steel specimen having an original diameter of 12.5 mm and gauge length of 50 mm. The data is listed in the table. Plot the stress–strain diagram and determine approximately the modulus of elasticity, the yield stress, the ultimate stress, and the rupture stress. Use a scale of 25 mm = 140 MPa and 25 mm= 0.05 mm/mm. Redraw the elastic region, using the same stress scale but a strain scale of 25mm=0.001 mm/mm.

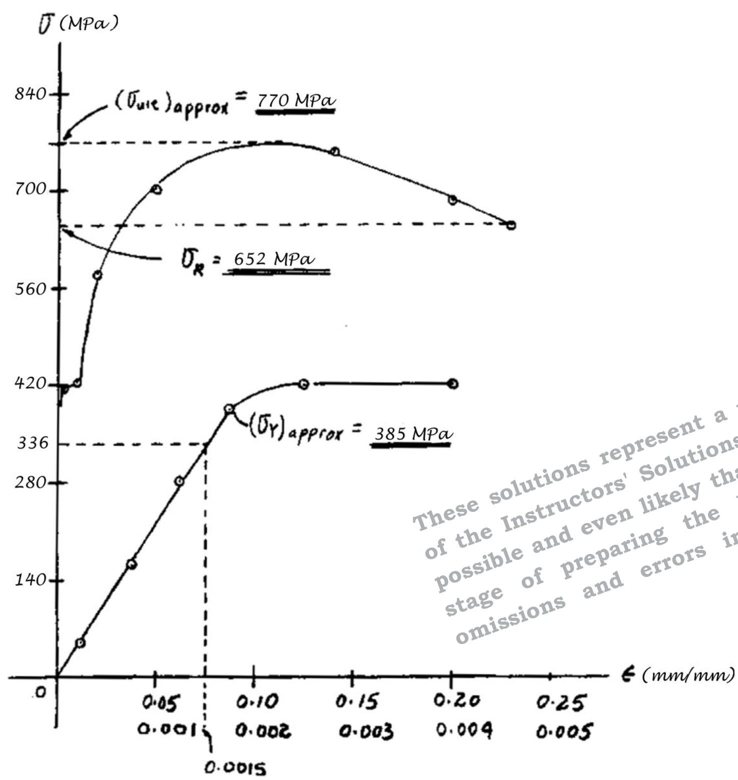
Load (kN)	Elongation (mm)
0	0
7.0	0.0125
21.0	0.0375
36.0	0.0625
50.0	0.0875
53.0	0.125
53.0	0.2
54.0	0.5
75.0	1.0
90.0	2.5
97.0	7.0
87.8	10.0
83.3	11.5

$$A = \frac{1}{4}\pi(0.0125^2) = 122.72(10^{-6})\ \text{m}^2$$

$L = 50$ mm.

σ(MPa)	ε(mm/mm)
0	0
57.07	0.00025
171.21	0.00075
293.51	0.00125
407.66	0.00175
432.12	0.0025
432.12	0.0040
440.27	0.010
611.49	0.020
733.79	0.050
790.86	0.140
715.85	0.200
679.16	0.230

$$E_{\text{approx}} = \frac{336(10^6)}{0.0015} = 224(10^9)\ \text{N/m}^2 = 224\ \text{GPa}$$ **Ans.**



Ans:
$(\sigma_{\text{ult}})_{\text{approx}} = 770$ MPa, $(\sigma_R)_{\text{approx}} = 652$ MPa,
$(\sigma_Y)_{\text{approx}} = 385$ MPa, $E_{\text{approx}} = 224$ GPa

3–2. Data taken from a stress–strain test for a ceramic are given in the table. The curve is linear between the origin and the first point. Plot the diagram, and determine the modulus of elasticity and the modulus of resilience.

σ (MPa)	ϵ (mm/mm)
0	0
232.4	0.0006
318.5	0.0010
345.8	0.0014
360.5	0.0018
373.8	0.0022

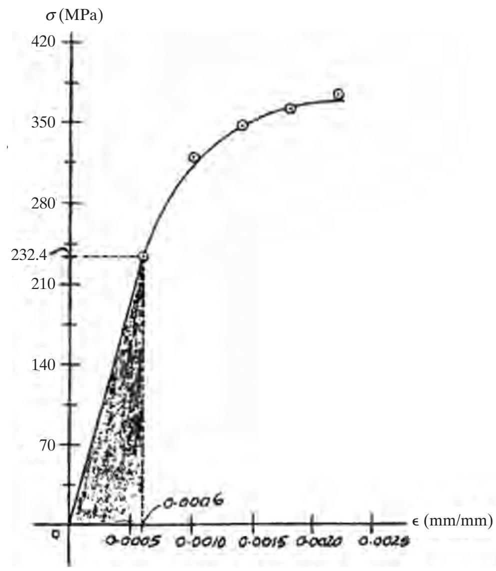
SOLUTION

Modulus of Elasticity: From the stress–strain diagram

$$E = \frac{232.4(10^6) - 0}{0.0006 - 0} = 387.33(10^9)\ \text{N/m}^2 = 387\ \text{GPa}$$ **Ans.**

Modulus of Resilience: The modulus of resilience is equal to the area under the *linear portion* of the stress–strain diagram (shown shaded).

$$(U_i)_r = \frac{1}{2}\left[232.4(10^6)\ \frac{\text{N}}{\text{m}^2}\right]\left(0.0006\ \frac{\text{m}}{\text{m}}\right) = 69.72(10^3)\ \frac{\text{N}\cdot\text{m}}{\text{m}^3} = 69.7\ \text{kJ/m}^3$$ **Ans.**



Ans:

$E = 387\ \text{GPa},\ u_r = 69.7\ \text{kJ/m}^3$

3–3. Data taken from a stress–strain test for a ceramic are given in the table. The curve is linear between the origin and the first point. Plot the diagram, and determine approximately the modulus of toughness. The rupture stress is $\sigma_r = 373.8$ MPa

σ (MPa)	ϵ (mm/mm)
0	0
232.4	0.0006
318.5	0.0010
345.8	0.0014
360.5	0.0018
373.8	0.0022

SOLUTION

Modulus of Toughness: The modulus of toughness is equal to the area under the stress–strain diagram (shown shaded).

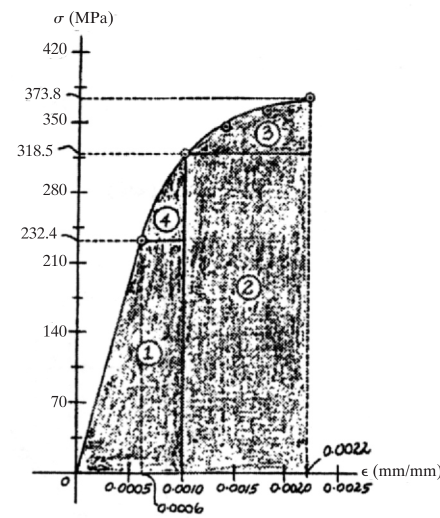
$$(U_i)_t = \frac{1}{2}\left[232.4(10^6)\ \frac{\text{N}}{\text{m}^2}\right]\left[(0.0004 + 0.0010)\ \frac{\text{m}}{\text{m}}\right]$$

$$+\left[318.5(10^6)\ \frac{\text{N}}{\text{m}^2}\right]\left(0.0012\ \frac{\text{m}}{\text{m}}\right)$$

$$+\frac{1}{2}\left[55.3(10^6)\ \frac{\text{N}}{\text{m}^2}\right]\left(0.0012\ \frac{\text{m}}{\text{m}}\right)$$

$$+\frac{1}{2}\left[86.1(10^6)\ \frac{\text{N}}{\text{m}^2}\right]\left(0.0004\ \frac{\text{m}}{\text{m}}\right)$$

$$= 595.28(10^3)\ \frac{\text{N}\cdot\text{m}}{\text{m}^3} = 595\ \text{kJ/m}^3$$ **Ans.**



Ans:
$(u_i)_t = 595$ kJ/m^3

***3–4.**

The stress–strain diagram for a metal alloy having an original diameter of 12 mm and a gauge length of 50 mm is given in the figure. Determine approximately the modulus of elasticity for the material, the load on the specimen that causes yielding, and the ultimate load the specimen will support.

SOLUTION

From the stress–strain diagram, Fig. *a*,

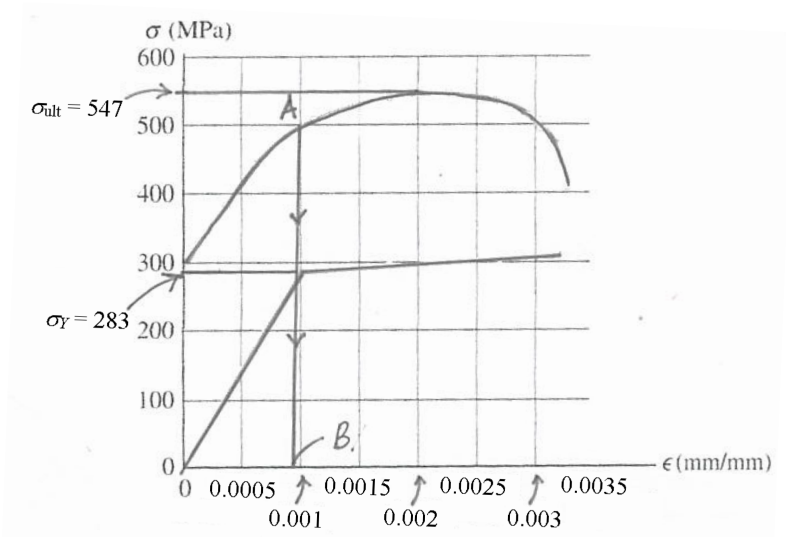
$$E = \frac{283(10^6)}{0.001} = 283(10^9)\ \text{N/m}^2 = 283\ \text{GPa} \qquad \textbf{Ans.}$$

$$\sigma_Y = 283\ \text{MPa} \qquad \sigma_{u/t} = 547\ \text{GPa}$$

Thus,

$$P_Y = \sigma_Y A = \left[283(10^6)\right]\left[\tfrac{\pi}{4}(0.012^2)\right] = 32.01(10^3)\ \text{N} = 32.0\ \text{kN} \qquad \textbf{Ans.}$$

$$P_{u/t} = \sigma_{u/t} A = \left[547(10^6)\right]\left[\tfrac{\pi}{4}(0.012^2)\right] = 61.86(10^3)\ \text{N} = 61.9\ \text{kN} \qquad \textbf{Ans.}$$



(a)

Ans:
$E = 283$ GPa, $P_Y = 32.0$ kN, $P_{ult} = 61.9$ kN

3–5.

The stress–strain diagram for a steel alloy having an original diameter of 12 mm and a gauge length of 50 mm is given in the figure. If the specimen is loaded until it is stressed to 500 MPa, determine the approximate amount of elastic recovery and the increase in the gauge length after it is unloaded.

SOLUTION

From the stress–strain diagram Fig. *a*, the modulus of elasticity for the steel alloy is

$$E = \frac{283(10^6)}{0.001} = 283(10^9)\ \text{N/m}^2 = 283\ \text{GPa}$$

when the specimen is unloaded, its normal strain recovered along line AB, Fig. *a*, which has a gradient of E. Thus

$$\text{Elasticity Recovery} = \frac{\sigma}{E} = \frac{500(10^6)}{283(10^9)} = 0.001767\ \text{mm/mm}$$

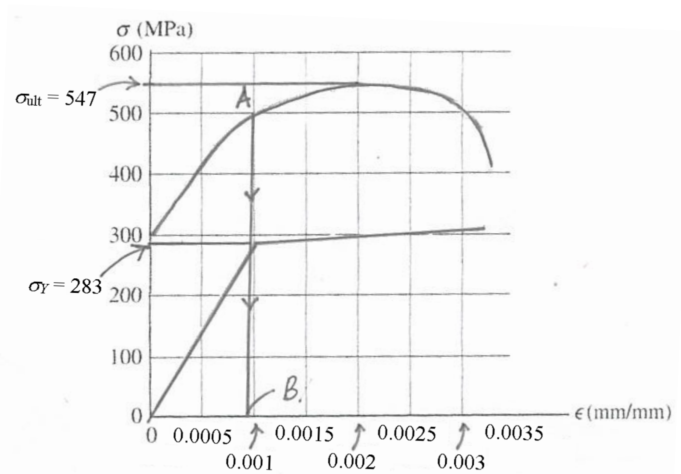
$$\text{Amount of Elastic Recovery} = (0.001767\ \text{mm/mm})(50\ \text{mm}) = 0.08834\ \text{mm} = 0.0883\ \text{mm} \qquad \textbf{Ans.}$$

Thus, the permanent set is

$$\epsilon_P = 0.08 - 0.001767 = 0.07823\ \text{mm/mm}$$

Then, the increase in gauge length is

$$\Delta L = \epsilon_P L = 0.07823(50\ \text{mm}) = 3.9137\ \text{mm} = 3.91\ \text{mm} \qquad \textbf{Ans.}$$



(a)

Ans:
Elastic Recovery = 0.0883 mm
ΔL = 3.91 mm

3-6.

The stress–strain diagram for a steel alloy having an original diameter of 12 mm and a gauge length of 50 mm is given in the figure. Determine approximately the modulus of resilience and the modulus of toughness for the material.

SOLUTION

The Modulus of resilience is equal to the area under the stress–strain diagram up to the proportional limit.

$$\sigma_{PL} = 283 \text{ MPa} \qquad \epsilon_{PL} = 0.001 \text{ m/m}$$

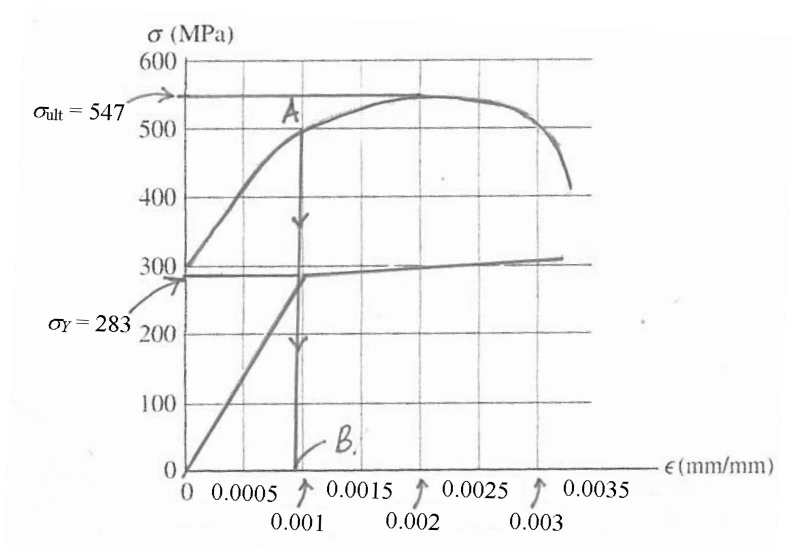
Thus,

$$(U_i)_r = \frac{1}{2}\sigma_{PL}\epsilon_{PL} = \frac{1}{2}\left[283(10^6)\ \frac{\text{N}}{\text{m}^2}\right]\left(0.001\ \frac{\text{m}}{\text{m}}\right) = 141.5(10^3)\ \frac{\text{N}\cdot\text{m}}{\text{m}^3}$$

$$= 141.5 \text{ kJ/m}^3 \qquad \textbf{Ans.}$$

The modulus of toughness is equal to the area under the entire stress–strain diagram. This area can be approximated by counting the number of squares. The total number is 32. Thus,

$$\left[(U_i)_{ult}\right]_{aprox} = 32\left[100(10^6)\ \frac{\text{N}}{\text{m}^2}\right]\left(0.04\ \frac{\text{m}}{\text{m}}\right) = 128(10^6)\ \frac{\text{N}\cdot\text{m}}{\text{m}^3} = 128 \text{ MJ/m}^3 \qquad \textbf{Ans.}$$



(a)

Ans:
$(u_i)_r = 141.5 \text{ kJ/m}^3$, $[(U_i)_{ult}]_{\text{approx}} = 128 \text{ MJ/m}^3$

3–7. A specimen is originally 300 mm long, has a diameter of 12 mm, and is subjected to a force of 2.5 kN. When the force is increased from 2.5 kN to 9 kN,, the specimen elongates 0.225 mm. Determine the modulus of elasticity for the material if it remains linear elastic.

SOLUTION

Normal Stress and Strain: Applying $\sigma = \dfrac{P}{A}$ and $\varepsilon = \dfrac{\delta L}{L}$.

$$\sigma_1 = \frac{2.5(10^3)}{\frac{\pi}{4}(0.012^2)} = 22.10(10^6)\ \text{N/m}^2 = 22.10\ \text{MPa}$$

$$\sigma_2 = \frac{9(10^3)}{\frac{\pi}{4}(0.012^2)} = 79.58(10^6)\ \text{N/m}^2 = 79.58\ \text{MPa}$$

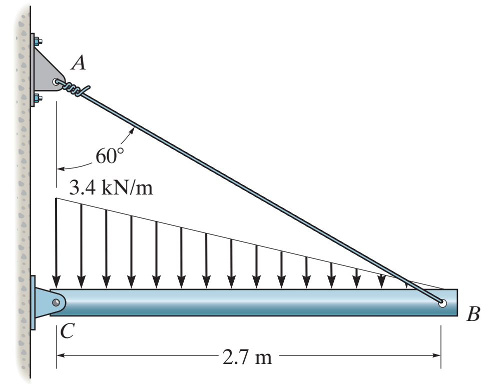
$$\Delta\varepsilon = \frac{0.225}{300} = 0.000750\ \text{mm/mm}$$

Modulus of Elasticity:

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{(79.58 - 22.10)(10^6)}{0.000750} = 76.63(10^9)\ \text{Pa} = 76.6\ \text{GPa} \qquad \textbf{Ans.}$$

Ans:
$E = 76.6$ GPa

***3–8.** The strut is supported by a pin at C and an A-36 steel guy wire AB. If the wire has a diameter of 5 mm, determine how much it stretches when the distributed load acts on the strut.



SOLUTION

Here, we are only interested in determining the force in wire AB.

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{AB}\cos 60°(2.7) - \frac{1}{2}(3.4)(2.7)(0.9) = 0 \quad F_{AB} = 3.06 \text{ kN}$$

The normal stress the wire is

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{3.06(10^3)}{\frac{\pi}{4}(0.005^2)} = 155.84(10^6)\text{ N/m}^2 = 155.84 \text{ MPa}$$

Since $\sigma_{AB} < \sigma_Y = 250$ MPa, Hooke's Law can be applied to determine the strain in wire.

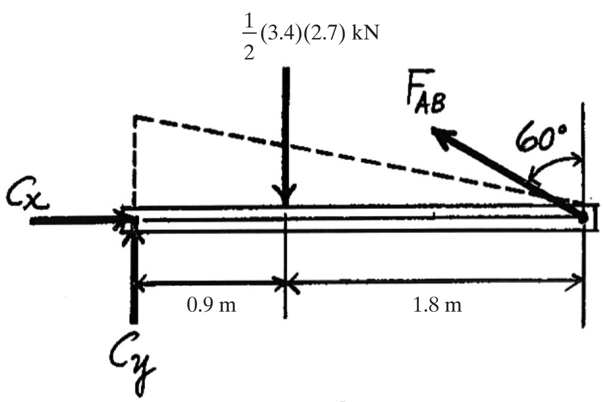
$$\sigma_{AB} = E\epsilon_{AB}; \qquad 155.84(10^6) = 200(10^9)\epsilon_{AB}$$

$$\epsilon_{AB} = 0.7792(10^{-3}) \text{ mm/mm}$$

The unstretched length of the wire is $L_{AB} = \dfrac{2.7(10^3)}{\sin 60°} = 3117.69$ mm. Thus, the wire stretches

$$\delta_{AB} = \epsilon_{AB}\, L_{AB} = 0.7792(10^{-3})(3117.69)$$

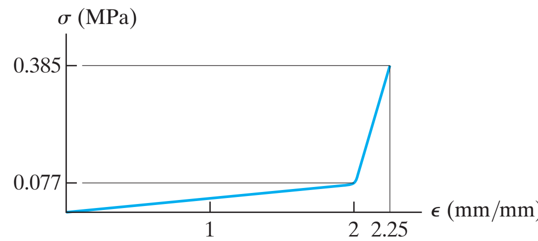
$$= 2.429 \text{ mm} = 2.43 \text{ mm}$$ **Ans.**



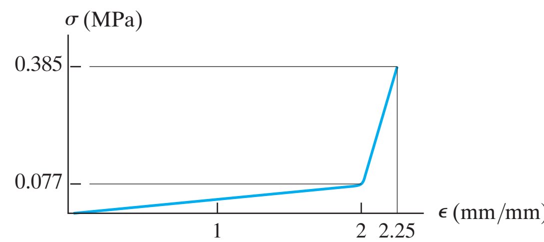
(a)

Ans:
$\delta_{AB} = 2.43$ mm

3–9. The $\sigma-\epsilon$ diagram for elastic fibers that make up human skin and muscle is shown. Determine the modulus of elasticity of the fibers and estimate their modulus of toughness and modulus of resilience.



SOLUTION



$$E = \frac{0.077}{2} = 0.0385 \text{ MPa}$$ **Ans.**

$$(U_i)_r = \frac{1}{2}\left[0.077(10^6)\ \text{N/m}^2\right](2\ \text{m/m})$$

$$= 77.0(10^3)\ \text{N}\cdot\text{m/m}^3 = 77.0\ \text{kJ/m}^3$$ **Ans.**

$$(U_i)_t = \frac{1}{2}\left[0.077(10^6)\ \text{N/m}^2\right](2\ \text{m/m})$$

$$+\frac{1}{2}\left\{\left[0.077(10^6) + 0.385(10^6)\right]\ \text{N/m}^2\right\}\left[(2.25-2)\ \text{m/m}\right]$$

$$= 134.75\ \text{N}\cdot\text{m/m}^3 = 135\ \text{kJ/m}^3$$ **Ans.**

Ans:

$E = 0.0385$ MPa, $(u_i)r = 77.0$ kJ/m^3, $(u_i)t = 135$ kJ/m^3

3–10.

A structural member in a nuclear reactor is made of a zirconium alloy. If an axial load of 20 kN is to be supported by the member, determine its required cross-sectional area. Use a factor of safety of 3 relative to yielding. What is the load on the member if it is 1 m long and its elongation is 0.5 mm? $E_{zr} = 100$ GPa, $\sigma_Y = 400$ MPa. The material has elastic behavior.

SOLUTION

Allowable Normal Stress:

$$\text{F.S.} = \frac{\sigma_y}{\sigma_{\text{allow}}}$$

$$3 = \frac{400}{\sigma_{\text{allow}}}$$

$$\sigma_{\text{allow}} = 133.33 \text{ MPa}$$

$$\sigma_{\text{allow}} = \frac{P}{A}$$

$$133.33 = \frac{20(10^3)}{A}$$

$$A = 150 \text{ mm}^2 \qquad \textbf{Ans.}$$

Stress–Strain Relationship: Applying Hooke's law with

$$\varepsilon = \frac{\delta}{L} = \frac{0.5}{1(10^3)} = 0.0005 \text{ mm/mm}$$

$$\sigma = E\varepsilon = 100(10^3)(0.0005) = 50 \text{ MPa}$$

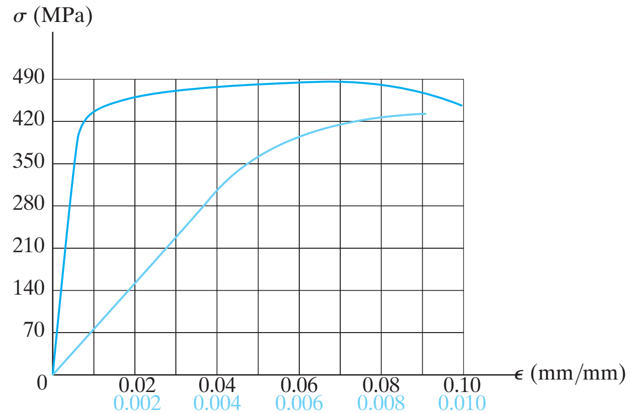
Normal Force: Applying equation $\sigma = \dfrac{P}{A}$.

$$P = \sigma A = 50(150) = 7500 \text{ N} = 7.5 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$A = 150 \text{ mm}^2, P = 7.5 \text{ kN}$

3–11.

A tension test was performed on an aluminum 2014-T6 alloy specimen. The resulting stress–strain diagram is shown in the figure. Estimate (a) the proportional limit,(b) the modulus of elasticity, and (c) the yield strength based on a 0.2% strain offset method.



SOLUTION

Proportional Limit and Yield Strength: From the stress–strain diagram, Fig. *a*,

$$\sigma_{pl} = 308 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_Y = 420 \text{ MPa} \qquad \textbf{Ans.}$$

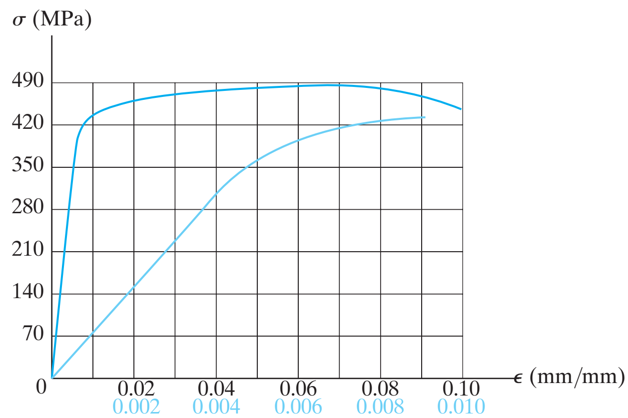
Modulus of Elasticity: From the stress–strain diagram, the corresponding strain for $\sigma_{PL} = 308$ MPa is $\varepsilon_{pl} = 0.004$ mm/mm. Thus,

$$E = \frac{308 - 0}{0.004 - 0} = 77.0(10^3) \text{ MPa} = 77.0 \text{ GPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma_{pl} = 308$ MPa
$\sigma_Y = 420$ MPa
$E = 77.0$ GPa

***3–12.**

A tension test was performed on an aluminum 2014-T6 alloy specimen. The resulting stress–strain diagram is shown in the figure. Estimate (a) the modulus of resilience; and (b) modulus of toughness.



SOLUTION

Modulus of Resilence The modulus of resilence is equal to the area under the stress–strain diagram up to the propotional limit. From the stress–strain diagram,

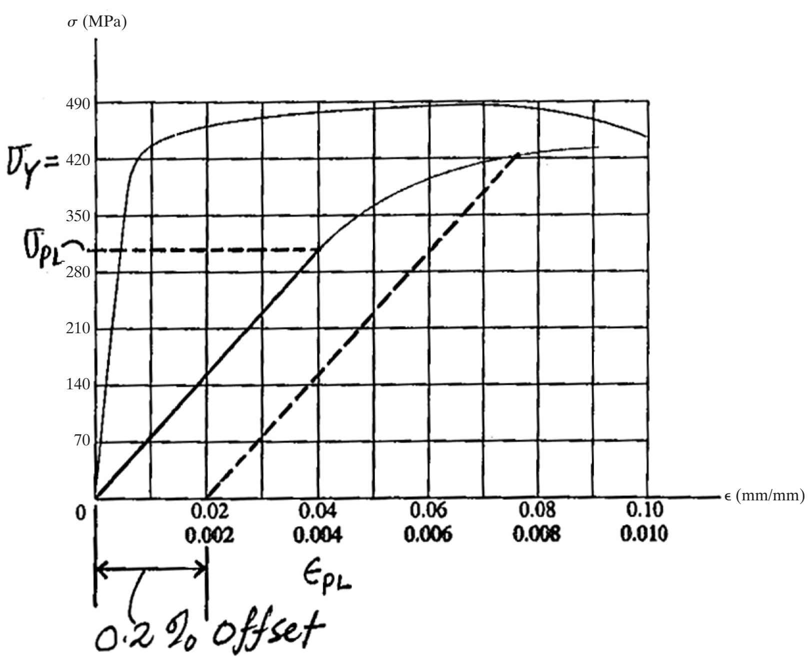
$$\sigma_{pl} = 308 \text{ MPa} \qquad \varepsilon_{pl} = 0.004 \text{ mm/mm}$$

Thus,

$$\left(U_i\right)_r = \frac{1}{2}\sigma_{pl}\varepsilon_{pl} = \frac{1}{2}[308(10^6)](0.004) = 0.616 \text{ MJ/m}^3$$ **Ans.**

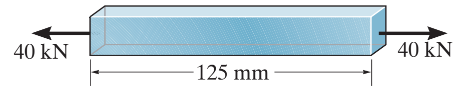
Modulus of Toughness: The modulus of toughness is equal to the area under the entire stress–strain diagram. This area can be approximated by counting the number of squares. The total number of squares is 65. Thus,

$$\left[\left(U_i\right)_t\right]_{\text{approx}} = 65[70(10^6)](0.01) = 45.5 \text{ MJ/m}^3$$ **Ans.**



Ans:
$\left(U_i\right)_r = 0.616 \text{ MJ/m}^3$
$\left[\left(U_i\right)_t\right]_{\text{approx}} = 45.5 \text{ MJ/m}^3$

3.13.



A bar having a length of 125 mm and cross-sectional area of 437.5 mm^2 is subjected to an axial force of 40 kN. If the bar streches 0.05 mm, determine the modulus of elasticity of the material. The material has linear-elastic behavior.

SOLUTION

Normal Stress and Strain:

$$\sigma = \frac{P}{A} = \frac{40(10^3)}{437.5(10^{-6})} = 91.43(10^6)\ \text{N/m}^2 = 91.43\ \text{MPa}$$

$$\varepsilon = \frac{\delta L}{L} = \frac{0.05}{125} = 0.000400\ \text{mm/mm}$$

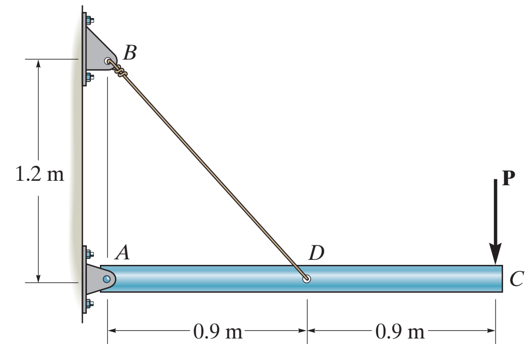
Modulus of Elasticity:

$$E = \frac{\sigma}{\varepsilon} = \frac{91.43(10^6)}{0.000400} = 228.57(10^9)\ \text{N/m}^2 = 229\ \text{GPa}$$ **Ans.**

Ans:

$E = 229$ GPa

3–14. The rigid pipe is supported by a pin at A and an A-36 steel guy wire BD. If the wire has a diameter of 6.5 mm, determine how much it streches when a load of $P = 3$ kN acts on the pipe.



SOLUTION

Here, we are only interested in determining the force in wire BD. Referring to the FBD in Fig. *a*

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BD}\left(\tfrac{4}{5}\right)(0.9) - 3(1.8) = 0 \qquad F_{BD} = 7.50 \text{ kN}$$

The normal stress developed in the wire is

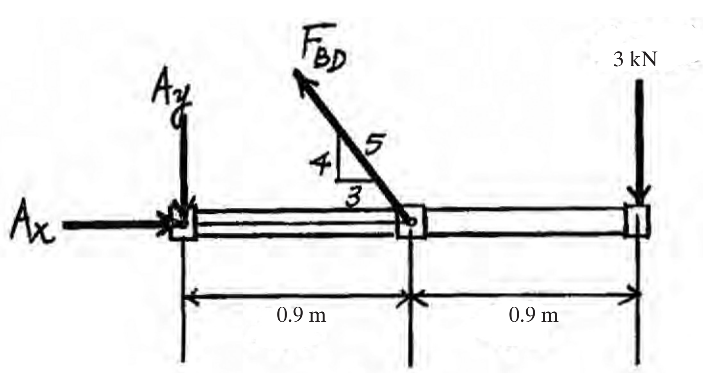
$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}} = \frac{7.50(10^3)}{\frac{\pi}{4}(0.0065^2)} = 226.02(10^6) \text{ N/m}^2 = 226.02 \text{ MPa}$$

Since $\sigma_{BD} < \sigma_y = 250$ MPa, Hooke's Law can be applied to determine the strain in the wire.

$$\sigma_{BD} = E\epsilon_{BD}; \qquad 226.02(10^6) = 200(10^9)\,\epsilon_{AB}$$
$$\epsilon_{BD} = 1.1301(10^{-3}) \text{ mm/mm}$$

The unstretched length of the wire is $L_{BD} = \sqrt{0.9^2 + 1.2^2} = 1.5$ m. Thus, the wire stretches

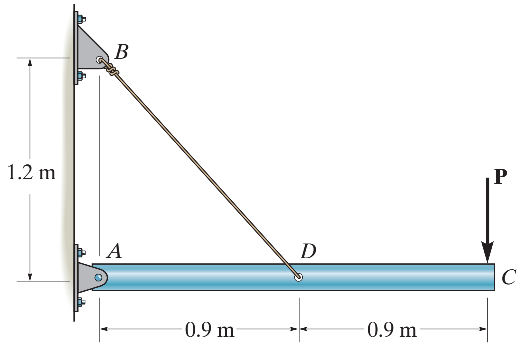
$$\delta_{BD} = \epsilon_{BD} L_{BD} = 1.1301(10^{-3})[1.5(10^3)]$$
$$= 1.6951 \text{ mm} = 1.70 \text{ mm}$$ **Ans.**



(a)

Ans:
$\delta_{BD} = 1.70$ mm

3–15. The rigid pipe is supported by a pin at A and an A-36 guy wire BD. If the wire has a diameter of 6 mm, determine the load P if the end C is displaced 1.875 mm downward.



SOLUTION

Here, we are only interested in determining the force in wire BD. Referring to the FBD in Fig. a

$$\zeta + \Sigma M_A = 0; \qquad F_{BD}\left(\tfrac{4}{5}\right)(0.9) - P(1.8) = 0 \qquad F_{BD} = 2.50\,P$$

The unstretched length for wire BD is $L_{BD} = \sqrt{0.9^2 + 1.2^2} = 1.5$ m. From the geometry shown in Fig. b, the stretched length of wire BD is

$$L_{BD'} = \sqrt{1500^2 + 1.675^2 - 2(1500)(1.675)\cos 143.13^\circ} = 1501.3403 \text{ mm}$$

Thus, the normal strain is

$$\epsilon_{BD} = \frac{L_{BD'} - L_{BD}}{L_{BD}} = \frac{1501.3403 - 1500}{1500} = 0.8936(10^{-3}) \text{ mm/mm}$$

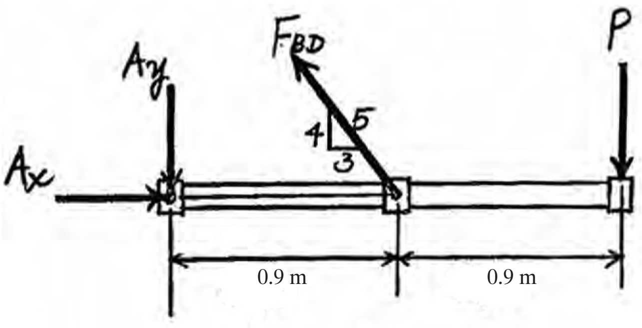
Then, the normal stress can be obtain by applying Hooke's Law.

$$\sigma_{BD} = E\epsilon_{BD} = 200(10^9)[0.8936(10^{-3})] = 178.71(10^6) \text{ N/m}^2 = 178.71 \text{ MPa}$$

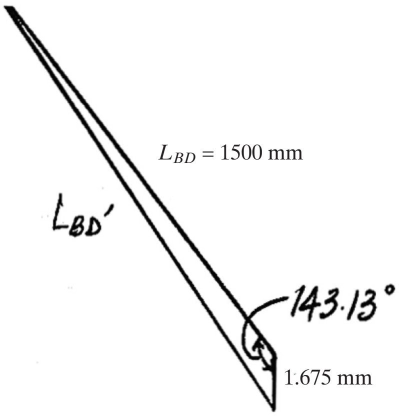
Since $\sigma_{BD} < \sigma_y = 250$ MPa, the result is valid.

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}}; \qquad 178.71(10^6) = \frac{2.50P}{\frac{\pi}{4}(0.0065^2)}$$

$$P = 2.3721(10^3) \text{ N} = 2.37 \text{ kN} \qquad \textbf{Ans.}$$



(a)

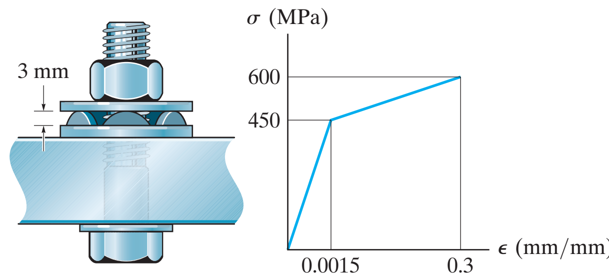


(b)

Ans:
$P = 2.37$ kN

***3–16.**

Direct tension indicators are sometimes used instead of torque wrenches to ensure that a bolt has a prescribed tension when used for connections. If a nut on the bolt is tightened so that the six 3-mm high heads of the indicator are strained 0.1 mm/mm, and leave a contact area on each head of 1.5 mm^2, determine the tension in the bolt shank. The material has the stress–strain diagram shown.



SOLUTION

Stress–Strain Relationship: From the stress–strain diagram with

$\epsilon = 0.1 \text{ mm/mm} > 0.0015 \text{ mm/mm}$

$$\frac{\sigma - 450}{0.1 - 0.0015} = \frac{600 - 450}{0.3 - 0.0015}$$

$$\sigma = 499.497 \text{ MPa}$$

Axial Force: For each head

$$P = \sigma A = 499.4971\left(10^6\right)(1.5)\left(10^{-6}\right) = 749.24 \text{ N}$$

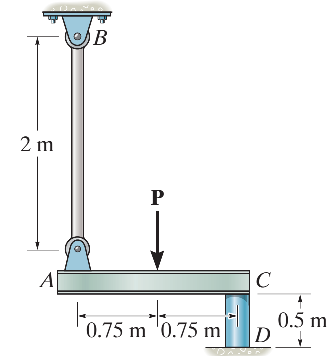
Thus, the tension in the bolt is

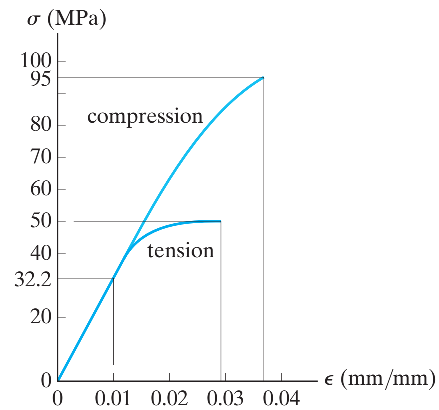
$$T = 6\,P = 6(749.24) = 4495 \text{ N} = 4.50 \text{ kN}$$ **Ans.**

Ans:
$T = 4.50$ kN

3–17.

The stress–strain diagram for a polyester resin is given in the figure. If the rigid beam is supported by a strut AB and post CD, both made from this material, and subjected to a load of $P = 80$ kN, determine the angle of tilt of the beam when the load is applied. The diameter of the strut is 40 mm and the diameter of the post is 80 mm.





SOLUTION

From the stress–strain diagram,

$$E = \frac{32.2(10)^6}{0.01} = 3.22(10^9)\ \text{Pa}$$

Thus,

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{40(10^3)}{\frac{\pi}{4}(0.04)^2} = 31.83\ \text{MPa}$$

$$\varepsilon_{AB} = \frac{\sigma_{AB}}{E} = \frac{31.83(10^6)}{3.22(10^9)} = 0.009885\ \text{mm/mm}$$

$$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{40(10^3)}{\frac{\pi}{4}(0.08)^2} = 7.958\ \text{MPa}$$

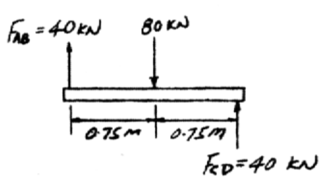
$$\varepsilon_{CD} = \frac{\sigma_{CD}}{E} = \frac{7.958(10^6)}{3.22(10^9)} = 0.002471\ \text{mm/mm}$$

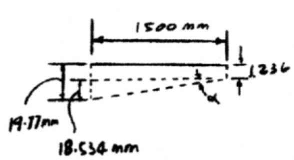
$$\delta_{AB} = \varepsilon_{AB} L_{AB} = 0.009885(2000) = 19.771\ \text{mm}$$

$$\delta_{CD} = \varepsilon_{CD} L_{CD} = 0.002471(500) = 1.236\ \text{mm}$$

Angle of tilt α:

$$\tan \alpha = \frac{18.535}{1500}; \qquad \alpha = 0.708^\circ$$ **Ans.**

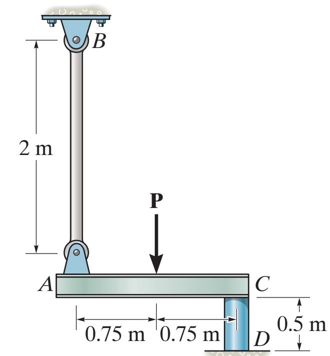


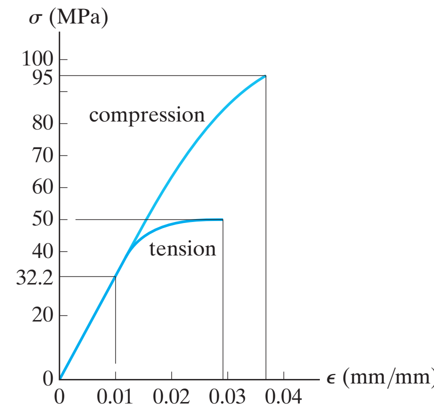


Ans.
$\alpha = 0.708^\circ$

3–18.

The stress–strain diagram for a polyester resin is given in the figure. If the rigid beam is supported by a strut AB and post CD made from this material, determine the largest load P that can be applied to the beam before it ruptures. The diameter of the strut is 12 mm and the diameter of the post is 40 mm.





SOLUTION

Rupture of strut AB:

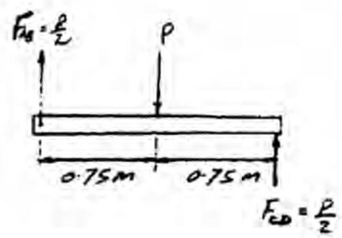
$$\sigma_R = \frac{F_{AB}}{A_{AB}}; \qquad 50(10^6) = \frac{P/2}{\frac{\pi}{4}(0.012)^2};$$

$$P = 11.3 \text{ kN (controls)}$$ **Ans.**

Rupture of post CD:

$$\sigma_R = \frac{F_{CD}}{A_{CD}}; \qquad 95(10^6) = \frac{P/2}{\frac{\pi}{4}(0.04)^2}$$

$$P = 239 \text{ kN}$$

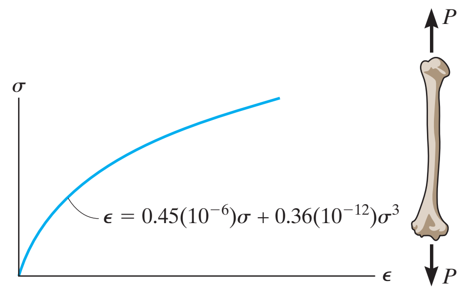


Ans.

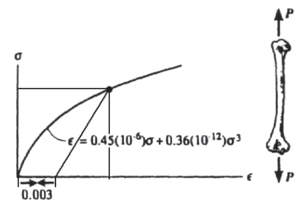
$P = 11.3$ kN (controls)

3–19.

The stress–strain diagram for a bone is shown, and can be described by the equation $\epsilon = 0.45(10^{-6})\,\sigma + 0.36(10^{-12})\,\sigma^3$, where σ is in kPa. Determine the yield strength assuming a 0.3% offset.



SOLUTION



$$\epsilon = 0.45(10^{-6})\sigma + 0.36(10^{-12})\sigma^3,$$

$$d\epsilon = \left(0.45(10^{-6}) + 1.08(10^{-12})\,\sigma^2\right)d\sigma$$

$$E = \left.\frac{d\sigma}{d\epsilon}\right|_{\sigma=0} = \frac{1}{0.45(10^{-6})} = 2.22(10^6)\text{ kPa} = 2.22\text{ GPa}$$

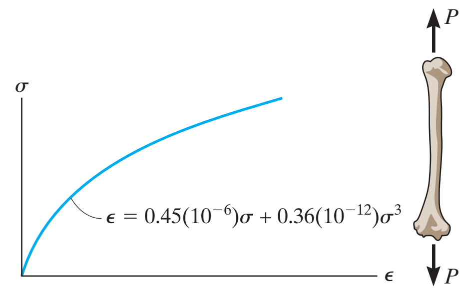
The equation for the recovery line is $\sigma = 2.22(10^6)(\epsilon - 0.003)$.

This line intersects the stress–strain curve at $\sigma_{YS} = 2027\text{ kPa} = 2.03\text{ MPa}$ **Ans.**

Ans:
$\sigma_{YS} = 2.03\text{ MPa}$

***3–20.**

The stress–strain diagram for a bone is shown and can be described by the equation $\epsilon = 0.45(10^{-6})\,\sigma + 0.36(10^{-12})\,\sigma^3$, where σ is in kPa. Determine the modulus of toughness and the amount of elongation of a 200-mm-long region just before it fractures if failure occurs at $\epsilon = 0.12$ mm/mm.



SOLUTION

When $\epsilon = 0.12$

$$120(10^{-3}) = 0.45\,\sigma + 0.36(10^{-6})\sigma^3$$

Solving for the real root:

$$\sigma = 6873.52 \text{ kPa}$$

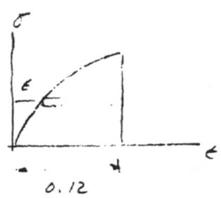
$$u_t = \int_A dA = \int_0^{6873.52} (0.12 - \epsilon)d\sigma$$

$$u_t = \int_0^{6873.52} (0.12 - 0.45(10^{-6})\sigma - 0.36(10^{-12})\sigma^3)d\sigma$$

$$= 0.12\,\sigma - 0.225(10^{-6})\sigma^2 - 0.09(10^{-12})\sigma^4\Big|_0^{6873.52}$$

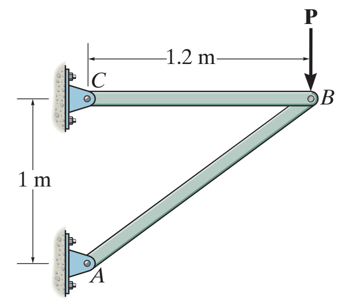
$$= 613 \text{ kJ/m}^3$$ **Ans.**

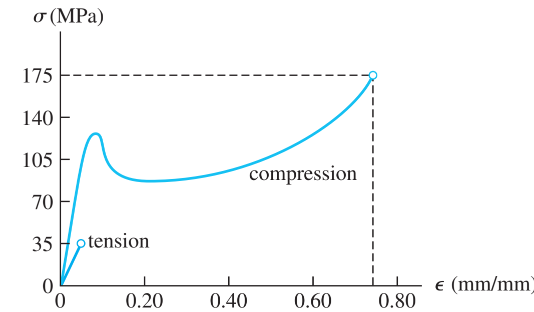
$$\delta = \epsilon L = 0.12(200) = 24 \text{ mm}$$ **Ans.**



Ans:
$u_t = 613$ kJ/m^3,
$\delta = 24$ mm

3–21. The two bars are made of polystyrene, which has the stress–strain diagram shown. If the cross-sectional area of bar AB is 975 mm^2 and BC is 2600 mm^2, determine the largest force P that can be supported before any member ruptures. Assume that buckling does not occur.





SOLUTION

$+\uparrow \Sigma F_y = 0;\quad \frac{5}{\sqrt{61}} F_{AB} - P = 0;\qquad F_{AB} = \frac{\sqrt{61}}{5} P \quad \text{(C)} \quad (1)$

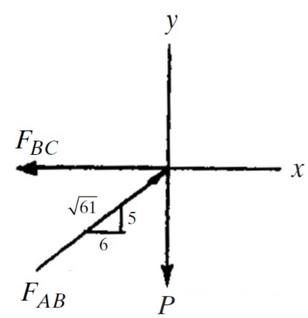
$\overset{+}{\leftarrow} \Sigma F_x = 0;\quad F_{BC} = \left(\frac{\sqrt{61}}{5} P\right)\left(\frac{6}{\sqrt{61}}\right) = 0 \qquad F_{BC} = 1.20P \quad \text{(T)} \quad (2)$

Assuming failure of bar BC:

From the stress–strain diagram $(\sigma_R)_t = 35$ MPa

$\sigma = \frac{F_{BC}}{A_{BC}};\qquad 35(10^6) = \frac{F_{BC}}{2600(10^{-6})};\qquad F_{BC} = 91.0(10^3)\text{ N} = 91.0\text{ kN}$

From Eq. (2), $P = 75.83$ kN



Assuming failure of bar AB:

From stress–strain diagram $(\sigma_R)_c = 175$ MPa

$\sigma = \frac{F_{AB}}{A_{AB}};\qquad 175(10^6) = \frac{F_{AB}}{975(10^{-6})};\qquad F_{AB} = 170.625(10^3)\text{ N} = 170.625\text{ kN}$

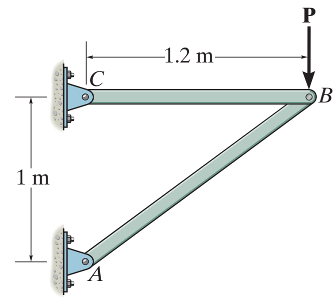
From Eq. (1), $P = 109.23$ kN

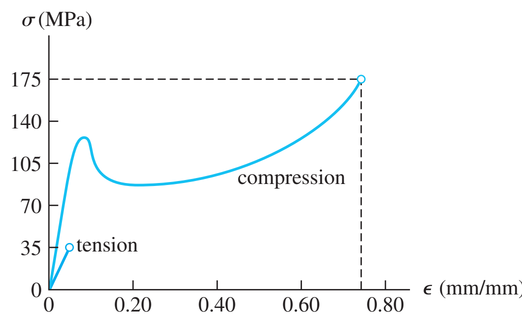
Choose the smallest value

$$P = 75.8\text{ kN} \qquad \textbf{Ans.}$$

Ans:
$P = 75.8$ kN

3–22. The two bars are made of polystyrene, which has the stress–strain diagram shown. Determine the cross-sectional area of each bar so that the bars rupture simultaneously when the load $P = 13.5$ kN. Assume that buckling does not occur.





SOLUTION

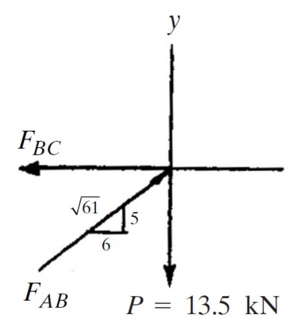
$+\uparrow \Sigma F_y = 0;\qquad F_{AB} = \left(\frac{5}{\sqrt{61}}\right) - 13.5 = 0 \qquad F_{AB} = 21.09 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad -F_{BC} + 21.06\left(\frac{6}{\sqrt{61}}\right) = 0 \qquad F_{BC} = 16.2 \text{ kN}$

For member BC:

$$(\sigma_{\max})_t = \frac{F_{BC}}{A_{BC}}; \qquad 35(10^6) = \frac{16.2(10^3)}{A_{BC}};$$

$$A_{BC} = 0.4629(10^{-3}) \text{ m}^2 = 462.9 \text{ mm}^2 = 463 \text{ mm}^2$$ **Ans.**



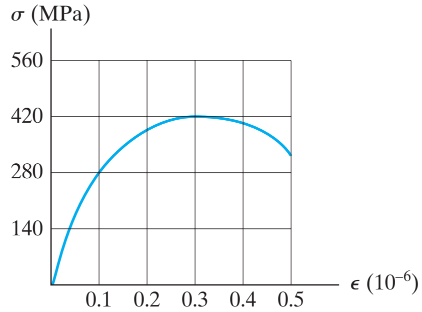
For member BA:

$$(\sigma_{\max})_c = \frac{F_{BA}}{A_{BA}}; \qquad 175(10^6) = \frac{21.09(10^3)}{A_{AB}};$$

$$A_{AB} = 0.1205(10^{-3}) \text{ m}^2 = 120.5 \text{ mm}^2 = 121 \text{ mm}^2$$ **Ans.**

Ans:
$A_{BC} = 463 \text{ mm}^2, A_{BA} = 121 \text{ mm}^2$

3–23. The stress–strain diagram for many metal alloys can be described analytically using the Ramberg-Osgood three parameter equation $\epsilon = \sigma/E + k\sigma^n$, where E, k, and n are determined from measurements taken from the diagram. Using the stress–strain diagram shown in the figure, take $E = 210$ GPa and determine the other two parameters k and n and thereby obtain an analytical expression for the curve.



SOLUTION

Choose,

$$\sigma = 280 \text{ MPa}, \quad \varepsilon = 0.1$$

$$\sigma = 420 \text{ MPa}, \quad \varepsilon = 0.3$$

$$0.1 = \frac{280}{210(10^3)} + k(280)^n$$

$$0.3 = \frac{420}{210(10^3)} + k(420)^n$$

$$0.098667 = k(280)^n$$

$$0.29800 = k(420)^n$$

$$0.3310962 = (0.6667)^n$$

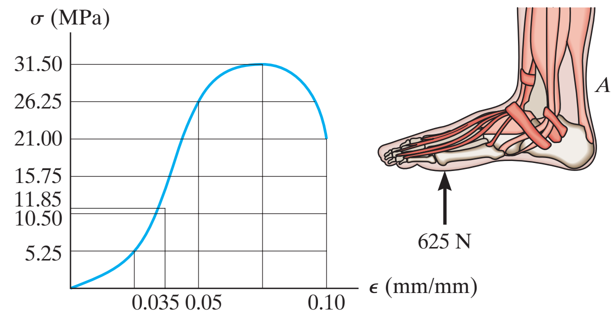
$$\ln(0.3310962) = n \ln(0.6667)$$

$$n = 2.73 \qquad \textbf{Ans.}$$

$$k = 21.0(10^{-9}) \qquad \textbf{Ans.}$$

Ans:
$n = 2.73$
$k = 21.0(10^{-9})$

***3–24.** The $\sigma - \epsilon$ diagram for a collagen fiber bundle from which a human tendon is composed is shown. If a segment of the Achilles tendon at A has a length of 165 mm and an approximate cross-sectional area of 145 mm^2, determine its elongation if the foot supports a load of 625 N, which causes a tension in the tendon of 1718.75 N.



SOLUTION

$$\sigma = \frac{P}{A} = \frac{1718.75}{145(10^{-6})} = 11.85 \text{ MPa}$$

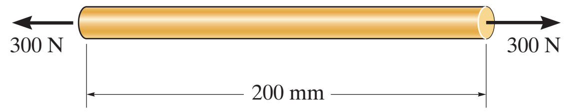
From the graph $\varepsilon = 0.035$ mm/mm

$$\delta = \varepsilon L = 0.035(165) = 5.775 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$\delta = 5.775$ mm

3–25.

The acrylic plastic rod is 200 mm long and 15 mm in diameter. If an axial load of 300 N is applied to it, determine the change in its length and the change in its diameter. $E_p = 2.70$ GPa, $\nu_p = 0.4$.



SOLUTION

$$\sigma = \frac{P}{A} = \frac{300}{\frac{\pi}{4}(0.015)^2} = 1.678 \text{ MPa}$$

$$\epsilon_{\text{long}} = \frac{\sigma}{E} = \frac{1.678\left(10^6\right)}{2.70\left(10^9\right)} = 0.0006288$$

$$\delta = \epsilon_{\text{long}} L = 0.0006288(200) = 0.126 \text{ mm}$$ **Ans.**

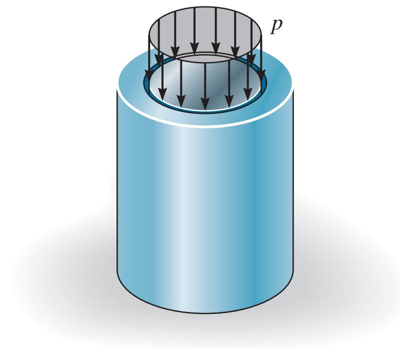
$$\epsilon_{\text{lat}} = -\nu\epsilon_{\text{long}} = -0.4(0.0006288) = -0.0002515$$

$$\Delta d = \epsilon_{\text{lat}} d = -0.0002515(15) = -0.00377 \text{ mm}$$ **Ans.**

Ans:
$\delta = 0.126$ mm, $\Delta d = -0.00377$ mm

3–26.

The plug has a diameter of 30 mm and fits within a rigid sleeve having an inner diameter of 32 mm. Both the plug and the sleeve are 50 mm long. Determine the axial pressure p that must be applied to the top of the plug to cause it to contact the sides of the sleeve. Also, how far must the plug be compressed downward in order to do this? The plug is made from a material for which $E = 5$ MPa, $\nu = 0.45$.



SOLUTION

$$\epsilon_{\text{lat}} = \frac{d' - d}{d} = \frac{32 - 30}{30} = 0.06667 \text{ mm/mm}$$

$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}}; \qquad \epsilon_{\text{long}} = -\frac{\epsilon_{\text{lat}}}{\nu} = -\frac{0.06667}{0.45} = -0.1481 \text{ mm/mm}$$

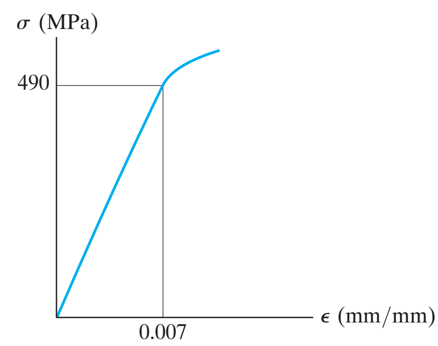
$$p = \sigma = E\,\epsilon_{\text{long}} = 5\left(10^6\right)(0.1481) = 741 \text{ kPa} \qquad \textbf{Ans.}$$

$$\delta = \left|\epsilon_{\text{long}} L\right| = \left|-0.1481(50)\right| = 7.41 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$p = 741$ kPa,
$\delta = 7.41$ mm

3–27.

The elastic portion of the stress–strain diagram for an aluminum alloy is shown in the figure. The specimen from which it was obtained has an original diameter of 12.7 mm and a gage length of 50.8 mm. When the applied load on the specimen is 50 kN, the diameter is 12.67494 mm. Determine Poisson's ratio for the material.



SOLUTION

Average Normal Stress:

$$\sigma = \frac{N}{A} = \frac{50(10^3)}{\frac{\pi}{4}(0.0127^2)} = 394.71(10^6)\text{ Pa} = 394.71\text{ MPa}$$

Average Normal Strain: Referring to the stress–strain diagram, the modulus of elasticity is $E = \dfrac{490(10^6)}{0.007} = 70.0(10^9)\text{ Pa} = 70.0\text{ GPa}.$

$$\epsilon_{\text{long}} = \frac{\sigma}{E} = \frac{394.71(10^6)}{70.0(10^9)} = 0.0056386\text{ mm/mm}$$

$$\epsilon_{\text{lat}} = \frac{d - d_0}{d_0} = \frac{12.67494 - 12.7}{12.7} = -0.0019732$$

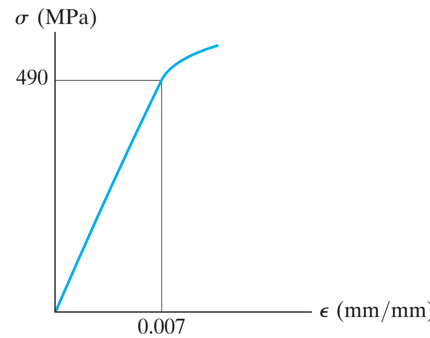
Poisson's Ratio: The lateral and longitudinal strain can be related using Poisson's ratio, that is

$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}} = -\frac{(-0.019732)}{0.0056386} = 0.350$$ **Ans.**

Ans:
$\nu = 0.350$

***3–28.**

The elastic portion of the stress–strain diagram for an aluminum alloy is shown in the figure. The specimen from which it was obtained has an original diameter of 12.7 mm and a gage length of 50.8 mm. If a load of $P = 60$ kN is applied to the specimen, determine its new diameter and length. Take $\nu = 0.35$.



SOLUTION

Average Normal Stress:

$$\sigma = \frac{N}{A} = \frac{60(10^3)}{\frac{\pi}{4}(0.0127^2)} = 473.65(10^6)\text{ Pa} = 473.65\text{ MPa}$$

Average Normal Strain: Referring to the stress–strain diagram, the modulus of elasticity is $E = \dfrac{490(10^6)}{0.007} = 70.0(10^9)$ Pa $= 70.0$ GPa.

$$\epsilon_{\text{long}} = \frac{\sigma}{E} = \frac{473.65(10^6)}{70.0(10^9)} = 0.0067664\text{ mm/mm}$$

Thus,

$$\delta L = \epsilon_{\text{long}} L_0 = 0.0067664(50.8) = 0.34373\text{ mm}$$

Then

$$L = L_0 + \delta L = 50.8 + 0.34373 = 51.1437\text{ mm}$$ **Ans.**

Poisson's Ratio: The lateral strain can be related to the longitudinal strain using Poisson's ratio.

$$\epsilon_{\text{lat}} = -\nu\epsilon_{\text{long}} = -0.35(0.0067664) = -0.0023682\text{ mm/mm}$$

Thus,

$$\delta d = \epsilon_{\text{lat}}\, d = -0.0023682(12.7) = -0.030077\text{ mm}$$

Then

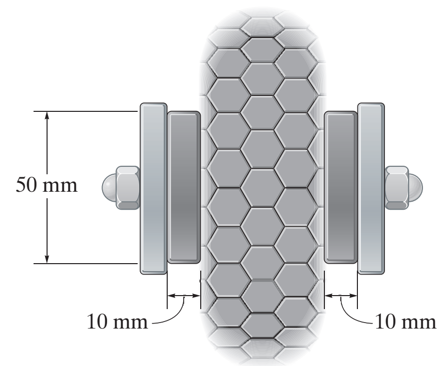
$$d = d_0 + \delta d = 12.7 + (-0.030077) = 12.66992\text{ mm}$$

$$= 12.67\text{ mm}$$ **Ans.**

Ans:
$L = 51.1437$ mm,
$d = 12.67$ mm

3–29.

The brake pads for a bicycle tire are made of rubber. If a frictional force of 50 N is applied to each side of the tires, determine the average shear strain in the rubber. Each pad has cross-sectional dimensions of 20 mm and 50 mm. $G_r = 0.20$ MPa.



SOLUTION

Average Shear Stress: The shear force is $V = 50$ N.

$$\tau = \frac{V}{A} = \frac{50}{0.02(0.05)} = 50.0 \text{ kPa}$$

Shear-Stress – Strain Relationship: Applying Hooke's law for shear

$$\tau = G\gamma$$

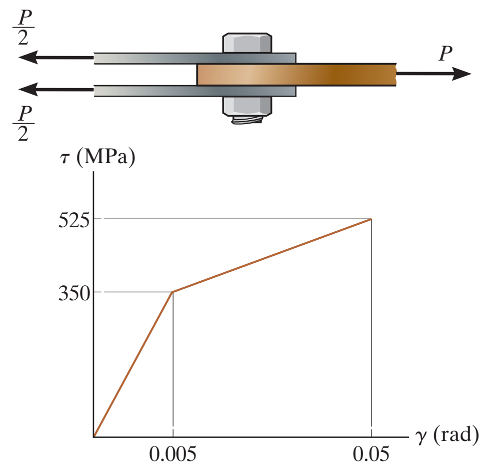
$$50.0(10^3) = 0.2(10^6)\,\gamma$$

$$\gamma = 0.250 \text{ rad} \qquad \textbf{Ans.}$$

Ans:
$\gamma = 0.250$ rad

3–30.

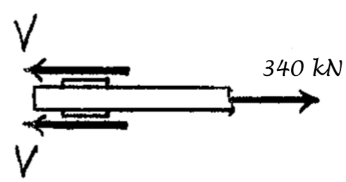
The lap joint is connected together using a 30 mm diameter bolt. If the bolt is made from a material having a shear stress–strain diagram that is approximated as shown, determine the shear strain developed in the shear plane of the bolt when $P = 340$ kN.



SOLUTION

Internal Loadings: The shear force developed in the shear planes of the bolt can be determined by considering the equilibrium of the free-body diagram shown in Fig. *a*.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 340 - 2V = 0 \qquad V = 170 \text{ kN}$$



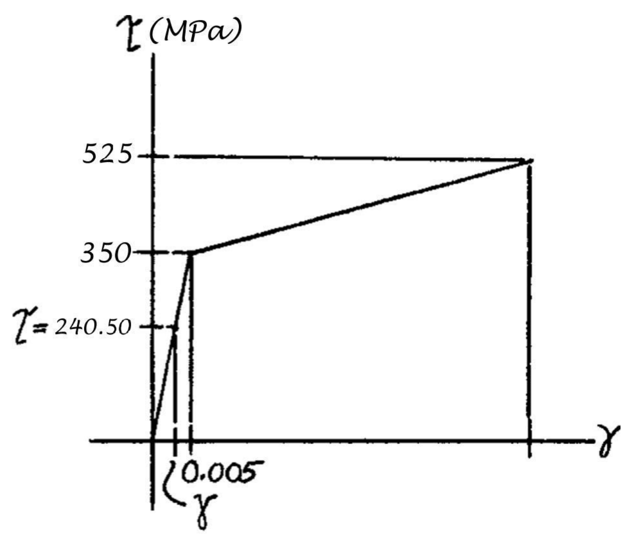
(a)

Shear Stress and Strain:

$$\tau = \frac{V}{A} = \frac{170(10^3)}{\frac{\pi}{4}(0.03^2)} = 240.50(10^6)\ \text{N/m}^2 = 240.50\ \text{MPa}$$

Using this result, the corresponding shear strain can be obtained from the shear stress–strain diagram, Fig. *b*.

$$\frac{240.50}{\gamma} = \frac{350}{0.005}; \qquad \gamma = 3.4357(10^{-3})\ \text{rad} = 3.44(10^{-3})\ \text{rad}$$ **Ans.**

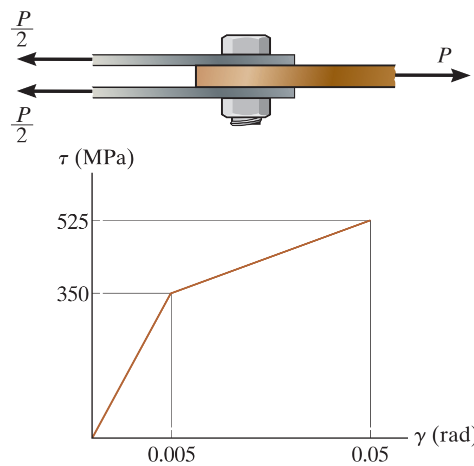


(b)

Ans:
$\gamma = 3.44(10^{-3})$ rad

3–31

The lap joint is connected together using a 30 mm diameter bolt. If the bolt is made from a material having a shear stress–strain diagram that is approximated as shown, determine the permanent shear strain in the shear plane of the bolt when the applied force $P = 680$ kN is removed.



SOLUTION

Internal Loadings: The shear force developed in the shear planes of the bolt can be determined by considering the equilibrium of the free-body diagram shown in Fig. *a*.

$\xrightarrow{+}\Sigma F_x = 0; \quad 680 - 2V = 0 \quad V = 340\text{ kN}$

Shear Stress and Strain:

$$\tau = \frac{V}{A} = \frac{340(10^3)}{\frac{\pi}{4}(0.03^2)} = 481.00(10^6)\text{ N/m}^2 = 481.00\text{ MPa}$$

Using this result, the corresponding shear strain can be obtained from the shear stress–strain diagram, Fig. *b*.

$$\frac{481.00 - 350}{\gamma - 0.005} = \frac{525 - 350}{0.05 - 0.005}; \qquad \gamma = 0.03869\text{ rad}$$

When force **P** is removed, the shear strain recovers linearly along line *BC*, Fig. *b*, with a slope that is the same as line *OA*. This slope represents the shear modulus.

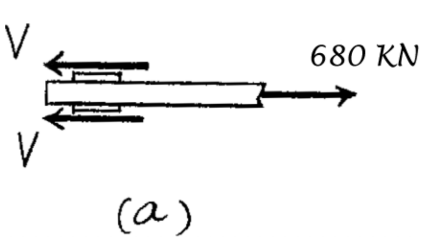
$$G = \frac{350(10^3)}{0.005} = 700.0(10^9)\text{ Pa} = 70.0\text{ GPa}$$

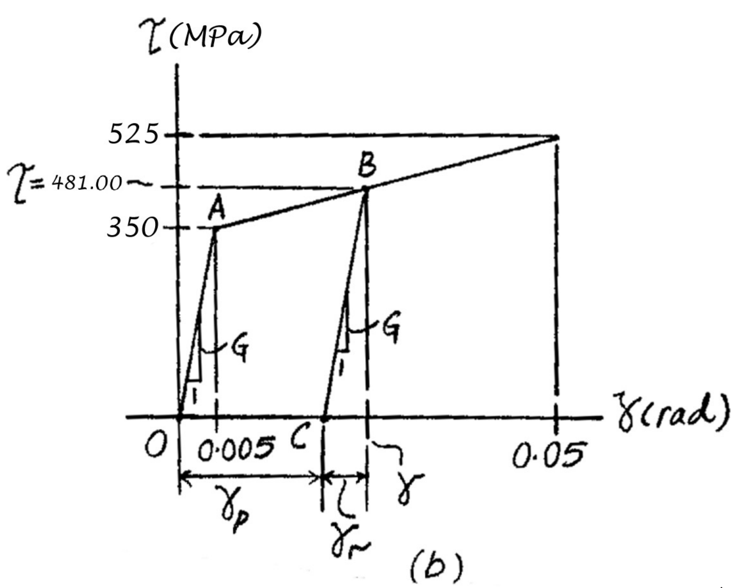
Thus, the elastic recovery of shear strain is

$$\tau = G\gamma_r; \qquad 481.00(10^6) = 70.0(10^9)\gamma_r \qquad \gamma_r = 6.8715(10^{-3})\text{ rad}$$

And the permanent shear strain is

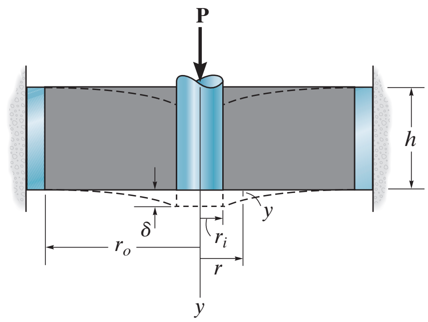
$$\gamma_P = \gamma - \gamma_r = 0.03869 - 6.874(10^{-3}) = 0.031815\text{ rad} = 0.0318\text{ rad}$$ **Ans.**





Ans:
$\gamma_P = 0.0318$ rad

***3–32.** A shear spring is made by bonding the rubber annulus to a rigid fixed ring and a plug. When an axial load **P** is placed on the plug, show that the slope at point y in the rubber is $dy/dr = -\tan\gamma = -\tan(P/(2\pi hGr))$. For small angles we can write $dy/dr = -P/(2\pi hGr)$. Integrate this expression and evaluate the constant of integration using the condition that $y = 0$ at $r = r_o$. From the result compute the deflection $y = \delta$ of the plug.



Shear Stress–Strain Relationship: Applying Hooke's law with $\tau_A = \dfrac{P}{2\pi\, r\, h}$.

$$\gamma = \frac{\tau_A}{G} = \frac{P}{2\pi\, h\, G\, r}$$

$$\frac{dy}{dr} = -\tan\gamma = -\tan\left(\frac{P}{2\pi\, h\, G\, r}\right) \qquad \text{(Q.E.D)}$$

If γ is small, then $\tan\gamma = \gamma$. Therefore,

$$\frac{dy}{dr} = -\frac{P}{2\pi\, h\, G\, r}$$

$$y = -\frac{P}{2\pi\, h\, G}\int\frac{dr}{r}$$

$$y = -\frac{P}{2\pi\, h\, G}\ln r + C$$

At $r = r_o$, $\quad y = 0$

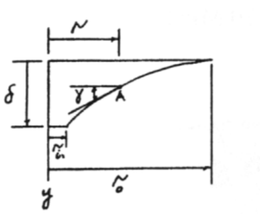
$$0 = -\frac{P}{2\pi\, h\, G}\ln r_o + C$$

$$C = \frac{P}{2\pi\, h\, G}\ln r_o$$

Then, $y = \dfrac{P}{2\pi\, h\, G}\ln\dfrac{r_o}{r}$

At $r = r_i$, $\quad y = \delta$

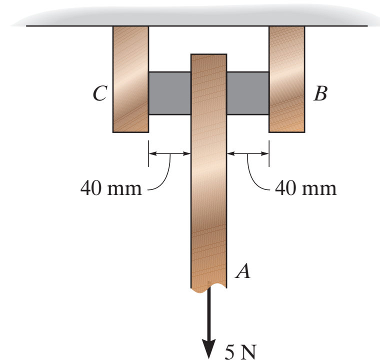
$$\delta = \frac{P}{2\pi\, h\, G}\ln\frac{r_o}{r_i} \qquad \textbf{Ans.}$$



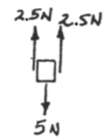
Ans.

$$\delta = \frac{P}{2\pi\, h\, G}\ln\frac{r_o}{r_i}$$

3–33. The support consists of three rigid plates, which are connected together using two symmetrically placed rubber pads. If a vertical force of 5 N is applied to plate A, determine the approximate vertical displacement of this plate due to shear strains in the rubber. Each pad has cross-sectional dimensions of 30 mm and 20 mm. $G_r = 0.20$ MPa.



SOLUTION



$$\tau_{avg} = \frac{V}{A} = \frac{2.5}{(0.03)(0.02)} = 4166.7 \text{ Pa}$$

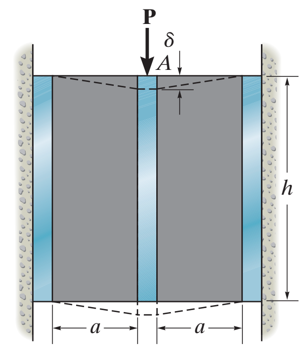
$$\gamma = \frac{\tau}{G} = \frac{4166.7}{0.2(10^6)} = 0.02083 \text{ rad}$$

$$\delta = 40(0.02083) = 0.833 \text{ mm}$$ **Ans.**

Ans.
$\delta = 0.833$ mm

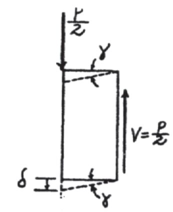
3–34.

A shear spring is made from two blocks of rubber, each having a height h, width b, and thickness a. The blocks are bonded to three plates as shown. If the plates are rigid and the shear modulus of the rubber is G, determine the displacement of plate A when the vertical load $\mathbf{P}$ is applied. Assume that the displacement is small so that $\delta = a \tan \gamma \approx a\gamma$.



SOLUTION

Average Shear Stress: The rubber block is subjected to a shear force of $V = \dfrac{P}{2}$.



$$\tau = \frac{V}{A} = \frac{\frac{P}{2}}{bh} = \frac{P}{2bh}$$

Shear Strain: Applying Hooke's law for shear

$$\gamma = \frac{\tau}{G} = \frac{\frac{P}{2bh}}{G} = \frac{P}{2bhG}$$

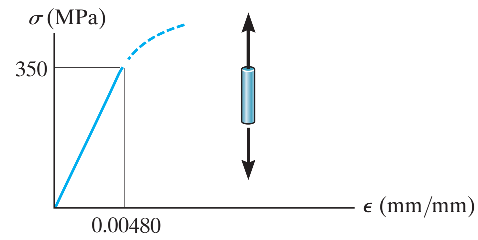
Thus,

$$\delta = a\gamma = \frac{Pa}{2bhG}$$ **Ans.**

Ans:
$\delta = \dfrac{Pa}{2bhG}$

R3–1.

The elastic portion of the tension stress–strain diagram for an aluminum alloy is shown in the figure. The specimen used for the test has a gauge length of 50 mm and a diameter of 12.5 mm. When the applied load is 45 kN, the new diameter of the specimen is 12.4780 mm. Calculate the shear modulus G_{al} for the aluminum.



SOLUTION

From the stress–strain diagram,

$$E_{al} = \frac{\sigma}{\varepsilon} = \frac{350(10^6)}{0.00480} = 72.92(10^9)\ \text{N/m}^2 = 72.92\ \text{GPa}$$

When specimen is loaded with a 45-kN load,

$$\sigma = \frac{P}{A} = \frac{45(10^3)}{\frac{\pi}{4}(0.0125^2)} = 366.69(10^6)\ \text{N/m}^2 = 366.69\ \text{MPa}$$

$$\varepsilon_{\text{long}} = \frac{\sigma}{E_{al}} = \frac{366.69(10^6)}{72.92(10^9)} = 5.0289(10^{-3})\ \text{mm/mm}$$

$$\varepsilon_{\text{lat}} = \frac{d' - d}{d} = \frac{12.4780 - 12.5}{12.5} = -1.76(10^{-3})\ \text{mm/mm}$$

$$V = \frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}} = \frac{-1.76(10^{-3})}{5.0289(10^{-3})} = 0.3500$$

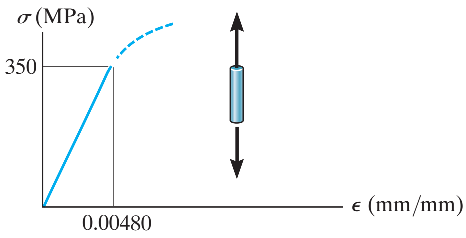
$$G_{al} = \frac{E_{at}}{2(1+v)} = \frac{72.92(10^9)}{2(1+0.3500)} = 27.00(10^9)\ \text{N/m}^2 = 27.0\ \text{GPa}$$ **Ans.**

Ans:

$G_{al} = 27.0$ GPa

R3–2.

The elastic portion of the tension stress–strain diagram for an aluminum alloy is shown in the figure. The specimen used for the test has a gauge length of 50 mm and a diameter of 12.5 mm. If the applied load is 40 kN, determine the new diameter of the specimen. The shear modulus is $G_{al} = 27$ GPa.



SOLUTION

$$\sigma = \frac{P}{A} = \frac{40(10^3)}{\frac{\pi}{4}(0.0125^2)} = 325.95(10^6)\ \text{N/m}^2 = 325.95\ \text{MPa}$$

From the stress–strain diagram,

$$E = \frac{350(10^6)}{0.00480} = 72.92(10^9)\ \text{N/m}^2 = 72.92\ \text{GPa}$$

$$\varepsilon_{\text{long}} = \frac{\sigma}{E} = \frac{325.95(10^6)}{72.92(10^9)} = 4.4702(10^{-3})\ \text{mm/mm}$$

$$G = \frac{E}{2(1+\nu)}; \quad 27(10^9) = \frac{72.92(10^9)}{2(1+\nu)}; \quad \nu = 0.3503$$

$$\varepsilon_{\text{lat}} = -\nu\varepsilon_{\text{long}} = -0.3503[4.4702(10^{-3})] = -1.5659(10^{-3})\ \text{mm/mm}$$

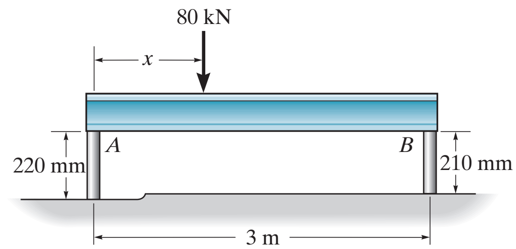
$$\Delta d = \varepsilon_{\text{lat}} d = [-1.5659(10^{-3})](12.5) = -0.01957\ \text{mm}$$

$$d' = d + \Delta d = 12.5 - 0.01957 = 12.4804\ \text{mm}$$ **Ans.**

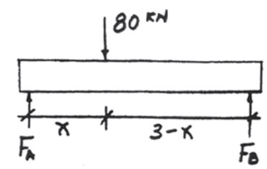
Ans:
$d' = 12.4804$ mm

R3–3.

The rigid beam rests in the horizontal position on two 2014-T6 aluminum cylinders having the *unloaded* lengths shown. If each cylinder has a diameter of 30 mm, determine the placement x of the applied 80-kN load so that the beam remains horizontal. What is the new diameter of cylinder A after the load is applied? $\nu_{al} = 0.35$.



SOLUTION



$$\zeta + \Sigma M_A = 0; \qquad F_B(3) - 80(x) = 0; \quad F_B = \frac{80x}{3} \qquad \textbf{(1)}$$

$$\zeta + \Sigma M_B = 0; \qquad -F_A(3) + 80(3 - x) = 0; \quad F_A = \frac{80(3 - x)}{3} \qquad \textbf{(2)}$$

Since the beam is held horizontally, $\delta_A = \delta_B$

$$\sigma = \frac{P}{A}; \qquad \epsilon = \frac{\sigma}{E} = \frac{\frac{P}{A}}{E}$$

$$\delta = \epsilon L = \left(\frac{\frac{P}{A}}{E}\right) L = \frac{PL}{AE}$$

$$\delta_A = \delta_B; \qquad \frac{\frac{80(3 - x)}{3}(220)}{AE} = \frac{\frac{80x}{3}(210)}{AE}$$

$$80(3 - x)(220) = 80x(210)$$

$$x = 1.53 \text{ m} \qquad \textbf{Ans.}$$

From Eq. (2),

$$F_A = 39.07 \text{ kN}$$

$$\sigma_A = \frac{F_A}{A} = \frac{39.07(10^3)}{\frac{\pi}{4}(0.03^2)} = 55.27 \text{ MPa}$$

$$\epsilon_{long} = \frac{\sigma_A}{E} = -\frac{55.27(10^6)}{73.1(10^9)} = -0.000756$$

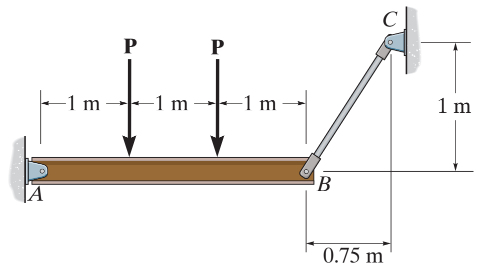
$$\epsilon_{lat} = -\nu\epsilon_{long} = -0.35(-0.000756) = 0.0002646$$

$$d'_A = d_A + d\,\epsilon_{lat} = 30 + 30(0.0002646) = 30.008 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$x = 1.53$ m, $d'_A = 30.008$ mm

*R3–4.

When the two forces are placed on the beam, the diameter of the A-36 steel rod BC decreases from 40 mm to 39.99 mm. Determine the magnitude of each force **P**.



SOLUTION

Equations of Equilibrium: The force developed in rod BC can be determined by writing the moment equation of equilibrium about A with reference to the free-body diagram of the beam shown in Fig. *a*.

$$\zeta + \Sigma M_A = 0; \qquad F_{BC}\left(\frac{4}{5}\right)(3) - P(2) - P(1) = 0 \qquad F_{BC} = 1.25P$$

Normal Stress and Strain: The lateral strain of rod BC is

$$\epsilon_{\text{lat}} = \frac{d - d_0}{d_0} = \frac{39.99 - 40}{40} = -0.25(10^{-3})\ \text{mm/mm}$$

$$\epsilon_{\text{lat}} = -\nu\epsilon_a; \qquad -0.25(10^{-3}) = -(0.32)\epsilon_a$$

$$\epsilon_a = 0.78125(10^{-3})\ \text{mm/mm}$$

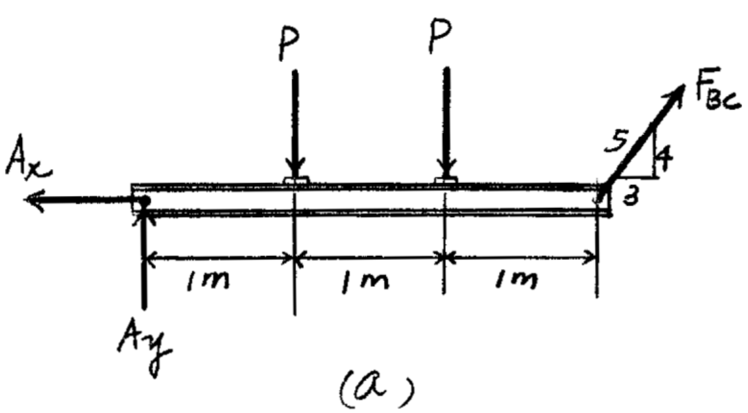
Assuming that Hooke's Law applies,

$$\sigma_{BC} = E\epsilon_a; \qquad \sigma_{BC} = 200(10^9)(0.78125)(10^{-3}) = 156.25\ \text{MPa}$$

Since $\sigma < \sigma_Y$, the assumption is correct.

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}}; \qquad 156.25(10^6) = \frac{1.25P}{\frac{\pi}{4}\left(0.04^2\right)}$$

$$P = 157.08(10^3)\text{N} = 157\ \text{kN}$$ **Ans.**

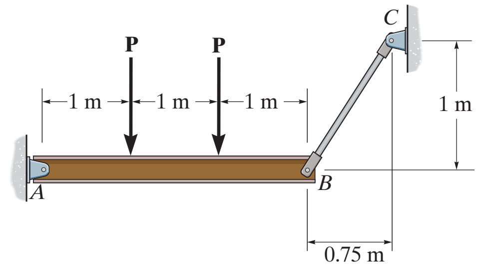


(a)

Ans:
$P = 157$ kN

R3–5.

If $P = 150$ kN, determine the elastic elongation of rod BC and the decrease in its diameter. Rod BC is made of A-36 steel and has a diameter of 40 mm.



SOLUTION

Equations of Equilibrium: The force developed in rod BC can be determined by writing the moment equation of equilibrium about A with reference to the free-body diagram of the beam shown in Fig. *a*.

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BC}\left(\frac{4}{5}\right)(3) - 150(2) - 150(1) = 0 \qquad F_{BC} = 187.5 \text{ kN}$$

Normal Stress and Strain: The lateral strain of rod BC is

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{187.5(10^3)}{\frac{\pi}{4}(0.04^2)} = 149.21 \text{ MPa}$$

Since $\sigma < \sigma_Y$, Hooke's Law can be applied. Thus,

$$\sigma_{BC} = E\epsilon_{BC}; \qquad 149.21(10^6) = 200(10^9)\epsilon_{BC}$$

$$\epsilon_{BC} = 0.7460(10^{-3}) \text{ mm/mm}$$

The unstretched length of rod BC is $L_{BC} = \sqrt{750^2 + 1000^2} = 1250$ mm. Thus the elongation of this rod is given by

$$\delta_{BC} = \epsilon_{BC}L_{BC} = 0.7460(10^{-3})(1250) = 0.933 \text{ mm} \qquad \textbf{Ans.}$$

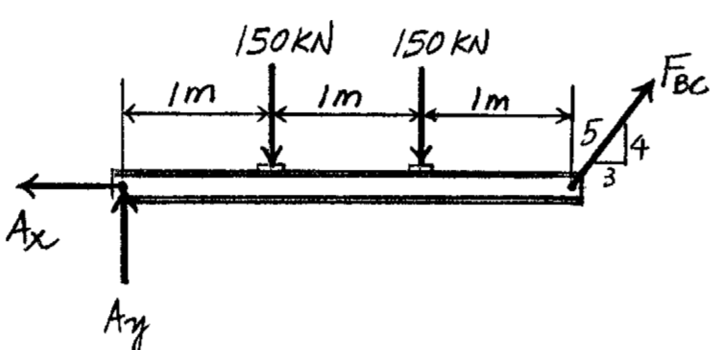
We obtain,

$$\epsilon_{\text{lat}} = -\nu\epsilon_a; \qquad \epsilon_{\text{lat}} = -(0.32)(0.7460)(10^{-3})$$

$$= -0.2387(10^{-3}) \text{ mm/mm}$$

Thus,

$$\delta d = \epsilon_{\text{lat}}\, d_{BC} = -0.2387(10^{-3})(40) = -9.55(10^{-3}) \text{ mm} \qquad \textbf{Ans.}$$



(a)

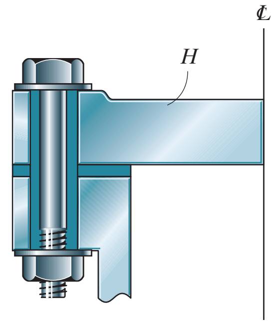
Ans.

$\delta_{BC} = 0.933$ mm

$\delta d = -9.55(10^{-3})$ mm

R3–6.

The head H is connected to the cylinder of a compressor using six steel bolts. If the clamping force in each bolt is 4 kN, determine the normal strain in the bolts. Each bolt has a diameter of 5 mm. If $\sigma_Y = 280$ MPa and $E_{st} = 200$ GPa, what is the strain in each bolt when the nut is unscrewed so that the clamping force is released?



SOLUTION

Normal Stress:

$$\sigma = \frac{P}{A} = \frac{4(10^3)}{\frac{\pi}{4}(0.005^2)} = 203.72(10^6)\ \text{N/m}^2 = 203.72\ \text{MPa} < \sigma_\gamma = 280\ \text{MPa}$$

Normal Strain: Since $\sigma < \sigma_\gamma$, Hooke's law is still valid.

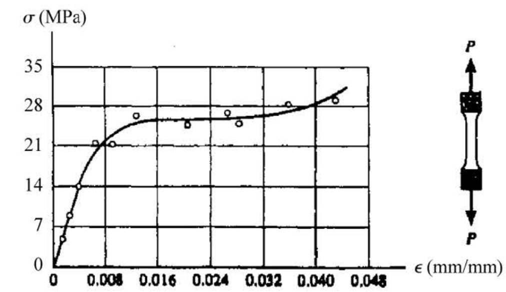
$$\varepsilon = \frac{\sigma}{E} = \frac{203.72(10^6)}{200(10^9)} = 1.0186(10^{-3})\ \text{mm/mm} = 1.02(10^{-3})\ \text{mm/mm}$$ **Ans.**

If the nut is unscrewed, the load is zero. Also, the nut is not stressed beyond σ_γ and so no permanent strain will be developed. Therefore, the strain $\varepsilon = 0$ **Ans.**

Ans:
$\varepsilon = 0.0010186$ mm/mm, $\varepsilon_{unscr} = 0$

R3–7.

The stress–strain diagram for polyethylene, which is used to sheath coaxial cables, is determined from testing a specimen that has a gauge length of 250 mm. If a load P on the specimen develops a strain of $\epsilon = 0.024$ mm/mm, determine the approximate length of the specimen, measured between the gauge points, when the load is removed. Assume the specimen recovers elastically.



SOLUTION

From the graph $\sigma = 14$ Mpa, $\epsilon = 0.004$ mm/mm,

$$E = \frac{\sigma}{\epsilon} = \frac{14(10^6)}{0.004} = 3.50(10^9)\text{ Pa} = 3.50\text{ GPa}$$

$\epsilon = 0.024$ mm/mm, $\sigma = 25.67$ Mpa

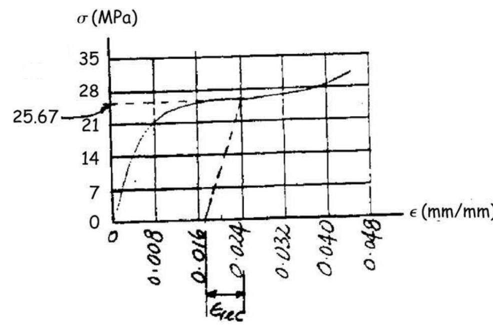
$L' = 250\text{ mm} + 0.024(250) = 256\text{ mm}$

Elastic strain recovery:

$$\epsilon_{\text{rec}} = \frac{\sigma}{E} = \frac{25.67(10^6)}{3.50(10^9)} = 7.3333(10^{-3})\text{ mm/mm}$$

$$\delta = \epsilon_{\text{rec}}L = [7.3333(10^{-3})](250)\text{ mm/mm} = 1.8333\text{ mm}$$

$L = L' - \delta = 256\text{ mm} - 1.8333\text{ mm} = 254.167\text{ mm}$ **Ans.**

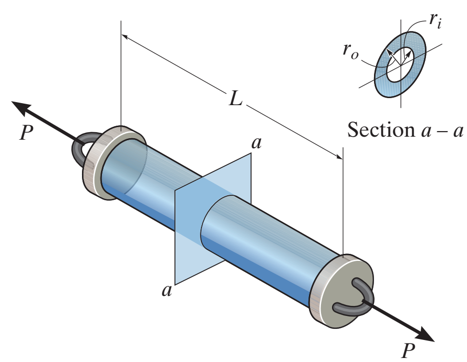


Ans:

254.167 mm

***R3–8.**

The pipe with two rigid caps attached to its ends is subjected to an axial force P. If the pipe is made from a material having a modulus of elasticity E and Poisson's ratio ν, determine the change in volume of the material.



SOLUTION

Normal Stress: The rod is subjected to uniaxial loading. Thus, $\sigma_{\text{long}} = \dfrac{P}{A}$ and $\sigma_{\text{lat}} = 0$.

$$\delta V = A\delta L + 2\pi r L \delta r$$

$$= A\epsilon_{\text{long}} L + 2\pi r L \epsilon_{\text{lat}} r$$

Using Poisson's ratio and noting that $AL = \pi r^2 L = V$,

$$\delta V = \epsilon_{\text{long}} V - 2\nu \epsilon_{\text{long}} V$$

$$= \epsilon_{\text{long}}(1 - 2\nu)V$$

$$= \frac{\sigma_{\text{long}}}{E}(1 - 2\nu)V$$

Since $\sigma_{\text{long}} = P/A$,

$$\delta V = \frac{P}{AE}(1 - 2\nu)AL$$

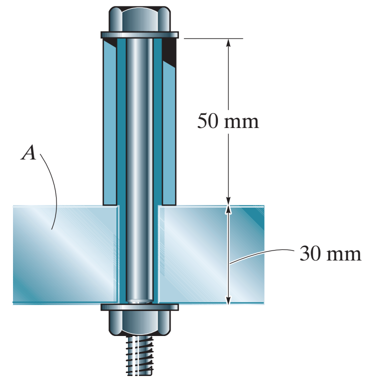
$$= \frac{PL}{E}(1 - 2\nu) \qquad \textbf{Ans.}$$

Ans:

$\delta V = \dfrac{PL}{E}(1 - 2\nu)$

R3–9.

The 8-mm-diameter bolt is made of an aluminum alloy. It fits through a magnesium sleeve that has an inner diameter of 12 mm and an outer diameter of 20 mm. If the original lengths of the bolt and sleeve are 80 mm and 50 mm, respectively, determine the strains in the sleeve and the bolt if the nut on the bolt is tightened so that the tension in the bolt is 8 kN. Assume the material at A is rigid. $E_{al} = 70$ GPa, $E_{mg} = 45$ GPa.



SOLUTION

Normal Stress:

$$\sigma_b = \frac{P}{A_b} = \frac{8(10^3)}{\frac{\pi}{4}(0.008^2)} = 159.15 \text{ MPa}$$

$$\sigma_s = \frac{P}{A_s} = \frac{8(10^3)}{\frac{\pi}{4}(0.02^2 - 0.012^2)} = 39.79 \text{ MPa}$$

Normal Strain: Applying Hooke's Law

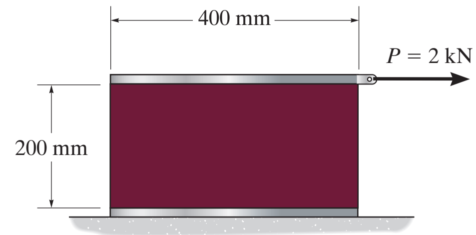
$$\epsilon_b = \frac{\sigma_b}{E_{al}} = \frac{159.15(10^6)}{70(10^9)} = 0.00227 \text{ mm/mm} \qquad \textbf{Ans.}$$

$$\epsilon_s = \frac{\sigma_s}{E_{mg}} = \frac{39.79(10^6)}{45(10^9)} = 0.000884 \text{ mm/mm} \qquad \textbf{Ans.}$$

Ans:
$\epsilon_b = 0.00227$ mm/mm, $\epsilon_s = 0.000884$ mm/mm

R3–10.

An acetal polymer block is fixed to the rigid plates at its top and bottom surfaces. If the top plate displaces 2 mm horizontally when it is subjected to a horizontal force $P = 2$ kN, determine the shear modulus of the polymer. The width of the block is 100 mm. Assume that the polymer is linearly elastic and use small angle analysis.



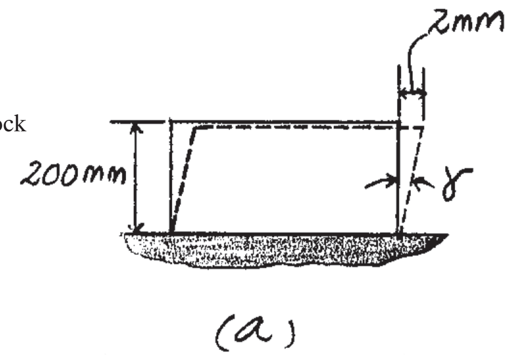
SOLUTION

Normal and Shear Stress:

$$\tau = \frac{V}{A} = \frac{2(10^3)}{0.4(0.1)} = 50 \text{ kPa}$$

Referring to the geometry of the undeformed and deformed shape of the block shown in Fig. *a*,

$$\gamma = \frac{2}{200} = 0.01 \text{ rad}$$



Applying Hooke's Law,

$\tau = G\gamma;\quad 50(10^3) = G(0.01)$

$$G = 5 \text{ MPa}$$ **Ans.**

Ans:
$G = 5$ MPa