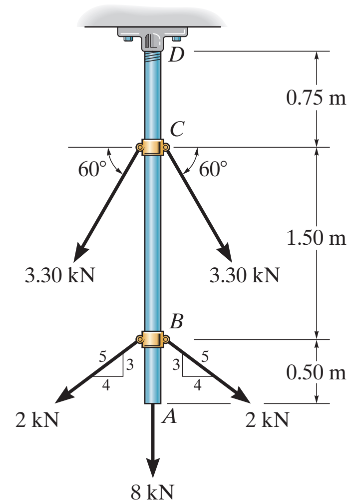
4–1. The A992 steel rod is subjected to the loading shown. If the cross-sectional area of the rod is 60 mm^2, determine the displacement of B and A, Neglect the size of the couplings at B, C, and D.

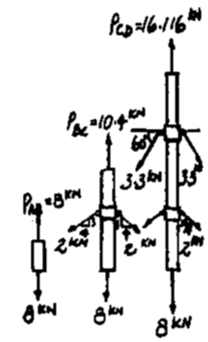


SOLUTION

$$\delta_B = \Sigma \frac{PL}{AE} = \frac{16.116(10^3)(0.75)}{60(10^{-6})(200)(10^9)} + \frac{10.4(10^3)(1.50)}{60(10^{-6})(200)(10^9)}$$

$$= 0.00231 \text{ m} = 2.31 \text{ mm}$$ **Ans.**

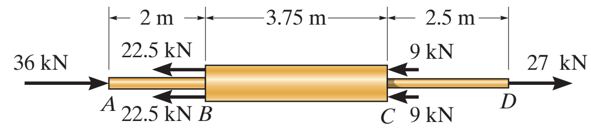
$$\delta_A = \delta_B + \frac{8(10^3)(0.5)}{60(10^{-6})(200)(10^9)} = 0.00264 \text{ m} = 2.64 \text{ mm}$$ **Ans.**



These solutions represent a preliminary version of the Instructors' Solutions Manual (ISM). It is possible and even likely that at this preliminary stage of preparing the ISM there are some omissions and errors in the draft solutions.

Ans:
$\delta_B = 2.31$ mm, $\delta_A = 2.64$ mm

4–2. The copper shaft is subjected to the axial loads shown. Determine the displacement of end A with respect to end D if the diameters of each segment are $d_{AB} = 20$ mm, $d_{BC} = 25$ mm, and $d_{CD} = 12$ mm. Take $E_{cu} = 126$ GPa.

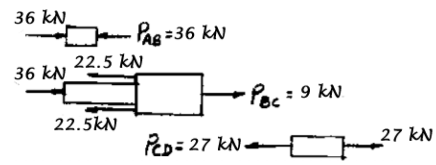


SOLUTION

$$\delta_{A/D} = \Sigma \frac{PL}{AE} = \frac{-36(10^3)(2000)}{\left[\frac{\pi}{4}(0.02^2)\right]\left[126(10^9)\right]} + \frac{9(10^3)(3750)}{\left[\frac{\pi}{4}(0.025^2)\right]\left[126(10^9)\right]} + \frac{27(10^3)(2500)}{\left[\frac{\pi}{4}(0.012^2)\right]\left[126(10^9)\right]}$$

$$= 3.4635 \text{ mm} = 3.46 \text{ mm}$$ **Ans.**

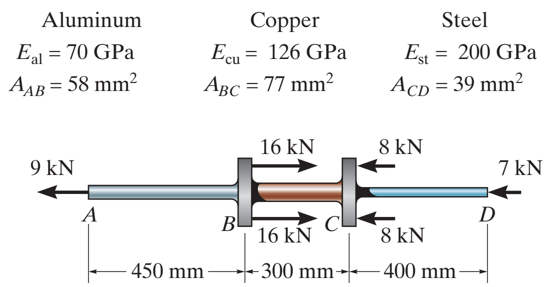
The positive sign indicates that end A moves away from end D.



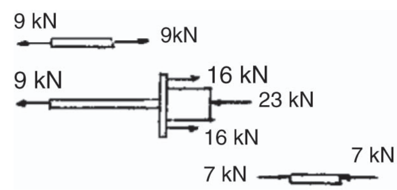
Ans:

$\delta_{A/D} = 3.46$ mm away from end D.

4–3. The composite shaft, consisting of aluminum, copper, and steel sections, is subjected to the loading shown. Determine the displacement of end A with respect to end D and the normal stress in each section. The cross-sectional area and modulus of elasticity for each section are shown in the figure. Neglect the size of the collars at B and C.



SOLUTION



$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{9(10^3)}{58(10^{-6})} = 155.17(10^6)\text{ N/m}^2 = 155\text{ MPa} \qquad \text{(T)} \qquad \textbf{Ans.}$$

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{23(10^3)}{77(10^{-6})} = 298.70(10^6)\text{ N/m}^2 = 299\text{ MPa} \qquad \text{(C)} \qquad \textbf{Ans.}$$

$$\sigma_{CD} = \frac{P_{BC}}{A_{BC}} = \frac{7(10^3)}{39(10^{-6})} = 179.49(10^6)\text{ N/m}^2 = 179\text{ MPa} \qquad \text{(C)} \qquad \textbf{Ans.}$$

$$\delta_{ND} = \Sigma\frac{PL}{AE} = \frac{9(10^3)(450)}{\left[58(10^{-6})\right]\left[70(10^9)\right]} + \frac{-23(10^3)(300)}{\left[77(10^{-6})\right]\left[126(10^9)\right]} + \frac{-7(10^3)(400)}{\left[39(10^{-6})\right]\left[200(10^9)\right]}$$

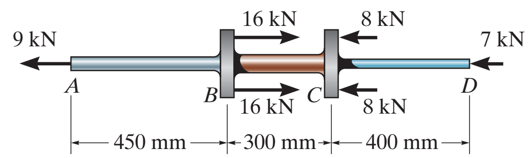
$$= -0.07263\text{ mm} = -0.0726\text{ mm} \qquad \textbf{Ans.}$$

The negative sign indicates end A moves towards end D.

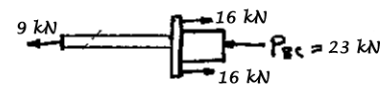
Ans:
$\sigma_{AB} = 155$ MPa (T), $\sigma_{BC} = 299$ MPa (C), $\sigma_{CD} = 179$ MPa (C), $\delta_{A/D} = 0.0726$ mm towards end D.

***4–4.** Determine the displacement of B with respect to C of the composite shaft in Prob. 4–3.

Aluminum	Copper	Steel
$E_{al} = 70$ GPa	$E_{cu} = 126$ GPa	$E_{st} = 200$ GPa
$A_{AB} = 58\ \text{mm}^2$	$A_{BC} = 77\ \text{mm}^2$	$A_{CD} = 39\ \text{mm}^2$



$$\delta_{B/C} = \frac{PL}{AE} = \frac{-23(10^3)(300)}{\left[77(10^{-6})\right]\left[126(10^9)\right]} = -0.7112\ \text{mm} = -0.711\ \text{mm}$$ **Ans.**



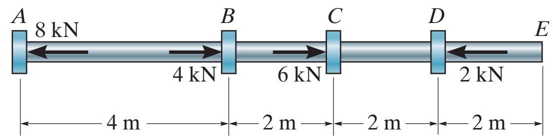
The negative sign indicates end B moves towards end C.

Ans.

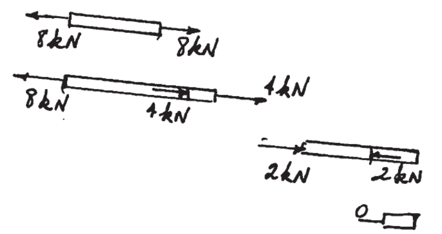
$\delta_{B/C} = -0.711$ mm

4–5.

The 2014-T6 aluminium rod has a diameter of 30 mm and supports the load shown. Determine the displacement of end A with respect to end E. Neglect the size of the couplings.



SOLUTION

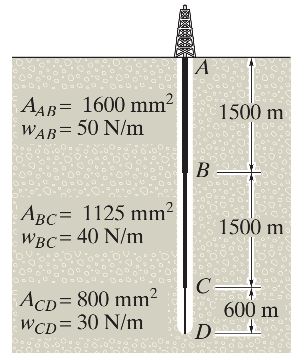


$$\delta_{A/E} = \Sigma \frac{PL}{AE} = \frac{1}{AE}[8(4) + 4(2) - 2(2) + 0(2)]\left(10^3\right)$$

$$= \frac{36\left(10^3\right)}{\frac{\pi}{4}(0.03)^2(73.1)\left(10^9\right)} = 0.697\left(10^{-3}\right) = 0.697 \text{ mm}$$ **Ans.**

Ans:
$\delta_{A/E} = 0.697$ mm

4–6. The A992 steel drill shaft of an oil well extends 3600 m into the ground. Assuming that the pipe used to drill the well is suspended freely from the derrick at A, determine the maximum average normal stress in each pipe segment and the elongation of its end D with respect to the fixed end at A. The shaft consists of three different sizes of pipe, AB, BC, and CD, each having the length, weight per unit length, and cross-sectional area indicated.

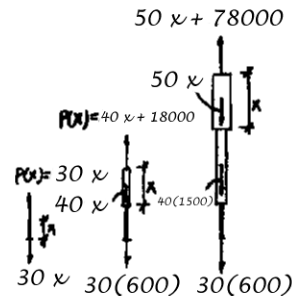


SOLUTION

$$\sigma_A = \frac{P}{A} = \frac{50(1500) + 78000}{1600(10^{-6})} = 95.625(10^6)\ \text{N/m}^2 = 95.6\ \text{MPa}$$ **Ans.**

$$\sigma_B = \frac{P}{A} = \frac{40(1500) + 18000}{1125(10^{-6})} = 69.33(10^6)\ \text{N/m}^2 = 69.3\ \text{MPa}$$ **Ans.**

$$\sigma_C = \frac{P}{A} = \frac{30(600)}{800(10^{-6})} = 22.5(10^6)\ \text{N/m}^2 = 22.5\ \text{MPa}$$ **Ans.**



$$\delta_D = \Sigma \int \frac{P(x)\,dx}{A(x)\,E} = \int_0^{600\text{ m}} \frac{30x\,dx}{\left[800(10^{-6})\right]\left[200(10^9)\right]} + \int_0^{1500\text{ m}} \frac{(40x + 18000)dx}{\left[1125(10^{-6})\right]\left[200(10^9)\right]} + \int_0^{1500\text{ m}} \frac{(50x + 78000)dx}{\left[1600(10^{-6})\right]\left[200(10^9)\right]}$$

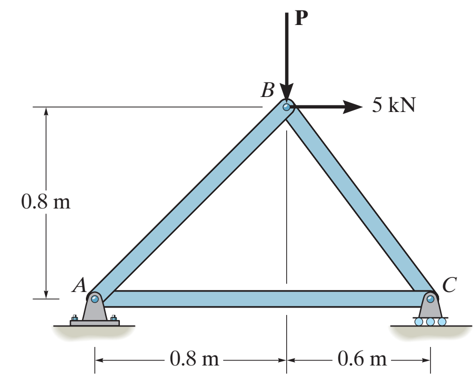
$$= 0.8952\ \text{m} = 0.895\ \text{m}$$ **Ans.**

Ans.

$\sigma_A = 95.6$ MPa, $\sigma_B = 69.3$ MPa,
$\sigma_C = 22.5$ MPa, $\delta_D = 0.895$ m

4–7.

The truss is made of three A-36 steel members, each having a cross-sectional area of 400 mm^2. Determine the horizontal displacement of the roller at C when $P = 8$ kN.

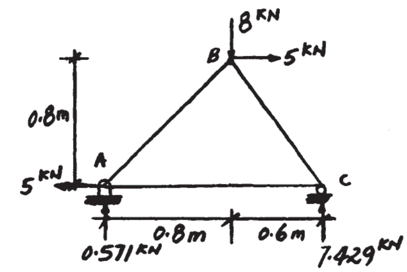


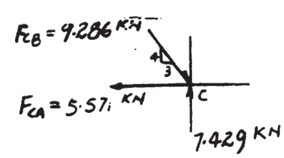
SOLUTION

By observation the horizontal displacement of roller C is equal to the displacement of point C obtained from member AC.

$F_{CA} = 5.571$ kN

$$\delta_C = \frac{F_{CA}L}{AE} = \frac{5.571(10^3)(1.40)}{(400)(10^{-6})(200)(10^6)} = 0.0975 \text{ mm} \rightarrow$$ **Ans.**

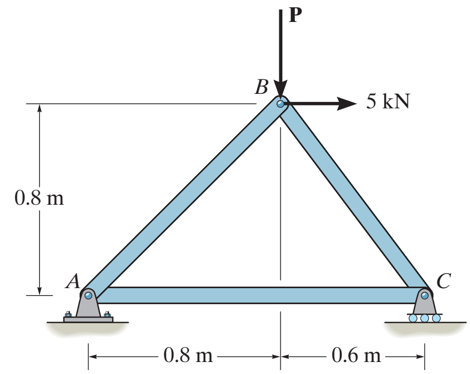




Ans:
$\delta_C = 0.0975$ mm $\rightarrow$

***4–8.**

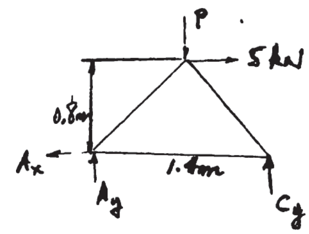
The truss is made of three A-36 steel members, each having a cross-sectional area of 400 mm^2. Determine the magnitude P required to displace the roller to the right 0.2 mm.



SOLUTION

$\circlearrowleft + M_A = 0; \qquad -P(0.8) - 5(0.8) + C_y(1.4) = 0$

$$C_y = 0.5714\,P + 2.857$$

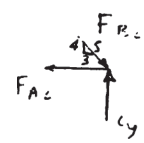


$+\uparrow \Sigma F_y = 0; \qquad C_y - F_{BC}\left(\frac{4}{5}\right) = 0$

$$F_{BC} = 1.25\,C_y$$

$\xrightarrow{+} \Sigma F_x = 0; \qquad -F_{AC} + 1.25\,C_y\,(0.6) = 0$

$$F_{AC} = 0.75\,C_y = 0.4286\,P + 2.14286$$



Require,

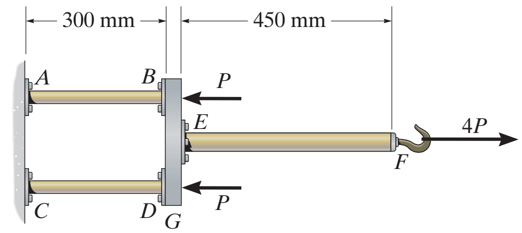
$$\delta_{C_A} = 0.0002 = \frac{(0.4286\,P + 2.14286)(10^3)(1.4)}{(400)(10^{-6})(200)(10^9)}$$

$P = 21.7$ kN **Ans.**

Ans:
$P = 21.7$ kN

4–9.

The assembly consists of two 10-mm diameter red brass C83400 copper rods AB and CD, a 15-mm diameter 304 stainless steel rod EF, and a rigid bar G. If $P = 5$ kN, determine the horizontal displacement of end F of rod EF.



SOLUTION

Internal Loading: The normal forces developed in rods EF, AB, and CD are shown on the free-body diagrams in Figs. *a* and *b*.

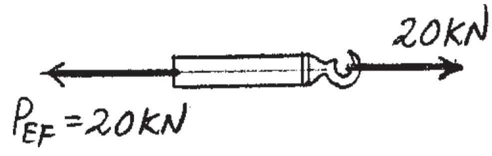
Displacement: The cross-sectional areas of rods EF and AB are $A_{EF} = \frac{\pi}{4}(0.015^2) = 56.25(10^{-6})\pi \text{ m}^2$ and $A_{AB} = \frac{\pi}{4}(0.01^2) = 25(10^{-6})\pi \text{ m}^2$.

$$\delta_F = \Sigma\frac{PL}{AE} = \frac{P_{EF}L_{EF}}{A_{EF}E_{\text{st}}} + \frac{P_{AB}L_{AB}}{A_{AB}E_{\text{br}}}$$

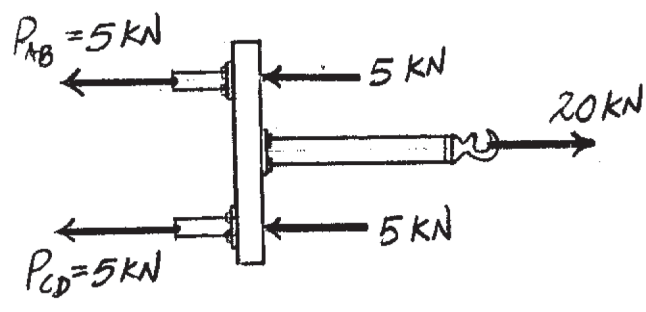
$$= \frac{20(10^3)(450)}{56.25(10^{-6})\pi(193)(10^9)} + \frac{5(10^3)(300)}{25(10^{-6})\pi(101)(10^9)}$$

$$= 0.453 \text{ mm}$$ **Ans.**

The positive sign indicates that end F moves away from the fixed end.



(a)

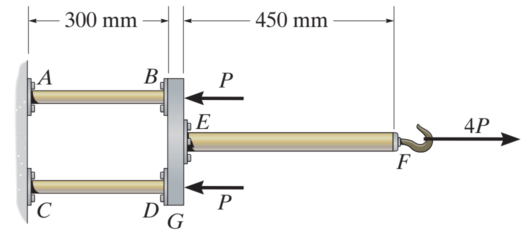


(b)

Ans:
$\delta_F = 0.453$ mm

4–10.

The assembly consists of two 10-mm diameter red brass C83400 copper rods AB and CD, a 15-mm diameter 304 stainless steel rod EF, and a rigid bar G. If the horizontal displacement of end F of rod EF is 0.45 mm, determine the magnitude of P.



SOLUTION

Internal Loading: The normal forces developed in rods EF, AB, and CD are shown on the free-body diagrams in Figs. a and b.

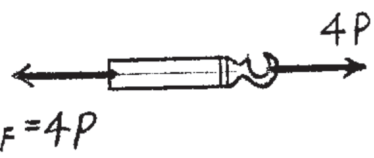
Displacement: The cross-sectional areas of rods EF and AB are $A_{EF} = \frac{\pi}{4}(0.015^2) = 56.25(10^{-6})\pi\ \text{m}^2$ and

$$A_{AB} = \frac{\pi}{4}(0.01^2) = 25(10^{-6})\pi\ \text{m}^2.$$

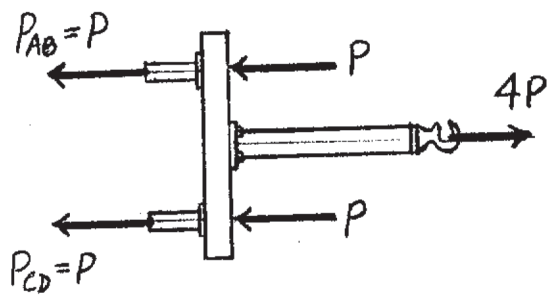
$$\delta_F = \Sigma\frac{PL}{AE} = \frac{P_{EF}L_{EF}}{A_{EF}E_{\text{st}}} + \frac{P_{AB}L_{AB}}{A_{AB}E_{\text{br}}}$$

$$0.45 = \frac{4P(450)}{56.25(10^{-6})\pi(193)(10^9)} + \frac{P(300)}{25(10^{-6})\pi(101)(10^9)}$$

$P = 4967\ \text{N} = 4.97\ \text{kN}$ **Ans.**



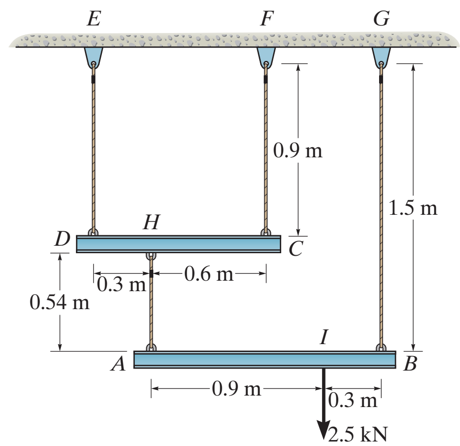
(a)



(b)

Ans:
$P = 4.97$ kN

4–11. The load is supported by the four 304 stainless steel wires that are connected to the rigid members AB and DC. Determine the vertical displacement of the 2.5-kN load if the members were originally horizontal when the load was applied. Each wire has a cross-sectional area of 16 mm^2.



SOLUTION

Internal Forces in the wires:

FBD (b)

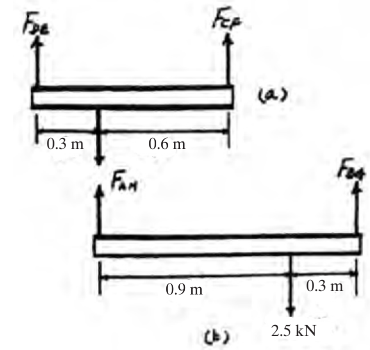
$\circlearrowleft +\Sigma M_A = 0; \quad F_{BG}(1.2) - 2.5(0.9) = 0 \quad F_{BC} = 1.875 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \quad F_{AH} + 1.875 - 2.5 = 0 \quad F_{AH} = 0.625 \text{ kN}$

FBD (a)

$\circlearrowleft +\Sigma M_D = 0; \quad F_{CF}(0.9) - 0.625(0.3) = 0 \quad F_{CF} = 0.2083 \text{ kN}$

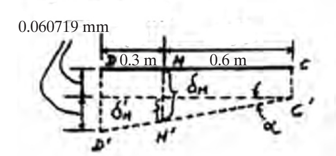
$+\uparrow \Sigma F_y = 0; \quad F_{DE} + 0.2083 - 0.625 = 0 \quad F_{DE} = 0.4167 \text{ kN}$



Displacement:

$$\delta_D = \frac{F_{DE}L_{DE}}{A_{DE}E} = \frac{0.4167(10^3)(0.9)(10^3)}{[16(10^{-6})][193(10^9)]} = 0.121438 \text{ mm}$$

$$\delta_C = \frac{F_{CF}L_{CF}}{A_{CF}E} = \frac{0.2083(10^3)(0.9)(10^3)}{[16(10^{-6})][193(10^9)]} = 0.060719 \text{ mm}$$



$$\frac{\delta_H'}{0.6} = \frac{0.060719}{0.9}; \quad \delta_H' = 0.040479 \text{ mm}$$

$$\delta_H = 0.040479 + 0.060719 = 0.101198 \text{ mm}$$

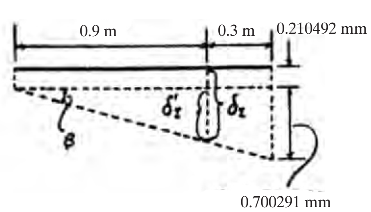
$$\delta_{A/H} = \frac{F_{AH}L_{AH}}{A_{AH}E} = \frac{0.625(10^3)(0.54)(10^3)}{[16(10^{-6})][193(10^9)]} = 0.109294 \text{ mm}$$

$$\delta_A = \delta_H + \delta_{A/H} = 0.101198 + 0.109294 = 0.210492 \text{ mm}$$

$$\delta_B = \frac{F_{BG}L_{BG}}{A_{BG}E} = \frac{1.875(10^3)(1.5)(10^3)}{[16(10^{-6})][193(10^9)]} = 0.910784 \text{ mm}$$

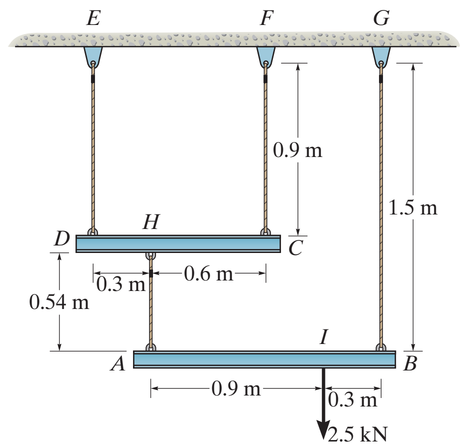
$$\frac{\delta_I'}{0.9} = \frac{0.700291}{1.2}; \quad \delta_I' = 0.525219 \text{ mm}$$

$$\delta_I = 0.210492 + 0.525219 = 0.7357 \text{ mm} = 0.736 \text{ mm}$$ **Ans.**



Ans:
$\delta_I = 0.736$ mm

***4–12.** The load is supported by the four 304 stainless steel wires that are connected to the rigid members AB and DC. Determine the angle of tilt of each member after the 2.5-kN load is applied. The members were originally horizontal, and each wire has a cross-sectional area of 16 mm^2.



SOLUTION

***Internal Forces in the wires*:**

FBD (b)

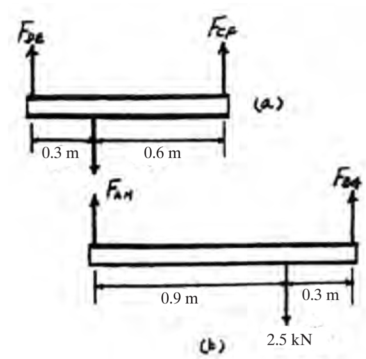
$\circlearrowleft + \Sigma M_A = 0; \quad F_{BG}(1.2) - 2.5(0.9) = 0 \quad F_{BG} = 1.875 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \quad F_{AH} + 1.875 - 2.5 = 0 \quad F_{AH} = 0.625 \text{ kN}$

FBD (a)

$\circlearrowleft + \Sigma M_D = 0; \quad F_{CF}(0.9) - 0.625(0.3) = 0 \quad F_{CF} = 0.2083 \text{ kN}$

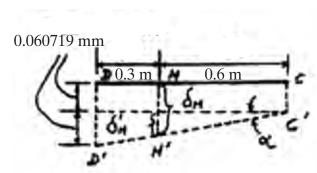
$+\uparrow \Sigma F_y = 0; \quad F_{DE} + 0.2083 - 0.625 = 0 \quad F_{DE} = 0.4167 \text{ kN}$



Displacement:

$$\delta_D = \frac{F_{DE}L_{DE}}{A_{DE}E} = \frac{0.4167(10^3)(0.9)(10^3)}{[16(10^{-6})][193(10^9)]} = 0.121438 \text{ mm}$$

$$\delta_C = \frac{F_{CF}L_{CF}}{A_{CF}E} = \frac{0.2083(10^3)(0.9)(10^3)}{[16(10^{-6})][193(10^9)]} = 0.060719 \text{ mm}$$



$$\frac{\delta'_H}{0.6} = \frac{0.060719}{0.9}; \quad \delta'_H = 0.040479 \text{ mm}$$

$$\delta_H = \delta'_H + \delta_C = 0.040479 + 0.060719 = 0.101198 \text{ mm}$$

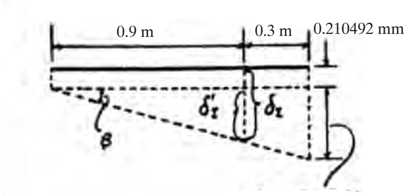
$$\tan\alpha = \frac{0.060719}{0.9(10^3)}; \quad \alpha = 0.003865° = 0.00387° \qquad \textbf{Ans.}$$

$$\delta_{A/H} = \frac{F_{AH}L_{AH}}{A_{AH}E} = \frac{0.625(10^3)(0.54)(10^3)}{[16(10^{-6})][193(10^9)]} = 0.109294 \text{ mm}$$

$$\delta_A = \delta_H + \delta_{A/H} = 0.101198 + 0.109294 = 0.210492 \text{ mm}$$

$$\delta_B = \frac{F_{BG}L_{BG}}{A_{BG}E} = \frac{1.875(10^3)(1.5)(10^3)}{[16(10^{-6})][193(10^9)]} = 0.910784 \text{ mm}$$

$$\tan\beta = \frac{0.700291}{1.2(10^3)}; \quad \beta = 0.03344° = 0.0334°$$

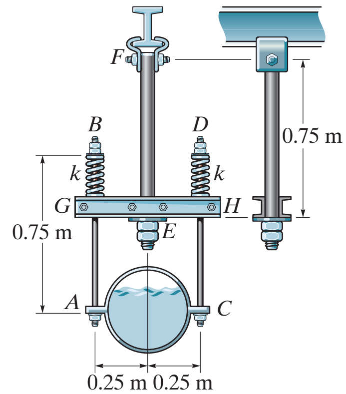


Ans.

$\alpha = 0.00387°$

$\beta = 0.0334°$

4–13. A spring-supported pipe hanger consists of two springs which are originally unstretched and have a stiffness of $k = 60$ kN/m, three 304 stainless steel rods, AB and CD, which have a diameter of 5 mm, and EF, which has a diameter of 12 mm, and a rigid beam GH. If the pipe and the fluid it carries have a total weight of 4 kN, determine the displacement of the pipe when it is attached to the support.



SOLUTION

***Internal Force in the Rods*:**

FBD (a)

$$\circlearrowleft +\Sigma M_A = 0; \qquad F_{CD}(0.5) - 4(0.25) = 0 \qquad F_{CD} = 2.00 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} + 2.00 - 4 = 0 \qquad F_{AB} = 2.00 \text{ kN}$$

FBD (b)

$$+\uparrow \Sigma F_y = 0; \qquad F_{EF} - 2.00 - 2.00 = 0 \qquad F_{EF} = 4.00 \text{ kN}$$

***Displacement*:**

$$\delta_D = \delta_E = \frac{F_{EF}L_{EF}}{A_{EF}E} = \frac{4.00(10^3)(750)}{\frac{\pi}{4}(0.012)^2(193)(10^9)} = 0.1374 \text{ mm}$$

$$\delta_{A/B} = \delta_{C/D} = \frac{P_{CD}L_{CD}}{A_{CD}E} = \frac{2(10^3)(750)}{\frac{\pi}{4}(0.005)^2(193)(10^9)} = 0.3958 \text{ mm}$$

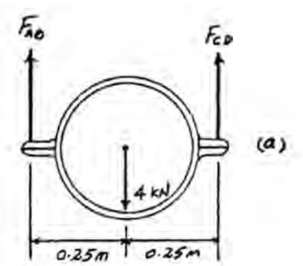
$$\delta_C = \delta_D + \delta_{C/D} = 0.1374 + 0.3958 = 0.5332 \text{ mm}$$

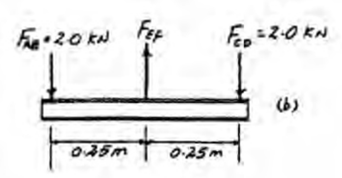
Displacement of the spring

$$\delta_{sp} = \frac{F_{sp}}{k} = \frac{2.00}{60} = 0.0333333 \text{ m} = 33.3333 \text{ mm}$$

$$\delta_{\text{lat}} = \delta_C + \delta_{sp}$$

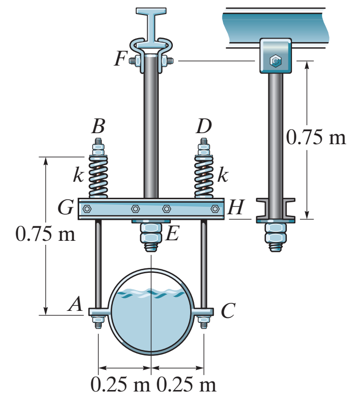
$$= 0.5332 + 33.3333 = 33.9 \text{ mm}$$ **Ans.**





Ans.
$\delta_D = 0.1374$ mm, $\delta_{A/B} = 0.3958$ mm,
$\delta_C = 0.5332$ mm, $\delta_{\text{tot}} = 33.9$ mm

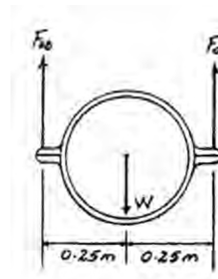
4–14. A spring-supported pipe hanger consists of two springs, which are originally unstretched and have a stiffness of $k = 60$ kN/m, three 304 stainless steel rods, AB and CD, which have a diameter of 5 mm, and EF, which has a diameter of 12 mm, and a rigid beam GH. If the pipe is displaced 82 mm when it is filled with fluid, determine the weight of the fluid.



SOLUTION

Internal Force in the Rods:

FBD (a)



$$\circlearrowleft +\Sigma M_A = 0; \qquad F_{CD}(0.5) - W(0.25) = 0 \qquad F_{CD} = \frac{W}{2}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} + \frac{W}{2} - W = 0 \qquad F_{AB} = \frac{W}{2}$$

FBD (b)

$$+\uparrow \Sigma F_y = 0; \qquad F_{EF} - \frac{W}{2} - \frac{W}{2} = 0 \qquad F_{EF} = W$$

Displacement:

$$\delta_D = \delta_E = \frac{F_{EF}L_{EF}}{A_{EF}E} = \frac{W(750)}{\frac{\pi}{4}(0.012)^2(193)(10^9)}$$

$$= 34.35988(10^{-6})\, W$$

$$\delta_{A/B} = \delta_{C/D} = \frac{F_{CD}L_{CD}}{A_{CD}E} = \frac{\frac{W}{2}(750)}{\frac{\pi}{4}(0.005)^2(193)(10^9)}$$

$$= 98.95644(10^{-6})\, W$$

$$\delta_C = \delta_D + \delta_{C/D}$$

$$= 34.35988(10^{-6})\, W + 98.95644(10^{-6})\, W$$

$$= 0.133316(10^{-3})\, W$$

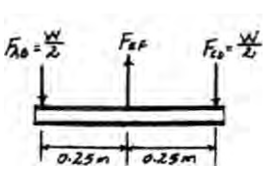
Displacement of the spring

$$\delta_{sp} = \frac{F_{sp}}{k} = \frac{\frac{W}{2}}{60(10^3)}(1000) = 0.008333\, W$$

$$\delta_{\text{lat}} = \delta_C + \delta_{sp}$$

$$82 = 0.133316(10^{-3})\, W + 0.008333W$$

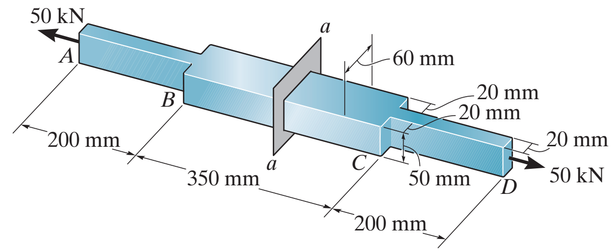
$$W = 9685 \text{ N} = 9.69 \text{ kN}$$ **Ans.**



Ans.
$W = 9.69$ kN

4–15.

The steel bar has the original dimensions shown in the figure. If it is subjected to an axial load of 50 kN, determine the change in its length and its new cross-sectional dimensions at section a–a. $E_{st} = 200$ GPa, $\nu_{st} = 0.29$.



SOLUTION

$$\delta_{A/D} = \Sigma\frac{PL}{AE} = \frac{2(50)(10^3)(200)}{(0.02)(0.05)(200)(10^9)} + \frac{50(10^3)(350)}{(0.06)(0.05)(200)(10^9)}$$

$$= 0.129 \text{ mm}$$ **Ans.**

$$\delta_{B/C} = \frac{PL}{AE} = \frac{50(10^3)(350)}{(0.06)(0.05)(200)(10^9)} = 0.02917 \text{ mm}$$

$$\epsilon_{BC} = \frac{\delta_{B/C}}{L_{BC}}\frac{0.02917}{350} = 0.00008333$$

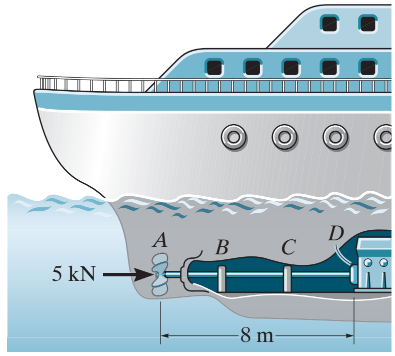
$$\epsilon_{lat} = -\nu\,\epsilon_{long} = -(0.29)(0.00008333) = -0.00002417$$

$$h' = 50 - 50(0.00002417) = 49.9988 \text{ mm}$$ **Ans.**

$$w' = 60 - 60(0.00002417) = 59.9986 \text{ mm}$$ **Ans.**

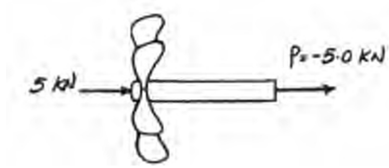
Ans:
$\delta_{A/D} = 0.129$ mm,
$h' = 49.9988$ mm,
$w' = 59.9986$ mm

***4–16.** The ship is pushed through the water using an A-36 steel propeller shaft that is 8 m long, measured from the propeller to the thrust bearing D at the engine. If it has an outer diameter of 400 mm and a wall thickness of 50 mm, determine the amount of axial contraction of the shaft when the propeller exerts a force on the shaft of 5 kN. The bearings at B and C are journal bearings.



SOLUTION

Internal Force: As shown on *FBD*.



Displacement:

$$\delta_A = \frac{PL}{AE} = \frac{-5.00\,(10^3)(8)}{\frac{\pi}{4}\,(0.4^2 - 0.3^2)\,200(10^9)}$$

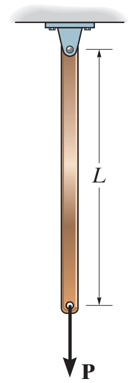
$$= -3.638(10^{-6})\text{ m}$$

$$= -3.64\left(10^{-3}\right)\text{ mm} \qquad \textbf{Ans.}$$

Negative sign indicates that end A moves towards end D.

Ans.
$\delta_A = -3.64(10^{-3})$ mm

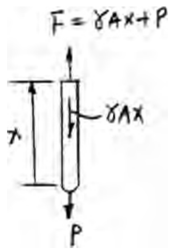
4–17. The bar has a length L and cross-sectional area A. Determine its elongation due to the force $\mathbf{P}$ and its own weight. The material has a specific weight γ (weight/volume) and a modulus of elasticity E.



SOLUTION

$$\delta = \int \frac{P(x)\,dx}{A(x)\,E} = \frac{1}{AE}\int_0^L (\gamma A x + P)\,dx$$

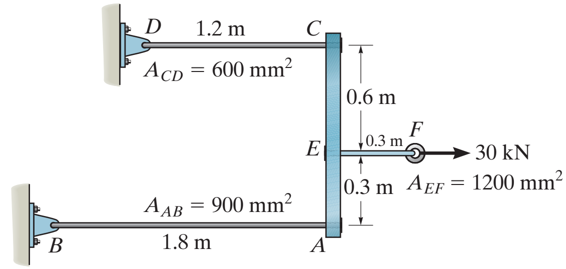
$$= \frac{1}{AE}\left(\frac{\gamma A L^2}{2} + PL\right) = \frac{\gamma L^2}{2E} + \frac{PL}{AE} \qquad \textbf{Ans.}$$



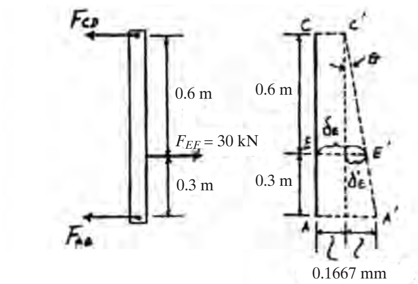
Ans.

$$\delta = \frac{1}{AE}\int_0^L (\gamma A x + P)\,dx = \frac{\gamma L^2}{2E} + \frac{PL}{AE}$$

4–18. The assembly consists of three titanium (Ti-6A1-4V) rods and a rigid bar AC. The cross-sectional area of each rod is given in the figure. If a force of 30 kN is applied to the ring F, determine the horizontal displacement of point F.



SOLUTION



Internal Force in the Rods:

$\zeta + \Sigma M_A = 0; \qquad F_{CD}(0.9) - 30(0.3) = 0 \qquad F_{CD} = 10\text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 30 - 10 - F_{AB} = 0 \qquad F_{AB} = 20\text{ kN}$

Displacement:

$$\delta_C = \frac{F_{CD}L_{CD}}{A_{CD}E} = \frac{10(10^3)(1.2)(10^3)}{[600(10^{-6})][120(10^9)]} = 0.1667\text{ mm}$$

$$\delta_A = \frac{F_{AB}L_{AB}}{A_{AB}E} = \frac{20(10^3)(1.8)(10^3)}{[900(10^{-6})][120(10^9)]} = 0.3333\text{ mm}$$

$$\delta_{F/E} = \frac{F_{EF}L_{EF}}{A_{EF}E} = \frac{30(10^3)(0.3)(10^3)}{[1200(10^{-6})][120(10^9)]} = 0.0625\text{ mm}$$

$$\frac{\delta'_E}{0.6} = \frac{0.1667}{0.9}, \qquad \delta'_E = 0.1111\text{ mm}$$

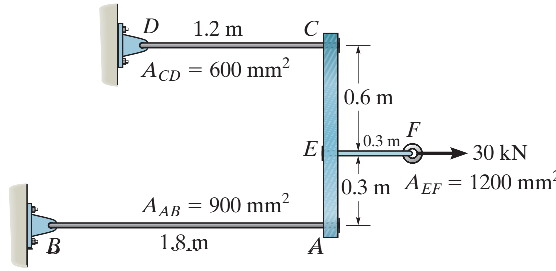
$$\delta_E = \delta_C + \delta'_E = 0.1667 + 0.1111 = 0.2778\text{ mm}$$

$$\delta_F = \delta_E + \delta_{F/E}$$
$$= 0.2778 + 0.0625 = 0.3403\text{ mm} = 0.340\text{ mm} \qquad \textbf{Ans.}$$

Ans.
$\delta_F = 0.3403$ mm

4–19.

The assembly consists of three titanium (Ti-6A1-4V) rods and a rigid bar AC. The cross-sectional area of each rod is given in the figure. If a forceof 30 kN is applied to the ring F, determine the angle of tilt of bar AC.

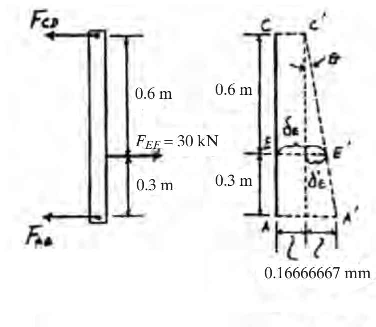


SOLUTION

Internal Force in the Rods:

$\circlearrowleft + \Sigma M_A = 0;\quad F_{CD}(0.9) - 30(0.3) = 0 \qquad F_{CD} = 10\text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad 30 - 10 - F_{AB} = 0 \qquad F_{AB} = 20\text{ kN}$



Displacement:

$$\delta_C = \frac{F_{CD}L_{CD}}{A_{CD}E} = \frac{10(10^3)(1.2)(10^3)}{[600(10^{-6})][120(10^9)]} = 0.1667\text{ mm}$$

$$\delta_A = \frac{F_{AB}L_{AB}}{A_{AB}E} = \frac{20(10^3)(1.8)(10^3)}{[900(10^{-6})][120(10^9)]} = 0.3333\text{ mm}$$

$$\theta = \tan^{-1}\frac{\delta_A - \delta_C}{L_{AC}} = \tan^{-1}\frac{0.3333 - 0.1667}{0.9(10^3)}$$

$$= 0.01061° = 0.0106°$$

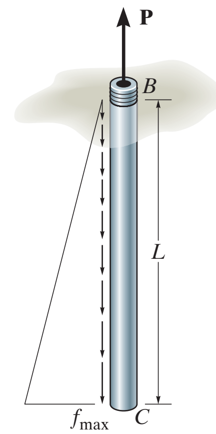
Ans.

Ans.

$\theta = 0.0106°$

***4–20.**

The pipe is stuck in the ground so that when it is pulled upward the frictional force along its length varies linearly from zero at B to f_{max} (force/length) at C. Determine the initial force P required to pull the pipe out and the pipe's elongation just before it starts to slip. The pipe has a length L, cross-sectional area A, and the material from which it is made has a modulus of elasticity E.

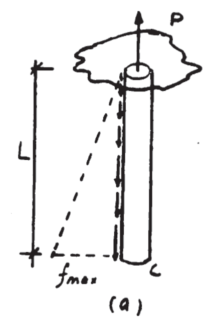


SOLUTION

From FBD (a)

$$+\uparrow \Sigma F_y = 0; \qquad P - \frac{1}{2}(F_{max} L) = 0$$

$$P = \frac{F_{max} L}{2} \qquad \textbf{Ans.}$$



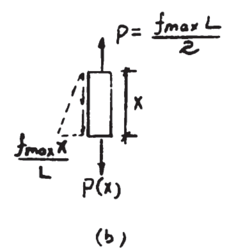
From FBD (b)

$$+\downarrow \Sigma F_y = 0; \qquad P(x) + \frac{1}{2}\left(\frac{F_{max}\, x}{L}\right)x - \frac{F_{max} L}{2} = 0$$

$$P(x) = \frac{F_{max} L}{2} - \frac{F_{max} x^2}{2L}$$

$$\delta = \int_0^L \frac{P(x)\,dx}{A(x)E} = \int_0^L \frac{F_{max} L}{2AE} dx - \int_0^L \frac{F_{max} x^2}{2AEL} dx$$

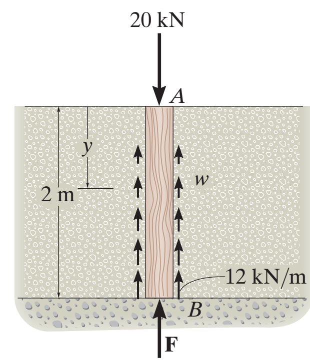
$$= \frac{F_{max} L^2}{3AE} \qquad \textbf{Ans.}$$



Ans:
$P = \frac{F_{max} L}{2}$,
$\delta = \frac{F_{max} L^2}{3AE}$

4–21.

The post is made of Douglas fir and has a diameter of 100 mm. If it is subjected to the load of 20 kN and the soil provides a frictional resistance distributed around the post that is triangular along its sides; that is, it varies from $w = 0$ at $y = 0$ to $w = 12$ kN/m at $y = 2$ m, determine the force **F** at its bottom needed for equilibrium. Also, what is the displacement of the top of the post A with respect to its bottom B? Neglect the weight of the post.



SOLUTION

Equation of Equilibrium: Referring to the FBD of the entire post, Fig. *a*,

$$+\uparrow\Sigma F_y = 0; \qquad F + \frac{1}{2}(12)(2) - 20 = 0 \qquad F = 8.00 \text{ kN} \qquad \textbf{Ans.}$$

Normal Force: Referring to the FBD of the upper segment of the post sectioned at arbitrary distance y, Fig. *b*

$$+\uparrow\Sigma F_y = 0; \qquad \frac{1}{2}(6y)(y) - 20 - P(y) = 0 \qquad Py = \left(3y^2 - 20\right) \text{ kN}$$

Displacement: For Douglas Fir, $E = 13.1$ GPa

$$\delta_{A/B} = \int_0^L \frac{N(y)dy}{A(y)E} = \frac{1}{AE}\int_0^{2 \text{ meters}} \left(3y^2 - 20\right)dy$$

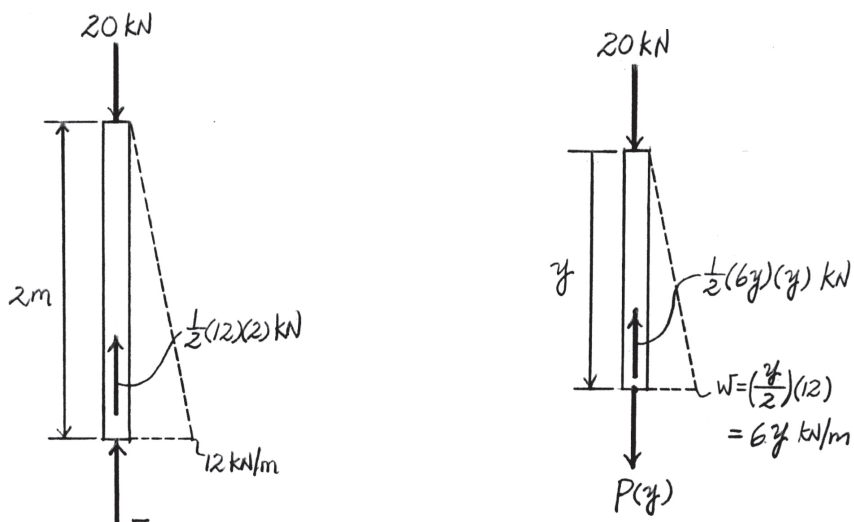
$$= \frac{1}{AE}\left(y^3 - 20y\right)\Big|_0^{2 \text{ meters}}$$

$$= -\frac{32 \text{ kN}\cdot\text{m}}{AE}$$

$$= -\frac{32\left(10^3\right)}{\frac{\pi}{4}\left(0.1^2\right)\left[13.1\left(10^9\right)\right]}$$

$$= -0.3110\left(10^{-3}\right) \text{ m} = -0.311 \text{ mm} \qquad \textbf{Ans.}$$

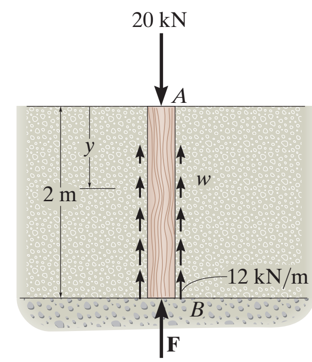
The sign indicates that end A moves toward end B.



Ans:
$F = 8.00$ kN,
$\delta_{A/B} = -0.311$ mm

4–22.

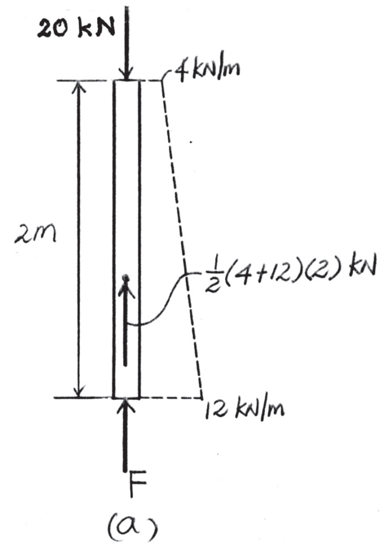
The post is made of Douglas fir and has a diameter of 100 mm. If it is subjected to the load of 20 kN and the soil provides a frictional resistance that is distributed along its length and varies linearly from $w = 4$ kN/m at $y = 0$ to $w = 12$ kN/m at $y = 2$ m, determine the force **F** at its bottom needed for equilibrium. Also, what is the displacement of the top of the post A with respect to its bottom B? Neglect the weight of the post.



SOLUTION

Equation of Equilibrium: Referring to the FBD of the entire post, Fig. *a*,

$+\uparrow\Sigma F_y = 0;\quad F + \frac{1}{2}(4 + 12)(2) - 20 = 0 \qquad F = 4.00\text{ kN}$ **Ans.**



Normal Force: Referring to the FBD of the upper segment of the post sectioned at arbitrary distance *y*, Fig. *b*,

$+\uparrow\Sigma F_y = 0;\quad (4 + 2y)y - 20 - P(y) = 0 \qquad P(y) = (2y^2 + 4y - 20)\text{ kN}$

Displacement: For Douglas Fir, $E = 13.1$ GPa

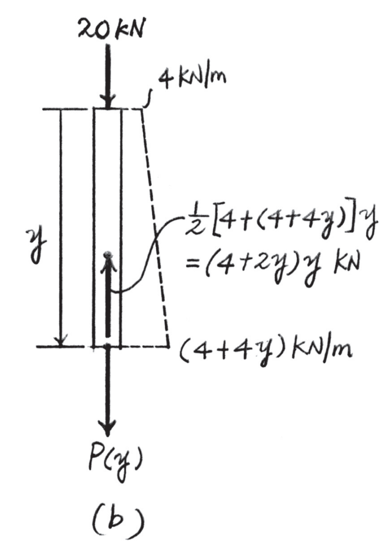
$$\delta_{A/B} = \int_0^L \frac{N(y)dy}{A(y)E} = \frac{1}{AE}\int_0^{2\text{ m}} (2y^2 + 4y - 20)dy$$

$$= \frac{1}{AE}\left(\frac{2}{3}y^3 + 2y^2 - 20y\right)\Big|_0^{2\text{ m}}$$

$$= -\frac{80\text{ kN}\cdot\text{m}}{3\,AE}$$

$$= -\frac{80(10^3)}{3\left[\frac{\pi}{4}(0.1^2)\right][13.1(10^9)]}$$

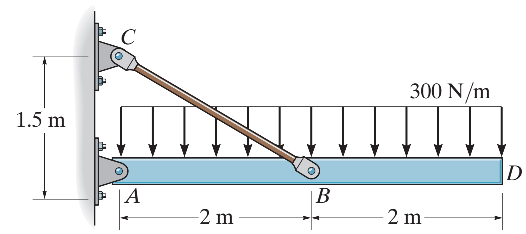
$$= -0.2592(10^{-3})\text{ m} = -0.259\text{ mm}$$ **Ans.**



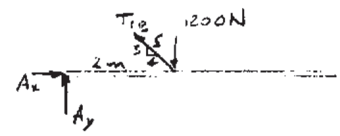
Ans:
$F = 4.00$ kN,
$\delta_{A/B} = -0.259$ mm

4–23.

The rigid bar is supported by the pin-connected rod CB that has a cross-sectional area of 14 mm^2 and is made from 6061-T6 aluminum. Determine the vertical deflection of the bar at D when the distributed load is applied.



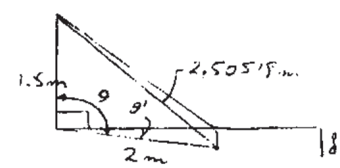
SOLUTION



$\circlearrowleft + \Sigma M_A = 0; \quad 1200(2) - T_{CB}(0.6)(2) = 0$

$$T_{CB} = 2000 \text{ N}$$

$$\delta_{B/C} = \frac{PL}{AE} = \frac{(2000)(2.5)}{14(10^{-6})(68.9)(10^9)} = 0.0051835$$



$$(2.5051835)^2 = (1.5)^2 + (2)^2 - 2(1.5)(2)\cos\theta$$

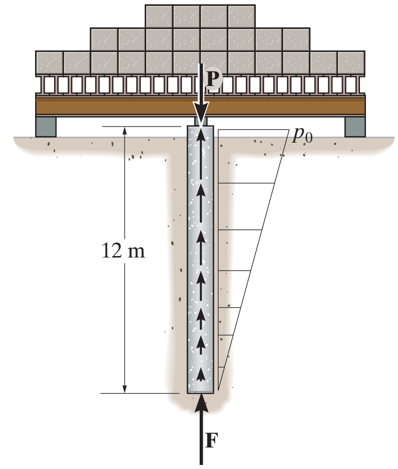
$$\theta = 90.248°$$

$$\theta = 90.248° - 90° = 0.2478° = 0.004324 \text{ rad}$$

$\delta_D = \theta\, r = 0.004324(4000) = 17.3 \text{ mm}$ **Ans.**

Ans:
$\delta_D = 17.3$ mm

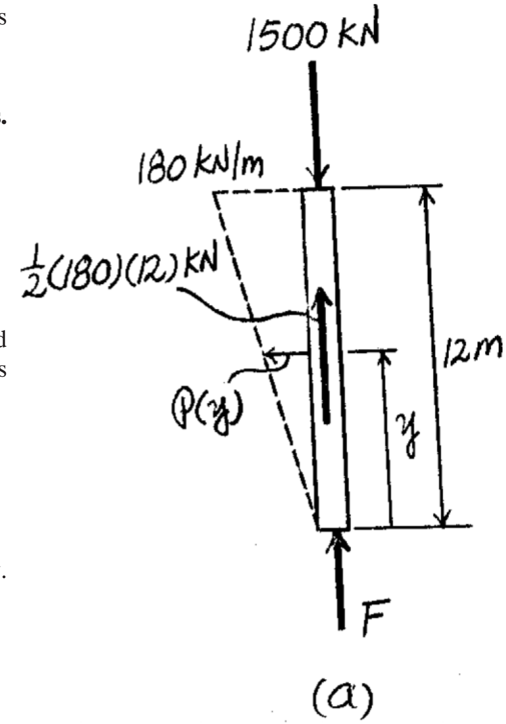
***4–24.** The weight of the kentledge exerts an axial force of $P = 1500$ kN on the 300-mm diameter high-strength concrete bore pile. If the distribution of the resisting skin friction developed from the interaction between the soil and the surface of the pile is approximated as shown, determine the resisting bearing force **F** for equilibrium. Take $p_0 = 180$ kN/m. Also, find the corresponding elastic shortening of the pile. Neglect the weight of the pile.



SOLUTION

Internal Loading: By considering the equilibrium of the pile with reference to its entire free-body diagram shown in Fig. *a*. We have

$$+\uparrow \Sigma F_y = 0; \qquad F + \frac{1}{2}(180)(12) - 1500 = 0 \qquad F = 420 \text{ kN} \qquad \textbf{Ans.}$$



Also,

$$p(y) = \frac{180}{12}y = 15y \text{ kN/m}$$

The normal force developed in the pile as a function of y can be determined by considering the equilibrium of the sectional of the pile with reference to its free-body diagram shown in Fig. *b*.

$$+\uparrow \Sigma F_y = 0; \qquad \frac{1}{2}(15y)y + 420 - P(y) = 0 \qquad P(y) = (7.5y^2 + 420) \text{ kN}$$

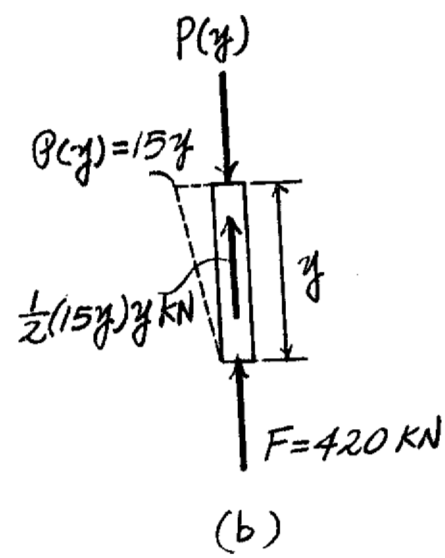
Displacement: The cross-sectional area of the pile is $A = \frac{\pi}{4}(0.3^2) = 0.0225\pi \text{ m}^2$. We have

$$\delta = \int_0^L \frac{P(y)dy}{A(y)E} = \int_0^{12\text{ m}} \frac{(7.5y^2 + 420)(10^3)dy}{0.0225\pi(29.0)(10^9)}$$

$$= \int_0^{12\text{ m}} \left[3.6587(10^{-6})y^2 + 0.2049(10^{-3})\right]dy$$

$$= \left[1.2196(10^{-6})y^3 + 0.2049(10^{-3})y\right]_0^{12\text{ m}}$$

$$= 4.566(10^{-3}) \text{ m} = 4.57 \text{ mm} \qquad \textbf{Ans.}$$

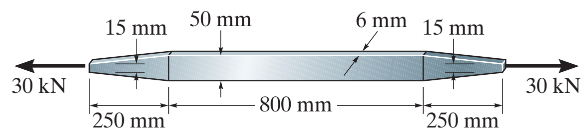


Ans:

$F = 420$ kN, $\delta = 4.57$ mm

4–25.

Determine the elongation of the aluminum strap when it is subjected to an axial force of 30 kN. $E_{al} = 70$ GPa.



SOLUTION

$$\delta = (2)\frac{Ph}{Et(d_2 - d_1)}\ln\frac{d_2}{d_1} + \frac{PL}{AE}$$

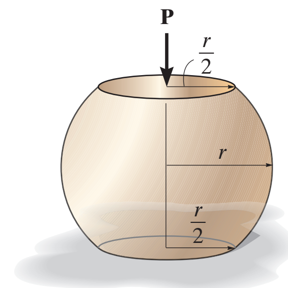
$$= \frac{2(30)(10^3)(250)}{(70)(10^9)(0.006)(0.05 - 0.015)}\left(\ln\frac{50}{15}\right) + \frac{30(10^3)(800)}{(0.006)(0.05)(70)(10^9)}$$

$$= 2.37 \text{ mm}$$ **Ans.**

Ans:
$\delta = 2.37$ mm

4–26.

The ball is truncated at its ends and is used to support the bearing load **P**. If the modulus of elasticity for the material is E, determine the decrease in the ball's height when the load is applied.



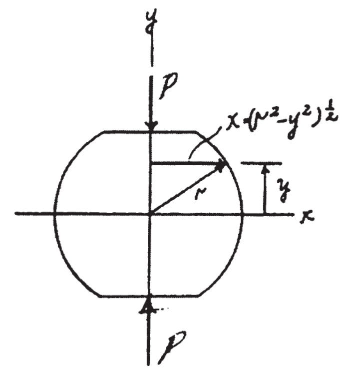
SOLUTION

Displacement:

Geometry:

$A(y) = \pi x^2 = \pi(r^2 - y^2)$

Displacement: When $x = \frac{r}{2}$, $\qquad y = \pm\frac{\sqrt{3}}{2}r$



$$\delta = \int_0^L \frac{P(y)\,dy}{A(y)\,E}$$

$$= \frac{P}{\pi E}\int_{-\frac{\sqrt{3}}{2}r}^{\frac{\sqrt{3}}{2}r} \frac{dy}{r^2 - y^2}$$

$$= \frac{P}{\pi\,E}\left[\frac{1}{2r}\ln\frac{r+y}{r-y}\right]\Bigg|_{-\frac{\sqrt{3}}{2}r}^{\frac{\sqrt{3}}{2}r}$$

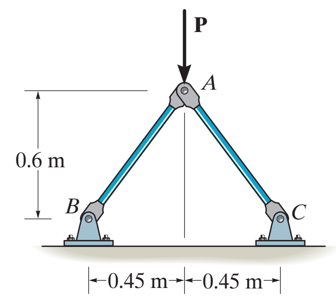
$$= \frac{P}{2\pi\,r\,E}[\,\ln 13.9282 - \ln 0.07180\,]$$

$$= \frac{2.63\,P}{\pi r E}$$

Ans.

Ans:
$\delta = \frac{2.63\,P}{\pi r E}$

4–27. The linkage is made of two pin-connected A-36 steel members, each having a cross-sectional area of 1000 mm^2. If a vertical force of $P = 250$ kN is applied to point A, determine its vertical displacement at A.



SOLUTION

Analysing the equilibrium of Joint A by referring to its FBD, Fig. a,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AC}\left(\frac{3}{5}\right) - F_{AB}\left(\frac{3}{5}\right) = 0 \qquad F_{AC} = F_{AB} = F$$

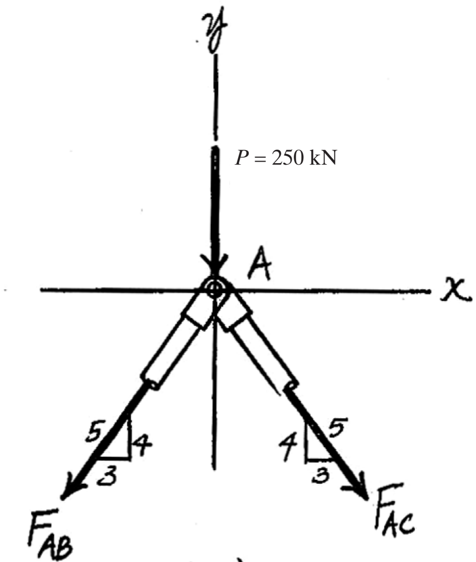
$$+\uparrow \Sigma F_y = 0 \qquad -2F\left(\frac{4}{5}\right) - 250 = 0 \qquad F = -156.25 \text{ kN}$$

The initial length of members AB and AC is $L = \sqrt{0.45^2 + 0.6^2} = 0.75$ m. The axial deformation of members AB and AC is

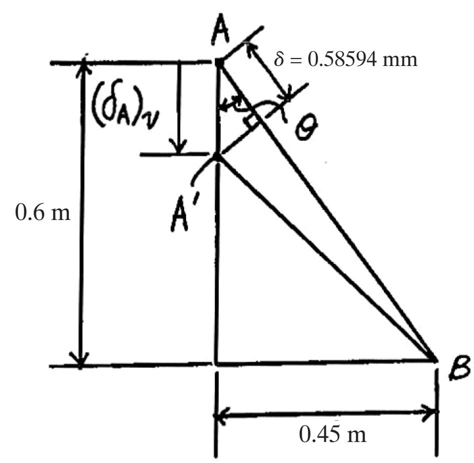
$$\delta = \frac{FL}{AE} = \frac{(-156.25)(10^3)(750)}{[1000(10^{-6})][200(10^9)]} = -0.58594 \text{ mm}$$

The negative sign indicates that end A moves toward B and C. From the geometry shown in Fig. b, $\theta = \tan^{-1}\left(\frac{0.45}{0.6}\right) = 36.87°$. Thus,

$$(\delta_A)_v = \frac{\delta}{\cos\theta} = \frac{0.58594}{\cos 36.87°} = 0.7324 \text{ mm} = 0.732 \text{ mm} \downarrow$$ **Ans.**



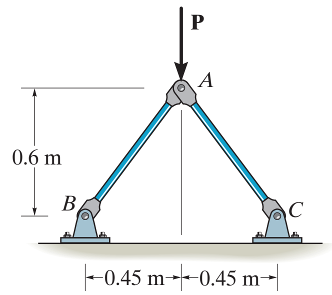
(a)



(b)

Ans.
$(\delta_A)_v = 0.732$ mm $\downarrow$

***4–28.** The linkage is made of two pin-connected A-36 steel members, each having a cross-sectional area of 1000 mm^2. Determine the magnitude of the force **P** needed to displace point *A* 0.625 mm downward.



SOLUTION

Analysing the equilibrium of joint *A* by referring to its *FBD*, Fig. *a*

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AC}\left(\frac{3}{5}\right) - F_{AB}\left(\frac{3}{5}\right) = 0 \qquad F_{AC} = F_{AB} = F$$

$$+\uparrow \Sigma F_y = 0; \quad -2F\left(\frac{4}{5}\right) - P = 0 \qquad F = -0.625\,P$$

The initial length of members *AB* and *AC* are $L = \sqrt{0.45^2 + 0.6^2} = 0.75$ m. The axial deformation of members *AB* and *AC* is

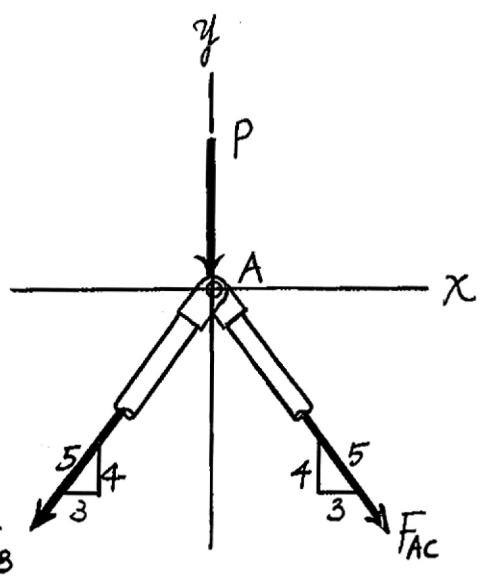
$$\delta = \frac{FL}{AE} = \frac{-0.625P(750)}{[1000(10^{-6})][200(10^9)]} = -2.34375(10^{-6})P$$

The negative sign indicates that end *A* moves toward *B* and *C*. From the geometry shown in Fig. *b*, we obtain $\theta = \tan^{-1}\left(\frac{0.45}{0.6}\right) = 36.87°$. Thus,

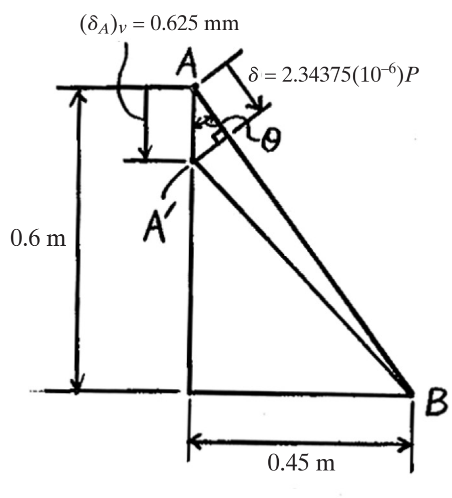
$$(\delta_A)_v = \frac{\delta}{\cos\theta}$$

$$0.625 = \frac{2.34375(10^{-6})P}{\cos 36.87°}$$

$$P = 213.33(10^3)\text{ N} = 213\text{ kN}$$ **Ans.**



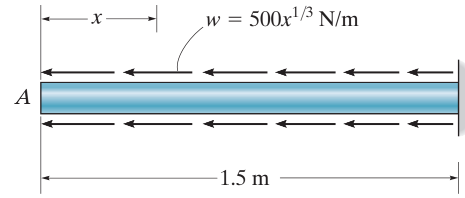
(a)



(b)

Ans.
$P = 213$ kN

4–29. The bar has a cross-sectional area of 1800 mm^2, and $E = 250$ GPa. Determine the displacement of its end A when it is subjected to the distributed loading.



SOLUTION

$$P(x) = \int_0^x w\,dx = 500\int_0^x x^{\frac{1}{3}}\,dx = \left(375x^{4/3}\right)\text{N}$$

$$\delta_A = \int_0^L \frac{P(x)\,dx}{AE} = \int_0^{1.5\text{ m}} \frac{\left(375x^{4/3}\right)dx}{[1800(10^{-6})][250(10^9)]} = 0.8333(10^{-6})\left(\frac{3}{7}x^{7/3}\right)\Bigg|_0^{1.5\text{ m}} = 0.9199(10^{-6})\text{ m} = 0.920\text{ mm}$$

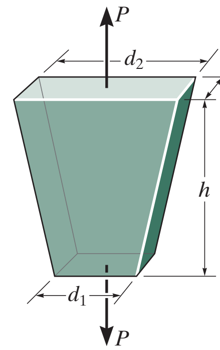
$\delta_A = 9.199(10^{-7})$ m **Ans.**

$= 0.00092$ mm

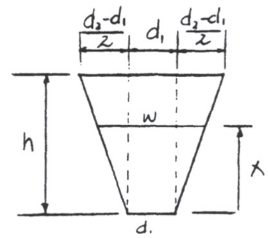
Ans:
$\delta_A = 2.990$ mm

4–30.

Determine the relative displacement of one end of the tapered plate with respect to the other end when it is subjected to an axial load P.



SOLUTION



$$w = d_1 + \frac{d_2 - d_1}{h}x = \frac{d_1 h + (d_2 - d_1)x}{h}$$

$$\delta = \int \frac{P(x)\,dx}{A(x)E} = \frac{P}{E}\int_0^h \frac{dx}{\frac{[d_1 h + (d_2 - d_1)x]t}{h}}$$

$$= \frac{Ph}{E\,t}\int_0^h \frac{dx}{d_1 h + (d_2 - d_1)x}$$

$$= \frac{Ph}{E\,t\,d_1\,h}\int_0^h \frac{dx}{1 + \frac{d_2 - d_1}{d_1 h}x} = \frac{Ph}{E\,t\,d_1\,h}\left(\frac{d_1\,h}{d_2 - d_1}\right)\left[\ln\left(1 + \frac{d_2 - d_1}{d_1\,h}x\right)\right]\Big|_0^h$$

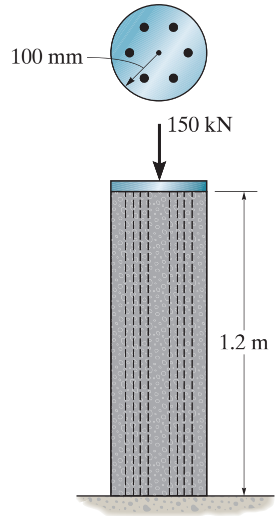
$$= \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\left(1 + \frac{d_2 - d_1}{d_1}\right)\right] = \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\left(\frac{d_1 + d_2 - d_1}{d_1}\right)\right]$$

$$= \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\frac{d_2}{d_1}\right] \qquad \textbf{Ans.}$$

Ans:

$$\delta = \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\frac{d_2}{d_1}\right]$$

4–31. The column is constructed from high-strength concrete and six A-36 steel reinforcing rods. If it is subjected to an axial force of 150 kN, determine the average normal stress in the concrete and in each rod. Each rod has a diameter of 20 mm.



SOLUTION

Equations of Equilibrium:

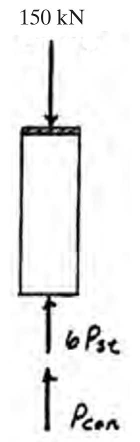
$+\uparrow \Sigma F_y = 0;\qquad 6P_{st} + P_{con} - 150 = 0$ **[1]**

Compatibility:

$$\delta_{st} = \delta_{con}$$

$$\frac{P_{st}(1.2)}{\left[\frac{\pi}{4}(0.02^2)\right][200(10^9)]} = \frac{P_{con}(1.2)}{\left[\frac{\pi}{4}(0.2^2) - 6\left(\frac{\pi}{4}\right)(0.02^2)\right][29.0(10^9)]}$$

$$P_{st} = 0.073368\ P_{con} \qquad \textbf{[2]}$$



Solving Eqs. [1] and [2] yields:

$P_{st} = 7.641$ kN $\qquad P_{con} = 104.152$ kN

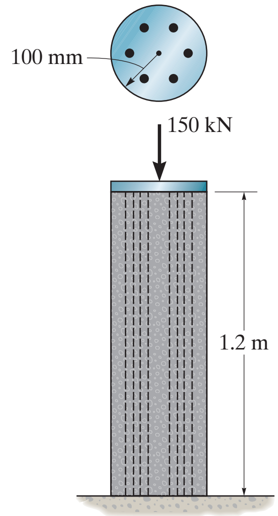
Average Normal Stress:

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{7.641(10^3)}{\frac{\pi}{4}(0.02^2)} = 24.32(10^6)\ \text{N/m}^2 = 24.3\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_{con} = \frac{P_{con}}{A_{con}} = \frac{104.15(10^3)}{\frac{\pi}{4}(0.2^2) - 6\left(\frac{\pi}{4}\right)(0.02^2)} = 3.527(10^6)\ \text{N/m}^2 = 3.53\ \text{MPa} \qquad \textbf{Ans.}$$

Ans.
$\sigma_{st} = 24.3$ MPa, $\sigma_{con} = 3.53$ MPa

***4–32.** The column is constructed from high-strength concrete and six A-36 steel reinforcing rods. If it is subjected to an axial force of 150 kN, determine the required diameter of each rod so that one-fourth of the load is carried by the concrete and three-fourths by the steel.



SOLUTION

Equilibrium: The force of 150 kN is required to distribute in such a manner that 3/4 of the force is carried by steel and 1/4 of the force is carried by concrete. Hence

$$P_{st} = \frac{3}{4}(150) = 112.5\text{ kN} \qquad P_{\text{con}} = \frac{1}{4}(150) = 37.5\text{ kN}$$

Compatibility:

$$\delta_{st} = \delta_{\text{con}}$$

$$\frac{P_{st}L}{A_{st}E_{st}} = \frac{P_{\text{con}}\,L}{A_{\text{con}}\,E_{\text{con}}}$$

$$A_{st} = \frac{P_{st}A_{\text{con}}E_{\text{con}}}{P_{\text{con}}E_{st}}$$

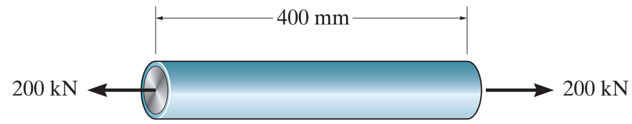
$$6\left(\frac{\pi}{4}\right)d^2 = \frac{(112.5)\left[\frac{\pi}{4}(0.2^2) - 6\left(\frac{\pi}{4}\right)(0.02^2)\right]\left[29(10^9)\right]}{(37.5)\left[200(10^9)\right]}$$

$$d = 0.05221\text{ m} = 52.2\text{ mm}$$ **Ans.**

Ans.
$d = 52.2$ mm

4–33.

The A-36 steel pipe has a 6061-T6 aluminum core. It is subjected to a tensile force of 200 kN. Determine the average normal stress in the aluminum and the steel due to this loading. The pipe has an outer diameter of 80 mm and an inner diameter of 70 mm.



SOLUTION

Equations of Equilibrium:

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad P_{al} + P_{st} - 200 = 0 \qquad \textbf{(1)}$$

Compatibility:

$$\delta_{al} = \delta_{st}$$

$$\frac{P_{al}(400)}{\frac{\pi}{4}(0.07^2)(68.9)(10^9)} = \frac{P_{st}(400)}{\frac{\pi}{4}(0.08^2 - 0.07^2)(200)(10^9)}$$

$$P_{al} = 1.125367\, P_{st} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$P_{st} = 94.10 \text{ kN} \qquad P_{al} = 105.90 \text{ kN}$$

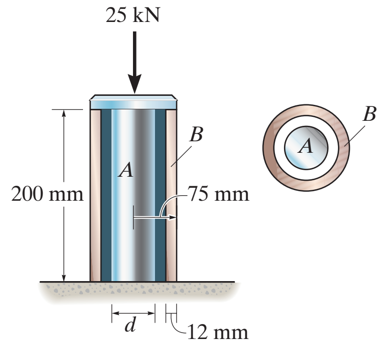
Average Normal Stress:

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{105.90(10^3)}{\frac{\pi}{4}(0.07^2)} = 27.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{94.10(10^3)}{\frac{\pi}{4}(0.08^2 - 0.07^2)} = 79.9 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma_{al} = 27.5$ MPa,
$\sigma_{st} = 79.9$ MPa

4–34. The 304 stainless steel post A has a diameter of $d = 50$ mm and is surrounded by a red brass C83400 tube B. Both rest on the rigid surface. If a force of 25 kN is applied to the rigid cap, determine the average normal stress developed in the post and the tube.



SOLUTION

Equations of Equilibrium:

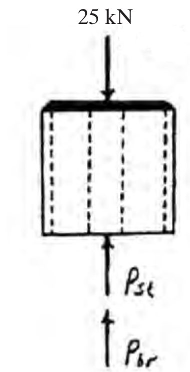
$$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{br} - 25 = 0 \qquad \textbf{[1]}$$

Compatibility:

$$\delta_{st} = \delta_{br}$$

$$\frac{P_{st}(200)}{\left[\frac{\pi}{4}(0.05^2)\right][193(10^9)]} = \frac{P_{b\pi}(200)}{\left[\frac{\pi}{4}(0.15^2 - 0.126^2)\right][101(10^9)]}$$

$$P_{st} = 0.72120\, P_{br} \qquad \textbf{[2]}$$



Solving Eqs. [1] and [2] yields:

$$P_{br} = 14.525 \text{ kN} \qquad P_{st} = 10.475 \text{ kN}$$

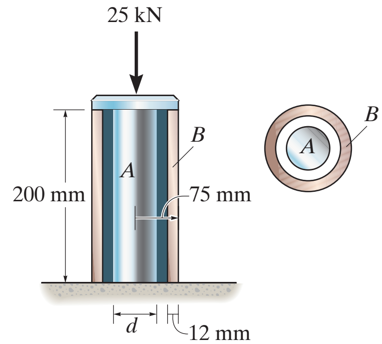
Average Normal Stress:

$$\sigma_{br} = \frac{P_{br}}{A_{br}} = \frac{14.525(10^3)}{\frac{\pi}{4}(0.15^2 - 0.126^2)} = 2.792(10^6)\text{ N/m}^2 = 2.79\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{10.475(10^3)}{\frac{\pi}{4}(0.05^2)} = 5.335(10^6)\text{ N/m}^2 = 5.34\text{ MPa} \qquad \textbf{Ans.}$$

Ans.
$\sigma_{br} = 2.79$ MPa, $\sigma_{st} = 5.34$ MPa

4–35. The 304 stainless steel post A is surrounded by a red brass C83400 tube B. Both rest on the rigid surface. If a force of 25 kN is applied to the rigid cap, determine the required diameter d of the steel post so that the load is shared equally between the post and tube.



SOLUTION

Equilibrium: The force of 25 kN is shared equally by the brass and steel. Hence

$$P_{st} = P_{br} = P = 12.5\text{ kN}$$

Compatibility:

$$\delta_{st} = \delta_{br}$$

$$\frac{PL}{A_{st}E_{st}} = \frac{PL}{A_{br}E_{br}}$$

$$A_{st} = \frac{A_{br}E_{br}}{E_{st}}$$

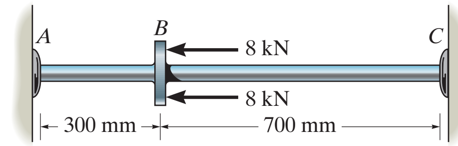
$$\left(\frac{\pi}{4}\right)d^2 = \frac{\left[\frac{\pi}{4}(0.15^2 - 0.126^2)\right]\left[101(10^9)\right]}{193(10^9)}$$

$$d = 0.05888\text{ m} = 58.9\text{ mm}$$ **Ans.**

Ans.
$d = 58.9$ mm

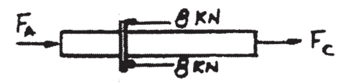
***4–36.**

The A-36 steel pipe has an outer radius of 20 mm and an inner radius of 15 mm. If it fits snugly between the fixed walls before it is loaded, determine the reaction at the walls when it is subjected to the load shown.



SOLUTION

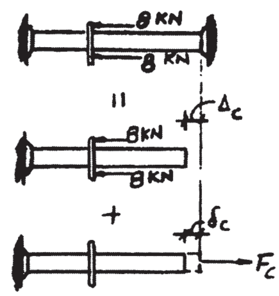
$\xrightarrow{+} \Sigma F_x = 0; \qquad F_A + F_C - 16 = 0$ **(1)**



By superposition:

$(\xrightarrow{+}) \qquad 0 = -\Delta_C + \delta_C$

$$0 = \frac{-16\,(300)}{AE} + \frac{F_C\,(1000)}{AE}$$



$F_C = 4.80$ kN **Ans.**

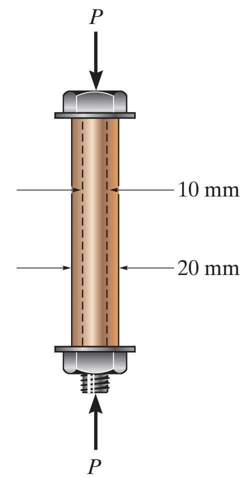
From Eq. (1),

$F_A = 11.2$ kN **Ans.**

Ans:
$F_C = 4.80$ kN,
$F_A = 11.2$ kN

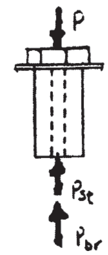
4–37.

The 10-mm-diameter steel bolt is surrounded by a bronze sleeve. The outer diameter of this sleeve is 20 mm, and its inner diameter is 10 mm. If the yield stress for the steel is $(\sigma_Y)_{st} = 640$ MPa, and for the bronze $(\sigma_Y)_{br} = 520$ MPa, determine the magnitude of the largest elastic load P that can be applied to the assembly. $E_{st} = 200$ GPa, $E_{br} = 100$ GPa.



SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{br} - P = 0 \qquad \textbf{(1)}$



Assume failure of bolt:

$$P_{st} = (\sigma_Y)_{st}(A) = 640(10^6)\left(\frac{\pi}{4}\right)(0.01^2)$$

$$= 50265.5 \text{ N}$$

$$\delta_{st} = \delta_{br}$$

$$\frac{P_{st}L}{\frac{\pi}{4}(0.01^2)(200)(10^9)} = \frac{P_{br}L}{\frac{\pi}{4}(0.02^2 - 0.01^2)(100)(10^9)}$$

$$P_{st} = 0.6667\, P_{br}$$

$$50\,265.5 = 0.6667\, P_{br}$$

$$P_{br} = 75398.2 \text{ N}$$

From Eq. (1)

$$P = 50265.5 + 75398.2$$

$$= 125663.7 \text{ N} = 126 \text{ kN} \qquad \text{(controls)} \qquad \textbf{Ans.}$$

Assume failure of sleeve:

$$P_{br} = (\sigma_Y)_{br}(A) = 520(10^6)\left(\frac{\pi}{4}\right)(0.02^2 - 0.01^2) = 122\,522.11 \text{ N}$$

$$P_{st} = 0.6667\, P_{br}$$

$$= 0.6667(122\,522.11)$$

$$= 81\,681.4 \text{ N}$$

From Eq. (1),

$$P = 122\,522.11 + 81\,681.4$$

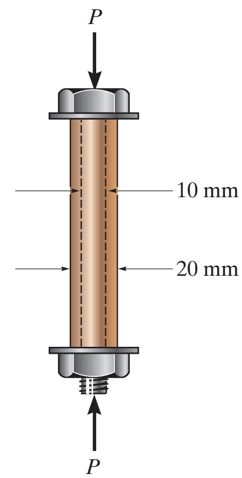
$$= 204\,203.52 \text{ N}$$

$$= 204 \text{ kN}$$

Ans:
$P = 126$ kN

4–38.

The 10-mm-diameter steel bolt is surrounded by a bronze sleeve. The outer diameter of this sleeve is 20 mm, and its inner diameter is 10 mm. If the bolt is subjected to a compressive force of $P = 20$ kN, determine the average normal stress in the steel and the bronze. $E_{st} = 200$ GPa, $E_{br} = 100$ GPa.



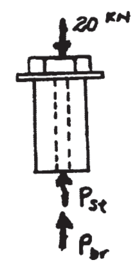
SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{br} - 20 = 0 \qquad$ **(1)**

$$\delta_{st} = \delta_{br}$$

$$\frac{P_{st}L}{\frac{\pi}{4}(0.01^2)(200)(10^9)} = \frac{P_{br}L}{\frac{\pi}{4}(0.02^2 - 0.01^2)(100)(10^9)}$$

$$P_{st} = 0.6667\,P_{br} \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

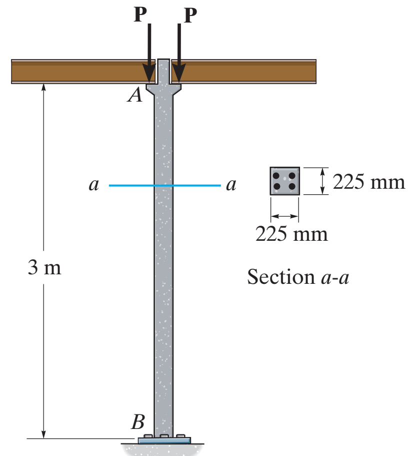
$P_{st} = 8$ kN $\qquad P_{br} = 12$ kN

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{8(10^3)}{\frac{\pi}{4}(0.01^2)} = 102 \text{ MPa}$$ **Ans.**

$$\sigma_{br} = \frac{P_{br}}{A_{br}} = \frac{12(10^3)}{\frac{\pi}{4}(0.02^2 - 0.01^2)} = 50.9 \text{ MPa}$$ **Ans.**

Ans:
$\sigma_{st} = 102$ MPa,
$\sigma_{br} = 50.9$ MPa

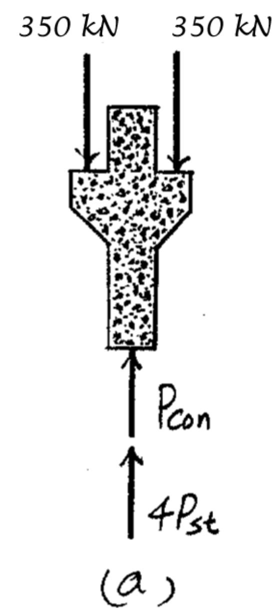
4–39. If column AB is made from high strength pre-cast concrete and reinforced with four 20 mm diameter A-36 steel rods, determine the average normal stress developed in the concrete and in each rod. Set $P = 350$ kN.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the cut part of the concrete column shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad P_{con} + 4P_{st} - 2(350) = 0 \qquad (1)$$



Compatibility Equation: Since the steel bars and the concrete are firmly bonded, their deformation must be the same. Thus,

$$\delta_{con} = \delta_{st}$$

$$\frac{P_{con}(3)}{\left[0.225(0.225) - 4\left(\frac{\pi}{4}\right)(0.02^2)\right][29.0(10^9)]} = \frac{P_{st}(3)}{\left[\frac{\pi}{4}(0.2^2)\right][200(10^9)]}$$

$$P_{con} = 22.7859 P_{st} \qquad (2)$$

Solving Eqs. (1) and (2),

$$P_{st} = 26.13 \text{ kN} \qquad P_{con} = 595.47 \text{ kN}$$

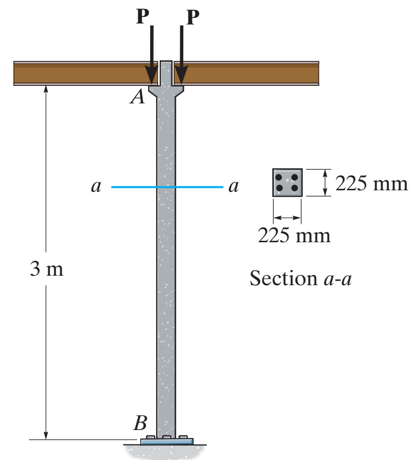
Normal Stress: Applying Eq. (1-6),

$$\sigma_{con} = \frac{P_{con}}{A_{con}} = \frac{595.47(10^3)}{0.225(0.225) - 4\left(\frac{\pi}{4}\right)(0.02^2)} = 12.06(10^6)\ \text{N/m}^2 = 12.1 \text{ Mpa} \qquad \textbf{Ans.}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{26.13(10^3)}{\left(\frac{\pi}{4}\right)(0.02^2)} = 83.18(10^6)\ \text{N/m}^2 = 83.2 \text{ Mpa} \qquad \textbf{Ans.}$$

Ans:
$\sigma_{con} = 12.1$ MPa, $\sigma_{st} = 83.2$ MPa

***4–40.** If column AB is made from high strength pre-cast concrete and reinforced with four 20 mm diameter A-36 steel rods, determine the maximum allowable floor loadings **P**. The allowable normal stress for the high strength concrete and the steel are $(\sigma_{allow})_{con} = 18$ MPa and $(\sigma_{allow})_{st} = 170$ MPa, respectively.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the cut part of the concrete column shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad P_{con} + 4P_{st} - 2P = 0 \qquad (1)$$

Compatibility Equation: Since the steel bars and the concrete are firmly bonded, their deformation must be the same. Thus,

$$\delta_{con} = \delta_{st}$$

$$\frac{P_{con}(3)}{\left[0.225(0.225) - 4\left(\frac{\pi}{4}\right)(0.02^2)\right][29.0(10^9)]} = \frac{P_{st}(3)}{\left[\frac{\pi}{4}(0.2^2)\right][200(10^9)]}$$

$$P_{con} = 22.7859P_{st} \qquad (2)$$

Solving Eqs. (1) and (2),

$$P_{st} = 0.07467P \qquad P_{con} = 1.7013P$$

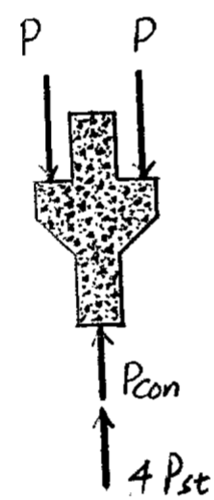
Allowable Normal Stress:

$$(\sigma_{con})_{allow} = \frac{P_{con}}{A_{con}}; \qquad 18(10^6) = \frac{1.7013P}{0.225(0.225) - 4\left(\frac{\pi}{4}\right)(0.02^2)}$$

$$P = 522.31(10^3)\text{ N} = 522\text{ kN (controls)} \qquad \textbf{Ans.}$$

$$(\sigma_{st})_{allow} = \frac{P_{st}}{A_{st}}; \qquad 170(10^6) = \frac{0.07467P}{\frac{\pi}{4}(0.02^2)}$$

$$P = 715.28(10^3)\text{ N} = 715.28\text{ kN}$$

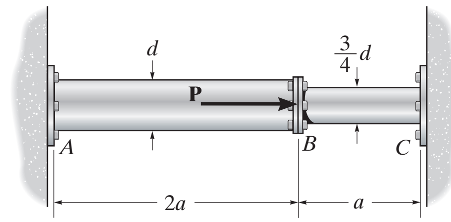


(a)

Ans:
$P = 522$ kN

4–41.

Determine the support reactions at the rigid supports A and C. The material has a modulus of elasticity of E.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. a,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad P - F_A - F_C = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Using the method of superposition, Fig. b,

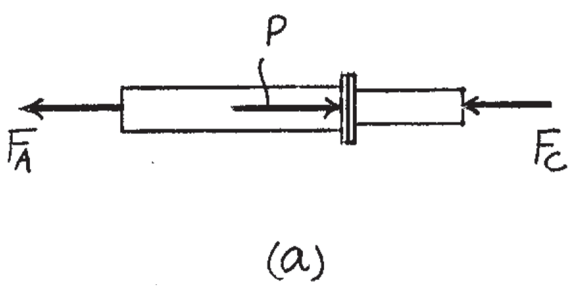
$$(\xrightarrow{+}) \qquad \delta = \delta_P - \delta_{F_C}$$

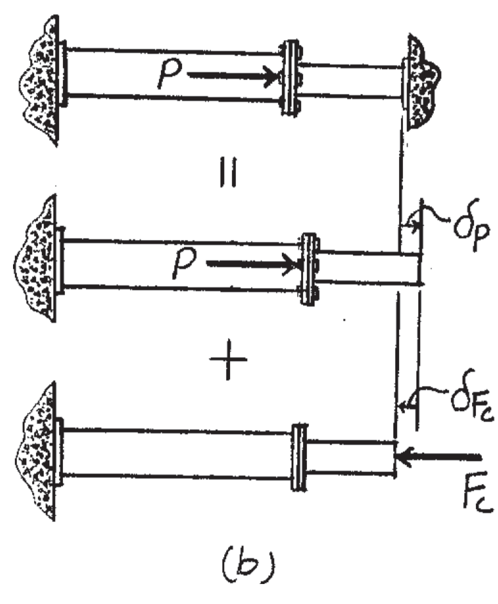
$$0 = \frac{P(2a)}{\left(\frac{\pi}{4}d^2\right)E} - \left[\frac{F_C a}{\frac{\pi}{4}\left(\frac{3}{4}d\right)^2 E} + \frac{F_C(2a)}{\left(\frac{\pi}{4}d^2\right)E}\right]$$

$$F_C = \frac{9}{17}P \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$F_A = \frac{8}{17}P \qquad \textbf{Ans.}$$

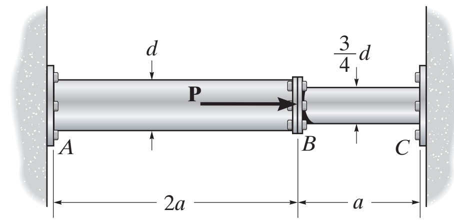




Ans:
$F_C = \frac{9}{17}P$,
$F_A = \frac{8}{17}P$

4–42.

If the supports at A and C are flexible and have a stiffness k, determine the support reactions at A and C. The material has a modulus of elasticity of E.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. a,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P - F_A - F_C = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Using the method of superposition, Fig. b,

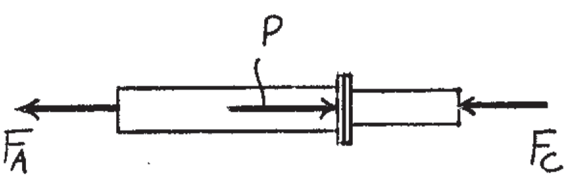
$$(\overset{+}{\rightarrow}) \qquad \delta_C = \delta_P - \delta_{F_C}$$

$$\frac{F_C}{k} = \left[\frac{P(2a)}{\left(\frac{\pi}{4}d^2\right)E} + \frac{P}{k}\right] - \left[\frac{F_C a}{\frac{\pi}{4}\left(\frac{3}{4}d\right)^2 E} + \frac{F_C(2a)}{\left(\frac{\pi}{4}d^2\right)E} + \frac{F_C}{k}\right]$$

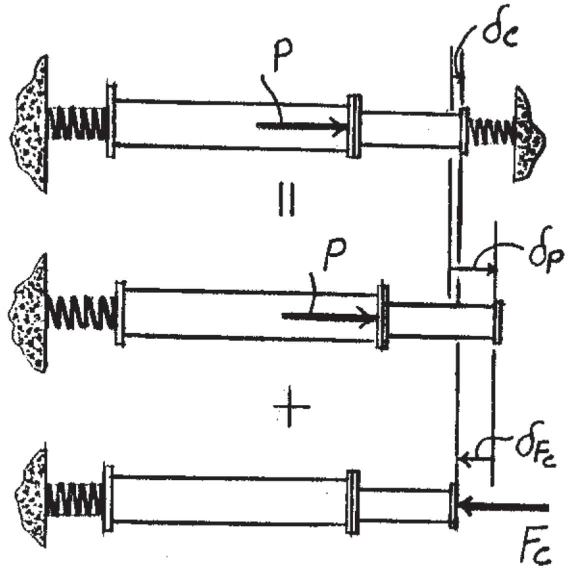
$$F_C = \left[\frac{9(8ka + \pi d^2 E)}{136ka + 18\pi d^2 E}\right]P \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$F_A = \left(\frac{64ka + 9\pi d^2 E}{136ka + 18\pi d^2 E}\right)P \qquad \textbf{Ans.}$$



(a)



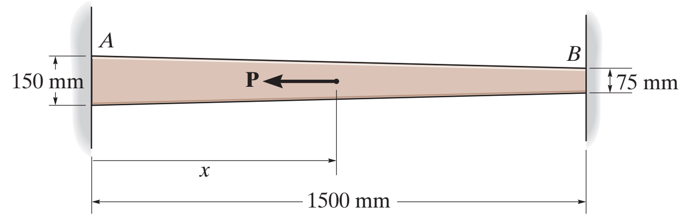
(b)

Ans:

$$F_C = \left[\frac{9(8ka + \pi d^2 E)}{136ka + 18\pi d^2 E}\right]P,$$

$$F_A = \left(\frac{64ka + 9\pi d^2 E}{136ka + 18\pi d^2 E}\right)P$$

4–43. The tapered member is fixed connected at its ends A and B and is subjected to a load $P = 35$ kN at $x = 750$ mm Determine the reactions at the supports. The material is 50 mm thick and is made froom 2014-T6 aluminum.



SOLUTION

Equilibrium. Refering to the FBD of the member, Fig. a

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_A + F_B - 35 = 0 \qquad \textbf{(1)}$$

Compatibility. From the geometry shown in Fig. b

$$y = 2\left[0.0375 + \left(\frac{1.5 - x}{1.5}\right)(0.0375)\right] = [0.05(3 - x)] \text{ m}$$

Thus, the cross-sectional area of the member as a function of x is

$$A = (0.05)[0.05(3 - x)] = [0.0025(3 - x)] \text{ m}^2$$

It is required that

$$\delta_{A/B} = 0$$

$$-\int_0^{0.75\text{ m}} \frac{F_A\,dx}{0.0025(3 - x)(E)} + \int_{0.75\text{ m}}^{1.5\text{ m}} \frac{F_B\,dx}{0.0025(3 - x)(E)} = 0$$

$$-\int_0^{0.75\text{ m}} \frac{F_A\,dx}{3 - x} + \int_{0.75\text{ m}}^{1.5\text{ m}} \frac{F_B dx}{3 - x} = 0$$

$$F_A \ln(3 - x)\Big|_0^{0.75\text{ m}} - F_B \ln(3 - x)\Big|_{0.75\text{ m}}^{1.5\text{ m}} = 0$$

$$F_A \ln\left(\frac{2.25}{3}\right) - F_B \ln\left(\frac{1.5}{2.25}\right) = 0$$

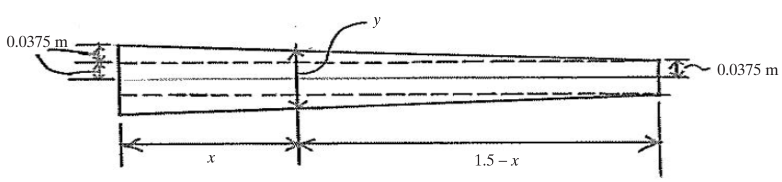
$$F_A = 1.4094 F_B \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$F_A = 20.47 = 20.5 \text{ kN} \qquad \textbf{Ans.}$$

$$F_B = 14.53 = 14.5 \text{ kN} \qquad \textbf{Ans.}$$

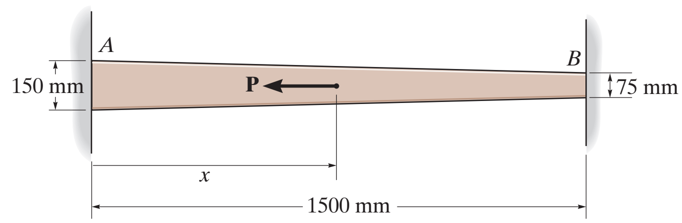
(a)



(b)

Ans.
$y = 75 - 0.025x,\ F_A = 20.47 \text{ kN},\ F_B = 14.53 \text{ kN}$

***4–44.** The tapered member is fixed connected at its ends A and B and is subjected to a load **P**. Determine the location x of the load and its greatest magnitude so that the average normal stress in the bar does not exceed $\sigma_{\text{allow}} = 28$ MPa. The number is 50 mm thick.



SOLUTION

Equilibrium. Refering to the FBD of the member, Fig. a

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_A + F_B - P = 0 \qquad \textbf{(1)}$$

Compatibility. From the geometry shown in Fig. b

$$y = 2\left[0.0375 + \left(\frac{1.5 - x}{1.5}\right)(0.0375)\right] = [0.05(3 - x)] \text{ m}$$

Thus, the cross-sectional area of the member as a function of x is

$$A = (0.05)[0.05(3 - x)] = [0.0025(3 - x)] \text{ m}^2$$

It is required that

$$\delta_{A/B} = 0$$

$$-\int_0^x \frac{F_A\,dx}{0.0025(3 - x)(E)} + \int_x^{1.5\text{ m}} \frac{F_B\,dx}{0.0025(3 - x)(E)} = 0$$

$$-\int_0^x \frac{F_A\,dx}{3 - x} + \int_x^{1.5\text{ m}} \frac{F_B dx}{3 - x} = 0$$

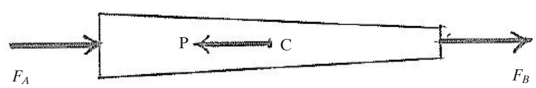
$$F_A \ln(3 - x)\Big|_0^x - F_B \ln(3 - x)\Big|_x^{1.5\text{ m}} = 0$$

$$F_A \ln\left(\frac{3 - x}{3}\right) - F_B \ln\left(\frac{1.5}{3 - x}\right) = 0 \qquad \textbf{(2)}$$

The maximum normal stress in segment AC occurs at position x and in segment BC occurs at B. Then

$$\sigma_{\text{allow}} = \frac{F_A}{A}; \quad 28(10^6) = \frac{F_A}{0.0025(3 - x)} \qquad F_A = [70(10^3)(3 - x)] \text{ N}$$

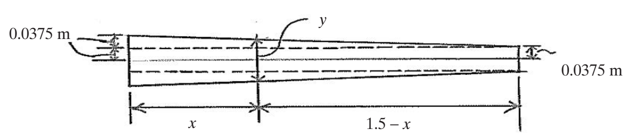
$$\sigma_{\text{allow}} = \frac{F_B}{A}; \quad 28(10^6) = \frac{F_B}{(0.05)(0.075)} \qquad F_B = 105(10^3) \text{ N}$$



(a)

Substitute these results into Eq. (2)

$$70(10^3)(3 - x)\ln\left(\frac{3 - x}{3}\right) - 105(10^3)\ln\left(\frac{1.5}{3 - x}\right) = 0$$



(b)

$$\ln\left(\frac{3 - x}{3}\right)^{3 - x} = \ln\left(\frac{1.5}{3 - x}\right)^{1.5}$$

$$\left(\frac{3 - x}{3}\right)^{3 - x} = \left(\frac{1.5}{3 - x}\right)^{1.5}$$

Solving numerically,

$x = 0.721698 \text{ m} = 722 \text{ m}$ **Ans.**

Then,

$$F_A = [70(10^3)(3 - 0.7216980) = 159.48(10^3) \text{ N}$$

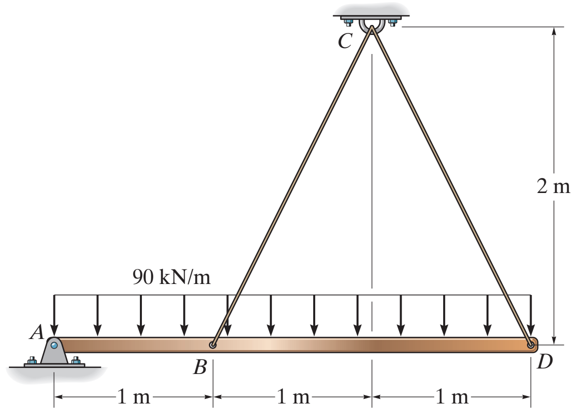
Substitute the results of F_A and F_B into Eq. (1)

$$159.48(10^3) + 105(10^3) - P = 0$$

$P = 264.48(10^3) = 264 \text{ N}$ **Ans.**

Ans.
$x = 722$ m, $P = 264$ N

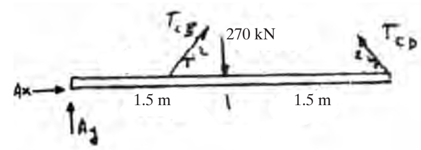
4–45. The rigid bar supports the uniform distributed load of 90 kN/m. Determine the force in each cable if each cable has a cross-sectional area of 36 mm^2, and $E = 200$ GPa.



SOLUTION

Equilibrium:

$$\zeta + \Sigma M_A = 0; \qquad T_{CB}\left(\frac{2}{\sqrt{5}}\right)(1) - 270(1.5) + T_{CD}\left(\frac{2}{\sqrt{5}}\right)(3) = 0 \qquad \textbf{(1)}$$



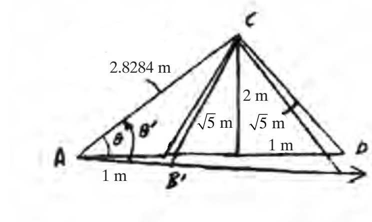
Geometry and compatibility:

$$\theta = \tan^{-1}\frac{2}{2} = 45^\circ$$

$$L^2_{B'C} = (1)^2 + (2.8284)^2 - 2(1)(2.8284)\cos\theta'$$

Also,

$$L^2_{D'C} = (3)^2 + (2.8284)^2 - 2(3)(2.8284)\cos\theta' \qquad \textbf{(2)}$$



Thus, eliminating $\cos\theta'$.

$$-L^2_{B'C}(0.176778) + 1.590978 = -L^2_{D'C}(0.058926) + 1.001735$$

$$-L^2_{B'C}(0.176778) = 0.058926L^2_{D'C} + 0.589243$$

$$-L^2_{B'C} = 0.3333L^2_{D'C} + 3.333$$

But,

$$L_{B'C} = \sqrt{5} + \delta_{BC}, \qquad L_{D'C} = \sqrt{5} + \delta_{DC},$$

Neglect squares or δ'_B since small strain occurs.

$$L^2_{B'C} = (\sqrt{5} + \delta_{BC})^2 = 5 + 2\sqrt{5}\,\delta_{BC}$$

$$L^2_{D'C} = (\sqrt{5} + \delta_{DC})^2 = 5 + 2\sqrt{5}\,\delta_{DC}$$

$$5 + 2\sqrt{5}\delta_{BC} = 0.3333(5 + 2\sqrt{5}\delta_{DC}) + 3.333$$

$$2\sqrt{5}\delta_{BC} = 0.3333(2\sqrt{5}\delta_{DC})$$

$$\delta_{DC} = 3\delta_{BC}$$

Thus,

$$\frac{T_{CD}\sqrt{5}}{AE} = 3\frac{T_{CB}\sqrt{5}}{AE}$$

$$T_{CD} = 3\,T_{CB}$$

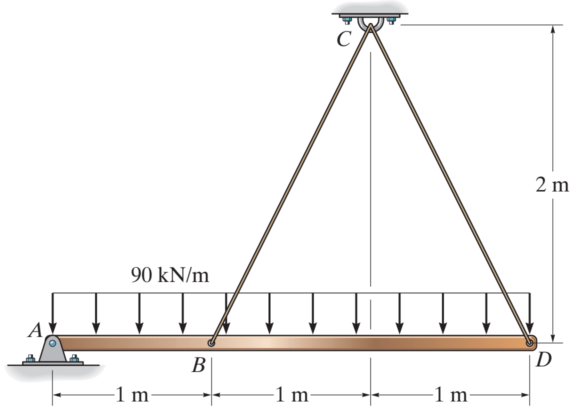
From Eq. (1).

$$T_{CD} = 135.84 \text{ kN} = 136 \text{ kN} \qquad \textbf{Ans.}$$

$$T_{CB} = 45.28 \text{ kN} = 45.3 \text{ kN} \qquad \textbf{Ans.}$$

Ans.
$T_{CD} = 136$ kN, $T_{CB} = 45.3$ kN

4–46. The rigid bar is originally horizontal and is supported by two cables each having a cross-sectional area of 36 mm^2, and E = 200 GPa. Determine the slight rotation of the bar when the uniform load is applied.



SOLUTION

See solution of Prob. 4-51.

$$T_{CD} = 135.84 \text{ kN}$$

$$\delta_{DC} = \frac{T_{CD}L_{CD}}{AE} = \frac{(135.84)(10^3)\sqrt{5}}{[36(10^{-6})][200(10^9)]} = 0.04219 \text{ m}$$

Using Eq. (2) of Prob. 4-51,

$$(\sqrt{5} + 0.04219)^2 = (3)^2 + (2.8284)^2 - 2(3)(2.8284)\cos\theta'$$

$$\theta' = 45.9023°$$

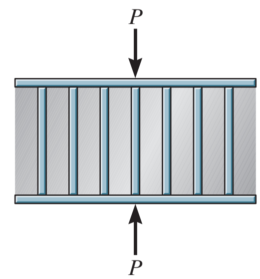
Thus,

$$\Delta\theta = 45.9023° - 45° = 0.9023° = 0.902° \qquad \textbf{Ans.}$$

Ans.
$\Delta\theta = 0.902°$

4–47.

The specimen represents a filament-reinforced matrix system made from plastic (matrix) and glass (fiber). If there are n fibers, each having a cross-sectional area of A_f and modulus of E_f, embedded in a matrix having a cross-sectional area of A_m and modulus of E_m, determine the stress in the matrix and in each fiber when the force P is applied on the specimen.

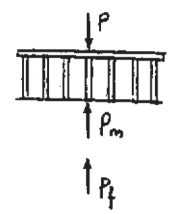


SOLUTION

$$+\uparrow \Sigma F_y = 0; \qquad -P + P_m + P_f = 0 \qquad \textbf{(1)}$$

$$\delta_m = \delta_f$$

$$\frac{P_m L}{A_m E_m} = \frac{P_f L}{n A_f E_f}; \qquad P_m = \frac{A_m E_m}{n A_f E_f} P_f \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

$$P_m = \frac{A_m E_m}{n A_f E_f + A_m E_m} P; \qquad P_f = \frac{n A_f E_f}{n A_f E_f + A_m E_m} P$$

Normal stress:

$$\sigma_m = \frac{P_m}{A_m} = \frac{\left(\frac{A_m E_m}{n A_f E_f + A_m E_m} - P\right)}{A_m} = \frac{E_m}{n A_f E_f + A_m E_m} P \qquad \textbf{Ans.}$$

$$\sigma_f = \frac{P_f}{n A_f} = \frac{\left(\frac{n A_f E_f}{n A_f E_f + A_m E_m} P\right)}{n A_f} = \frac{E_f}{n A_f E_f + A_m E_m} P \qquad \textbf{Ans.}$$

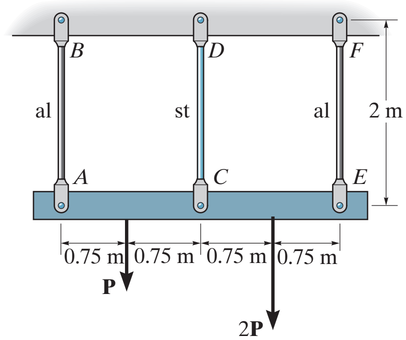
Ans:

$$\sigma_m = \frac{E_m}{n A_f E_f + A_m E_m} P,$$

$$\sigma_f = \frac{E_f}{n A_f E_f + A_m E_m} P$$

***4–48.**

The rigid beam is supported by the three suspender bars. Bars AB and EF are made of aluminum and bar CD is made of steel. If each bar has a cross-sectional area of 450 mm^2, determine the maximum value of P if the allowable stress is $(\sigma_{\text{allow}})_{\text{st}} = 200$ MPa for the steel and $(\sigma_{\text{allow}})_{\text{al}} = 150$ MPa for the aluminum. $E_{\text{st}} = 200$ GPa, $E_{\text{al}} = 70$ GPa.



SOLUTION

Equation of Equilibrium: Referring to the FBD of the rigid beam Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{CD}(1.5) + F_{EF}(3) - P(0.75) - 2P(2.25) = 0$$

$$1.5\,F_{CD} + 3\,F_{EF} = 5.25\,P \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_E = 0; \qquad 2\,P(0.75) + P(2.25) - F_{CD}(1.5) - F_{AB}(3) = 0$$

$$1.5\,F_{CD} + 3\,F_{AB} = 3.75\,P \qquad \textbf{(2)}$$

Compatibility: Referring to the displacement diagram of the rigid beam, Fig. *b*,

$$\frac{\delta_{CD} - \delta_{AB}}{1.5} = \frac{\delta_{EF} - \delta_{AB}}{3}$$

$$2\delta_{CD} = \delta_{EF} + \delta_{AB}$$

$$2\left(\frac{F_{CD}L}{A\left[200(10^9)\right]}\right) = \frac{F_{EF}L}{A\left[70(10^9)\right]} + \frac{F_{AB}L}{A\left[70(10^9)\right]}$$

$$F_{CD} = \frac{10}{7}(F_{EF} + F_{AB}) \qquad \textbf{(3)}$$

Solving Eqs. (1), (2) and (3),

$$F_{EF} = 0.8676\,P \qquad F_{AB} = 0.3676\,P \qquad F_{CD} = 1.7647\,P$$

Assuming that bar EF fails. Then

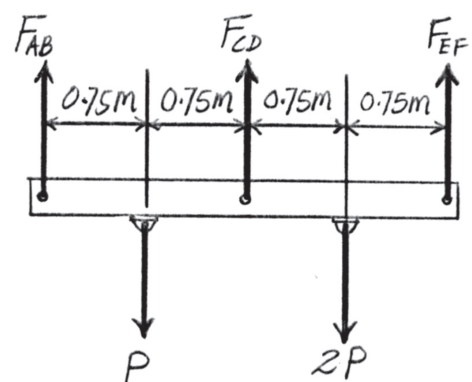
$$(\sigma_{\text{allow}})_{\text{al}} = \frac{F_{EF}}{A}; \qquad 150(10^6) = \frac{0.8676\,P}{450(10^{-6})}$$

$$P = 77.80(10^3)\text{ N} = 77.80\text{ kN}$$

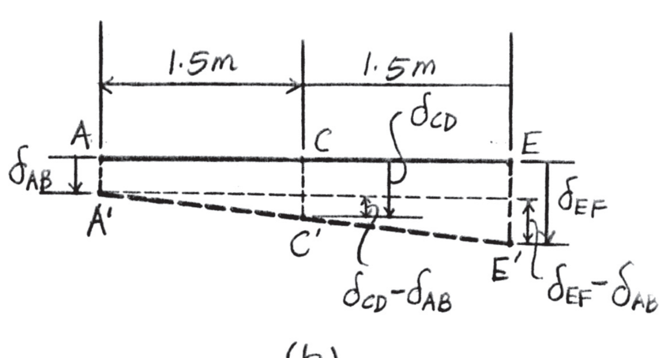
Assuming that bar CD fails. Then

$$(\sigma_{\text{allow}})_{\text{st}} = \frac{F_{CD}}{A}; \qquad 200(10^6) = \frac{1.7647\,P}{450(10^{-6})}$$

$$P = 51.0(10^3)\text{ N} = 51.0\text{ kN} \qquad \text{(control!)} \ \textbf{Ans.}$$



(a)

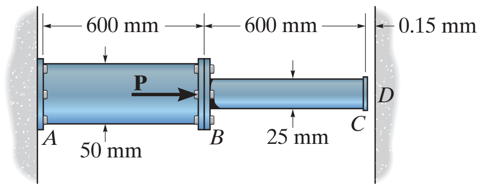


(b)

Ans:
$P = 51.0$ kN

4–49.

If the gap between C and the rigid wall at D is initially 0.15 mm, determine the support reactions at A and D when the force $P = 200$ kN is applied. The assembly is made of solid A-36 steel cylinders.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. *a*,

$\xrightarrow{+} \Sigma F_x = 0; \qquad 200(10^3) - F_D - F_A = 0 \qquad$ **(1)**

Compatibility Equation: Using the method of superposition, Fig. *b*,

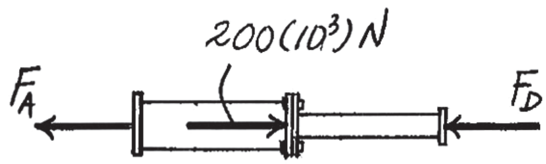
$(\xrightarrow{+}) \qquad \delta = \delta_P - \delta_{F_D}$

$$0.15 = \frac{200(10^3)(600)}{\frac{\pi}{4}(0.05^2)(200)(10^9)} - \left[\frac{F_D\,(600)}{\frac{\pi}{4}(0.05^2)(200)(10^9)} + \frac{F_D\,(600)}{\frac{\pi}{4}(0.025^2)(200)(10^9)}\right]$$

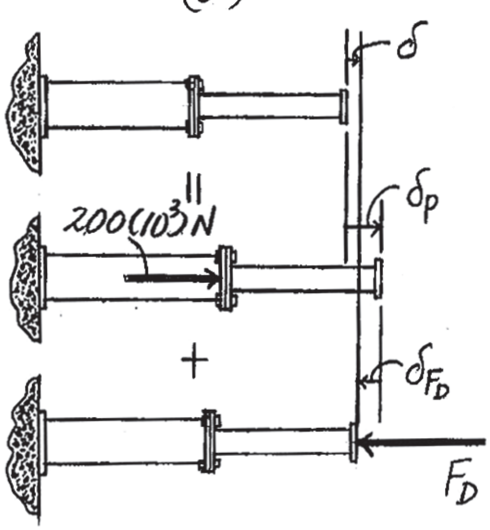
$F_D = 20\,365.05 \text{ N} = 20.4 \text{ kN}$ **Ans.**

Substituting this result into Eq. (1),

$F_A = 179\,634.95 \text{ N} = 180 \text{ kN}$ **Ans.**



(a)

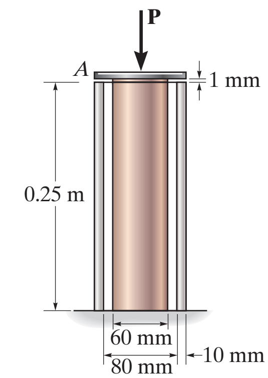


(b)

Ans:
$F_D = 20.4$ kN, $F_A = 180$ kN

4–50.

The support consists of a solid red brass C83400 copper post surrounded by a 304 stainless steel tube. Before the load is applied the gap between these two parts is 1 mm. Given the dimensions shown, determine the greatest axial load that can be applied to the rigid cap A without causing yielding of any one of the materials.

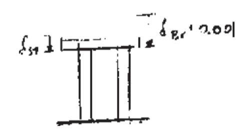


SOLUTION

Require,

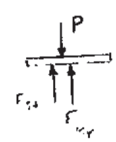
$$\delta_{st} = \delta_{br} + 0.001$$

$$\frac{F_{st}(0.25)}{\pi[(0.05)^2 - (0.04)^2]193(10^9)} = \frac{F_{br}(0.25)}{\pi(0.03)^2(101)(10^9)} + 0.001$$



$$0.45813\,F_{st} = 0.87544\,F_{br} + 10^6 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = 0; \qquad F_{st} + F_{br} - P = 0 \qquad \textbf{(2)}$$



Assume brass yields, then

$$(F_{br})_{max} = \sigma_\gamma A_{br} = 70(10^6)(\pi)(0.03)^2 = 197\,920.3\text{ N}$$

$$(\epsilon_\gamma)_{br} = \sigma_\gamma/E = \frac{70.0(10^6)}{101(10^9)} = 0.6931(10^{-3})\text{ mm/mm}$$

$$\delta_{br} = (\epsilon_\gamma)_{br}L = 0.6931(10^{-3})(0.25) = 0.1733\text{ mm} < 1\text{ mm}$$

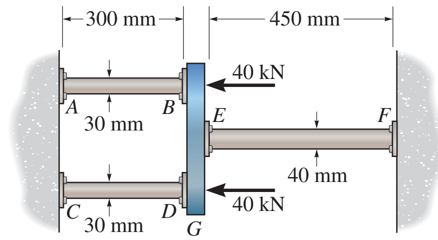
Thus only the brass is loaded.

$$P = F_{br} = 198\text{ kN} \qquad \textbf{Ans.}$$

Ans:
$P = 198$ kN

4–51.

The assembly consists of two red brass C83400 copper rods AB and CD of diameter 30 mm, a stainless 304 steel alloy rod EF of diameter 40 mm, and a rigid cap G. If the supports at A, C, and F are rigid, determine the average normal stress developed in the rods.



SOLUTION

Equation of Equilibrium: Due to symmetry, $F_{AB} = F_{CD} = F$. Referring to the free-body diagram of the assembly shown in Fig. a,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 2F + F_{EF} - 2\left[40(10^3)\right] = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Using the method of superposition, Fig. b,

$$(\xrightarrow{+}) \quad 0 = -\delta_P + \delta_{EF}$$

$$0 = -\frac{40(10^3)(300)}{\frac{\pi}{4}(0.03^2)(101)(10^9)} + \left[\frac{F_{EF}(450)}{\frac{\pi}{4}(0.04^2)(193)(10^9)} + \frac{\left(F_{EF}/2\right)(300)}{\frac{\pi}{4}(0.03^2)(101)(10^9)}\right]$$

$$F_{EF} = 42\,483.23 \text{ N}$$

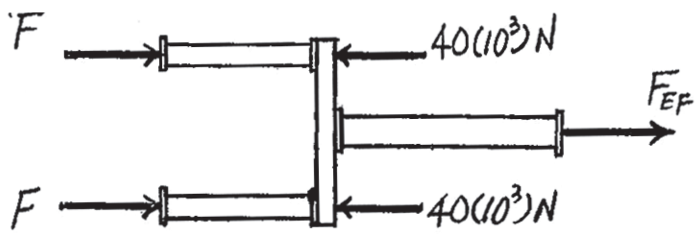
Substituting this result into Eq. (1),

$$F = 18\,758.38 \text{ N}$$

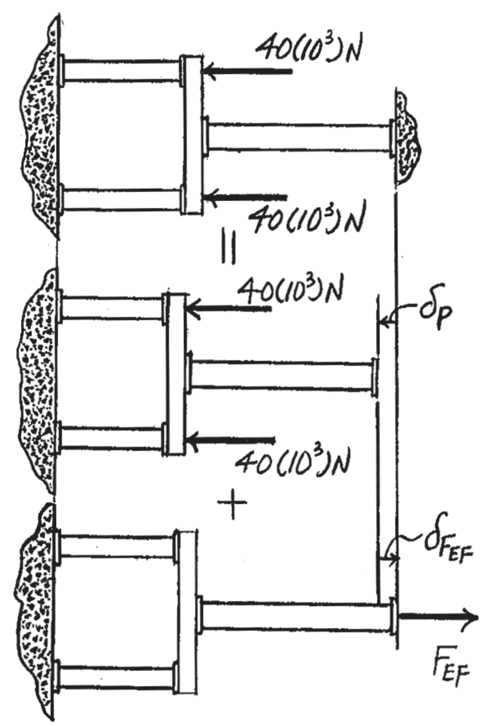
Normal Stress: We have,

$$\sigma_{AB} = \sigma_{CD} = \frac{F}{A_{CD}} = \frac{18\,758.38}{\frac{\pi}{4}(0.03^2)} = 26.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{EF} = \frac{F_{EF}}{A_{EF}} = \frac{42\,483.23}{\frac{\pi}{4}(0.04^2)} = 33.8 \text{ MPa} \qquad \textbf{Ans.}$$



(a)

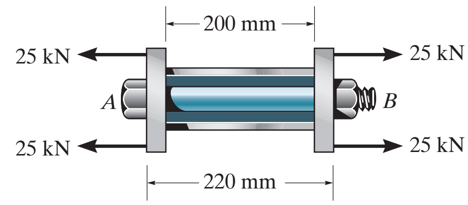


(b)

Ans:
$\sigma_{AB} = \sigma_{CD} = 26.5 \text{ MPa}, \sigma_{EF} = 33.8 \text{ MPa}$

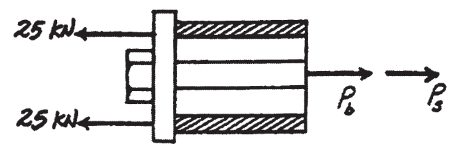
***4–52.**

The bolt AB has a diameter of 20 mm and passes through a sleeve that has an inner diameter of 40 mm and an outer diameter of 50 mm. The bolt and sleeve are made of A-36 steel and are secured to the rigid brackets as shown. If the bolt length is 220 mm and the sleeve length is 200 mm, determine the tension in the bolt when a force of 50 kN is applied to the brackets.



SOLUTION

Equation of Equilibrium:



$\xrightarrow{+} \Sigma F_x = 0; \qquad P_b + P_s - 25 - 25 = 0$

$$P_b + P_s - 50 = 0 \qquad \textbf{(1)}$$

Compatibility:

$$\delta_b = \delta_s$$

$$\frac{P_b(220)}{\frac{\pi}{4}(0.02^2)200(10^9)} = \frac{P_s(200)}{\frac{\pi}{4}(0.05^2 - 0.04^2)(200)(10^9)}$$

$$P_b = 0.40404\, P_s \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

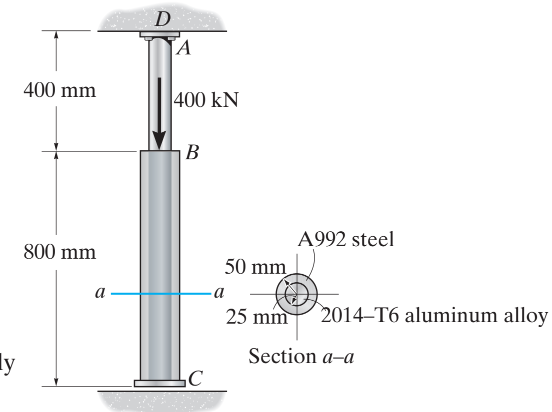
$$P_s = 35.61 \text{ kN}$$

$$P_b = 14.4 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$P_b = 14.4$ kN

4–53.

The 2014-T6 aluminum rod AC is reinforced with the firmly bonded A992 steel tube BC. If the assembly fits snugly between the rigid supports so that there is no gap at C, determine the support reactions when the axial force of 400 kN is applied. The assembly is attached at D.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad F_D + (F_C)_{al} + (F_C)_{st} - 400(10^3) = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Using the method of superposition, Fig. b,

$$(+\downarrow) \qquad 0 = \delta_p - \delta_{FC}$$

$$0 = +\frac{400(10^3)(400)}{\pi(0.025^2)(73.1)(10^9)} - \left[\frac{(F_C)_{al}(800)}{\pi(0.025^2)(73.1)(10^9)} + \frac{[(F_C)_{al} + (F_C)_{st}](400)}{\pi(0.025^2)(73.1)(10^9)}\right]$$

$$400(10^3) = 3(F_C)_{al} + (F_C)_{st} \qquad \textbf{(2)}$$

Also, since the aluminum rod and steel tube of segment BC are firmly bonded, their deformation must be the same. Thus,

$$(\delta_{BC})_{st} = (\delta_{BC})_{al}$$

$$\frac{(F_C)_{st}(800)}{\pi(0.05^2 - 0.025^2)(200)(10^9)} = \frac{(F_C)_{al}(800)}{\pi(0.025^2)(73.1)(10^9)}$$

$$(F_C)_{st} = 8.2079(F_C)_{al} \qquad \textbf{(3)}$$

Solving Eqs. (1) and (2),

$$(F_C)_{al} = 35.689 \text{ kN} \qquad (F_C)_{st} = 292.93 \text{ kN}$$

Substituting these results into Eq. (1),

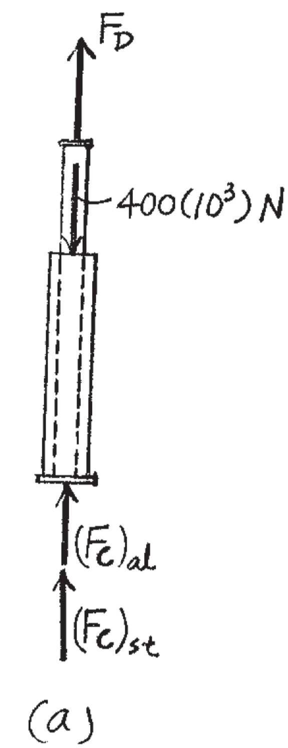
$$F_D = 71.4 \text{ kN} \qquad \textbf{Ans.}$$

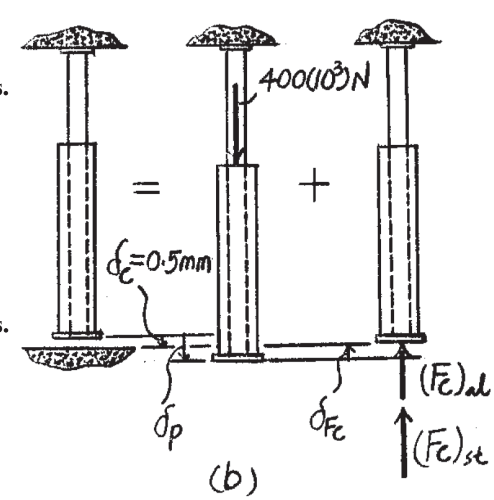
Also,

$$F_C = (F_C)_{st} + (F_C)_{al}$$

$$= 35.689 + 292.93$$

$$= 329 \text{ kN} \qquad \textbf{Ans.}$$

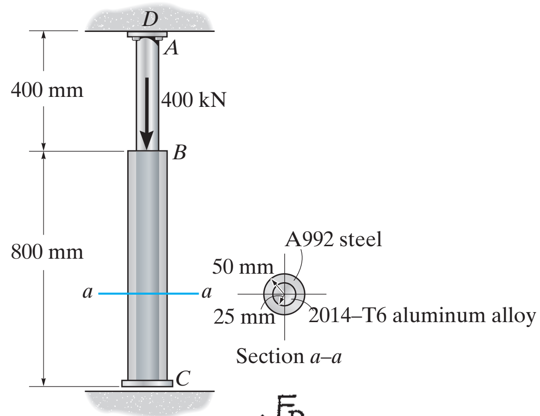




Ans:
$F_D = 71.4$ kN, $F_C = 329$ kN

4–54.

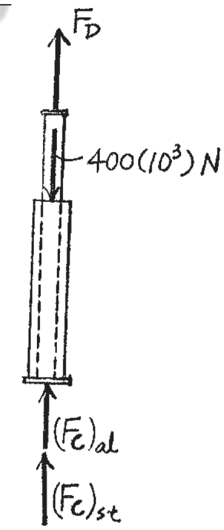
The 2014-T6 aluminum rod AC is reinforced with the firmly bonded A992 steel tube BC. When no load is applied to the assembly, the gap between end C and the rigid support is 0.5 mm. Determine the support reactions when the axial force of 400 kN is applied.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad F_D + (F_C)_{al} + (F_C)_{st} - 400(10^3) = 0 \qquad \textbf{(1)}$$



(a)

Compatibility Equation: Using the method of superposition, Fig. b,

$$(+\downarrow) \qquad \delta_C = \delta_P - \delta_{FC}$$

$$0.5 = +\frac{400(10^3)(400)}{\pi(0.025^2)(73.1)(10^9)} - \left[\frac{(F_C)_{al}(800)}{\pi(0.025^2)(73.1)(10^9)} + \frac{[(F_C)_{al} + (F_C)_{st}](400)}{\pi(0.025^2)(73.1)(10^9)}\right]$$

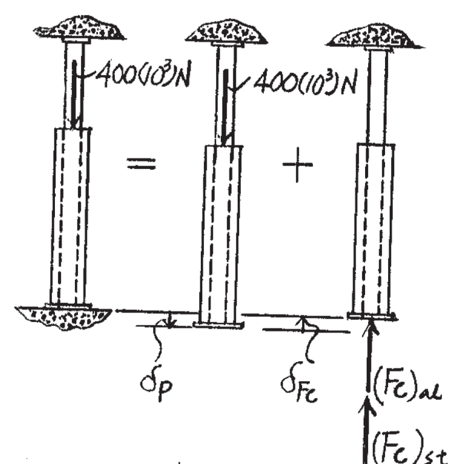
$$220.585(10^3) = 3(F_C)_{al} + (F_C)_{st} \qquad \textbf{(2)}$$

Also, since the aluminum rod and steel tube of segment BC are firmly bonded, their deformation must be the same. Thus,

$$(\delta_{BC})_{st} = (\delta_{BC})_{al}$$

$$\frac{(F_C)_{st}(800)}{\pi(0.05^2 - 0.025^2)(200)(10^9)} = \frac{(F_C)_{al}(800)}{\pi(0.025^2)(73.1)(10^9)}$$

$$(F_C)_{st} = 8.2079(F_C)_{al} \qquad \textbf{(3)}$$



(b)

Solving Eqs. (2) and (3),

$$(F_C)_{al} = 19.681 \text{ kN} \qquad (F_C)_{st} = 161.54 \text{ kN}$$

Substituting these results into Eq. (1),

$$F_D = 218.777 \text{ kN} = 219 \text{ kN} \qquad \textbf{Ans.}$$

Also,

$$F_C = (F_C)_{al} + (F_C)_{st}$$

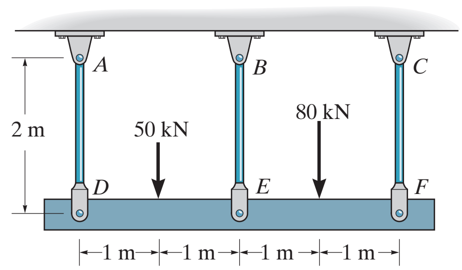
$$= 19.681 + 161.54$$

$$= 181 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$F_D = 219$ kN, $F_C = 181$ kN

4–55.

The three suspender bars are made of A992 steel and have equal cross-sectional areas of 450 mm^2. Determine the average normal stress in each bar if the rigid beam is subjected to the loading shown.



SOLUTION

Referring to the *FBD* of the rigid beam, Fig. *a*,

$$+\uparrow\Sigma F_y = 0; \qquad F_{AD} + F_{BE} + F_{CF} - 50(10^3) - 80(10^3) = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft+\Sigma M_D = 0; \qquad F_{BE}(2) + F_{CF}(4) - 50(10^3)(1) - 80(10^3)(3) = 0 \qquad \textbf{(2)}$$

Referring to the geometry shown in Fig. *b*,

$$\delta_{BE} = \delta_{AD} + \left(\frac{\delta_{CF} - \delta_{AD}}{4}\right)(2)$$

$$\delta_{BE} = \frac{1}{2}\left(\delta_{AD} + \delta_{CF}\right)$$

$$\frac{F_{BE}\cancel{L}}{\cancel{AE}} = \frac{1}{2}\left(\frac{F_{AD}\cancel{L}}{\cancel{AE}} + \frac{F_{CF}\cancel{L}}{\cancel{AE}}\right)$$

$$F_{AD} + F_{CF} = 2\,F_{BE} \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3) yields

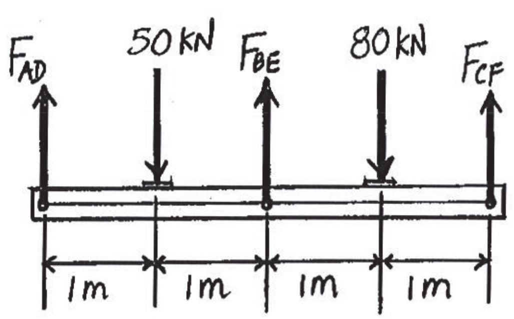
$$F_{BE} = 43.33(10^3)\text{ N} \qquad F_{AD} = 35.83(10^3)\text{ N} \qquad F_{CF} = 50.83(10^3)\text{ N}$$

Thus,

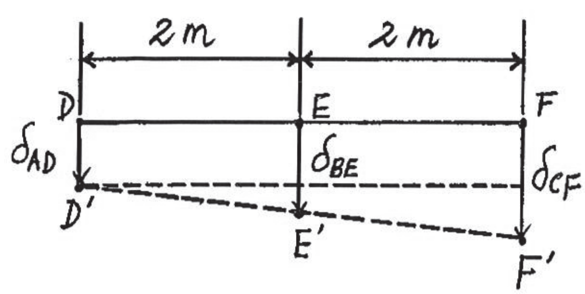
$$\sigma_{BE} = \frac{F_{BE}}{A} = \frac{43.33(10^3)}{0.45(10^{-3})} = 96.3\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{AD} = \frac{F_{AD}}{A} = \frac{35.83(10^3)}{0.45(10^{-3})} = 79.6\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{CF} = 113\text{ MPa} \qquad \textbf{Ans.}$$



(a)

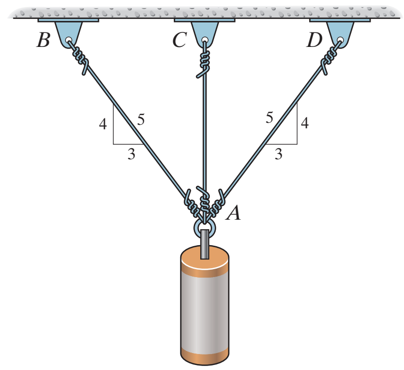


(b)

Ans:
$\sigma_{BE} = 96.3$ MPa, $\sigma_{AD} = 79.6$ MPa,
$\sigma_{CF} = 113$ MPa

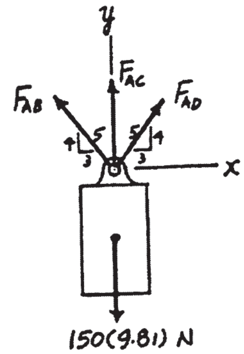
***4–56.**

The three A-36 steel wires each have a diameter of 2 mm and unloaded lengths of $L_{AC} = 1.60$ m and $L_{AB} = L_{AD} = 2.00$ m. Determine the force in each wire after the 150-kg mass is suspended from the ring at A.



SOLUTION

Equations of Equilibrium:



$$\xrightarrow{+} \Sigma F_x = 0; \qquad \frac{3}{5}F_{AD} - \frac{3}{5}F_{AB} = 0 \qquad F_{AD} = F_{AB} = F$$

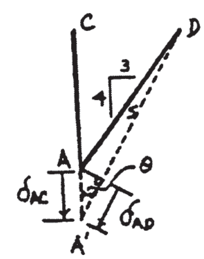
$$+\uparrow \Sigma F_y = 0; \qquad 2\left(\frac{4}{5}F\right) + F_{AC} - 150(9.81) = 0$$

$$1.6F + F_{AC} - 1471.5 = 0 \qquad \textbf{(1)}$$

Compatibility:

$$\delta_{AD} = \delta_{AC} \cos \theta$$

Since the displacement is very small, $\cos \theta = \dfrac{4}{5}$



$$\delta_{AD} = \frac{4}{5}\delta_{AC}$$

$$\frac{F(2)}{AE} = \frac{4}{5}\left[\frac{F_{AC}(1.6)}{AE}\right]$$

$$F = 0.640\, F_{AC} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

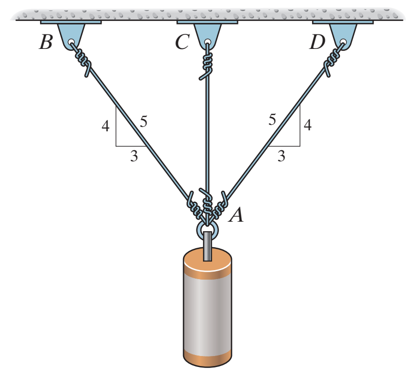
$$F_{AC} = 727 \text{ N} \qquad \textbf{Ans.}$$

$$F_{AB} = F_{AD} = F = 465 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$F_{AC} = 727$ N,
$F_{AB} = 465$ N

4–57.

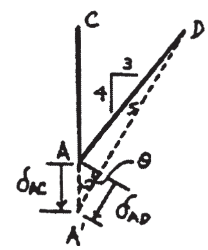
The A-36 steel wires AB and AD each have a diameter of 2 mm and the unloaded lengths of each wire are $L_{AC} = 1.60$ m and $L_{AB} = L_{AD} = 2.00$ m. Determine the required diameter of wire AC so that each wire is subjected to the same force when the 150-kg mass is suspended from the ring at A.



SOLUTION

Equations of Equilibrium: Each wire is required to carry the same amount of load. Hence

$$F_{AB} = F_{AC} = F_{AD} = F$$



Compatibility:

$$\delta_{AD} = \delta_{AC} \cos \theta$$

Since the displacement is very small, $\cos \theta = \dfrac{4}{5}$

$$\delta_{AD} = \frac{4}{5} \delta_{AC}$$

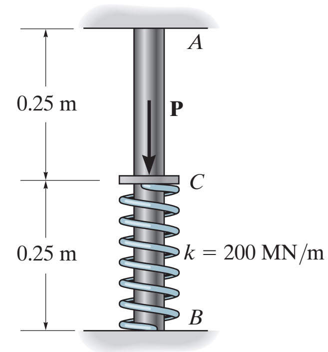
$$\frac{F(2)}{\frac{\pi}{4}(0.002^2)E} = \frac{F(1.6)}{\frac{\pi}{4}d_{AC}^2 E}$$

$$d_{AC} = 0.001789 \text{ m} = 1.79 \text{ mm}$$ **Ans.**

Ans:
$d_{AC} = 1.79$ mm

4–58.

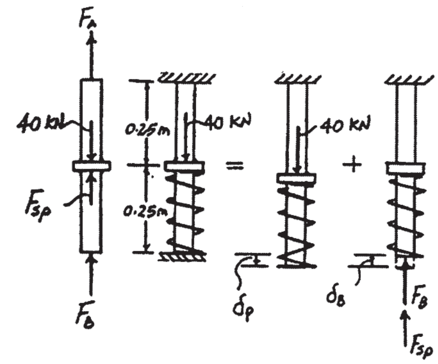
The post is made from 606l-T6 aluminum and has a diameter of 50 mm. It is fixed supported at A and B, and at its center C there is a coiled spring attached to the rigid collar. If the spring is originally uncompressed, determine the reactions at A and B when the force $P = 40$ kN is applied to the collar.



SOLUTION

Equations of Equilibrium:

$$+\uparrow \Sigma F_y = 0; \qquad F_A + F_B + F_{sp} - 40(10^3) = 0 \qquad \textbf{(1)}$$



Compatibility:

$$0 = \delta_p - \delta_B$$

$$0 = \frac{40(10^3)(0.25)}{\frac{\pi}{4}(0.05^2)68.9(10^9)} - \left[\frac{(F_B + F_{sp})(0.25)}{\frac{\pi}{4}(0.05^2)68.9(10^9)} + \frac{F_B + F_{sp}}{\frac{\frac{\pi}{4}(0.05^2)68.9(10^9)}{0.25} + 200(10^6)}\right]$$

$$F_B + F_{sp} = 23119.45 \qquad \textbf{(2)}$$

Also,

$$\delta_{sp} = \delta_{BC}$$

$$\frac{F_{sp}}{200(10^6)} = \frac{F_B + F_{sp}}{\frac{\frac{\pi}{4}(0.05^2)68.9(10^9)}{0.25} + 200(10^6)}$$

$$F_B = 2.7057\,F_{sp} \qquad \textbf{(3)}$$

Solving Eq. (2) and (3) yields

$$F_{sp} = 6238.9 \text{ N}$$

$$F_B = 16880.6 \text{ N} = 16.9 \text{ kN} \qquad \textbf{Ans.}$$

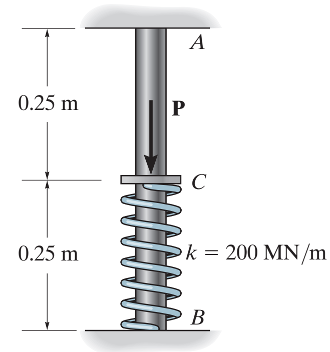
Substitute the results into Eq. (1)

$$F_A = 16880.6 \text{ N} = 16.9 \text{ kN} \qquad \textbf{Ans.}$$

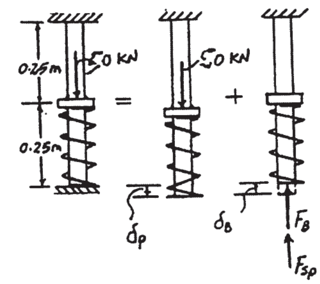
Ans:
$F_B = 16.9$ kN,
$F_A = 16.9$ kN

4–59.

The post is made from 606l-T6 aluminum and has a diameter of 50 mm. It is fixed supported at A and B, and at its center C there is a coiled spring attached to the rigid collar. If the spring is originally uncompressed, determine the compression in the spring when the load of $P = 50$ kN is applied to the collar.



SOLUTION



Compatibility:

$$0 = \delta_p - \delta_B$$

$$0 = \frac{50(10^3)(0.25)}{\frac{\pi}{4}(0.05^2)68.9(10^9)} - \left[\frac{(F_B + F_{sp})(0.25)}{\frac{\pi}{4}(0.05^2)68.9(10^9)} + \frac{F_B + F_{sp}}{\frac{\frac{\pi}{4}(0.05^2)68.9(10^9)}{0.25} + 200(10^6)}\right]$$

$$F_B + F_{sp} = 28899.31 \qquad \textbf{(1)}$$

Also,

$$\delta_{sp} = \delta_{BC}$$

$$\frac{F_{sp}}{200(10^6)} = \frac{F_B + F_{sp}}{\frac{\frac{\pi}{4}(0.05^2)68.9(10^9)}{0.25} + 200(10^6)}$$

$$F_B = 2.7057\,F_{sp} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yield

$$F_{sp} = 7798.6\text{ N} \qquad F_B = 21100.7\text{ N}$$

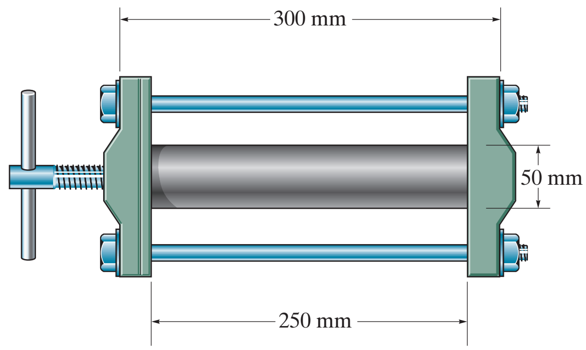
Thus,

$$\delta_{sp} = \frac{F_{sp}}{k} = \frac{7798.6}{200(10^6)}$$

$$= 0.0390(10^{-3})\text{ m} = 0.0390\text{ mm} \qquad \textbf{Ans.}$$

Ans:
$\delta_{sp} = 0.0390$ mm

***4–60.** The press consists of two rigid heads that are held together by the two A-36 steel 12-mm-diameter rods. A 6061-T6-solid-aluminum cylinder is placed in the press and the screw is adjusted so that it just presses up against the cylinder. If it is then tightened one-half turn, determine the average normal stress in the rods and in the cylinder. The single-threaded screw on the bolt has a lead of 0.25 mm *Note*: The lead represents the distance the screw advances along its axis for one complete turn of the screw.



SOLUTION

Equilibrium:

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 2F_{st} - F_{al} = 0$$

Compatibility:

$$\delta_{st} = 0.125(10^{-3}) - \delta_{al}$$

$$\frac{F_{st}(0.3)}{\left[\frac{\pi}{4}(0.012^2)\right][200(10^9)]} = 0.125(10^{-3}) - \frac{F_{al}(0.25)}{\left[\frac{\pi}{4}(0.05^2)\right][68.9(10^9)]}$$

$$13.2629F_{st} = 125(10^3) - 1.8480F_{al}$$

Solving,

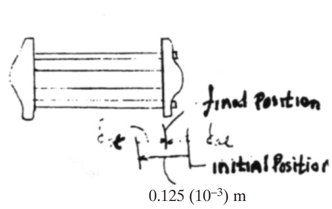
$$F_{st} = 7.3708(10^3)\text{ N}$$

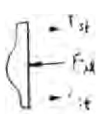
$$F_{al} = 14.7416(10^3)\text{ N}$$

Normal stress:

$$\sigma_{rod} = \frac{F_{st}}{A_{st}} = \frac{7.3708(10^3)}{\left(\frac{\pi}{4}\right)(0.012^2)} = 65.17(10^6)\text{ N/m}^2 = 65.2\text{ MPa}$$ **Ans.**

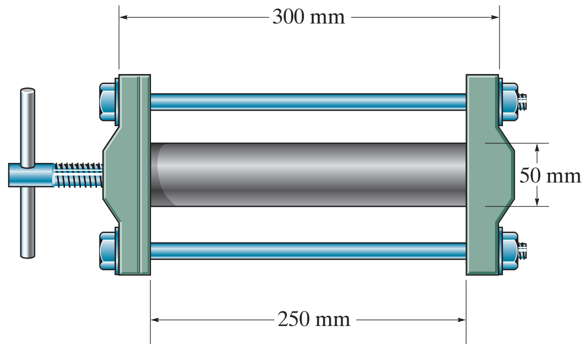
$$\sigma_{cyl} = \frac{F_{al}}{A_{al}} = \frac{14.7416(10^3)}{\left(\frac{\pi}{4}\right)(0.05^2)} = 7.508(10^6)\text{ N/m}^2 = 7.51\text{ MPa}$$ **Ans.**





Ans.
$\sigma_{rod} = 65.2$ MPa, $\sigma_{cyl} = 7.51$ MPa

4–61. The press consists of two rigid heads that are held together by the two A-36 steel 12-mm-diameter rods. A 6061-T6-solid-aluminum cylinder is placed in the press and the screw is adjusted so that it just presses up against the cylinder. Determine the angle through which the screw can be turned before the rods or the specimen begin to yield. The single-threaded screw on the bolt has a lead of 0.25 mm. *Note*: The lead represents the distance the screw advances along its axis for one complete turn of the screw.



SOLUTION

Equilibrium:

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 2F_{st} - F_{al} = 0$$

Compatibility:

$$\delta_{st} = d - \delta_{al}$$

$$\frac{F_{st}(0.3)}{\left[\frac{\pi}{4}(0.012^2)\right][200(10^9)]} = d - \frac{F_{al}(0.25)}{\left[\frac{\pi}{4}(0.05^2)\right][68.9(10^9)]}$$

$$13.2629F_{st} = (10^9)d - 1.8480F_{al} \qquad \textbf{(1)}$$

Assume steel yields first,

$$\sigma_Y = 250(10^6) = \frac{F_{st}}{\left(\frac{\pi}{4}\right)(0.012^2)}; \qquad F_{st} = 28.27(10^3)\text{ N}$$

Then $F_{al} = 56.55(10^3)$ N

$$\sigma_{al} = \frac{56.55(10^3)}{\left(\frac{\pi}{4}\right)(0.05^2)} = 28.80(10^6)\text{ N/m}^2 = 28.80\text{ MPa}$$

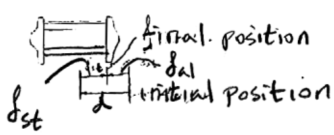
28.80 MPa < 255 MPa, steel yields first as assumed. From Eq. (1),

$$d = 0.4795(10^{-3})\text{ m} = 0.4795\text{ mm}$$

Thus,

$$\frac{\theta}{360°} = \frac{0.4795}{0.25}$$

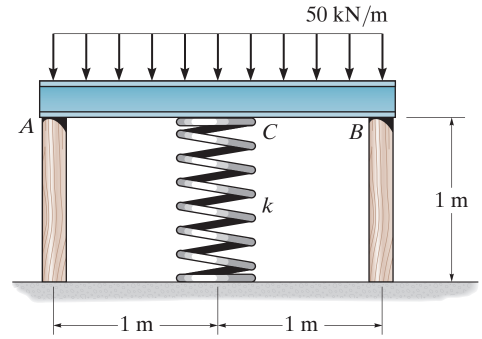
$$\theta = 690.48° = 690° \qquad \textbf{Ans.}$$



Ans.
$\theta = 690°$

4–62.

The rigid bar is supported by the two short white spruce wooden posts and a spring. If each of the posts has an unloaded length of 1 m and a cross-sectional area of 600 mm^2, and the spring has a stiffness of $k = 2$ MN/m and an unstretched length of 1.02 m, determine the force in each post after the load is applied to the bar.



SOLUTION

Equations of Equilibrium:

$\circlearrowleft + \Sigma M_C = 0; \quad F_B(1) - F_A(1) = 0 \quad F_A = F_B = F$

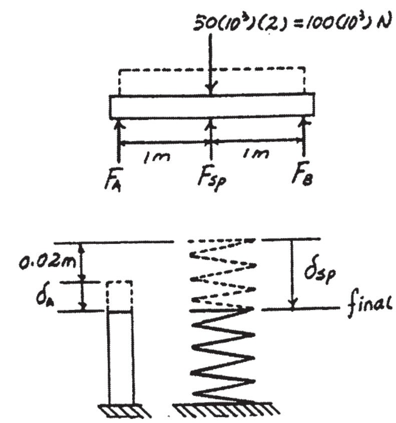
$+\uparrow \Sigma F_y = 0; \quad 2F + F_{sp} - 100(10^3) = 0 \quad$ **(1)**

Compatibility:

$(+\downarrow) \quad \delta_A + 0.02 = \delta_{sp}$

$$\frac{F(1)}{600(10^{-6})9.65(10^9)} + 0.02 = \frac{F_{sp}}{2.0(10^6)}$$

$0.1727F + 20(10^3) = 0.5\,F_{sp} \quad$ **(2)**



Solving Eqs. (1) and (2) yields:

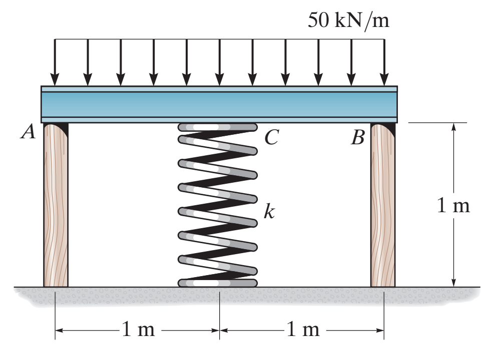
$F_A = F_B = F = 25581.7 \text{ N} = 25.6 \text{ kN} \quad$ **Ans.**

$F_{sp} = 48\,836.5 \text{ N}$

Ans:
$F_A = F_B = 25.6$ kN

4–63.

The rigid bar is supported by the two short white spruce wooden posts and a spring. If each of the posts has an unloaded length of 1 m and a cross-sectional area of 600 mm^2, and the spring has a stiffness of $k = 2$ MN/m and an unstretched length of 1.02 m, determine the vertical displacement of A and B after the load is applied to the bar.

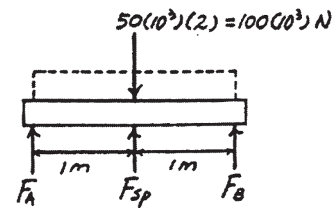


SOLUTION

Equations of Equilibrium:

$\circlearrowleft + \Sigma M_C = 0;\quad F_B(1) - F_A(1) = 0 \quad F_A = F_B = F$

$+\uparrow \Sigma F_y = 0;\quad 2F + F_{sp} - 100(10^3) = 0 \quad$ **(1)**

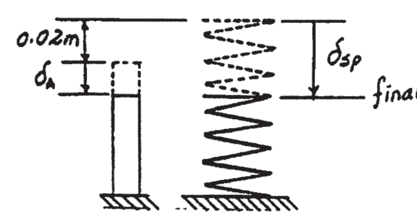


Compatibility:

$$(+\downarrow) \qquad \delta_A + 0.02 = \delta_{sp}$$

$$\frac{F(1)}{600(10^{-6})9.65(10^9)} + 0.02 = \frac{F_{sp}}{2.0(10^6)}$$

$$0.1727F + 20(10^3) = 0.5\,F_{sp} \quad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields:

$$F = 25\,581.7\text{ N} \qquad F_{sp} = 48\,836.5\text{ N}$$

Displacement:

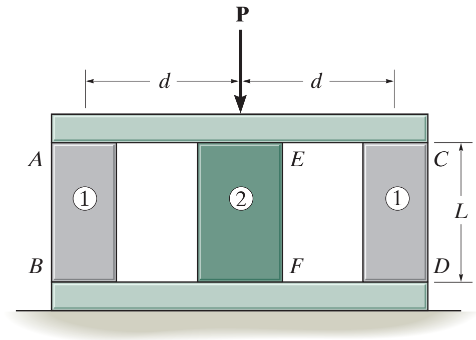
$$\delta_A = \delta_B = \frac{FL}{AE}$$

$$= \frac{25\,581.7(1000)}{600(10^{-6})(9.65)(10^9)} = 4.42\text{ mm} \qquad \textbf{Ans.}$$

Ans:
$\delta_A = \delta_B = 4.42$ mm

***4–64.**

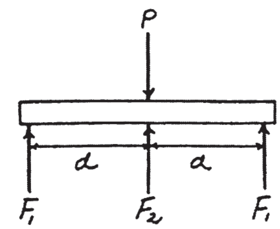
The assembly consists of two posts AB and CD each made from material 1 having a modulus of elasticity of E_1 and a cross-sectional area A_1, and a central post made from material 2 having a modulus of elasticity E_2 and cross-sectional area A_2. If a load **P** is applied to the rigid cap, determine the force in each material.



SOLUTION

Equilibrium:

$$+\uparrow \Sigma F_y = 0; \qquad 2F_1 + F_2 - P = 0 \qquad \textbf{(1)}$$



Compatibility:

$$\delta = \delta_1 = \delta_2$$

$$\frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2 E_2} \qquad F_1 = \left(\frac{A_1 E_1}{A_2 E_2}\right) F_2 \qquad \textbf{(2)}$$

Solving Eq. (1) and (2) yields:

$$F_1 = \left(\frac{A_1 E_1}{2\, A_1 E_1 + A_2 E_2}\right) P \qquad \textbf{Ans.}$$

$$F_2 = \left(\frac{A_2 E_2}{2 A_1 E_1 + A_2 E_2}\right) P \qquad \textbf{Ans.}$$

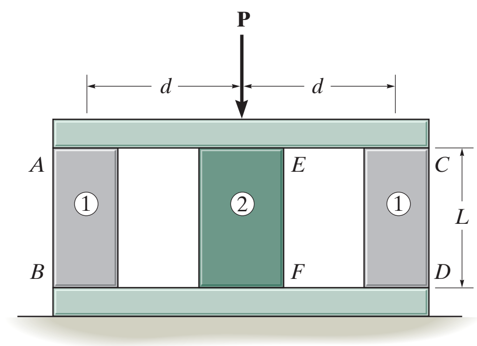
Ans:

$F_1 = \left(\dfrac{A_1 E_1}{2\, A_1 E_1 + A_2 E_2}\right) P,$

$F_2 = \left(\dfrac{A_2 E_2}{2 A_1 E_1 + A_2 E_2}\right) P$

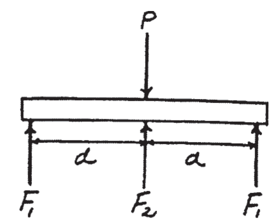
4–65.

The assembly consists of two posts AB and CD each made from material 1 having a modulus of elasticity of E_1 and a cross-sectional area A_1, and a central post EF made from material 2 having a modulus of elasticity E_2 and a cross-sectional area A_2. If posts AB and CD are to be replaced by those having a material 2, determine the required cross-sectional area of these new posts so that both assemblies deform the same amount when loaded.



SOLUTION

$$+\uparrow \Sigma F_y = 0; \qquad 2F_1 + F_2 - P = 0 \qquad \textbf{(1)}$$



Compatibility:

$$\delta_{\text{in}} = \delta_1 = \delta_2$$

$$\frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2 E_2} \qquad F_1 = \left(\frac{A_1 E_1}{A_2 E_2}\right) F_2 \qquad \textbf{(2)}$$

Solving Eq. (1) and (2) yields:

$$F_1 = \left(\frac{A_1 E_1}{2\,A_1 E_1 + A_2 E_2}\right) P \qquad F_2 = \left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) P$$

$$\delta_{\text{in}} = \frac{F_2 L}{A_2 E_2} = \frac{\left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) PL}{A_2 E_2} = \frac{PL}{2A_1 E_1 + A_2 E_2}$$

Compatibility: When material 1 has been replaced by material 2 for two side posts, then

$$\delta_{\text{final}} = \delta_1 = \delta_2$$

$$\frac{F_1 L}{A'_1 E_2} = \frac{F_2 L}{A_2 E_2} \qquad F_1 = \left(\frac{A'_1}{A_2}\right) F_2 \qquad \textbf{(3)}$$

Solving for F_2 from Eq. (1) and (3)

$$F_2 = \left(\frac{A_2}{2A'_1 + A_2}\right) P$$

$$\delta_{\text{final}} = \frac{F_2 L}{A_2 E_2} = \frac{\left(\frac{A_2}{2A'_1 + A_2}\right) PL}{A_2 E_2} = \frac{PL}{E_2(2A'_1 + A_2)}$$

Requires, $\qquad \delta_{\text{in}} = \delta_{\text{final}}$

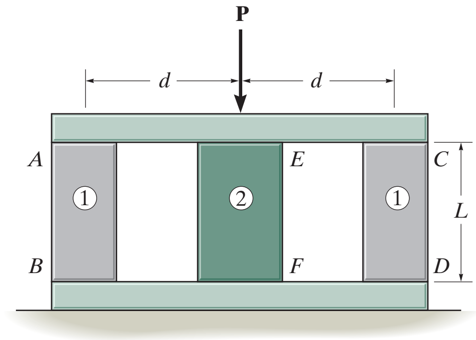
$$\frac{PL}{2A_1 E_1 + A_2 E_2} = \frac{PL}{E_2\,(2A'_1 + A_2)}$$

$$A'_1 = \left(\frac{E_1}{E_2}\right) A_1 \qquad \textbf{Ans.}$$

Ans:
$A'_1 = \left(\frac{E_1}{E_2}\right) A_1$

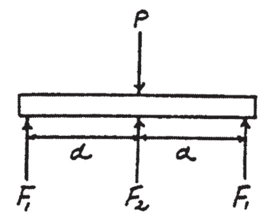
4–66.

The assembly consists of two posts AB and CD each made from material 1 having a modulus of elasticity of E_1 and a cross-sectional area A_1, and a central post EF made from material 2 having a modulus of elasticity E_2 and a cross-sectional area A_2. If post EF is to be replaced by one having a material 1, determine the required cross-sectional area of this new post so that both assemblies deform the same amount when loaded.



SOLUTION

$$+\uparrow \Sigma F_y = 0; \qquad 2F_1 + F_2 - P = 0 \qquad \textbf{(1)}$$



Compatibility:

$$\delta_{in} = \delta_1 = \delta_2$$

$$\frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2 E_2} \qquad F_1 = \left(\frac{A_1 E_1}{A_2 E_2}\right) F_2 \qquad \textbf{(2)}$$

Solving Eq. (1) and (2) yields:

$$F_1 = \left(\frac{A_1 E_1}{2\,A_1 E_1 + A_2 E_2}\right) P \qquad F_2 = \left(\frac{A_2 E_2}{2 A_1 E_1 + A_2 E_2}\right) P$$

$$\delta_{in} = \frac{F_2 L}{A_2 E_2} = \frac{\left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) P}{A_2 E_2} = \frac{PL}{2A_1 E_1 + A_2 E_2}$$

Compatibility: When material 2 has been replaced by material 1 for central posts, then

$$\delta_{final} = \delta_1 = \delta_2$$

$$\frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2' E_1} \qquad F_2 = \left(\frac{A_2'}{A_2}\right) F_1 \qquad \textbf{(3)}$$

Solving for F_1 from Eq. (1) and (3)

$$F_1 = \left(\frac{A_1}{2A_1 + A_2'}\right) P$$

$$\delta_{final} = \frac{F_1 L}{A_1 E_1} = \frac{\left(\frac{A_1}{2A_1 + A_2'}\right) PL}{A_1 E_1} = \frac{PL}{E_1(2A_1 + A_2')}$$

Requires, $\qquad \delta_{in} = \delta_{final}$

$$\frac{PL}{2A_1 E_1 + A_2 E_2} = \frac{PL}{E_1\,(2A_1 + A_2')}$$

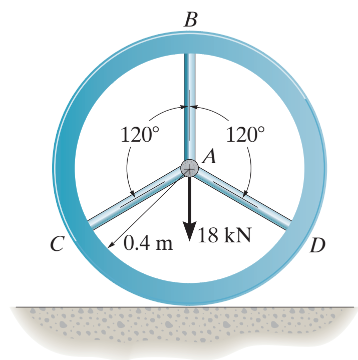
$$A_2' = \left(\frac{E_2}{E_1}\right) A_2 \qquad \textbf{Ans.}$$

Ans:

$$A_2' = \left(\frac{E_2}{E_1}\right) A_2$$

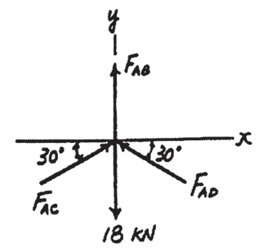
4–67.

The wheel is subjected to a force of 18 kN from the axle. Determine the force in each of the three spokes. Assume the rim is rigid and the spokes are made of the same material, and each has the same cross-sectional area.



SOLUTION

Equations of Equilibrium:



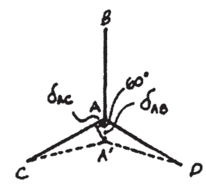
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{AC} \cos 30^\circ - F_{AD} \cos 30^\circ = 0$$

$$F_{AC} = F_{AD} = F$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} + 2F \sin 30^\circ - 18 = 0$$

$$F_{AB} + F = 18 \qquad \textbf{(1)}$$

Compatibility:



$$\delta_{AC} = \delta_{AB} \cos 60^\circ$$

$$\frac{F(0.4)}{AE} = \frac{F_{AB}\,(0.4)}{AE} \cos 60^\circ$$

$$F = 0.5F_{AB} \qquad \textbf{(2)}$$

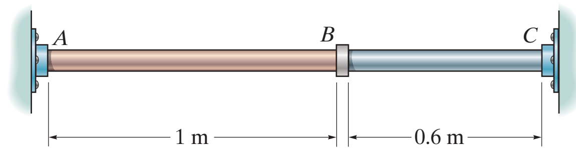
Solving Eq. (1) and (2) yields:

$F_{AB} = 12.0\text{ kN (T)}$ **Ans.**

$F_{AC} = F_{AD} = F = 6.00\text{ kN (C)}$ **Ans.**

Ans:
$F_{AB} = 12.0\text{ kN (T)}$
$F_{AC} = F_{AD} = 6.00\text{ kN (C)}$

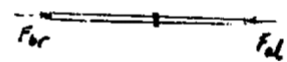
***4–68.** The C83400-red-brass rod AB and 2014-T6-aluminum rod BC are joined at the collar B and fixed connected at their ends. If there is no load in the members when $T_1 = 10°\text{C}$, determine the average normal stress in each member when $T_2 = 45°\text{C}$. Also, how far will the collar be displaced? The cross-sectional area of each member is 1130 mm².



SOLUTION

Equilibrium:

$\Sigma F_x = 0; \qquad F_{br} = F_{al} = F$



Compatibility:

$$\delta_{A/C} = 0$$

$$-\frac{F_{br} L_{AB}}{A_{AB} E_{br}} + \alpha_{br}\Delta T\, L_{AB} - \frac{F_{al} L_{BC}}{A_{BC} E_{al}} + \alpha_{al}\Delta T\, L_{BC} = 0$$

$$-\frac{F(1)}{[(1130)(10^{-6})][101(10^9)]} + 18(10^{-6})(45 - 10)(1)$$

$$-\frac{F(0.6)}{[(1130)(10^{-6})][73.1(10^9)]} + 23(10^{-6})(45 - 10)(0.6) = 0$$

$$F = 69.45(10^3)\text{ N}$$

$$\sigma_{br} = \sigma_{al} = \frac{69.45(10^3)}{1130(10^{-6})} = 61.46(10^6)\text{ N/m}^2 = 61.5\text{ MPa}$$ **Ans.**

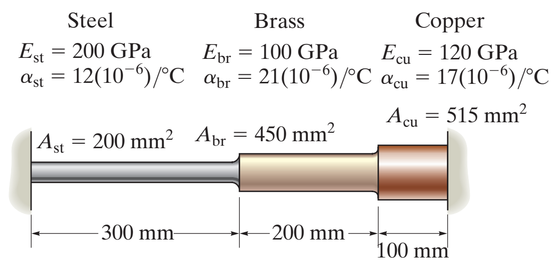
$61.46\text{ MPa} < (\sigma_Y)_{al}$ and $(\sigma_Y)_{br}$ **OK**

$$\delta_B = -\frac{[69.45(10^3)][(1)(10^3)]}{[1130(10^{-6})][101(10^9)]} + 18(10^{-6})(45 - 10)[1(10^3)]$$

$$\delta_B = 0.02147\text{ mm} = 0.0215\text{ mm} \rightarrow$$ **Ans.**

Ans.
$\sigma_{br} = 61.5\text{ MPa}$
$\delta_B = 0.0215\text{ mm}$

4–69. Three bars each made of different materials are connected together and placed between two walls when the temperature is $T_1 = 12°\text{C}$. Determine the force exerted on the (rigid) supports when the temperature becomes $T_2 = 18°\text{C}$. The material properties and cross-sectional area of each bar are given in the figure.



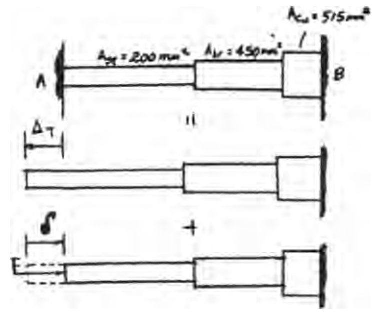
SOLUTION

$(\overset{+}{\leftarrow})\qquad 0 = \Delta_T - \delta$

$$0 = 12(10^{-6})(6)(0.3) + 21\,(10^{-6})(6)(0.2) + 17\,(10^{-6})(6)(0.1)$$

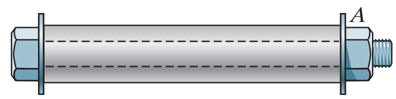
$$-\frac{F(0.3)}{200(10^{-6})(200)(10^9)} - \frac{F(0.2)}{450(10^{-6})(100)(10^9)} - \frac{F(0.1)}{515(10^{-6})(120)(10^9)}$$

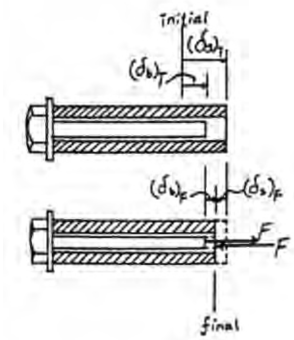
$F = 4203\text{ N} = 4.20\text{ kN}$ **Ans.**



Ans.
$0 = \Delta_T - \delta, F = 4.20\text{ kN}$

4–70. The steel bolt has a diameter of 7 mm and fits through an aluminum sleeve as shown. The sleeve has an inner diameter of 8 mm and an outer diameter of 10 mm. The nut at A is adjusted so that it just presses up against the sleeve. If the assembly is originally at a temperature of $T_1 = 20°\text{C}$ and then is heated to a temperature of $T_2 = 100°\text{C}$, determine the average normal stress in the bolt and the sleeve. $E_{st} = 200\text{ GPa}$, $E_{al} = 70\text{ GPa}$, $\alpha_{st} = 14(10^{-6})/°\text{C}$, $\alpha_{al} = 23(10^{-6})/°\text{C}$.





Compatibility:

$$(\delta_s)_T - (\delta_b)_T = (\delta_s)_F + (\delta_b)_F$$

$$23(10^{-6})(100 - 20)L - 14(10^{-6})(100 - 20)L$$

$$= \frac{FL}{\frac{\pi}{4}(0.01^2 - 0.008^2)70(10^9)} + \frac{FL}{\frac{\pi}{4}(0.007^2)200(10^9)}$$

$$F = 1133.54\text{ N}$$

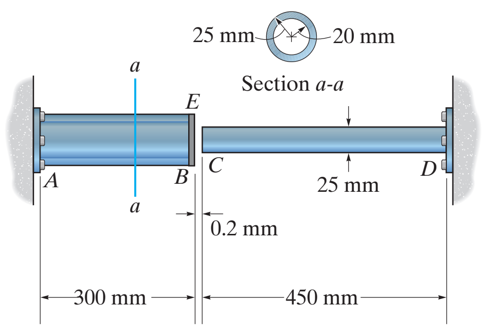
Average Normal Stress:

$$\sigma_s = \frac{F}{A_s} = \frac{1133.54}{\frac{\pi}{4}(0.01^2 - 0.008^2)} = 40.1\text{ MPa}$$ **Ans.**

$$\sigma_b = \frac{F}{A_b} = \frac{1133.54}{\frac{\pi}{4}(0.007^2)} = 29.5\text{ MPa}$$ **Ans.**

Ans.
$\sigma_s = 40.1\text{ MPa}, \sigma_b = 29.5\text{ MPa}$

4–71. The AM1004-T61 magnesium alloy tube AB is capped with a rigid plate E. The gap between E and end C of the 6061-T6 aluminum alloy solid circular rod CD is 0.2 mm when the temperature is at 30° C. Determine the normal stress developed in the tube and the rod if the temperature rises to 80° C. Neglect the thickness of the rigid cap.



SOLUTION

Compatibility Equation: If tube AB and rod CD are unconstrained, they will have a free expansion of $\left(\delta_T\right)_{AB} = \alpha_{mg}\,\Delta T L_{AB} = 26(10^{-6})(80 - 30)(300) = 0.39$ mm and $\left(\delta_T\right)_{CD} = \alpha_{al}\Delta T L_{CD} = 24(10^{-6})(80 - 30)(450) = 0.54$ mm. Referring to the deformation diagram of the tube and the rod shown in Fig. a,

$$\delta = \left[\left(\delta_T\right)_{AB} - \left(\delta_F\right)_{AB}\right] + \left[\left(\delta_T\right)_{CD} - \left(\delta_F\right)_{CD}\right]$$

$$0.2 = \left[0.39 - \frac{F(300)}{\pi\left(0.025^2 - 0.02^2\right)(44.7)(10^9)}\right] + \left[0.54 - \frac{F(450)}{\frac{\pi}{4}\left(0.025^2\right)(68.9)(10^9)}\right]$$

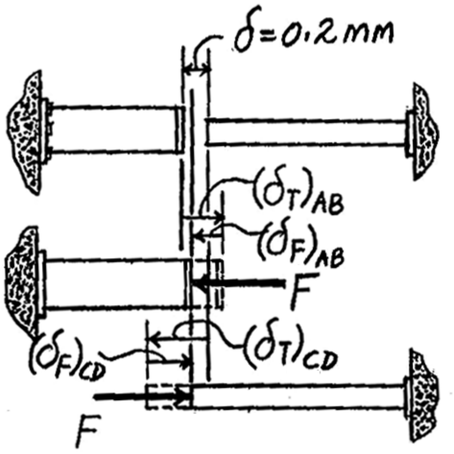
$$F = 32\,017.60 \text{ N}$$

Normal Stress:

$$\sigma_{AB} = \frac{F}{A_{AB}} = \frac{32\,017.60}{\pi\left(0.025^2 - 0.02^2\right)} = 45.3 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{CD} = \frac{F}{A_{CD}} = \frac{32\,017.60}{\frac{\pi}{4}\left(0.025^2\right)} = 65.2 \text{ MPa} \qquad \textbf{Ans.}$$

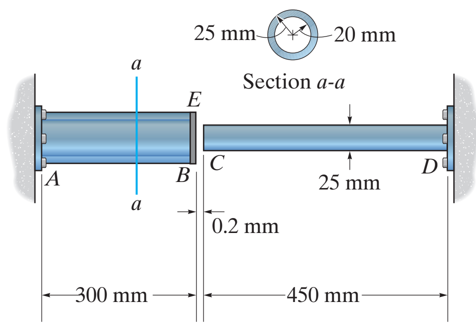
$$F = 107\,442.47 \text{ N}$$



(a)

Ans.
$\sigma_{AB} = 45.3$ MPa, $\sigma_{CD} = 65.2$ MPa,
$F = 107{,}442.47$ N

***4–72.** The AM1004-T61 magnesium alloy tube AB is capped with a rigid plate. The gap between E and end C of the 6061-T6 aluminum alloy solid circular rod CD is 0.2 mm when the temperature is at 30° C. Determine the highest temperature to which it can be raised without causing yielding either in the tube or the rod. Neglect the thickness of the rigid cap.



SOLUTION

Then

$$\sigma_{CD} = \frac{F}{A_{CD}} = \frac{107\,442.47}{\frac{\pi}{4}\left(0.025^2\right)} = 218.88\text{MPa} < (\sigma_Y)_{\text{al}} \qquad \textbf{(O.K.!)}$$

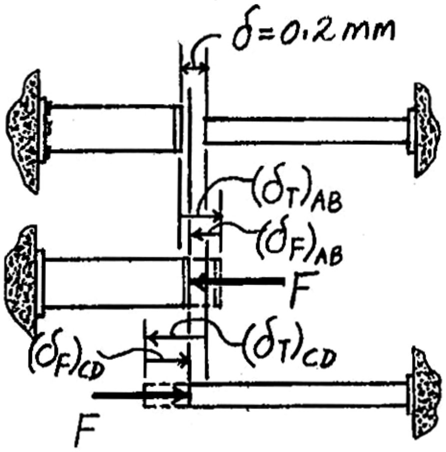
Compatibility Equation: If tube AB and rod CD are unconstrained, they will have a free expansion of $\left(\delta_T\right)AB = \alpha_{\text{mg}}\,\Delta T L_{AB} = 26(10^{-6})(T - 30)(300) = 7.8(10^{-6})$ $(T - 30)$ and $\left(\delta_T\right)_{CD} = \alpha_{\text{al}}\,\Delta T L_{CD} = 24(10^{-6})(T - 30)(450) = 0.0108(T - 30)$. Referring to the deformation diagram of the tube and the rod shown in Fig. a,

$$\delta = \left[\left(\delta_T\right)_{AB} - \left(\delta_F\right)_{AB}\right] + \left[\left(\delta_T\right)_{CD} - \left(\delta_F\right)_{CD}\right]$$

$$0.2 = \left[7.8(10^{-3})(T - 30) - \frac{107\,442.47(300)}{\pi\left(0.025^2 - 0.02^2\right)(44.7)(10^9)}\right]$$

$$+\left[0.0108(T - 30) - \frac{107\,442.47(450)}{\frac{\pi}{4}\left(0.025^2\right)(68.9)(10^9)}\right]$$

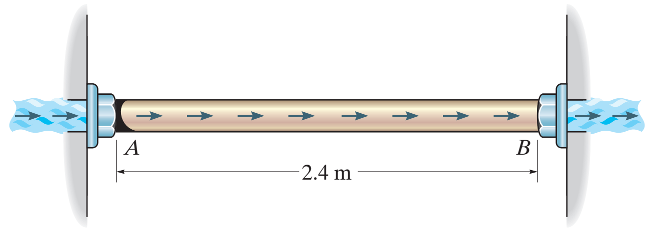
$T = 172°$ C **Ans.**



(a)

Ans.
$T = 172°$ C

4–73. The pipe is made of A992 steel and is connected to the collars at A and B. When the temperature is 15°C, there is no axial load in the pipe. If hot gas traveling through the pipe causes its temperature to vary along the pipe defined by $T = (35 + 30x)$°C, where x is in meter, determine the average normal stress in the pipe. The inner diameter is 50 mm, the wall thickness is 4 mm.



SOLUTION

Compatibility:

The cross-sectional area is $A = \frac{\pi}{4}(.,.38^2 - .,.3^2) = 216(10^{-6})\,\pi\ \text{m}^2$.

$0 = \delta_T - \delta_F$ Where $\delta_T = \int_0^L \alpha\,\Delta T\,dx$ and ΔT ; &13) 1.x' 1 /3 ; &0.) 1.x'°C.

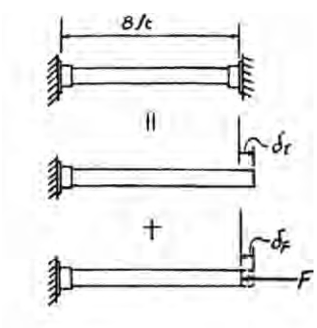
$$0 = 12(10^{-6})\int_0^{2.4\text{m}} (20 + 30x)\,dx - \frac{F(2.4)}{[216(10^{-6})\pi][200(10^9)]}$$

$$0 = 12(10^{-6})\left[20(2.4) + \frac{30(2.4)^2}{2}\right] - \frac{F(2.4)}{[216(10^{-6})\pi][200(10^9)]}$$

$F = 91.20(10^3)$ N

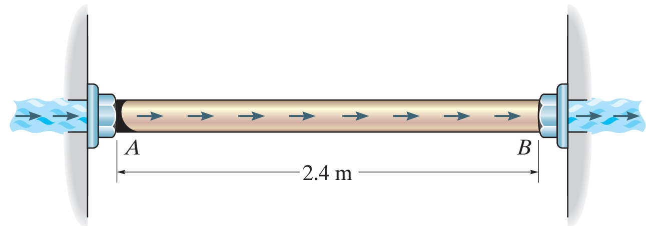
Average Normal Stress:

$$\sigma = \frac{91.20(10^3)}{216(10^{-6})\pi} = 134.40(10^6)\ \text{N/m}^2 = 134\ \text{MPa}$$ **Ans.**



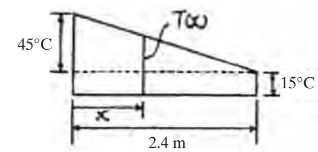
Ans:
$\sigma = 134$ MPa

4–74. The bronze C86100 pipe has an inner radius of 12.5 mm and a wall thickness of 5 mm. If the gas flowing through it changes the temperature of the pipe uniformly from $T_A = 60°\text{C}$ at A to $T_B = 15°\text{C}$ at B, determine the axial force it exerts on the walls. The pipe was fitted between the walls when $T = 15°\text{C}$.



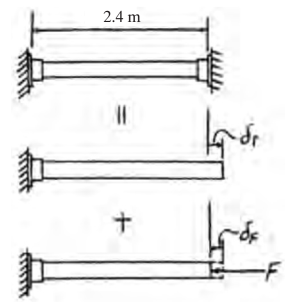
SOLUTION

Temperature Gradient:



$$T(x) = 15 + \left(\frac{2.4 - x}{2.4}\right)(45) = 60 - 18.75x$$

Compatibility:



$$0 = \delta_T - \delta_F \qquad \text{Where} \qquad \delta_T = \int \alpha \Delta T\, dx$$

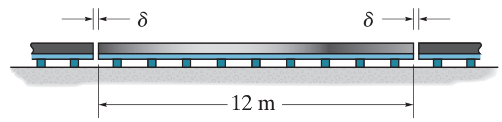
$$0 = 17(10^{-6})\int_0^{2.4\text{m}} [(60 - 18.75x) - 15]\, dx - \frac{F(2.4)}{\left[\frac{\pi}{4}(0.035^2 - 0.025^2)\right][103(10^9)]}$$

$$0 = 17(10^{-6})\int_0^{2.4\text{m}} (45 - 18.75x)\, dx - \frac{F(2.4)}{\left[\frac{\pi}{4}(0.035^2 - 0.025^2)\right][103(10^9)]}$$

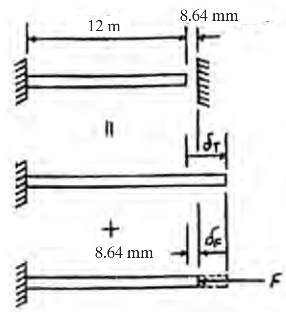
$F = 18.57(10^3)\ \text{N} = 18.6\ \text{kN}$ **Ans.**

Ans:
$F = 18.6\ \text{kN}$

4–75. The 12-m-long A-36 steel rails on a train track are laid with a small gap between them to allow for thermal expansion. Determine the required gap δ so that the rails just touch one another when the temperature is increased from $T_1 = -30°\text{C}$ to $T_2 = 30°\text{C}$. Using this gap, what would be the axial force in the rails if the temperature were to rise to $T_3 = 40°\text{C}$? The cross-sectional area of each rail is 3200 mm^2.



SOLUTION



Thermal Expansion: Note that since adjacent rails expand, each rail will be required to expand $\frac{\delta}{2}$ on each end, or δ for the entine rail.

$$\delta = \alpha \Delta T L = 12(10^{-6})[30 - (-30)](12)(10^3)$$

$$= 8.64 \text{ mm} \qquad \textbf{Ans.}$$

Compatibility:

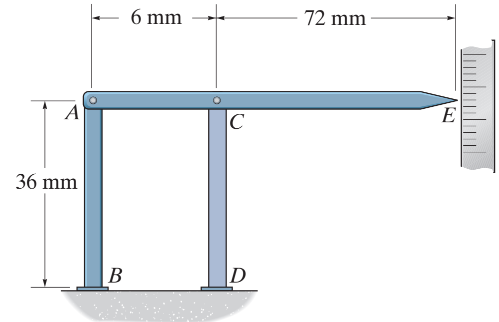
$$(\overset{+}{\rightarrow}) \qquad 8.64 \text{ mm} = \delta_T - \delta_F$$

$$8.64 = 12(10^{-6})[40 - (-30)]12(10^3) - \frac{F(12)(10^3)}{[3200(10^{-6})][200(10^9)]}$$

$$F = 76.8(10^3) \text{ N} = 76.8 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$\delta = 8.64$ mm, $F = 76.8$ kN

***4–76.** The device is used to measure a change in temperature. Bars AB and CD are made of A-36 steel and 2014-T6 aluminum alloy respectively. When the temperature is at 40°C, ACE is in the horizontal position. Determine the vertial displacement of the pointer at E when the temperature rises to 80°C.



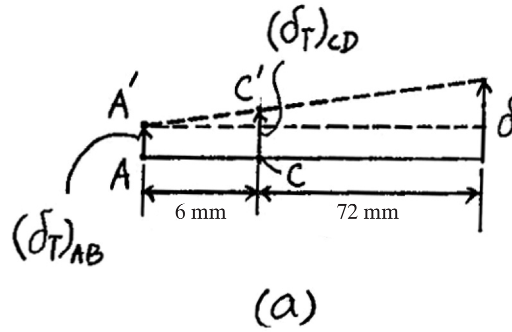
SOLUTION

Thermal Expansion:

$$\left(\delta_T\right)_{CD} = \alpha_{al}\Delta T L_{CD} = 23(10^{-6})(80 - 40)(36) = 33.12(10^{-3})\text{ mm}$$

$$\left(\delta_T\right)_{AB} = \alpha_{st}\Delta T L_{AB} = 12(10^{-6})(80 - 40)(36) = 17.28(10^{-3})\text{ mm}$$

From the geometry of the deflected bar AE shown Fig. b,



$$\delta_E = \left(\delta_T\right)_{AB} + \left[\frac{\left(\delta_T\right)_{CD} - \left(\delta_T\right)_{AB}}{6}\right](78)$$

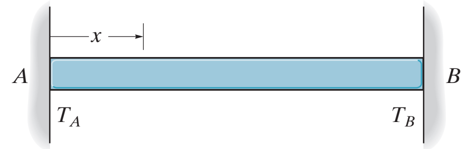
$$= 17.28(10^{-3}) + \left[\frac{32.12(10^{-3}) - 17.28(10^{-3})}{6}\right](78)$$

$$= 0.2102\text{ mm}$$ **Ans.**

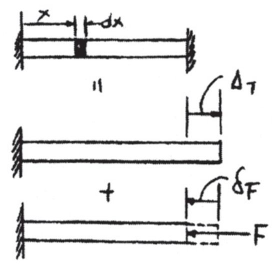
Ans.
$\delta_E = 0.2102$ mm

4–77.

The bar has a cross-sectional area A, length L, modulus of elasticity E, and coefficient of thermal expansion α. The temperature of the bar changes uniformly along its length from T_A at A to T_B at B so that at any point x along the bar $T = T_A + x(T_B - T_A)/L$. Determine the force the bar exerts on the rigid walls. Initially no axial force is in the bar and the bar has a temperature of T_A.



SOLUTION



$\xrightarrow{+}$ $0 = \Delta_T - \delta_F$ **(1)**

However,

$$d\Delta_T = \alpha \Delta_T \, dx = \alpha\left(T_A + \frac{T_B - T_A}{L}x - T_A\right)dx$$

$$\Delta_T = \alpha \int_0^L \frac{T_B - T_A}{L} x \, dx = \alpha\left[\frac{T_B - T_A}{2L}x^2\right]\Bigg|_0^L$$

$$= \alpha\left[\frac{T_B - T_A}{2}L\right] = \frac{\alpha L}{2}(T_B - T_A)$$

From Eq. (1).

$$0 = \frac{\alpha L}{2}(T_B - T_A) - \frac{FL}{AE}$$

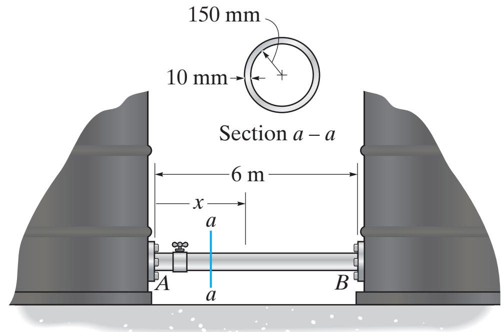
$$F = \frac{\alpha\, AE}{2}(T_B - T_A)$$ **Ans.**

Ans:

$F = \dfrac{\alpha AE}{2}(T_B - T_A)$

4–78.

When the temperature is at 30°C, the A-36 steel pipe fits snugly between the two fuel tanks. When fuel flows through the pipe, the temperatures at ends A and B rise to 130°C and 80°C, respectively. If the temperature drop along the pipe is linear, determine the average normal stress developed in the pipe. Assume each tank provides a rigid support at A and B.



SOLUTION

Temperature Gradient: Since the temperature varies linearly along the pipe, Fig. *a*, the temperature gradient can be expressed as a function of x as

$$T(x) = 80 + \frac{50}{6}(6 - x) = \left(130 - \frac{50}{6}x\right)°\text{C}$$

Thus, the change in temperature as a function of x is

$$\Delta T = T(x) - 30° = \left(130 - \frac{50}{6}x\right) - 30 = \left(100 - \frac{50}{6}x\right)°\text{C}$$

Compatibility Equation: If the pipe is unconstrained, it will have a free expansion of

$$\delta_T = \alpha \int \Delta T dx = 12(10^{-6}) \int_0^{6\text{m}} \left(100 - \frac{50}{6}x\right)dx = 0.0054 \text{ m} = 5.40 \text{ mm}$$

Using the method of superposition, Fig. *b*,

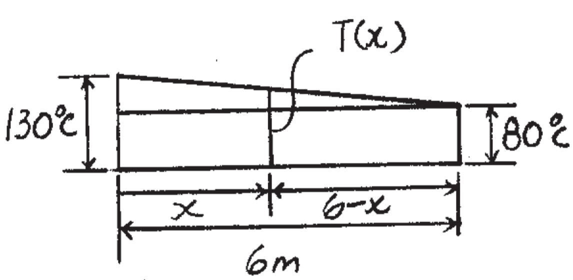
$(\overset{+}{\rightarrow})$ $\qquad 0 = \delta_T - \delta_F$

$$0 = 5.40 - \frac{F(6000)}{\pi(0.16^2 - 0.15^2)(200)(10^9)}$$

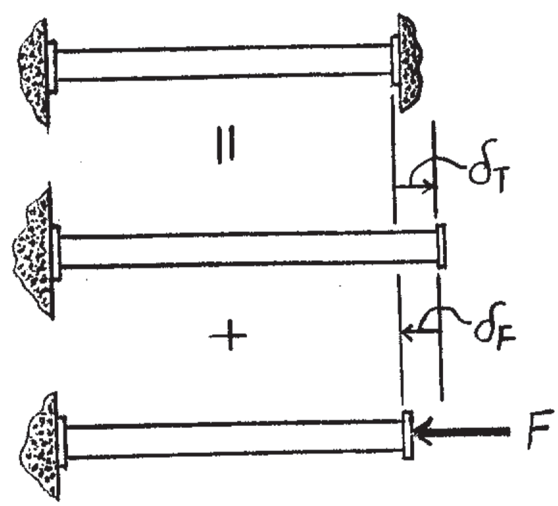
$$F = 1\,753\,008 \text{ N}$$

Normal Stress:

$$\sigma = \frac{F}{A} = \frac{1\,753\,008}{\pi(0.16^2 - 0.15^2)} = 180 \text{ MPa}$$ **Ans.**



(a)

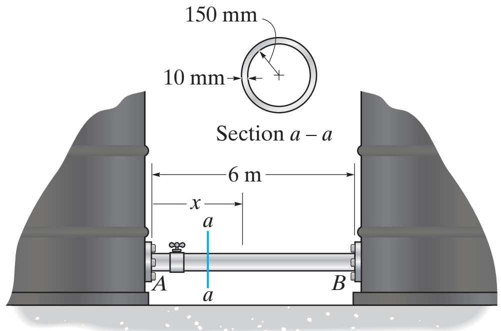


(b)

Ans:
$\sigma = 180$ MPa

4–79.

When the temperature is at 30°C, the A-36 steel pipe fits snugly between the two fuel tanks. When fuel flows through the pipe, the temperatures at ends A and B rise to 130°C and 80°C, respectively. If the temperature drop along the pipe is linear, determine the average normal stress developed in the pipe. Assume the walls of each tank act as a spring, each having a stiffness of $k = 900$ MN/m.



SOLUTION

Temperature Gradient: Since the temperature varies linearly along the pipe, Fig. *a*, the temperature gradient can be expressed as a function of x as

$$T(x) = 80 + \frac{50}{6}(6 - x) = \left(130 - \frac{50}{6}x\right)°\text{C}$$

Thus, the change in temperature as a function of x is

$$\Delta T = T(x) - 30° = \left(130 - \frac{50}{6}x\right) - 30 = \left(100 - \frac{50}{6}x\right)°\text{C}$$

Compatibility Equation: If the pipe is unconstrained, it will have a free expansion of

$$\delta_T = \alpha\int \Delta T dx = 12(10^{-6})\int_0^{6\text{ m}}\left(100 - \frac{50}{6}x\right)dx = 0.0054 \text{ m} = 5.40 \text{ mm}$$

Using the method of superposition, Fig. *b*,

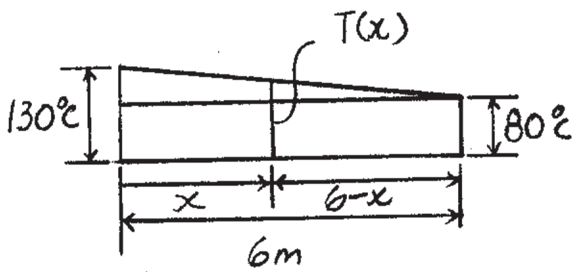
$$(\overset{+}{\rightarrow}) \qquad \delta = \delta_T - \delta_F$$

$$\frac{F}{900(10^6)}(1000) = 5.40 - \left[\frac{F(6000)}{\pi(0.16^2 - 0.15^2)(200)(10^9)} + \frac{F}{900(10^6)}(1000)\right]$$

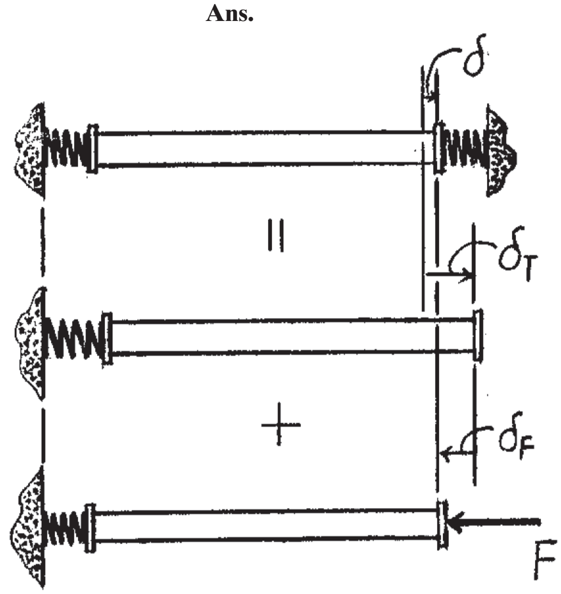
$$F = 1\,018\,361 \text{ N}$$

Normal Stress:

$$\sigma = \frac{F}{A} = \frac{1\,018\,361}{\pi(0.16^2 - 0.15^2)} = 105 \text{ MPa}$$ **Ans.**



(a)

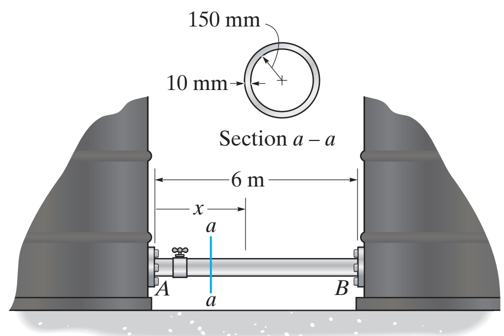


(b)

Ans:
$\sigma = 105$ MPa

***4–80.**

When the temperature is at 30°C, the A-36 steel pipe fits snugly between the two fuel tanks. When fuel flows through the pipe, it causes the temperature to vary along the pipe as $T = (\frac{5}{3}x^2 - 20x + 120)$°C, where x is in meters. Determine the normal stress developed in the pipe. Assume each tank provides a rigid support at A and B.



SOLUTION

Compatibility Equation: The change in temperature as a function of x is $\Delta T = T - 30° = \left(\frac{5}{3}x^2 - 20x + 120\right) - 30 = \left(\frac{5}{3}x^2 - 20x + 90\right)$°C. If the pipe is unconstrained, it will have a free expansion of

$$\delta_T = \alpha \int \Delta T dx = 12(10^{-6}) \int_0^{6\text{ m}} \left(\frac{5}{3}x^2 - 20x + 90\right) dx = 0.0036 \text{ m} = 3.60 \text{ mm}$$

Using the method of superposition, Fig. b,

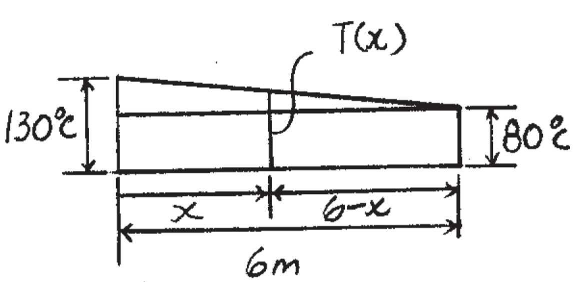
$$(\overset{+}{\rightarrow}) \qquad 0 = \delta_T - \delta_F$$

$$0 = 3.60 - \frac{F(6000)}{\pi(0.16^2 - 0.15^2)(200)(10^9)}$$

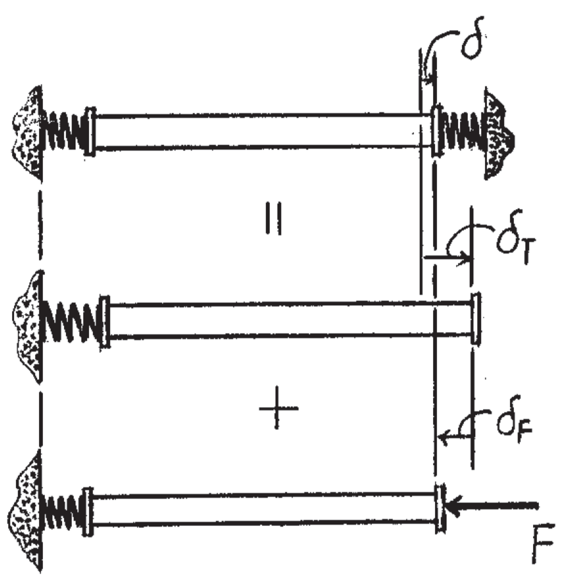
$$F = 1\,168\,672.47 \text{ N}$$

Normal Stress:

$$\sigma = \frac{F}{A} = \frac{1\,168\,672.47}{\pi\left(0.16^2 - 0.15^2\right)} = 120 \text{ MPa}$$ **Ans.**



(a)

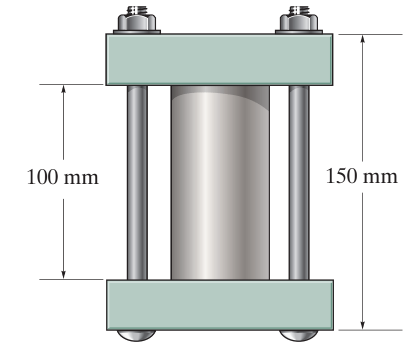


(b)

Ans:
$\sigma = 120$ MPa

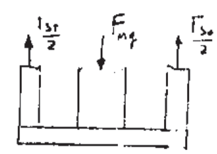
4–81.

The 50-mm-diameter cylinder is made from Am 1004-T61 magnesium and is placed in the clamp when the temperature is $T_1 = 20°$ C. If the 304-stainless-steel carriage bolts of the clamp each have a diameter of 10 mm, and they hold the cylinder snug with negligible force against the rigid jaws, determine the force in the cylinder when the temperature rises to $T_2 = 130°$C.



SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad F_{st} = F_{mg} = F$



$$\delta_{mg} = \delta_{st}$$

$$\alpha_{mg} L_{mg} \Delta T - \frac{F_{mg} L_{mg}}{E_{mg} A_{mg}} = \alpha_{st} L_{st} \Delta T + \frac{F_{st} L_{st}}{E_{st} A_{st}}$$

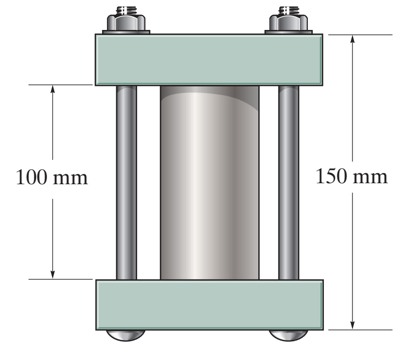
$$26(10^{-6})(0.1)(110) - \frac{F(0.1)}{44.7(10^9)\frac{\pi}{4}(0.05)^2} = 17(10^{-6})(0.150)(110) + \frac{F(0.150)}{193(10^9)(2)\frac{\pi}{4}(0.01)^2}$$

$F = 904$ N **Ans.**

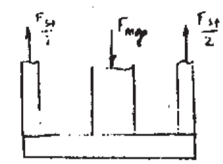
Ans:
$F = 904$ N

4–82.

The 50-mm-diameter cylinder is made from Am 1004-T61 magnesium and is placed in the clamp when the temperature is $T_1 = 15°\text{C}$. If the two 304-stainless-steel carriage bolts of the clamp each have a diameter of 10 mm, and they hold the cylinder snug with negligible force against the rigid jaws, determine the temperature at which the average normal stress in either the magnesium or the steel first becomes 12 MPa.



SOLUTION



$+\uparrow \Sigma F_y = 0; \qquad F_{st} = F_{mg} = F$

$$\delta_{mg} = \delta_{st}$$

$$\alpha_{mg} L_{mg} \Delta T - \frac{F_{mg} L_{mg}}{E_{mg} A_{mg}} = \alpha_{st} L_{st} \Delta T + \frac{F_{st} L_{st}}{E_{st} A_{st}}$$

$$26(10^{-6})(0.1)(\Delta T) - \frac{F(0.1)}{44.7(10^9)\frac{\pi}{4}(0.05)^2} = 17(10^{-6})(0.150)(\Delta T) + \frac{F(0.150)}{193(10^9)(2)\frac{\pi}{4}(0.01)^2}$$

The steel has the smallest cross-sectional area.

$$F = \sigma A = 12(10^6)(2)\left(\frac{\pi}{4}\right)(0.01)^2 = 1885.0 \text{ N}$$

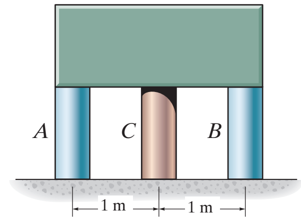
Thus,

$$\Delta T = 229°$$

$$T_2 = 229° + 15° = 244° \text{ C}$$ **Ans.**

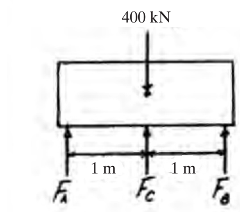
Ans:
$T_2 = 244° \text{ C}$

4–83. The rigid block has a weight of 400 kN and is to be supported by posts A and B, which are made of A-36 steel, and the post C, which is made of C83400 red brass. If all the posts have the same original length before they are loaded, determine the average normal stress developed in each post when post C is heated so that its temperature is increased by 10°C. Each post has a cross-sectional area of 5000 mm^2.



SOLUTION

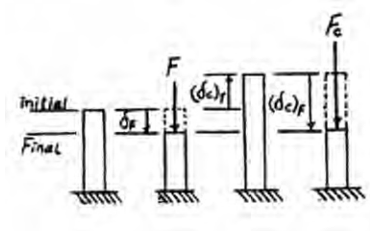
Equations of Equilibrium:



$$\zeta+\Sigma M_C = 0; \qquad F_B(1) - F_A(1) = 0 \qquad F_A = F_B = F$$

$$+\uparrow\Sigma F_y = 0; \qquad 2F + F_C - 400(10^3) = 0 \qquad \textbf{[1]}$$

Compatibility:



$$(+\downarrow) \qquad (\delta_C)_F - (\delta_C)_T = \delta_F$$

$$\frac{F_C L}{\left[5000(10^{-6})\right][101(10^9)]} - 18(10^{-6})(10)L = \frac{FL}{\left[5000(10^{-6})\right][200(10^9)]}$$

$$1.9802F_C - F = 180(10^3) \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields:

$$F = 123.393(10^3)\text{ N} \qquad F_C = 153.214(10^3)\text{ N}$$

average Normal Sress:

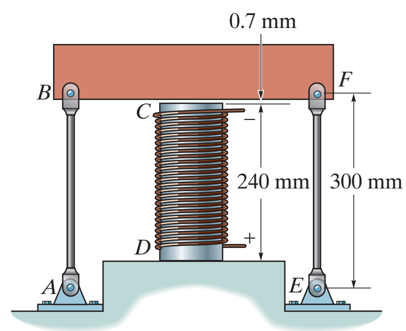
$$\sigma_A = \sigma_B = \frac{F}{A} = \frac{123.393(10^3)}{5000(10^{-6})} = 24.68(10^6)\text{ N/m}^2 = 24.7\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_C = \frac{F_C}{A} = \frac{153.214(10^3)}{5000(10^{-6})} = 30.64(10^6)\text{ N/m}^2 = 30.6\text{ MPa} \qquad \textbf{Ans.}$$

Ans.
$\sigma_A = \sigma_B = 24.7$ MPa, $\sigma_C = 30.6$ MPa

***4–84.**

The cylinder CD of the assembly is heated from $T_1 = 30°\text{C}$ to $T_2 = 180°\text{C}$ using electrical resistance. At the lower temperature T_1 the gap between C and the rigid bar is 0.7 mm. Determine the force in rods AB and EF caused by the increase in temperature. Rods AB and EF are made of steel, and each has a cross-sectional area of 125 mm². CD is made of aluminum and has a cross-sectional area of 375 mm². $E_{st} = 200$ GPa, $E_{al} = 70$ GPa, and $\alpha_{al} = 23(10^{-6})/°\text{C}$.



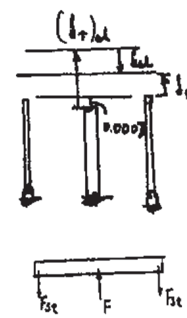
SOLUTION

$$\delta_{st} = (\delta_{\gamma})_{al} - \delta_{al} - 0.0007$$

$$\frac{F_{st}(0.3)}{(125)(10^{-6})(200)(10^9)} = 23(10^{-6})(150)(0.24) - \frac{F(0.24)}{(375)(10^{-6})(70)(10^9)} - 0.0007$$

$$12F_{st} = 128\,000 - 9.1428F \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F - 2F_{st} = 0 \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields,

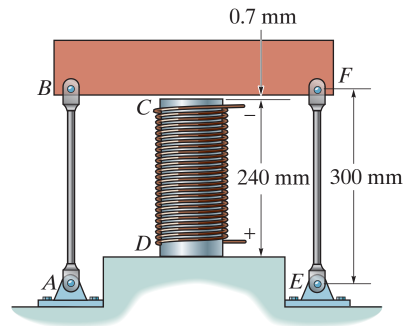
$F_{AB} = F_{EF} = F_{st} = 4.23$ kN

$F_{CD} = F = 8.45$ kN **Ans.**

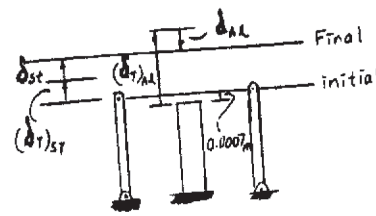
Ans:
$F_{CD} = 8.45$ kN

4–85.

The cylinder CD of the assembly is heated from $T_1 = 30°\text{C}$ to $T_2 = 180°\text{C}$ using electrical resistance. Also, the two end rods AB and EF are heated from $T_1 = 30°\text{C}$ to $T_2 = 50°\text{C}$. At the lower temperature T_1 the gap between C and the rigid bar is 0.7 mm. Determine the force in rods AB and EF caused by the increase in temperature. Rods AB and EF are made of steel, and each has a cross-sectional area of 125 mm^2. CD is made of aluminum and has a cross-sectional area of 375 mm^2. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa, $\alpha_{st} = 12(10^{-6})/°\text{C}$, and $\alpha_{al} = 23(10^{-6})/°\text{C}$.



SOLUTION

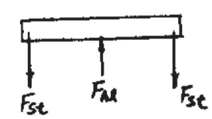


$$\delta_{st} + (\delta_T)_{st} = (\delta_T)_{al} - \delta_{al} - 0.0007$$

$$\frac{F_{st}(0.3)}{(125)(10^{-6})(200)(10^9)} + 12(10^{-6})(50 - 30)(0.3)$$

$$= 23(10^{-6})(180 - 30)(0.24) - \frac{F_{al}(0.24)}{375(10^{-6})(70)(10^9)} - 0.0007$$

$$12.0F_{st} + 9.14286F_{al} = 56000 \qquad \textbf{(1)}$$



$$+\uparrow \Sigma F_y = 0; \qquad F_{al} - 2F_{st} = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

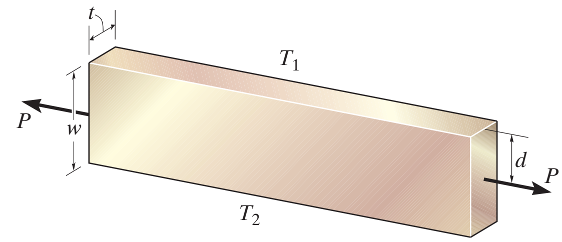
$$F_{AB} = F_{EF} = F_{st} = 1.85 \text{ kN} \qquad \textbf{Ans.}$$

$$F_{CD} = F_{al} = 3.70 \text{ kN}$$

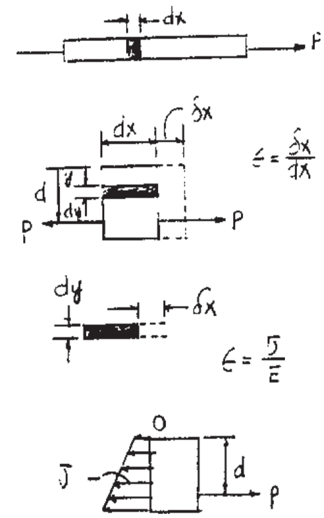
Ans:
$F_{AB} = F_{EF} = 1.85$ kN

4–86.

The metal strap has a thickness t and width w and is subjected to a temperature gradient T_1 to T_2 ($T_1 < T_2$). This causes the modulus of elasticity for the material to vary linearly from E_1 at the top to a smaller amount E_2 at the bottom. As a result, for any vertical position y, measured from the top surface, $E = [(E_2 - E_1)/w]y + E_1$. Determine the position d where the axial force P must be applied so that the bar stretches uniformly over its cross section.



SOLUTION



$$\epsilon = \text{constant} = \epsilon_0$$

$$\epsilon_0 = \frac{\sigma}{E} = \frac{\sigma}{\left(\left(\frac{E_2 - E_1}{w}\right)y + E_1\right)}$$

$$\sigma = \epsilon_0\left(\frac{E_2 - E_1}{w}y + E_1\right)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad P - \int_A \sigma\, dA = 0$$

$$P = \int_0^w \sigma t\, dy = \int_0^m \epsilon_0\left(\frac{E_2 - E_1}{w}y + E_1\right) t\, dy$$

$$P = \epsilon_0 t\left(\frac{E_2 - E_1}{2} + E_1 w\right) = \epsilon_0 t\left(\frac{E_2 + E_1}{2}\right) w$$

$$\circlearrowleft + \Sigma M_0 = 0: \quad P(d) - \int_A y\,\sigma\, dA = 0$$

$$\epsilon_0 t\left(\frac{E_2 + E_1}{2}\right)wd = \int_0^w \epsilon_0\left(\left(\frac{E_2 - E_1}{w}\right)y^2 + E_1 y\right) t\, dy$$

$$\epsilon_0 t\left(\frac{E_2 + E_1}{2}\right)wd = \epsilon_0 t\left(\frac{E_2 - E_1}{3}w^2 + \frac{E_1}{2}w^2\right)$$

$$\left(\frac{E_2 + E_1}{2}\right)d = \frac{1}{6}(2E_2 + E_1)w$$

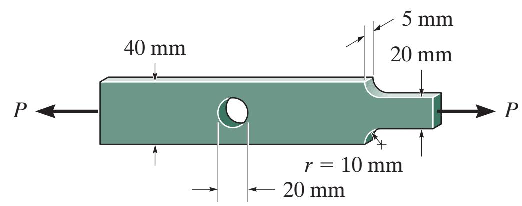
$$d = \left(\frac{2E_2 + E_1}{3(E_2 + E_1)}\right)w \qquad \textbf{Ans.}$$

Ans:

$$d = \left(\frac{2E_2 + E_1}{3(E_2 + E_1)}\right)w$$

4–87.

Determine the maximum normal stress developed in the bar when it is subjected to a tension of $P = 8$ kN.



SOLUTION

For the fillet:

$$\frac{w}{h} = \frac{40}{20} = 2 \qquad \frac{r}{h} = \frac{10}{20} = 0.5$$

From Fig. 4–23, $K = 1.4$

$$\sigma_{\max} = K\sigma_{\text{avg}}$$

$$= 1.4\left(\frac{8\,(10^3)}{0.02\,(0.005)}\right)$$

$$= 112 \text{ MPa}$$

For the hole:

$$\frac{2r}{w} = \frac{20}{40} = 0.5$$

From Fig. 4–24, $K = 2.1$

$$\sigma_{\max} = K\sigma_{\text{avg}}$$

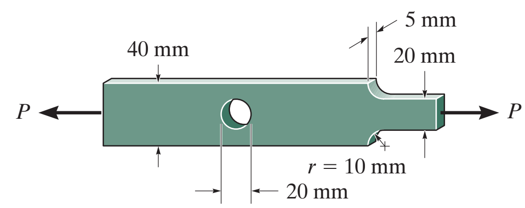
$$= 2.1\left(\frac{8\,(10^3)}{(0.04 - 0.02)(0.005)}\right)$$

$$= 168 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma_{\max} = 168$ MPa

***4–88.**

If the allowable normal stress for the bar is $\sigma_{allow} = 120$ MPa, determine the maximum axial force P that can be applied to the bar.



SOLUTION

Assume failure of the fillet.

$$\frac{w}{h} = \frac{40}{20} = 2; \qquad \frac{r}{h} = \frac{10}{20} = 0.5$$

From Fig. 4–23. $\quad K = 1.4$

$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$120(10^6) = 1.4\left(\frac{P}{0.02(0.005)}\right)$$

$$P = 8.57 \text{ kN}$$

Assume failure of the hole.

$$\frac{2r}{w} = \frac{20}{40} = 0.5$$

From Fig. 4–24. $\quad K = 2.1$

$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

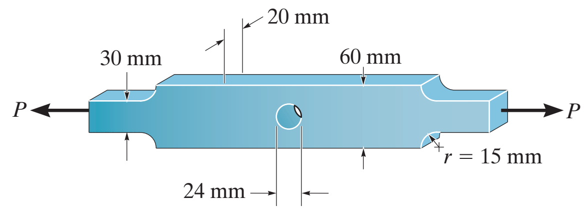
$$120(10^4) = 2.1\left(\frac{P}{(0.04 - 0.02)(0.005)}\right)$$

$$P = 5.71 \text{ kN (controls)} \qquad \textbf{Ans.}$$

Ans:
$P = 5.71$ kN

4–89.

The steel bar has the dimensions shown. Determine the maximum axial force P that can be applied so as not to exceed an allowable tensile stress of $\sigma_{allow} = 150$ MPa.



SOLUTION

Assume failure occurs at the fillet:

$$\frac{w}{h} = \frac{60}{30} = 2 \quad \text{and} \quad \frac{r}{h} = \frac{15}{30} = 0.5$$

From the text, $K = 1.4$

$$\sigma_{max} = \sigma_{allow} = K\sigma_{avg}$$

$$150(10^6) = 1.4\left[\frac{P}{0.03(0.02)}\right]$$

$$P = 64.3 \text{ kN}$$

Assume failure occurs at the hole:

$$\frac{2r}{w} = \frac{24}{60} = 0.4$$

From the text, $K = 2.2$

$$\sigma_{max} = \sigma_{allow} = K\sigma_{avg}$$

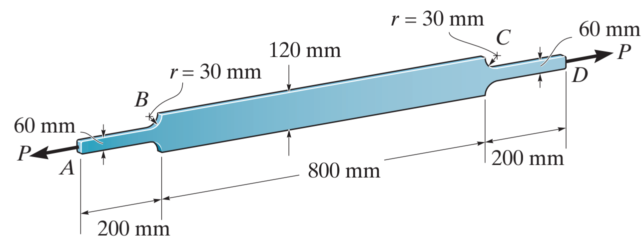
$$150(10^6) = 2.2\left[\frac{P}{(0.06 - 0.024)(0.02)}\right]$$

$$P = 49.1 \text{ kN (controls!)}$$ **Ans.**

Ans:
$P = 49.1$ kN

4–90.

The A-36 steel plate has a thickness of 12 mm. If $\sigma_{allow} = 150$ MPa, determine the maximum axial load P that it can support. Calculate its elongation, neglecting the effect of the fillets.



SOLUTION

Maximum Normal Stress at fillet:

$$\frac{r}{h} = \frac{30}{60} = 0.5 \quad \text{and} \quad \frac{w}{h} = \frac{120}{60} = 2$$

From the text, $K = 1.4$

$$\sigma_{max} = \sigma_{allow} = K\sigma_{avg}$$

$$150(10^6) = 1.4\left[\frac{P}{0.06(0.012)}\right]$$

$$P = 77142.86 \text{ N} = 77.1 \text{ kN}$$ **Ans.**

Displacement:

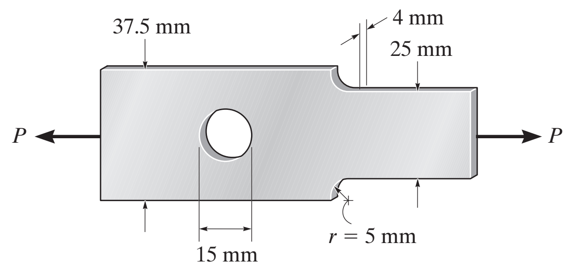
$$\delta = \Sigma\frac{PL}{AE}$$

$$= \frac{77142.86(400)}{(0.06)(0.012)(200)(10^9)} + \frac{77142.86(800)}{(0.12)(0.012)(200)(10^9)}$$

$$= 0.429 \text{ mm}$$ **Ans.**

Ans:
$P = 77.1$ kN,
$\delta = 0.429$ mm

4–91. Determine the maximum axial force P that can be applied to the bar. The bar is made from steel and has an allowable stress of $\sigma_{\text{allow}} = 147$ MPa.



SOLUTION

Assume failure of the fillet.

$$\frac{r}{h} = \frac{5}{25} = 0.2 \qquad \frac{w}{h} = \frac{37.5}{25} = 1.5$$

From Fig. 4-24, $K = 1.73$

$$\sigma_{\text{allow}} = \sigma_{\text{max}} = K\sigma_{\text{avg}}$$

$$147(10^6) = 1.73\left[\frac{P}{(0.025)(0.004)}\right]$$

$$P = 8.497(10^3)\text{ N}$$

Assume failure of the hole.

$$\frac{r}{w} = \frac{7.5}{37.5} = 0.20$$

From Fig. 4-25, $K = 2.45$

$$\sigma_{\text{allow}} = \sigma_{\text{max}} = K\sigma_{\text{avg}}$$

$$147(10^6) = 2.45\left[\frac{P}{(0.0375 - 0.015)(0.004)}\right]$$

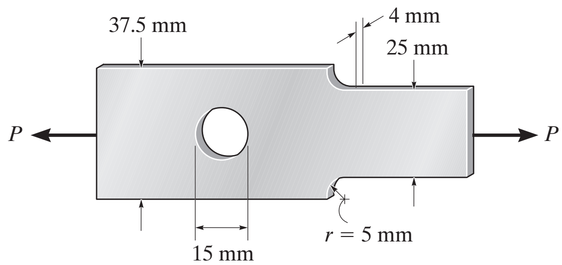
$P = 5.400(10^3)$ N (controls) **Ans.**

$= 5.40$ kN **Ans.**

Ans.
$P = 5.40$ kN

***4–92.**

Determine the maximum normal stress developed in the bar when it is subjected to a tension of $P = 8$ kN.



SOLUTION

At fillet:

$$\frac{r}{h} = \frac{5}{25} = 0.2 \qquad \frac{w}{h} = \frac{37.5}{25} = 1.5$$

From Fig. 4-24, $K = 1.73$

$$\sigma_{\max} = K\left(\frac{P}{A}\right) = 1.73\left[\frac{8(10^3)}{(0.025)(0.004)}\right] = 138.4(10^6)\ \text{N/m}^2 = 138\ \text{MPa}$$

At hole:

$$\frac{r}{w} = \frac{7.5}{37.5} = 0.20$$

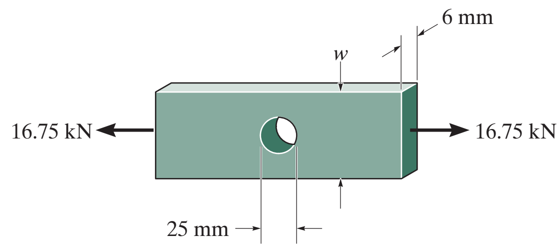
From Fig. 4-25, $K = 2.45$

$$\sigma_{\max} = 2.45\left[\frac{8(10^3)}{(0.0375-0.015)(0.004)}\right] = 217.78(10^6)\ \text{N/m}^2 = 218\ \text{MPa} \quad \text{(Controls)}$$

Ans.

Ans.
$\sigma_{\max} = 2.45$ MPa

4–93. The member is to be made from a steel plate that is 6 mm thick. If a 25-mm hole is drilled through its center, determine the approximate width w of the plate so that it can support an axial force of 16.75 kN. The allowable stress is $\sigma_{\text{allow}} = 150$ MPa.



SOLUTION

$$\sigma_{\text{allow}} = \sigma_{\max} = K\sigma_{\text{avg}}$$

$$150(10^6) = K\left[\frac{16.75(10^3)}{(w-0.025)(0.006)}\right]$$

$$w = 0.025 + 0.01861K$$

By trial and error, from Fig. 4-25, choose $\frac{r}{w} = 0.17$; $\quad K = 2.50$

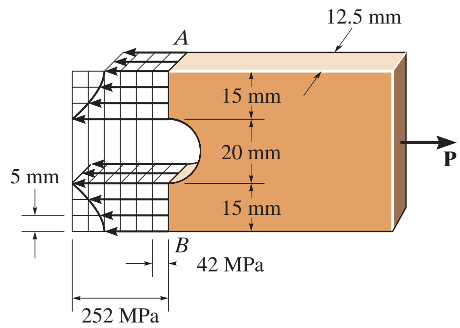
$$w = 0.025 + 0.01861(2.50) = 0.07153 \text{ m} = 71.5 \text{ mm}$$ **Ans.**

Since $\frac{r}{w} = \frac{12.5}{71.53} = 0.17$ OK

Ans.
$K = 2.50,\ w = 71.5$ mm

4–94.

The resulting stress distribution along section AB for the bar is shown. From this distribution, determine the approximate resultant axial force P applied to the bar. Also, what is the stress-concentration factor for this geometry?



SOLUTION

Number of squares = 28

$P = 28[42(10^6)](0.005)(0.0125) = 73.5(10^3)\text{ N} = 73.5\text{ kN}$ **Ans.**

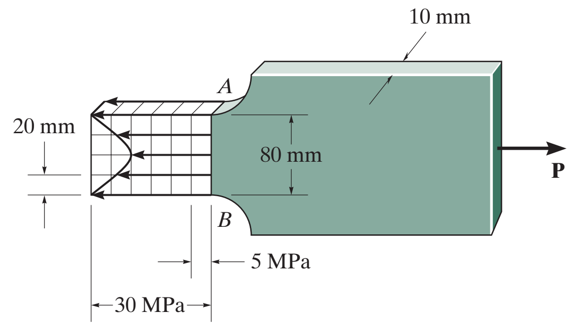
$$\sigma_{avg} = \frac{P}{A} = \frac{73.5(10^3)}{2(0.015)(0.0125)} = 196(10^6)\text{ N/m}^2 = 196\text{ MPa}$$

$$K = \frac{\sigma_{max}}{\sigma_{avg}} = \frac{252}{196} = 1.286 = 1.29$$ **Ans.**

Ans.
$P = 73.5\text{ kN}, K = 1.29$

4–95.

The resulting stress distribution along section AB for the bar is shown. From this distribution, determine the approximate resultant axial force P applied to the bar. Also, what is the stress concentration factor?



SOLUTION

Number of squares = 19

$P = 19(5)(10^6)(0.02)(0.01) = 19 \text{ kN}$ **Ans.**

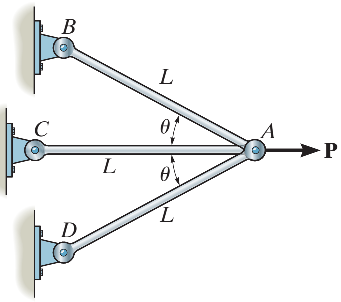
$$\sigma_{avg} = \frac{P}{A} = \frac{19(10^3)}{0.08(0.01)} = 23.75 \text{ MPa}$$

$$K = \frac{\sigma_{max}}{\sigma_{avg}} = \frac{30 \text{ MPa}}{23.75 \text{ MPa}} = 1.26$$ **Ans.**

Ans:
$P = 19$ kN,
$K = 1.26$

***4–96.**

The three bars are pinned together and subjected to the load **P**. If each bar has a cross-sectional area A, length L, and is made from an elastic perfectly plastic material, for which the yield stress is σ_Y, determine the largest load (ultimate load) that can be supported by the bars, i.e., the load P that causes all the bars to yield. Also, what is the horizontal displacement of point A when the load reaches its ultimate value? The modulus of elasticity is E.

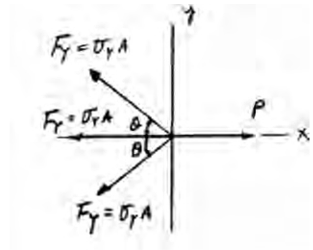


SOLUTION

When all bars yield, the force in each bar is, $F_Y = \sigma_Y A$

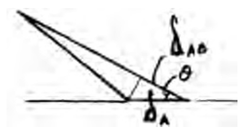
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P - 2\sigma_Y A \cos\theta - \sigma_Y A = 0$

$P = \sigma_Y A(2\cos\theta + 1)$ **Ans.**



Bar AC will yield first followed by bars AB and AD.

$$\delta_{AB} = \delta_{AD} = \frac{F_Y(L)}{AE} = \frac{\sigma_Y AL}{AE} = \frac{\sigma_Y L}{E}$$



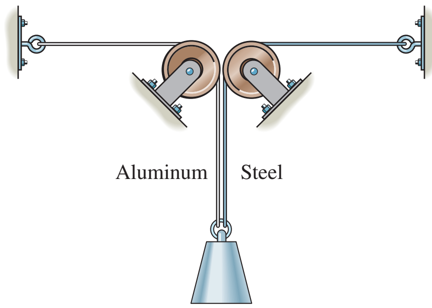
$$\delta_A = \frac{\delta_{AB}}{\cos\theta} = \frac{\sigma_Y L}{E\cos\theta}$$ **Ans.**

Ans.

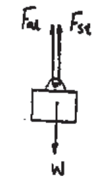
$$\delta_A = \frac{\delta_{AB}}{\cos\theta} = \frac{\sigma_Y L}{E\cos\theta}$$

4–98.

The weight is suspended from steel and aluminum wires, each having the same initial length of 3 m and cross-sectional area of 4 mm^2. If the materials can be assumed to be elastic perfectly plastic, with $(\sigma_Y)_{st} = 120$ MPa and $(\sigma_Y)_{al} = 70$ MPa, determine the force in each wire if the weight is (a) 600 N and (b) 720 N. $E_{al} = 70$ GPa, $E_{st} = 200$ GPa.



SOLUTION



$+\uparrow \Sigma F_y = 0; \quad F_{al} + F_{st} - W = 0$ **(1)**

Assume both wires behave elastically.

$$\delta_{al} = \delta_{st}; \quad \frac{F_{al} L}{A(70)} = \frac{F_{st} L}{A(200)}$$

$F_{al} = 0.35\, F_{st}$ **(2)**

(a) When $W = 600$ N, solving Eqs. (1) and (2) yields:

$F_{st} = 444.44 \text{ N} = 444 \text{ N}$ **Ans.**

$F_{al} = 155.55 \text{ N} = 156 \text{ N}$ **Ans.**

$$\sigma_{al} = \frac{F_{al}}{A_{st}} = \frac{155.55}{4(10^{-6})} = 38.88 \text{ MPa} < (\sigma_\gamma)_{al} = 70 \text{ MPa}$$ **OK**

$$\sigma_{st} = \frac{F_{st}}{A_{st}} = \frac{444.44}{4(10^{-6})} = 111.11 \text{ MPa} < (\sigma_\gamma)_{st} = 120 \text{ MPa}$$ **OK**

The elastic analysis is valid for both wires.

(b) When $W = 720$ N, solving Eqs. (1) and (2) yields:

$F_{st} = 533.33 \text{ N}: \quad F_{st} = 186.67 \text{ N}$

$$\sigma_{al} = \frac{F_{al}}{A_{al}} = \frac{186.67}{4(10^{-6})} = 46.67 \text{ MPa} < (\sigma_\gamma)_{al} = 70 \text{ MPa}$$ **OK**

$$\sigma_{st} = \frac{F_{st}}{A_{st}} = \frac{533.33}{4(10^{-6})} = 133.33 \text{ MPa} > (\sigma_\gamma)_{st} = 120 \text{ MPa}$$

Therefore, the steel wire yields. Hence,

$F_{st} = (\sigma_\gamma)_{st} A_{st} = 120(10^6)(4)(10^{-6}) = 480 \text{ N}$ **Ans.**

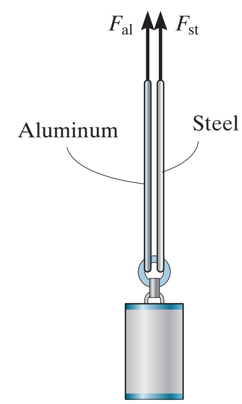
From Eq. (1), $F_{al} = 240$ N **Ans.**

$$\sigma_{al} = \frac{240}{4(10^{-6})} = 60 \text{ MPa} < (\sigma_\gamma)_{al}$$ **OK**

Ans:
(a) $F_{st} = 444$ N, $F_{al} = 156$ N,
(b) $F_{st} = 480$ N, $F_{al} = 240$ N

4–99.

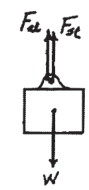
The weight is suspended from steel and aluminum wires, each having the same initial length of 3 m and cross-sectional area of 4 mm^2. If the materials can be assumed to be elastic perfectly plastic, with $(\sigma_Y)_{st} = 120$ MPa and $(\sigma_Y)_{al} = 70$ MPa, determine the force in each wire if the weight is (a) 600 N and (b) 720 N. $E_{al} = 70$ GPa, $E_{st} = 200$ GPa.



SOLUTION

Equations of Equilibrium:

$$+\uparrow \Sigma F_y = 0; \qquad F_{al} + F_{st} - w = 0 \qquad \textbf{(1)}$$



Elastic Analysis: Assume both wires behave elastically.

$$\delta_{al} = \delta_{st}$$

$$\frac{F_{al} L}{A(70)(10^9)} = \frac{F_{st} L}{A(200)(10^9)}$$

$$F_{al} = 0.350\, F_{st} \qquad \textbf{(2)}$$

a) When $W = 600$ N, solving Eq. (1) and (2) yields:

$$F_{st} = 444.44 \text{ N} = 444 \text{ N} \qquad \textbf{Ans.}$$

$$F_{al} = 155.55 \text{ N} = 156 \text{ N} \qquad \textbf{Ans.}$$

Average Normal Stress:

$$\sigma_{al} = \frac{F_{al}}{A_{sl}} = \frac{155.55}{4.00(10^{-6})} = 38.88 \text{ MPa} < (\sigma_\gamma)_{al} = 70.0 \text{ MPa} \qquad \textbf{(OK!)}$$

$$\sigma_{st} = \frac{F_{st}}{A_{st}} = \frac{444.44}{4.00(10^{-6})} = 111.11 \text{ MPa} < (\sigma_\gamma)_{st} = 120 \text{ MPa} \qquad \textbf{(OK!)}$$

The average normal stress for both wires do not exceed their respective yield stress. Therefore, the elastic analysis is vaild for both wires

b) When $W = 720$ N, solving Eq. (1) and (2) yields:

$$F_{st} = 533.33 \text{ N} \qquad F_{al} = 186.67 \text{ N}$$

Average Normal Stress:

$$\sigma_{al} = \frac{F_{al}}{A_{al}} = \frac{186.67}{4.00(10^{-6})} = 46.67 \text{ MPa} < (\sigma_\gamma)_{al} = 70.0 \text{ MPa} \qquad \textbf{(OK!)}$$

$$\sigma_{st} = \frac{F_{st}}{A_{st}} = \frac{533.33}{4.00(10^{-6})} = 133.33 \text{ MPa} > (\sigma_\gamma)_{st} = 120 \text{ MPa}$$

Therefore, the steel wire yields. Hence,

$$F_{st} = (\sigma_\gamma)_{st} A_{st} = 120(10^6)(4.00)(10^{-6}) = 480 \text{ N} \qquad \textbf{Ans.}$$

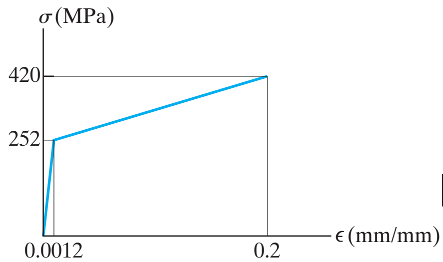
From Eq. (1), $\qquad F_{al} = 240$ N $\qquad$ **Ans.**

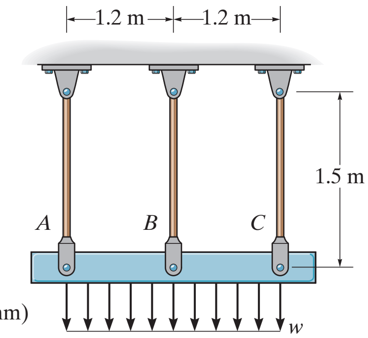
$$\sigma_{al} = \frac{240}{4.00(10^{-6})} = 60.00 \text{ MPa} < (\sigma_\gamma)_{al} \qquad \textbf{(OK!)}$$

Ans:
$F_{st} = 444$ N,
$F_{al} = 156$ N,
$F_{st} = 480$ N,
$F_{al} = 240$ N

***4–100.**

The distributed loading is applied to the rigid beam, which is supported by the three bars. Each bar has a cross-sectional area of 780 mm^2 and is made from a material having a stress–strain diagram that can be approximated by the two line segments shown. If a load of $w = 400$ kN/m is applied to the beam, determine the stress in each bar and the vertical displacement of the beam.





SOLUTION

$\circlearrowleft +\Sigma M_B = 0; \qquad F_C(1.2) - F_A(1.2) = 0;$

$F_A = F_C = F$

$+\uparrow \Sigma F_y = 0; \qquad 2F + F_B - 960 = 0 \qquad$ **(1)**

Since the loading and geometry are symmetrical, the bar will remain horizontal. Therefore, the displacement of the bars is the same and hence, the force in each bar is the same. From Eq. (1).

$F = F_B = 320$ kN

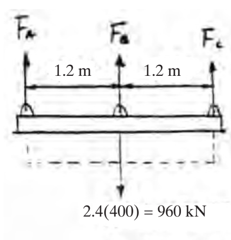
Thus,

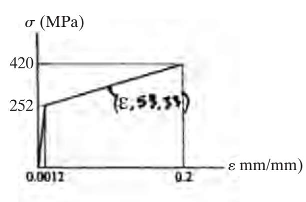
$$\sigma_A = \sigma_B = \sigma_C = \frac{320(10^3)}{780(10^{-6})} = 410.26(10^6)\ \text{N/m}^2 = 410\ \text{MPa}$$ **Ans.**

From the stress-strain diagram:

$$\frac{410.26 - 252}{\varepsilon - 0.0012} = \frac{420 - 252}{0.2 - 0.0012}; \quad \varepsilon = 0.18847\ \text{mm/mm}$$

$$\delta = \varepsilon L = 0.18847(1.5)(10^3) = 282.71\ \text{mm} = 283\ \text{mm}$$ **Ans.**

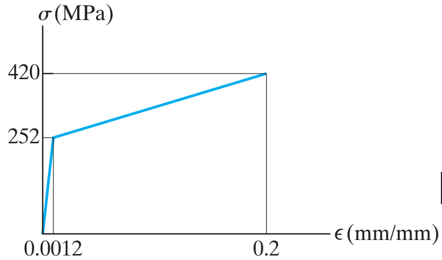


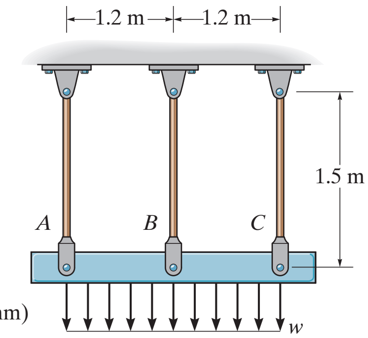


Ans.
$\delta = 283$ mm

4–101.

The distributed loading is applied to the rigid beam, which is supported by the three bars. Each bar has a cross-sectional area of 468 mm^2 and is made from a material having a stress–strain diagram that can be approximated by the two line segments shown. Determine the intensity of the distributed loading w needed to cause the beam to be displaced downard 37.5 mm.





SOLUTION

$\circlearrowleft + \Sigma M_B = 0; \qquad F_C(1.2) - F_A(1.2) = 0; \qquad F_A = F_C = F$

$+\uparrow \Sigma F_y = 0; \qquad 2F + F_B - 2.4w = 0$ **(1)**

Since the system and the loading are symmetrical, the bar will remain horizontal. Hence the displacement of the bars is the same and the force supported by each bar is the same.

From Eq. (1),

$F_B = F = 0.8\,w$ **(2)**

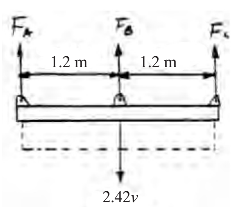
From the stress-strain diagram:

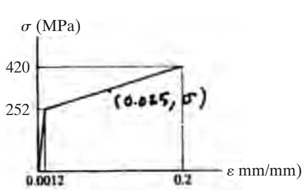
$$\varepsilon = \frac{37.5}{1.5(10^3)} = 0.025 \text{ mm/mm}$$

$$\frac{\sigma - 252}{0.025 - 0.0012} = \frac{420 - 252}{0.2 - 0.0012}; \qquad \sigma = 272.11 \text{ MPa}$$

Hence $F = \sigma A = [272.11(10^6)][468(10^{-6})] = 127.35(10^3)$ N

From Eq. (2), $w = 159.19(10^3)$ N/m $= 159$ kN/m **Ans.**

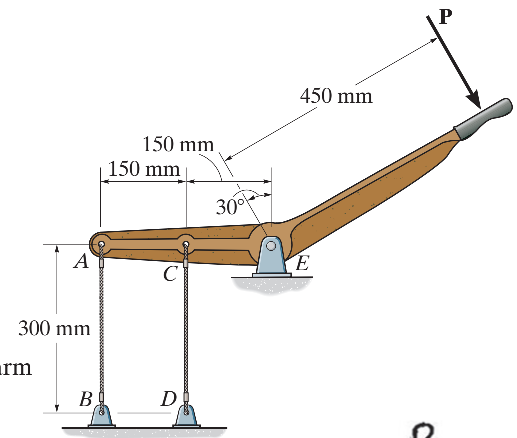




Ans.
$w = 159$ kN/m

4–102.

The rigid lever arm is supported by two A-36 steel wires having the same diameter of 4 mm. Determine the smallest force **P** that will cause (a) only one of the wires to yield; (b) both wires to yield. Consider A-36 steel as an elastic perfectly plastic material.

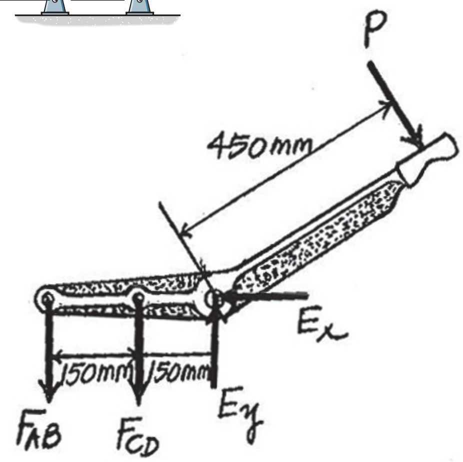


SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the lever arm shown in Fig. *a*,

$\circlearrowleft + \Sigma M_E = 0;\qquad F_{AB}(300) + F_{CD}(150) - P(450) = 0$

$$2F_{AB} + F_{CD} = 3P \qquad \textbf{(1)}$$



(a)

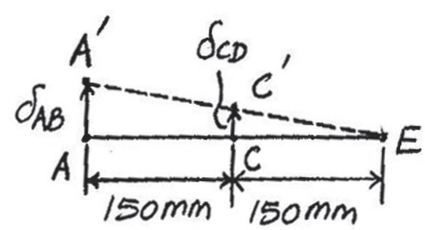
(a) Elastic Analysis: The compatibility equation can be written by referring to the geometry of Fig. *b*.

$$\delta_{AB} = \left(\frac{300}{150}\right)\delta_{CD}$$

$$\delta_{AB} = 2\delta_{CD}$$

$$\frac{F_{AB}L}{AE} = 2\left(\frac{F_{CD}L}{AE}\right)$$

$$F_{CD} = \frac{1}{2}F_{AB} \qquad \textbf{(2)}$$



(b)

Assuming that wire AB is about to yield first,

$$F_{AB} = (\sigma_Y)_{\text{st}}A_{AB} = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.004^2\right)\right] = 3141.59\,\text{N}$$

From Eq. (2),

$$F_{CD} = \frac{1}{2}(3141.59) = 1570.80\,\text{N}$$

Substituting the result of F_{AB} and F_{CD} into Eq. (1),

$$P = 2618.00\,\text{N} = 2.62\,\text{kN} \qquad \textbf{Ans.}$$

(b) Plastic Analysis: Since both wires AB and CD are required to yield,

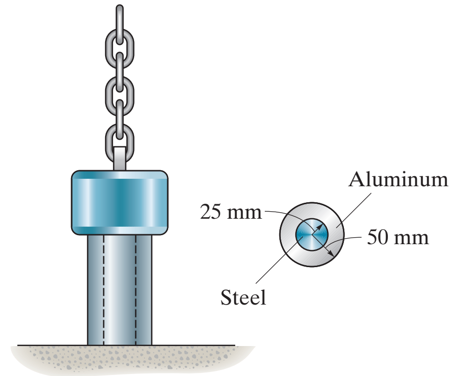
$$F_{AB} = F_{CD} = (\sigma_Y)_{\text{st}}A = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.004^2\right)\right] = 3141.59\,\text{N}$$

Substituting this result into Eq. (1),

$$P = 3141.59\text{ N} = 3.14\text{ kN} \qquad \textbf{Ans.}$$

Ans:
(a) $P = 2.62\,\text{kN}$, (b) $P = 3.14$ kN

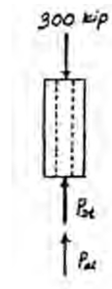
4–103. The 1500-kN weight is slowly set on the top of a post made of 2014-T6 aluminum with an A-36 steel core. If both materials can be considered elastic perfectly plastic, determine the stress in each material.



SOLUTION

Equations of Equilibrium:

$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{al} - 1500 = 0$ **[1]**



Elastic Analysis: Assume both materials still behave elastically under the load.

$$\delta_{st} = \delta_{al}$$

$$\frac{P_{st}L}{\left[\frac{\pi}{4}(0.05^2)\right][200(10^9)]} = \frac{P_{al}L}{\left[\frac{\pi}{4}(0.1^2 - 0.05^2)\right][73.1(10^9)]}$$

$$P_{st} = 0.9120P_{al}$$

Solving Eqs. [1] and [2] yields:

$$P_{al} = 784.52 \text{ kN} \qquad P_{st} = 715.48 \text{ kN}$$

Average Normal Stress:

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{784.52(10^3)}{\frac{\pi}{4}(0.1^2 - 0.05^2)} = 133.18 \text{ MPa} < (\sigma_\gamma)_{al} = 414 \text{ MPa} \qquad \textbf{(OK!)}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{715.48(10^3)}{\frac{\pi}{4}(0.05^2)} = 364.39 \text{ MPa} > (\sigma_\gamma)_{st} = 250 \text{ MPa}$$

Therefore, the steel core yields and so the elastic analysis is invalid. The stress in the steel is

$\sigma_{st} = (\sigma_\gamma)_{st} = 250 \text{ MPa}$ **Ans.**

$$P_{st} = (\sigma_\gamma)_{st}A_{st} = \left[250(10^6)\right]\left[\frac{\pi}{4}(0.05^2)\right] = 490.87(10^3) \text{ N} = 490.87 \text{ kN}$$

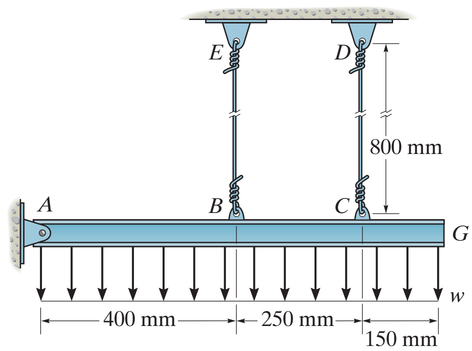
From Eq. [1] $P_{al} = 1009.13 \text{ kN}$

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{1009.13(10^3)}{\frac{\pi}{4}(0.1^2 - 0.05^2)} = 171.31(10^6) \text{ N/m}^2 = 171.31 \text{ MPa} < (\sigma_\gamma)_{al} = 414 \text{ MPa}$$

Then $\sigma_{al} = 171.31 \text{ MPa} = 171 \text{ MPa}$ **Ans.**

Ans.
$\sigma_{st} = 250 \text{ MPa}, \sigma_{al} = 171 \text{ MPa}$

***4–104.** The rigid bar is supported by a pin at A and two steel wires, each having a diameter of 4 mm. If the yield stress for the wires is $\sigma_Y = 530$ MPa, and $E_{st} = 200$ GPa, determine (a) the intensity of the distributed load w that can be placed on the beam that will cause only one of the wires to start to yield and (b) the smallest intensity of the distributed load that will cause both wires to yield. For the calculation, assume that the steel is elastic perfectly plastic.



SOLUTION

Equations of Equilibrium:

$$\zeta + \Sigma M_A = 0; \qquad F_{BE}(0.4) + F_{CD}(0.65) - 0.8w\,(0.4) = 0$$

$$0.4\,F_{BE} + 0.65\,F_{CD} = 0.32w \qquad \textbf{[1]}$$

(a) By observation, wire CD will yield first.

Then $F_{CD} = \sigma_\gamma A = 530\left(10^6\right)\left(\frac{\pi}{4}\right)\left(0.004^2\right) = 6.660$ kN.

From the geometry

$$\frac{\delta_{BE}}{0.4} = \frac{\delta_{CD}}{0.65}; \qquad \delta_{CD} = 1.625\delta_{BE}$$

$$\frac{F_{CD}L}{AE} = 1.625\,\frac{F_{BE}L}{AE}$$

$$F_{CD} = 1.625\,F_{BE} \qquad \textbf{[2]}$$

Using $F_{CD} = 6.660$ kN and solving Eqs. [1] and [2] yields:

$$F_{BE} = 4.099 \text{ kN}$$

$$w = 18.7 \text{ kN/m} \qquad \textbf{Ans.}$$

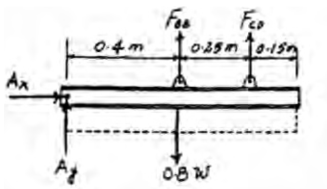
(b) When both wires yield

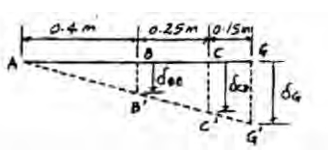
$$F_{BE} = F_{CD} = (\sigma_\gamma)A$$

$$= 530\left(10^6\right)\left(\frac{\pi}{4}\right)\left(0.004^2\right) = 6.660 \text{ kN}$$

Substituting the results into Eq. [1] yields:

$$w = 21.9 \text{ kN/m} \qquad \textbf{Ans.}$$

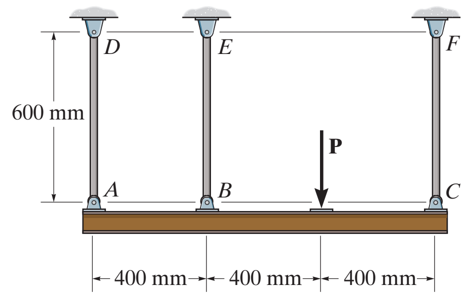




Ans.
$w = 21.9$ kN/m

4–105.

The rigid beam is supported by three 25-mm diameter A-36 steel rods. If the beam supports the force of $P = 230$ kN, determine the force developed in each rod. Consider the steel to be an elastic perfectly plastic material.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the beam shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad F_{AD} + F_{BE} + F_{CF} - 230(10^3) = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BE}(400) + F_{CF}(1200) - 230(10^3)(800) = 0$$

$$F_{BE} + 3F_{CF} = 460(10^3) \qquad \textbf{(2)}$$

Elastic Analysis: Referring to the deflection diagram of the beam shown in Fig. *b*, the compatibility equation can be written as

$$\delta_{BE} = \delta_{AD} + \left(\frac{\delta_{CF} - \delta_{AD}}{1200}\right)(400)$$

$$\delta_{BE} = \frac{2}{3}\delta_{AD} + \frac{1}{3}\delta_{CF}$$

$$\frac{F_{BE}L}{AE} = \frac{2}{3}\left(\frac{F_{AD}L}{AE}\right) + \frac{1}{3}\left(\frac{F_{CF}L}{AE}\right)$$

$$F_{BE} = \frac{2}{3}F_{AD} + \frac{1}{3}F_{CF} \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3)

$F_{CF} = 131\,428.57$ N $\qquad F_{BE} = 65\,714.29$ N $\quad F_{AD} = 32\,857.14$ N

Normal Stress:

$$\sigma_{CF} = \frac{F_{CF}}{A_{CF}} = \frac{131428.57}{\frac{\pi}{4}(0.025^2)} = 267.74 \text{ MPa} > (\sigma_Y)_{st} \qquad \textbf{(N.G.)}$$

$$\sigma_{BE} = \frac{F_{BE}}{A_{BE}} = \frac{65714.29}{\frac{\pi}{4}(0.025^2)} = 133.87 \text{ MPa} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

$$\sigma_{AD} = \frac{F_{AD}}{A_{AD}} = \frac{32857.14}{\frac{\pi}{4}(0.025^2)} = 66.94 \text{ MPa} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

Since rod CF yields, the elastic analysis is not valid. The solution must be reworked using

$$F_{CF} = (\sigma_Y)_{st}\,A_{CF} = 250\,(10^6)\left[\frac{\pi}{4}(0.025^2)\right] = 122\,718.46\,\text{N} = 123\,\text{kN} \qquad \textbf{Ans.}$$

4–105. Continued

Substituting this result into Eq. (2),

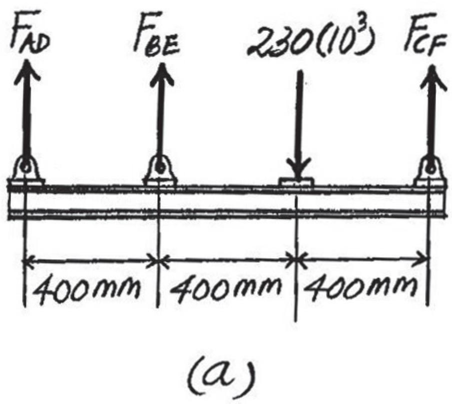
$$F_{BE} = 91844.61 \text{ N} = 91.8 \text{ kN} \qquad \textbf{Ans.}$$

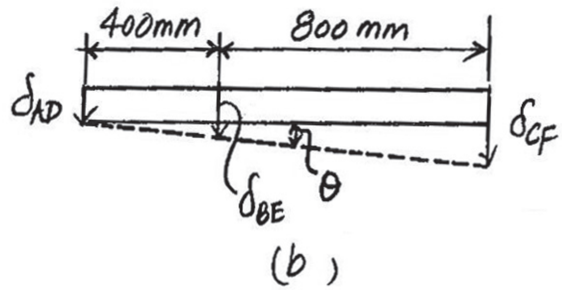
Substituting the result for F_{CF} and F_{BE} into Eq. (1),

$$F_{AD} = 15436.93 \text{ N} = 15.4 \text{ kN} \qquad \textbf{Ans.}$$

$$\sigma_{BE} = \frac{F_{BE}}{A_{BE}} = \frac{91844.61}{\frac{\pi}{4}\left(0.025^2\right)} = 187.10 \text{ MPa} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

$$\sigma_{AD} = \frac{F_{AD}}{A_{AD}} = \frac{15436.93}{\frac{\pi}{4}\left(0.025^2\right)} = 31.45 \text{ MPa} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

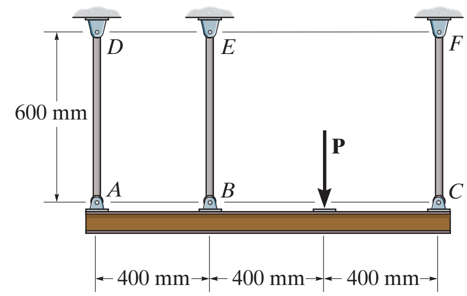




Ans:
$F_{CF} = 123$ kN,
$F_{BE} = 91.8$ kN,
$F_{AD} = 15.4$ kN

4–106.

The rigid beam is supported by three 25-mm diameter A-36 steel rods. If the force of $P = 230$ kN is applied on the beam and removed, determine the residual stresses in each rod. Consider the steel to be an elastic perfectly plastic material.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the beam shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad F_{AD} + F_{BE} + F_{CF} - 230(10^3) = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BE}(400) + F_{CF}(1200) - 230(10^3)(800) = 0$$

$$F_{BE} + 3F_{CF} = 460(10^3) \qquad \textbf{(2)}$$

Elastic Analysis: Referring to the deflection diagram of the beam shown in Fig. *b*, the compatibility equation can be written as

$$\delta_{BE} = \delta_{AD} + \left(\frac{\delta_{CF} - \delta_{AD}}{1200}\right)(400)$$

$$\delta_{BE} = \frac{2}{3}\delta_{AD} + \frac{1}{3}\delta_{CF} \qquad \textbf{(3)}$$

$$\frac{F_{BE}L}{AE} = \frac{2}{3}\left(\frac{F_{AD}L}{AE}\right) + \frac{1}{3}\left(\frac{F_{CF}L}{AE}\right)$$

$$F_{BE} = \frac{2}{3}F_{AD} + \frac{1}{3}F_{CF} \qquad \textbf{(4)}$$

Solving Eqs. (1), (2), and (4)

$$F_{CF} = 131428.57 \text{ N} \qquad F_{BE} = 65714.29 \text{ N} \quad F_{AD} = 32857.14 \text{ N}$$

Normal Stress:

$$\sigma_{CF} = \frac{F_{CF}}{A_{CF}} = \frac{131428.57}{\frac{\pi}{4}(0.025^2)} = 267.74 \text{ MPa (T)} > (\sigma_Y)_{st} \qquad \textbf{(N.G.)}$$

$$\sigma_{BE} = \frac{F_{BE}}{A_{BE}} = \frac{65714.29}{\frac{\pi}{4}(0.025^2)} = 133.87 \text{ MPa (T)} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

$$\sigma_{AD} = \frac{F_{AD}}{A_{AD}} = \frac{32857.14}{\frac{\pi}{4}(0.025^2)} = 66.94 \text{ MPa (T)} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

Since rod *CF* yields, the elastic analysis is not valid. The solution must be reworked using

$$\sigma_{CF} = (\sigma_Y)_{st} = 250 \text{ MPa (T)}$$

$$F_{CF} = \sigma_{CF}A_{CF} = 250(10^6)\left[\frac{\pi}{4}(0.025^2)\right] = 122718.46 \text{ N}$$

4–106. Continued

Substituting this result into Eq. (2),

$$F_{BE} = 91844.61 \text{ N}$$

Substituting the result for F_{CF} and F_{BE} into Eq. (1),

$$F_{AD} = 15436.93 \text{ N}$$

$$\sigma_{BE} = \frac{F_{BE}}{A_{BE}} = \frac{91844.61}{\frac{\pi}{4}(0.025^2)} = 187.10 \text{ MPa (T)} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

$$\sigma_{AD} = \frac{F_{AD}}{A_{AD}} = \frac{15436.93}{\frac{\pi}{4}(0.025^2)} = 31.45 \text{ MPa (T)} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

Residual Stresses: The process of removing **P** can be represented by applying the force **P′**, which has a magnitude equal to that of **P** but is opposite in sense. Since the process occurs in a linear manner, the corresponding normal stress must have the same magnitude but opposite sense to that obtained from the elastic analysis. Thus,

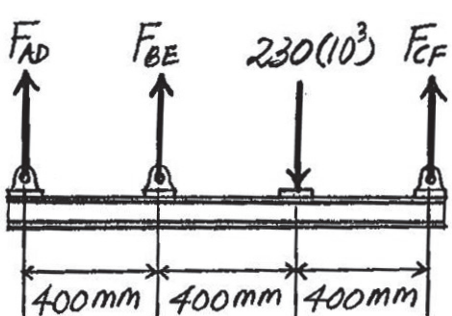
$\sigma'_{CF} = 267.74$ MPa (C) $\quad \sigma'_{BE} = 133.87$ MPa (C) $\quad \sigma'_{AD} = 66.94$ MPa (C)

Considering the tensile stress as positive and the compressive stress as negative,

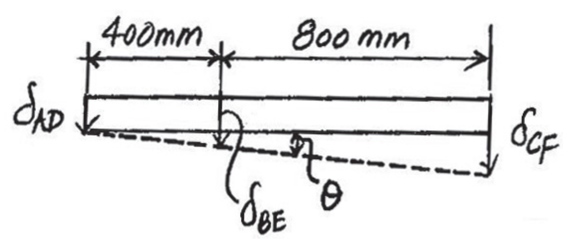
$(\sigma_{CF})_r = \sigma_{CF} + \sigma'_{CF} = 250 + (-267.74) = -17.7 \text{ MPa} = 17.7 \text{ MPa (C)}$ **Ans.**

$(\sigma_{BE})_r = \sigma_{BE} + \sigma'_{BE} = 187.10 + (-133.87) = 53.2 \text{ MPa (T)}$ **Ans.**

$(\sigma_{AD})_r = \sigma_{AD} + \sigma'_{AD} = 31.45 + (-66.94) = -35.5 \text{ MPa} = 35.5 \text{ MPa (C)}$ **Ans.**



(a)

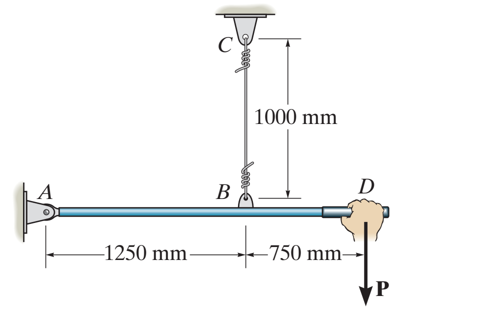


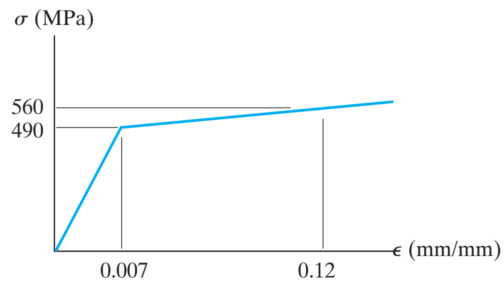
(b)

Ans:
$(\sigma_{CF})_r = 17.7$ MPa (C),
$(\sigma_{BE})_r = 53.2$ MPa (T),
$(\sigma_{AD})_r = 35.5$ MPa (C)

4–107.

The wire BC has a diameter of 3.4 mm and the material has the stress–strain characteristics shown in the figure. Determine the vertical displacement of the handle at D if the pull at the grip is slowly increased and reaches a magnitude of (a) $P = 2250$ N, (b) $P = 3000$ N.





SOLUTION

Equations of Equilibrium:

$$\zeta + \Sigma M_A = 0; \qquad F_{BC}(1250) - P(2000) = 0 \qquad \textbf{[1]}$$

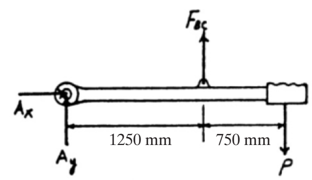
(a) From Eq. [1] when $P = 2250$ N, $\quad F_{BC} = 3600$ N

Average Normal Stress and Strain:

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{3600}{\frac{\pi}{4}(0.0034^2)} = 396.51(10^6)\ \text{N/m}^2 = 396.51\ \text{MPa}$$

From the Stress–Strain diagram

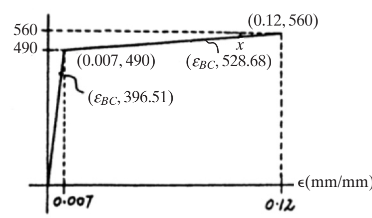
$$\frac{396.51}{\varepsilon_{BC}} = \frac{490}{0.007}; \qquad \varepsilon_{BC} = 0.005664\ \text{mm/mm}$$



Displacement:

$$\delta_{BC} = \varepsilon_{BC} L_{BC} = 0.005664(1000) = 5.6644\ \text{mm}$$

$$\frac{\delta_D}{2000} = \frac{\delta_{BC}}{1250}; \qquad \delta_D = (1.6)(5.664) = 9.0631\ \text{mm} = 9.06\ \text{mm} \qquad \textbf{Ans.}$$



(b) From Eq. [1] when $P = 3000$ N, $\quad F_{BC} = 4800$ N

Average Normal Stress and Strain:

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{4800}{\frac{\pi}{4}(0.0034^2)} = 528.68(10^6)\ \text{N/m}^2 = 528.68\ \text{MPa}$$

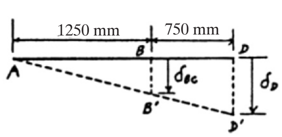
From Stress–Strain diagram

$$\frac{528.68 - 490}{\varepsilon_{BC} - 0.007} = \frac{560 - 490}{0.12 - 0.007} \qquad \varepsilon_{BC} = 0.06944\ \text{mm/mm}$$

Displacement:

$$\delta_{BC} = \varepsilon_{BC} L_{BC} = 0.06944(1000) = 69.44\ \text{mm}$$

$$\frac{\delta_D}{2000} = \frac{\delta_{BC}}{1250}; \qquad \delta_D = (1.6)(69.44) = 111.11\ \text{mm} = 111\ \text{mm} \qquad \textbf{Ans.}$$

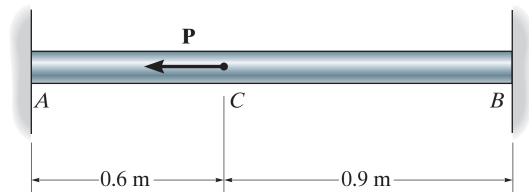


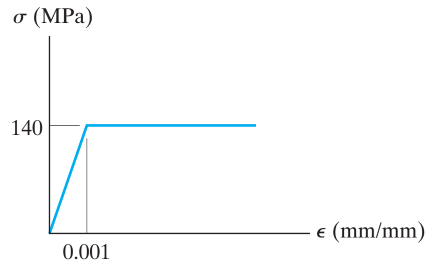
Ans.

(a) $\delta_D = 9.06$ mm, (b) $\delta_D = 111$ mm

***4–108.**

The bar having a diameter of 50 mm is fixed connected at its ends and supports the axial load **P**. If the material is elastic perfectly plastic as shown by the stress–strain diagram, determine the smallest load P needed to cause segment CB to yield. If this load is released, determine the permanent displacement of point C.





SOLUTION

When P is increased, region AC will become plastic first, then CB will become plastic. Thus,

$$F_A = F_B = \sigma A = \left[140(10^6)\right]\left[\tfrac{\pi}{4}(0.05^2)\right] = 274.89(10^3)\text{ N} = 274.89\text{ kN}$$

$$\xrightarrow{+}\ \Sigma F_x = 0; \qquad F_A + F_B - P = 0 \qquad \textbf{(1)}$$

$$P = 2(274.89) = 549.78\text{ kN}$$

$$P = 550\text{ kN}$$ **Ans.**

The deflection of point C is,

$$\delta_C = \varepsilon L = (0.001)(0.9)(10^3) = 0.9\text{ mm} \leftarrow$$

Consider the reverse of P on the bar.

$$\frac{F_B'(0.6)}{AE} = \frac{F_B'(0.9)}{AE}$$

$$F_A' = 1.5\,F_B'$$

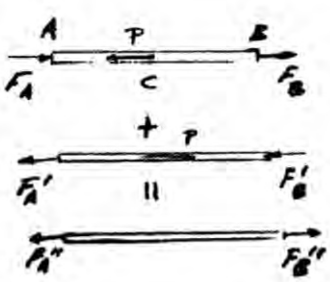
So that from Eq. (1)

$$F_B' = 0.4P$$

$$F_A' = 0.6P$$

$$\delta_C' = \frac{F_{B'}L}{AE} = \frac{0.4\left[549.78(10^3)\right]\left[0.9(10^3)\right]}{\left[\frac{\pi}{4}(0.05^2)\right]\left[140(10^6)/0.001\right]} = 0.720\text{ mm} \rightarrow$$

$$\Delta\delta = 0.9 - 0.720 = 0.180\text{ mm} \leftarrow$$ **Ans.**

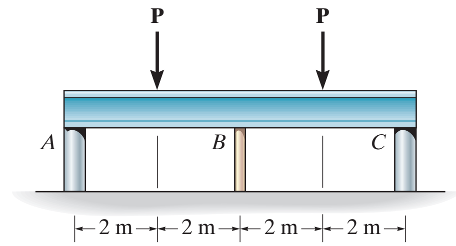


Ans.

$P = 550$ kN, $\Delta\delta = 0.180$ mm. $\leftarrow$

4–109.

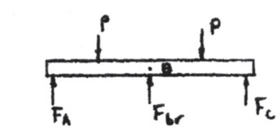
The rigid beam is supported by the three posts A, B, and C of equal length. Posts A and C have a diameter of 75 mm and are made of a material for which $E = 70$ GPa and $\sigma_Y = 20$ MPa. Post B has a diameter of 20 mm and is made of a material for which $E' = 100$ GPa and $\sigma_Y' = 590$ MPa. Determine the smallest magnitude of **P** so that (a) only rods A and C yield and (b) all the posts yield.



SOLUTION

$\Sigma M_B = 0; \qquad F_A = F_C = F_{al}$

$+\uparrow \Sigma F_y = 0; \qquad F_{at} + 2F_{at} - 2P = 0$ **(1)**



(a) Post A and C will yield,

$$F_{al} = (\sigma_t)_{al}A$$

$$= 20(10^4)\left(\frac{\pi}{a}\right)(0.075)^2$$

$$= 88.36 \text{ kN}$$

$$(E_{al})_r = \frac{(\sigma_r)_{al}}{E_{al}} = \frac{20(10^4)}{70(10^4)} = 0.0002857$$

Compatibility condition:

$$\delta_{br} = \delta_{al}$$

$$= 0.0002857(L)$$

$$\frac{F_{br}\,(L)}{\frac{\pi}{4}(0.02)^2(100)(10^4)} = 0.0002857\,L$$

$$F_{br} = 8.976 \text{ kN}$$

$$\sigma_{br} = \frac{8.976(10^3)}{\frac{\pi}{4}(0.02^3)} = 28.6 \text{ MPa} < \sigma_\gamma$$ **OK.**

From Eq. (1),

$$8.976 + 2(88.36) - 2P = 0$$

$$P = 92.8 \text{ kN}$$ **Ans.**

(b) All the posts yield:

$$F_{br} = (\sigma_\gamma)_{br}A$$

$$= (590)(10^4)\left(\frac{\pi}{4}\right)(0.02^2)$$

$$= 185.35 \text{ kN}$$

$$F_{al} = 88.36 \text{ kN}$$

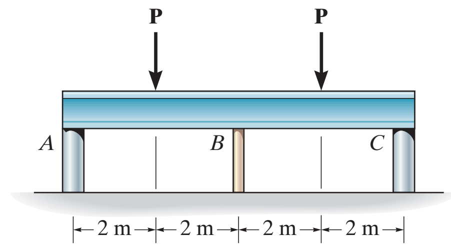
From Eq. (1); $185.35 + 2(88.36) - 2P = 0$

$$P = 181 \text{ kN}$$ **Ans.**

Ans:
$P = 92.8$ kN,
$P = 181$ kN

4–110.

The rigid beam is supported by the three posts A, B, and C. Posts A and C have a diameter of 60 mm and are made of a material for which $E = 70$ GPa and $\sigma_Y = 20$ MPa. Post B is made of a material for which $E' = 100$ GPa and $\sigma_Y' = 590$ MPa. If $P = 130$ kN, determine the diameter of post B so that all three posts are about to yield.



SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad 2(F_\gamma)_{\text{al}} + F_{\text{br}} - 260 = 0$ **(1)**

$$(F_{\text{al}})_\gamma = (\sigma_\gamma)_{\text{al}}\, A$$

$$= 20(10^6)\left(\frac{\pi}{4}\right)(0.06)^2 = 56.55 \text{ kN}$$

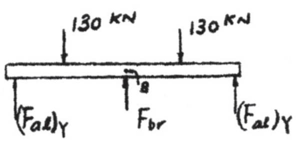
From Eq. (1),

$$2(56.55) + F_{\text{br}} - 260 = 0$$

$$F_{\text{br}} = 146.9 \text{ kN}$$

$$(\sigma_\gamma)_{\text{br}} = 590\left(10^6\right) = \frac{146.9\left(10^3\right)}{\frac{\pi}{4}(d_B)^3}$$

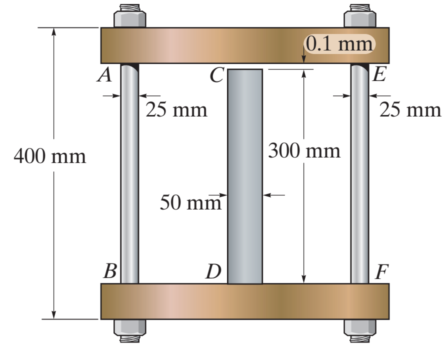
$d_B = 0.01779 \text{ m} = 17.8 \text{ mm}$ **Ans.**



Ans:
$d_B = 17.8$ mm

R4–1.

The assembly consists of two A992 steel bolts AB and EF and an 6061-T6 aluminum rod CD. When the temperature is at 30° C, the gap between the rod and rigid member AE is 0.1 mm. Determine the normal stress developed in the bolts and the rod if the temperature rises to 130° C. Assume BF is also rigid.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the rigid cap shown in Fig. a,

$$+\uparrow \Sigma F_y = 0; \quad F_r - 2F_b = 0 \qquad \textbf{(1)}$$

Compatibility Equation: If the bolts and the rod are unconstrained, they will have a free expansion of $(\delta_T)_b = \alpha_{st}\Delta T L_b = 12(10^{-6})(130 - 30)(400) = 0.48$ mm and $(\delta\gamma)_r = \alpha_{al}\Delta T L_r = 24(10^{-6})(130 - 30)(300) = 0.72$ mm. Referring to the initial and final position of the assembly shown in Fig. b,

$$(\delta_T)_r - \delta_{Fr} - 0.1 = (\delta_T)_b + \delta_{Fb}$$

$$0.72 - \frac{F_r(300)}{\frac{\pi}{4}(0.05^2)(68.9)(10^9)} - 0.1 = 0.48 + \frac{F_b(400)}{\frac{\pi}{4}(0.025^2)(200)(10^9)}$$

$$F_b + 0.5443F_r = 34361.17 \qquad \textbf{(2)}$$

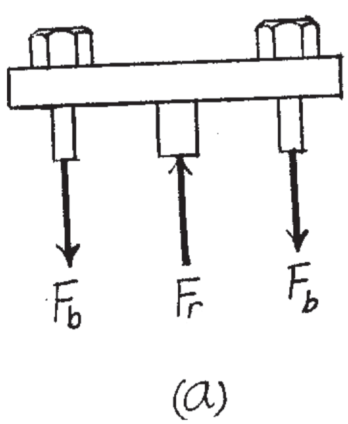
Solving Eqs. (1) and (2).

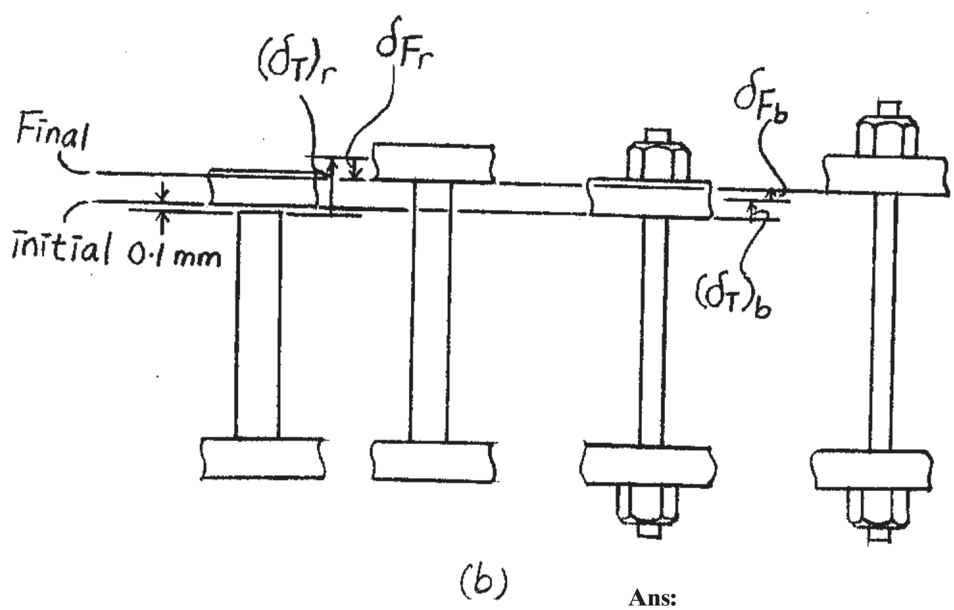
$$F_b + 16\,452.29 \text{ N} \qquad F_r = 32\,904.58 \text{ N}$$

Normal Stress:

$$\sigma_b = \frac{F_b}{A_b} = \frac{16\,452.29}{\frac{\pi}{4}(0.025^2)} = 33.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_r = \frac{F_r}{A_r} = \frac{32\,904.58}{\frac{\pi}{4}(0.05^2)} = 16.8 \text{ MPa} \qquad \textbf{Ans.}$$

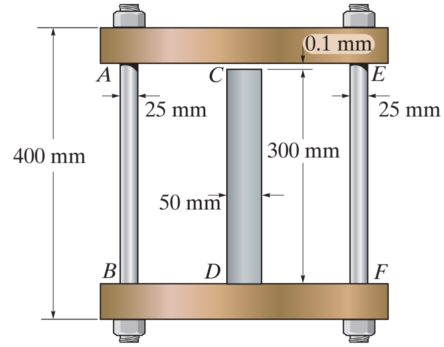




Ans:
$\sigma_b = 33.5$ MPa, $\sigma_r = 16.8$ MPa

R4–2.

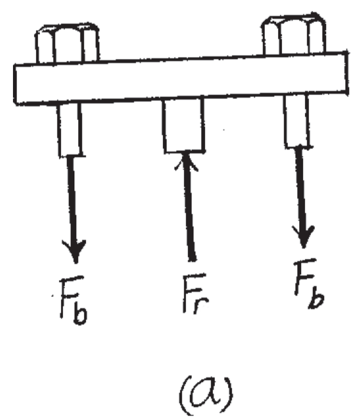
The assembly shown consists of two A992 steel bolts AB and EF and an 6061-T6 aluminum rod CD. When the temperature is at 30° C, the gap between the rod and rigid member AE is 0.1 mm. Determine the highest temperature to which the assembly can be raised without causing yielding either in the rod or the bolts. Assume BF is also rigid.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the rigid cap shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \quad F_p - 2F_b = 0 \qquad \textbf{(1)}$$



Normal Stress: Assuming that the steel bolts yield first, then

$$F_b = (\sigma_Y)_{st}A_b = 250(10^6)\left[\frac{\pi}{4}(0.025^2)\right] = 122\ 718.46\ \text{N}$$

Substituting this result into Eq. (1),

$$F_p = 245\ 436.93\ \text{N}$$

Then,

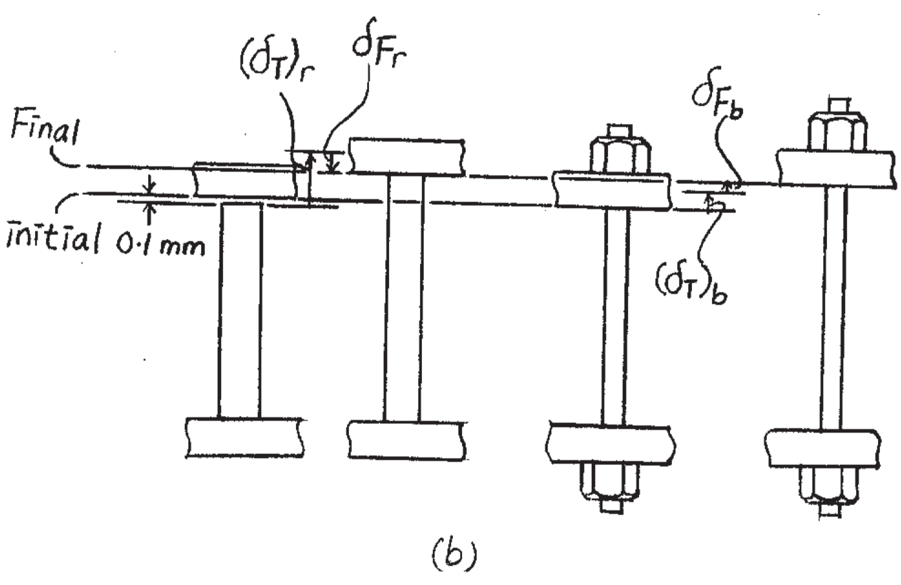
$$\sigma_p = \frac{F_p}{A_p} = \frac{245\ 436.93}{\frac{\pi}{4}(0.05^2)} = 125\ \text{MPa} < (\sigma_Y)_{al} \qquad \textbf{(O.K!)}$$

Compatibility Equation: If the assembly is unconstrained, the bolts and the post will have free expansion of $(\delta_T)_b = \alpha_{st}\Delta T L_b = 12(10^{-6})(T-30)(400) = 4.8(10^{-3})(T-30)$ and $(\delta_T)_p = \alpha_{al}\Delta T L_p = 24(10^{-6})T - 30)(300) = 7.2(10^{-3})(T - 30)$. Referring to the initial and final position of the assembly shown in Fig. *b*,

$$(\delta_T)_p - \delta_{Fp} - 0.1 = (\delta_T)_b + \delta_{Fb}$$

$$7.2(10^{-3})(T-30) - \frac{245\ 436.93(300)}{\frac{\pi}{4}(0.05^2)(68.9)(10^9)} - 0.1 = 4.8(10^{-3})(T-30) + \frac{122\ 718.46(400)}{\frac{\pi}{4}(0.025^2)(200)(10^9)}$$

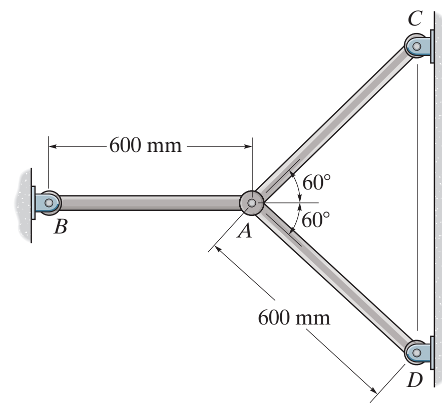
$$T = 506.78°\ \text{C} = 507°\text{C} \qquad \textbf{Ans.}$$



Ans:
$T = 507°\text{C}$

R4–3.

The rods each have the same 25-mm diameter and 600-mm length. If they are made of A992 steel, determine the forces developed in each rod when the temperature increases by 50° C.



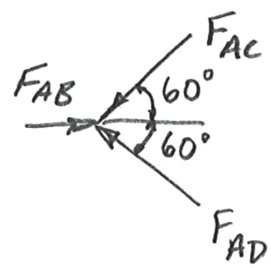
SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of joint A shown in Fig. a,

$$+\uparrow \Sigma F_x = 0; \qquad F_{AD}\sin 60^\circ - F_{AC}\sin 60^\circ = 0 \qquad F_{AC} = F_{AD} = F$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{AB} - 2F\cos 60^\circ = 0$$

$$F_{AB} = F \qquad \textbf{(1)}$$



(a)

Compatibility Equation: If AB and AC are unconstrained, they will have a free expansion of $\left(\delta_T\right)_{AB} = \left(\delta_T\right)_{AC} = \alpha_{st}\Delta TL = 12(10^{-6})(50)(600) = 0.36$ mm. Referring to the initial and final position of joint A,

$$\delta_{F_{AB}} - \left(\delta_T\right)_{AB} = \left(\delta_T{}'\right)_{AC} - \delta_{F_{AC}}{}'$$

Due to symmetry, joint A will displace horizontally, and $\delta_{AC}{}' = \dfrac{\delta_{AC}}{\cos 60^\circ} = 2\delta_{AC}$. Thus, $\left(\delta_T{}'\right)_{AC} = 2(\delta_T)_{AC}$ and $\delta_{F_{AC}}{}' = 2\delta_{F_{AC}}$. Thus, this equation becomes

$$\delta_{F_{AB}} - \left(\delta_T\right)_{AB} = 2\left(\delta_T\right)_{AC} - 2\delta_{AC}$$

$$\frac{F_{AB}(600)}{\frac{\pi}{4}\left(0.025^2\right)(200)(10^9)} - 0.36 = 2(0.36) - 2\left[\frac{F(600)}{\frac{\pi}{4}\left(0.025^2\right)(200)(10^9)}\right]$$

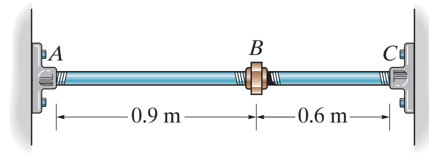
$$F_{AB} + 2F = 176\,714.59 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

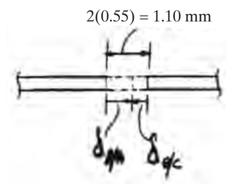
$$F_{AB} = F_{AC} = F_{AD} = 58\,904.86\text{ N} = 58.9\text{ kN (C)} \qquad \textbf{Ans.}$$

Ans:
$F_{AB} = F_{AC} = F_{AD} = 58.9$ kN (C)

***R4–4.**



Two A-36 steel pipes, each having a cross-sectional area of 200 mm^2, are screwed together using a union at B as shown. Originally the assembly is adjusted so that no load is on the pipe. If the union is then tightened so that its screw, having a lead of 0.55 mm, undergoes two full turns, determine the average normal stress developed in the pipe. Assume that the union at B and couplings at A and C are rigid. Neglect the size of the union. *Note:* The lead would cause the pipe, when *unloaded*, to shorten shorten 0.55 mm when the union is rotated one revolution.



The loads acting on both segments AB and BC are the same since no external load acts on the system.

$$1.10 = \delta_{B/A} + \delta_{B/C}$$

$$1.10 = \frac{P(0.9)(1000)}{[200(10^{-6})][200(10^9)]} + \frac{P(0.6)(1000)}{[200(10^{-6})][200(10^9)]}$$

$$P = 29.33(10^3) = 29.33 \text{ kN}$$

$$\sigma_{AB} = \sigma_{BC} = \frac{P}{A} = \frac{29.33(10^3)}{200(10^{-6})} = 146.67(10^6) \text{ N/m}^2 = 147 \text{ MPa}$$ **Ans.**

Ans.
$\sigma_{AB} = \sigma_{BC} = 147$ MPa

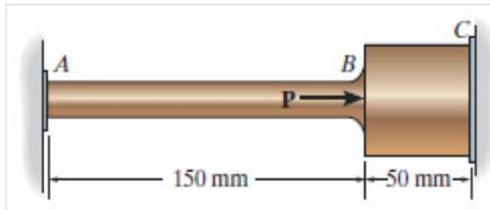
R4-5.

The force **P** is applied to the bar, which is composed of an elastic perfectly plastic material. Construct a graph to show how the force in each section *AB* and *BC* (ordinate) varies as *P* (abscissa) is increased. The bar has cross-sectional areas of 625 mm^2 in region *AB* and 2500 mm^2 in region *BC*, and $\sigma_Y = 210$ MPa.

Given: $L_{AB} := 150\text{mm}$ $\quad A_{AB} := 625\text{mm}^2$

$L_{BC} := 50\text{mm}$ $\quad A_{BC} := 2500\text{mm}^2$

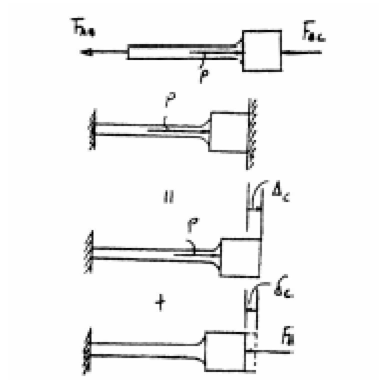
$\sigma_Y := 210\text{MPa}$



Solution:

Equations of equilibrium:

$\xrightarrow{+} \Sigma F_x = 0;\qquad P - F_A - F_C = 0$ **[1]**



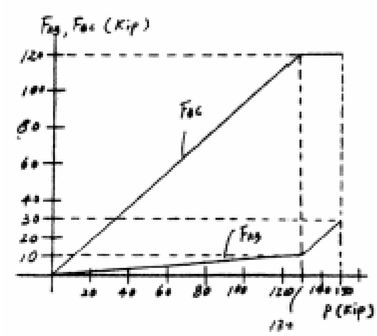
Elastic behavior:

$\xrightarrow{+}\quad 0 = \Delta_C - \delta_C$

$$0 = \frac{(P)\cdot L_{AB}}{E\cdot A_{AB}} - \left[\frac{(F_C)\cdot L_{BC}}{E\cdot A_{BC}} + \frac{(F_C)\cdot L_{AB}}{E\cdot A_{AB}}\right]$$

$0 = 6P - (F_C)(0.5 + 6)\qquad F_C = \frac{12}{13}P$ **[2]**

Substituting **[2]** into **[1]**: $\qquad F_A = \frac{1}{13}P$ **[3]**



By comparison, segment *BC* will yield first. Hence,

$F_C := (\sigma_Y)\cdot A_{BC}\qquad F_C = 525\,\text{kN}$

From **[2]**: $\quad P := \frac{13}{12}\cdot F_C\qquad P = 568.75\,\text{kN}$

From **[3]**: $\quad F_A := \frac{1}{13}\cdot P\qquad F_A = 43.75\,\text{kN}$

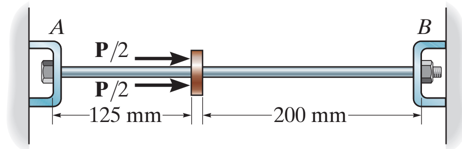
When segment *AB* yields,

$F_A := (\sigma_Y)\cdot A_{AB}\qquad F_A = 131.25\,\text{kN}$

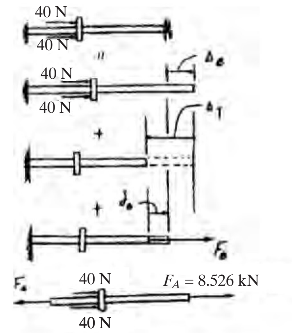
$F_C := (\sigma_Y)\cdot A_{BC}\qquad F_C = 525\,\text{kN}$

From **[1]**: $\quad P := F_A + F_C\qquad P = 656.25\,\text{kN}$

R4–6. The 2014-T6 aluminum rod has a diameter of 12 mm and is lightly attached to the rigid supports at A and B when $T_1 = 25°\text{C}$. If the temperature becomes $T_2 = -20°$, and an axial force of $P = 80$ N is applied to the rigid collar as shown, determine the reactions at A and B.



SOLUTION



Compatibility:

$$\xrightarrow{+} \; 0 = \Delta_B - \Delta_T + \delta_B$$

$$0 = \frac{80(125)}{\left[\frac{\pi}{4}(0.012^2)\right][73.1(10^9)]} - 23(10^{-6})[25° - (-20°)](325) + \frac{F_B(325)}{\left[\frac{\pi}{4}(0.012^2)\right][73.1(10^9)]}$$

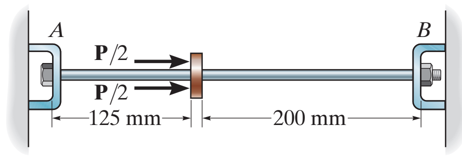
$F_B = 8.526(10^3) = 8.53$ kN **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \qquad 2(0.040) + 8.526 - F_A = 0$

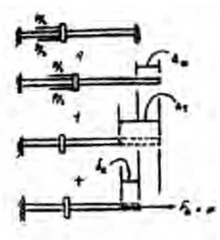
$F_A = 8.606$ kN $= 8.61$ kN **Ans.**

Ans.
$F_B = 8.53$ kN, $F_A = 8.61$ kN

R4–7. The 2014-T6 aluminum rod has a diameter of 12 mm and is lightly attached to the rigid supports at A and B when $T_1 = 40°C$. Determine the force P that must be applied to the collar so that, when $T = 0°C$, the reaction at B is zero.



SOLUTION



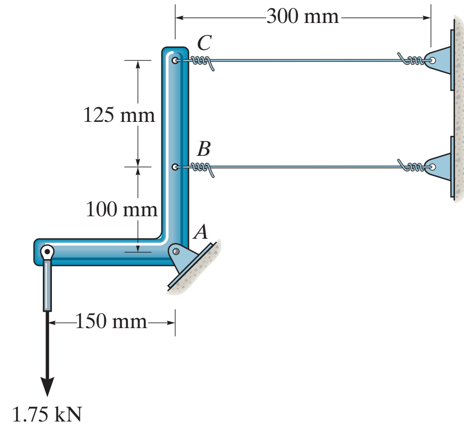
$$\overset{+}{\rightarrow} \quad 0 = \Delta_B - \Delta_T + \delta_B$$

$$0 = \frac{P(125)}{\left[\frac{\pi}{4}(0.012^2)\right][73.1(10^9)]} - 23(10^{-6})(40)(325) + 0$$

$P = 19.776(10^3)\text{ N} = 19.8\text{ kN}$ **Ans.**

Ans.
$P = 19.8$ kN

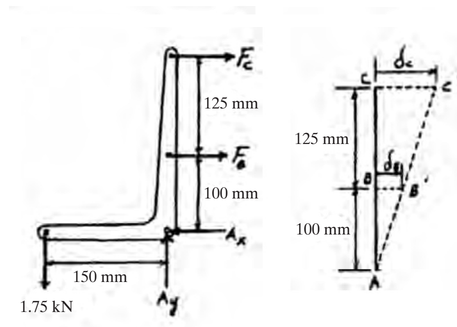
*R4–8. The rigid link is supported by a pin at A and two A-36 steel wires, each having an unstretched length of 300 mm and cross-sectional area of 7.8 mm^2. Determine the force developed in the wires when the link supports the vertical load of 1.75 kN.



SOLUTION

Equations of Equilibrium:

$\zeta + \Sigma M_A = 0; \qquad -F_C(225) - F_B(100) + 1.75(150) = 0$ **[1]**



Compatibility:

$$\frac{\delta_B}{100} = \frac{\delta_C}{225}$$

$$\frac{F_B(L)}{100AE} = \frac{F_C(L)}{225AE}$$

$$2.25F_B - F_C = 0, \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields:

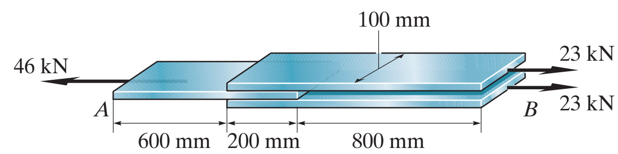
$F_B = 0.4330 \text{ kN} = 433 \text{ N}$ **Ans.**

$F_C = 0.9742 \text{ kN} = 974 \text{ N}$ **Ans.**

Ans.
$F_B = 433$ N, $F_C = 974$ N,

R4–9.

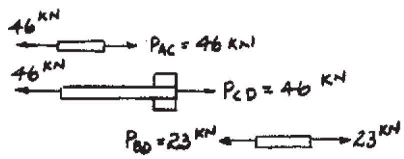
The joint is made from three A992 steel plates that are bonded together at their seams. Determine the displacement of end A with respect to end B when the joint is subjected to the axial loads. Each plate has a thickness of 5 mm.



SOLUTION

$$\delta_{A/B} = \Sigma\frac{PL}{AE} = \frac{46(10^3)(600)}{(0.005)(0.1)(200)(10^9)} + \frac{46(10^3)(200)}{3(0.005)(0.1)(200)(10^9)} + \frac{23(10^3)(800)}{(0.005)(0.1)(200)(10^9)}$$

$= 0.491$ mm **Ans.**



Ans:
$\delta_{A/B} = 0.491$ mm