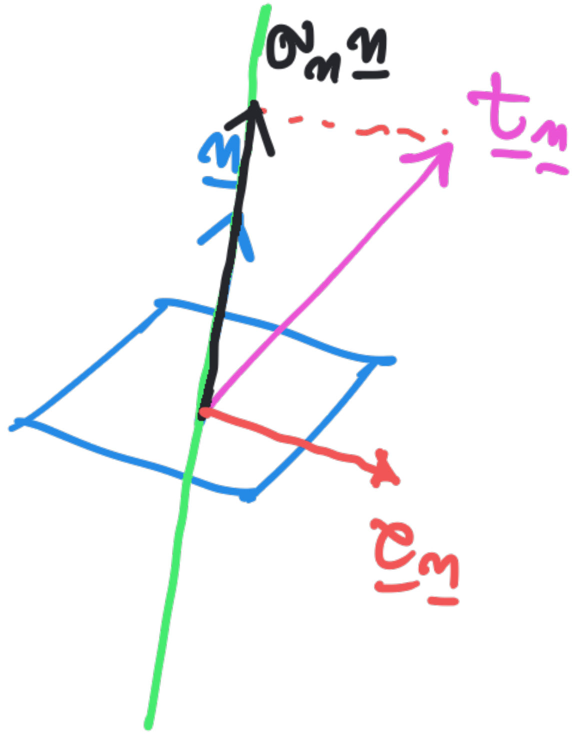
MEMO:

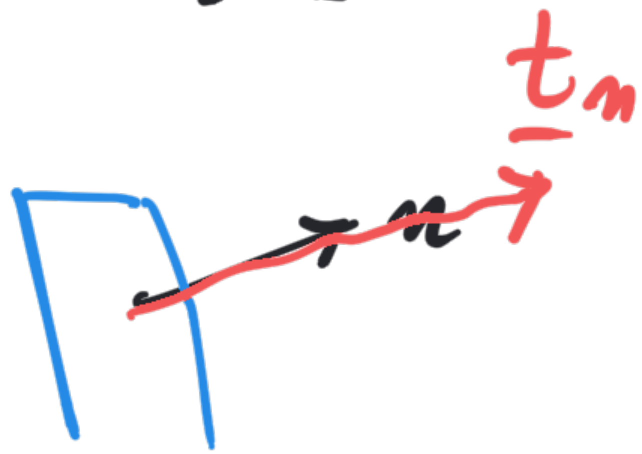


$\sigma_n = \underline{t}_n \cdot \underline{n}$ (tensione normale)

$$\underline{t}_n = \underbrace{\sigma_n \underline{n}}_{\text{comp. normale}} + \underbrace{\underline{\tau}_n}_{\text{comp. tang.}} = \underline{T}\,\underline{n}$$

$$[\underline{T}] = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

Problema: cerchiamo le giaciture n tali che $\underline{\tau}_n = \underline{0}$



$\Leftrightarrow \quad \underline{T}\,\underline{n} = \sigma_n \underline{n}$

$\underline{n}$ autovettore di $\underline{T}$ $\quad\quad$ $\underline{n}$ direz. princ.

σ_n autovalore di $\underline{T}$ $\quad\quad$ σ_n tens. princ.

Problema: cerchiamo le direzioni $\underline{n}$ tali che $\underline{\tau}_{\underline{n}} = \underline{0}$

$\Leftrightarrow \quad \underline{T}\,\underline{n} = \sigma_{\underline{n}}\,\underline{n}$

$\underline{n}$ autovettore di $\underline{T}$ $\qquad$ $\underline{n}$ direz. princ.

$\sigma_{\underline{n}}$ autovalo. di $\underline{T}$ $\qquad$ σ_n tens. princ.

$|\underline{n}| = 1$

- $\underline{T}\,\underline{n} = \sigma_n\,\underline{I}\,\underline{n}$ $\qquad$ $\begin{cases} (\underline{T} - \sigma_n \underline{I})\,\underline{n} = \underline{0} \\ \alpha^2 + \beta^2 + \gamma^2 = 1 \end{cases}$ $\qquad$ $[\underline{n}] = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$

incognite: $\alpha, \beta, \gamma, \sigma_n$

$$\begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

$$\det(\underline{T}-\sigma_n\underline{I})=0$$

$$I_1=\mathrm{tr}\,\underline{T}$$

$$I_2=\tfrac{1}{2}\left[(\mathrm{tr}\,\underline{T})^2-\mathrm{tr}\,\underline{T}^2\right]$$

$$I_3=\det\underline{T}$$

(14.32)

$\underline{T}^2=\underline{T}\,\underline{T}$.

$$(*)\qquad \sigma_n^3-I_1\sigma_n^2+I_2\sigma_n-I_3=0$$

invarianti di $\underline{T}$

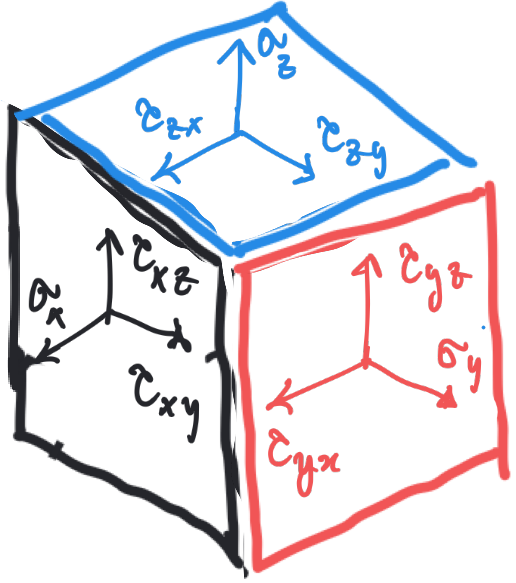
$\underline{T}=\underline{T}^T \Rightarrow$ il polinomio (*) ha radici reali.

autovettori relativi ad autov. distinti sono fra loro ortogonali.

Esiste una base ortonormale $\underline{n}_1, \underline{n}_2, \underline{n}_3$ di autov.

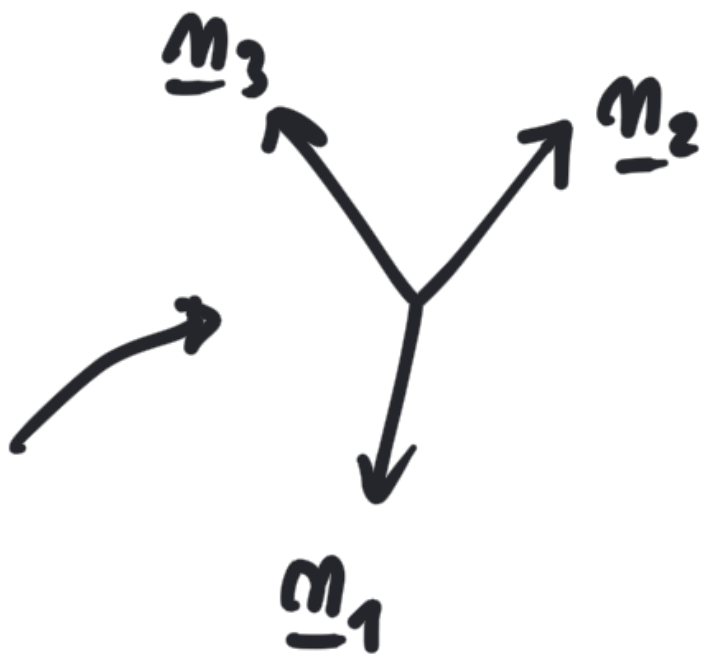
$\sigma_1, \sigma_2, \sigma_3$ autovalori.

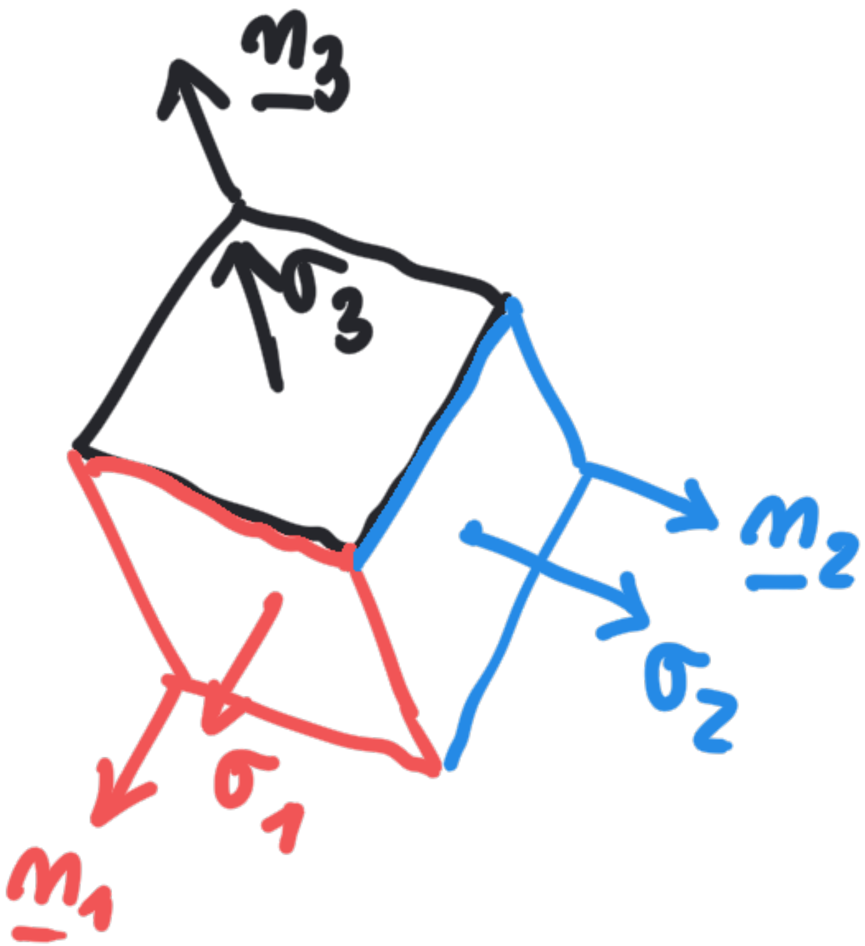
$$\begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$



$$[T(P)] = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

rifer.
princip.





$$[T]_* = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

Oss: il r.f. princ. non è unico.

Es: $[\underline{T}] = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{bmatrix}$ $\qquad [\underline{T}]_* = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{bmatrix}$

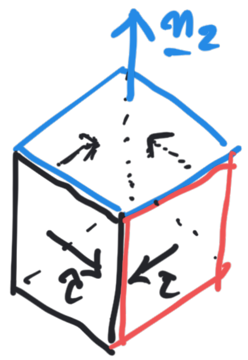
stato di sforzo sferico

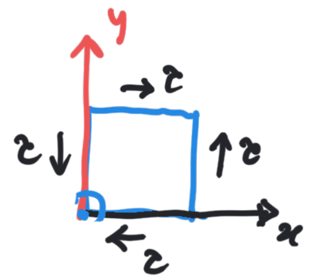
qualsiasi altra base

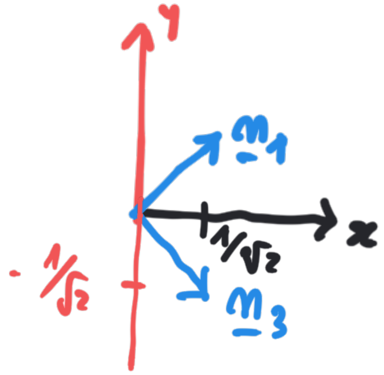
ogni r.f. è principale!

ESEMPIO: taglio puro

$$[\underline{T}] = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$







$$[\underline{T} - \sigma \underline{I}] = \begin{bmatrix} -\sigma & \tau & 0 \\ \tau & -\sigma & 0 \\ 0 & 0 & -\sigma \end{bmatrix}$$

$$\det(\underline{T} - \sigma \underline{I}) = -\sigma \det\begin{pmatrix} -\sigma & \tau \\ \tau & -\sigma \end{pmatrix} = -\sigma(\sigma^2 - \tau^2)$$
$$= -\sigma(\sigma + \tau)(\sigma - \tau)$$

$\sigma_1 = \tau \qquad \sigma_2 = 0 \qquad \sigma_3 = -\tau$

$$\begin{bmatrix} \tau & \tau & 0 \\ \tau & \tau & 0 \\ 0 & 0 & \tau \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[\underline{n}_3] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\tau & \tau & 0 \\ \tau & -\tau & 0 \\ 0 & 0 & -\tau \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

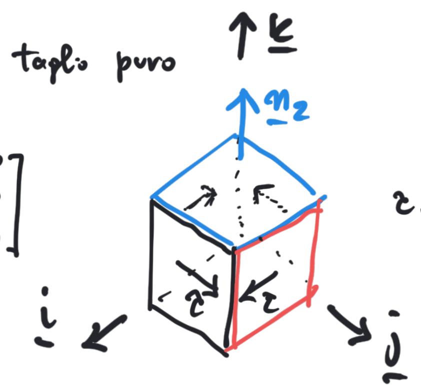
$[\underline{n}_1] = 1/\sqrt{2} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \qquad |\underline{n}_1| = 1$

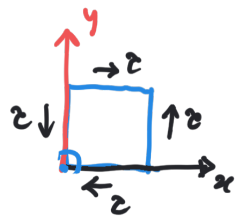
$[\underline{n}_2] = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \underline{n}_2 = \underline{k}$

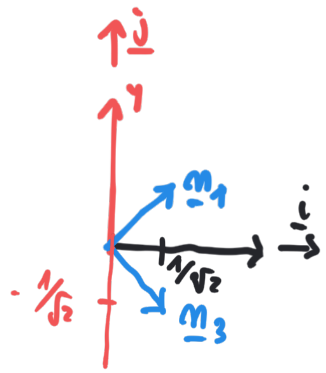
att: anche $-\underline{n}_1$ è una direz. princ.

ESEMPIO: taglio puro

$$[\underline{T}] = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$





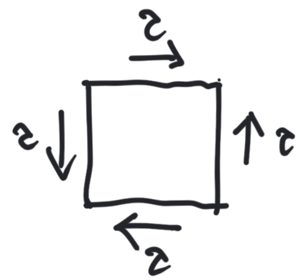


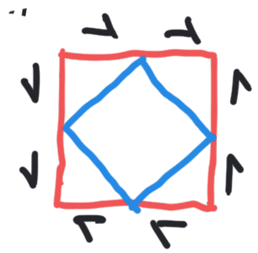
$\sigma_1 = \tau \quad \sigma_2 = 0 \quad \sigma_3 = -\tau$

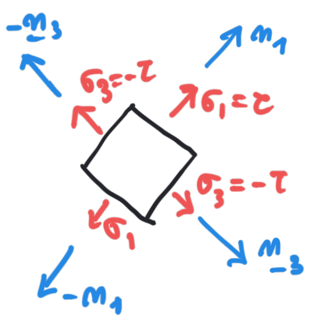
$\underline{n}_1 = \frac{1}{\sqrt{2}}\,\underline{i} + \frac{1}{\sqrt{2}}\,\underline{j}$

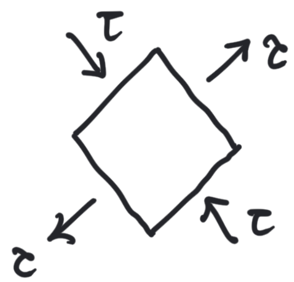
$\underline{n}_2 = \underline{k}$

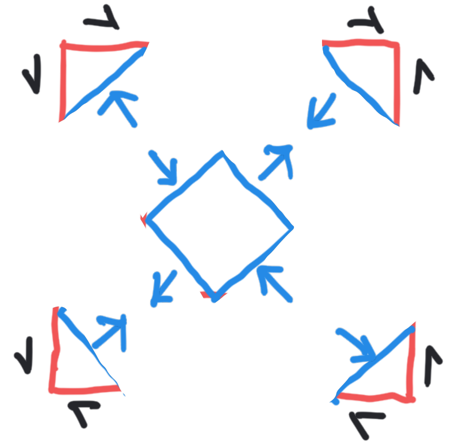
$\underline{n}_3 = \frac{1}{\sqrt{2}}\,\underline{i} - \frac{1}{\sqrt{2}}\,\underline{j}$











Stati tensionali

- stato triassiale $\sigma_1 > \sigma_2 > \sigma_3$
- stato cilindrico $\sigma_1 = \sigma_2 \neq \sigma_3$
- stato sferico $\sigma_1 = \sigma_2 = \sigma_3$

Arbelo di Mohr

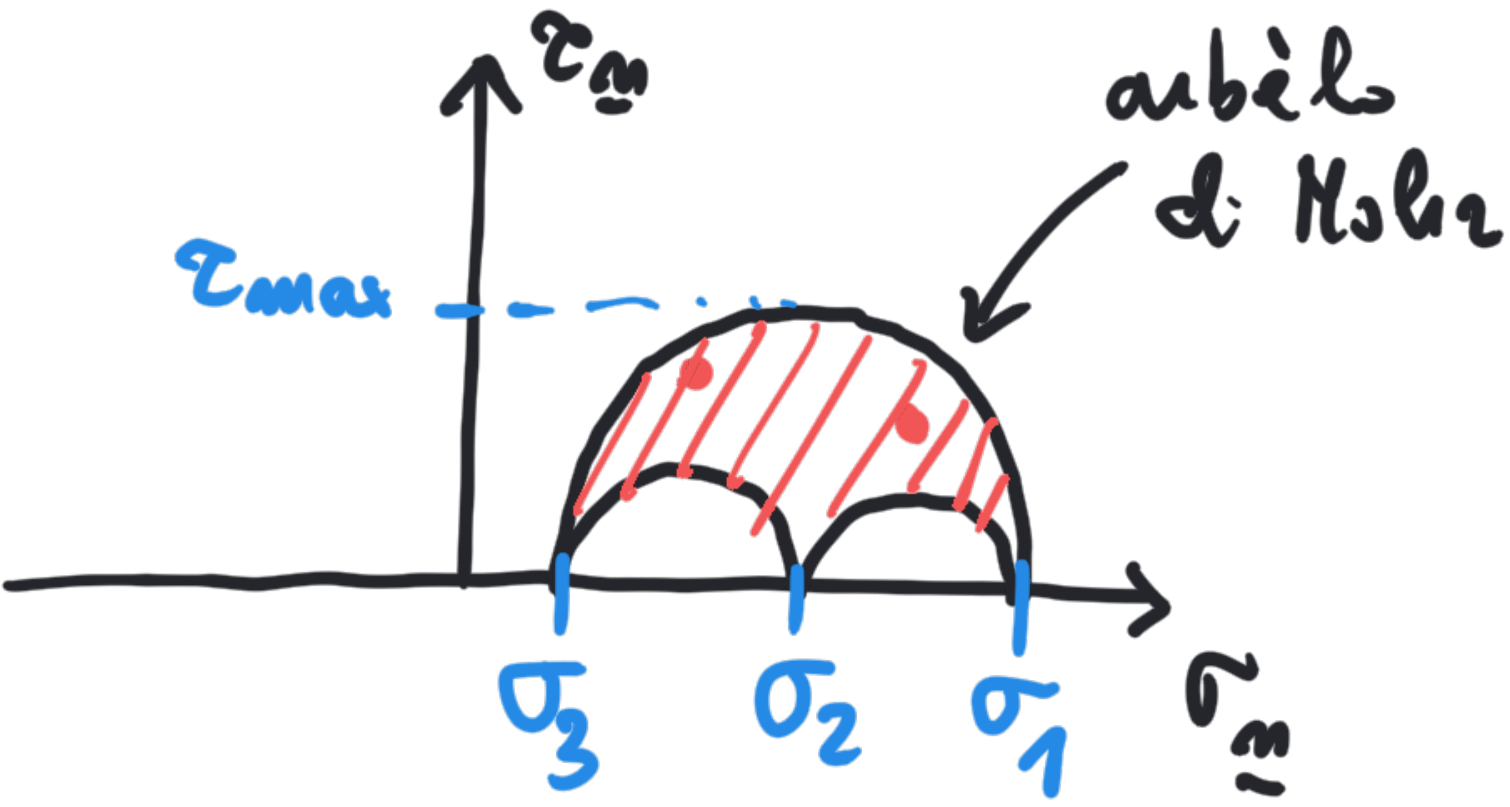
Problema: massimo tensione tang.

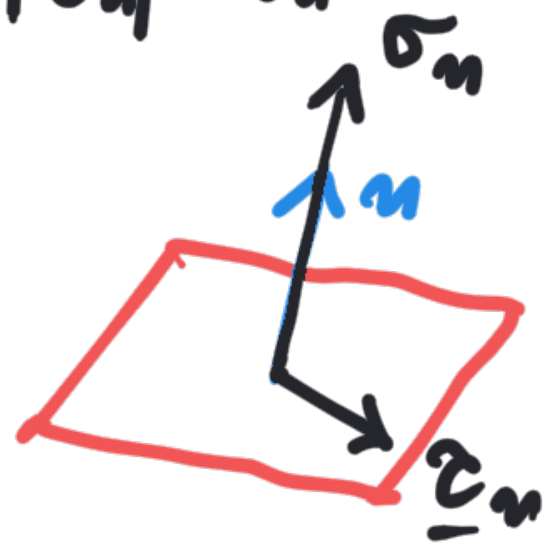
$$\sigma_n = \underline{n} \cdot \underline{t}_n = \underline{n} \cdot \underline{\underline{T}}\,\underline{n}$$

tensione normale

$$0 \leq \tau_n = |\underline{\tau}_n| = \sqrt{|t_n|^2 - \sigma_n^2}$$

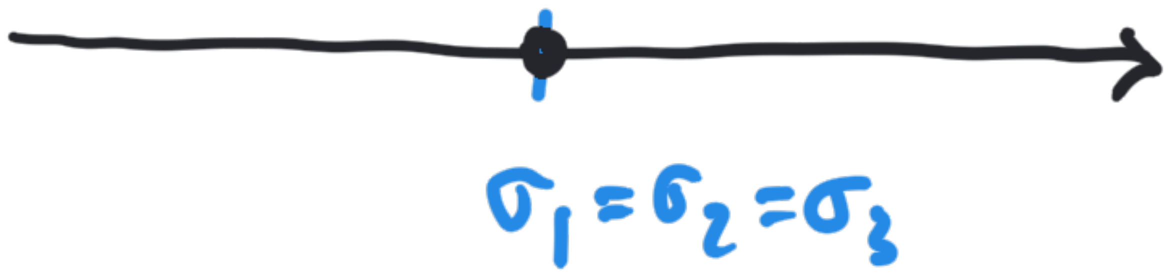
intensità tensione tang.

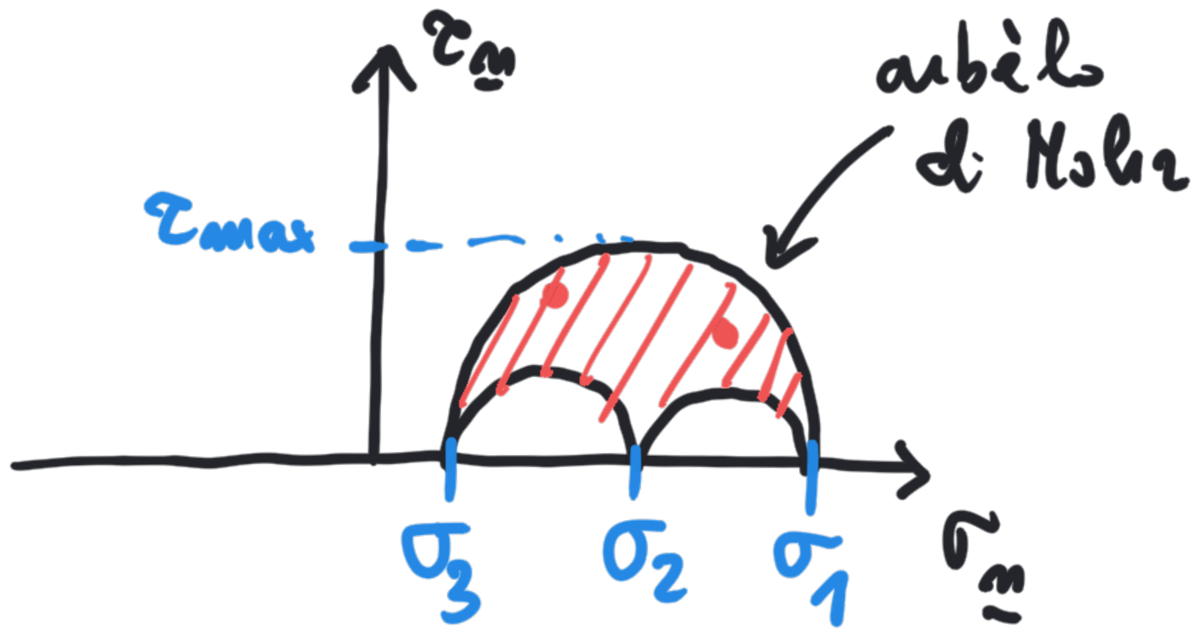




$$\tau_{max} = \frac{1}{2} |\sigma_1 - \sigma_3|$$

Stato sferico





stato cilindrico