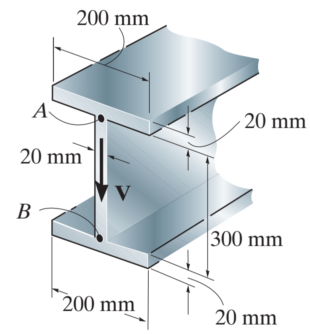
7–1.

If the wide-flange beam is subjected to a shear of $V = 20$ kN, determine the shear stress on the web at A. Indicate the shear-stress components on a volume element located at this point.



SOLUTION

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.34^3) - \frac{1}{12}(0.18)(0.3^3) = 0.2501(10^{-3})\ \text{m}^4$$

From Fig. a,

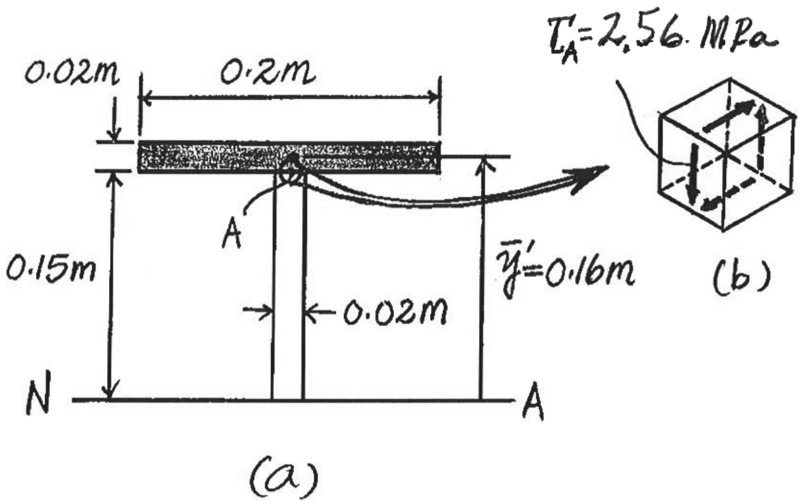
$$Q_A = \bar{y}'A' = 0.16\,(0.02)(0.2) = 0.64(10^{-3})\ \text{m}^3$$

Applying the shear formula,

$$\tau_A = \frac{VQ_A}{It} = \frac{20(10^3)[0.64(10^{-3})]}{0.2501(10^{-3})(0.02)}$$

$$= 2.559(10^6)\ \text{Pa} = 2.56\ \text{MPa} \qquad \textbf{Ans.}$$

The shear stress component at A is represented by the volume element shown in Fig. b.

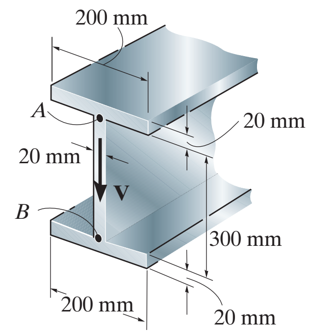


These solutions represent a preliminary version of the Instructors' Solutions Manual (ISM). It is possible and even likely that at this preliminary stage of preparing the ISM there are some omissions and errors in the draft solutions.

Ans:
$\tau_A = 2.56$ MPa

7–2.

If the wide-flange beam is subjected to a shear of $V = 20$ kN, determine the maximum shear stress in the beam.



SOLUTION

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.34^3) - \frac{1}{12}(0.18)(0.3^3) = 0.2501(10^{-3})\ \text{m}^4$$

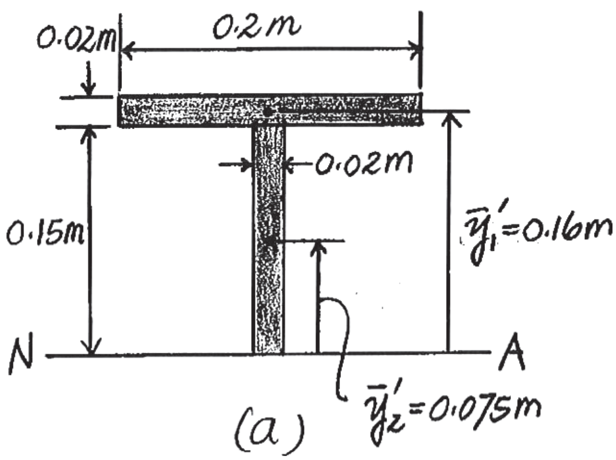
From Fig. *a*.

$$Q_{max} = \Sigma \bar{y}'A' = 0.16\,(0.02)(0.2) + 0.075\,(0.15)(0.02) = 0.865(10^{-3})\ \text{m}^3$$

The maximum shear stress occurs at the points along neutral axis since Q is maximum and thickness t is the smallest.

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{20(10^3)\,[0.865(10^{-3})]}{0.2501(10^{-3})\,(0.02)}$$

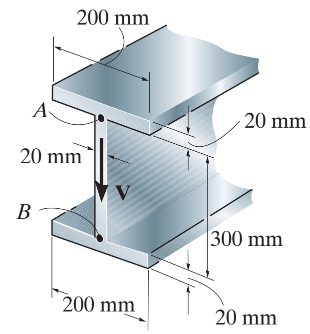
$$= 3.459(10^6)\ \text{Pa} = 3.46\ \text{MPa}$$ **Ans.**



Ans:
$\tau_{max} = 3.46$ MPa

7–3.

If the wide-flange beam is subjected to a shear of $V = 20$ kN, determine the shear force resisted by the web of the beam.



SOLUTION

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.34^3) - \frac{1}{12}(0.18)(0.3^3) = 0.2501(10^{-3})\ \text{m}^4$$

For $0 \leq y < 0.15$ m, Fig. a, Q as a function of y is

$$Q = \Sigma \bar{y}'A' = 0.16\,(0.02)(0.2) + \frac{1}{2}(y + 0.15)(0.15 - y)(0.02)$$

$$= 0.865(10^{-3}) - 0.01y^2$$

For $0 \leq y < 0.15$ m, $t = 0.02$ m. Thus,

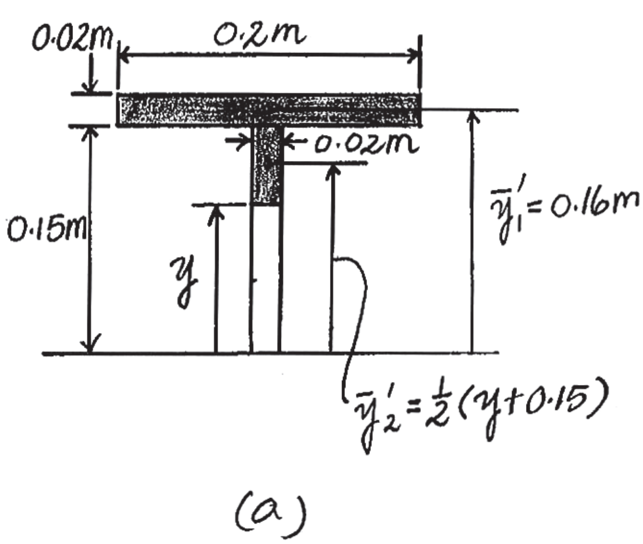
$$\tau = \frac{VQ}{It} = \frac{20(10^3)\left[0.865(10^{-3}) - 0.01y^2\right]}{0.2501(10^{-3})\,(0.02)}$$

$$= \left\{3.459(10^6) - 39.99(10^6)\,y^2\right\}\ \text{Pa}$$

The sheer force resisted by the web is

$$V_w = 2\int_0^{0.15\text{ m}} \tau dA = 2\int_0^{0.15\text{ m}} \left[3.459(10^6) - 39.99(10^6)\,y^2\right](0.02\,dy)$$

$$= 18.95\,(10^3)\ \text{N} = 19.0\ \text{kN}$$ **Ans.**

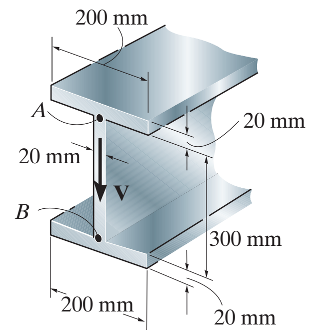


(a)

Ans:
$V_w = 19.0$ kN

***7–4.**

If the beam is subjected to a shear of $V = 30$ kN, determine the web's shear stress at A and B. Indicate the shear-stress components on a volume element located at these points. Set $w = 200$ mm. Show that the neutral axis is located at $\bar{y} = 0.2433$ m from the bottom and $I = 0.5382(10^{-3})$ m^4.



SOLUTION

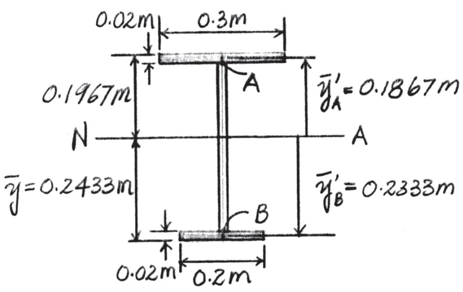
Section Properties: The location of the centroid measured from the bottom is

$$\bar{y} = \frac{(0.01)(0.2)(0.02) + 0.22\,(0.02)(0.4) + 0.430\,(0.3)(0.02)}{0.2\,(0.02) + 0.02\,(0.4) + 0.3\,(0.02)}$$

$$= 0.2433 \text{ m}$$

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.2)\left(0.02^3\right) + 0.2\,(0.02)(0.2433 - 0.01)^2$$
$$+ \frac{1}{12}(0.02)\left(0.4^3\right) + 0.02\,(0.4)(0.2433 - 0.22)^2$$
$$+ \frac{1}{12}(0.3)\left(0.02^3\right) + 0.3(0.02)(0.43 - 0.2433)^2$$
$$= 0.5382\left(10^{-3}\right) \text{ m}^4$$



(a)

Referring to Fig. *a*,

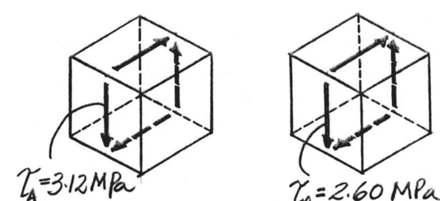
$$Q_A = \bar{y}'_A A'_A = 0.1867\,[0.3(0.02)] = 1.12\left(10^{-3}\right) \text{ m}^3$$

$$Q_B = \bar{y}'_B A'_B = 0.2333\,[0.2(0.02)] = 0.9333\left(10^{-3}\right) \text{ m}^3$$

Shear Stress: Applying the shear formula,

$$\tau_A = \frac{VQ_A}{It_A} = \frac{30\left(10^3\right)\left[1.12\left(10^{-3}\right)\right]}{0.5382\left(10^{-3}\right)(0.02)} = 3.122\left(10^6\right)\text{Pa} = 3.12 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_B = \frac{VQ_B}{It_B} = \frac{30\left(10^3\right)\left[0.9333\left(10^{-3}\right)\right]}{0.5382\left(10^{-3}\right)(0.02)} = 2.601\left(10^6\right)\text{Pa} = 2.60 \text{ MPa} \qquad \textbf{Ans.}$$



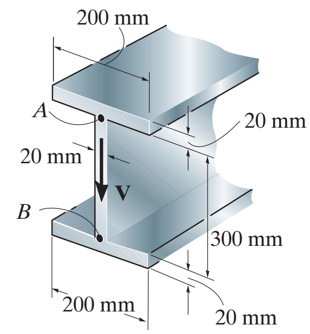
(b)

These shear stresses on the volume element at points A and B are shown in Fig. *b*.

Ans:
$\tau_A = 3.12$ MPa,
$\tau_B = 2.60$ MPa

7–5.

If the wide-flange beam is subjected to a shear of $V = 30$ kN, determine the maximum shear stress in the beam. Set $w = 300$ mm.



SOLUTION

Section Properties: The moment of inertia of the cross section about the neutral axis is

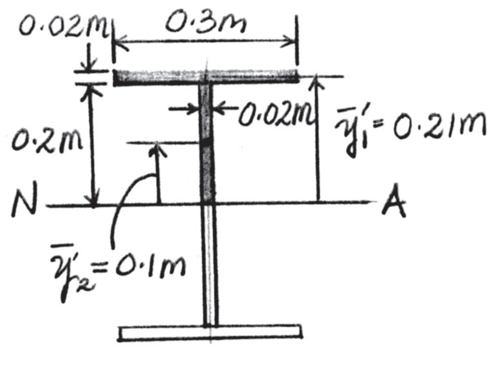
$$I = \frac{1}{12}(0.3)\left(0.44^3\right) - \frac{1}{12}(0.28)\left(0.4^3\right) = 0.63627\left(10^{-3}\right)\text{ m}^4$$

Maximum shear stress occurs at the neutral axis. Referring to Fig. *a*,

$$Q_{max} = \Sigma\bar{y}'A' = 0.21[0.3(0.02)] + 0.1[0.2(0.02)]$$
$$= 1.66\left(10^{-3}\right)\text{ m}^3$$

Maximum Shear Stress: Applying the shear formula,

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{30\left(10^3\right)\left[1.66\left(10^{-3}\right)\right]}{0.63627\left(10^{-3}\right)(0.02)}$$
$$= 3.913\left(10^6\right)\text{Pa} = 3.91\text{ MPa}$$ **Ans.**

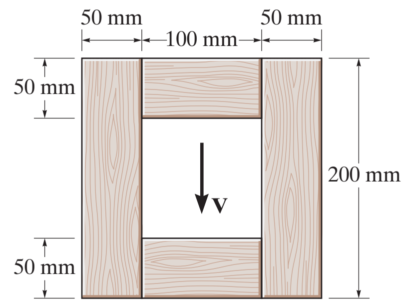


(a)

Ans:
$\tau_{max} = 3.91$ MPa

7–6.

The wood beam has an allowable shear stress of $\tau_{\text{allow}} = 7$ MPa. Determine the maximum shear force V that can be applied to the cross section.



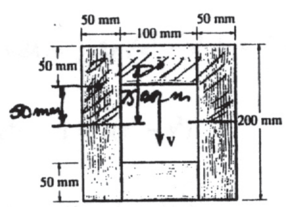
SOLUTION

$$I = \frac{1}{12}(0.2)(0.2)^3 - \frac{1}{12}(0.1)(0.1)^3 = 125(10^{-6})\text{ m}^4$$

$$\tau_{\text{allow}} = \frac{VQ_{\text{max}}}{It}$$

$$7(10^6) = \frac{V[(0.075)(0.1)(0.05) + 2(0.05)(0.1)(0.05)]}{125(10^{-6})(0.1)}$$

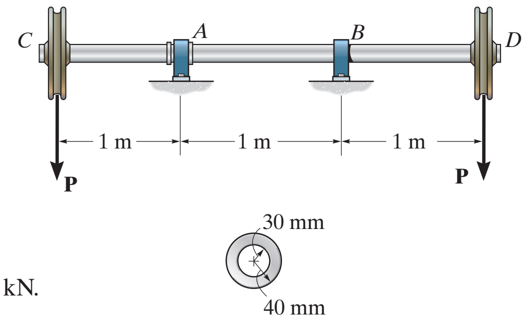
$V = 100$ kN **Ans.**



Ans:
$V_{\text{max}} = 100$ kN

7–7.

The shaft is supported by a thrust bearing at A and a journal bearing at B. If $P = 20$ kN, determine the absolute maximum shear stress in the shaft.



SOLUTION

Support Reactions: As shown on the free-body diagram of the beam, Fig. *a*.

Maximum Shear: The shear diagram is shown in Fig. *b*. As indicated, $V_{max} = 20$ kN.

Section Properties: The moment of inertia of the hollow circular shaft about the neutral axis is

$$I = \frac{\pi}{4}(0.04^4 - 0.03^4) = 0.4375(10^{-6})\pi \text{ m}^4$$

Q_{max} can be computed by taking the first moment of the shaded area in Fig. *c* about the neutral axis.

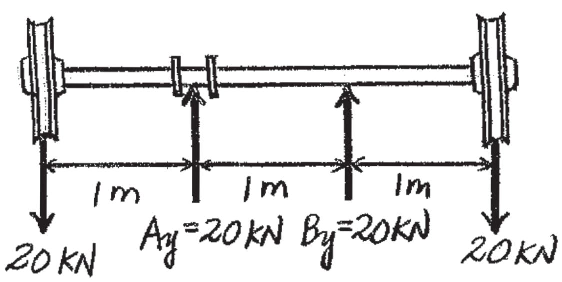
Here, $\bar{y}'_1 = \dfrac{4(0.04)}{3\pi} = \dfrac{4}{75\pi}$ m and $\bar{y}'_2 = \dfrac{4(0.03)}{3\pi} = \dfrac{1}{25\pi}$ m. Thus,

$$Q_{max} = \bar{y}'_1 A'_1 - \bar{y}'_2 A'_2$$

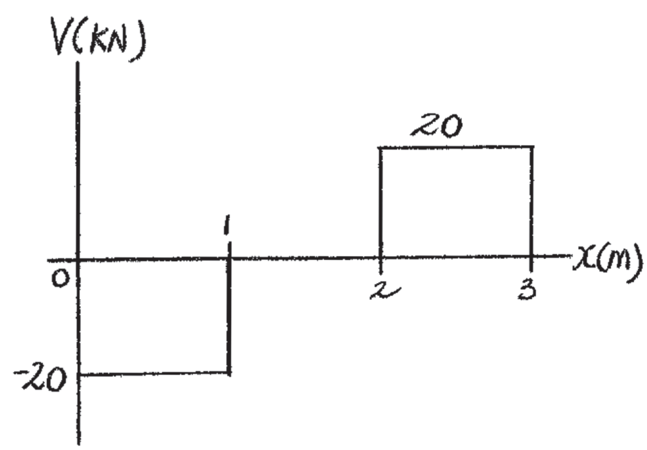
$$= \frac{4}{75\pi}\left[\frac{\pi}{2}(0.04^2)\right] - \frac{1}{25\pi}\left[\frac{\pi}{2}(0.03^2)\right] = 24.667(10^{-6}) \text{ m}^3$$

Shear Stress: The maximum shear stress occurs at points on the neutral axis since Q is maximum and the thickness $t = 2(0.04 - 0.03) = 0.02$ m is the smallest.

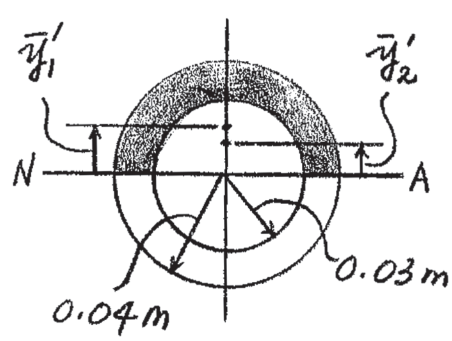
$$\tau_{max} = \frac{V_{max} Q_{max}}{It} = \frac{20(10^3)(24.667)(10^{-6})}{0.4375(10^{-6})\pi(0.02)} = 17.9 \text{ MPa} \qquad \textbf{Ans.}$$



(a)



(b)

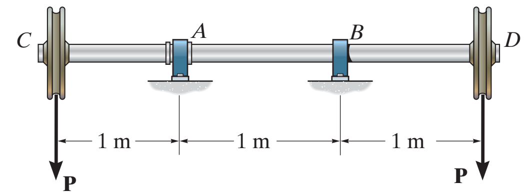


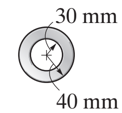
(c)

Ans:
$\tau_{max} = 17.9$ MPa

***7–8.**

The shaft is supported by a thrust bearing at A and a journal bearing at B. If the shaft is made from a material having an allowable shear stress of $\tau_{\text{allow}} = 75$ MPa, determine the maximum value for P.





SOLUTION

Support Reactions: As shown on the free-body diagram of the shaft, Fig. *a*.

Maximum Shear: The shear diagram is shown in Fig. *b*. As indicated, $V_{\text{max}} = P$.

Section Properties: The moment of inertia of the hollow circular shaft about the neutral axis is

$$I = \frac{\pi}{4}(0.04^4 - 0.03^4) = 0.4375(10^{-6})\pi \text{ m}^4$$

Q_{max} can be computed by taking the first moment of the shaded area in Fig. *c* about the neutral axis.

Here, $\bar{y}_1' = \dfrac{4(0.04)}{3\pi} = \dfrac{4}{75\pi}$ m and $\bar{y}_2' = \dfrac{4(0.03)}{3\pi} = \dfrac{1}{25\pi}$ m. Thus,

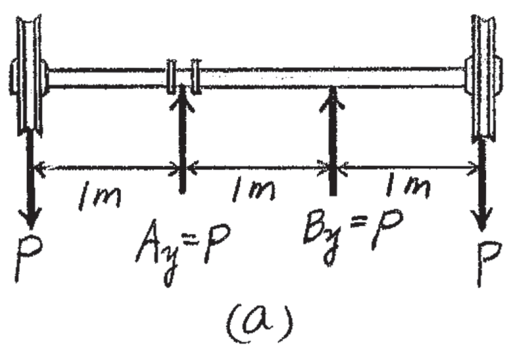
$$Q_{\text{max}} = \bar{y}_1' A_1' - \bar{y}_2' A_2'$$

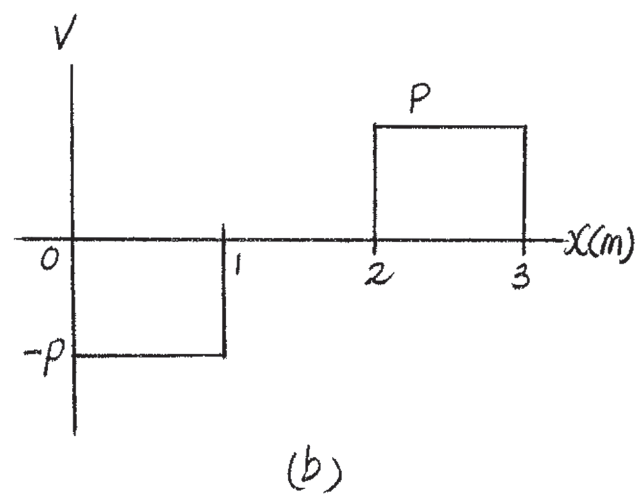
$$= \frac{4}{75\pi}\left[\frac{\pi}{2}(0.04^2)\right] - \frac{1}{25\pi}\left[\frac{\pi}{2}(0.03^2)\right] = 24.667(10^{-6}) \text{ m}^3$$

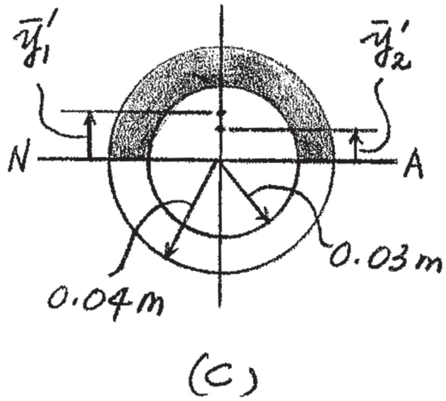
Shear Stress: The maximum shear stress occurs at points on the neutral axis since Q is maximum and the thickness $t = 2(0.04 - 0.03) = 0.02$ m.

$$\tau_{\text{allow}} = \frac{V_{\text{max}} Q_{\text{max}}}{It}; \qquad 75(10^6) = \frac{P(24.667)(10^{-6})}{0.4375(10^{-6})\pi(0.02)}$$

$$P = 83\,581.22 \text{ N} = 83.6 \text{ kN} \qquad \textbf{Ans.}$$

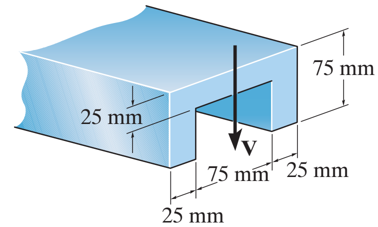






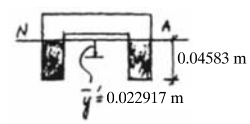
Ans:
$P = 83.6$ kN

7–9. Determine the largest shear force V that the member can sustain if the allowable shear stress is $\tau_{\text{allow}} = 56$ MPa.



SOLUTION

$$\bar{y} = \frac{(0.0125)(0.125)(0.025) + 2[0.05(0.025)(0.05)]}{0.125(0.025) + 2(0.025)(0.05)} = 0.029167 \text{ m}$$



$$I = \frac{1}{12}(0.125)(0.025^3) + 0.125(0.025)(0.029167 - 0.0125)^2$$

$$+ 2\left[\frac{1}{12}(0.025)(0.05^3) + 0.025(0.05)(0.05 - 0.029167)^2\right] = 2.6367(10^{-6}) \text{ m}^4$$

$$Q_{\max} = \Sigma \bar{y}' A' = 2[(0.022917)(0.025)(0.04583)] = 52.5174(10^{-6}) \text{ m}^3$$

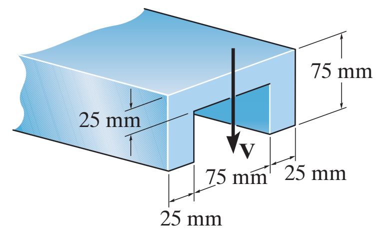
$$\tau_{\max} = \tau_{\text{allow}} = \frac{V Q_{\max}}{I\, t}$$

$$56(10^6) = \frac{V[52.5174(10^{-6})]}{[2.6367(10^{-6})][2(0.025)]}$$

$$V = 140.58(10^3) \text{ N} = 141 \text{ kN}$$ **Ans.**

Ans:
$V = 141$ kN

7–10. If the applied shear force $V = 90$ kN, determine the maximum shear stress in the member.



SOLUTION

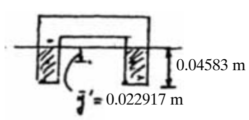
$$\bar{y} = \frac{(0.0125)(0.125)(0.025) + 2[0.05(0.025)(0.05)]}{0.125(0.025) + 2(0.025)(0.05)} = 0.029167 \text{ m}$$

$$I = \frac{1}{12}(0.125)(0.025^3) + 0.125(0.025)(0.029167 - 0.0125)^2$$

$$+ 2\left[\frac{1}{12}(0.025)(0.05^3) + 0.025(0.05)(0.05 - 0.029167)^2\right] = 2.6367(10^{-6}) \text{ m}^4$$

$$Q_{max} = \Sigma \bar{y}' A' = 2[(0.022917)(0.025)(0.04583)] = 52.5174(10^{-6}) \text{ m}^3$$

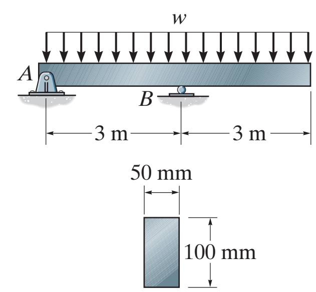
$$\tau_{max} = \frac{VQ_{max}}{I\,t} = \frac{[90(10^3)][52.5174(10^{-6}) \text{ m}^3]}{[2.6367(10^{-6})][2(0.025)]} = 35.85(10^6) \text{ N/m}^3 = 35.9 \text{ MPa} \quad \textbf{Ans.}$$



Ans:
$\tau_{max} = 35.9$ MPa

7–11.

The overhang beam is subjected to the uniform distributed load having an intensity of $w = 50$ kN/m. Determine the maximum shear stress in the beam.



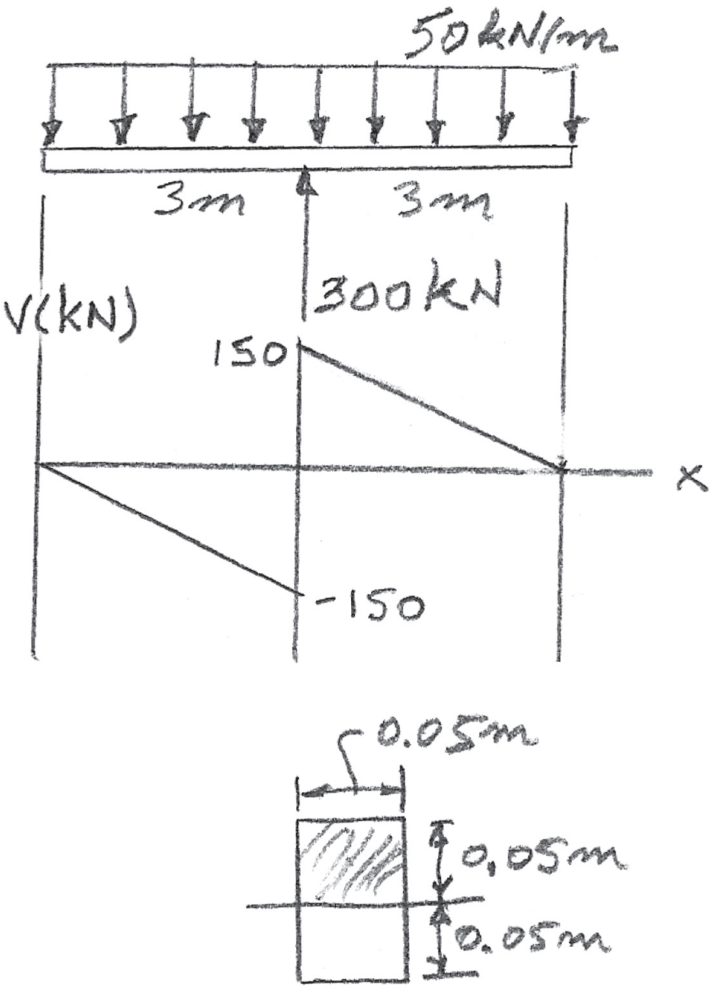
SOLUTION

$$\tau_{max} = \frac{VQ}{It} = \frac{150(10^3)\text{ N }(0.025\text{ m})(0.05\text{ m})(0.05\text{ m})}{\frac{1}{12}(0.05\text{ m})(0.1\text{ m})^3(0.05\text{ m})}$$

$\tau_{max} = 45.0$ MPa **Ans.**

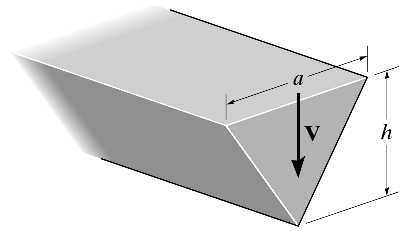
Because the cross section is a rectangle, then also,

$\tau_{max} = 1.5\dfrac{V}{A} = 1.5\dfrac{150(10^3)\text{ N}}{(0.05\text{ m})(0.1\text{ m})} = 45.0$ MPa **Ans.**



Ans:
$\tau_{max} = 45.0$ MPa

***7–12.** A member has a cross section in the form of an equilateral triangle. If it is subjected to a shear force **V**, determine the maximum average shear stress in the member using the shear formula. Should the shear formula actually be used to predict this value? Explain.



SOLUTION

$$I = \frac{1}{36}(a)(h)^3$$

$$\frac{y}{x} = \frac{h}{a/2}; \qquad y = \frac{2h}{a}x$$

$$Q = \int_{A'} y\,dA = 2\left[\left(\frac{1}{2}\right)(x)(y)\left(\frac{2}{3}h - \frac{2}{3}y\right)\right]$$

$$Q = \left(\frac{4h^2}{3a}\right)(x^2)\left(1 - \frac{2x}{a}\right)$$

$$t = 2x$$

$$\tau = \frac{VQ}{It} = \frac{V(4h^2/3a)(x^2)(1 - \frac{2x}{a})}{((1/36)(a)(h^3))(2x)}$$

$$\tau = \frac{24V(x - \frac{2}{a}x^2)}{a^2h}$$

$$\frac{d\tau}{dx} = \frac{24V}{a^2h^2}\left(1 - \frac{4}{a}x\right) = 0$$

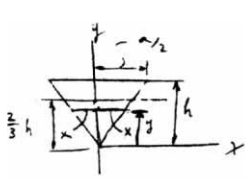
$$\text{At } x = \frac{a}{4}$$

$$y = \frac{2h}{a}\left(\frac{a}{4}\right) = \frac{h}{2}$$

$$\tau_{max} = \frac{24V}{a^2h}\left(\frac{a}{4}\right)\left(1 - \frac{2}{a}\left(\frac{a}{4}\right)\right)$$

$$\tau_{max} = \frac{3V}{ah}$$ **Ans.**

No, because the shear stress is not perpendicular to the boundary. See Sec. 7-3.

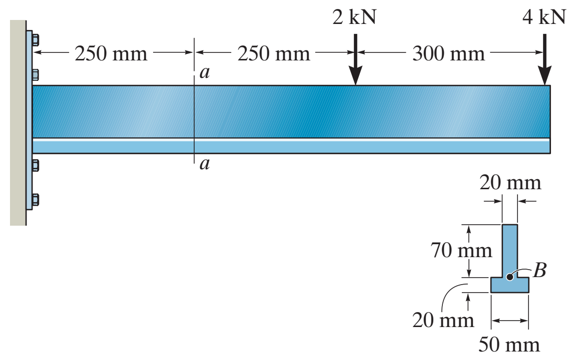


Ans.

$$\tau_{max} = \frac{3V}{ah}$$

7–13.

Determine the shear stress at point B on the web of the cantilevered strut at section a–a.

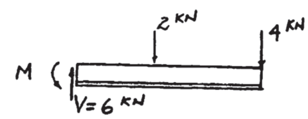


SOLUTION

$$\bar{y} = \frac{(0.01)(0.05)(0.02) + (0.055)(0.07)(0.02)}{(0.05)(0.02) + (0.07)(0.02)} = 0.03625 \text{ m}$$

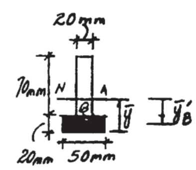
$$I = \frac{1}{12}(0.05)\left(0.02^3\right) + (0.05)(0.02)(0.03625 - 0.01)^2$$

$$+ \frac{1}{12}(0.02)\left(0.07^3\right) + (0.02)(0.07)(0.055 - 0.03625)^2 = 1.78625\left(10^{-6}\right) \text{ m}^4$$



$$\bar{y}'_B = 0.03625 - 0.01 = 0.02625 \text{ m}$$

$$Q_B = (0.02)(0.05)(0.02625) = 26.25\left(10^{-6}\right) \text{ m}^3$$



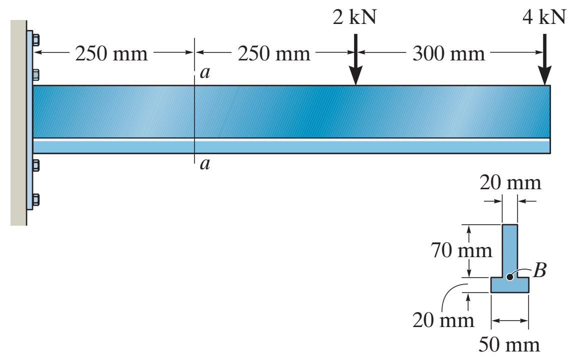
$$\tau_B = \frac{VQ_B}{It} = \frac{6\left(10^3\right)(26.25)\left(10^{-6}\right)}{1.78622\left(10^{-6}\right)(0.02)}$$

$$= 4.41 \text{ MPa}$$ **Ans.**

Ans:
$\tau_B = 4.41$ MPa

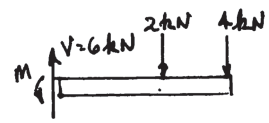
7–14.

Determine the maximum shear stress acting at section *a–a* of the cantilevered strut.



SOLUTION

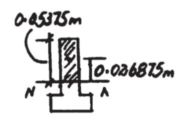
$$\bar{y} = \frac{(0.01)(0.05)(0.02) + (0.055)(0.07)(0.02)}{(0.05)(0.02) + (0.07)(0.02)} = 0.03625 \text{ m}$$



$$I = \frac{1}{12}(0.05)\left(0.02^3\right) + (0.05)(0.02)(0.03625 - 0.01)^2$$

$$+ \frac{1}{12}(0.02)\left(0.07^3\right) + (0.02)(0.07)(0.055 - 0.03625)^2 = 1.78625\left(10^{-6}\right) \text{m}^4$$

$$Q_{\max} = \bar{y}'A' = (0.026875)(0.05375)(0.02) = 28.8906\left(10^{-6}\right) \text{m}^3$$

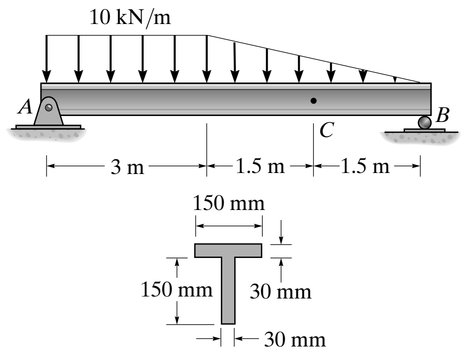


$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{6\left(10^3\right)(28.8906)\left(10^{-6}\right)}{1.78625\left(10^{-6}\right)(0.02)}$$

$$= 4.85 \text{ MPa}$$ **Ans.**

Ans:
$\tau_{\max} = 4.85$ MPa

7–15. Determine the maximum shear stress in the T-beam at the critical section where the internal shear force is maximum.



The FBD of the beam is shown in Fig. *a*,

The shear diagram is shown in Fig. *b*. As indicated, $V_{\max} = 27.5$ kN

The neutral axis passes through centroid *c* of the cross-section, Fig. *c*.

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.075(0.15)(0.03) + 0.165(0.03)(0.15)}{0.15(0.03) + 0.03(0.15)}$$

$$= 0.12 \text{ m}$$

$$I = \frac{1}{12}(0.03)(0.15^3) + 0.03(0.15)(0.12 - 0.075)^2$$

$$+ \frac{1}{12}(0.15)(0.03^3) + 0.15(0.03)(0.165 - 0.12)^2$$

$$= 27.0(10^{-6}) \text{ m}^4$$

From Fig. *d*,

$$Q_{\max} = \bar{y}'A' = 0.06(0.12)(0.03)$$

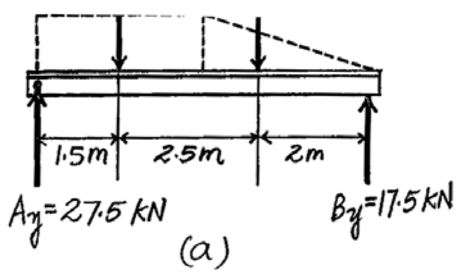
$$= 0.216\,(10^{-3}) \text{ m}^3$$

The maximum shear stress occurs at points on the neutral axis since Q is maximum and thickness $t = 0.03$ m is the smallest.

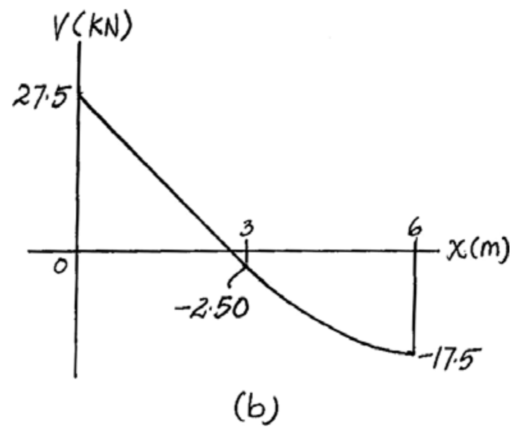
$$\tau_{\max} = \frac{V_{\max} Q_{\max}}{I\,t} = \frac{27.5(10^3)\left[0.216(10^{-3})\right]}{27.0(10^{-6})(0.03)}$$

$$= 7.333(10^6) \text{ Pa}$$

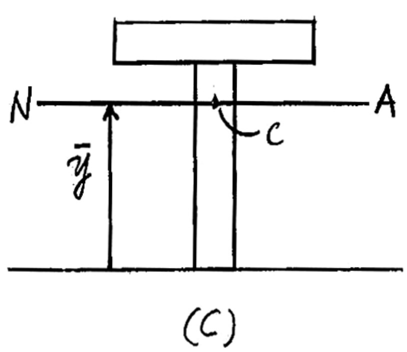
$$= 7.33 \text{ MPa}$$ **Ans.**



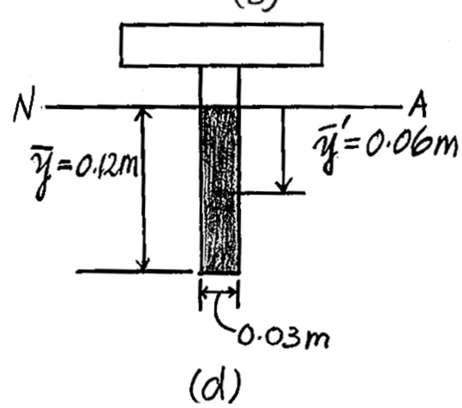
(a)



(b)



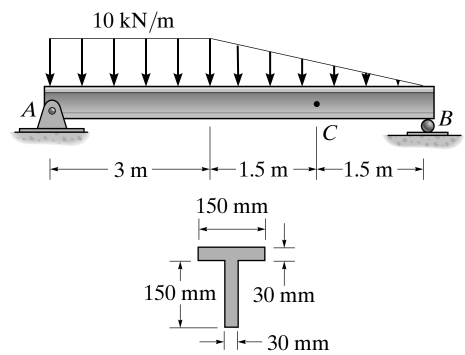
(c)



(d)

Ans.
$\tau_{\max} = 7.33$ MPa

***7–16.** Determine the maximum shear stress in the T-beam at point C. Show the result on a volume element at this point.



SOLUTION

using the method of sections,

$$+\uparrow \Sigma F_y = 0; \qquad V_C + 17.5 - \frac{1}{2}(5)(1.5) = 0$$

$$V_C = -13.75 \text{ kN}$$

The neutral axis passes through centroid C of the cross-section,

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.075\,(0.15)(0.03) + 0.165(0.03)(0.15)}{0.15(0.03) + 0.03(0.15)}$$

$$= 0.12 \text{ m}$$

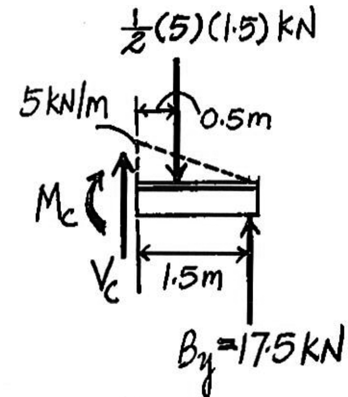
$$I = \frac{1}{12}(0.03)(0.15) + 0.03(0.15)(0.12 - 0.075)^2$$

$$+ \frac{1}{12}(0.15)(0.03^3) + 0.15(0.03)(0.165 - 0.12)^2$$

$$= 27.0\,(10^{-6}) \text{ m}^4$$

$$Q_{\max} = \bar{y}' A' = 0.06\,(0.12)(0.03)$$

$$= 0.216\,(10^{-3}) \text{ m}^3 \; 490$$

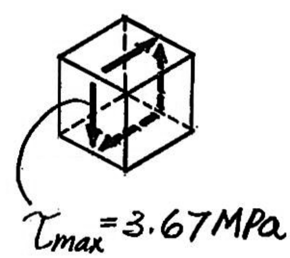


(b)

The maximum shear stress occurs at points on the neutral axis since Q is maximum and thickness $t = 0.03$ m is the smallest.

$$\tau_{\max} = \frac{V_C Q_{\max}}{It} = \frac{13.75(10^3)\left[0.216(10^{-3})\right]}{27.0(10^{-6})\,(0.03)}$$

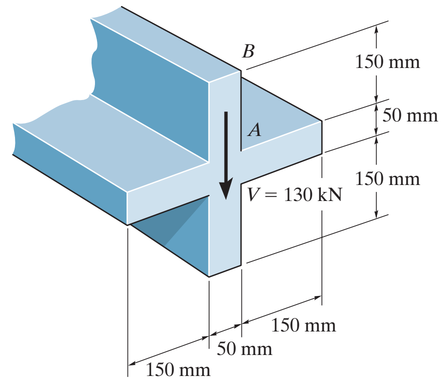
$$= 3.667(10^6) \text{ Pa} = 3.67 \text{ MPa} \qquad \textbf{Ans.}$$



(e)

Ans.
$\tau_{\max} = 3.67$ MPa

7–17. The strut is subjected to a vertical shear of $V = 130$ kN. Plot the intensity of the shear-stress distribution acting over the cross-sectional area, and compute the resultant shear force developed in the vertical segment AB.



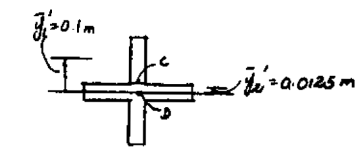
SOLUTION

$$I = \frac{1}{12}(0.05)(0.35^3) + \frac{1}{12}(0.3)(0.05^3) = 0.18177083(10^{-3})\ \text{m}^4$$

$$Q_C = \bar{y}'A' = (0.1)(0.05)(0.15) = 0.75(10^{-3})\ \text{m}^3$$

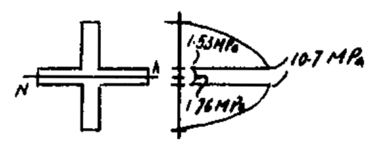
$$Q_D = \Sigma\bar{y}'A' = (0.1)(0.05)(0.15) + (0.0125)(0.35)(0.025)$$

$$= 0.859375(10^{-3})\ \text{m}^3$$



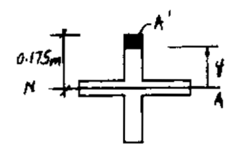
$$\tau = \frac{VQ}{It}$$

$$(\tau_C)_{t = 0.05\text{ m}} = \frac{130(10^3)(0.75)(10^{-3})}{0.18177083(10^{-3})(0.05)} = 10.7\ \text{MPa}$$



$$(\tau_C)_{t = 0.35\text{ m}} = \frac{130(10^3)(0.75)(10^{-3})}{0.18177083(10^{-3})(0.35)} = 1.53\ \text{MPa}$$

$$\tau_D = \frac{130(10^3)(0.859375)(10^{-3})}{0.18177083(10^{-3})(0.35)} = 1.76\ \text{MPa}$$



$$A' = (0.05)(0.175 - y)$$

$$\bar{y}' = y + \frac{(0.175 - y)}{2} = \frac{1}{2}(0.175 + y)$$

$$Q = \bar{y}'A' = 0.025(0.030625 - y^2)$$

$$\tau = \frac{VQ}{It}$$

$$= \frac{130(0.025)(0.030625 - y^2)}{0.18177083(10^{-3})(0.05)}$$

$$= 10951.3 - 357593.1\, y^2$$

$$V_{AB} = \int \tau\, dA \qquad dA = 0.05\, dy$$

$$= \int_{0.025}^{0.175} (10951.3 - 357593.1y^2)(0.05\, dy)$$

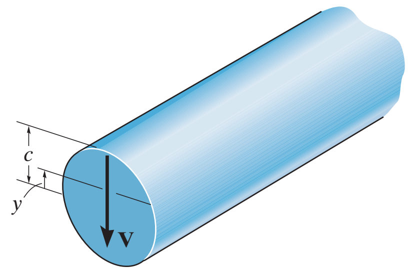
$$= \int_{0.025}^{0.175} (547.565 - 17879.66y^2)\, dy$$

$$= 50.3\ \text{kN}$$ **Ans.**

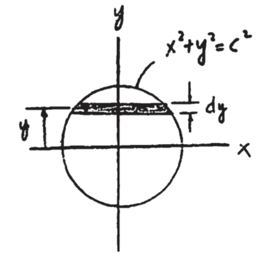
Ans:
$V_{AB} = 50.3$ kN

7–18.

Plot the shear-stress distribution over the cross section of a rod that has a radius c. By what factor is the maximum shear stress greater than the average shear stress acting over the cross section?



SOLUTION



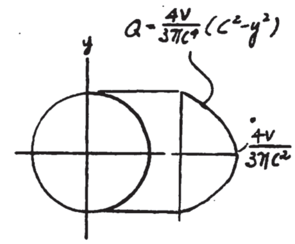
$$x = \sqrt{c^2 - y^2}; \qquad I = \frac{\pi}{4}c^4$$

$$t = 2x = 2\sqrt{c^2 - y^2}$$

$$dA = 2x\,dy = 2\sqrt{c^2 - y^2}\,dy$$

$$dQ = y\,dA = 2y\sqrt{c^2 - y^2}\,dy$$

$$Q = \int_y^c 2y\sqrt{c^2 - y^2}\,dy = -\frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}\Big|_y^c = \frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}$$



$$\tau = \frac{VQ}{It} = \frac{V\left[\frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}\right]}{\left(\frac{\pi}{4}c^4\right)\left(2\sqrt{c^2 - y^2}\right)} = \frac{4V}{3\pi c^4}(c^2 - y^2)$$

The maximum shear stress occur when $y = 0$

$$\tau_{\max} = \frac{4V}{3\pi c^2}$$

$$\tau_{\text{avg}} = \frac{V}{A} = \frac{V}{\pi c^2}$$

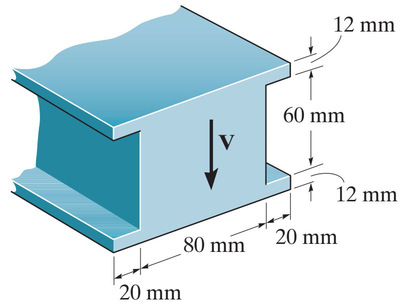
$$\text{The factor} = \frac{\tau_{\max}}{\tau_{\text{avg}}} = \frac{\frac{4V}{3\pi c^2}}{\frac{V}{\pi c^2}} = \frac{4}{3}$$ **Ans.**

Ans:

The factor $= \frac{4}{3}$

7–19.

Determine the maximum shear stress in the strut if it is subjected to a shear force of $V = 20$ kN.

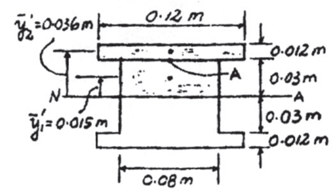


SOLUTION

Section Properties:

$$I_{NA} = \frac{1}{12}(0.12)(0.084^3) - \frac{1}{12}(0.04)(0.06^3)$$

$$= 5.20704\left(10^{-6}\right)\text{ m}^4$$



$$Q_{max} = \Sigma \bar{y}'A'$$

$$= 0.015(0.08)(0.03) + 0.036(0.012)(0.12)$$

$$= 87.84\left(10^{-6}\right)\text{ m}^3$$

Maximum Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section.

Applying the shear formula

$$\tau_{max} = \frac{VQ_{max}}{It}$$

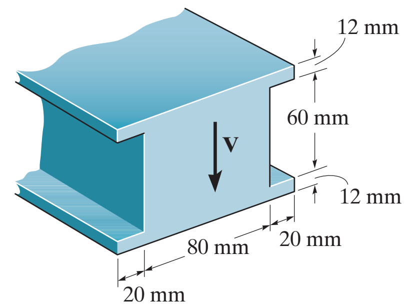
$$= \frac{20(10^3)(87.84)(10^{-6})}{5.20704(10^{-6})(0.08)}$$

$$= 4.22 \text{ MPa}$$ **Ans.**

Ans:
$\tau_{max} = 4.22$ MPa

***7–20.**

Determine the maximum shear force V that the strut can support if the allowable shear stress for the material is $\tau_{allow} = 40$ MPa.



SOLUTION

Section Properties:

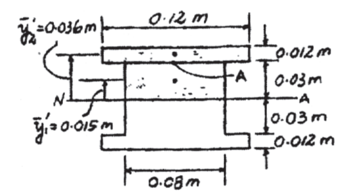
$$I_{NA} = \frac{1}{12}(0.12)(0.084^3) - \frac{1}{12}(0.04)(0.06^3)$$

$$= 5.20704(10^{-6})\ \text{m}^4$$

$$Q_{max} = \Sigma \bar{y}'A'$$

$$= 0.015(0.08)(0.03) + 0.036(0.012)(0.12)$$

$$= 87.84(10^{-6})\ \text{m}^3$$



Allowable Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section.

Applying the shear formula

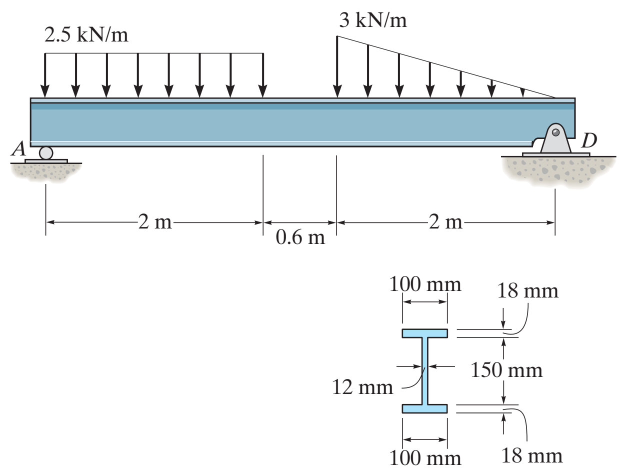
$$\tau_{max} = \tau_{allow} = \frac{VQ_{max}}{It}$$

$$40(10^6) = \frac{V(87.84)(10^{-6})}{5.20704(10^{-6})(0.08)}$$

$$V = 189\,692\ \text{N} = 190\ \text{kN}$$ **Ans.**

Ans:
$V_{max} = 190$ kN

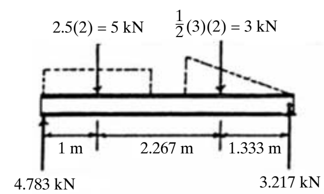
7–21. Determine the maximum shear stress acting in the fiberglass beam at the section where the internal shear force is maximum.



SOLUTION

Support Reactions: As shown on FBD.

Internal Shear Force: As shown on shear diagram, $V_{max} = 4.783$ kN.

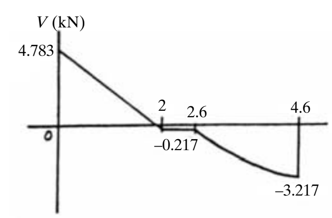


Section Properties:

$$I_{NA} = \frac{1}{12}(0.1)(0.186^3) - \frac{1}{12}(0.088)(0.15^3) = 28.8738(10^{-6})\ \text{m}^4$$

$$Q_{max} = \Sigma y' A'$$

$$= 0.084(0.1)(0.018) + 0.0375(0.012)(0.075) = 0.18495(10^{-3})\ \text{m}^3$$

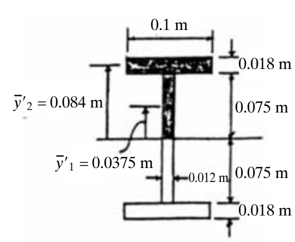


Maximum Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section.

Applying the shear formula

$$\tau_{max} = \frac{VQ_{max}}{It}$$

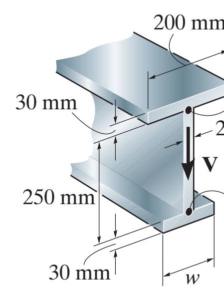
$$= \frac{[4.783(10^3)][0.18495(10^{-3})]}{[28.8738(10^{-6})](0.012)} = 2.553(10^6)\ \text{N/m}^3 = 2.55\ \text{MPa} \qquad \textbf{Ans.}$$



Ans.
$\tau_{max} = 2.55$ MPa

7–22.

If the beam is subjected to a shear of $V = 15$ kN, determine the web's shear stress at A and B. Indicate the shear-stress components on a volume element located at these points. Set $w = 125$ mm. Show that the neutral axis is located at $\bar{y} = 0.1747$ m from the bottom and $I_{NA} = 0.2182(10^{-3})$ m^4.



SOLUTION

$$\bar{y} = \frac{(0.015)(0.125)(0.03) + (0.155)(0.025)(0.25) + (0.295)(0.2)(0.03)}{0.125(0.03) + (0.025)(0.25) + (0.2)(0.03)} = 0.1747 \text{ m}$$

$$I = \frac{1}{12}(0.125)(0.03^3) + 0.125(0.03)(0.1747 - 0.015)^2$$

$$+ \frac{1}{12}(0.025)(0.25^3) + 0.25(0.025)(0.1747 - 0.155)^2$$

$$+ \frac{1}{12}(0.2)(0.03^3) + 0.2(0.03)(0.295 - 0.1747)^2 = 0.218182(10^{-3}) \text{ m}^4$$

$$Q_A = \bar{y}A'_A = (0.310 - 0.015 - 0.1747)(0.2)(0.03) = 0.7219(10^{-3}) \text{ m}^3$$

$$Q_B = \bar{y}A'_B = (0.1747 - 0.015)(0.125)(0.03) = 0.59883(10^{-3}) \text{ m}^3$$

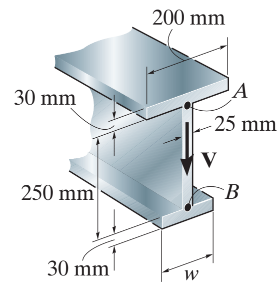
$$\tau_A = \frac{VQ_A}{It} = \frac{15(10^3)(0.7219)(10^{-3})}{0.218182(10^{-3})0.025} = 1.99 \text{ MPa}$$ **Ans.**

$$\tau_B = \frac{VQ_B}{It} = \frac{15(10^3)(0.59883)(10^{-3})}{0.218182(10^{-3})0.025} = 1.65 \text{ MPa}$$ **Ans.**

Ans:
$\tau_A = 1.99$ MPa, $\tau_B = 1.65$ MPa

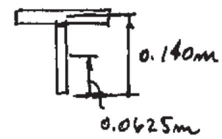
7–23.

If the wide-flange beam is subjected to a shear of $V = 30$ kN, determine the maximum shear stress in the beam. Set $w = 200$ mm.



SOLUTION

Section Properties:



$$I = \frac{1}{12}(0.2)(0.310)^3 - \frac{1}{12}(0.175)(0.250)^3 = 268.652(10)^{-6}\ \text{m}^4$$

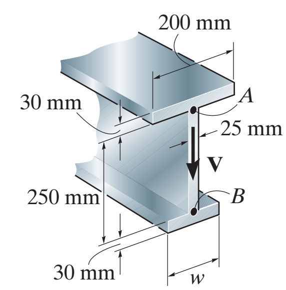
$$Q_{\max} = \Sigma\bar{y}A = 0.0625(0.125)(0.025) + 0.140(0.2)(0.030) = 1.0353(10)^{-3}\ \text{m}^3$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{30(10)^3(1.0353)(10)^{-3}}{268.652(10)^{-6}(0.025)} = 4.62\ \text{MPa}$$ **Ans.**

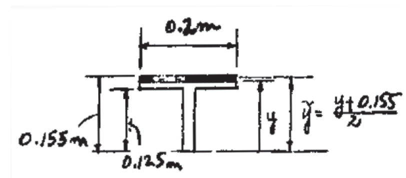
Ans:
$\tau_{\max} = 4.62$ MPa

***7–24.**

If the wide-flange beam is subjected to a shear of $V = 30$ kN, determine the shear force resisted by the web of the beam. Set $w = 200$ mm.



SOLUTION



$$I = \frac{1}{12}(0.2)(0.310)^3 - \frac{1}{12}(0.175)(0.250)^3 = 268.652(10)^{-6}\text{ m}^4$$

$$Q = \left(\frac{0.155 + y}{2}\right)(0.155 - y)(0.2) = 0.1(0.024025 - y^2)$$

$$\tau_f = \frac{30(10)^3(0.1)(0.024025 - y^2)}{268.652(10)^{-6}(0.2)}$$

$$V_f = \int \tau_f \, dA = 55.8343(10)^6 \int_{0.125}^{0.155} (0.024025 - y^2)(0.2\,dy)$$

$$= 11.1669(10)^6 \left[0.024025y - \frac{1}{3}y^3\right]_{0.125}^{0.155}$$

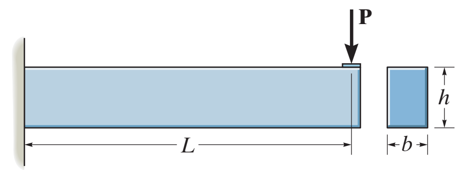
$$V_f = 1.457\text{ kN}$$

$$V_w = 30 - 2(1.457) = 27.1\text{ kN}$$ **Ans.**

Ans:
$V_w = 27.1$ kN

7–25.

Determine the length of the cantilevered beam so that the maximum bending stress in the beam is equivalent to the maximum shear stress.



SOLUTION

$V_{max} = P$

$M_{max} = PL$

$$\sigma_{max} = \frac{Mc}{I} = \frac{PL(h/2)}{I} = \frac{PLh}{2I}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{P(h/2)(b)(h/4)}{Ib} = \frac{Ph^2}{8I}$$

Require,

$$\sigma_{max} = \tau_{max}$$

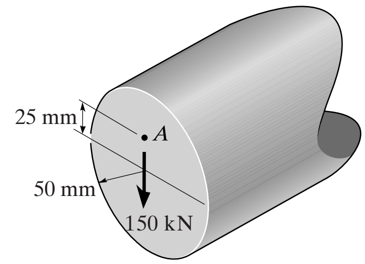
$$\frac{PLh}{2I} = \frac{Ph^2}{8I}$$

$$L = \frac{h}{4}$$ **Ans.**

Ans:

$L = \dfrac{h}{4}$

7–26. The steel rod is subjected to a shear of 150 kN. Determine the shear stress at point A. Show the result on a volume element at this point.



The moment of inertia of the circular cross-section about the neutral axis (x axis) is

$$I = \frac{\pi}{4}r^4 = \frac{\pi}{4}(50^4) = 1.5625\,\pi(10^6)\text{ mm}^4$$

Q for the differential area shown in Fig. a is

$$dQ = y\,dA = y\,(2x\,dy) = 2xy\,dy$$

However, from the equation of the circle, $x = (2500 - y^2)^{\frac{1}{2}}$, Then

$$dQ = 2y(2500 - y^2)^{\frac{1}{2}}\,dy$$

Thus, Q for the area above y is

$$Q = \int_y^{50\text{ mm}} 2y(2500 - y^2)^{\frac{1}{2}}\,dy$$

$$= -\frac{2}{3}(2500 - y^2)^{\frac{3}{2}}\Big|_y^{50\text{ mm}} = \frac{2}{3}(2500 - y^2)^{\frac{3}{2}}$$

Here $t = 2x = 2(2500 - y^2)^{\frac{1}{2}}$. Thus

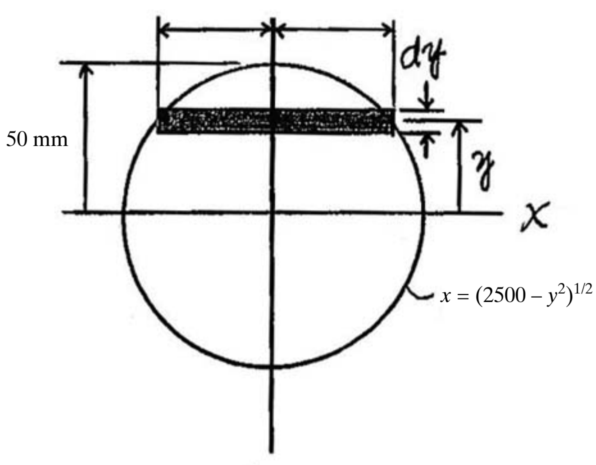
$$\tau = \frac{VQ}{It} = \frac{150(10^3)\left[\frac{2}{3}(2500 - y^2)^{\frac{3}{2}}\right]}{1.5625\pi(10^6)\left[2(2500 - y^2)^{\frac{1}{2}}\right]}$$

$$\tau = \frac{4}{125\pi}(2500 - y^2)\text{ Ksi}$$

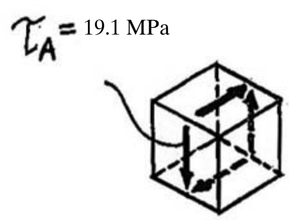
For point A, $y = 25$ mm. Thus

$$\tau_A = \frac{4}{125\pi}(2500 - 25^2) = 19.1\text{ MPa}$$ **Ans.**

The state of shear stress at point A can be represented by the volume element shown in Fig. b.



(a)

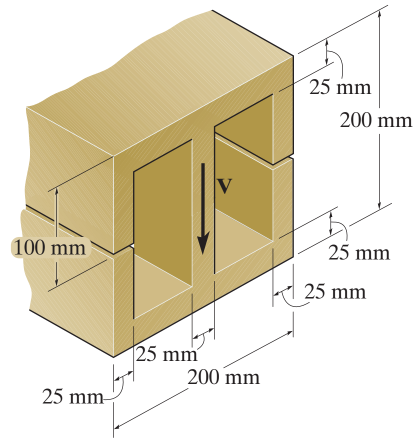


(b)

Ans:
$P = 5.9988$ kN

7–27.

The beam is slit longitudinally along both sides. If it is subjected to a shear of $V = 250$ kN, compare the maximum shear stress in the beam before and after the cuts were made.



SOLUTION

Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.2^3) - \frac{1}{12}(0.125)(0.15^3) = 98.1771(10^{-6})\text{ m}^4$$

Q_{max} is the first moment of the shaded area shown in Fig. *a* about the neutral axis. Thus,

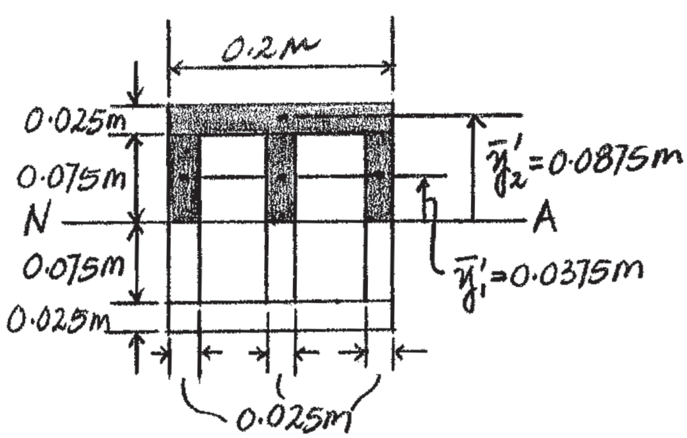
$$\begin{aligned} Q_{max} &= 3\bar{y}_1'A_1' + \bar{y}_2'A_2' \\ &= 3\left[0.0375(0.075)(0.025)\right] + 0.0875(0.025)(0.2) \\ &= 0.6484375(10^{-3})\text{ m}^3 \end{aligned}$$

Maximum Shear Stress: The maximum shear stress occurs at the points on the neutral axis since Q is maximum and t is minimum. Before the cross section is slit, $t = 3(0.025) = 0.075$ m.

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{250(10^3)(0.6484375)(10^{-3})}{98.1771(10^{-6})(0.075)} = 22.0\text{ MPa} \qquad \textbf{Ans.}$$

After the cross section is slit, $t = 0.025$ m.

$$(\tau_{max})_s = \frac{VQ_{max}}{It} = \frac{250(10^3)(0.6484375)(10^{-3})}{98.1771(10^{-6})(0.025)} = 66.0\text{ MPa} \qquad \textbf{Ans.}$$

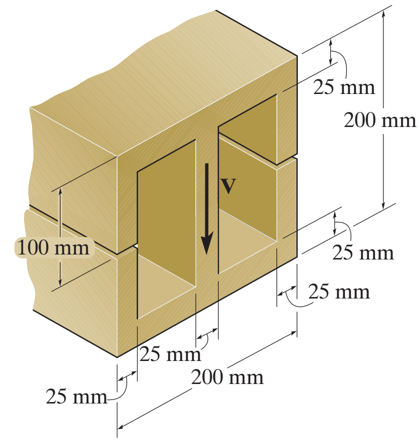


(a)

Ans:
$\tau_{max} = 22.0$ MPa, $(\tau_{max})_s = 66.0$ MPa

***7–28.**

The beam is to be cut longitudinally along both sides as shown. If it is made from a material having an allowable shear stress of $\tau_{allow} = 75$ MPa, determine the maximum allowable shear force **V** that can be applied before and after the cut is made.



SOLUTION

Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.2^3) - \frac{1}{12}(0.125)(0.15^3) = 98.1771(10^{-6})\ \text{m}^4$$

Q_{max} is the first moment of the shaded area shown in Fig. *a* about the neutral axis. Thus,

$$\begin{aligned} Q_{max} &= 3\bar{y}_1'A_1' + \bar{y}_2'A_2' \\ &= 3(0.0375)(0.075)(0.025) + 0.0875(0.025)(0.2) \\ &= 0.6484375(10^{-3})\ \text{m}^3 \end{aligned}$$

Shear Stress: The maximum shear stress occurs at the points on the neutral axis since Q is maximum and thickness t is minimum. Before the cross section is slit, $t = 3(0.025) = 0.075$ m.

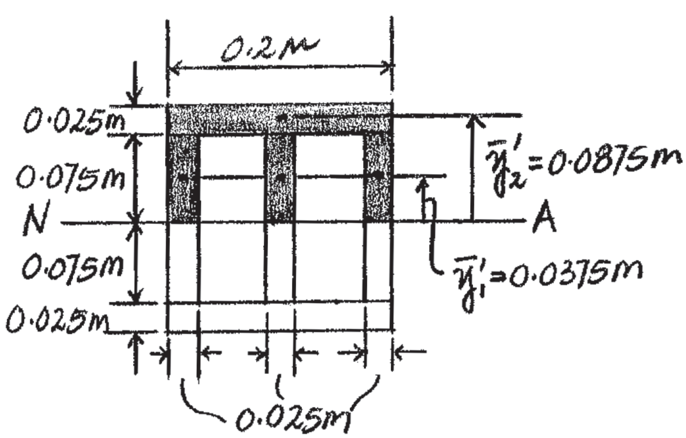
$$\tau_{allow} = \frac{VQ_{max}}{It}; \qquad 75(10^6) = \frac{V(0.6484375)(10^{-3})}{98.1771(10^{-6})(0.075)}$$

$$V = 851\,656.63\ \text{N} = 852\ \text{kN} \qquad \textbf{Ans.}$$

After the cross section is slit, $t = 0.025$ m.

$$\tau_{allow} = \frac{VQ_{max}}{It}; \qquad 75(10^6) = \frac{V_s(0.6484375)(10^{-3})}{98.1771(10^{-6})(0.025)}$$

$$V_s = 283\,885.54\ \text{N} = 284\ \text{kN} \qquad \textbf{Ans.}$$

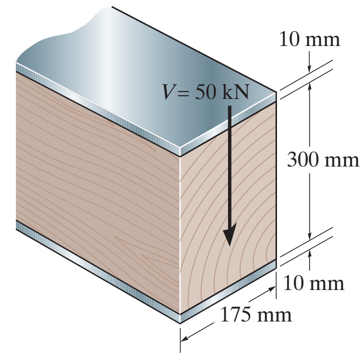


(a)

Ans:
$V = 852$ kN,
$V_s = 284$ kN

7–29.

The composite beam is constructed from wood and reinforced with a steel strap. Use the method of Sec. 6.6 and calculate the maximum shear stress in the beam when it is subjected to a shear of $V = 50$ kN. Take $E_{st} = 200$ GPa, $E_w = 15$ GPa.

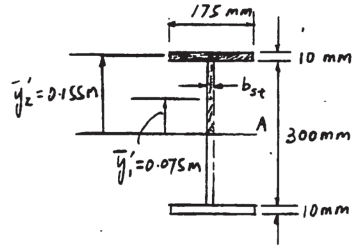


SOLUTION

$$b_{st} = nb_w = \frac{15}{200}(0.175) = 0.013125 \text{ m}$$

$$I = \frac{1}{12}(0.175)(0.32^3) - \frac{1}{12}(0.175 - 0.013125)(0.3^3) = 0.113648(10^{-3}) \text{ m}^4$$

$$Q_{max} = \Sigma \bar{y}'A' = 0.075(0.013125)(0.15) + 0.155(0.175)(0.01) = 0.4189(10^{-3}) \text{ m}^3$$



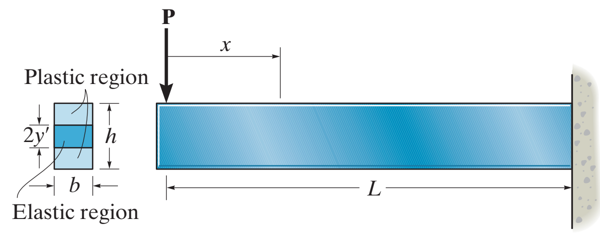
$$\tau_{max} = n\frac{VQ_{max}}{It} = \left(\frac{15}{200}\right)\frac{50(10^3)(0.4189)(10^{-3})}{0.113648(10^{-3})(0.013125)}$$

$$= 1.05 \text{ MPa}$$ **Ans.**

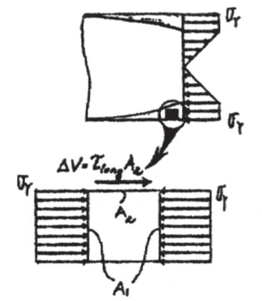
Ans:
$\tau_{max} = 1.05$ MPa

7–30.

The beam has a rectangular cross section and is subjected to a load P that is just large enough to develop a fully plastic moment $M_p = PL$ at the fixed support. If the material is elastic perfectly plastic, then at a distance $x < L$ the moment $M = Px$ creates a region of plastic yielding with an associated elastic core having a height $2y'$. This situation has been described by Eq. 6–30 and the moment **M** is distributed over the cross section as shown in Fig. 6–48*e*. Prove that the maximum shear stress in the beam is given by $\tau_{\max} = \frac{3}{2}(P/A')$, where $A' = 2y'b$, the cross-sectional area of the elastic core.



SOLUTION



Force Equilibrium: The shaded area indicates the plastic zone. Isolate an element in the plastic zone and write the equation of equilibrium.

$$\pm \Sigma F_x = 0; \qquad \tau_{\text{long}} A_2 + \sigma_Y A_1 - \sigma_Y A_1 = 0$$

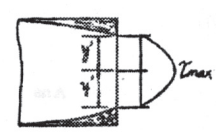
$$\tau_{\text{long}} = 0$$

This proves that the longitudinal shear stress. τ_{long}, is equal to zero. Hence the corresponding transverse stress, $\tau_{\max}$, is also equal to zero in the plastic zone. Therefore, the shear force $V = P$ is carried by the material only in the elastic zone.

Section Properties:

$$I_{NA} = \frac{1}{12}(b)(2y')^3 = \frac{2}{3} b\, y'^3$$

$$Q_{\max} = \bar{y}'\, A' = \frac{y'}{2}(y')(b) = \frac{y'^2 b}{2}$$



Maximum Shear Stress: Applying the shear formula

$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{V\left(\frac{y'^2 b}{2}\right)}{\left(\frac{2}{3} b y'^3\right)(b)} = \frac{3P}{4by'}$$

However, $\qquad A' = 2by' \qquad$ hence

$$\tau_{\max} = \frac{3P}{2A'}, \qquad \text{(Q.E.D.)}$$

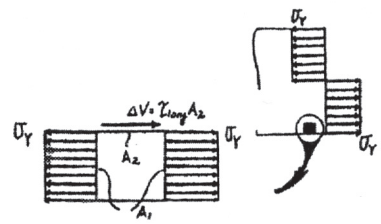
Ans:
N/A

7–31.

The beam in Fig. 6–48*f* is subjected to a fully plastic moment $\mathbf{M}_p$. Prove that the longitudinal and transverse shear stresses in the beam are zero. *Hint:* Consider an element of the beam shown in Fig. 7–4*d*.

SOLUTION

Force Equilibrium: If a fully plastic moment acts on the cross section, then an element of the material taken from the top or bottom of the cross section is subjected to the loading shown. For equilibrium



$$\pm\ \Sigma F_x = 0; \qquad \sigma_\gamma A_1 + \tau_{\text{long}} A_2 - \sigma_\gamma A_1 = 0$$

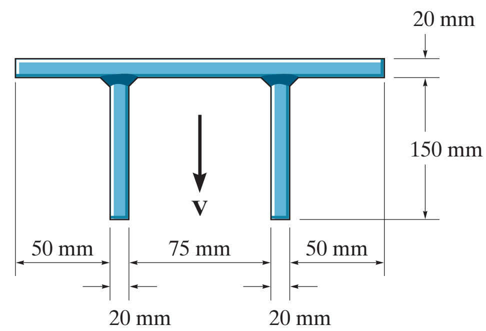
$$\tau_{\text{long}} = 0$$

Thus no shear stress is developed on the longitudinal or transverse plane of the element. (*Q. E. D.*)

Ans:
N/A

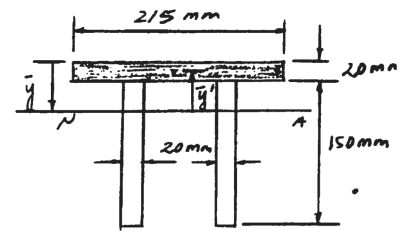
***7–32.**

The double T-beam is fabricated by welding the three plates together as shown. Determine the shear stress in the weld necessary to support a shear force of $V = 80$ kN.



SOLUTION

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{0.01(0.215)(0.02) + 2[0.095(0.15)(0.02)]}{0.215(0.02) + 2(0.15)(0.02)} = 0.059515 \text{ m}$$



$$I = \frac{1}{12}(0.215)\left(0.02^3\right) + 0.215(0.02)(0.059515 - 0.01)^2$$

$$+ 2\left[\frac{1}{12}(0.02)\left(0.15^3\right) + 0.02(0.15)(0.095 - 0.059515)^2\right] = 29.4909\left(10^{-6}\right) \text{ m}^4$$

$$\bar{y}' = 0.059515 - 0.01 = 0.049515 \text{ m}$$

$$Q = \bar{y}'A' = 0.049515(0.215)(0.02) = 0.2129\left(10^{-3}\right) \text{ m}^3$$

Shear stress:

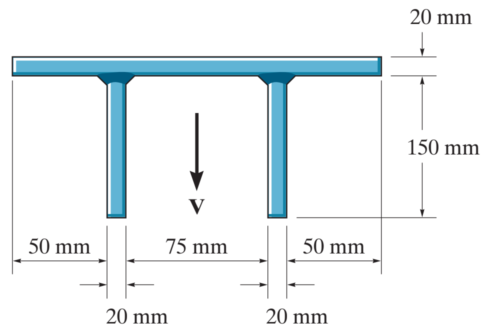
$$\tau = \frac{VQ}{It} = \frac{80\left(10^3\right)(0.2129)\left(10^{-3}\right)}{29.4909\left(10^{-6}\right)(2)(0.02)}$$

$$= 14.4 \text{ MPa}$$ **Ans.**

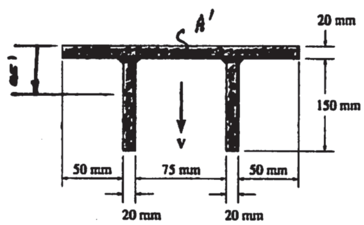
Ans:
$\tau = 14.4$ MPa

7–33.

The double T-beam is fabricated by welding the three plates together as shown. If the weld can resist a shear stress $\tau_{\text{allow}} = 90$ MPa, determine the maximum shear V that can be applied to the beam.



SOLUTION



$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.01(0.215)(0.02) + 2[0.095(0.15)(0.02)]}{0.215(0.02) + 2(0.15)(0.02)} = 0.059515 \text{ m}$$

$$I = \frac{1}{12}(0.215)(0.02^3) + 0.215(0.02)(0.059515 - 0.01)^2$$

$$+ 2\left[\frac{1}{12}(0.02)(0.15^3) + 0.02(0.15)(0.095 - 0.059515)^2\right] = 29.4909(10^{-6}) \text{ m}^4$$

$$\bar{y}' = 0.059515 - 0.01 = 0.049515 \text{ m}$$

$$Q = \bar{y}'A' = 0.049515(0.215)(0.02) = 0.2129(10^{-3}) \text{ m}^3$$

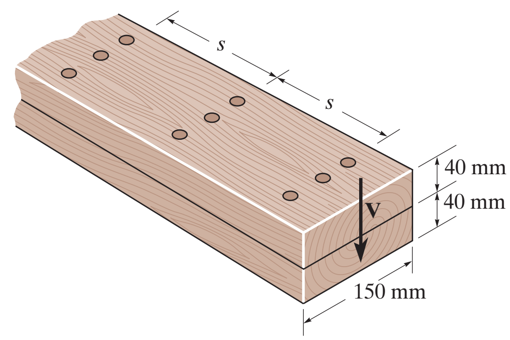
$$\tau = \frac{VQ}{It}$$

$$90(10^6) = \frac{V(0.2129)(10^{-3})}{29.491(10^{-6})(2)(0.02)}$$

$$V = 499 \text{ kN}$$ **Ans.**

Ans:
$V = 499$ kN

7–34. The beam is constructed from two boards fastened together with three rows of nails spaced $s = 50$ mm apart. If each nail can support a 2.25-kN shear force, determine the maximum shear force V that can be applied to the beam. The allowable shear stress for the wood is $\tau_{allow} = 2.1$ MPa.



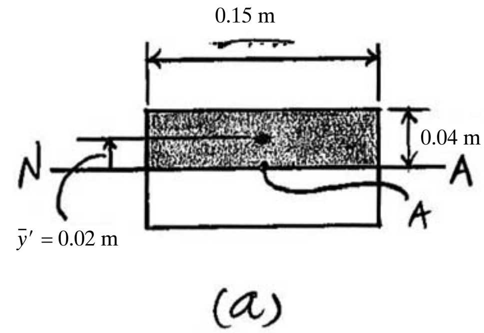
SOLUTION

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.15)(0.08^3) = 6.40(10^{-6})\ \text{m}^4$$

Refering to Fig. *a*,

$$Q_A = Q_{max} = \bar{y}'A' = 0.02(0.15)(0.04) = 0.12(10^{-3})\ \text{m}^3$$



The maximum shear stress occurs at the points on the neutral axis where Q is maximum and $t = 0.15$ m.

$$\tau_{allow} = \frac{VQ_{max}}{It}; \qquad 2.1(10^6) = \frac{V[0.12(10^{-3})]}{[6.40(10^{-6})](0.15)}$$

$$V = 16.8(10^3)\ \text{N} = 16.8\ \text{kN}$$

Shear Flow: Since there are three rows of nails,

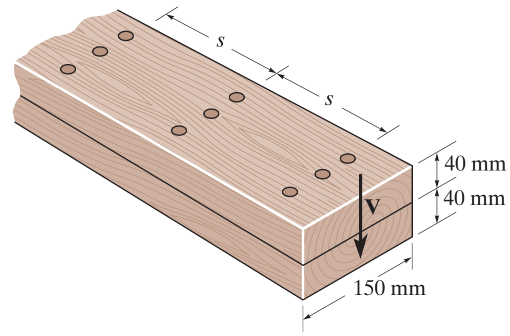
$$q_{allow} = 3\left(\frac{F}{s}\right) = 3\left[\frac{2.25(10^3)}{0.05}\right] = 135(10^3)\ \text{N/m}$$

$$q_{allow} = \frac{VQ_A}{I}; \qquad 135(10^3) = \frac{V[0.12(10^{-3})]}{6.40(10^{-6})}$$

$$V = 7.20(10^3)\ \text{N} = 7.20\ \text{kN} \qquad \textbf{Ans.}$$

Ans.
$V = 7.20$ kN

7–35. The beam is constructed from two boards fastened together with three rows of nails. If the allowable shear stress for the wood is $\tau_{\text{allow}} = 1$ MPa. determine the maximum shear force V that can be applied to the beam. Also, find the maximum spacing s of the nails if each nail can resist 3.25 kN in shear.



SOLUTION

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.15)(0.08^3) = 6.40(10^{-6})\ \text{m}^4$$

Refering to Fig. *a*,

$$Q_A = Q_{\max} = \bar{y}'A' = 0.02(0.15)(0.04) = 0.12(10^{-3})\ \text{m}^3$$

The maximum shear stress occurs at the points on the neutral axis where Q is maximum and $t = 0.15$ m.

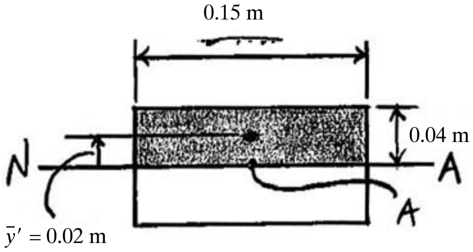
$$\tau_{\text{allow}} = \frac{VQ_{\max}}{It}; \qquad 1(10^6) = \frac{V[0.12(10^{-3})]}{[6.40(10^{-6})](0.15)}$$

$$V = 8.00(10^3)\ \text{N} = 8.00\ \text{kN} \qquad \textbf{Ans.}$$

Since there are three rows of nails, $q_{\text{allow}} = 3\left[\frac{2.25(10^3)}{s}\right] = \left(\frac{9750}{s}\right)$ N/m

$$q_{\text{allow}} = \frac{VQ_A}{I}; \qquad \frac{9750}{s} = \frac{[8(10^3)][0.12(10^{-3})]}{6.40(10^{-6})}$$

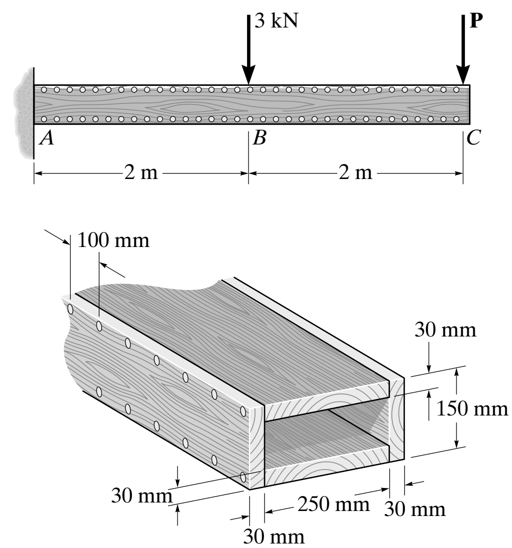
$$s = 0.0650\ \text{m} = 65.0\ \text{mm} \qquad \textbf{Ans.}$$



(a)

Ans.
$V = 8.00$ kN, $s = 65.0$ mm

***7–36.** The beam is constructed from four boards which are nailed together. If the nails are on both sides of the beam and each can resist a shear of 3 kN, determine the maximum load P that can be applied to the end of the beam.



SOLUTION

Support Reactions: As shown on FBD.

Internal Shear Force: As shown on shear diagram, $V_{AB} = (P + 3)$ kN.

Section Properties:

$$I_{NA} = \frac{1}{12}(0.31)\left(0.15^3\right) - \frac{1}{12}(0.25)\left(0.09^3\right)$$
$$= 72.0\left(10^{-6}\right)\text{ m}^4$$

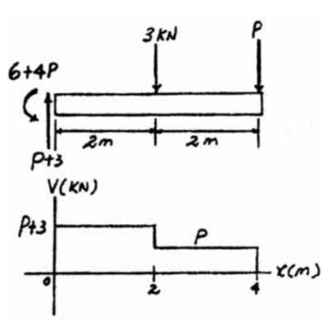
$$Q = \bar{y}'A' = 0.06(0.25)(0.03) = 0.450\left(10^{-3}\right)\text{ m}^3$$

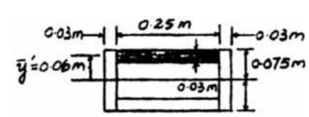
Shear Flow: There are two rows of nails. Hence the allowable shear flow is $q = \dfrac{3(2)}{0.1} = 60.0$ kN/m.

$$q = \frac{VQ}{I}$$

$$60.0\left(10^3\right) = \frac{(P + 3)(10^3)0.450(10^{-3})}{72.0(10^{-6})}$$

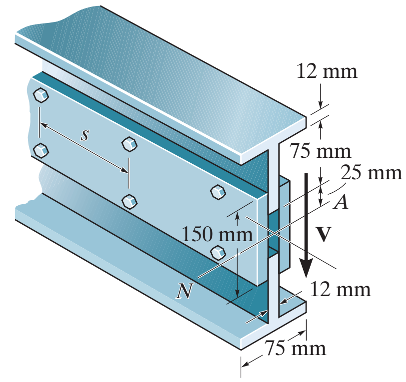
$$P = 6.60\text{ kN}$$ **Ans.**





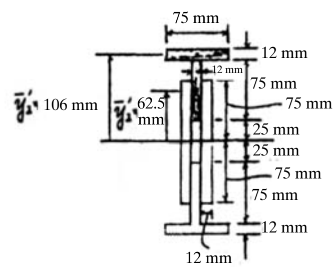
Ans.
$P = 6.60$ kN

7–37. The beam is fabricated from two equivalent structural tees and two plates. Each plate has a height of 150 mm and a thickness of 12 mm. If a shear of $V = 250$ kN is applied to the cross section, determine the maximum spacing of the bolts. Each bolt can resist a shear force of 75 kN.



SOLUTION

Section Properties:



$$I_{NA} = \frac{1}{12}(0.075)(0.224^3) - \frac{1}{12}(0.063)(0.2^3) - \frac{1}{12}(0.012)(0.05^3) + \frac{1}{12}(0.024)(0.15^3)$$

$$= 34.8714(10^{-6})\text{ m}^4$$

$$Q = \Sigma \bar{y}' A' = 0.0625(0.012)(0.075) + 0.106(0.075)(0.012) = 0.15165(10^{-3})\text{ m}^3$$

Shear Flow: Since there are two shear planes on the bolt, the allowable shear flow is $q = \frac{2(75)(10^3)}{s} = \frac{150(10^3)}{s}$.

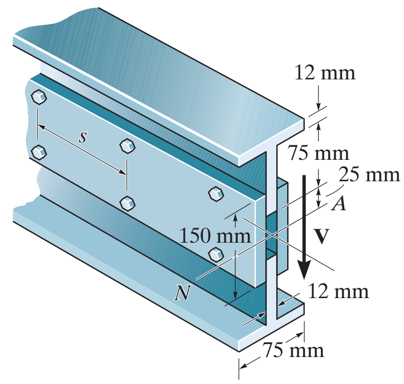
$$q = \frac{VQ}{I}$$

$$\frac{150(10^3)}{s} = \frac{[250(10^3)][0.15165(10^{-3})]}{34.8714(10^{-6})}$$

$$s = 0.1380\text{ m} = 138\text{ mm}$$ **Ans.**

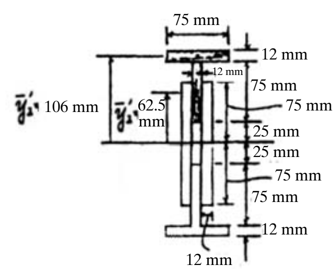
Ans.
$s = 138$ mm

7–38. The beam is fabricated from two equivalent structural tees and two plates. Each plate has a height of 150 mm and a thickness of 12 mm. If the bolts are spaced at $s = 200$ mm, determine the maximum shear force V that can be applied to the cross section. Each bolt can resist a shear force of 75 kN.



SOLUTION

Section Properties:



$$I_{NA} = \frac{1}{12}(0.075)(0.224^3) - \frac{1}{12}(0.063)(0.2^3)$$
$$-\frac{1}{12}(0.012)(0.05^3) + \frac{1}{12}(0.024)(0.15^3)$$
$$= 34.8714(10^{-6})\text{ m}^4$$
$$Q = \Sigma\bar{y}'A' = 0.0625(0.012)(0.075) + 0.106(0.075)(0.012) = 0.15165(10^{-3})\text{ m}^3$$

Shear Flow: Since there are two shear planes on the bolt, the allowable shear flow is

$$q = \frac{2(75)(10^3)}{0.2} = 750(10^3)\text{ N/m}$$

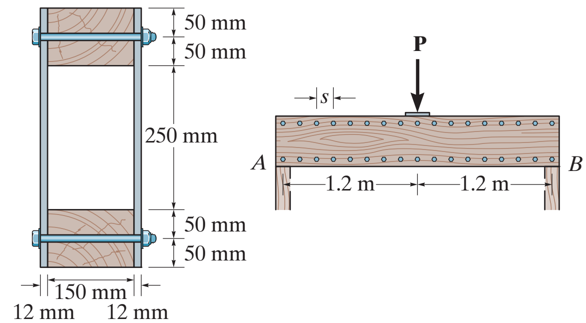
$$q = \frac{VQ}{I}$$

$$750(10^3) = \frac{V[0.15165(10^{-3})]}{34.8714(10^{-6})}$$

$$V = 172.46(10^3)\text{ N} = 172\text{ kN} \qquad \textbf{Ans.}$$

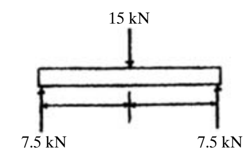
Ans.
$I_{NA} = 34.8714(10^{-6})\text{ m}^4$, $Q = 0.15165\ (10^{-3})\text{ m}^3$
$V = 172$ kN

7–39. The double-web girder is constructed from two plywood sheets that are secured to wood members at its top and bottom. If each fastener can support 3 kN in single shear, determine the required spacing s of the fasteners needed to support the loading P = 15 kN. Assume A is pinned and B is a roller.

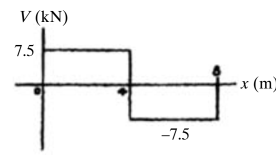


SOLUTION

Support Reactions: As shown on FBD.



Internal Shear Force: As shown on shear diagram, $V_{\max} = 7.5$ kN.

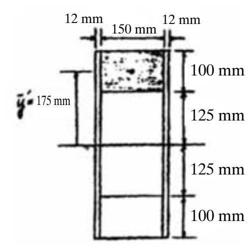


Section Properties:

$$I_{NA} = \frac{1}{12}(0.174)(0.45^3) - \frac{1}{12}(0.15)(0.25^3) = 1.126(10^{-3})\ \text{m}^4$$

$$Q = \bar{y}'A' = 0.175(0.15)(0.1) = 2.625(10^{-3})\ \text{m}^3$$

Shear Flow: Since there are two shear planes on the bolt, the allowable shear flow is $q = \dfrac{2(3)(10^3)}{s} = \dfrac{6000}{s}$.



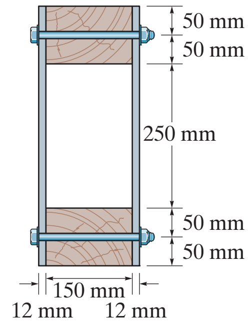
$$q = \frac{VQ}{I}$$

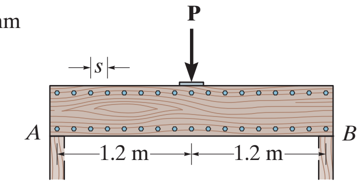
$$\frac{6000}{s} = \frac{7.5(10^3)[2.625(10^{-3})]}{1.126(10^{-3})}$$

$$s = 0.3432\ \text{m} = 343\ \text{mm}$$ **Ans.**

Ans.
$s = 343$ mm

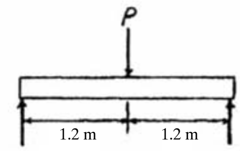
***7–40.** The double-web girder is constructed from two plywood sheets that are secured to wood members at its top and bottom. The allowable bending stress for the wood is $\sigma_{allow} = 56$ MPa and the allowable shear stress is $\tau_{allow} = 21$ MPa. If the fasteners are spaced $s = 150$ mm and each fastener can support 3 kN in single shear, determine the maximum load P that can be applied to the beam.



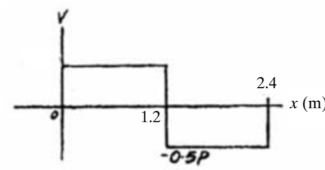


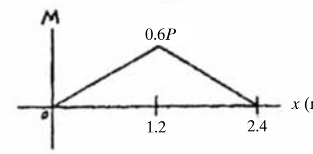
SOLUTION

Support Reactions: As shown on FBD.



Internal Shear Force and Moment: As shown on shear and moment diagram, $V_{max} = 0.500P$ and $M_{max} = 2.00P$.



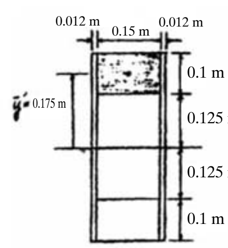


Section Properties:

$$I_{NA} = \frac{1}{12}(0.174)(0.45^3) - \frac{1}{12}(0.15)(0.25^3) = 1.126(10^{-3})\ \text{m}^4$$

$$Q = \bar{y}_2' A' = 0.175(0.15)(0.1) = 2.625(10^{-3})\ \text{m}^3$$

$$Q_{max} = \Sigma \bar{y}' A' = 0.175(0.15)(0.1) + 0.1125(0.024)(0.225) = 3.2325(10^{-3})\ \text{m}^3$$



Shear Flow: Assume bolt failure. Since there are two shear planes on the bolt, the allowable shear flow is $q = \dfrac{2(3)(10^3)}{0.15} = 40(10^3)$ N/m

$$q = \frac{VQ}{I}$$

$$40(10^3) = \frac{0.5P[2.625(10^{-3})]}{1.126(10^{-3})}$$

$$P = 34.32(10^3)\ \text{N} = 34.3\ \text{kN} \qquad \text{(Controls!)}$$ **Ans.**

Shear Stress: Assume failure due to shear stress.

$$\tau_{max} = \tau_{allow} = \frac{VQ_{max}}{It}$$

$$21(10^6) = \frac{0.5P[3.2325(10^{-3})]}{[1.126(10^{-3})](0.024)}$$

$$P = 351.12(10^3)\ \text{N} = 351\ \text{kN}$$

Bending Stress: Assume failure due to bending stress.

$$\sigma_{max} = \sigma_{allow} = \frac{Mc}{I}$$

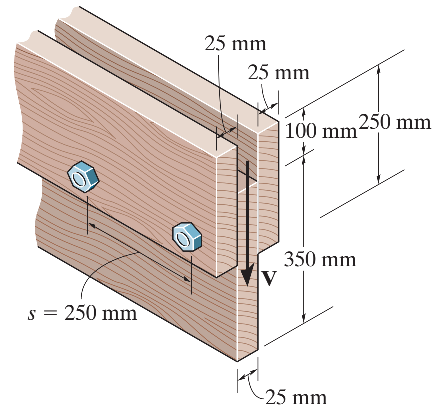
$$56(10^6) = \frac{0.6P(0.225)}{1.126(10^{-3})}$$

$$P = 467.08(10^3)\ \text{N} = 467\ \text{kN}$$

Ans.
$P = 34.3$ kN

7–41.

A beam is constructed from three boards bolted together as shown. Determine the shear force in each bolt if the bolts are spaced $s = 250$ mm apart and the shear is $V = 35$ kN.



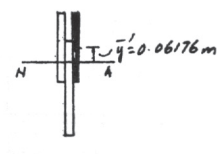
SOLUTION

$$\bar{y} = \frac{2\,(0.125)(0.25)(0.025) + 0.275\,(0.35)(0.025)}{2\,(0.25)(0.025) + 0.35\,(0.025)} = 0.18676 \text{ m}$$

$$I = (2)\left(\frac{1}{12}\right)(0.025)(0.25^3) + 2\,(0.025)(0.25)(0.18676 - 0.125)^2$$

$$+ \frac{1}{12}(0.025)(0.35)^3 + (0.025)(0.35)(0.275 - 0.18676)^2$$

$$= 0.270236\,(10^{-3})\text{ m}^4$$



$$Q = \bar{y}'A' = 0.06176(0.025)(0.25) = 0.386(10^{-3})\text{ m}^3$$

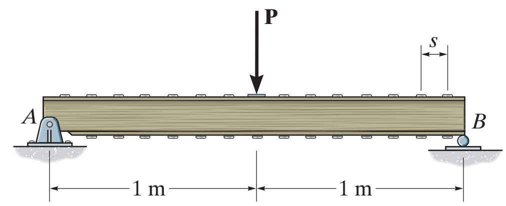
$$q = \frac{VQ}{I} = \frac{35(0.386)(10^{-3})}{0.270236(10^{-3})} = 49.997 \text{ kN/m}$$

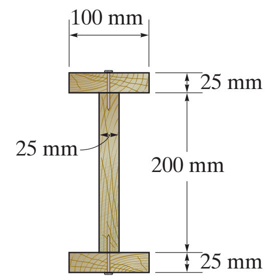
$F = q(s) = 49.997\,(0.25) = 12.5$ kN **Ans.**

Ans:
$F = 12.5$ kN

7–42.

The simply supported beam is built up from three boards by nailing them together as shown. The wood has an allowable shear stress of $\tau_{\text{allow}} = 1.5$ MPa, and an allowable bending stress of $\sigma_{\text{allow}} = 9$ MPa. The nails are spaced at $s = 75$ mm, and each has a shear strength of 1.5 kN. Determine the maximum allowable force **P** that can be applied to the beam.





SOLUTION

Support Reactions: As shown on the free-body diagram of the beam shown in Fig. *a*.

Maximum Shear and Moment: The shear diagram is shown in Fig. *b*. As indicated, $V_{\max} = \dfrac{P}{2}$.

Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.1)(0.25^3) - \frac{1}{12}(0.075)(0.2^3)$$

$$= 80.2083(10^{-6})\text{ m}^4$$

Referring to Fig. *d*,

$$Q_B = \bar{y}_2' A_2' = 0.1125(0.025)(0.1) = 0.28125(10^{-3})\text{ m}^3$$

Shear Flow: Since there is only one row of nails, $q_{\text{allow}} = \dfrac{F}{s} = \dfrac{1.5(10^3)}{0.075} = 20(10^3)\text{ N/m}$.

$$q_{\text{allow}} = \frac{V_{\max} Q_B}{I}; \qquad 20(10^3) = \frac{\frac{P}{2}\left[0.28125(10^{-3})\right]}{80.2083(10^{-6})}$$

$$P = 11417.41\text{ N} = 11.4\text{ kN (controls)} \qquad \textbf{Ans.}$$

Bending,

$$\sigma_{\max} = \frac{Mc}{I}$$

$$\sigma(10^6)\text{ N/m}^2 = \frac{\left(\frac{P}{2}\right)(0.125\text{ m})}{80.2083(10^{-6})\text{ m}^4}$$

$$P = 11.550\text{ N} = 11.55\text{ kN}$$

Shear,

$$\tau_{\max} = \frac{VQ}{It}$$

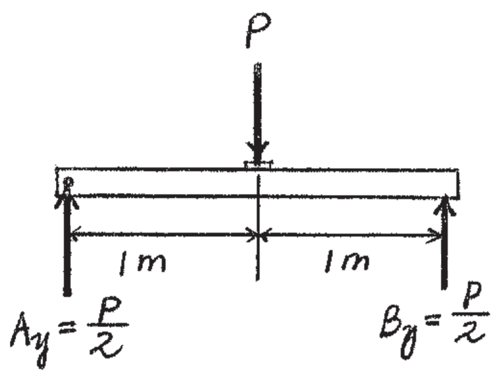
$$Q = (0.1125)(0.025)(0.1) + (0.05)(0.1)(0.025)$$

$$= 0.40625(10^{-3})\text{ m}^3$$

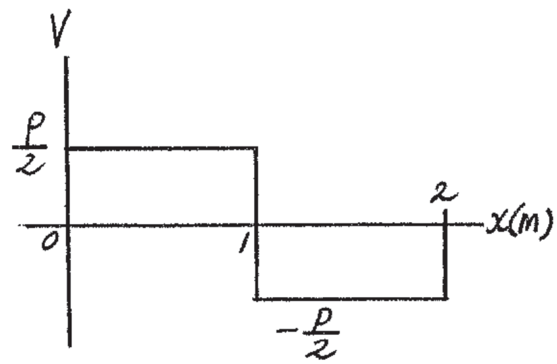
$$1.5\left(10^6\right) = \frac{\left(\frac{P}{2}\right)(0.40625)(10^{-3})\text{ m}^3}{80.2083(10^{-6})\text{ m}^4(0.025\text{ m})}$$

$$P = 14.808\text{ N} = 14.8\text{ kN}$$

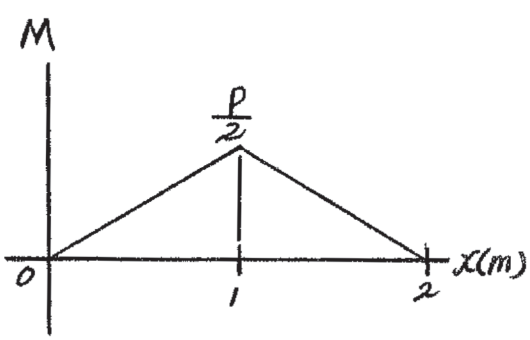
7–42. Continued



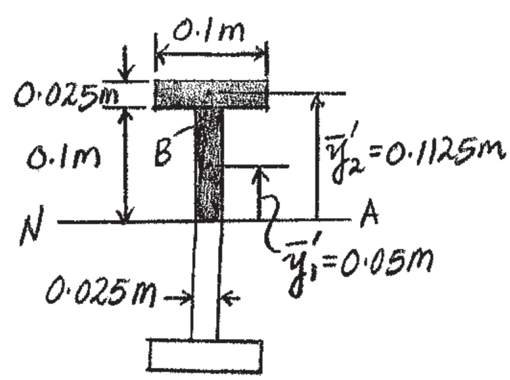
(a)



(b)



(c)

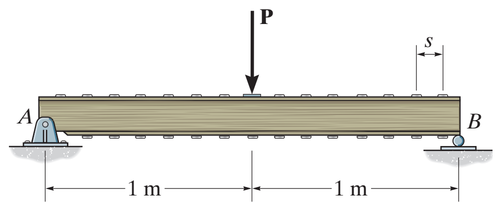


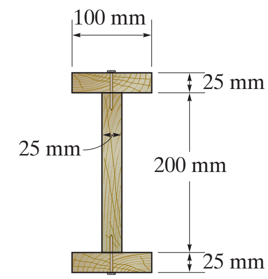
(d)

Ans:
$P = 11.4$ kN

7–43.

The simply supported beam is built up from three boards by nailing them together as shown. If $P = 12$ kN, determine the maximum allowable spacing s of the nails to support that load, if each nail can resist a shear force of 1.5 kN.





SOLUTION

Support Reactions: As shown on the free-body diagram of the beam shown in Fig. a.

Maximum Shear and Moment: The shear diagram is shown in Fig. b. As indicated,

$$V_{\max} = \frac{P}{2} = \frac{12}{2} = 6 \text{ kN}.$$

Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.1)(0.25^3) - \frac{1}{12}(0.075)(0.2^3)$$
$$= 80.2083(10^{-6}) \text{ m}^4$$

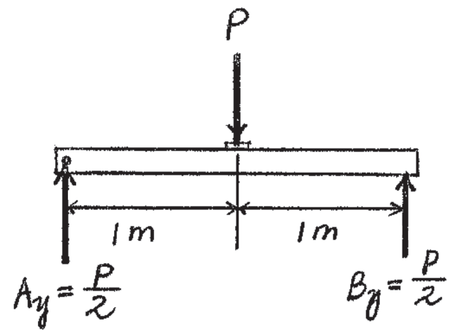
Referring to Fig. d,

$$Q_B = \bar{y}'_2 A'_2 = 0.1125(0.025)(0.1) = 0.28125(10^{-3}) \text{ m}^3$$

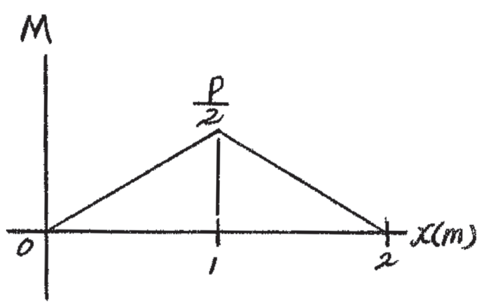
Shear Flow: Since there is only one row of nails, $q_{\text{allow}} = \dfrac{F}{s} = \dfrac{1.5(10^31)}{s}$.

$$q_{\text{allow}} = \frac{V_{\max} Q_B}{I}; \qquad \frac{1.5(10^31)}{s} = \frac{6000\left[0.28125(10^{-3})\right]}{80.2083(10^{-6}1)}$$

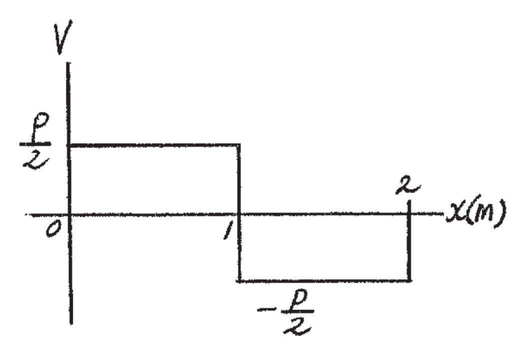
$$s = 0.07130 \text{ m} = 71.3 \text{ mm} \qquad \textbf{Ans.}$$



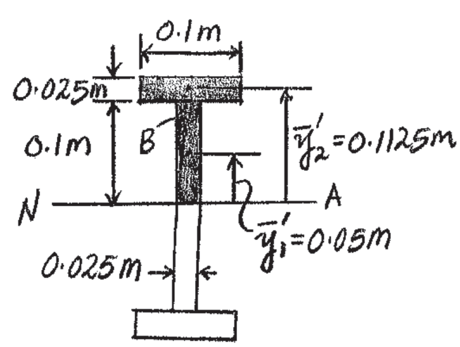
(a)



(c)



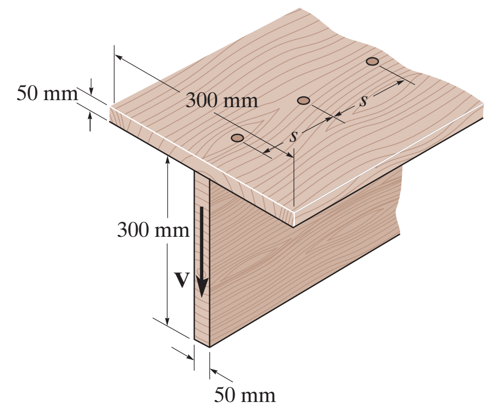
(b)



(d)

Ans:
$s = 71.3$ mm

***7–44.** The T-beam is nailed together as shown. If the nails can each support a shear force of 4.5 kN, determine the maximum shear force V that the beam can support and the corresponding maximum nail spacing s to the nearest multiples of 5 mm. The allowable shear stress for the wood is $\tau_{allow} = 3$ MPa.



SOLUTION

The neutral axis passes through the centroid c of the cross-section as shown in Fig. a.

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.325(0.3)(0.05) + 0.15(0.05)(0.3)}{0.3(0.05) + 0.05(0.3)} = 0.2375 \text{ m}$$

$$I = \frac{1}{12}(0.05)(0.3^3) + 0.05(0.3)(0.2375 - 0.15)^2 + \frac{1}{12}(0.3)(0.05^3) + 0.3(0.05)(0.325 - 0.2375)^2 = 0.3453125(10^{-3}) \text{ m}^4$$

Refering to Fig. a, Q_{max} and Q_A are

$$Q_{max} = \bar{y}_1' A_1' = 0.11875(0.05)(0.2375) = 1.41016(10^{-3}) \text{ m}^3$$

$$Q_A = \bar{y}_2' A_2' = 0.0875(0.3)(0.05) = 1.3125(10^{-3}) \text{ m}^3$$

The maximum shear stress occurs at the points on the neutral axis where Q is maximum and $t = 0.05$ m.

$$\tau_{allow} = \frac{VQ_{max}}{It}; \qquad 3(10^6) = \frac{V[1.41016(10^{-3})]}{[0.3453125(10^{-3})](0.05)}$$

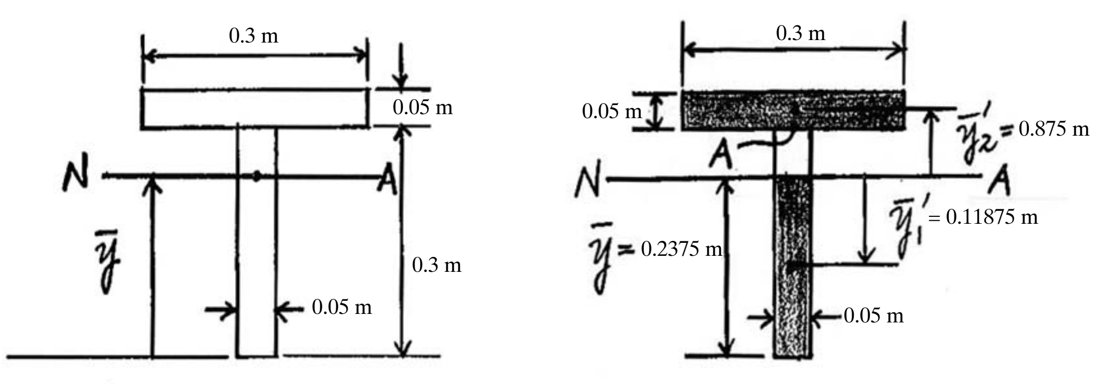
$$V = 36.73(10^3) \text{ N} = 36.7 \text{ kN} \qquad \textbf{Ans.}$$

Here, $q_{allow} = \frac{F}{s} = \left[\frac{4.5(10^3)}{s}\right]$ N/m. Then

$$q_{allow} = \frac{VQ_A}{I}; \qquad \left[\frac{4.5(10^3)}{s}\right] = \frac{[36.73(10^3)][1.3125(10^{-3})]}{0.3453125(10^{-3})}$$

$$s = 0.03223 \text{ m} = 32.23 \text{ mm} \qquad \textbf{Ans.}$$

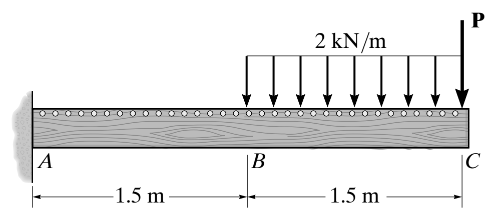
Use $s = 35$ mm

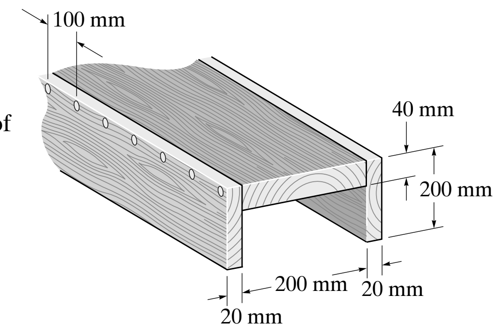


(a)

Ans.
$V = 36.7$ kN, $s = 32.23$ mm

7–45. The nails are on both sides of the beam and each can resist a shear of 2 kN. In addition to the distributed loading, determine the maximum load P that can be applied to the end of the beam. The nails are spaced 100 mm apart and the allowable shear stress for the wood is $\tau_{allow} = 3$ MPa.





SOLUTION

The FBD is shown in Fig. *a*.

As indicated the shear diagram, Fig. *b*, the maximum shear occurs in region AB of Constant value, $V_{max} = (P + 3)$ kN.

The neutral axis passes through Centroid C of the cross-section as shown in Fig. *c*.

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.18(0.04)(0.2) + 0.1(0.2)(0.04)}{0.04(0.2) + 0.2(0.04)}$$
$$= 0.14 \text{ m}$$

$$I = \frac{1}{12}(0.04)(0.2^3) + 0.04(0.2)(0.14 - 0.1)^2$$
$$+ \frac{1}{12}(0.2)(0.04^3) + 0.2(0.04)(0.18 - 0.14^2)$$
$$= 53.333(10^{-6}) \text{ m}^4$$

Refering to Fig. *d*,

$$Q_{max} = \bar{y}_1' A_1' = 0.07(0.14)(0.04) = 0.392(10^{-3}) \text{ m}^3$$

$$Q_A = \bar{y}_2' A_2' = 0.04(0.04)(0.2) = 0.32(10^{-3}) \text{ m}^3$$

The maximum shear stress occurs at the points on Neutral axis where Q is maximum and $t = 0.04$ m.

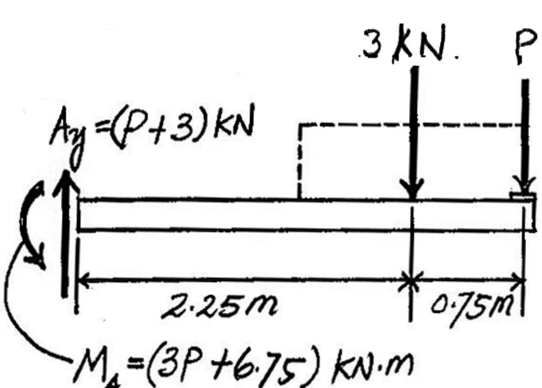
$$\tau_{allow} = \frac{V_{max} Q_{max}}{It}; \qquad 3(10^6) = \frac{(P + 3)(10^3)\left[0.392(10^{-3})\right]}{53.333(10^{-6})(0.04)}$$

$$P = 13.33 \text{ kN}$$

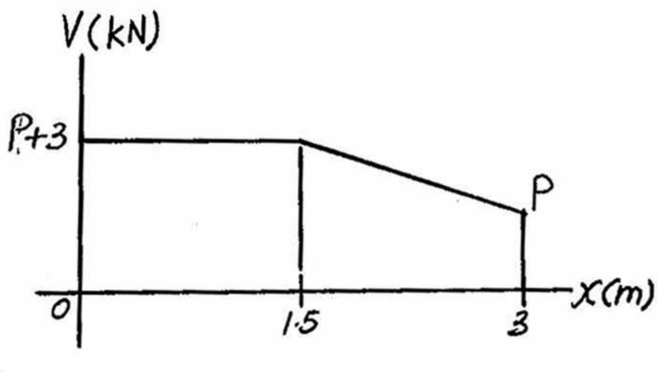
Since there are two rows of nails $q_{allow} = 2\left(\frac{F}{s}\right) = 2\left[\frac{2(10^3)}{0.1}\right] = 40\,000$ N/m.

$$q_{allow} = \frac{V_{max} Q_A}{I}; \qquad 40\,000 = \frac{(P + 3)(10^3)\left[0.32(10^{-3})\right]}{53.333(10^{-6})}$$

$$P = 3.67 \text{ kN (Controls!)}$$ **Ans.**

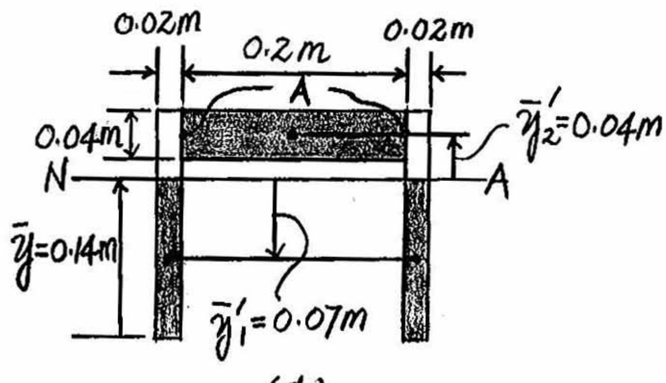


(a)

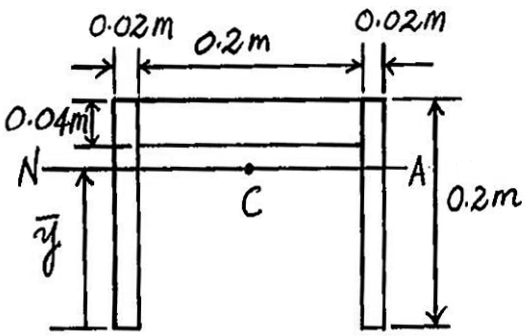


(b)

7–45. Continued



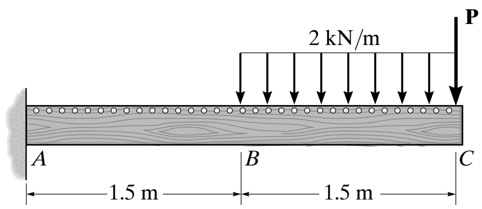
(d)

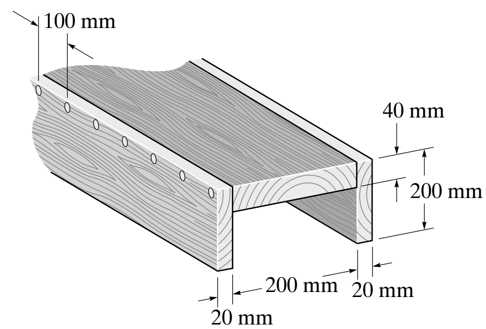


(c)

Ans.
$P = 3.67 \text{ kN}$

7–46. Determine the average shear stress developed in the nails within region AB of the beam. The nails are located on each side of the beam and are spaced 100 mm apart. Each nail has a diameter of 4 mm. Take $P = 2$ kN.





The FBD is shown in Fig. *a*.

As indicated in Fig. *b*, the internal shear force on the cross-section within region AB is constant that is $V_{AB} = 5$ kN.

The neutral axis passes through centroid C of the cross section as shown in Fig. *c*.

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.18(0.04)(0.2) + 0.1(0.2)(0.04)}{0.04(0.2) + 0.2(0.04)}$$

$$= 0.14 \text{ m}$$

$$I = \frac{1}{12}(0.04)(0.2^3) + 0.04(0.2)(0.14 - 0.1)^2$$

$$+ \frac{1}{12}(0.2)(0.04^3) + 0.2(0.04)(0.18 - 0.14)^2$$

$$= 53.333(10^{-6}) \text{ m}^4$$

Q for the shaded area shown in Fig. *d* is

$$Q = \bar{y}'A' = 0.04(0.04)(0.2) = 0.32(10^{-3}) \text{ m}^3$$

Since there are two rows of nail, $q = 2\left(\frac{F}{s}\right) = 2\left(\frac{F}{0.1}\right) = 20F$ N/m.

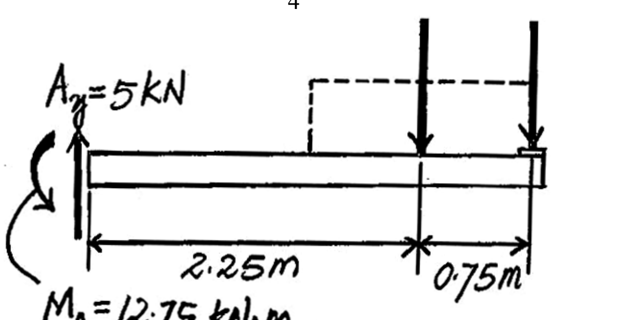
$$q = \frac{V_{AB}Q}{I}; \qquad 20F = \frac{5(10^3)\left[0.32(10^{-3})\right]}{53.333(10^{-6})}$$

$$F = 1500 \text{ N}$$

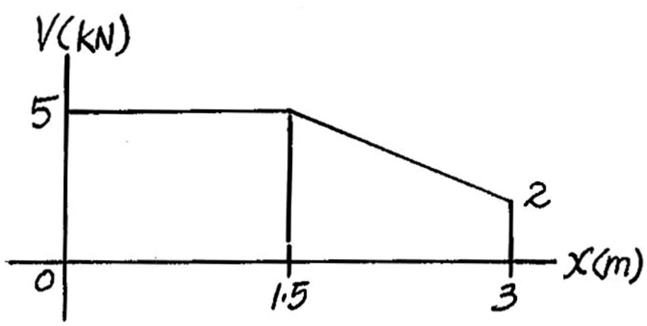
Thus, the average shear stress developed in each nail is

$$\left(\tau_{\text{nail}}\right)_{\text{avg}} = \frac{F}{A_{\text{nail}}} = \frac{1500}{\frac{\pi}{4}(0.004^2)} = 119.37(10^6)\text{Pa} = 119 \text{ MPa}$$

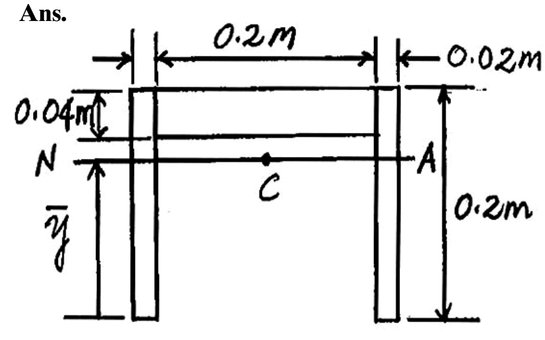
Ans.



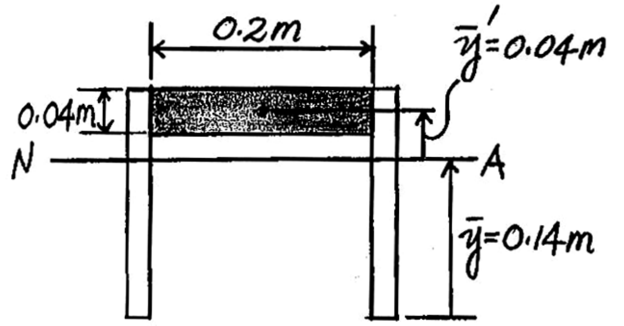
(a)



(b)

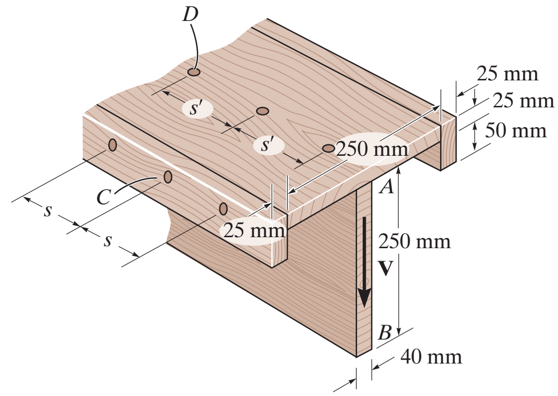


(c)



(d)

7–47. The beam is made from four boards nailed together as shown. If the nails can each support a shear force of 500 N, determine their required spacing s and s' if the beam is subjected to a shear of $V = 3.5$ kN.



SOLUTION

Section Properties:

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{0.0125(0.25)(0.025) + 0.0375(0.05)(0.75) + 0.15(0.04)(0.25)}{0.25(0.025) + 0.05(0.075) + 0.04(0.25)}$$

$$= 0.0859375 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.25)(0.025^3) + 0.25(0.025)(0.0859375 - 0.0125)^2$$

$$+\frac{1}{12}(0.05)(0.075^3) + 0.05(0.075)(0.0859375 - 0.0375)^2$$

$$+\frac{1}{12}(0.04)(0.25^3) + 0.04(0.25)(0.15 - 0.0859375)^2$$

$$= 0.137712(10^{-3}) \text{ m}^4$$

$$Q_C = \bar{y}_1'A' = (0.0859375 - 0.0375)(0.25)(0.075) = 90.8203(10^{-6}) \text{ m}^3$$

$$Q_D = \bar{y}_2'A' = (0.0859375 - 0.0125)(0.25)(0.025) + [(0.0859375 - 0.0375)](0.25)(0.075) = 0.640625(10^{-3}) \text{ m}^3$$

Shear Flow: The allowable shear flow at points C and D is $q_C = \frac{500}{s}$ and $q_B = \frac{500}{s'}$, respectively.

$$q_C = \frac{VQ_C}{I}$$

$$\frac{500}{s} = \frac{[3.5(10^3)][90.8203(10^{-6})]}{0.137712(10^{-3})}$$

$$s = 0.2166 \text{ m} = 216 \text{ mm}$$ **Ans.**

$$q_D = \frac{VQ_D}{I}$$

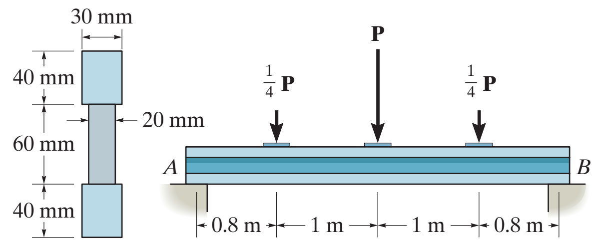
$$\frac{500}{s'} = \frac{[3.5(10^3)][0.640625(10^{-3})]}{0.137712(10^{-3})}$$

$$s' = 0.03071 \text{ m} = 30 \text{ mm}$$ **Ans.**

Ans.
$s = 216$ mm, $s' = 30$ mm

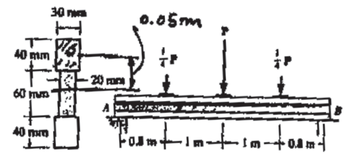
***7–48.**

The beam is made from three polystyrene strips that are glued together as shown. If the glue has a shear strength of 80 kPa, determine the maximum load P that can be applied without causing the glue to lose its bond.



SOLUTION

Maximum shear is at supports.



$$V_{\max} = \frac{3P}{4}$$

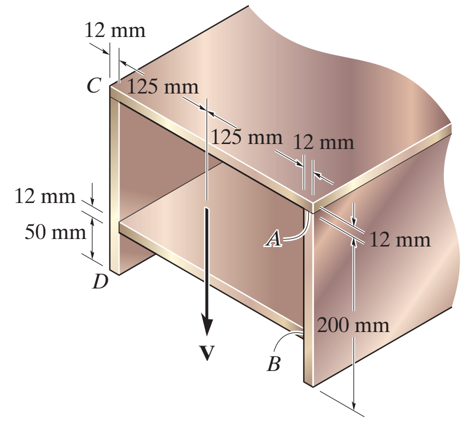
$$I = \frac{1}{12}(0.02)(0.06)^3 + 2\left[\frac{1}{12}(0.03)(0.04)^3 + (0.03)(0.04)(0.05)^2\right] = 6.68(10^{-6})\ \text{m}^4$$

$$\tau = \frac{VQ}{It}; \qquad 80(10^3) = \frac{(3P/4)(0.05)(0.04)(0.03)}{6.68(10^{-6})(0.02)}$$

$$P = 238\ \text{N}$$ **Ans.**

Ans:
$P = 238$ N

7–50. The beam is subjected to a shear force of $V = 25$ kN. Determine the shear flow at points A and B.

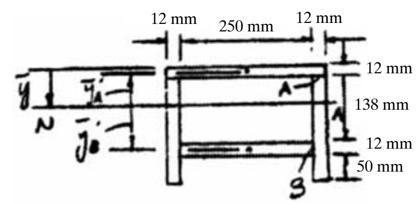


SOLUTION

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.006(0.274)(0.012) + 2[0.112(0.012)(0.2)] + 0.156(0.25)(0.012)}{0.274(0.012) + 2(0.012)(0.2) + 0.25(0.012)}$$

$$= 0.092472 \text{ m}$$

$$I = \frac{1}{12}(0.274)(0.012^3) + 0.274(0.012)(0.092472 - 0.006)^2$$
$$+ 2\left[\frac{1}{12}(0.012)(0.2^3) + (0.012)(0.2)(0.112 - 0.092472)^2\right]$$
$$+ \frac{1}{12}(0.25)(0.012^3) + 0.25(0.012)(0.156 - 0.092472)^2$$
$$= 54.5990(10^{-6}) \text{ m}^4$$



$$\bar{y}'_A = 0.092472 - 0.006 = 0.086472 \text{ m}$$

$$\bar{y}'_B = 0.156 - 0.092472 = 0.063528 \text{ m}$$

$$Q_A = \bar{y}'_A A' = 0.086472(0.274)(0.012) = 0.28432(10^{-3})$$

$$Q_B = \bar{y}'_B A' = 0.063528(0.25)(0.012) = 0.19058(10^{-3})$$

$$q_A = \frac{1}{2}\left(\frac{VQ_A}{I}\right) = \frac{1}{2}\left\{\frac{[25(10^3)][0.28432(10^{-3})]}{54.5990(10^{-6})}\right\} = 65.09(10^3) \text{ N/m}$$

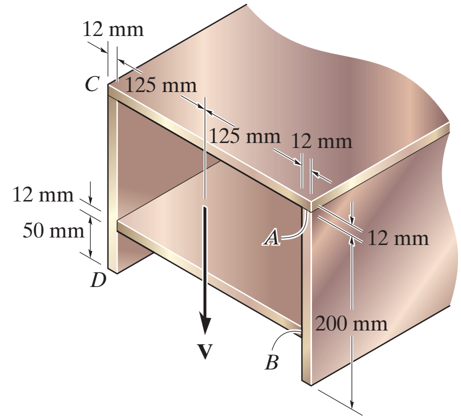
$$= 65.1 \text{ kN/m}$$ **Ans.**

$$q_B = \frac{1}{2}\left(\frac{VQ_B}{I}\right) = \frac{1}{2}\left\{\frac{[25(10^3)][0.19058(10^{-3})]}{54.5990(10^{-6})}\right\} = 43.63(10^3) \text{ N/m}$$

$$= 43.6 \text{ kN/m}$$ **Ans.**

Ans.
$q_A = 65.1$ kN/m, $q_B = 43.6$ kN/m

7–51. The beam is constructed from four plates and is subjected to a shear force of $V = 25$ kN. Determine the maximum shear flow in the cross section.

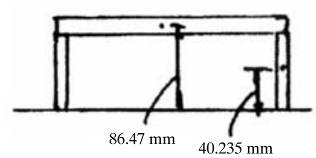


SOLUTION

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.006(0.274)(0.012) + 2[0.112(0.012)(0.2)] + 0.156(0.25)(0.012)}{0.274(0.012) + 2(0.012)(0.2) + 0.25(0.012)}$$

$$= 0.092472 \text{ m}$$

$$I = \frac{1}{12}(0.274)(0.012^3) + 0.274(0.012)(0.092472 - 0.006)^2$$
$$+ 2\left[\frac{1}{12}(0.012)(0.2^3) + (0.012)(0.2)(0.112 - 0.092472)^2\right]$$
$$+ \frac{1}{12}(0.25)(0.012^3) + 0.25(0.012)(0.156 - 0.092472)^2$$
$$= 54.5990(10^{-6}) \text{ m}^4$$



$$Q_{max} = 0.086472(0.274)(0.012) + 0.040236(0.012)(0.086472) = 0.36203(10^{-3}) \text{ m}^3$$

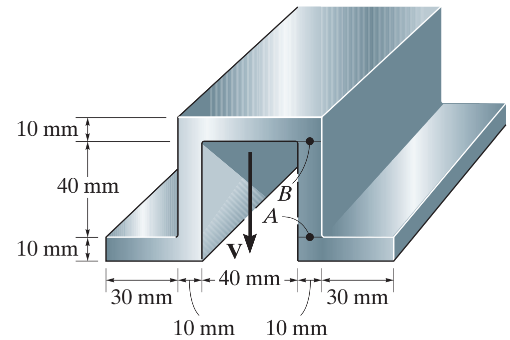
$$q_{max} = \frac{1}{2}\left(\frac{VQ_{max}}{I}\right) = \frac{1}{2}\left\{\frac{[25(10^3)][0.36203(10^{-3})]}{54.5990(10^{-6})}\right\} = 82.88(10^3) \text{ N/m}$$

$$= 82.9 \text{ kN/m}$$ **Ans.**

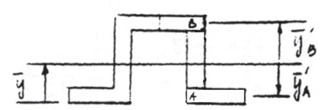
Ans.
$\bar{y} = 0.092472$ m, $I = 54.5990(10^{-6})$ m^4,
$q_{max} = 82.9$ kN/m

***7–52.**

The aluminum strut is 10 mm thick and has the cross section shown. If it is subjected to a shear of $V = 150$ N, determine the shear flow at points A and B.



SOLUTION



$$\bar{y} = \frac{2[0.005(0.03)(0.01)] + 2[0.03(0.06)(0.01)] + 0.055(0.04)(0.01)}{2(0.03)(0.01) + 2(0.06)(0.01) + 0.04(0.01)} = 0.027727 \text{ m}$$

$$I = 2\left[\frac{1}{12}(0.03)(0.01)^3 + 0.03(0.01)(0.027727 - 0.005)^2\right]$$

$$+ 2\left[\frac{1}{12}(0.01)(0.06)^3 + 0.01(0.06)(0.03 - 0.027727)^2\right]$$

$$+ \frac{1}{12}(0.04)(0.01)^3 + 0.04(0.01)(0.055 - 0.027727)^2 = 0.98197(10^{-6}) \text{ m}^4$$

$$\bar{y}_B' = 0.055 - 0.027727 = 0.027272 \text{ m}$$

$$\bar{y}_A' = 0.027727 - 0.005 = 0.022727 \text{ m}$$

$$Q_A = \bar{y}_A'A' = 0.022727(0.04)(0.01) = 9.0909(10^{-6}) \text{ m}^3$$

$$Q_B = \bar{y}_B'A' = 0.027272(0.03)(0.01) = 8.1818(10^{-6}) \text{ m}^3$$

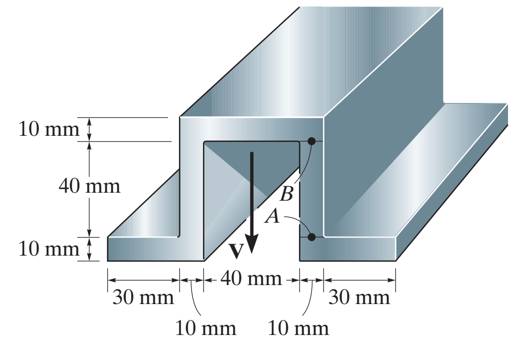
$$q_A = \frac{VQ_A}{I} = \frac{150(9.0909)(10^{-6})}{0.98197(10^{-6})} = 1.39 \text{ kN/m}$$ **Ans.**

$$q_B = \frac{VQ_B}{I} = \frac{150(8.1818)(10^{-6})}{0.98197(10^{-6})} = 1.25 \text{ kN/m}$$ **Ans.**

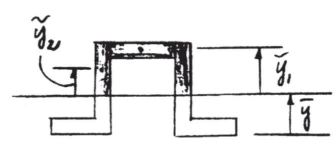
Ans:
$q_A = 1.39$ kN/m, $q_B = 1.25$ kN/m

7–53.

The aluminum strut is 10 mm thick and has the cross section shown. If it is subjected to a shear of $V = 150$ N, determine the maximum shear flow in the strut.



SOLUTION



$$\bar{y} = \frac{2[0.005(0.03)(0.01)] + 2[0.03(0.06)(0.01)] + 0.055(0.04)(0.01)}{2(0.03)(0.01) + 2(0.06)(0.01) + 0.04(0.01)}$$

$$= 0.027727 \text{ m}$$

$$I = 2\left[\frac{1}{12}(0.03)(0.01)^3 + 0.03(0.01)(0.027727 - 0.005)^2\right]$$

$$+ 2\left[\frac{1}{12}(0.01)(0.06)^3 + 0.01(0.06)(0.03 - 0.027727)^2\right]$$

$$+ \frac{1}{12}(0.04)(0.01)^3 + 0.04(0.01)(0.055 - 0.027727)^2$$

$$= 0.98197(10^{-6}) \text{ m}^4$$

$$Q_{\max} = (0.055 - 0.027727)(0.04)(0.01) + 2[(0.06 - 0.027727)(0.01)]\left(\frac{0.06 - 0.0277}{2}\right)$$

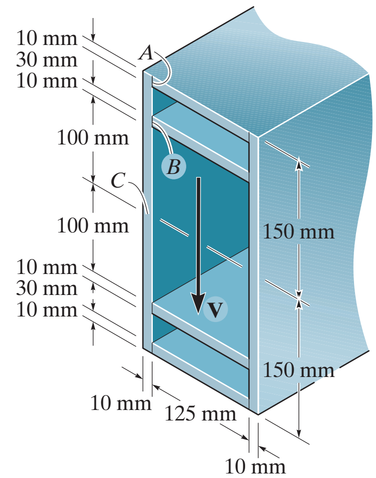
$$= 21.3(10^{-6}) \text{ m}^3$$

$$q_{\max} = \frac{1}{2}\left(\frac{VQ_{\max}}{I}\right) = \frac{1}{2}\left(\frac{150(21.3(10^{-6}))}{0.98197(10^{-6})}\right) = 1.63 \text{ kN/m}$$ **Ans.**

Ans:
$q_{\max} = 1.63$ kN/m

7–54.

A shear force of $V = 18$ kN is applied to the box girder. Determine the shear flow at points A and B.



SOLUTION

Section Properties:

$$I_{NA} = \frac{1}{12}(0.145)(0.3^3) - \frac{1}{12}(0.125)(0.28^3)$$

$$+ 2\left[\frac{1}{12}(0.125)(0.01^3) + 0.125(0.01)(0.105^2)\right]$$

$$= 125.17(10^{-6})\text{ m}^4$$

$$Q_A = \bar{y}'_2 A' = 0.145(0.125)(0.01) = 0.18125(10^{-3})\text{ m}^3$$

$$Q_B = \bar{y}'_1 A' = 0.105(0.125)(0.01) = 0.13125(10^{-3})\text{ m}^3$$

Shear Flow:

$$q_A = \frac{1}{2}\left[\frac{VQ_A}{I}\right]$$

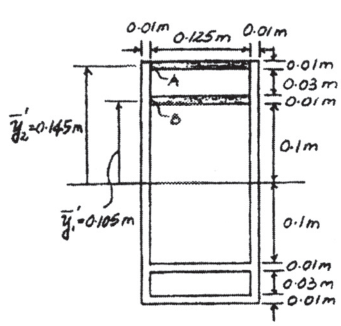
$$= \frac{1}{2}\left[\frac{18(10^3)(0.18125)(10^{-3})}{125.17(10^{-6})}\right]$$

$$= 13033\text{ N/m} = 13.0\text{ kN/m}$$ **Ans.**

$$q_B = \frac{1}{2}\left[\frac{VQ_B}{I}\right]$$

$$= \frac{1}{2}\left[\frac{18(10^3)(0.13125)(10^{-3})}{125.17(10^{-6})}\right]$$

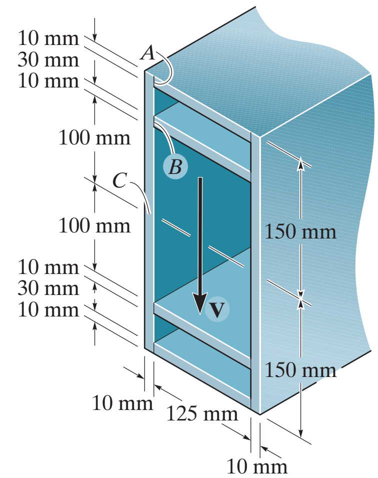
$$= 9437\text{ N/m} = 9.44\text{ kN/m}$$ **Ans.**



Ans:
$q_A = 13.0\text{ kN/m}$, $q_B = 9.44\text{ kN/m}$

7–55.

A shear force of $V = 18$ kN is applied to the box girder. Determine the shear flow at point C.



SOLUTION

Section Properties:

$$I_{NA} = \frac{1}{12}(0.145)(0.3^3) - \frac{1}{12}(0.125)(0.28^3)$$

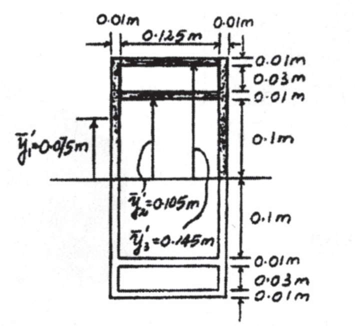
$$+2\left[\frac{1}{12}(0.125)(0.01^3) + 0.125(0.01)(0.105^2)\right]$$

$$= 125.17(10^{-6})\ \text{m}^4$$

$$Q_C = \Sigma \bar{y}'A'$$

$$= 0.145(0.125)(0.01) + 0.105(0.125)(0.01) + 0.075(0.15)(0.02)$$

$$= 0.5375(10^{-3})\ \text{m}^3$$



Shear Flow:

$$q_C = \frac{1}{2}\left[\frac{VQ_C}{I}\right]$$

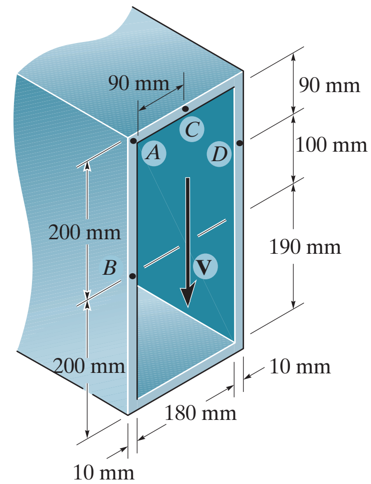
$$= \frac{1}{2}\left[\frac{18(10^3)(0.5375)(10^{-3})}{125.17(10^{-4})}\right]$$

$$= 38648\ \text{N/m} = 38.6\ \text{kN/m} \qquad \textbf{Ans.}$$

Ans:
$q_C = 38.6$ kN/m

***7–56.**

A shear force of $V = 300$ kN is applied to the box girder. Determine the shear flow at points A and B.



SOLUTION

The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.4^3) - \frac{1}{12}(0.18)(0.38^3) = 0.24359(10^{-3})\ \text{m}^4$$

Referring to Fig. a, Fig. b,

$$Q_A = \bar{y}'_1 A'_1 = 0.195(0.01)(0.19) = 0.3705(10^{-3})\ \text{m}^3$$

$$Q_B = 2\bar{y}'_2 A'_2 + \bar{y}'_3 A'_3 = 2[0.1(0.2)(0.01)] + 0.195(0.01)(0.18) = 0.751(10^{-3})\ \text{m}^3$$

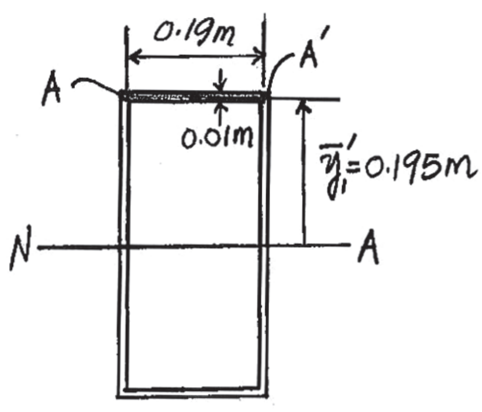
Due to symmetry, the shear flow at points A and A', Fig. a, and at points B and B', Fig. b, are the same. Thus

$$q_A = \frac{1}{2}\left(\frac{VQ_A}{I}\right) = \frac{1}{2}\left\{\frac{300(10^3)\left[0.3705(10^{-3})\right]}{0.24359(10^{-3})}\right\}$$

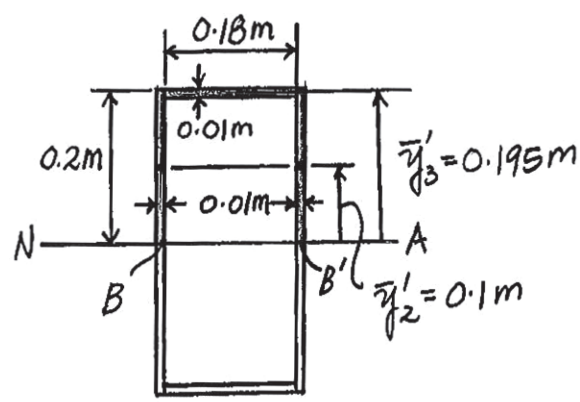
$$= 228.15(10^3)\ \text{N/m} = 228\ \text{kN/m}$$ **Ans.**

$$q_B = \frac{1}{2}\left(\frac{VQ_B}{I}\right) = \frac{1}{2}\left\{\frac{300(10^3)\left[0.751(10^{-3})\right]}{0.24359(10^{-3})}\right\}$$

$$= 462.46(10^3)\ \text{N/m} = 462\ \text{kN/m}$$ **Ans.**



(a)

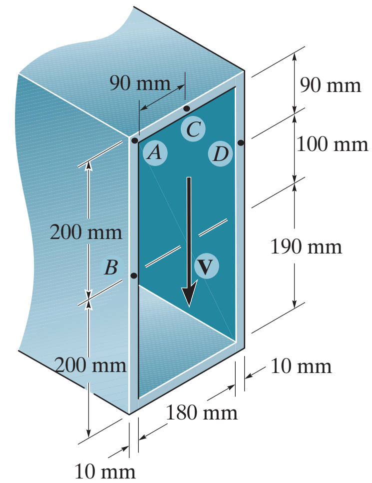


(b)

Ans:
$q_A = 228$ kN/m, $q_B = 462$ kN/m

7–57.

A shear force of $V = 450$ kN is applied to the box girder. Determine the shear flow at points C and D.



SOLUTION

The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.4^3) - \frac{1}{12}(0.18)(0.38^3) = 0.24359(10^{-3})\ \text{m}^4$$

Referring to Fig. a, due to symmetry $A'_C = 0$. Thus

$$Q_C = 0$$

Then referring to Fig. b,

$$Q_D = \bar{y}'_1 A'_1 + \bar{y}'_2 A'_2 = 0.195(0.01)(0.09) + 0.15(0.1)(0.01)$$

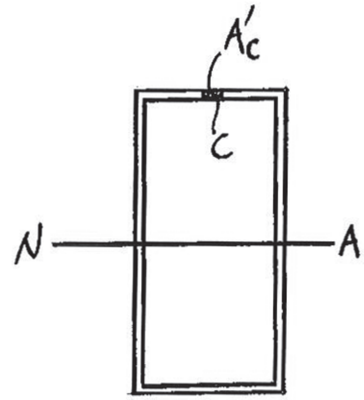
$$= 0.3255(10^{-3})\ \text{m}^3$$

Thus,

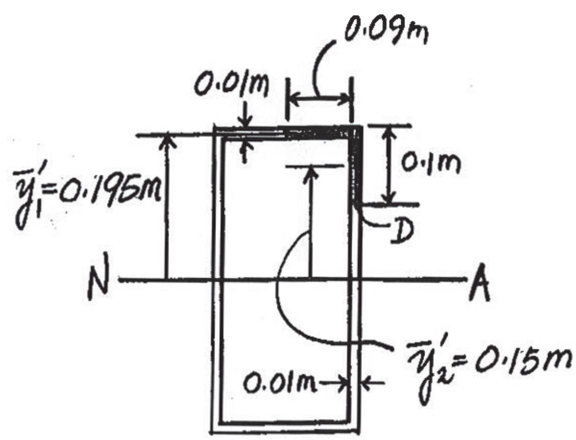
$$q_C = \frac{VQ_C}{I} = 0$$ **Ans.**

$$q_D = \frac{VQ_D}{I} = \frac{450(10^3)\left[0.3255(10^{-3})\right]}{0.24359(10^{-3})}$$

$$= 601.33(10^3)\ \text{N/m} = 601\ \text{kN/m}$$ **Ans.**



(a)

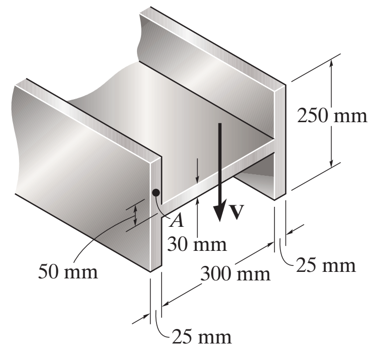


(b)

Ans:
$q_C = 0$, $q_D = 601$ kN/m

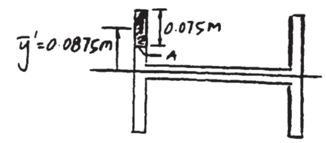
7–58.

The H-beam is subjected to a shear of $V = 80$ kN. Determine the shear flow at point A.



SOLUTION

$$I = 2\left[\frac{1}{12}(0.025)\left(0.25^3\right) + \frac{1}{12}(0.3)\left(0.03^3\right)\right] = 65.7792\left(10^{-6}\right)\ \text{m}^4$$



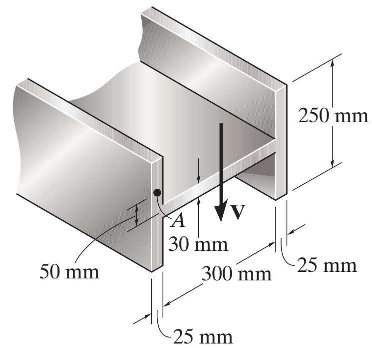
$$Q_A = \bar{y}'A' = 0.0875(0.075)(0.025) = 0.1641(10^{-3})\ \text{m}^3$$

$$q_A = \frac{VQ_A}{I} = \frac{80\left(10^3\right)(0.1641)\left(10^{-3}\right)}{65.7792\left(10^{-6}\right)} = 200\ \text{kN/m}$$ **Ans.**

Ans:
$q_A = 200$ kN/m

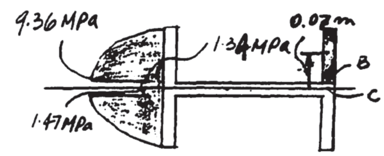
7–59.

The H-beam is subjected to a shear of $V = 80$ kN. Sketch the shear-stress distribution acting along one of its side segments. Indicate all peak values.



SOLUTION

$$I = 2\left[\frac{1}{12}(0.025)(0.25^3)\right] + \frac{1}{12}(0.3)(0.03^3) = 65.7792(10^{-6})\ \text{m}^4$$



$$Q_B = (0.070)(0.025)(0.110) = 0.1925(10^{-3})\ \text{m}^3$$

$$\tau_B = \frac{VQ}{It} = \frac{80(10^3)(0.1925)(10^{-3})}{65.7792(10^{-6})(0.025)} = 9.36\ \text{MPa}$$

$$\tau_B = \frac{VQ}{It} = \frac{80(10^3)\left[2(0.1925)(10^{-3})\right]}{65.7792(10^{-6})(0.35)} = 1.3378\ \text{MPa}$$

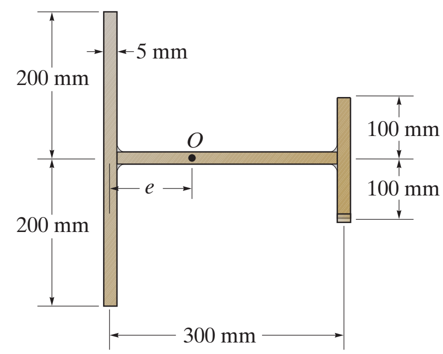
$$Q_{\max} = 2(0.07)(0.025)(0.110) + (0.0075)(0.35)(0.015) = 0.4244(10^{-3})\ \text{m}^3$$

$$\tau_C = \frac{VQ}{It} = \frac{80(10^3)(0.4244)(10^{-3})}{65.7792(10^{-6})(0.35)} = 1.47\ \text{MPa}$$

Ans:
$\tau_{\max} = 9.36$ MPa

***7–60.**

The built-up beam is formed by welding together the thin plates of thickness 5 mm. Determine the location of the shear center O.



SOLUTION

Shear Center: Referring to Fig. a and summing moments about point A, we have

$$\circlearrowleft + \Sigma(M_R)_A = \Sigma M_A; \qquad -Pe = -(F_w)_1(0.3)$$

$$e = \frac{0.3(F_w)_1}{P} \qquad (1)$$

Section Properties: The moment of inertia of the cross section about the axis of symmetry is

$$I = \frac{1}{12}(0.005)(0.4^3) + \frac{1}{12}(0.005)(0.2^3) = 30(10^{-6})\ \text{m}^4$$

Referring to Fig. b, $\bar{y}' = (0.1 - s) + \frac{s}{2} = (0.1 - 0.5s)$ m. Thus, Q as a function of s is

$$Q = \bar{y}'A' = (0.1 - 0.5s)(0.005s) = [0.5(10^{-3})s - 2.5(10^{-3})s^2]\ \text{m}^3$$

Shear Flow:

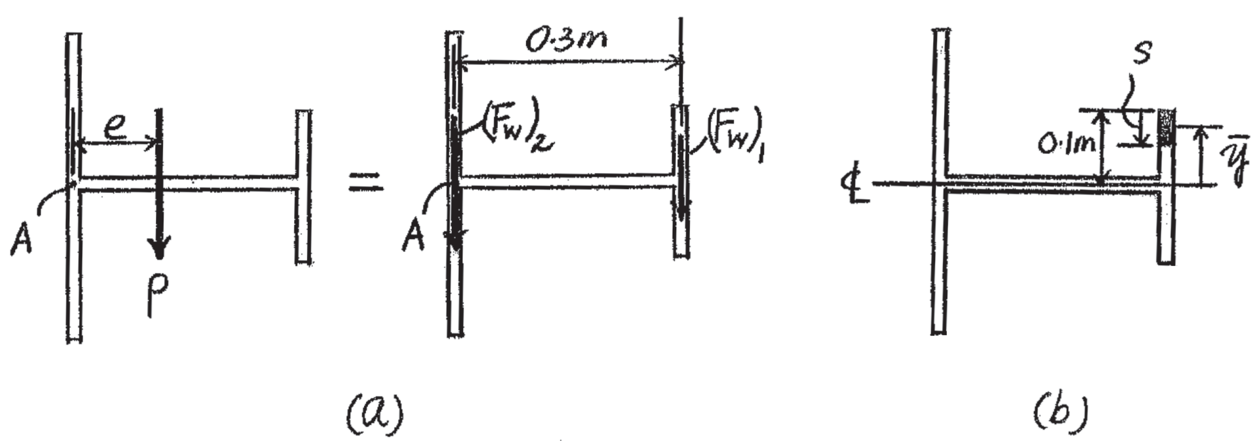
$$q = \frac{VQ}{I} = \frac{P[0.5(10^{-3})s - 2.5(10^{-3})s^2]}{30(10^{-6})} = P(16.6667s - 83.3333s^2)$$

Resultant Shear Force: The shear force resisted by the shorter web is

$$(F_w)_1 = 2\int_0^{0.1\text{ m}} q\,ds = 2\int_0^{0.1\text{ m}} P(16.6667s - 83.3333s^2)ds = 0.1111P$$

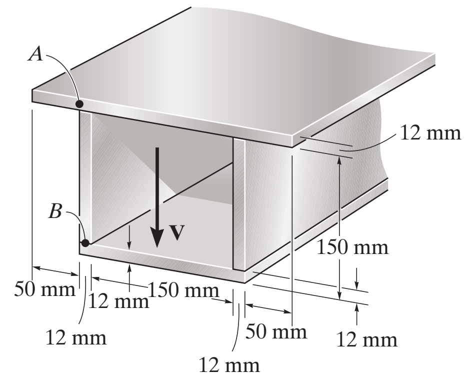
Substituting this result into Eq. (1),

$$e = 0.03333\text{ m} = 33.3\text{ m} \qquad \textbf{Ans.}$$



Ans:
$e = 33.3$ m

7–61. The assembly is subjected to a vertical shear of $V = 35$ kN. Determine the shear flow at points A and B and the maximum shear flow in the cross section.



SOLUTION

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.006(0.274)(0.012) + 2[0.081(0.012)(0.138)] + 0.156(0.174)(0.012)}{0.274(0.012) + 2(0.012)(0.138) + 0.174(0.012)}$$

$$= 0.070641 \text{ m}$$

$$I = \frac{1}{12}(0.274)(0.012^3) + 0.274(0.012)(0.070641 - 0.006)^2$$

$$+ 2\left[\frac{1}{12}(0.012)(0.138^3) + (0.012)(0.138)(0.081 - 0.070641)^2\right]$$

$$+ \frac{1}{12}(0.174)(0.012^3) + 0.174(0.012)(0.156 - 0.070641)^2$$

$$= 34.6283(10^{-6}) \text{ m}^4$$

$$Q_A = \bar{y}_1' A_1' = 0.06464(0.05)(0.012) = 0.28432(10^{-3}) \text{ m}^3$$

$$Q_B = \bar{y}_2' A_2' = 0.08536(0.174)(0.012) = 0.17823(10^{-3}) \text{ m}^3$$

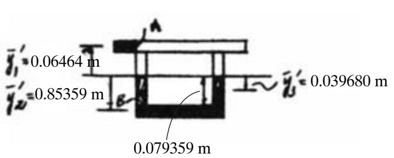
$$Q_{\max} = \Sigma \bar{y}' A' = 0.08536(0.174)(0.012) + 2[0.039680(0.012)(0.079359)] = 0.25380(10^{-3}) \text{ m}^3$$

$$q = \frac{VQ}{I}$$

$$q_A = \frac{[35(10^3)][38.7845(10^{-6})]}{34.6283(10^{-6})} = 39.20(10^3) \text{ N/m} = 39.2 \text{ kN/m}$$ **Ans.**

$$q_B = \frac{1}{2}\left\{\frac{[35(10^3)][0.17823(10^{-3})]}{34.6283(10^{-6})}\right\} = 90.07(10^3) \text{ N/m} = 90.1 \text{ kN/m}$$ **Ans.**

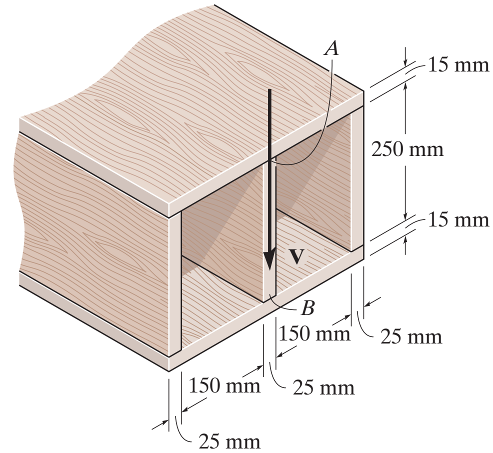
$$q_{\max} = \frac{1}{2}\left\{\frac{[35(10^3)][0.25380(10^{-3})]}{34.6283(10^{-6})}\right\} = 128.26(10^3) \text{ N/m} = 128 \text{ kN/m}$$ **Ans.**



Ans.
$\bar{y} = 0.070641$ m, $I = 34.6283(10^{-6})$ m^4,
$q_A = 39.2$ kN/m, $q_B = 90.1$ kN/m,
$q_{\max} = 128$ kN/m

7–62.

The box girder is subjected to a shear of $V = 15$ kN. Determine the shear flow at point B and the maximum shear flow in the girder's web AB.

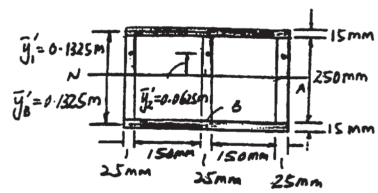


SOLUTION

$$I = \frac{1}{12}(0.375)(0.28^3) - \frac{1}{12}(0.3)(0.25^3) = 0.295375(10^{-3})\ \text{m}^4$$

$$Q_B = \bar{y}'_B A' = 0.1325(0.375)(0.015) = 0.7453125(10^{-3})\ \text{m}^3$$

$$Q_{max} = \Sigma\bar{y}'A' = 0.1325(0.375)(0.015) + 3[(0.0625)(0.125)(0.025)] = 1.33125(10^{-3})\ \text{m}^3$$



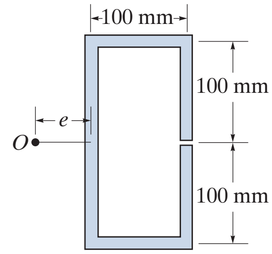
$$q_B = \frac{1}{3}\left[\frac{VQ_B}{I}\right] = \frac{1}{3}\left[\frac{15(10^3)(0.7453125)(10^{-3})}{0.295375(10^{-3})}\right] = 12.6\ \text{kN/m}$$ **Ans.**

$$q_{max} = \frac{1}{3}\left[\frac{VQ_{max}}{I}\right] = \frac{1}{3}\left[\frac{15(10^3)(1.33125)(10^{-3})}{0.295375(10^{-3})}\right] = 22.5\ \text{kN/m}$$ **Ans.**

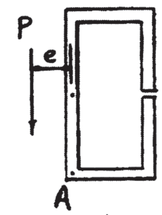
Ans:
$q_B = 12.6$ kN/m, $q_{max} = 22.5$ kN/m

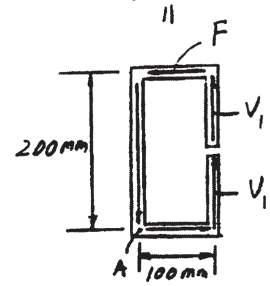
7–63.

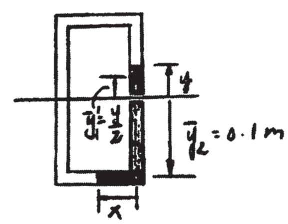
Determine the location e of the shear center, point O, for the thin-walled member having a slit along its section.



SOLUTION







Suming moments about A,

$$Pe = 2V_1(100) + F(200) \qquad (1)$$

$$I = 2\left[\frac{1}{12}t(0.2^3)\right] + 2\left[(0.1)(t)(0.1^2)\right] = 3.3333(10^{-3})t \text{ m}^4$$

$$Q_1 = \bar{y}_1'A' = \frac{y}{2}(y)t = 0.5y^2 t$$

$$Q_2 = \Sigma\bar{y}'A = 0.05(0.1)(t) + 0.1(x)(t) = 0.005t + 0.1xt$$

$$q_1 = \frac{VQ_1}{I} = \frac{P(0.5y^2 t)}{3.3333(10^{-3})t} = 150P\,y^2$$

$$q_2 = \frac{VQ_2}{I} = \frac{P(0.005t + 0.1xt)}{3.3333(10^{-3})t} = 300P(0.005 + 0.1x)$$

$$V_1 = \int_0^{0.1} q_1\,dy = 150P\int_0^{0.1} y^2 dy = 150P\left[\frac{y^3}{3}\right]_0^{0.1} = 0.05P$$

$$F = \int_0^{0.1} q_2\,dx = 300P\int_0^{0.1}(0.005 + 0.1x)\,dx$$

$$= 300P\left[0.005x + \frac{0.1x^2}{2}\right]\Bigg|_0^{0.1} = 0.3P$$

From Eq. (1);

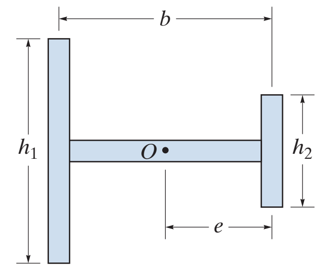
$$Pe = 2(0.05P)(100) + 0.3P(200)$$

$$e = 70 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$e = 70$ mm

***7–64.**

Determine the location e of the shear center, point O, for the thin-walled member. The member segments have the same thickness t.



SOLUTION

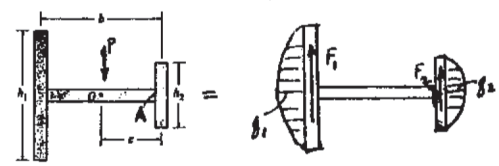
Summing moments about A,

$$eP = bF_1 \qquad (1)$$

$$I = \frac{1}{12}(t)(h_1)^3 + \frac{1}{12}(t)(h_2)^3 = \frac{1}{12}t\,(h_1^3 + h_2^3)$$

$$q_1 = \frac{P(h_1/2)(t)(h_1/4)}{I} = \frac{Ph_1^2 t}{8I}$$

$$F_1 = \frac{2}{3}q_1(h_1) = \frac{Ph_1^3 t}{12I}$$



From Eq. (1),

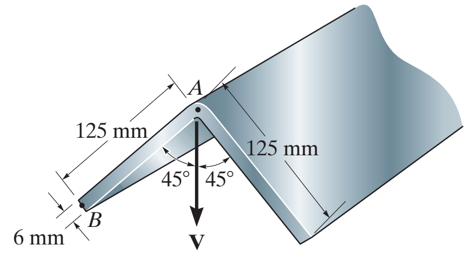
$$e = \frac{b}{P}\left(\frac{Ph_1^3 t}{12I}\right)$$

$$= \frac{h_1^3 b}{(h_1^3 + h_2^3)}$$

$$= \frac{b}{1 + (h_2/h_1)^3} \qquad \textbf{Ans.}$$

Ans:
$$e = \frac{b}{1 + (h_2/h_1)^3}$$

7–65. The angle is subjected to a shear of $V = 10$ kN. Sketch the distribution of shear flow along the leg AB. Indicate numerical values at all peaks.



SOLUTION

Section Properties:

$$b = \frac{0.006}{\sin 45°} = 0.00848528 \text{ m}$$

$$h = 0.125 \cos 45° = 0.08839 \text{ m}$$

$$I_{NA} = 2\left[\frac{1}{12}(0.00848528)(0.08839^3)\right] = 0.97656(10^{-6}) \text{ m}^4$$

$$Q = \bar{y}'A' = [0.25(0.08839) + 0.5y]\left(0.625 - \frac{y}{\sin 45°}\right)(0.006)$$

$$= 8.2864(10^{-6}) - 4.2426(10^{-3})y^2$$

Shear Flow:

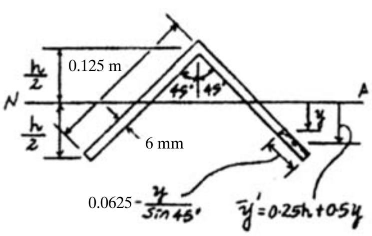
$$q = \frac{VQ}{I}$$

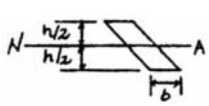
$$= \frac{[10(10^3)][8.2864(10^{-6}) - 4.2426(10^{-3})y^2]}{0.97656(10^{-6})}$$

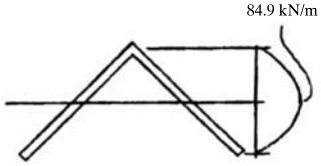
$$= [84.8528(10^3) - 43.4446(10^6)y^2] \text{ N/m}$$ **Ans.**

$$= [84.9 - 43.4y^2] \text{ kN/m}$$

At $y = 0$, $q = q_{\max} = 84.9$ kN/m **Ans.**

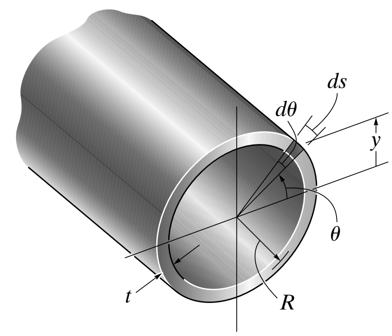






Ans.
At $y = 0, q = q_{\max} = 84.9$ kN/m

7–66. Determine the shear-stress variation over the cross section of the thin-walled tube as a function of elevation y and show that $\tau_{\max} = 2V/A$, where $A = 2\pi r t$. *Hint:* Choose a differential area element $dA = Rt\,d\theta$. Using $dQ = y\,dA$, formulate Q for a circular section from θ to $(\pi - \theta)$ and show that $Q = 2R^2 t\cos\theta$, where $\cos\theta = \sqrt{R^2 - y^2}/R$.



SOLUTION

$dA = R\,t\,d\theta$

$dQ = y\,dA = yR\,t\,d\theta$

Here $y = R\sin\theta$

Therefore $dQ = R^2\,t\sin\theta\,d\theta$

$$Q = \int_{\theta}^{\pi-\theta} R^2\,t\sin\theta\,d\theta = R^2\,t(-\cos\theta)\Big|_{\theta}^{\pi-\theta}$$

$$= R^2 t\,[-\cos(\pi - \theta) - (-\cos\theta)] = 2R^2 t\cos\theta$$

$dI = y^2\,dA = y^2\,R\,t\,d\theta = R^3\,t\sin^2\theta\,d\theta$

$$I = \int_0^{2\pi} R^3\,t\sin^2\theta\,d\theta = R^3\,t\int_0^{2\pi}\frac{(1 - \cos 2\theta)}{2}\,d\theta$$

$$= \frac{R^3\,t}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]\Big|_0^{2\pi} = \frac{R^3\,t}{2}[2\pi - 0] = \pi R^3\,t$$

$$\tau = \frac{VQ}{I\,t} = \frac{V(2R^2 t\cos\theta)}{\pi R^3\,t(2t)} = \frac{V\cos\theta}{\pi R\,t}$$

Here $\cos\theta = \dfrac{\sqrt{R^2 - y^2}}{R}$

$$\tau = \frac{V}{\pi R^2 t}\sqrt{R^2 - y^2}$$ **Ans.**

$\tau_{\max}$ occurs at $y = 0$; therefore

$$\tau_{\max} = \frac{V}{\pi R\,t}$$

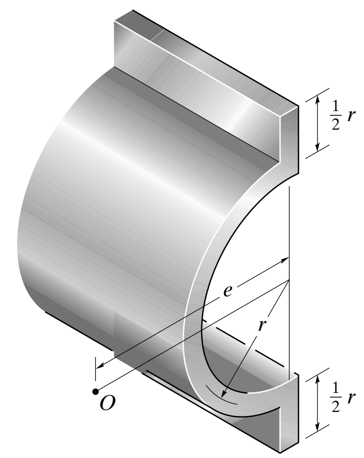
$A = 2\pi Rt$; therefore

$$\tau_{\max} = \frac{2V}{A}$$ **QED**

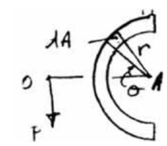
Ans.

$$\tau = \frac{V}{\pi R^2 t}\sqrt{R^2 - y^2}$$

7–67. Determine the location e of the shear center, point O, for the beam having the cross section shown. The thickness is t.



SOLUTION

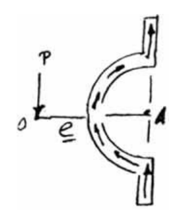


$$I = (2)\left[\frac{1}{12}(t)(r/2)^3 + (r/2)(t)\left(r + \frac{r}{4}\right)^2\right] + I_{\text{semi-circle}}$$

$$= 1.583333t\, r^3 + I_{\text{semi-circle}}$$

$$I_{\text{semi-circle}} = \int_{-\pi/2}^{\pi/2} (r \sin\theta)^2\, t\, r\, d\theta = t\, r^3 \int_{-\pi/2}^{\pi/2} \sin^2\theta\, d\theta$$

$$I_{\text{semi-circle}} = t\, r^3\left(\frac{\pi}{2}\right)$$



Thus,

$$I = 1.583333t\, r^3 + t\, r^3\left(\frac{\pi}{2}\right) = 3.15413t\, r^3$$

$$Q = \left(\frac{r}{2}\right)t\left(\frac{r}{4} + r\right) + \int_{\theta}^{\pi/2} r\sin\theta\,(t\, r\, d\theta)$$

$$Q = 0.625\, t\, r^2 + t\, r^2\cos\theta$$

$$q = \frac{VQ}{I} = \frac{P(0.625 + \cos\theta)t\, r^2}{3.15413\, t\, r^3}$$

Summing moments about A:

$$Pe = \int_{-\pi/2}^{\pi/2} (q\, r\, d\theta)r$$

$$Pe = \frac{Pr}{3.15413}\int_{-\pi/2}^{\pi/2} (0.625 + \cos\theta)d\theta$$

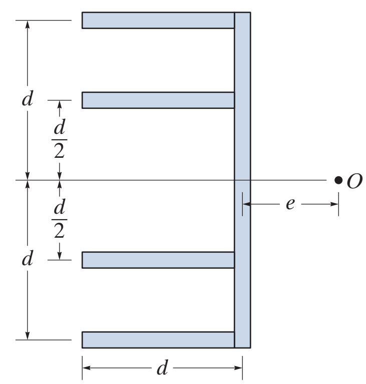
$$e = \frac{r\,(1.9634 + 2)}{3.15413}$$

$$e = 1.26\, r$$ **Ans.**

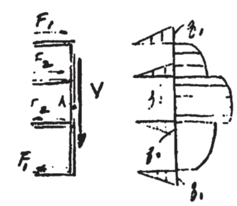
Ans.
$e = 1.26\, r$

***7–68.**

Determine the location e of the shear center, point O, for the thin-walled member. The member segments have the same thickness t.



SOLUTION



Summing moments about point A:

$$Pe = F_2 d + F_1(2d) \qquad \textbf{(1)}$$

$$I = 2[dt(d)^2] + 2[dt(d/2)^2] = \frac{1}{12}t(2d)^3 = \frac{19}{6}td^3$$

$$q_1 = \frac{P(dt)(d)}{\frac{19}{6}td^3} = \frac{6P}{19d}$$

$$F_1 = \frac{1}{2}\left(\frac{6P}{19d}\right)(d) = \frac{3}{19}P$$

$$q_2 = \frac{P(dt)(d/2)}{\frac{19}{6}td^3} = \frac{3P}{19d}$$

$$F_2 = \frac{1}{2}\left(\frac{3P}{19d}\right)d = \frac{1.5P}{19}$$

From Eq. (1):

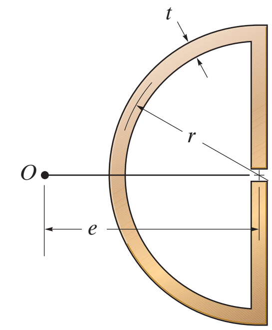
$$Pe = 2d\left(\frac{3}{19}P\right) + d\left(\frac{1.5P}{19}\right)$$

$$e = \frac{7.5}{19}d = \frac{15}{38}d \qquad \textbf{Ans.}$$

Ans:
$e = \dfrac{15}{38}d$

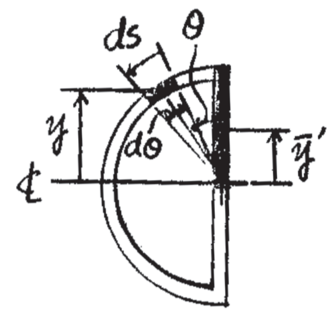
7–69.

A thin plate of thickness t is bent to form the beam having the cross section shown. Determine the location of the shear center O.



SOLUTION

Section Properties: For the arc segment, Fig. a, $y = r\cos\theta$ and the area of the differential element shown shaded is $dA = t\,ds = tr\,d\theta$. Then, the moment of inertia of the entire cross section about the axis of symmetry is



(a)

$$I = \frac{1}{12}(t)(2r)^3 + \int y^2\,dA$$

$$= \frac{2}{3}r^3t + \int_0^{\pi}(r\cos\theta)^2\,trd\theta$$

$$= \frac{2}{3}r^3t + \frac{r^3t}{2}\int_0^{\pi}(\cos 2\theta + 1)\,d\theta$$

$$= \frac{2}{3}r^3t + \frac{r^3t}{2}\left(\frac{1}{2}\sin 2\theta + \theta\right)\Big|_0^{\pi}$$

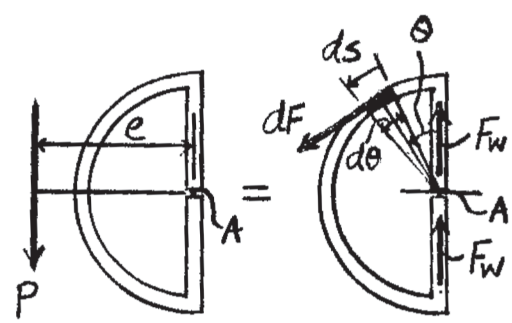
$$= \frac{r^3t}{6}(4 + 3\pi)$$

Referring to Fig. a, $\bar{y}' = \dfrac{r}{2}$. Thus, Q as a function of s is

$$Q = \bar{y}'A' + \int y dA = \frac{r}{2}(rt) + \int_0^{\theta} r\cos\theta(trd\theta)$$

$$= \frac{1}{2}r^2t + r^2t\int_0^{\theta}\cos\theta d\theta$$

$$= \frac{r^2t}{2}(1 + 2\sin\theta)$$



(b)

Shear Flow:

$$q = \frac{VQ}{I} = \frac{P\left[\frac{r^2t}{2}(1 + 2\sin\theta)\right]}{\frac{r^3t}{6}(4 + 3\pi)} = \frac{3P}{(4 + 3\pi)r}(1 + 2\sin\theta)$$

Resultant Shear Force: The shear force resisted by the arc segment is

$$F = \int q ds = \int_0^{\pi} qrd\theta = \int_0^{\pi}\frac{3P}{(4 + 3\pi)r}(1 + 2\sin\theta)rd\theta$$

$$= \frac{3P}{4 + 3\pi}(\theta - 2\cos\theta)\Big|_0^{\pi}$$

$$= \frac{3P(\pi + 4)}{4 + 3\pi}$$

Shear Center: Referring to Fig. b and summing the moments about point A,

$$\circlearrowleft + \Sigma(M_R)_A = \Sigma M_A; \qquad Pe = r\int dF$$

$$Pe = r\left[\frac{3P(\pi + 4)}{4 + 3\pi}\right]$$

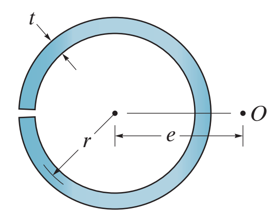
$$e = \left[\frac{3(\pi + 4)}{4 + 3\pi}\right]r \qquad \textbf{Ans.}$$

Ans:

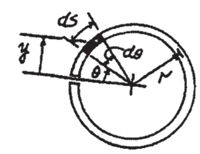
$$e = \left[\frac{3(\pi + 4)}{4 + 3\pi}\right]r$$

7–70.

Determine the location e of the shear center, point O, for the tube having a slit along its length.



SOLUTION



Section Properties:

$$dA = t\,ds = t\,r\,d\theta \qquad y = r\sin\theta$$

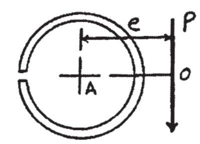
$$dI = y^2\,dA = r^2\sin^2\theta(t\,r\,d\theta) = r^3 t\sin^2\theta\,d\theta$$

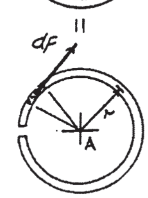
$$I = r^3 t\int_0^{2\pi}\sin^2\theta\,d\theta$$

$$= r^3 t\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta = \pi r^3 t$$

$$dQ = \bar{y}'A' = y\,dA = r\sin\theta(t\,r\,d\theta) = r^2\,t\sin\theta\,d\theta$$

$$Q = r^2\,t\int_0^{\theta}\sin\theta\,d\theta = r^2\,t\,(1-\cos\theta)$$





Shear Flow Resultant:

$$q = \frac{VQ}{I} = \frac{P\,r^2 t(1-\cos\theta)}{\pi r^3 t} = \frac{P}{\pi r}(1-\cos\theta)$$

$$F = \int_0^{2\pi} q\,ds = \int_0^{2\pi}\frac{P}{\pi r}(1-\cos\theta)\,r\,d\theta$$

$$= \frac{P}{\pi}\int_0^{2\pi}(1-\cos\theta)\,d\theta$$

$$= 2P$$

Shear Center: Summing moments about point A.

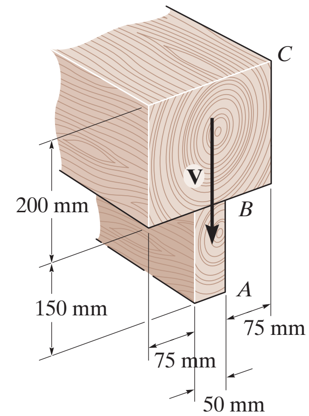
$$Pe = Fr$$

$$Pe = 2Pr$$

$$e = 2r \qquad \textbf{Ans.}$$

Ans:
$e = 2r$

R7–1. Sketch the intensity of the shear-stress distribution acting over the beam's cross-sectional area, and determine the resultant shear force acting on the segment AB. The shear acting at the section is $V = 175$ kN. Show that $I_{NA} = 0.3048(10^{-3})$ m^4.



SOLUTION

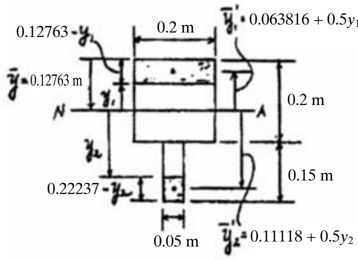
Section Properties:

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.1(0.2)(0.2) + 0.275(0.05)(0.15)]}{0.2(0.2) + 0.05(0.15)} = 0.12763 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.2)(0.2^3) + 0.2(0.2)(0.12763 - 0.1)^2$$

$$+\frac{1}{12}(0.05)(0.15^3) + (0.05)(0.15)(0.275 - 0.12763)^2$$

$$= 0.3408(10^{-3}) \text{ m}^4 \quad (Q.E.D)$$

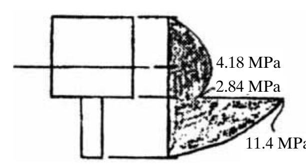


$$Q_1 = \bar{y}_1' A' = (0.063816 + 0.5y_1)(0.2)(0.12763 - y_1)$$

$$= 1.62898(10^{-3}) - 0.1y_1^2$$

$$Q_2 = \bar{y}_2' A' = (0.11118 + 0.5y_2)(0.05)(0.22237 - y_2)$$

$$= 1.23619(10^{-3}) - 0.25y_2^2$$



Shear Stress: Applying the shear formula $\tau = \dfrac{VQ}{It}$,

$$\tau_{CB} = \frac{VQ_1}{It} = \frac{[175(10^3)][1.62898(10^{-3}) - 0.1y_1^2]}{[0.3408(10^{-3})](0.2)}$$

$$= \left(4.18218 - 256.74y_1^2\right)(10^6)$$

At $y_1 = 0$, $\quad \tau_{CB} = 4.18$ MPa

At $y_1 = -0.07237$ m $\quad \tau_{CB} = 2.84$ MPa

$$\tau_{AB} = \frac{VQ_2}{It} = \frac{[175(10^3)][1.23619(10^{-3}) - 0.25y_2^2]}{[0.3408(10^{-3})](0.05)}$$

$$= \left(12.6950 - 256.74y_2^2\right)(10^6)$$

At $y_2 = 0.07237$ m $\quad \tau_{AB} = 11.4$ MPa

Resultant Shear Force: For segment AB.

$$V_{AB} = \int \tau_{AB}\, dA$$

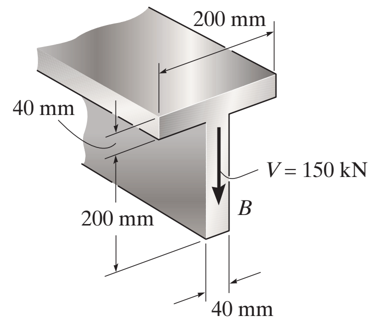
$$= \int_{0.07237 \text{ m}}^{0.22237 \text{ m}} \left[\left(12.6950 - 256.74y_2^2\right)(10^6)\right](0.05dy)$$

$$= 49.78(10^3) \text{ N} = 49.8 \text{ kN}$$ **Ans.**

Ans.
$V_{AB} = 49.78$ kN

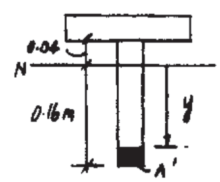
R7–2.

The T-beam is subjected to a shear of $V = 150$ kN. Determine the amount of this force that is supported by the web B.



SOLUTION

$$\bar{y} = \frac{(0.02)(0.2)(0.04) + (0.14)(0.2)(0.04)}{0.2(0.04) + 0.2(0.04)} = 0.08 \text{ m}$$



$$I = \frac{1}{12}(0.2)(0.04^3) + 0.2(0.04)(0.08 - 0.02)^2$$

$$+ \frac{1}{12}(0.04)(0.2^3) + 0.2(0.04)(0.14 - 0.08)^2 = 85.3333(10^{-6}) \text{ m}^4$$

$$A' = 0.04(0.16 - y)$$

$$\bar{y}' = y + \frac{(0.16 - y)}{2} = \frac{(0.16 + y)}{2}$$

$$Q = \bar{y}'A' = 0.02(0.0256 - y^2)$$

$$\tau = \frac{VQ}{It} = \frac{150(10^3)(0.02)(0.0256 - y^2)}{85.3333(10^{-6})(0.04)} = 22.5(10^6) - 878.9(10^6)y^2$$

$$V = \int \tau \, dA, \qquad dA = 0.04\, dy$$

$$V = \int_{-0.04}^{0.16} (22.5(10^6) - 878.9(10^6)y^2)\, 0.04\, dy$$

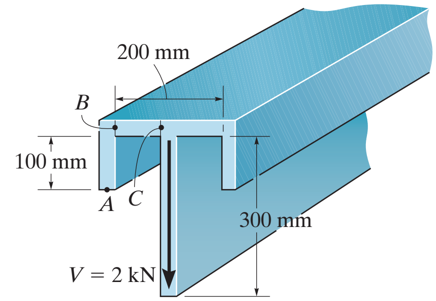
$$= \int_{-0.04}^{0.16} (900(10^3) - 35.156(10^6)y^2) dy$$

$$= 131\,250 \text{ N} = 131 \text{ kN}$$ **Ans.**

Ans:
N/A

R7–3.

The member is subject to a shear force of $V = 2$ kN. Determine the shear flow at points A, B, and C. The thickness of each thin-walled segment is 15 mm.



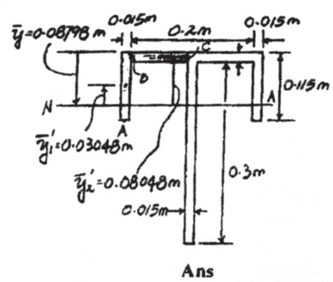
SOLUTION

Section Properties:

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A}$$

$$= \frac{0.0075(0.2)(0.015) + 0.0575(0.115)(0.03) + 0.165(0.3)(0.015)}{0.2(0.015) + 0.115(0.03) + 0.3(0.015)}$$

$$= 0.08798 \text{ m}$$



$$I_{NA} = \frac{1}{12}(0.2)(0.015^3) + 0.2(0.015)(0.08798 - 0.0075)^2$$

$$+ \frac{1}{12}(0.03)(0.115^3) + 0.03(0.115)(0.08798 - 0.0575)^2$$

$$+ \frac{1}{12}(0.015)(0.3^3) + 0.015(0.3)(0.165 - 0.08798)^2$$

$$= 86.93913(10^{-6}) \text{ m}^4$$

$$Q_A = 0$$ **Ans.**

$$Q_B = \bar{y}_1'A' = 0.03048(0.115)(0.015) = 52.57705(10^{-6}) \text{ m}^{-3}$$

$$Q_C = \Sigma \overline{y1}'A'$$

$$= 0.03048(0.115)(0.015) + 0.08048(0.0925)(0.015)$$

$$= 0.16424(10^{-3}) \text{ m}^3$$

Shear Flow:

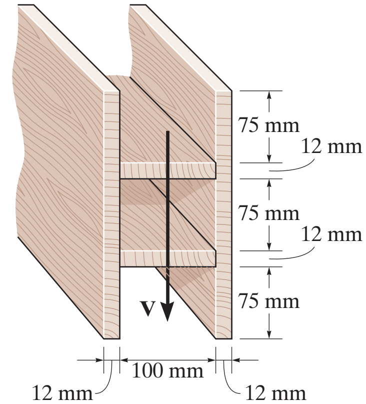
$$q_A = \frac{VQ_A}{I} = 0$$

$$q_B = \frac{VQ_B}{I} = \frac{2(10^3)(52.57705)(10^{-6})}{86.93913(10^{-6})} = 1.21 \text{ kN/m}$$ **Ans.**

$$q_C = \frac{VQ_C}{I} = \frac{2(10^3)(0.16424)(10^{-3})}{86.93913(10^{-6})} = 3.78 \text{ kN/m}$$ **Ans.**

Ans:
$q_A = 0, q_B = 1.21$ kN/m, $q_C = 3.78$ kN/m

***R7–4.** The beam is constructed from four boards glued together at their seams. If the glue can withstand 15 kN/m, what is the maximum vertical shear V that the beam can support?



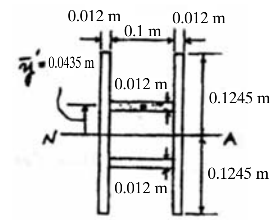
SOLUTION

Section Properties:

$$I_{NA} = 2\left[\frac{1}{12}(0.012)(0.249^3)\right] + 2\left[\frac{1}{12}(0.1)(0.012^3) + 0.1(0.012)(0.0435^2)\right]$$

$$= 35.4467(10^{-6})\ \text{m}^4$$

$$Q = \bar{y}'A' = 0.0435(0.1)(0.012) = 52.2(10^{-6})\ \text{m}^3$$



Shear Flow: There are two glue joints in this case, hence the allowable shear flow is $2[15(10^3)] = 30(10^3)$ N/m.

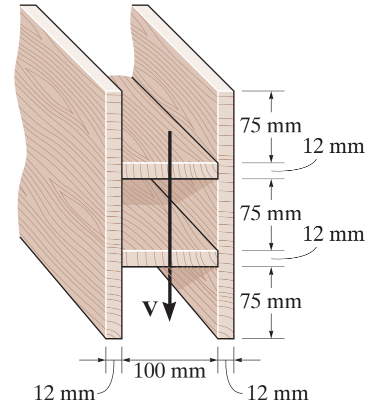
$$q = \frac{VQ}{I}$$

$$30(10^3) = \frac{V[52.2(10^{-6})]}{35.4467(10^{-6})}$$

$$V = 20.37(10^3)\ \text{N} = 20.4\ \text{kN}$$ **Ans.**

Ans.
$V = 20.4$ kN

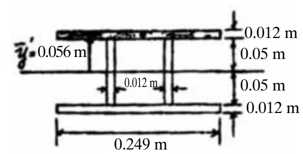
R7–5. Solve Prob. R7–4 if the beam is rotated 90° from the position shown.



SOLUTION

Section Properties:

$$I_{NA} = \frac{1}{12}(0.249)(0.124^3) - \frac{1}{12}(0.225)(0.1^3) = 20.8124(10^{-6})\ \text{m}^4$$



$$Q = \bar{y}'A' = 0.056(0.249)(0.012) = 1.167328(10^{-3})\ \text{m}^3$$

Shear Flow: There are two glue joints in this case, hence the allowable shear flow is $2[15(10^3)] = 30(10^3)$ N/m.

$$q = \frac{VQ}{I}$$

$$30(10^3) = \frac{V[1.167328(10^{-3})]}{20.8124(10^{-6})}$$

$$V = 3.731(10^3)\ \text{N} = 3.73\ \text{kN}$$ **Ans.**

Ans.
$V = 3.73$ kN