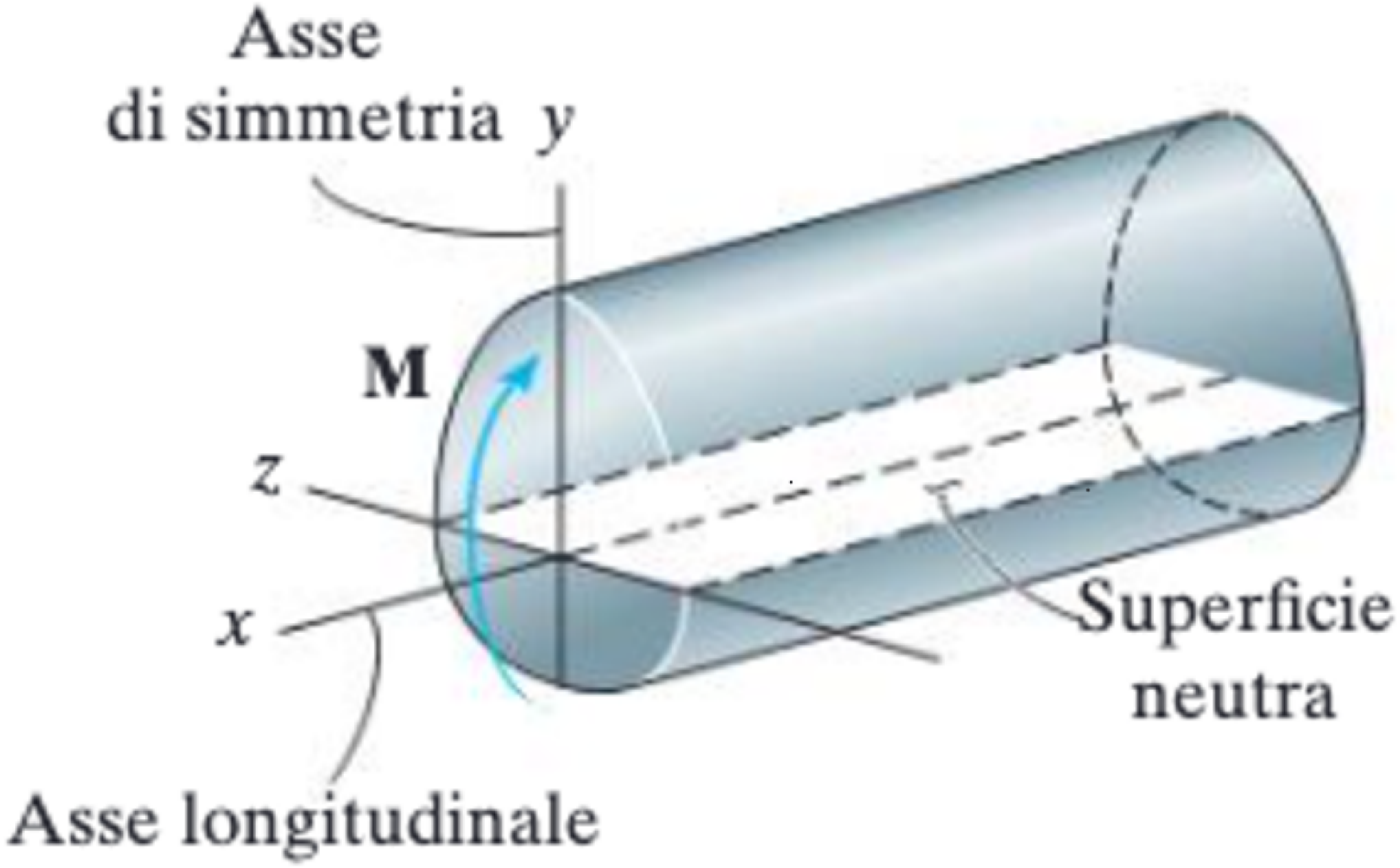
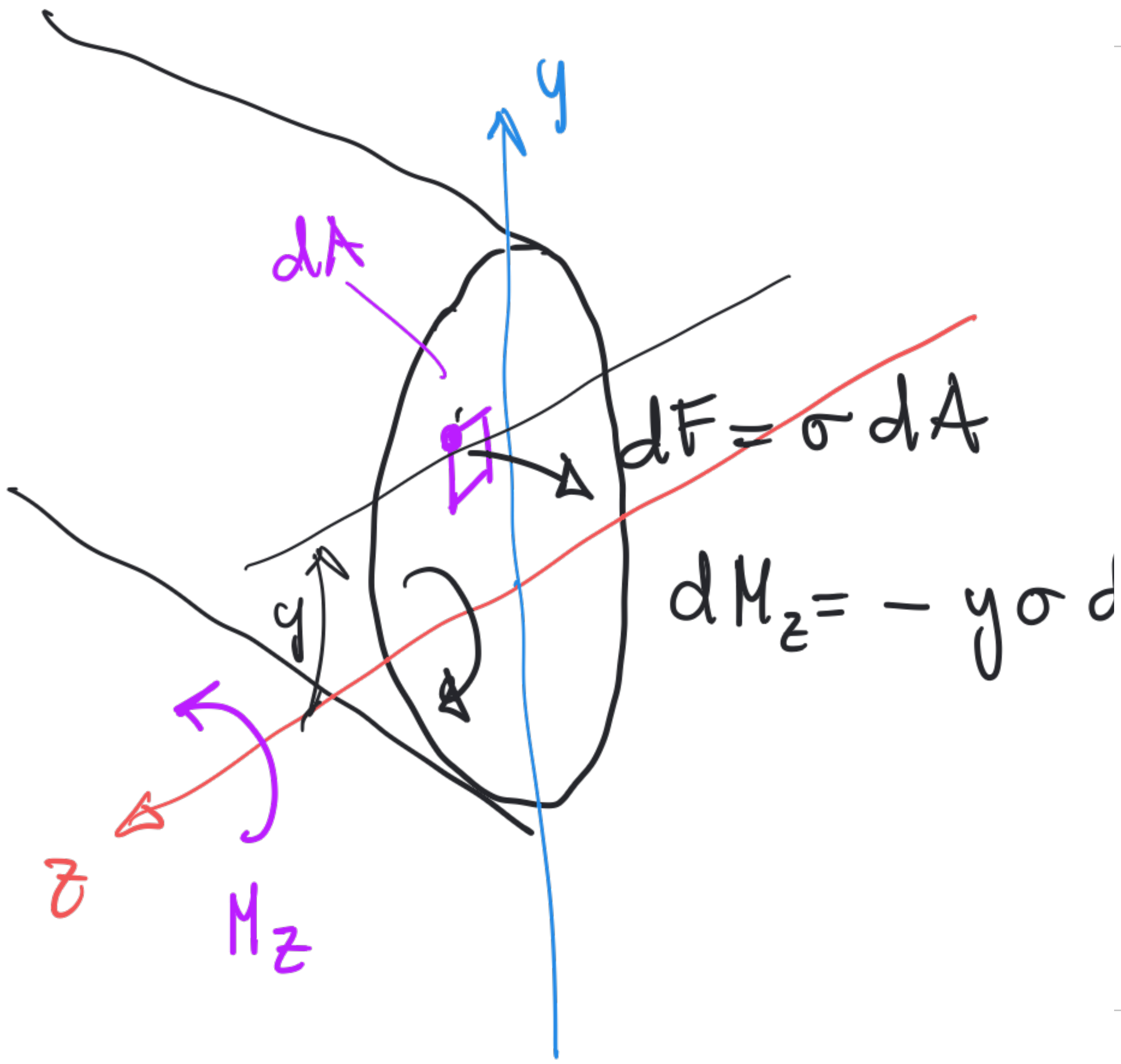
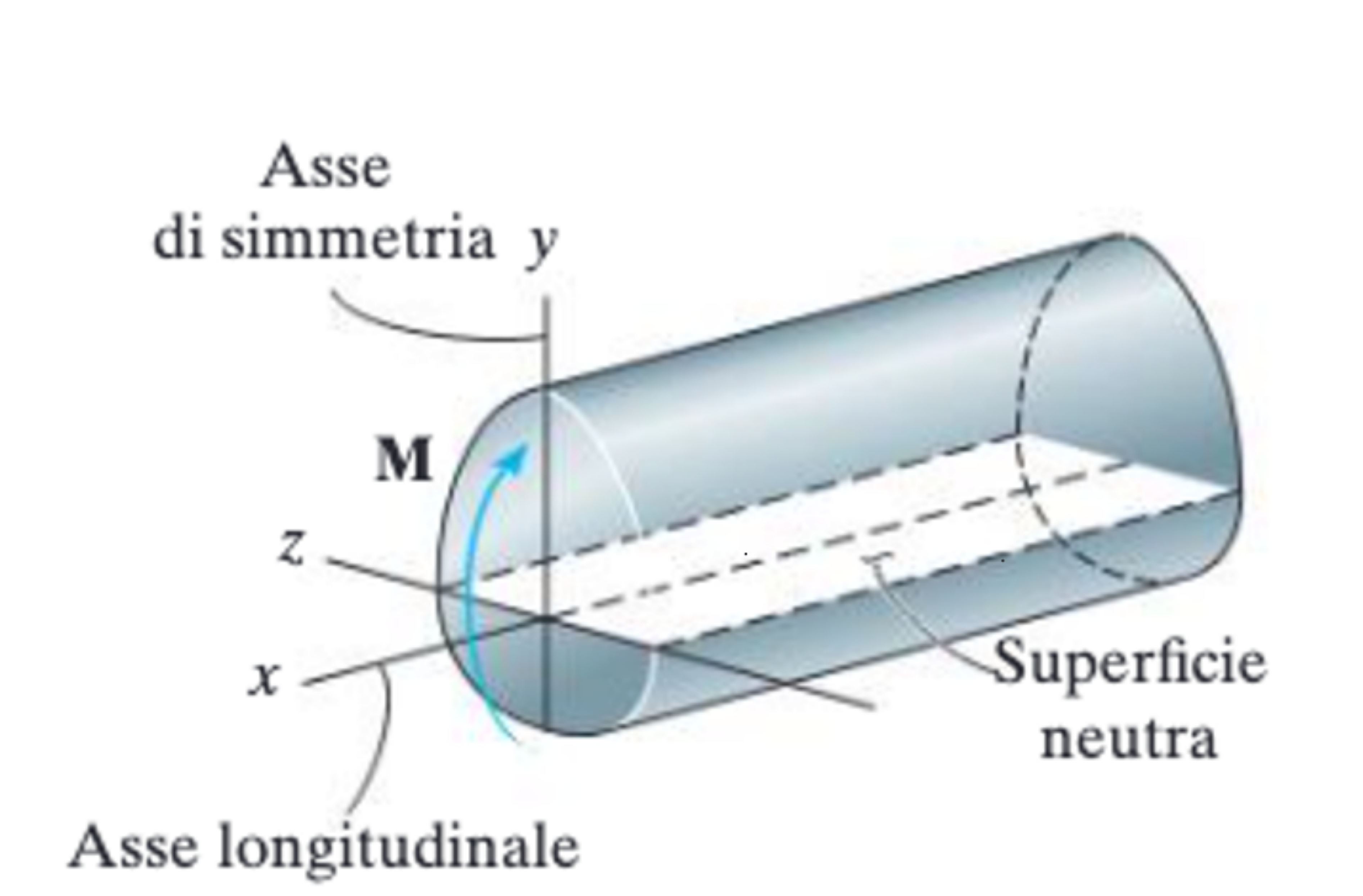
DEFORMAZIONI NELLE TRAVI INFLESSE

SISTEMA DI RIFERIMENTO







$$M_z = -\int_A y\sigma \, dA \qquad M_y = \int_A z\sigma \, dA$$

RELAZIONE MOMENTO - CURVATURA

Dobbiamo rivedere le relazioni che intercorrono tra tensioni e caratteristiche della sollecitazione

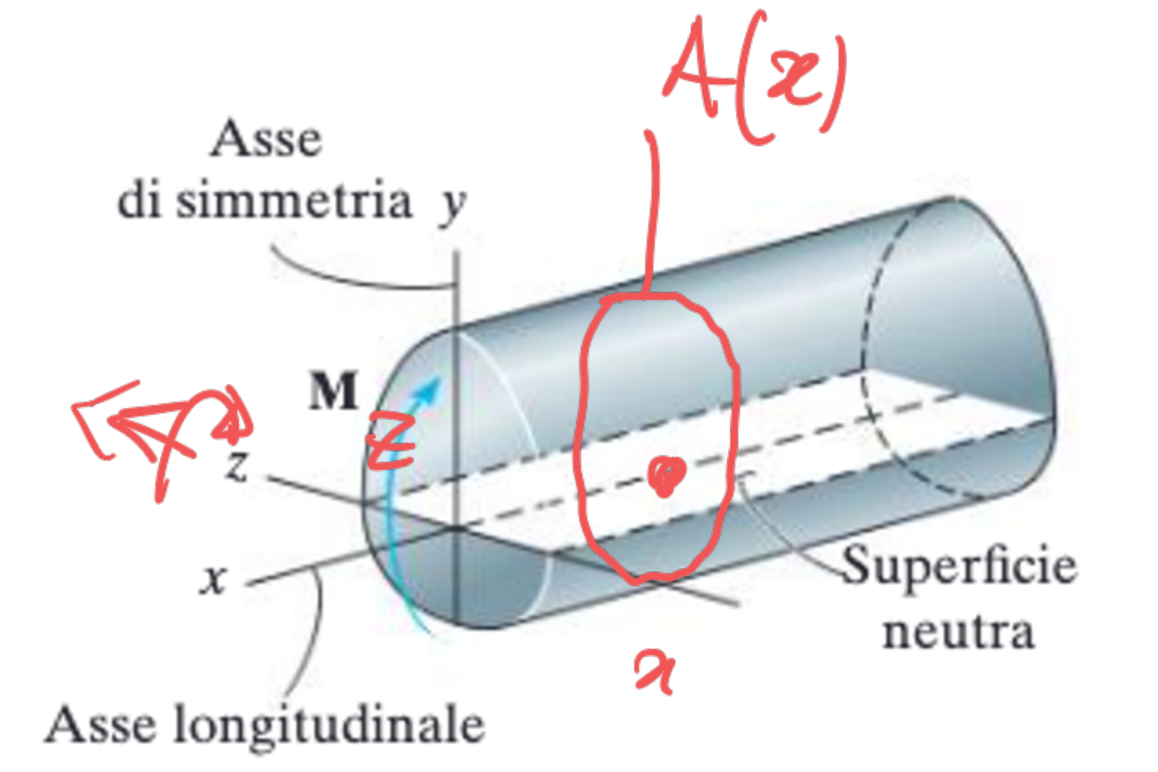
$$N(x) = \int_{A(x)} \sigma(x,y,z) \underbrace{dy\,dz}_{dA}$$

$$M_z(x) = \int_{A(x)} -y\,\sigma(x,y,z)\,dy\,dz$$

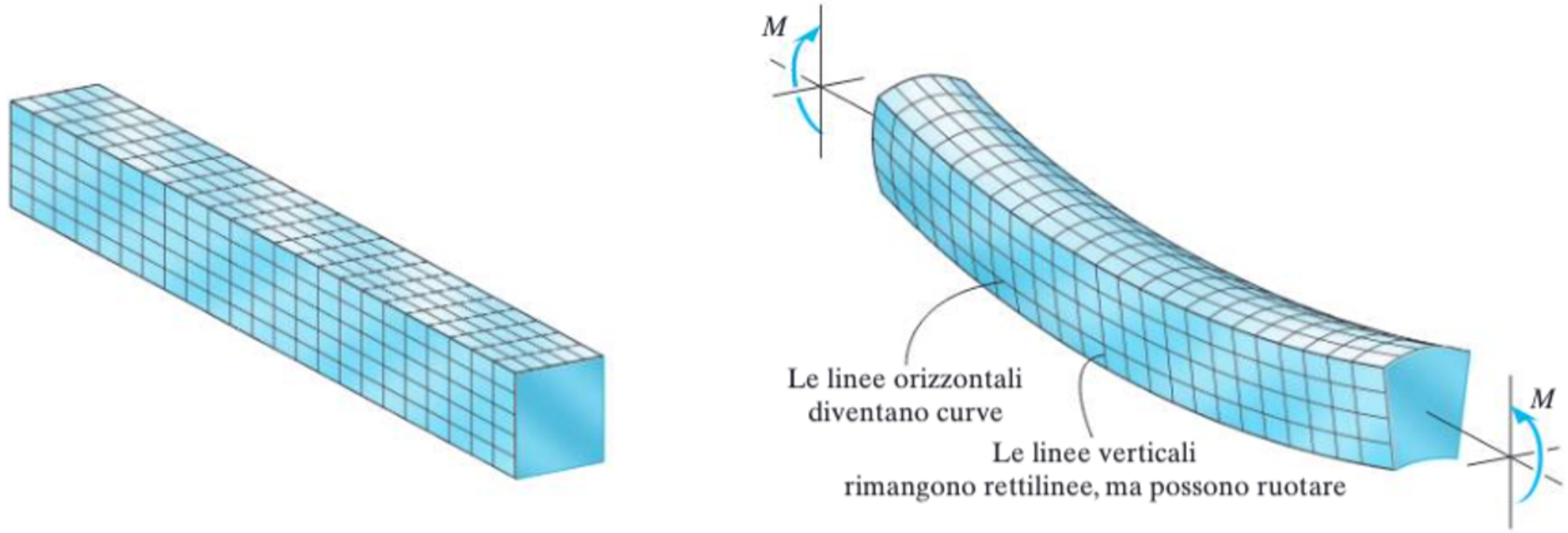
$$M_y(x) = \int_{A(x)} z\,\sigma(x,y,z)\,dy\,dz$$

oss: nel caso $\sigma(x,y,z) = \sigma(x)$

$N(x) = \sigma(x)\,A(x$



DEFORMAZIONI NELLA TRAVE INFLESSA



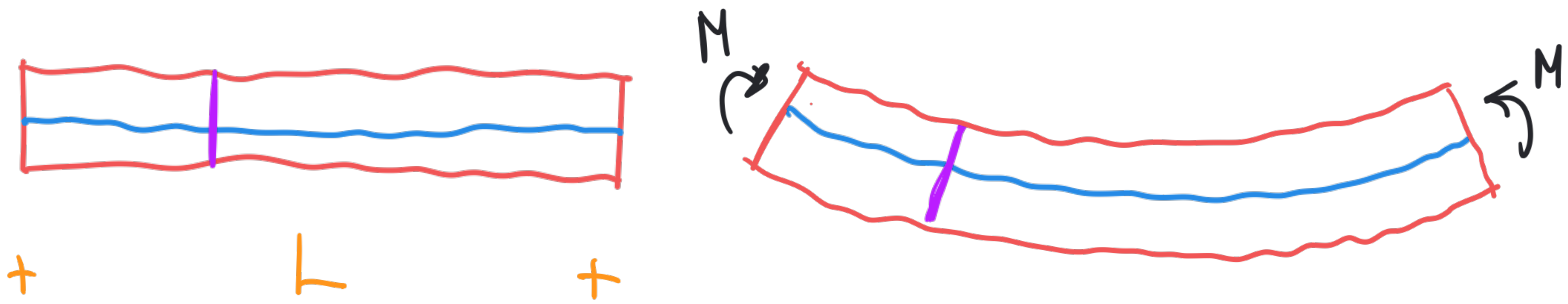
Prima della deformazione

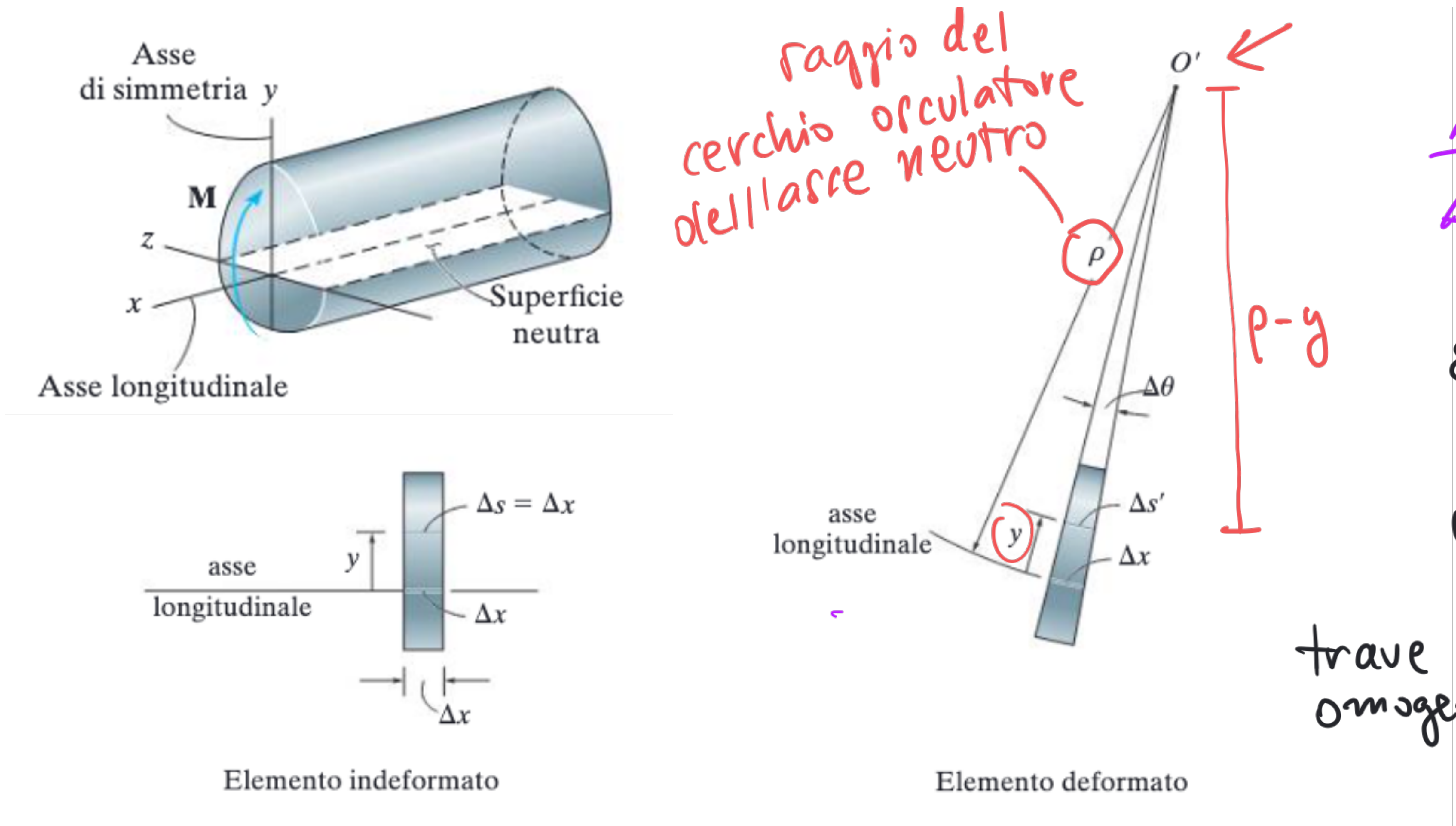
(a)

Dopo la deformazione

(b)

- I segmenti longitudinali della trave diventano archi di circonferenza
- le sezioni rimangono piane
- La dilatazione delle fibre disposte sull'asse x è nulla.



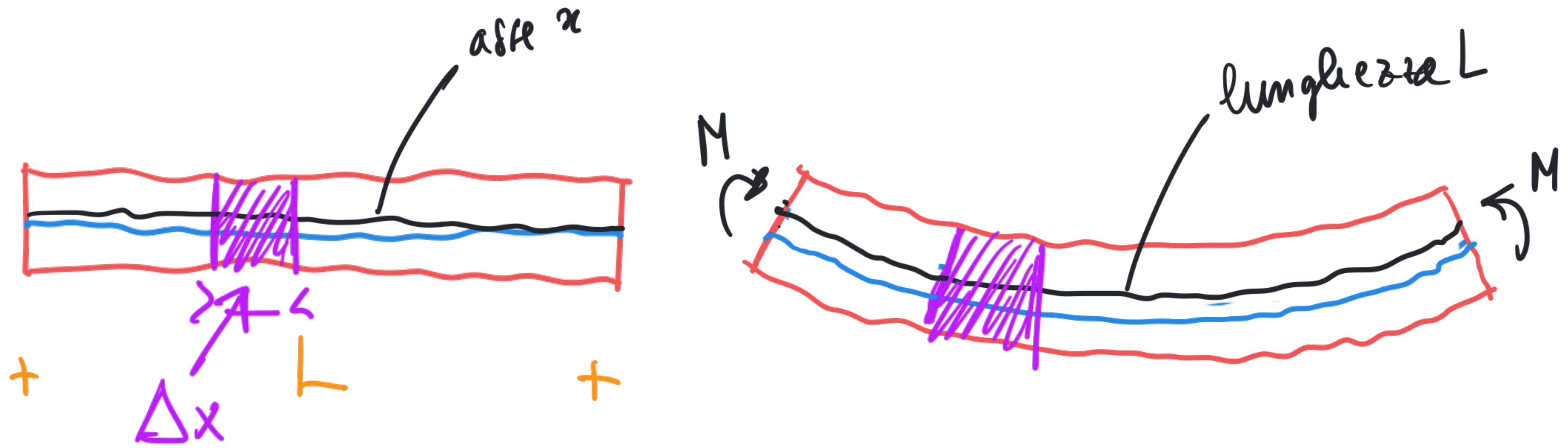


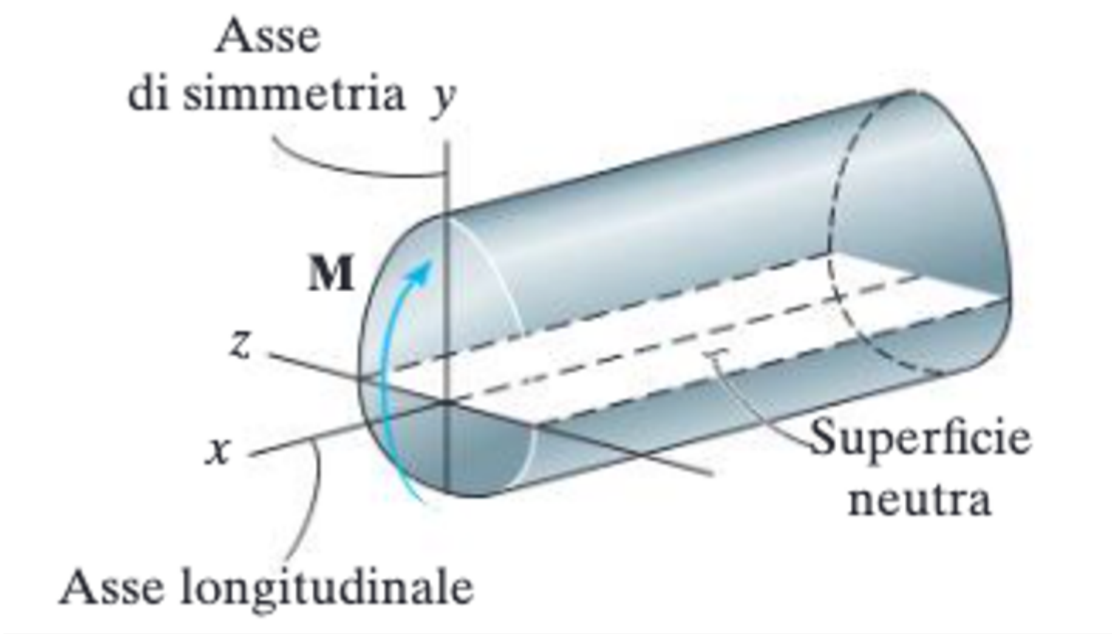
$$\frac{\Delta s'}{\Delta s} = \frac{\Delta s'}{\Delta x} = \frac{\rho - y}{\rho} = 1 - \frac{y}{\rho}$$

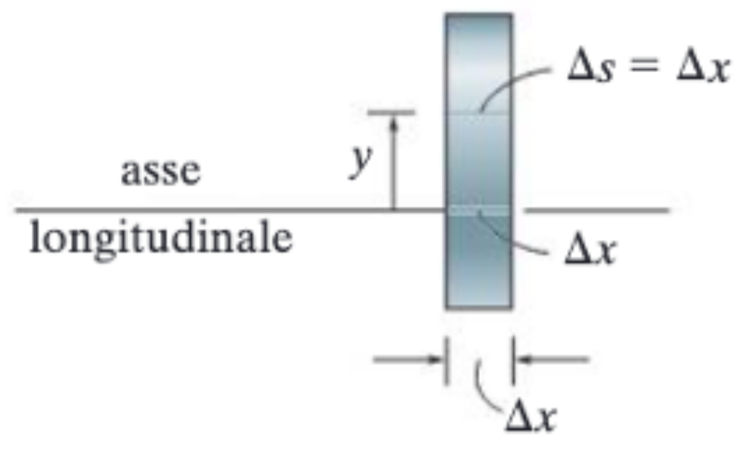
$$\varepsilon(y) = \frac{\Delta s' - \Delta s}{\Delta s} = \frac{\Delta s'}{\Delta s} - 1 = -\frac{y}{\rho}$$

$$\sigma(y) = E\,\varepsilon(y) = -E\, y/\rho$$

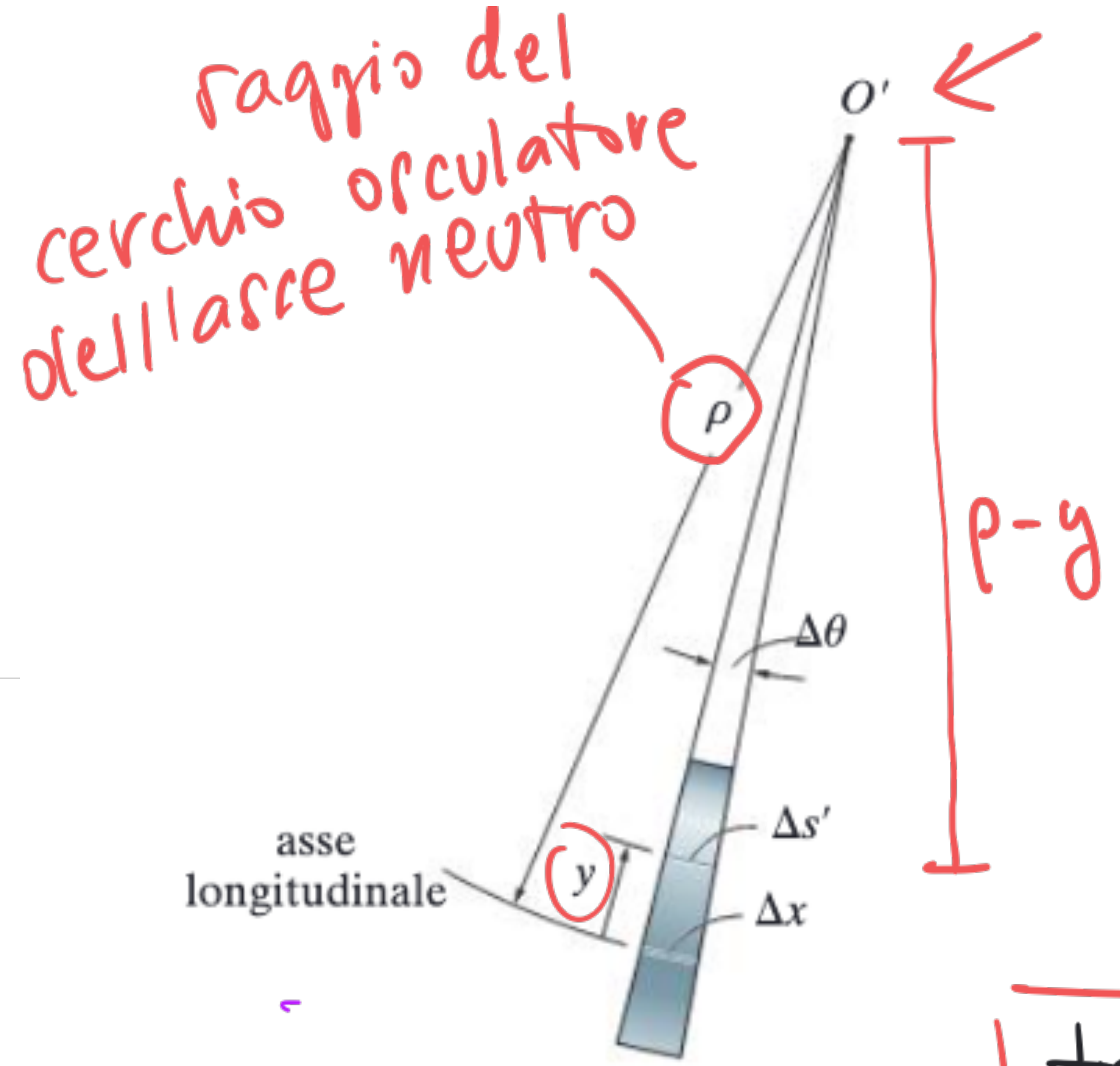
trave omogenea







Elemento indeformato



Elemento deformato

$$\frac{\Delta s'}{\Delta s} = \frac{\Delta s'}{\Delta x} = \frac{\rho - y}{\rho} = 1 - \frac{y}{\rho}$$

$$\varepsilon(y) = \frac{\Delta s' - \Delta s}{\Delta s} = \frac{\Delta s'}{\Delta s} - 1 = -\frac{y}{\rho}$$

$$\sigma(y) = \boxed{E}\,\varepsilon(y) = -E\,y/\rho$$

trave omogenea

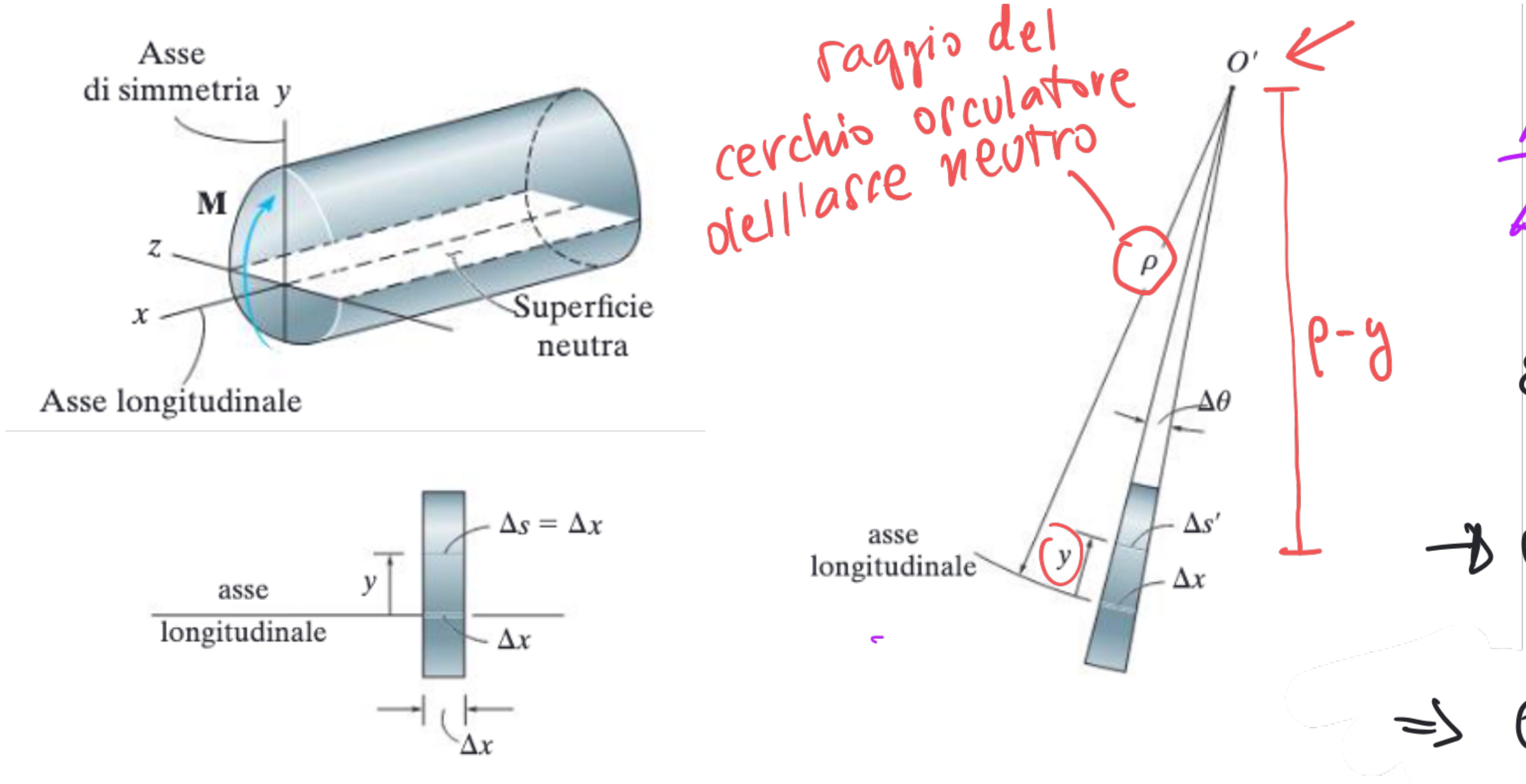
$$M_z = \int_A -y\sigma\, dA = \frac{E}{\rho}\int_A y^2\, dA = \frac{EI}{\rho}$$

$$I = I_z = \int_A y^2\, dA$$

$$\int_A y\, dA = A\, y_G = 0$$

y asse di simmetria

$$N = \int_A \sigma\, dA = -\frac{E}{\rho}\int_A y\, dA = 0 \qquad M_y = \int_A z\sigma\, dA = -\frac{E}{\rho}\overbrace{\int_A zy\, dA}^{0}$$



Elemento indeformato

Elemento deformato

$$\frac{\Delta s'}{\Delta s} = \frac{\Delta s'}{\Delta x} = \frac{\rho - y}{\rho} = 1 - \frac{y}{\rho}$$

$$\varepsilon(y) = \frac{\Delta s' - \Delta s}{\Delta s} = \frac{\Delta s'}{\Delta s} - 1 = -\frac{y}{\rho}$$

$$\rightarrow \sigma(y) = E\,\varepsilon(y) = -E\,y/\rho$$

$$\Rightarrow \sigma(y) = -\frac{M_z y}{I_z}$$

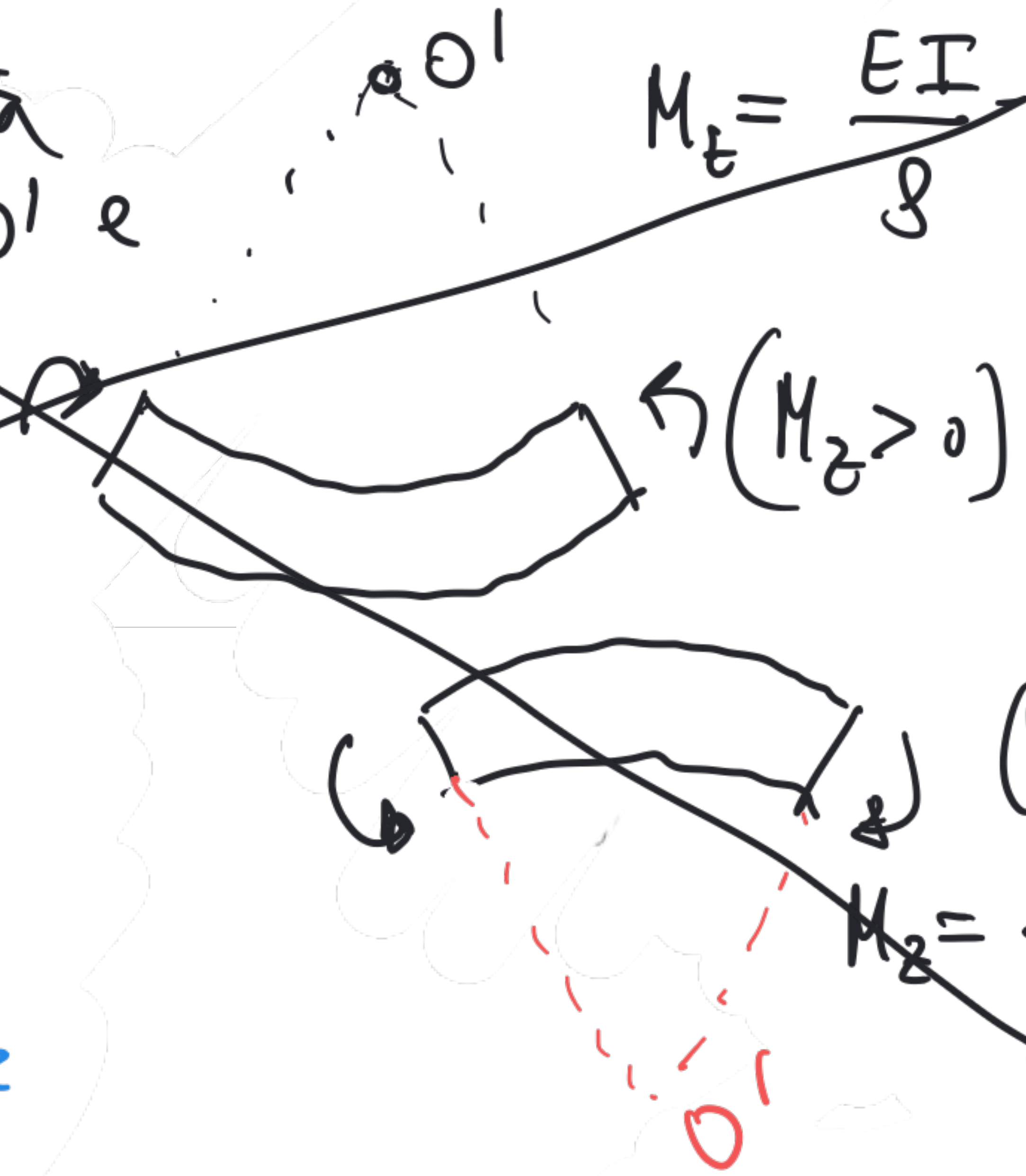
RISULTA DIMOSTRATA LA FORMULA DELLA FLESSIONE

$$I = I_z = \int_A y^2\,dA$$

$$(*)\quad M_z = \int_A -y\sigma\,dA = \frac{E}{\rho}\int_A y^2\,dA = \frac{EI}{\rho}$$

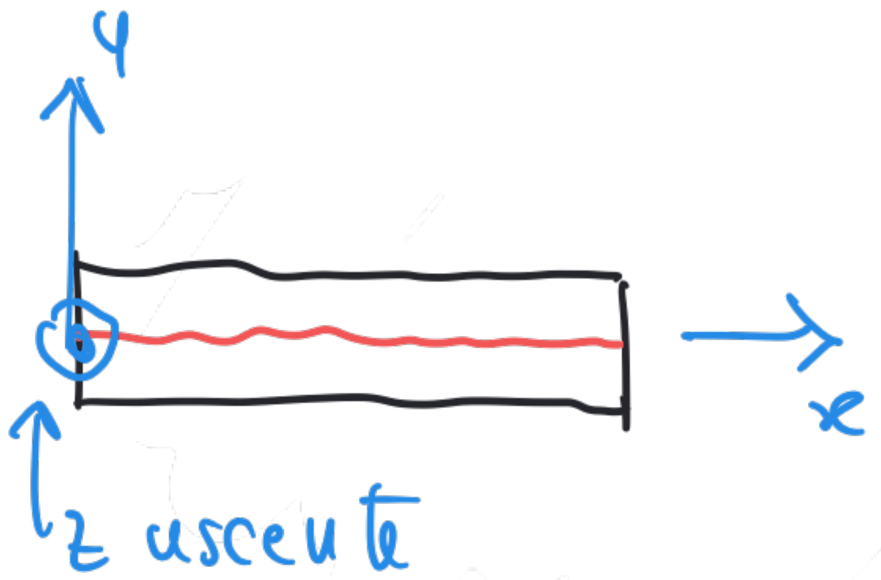
Se M_z e' noto, allora dalla $(*)$ $\Rightarrow \dfrac{1}{\rho} = \dfrac{M_z}{EI_z}$

~~Se ρ rappresenta la distanza tra O' e l'asse neutro.~~



~~$M_z = \frac{EI}{\rho}$~~

~~$M_z = -\frac{EI}{\rho}$~~



Curvature con segno

$$k = 1/\rho$$

$$M_z = EIk$$

Convenzione alternativa

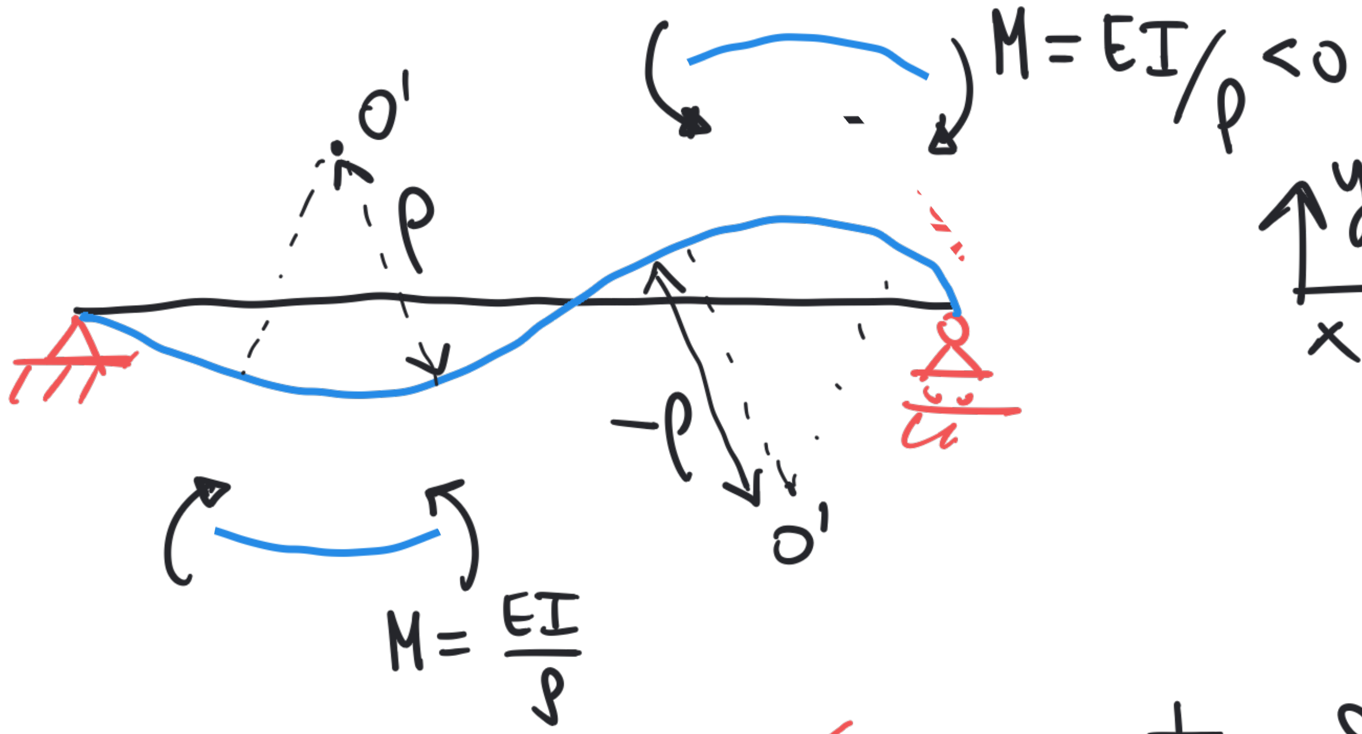
$\rho > 0$ se O' si trova sopra l'asse neutro

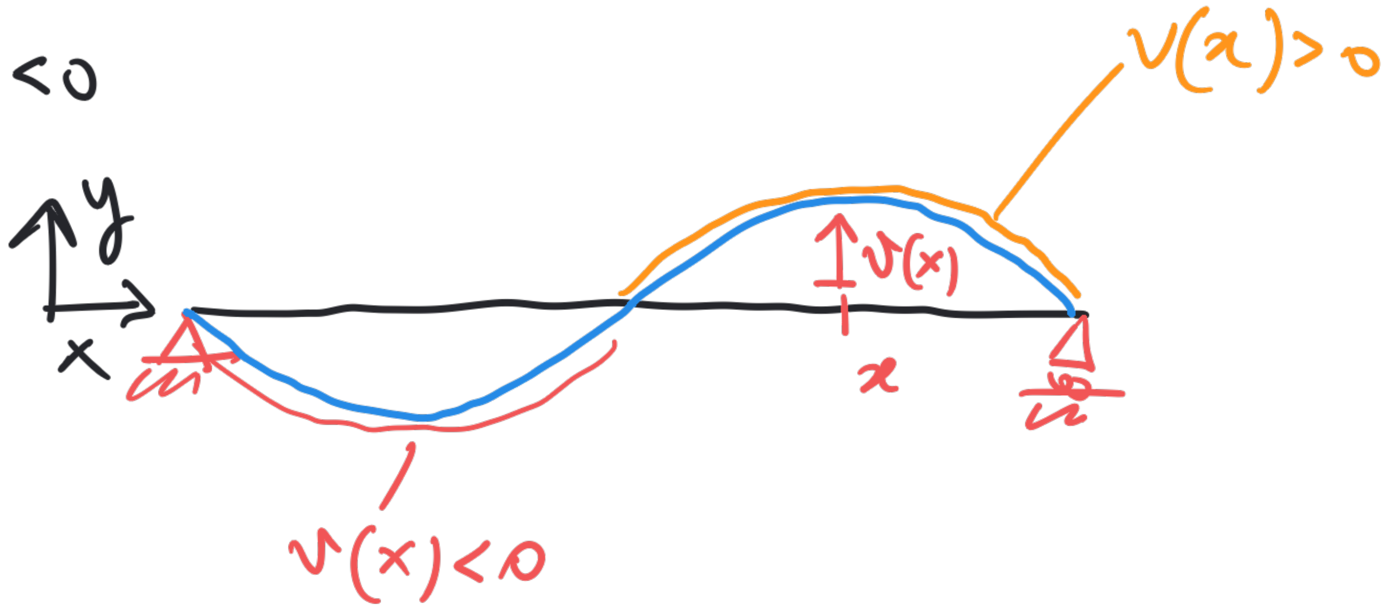
$\rho < 0$ se O' si trova sotto.

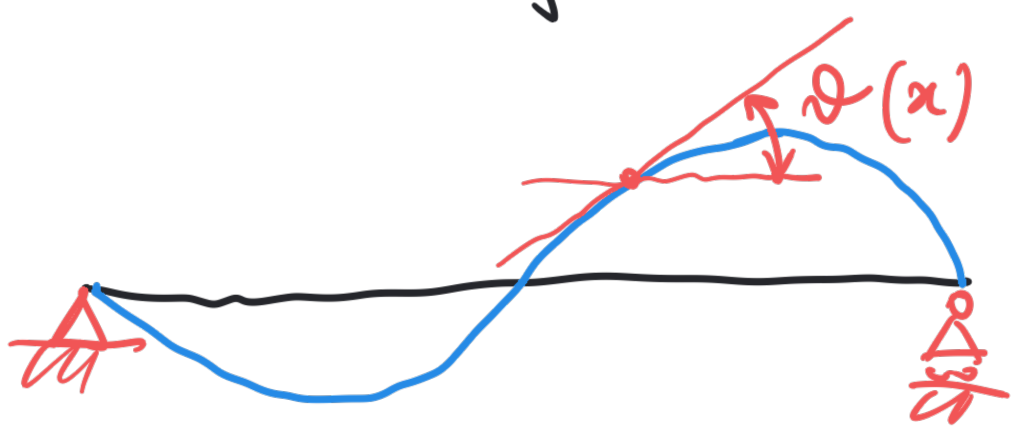
$$M_z = \frac{EI}{\rho}$$

Caso in cui E non sia costante sulla sezione

- Attenzione: se il modulo di Young non e’ costante sulla sezione non e’ piu’ vero che l’asse neutro e’ quello baricentrico.
- In tal caso, occorrera’ individuare un “baricentro equivalente”, ottenuto pesando l’area con il modulo di Young.



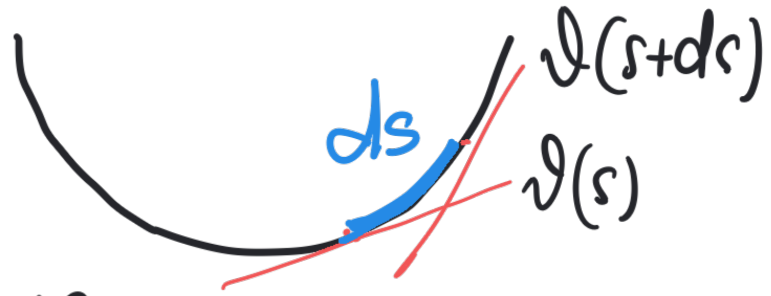


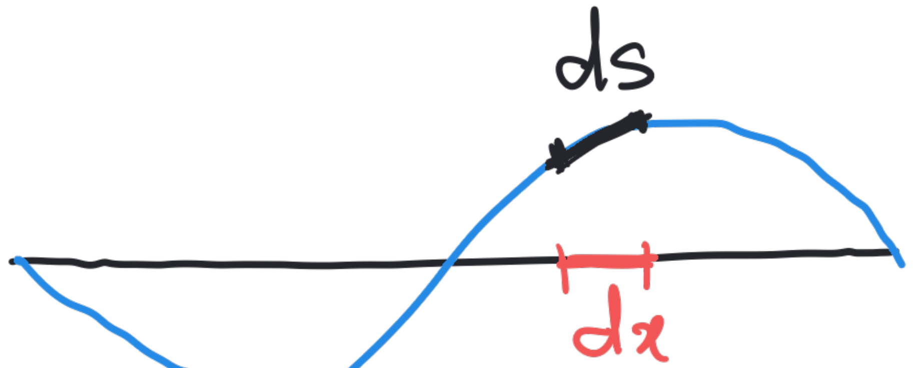


$$\tan \vartheta = \frac{dv}{dx}$$

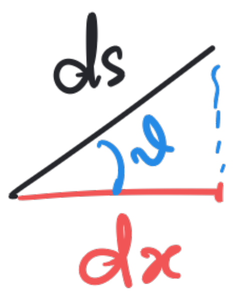
$$|\vartheta| \ll 1 \Rightarrow \vartheta \simeq \frac{dv}{dx}$$

$$1/\rho = k = \frac{d\vartheta}{ds} \simeq \frac{d\vartheta}{dx} = \frac{d^2 v}{dx^2}$$



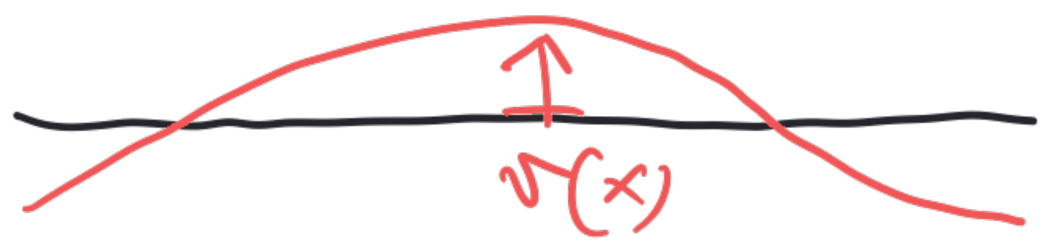


$$\boxed{M = \frac{EI}{\rho} = EI \frac{d^2 v}{dx^2}}$$



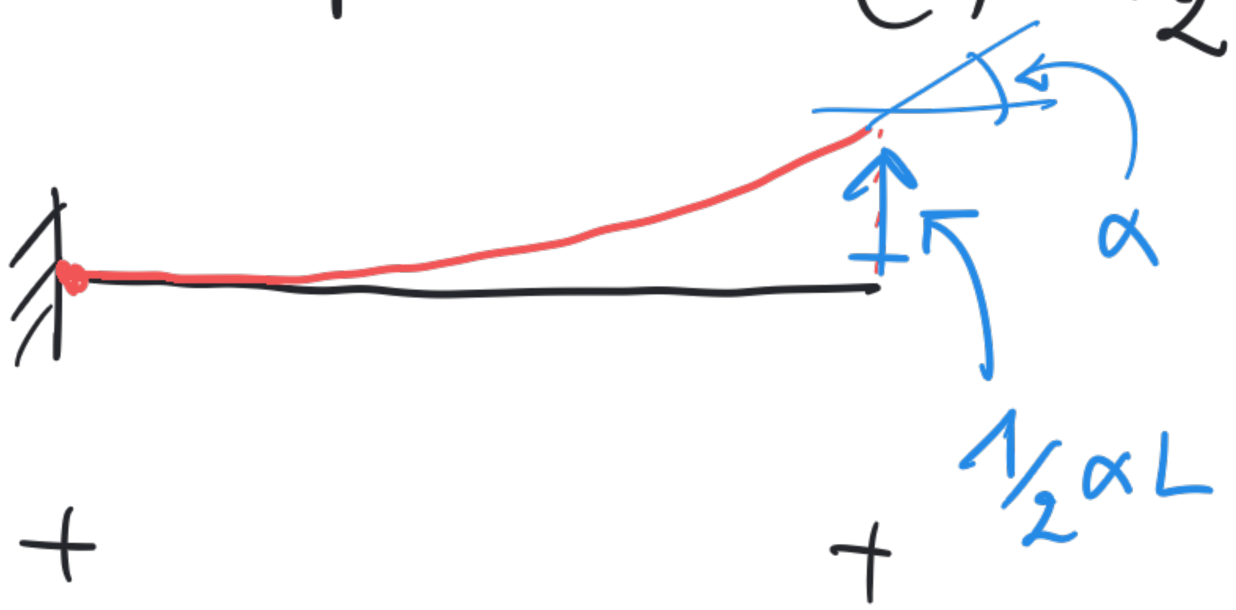
$$ds = \sqrt{dx^2 + \vartheta^2 dx^2}$$
$$= dx\sqrt{1+\vartheta^2} \simeq dx \quad \text{se } |\vartheta| \ll 1$$

$$M = \frac{EI}{\rho} = EI\,k = EI\,\frac{d^2v}{dx^2}$$



$$\frac{1}{\rho} = k = \frac{d\vartheta}{dx} = \frac{d^2v}{dx^2}$$

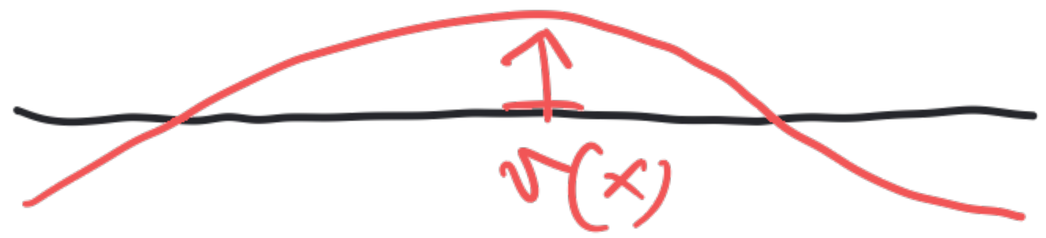
Esempio: $v(x) = \frac{1}{2}\alpha\,\frac{x^2}{L}$ $\qquad \vartheta(x) = \alpha\,\frac{x}{L}$ $\qquad \left(\alpha = \vartheta(L)\right)$



$$\vartheta(0) = 0$$
$$v(0) = 0$$

$k(x) = \alpha/L$ <u>costante</u>

Problema: dato k calcolare v



$$\frac{1}{\rho} = k = \frac{d\vartheta}{dx} = \frac{d^2 v}{dx^2}$$

__NB__: i vincoli permettono di ricostruire gli spostamenti note che siano le deformazioni.

Esempio:

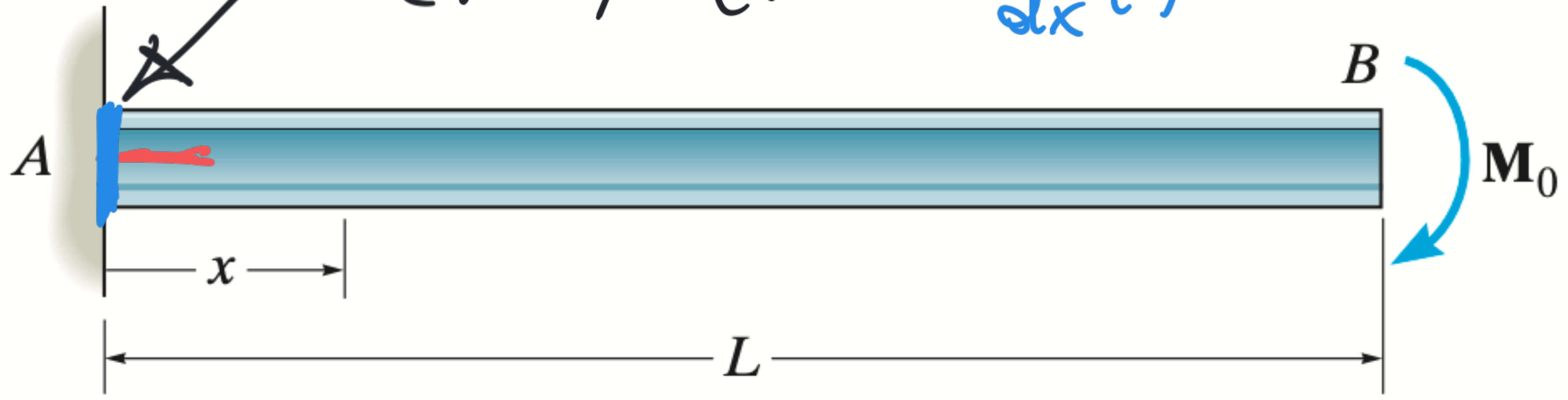
$$\frac{d^2 v}{dx^2} = \frac{\alpha}{L} + \frac{\beta x}{L^2} \Rightarrow v(x) = \frac{\alpha}{L}x + \frac{\beta x^2}{2L} + Cx + D$$

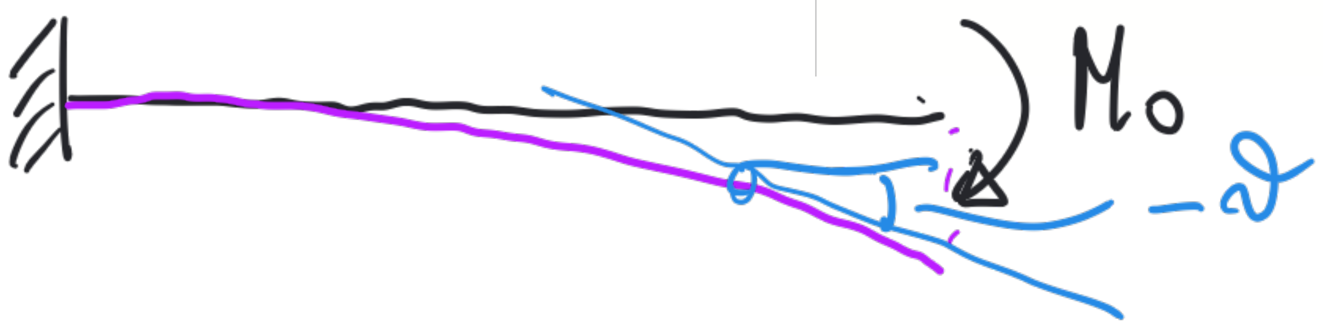
$$\begin{aligned} v(0) &= 0 \\ \vartheta(0) &= 0 \end{aligned} \Rightarrow C = 0,\ D = 0$$

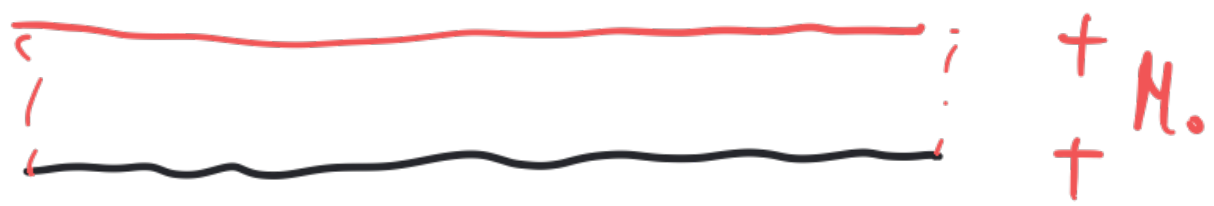
$$k(x) = \alpha/L + \beta x/L^2$$

ESEMPIO

$v(0)=0\,,\ \vartheta(0)=0 \qquad \frac{dv}{dx}(0)=0$







$(M<0)$

$$(*) \Rightarrow \quad v(x) = -\frac{1}{2}\frac{M_0}{EI}x^2 + Cx + D$$

$$v(0)=0 \Rightarrow D=0$$

$$\frac{dv}{dx}(0)=0 \Rightarrow C=0$$

$$M = EI\frac{d^2v}{dx}$$

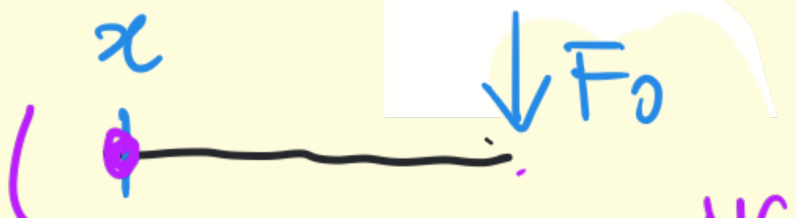
$$\frac{d^2v}{dx} = \frac{M}{EI} = -\frac{M_0}{EI} \quad (*)$$

$$v(x) = -\frac{1}{2}\frac{M_0}{EI}x^2$$

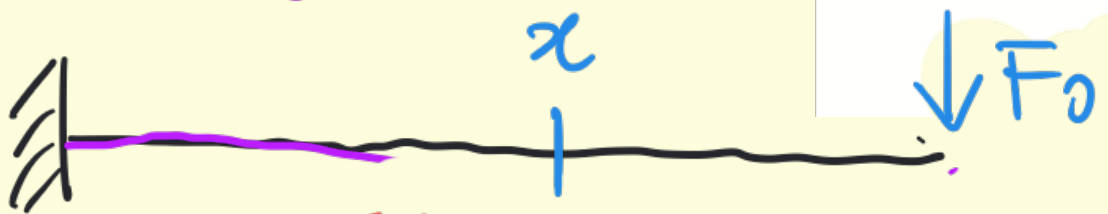
$$v(L) = -\frac{1}{2}\frac{M_0 L^2}{EI}$$

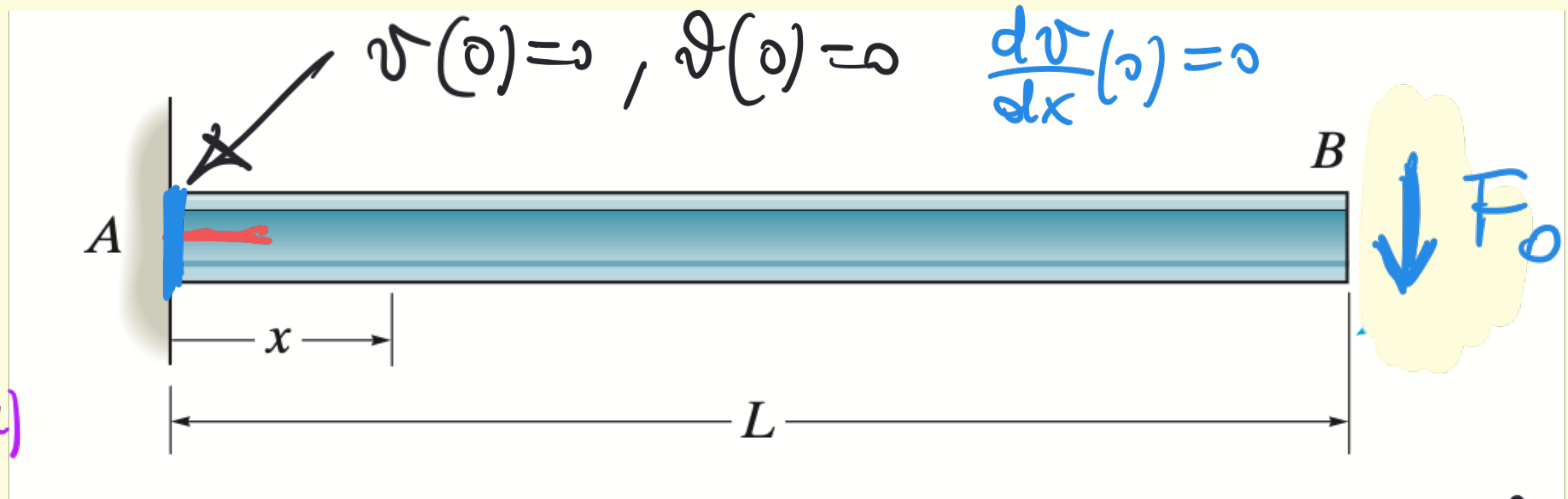
$$\vartheta(x) = \frac{dv}{dx} = -\frac{M_0}{EI}x$$

ESEMPIO



$$F_0(L-x) = -M(x)$$





$$v(0)=0\ ,\ \vartheta(0)=0 \qquad \frac{dv}{dx}(0)=0$$

$M<0$, $M=-F_0L$

$-F_0(L-x)$

$+F_0 L$ $(M<0)$ +

$$(x) \Rightarrow v(x) = -\frac{1}{2}\frac{M_0}{EI}x^2 + Cx + D$$

$$v(0)=0 \Rightarrow D=0$$

$$\frac{dv}{dx}(0)=0 \Rightarrow C=0$$

$$M = EI\frac{d^2v}{dx}$$

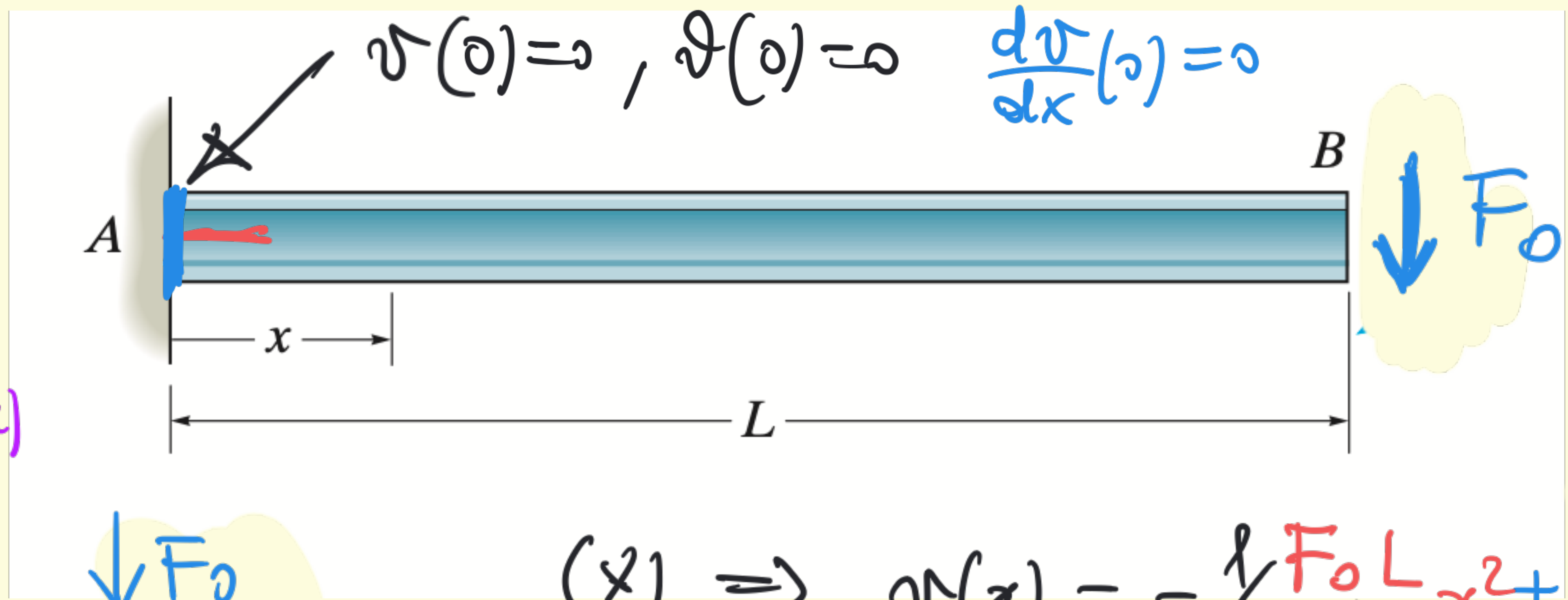
$$\boxed{\frac{d^2v}{dx} = \frac{M}{EI} = -\frac{F_0(L-x)}{EI} = -\frac{F_0L}{EI} + \frac{F_0}{EI}x}$$

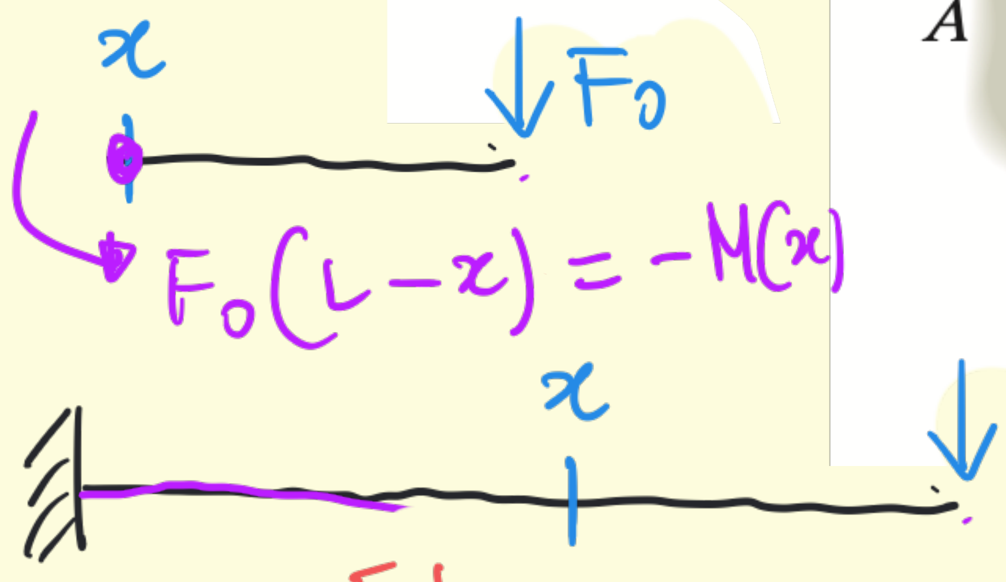
$$v(x) = -\frac{1}{2}\frac{M_0}{EI}x^2$$

$$v(L) = -\frac{1}{2}\frac{M_0L^2}{EI}$$

ESEMPIO

$v(0)=0$, $\vartheta(0)=0$ $\quad \frac{dv}{dx}(0)=0$





$F_0(L-x) = -M(x)$

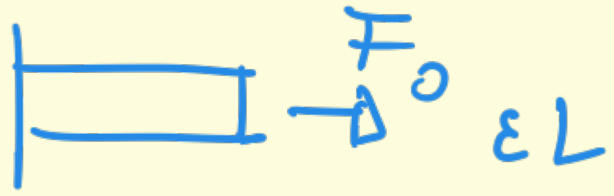
$M<0$, $M=-F_0L$

$-F_0(L-x)$

$+F_0L \quad (M<0)$

$$M = EI \frac{d^2v}{dx}$$

$$\frac{d^2v}{dx} = -\frac{F_0L}{EI} + \frac{F_0x}{EI}$$



$$\delta = \frac{F_0L}{EA} = \frac{\sigma}{E}L$$

$$(x) \Rightarrow v(x) = -\frac{1}{2}\frac{F_0L}{EI}x^2 + \frac{1}{6}\frac{F_0x}{EI}x^2 + Cx + D$$

$$v(0)=0 \Rightarrow D=0$$

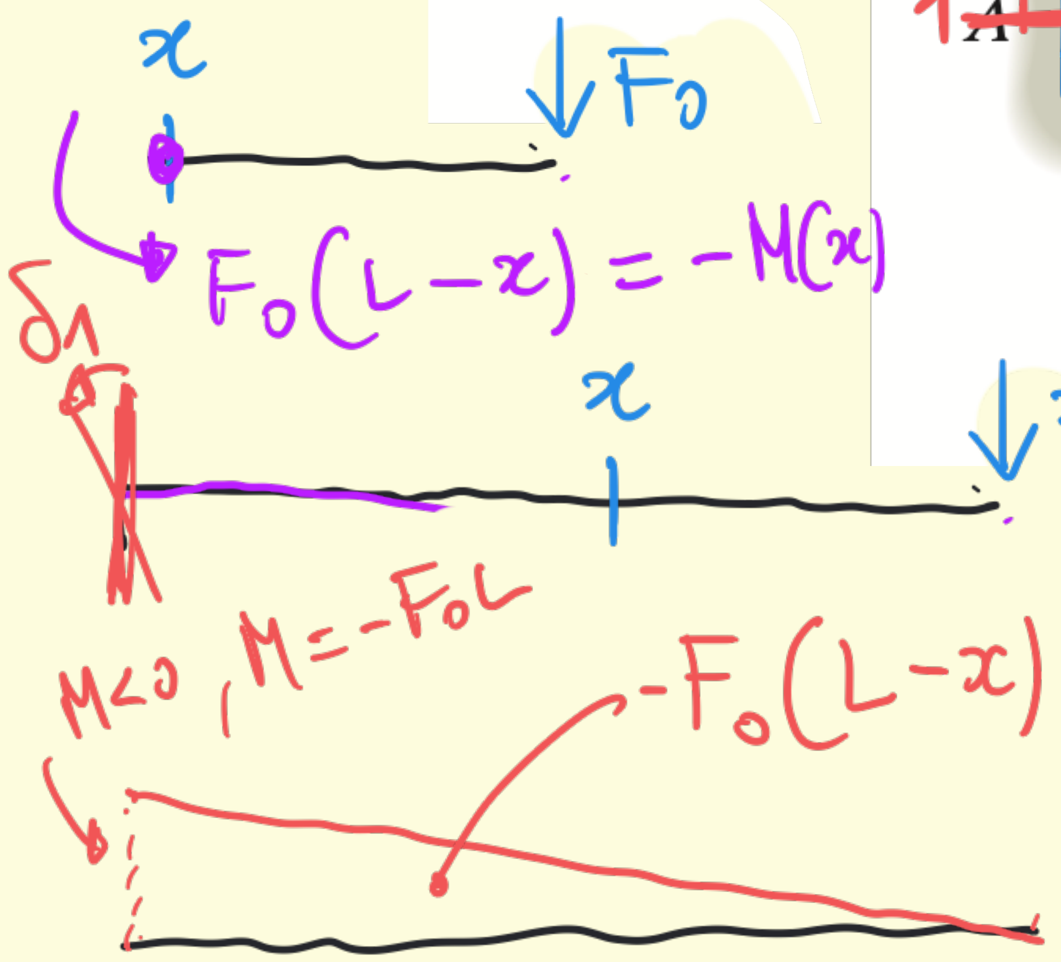
$$\frac{dv}{dx}(0)=0 \Rightarrow C=0$$

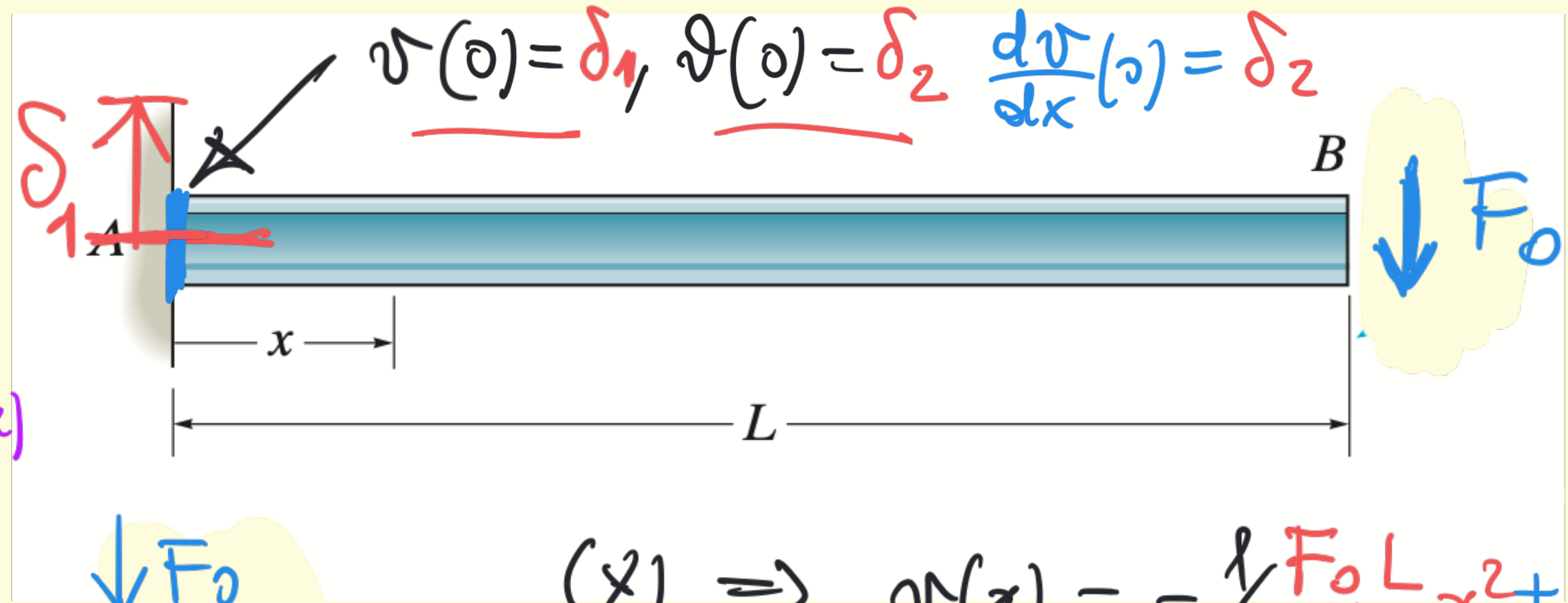
$$v(x) = -\frac{1}{2}\frac{F_0L}{EI}x^2 + \frac{1}{6}\frac{F_0x}{EI}x^2$$

$$v(L) = -\frac{1}{3}F_0L^3/EI$$

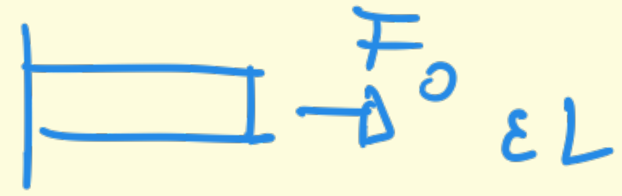
$$\vartheta(x) = \frac{F_0L}{EI}x + \frac{1}{2}\frac{F_0x^2}{EI}$$

CEDIMENTO





$v(0)=\delta_1$, $\vartheta(0)=\delta_2$ $\quad \frac{dv}{dx}(0)=\delta_2$



$$\delta = \frac{F_0 L}{EA} = \frac{\sigma}{E}L$$

$$M = EI\,\frac{d^2 v}{dx}$$

$$\frac{d^2 v}{dx} = -\frac{F_0 L}{EI} + \frac{F_0 x}{EI}$$

$$(*) \Rightarrow \quad v(x) = -\frac{1}{2}\frac{F_0 L}{EI}x^2 + \frac{1}{6}\frac{F_0 x}{EI}x^2 + Cx + D$$

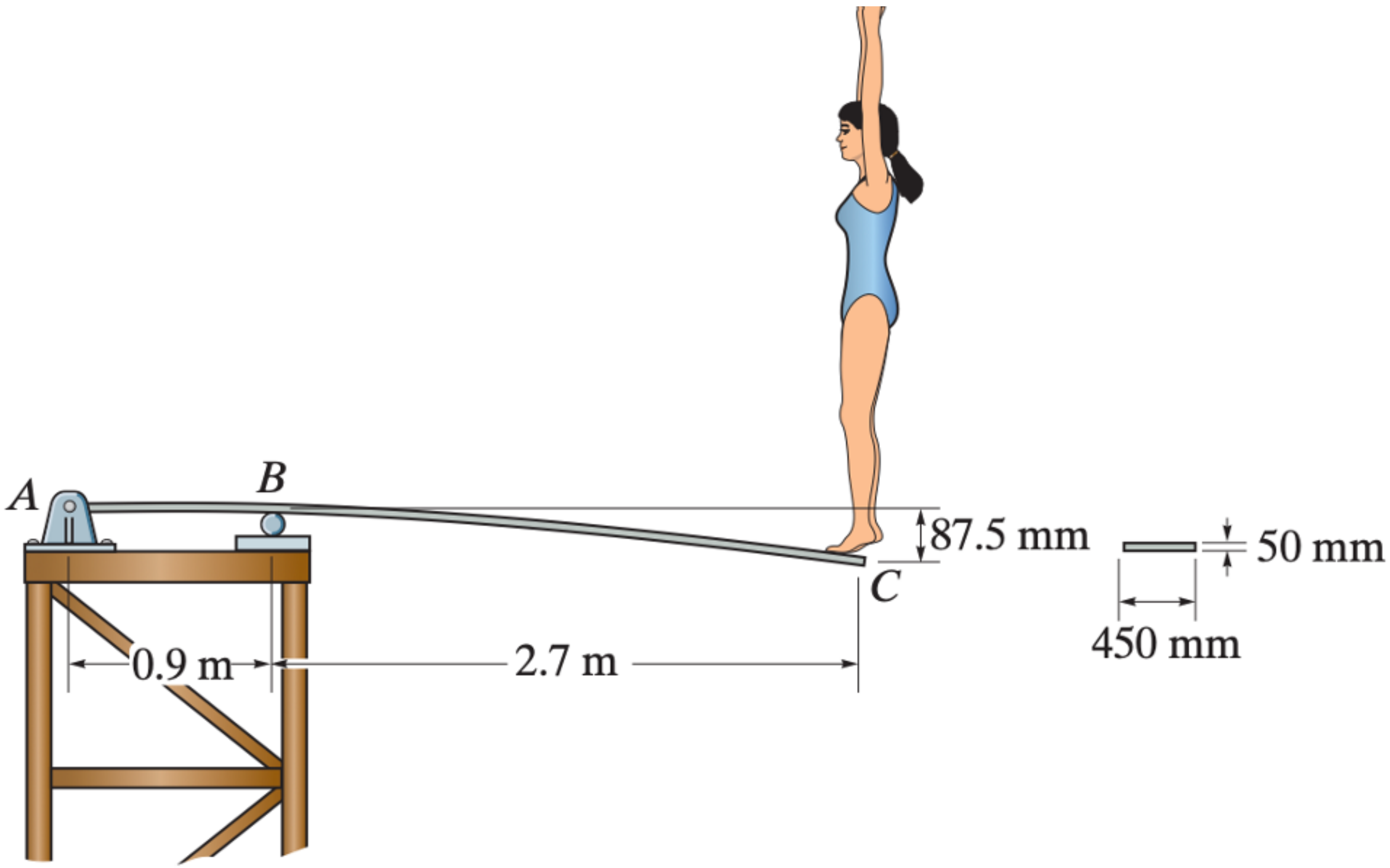
$$v(0)=\delta_1 \Rightarrow D = \delta_1$$

$$\frac{dv}{dx}(0)=\delta_2 \Rightarrow C = \delta_2$$

$$v(x) = -\frac{1}{2}\frac{F_0 L}{EI}x^2 + \frac{1}{6}\frac{F_0 x}{EI}x^2 + \delta_2 x + \delta_1$$

$$v(L) = -\frac{1}{3}F_0 L^3/EI + \delta_2 L + \delta_1$$

$$\vartheta(x) = \frac{F_0 L}{EI}x + \frac{1}{2}\frac{F_0 x^2}{EI}$$


A
B
87.5 mm
50 mm
C
450 mm
0.9 m
2.7 m