12–1. An A-36 steel strap havingathickness of 10 mm and a width of 20 mm is bent into a circular arc of radius $\rho = 10$ m. Determine the maximum bending stress in the strap.

SOLUTION

Moment-Curvature Relationship:

$$\frac{1}{\rho} = \frac{M}{EI} \qquad \text{however,} \qquad M = \frac{I}{c}\sigma$$

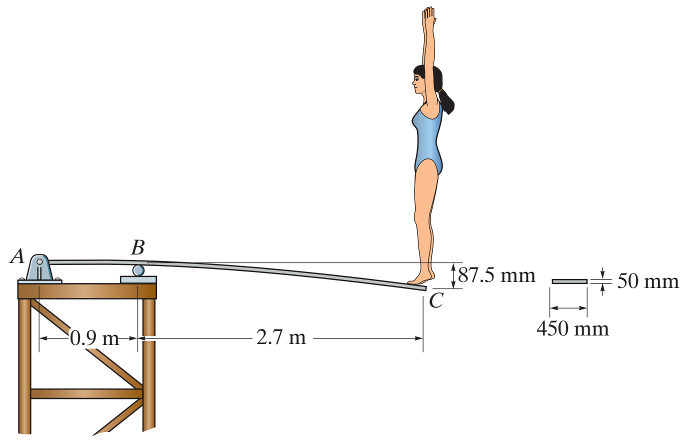
$$\frac{1}{\rho} = \frac{\frac{1}{c}\sigma}{EI}$$

$$\sigma = \frac{c}{\rho}E = \left(\frac{0.005}{10}\right)\left[200\left(10^9\right)\right] = 100 \text{ MPa}$$

Ans:

$\sigma = 100$ MPa

12–2. When the diver stands at end C of the diving board, it deflects downward 87.5 mm. Determine the mass of the diver. The board is made of material having a modulus of elasticity of $E = 10$ GPa.



SOLUTION

Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Functions. Referring to the free-body diagrams of the diving board's cut segments, Fig. *b*, $M(x_1)$ is

$$\zeta + \Sigma M_O = 0; \qquad M(x_1) + 3Wx_1 = 0 \qquad M(x_1) = -3Wx_1$$

and $M(x_2)$ is

$$\zeta + \Sigma M_O = 0; \qquad -M(x_2) - Wx_2 = 0 \qquad M(x_2) = -Wx_2$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1{}^2} = -3Wx_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{3}{2}Wx_1{}^2 + C_1 \qquad (1)$$

$$EIv_1 = -\frac{1}{2}Wx_1{}^3 + C_1x_1 + C_2 \qquad (2)$$

For coordinate x_2

$$EI\frac{d^2v_2}{dx_2{}^2} = -Wx_2$$

$$EI\frac{dv_2}{dx_2} = -\frac{1}{2}Wx_2{}^2 + C_3 \qquad (3)$$

$$EIv_2 = -\frac{1}{6}Wx_2{}^3 + C_3x_2 + C_4 \qquad (4)$$

Boundary Conditions. At $x_1 = 0, v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{1}{2}W(0^3) + C_1(0) + C_2 \qquad C_2 = 0$$

At $x_1 = 0.9$ m, $v_1 = 0$. Then Eq.(2) gives

$$EI(0) = -\frac{1}{2}W(0.9^3) + C_1(0.9) + 0 \qquad C_1 = 0.405W$$

12-2. Continued

At $x_2 = 2.7$ m, $v_2 = 0$. Then Eq.(4) gives

$$EI(0) = -\frac{1}{6}W(2.7^3) + C_3(2.7) + C_4$$

$$2.7C_3 + C_4 = 3.2805W \qquad (5)$$

Continuity Conditions. At $x_1 = 0.9$ m and $x_2 = 2.7$ m, $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. Thus Eqs. (1) and (3) give

$$-\frac{3}{2}W(0.9^2) + 0.405W = -\left[-\frac{1}{2}W(2.7^2) + C_3\right] \qquad C_3 = 4.455W$$

Substituting the value of C_3 into Eq. (5),

$$C_4 = -8.748W$$

Substituting the values of C_3 and C_4 into Eq. (4),

$$v_2 = \frac{1}{EI}\left(-\frac{1}{6}Wx_2^{\,3} + 4.455Wx_2 - 8.748W\right)$$

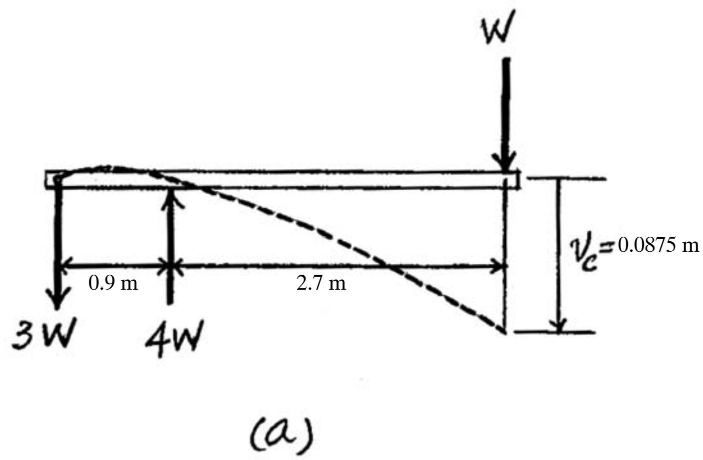
At $x_2 = 0$, $v_2 = -0.0875$ m. Then,

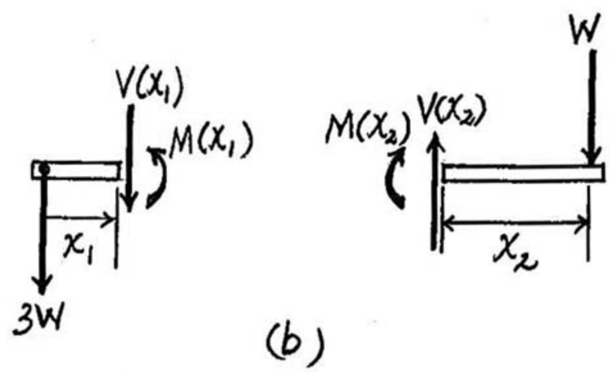
$$-0.0875 = \frac{-8.748W}{[10(10^9)]\left[\frac{1}{12}(0.45)(0.05^3)\right]}$$

$W = 468.86$ N

Then

$$m = \frac{W}{g} = \frac{468.86}{9.81} = 47.8 \text{ kg}$$ **Ans.**

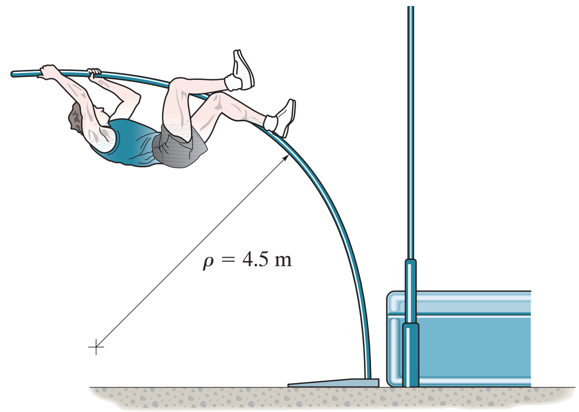




Ans:
$W = 47.8$ kg

12–3.

A picture is taken of a man performing a pole vault, and the minimum radius of curvature of the pole is estimated by measurement to be 4.5 m. If the pole is 40 mm in diameter and it is made of a glass-reinforced plastic for which $E_g = 131$ GPa, determine the maximum bending stress in the pole.



SOLUTION

Moment-Curvature Relationship:

$$\frac{1}{\rho} = \frac{M}{EI} \qquad \text{however,} \qquad M = \frac{I}{c}\sigma$$

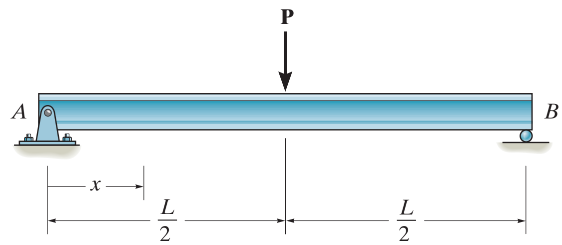
$$\frac{1}{\rho} = \frac{\frac{I}{c}\sigma}{EI}$$

$$\sigma = \frac{c}{\rho}E = \left(\frac{0.02}{4.5}\right)\left[131\left(10^9\right)\right] = 582\text{ MPa} \qquad \textbf{Ans.}$$

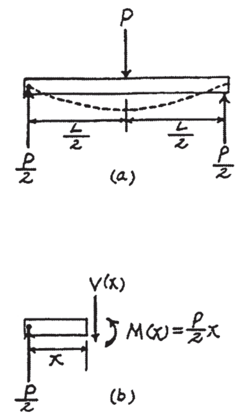
Ans:
$\sigma = 582$ MPa

***12–4.**

Determine the equation of the elastic curve for the beam using the x coordinate that is valid for $0 \le x < L/2$. Specify the slope at A and the beam's maximum deflection. EI is constant.



SOLUTION



Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b).

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{P}{2}x$$

$$EI\frac{dv}{dx} = \frac{P}{4}x^2 + C_1 \qquad \textbf{(1)}$$

$$EI\,v = \frac{P}{12}x^3 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions: Due to symmetry, $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$.

Also, $v = 0$ at $x = 0$.

From Eq. (1) $\quad 0 = \frac{P}{4}\left(\frac{L}{2}\right)^2 + C_1 \qquad C_1 = -\frac{PL^2}{16}$

From Eq. (2) $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

The Slope: Substitute the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{P}{16EI}\left(4x^2 - L^2\right)$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = -\frac{PL^2}{16EI} \qquad \textbf{Ans.}$$

The negative sign indicates clockwise rotation.

The Elastic Curve: Substitute the values of C_1 and C_2 into Eq. (2),

$$v = \frac{Px}{48EI}\left(4x^2 - 3L^2\right) \qquad \textbf{Ans.}$$

v_{max} occurs at $x = \frac{L}{2}$,

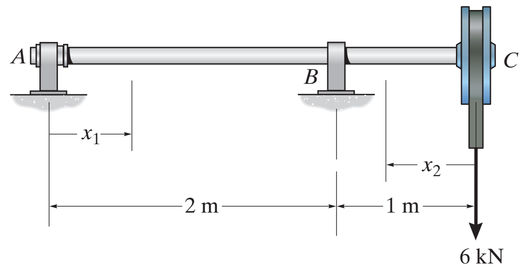
$$v_{max} = -\frac{PL^3}{48EI} \qquad \textbf{Ans.}$$

The negative sign indicates downward displacement.

Ans:
$\theta_A = -\frac{PL^2}{16EI}$,
$v = \frac{Px}{48EI}\left(4x^2 - 3L^2\right)$,
$v_{max} = -\frac{PL^3}{48EI}$

12–5.

Determine the deflection of end C of the 100-mm-diameter solid circular shaft. Take $E = 200$ GPa.



SOLUTION

Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Functions. Referring to the free-body diagrams of the shaft's cut segments, Fig. *b*, $M(x_1)$ is

$$\circlearrowleft + \Sigma M_O = 0; \qquad M(x_1) + 3x_1 = 0 \qquad M(x_1) = -3x_1 \text{ kN}\cdot\text{m}$$

and $M(x_2)$ is

$$\circlearrowleft + \Sigma M_O = 0; \qquad -M(x_2) - 6x_2 = 0 \qquad M(x_2) = -6x_2 \text{ kN}\cdot\text{m}$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1^2} = -3x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{3}{2}x_1^2 + C_1 \qquad \textbf{(1)}$$

$$EIv_1 = -\frac{1}{2}x_1^3 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For coordinate x_2,

$$EI\frac{d^2v_2}{dx_2^2} = -6x_2$$

$$EI\frac{dv_2}{dx_2} = -3x_2^2 + C_3 \qquad \textbf{(3)}$$

$$EIv_2 = -x_2^3 + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions. At $x_1 = 0$, $v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{1}{2}(0^3) + C_1(0) + C_2 \qquad C_2 = 0$$

At $x_1 = 2$ m, $v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{1}{2}(2^3) + C_1(2) + 0 \qquad C_1 = 2 \text{ kN}\cdot\text{m}^2$$

12–5. Continued

At $x_2 = 1$ m, $v_2 = 0$. Then, Eq. (4) gives

$$EI(0) = -(1^3) + C_3(1) + C_4$$

$$C_3 + C_4 = 1 \qquad \textbf{(5)}$$

Continuity Conditions. At $x_1 = 2$ m and $x_2 = 1$, $\dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$. Thus, Eqs. (1) and (3) give

$$-\frac{3}{2}(2^2) + 2 = -[-3(1^2) + C_3] \qquad C_3 = 7\ \text{kN}\cdot\text{m}^2$$

Substituting the values of C_3 into Eq. (5),

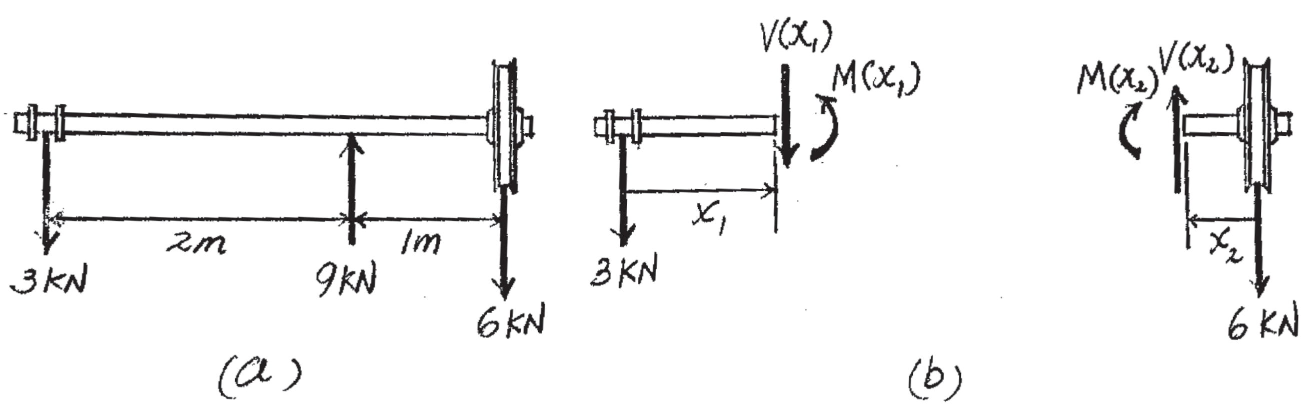
$$C_4 = -6\ \text{kN}\cdot\text{m}^3$$

Substituting the values of C_3 and C_4 into Eq. (4),

$$v_2 = \frac{1}{EI}(-x_2^3 + 7x_2 - 6)$$

$$v_C = v_2|_{x_2=0} = -\frac{6\ \text{kN}\cdot\text{m}^3}{EI}$$

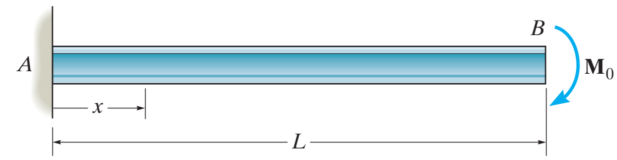
$$= -\frac{6(10^3)}{200(10^9)\left[\frac{\pi}{4}(0.05^4)\right]} = -0.006112\ \text{m} = -6.11\ \text{mm} \qquad \textbf{Ans.}$$



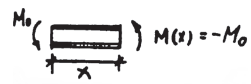
Ans:
$v_C = -6.11$ mm

12–6.

Determine the elastic curve for the cantilevered beam, which is subjected to the couple moment M_0. Also calculate the maximum slope and maximum deflection of the beam. EI is constant.



SOLUTION



Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = -M_0$$

$$EI\frac{dv}{dx} = -M_0x + C_1 \qquad \textbf{(1)}$$

$$EIv = \frac{-M_0x^2}{2} + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions:

$$\frac{dv}{dx} = 0 \quad \text{at} \quad x = 0$$

From Eq. (1), $C_1 = 0$

$$v = 0 \quad \text{at} \quad x = 0$$

From Eq. (2), $C_2 = 0$

$$\frac{dv}{dx} = -\frac{M_0x}{EI}$$

$$\theta_{max} = \left.\frac{dv}{dx}\right|_{x=L} = -\frac{M_0L}{EI} \qquad \textbf{Ans.}$$

The negative sign indicates clockwise rotation.

$$v = -\frac{M_0x^2}{2EI} \qquad \textbf{Ans.}$$

$$v_{max} = v\Big|_{x=L} = -\frac{M_0L^2}{2EI} \qquad \textbf{Ans.}$$

Negative sign indicates downward displacement.

Ans:

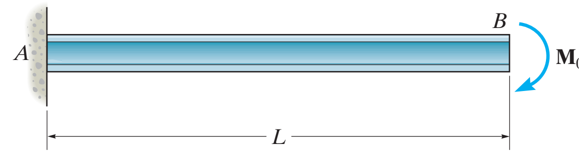
$\theta_{max} = -\frac{M_0L}{EI}$,

$v = -\frac{M_0x^2}{2EI}$,

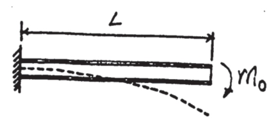
$v_{max} = -\frac{M_0L^2}{2EI}$

12–7.

The A-36 steel beam has a depth of 10 in. and is subjected to a constant moment $\mathbf{M}_0$, which causes the stress at the outer fibers to become $\sigma_Y = 36$ ksi. Determine the radius of curvature of the beam and the beam's maximum slope and deflection.



SOLUTION



Moment-Curvature Relationship:

$$\frac{1}{\rho} = \frac{M_0}{EI} \qquad \text{however,} \qquad M_0 = \frac{I}{c}\sigma_Y$$

$$\frac{1}{\rho} = \frac{\frac{I}{c}\sigma_Y}{EI}$$

$$\rho = \frac{Ec}{\sigma_Y} = \frac{29.0\left(10^3\right)(5)}{36} = 4027.78 \text{ in.} = 336 \text{ ft}$$ **Ans.**

Elastic Curve: As shown.

Moment Function: As shown on FBD.

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = -M_0$$

$$EI\frac{dv}{dx} = -M_0x + C_1 \qquad \textbf{(1)}$$

$$EI\,v = -\frac{M_0}{2}x^2 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions: $\dfrac{dv}{dx} = 0$ at $x = L$, and $v = 0$ at $x = L$.

From Eq. (1), $\quad 0 = -M_0L + C_1 \qquad C_1 = M_0L$

From Eq. (2), $\quad 0 = -\dfrac{M_0}{2}\left(L^2\right) + M_0L^2 + C_2 \qquad C_2 = -\dfrac{M_0L^2}{2}$

The Slope: Substitute the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{M_0}{EI}(-x + L)$$

The maximum slope occurs at $x = 0$.

$$\theta_{max} = \left.\frac{dv}{dx}\right|_{x=0} = \frac{M_0L}{EI}$$ **Ans.**

The Elastic Curve: Substitute the values of C_1 and C_2 into Eq. (2),

$$v = \frac{M_0}{2EI}\left(-x^2 + 2Lx - L^2\right)$$

The maximum displacement occurs at $x = 0$.

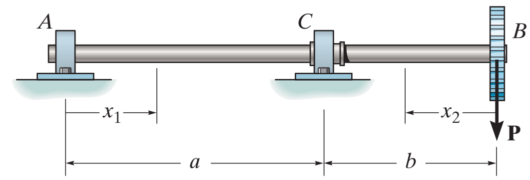
$$v_{max} = -\frac{M_0L^2}{2EI}$$ **Ans.**

The negative sign indicates downward displacement.

Ans:
$\rho = 336$ ft,
$\theta_{max} = \dfrac{M_0L}{EI}$,
$v_{max} = -\dfrac{M_0L^2}{2EI}$

***12–8.**

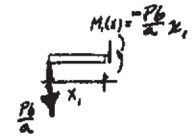
Determine the equations of the elastic curve using the coordinates x_1 and x_2. EI is constant.



SOLUTION

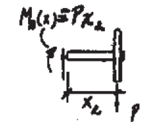
Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$



For $M_1(x) = -\dfrac{Pb}{a}x_1$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{Pb}{a}x_1$$



$$EI\frac{dv_1}{dx_1} = -\frac{Pb}{2a}x_1^2 + C_1 \qquad \textbf{(1)}$$

$$EIv_1 = -\frac{Pb}{6a}x_1^3 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M_2(x) = -Px_2$

$$EI\frac{d^2v_2}{dx_2^2} = -Px_2$$

$$EI\frac{dv_2}{dx_2} = \frac{-Px_2^2}{2} + C_3 \qquad \textbf{(3)}$$

$$EIv_2 = \frac{-Px_2^3}{6} + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

$v_1 = 0$ at $x = 0$

From Eq. (2), $C_2 = 0$

$v_1 = 0$ at $x_1 = a$

From Eq. (2),

$$0 = -\frac{Pb}{6a}a^3 + C_1a$$

$$C_1 = \frac{Pab}{6}$$

$v_2 = 0$ at $x_2 = b$

From Eq. (4),

$$0 = \frac{Pb^3}{6} + C_3b + C_4$$

$$C_3b + C_4 = \frac{Pb^3}{6} \qquad \textbf{(5)}$$

***12–8. Continued**

Continuity Conditions:

$$\frac{dv_1}{dx_1} = \frac{-dv_2}{dx_2} \quad \text{at} \quad x_1 = a \qquad x_2 = b$$

From Eqs. (1) and (3)

$$-\frac{Pb}{2a}(a^2) + \frac{Pab}{6} = \frac{Pb^2}{2} - C_3$$

$$C_3 = \frac{Pab}{3} + \frac{Pb^2}{2}$$

Substitute C_3 into Eq. (5)

$$C_4 = \frac{Pb^3}{3} - \frac{Pab^2}{3}$$

$$v_1 = \frac{-Pb}{6aEI}(x_1^3 - a^2x_1) \qquad \textbf{Ans.}$$

$$v_2 = \frac{P}{6EI}[-x_2^3 + b(2a + 3b)x_2 - 2b^2(a + b)] \qquad \textbf{Ans.}$$

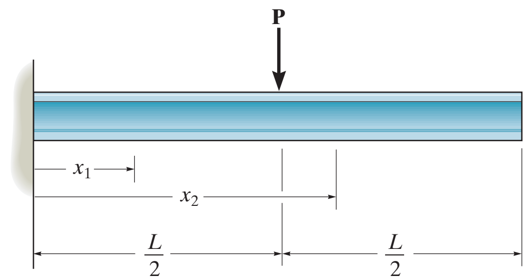
Ans:

$v_1 = \dfrac{-Pb}{6aEI}(x_1^3 - a^2x_1),$

$v_2 = \dfrac{P}{6EI}[-x_2^3 + b(2a + 3b)x_2 - 2b^2(a + b)]$

12–9.

Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. EI is constant.



SOLUTION

Referring to the FBDs of the beam's cut segments shown in Fig. b and c,

$$\circlearrowleft + \Sigma M_O = 0; \qquad M(x_1) + \frac{PL}{2} - Px_1 = 0 \qquad M(x_1) = Px_1 - \frac{PL}{2}$$

And

$$\circlearrowleft + \Sigma M_O = 0; \qquad M(x_2) = 0$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1^2} = Px_1 - \frac{PL}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{P}{2}x_1{}^2 - \frac{PL}{2}x_1 + C_1 \qquad \textbf{(1)}$$

$$EI\,v_1 = \frac{P}{6}x_1{}^3 - \frac{PL}{4}x_1{}^2 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For coordinate x_2,

$$EI\frac{d^2v_2}{dx_2^2} = 0$$

$$EI\frac{dv_2}{dx_2} = C_3 \qquad \textbf{(3)}$$

$$EI\,v_2 = C_3x_2 = C_4 \qquad \textbf{(4)}$$

At $x_1 = 0$, $\frac{dv_1}{dx_1} = 0$. Then, Eq. (1) gives

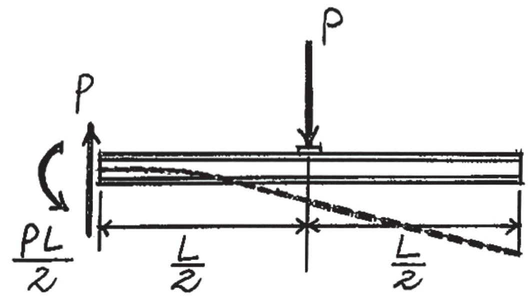
$$EI(0) = \frac{P}{2}(0^2) - \frac{PL}{2}(0) + C_1 \qquad C_1 = 0$$

At $x_1 = 0$, $v_1 = 0$. Then, Eq(2) gives

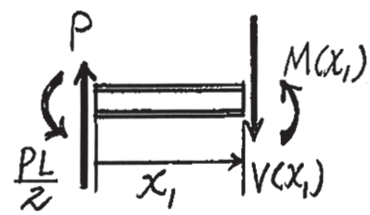
$$EI(0) = \frac{P}{6}(0^3) - \frac{PL}{4}(0^2) + 0 + C_2 \qquad C_2 = 0$$

At $x_1 = x_2 = \frac{L}{2}$, $\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$. Thus, Eqs.(1) and (3) gives

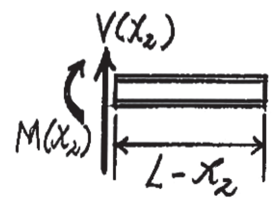
$$\frac{P}{2}\left(\frac{L}{2}\right)^2 - \frac{PL}{2}\left(\frac{L}{2}\right) = C_3 \qquad C_3 = -\frac{PL^2}{8}$$



(a)



(b)



(c)

12–9. Continued

Also, at $x_1 = x_2 = \frac{L}{2}$, $v_1 = v_2$. Thus, Eqs, (2) and (4) gives

$$\frac{P}{6}\left(\frac{L}{2}\right)^3 - \frac{PL}{4}\left(\frac{L}{2}\right)^2 = \left(-\frac{PL^2}{8}\right)\left(\frac{L}{2}\right) + C_4 \qquad C_4 = \frac{PL^3}{48}$$

Substitute the values of C_1 and C_2 into Eq. (2) and C_3 and C_4 into Eq (4),

$$v_1 = \frac{P}{12EI}\left(2x_1^3 - 3Lx_1^2\right) \qquad \textbf{Ans.}$$

$$v_2 = \frac{PL^2}{48EI}(-6x_2 + L) \qquad \textbf{Ans.}$$

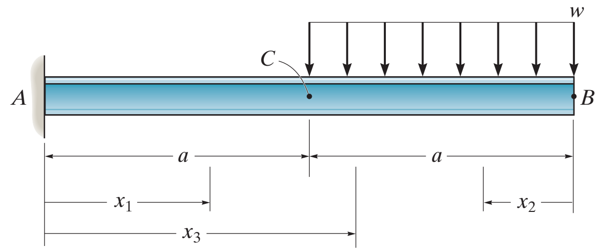
Ans:

$v_1 = \frac{P}{12EI}(2x_1^3 - 3Lx_1^2)$,

$v_2 = \frac{PL^2}{48EI}(-6x_2 + L)$

12–10.

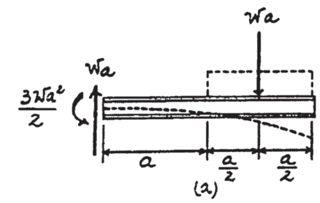
Determine the equations of the elastic curve using the coordinates x_1 and x_2. What is the slope at C and displacement at B? EI is constant.



SOLUTION

Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b) and (c).

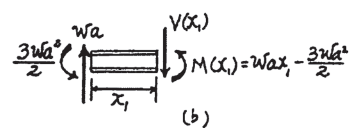


Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = w\,ax_1 = -\dfrac{3wa^2}{2}$,

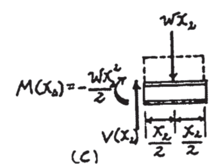
$$EI\frac{d^2v_1}{dx_1^2} = wax_1 - \frac{3wa^2}{2}$$



$$EI\frac{dv_1}{dx_1} = \frac{wa}{2}x_1^2 - \frac{3wa^2}{2}x_1 + C_1 \qquad \textbf{(1)}$$

$$EI\,v_1 = \frac{wa}{6}x_1^3 - \frac{3wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M(x_2) = -\dfrac{w}{2}x_2^2$,



$$EI\frac{d^2v_2}{dx_2^2} = -\frac{w}{2}x_2^2$$

$$EI\frac{dv_2}{dx_2} = -\frac{w}{6}x_2^3 + C_3 \qquad \textbf{(3)}$$

$$EI\,v_2 = -\frac{w}{24}x_2^4 + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

$\dfrac{dv_1}{dx_1} = 0$ at $x_1 = 0$. From Eq. (1), $C_1 = 0$

$v_1 = 0$ at $x_1 = 0$. From Eq. (2), $C_2 = 0$

Continuity Conditions:

At $x_1 = a$ and $x_2 = a$, $\dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$. From Eqs. (1) and (3),

$$\frac{wa^3}{2} - \frac{3wa^3}{2} = -\left(-\frac{wa^3}{6} + C_3\right) \qquad C_3 = \frac{7wa^3}{6}$$

At $x_1 = a$ and $x_2 = a$, $v_1 = v_2$. From Eqs. (2) and (4),

$$\frac{wa^4}{6} - \frac{3wa^4}{4} = -\frac{wa^4}{24} + \frac{5wa^4}{6} + C_4 \qquad C_4 = -\frac{11wa^4}{8}$$

12–10. Continued

The Slope: Substituting into Eq. (1),

$$\frac{dv_1}{dx_1} = \frac{wax_1}{2EI}(x_1 - 3a)$$

$$\theta_C = \left.\frac{dv_1}{dx_1}\right|_{x_1=a} = -\frac{wa^3}{EI} \qquad \textbf{Ans.}$$

The Elastic Curve: Substituting the values of C_1, C_2, C_3, and C_4 into Eqs. (2) and (4), respectively,

$$v_1 = \frac{wax_1}{12EI}\left(2x_1^2 - 9ax_1\right) \qquad \textbf{Ans.}$$

$$v_2 = \frac{w}{24EI}\left(-x_2^4 + 28a^3x_2 - 41a^4\right) \qquad \textbf{Ans.}$$

$$v_B = v_2|_{x_2=0} = -\frac{41wa^4}{24EI} \qquad \textbf{Ans.}$$

Ans:

$$\theta_C = \left.\frac{dv_1}{dx_1}\right|_{x_1=a} = -\frac{wa^3}{EI},$$

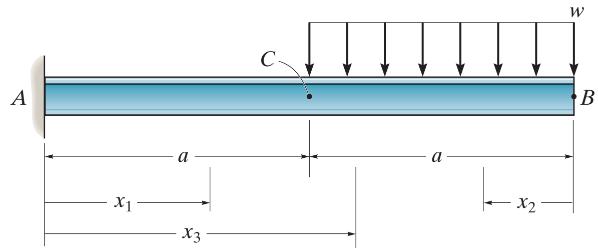
$$v_1 = \frac{wax_1}{12EI}\left(2x_1^2 - 9ax_1\right),$$

$$v_2 = \frac{w}{24EI}\left(-x_2^4 + 28a^3x_2 - 41a^4\right),$$

$$v_B = v_2|_{x_2=0} = -\frac{41\,wa^4}{24\,EI}$$

12–11.

Determine the equations of the elastic curve using the coordinates x_1 and x_3. What is the slope at B and deflection at C? EI is constant.



SOLUTION

Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b) and (c).

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = wax_1 - \dfrac{3wa^2}{2}$,

$$EI\frac{d^2v_1}{dx_1^2} = wax_1 - \frac{3wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{wa}{2}x_1^2 - \frac{3wa^2}{2}x_1 + C_1 \qquad \textbf{(1)}$$

$$EI\,v_1 = \frac{wa}{6}x_1^3 - \frac{3wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M(x_3) = 2wax_3 - \dfrac{w}{2}x_3^2 - 2wa^2$,

$$EI\frac{d^2v_3}{dx_3^2} = 2wax_3 - \frac{w}{2}x_3^2 - 2wa^2$$

$$EI\frac{dv_3}{dx_3} = wax_3^2 - \frac{w}{6}x_3^3 - 2wa^2x_3 + C_3 \qquad \textbf{(3)}$$

$$EI\,v_3 = \frac{wa}{3}x_3^3 - \frac{w}{24}x_3^4 - wa^2x_3^2 + C_3x_3 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

$\dfrac{dv_1}{dx_1} = 0$ at $x_1 = 0$. From Eq. (1), $C_1 = 0$

$v_1 = 0$ at $x_1 = 0$. From Eq. (2), $C_2 = 0$

Continuity Conditions:

At $x_1 = a$ and $x_3 = a$, $\dfrac{dv_1}{dx_1} = \dfrac{dv_3}{dx_3}$. From Eqs. (1) and (3),

$$\frac{wa^3}{2} - \frac{3wa^3}{2} = wa^3 - \frac{wa^3}{6} - 2wa^3 + C_3 \qquad C_3 = \frac{wa^3}{6}$$

At $x_1 = a$ and $x_3 = a$, $v_1 = v_3$. From Eqs. (2) and (4),

$$\frac{wa^4}{6} - \frac{3wa^4}{4} = \frac{wa^4}{3} - \frac{wa^4}{24} - wa^4 + \frac{wa^4}{6} + C_4 \qquad C_4 = -\frac{wa^4}{24}$$

12–11. Continued

The Slope: Substituting the value of C_1 into Eq. (1),

$$\frac{dv_3}{dx_3} = \frac{w}{6EI}(6ax_3^2 - x_3^3 - 12a^2x_3 + a^3)$$

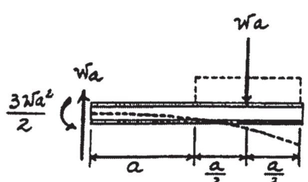
$$\theta_B = \left.\frac{dv_3}{dx_3}\right|_{x_3=2a} = -\frac{7wa^3}{6EI} \qquad \textbf{Ans.}$$

The Elastic Curve: Substituting the values of C_1, C_2, C_3, and C_4 into Eqs. (2) and (4), respectively,

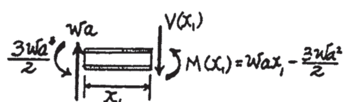
$$v_1 = \frac{wax_1}{12EI}\left(2x_1^2 - 9ax_1\right) \qquad \textbf{Ans.}$$

$$v_C = v_1\big|_{x_1=a} = -\frac{7\,wa^4}{12EI} \qquad \textbf{Ans.}$$

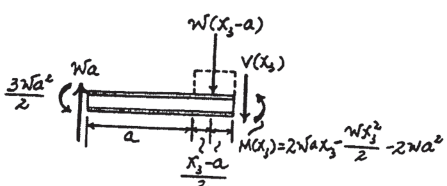
$$v_3 = \frac{w}{24EI}\left(-x_3^4 + 8ax_3^3 - 24a^2x_3^2 + 4a^3x_3 - a^4\right) \qquad \textbf{Ans.}$$



(a)



(b)



(c)

Ans:

$\theta_B = -\dfrac{7wa^3}{6EI}$,

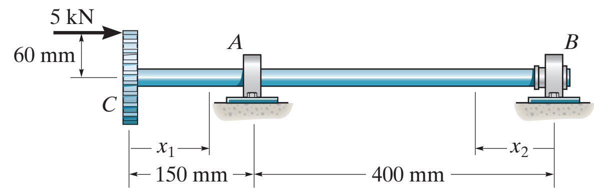
$v_1 = \dfrac{wax_1}{12EI}\left(2x_1^2 - 9ax_1\right)$,

$v_C = v_1\big|_{x_1=a} = -\dfrac{7\,wa^4}{12EI}$,

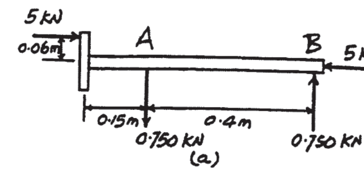
$v_3 = \dfrac{w}{24EI}\left(-x_3^4 + 8ax_3^3 - 24a^2x_3^2 + 4a^3x_3 - a^4\right)$

***12–12.**

Draw the bending-moment diagram for the shaft and then, from this diagram, sketch the deflection or elastic curve for the shaft's centerline. Determine the equations of the elastic curve using the coordinates x_1 and x_2. EI is constant.



SOLUTION

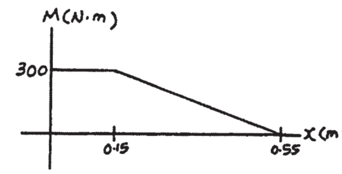


Elastic Curve: As shown.

Moment Function: As shown on FBD(b) and (c).

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$



For $M(x_1) = 300\ \text{N}\cdot\text{m}$,

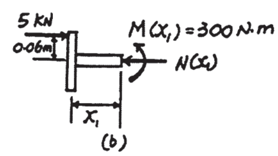
$$EI\frac{d^2v_1}{dx_1^2} = 300$$

$$EI\frac{dv_1}{dx_1} = 300x_1 + C_1 \qquad \textbf{(1)}$$

$$EI\,v_1 = 150x_1^2 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M(x_2) = 750x_2$,

$$EI\frac{d^2v_2}{dx_2^2} = 750x_2$$

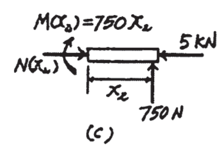


$$EI\frac{dv_2}{dx_2} = 375x_2^2 + C_3 \qquad \textbf{(3)}$$

$$EI\,v_2 = 125x_2^3 + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

$v_1 = 0$ at $x_1 = 0.15$ m. From Eq. (2),



$$0 = 150(0.15^2) + C_1(0.15) + C_2 \qquad \textbf{(5)}$$

$v_2 = 0$ at $x_2 = 0$. From Eq. (4), $C_4 = 0$

$v_2 = 0$ at $x_2 = 0.4$ m. From Eq. (4),

$$0 = 125(0.4^3) + C_3(0.4) \qquad C_3 = -20.0$$

Continuity Condition:

At $x_1 = 0.15$ m and $x_2 = 0.4$ m, $\dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$. From Eqs. (1) and (3),

$$300(0.15) + C_1 = -\left[375(0.4^2) - 20\right] \qquad C_1 = -85.0$$

From Eq. (5), $\qquad C_2 = 9.375$

The Elastic Curve: Substitute the values of C_1, C_2, C_3, and C_4 into Eqs. (2) and (4), respectively.

$$v_1 = \frac{1}{EI}\left(150x_1^2 - 85.0x_1 + 9.375\right)\ \text{N}\cdot\text{m}^3 \qquad \textbf{Ans.}$$

$$v_2 = \frac{1}{EI}\left(125x_2^3 - 20.0x_2\right)\ \text{N}\cdot\text{m}^3 \qquad \textbf{Ans.}$$

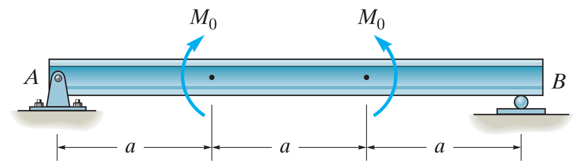
Ans:

$v_1 = \dfrac{1}{EI}\left(150x_1^2 - 85.0x_1 + 9.375\right)\ \text{N}\cdot\text{m}^3$,

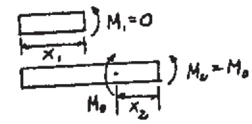
$v_2 = \dfrac{1}{EI}\left(125x_2^3 - 20.0x_2\right)\ \text{N}\cdot\text{m}^3$

12–13.

Determine the maximum deflection of the beam and the slope at A. EI is constant.



SOLUTION



$M_1 = 0$

$$EI\frac{d^2v_1}{dx_1^2} = 0; \qquad EI\frac{dv_1}{dx_1} = C_1$$

$$EIv_1 = C_1x_1 + C_2$$

At $x_1 = 0$, $\quad v_1 = 0; \qquad C_2 = 0$

$$M_2 = M_0; \qquad EI\frac{d^2v_1}{dx_2^2} = M_0$$

$$EI\frac{dv_2}{dx_2} = M_0x_2 + C_2$$

$$EI\,v_2 = \frac{1}{2}M_0x_2^2 + C_3x_2 + C_4$$

At $x_2 = \frac{a}{2}$, $\quad \frac{dv_2}{dx_2} = 0; \qquad C_1 = \frac{-M_0\,a}{2}$

At $x_1 = a$, $\quad x_2 = 0$, $\quad v_1 = v_2$, $\quad \frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$

$$C_1a = C_a$$

$$C_1 = \frac{-M_0\,a}{2}, \qquad C_a = \frac{-M_0\,a^2}{2}$$

At $x_1 = 0$,

$$EI\frac{dv_1}{dx_1} = -\frac{M_0\,a}{2}$$

$$\boldsymbol{\theta_A} = -\frac{M_0\,a}{2EI} \qquad \textbf{Ans.}$$

At $x_2 = \frac{a}{2}$,

$$EI\,v_{\max} = \frac{1}{2}M_0\left(\frac{a^2}{4}\right) - \frac{M_0\,a}{2}\left(\frac{a}{2}\right) - \frac{M_0\,a^2}{2}$$

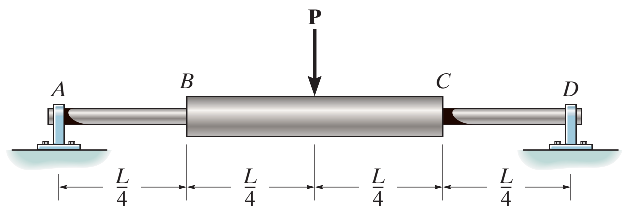
$$v_{\max} = -\frac{5M_0\,a^2}{8EI} \qquad \textbf{Ans.}$$

Ans:

$$\theta_A = -\frac{M_0a}{2EI}, \; v_{\max} = -\frac{5M_0\,a^2}{8EI}$$

12–14.

The simply supported shaft has a moment of inertia of $2I$ for region BC and a moment of inertia I for regions AB and CD. Determine the maximum deflection of the shaft due to the load **P**.



SOLUTION

$$M_1(x) = \frac{P}{2}x_1$$

$$M_2(x) = \frac{P}{2}x_2$$

Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v_1}{dx_1{}^2} = \frac{P}{2}x_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{4} + C_1 \qquad \textbf{(1)}$$

$$EIv_1 = \frac{Px_1^3}{12} + C_1x_1 + C_2 \qquad \textbf{(2)}$$

$$2EI\frac{d^2v_2}{dx_2{}^2} = \frac{P}{2}x_2$$

$$2EI\frac{dv_2}{dx_1} = \frac{Px_2^2}{4} + C_3 \qquad \textbf{(3)}$$

$$2EIv_2 = \frac{Px_2^3}{12} + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

$$v_1 = 0 \text{ at } x_1 = 0$$

From Eq. (2), $C_2 = 0$

$$\frac{dv_2}{dx_2} = 0 \text{ at } x_2 = \frac{L}{2}$$

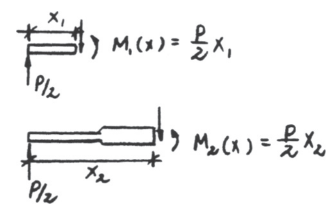
From Eq. (3),

$$0 = \frac{PL^2}{16} + C_3$$

$$C_3 = -\frac{PL^2}{16}$$

12–14. Continued

Continuity Conditions:



$$\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2} \text{ at } x_1 = x_2 = \frac{L}{4}$$

From Eqs. (1) and (3),

$$\frac{PL^2}{64} + C_1 = \frac{PL^2}{128} - \frac{1}{2}\left(\frac{PL^2}{16}\right)$$

$$C_1 = \frac{-5PL^2}{128}$$

$$v_1 = v_2 \text{ at } x_1 = x_2 = \frac{L}{4}$$

From Eqs. (2) and (4)

$$\frac{PL^3}{768} - \frac{5PL^2}{128}\left(\frac{L}{4}\right) = \frac{PL^3}{1536} - \frac{1}{2}\left(\frac{PL^2}{16}\right)\left(\frac{L}{4}\right) + \frac{1}{2}C_4$$

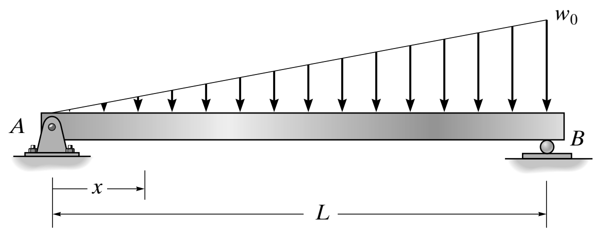
$$C_4 = \frac{-PL^3}{384}$$

$$v_2 = \frac{P}{768EI}\left(32x_2^3 - 24L^2x_2 - L^3\right)$$

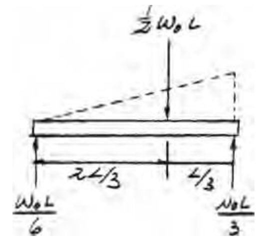
$$v_{\max} = v_2\Big|_{x_2=\frac{L}{2}} = \frac{-3PL^3}{256EI} \qquad \textbf{Ans.}$$

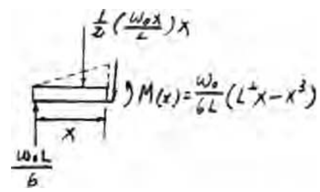
Ans:
$v_{\max} = -\dfrac{3PL^3}{256EI}$

12–15. The beam is subjected to the linearly varying distributed load. Determine the maximum deflection of the beam. EI is constant.



SOLUTION





$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{w_0}{6L}\left(L^2x - x^3\right)$$

$$EI\frac{dv}{dx} = \frac{w_0}{6L}\left(\frac{L^2x^2}{2} - \frac{x^4}{4}\right) + C_1 \qquad (1)$$

$$EI\,v = \frac{w_0}{6L}\left(\frac{L^2x^3}{6} - \frac{x^5}{20}\right) + C_1x + C_2 \qquad (2)$$

Boundary conditions:

$v = 0$ at $x = 0$.

From Eq. (2), $C_2 = 0$

$v = 0$ at $x = L$.

From Eq. (2),

$$0 = \frac{w_0}{6L}\left(\frac{L^2}{6} - \frac{L^5}{20}\right) + C_1L; \qquad C_1 = -\frac{7w_0L^3}{360}$$

$$\frac{dv}{dx} = \frac{w_0}{6EIL}\left(\frac{L^2x^2}{2} - \frac{x^4}{4} - \frac{7L^4}{60}\right)$$

$$\frac{dv}{dx} = 0 = \left(\frac{L^2x^2}{2} - \frac{x^4}{4} - \frac{7L^4}{60}\right)$$

$$15x^4 - 30L^2x^2 + 7L^4 = 0; \qquad x = 0.5193L$$

$$v = \frac{w_0x}{360EIL}\left(10L^2x^2 - 3x^4 - 7L^4\right)$$

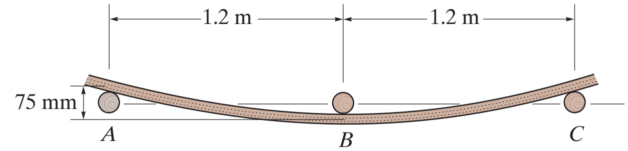
Substitute $x = 0.5193L$ into v,

$$v_{max} = -\frac{0.00652w_0L^4}{EI} \qquad \textbf{Ans.}$$

Ans:

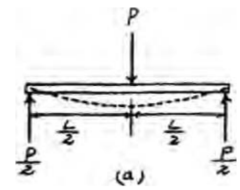
$$v_{max} = -\frac{0.00652w_0L^4}{EI}$$

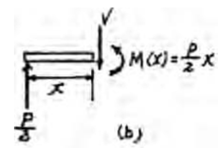
***12–16.** The fence board weaves between the three smooth fixed posts. If the posts remain along the same line, determine the maximum bending stress in the board. The board has a width of 150 mm and a thickness of 12 mm. $E = 12$ GPa Assume the displacement of each end of the board relative to its center is 75 mm.



Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b).





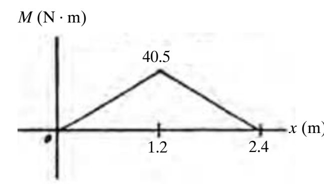
Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{P}{2}x$$

$$EI\frac{dv}{dx} = \frac{P}{4}x^2 + C_1 \qquad [1]$$

$$EI\,v = \frac{P}{12}x^3 + C_1x + C_2 \qquad [2]$$



Boundary Conditions: Due to symmetry, $\dfrac{dv}{dx} = 0$ at $x = \dfrac{L}{2}$.

Also, $v = 0$ at $x = 0$.

From Eq. [1] $0 = \dfrac{P}{4}\left(\dfrac{L}{2}\right)^2 + C_1 \qquad C_1 = -\dfrac{PL^2}{16}$

From Eq. [2] $0 = 0 + 0 + C_2 \qquad C_2 = 0$

The Elastic Curve: Substitute the values of C_1 and C_2 into Eq. [2],

$$v = \frac{Px}{48EI}\left(4x^2 - 3L^2\right) \qquad [1]$$

Require $x = 1.2$, $v = -0.075$ m. From Eq. [1],

$$-0.075 = \frac{P(1.2)}{48[12(10^9)]\left[\frac{1}{12}(0.150)(0.012^3)\right]}[4(1.2^2) - 3(2.4^2)]$$

$$P = 67.5 \text{ N}$$

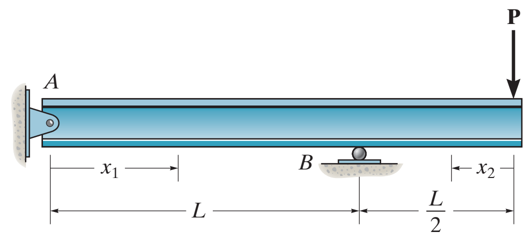
Maximum Bending Stress: From the moment diagram, the maximum moment is $M_{max} = 40.5$ N · m. Applying the flexure formula,

$$\sigma_{max} = \frac{Mc}{I} = \frac{40.5(0.006)}{\frac{1}{12}(0.15)(0.012^3)} = 11.25(10^6) \text{ N/m}^2 = 11.25 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma_{max} = 11.25$ MPa

12–17.

Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. Specify the beam's maximum deflection. EI is constant.

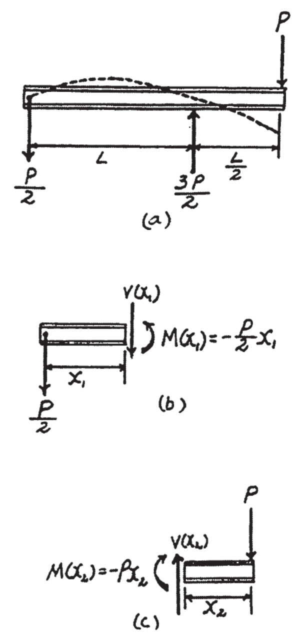


SOLUTION

Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b) and (c).

Slope and Elastic Curve:



$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = -\frac{P}{2}x_1$,

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{P}{2}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{P}{4}x_1^2 + C_1 \quad \textbf{(1)}$$

$$EI\,v_1 = -\frac{P}{12}x_1^3 + C_1x_1 + C_2 \quad \textbf{(2)}$$

For $M(x_2) = -Px_2$,

$$EI\frac{d^2v_2}{dx_2^2} = -Px_2$$

$$EI\frac{dv_2}{dx_2} = -\frac{P}{2}x_2^2 + C_3 \quad \textbf{(3)}$$

$$EI\,v_2 = -\frac{P}{6}x_2^3 + C_3x_2 + C_4 \quad \textbf{(4)}$$

Boundary Conditions:

$v_1 = 0$ at $x_1 = 0$. From Eq. (2), $C_2 = 0$

$v_1 = 0$ at $x_1 = L$. From Eq. (2),

$$0 = -\frac{PL^3}{12} + C_1L \qquad C_1 = \frac{PL^2}{12}$$

$v_2 = 0$ at $x_2 = \frac{L}{2}$. From Eq. (4),

$$0 = -\frac{PL^3}{48} + \frac{L}{2}C_3 + C_4 \quad \textbf{(5)}$$

12–17. Continued

Continuity Conditions:

At $x_1 = L$ and $x_2 = \frac{L}{2}$, $\quad \frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. From Eqs. (1) and (3),

$$-\frac{PL^2}{4} + \frac{PL^2}{12} = -\left(-\frac{PL^2}{8} + C_3\right) \qquad C_3 = \frac{7PL^2}{24}$$

From Eq. (5), $\qquad C_4 = -\frac{PL^3}{8}$

The Slope: Substitute the value of C_1 into Eq. (1),

$$\frac{dv_1}{dx_1} = \frac{P}{12EI}\left(L^2 - 3x_1^2\right)$$

$$\frac{dv_1}{dx_1} = 0 = \frac{P}{12EI}\left(L^2 - 3x_1^2\right) \qquad x_1 = \frac{L}{\sqrt{3}}$$

The Elastic Curve: Substitute the values of C_1, C_2, C_3, and C_4 into Eqs. (2) and (4), respectively,

$$v_1 = \frac{Px_1}{12EI}\left(-x_1^2 + L^2\right) \qquad \textbf{Ans.}$$

$$v_D = v_1\big|_{x_1 = \frac{L}{\sqrt{3}}} = \frac{P\left(\frac{L}{\sqrt{3}}\right)}{12EI}\left(-\frac{L^2}{3} + L^2\right) = \frac{0.0321PL^3}{EI}$$

$$v_2 = \frac{P}{24EI}\left(-4x_2^3 + 7L^2x_2 - 3L^3\right) \qquad \textbf{Ans.}$$

$$v_C = v_2\big|_{x_2 = 0} = -\frac{PL^3}{8EI}$$

Hence, $\qquad v_{max} = v_C = \frac{PL^3}{8EI} \qquad$ **Ans.**

Ans:

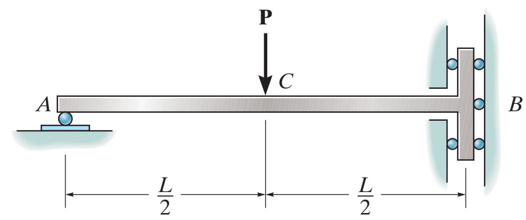
$v_1 = \frac{Px_1}{12EI}\left(-x_1^2 + L^2\right)$,

$v_2 = \frac{P}{24EI}\left(-4x_2^3 + 7L^2x_2 - 3L^3\right)$,

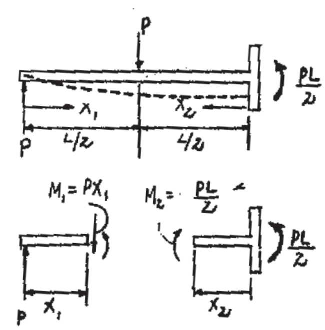
$v_{max} = \frac{PL^3}{8EI}$

12–18.

The bar is supported by a roller constraint at B, which allows vertical displacement but resists axial load and moment. If the bar is subjected to the loading shown, determine the slope at A and the deflection at C. EI is constant.



SOLUTION



$$EI\frac{d^2v_1}{dx_1^2} = M_1 = Px_1$$

$$EI\,v_1 = \frac{Px_1^2}{6} + C_1$$

$$EI\,v_1 = \frac{Px_1^2}{6} + C_1x_1 + C_2$$

$$EI\frac{d^2v_2}{dx_2} = M_2 = \frac{PL}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{PL}{2}x_2 + C_3$$

$$EI\,v_2 = \frac{PL}{4}x_2^2 + C_3x_2 + C_4$$

Boundary Conditions:

At $x_1 = 0$, $v_1 = 0$

$0 = 0 + 0 + C_2; \quad C_2 = 0$

At $x_2 = 0, \quad \dfrac{dv_2}{dx_2} = 0$

$0 + C_3 = 0; \quad C_3 = 0$

At $x_1 = \dfrac{L}{2}, \quad x_2 = \dfrac{L}{2}, v_1 = v_2, \dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$

$$\frac{P\left(\frac{1}{2}\right)^2}{6} + C_1\left(\frac{L}{2}\right) = \frac{PL\left(\frac{1}{2}\right)^2}{4} + C_4$$

$$\frac{P\left(\frac{1}{2}\right)^2}{2} + C_1 = -\frac{PL\left(\frac{1}{2}\right)}{2}; \quad C_1 = -\frac{3}{8}PL^2$$

$$C_4 = -\frac{11}{48}PL^3$$

At $x_1 = 0$

$$\frac{dv_1}{dx_1} = \theta_A = -\frac{3}{8}\frac{PL^2}{EI} \qquad \textbf{Ans.}$$

At $x_1 = \dfrac{L}{2}$

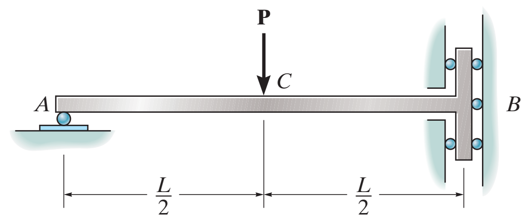
$$v_C = \frac{P\left(\frac{1}{2}\right)^3}{6EI} - \left(\frac{3}{8EI}PL^3\right)\left(\frac{L}{2}\right) + 0$$

$$v_C = -\frac{PL^3}{6EI} \qquad \textbf{Ans.}$$

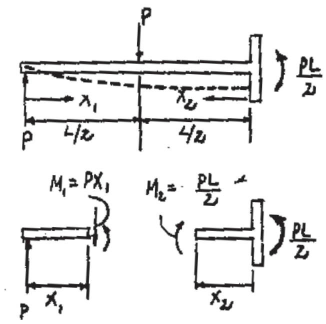
Ans:
$\theta_A = -\dfrac{3PL^2}{8EI}, v_C = -\dfrac{PL^3}{6EI}$

12–19.

Determine the deflection at B of the bar in Prob. 12–18.



SOLUTION



$$EI\frac{d^2v_1}{dx_1^2} = M_1 = Px_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{2} + C_1$$

$$EI\,v_1 = \frac{Px_1^2}{6} + C_1x_1 + C_2$$

$$EI\frac{d^2v_2}{dx_2} = M_2 = \frac{PL}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{PL}{2}x_2 + C_2$$

$$EI\,v_2 = \frac{PL}{4}x_2^2 + C_3x_2 + C_4$$

Boundary Conditions:

At $x_1 = 0, v_1 = 0$

$0 = 0 + 0 + C_2; \quad C_2 = 0$

At $x_2 = 0, \quad \dfrac{dv_2}{dx_2} = 0$

$0 + C_3 = 0; \quad C_3 = 0$

At $x_1 = \dfrac{L}{2}, \quad x_2 = \dfrac{L}{2}, v_1 = v_2, \dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$

$$\frac{P\left(\frac{1}{2}\right)^2}{6} + C_1\left(\frac{L}{2}\right) = \frac{PL\left(\frac{1}{2}\right)^2}{4} + C_4$$

$$\frac{P\left(\frac{1}{2}\right)^2}{2} + C_1 = -\frac{PL\left(\frac{1}{2}\right)}{2}; \quad C_1 = -\frac{3}{8}PL^2$$

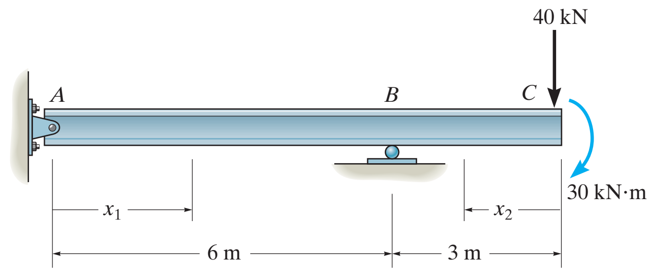
$$C_4 = -\frac{11}{48}PL^3$$

At $x_2 = 0$.

$$v_2 = v_B = -\frac{11PL^3}{48EI} \qquad \textbf{Ans.}$$

Ans:
$v_B = -\dfrac{11PL^3}{48EI}$

***12–20.** Determine the equations of the elastic curve using the x_1 and x_2 coordinates, and specify the slope at A and the deflection at C. EI is constant.



SOLUTION

Referring to the FBDs of the beam's cut segments shown in Fig. b, and c,

$$\zeta + \Sigma M_o = 0; \qquad M(x_1) + 25x_1 = 0 \qquad M(x_1) = (-25x_1)\ \text{kN}\cdot\text{m}$$

And

$$\zeta + \Sigma M_o = 0; \qquad -M(x_2) - 40x_2 - 30 = 0 \qquad M(x_2) = (-40x_2 - 30)\ \text{kN}\cdot\text{m}$$

$$EI\,\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\,\frac{d^2v_1}{dx_1^2} = (-25x_1)\ \text{kN}\cdot\text{m}$$

$$EI\,\frac{dv_1}{dx_1} = \left(-\frac{25}{2}x_1^2 + C_1\right)\text{kN}\cdot\text{m}^2 \qquad (1)$$

$$EI\,v_1 = \left(-\frac{25}{6}x_1{}^3 + C_1x_1 + C_2\right)\text{kN}\cdot\text{m}^3 \qquad (2)$$

For coordinate x_2,

$$EI\,\frac{d^2v_2}{dx_2{}^2} = (-40x_2 - 30)\ \text{kN}\cdot\text{m}$$

$$EI\,\frac{dv_2}{dx_2} = (-20x_2^2 - 30x_2 + C_3)\ \text{kN}\cdot\text{m}^2 \qquad (3)$$

$$EI\,v_2 = \left(-\frac{20}{3}x_2{}^3 - 15\,x_2{}^2 + C_3x_2 + C_4\right)\text{kN}\cdot\text{m}^3 \qquad (4)$$

At $x_1 = 0, v_1 = 0$. Then, Eq (2) gives

$$EI(0) = -\frac{25}{6}(0^3) + C_1(0) + C_2 \qquad C_2 = 0$$

Also, at $x_1 = 6$ m, $v_1 = 0$. Then, Eq (2) gives

$$EI(0) = -\frac{25}{6}(6^3) + C_1(6) + 0 \qquad C_1 = 150\ \text{kN}\cdot\text{m}^2$$

Also, at $x_2 = 3$ m, $v_2 = 0$. Then, Eq. (4) gives

$$EI(0) = -\frac{20}{3}(3^3) - 15(3^2) + C_3(3) + C_4$$

$$3C_3 + C_4 = 315 \qquad (5)$$

***12–20. Continued**

At $x_1 = 6$ m and $x_2 = 3$ m, $\dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$. Then Eq. (1) and (3) gives

$$-\frac{25}{2}(6^2) + 150 = -[-20(3^2) - 30(3) + C_3]$$

$$C_3 = 570 \text{ kN} \cdot \text{m}^2$$

Substitute the value of C_3 into Eq (5),

$$C_4 = -1395 \text{ kN} \cdot \text{m}^3$$

Substitute the value of C_1 into Eq. (1),

$$\frac{dv_2}{dx_1} = \frac{1}{EI}\left(-\frac{25}{2}x_1^2 + 150\right) \text{ kN} \cdot \text{m}^2$$

At A, $x_1 = 0$. Thus,

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1=0} = \frac{150 \text{ kN} \cdot \text{m}^2}{EI} \quad \nearrow \theta_A \qquad \textbf{Ans.}$$

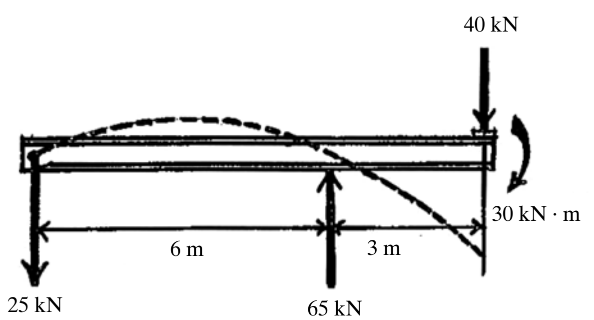
Substitute the values of C_1 and C_2 into Eq. (2) and C_3 and C_4 into Eq (4),

$$v_1 = \frac{1}{EI}\left(-\frac{25}{6}x_1{}^3 + 150x_1\right) \text{kN} \cdot \text{m}^3 \qquad \textbf{Ans.}$$

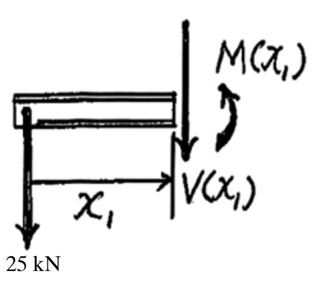
$$v_2 = \frac{1}{EI}\left(-\frac{20}{3}x_2{}^3 - 15x_2{}^2 + 570x_2 - 1395\right) \text{kN} \cdot \text{m}^3 \qquad \textbf{Ans.}$$

At C, $x_2 = 0$. Thus

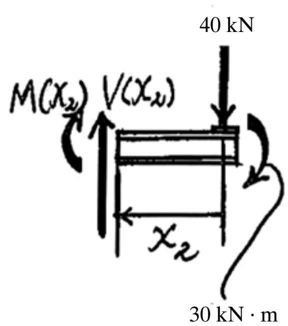
$$v_C = v_2\big|_{x_2=0} = -\frac{1395 \text{ kN} \cdot \text{m}^3}{EI} = \frac{1395 \text{ kN} \cdot \text{m}^3}{EI} \downarrow \qquad \textbf{Ans.}$$



(a)



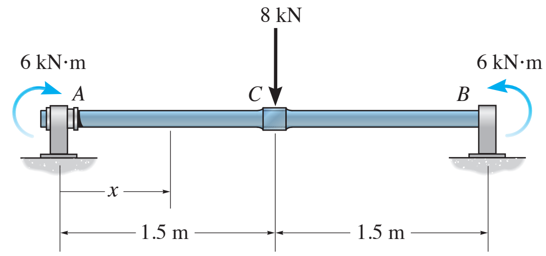
(b)



(c)

12–21.

Determine the maximum deflection of the solid circular shaft. The shaft is made of steel having $E = 200$ GPa. It has a diameter of 100 mm.



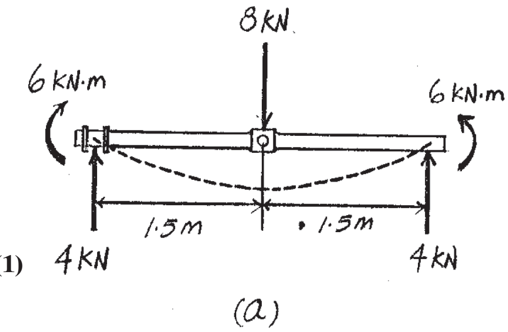
SOLUTION

Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Function. Referring to the free-body diagram of the beam's cut segment, Fig. *b*,

$\circlearrowleft + \Sigma M_O = 0; \qquad M(x) - 4x - 6 = 0 \qquad M(x) = (4x + 6)\ \text{kN} \cdot \text{m}$

Equations of Slope and Elastic Curve.



$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = 4x + 6$$

$$EI\frac{dv}{dx} = 2x^2 + 6x + C_1 \qquad \textbf{(1)}$$

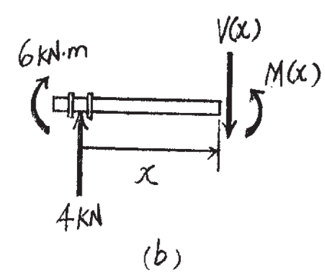
$$EIv = \frac{2}{3}x^3 + 3x^2 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions. Due to symmetry, $\frac{dv}{dx} = 0$ at $x = 1.5$ m. Then Eq. (1) gives

$$EI(0) = 2(1.5^2) + 6(1.5) + C_1 \qquad C_1 = -13.5\ \text{kN} \cdot \text{m}^2$$

Also, at $x = 0$, $v = 0$. Then Eq. (2) gives

$$EI(0) = \frac{2}{3}(0^3) + 3(0^2) + C_1(0) + C_2 \qquad C_2 = 0$$



Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{1}{EI}\left(\frac{2}{3}x^3 + 3x^2 - 13.5x\right)$$

v_{max} occurs at $x = 1.5$ m, where $\frac{dv}{dx} = 0$. Thus,

$$v_{max} = v|_{x=1.5\text{ m}} = \frac{1}{EI}\left[\frac{2}{3}(1.5^3) + 3(1.5^2) - 13.5(1.5)\right]$$

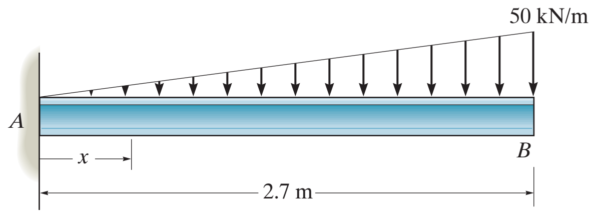
$$= -\frac{11.25\ \text{kN} \cdot \text{m}^3}{EI}$$

$$= -\frac{11.25(10^3)}{200(10^9)\left[\frac{\pi}{4}(0.05^4)\right]}$$

$$= -0.01146\text{ m} = -11.5\text{ mm} \qquad \textbf{Ans.}$$

Ans:
$v_{max} = -11.5$ mm

12–22. Determine the elastic curve for the cantilevered W360 × 45 beam using the x coordinate. Specify the maximum slope and maximum deflection. $E = 200$ GPa.



Referring to the FBD of the beam's cut segment shown in Fig. b,

$$\zeta + \Sigma M_o = 0; \qquad M(x) + 121.5 + \frac{1}{2}\left(\frac{50}{2.7}x\right)(x)\left(\frac{x}{3}\right) - 67.5x = 0$$

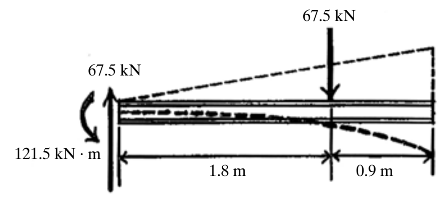
$$M(x) = (67.5x - 3.0864x^3 - 121.5)\ \text{kN}\cdot\text{m}$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

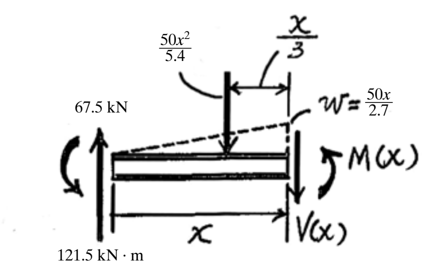
$$EI\frac{d^2v}{dx^2} = (67.5x - 3.0864x^3 - 121.5)\ \text{kN}\cdot\text{m}$$

$$EI\frac{dv}{dx} = (33.75x^2 - 0.7716x^4 - 121.5x + C_1)\ \text{kN}\cdot\text{m}^2 \qquad (1)$$

$$EI\,v = (11.25x^3 - 0.15432x^5 - 60.75x^2 + C_1x + C_2)\ \text{kN}\cdot\text{m}^3 \qquad (2)$$



(a)



(b)

At $x = 0, \dfrac{dv}{dx} = 0$. Then, Eq (1) gives

$$EI(0) = 33.75(0^2) - 0.7716(0^4) - 121.5(0) + C_1 \qquad C_1 = 0$$

Also, at $x = 0, v = 0$. Then Eq. (2) gives

$$EI(0) = 11.25(0^3) - 0.15432(0^5) - 60.75(0^2) + 0 + C_2 \qquad C_2 = 0$$

Substitute the value of C_1 into Eq (1) gives.

$$\frac{dv}{dx} = \frac{1}{EI}(33.75x^2 - 0.7716x^4 - 121.5x)\ \text{kN}\cdot\text{m}^2$$

The Maximum Slope occurs at $x = 2.7$ m. Thus,

$$\theta_{max} = \left.\frac{dv}{dx}\right|_{x=2.7\text{m}} = -\frac{123.02\ \text{kN}\cdot\text{m}^2}{EI}$$

$$= \frac{123.02\ \text{kN}\cdot\text{m}^2}{EI}$$

For W360 × 45, $I = 121(10^6)\ \text{mm}^4 = 121(10^{-6})\ \text{m}^4$. Thus

$$\theta_{max} = \frac{123.02(10^3)\ \text{N}\cdot\text{m}^2}{[200(10^9)\ \text{N/m}^2][121(10^{-6})\ \text{m}^4]} = 0.00508\ \text{rad} \quad \searrow \theta_{max}$$ **Ans.**

Substitute the values of C_1 and C_2 into Eq (2),

$$v = \frac{1}{EI}(11.25x^3 - 0.15432x^5 - 60.75x^2)\ \text{kN}\cdot\text{m}^3$$

The maximum deflection occurs at $x = 2.7$ m. Thus,

$$v_{max} = v|_{x=2.7\text{m}} = -\frac{243.58\ \text{kN}\cdot\text{m}^3}{EI}$$

$$= \frac{243.58\ \text{kN}\cdot\text{m}^3}{EI}\downarrow$$

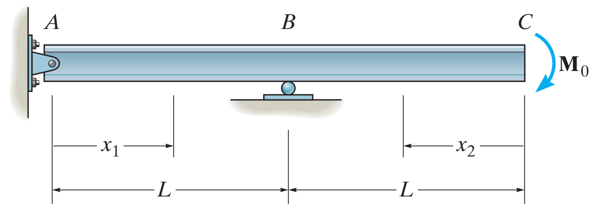
$$= \frac{243.58(10^3)\ \text{N}\cdot\text{m}^3}{[200(10^9)\ \text{N/m}^2][121(10^{-6})\ \text{m}^4]}$$

$$= 0.01007\ \text{m} = 10.1\ \text{mm}\downarrow$$ **Ans.**

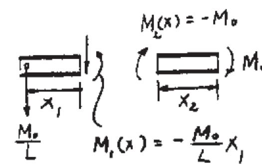
Ans:
$\theta_{max} = -0.00508$

12–23.

Determine the equations of the elastic curve using the coordinates x_1 and x_2. What is the deflection and slope at C? EI is constant.



SOLUTION



$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x_1) = -\dfrac{M_0}{L}x_1$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{M_0}{L}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{M_0}{2L}x_1^2 + C_1 \qquad \textbf{(1)}$$

$$EIv_1 = -\frac{M_0}{6L}x_1^3 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M_2(x) = -M_0;$ $\qquad EI\dfrac{d^2v_2}{dx_2^2} = -M_0$

$$EI\frac{dv_2}{dx_2} = -M_0x_2 + C_3 \qquad \textbf{(3)}$$

$$EIv_2 = -\frac{M_0}{2}x_2^2 + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

At $x_1 = 0, v_1 = 0$

From Eq. (2),

$0 = 0 + 0 + C_2;\ C_2 = 0$

At $x_1 = x_2 = L, v_1 = v_2 = 0$

From Eq. (2),

$$0 = -\frac{M_0L^2}{6} + C_1L; \qquad C_1 = \frac{M_0L}{6}$$

From Eq. (4),

$$0 = -\frac{M_0L^2}{2} + C_3L + C_4 \qquad \textbf{(5)}$$

12–23. Continued

Continuity Condition:

At $x_1 = x_2 = L, \dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$$-\frac{M_0L}{2} + \frac{M_0L}{6} = -(-M_0L + C_3); \qquad C_3 = \frac{4M_0L}{3}$$

Substituting C_3 into Eq. (5) yields,

$$C_4 = -\frac{5M_0L^2}{6}$$

The slope:

$$\frac{dv_2}{dx_2} = \frac{1}{EI}\left[-M_0x_2 + \frac{4M_0L}{3}\right]$$

$$\theta_C = \left.\frac{dv_2}{dx_2}\right|_{x_2=0} = \frac{4M_0L}{3EI} \circlearrowright \qquad \textbf{Ans.}$$

The elastic Curve:

$$v_1 = \frac{M_0}{6EIL}(-x_1^3 + L^2x_1) \qquad \textbf{Ans.}$$

$$v_2 = \frac{M_0}{6EIL}(-3Lx_2^2 + 8L^2x_2 - 5L^3) \qquad \textbf{Ans.}$$

$$v_C = v_2|_{x_2=0} = -\frac{5M_0L^2}{6EI} \qquad \textbf{Ans.}$$

The negative sign indicates downward deflection.

Ans:

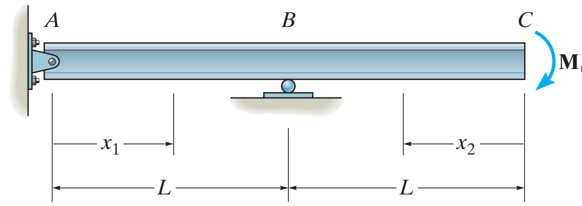
$\theta_C = \dfrac{4M_0L}{3EI} \circlearrowright, v_1 = \dfrac{M_0}{6EIL}(-x_1^3 + L^2x_1)$,

$v_2 = \dfrac{M_0}{6EIL}(-3Lx_2^2 + 8L^2x_2 - 5L^3)$,

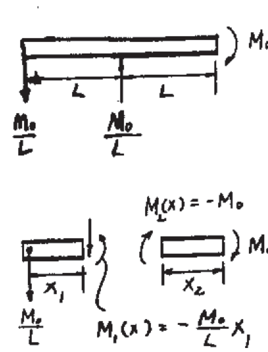
$v_C = -\dfrac{5M_0L^2}{6EI}$

***12–24.**

Determine the equations of the elastic curve using the coordinates x_1 and x_2. What is the slope at A? EI is constant.



SOLUTION



$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x_1) = -\frac{M_0}{L}x_1$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{M_0}{L}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{M_0}{2L}x_1^2 + C_1 \qquad \textbf{(1)}$$

$$EIv_1 = -\frac{M_0}{6L}x_1^3 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M_2(x) = -M_0$; $\quad EI\frac{d^2v_2}{dx_2^2} = -M_0$

$$EI\frac{dv_2}{dx_2} = -M_0x_2 + C_3 \qquad \textbf{(3)}$$

$$EIv_2 = -\frac{M_0}{2}x_2^2 + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

At $x_1 = 0$, $v_1 = 0$

From Eq. (2),

$$0 = 0 + 0 + C_2; \quad C_2 = 0$$

At $x_1 = x_2 = L$, $v_1 = v_2 = 0$

From Eq. (2),

$$0 = -\frac{M_0L^2}{6} + C_1L; \quad C_1 = \frac{M_0L}{6}$$

From Eq. (4),

$$0 = -\frac{M_0L^2}{2} + C_3L + C_4 \qquad \textbf{(5)}$$

***12–24. Continued**

Continuity Condition:

At $x_1 = x_2 = L$, $\quad \dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$$-\frac{M_0L}{2} + \frac{M_0L}{6} = -(-M_0L + C_3); \qquad C_3 = \frac{4M_0L}{3}$$

Substiuting C_3 into Eq. (5) yields,

$$C_4 = -\frac{5M_0L^2}{6}$$

The Elastic Curve:

$$v_1 = \frac{M_0}{6EIL}(-x_1^3 + L^2x_1) \qquad \textbf{Ans.}$$

$$v_2 = \frac{M_0}{6EIL}(-3Lx_2^2 + 8L^2x_2 - 5L^3) \qquad \textbf{Ans.}$$

From Eq. (1),

$$EI\frac{dv_1}{dx_1} = 0 + C_1 = \frac{M_0L}{6}$$

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1=0} = \frac{M_0L}{6EI} \qquad \textbf{Ans.}$$

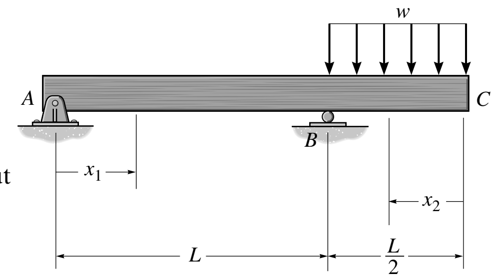
Ans:

$v_1 = \dfrac{M_0}{6EIL}(-x_1^3 + L^2x_1)$,

$v_2 = \dfrac{M_0}{6EIL}(-3Lx_2^2 + 8L^2x_2 - 5L^3)$,

$\theta_A = \dfrac{M_0L}{6EI}$

12–25. Determine the elastic curve in terms of the and coordinates and the deflection of end C of the overhang beam. EI is constant.



Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Functions. Referring to the free-body diagrams of the beam's cut segments, Fig. *b*, $M(x_1)$ is

$$\zeta + \Sigma M_O = 0; \qquad M(x_1) + \frac{wL}{8}x_1 = 0 \qquad M(x_1) = -\frac{wL}{8}x_1$$

and $M(x_2)$ is

$$\zeta + \Sigma M_O = 0; \qquad -M(x_2) - wx_2\left(\frac{x_2}{2}\right) = 0 \qquad M(x_2) = -\frac{w}{2}x_2^{\,2}$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1^{\,2}} = -\frac{wL}{8}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{wL}{16}x_1^{\,2} + C_1 \tag{1}$$

$$EIv_1 = -\frac{wL}{48}x_1^{\,3} + C_1x_1 + C_2 \tag{2}$$

For coordinate x_2,

$$EI\frac{d^2v_2}{dx_2^{\,2}} = -\frac{w}{2}x_2^{\,2}$$

$$EI\frac{dv_2}{dx_2} = -\frac{w}{6}x_2^{\,3} + C_3 \tag{3}$$

$$EIv_2 = -\frac{w}{24}x_2^{\,4} + C_3x_2 + C_4 \tag{4}$$

Boundary Conditions. At $x_1 = 0, v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{wL}{48}(0^3) + C_1(0) + C_2 \qquad C_2 = 0$$

At $x_1 = L, v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{wL}{48}(L^3) + C_1L + 0 \qquad C_1 = \frac{wL^3}{48}$$

At $x_2 = \frac{L}{2}, v_2 = 0$. Then, Eq. (4) gives

$$EI(0) = -\frac{w}{24}\left(\frac{L}{2}\right)^4 + C_3\left(\frac{L}{2}\right) + C_4$$

$$\frac{L}{2}C_3 + C_4 = \frac{wL^4}{384} \tag{5}$$

12–25. Continued

Continuity Conditions. At $x_1 = L$ Land $x_2 = \frac{L}{2}, \frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. Thus, Eqs. (1) and (3) give

$$-\frac{wL}{16}\left(L^2\right) + \frac{wL^3}{48} = -\left[-\frac{w}{6}\left(\frac{L}{2}\right)^3 + C_3\right] \qquad C_3 = \frac{wL^3}{16}$$

Substituting the value of C_3 into Eq. (5),

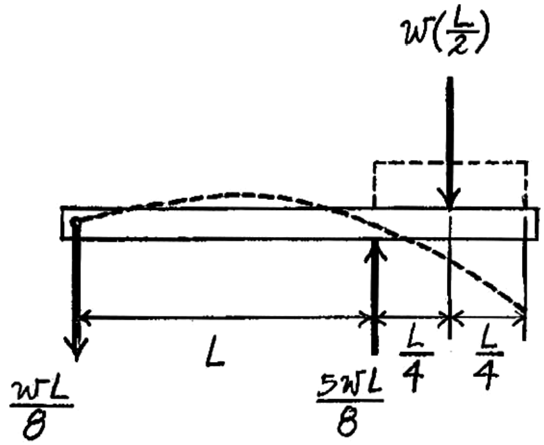
$$C_4 = -\frac{11wL^4}{384}$$

Substituting the values of C_3 and C_4 into Eq. (4),

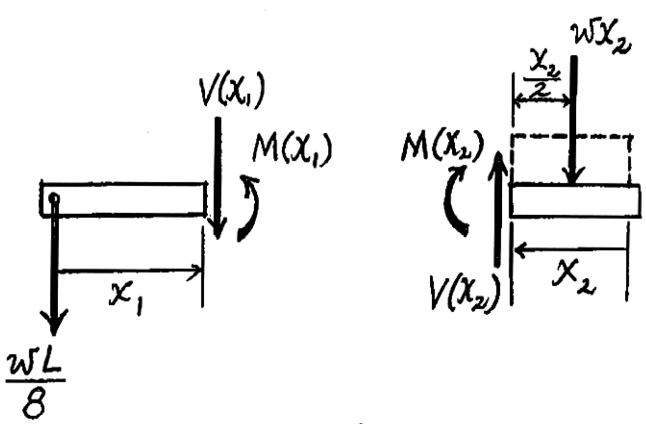
$$v_2 = \frac{w}{384EI}\left(-16x_2{}^4 + 24L^3x_2 - 11L^4\right)$$

At C, $x_2 = 0$. Thus,

$$v_C = v_2\big|_{x_2=0} = -\frac{11wL^4}{384EI} = \frac{11wL^4}{384EI} \downarrow \qquad \textbf{Ans.}$$



(a)

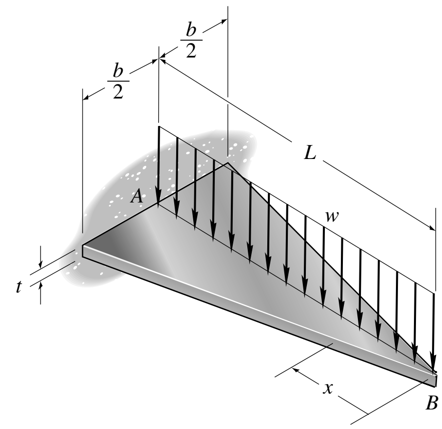


(b)

Ans:

$v_C = \frac{11wL^4}{384EI} \downarrow$

12–26. Determine the slope at end B and the maximum deflection of the cantilevered triangular plate of constant thickness t. The plate is made of material having a modulus of elasticity E.

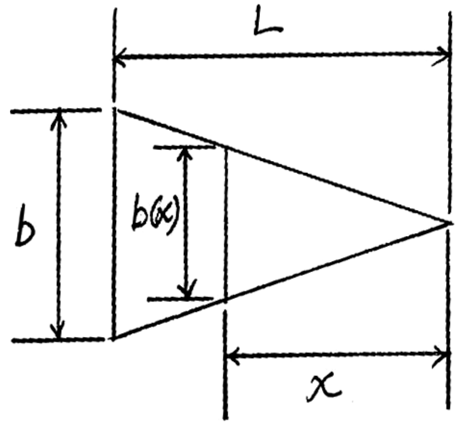


Section Properties. Referring to the geometry shown in Fig. *a*,

$$\frac{b(x)}{x} = \frac{b}{L}; \qquad b(x) = \frac{b}{L}x$$

Thus, the moment of the plate as a function of x is

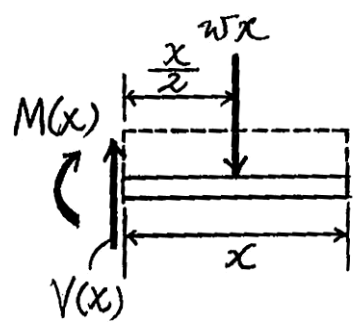
$$I(x) = \frac{1}{12}\left[b(x)\right]t^3 = \frac{bt^3}{12L}x$$



(a)

Moment Functions. Referring to the free-body diagram of the plate's cut segments, Fig. b,

$$+\Sigma M_O = 0; \qquad -M(x) - w(x)\left(\frac{x}{2}\right) = 0 \qquad M(x) = -\frac{w}{2}x^2$$



(b)

Equations of Slope and Elastic Curve.

$$E\frac{d^2v}{dx^2} = \frac{M(x)}{I(x)}$$

$$E\frac{d^2v}{dx^2} = \frac{-\frac{w}{2}x^2}{\frac{bt^3}{12L}x} = -\frac{6wL}{bt^3}x$$

$$E\frac{dv}{dx} = -\frac{3wL}{bt^3}x^2 + C_1 \tag{1}$$

$$Ev = -\frac{wL}{bt^3}x^3 + C_1x + C_2 \tag{2}$$

Boundary Conditions. At $x = L, \dfrac{dv}{dx} = 0$. Then Eq. (1) gives

$$E(0) = -\frac{3wL}{bt^3}\left(L^2\right) + C_1 \qquad C_1 = \frac{3wL^3}{bt^3}$$

At $x = L, v = 0$. Then Eq. (2) gives

$$E(0) = -\frac{wL}{bt^3}\left(L^3\right) + C_1(L) + C_2 \qquad C_2 = -\frac{2wL^4}{bt^3}$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{3wL}{Ebt^3}\left(-x^2 + L^2\right)$$

At $B, x = 0$. Thus,

$$\theta_B = \frac{dv}{dx}\bigg|_{x=0} = \frac{3wL^3}{Ebt^3}$$

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{wL}{Ebt^3}\left(-x^3 + 3L^2x - 2L^3\right)$$

v_{max} occurs at $x = 0$. Thus,

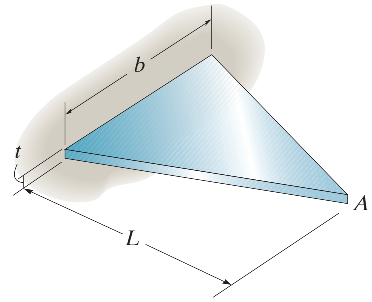
$$v_{max} = v\big|_{x=0} = -\frac{2wL^4}{Ebt^3} = \frac{2wL^4}{Ebt^3} \downarrow \qquad \textbf{Ans.}$$

Ans:

$$v_{max} = \frac{2wL^4}{Ebt^3}\downarrow$$

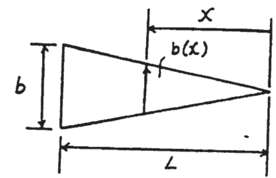
12–27.

The beam is made of a material having a specific weight γ. Determine the displacement and slope at its end A due to its weight. The modulus of elasticity for the material is E.



SOLUTION

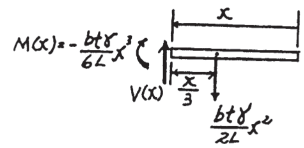
Section Properties:



$$b(x) = \frac{b}{L}x \qquad V(x) = \frac{1}{2}\left(\frac{b}{L}x\right)(x)(t) = \frac{bt}{2L}x^2$$

$$I(x) = \frac{1}{12}\left(\frac{b}{L}x\right)t^3 = \frac{bt^3}{12L}x$$

Moment Function: As shown on FBD.



Slope and Elastic Curve:

$$E\frac{d^2v}{dx^2} = \frac{M(x)}{I(x)}$$

$$E\frac{d^2v}{dx^2} = -\frac{\frac{bt\gamma}{6L}x^3}{\frac{bt^3}{12L}x} = -\frac{2\gamma}{t^2}x^2$$

$$E\frac{dv}{dx} = -\frac{2\gamma}{3t^2}x^3 + C_1 \qquad \textbf{(1)}$$

$$Ev = -\frac{\gamma}{6t^2}x^4 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions: $\frac{dv}{dx} = 0$ at $x = L$ and $v = 0$ at $x = L$.

From Eq. (1), $\quad 0 = -\frac{2\gamma}{3t^2}(L^3) + C_1 \qquad C_1 = \frac{2\gamma L^3}{3t^2}$

From Eq. (2), $\quad 0 = -\frac{\gamma}{6t^2}(L^4) + \left(\frac{2\gamma L^3}{3t^2}\right)(L) + C_2$

$$C_2 = -\frac{\gamma L^4}{2^2}$$

The Slope: Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{2\gamma}{3t^2E}\left(-x^3 + L^3\right)$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{2\gamma L^3}{3t^2E} \qquad \textbf{Ans.}$$

The Elastic Curve: Substituting the value of C_1 and C_2 into Eq. (2),

$$v = \frac{\gamma}{6t^2E}\left(-x^4 + 4L^3x - 3L^4\right)$$

$$v_A = v|_{x=0} = -\frac{\gamma L^4}{2t^2E} \qquad \textbf{Ans.}$$

The negative sign indicates downward displacement.

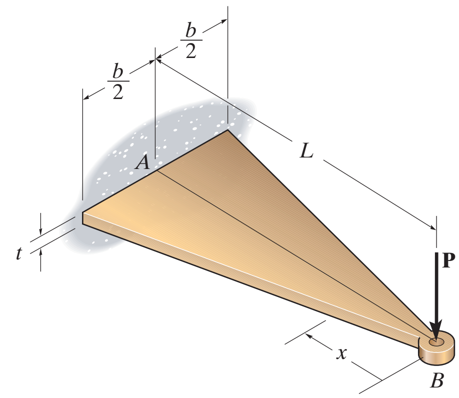
Ans:

$\theta_A = \frac{2\gamma L^3}{3t^2E}$,

$v_A = -\frac{\gamma L^4}{2t^2E}$

***12–28.**

Determine the slope at end B and the maximum deflection of the cantilever triangular plate of constant thickness t. The plate is made of material having a modulus of elasticity of E.



SOLUTION

Section Properties: Referring to the geometry shown in Fig. a,

$$\frac{b(x)}{x} = \frac{b}{L}; \qquad b(x) = \frac{b}{L}x$$

Thus, the moment of the plate as a function of x is

$$I(x) = \frac{1}{12}[b(x)]t^3 = \frac{bt^3}{12L}x$$

Moment Functions. Referring to the free–body diagram of the plate's cut segment, Fig. b,

$$\circlearrowleft +\Sigma M_O = 0; \qquad -M(x) - Px = 0 \qquad M(x) = -Px$$

Equations of Slope and Elastic Curve.

$$E\frac{d^2v}{dx^2} = \frac{M(x)}{I(x)}$$

$$E\frac{d^2v}{dx^2} = \frac{-Px}{\dfrac{bt^3}{12L}x} = -\frac{12PL}{bt^3}$$

$$E\frac{dv}{dx} = -\frac{12PL}{bt^3}x + C_1 \qquad \textbf{(1)}$$

$$Ev = -\frac{6PL}{bt^3}x^2 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions. At $x = L$, $\dfrac{dv}{dx} = 0$. Then Eq. (1) gives

$$E(0) = -\frac{12PL}{bt^3}(L) + C_1 \qquad C_1 = \frac{12PL^2}{bt^3}$$

At $x = L$, $v = 0$. Then Eq. (2) gives

$$E(0) = -\frac{6PL}{bt^3}(L^2) + C_1(L) + C_2 \qquad C_2 = -\frac{6PL^3}{bt^3}$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{12PL}{bt^3E}(-x + L)$$

At B, $x = 0$. Thus,

$$\theta_B = \left.\frac{dv}{dx}\right|_{x=0} = \frac{12PL^2}{bt^3E} \qquad \textbf{Ans.}$$

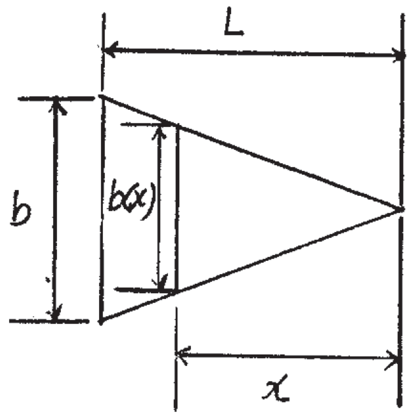
***12–28. Continued**

Substituting the values of C_1 and C_2 into Eq. (2),

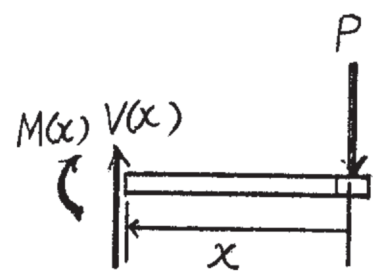
$$v = \frac{6PL}{Ebt^3}(-x^2 + 12Lx - L^2)$$

v_{max} occurs at $x = 0$. Thus,

$$v_{max} = v|_{x=0} = --\frac{6PL^3}{Ebt^3} = \frac{6PL^3}{Ebt^3} \downarrow$$ **Ans.**



(a)



(b)

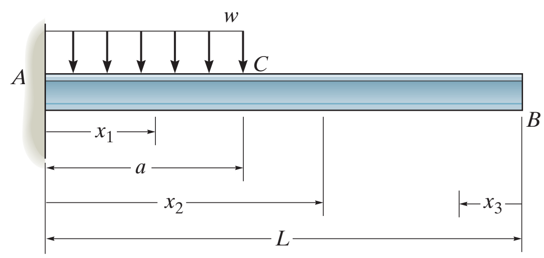
Ans:

$\theta_B = \dfrac{12PL^2}{bt^3E}$,

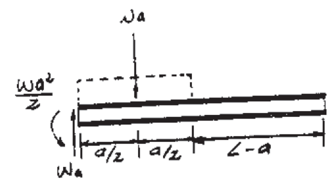
$v_{max} = \dfrac{6PL^3}{Ebt^3} \downarrow$

12–29.

Determine the equation of the elastic curve using the coordinates x_1 and x_2. What is the slope and deflection at B? EI is constant.



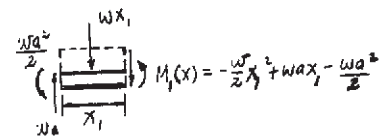
SOLUTION



$$EI\frac{d^2v}{dx^2} = M(x)$$

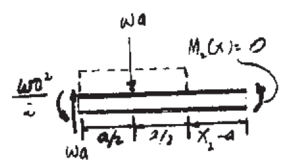
For $M_1(x) = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$$



$$EI\frac{dv_1}{dx_1} = -\frac{w}{6}x_1^3 + \frac{wa}{2}x_1^2 - \frac{wa^2}{2}x_1 + C_1 \qquad \textbf{(1)}$$

$$EI\, v_1 = -\frac{w}{24}x_1^4 + \frac{wa}{6}x_1^3 - \frac{wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad \textbf{(2)}$$



For $M_2(x) = 0;\qquad EI\frac{d^2v_2}{dx_2^{\,2}} = 0$

$$EI\frac{dv_2}{dx_2} = C_3 \qquad \textbf{(3)}$$

$$EI\, v_2 = C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

At $x_1 = 0,\qquad \frac{dv_1}{dx_1} = 0$

From Eq. (1), $\qquad C_1 = 0$

At $x_1 = 0$, $v_1 = 0$

From Eq. (2); $\qquad C_2 = 0$

Continuity Conditions:

At $x_1 = a,\qquad x_2 = a;\qquad \frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$$-\frac{wa^3}{6} + \frac{wa^3}{2} - \frac{wa^3}{2} = C_3;\qquad C_3 = -\frac{wa^3}{6}$$

12–29. Continued

From Eqs. (2) and (4),

At $x_1 = a$, $\quad x_2 = a \qquad v_1 = v_2$

$$-\frac{wa^4}{24} + \frac{wa^4}{6} - \frac{wa^4}{4} = -\frac{wa^4}{6} + C_4; \qquad C_4 = \frac{wa^4}{24}$$

The slope, from Eq. (3),

$$\theta_B = \frac{dv_1}{dx_2} = -\frac{wa^3}{6EI}$$ **Ans.**

The Elastic Curve:

$$v_1 = \frac{w}{24EI}(-x_1^4 + 4ax_1^3 - 6a^2x_1^2)$$ **Ans.**

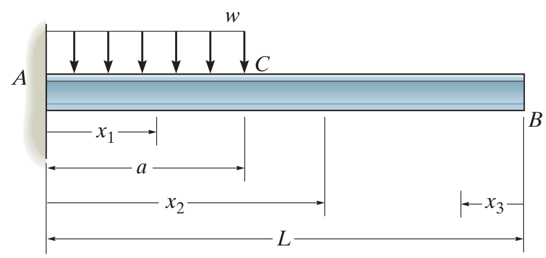
$$v_2 = \frac{wa^3}{24EI}(-4x_2 + a)$$ **Ans.**

$$v_B = v_2\Big|_{x_2=L} = \frac{wa^3}{24EI}(-4L + a)$$ **Ans.**

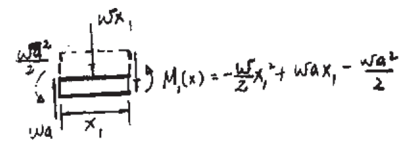
Ans:
$\theta_B = -\dfrac{wa^3}{6EI}$,
$v_1 = \dfrac{w}{24EI}(-x_1^4 + 4ax_1^3 - 6a^2x_1^2)$,
$v_2 = \dfrac{wa^3}{24EI}(-4x_2 + a)$,
$v_B = \dfrac{wa^3}{24EI}(-4L + a)$

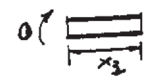
12–30.

Determine the equations of the elastic curve using the coordinates x_1 and x_3. What is the slope and deflection at point B? EI is constant.



SOLUTION





$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = -\frac{w}{6}x_1^3 + \frac{wa}{2}x_1^2 - \frac{wa^2}{2}x_1 + C_1 \qquad \textbf{(1)}$$

$$EI\,v_1 = -\frac{w}{24}x_1^4 + \frac{wa}{6}x_1^3 - \frac{wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M_3(x) = 0;$ $\qquad EI\frac{d^2v_3}{dx_3^2} = 0$ **(3)**

$$EI\frac{dv_3}{dx_3} = C_3 \qquad \textbf{(4)}$$

$$EI\,v_3 = C_3x_3 + C_4$$

Boundary Conditions:

At $x_1 = 0, \dfrac{dv_1}{dx_1} = 0$

From Eq. (1),

$0 = -0 + 0 - 0 + C_1; \quad C_1 = 0$

At $x_1 = 0, \quad v_1 = 0$

From Eq. (2),

$0 = -0 - 0 - 0 + 0 + C_2; \qquad C_2 = 0$

Continuity Conditions:

At $x_1 = a, \qquad x_3 = L - a; \qquad \dfrac{dv_1}{dx_1} = -\dfrac{dv_3}{dx_3}$

$$-\frac{wa^3}{6} + \frac{wa^3}{2} - \frac{wa^3}{2} = -C_3; \quad C_3 = +\frac{wa^3}{6}$$

12–30. Continued

At $x_1 = a, x_3 = L - a \qquad v_1 = v_3$

$$-\frac{wa^4}{24} + \frac{wa^4}{6} - \frac{wa^4}{4} = \frac{wa^3}{6}(L - a) + C_4; \quad C_4 = \frac{wa^4}{24} - \frac{wa^3L}{6}$$

The Slope:

$$\frac{dv_1}{dx_3} = \frac{wa^3}{6EI}$$

$$\theta_B = \left.\frac{dv_1}{dx_3}\right|_{x_3=0} = \frac{wa^3}{6EI}$$ **Ans.**

The Elastic Curve:

$$v_1 = \frac{wx_1^2}{24EI}(-x_1^2 + 4ax_1 - 6a^2)$$ **Ans.**

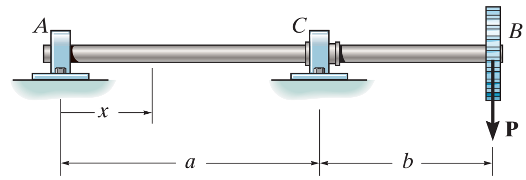
$$v_2 = \frac{wa^3}{24EI}(4x_3 + a - 4L)$$ **Ans.**

$$v_B = \left. v_3 \right|_{x_3=a} = \frac{wa^3}{24EI}(a - 4L)$$ **Ans.**

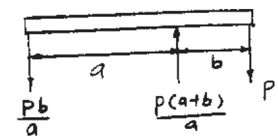
Ans:
$\theta_B = -\frac{wa^3}{6EI}, v_1 = \frac{wx_1^2}{24EI}(-x_1^2 + 4ax_1 - 6a^2),$
$v_2 = \frac{wa^3}{24EI}(4x_3 + a - 4L), v_B = \frac{wa^3}{24EI}(a - 4L)$

12–31.

The shaft is supported at A by a journal bearing and at C by a thrust bearing. Determine the equation of the elastic curve. EI is constant.



SOLUTION



$$M = -\frac{Pb}{a}\langle x - 0 \rangle - \left(-\frac{P(a+b)}{a}\langle x - a \rangle\right) = -\frac{Pb}{a}x + \frac{P(a+b)}{a}\langle x - a \rangle$$

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -\frac{Pb}{a}x + \frac{P(a+b)}{a}\langle x - a \rangle$$

$$EI\frac{dv}{dx} = -\frac{Pb}{2a}x^2 + \frac{P(a+b)}{2a}\langle x - a \rangle^2 + C_1 \qquad \textbf{(1)}$$

$$EIv = -\frac{Pb}{6a}x^3 + \frac{P(a+b)}{6a}\langle x - a \rangle^3 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions:

At $x = 0, v = 0$

From Eq. (2)

$0 = -0 + 0 + 0 + C_2; \qquad C_2 = 0$

At $x = a, v = 0$

From Eq. (2)

$$0 = -\frac{Pb}{6a}(a^3) + 0 + C_1a + 0; \qquad C_1 = \frac{Pab}{6}$$

From Eq. (2),

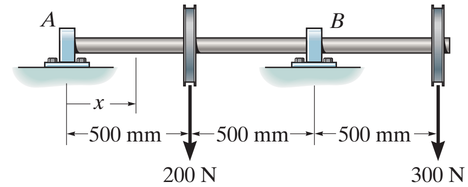
$$v = \frac{1}{EI}\left[-\frac{Pb}{6a}x^3 + \frac{P(a+b)}{6a}\langle x - a \rangle^3 + \frac{Pab}{6}x\right] \qquad \textbf{Ans.}$$

Ans:

$$v = \frac{1}{EI}\left[-\frac{Pb}{6a}x^3 + \frac{P(a+b)}{6a}\langle x - a \rangle^3 + \frac{Pab}{6}x\right]$$

***12–32.**

The shaft supports the two pulley loads shown. Determine the equation of the elastic curve. The bearings at A and B exert only vertical reactions on the shaft. EI is constant.



SOLUTION

$M = -50 < x - 0 > - 200 < x - 0.5 > + 550 < x - 1.0 >$

$M = -50x - 200 < x - 0.5 > + 550 < x - 1.0 >$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -50x - 200 < x - 0.5 > + 550 < x - 1.0 >$$

$$EI\frac{dv}{dx} = -25x^2 - 100 < x - 0.5 >^2 + 275 < x - 1.0 >^2 + C_1$$

$$EIv = -8.333x^3 - 33.333 < x - 0.5 >^3 + 91.67 < x - 1.0 >^3 + C_1x + C_2 \qquad (1)$$

Boundary conditions:

$v = 0$ at $x = 0$

From Eq. (1):

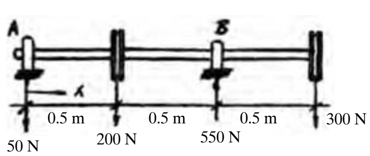
$C_2 = 0$

$v = 0$ at $x = 1.0$ m

$0 = -8.333 - 4.167 + 0 + C_1$

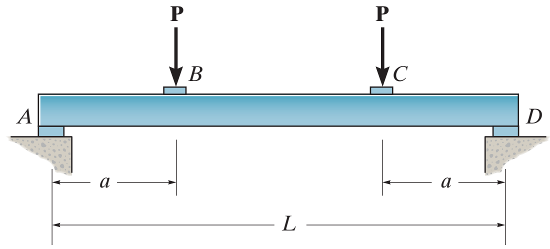
$C_1 = 12.5$

$$v = \frac{1}{EI} = [-8.33x^3 - 33.3 < x - 0.5 >^3 + 91.7 < x - 1.0 >^3 + 12.5x]\ \mathrm{N \cdot m^3}$$ **Ans.**

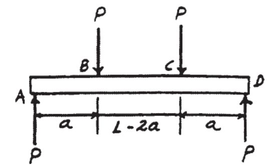


12–33.

The beam is made of a ceramic material. If it is subjected to the elastic loading shown, and the moment of inertia is I and the beam has a measured maximum deflection Δ at its center, determine the modulus of elasticity, E. The supports at A and D exert only vertical reactions on the beam.



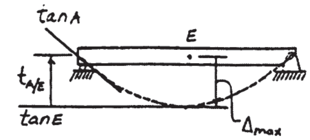
SOLUTION



Moment-Area Theorems: Due to symmetry, the slope at midspan (point E) is zero. Hence the maximum displacement is

$$\Delta_{\max} = t_{A/E} = \left(\frac{Pa}{EI}\right)\left(\frac{L-2a}{2}\right)\left(a + \frac{L-2a}{4}\right) + \frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\left(\frac{2}{3}a\right)$$

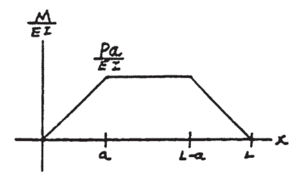
$$= \frac{Pa}{24EI}(3L^2 - 4a^2)$$



Require, $\Delta_{\max} = \Delta$, then

$$\Delta = \frac{Pa}{24EI}(3L^2 - 4a^2)$$

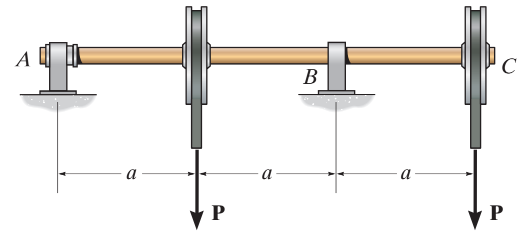
$$E = \frac{Pa}{24\Delta I}(3L^2 - 4a^2) \qquad \textbf{Ans.}$$



Ans:
$E = \dfrac{Pa}{24\Delta I}(3L^2 - 4a^2)$

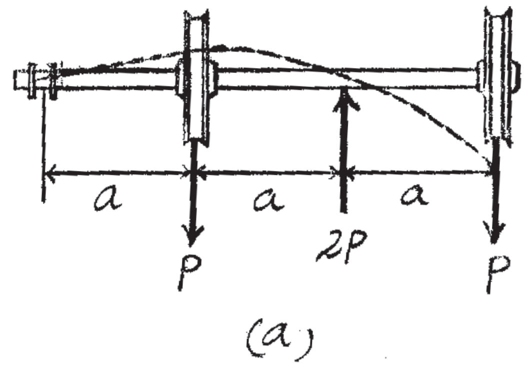
12–34.

Determine the equation of the elastic curve, the maximum deflection in region AB, and the deflection of end C. EI is constant.



SOLUTION

Support Reactions and Elastic Curve: As shown in Fig. a.



Moment Function:

$$M = -P\langle x - a\rangle - (-2P)\langle x - 2a\rangle$$
$$= -P\langle x - a\rangle + 2P\langle x - 2a\rangle$$

Equations of Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -P\langle x - a\rangle + 2P\langle x - 2a\rangle$$

$$EI\frac{dv}{dx} = \frac{-P}{2}\langle x - a\rangle^2 + P\langle x - 2a\rangle^2 + C_1 \qquad \textbf{(1)}$$

$$EIv = \frac{-P}{6}\langle x - a\rangle^3 + \frac{P}{3}\langle x - 2a\rangle^3 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions: At $x = 0, v = 0$. Then Eq. (2) gives

$$EI(0) = -0 + 0 + C_1(0) + C_2 \qquad C_2 = 0$$

At $x = 2a, v = 0$. Then Eq. (2) gives

$$EI(0) = -\frac{P}{6}(2a - a)^3 + \frac{P}{3}(2a - 2a)^3 + C_1(2a) + 0 \qquad C_1 = \frac{Pa^2}{12}$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{P}{12EI}[-6\langle x - a\rangle^2 + 12\langle x - 2a\rangle^2 + a^2]$$

Assuming that $\frac{dv}{dx} = 0$ occurs in the region $a < x < 2a$,

$$-6(x - a)^2 + 0 + a^2 = 0 \qquad x = 1.4082a \qquad \textbf{(O.K.)}$$

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{P}{12EI}[-2\langle x - a\rangle^3 + 4\langle x - 2a\rangle^3 + a^2x] \qquad \textbf{Ans.}$$

$(v_{\max})_{AB}$ occurs at $x = 1.4082a$, where $\frac{dv}{dx} = 0$. Thus,

$$(v_{\max})_{AB} = v|_{x=1.4082a} = \frac{P}{12EI}[-2\langle 1.4082a - a\rangle^3 + 0 + a^2(1.4082a)]$$

$$= \frac{0.106Pa^3}{EI} \qquad \textbf{Ans.}$$

At $C, x = 3a$. Thus,

$$v_C = v|_{x=3a} = \frac{P}{12EI}[-2\langle 3a - a\rangle^3 + 4(3a - 2a)^3 + a^2(3a)]$$

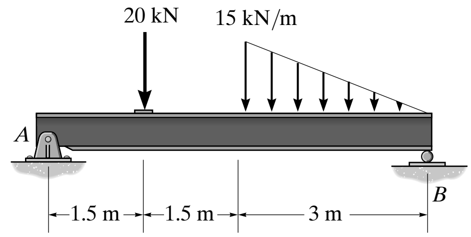
$$= -\frac{3Pa^3}{4EI} \qquad \textbf{Ans.}$$

Ans:

$v = \frac{P}{12EI}[-2\langle x - a\rangle^3 + 4\langle x - 2a\rangle^3 + a^2x]$,

$(v_{\max})_{AB} = \frac{0.106Pa^3}{EI}, v_C = -\frac{3Pa^3}{4EI}$

12–35. Determine the maximum deflection of the simply supported beam. $E = 200$ GPa and $I = 65.0(10^6)$ mm^4.



Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Function. From Fig. *b*, we obtain

$$M = -(-22.5)\langle x - 0\rangle - 20\langle x - 1.5\rangle - \frac{15}{2}\langle x - 3\rangle^2 - \left(-\frac{5}{6}\right)\langle x - 3\rangle^3$$

$$= 22.5x - 20\langle x - 1.5\rangle - 7.5\langle x - 3\rangle^2 + \frac{5}{6}\langle x - 3\rangle^3$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = 22.5x - 20\langle x - 1.5\rangle - 7.5\langle x - 3\rangle^2 + \frac{5}{6}\langle x - 3\rangle^3$$

$$EI\frac{dv}{dx} = 11.25x^2 - 10\langle x - 1.5\rangle^2 - 2.5\langle x - 3\rangle^3 + \frac{5}{24}\langle x - 3\rangle^4 + C_1 \qquad (1)$$

$$EIv = 3.75x^3 - \frac{10}{3}\langle x - 1.5\rangle^3 - 0.625\langle x - 3\rangle^4 + \frac{1}{24}\langle x - 3\rangle^5 + C_1x + C_2 \; (2)$$

Boundary Conditions. At $x = 0, v = 0$. Then, Eq. (2) gives

$$0 = 0 - 0 - 0 + 0 + C_1(0) + C_2 \qquad C_2 = 0$$

At $x = 6$ m, $v = 0$. Then Eq. (2) gives

$$0 = 3.75\left(6^3\right) - \frac{10}{3}\langle 6 - 1.5\rangle^3 - 0.625\langle 6 - 3\rangle^4 + \frac{1}{24}\langle 6 - 3\rangle^5 + C_1(6) + C_2$$

$$C_1 = -77.625\,\text{kN}\cdot\text{m}^2$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{1}{EI}\left[11.25x^2 - 10\langle x - 1.5\rangle^2 - 2.5\langle x - 3\rangle^3 + \frac{5}{24}\langle x - 3\rangle^4 - 77.625\right]$$

Assuming that $\frac{dv}{dx} = 0$ occurs in the region 1.5 m $< x <$ 3 m, then

$$\frac{dv}{dx} = 0 = \frac{1}{EI}\left[11.25x^2 - 10\langle x - 1.5\rangle^2 - 0 + 0 - 77.625\right]$$

Solving for the root 1.5 m $< x <$ 3 m,

$$x = 2.970 \text{ m O.K.}$$

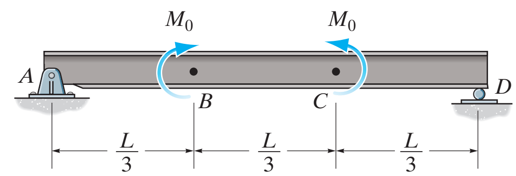
Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{1}{EI}\left[3.75x^3 - \frac{10}{3}\langle x - 1.5\rangle^3 - 0.625\langle x - 3\rangle^4 + \frac{1}{24}\langle x - 3\rangle^5 - 77.625x\right] \quad \textbf{Ans.}$$

v_{max} occurs at $x = 2.970$ m, where $\frac{dv}{dx} = 0$. Thus,

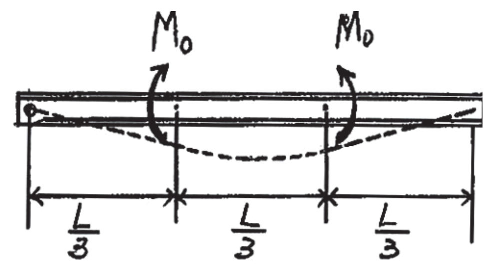
***12–36.**

Determine the equation of the elastic curve, the slope at A, and the deflection at B. EI is constant.



SOLUTION

Support Reactions and Elastic Curve: As shown in Fig. *a*.



(a)

Moment Function:

$$M = -(-M_0)\left\langle x - \frac{L}{3}\right\rangle^0 - M_0\left\langle x - \frac{2}{3}L\right\rangle^0$$

$$= M_0\left\langle x - \frac{L}{3}\right\rangle^0 - M_0\left\langle x - \frac{2}{3}L\right\rangle^0$$

Equations of Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = M_0\left\langle x - \frac{L}{3}\right\rangle^0 - M_0\left\langle x - \frac{2}{3}L\right\rangle^0$$

$$EI\frac{dv}{dx} = M_0\left\langle x - \frac{L}{3}\right\rangle - M_0\left\langle x - \frac{2}{3}L\right\rangle + C_1 \qquad \textbf{(1)}$$

$$EIv\frac{M_0}{2}\left\langle x - \frac{L}{3}\right\rangle^2 - \frac{M_0}{2}\left\langle x - \frac{2}{3}L\right\rangle^2 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions: Due to symmetry, $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$. Then Eq. (1) gives

$$EI(0) = M_0\left(\frac{L}{2} - \frac{L}{3}\right) - 0 + C_1 \qquad C_1 = -\frac{M_0L}{6}$$

At $x = 0, v = 0$. Then, Eq. (2) gives

$$EI(0) = 0 - 0 + C_1(0) + C_2 \qquad C_2 = 0$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{M_0}{6EI}\left[6\left\langle x - \frac{L}{3}\right\rangle - 6\left\langle x - \frac{2}{3}L\right\rangle - L\right]$$

At $A, x = 0$. Thus,

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{M_0}{6EI}[6(0) - 6(0) - L] = -\frac{M_0L}{6EI} \qquad \textbf{Ans.}$$

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{M_0}{6EI}\left[3\left\langle x - \frac{L}{3}\right\rangle^2 - 3\left\langle x - \frac{2}{3}L\right\rangle^2 - Lx\right] \qquad \textbf{Ans.}$$

At $B, x = \frac{L}{3}$. Thus,

$$v_B = v|_{x=\frac{L}{3}} = \frac{M_0}{6EI}\left[3(0) - 3(0) - L\left(\frac{L}{3}\right)\right]$$

$$= -\frac{M_0L^2}{18EI} \qquad \textbf{Ans.}$$

Ans:

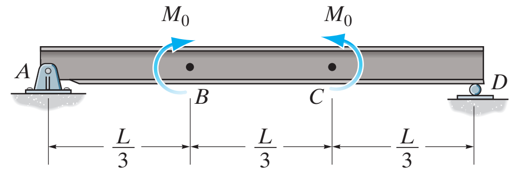
$\theta_A = -\frac{M_0L}{6EI}$,

$v = \frac{M_0}{6EI}\left[3\left\langle x - \frac{L}{3}\right\rangle^2 - 3\left\langle x - \frac{2}{3}L\right\rangle^2 - Lx\right]$,

$v_B = -\frac{M_0L^2}{18EI}$

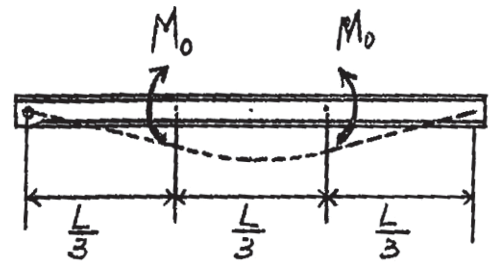
12–37.

Determine the equation of the elastic curve and the maximum deflection of the simply supported beam. EI is constant.



SOLUTION

Support Reactions and Elastic Curve: As shown in Fig. *a*.



(a)

Moment Function:

$$M = -(-M_0)\left\langle x - \frac{L}{3}\right\rangle^0 - M_0\left\langle x - \frac{2}{3}L\right\rangle^0$$

$$= M_0\left\langle x - \frac{L}{3}\right\rangle^0 - M_0\left\langle x - \frac{2}{3}L\right\rangle^0$$

Equations of Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = M_0\left\langle x - \frac{L}{3}\right\rangle^0 - M_0\left\langle x - \frac{2}{3}L\right\rangle^0$$

$$EI\frac{dv}{dx} = M_0\left\langle x - \frac{L}{3}\right\rangle - M_0\left\langle x - \frac{2}{3}L\right\rangle + C_1 \quad \textbf{(1)}$$

$$EIv\frac{M_0}{2}\left\langle x - \frac{L}{3}\right\rangle^2 - \frac{M_0}{2}\left\langle x - \frac{2}{3}L\right\rangle^2 + C_1x + C_2 \quad \textbf{(2)}$$

Boundary Conditions: Due to symmetry, $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$. Then Eq. (1) gives

$$EI(0) = M_0\left(\frac{L}{2} - \frac{L}{3}\right) - 0 + C_1 \qquad C_1 = -\frac{M_0L}{6}$$

At $x = 0, v = 0$. Then, Eq. (2) gives

$$EI(0) = 0 - 0 + C_1(0) + C_2 \qquad C_2 = 0$$

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{M_0}{6EI}\left[3\left\langle x - \frac{L}{3}\right\rangle^2 - 3\left\langle x - \frac{2}{3}L\right\rangle^2 - Lx\right] \quad \textbf{Ans.}$$

v_{max} occurs at $x = \frac{L}{2}$, where $\frac{dv}{dx} = 0$. Then,

$$v_{max} = v|_{x=\frac{L}{2}} = \frac{M_0}{6EI}\left[3\left(\frac{L}{2} - \frac{L}{3}\right)^2 - 0 - L\left(\frac{L}{2}\right)\right]$$

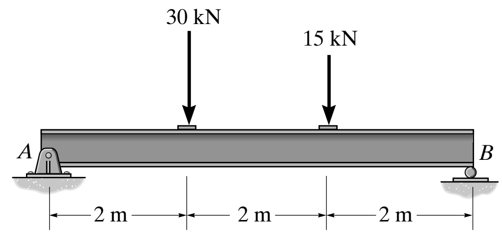
$$= -\frac{5M_0L^2}{72EI} \quad \textbf{Ans.}$$

Ans:

$v = \frac{M_0}{6EI}\left[3\left\langle x - \frac{L}{3}\right\rangle^2 - 3\left\langle x - \frac{2}{3}L\right\rangle^2 - Lx\right]$,

$v_{max} = -\frac{5M_0L^2}{72EI}$

12–38. Determine the maximum deflection of the simply supported beam. $E = 200$ GPa and $I = 65.0(10^6)$ mm^4.



Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Function. From Fig. *a*, we obtain

$$M = -(-25)\langle x - 0\rangle - 30\langle x - 2\rangle - 15\langle x - 4\rangle$$

$$= 25x - 30\langle x - 2\rangle - 15\langle x - 4\rangle$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = 25x - 30\langle x - 2\rangle - 15\langle x - 4\rangle$$

$$EI\frac{dv}{dx} = 12.5x^2 - 15\langle x - 2\rangle^2 - 7.5\langle x - 4\rangle^2 + C_1 \quad (1)$$

$$EIv = 4.1667x^3 - 5\langle x - 2\rangle^3 - 2.5\langle x - 4\rangle^3 + C_1x + C_2 \quad (2)$$

Boundary Conditions. At $x = 0, v = 0$. Then, Eq. (2) gives

$$0 = 0 - 0 - 0 + C_1(0) + C_2 \qquad C_2 = 0$$

At $x = 6$ m, $v = 0$. Then Eq. (2) gives

$$0 = 4.1667\left(6^3\right) - 5\langle 6 - 2\rangle^3 - 2.5\langle 6 - 4\rangle^3 + C_1(6) + C_2$$

$$C_1 = -93.333 \text{ kN}\cdot\text{m}^3$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{1}{EI}\left[12.5x^2 - 15\langle x - 2\rangle^2 - 7.5\langle x - 4\rangle^2 - 93.333\right]$$

Assuming that $\frac{dv}{dx} = 0$ occurs in the region $2\text{ m} < x < 4$ m. Then

$$\frac{dv}{dx} = 0 = \frac{1}{EI}\left[12.5x^2 - 15\langle x - 2\rangle^2 - 93.333\right]$$

$$12.5x^2 - 15(x - 2)^2 - 93.333 = 0$$

$$2.5x^2 - 60x + 153.333 = 0$$

Solving for the root $2\text{ m} < x < 4$ m,

$x = 2.9079$ m O.K.

12–38. Continued

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{1}{EI}\left[4.1667x^3 - 5\langle x-2\rangle^3 - 2.5\langle x-4\rangle^3 - 93.333x\right]$$ **Ans.**

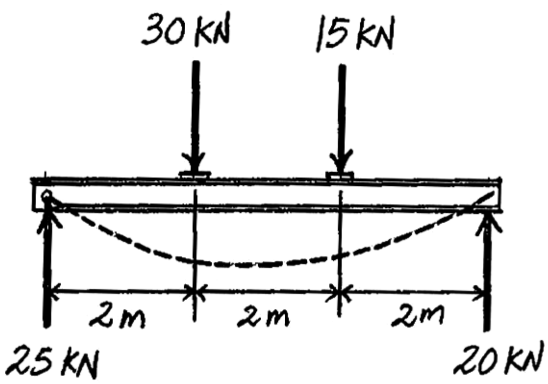
$v_{\max}$ occurs at $x = 2.9079$ m, where $\frac{dv}{dx} = 0$. Thus,

$$v_{\max} = v|_{x=2.9079\text{ ft}}$$

$$= \frac{1}{EI}\left[4.1667\left(2.9079^3\right) - 5\langle 2.9079-2\rangle^3 - 0 - 93.333(2.9079)\right]$$

$$= -\frac{172.69\text{kN}\cdot\text{m}^3}{EI} = -\frac{172.69\left(10^3\right)}{200\left(10^9\right)\left[65.0\left(10^{-6}\right)\right]}$$

$$= -0.01328\text{ m} = 13.3\text{ mm}\downarrow$$ **Ans.**



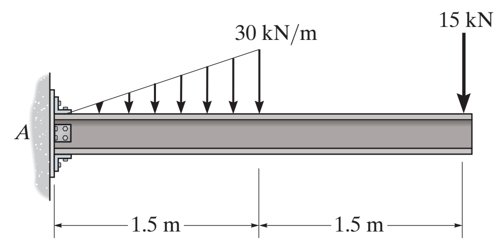
(a)

Ans:

$v = \frac{1}{EI}[4.1667x^3 - 5(x-2)^3 - 2.5(x-4)^3 - 93.333x]$, $v_{\max} = 13.3\text{ mm}\downarrow$

12–39.

Determine the maximum deflection of the cantilevered beam. Take $E = 200$ GPa and $I = 65.0(10^6)$ mm^4.



SOLUTION

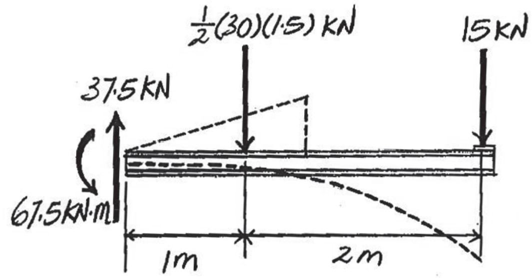
Support Reactions and Elastic Curve: As shown in Fig. *a*.

Moment Function: From Fig. *b*, we obtain

$$M = -(-37.5)\langle x-0\rangle - 67.5\langle x-0\rangle^0 - \frac{20}{6}\langle x-0\rangle^3$$

$$-\left(-\frac{20}{6}\right)\langle x-1.5\rangle^3 - \left(-\frac{30}{2}\right)\langle x-1.5\rangle^2$$

$$= 37.5x - 67.5 - \frac{10}{3}x^3 + \frac{10}{3}\langle x-1.5\rangle^3 + 15\langle x-1.5\rangle^2$$



(a)

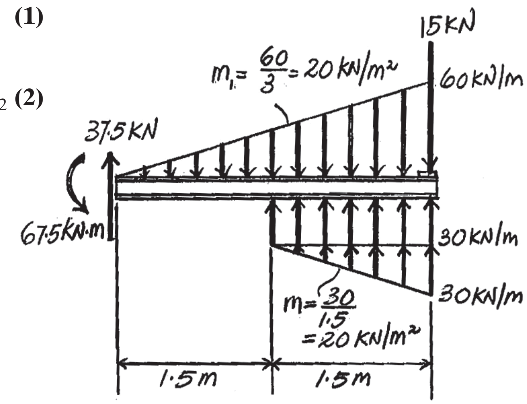
Equations of Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = 37.5x - 67.5 - \frac{10}{3}x^3 + \frac{10}{3}\langle x-1.5\rangle^3 + 15\langle x-1.5\rangle^2$$

$$EI\frac{dv}{dx} = 18.75x^2 - 67.5x - \frac{5}{6}x^4 + \frac{5}{6}\langle x-1.5\rangle^4 + 5\langle x-1.5\rangle^3 + C_1 \quad \textbf{(1)}$$

$$EIv = 6.25x^3 - 33.75x^2 - \frac{1}{6}x^5 + \frac{1}{6}\langle x-1.5\rangle^5 + \frac{5}{4}\langle x-1.5\rangle^4 + C_1x + C_2 \quad \textbf{(2)}$$



(b)

Boundary Conditions: At $x = 0$, $\frac{dv}{dx} = 0$. Then Eq. (1) gives

$$0 = 0 - 0 - 0 + 0 + 0 + C_1 \qquad C_1 = 0$$

At $x = 0$, $v = 0$. Then Eq. (2) gives

$$0 = 0 - 0 - 0 + 0 + 0 + 0 + C_2 \qquad C_2 = 0$$

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{1}{EI}\left[6.25x^3 - 33.75x^2 - \frac{1}{6}x^5 + \frac{1}{6}\langle x-1.5\rangle^5 + \frac{5}{4}\langle x-1.5\rangle^4\right]$$

v_{max} occurs at $x = 3$ m. Thus

$$v_{max} = v|_{x=3\text{ m}}$$

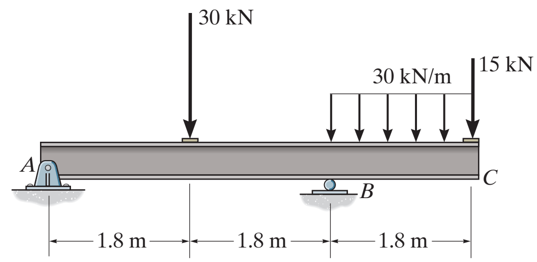
$$= \frac{1}{EI}\left[6.25(3^3) - 33.75(3^2) - \frac{1}{6}(3^5) + \frac{1}{6}(3-1.5)^5 + \frac{5}{4}(3-1.5)^4\right]$$

$$= -\frac{167.91\text{ kN}\cdot\text{m}^3}{EI} = -\frac{167.91(10^3)}{200(10^9)\left[65.0(10^{-6})\right]}$$

$$= -0.01292\text{ m} = -12.9\text{ mm} \qquad \textbf{Ans.}$$

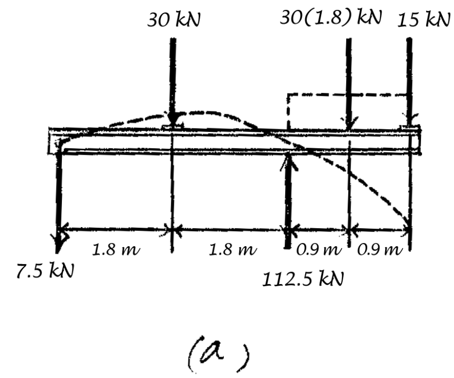
Ans:
$v_{max} = -12.9$ mm

*12–40. Determine the slope at A and the deflection of end C of the overhang beam. $E = 200$ GPa and $I = 84.9(10^{-6})\ \text{m}^4$.



SOLUTION

Support Reactions and Elastic Curve. As shown in Fig. *a*.



Moment Function. From Fig. *a*, we obtain

$$M = -6\langle x-0\rangle - 30\langle x-1.8\rangle + 105\langle x-3.6\rangle - \frac{30}{2}\langle x-3.6\rangle^2$$

$$= -6x - 30\langle x-1.8\rangle + 105\langle x-3.6\rangle - 15\langle x-3.6\rangle^2$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx_2} = M$$

$$EI\frac{dv_2}{dx^2} = -6x - 30\langle x-1.8\rangle + 105\langle x-3.6\rangle - 15\langle x-3.6\rangle^2$$

$$EI\frac{dv}{dx} = -3x^2 - 15\langle x-1.8\rangle^2 + 52.5\langle x-3.6\rangle^2 - 5\langle x-3.6\rangle^3 + C_1 \tag{1}$$

$$EIv = -x^3 - 5\langle x-1.8\rangle^3 + 17.5\langle x-3.6\rangle^3 - 1.25\langle x-3.6\rangle^4 + C_1x + C_2 \tag{2}$$

Boundary Conditions. At $x = 0$, $v = 0$. Then Eq. (2) gives

$$0 = -0 - 0 + 0 - 0 + C_1(0) + C_2 \qquad C_2 = 0$$

At $x = 3.6$ m, $v = 0$. Then Eq. (2) gives

$$0 = -3.6^3 - 5\langle 3.6-1.8\rangle^3 + 0 - 0 + C_1(3.6) + 0 \qquad C_1 = 21.06\ \text{kN}\cdot\text{m}^2$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{1}{EI}\left[-3x^2 - 15\langle x-1.8\rangle^2 + 52.5\langle x-3.6\rangle^2 - 5\langle x-3.6\rangle^3 + 21.06\right]\ \text{kN}\cdot\text{m}^2$$

At A, $x = 0$. Thus,

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{1}{EI}[-0 - 0 + 0 - 0 + 21.06]$$

$$= \frac{21.06\ \text{kN}\cdot\text{m}^2}{EI} = \frac{21.06(10^3)\ \text{N}\cdot\text{m}^2}{[200(10^9)\ \text{N/m}^2][84.9(10^{-6})\ \text{m}^4]} = 0.00124\ \text{rad} \ \measuredangle\theta_A \qquad \textbf{Ans.}$$

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{1}{EI}\left[-x^3 - 5\langle x-1.8\rangle^3 + 17.5\langle x-3.6\rangle^3 - 1.25\langle x-3.6\rangle^4 + 21.06x\right]\ \text{kN}\cdot\text{m}^3 \qquad \textbf{Ans.}$$

12–40. Continued

At C, $x = 5.4$ m. Thus,

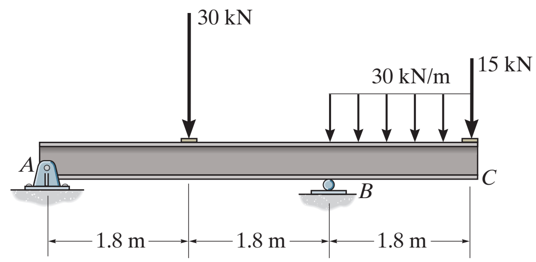
$$vC = v\big|_{x=5.4\text{ m}} = \frac{1}{EI}\left[-5.4x^3 - 5(5.4-1.8)^3 + 17.5(5.4-3.6)^3 - 1.25(5.4-3.6)^4 + 21.06(5.4)\right] \text{ kN}\cdot\text{m}^3$$

$$= \frac{188.08 \text{ kN}\cdot\text{m}^3}{EI} = \frac{188.08(10^3)\text{ N}\cdot\text{m}^3}{[200(10^9)\text{ N/m}^2][84.9(10^{-6})\text{ m}^4]}$$

$$= 0.1108 \text{ m} = 11.1 \text{ mm} \downarrow$$ **Ans.**

Ans:
$vC = 11.1$ mm $\downarrow$

12–41. Determine the maximum deflection in region AB of the overhang beam. $E = 200$ GPa and $I = 84.9(10^{-6})$ m^4.



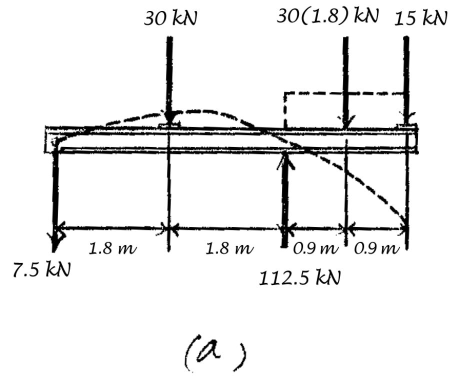
SOLUTION

Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Function. From Fig. *a*, we obtain

$$M = -6\langle x-0\rangle - 30\langle x-1.8\rangle + 105\langle x-3.6\rangle - \frac{30}{2}\langle x-3.6\rangle^2$$

$$= -6x - 30\langle x-1.8\rangle + 105\langle x-3.6\rangle - 15\langle x-3.6\rangle^2$$



Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx_2} = M$$

$$EI\frac{d^2v}{dx^2} = -6x - 30\langle x-1.8\rangle + 105\langle x-3.6\rangle - 15\langle x-3.6\rangle^2$$

$$EI\frac{dv}{dx} = -3x^2 - 15\langle x-1.8\rangle^2 + 52.5\langle x-3.6\rangle^2 - 5\langle x-3.6\rangle^3 + C_1 \qquad (1)$$

$$EIv = -x^3 - 5\langle x-1.8\rangle^3 + 17.5\langle x-3.6\rangle^3 - 5\langle x-3.6\rangle^4 + C_1x + C_2 \qquad (2)$$

Boundary Conditions. At $x = 0, v = 0$. Then Eq. (2) gives

$$0 = -0 - 0 + 0 - 0 + C_1(0) + C_2 \qquad C_2 = 0$$

At $x = 3.6$ m, $v = 0$. Then Eq. (2) gives

$$0 = -3.6^3 - 5\langle 3.6-1.8\rangle^3 + 0 - 0 + C_1(3.6) + 0 \qquad C_1 = 21.06 \text{ kN}\cdot\text{m}^2$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{1}{EI}\left[-3x^2 - 15\langle x-1.8\rangle^2 + 52.5\langle x-3.6\rangle^2 - 5\langle x-3.6\rangle^3 + 21.06\right] \text{kN}\cdot\text{m}^2$$

Assuming that $\frac{dv}{dx} = 0$ occurs in the region $1.8 \text{ m} < x < 3.6$ m, then

$$\frac{dv}{dx} = 0 = \frac{1}{EI}\left[-3x^2 - 15(x-1.8)^2 + 0 - 0 + 21.06\right]$$

$$= -3x^2 - 15(x-1.8)^2 + 21.06 = 0$$

$$18x^2 - 54x + 27.54 = 0$$

Solving for the root $1.8 \text{ m} < x < 3.6$ m,

$$x = 2.3485$$

12–41. Continued

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{1}{EI}\left[-x^3 - 5\langle x-1.8\rangle^3 + 17.5\langle x-3.6\rangle^3 - 1.25\langle x-3.6\rangle^4 + 21.06x\right] \text{ kN}\cdot\text{m}^3$$

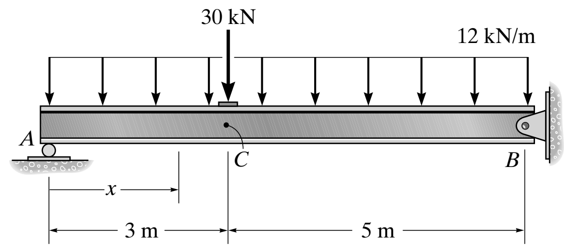
$$(v_{\max})_{AB} = v\big|_{x=2.3485\text{ m}} = \frac{1}{EI}\left[-2.3485^3 - 5(2.3485-1.8)^3 + 0 - 0 + 21.06(2.3485)\right] \text{ kN}\cdot\text{m}^3$$

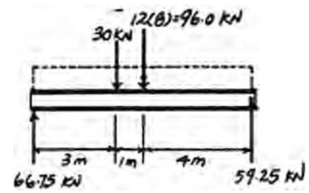
$$= \frac{35.68 \text{ kN}\cdot\text{m}^3}{EI} = \frac{35.68(10^3)\text{ N}\cdot\text{m}^3}{[200(10^9)\text{ N/m}^2][84.9(10^{-6})\text{ m}^4]}$$

$$= 0.002101 \text{ m} = 2.10 \text{ mm} \uparrow$$ **Ans.**

Ans:
$(v_{\max})_{AB} = 3.128 \text{ mm}\uparrow$

12–42. The beam is subjected to the load shown. Determine the slopes at A and B and the displacement at C. EI is constant.





The negative sign indicates downward displacement.

Support Reactions and Elastic Curve: As shown on FBD.

Moment Function: Using the discontinuity function,

$$M = 66.75 < x - 0 > -6 < x - 0 >^2 - 30 < x - 3 >$$

$$= 66.75x - 6x^2 - 30 < x - 3 >$$

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = 66.75x - 6x^2 - 30 < x - 3 >$$

$$EI\frac{dv}{dx} = 33.375x^2 - 2x^3 - 15 < x - 3 >^2 + C_1 \quad [1]$$

$$EI\,v = 11.125x^3 - 0.5x^4 - 5 < x - 3 >^3 + C_1x + C_2 \quad [2]$$

Boundary Conditions:

$v = 0$ at $x = 0$. From Eq.[2], $C_2 = 0$

$v = 0$ at $x = 8$ m. From Eq.[2],

$$0 = 11.125(8^3) - 0.5(8^4) - 5(8 - 3)^3 + C_1(8)$$

$$C_1 = -377.875$$

The Slope: Substituting the value of C_1 into Eq.[1],

$$\frac{dv}{dx} = \frac{1}{EI}\left\{33.375x^2 - 2x^3 - 15 < x - 3 >^2 - 377.875\right\}\ \text{kN}\cdot\text{m}^2$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{1}{EI}\{0 - 0 - 0 - 377.875\} = -\frac{378\ \text{kN}\cdot\text{m}^2}{EI} \quad \textbf{Ans.}$$

$$\theta_B = \left.\frac{dv}{dx}\right|_{x=8\text{m}}$$

$$= \frac{1}{EI}\left\{33.375(8^2) - 2(8^3) - 15(8 - 3)^2 - 377.875\right\}$$

$$= \frac{359\ \text{kN}\cdot\text{m}^2}{EI} \quad \textbf{Ans.}$$

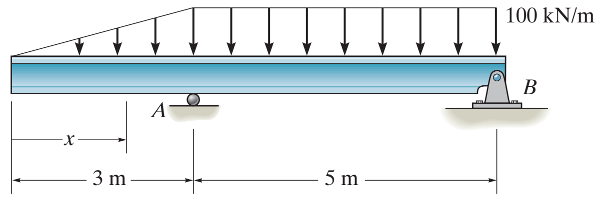
The Elastic Curve: Substituting the values of C_1 and C_2 into Eq.[2],

$$v = \frac{1}{EI}\left\{11.125x^3 - 0.5x^4 - 5 < x - 3 >^3 - 377.875\,x\right\}\ \text{kN}\cdot\text{m}^3$$

$$v_C = v|_{x=3\text{m}} = \frac{1}{EI}\left\{11.125(3^3) - 0.5(3^4) - 0 - 377.875(3)\right\}$$

$$= -\frac{874\ \text{kN}\cdot\text{m}^3}{EI} \quad \textbf{Ans.}$$

12–43. The beam is subjected to the load shown. Determine the equation of the elastic curve. EI is constant.



SOLUTION

$$M = \frac{1}{6}\left(-\frac{100}{3}\right)\langle x-0\rangle^3 + 430\langle x-3\rangle + \frac{1}{6}\left(-\frac{100}{3}\right)\langle x-3\rangle^3$$

$$M = \left(\frac{-50}{9}\right)x^3 + 430 < x - 3 > + \left(\frac{50}{9}\right) < x - 3 >^3$$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M = \left(-\frac{50}{9}\right)x^3 + 430 < x - 3 > + \left(\frac{50}{9}\right) < x - 3 >^3$$

$$EI\frac{dv}{dx} = -\left(\frac{25}{18}\right)x^4 + 215 < x - 3 >^2 + \left(\frac{25}{18}\right) < x - 3 >^4 + C_1$$

$$EIv = -\left(\frac{5}{18}\right)x^5 + \left(\frac{215}{3}\right) < x - 3 >^3 + \left(\frac{5}{18}\right) < x - 3 >^5 + C_1x + C_2 \qquad (1)$$

Boundary conditions:

$v = 0$ at $x = 3$ m

From Eq.(1)

$$0 = -67.5 + 0 + 0 + 3C_1 + C_2$$

$$3C_1 + C_2 = 67.5 \qquad (2)$$

$v = 0$ at $x = 8$ m

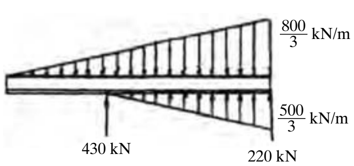
$$0 = -\left(\frac{81920}{9}\right) + \left(\frac{26875}{3}\right) + \left(\frac{15625}{18}\right) + 8C_1 + C_2$$

$$48C_1 + 6C_2 = -4345 \qquad (3)$$

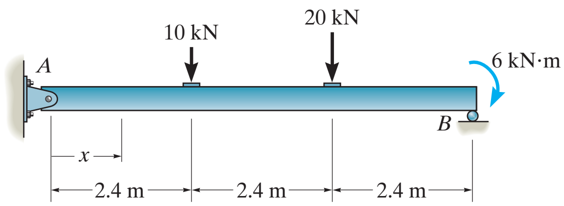
Solving Eqs. (2) and (3) yields,

$C_1 = -158.33\ \text{kN}\cdot\text{m}^2 \qquad C_2 = 542.5\ \text{kN}\cdot\text{m}^3$

$$v = \frac{1}{EI}\left[-0.278x^5 + 71.7\langle x-3\rangle^3 + 0.278\langle x-3\rangle^5 - 158x + 542.5\right]\ \text{kN}\cdot\text{m}^3$$ **Ans.**



***12–44.** The beam is subjected to the loads shown. Determine the equation of the elastic curve. EI is constant.



SOLUTION

$M = 12.5 < x - 0 > -10 < x - 2.4 > -20 < x - 4.8 >$

$M = 12.5x - 10 < x - 2.4 > -20 < x - 4.8 >$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M = 12.5x - 10 < x - 2.4 > - 20 < x - 4.8 >$$

$$EI\frac{dv}{dx} = 6.25x^2 - 5 < x - 2.4 >^2 - 10 < x - 4.8 >^2 + C_1$$

$$EIv = 2.0833x^3 - 1.6667 < x - 2.4 >^3 - 3.3333 < x - 4.8 >^3 + C_1x + C_2 \qquad (1)$$

Boundary conditions:

$v = 0 \quad$ at $\quad x = 0$

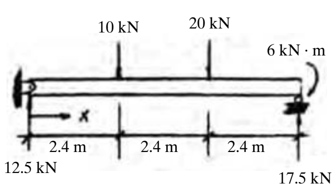
From Eq. (1), $C_2 = 0$

$v = 0 \quad$ at $\quad x = 7.2$ m

$$0 = 777.60 - 184.32 - 46.06 + 7.2C_1$$

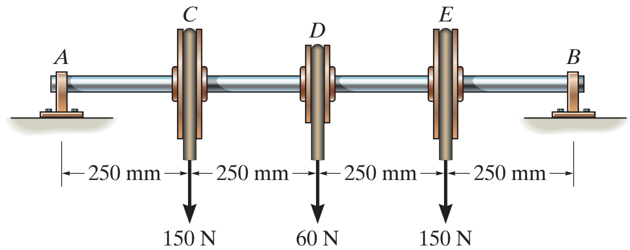
$$C_1 = -76.00$$

$$v = \frac{1}{EI}\left[2.08x^3 - 1.67 < x - 2.4 >^3 - 3.33 < x - 4.8 >^3 - 76.0x\right] \text{ kN}\cdot\text{m}^3 \qquad \textbf{Ans.}$$



12–45.

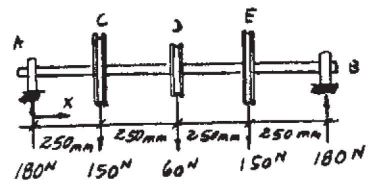
Determine the deflection at each of the pulleys C, D, and E. The shaft is made of steel and has a diameter of 30 mm. $E_{st} = 200$ GPa.



SOLUTION

$$M = -(-180)\langle x - 0\rangle - 150\langle x - 0.25\rangle - 60\langle x - 0.5\rangle - 150\langle x - 0.75\rangle$$

$$M = 180x - 150\langle x - 0.25\rangle - 60\langle x - 0.5\rangle - 150\langle x - 0.75\rangle$$



Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M = 180x - 150\langle x - 0.25\rangle - 60\langle x - 0.5\rangle - 150\langle x - 0.75\rangle$$

$$EI\frac{dv}{dx} = 90x^2 - 75\langle x - 0.25\rangle^2 - 30\langle x - 0.50\rangle^2 - 75\langle x - 0.75\rangle^2 + C_1 \quad \textbf{(1)}$$

$$EIv = 30x^3 - 25\langle x - 0.25\rangle^3 - 10\langle x - 0.50\rangle^3 - 25\langle x - 0.75\rangle^3 + C_1x + C_2 \quad \textbf{(2)}$$

Boundary Conditions:

$v = 0 \quad$ at $\quad x = 0$

From Eq. (2),

$$C_2 = 0$$

$v = 0 \quad$ at $\quad x = 1.0$ m

$$0 = 30 - 10.55 - 1.25 - 0.39 + C_1$$

$$C_1 = -17.8125$$

$$v = \frac{1}{EI}[30x^3 - 25\langle x - 0.25\rangle^3 - 10\langle x - 0.5\rangle^3 - 25\langle x - 0.75\rangle^3 - 17.8125x]$$

$$v_C = v\Big|_{x=0.25\text{ m}} = \frac{-3.984}{EI} = \frac{-3.984}{200(10^9)\frac{\pi}{4}(0.015)^4}$$

$$= -0.000501 \text{ m} = -0.501 \text{ mm} \qquad \textbf{Ans.}$$

$$v_D = v\Big|_{x=0.5\text{ m}} = \frac{-5.547}{200(10^9)\frac{\pi}{4}(0.015)^4} = -0.000698 \text{ m} = -0.698 \text{ mm} \qquad \textbf{Ans.}$$

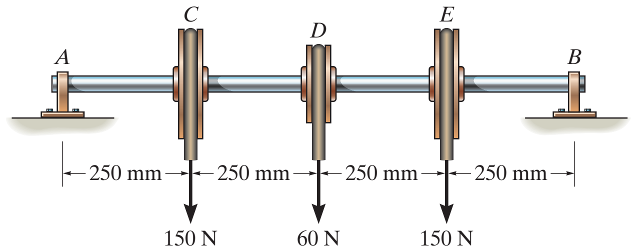
$$v_E = v\Big|_{x=0.75\text{ m}} = \frac{-3.984}{EI} = -0.501 \text{ mm} \quad \text{(symmetry check !)} \qquad \textbf{Ans.}$$

The negative signs indicate downward displacement.

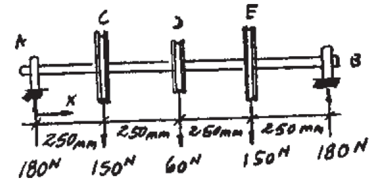
Ans:
$v_C = -0.501$ mm, $v_D = -0.698$ mm,
$v_E = -0.501$ mm

12–46.

Determine the slope of the shaft at A and B. The shaft is made of steel and has a diameter of 30 mm. The bearings only exert vertical reactions on the shaft. $E_{st} = 200$ GPa.



SOLUTION



$$M = -(-180)\langle x - 0\rangle - 150\langle x - 0.25\rangle - 60\langle x - 0.5\rangle - 150\langle x - 0.75\rangle$$

$$M = 180x - 150\langle x - 0.25\rangle - 60\langle x - 0.5\rangle - 150\langle x - 0.75\rangle$$

Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M = 180x - 150\langle x - 0.25\rangle - 60\langle x - 0.5\rangle - 150\langle x - 0.75\rangle$$

$$EI\frac{dv}{dx} = 90x^2 - 75\langle x - 0.25\rangle^2 - 30\langle x - 0.50\rangle^2 - 75\langle x - 0.75\rangle^2 + C_1 \qquad \textbf{(1)}$$

$$EIv = 30x^3 - 25\langle x - 0.25\rangle^3 - 10\langle x - 0.50\rangle^3 - 25\langle x - 0.75\rangle^3 + C_1x + x_2 \quad \textbf{(2)}$$

Boundary Conditions:

$$v = 0 \quad \text{at} \quad x = 0$$

From Eq. (2)

$$C_2 = 0$$

$$v = 0 \quad \text{at} \quad x = 1.0 \text{ m}$$

$$0 = 30 - 10.55 - 1.25 - 0.39 + C_1$$

$$C_1 = -17.8125$$

$$\frac{dv}{dx} = \frac{1}{EI}[90x^2 - 75\langle x - 0.25\rangle^2 - 30\langle x - 0.5\rangle^2 - 75\langle x - 0.75\rangle^2 - 17.8125] \quad \textbf{(3)}$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{-17.8125}{EI} = \frac{-17.8125}{200(10^9)\frac{\pi}{4}(0.015)^4} = -0.00224 \text{ rad} = -0.128° \qquad \textbf{Ans.}$$

The negative sign indicates clockwise rotation.

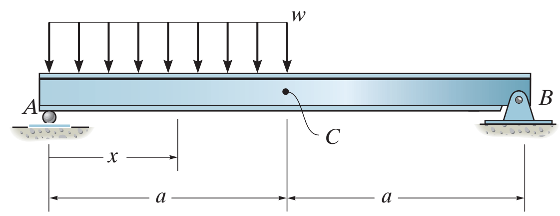
$$\theta_B = \left.\frac{dv}{dx}\right|_{x=1\text{ m}} = \frac{17.8125}{EI} = 0.128° \qquad \textbf{Ans.}$$

The positive result indicates counterclockwise rotation.

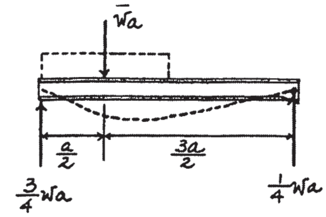
Ans:
$\theta_A = -0.128°, \theta_B = 0.128°$

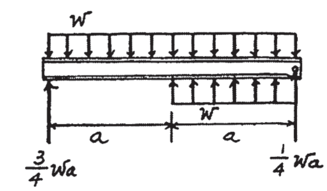
12–47.

Determine the equation of the elastic curve. Specify the slopes at A and B. EI is constant.



SOLUTION





Support Reactions and Elastic Curve: As shown on FBD.

Moment Function: Using the discontinuity function,

$$M = \frac{3}{4}wa\langle x - 0\rangle - \frac{w}{2}\langle x - 0\rangle^2 - \left(-\frac{w}{2}\right)\langle x - a\rangle^2$$

$$= \frac{3wa}{4}x - \frac{w}{2}x^2 + \frac{w}{2}\langle x - a\rangle^2$$

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = \frac{3wa}{4}x - \frac{w}{2}x^2 + \frac{w}{2}\langle x - a\rangle^2$$

$$EI\frac{dv}{dx} = \frac{3wa}{8}x^2 - \frac{w}{6}x^3 + \frac{w}{6}\langle x - a\rangle^3 + C_1 \qquad \textbf{(1)}$$

$$EI\,v = \frac{wa}{8}x^3 - \frac{w}{24}x^4 + \frac{w}{24}\langle x - a\rangle^4 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions:

$v = 0$ at $x = 0$. From Eq. (2), $C_2 = 0$

$v = 0$ at $x = 2a$. From Eq. (2),

$$0 = \frac{wa}{8}(2a)^3 - \frac{w}{24}(2a)^4 + \frac{w}{24}(2a - a)^4 + C_1(2a)$$

$$C_1 = -\frac{3wa^3}{16}$$

The Slope: Substituting the value C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{w}{48EI}\left\{18ax^2 - 8x^3 + 8\langle x - a\rangle^3 - 9a^3\right\}$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{w}{48EI}\left\{0 - 0 + 0 - 9a^3\right\} = -\frac{3wa^3}{16EI} \qquad \textbf{Ans.}$$

$$\theta_B = \left.\frac{dv}{dx}\right|_{x=2a} = \frac{w}{48EI}\left\{18a(2a)^2 - 8(2a)^3 + 8(2a - a)^3 - 9a^3\right\}$$

$$= \frac{7wa^3}{48EI} \qquad \textbf{Ans.}$$

The Elastic Curve: Substituting the value of C_1 and C_2 into Eq. (2),

$$v = \frac{w}{48EI}\left[6ax^3 - 2x^4 + 2\langle x - a\rangle^4 - 9a^3x\right] \qquad \textbf{Ans.}$$

Ans:

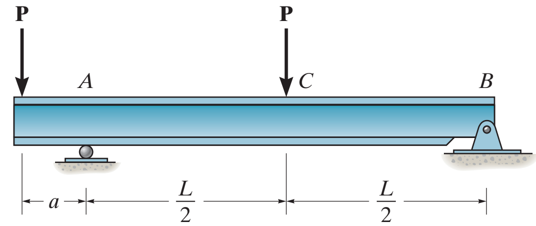
$$\theta_A = -\frac{3wa^3}{16EI}$$

$$\theta_B = \frac{7wa^3}{48EI}$$

$$v = \frac{w}{48EI}\left[6ax^3 - 2x^4 + 2\langle x - a\rangle^4 - 9a^3x\right]$$

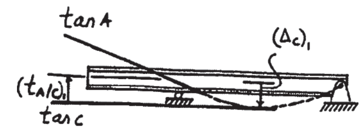
***12–48.**

Determine the value of a so that the displacement at C is equal to zero. EI is constant.



SOLUTION

Moment-Area Theorems:

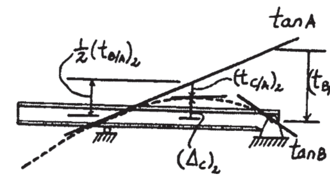


$$(\Delta_C)_1 = (t_{A/C})_1 = \frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) = \frac{PL^3}{48EI}$$

$$(t_{B/A})_2 = \frac{1}{2}\left(-\frac{Pa}{EI}\right)(L)\left(\frac{2}{3}L\right) = -\frac{PaL^2}{3EI}$$

$$(t_{C/A})_2 = \left(-\frac{Pa}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) + \frac{1}{2}\left(-\frac{Pa}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) = -\frac{5PaL^2}{48EI}$$

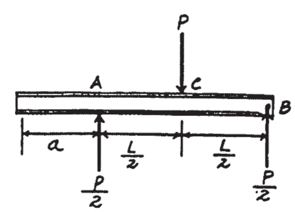
$$(\Delta_C)_2 = \frac{1}{2}|(t_{B/A})_2| - |(t_{C/A})_2| = \frac{1}{2}\left(\frac{PaL^2}{3EI}\right) - \frac{5PaL^2}{48EI} = \frac{PaL^2}{16EI}$$

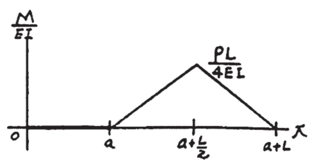


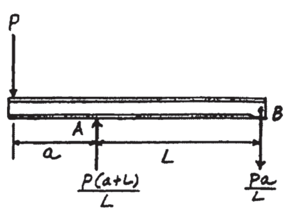
Require, $\Delta_C = 0 = (\Delta_C)_1 - (\Delta_C)_2$

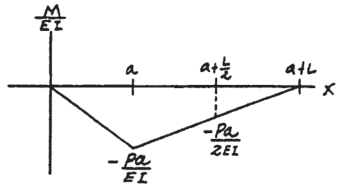
$$0 = \frac{PL^3}{48EI} - \frac{PaL^2}{16EI}$$

$$a = \frac{L}{3} \qquad \textbf{Ans.}$$



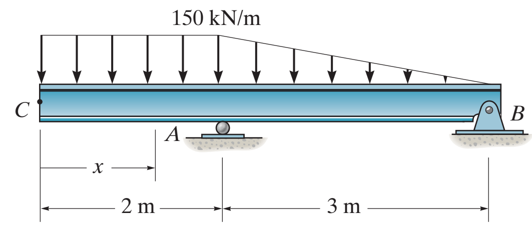






Ans:
$a = \dfrac{L}{3}$

12–49. Determine the displacement at C and the slope at A of the beam.

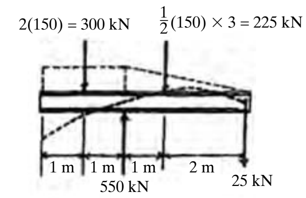


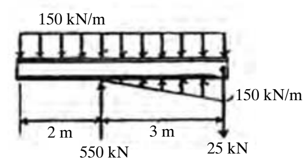
Support Reactions and Elastic Curve: As shown on FBD.

Moment Function: Using the discontinuity function,

$$M = -\frac{1}{2}(150) < x - 0 >^2 - \frac{1}{6}\left(-\frac{150}{3}\right) < x - 2 >^3 - (-550) < x - 2 >$$

$$= -75x^2 + \frac{25}{3} < x - 2 >^3 + 550 < x - 2 >$$





Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -75x^2 + \left(\frac{25}{3}\right) < x - 2 >^3 + 550 < x - 2 >$$

$$EI\frac{dv}{dx} = -25x^3 + \left(\frac{25}{12}\right) < x - 2 >^4 + 275 < x - 2 >^2 + C_1 \qquad [1]$$

$$EIv = -6.25x^4 + \left(\frac{5}{12}\right) < x - 2 >^5 + \left(\frac{275}{3}\right) < x - 2 >^3 + C_1x + C_2 \qquad [2]$$

Boundary Conditions:

$v = 0$ at $x = 2$ m. From Eq.[2],

$$0 = -6.25(2^4) + 0 + 0 + C_1(2) + C_2$$

$$100 = 2C_1 + C_2 \qquad [3]$$

$v = 0$ at $x = 5$ m. From Eq.[2],

$$0 = -6.25(5)^4 + \left(\frac{5}{12}\right)(5-2)^5 + \left(\frac{275}{3}\right)(5-2)^3 + C_1(5) + C_2$$

$$1330 = 5C_1 + C_2 \qquad [4]$$

Solving Eqs. [3] and [4] yields,

$$C_1 = 410 \text{ kN}\cdot\text{m}^2 \qquad C_2 = -720 \text{ kN}\cdot\text{m}^3$$

The Slope: Substitute the value of C_1 into Eq.[1],

$$\frac{dv}{dx} = \frac{1}{EI}\left\{-25x^3 + \left(\frac{25}{12}\right) < x - 2 >^4 + 275 < x - 2 >^2 + 410\right\} \text{ kN}\cdot\text{m}^2$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=2\text{ m}} = \frac{1}{EI}\{-25(2^3) + 0 + 0 + 410\} = \frac{210 \text{ kN}\cdot\text{m}^3}{EI} \quad \theta_A \qquad \textbf{Ans.}$$

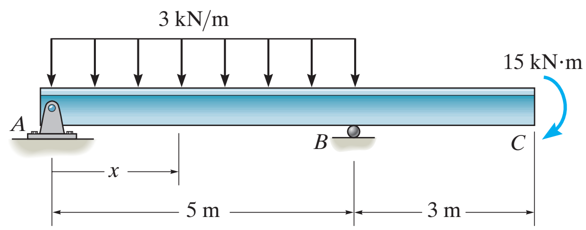
The Elastic Curve: Substitute the values of C_1 and C_2 into Eq. [2],

$$v = \frac{1}{EI}\left\{-6.25x^4 + \left(\frac{5}{12}\right) < x - 2 >^5 + \left(\frac{275}{3}\right) < x - 2 >^3 + 410x - 720\right\} \text{ kN}\cdot\text{m}^3$$

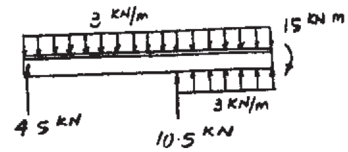
$$v_C = v|_{x=0} = \frac{1}{EI}\{-0 + 0 + 0 + 0 - 720\} \text{ kN}\cdot\text{m}^3 = \frac{720 \text{ kN}\cdot\text{m}^3}{EI} \quad \downarrow \qquad \textbf{Ans.}$$

12–50.

Determine the equations of the slope and elastic curve. EI is constant.



SOLUTION



$$M = -(-4.5)\langle x - 0\rangle - \frac{3}{2}\langle x - 0\rangle^2 - (-10.5)\langle x - 5\rangle - \left(\frac{-3}{2}\right)\langle x - 5\rangle^2$$

$$M = 4.5x - 1.5x^2 + 10.5\langle x - 5\rangle + 1.5\langle x - 5\rangle^2$$

Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M = 4.5x - 1.5x^2 + 10.5\langle x - 5\rangle + 1.5\langle x - 5\rangle^2$$

$$EI\frac{dv}{dx} = 2.25x^2 - 0.5x^3 + 5.25\langle x - 5\rangle^2 + 0.5\langle x - 5\rangle^3 + C_1 \qquad \textbf{(1)}$$

$$EIv = 0.75x^3 - 0.125x^4 + 1.75\langle x - 5\rangle^3 + 0.125\langle x - 5\rangle^4 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions:

$v = 0 \quad$ at $\quad x = 0$

From Eq. (2), $C_2 = 0$

$v = 0 \quad$ at $\quad x = 5$

$$0 = 93.75 - 78.125 + 5C_1$$

$$C_1 = -3.125$$

$$\frac{dv}{dx} = \frac{1}{EI}[2.25x^2 - 0.5x^3 + 5.25\langle x - 5\rangle^2 + 0.5\langle x - 5\rangle^3 - 3.125]\ \text{kN}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

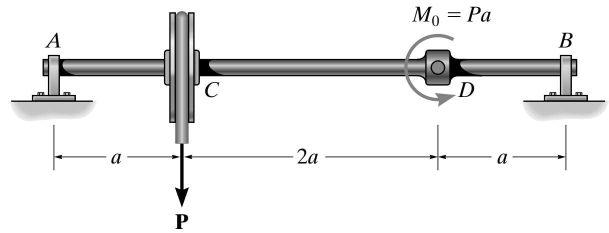
$$v = \frac{1}{EI}[0.75x^3 - 0.125x^4 + 1.75\langle x - 5\rangle^3 + 0.125\langle x - 5\rangle^4 - 3.125x]\ \text{kN}\cdot\text{m}^3 \quad \textbf{Ans.}$$

Ans:

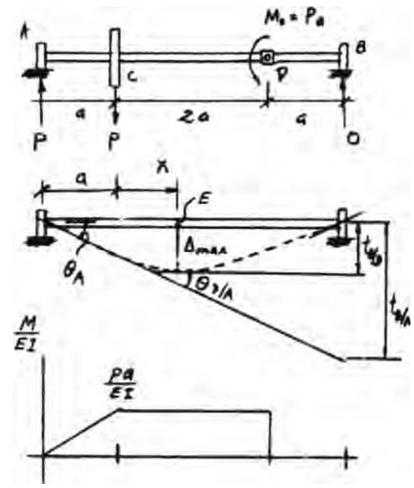
$$\frac{dv}{dx} = \frac{1}{EI}[2.25x^2 - 0.5x^3 + 5.25\langle x - 5\rangle^2 + 0.5\langle x - 5\rangle^3 - 3.125]\ \text{kN}\cdot\text{m}^2,$$

$$v = \frac{1}{EI}[0.75x^3 - 0.125x^4 + 1.75\langle x - 5\rangle^3 + 0.125\langle x - 5\rangle^4 - 3.125x]\ \text{kN}\cdot\text{m}^3$$

12–51. If the bearings at A and B exert only vertical reactions on the shaft, determine the slope at A and the maximum deflection.



SOLUTION



$$t_{B/A} = \frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\left(3a + \frac{a}{3}\right) + \left(\frac{Pa}{EI}\right)(2a)(a + a) = \frac{17Pa^3}{3EI}$$

$$\theta_A = \frac{|t_{B/A}|}{4a} = \frac{17Pa^2}{12EI}$$ **Ans.**

Assume Δ_{max} is at point E located at $0 < x < 2a$

$$\theta_{E/A} = \frac{1}{2}\left(\frac{Pa}{EI}\right)(a) + \left(\frac{Pa}{EI}\right)(x) = \frac{Pa^2}{2EI} + \frac{Pax}{EI}$$

$$\theta_E = 0 = \theta_{E/A} + \theta_A$$

$$0 = \frac{Pa^2}{2EI} + \frac{Pax}{EI} + \left(\frac{-17Pa^2}{12EI}\right)$$

$$x = \frac{11}{12}a$$

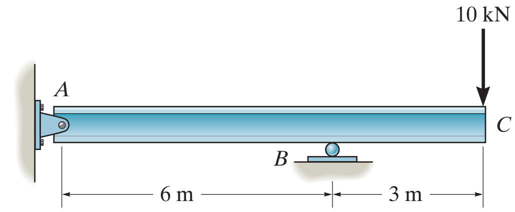
$$\Delta_{max} = |t_{B/E}| = \left(\frac{Pa}{EI}\right)\left(2a - \frac{11}{12}a\right)\left[\frac{(2a - \frac{11}{12}a)}{2} + a\right] = \frac{481Pa^3}{288EI}$$ **Ans.**

Ans:

$$\theta_A = \frac{17\,Pa^2}{12EI},\ \Delta_{\max} = \frac{481\,Pa^3}{288EI}$$

***12–52.**

Determine the slope and deflection at C. EI is constant.



SOLUTION

Referring to Fig. b,

$$|\theta_{C/A}| = \frac{1}{2}\left(\frac{30}{EI}\right)(9) = \frac{135 \text{ kN} \cdot \text{m}^2}{EI}$$

$$|t_{B/A}| = \frac{6}{3}\left[\frac{1}{2}\left(\frac{30}{EI}\right)(6)\right] = \frac{180 \text{ kN} \cdot \text{m}^3}{EI}$$

$$|t_{C/A}| = \left(\frac{6}{3} + 3\right)\left[\frac{1}{2}\left(\frac{30}{EI}\right)(6)\right] + \left[\frac{2}{3}(3)\right]\left[\frac{1}{2}\left(\frac{30}{EI}\right)(3)\right]$$

$$= \frac{540 \text{ kN} \cdot \text{m}^3}{EI}$$

From the geometry shown in Fig. b,

$$\theta_A = \frac{|t_{B/A}|}{6} = \frac{180/EI}{6} = \frac{30 \text{ kN} \cdot \text{m}^2}{EI}$$

Here,

$$+\circlearrowright \theta_C = \theta_A + \theta_{C/A}$$

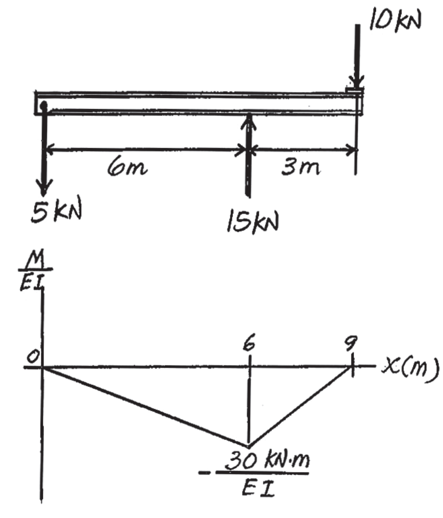
$$\theta_C = -\frac{30}{EI} + \frac{135}{EI}$$

$$\theta_C = \frac{105 \text{ kN} \cdot \text{m}^2}{EI}$$ **Ans.**

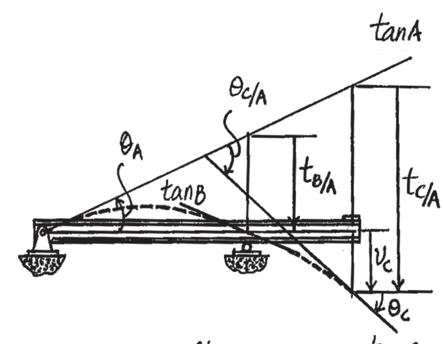
$$v_C = \left|t_{C/A}\right| - \left|t_{B/A}\right|\left(\frac{9}{6}\right)$$

$$= \frac{540}{EI} - \frac{180}{EI}\left(\frac{9}{6}\right)$$

$$= \frac{270 \text{ kN} \cdot \text{m}^3}{EI} \downarrow$$ **Ans.**



(a)



(b)

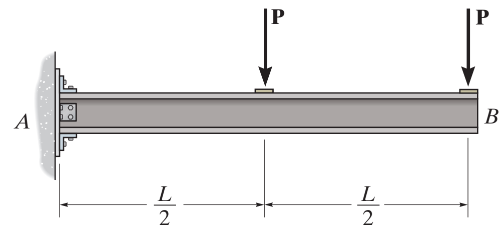
Ans:

$$\theta_C = \frac{105 \text{ kN} \cdot \text{m}^2}{EI},$$

$$v_C = \frac{270 \text{ kN} \cdot \text{m}^3}{EI} \downarrow$$

12–53.

Determine the deflection of end B of the cantilever beam. EI is constant.



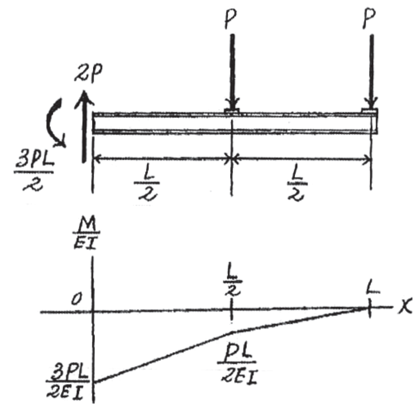
SOLUTION

Support Reactions and $\frac{M}{EI}$ Diagram: As shown in Fig. *a*.

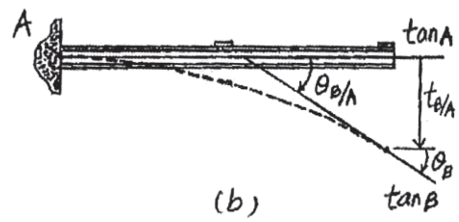
Moment Area Theorem: Since A is a fixed support, $\theta_A = 0$. Referring to the geometry of the elastic curve, Fig. *b*,

$$v_B = |t_{B/A}| = \left(\frac{3L}{4}\right)\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right) + \frac{5L}{6}\left[\frac{1}{2}\left(\frac{PL}{EI}\right)\left(\frac{L}{2}\right)\right] + \frac{L}{3}\left[\frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right)\right]$$

$$= \frac{7PL^3}{16EI} \downarrow \qquad \textbf{Ans.}$$



(a)



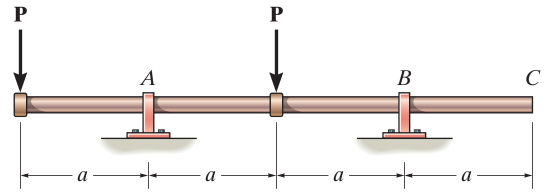
(b)

Ans:

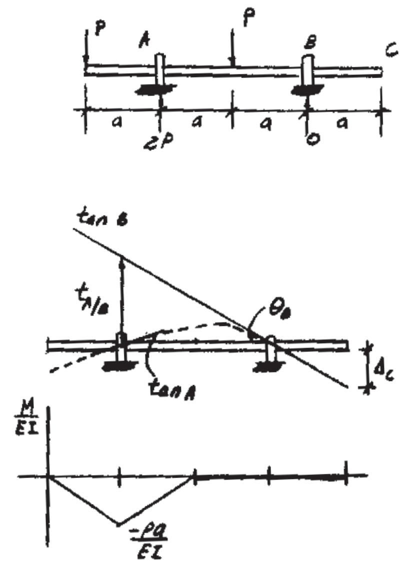
$v_B = \frac{7PL^3}{16EI}\downarrow$

12–54.

Determine the slope at B and the deflection at C. EI is constant.



SOLUTION



$$t_{A/B} = \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{a}{3}\right) = \frac{Pa^3}{6EI}$$

$$\theta_B = -\frac{|t_{A/B}|}{2a} = -\frac{Pa^3/6EI}{2a} = -\frac{Pa^2}{12EI}$$ **Ans.**

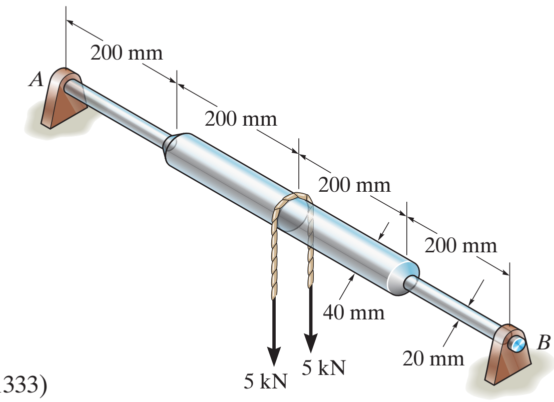
$$v_C = \theta_B a = \frac{Pa^2}{12EI}(a) = \frac{Pa^3}{12EI}$$ **Ans.**

Ans:

$$\theta_B = -\frac{Pa^2}{12EI}, v_C = \frac{Pa^3}{12EI}$$

12–55.

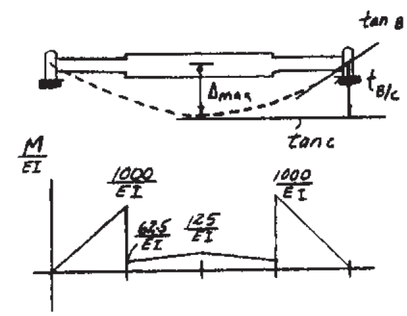
The composite simply supported steel shaft is subjected to a force of 10 kN at its center. Determine its maximum deflection. $E_{st} = 200$ GPa.



SOLUTION

$$v_{max} = |t_{B/C}| = \frac{62.5}{EI}(0.2)(0.3) + \frac{1}{2}\left(\frac{62.5}{EI}\right)(0.2)(0.3333) + \frac{1}{2}\left(\frac{1000}{EI}\right)(0.2)(0.1333)$$

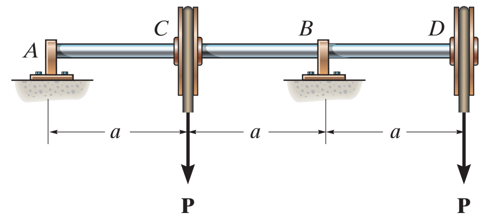
$$= \frac{19.167}{EI} = \frac{19.167}{200(10^9)(7.8540)(10^{-9})} = 0.0122 \text{ m} = 12.2 \text{ mm} \qquad \textbf{Ans.}$$



Ans:
$v_{max} = 12.2$ mm

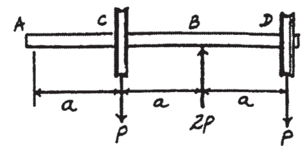
***12–56.**

Determine the slope of the shaft at A and the displacement at D. EI is constant.

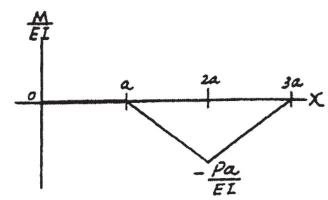


SOLUTION

Support Reactions and Elastic Curve: As shown.



M/EI Diagram: As shown.



Moment-Area Theorems:

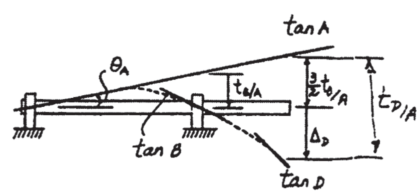
$$t_{B/A} = \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{a}{3}\right) = -\frac{Pa^3}{6EI}$$

$$t_{D/A} = \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(a + \frac{a}{3}\right) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{2}{3}a\right) = -\frac{Pa^3}{EI}$$

The slope at A is

$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{\frac{Pa^3}{6EI}}{2a} = \frac{Pa^2}{12EI}$$ **Ans.**

The displacement at D is



$$v_D = |t_{D/A}| - \left|\frac{3}{2}t_{B/A}\right|$$

$$= \frac{Pa^3}{EI} - \frac{3}{2}\left(\frac{Pa^3}{6EI}\right)$$

$$= \frac{3Pa^3}{4EI}$$ **Ans.**

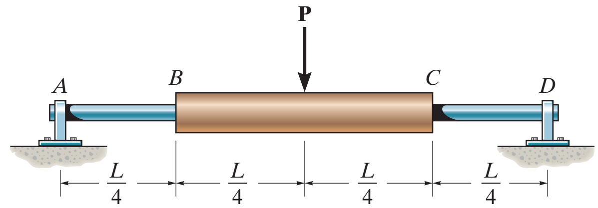
Ans:

$$\theta_A = \frac{Pa^2}{12EI},$$

$$v_D = \frac{3Pa^3}{4EI}$$

12–57.

The simply supported shaft has a moment of inertia of $2I$ for region BC and a moment of inertia I for regions AB and CD. Determine the maximum deflection of the shaft due to the load $\mathbf{P}$. The modulus of elasticity is E.



SOLUTION

Support Reactions and Elastic Curve: As shown.

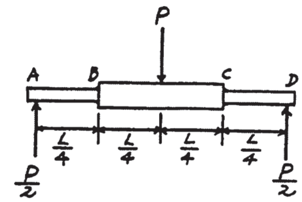
M/EI Diagram: As shown.

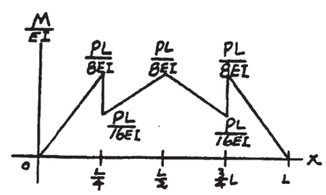
Moment-Area Theorems: Due to symmetry, the slope at midspan (point E) is zero. Hence,

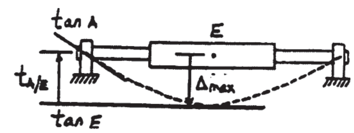
$$v_{\max} = t_{A/E} = \frac{1}{2}\left(\frac{PL}{16EI}\right)\left(\frac{L}{4}\right)\left(\frac{L}{4}+\frac{L}{6}\right) + \left(\frac{PL}{16EI}\right)\left(\frac{L}{4}\right)\left(\frac{L}{4}+\frac{L}{8}\right)$$

$$+ \frac{1}{2}\left(\frac{PL}{8EI}\right)\left(\frac{L}{4}\right)\left(\frac{L}{6}\right)$$

$$= \frac{3PL^3}{256EI} \downarrow \qquad \textbf{Ans.}$$





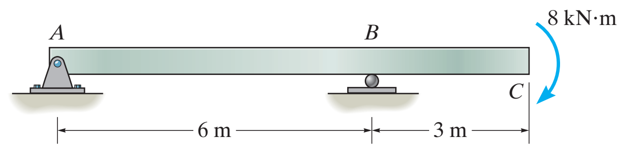


Ans:

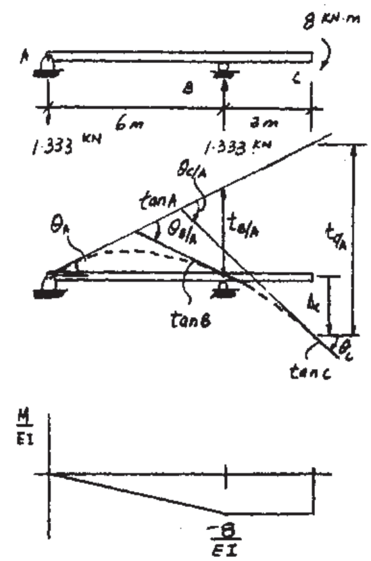
$$v_{\max} = \frac{3PL^3}{256EI} \downarrow$$

12–58.

Determine the deflection at C and the slope of the beam at A, B, and C. EI is constant.



SOLUTION



$$t_{B/A} = \frac{1}{2}\left(\frac{-8}{EI}\right)(6)(2) = \frac{-48}{EI}$$

$$t_{C/A} = \frac{1}{2}\left(\frac{-8}{EI}\right)(6)(3 + 2) + \left(\frac{-8}{EI}\right)(3)(1.5) = \frac{-156}{EI}$$

$$v_C = |t_{C/A}| - \frac{9}{6}|t_{B/A}| = \frac{156}{EI} - \frac{9(48)}{6(EI)} = \frac{84}{EI} \qquad \textbf{Ans.}$$

$$\theta_A = \frac{|t_{B/A}|}{6} = \frac{8}{EI} \qquad \textbf{Ans.}$$

$$\theta_{B/A} = \frac{1}{2}\left(\frac{-8}{EI}\right)(6) = \frac{-24}{EI}$$

$$\theta_B = \theta_{B/A} + \theta_A$$

$$\theta_B = -\frac{24}{EI} + \frac{8}{EI} = -\frac{16}{EI} \qquad \textbf{Ans.}$$

$$\theta_{C/A} = \frac{1}{2}\left(\frac{-8}{EI}\right)(6) + \left(\frac{-8}{EI}\right)(3) = \frac{-48}{EI}$$

$$\theta_C = \theta_{C/A} + \theta_A$$

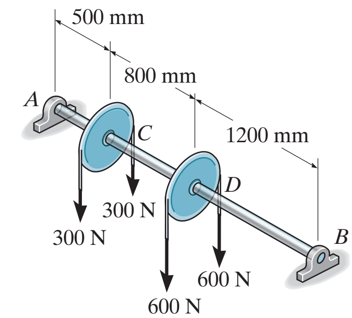
$$\theta_C = -\frac{48}{EI} + \frac{8}{EI} = -\frac{40}{EI} \qquad \textbf{Ans.}$$

Ans:

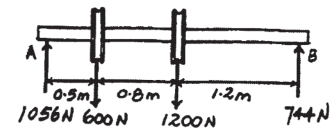
$$v_C = -\frac{84}{EI}, \theta_A = \frac{8}{EI}, \theta_B = -\frac{16}{EI}, \theta_C = -\frac{40}{EI}$$

12–59.

Determine the maximum deflection of the 50-mm-diameter A-36 steel shaft.



SOLUTION

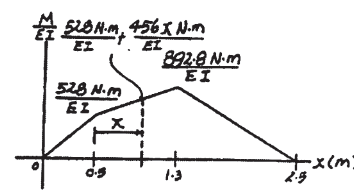


Moment-Area Theorems:

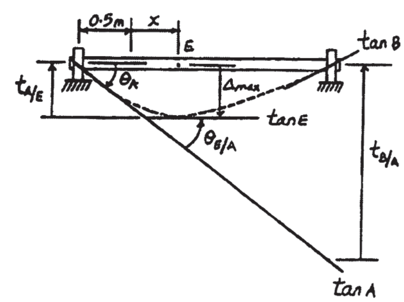
$$t_{B/A} = \frac{1}{2}\left(\frac{892.8}{EI}\right)(1.2)(0.8) + \frac{1}{2}\left(\frac{364.8}{EI}\right)(0.8)(1.4667) + \left(\frac{528}{EI}\right)(0.8)(1.6) + \frac{1}{2}\left(\frac{528}{EI}\right)(0.5)(2.1667)$$

$$= \frac{1604.4 \text{ N}\cdot\text{m}^3}{EI}$$

$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{\frac{1604.4 \text{ N}\cdot\text{m}^3}{EI}}{2.5 \text{ m}} = \frac{641.76 \text{ N}\cdot\text{m}^2}{EI}$$



The maximum displacement occurs at point E, where $\theta_E = 0$.



$$\theta_{E/A} = \frac{1}{2}\left(\frac{528}{EI}\right)(0.5) + \left(\frac{528}{EI}\right)x + \frac{1}{2}\left(\frac{456}{EI}x\right)x$$

$$= \frac{1}{EI}(228x^2 + 528x + 132)$$

$$\theta_E = \theta_A + \theta_{E/A}$$

$$0 = -\frac{641.76}{EI} + \frac{1}{EI}(228x^2 + 528x + 132)$$

$$x = 0.7333 \text{ m} < 0.8 \text{ m} \qquad \textbf{(O.K!)}$$

The maximum displacement is,

$$v_{max} = |t_{A/E}| = \frac{1}{2}\left(\frac{528}{EI}\right)(0.5)(0.3333) + \left(\frac{528}{EI}\right)(0.7333)(0.8666) + \frac{1}{2}\left(\frac{456}{EI}\right)(0.7333^2)(0.9888)$$

$$= \frac{500.76 \text{ N}\cdot\text{m}^3}{EI}$$

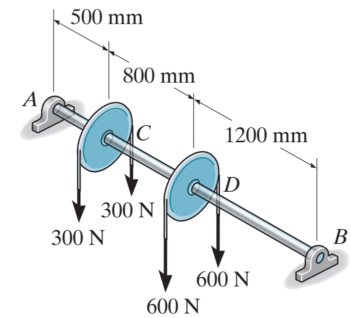
$$= \frac{500.76}{200(10^9)\left(\frac{\pi}{4}\right)(0.025^4)}$$

$$= 0.008161 \text{ m} = 8.16 \text{ mm}\downarrow \qquad \textbf{Ans.}$$

Ans:
$v_{max} = 8.16 \text{ mm}\downarrow$

***12–60.**

Determine the slope of the 50-mm-diameter A-36 steel shaft at the journal bearings at A and B. The bearings exert only vertical reactions on the shaft.

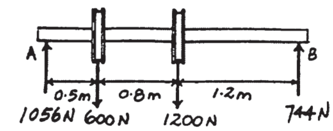


SOLUTION

Support Reactions and Elastic Curve: As shown.

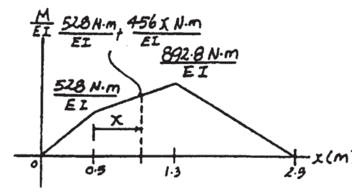
M/EI Diagram: As shown.

Moment-Area Theorems:



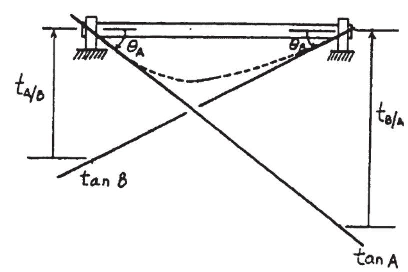
$$t_{B/A} = \frac{1}{2}\left(\frac{892.8}{EI}\right)(1.2)(0.8) + \frac{1}{2}\left(\frac{364.8}{EI}\right)(0.8)(1.4667) + \left(\frac{528}{EI}\right)(0.8)(1.6) + \frac{1}{2}\left(\frac{528}{EI}\right)(0.5)(2.1667)$$

$$= \frac{1604.4\ \text{N}\cdot\text{m}^3}{EI}$$



$$t_{A/B} = \frac{1}{2}\left(\frac{892.8}{EI}\right)(1.2)(1.7) + \frac{1}{2}\left(\frac{364.8}{EI}\right)(0.8)(1.0333) + \left(\frac{528}{EI}\right)(0.8)(0.9) + \frac{1}{2}\left(\frac{528}{EI}\right)(0.5)(0.3333)$$

$$= \frac{1485.6\ \text{N}\cdot\text{m}^3}{EI}$$



The slopes at A and B are,

$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{\frac{1604.4\ \text{N}\cdot\text{m}^3}{EI}}{2.5\ \text{m}}$$

$$= \frac{641.76\ \text{N}\cdot\text{m}^2}{EI}$$

$$= \frac{641.76}{200(10^9)\left(\frac{\pi}{4}\right)(0.025^4)} = 0.0105\ \text{rad} \qquad \textbf{Ans.}$$

$$\theta_B = \frac{|t_{A/B}|}{L} = \frac{\frac{1485.6\ \text{N}\cdot\text{m}^3}{EI}}{2.5\ \text{m}}$$

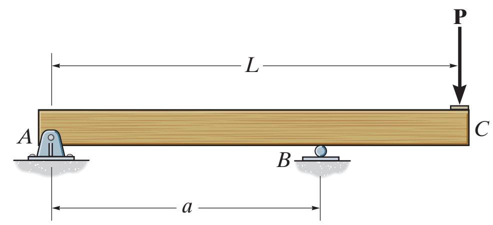
$$= \frac{594.24\ \text{N}\cdot\text{m}^2}{EI}$$

$$= \frac{594.24}{200(10^9)\left(\frac{\pi}{4}\right)(0.025^4)} = 0.00968\ \text{rad} \qquad \textbf{Ans.}$$

Ans:
$\theta_A = 0.0105$ rad

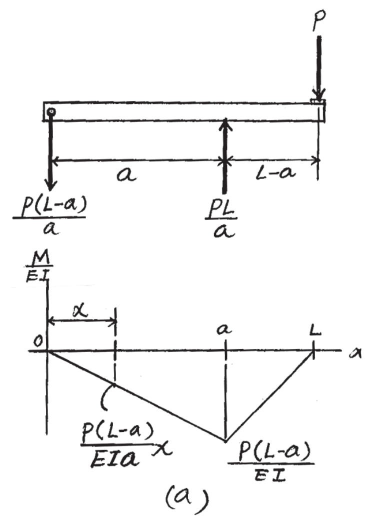
12–61.

Determine the position a of the roller support B in terms of L so that the deflection at end C is the same as the maximum deflection of region AB of the overhang beam. EI is constant.



SOLUTION

Support Reactions and $\frac{M}{EI}$ Diagram. As shown in Fig. a.



Moment Area Theorem. Referring to Fig. b,

$$|t_{B/A}| = \frac{a}{3}\left[\frac{1}{2}\left(\frac{P(L-a)}{EI}\right)(a)\right] = \frac{Pa^2(L-a)}{6EI}$$

$$|t_{C/A}| = \left(L - \frac{2}{3}a\right)\left[\frac{1}{2}\left(\frac{P(L-a)}{EI}\right)(a)\right] + \frac{2(L-a)}{3}\left[\frac{1}{2}\left(\frac{P(L-a)}{EI}\right)(L-a)\right]$$

$$= \frac{P(L-a)\left(2L^2 - aL\right)}{6EI}$$

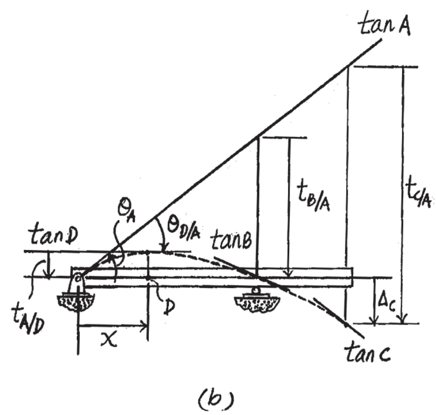
From the geometry shown in Fig. b,

$$v_C = |t_{C/A}| - \frac{|t_{B/A}|}{a}L$$

$$= \frac{PL(L-a)\left(2L-a\right)}{6EI} - \frac{Pa^2(L-a)}{6EI}\left(\frac{L}{a}\right)$$

$$= \frac{PL(L-a)^2}{3EI}$$

$$\theta_A = \frac{|t_{B/A}|}{a} = \frac{\frac{Pa^2(L-a)}{6EI}}{a} = \frac{Pa(L-a)}{6EI}$$



The maximum deflection in region AB occurs at point D, where the slope of the elastic curve is zero ($\theta_D = 0$).

Thus,

$$|\theta_{D/A}| = \theta_A$$

$$\frac{1}{2}\left[\frac{P(L-a)}{EIa}x\right](x) = \frac{Pa(L-a)}{6EI}$$

$$x = \frac{\sqrt{3}}{3}a$$

Also,

$$v_D = |t_{4/D}| = \left(\frac{2\sqrt{3}}{9}a\right)\left[\frac{1}{2}\left[\frac{P(L-a)}{EIa}\left(\frac{\sqrt{3}}{3}a\right)\right]\right]\left(\frac{\sqrt{3}}{3}a\right) = \frac{\sqrt{3}Pa^2(L-a)}{27EI}$$

It is required that

$$v_C = \Delta_D$$

$$\frac{PL(L-a)^2}{3EI} = \frac{\sqrt{3}Pa^2(L-a)}{27EI}$$

$$\frac{\sqrt{3}}{9}a^2 + La - L^2 = 0$$

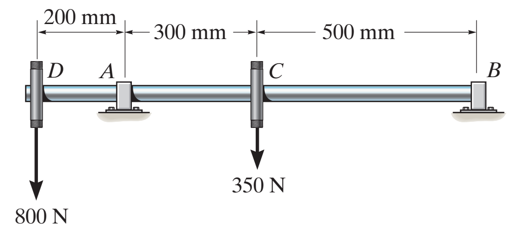
Solving for the positive root,

$$a = 0.858L \qquad \textbf{Ans.}$$

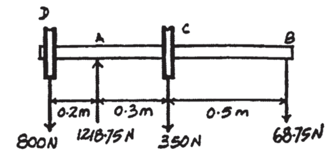
Ans:
$a = 0.858L$

12–62.

Determine the slope of the 20-mm-diameter A-36 steel shaft at the journal bearings A and B.



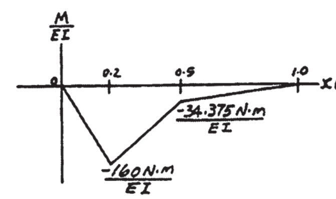
SOLUTION



Moment-Area Theorems:

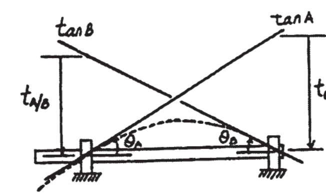
$$t_{B/A} = \frac{1}{2}\left(-\frac{34.375}{EI}\right)(0.5)(0.3333) + \frac{1}{2}\left(-\frac{125.625}{EI}\right)(0.3)(0.7) + \left(-\frac{34.375}{EI}\right)(0.3)(0.65)$$

$$= -\frac{22.75833 \text{ N}\cdot\text{m}^3}{EI}$$



$$t_{A/B} = \frac{1}{2}\left(-\frac{34.375}{EI}\right)(0.5)(0.4667) + \frac{1}{2}\left(-\frac{125.625}{EI}\right)(0.3)(0.1) + \left(-\frac{34.375}{EI}\right)(0.3)(0.15)$$

$$= -\frac{7.44167 \text{ N}\cdot\text{m}^3}{EI}$$



The slopes at A and B are,

$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{\frac{22.75833 \text{ N}\cdot\text{m}^3}{EI}}{0.8 \text{ m}}$$

$$= \frac{28.448 \text{ N}\cdot\text{m}^2}{EI}$$

$$= \frac{28.448}{200(10^9)\left(\frac{\pi}{4}\right)(0.01^4)} = 0.0181 \text{ rad}$$ **Ans.**

$$\theta_B = \frac{|t_{A/B}|}{L} = \frac{\frac{7.44167 \text{ N}\cdot\text{m}^3}{EI}}{0.8 \text{ m}}$$

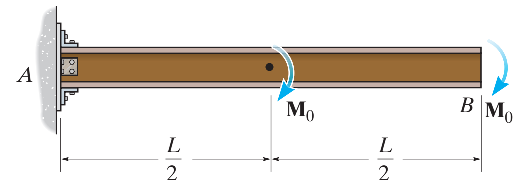
$$= \frac{9.302 \text{ N}\cdot\text{m}^2}{EI}$$

$$= \frac{9.302}{200(10^9)\left(\frac{\pi}{4}\right)(0.01^4)} = 0.00592 \text{ rad}$$ **Ans.**

Ans:
$\theta_A = 0.0181$ rad,
$\theta_B = 0.00592$ rad

12–63.

Determine the slope and the deflection of end B of the cantilever beam. EI is constant.



SOLUTION

Support Reactions and $\frac{M}{EI}$ Diagram. As shown in Fig. *a*.

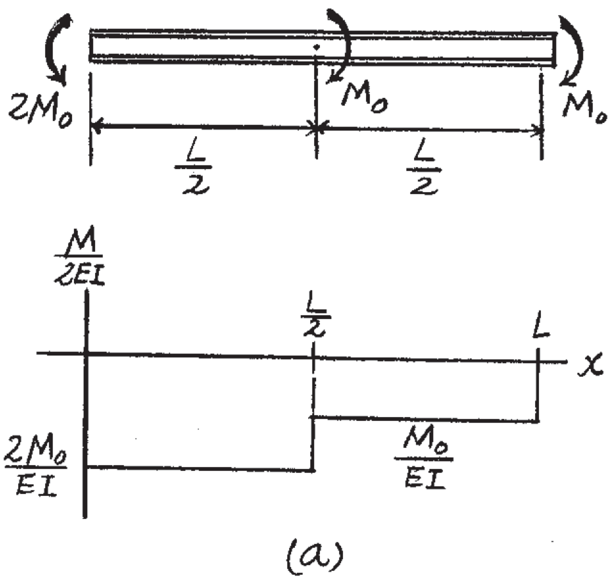
Moment Area Theorem: Since A is a fixed support, $\theta_A = 0$. Referring to the geometry of the elastic curve, Fig. *b*,

$$\theta_B = \theta_{B/A} = -\frac{2M_O}{EI}\left(\frac{L}{2}\right) + \left(-\frac{M_O}{EI}\right)\left(\frac{L}{2}\right)$$

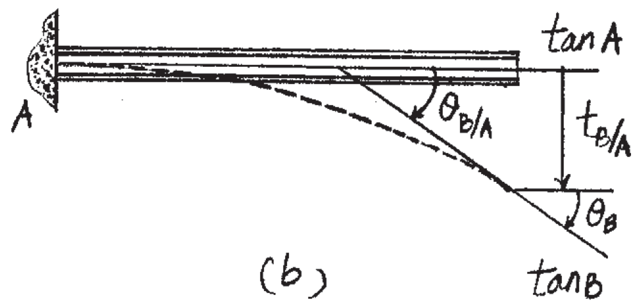
$$= -\frac{3M_O L}{2EI} \qquad \textbf{Ans.}$$

$$v_B = |t_{B/A}| = \frac{3L}{2}\left[\frac{2M_O}{EI}\left(\frac{L}{2}\right)\right] + \frac{L}{4}\left[\frac{M_O}{EI}\left(\frac{L}{2}\right)\right]$$

$$= \frac{7M_O L^2}{8EI} \downarrow \qquad \textbf{Ans.}$$



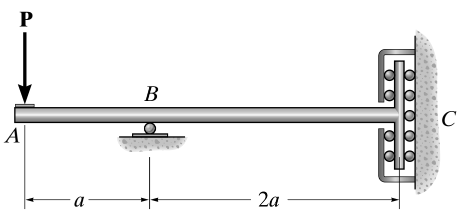
(a)



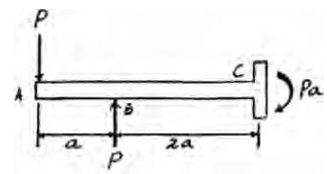
(b)

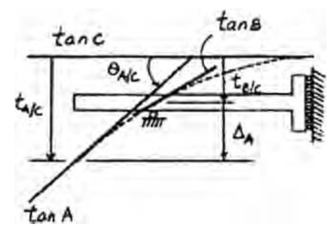
Ans:
$\theta_B = -\frac{3M_0 L}{2EI}, v_B = \frac{7M_0 L^2}{8EI} \downarrow$

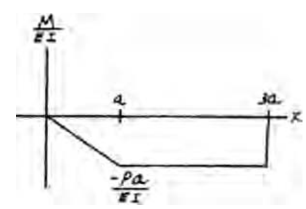
***12–64.** The bar is supported by the roller constraint at C, which allows vertical displacement but resists axial load and moment. If the bar is subjected to the loading shown, determine the slope and displacement at A. EI is constant.



SOLUTION







Support Reactions and Elastic Curve: As shown.

M/EI Diagram: As shown.

Moment-Area Theorems:

$$\theta_{A/C} = \left(-\frac{Pa}{EI}\right)(2a) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a) = -\frac{5Pa^2}{2EI}$$

$$t_{B/C} = \left(-\frac{Pa}{EI}\right)(2a)(a) = \frac{2Pa^3}{EI}$$

$$t_{A/C} = \left(-\frac{Pa}{EI}\right)(2a)(2a) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{2}{3}a\right) = -\frac{13Pa^3}{3EI}$$

Due to the moment constraint, the slope at support C is zero. Hence, the slope at A is

$$\theta_A = |\theta_{A/C}| = \frac{5Pa^2}{2EI}$$ **Ans.**

and the displacement at A is

$$\Delta_A = |t_{A/C}| - |t_{B/C}|$$

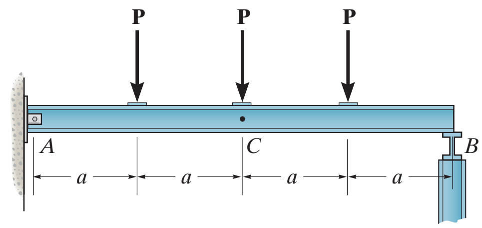
$$= \frac{13Pa^3}{3EI} - \frac{2Pa^3}{EI} = \frac{7Pa^3}{3EI} \downarrow$$ **Ans.**

Ans:

$\theta_A = |\theta_{A/C}| = \dfrac{5Pa^2}{2EI}$; $\Delta_A = \dfrac{7Pa^3}{3EI}$ $\downarrow$

12–65.

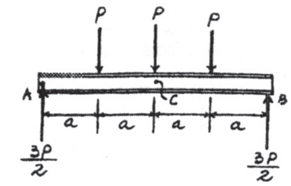
Determine the slope at A and the displacement at C. Assume the support at A is a pin and B is a roller. EI is constant.

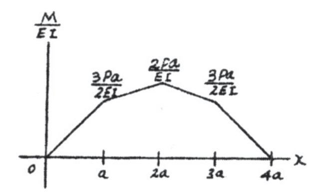


SOLUTION

Support Reactions and Elastic Curve: As shown.

M/EI Diagram: As shown.





Moment - Area Theorems: Due to symmetry, the slope at midspan (point C) is zero. Hence the slope at A is

$$|\theta_A| = \theta_{A/C} = \frac{1}{2}\left(\frac{3Pa}{2EI}\right)(a) + \left(\frac{3Pa}{2EI}\right)(a) + \frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)$$

$$\theta_A = -\frac{5Pa^2}{2EI} \qquad \textbf{Ans.}$$

The displacement at C is

$$v_C = t_{A/C} = \frac{1}{2}\left(\frac{3Pa}{2EI}\right)(a)\left(\frac{2a}{3}\right) + \left(\frac{3Pa}{2EI}\right)\left(a + \frac{a}{2}\right) + \frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)\left(a + \frac{2a}{3}\right)$$

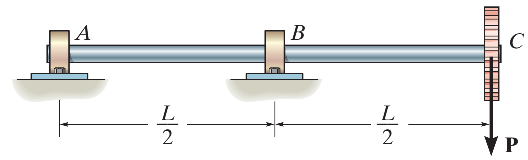
$$= \frac{19Pa^3}{6EI}\downarrow \qquad \textbf{Ans.}$$

Ans:

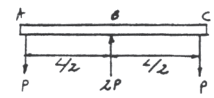
$$\theta_A = -\frac{5Pa^2}{2EI},\ v_C = \frac{19Pa^3}{6EI}\downarrow$$

12–66.

Determine the deflection at C and the slopes at the bearings A and B. EI is constant.

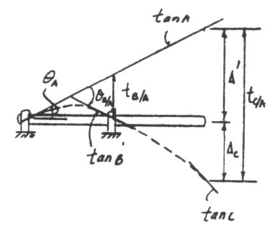


SOLUTION



$$t_{B/A} = \frac{1}{2}\left(\frac{-PL}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{6}\right) = \frac{-PL^3}{48EI}$$

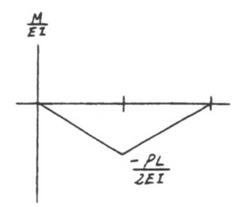
$$t_{C/A} = \frac{1}{2}\left(\frac{-PL}{2EI}\right)(L)\left(\frac{L}{2}\right) = \frac{-PL^3}{8EI}$$



$$|v_C| = |t_{C/A}| - \left(\frac{L}{\frac{L}{2}}\right)|t_{B/A}|$$

$$v_C = -\frac{PL^3}{8EI} + 2\left(\frac{PL^3}{48EI}\right) = -\frac{PL^3}{12EI} \qquad \textbf{Ans.}$$

$$\theta_A = \frac{|t_{B/A}|}{\frac{L}{2}} = \frac{\frac{PL^3}{48\,EI}}{\frac{L}{2}} = \frac{PL^2}{24EI} \qquad \textbf{Ans.}$$



$$\theta_{B/A} = \frac{1}{2}\left(\frac{-PL}{2EI}\right)\left(\frac{L}{2}\right) = \frac{-PL^2}{8EI}$$

$$\theta_B = \theta_{B/A} + \theta_A$$

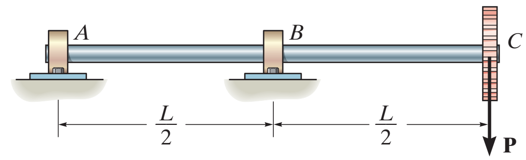
$$\theta_B = -\frac{PL^2}{8EI} + \frac{PL^2}{24EI} = -\frac{PL^2}{12EI} \qquad \textbf{Ans.}$$

Ans:

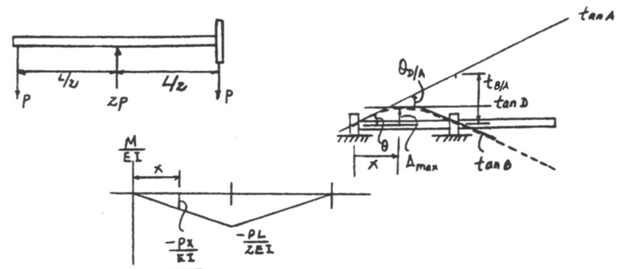
$v_C = -\dfrac{PL^3}{12EI},\ \theta_A = \dfrac{PL^2}{24EI},\ \theta_B = -\dfrac{PL^2}{12EI}$

12–67.

Determine the maximum deflection within region AB. EI is constant.



SOLUTION



$$\theta_{D/A} = \frac{t_{B/A}}{\left(\frac{L}{2}\right)}$$

$$\frac{1}{2}\left(\frac{Px}{EI}\right)x = \frac{\frac{1}{2}\left(\frac{L}{2}\right)\left(\frac{PL}{2EI}\right)\left(\frac{1}{3}\right)\left(\frac{L}{2}\right)}{\left(\frac{L}{2}\right)}; \quad x = 0.288675\,L$$

$$v_{\max} = \frac{1}{2}\left(\frac{P(0.288675\,L)}{EI}\right)(0.288675\,L)\left(\frac{2}{3}\right)(0.288675\,L)$$

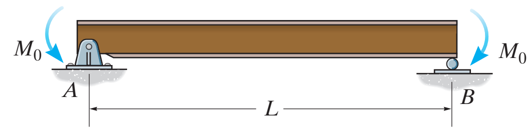
$$v_{\max} = \frac{0.00802PL^3}{EI}$$ **Ans.**

Ans:

$$v_{\max} = \frac{0.00802PL^3}{EI}$$

***12–68.**

Determine the slope at A and the maximum deflection of the simply supported beam. EI is constant.



SOLUTION

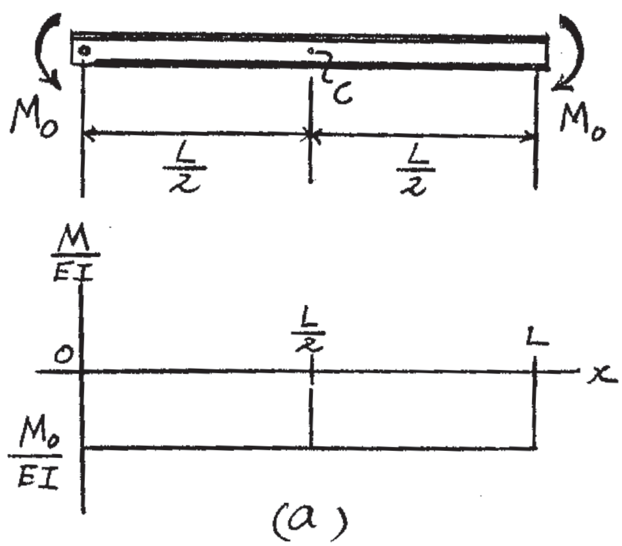
Support Reactions and $\frac{M}{EI}$ Diagram. As shown in Fig. *a*.

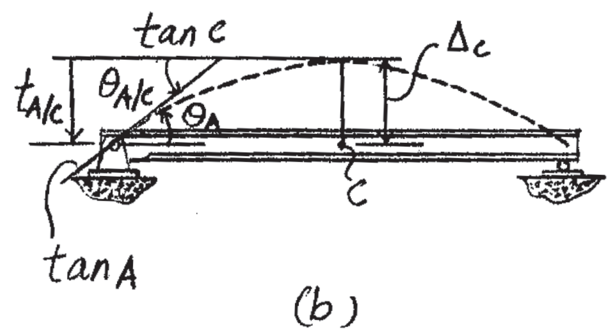
Moment Area Theorem. Due to symmetry, the slope at the midspan of the beam, i.e., point C is zero ($\theta_C = 0$). Thus, the maximum deflection of the beam occurs here. Referring to the geometry of the elastic curve, Fig. *b*,

$$\theta_A = |\theta_{A/C}| = \frac{M_O}{EI}\left(\frac{L}{2}\right) = \frac{M_O L}{2EI} \qquad \textbf{Ans.}$$

$$v_{max} = v_C = |t_{A/C}| = \frac{L}{4}\left[\frac{M_O}{EI}\left(\frac{L}{2}\right)\right]$$

$$= \frac{M_O L^2}{8EI} \uparrow \qquad \textbf{Ans.}$$





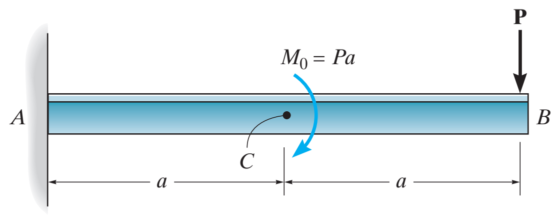
Ans:

$\theta_A = \frac{M_O L}{2EI}$,

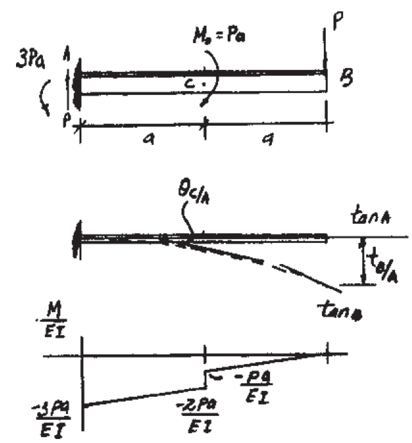
$v_{max} = \frac{M_O L^2}{8EI} \uparrow$

12–69.

Determine the slope at C and the deflection at B. EI is constant.



SOLUTION



$$\theta_{C/A} = \left(-\frac{2Pa}{EI}\right)(a) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)$$

$$= -\frac{5Pa^2}{2EI}$$ **Ans.**

$$\theta_C = \theta_{C/A}$$

$$\circlearrowleft + \quad \theta_C = +\frac{5Pa^2}{2EI} \quad \measuredangle$$

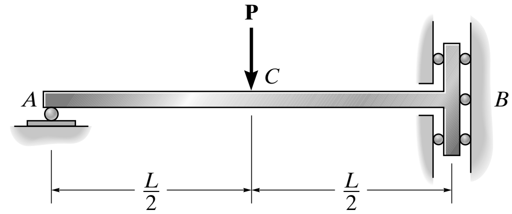
$$v_B = |t_{B/A}| = \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{2a}{3}\right) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(a + \frac{2a}{3}\right) + \left(-\frac{2Pa}{EI}\right)(a)\left(a + \frac{a}{2}\right)$$

$$= \frac{25Pa^3}{6EI}\downarrow$$ **Ans.**

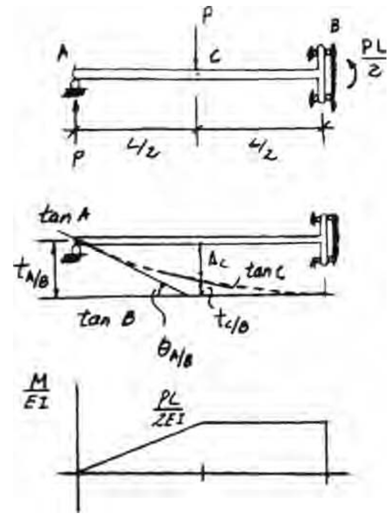
Ans:

$\theta_C = -\frac{5Pa^2}{2EI}$, $v_B = \frac{25Pa^3}{6EI}\downarrow$

12–70. The bar is supported by a roller constraint at B, which allows vertical displacement but resists axial load and moment. If the bar is subjected to the loading shown, determine the slope at A and the deflection at C. EI is constant.



SOLUTION



$$\theta_{A/B} = \frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right) + \frac{PL}{2EI}\left(\frac{L}{2}\right) = \frac{3PL^2}{8EI}$$

$$\theta_A = \theta_{A/B}$$

$$\theta_A = \frac{3PL^2}{8EI}$$ **Ans.**

$$t_{A/B} = \frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) + \frac{PL}{2EI}\left(\frac{L}{2}\right)\left(\frac{L}{2} + \frac{L}{4}\right) = \frac{11PL^3}{48EI}$$

$$t_{C/B} = \frac{PL}{2EI}\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) = \frac{PL^3}{16EI}$$

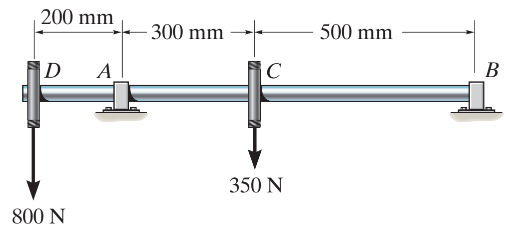
$$\Delta_C = t_{A/B} - t_{C/B} = \frac{11PL^3}{48EI} - \frac{PL^3}{16EI} = \frac{PL^3}{6EI}$$ **Ans.**

Ans:

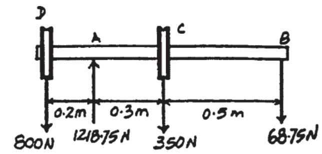
$\theta_A = \dfrac{3PL^2}{8EI}$, $\Delta_C = \dfrac{PL^3}{6EI}$

12–71.

Determine the displacement of the 20-mm-diameter A-36 steel shaft at D.

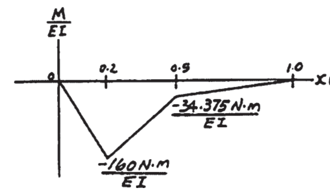


SOLUTION



Support Reactions and Elastic Curve: As shown.

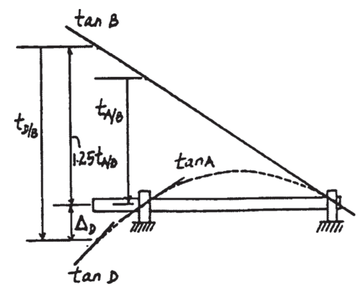
M/EI Diagram: As shown.



Moment-Area Theorems:

$$t_{D/B} = \frac{1}{2}\left(-\frac{34.375}{EI}\right)(0.5)(0.6667) + \frac{1}{2}\left(-\frac{125.625}{EI}\right)(0.3)(0.3)$$
$$+ \left(-\frac{34.375}{EI}\right)(0.3)(0.35) + \frac{1}{2}\left(-\frac{160}{EI}\right)(0.2)(0.1333)$$
$$= -\frac{17.125 \text{ N}\cdot\text{m}^3}{EI}$$

$$t_{A/B} = \frac{1}{2}\left(-\frac{34.375}{EI}\right)(0.5)(0.4667) + \frac{1}{2}\left(-\frac{125.625}{EI}\right)(0.3)(0.1)$$
$$+ \left(-\frac{34.375}{EI}\right)(0.3)(0.15)$$
$$= -\frac{7.44167 \text{ N}\cdot\text{m}^3}{EI}$$

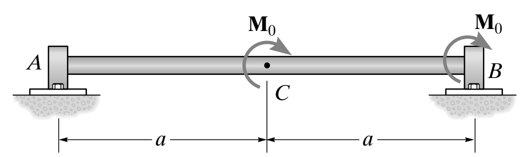


The displacement at D is,

$$v_D = |t_{D/B}| - |1.25t_{A/B}|$$
$$= \frac{17.125}{EI} - 1.25\left(\frac{7.44167}{EI}\right)$$
$$= \frac{7.823 \text{ N}\cdot\text{m}^3}{EI}$$
$$= \frac{7.823}{200(10^9)\left(\frac{\pi}{4}\right)(0.01^4)}$$
$$= 0.00498 \text{ m} = 4.98 \text{ mm}\downarrow \qquad \textbf{Ans.}$$

Ans:
$v_D = 4.98 \text{ mm}\downarrow$

***12–72.** The shaft is subjected to the loading shown. If the bearings at A and B only exert vertical reactions on the shaft, determine the slope at A and the displacement at C. EI is constant.



M/EI Diagram: As shown.

Moment-Area Theorems:

$$t_{B/A} = \frac{1}{2}\left(-\frac{M_0}{EI}\right)(a)\left(\frac{a}{3}\right) + \frac{1}{2}\left(-\frac{M_0}{EI}\right)(a)\left(a + \frac{a}{3}\right)$$

$$= -\frac{5M_0\,a^2}{6EI}$$

$$t_{C/A} = \frac{1}{2}\left(-\frac{M_0}{EI}\right)(a)\left(\frac{a}{3}\right) = -\frac{M_0\,a^2}{6EI}$$

The slope at A is

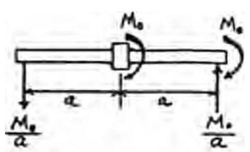
$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{\frac{5M_0a^2}{6EI}}{2a} = \frac{5M_0a}{12EI}$$ **Ans.**

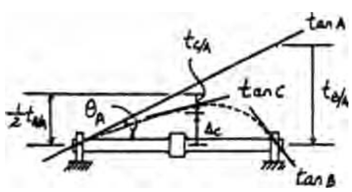
The displacement at C is,

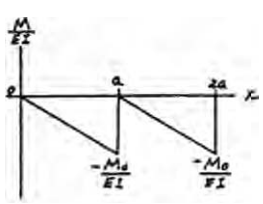
$$\Delta_C = \left|\frac{1}{2}\,t_{B/A}\right| - |t_{C/A}|$$

$$= \frac{1}{2}\left(\frac{5M_0\,a^2}{6EI}\right) - \frac{M_0\,a^2}{6EI}$$

$$= \frac{M_0\,a^2}{4EI} \quad \uparrow$$ **Ans.**





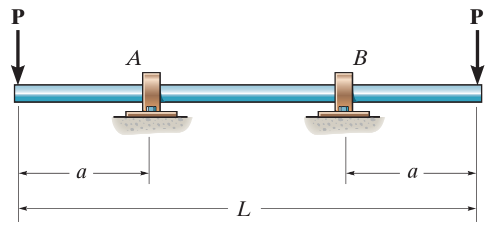


Ans:

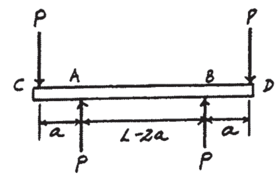
$$\theta_A = \frac{5M_0a}{12EI}\;;\quad \Delta_C = \frac{M_0\,a^2}{4EI}\quad \uparrow$$

12–73.

At what distance a should the journal bearing supports at A and B be placed so that the deflection at the center of the shaft is equal to the deflection at its ends? EI is constant.



SOLUTION

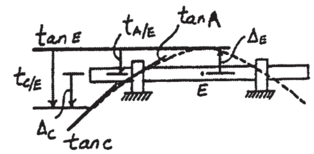


Support Reactions and Elastic Curve: As shown.

M/EI Diagram: As shown.

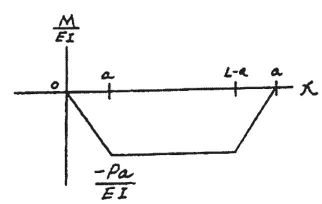
Moment-Area Theorems: Due to symmetry, the slope at midspan (point E) is zero.

$$v_E = |t_{A/E}| = \left(-\frac{Pa}{EI}\right)\left(\frac{L-2a}{2}\right)\left(\frac{L-2a}{4}\right) = \frac{Pa}{8EI}(L-2a)^2$$



$$t_{C/E} = \left(-\frac{Pa}{EI}\right)\left(\frac{L-2a}{2}\right)\left(a + \frac{L-2a}{4}\right) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{2}{3}a\right)$$

$$= -\frac{Pa}{24EI}\left(3L^2 - 4a^2\right)$$



$$v_C = |t_{C/E}| - |t_{A/E}|$$

$$= \frac{Pa}{24EI}\left(3L^2 - 4a^2\right) - \frac{Pa}{8EI}(L-2a)^2$$

$$= \frac{Pa^2}{6EI}(3L - 4a)$$

Require $v_E = v_C$, then,

$$\frac{Pa}{8EI}(L-2a)^2 = \frac{Pa^2}{6EI}(3L - 4a)$$

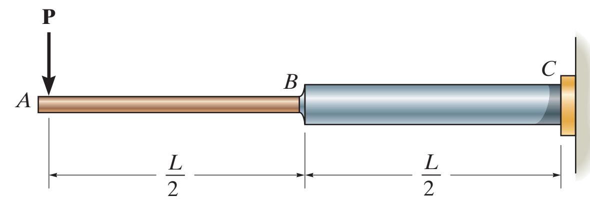
$$28a^2 - 24aL + 3L^2 = 0$$

$$a = 0.152L \qquad \textbf{Ans.}$$

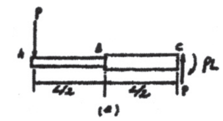
Ans:
$a = 0.152L$

12–74.

The rod is constructed from two shafts for which the moment of inertia of AB is I and for BC it is $2I$. Determine the maximum slope and deflection of the rod due to the loading. The modulus of elasticity is E.

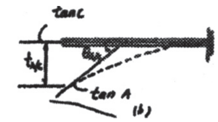


SOLUTION



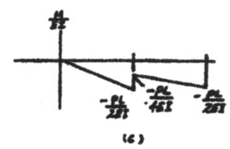
$$\theta_{A/C} = \frac{1}{2}\left(\frac{-PL}{2EI}\right)\left(\frac{L}{2}\right) + \frac{1}{2}\left(\frac{-PL}{4EI}\right)\left(\frac{L}{2}\right) + \left(\frac{-PL}{4EI}\right)\left(\frac{L}{2}\right) = \frac{-5PL^2}{16EI} = \frac{5PL^2}{16EI}$$

$$\theta_A = \theta_{A/C} + \theta_C$$



$$\theta_{max} = \theta_A = \frac{5PL^2}{16EI} + 0 = \frac{5PL^2}{16EI}$$ **Ans.**

$$v_{max} = v_A = |t_{A/C}|$$



$$= \left|\frac{1}{2}\left(\frac{-PL}{2\,EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) + \frac{1}{2}\left(\frac{-PL}{4EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2} + \frac{L}{3}\right) + \left(\frac{-PL}{4EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2} + \frac{L}{4}\right)\right|$$

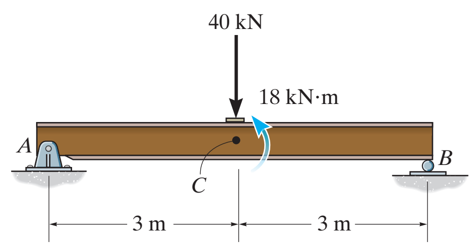
$$= \frac{3PL^3}{16EI}\downarrow$$ **Ans.**

Ans:

$$\theta_{max} = \frac{5PL^2}{16EI}, \; v_{max} = \frac{3PL^3}{16EI}\downarrow$$

12–75.

Determine the slope at B and the deflection at C of the beam. $E = 200$ GPa and $I = 65.0(10^6)$ mm^4.



SOLUTION

Support Reactions and $\frac{M}{EI}$ Diagram: As shown in Fig. *a*.

Moment Area Theorem: Referring to Fig. *b*,

$$|t_{C/B}| = \left[\frac{1}{3}(3)\right]\left[\frac{1}{2}\left(\frac{51}{EI}\right)(3)\right]$$

$$= \frac{76.5 \text{ kN}\cdot\text{m}^3}{EI}$$

$$|t_{A/B}| = \left[\frac{1}{3}(3) + 3\right]\left[\frac{1}{2}\left(\frac{51}{EI}\right)(3)\right] + \left[\frac{2}{3}(3)\right]\left[\frac{1}{2}\left(\frac{69}{EI}\right)(3)\right]$$

$$= \frac{513 \text{ kN}\cdot\text{m}^3}{EI}$$

From the geometry of the elastic curve, Fig. *b*,

$$\theta_B = \frac{|t_{A/B}|}{L} = \frac{513/EI}{6} = \frac{85.5 \text{ kN}\cdot\text{m}^2}{EI}$$

$$= \frac{85.5(10^3)}{200(10^9)[65.0(10^{-6})]} = 0.00658 \text{ rad} \qquad \textbf{Ans.}$$

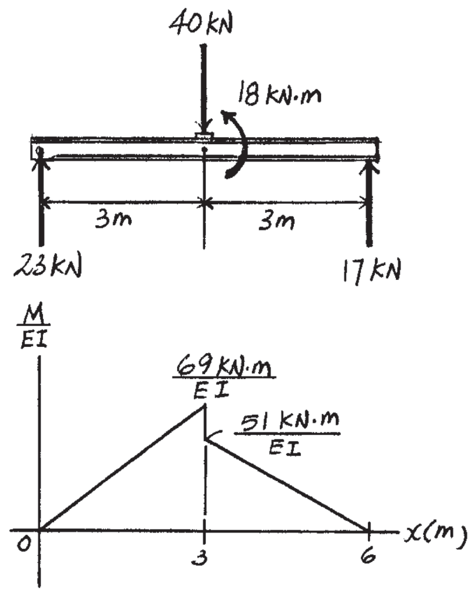
and

$$v_C = |t_{A/B}|\left(\frac{L_{BC}}{L}\right) - |t_{C/B}|$$

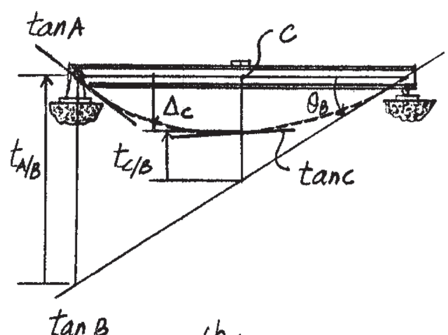
$$= \frac{513}{EI}\left(\frac{3}{6}\right) - \frac{76.5}{EI}$$

$$= \frac{180 \text{ kN}\cdot\text{m}^3}{EI} = \frac{180(10^3)}{200(10^9)[65.0(10^{-6})]}$$

$$= 0.0138 \text{ m} = 13.8 \text{ mm} \downarrow \qquad \textbf{Ans.}$$



(a)

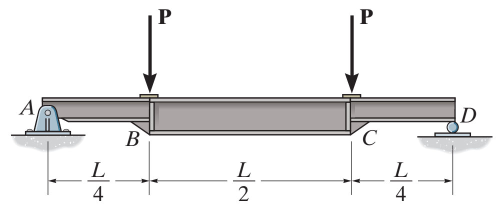


(b)

Ans:
$\theta_B = 0.00658$ rad, $v_C = 13.8$ mm$\downarrow$

***12–76.**

Determine the slope at point A and the maximum deflection of the simply supported beam. The beam is made of material having a modulus of elasticity E. The moment of inertia of segments AB and CD of the beam is I, and the moment of inertia of segment BC is $2I$.



SOLUTION

Support Reactions and $\frac{M}{EI}$ Diagram: As shown in Fig. *a*.

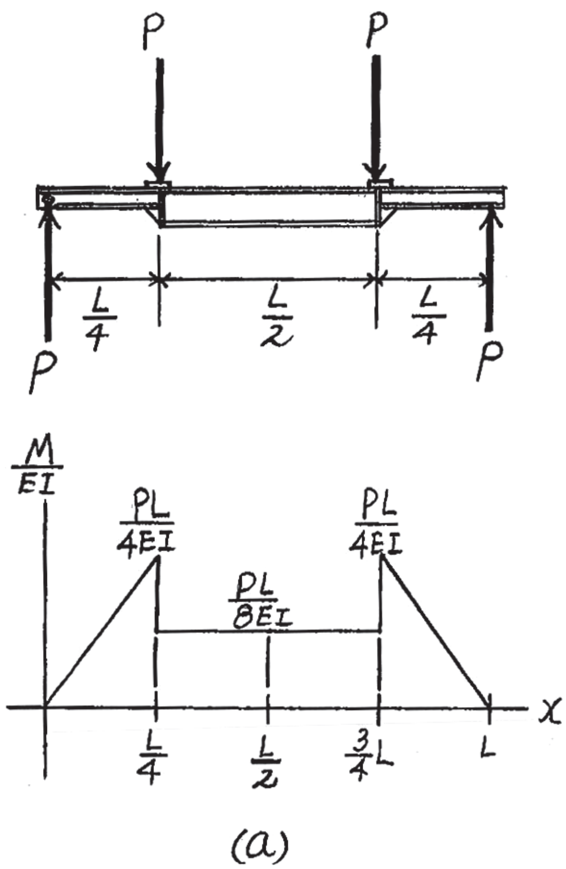
Moment Area Theorem: Due to symmetry, the slope at the midspan of the beam, i.e., point E, is zero ($\theta_E = 0$). Thus the maximum deflection occurs here. Referring to the geometry of the elastic curve, Fig. *b*,

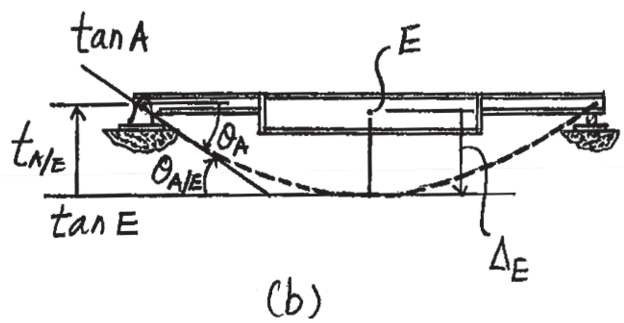
$$|\theta_A| = |\theta_{A/E}| = \frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{L}{4}\right) + \frac{PL}{8EI}\left(\frac{L}{4}\right)$$

$$\theta_A = -\frac{PL^2}{16EI} \qquad \textbf{Ans.}$$

$$v_{max} = v_E = |t_{A/E}| = \frac{3}{8}L\left[\frac{PL}{8EI}\left(\frac{L}{4}\right)\right] + \frac{L}{6}\left[\frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{L}{4}\right)\right]$$

$$= \frac{13PL^3}{768EI} \downarrow \qquad \textbf{Ans.}$$





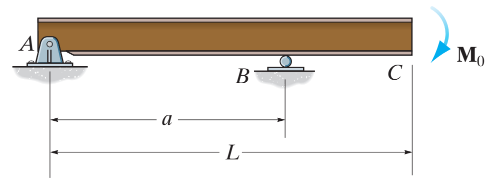
Ans:

$\theta_A = -\frac{PL^2}{16EI}$,

$v_{max} = \frac{13PL^3}{768EI} \downarrow$

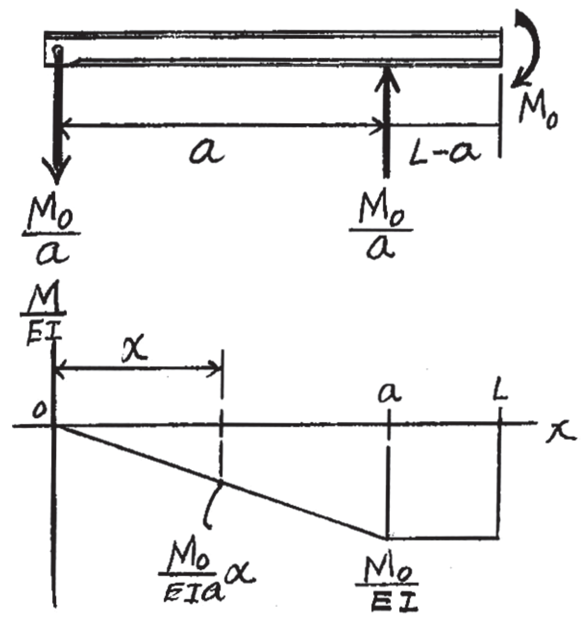
12–77.

Determine the position a of roller support B in terms of L so that deflection at end C is the same as the maximum deflection in region AB of the overhang beam. EI is constant.



SOLUTION

Support Reactions and $\frac{M}{EI}$ Diagram: As shown in Fig. a.



(a)

Moment Area Theorem: Referring to Fig. b,

$$|t_{B/A}| = \frac{a}{3}\left[\frac{1}{2}\left(\frac{M_O}{EI}\right)(a)\right] = \frac{M_O a^2}{6EI}$$

$$|t_{C/A}| = \left(L - \frac{2}{3}a\right)\left[\frac{1}{2}\left(\frac{M_O}{EI}\right)(a)\right] + \left(\frac{L-a}{2}\right)\left[\frac{M_O}{EI}(L-a)\right]$$

$$= \frac{M_O}{6EI}\left(a^2 + 3L^2 - 3La\right)$$

From the geometry shown in Fig. b,

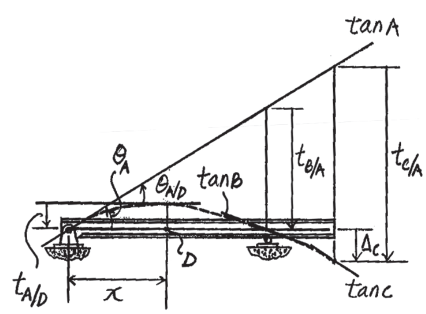
$$v_C = |t_{C/A}| - \frac{|t_{B/A}|}{a}L$$

$$= \frac{M_O}{6EI}\left(a^2 + 3L^2 - 3La\right) - \frac{M_O a^2}{6EI}\left(\frac{L}{a}\right)$$

$$= \frac{M_O}{6EI}\left(a^2 + 3L^2 - 4La\right)$$

$$\theta_A = \frac{|t_{B/A}|}{a} = \frac{\frac{M_O a^2}{6EI}}{a} = \frac{M_O a}{6EI}$$

The maximum deflection in region AB occurs at point D, where the slope of the elastic curve is zero ($\theta_D = 0$).



(b)

Thus,

$$|\theta_{D/A}| = \theta_A$$

$$\frac{1}{2}\left(\frac{M_O}{EIa}\right)(x)^2 = \frac{M_O a}{6EI}$$

$$x = \frac{\sqrt{3}}{3}a$$

Also,

$$v_D = |t_{A/D}| = \frac{2}{3}\left(\frac{\sqrt{3}}{3}a\right)\left[\frac{1}{2}\left(\frac{M_O}{EIa}\right)\left(\frac{\sqrt{3}}{3}a\right)\right]\left(\frac{\sqrt{3}}{3}a\right) = \frac{\sqrt{3}M_O a^2}{27EI}$$

It is required that

$$v_C = v_D$$

$$\frac{M_O}{6EI}\left(a^2 + 3L^2 - 4La\right) = \frac{\sqrt{3}M_O a^2}{27EI}$$

$$0.6151a^2 - 4La + 3L^2 = 0$$

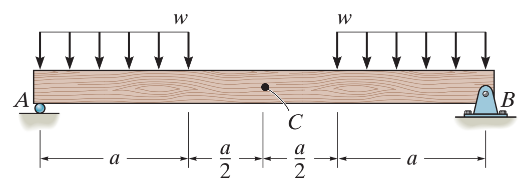
Solving for the root $< L$,

$$a = 0.865\,L$$ **Ans.**

Ans:
$a = 0.865\,L$

12–78.

Determine the slope at B and deflection at C. EI is constant.



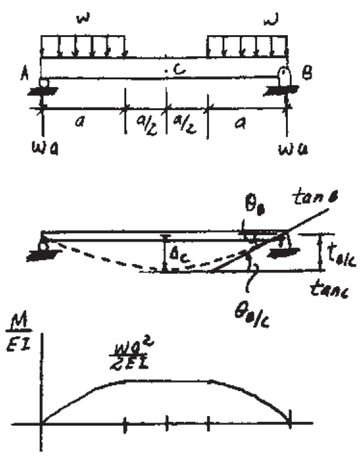
SOLUTION

$$\theta_{B/C} = \frac{wa^2}{2EI}\left(\frac{a}{2}\right) + \frac{2}{3}\left(\frac{wa^2}{2EI}\right)(a) = \frac{7wa^3}{12EI}$$

$$\theta_B = \theta_{B/C} = \frac{7wa^3}{12EI}$$ **Ans.**

$$v_C = t_{B/C} = \frac{wa^2}{2EI}\left(\frac{a}{2}\right)\left(a + \frac{a}{4}\right) + \frac{2}{3}\left(\frac{wa^2}{2EI}\right)(a)\left(\frac{5}{8}a\right)$$

$$= \frac{25wa^4}{48EI} \downarrow$$ **Ans.**

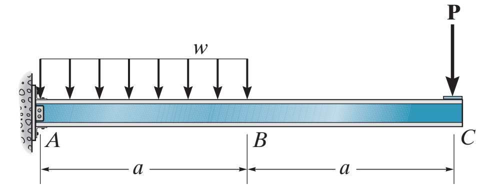


Ans:

$\theta_B = \dfrac{7wa^3}{12EI}$, $v_C = \dfrac{25wa^4}{48EI}\downarrow$

12–79.

Determine the slope and displacement at C. EI is constant.



SOLUTION

Support Reactions and Elastic Curve: As shown.

M/EI Diagrams: The M/EI diagrams for the uniform distributed load and concentrated load are drawn separately as shown.

Moment-Area Theorems: The slope at support A is zero. The slope at C is

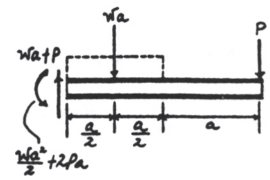
$$\theta_C = |\theta_{C/A}| = \frac{1}{2}\left(-\frac{2Pa}{EI}\right)(2a) + \frac{1}{3}\left(-\frac{wa^2}{2EI}\right)(a)$$

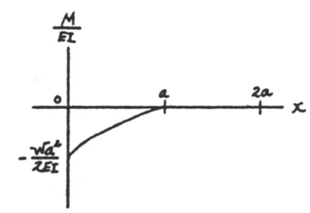
$$= \frac{a^2}{6EI}(12P + wa) \qquad \textbf{Ans.}$$

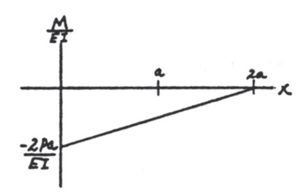
The displacement at C is

$$v_C = |t_{C/A}| = \frac{1}{2}\left(-\frac{2Pa}{EI}\right)(2a)\left(\frac{4}{3}a\right) + \frac{1}{3}\left(-\frac{wa^2}{2EI}\right)(a)\left(a + \frac{3}{4}a\right)$$

$$= \frac{a^3}{24EI}(64P + 7wa) \downarrow \qquad \textbf{Ans.}$$







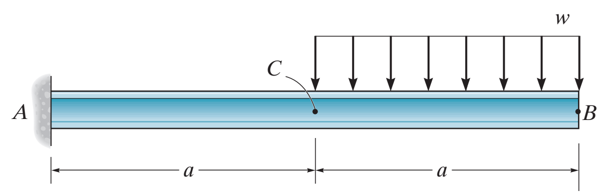
Ans:

$\theta_C = -\frac{a^2}{6EI}(12P + wa)$,

$v_C = \frac{a^3}{24EI}(64P + 7wa)\downarrow$

***12–80.**

Determine the slope at C and deflection at B. EI is constant.

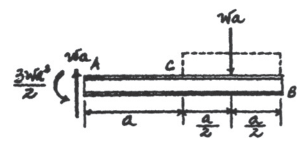


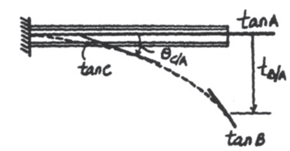
SOLUTION

Support Reactions and Elastic Curve: As shown.

M/EI Diagram: As shown.

Moment-Area Theorems: The slope at support A is zero. The slope at C is

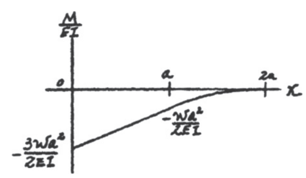




$$|\theta_C| = |\theta_{C/A}| = \frac{1}{2}\left(-\frac{wa^2}{EI}\right)(a) + \left(-\frac{wa^2}{2EI}\right)(a)$$

$$\theta_A = -\frac{wa^3}{EI} \qquad \textbf{Ans.}$$

The displacement at B is



$$v_B = |t_{B/A}|$$

$$= \frac{1}{2}\left(-\frac{wa^2}{EI}\right)(a)\left(a + \frac{2}{3}a\right) + \left(-\frac{wa^2}{2EI}\right)(a)\left(a + \frac{a}{2}\right) + \frac{1}{3}\left(-\frac{wa^2}{2EI}\right)(a)\left(\frac{3}{4}a\right)$$

$$= \frac{41wa^4}{24EI} \downarrow \qquad \textbf{Ans.}$$

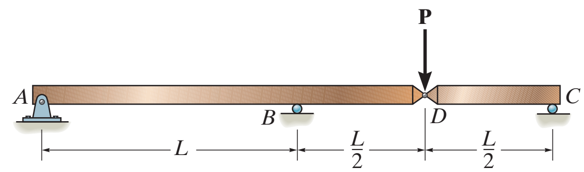
Ans:

$$\theta_A = -\frac{wa^3}{EI},$$

$$v_B = \frac{41wa^4}{24EI} \downarrow$$

12–81.

The two bars are pin connected at D. Determine the slope at A and the deflection at D. EI is constant.



SOLUTION

$$t_{B/A} = \frac{1}{2}\left(\frac{-PL}{2EI}\right)(L)\left(\frac{L}{3}\right) = \frac{-PL^3}{12EI}$$

$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{PL^2}{12EI}$$ **Ans.**

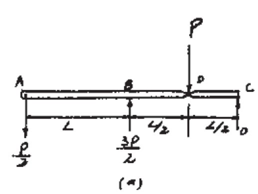
The Deflection:

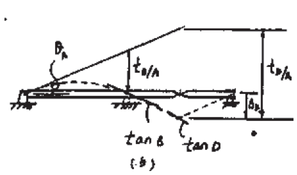
$$t_{D/A} = \frac{1}{2}\left(\frac{-PL}{2EI}\right)(L)\left(\frac{L}{2} + \frac{L}{3}\right) + \frac{1}{2}\left(\frac{-PL}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right)$$

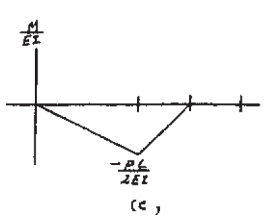
$$= -\frac{PL^3}{4EI}$$

$$v_D = |t_{D/A}| - \left(\frac{\frac{3}{2}L}{L}\right)|t_{B/A}|$$

$$= \frac{PL^3}{4EI} - \frac{3}{2}\left(\frac{PL^3}{12EI}\right) = \frac{PL^3}{8EI}\downarrow$$ **Ans.**



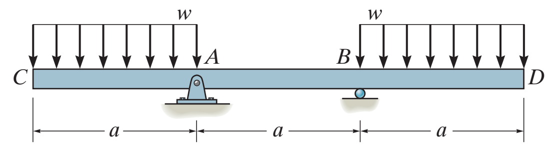




Ans:
$\theta_A = \frac{PL^2}{12EI}$, $v_D = \frac{PL^3}{8EI}\downarrow$

12–82.

Determine the maximum deflection of the beam. *EI* is constant.



SOLUTION

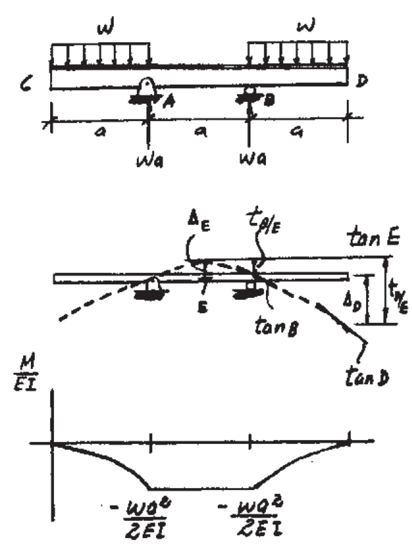
$$t_{B/E} = \left(\frac{-wa^2}{2EI}\right)\left(\frac{a}{2}\right)\left(\frac{a}{4}\right) = \frac{-wa^4}{16EI}$$

$$v_E = |t_{B/E}| = \frac{wa^4}{16EI}\uparrow$$

$$t_{D/E} = \left(\frac{-wa^2}{2EI}\right)\left(\frac{a}{2}\right)\left(a + \frac{a}{4}\right) + \frac{1}{3}\left(\frac{-wa^2}{2EI}\right)(a)\left(\frac{3a}{4}\right) = -\frac{7wa^4}{16EI}$$

$$v_D = |t_{D/E}| - |t_{B/E}| = \frac{7wa^4}{16EI} - \frac{wa^4}{16EI} = \frac{3wa^4}{8EI}$$

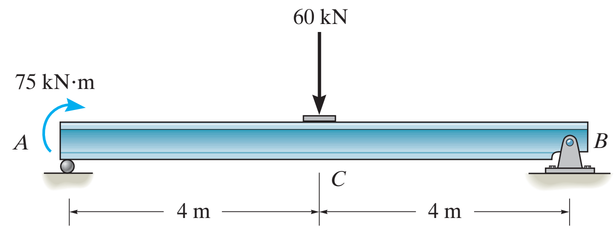
$$v_{\max} = v_D = \frac{3wa^4}{8EI}$$ **Ans.**



Ans:

$$v_{\max} = \frac{3wa^4}{8EI}$$

12–83. The W310 × 67 imply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the deflection at its center *C*.



SOLUTION

$$(\Delta_C)_1 = \frac{PL^3}{48EI} = \frac{60(4^3)}{48EI} = \frac{80}{EI}\ \text{kN}\cdot\text{m}^3$$

$$\Delta_2(x) = \frac{Mx}{6EIL}(L^2 - x^2)$$

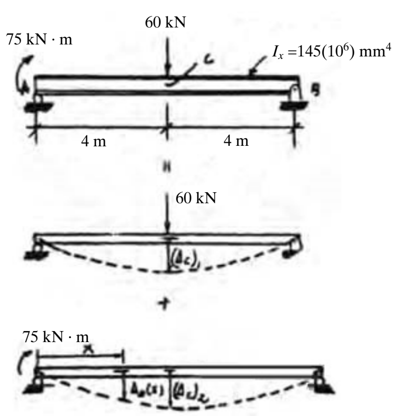
At point C, $x = \dfrac{L}{2}$

$$(\Delta_C)_2 = \frac{M\left(\frac{L}{2}\right)}{6EIL}\left[L^2 - \left(\frac{L}{2}\right)^2\right]$$

$$= \frac{ML^2}{16EI} = \frac{75(8)^2}{16EI} = \frac{300}{EI}\ \text{kN}\cdot\text{m}^3 \downarrow$$

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2 = \frac{80}{EI} + \frac{300}{EI} = \frac{380}{EI}\ \text{kN}\cdot\text{m}^3$$

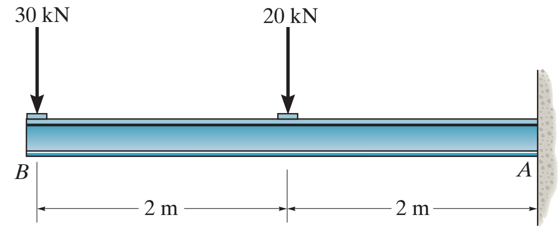
$$= \frac{380(10^3)\ \text{N}\cdot\text{m}^3}{[200(10^9)\ \text{N/m}^2][145(10^{-6})\ \text{m}^4]} = 0.01310\ \text{m} = 13.1\ \text{mm} \downarrow$$ **Ans.**



Ans:

$\Delta_C = 13.1\ \text{mm} \downarrow$

***12–84.** The W250 × 22 cantilevered beam is made of A-36 steel and is subjected to the loading shown. Determine the displacement at B and the slope at B.



SOLUTION

Using the table in appendix, the required slopes and deflections for each load case are computed as follow:

$$(\Delta_B)_1 = \frac{5PL^3}{48EI} = \frac{5(20)(4^3)}{48EI} = \frac{133.33\ \text{kN}\cdot\text{m}^3}{EI}\ \downarrow$$

$$(\theta_B)_1 = \frac{PL^2}{8EI} = \frac{20(4^2)}{8EI} = \frac{40\ \text{kN}\cdot\text{m}^2}{EI}$$

$$(\Delta_B)_2 = \frac{PL^3}{3EI} = \frac{30(4^3)}{3EI} = \frac{640\ \text{kN}\cdot\text{m}^3}{EI}\ \downarrow$$

$$(\theta_B)_2 = \frac{PL^2}{2EI} = \frac{30(4^2)}{2EI} = \frac{240\ \text{kN}\cdot\text{m}^2}{EI}$$

Then the slope and deflection at B are

$$\theta_B = (\theta_B)_1 + (\theta_B)_2$$

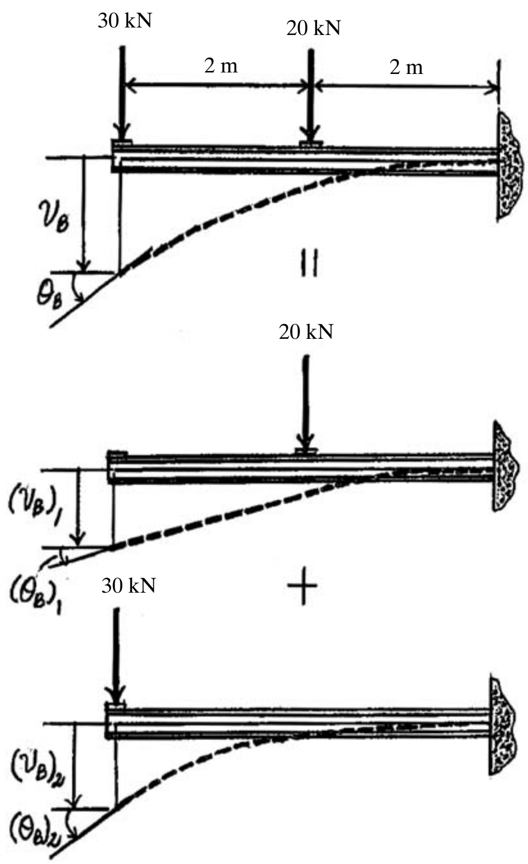
$$= \frac{40}{EI} + \frac{240}{EI}$$

$$= \frac{280\ \text{kN}\cdot\text{m}^2}{EI}$$

$$\Delta_B = (\Delta_B)_1 + (\Delta_B)_2$$

$$= \frac{133.33}{EI} + \frac{640}{EI}$$

$$= \frac{773.33\ \text{kN}\cdot\text{m}^3}{EI}$$



(a)

For A36 steel W250 × 22, $I = 28.8(10^6)\ \text{mm}^4 = 28.8(10^{-6})\ \text{m}^4$ and $E = 200$ GPa

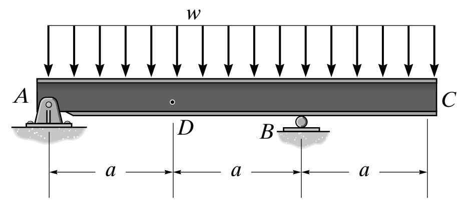
$$\theta_B = \frac{280(10^3)\ \text{N}\cdot\text{m}^2}{[200(10^9)\ \text{N/m}^2][28.8(10^{-6})\ \text{m}^4]} = 0.0486\ \text{rad}$$ **Ans.**

$$\Delta_B = \frac{773.33(10^3)\ \text{N}\cdot\text{m}^3}{[200(10^9)\ \text{N/m}^2][28.8(10^{-6})\ \text{m}^4]} = 0.1343\ \text{m} = 134\ \text{mm}$$ **Ans.**

Ans:

$\theta_B = 0.0486$ rad; $\Delta_B = 134$ mm

12–85. Determine the slope and deflection at end C of the overhang beam. EI is constant.



SOLUTION

Elastic Curves. The uniform distributed load on the beam is equivalent to the sum of the seperate loadings shown in Fig. *a*. The elastic curve for each seperate loading is shown Fig. *a*.

Method of Superposition. Using the table in the appendix, the required slopes and deflections are

$$(\theta_C)_1 = (\theta_B)_1 = \frac{wL^3}{24EI} = \frac{w(2a)^3}{24EI} = \frac{wa^3}{3EI}$$

$$(\Delta_C)_1 = (\theta_B)_1(a) = \frac{wa^3}{3EI}(a) = \frac{wa^4}{3EI} \quad \uparrow$$

$$(\theta_C)_2 = \frac{wL^3}{6EI} = \frac{wa^3}{6EI}$$

$$(\Delta_C)_2 = \frac{wL^4}{8EI} = \frac{wa^4}{8EI} \quad \downarrow$$

$$(\theta_C)_3 = (\theta_B)_3 = \frac{M_O L}{3EI} = \frac{\left(\frac{wa^2}{2}\right)(2a)}{3EI} = \frac{wa^3}{3EI}$$

$$(\Delta_C)_3 = (\theta_B)_3\,(a) = \frac{wa^3}{3EI}(a) = \frac{wa^4}{3EI} \quad \downarrow$$

Then the slope and deflection of C are

$$\theta_C = (\theta_C)_1 + (\theta_C)_2 + (\theta_C)_3$$

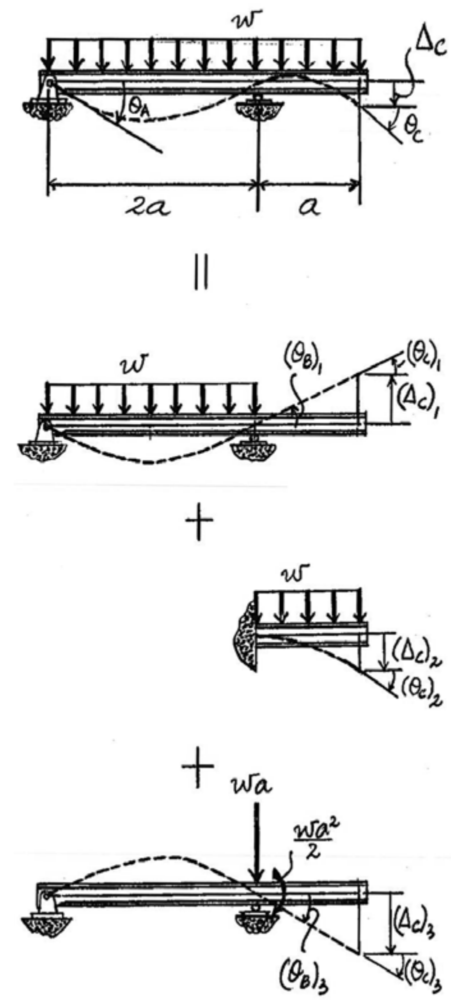
$$= -\frac{wa^3}{3EI} + \frac{wa^3}{6EI} + \frac{wa^3}{3EI}$$

$$= \frac{wa^3}{6EI} \qquad \textbf{Ans.}$$

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2 + (\Delta_C)_3$$

$$= -\frac{wa^4}{3EI} + \frac{wa^4}{8EI} + \frac{wa^4}{3EI}$$

$$= \frac{wa^4}{8EI} \downarrow \qquad \textbf{Ans.}$$

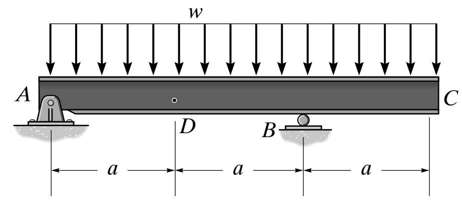


(a)

Ans:

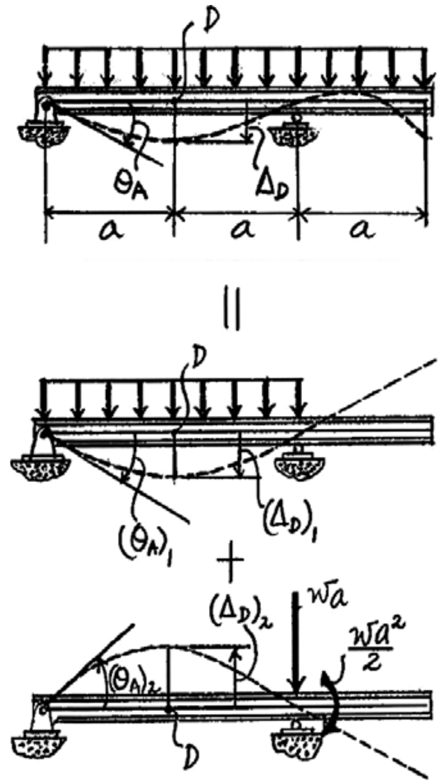
$\theta_C = \frac{wa^3}{6EI}$, $\Delta_C = \frac{wa^4}{8EI} \downarrow$

12–86. Determine the slope at A and the deflection at point D of the overhang beam. EI is constant.



SOLUTION

Elastic Curves. The uniform distributed load on the deformation of span AB is equivalent to the sum of the seperate loadings shown in Fig. a. The elastic curve for each seperate loading is shown in Fig. a.



(a)

Method of Superposition. Using the table in the appendix, the required slope and deflections are

$$(\theta_A)_1 = \frac{wL^3}{24EI} = \frac{w(2a)^3}{24EI} = \frac{wa^3}{3EI}$$

$$(\Delta_D)_1 = \frac{5wL^4}{384EI} = \frac{5w(2a)^4}{384EI} = \frac{5wa^4}{24EI} \quad \downarrow$$

$$(\theta_A)_2 = \frac{M_O L}{6EI} = \frac{\frac{wa^2}{2}(2a)}{6EI} = \frac{wa^3}{6EI}$$

$$(\Delta_D)_2 = \frac{M_O x}{6EIL}\left(L^2 - x^2\right) = \frac{\left(\frac{wa^2}{2}\right)(a)}{6EI(2a)}\left[(2a)^2 - a^2\right]$$

$$= \frac{wa^4}{8EI} \quad \uparrow$$

Then the slope and deflection of point D are

$$\theta_A = (\theta_A)_1 + (\theta_A)_2$$

$$= \frac{wa^3}{3EI} - \frac{wa^3}{6EI} = \frac{wa^3}{6EI}$$ **Ans.**

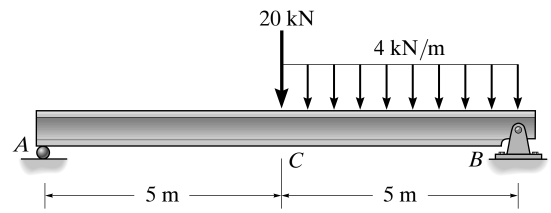
$$\Delta_D = (\Delta_D)_1 + (\Delta_D)_2$$

$$= \frac{5wa^4}{24EI} - \frac{wa^4}{8EI} = \frac{wa^4}{12EI}\downarrow$$ **Ans.**

Ans:

$\theta_A = \dfrac{wa^3}{6EI}$, $\Delta_D = \dfrac{wa^4}{12EI} \downarrow$

12–87. The simply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the deflection at its center C. $I = 0.1457(10^{-3})\ \text{m}^4$.



SOLUTION

Using the table in appendix, the required deflections for each load case are computed as follow:

$$(v_C)_1 = \frac{5wL^4}{768EI} = \frac{5(4)\left(10^4\right)}{768\ EI}$$

$$= \frac{260.42\ \text{kN}\cdot\text{m}^3}{EI} \downarrow$$

$$(v_C)_2 = \frac{PL^3}{48EI} = \frac{20N\left(10^3\right)}{48EI} = \frac{416.67\ \text{kN}\cdot\text{m}^3}{EI} \downarrow$$

Then the deflection of point C is

$$v_C = (v_C)_1 + (v_C)_2$$

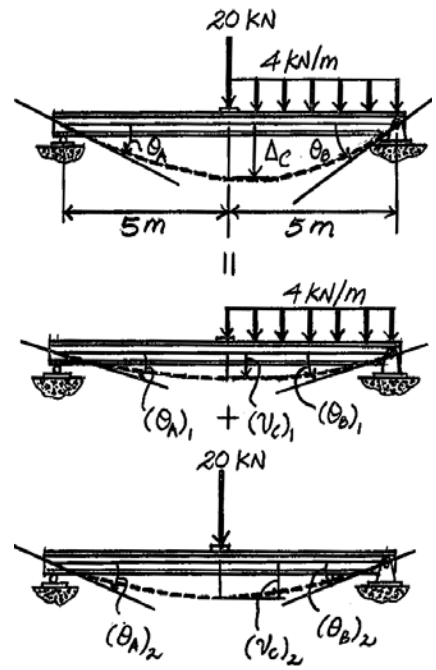
$$= \frac{260.42}{EI} + \frac{416.67}{EI}$$

$$= \frac{677.08\ \text{kN}\cdot\text{m}^3}{EI} \downarrow$$

$$= 0.1457\left(10^{-3}\right)\text{m}^4$$

and $E = 200GPa$

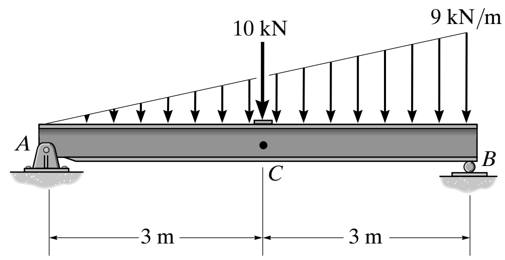
$$\Delta_C = \frac{677.08\left(10^3\right)}{200\left(10^9\right)\left[0.1457\left(10^{-3}\right)\right]} = 0.0232\ \text{m} = 23.2\ \text{m} \downarrow$$ **Ans.**



(a)

Ans:
$\Delta_C = 23.2\ \text{m} \downarrow$

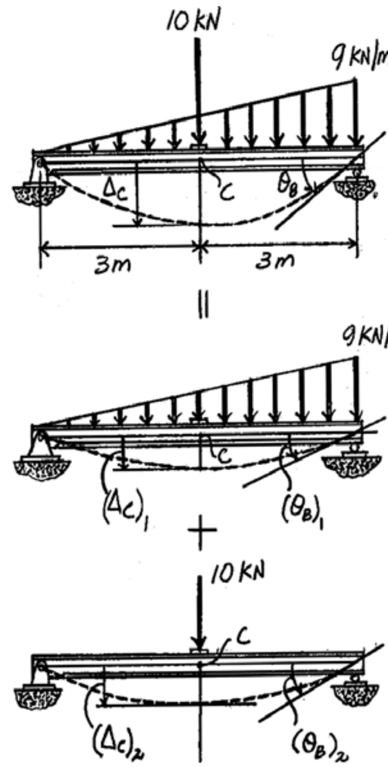
***12–88.** Determine the slope at B and the deflection at point C of the simply supported beam. $E = 200$ GPa and $I = 45.5(10^6)$ mm^4.



SOLUTION

Elastic Curves. The loading system on the beam is equivalent to the sum of the seperate loadings shown in Fig. *a*. The elastic curves for each loading are shown in Fig. *a*.

Method of Superposition. Using the table in the appendix, the required slope and deflections are



$$(\theta_B)_1 = \frac{w_O L^3}{45EI} = \frac{9(6^3)}{45EI} = \frac{43.2\text{kN}\cdot\text{m}^2}{EI}$$

$$(\Delta_C)_1 = \frac{w_O x}{360EIL}\left(3x^4 - 10L^2x^2 + 7L^4\right)$$

$$= \frac{9(3)}{360EI(6)}\left[3(3^4) - 10(6^2)(3^2) + 7(6^4)\right]$$

$$= \frac{75.9375\text{kN}\cdot\text{m}^3}{EI} \quad \downarrow$$

$$(\theta_B)_2 = \frac{PL^2}{16EI} = \frac{10(6^2)}{16EI} = \frac{22.5\text{kN}\cdot\text{m}^2}{EI}$$

$$(\Delta_C)_2 = \frac{PL^3}{48EI} = \frac{10(6^3)}{48EI} = \frac{45\text{kN}\cdot\text{m}^3}{EI} \quad \downarrow$$

Then the slope at B and deflection at C are

$$\theta_B = (\theta_B)_1 + (\theta_B)_2$$

$$= \frac{43.2}{EI} + \frac{22.5}{EI} = \frac{65.7\text{kN}\cdot\text{m}^2}{EI} = \frac{65.7(10^3)}{200(10^9)\left[45.5(10^{-6})\right]} = 0.00722 \text{ rad} \qquad \textbf{Ans.}$$

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2$$

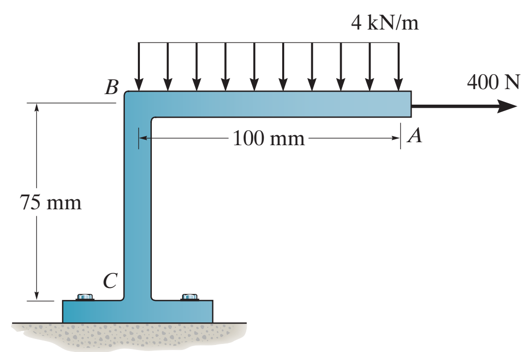
$$= \frac{75.9375}{EI} + \frac{45}{EI} = \frac{120.9375\,\text{kN}\cdot\text{m}^3}{EI} = \frac{120.9375(10^3)}{200(10^9)\left[45.5(10^{-6})\right]}$$

$$= 0.01329 \text{ m} = 13.3 \text{ mm} \downarrow \qquad \textbf{Ans.}$$

Ans:

$\theta_B = 0.00722$ rad;
$\Delta_C = 13.3$ mm $\downarrow$

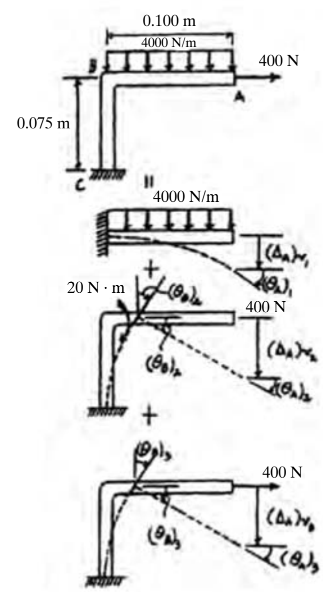
12–89. Determine the vertical deflection and slope at the end A of the bracket. Assume that the bracket is fixed supported at its base, and neglect the axial deformation of segment AB. EI is constant.



SOLUTION

Elastic Curve: The elastic curves for the concentrated load, uniform distibuted load, and couple moment are drawn separately as shown.

Method of Superposition: Using the table in Appendix C, the required slopes and displacements are



$$(\theta_A)_1 = \frac{wL_{AB}^3}{6EI} = \frac{4(10^3)(0.100)^3}{6EI} = \frac{0.6667\ \text{N}\cdot\text{m}^2}{EI}$$

$$(\theta_A)_2 = (\theta_B)_2 = \frac{M_0\,L_{BC}}{EI} = \frac{20(0.075)}{EI} = \frac{1.5\ \text{N}\cdot\text{m}^2}{EI}$$

$$(\theta_A)_3 = (\theta_B)_3 = \frac{PL_{BC}^2}{2EI} = \frac{400(0.075)^2}{2EI} = \frac{1.125\ \text{N}\cdot\text{m}^2}{EI}$$

$$(\Delta_A)_{v_1} = \frac{wL_{AB}^4}{8EI} = \frac{4(0.1)^4(10^3)}{8EI} = \frac{0.05\ \text{N}\cdot\text{m}^3}{EI}\ \downarrow$$

$$(\Delta_A)_{v_2} = (\theta_B)_2\,(L_{AB}) = \frac{1.5}{EI}(0.1) = \frac{0.15\ \text{N}\cdot\text{m}^3}{EI}\ \downarrow$$

$$(\Delta_A)_{v_3} = (\theta_B)_3\,(L_{AB}) = \frac{1.125}{EI}(0.1) = \frac{0.1125\ \text{N}\cdot\text{m}^3}{EI}\ \downarrow$$

The slope at A is

$$\theta_A = (\theta_A)_1 + (\theta_A)_2 + (\theta_A)_3$$

$$= \frac{0.6667}{EI} + \frac{1.5}{EI} + \frac{1.125}{EI}$$

$$= \frac{3.29\ \text{N}\cdot\text{m}^2}{EI}$$ **Ans.**

The vertical displacement at A is

$$(\Delta_A)_v = (\Delta_A)_{v_1} + (\Delta_A)_{v_2}\,(\Delta_A)_{v_3}$$

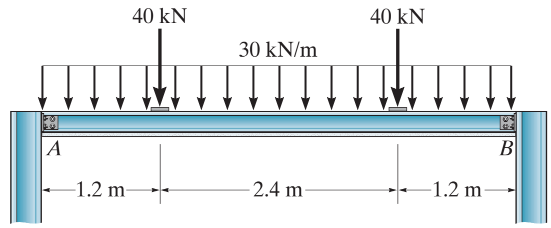
$$= \frac{0.05}{EI} + \frac{0.15}{EI} + \frac{0.1125}{EI}$$

$$= \frac{0.3125\ \text{N}\cdot\text{m}^3}{EI}\ \downarrow$$ **Ans.**

Ans:

$$\theta_A = \frac{3.29\ \text{N}\cdot\text{m}^2}{EI},\ (\Delta_A)_v = \frac{0.3125\ \text{N}\cdot\text{m}^3}{EI}\ \downarrow$$

12–90. The simply supported beam carries a uniform load of 30 kN/m. Code restrictions, due to a plaster ceiling, require the maximum deflection not to exceed 1/360 of the span length. Select the lightest-weight A-36 steel wide-flange beam from Appendix B that will satisfy this requirement and safely support the load. The allowable bending stress is σ_{allow} = 168 MPa and the allowable shear stress is τ_{allow} = 100 MPa. Assume A is a pin and B a roller support.



$M_{max} = 134.4\ \text{kN}\cdot\text{m}$

Strength criterion:

$$\sigma_{allow} = \frac{M}{S_{req'd}}; \quad 168(10^6) = \frac{134.4(10^3)}{S_{req'd}}$$

$$S_{req'd} = 0.8(10^{-3})\ \text{m}^3 = 800(10^3)\ \text{mm}^3$$

Choose W460 × 52 $\quad S = 942(10^3)\ \text{mm}^3 \quad t_w = 7.62\ \text{mm}, d = 450\ \text{mm}, I = 212(10^6)\ \text{mm}^4$

$$\tau_{allow} = \frac{V}{A_{web}}$$

$$100 \geq \frac{112(10^3)}{(0.00749)(0.403)} = 37.1\ \text{MPa} \quad \text{O.K.}$$

Deflection criterion:

Maximum is at center.

$$v_{max} = \frac{5wL^4}{384EI} + 2\left[\frac{Pbx}{6EIL}(L^2 - b^2 - x^2)\right]$$

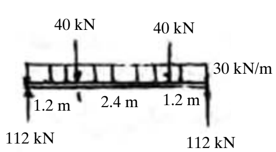
$$= \frac{5(30)(4.8)^4}{384EI} + 2\left\{\left[\frac{40(1.2)(2.4)}{6EI(4.8)}\right](4.8^2 - 1.2^2 - 2.4^2)\right\}$$

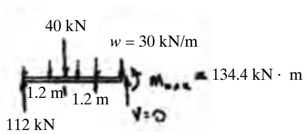
$$= \frac{334.08\ \text{kN}\cdot\text{m}^3}{EI} = \frac{334.08(10^3)\ \text{N}\cdot\text{m}^3}{[200(10^9)\ \text{N/m}^2][212(10^{-6})\ \text{m}^4]}$$

$$= 0.007879\ \text{m} = 7.88\ \text{mm} < \frac{1}{360}[4.8(10^3)]$$

$$= 13.3\ \text{mm} \quad \text{(O.K.)}$$

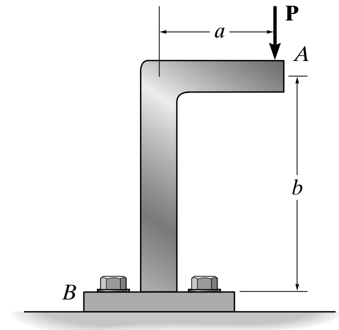
Use W460 × 52 **Ans.**





Ans:
Use W460 × 52

12–91. Determine the vertical deflection at the end A of the bracket. Assume that the bracket is fixed supported at its base B and neglect axial deflection. EI is constant.

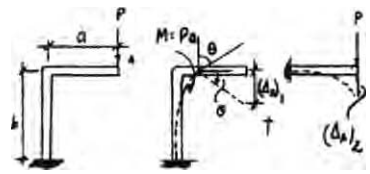


SOLUTION

$$\theta = \frac{ML}{EI} = \frac{Pab}{EI}$$

$$(\Delta_A)_1 = \theta(a) = \frac{Pa^2b}{EI}$$

$$(\Delta_A)_2 = \frac{PL^3}{3EI} = \frac{Pa^3}{3EI}$$

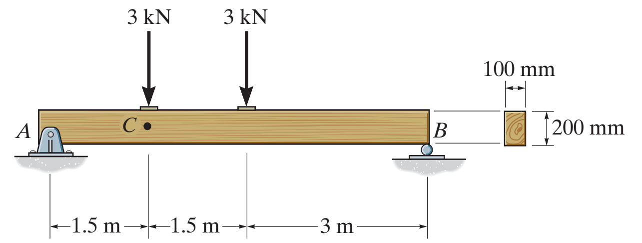


$$\Delta_A = (\Delta_A)_1 + (\Delta_A)_2 = \frac{Pa^2b}{EI} + \frac{Pa^3}{3EI} = \frac{Pa^2\,(3b + a)}{3EI}$$ **Ans.**

Ans:

$$\Delta_A = \frac{Pa^2\,(3b + a)}{3EI}$$

*12–92. Determine the slope at A and the deflection at point C of the simply supported beam. The modulus of elasticity of the wood is $E = 10$ GPa.



SOLUTION

Elastic Curves. The two concentrated forces **P** are applied separately on the beam and the resulting elastic curves are shown in Fig. *a*.

Method of Superposition. Using the table in the appendix, the required slopes and deflections are

$$(\theta_A)_1 = \frac{Pab(L + b)}{6EIL} = \frac{3(1.5)(4.5)(6 + 4.5)}{6EI(6)} = \frac{5.90625 \text{ kN}\cdot\text{m}^2}{EI}$$

$$(\Delta_C)_1 = \frac{Pbx}{6EIL}\left(L^2 - b^2 - x^2\right) = \frac{3(4.5)(1.5)}{6EI(6)}\left(6^2 - 4.5^2 - 1.5^2\right)$$

$$= \frac{7.594 \text{ kN}\cdot\text{m}^3}{EI} \downarrow$$

$$(\theta_A)_2 = \frac{PL^2}{16EL} = \frac{3\left(6^2\right)}{16EI} = \frac{6.75 \text{ kN}\cdot\text{m}^2}{EI}$$

$$(\Delta_C)_2 = \frac{Px}{48EI}\left(3L^2 - 4x^2\right) = \frac{3(1.5)}{48EI}\left(3(6)^2 - 4(1.5)^2\right) = \frac{9.281}{EI}$$

Then the slope at A and deflection at C are

$$\theta_A = (\theta_A)_1 + (\theta_A)_2$$

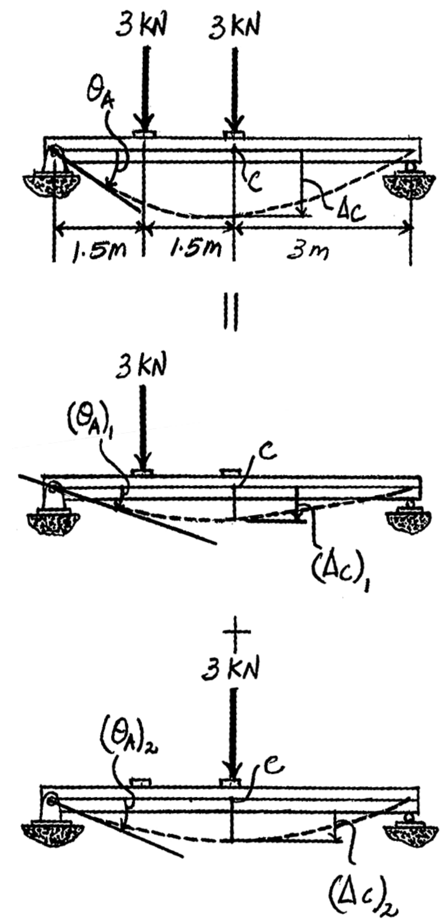
$$= \frac{5.90625}{EI} + \frac{6.75}{EI}$$

$$= \frac{12.65625 \text{ kN}\cdot\text{m}^2}{EI} = \frac{12.6525\left(10^3\right)}{10\left(10^9\right)\left[\frac{1}{12}(0.1)\left(0.2^3\right)\right]} = 0.0190 \text{ rad}$$ **Ans.**

and

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2$$

$$= \frac{7.594}{EI} + \frac{9.281}{EI} = \frac{16.88\left(10^3\right)}{10\left(10^9\right)\left[\frac{1}{12}(0.1)\left(0.2^3\right)\right]} = 0.0253 \text{ m} = 25.3 \text{ mm}$$ **Ans.**

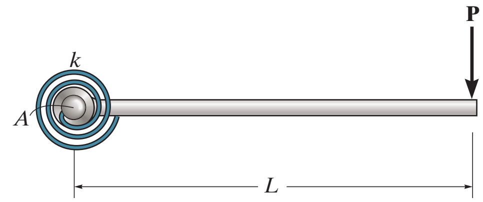


Ans:

$\theta_A = 0.0190$ rad; $\Delta_C = 25.3$ mm

12–93.

The rod is pinned at its end A and attached to a torsional spring having a stiffness k, which measures the torque per radian of rotation of the spring. If a force **P** is always applied perpendicular to the end of the rod, determine the displacement of the force. EI is constant.



SOLUTION

In order to maintain equilibrium, the rod has to rotate through an angle θ.

$$\circlearrowleft + \Sigma M_A = 0; \quad k\theta - PL = 0; \quad \theta = \frac{PL}{k}$$

Hence,

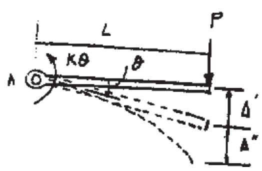
$$v' = L\theta = L\left(\frac{PL}{k}\right) = \frac{PL^2}{k}$$

Elastic deformation:

$$v'' = \frac{PL^3}{3EI}$$

Therefore,

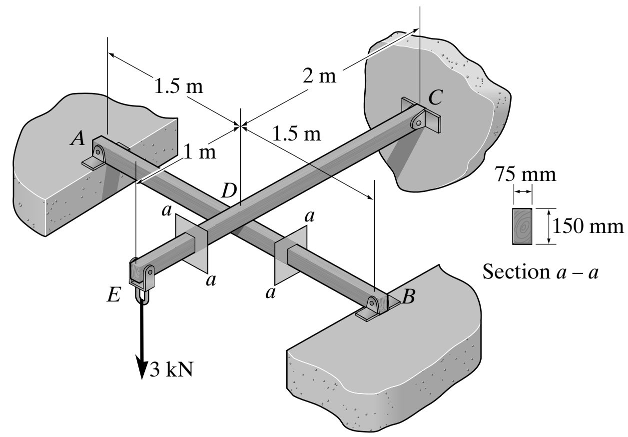
$$v = v' + v'' = \frac{PL^2}{k} + \frac{PL^3}{3EI} = PL^2\left(\frac{1}{k} + \frac{L}{3EI}\right)$$ **Ans.**



Ans:

$$v = PL^2\left(\frac{1}{k} + \frac{L}{3EI}\right)$$

12–94. Determine the deflection at end E of beam CDE. The beams are made of wood having a modulus of elasticity of $E = 10$ GPa.



SOLUTION

Method of Superposition. Referring to the table in the appendix, the deflection of point D is

$$\Delta_D = \frac{PL^3}{48EI} = \frac{4.5(3^3)}{48EI} = \frac{2.53125\text{ kN}\cdot\text{m}^3}{EI} \downarrow$$

Subsequently,

$$(\Delta_E)_1 = \Delta_D\left(\frac{3}{2}\right) = \frac{2.53125}{EI}\left(\frac{3}{2}\right) = \frac{3.796875\text{ kN}\cdot\text{m}^3}{EI} \downarrow$$

Also,

$$(\Delta_E)_2 = \frac{PL^3}{3EI} = \frac{3(1^3)}{3EI} = \frac{1\text{ kN}\cdot\text{m}^3}{EI} \downarrow$$

$$(\theta_D)_3 = \frac{M_O L}{3EI} = \frac{3(2)}{3EI} = \frac{2\text{ kN}\cdot\text{m}^2}{EI}$$

$$(\Delta_E)_3 = (\theta_D)_3 L = \frac{2}{EI}(1) = \frac{2\text{ kN}\cdot\text{m}^3}{EI} \downarrow$$

Thus, the deflection of end E is

$$\Delta_E = (\Delta_E)_1 + (\Delta_E)_2 + (\Delta_E)_3$$

$$= \frac{3.796875}{EI} + \frac{1}{EI} + \frac{2}{EI} = \frac{6.796875\text{kN}\cdot\text{m}^3}{EI} = \frac{6.796875(10^3)}{10(10^9)\left[\frac{1}{12}(0.075)(0.15^3)\right]}$$

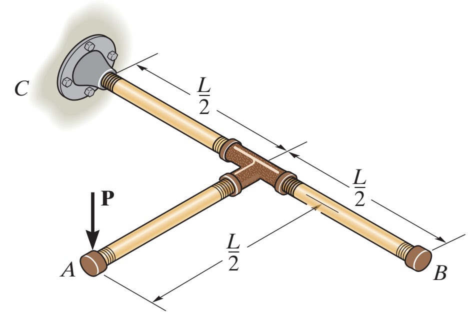
$$= 0.03222\text{ m} = 32.2\text{ mm} \downarrow$$ **Ans.**

Ans:

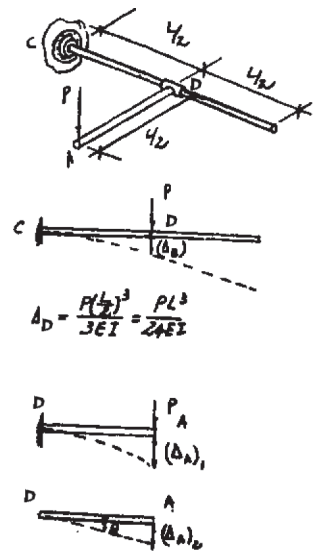
$\Delta_E = 32.2\text{ mm} \downarrow$

12–95.

The pipe assembly consists of three equal-sized pipes with flexibility stiffness EI and torsional stiffness GJ. Determine the vertical deflection at A.



SOLUTION



$$v_D = \frac{P\left(\frac{L}{2}\right)^3}{3EI} = \frac{PL^3}{24EI}$$

$$(v_A)_1 = \frac{P\left(\frac{L}{2}\right)^3}{3EI} = \frac{PL^3}{24EI}$$

$$\theta = \frac{TL}{GJ} = \frac{(PL/2)\left(\frac{L}{2}\right)}{GJ} = \frac{PL^2}{4GJ}$$

$$(v_A)_2 = \theta\left(\frac{L}{2}\right) = \frac{PL^3}{8GJ}$$

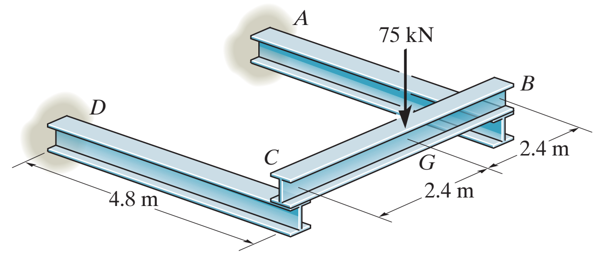
$$v_A = v_D + (v_A)_1 + (v_A)_2$$

$$= \frac{PL^3}{24EI} + \frac{PL^3}{24EI} + \frac{PL^3}{8GJ}$$

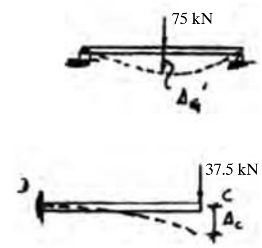
$$= PL^3\left(\frac{1}{12EI} + \frac{1}{8GJ}\right)\downarrow$$ **Ans.**

Ans:
$v_A = PL^3\left(\dfrac{1}{12EI} + \dfrac{1}{8GJ}\right)\downarrow$

***12–96.** The framework consists of two A-36 steel cantilevered beams CD and BA and a simply supported beam CB. If each beam is made of steel and has a moment of inertia about its principal axis of $I_x = 46(10^6)$ mm^4, determine the deflection at the center G of beam CB.



SOLUTION



$$\Delta_C = \frac{PL^3}{3EI} = \frac{37.5(4.8^3)}{3EI} = \frac{1382.4 \text{ kN}\cdot\text{m}^3}{EI} \downarrow$$

$$\Delta'_G = \frac{PL^3}{48EI} = \frac{75(4.8^3)}{48EI} = \frac{172.8 \text{ kN}\cdot\text{m}^3}{EI} \downarrow$$

$$\Delta_G = \Delta_C + \Delta'_G$$

$$= \frac{1382.4}{EI} + \frac{172.8}{EI} = \frac{1555.2 \text{ kN}\cdot\text{m}^3}{EI}$$

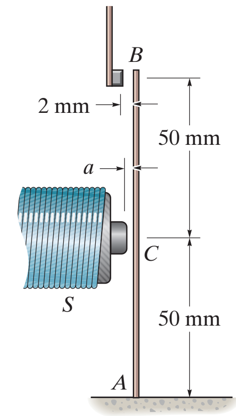
$$= \frac{1555.2(10^3)\text{ N}\cdot\text{m}^3}{[200(10^9)\text{ N/m}^2][46(10^{-6})\text{ m}^4]} = 0.1690 \text{ m} = 169 \text{ mm} \downarrow$$ **Ans.**

Ans:

$\Delta_G = 169$ mm $\downarrow$

12–97.

The relay switch consists of a thin metal strip or armature AB that is made of red brass C83400 and is attracted to the solenoid S by a magnetic field. Determine the smallest force F required to attract the armature at C in order that contact is made at the free end B. Also, what should the distance a be for this to occur? The armature is fixed at A and has a moment of inertia of $I = 0.18(10^{-12})\ \text{m}^4$.



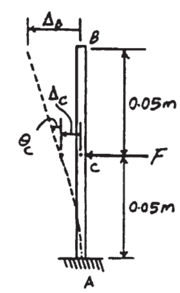
SOLUTION

Elastic Curve: As shown.

Method of Superposition: Using the table in Appendix C, the required slopes and displacements are

$$\theta_C = \frac{PL_{AC}^2}{2EI} = \frac{F(0.05^2)}{2EI} = \frac{0.00125F\ \text{m}^2}{EI}$$

$$\Delta_C = \frac{PL_{AC}^3}{3EI} = \frac{F(0.05^3)}{3EI} = \frac{41.667(10^{-6})F\ \text{m}^3}{EI} \qquad \textbf{(1)}$$



$$\Delta_B = \Delta_C + \theta_C L_{CB}$$

$$= \frac{41.667(10^{-6})F}{EI} + \frac{0.00125(10^{-6})F}{EI}(0.05)$$

$$= \frac{104.167(10^{-6})F\ \text{m}^3}{EI} \qquad \textbf{(2)}$$

Required the displacement $\Delta_B = 0.002$ m. From Eq. (2),

$$0.002 = \frac{104.167(10^{-6})F}{101(10^9)(0.18)(10^{-12})}$$

$$F = 0.349056\ \text{N} = 0.349\ \text{N} \qquad \textbf{Ans.}$$

From Eq. (1),

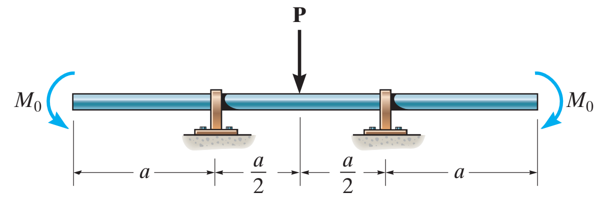
$$a = \Delta_C = \frac{41.667(10^{-6})(0.349056)}{101(10^9)(0.18)(10^{-12})}$$

$$= 0.800(10^{-3})\ \text{m} = 0.800\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$F = 0.349$ N,
$a = 0.800$ mm

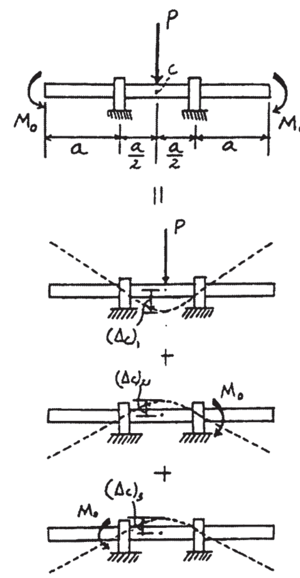
12–98.

Determine the moment M_0 in terms of the load P and dimension a so that the deflection at the center of the shaft is zero. EI is constant.



SOLUTION

Elastic Curve: The elastic curves for the concentrated load and couple moment are drawn separately as shown.



Method of Superposition: Using the table in Appendix C, the required slope and displacement are

$$(\Delta_C)_1 = \frac{Pa^3}{48EI} \downarrow$$

$$(\Delta_C)_2 = (\Delta_C)_3 = \frac{M_0 x}{6EIL}(x^2 - 3Lx + 2L^2)$$

$$= \frac{M_0 \left(\frac{a}{2}\right)}{6EIa}\left[\left(\frac{a}{2}\right)^2 - 3(a)\left(\frac{a}{2}\right) + 2a^2\right]$$

$$= \frac{M_0\, a^2}{16EI} \uparrow$$

Require the displacement at C to equal zero.

$$(+\uparrow) \qquad \Delta_C = 0 = (\Delta_C)_1 + (\Delta_C)_2 + (\Delta_C)_3$$

$$0 = -\frac{Pa^3}{48EI} + \frac{M_0\, a^2}{16EI} + \frac{M_0\, a^2}{16EI}$$

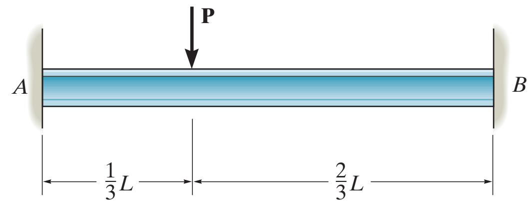
$$M_0 = \frac{Pa}{6}$$ **Ans.**

Ans:

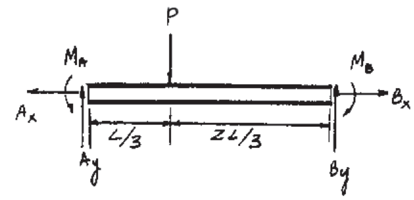
$$M_0 = \frac{Pa}{6}$$

12–99.

Determine the reactions at the supports A and B, then draw the shear and moment diagram. EI is constant. Neglect the effect of axial load.



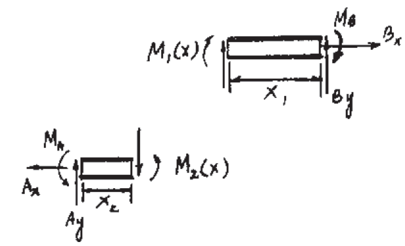
SOLUTION



$$\circlearrowleft + \Sigma M_A = 0; \qquad M_A + B_y L - P\left(\frac{L}{3}\right) - M_B = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y - P = 0 \qquad (2)$$

Moment Functions:



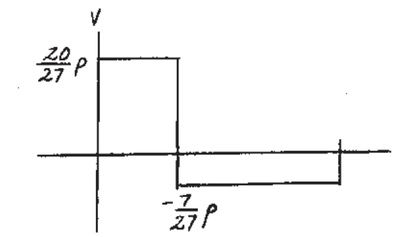
$M_1(x_1) = B_y x_1 - M_B$

$M_2(x_2) = A_y x_2 - M_A$

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

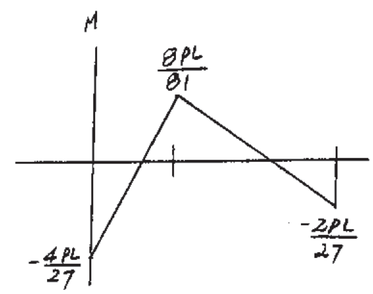
For $M_1(x) = B_y x_1 - M_B$; $\qquad EI\dfrac{d^2v_1}{dx_1^2} = B_y x_1 - M_B$



$$EI\frac{dv_2}{dx_2} = \frac{B_y x_1^2}{2} - M_B x_1 + C_1 \qquad (3)$$

$$EIv_1 = \frac{B_y x_1^3}{6} - \frac{M_B x_1^2}{2} + C_1 x + C_2 \qquad (4)$$

For $M_2(x) = A_y x_2 - M_A$



$$EI\frac{d^2v_2}{dx_2^2} = A_y x_2 - M_A$$

$$EI\frac{dv_2}{dx_2^2} = \frac{A_y x_2^2}{2} - M_A x_2 + C_3 \qquad (5)$$

$$EIv_2 = \frac{A_y x_2^3}{6} - \frac{M_A x_2^3}{2} + C_3 x_2 + C_4 \qquad (6)$$

Boundary Conditions:

At $x_1 = 0$, $\dfrac{dv_1}{dx_1} = 0$

From Eq. (3),

$0 = 0 - 0 + C_1; \quad C_1 = 0$

At $x_1 = 0$, $v_1 = 0$

From Eq. (4),

$0 = 0 - 0 + 0 + C_2; \quad C_2 = 0$

Similarly, from Eqs. (5) and (6), $C_3 = C_4 = 0$.

12–99. Continued

At $x_1 = \dfrac{2}{3}L$, $x_2 = \dfrac{1}{3}L$, $v_1 = v_2$ and $\dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$.

From Eqs. (4) and (6),

$$\frac{B_y}{6}\left(\frac{2}{3}L\right)^3 - \frac{M_B}{2}\left(\frac{2}{3}L\right)^2 = \frac{A_y}{6}\left(\frac{1}{3}L\right)^3 - \frac{M_A}{2}\left(\frac{1}{3}L\right)^2$$

$$8B_yL - 36M_B = A_yL - 9M_A \qquad \textbf{(7)}$$

From Eqs. (3) and (5),

$$\frac{B_y}{2}\left(\frac{2}{3}L\right)^2 - M_B\left(\frac{2}{3}L\right) = -\frac{A_y}{2}\left(\frac{1}{3}L\right)^2 + M_A\left(\frac{1}{3}L\right)$$

$$4B_yL - 12M_B = -A_yL + 6M_A \qquad \textbf{(8)}$$

Solving Eqs. (1), (2), (7) and (8) simultaneously,

$$A_y = \frac{20}{27}P \qquad \textbf{Ans.}$$

$$M_A = \frac{4}{27}PL \qquad \textbf{Ans.}$$

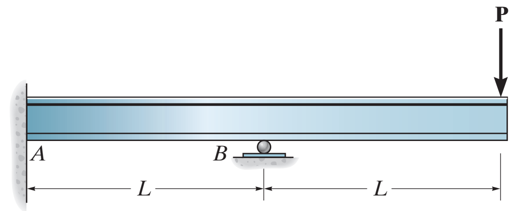
$$B_y = \frac{7}{27}P \qquad \textbf{Ans.}$$

$$M_B = \frac{2}{27}PL \qquad \textbf{Ans.}$$

Ans:
$A_x = B_x = 0$,
$A_y = \dfrac{20}{27}P$,
$M_A = \dfrac{4}{27}PL$,
$B_y = \dfrac{7}{27}P$,
$M_B = \dfrac{2}{27}PL$

***12–100.**

Determine the reactions at the supports, then draw the shear and moment diagram. EI is constant.



SOLUTION

Support Reactions: FBD(a).

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y - A_y - P = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad A_y L - M_A - PL = 0 \qquad \textbf{(2)}$$

Moment Functions: FBD(b) and (c).

$$M(x_1) = -Px_1$$

$$M(x_2) = M_A - A_y x_2$$

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = -Px_1$,

$$EI\frac{d^2v_1}{dx_1^2} = -Px_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{P}{2}x_1^2 + C_1 \qquad \textbf{(3)}$$

$$EI\,v_1 = -\frac{P}{6}x_1^3 + C_1x_1 + C_2 \qquad \textbf{(4)}$$

For $M(x_2) = M_A - A_y x_2$,

$$EI\frac{d^2v_2}{dx_2^2} = M_A - A_y x_2$$

$$EI\frac{dv_2}{dx_2} = M_A x_2 - \frac{A_y}{2}x_2^2 + C_3 \qquad \textbf{(5)}$$

$$EI\,v_2 = \frac{M_A}{2}x_2^2 - \frac{A_y}{6}x_2^3 + C_3x_2 + C_4 \qquad \textbf{(6)}$$

***12–100. Continued**

Boundary Conditions:

$v_2 = 0$ at $x_2 = 0$. From Eq. (6), $C_4 = 0$

$\dfrac{dv_2}{dx_2} = 0$ at $x_2 = 0$. From Eq. (5), $C_3 = 0$

$v_2 = 0$ at $x_2 = L$. From Eq. (6),

$$0 = \frac{M_A L^2}{2} - \frac{A_y L^3}{6} \qquad \textbf{(7)}$$

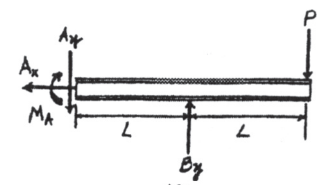
Solving Eqs. (2) and (7) yields,

$$M_A = \frac{PL}{2} \qquad A_y = \frac{3P}{2} \qquad \textbf{Ans.}$$

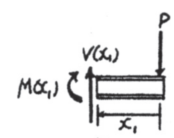
Substituting the value of A_y into Eqs. (1),

$$B_y = \frac{5P}{2} \qquad \textbf{Ans.}$$

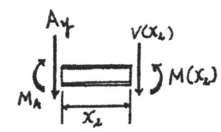
Note: The other boundary and continuity condition can be used to determine the constants C_1 and C_2 which are not needed here.



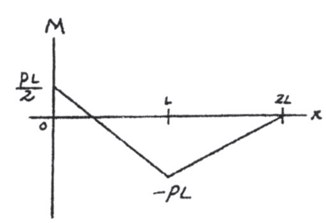
(a)



(b)



(c)



Ans:

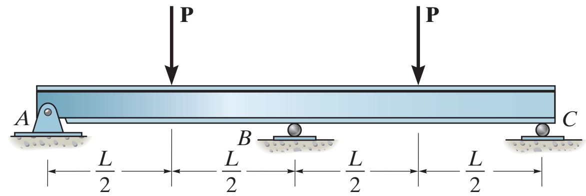
$M_A = \dfrac{PL}{2}$,

$A_y = \dfrac{3P}{2}$,

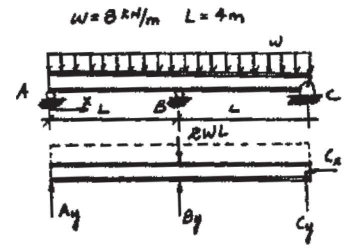
$B_y = \dfrac{5P}{2}$

12–101.

Determine the reactions at the supports A, B, and C, then draw the shear and moment diagrams. EI is constant.



SOLUTION



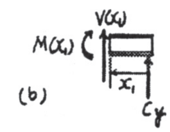
Support Reactions: FBD(a).

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y + C_y - 2P = 0 \qquad \textbf{(1)}$$

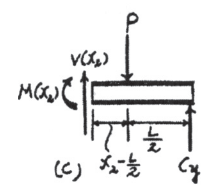
$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y L + C_y (2L) - P\left(\frac{L}{2}\right) - P\left(\frac{3L}{2}\right) = 0 \qquad \textbf{(2)}$$

Moment Function: FBD(b) and (c).



$$M(x_1) = C_y x_1$$

$$M(x_2) = C_y x_2 - Px_2 + \frac{PL}{2}$$



Slope and Elastic Curve:

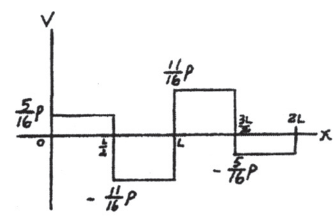
$$EI\frac{d^2v}{dx^2} = M(x)$$

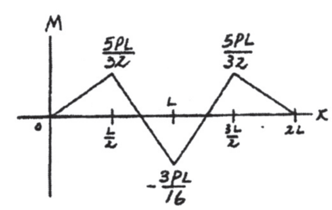
For $M(x_1) = C_y x_1$,

$$EI\frac{d^2v_1}{dx_1^2} = C_y x_1$$

$$EI\frac{dv_1}{dx_1} = \frac{C_y}{2}x_1^2 + C_1 \qquad \textbf{(3)}$$

$$EI\, v_1 = \frac{C_y}{6}x_1^3 + C_1 x_1 + C_2 \qquad \textbf{(4)}$$





For $M(x_2) = C_y x_2 - Px_2 + \dfrac{PL}{2}$,

$$EI\frac{d^2v_2}{dx_2^2} = C_y x_2 - Px_2 + \frac{PL}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{C_y}{2}x_2^2 - \frac{P}{2}x_2^2 + \frac{PL}{2}x_2 + C_3 \qquad \textbf{(5)}$$

$$EI\, v_2 = \frac{C_y}{6}x_2^3 - \frac{P}{6}x_2^3 + \frac{PL}{4}x_2^2 + C_3 x_2 + C_4 \qquad \textbf{(6)}$$

Boundary Conditions:

$v_1 = 0$ at $x_1 = 0$. From Eq. (4), $C_2 = 0$

Due to symmetry, $\dfrac{dv_2}{dx_2} = 0$ at $x_2 = L$. From Eq. (5),

$$0 = \frac{C_y L^2}{2} - \frac{PL^2}{2} + \frac{PL^2}{2} + C_3 \qquad C_3 = -\frac{C_y L^2}{2}$$

12–101. Continued

$v_2 = 0$ at $x_2 = L$. From Eq. (6),

$$0 = \frac{C_y L^3}{6} - \frac{PL^3}{6} + \frac{PL^3}{4} + \left(-\frac{C_y L^2}{2}\right)L + C_4$$

$$C_4 = \frac{C_y L^3}{3} - \frac{PL^3}{12}$$

Continuity Conditions:

At $x_1 = x_2 = \frac{L}{2}, \frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$. From Eqs. (3) and (5),

$$\frac{C_y}{2}\left(\frac{L}{2}\right)^2 + C_1 = \frac{C_y}{2}\left(\frac{L}{2}\right)^2 - \frac{P}{2}\left(\frac{L}{2}\right)^2 + \frac{PL}{2}\left(\frac{L}{2}\right) - \frac{C_y L^2}{2}$$

$$C_1 = \frac{PL^2}{8} - \frac{C_y L^2}{2}$$

At $x_1 = x_2 = \frac{L}{2}, v_1 = v_2$. From Eqs. (4) and (6),

$$\frac{C_y}{6}\left(\frac{L}{2}\right)^3 + \left(\frac{PL^2}{8} - \frac{C_y L^2}{2}\right)\left(\frac{L}{2}\right)$$

$$= \frac{C_y}{6}\left(\frac{L}{2}\right)^3 - \frac{P}{6}\left(\frac{L}{2}\right)^3 + \frac{PL}{4}\left(\frac{L}{2}\right)^2 + \left(-\frac{C_y L^2}{2}\right)\left(\frac{L}{2}\right) + \frac{C_y L^3}{3} - \frac{PL^3}{12}$$

$$C_y = \frac{5}{16}P$$ **Ans.**

Substituting C_y into Eqs. (1) and (2),

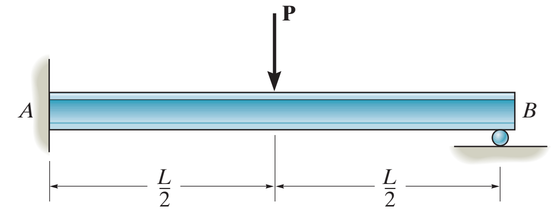
$$B_y = \frac{11}{8}P \qquad A_y = \frac{5}{16}P$$ **Ans.**

Ans:

$A_x = 0, C_y = \frac{5}{16}P, B_y = \frac{11}{8}P, A_y = \frac{5}{16}P$

12–102.

Determine the reactions at the supports A and B, then draw the shear and moment diagrams. EI is constant.



SOLUTION

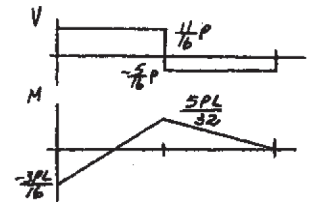
$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0 \qquad A_y + B_y - P = 0$

$A_y = P - B_y$ **(1)**

$\circlearrowleft + \Sigma M_A = 0 \qquad M_A + B_y(L) - P(L/2) = 0$

$$M_A = \frac{PL}{2} - B_y L \qquad \textbf{(2)}$$



Bending Moment $M(x)$:

$$M(x) = -(-B_y)\langle x - 0\rangle - P\left\langle x - \frac{L}{2}\right\rangle = B_y x - P\left\langle x - \frac{L}{2}\right\rangle$$

Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = B_y x - P\left\langle x - \frac{L}{2}\right\rangle$$

$$EI\frac{dv}{dx} = \frac{B_y x^2}{2} - \frac{P}{2}\left\langle x - \frac{L}{2}\right\rangle^2 + C_1 \qquad \textbf{(3)}$$

$$EIv = \frac{B_y x^3}{6} - \frac{P}{6}\left\langle x - \frac{L}{2}\right\rangle^3 + C_1 x + C_3 \qquad \textbf{(4)}$$

Boundary Conditions:

$v = 0 \qquad \text{at} \qquad x = 0$

From Eq. (4)

$C_1 = 0$

$v = 0 \qquad \text{at} \qquad x = L$

From Eq. (4)

$$0 = \frac{B_y L^3}{6} - \frac{PL^3}{48} + C_1 L \qquad \textbf{(5)}$$

$\frac{dv}{dx} = 0 \qquad \text{at} \qquad x = L$

From Eq. (3),

$$0 = \frac{B_y L^2}{2} - \frac{PL^2}{8} + C_1 \qquad \textbf{(6)}$$

Solving Eqs. (5) and (6) yields;

$B_y = \frac{5}{16}P$ **Ans.** $\qquad C_1 = \frac{-PL^3}{32}$

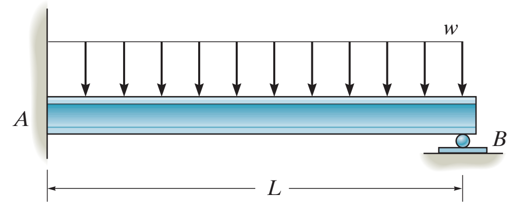
Substitute: $B_y = \frac{5}{16}P$ into Eqs. (1) and (2),

$A_y = \frac{11}{16}P$ **Ans.** $\qquad M_A = \frac{3PL}{16}$ **Ans.**

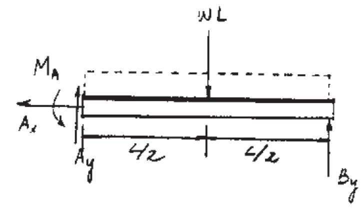
Ans:
$B_y = \frac{5}{16}P, A_y = \frac{11}{16}P, M_A = \frac{3PL}{16}$

12–103.

Determine the reactions at the supports A and B, then draw the shear and moment diagrams. EI is constant.



SOLUTION



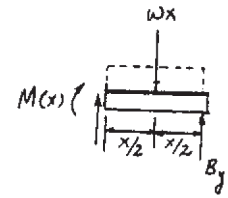
$\leftarrow \Sigma F_a = 0; \qquad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y - wL = 0$ **(1)**

$\zeta + \Sigma M_A = 0; \qquad M_A + B_y L - wL\left(\frac{L}{2}\right) = 0$ **(2)**

$\zeta + \Sigma M_{NA} = 0; \qquad B_y(x) - wx\left(\frac{x}{2}\right) - M(x) = 0$

$$M(x) + B_y x - \frac{wx^2}{2}$$

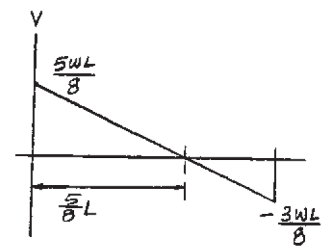


$EI\frac{d^2v}{dx^2} = M(x)$

$EI\frac{d^2v}{dx^2} = B_y x - \frac{wx^2}{2}$

$EI\frac{dv}{dx} = \frac{B_y x^2}{2} - \frac{wx^3}{6} + C_1$ **(3)**

$EIv = \frac{B_y x^3}{6} - \frac{wx^4}{24} + C_1 x + C_2$ **(4)**



Boundary Conditions:

At $x = 0, \qquad v = 0$

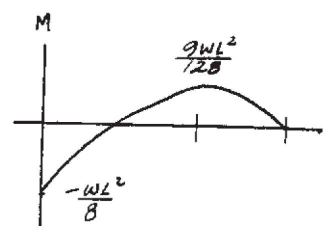
From Eq. (4),

$0 = 0 - 0 + 0 + C_2; \qquad C_2 = 0$

At $x = L, \quad \frac{dv}{dx} = 0$

From Eq. (3),

$0 = \frac{B_y L^2}{2} - \frac{wL^3}{6} + C_1$ **(5)**



At $x = L, \quad v = 0$

From Eq. (4),

$0 = \frac{B_y L^3}{6} - \frac{wL^4}{24} + C_1 L$ **(6)**

Solving Eqs. (5) and (6) yields:

$B_y = \frac{3wL}{8}$ **Ans.**

$C_1 = -\frac{wL^3}{48}$

Substituting B_y into Eqs. (1) and (2) yields:

$A_y = \frac{5wL}{8}$ **Ans.**

$M_A = \frac{wL^2}{8}$ **Ans.**

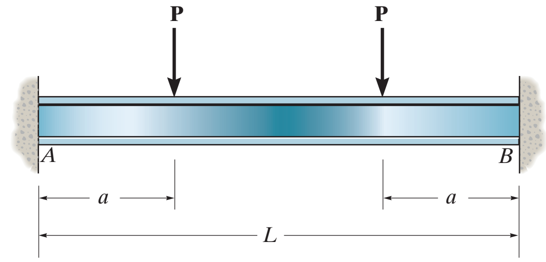
Ans:

$A_x = 0, B_y = \frac{3wL}{8},$

$A_y = \frac{5wL}{8}, M_A = \frac{wL^2}{8}$

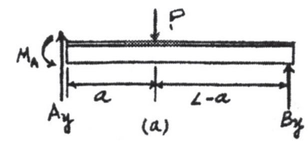
***12–104.**

Determine the moment reactions at the supports A and B. EI is constant.



SOLUTION

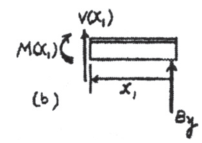
Support Reactions: FBD(a).



$$\circlearrowleft + \Sigma M_B = 0; \quad Pa + P(L - a) + M_A - A_y L - M_B = 0$$

$$PL + M_A - A_y L - M_B = 0 \qquad \textbf{(1)}$$

Moment Functions: FBD(b) and (c).

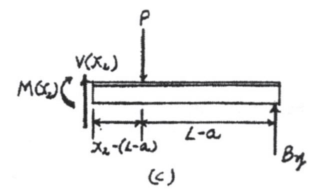


$$M(x_1) = A_y x_1 - M_A$$

$$M(x_2) = A_y x_2 - Px_2 + Pa - M_A$$

Slope and Elastic Curve:

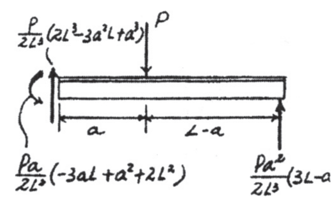
$$EI\frac{d^2v}{dx^2} = M(x)$$



For $M(x_1) = A_y x_1 - M_A$,

$$EI\frac{d^2v_1}{dx_1^2} = A_y x_1 - M_A$$

$$EI\frac{dv_1}{dx_1} = \frac{A_y}{2}x_1^2 - M_A x_1 + C_1 \qquad \textbf{(2)}$$

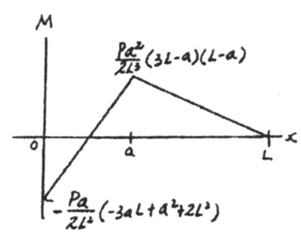


$$EI\,v_1 = \frac{A_y}{6}x_1^3 - \frac{M_A}{2}x_1^2 + C_1 x_1 + C_2 \qquad \textbf{(3)}$$

For $M(x_2) = A_y x_2 - Px_2 + Pa - Ma$,

$$EI\frac{d^2v}{dx_2^2} = A_y x_2 - Px_2 + Pa - M_A$$

$$EI\frac{dv_2}{dx_2} = \frac{A_y}{2}x_2^2 - \frac{P}{2}x_2^2 + Pax_2 - M_A x_2 + C_3 \qquad \textbf{(4)}$$



$$EI\,v_2 = \frac{A_y}{6}x_2^3 - \frac{P}{6}x_2^3 + \frac{Pa}{2}x_2^2 - \frac{M_A}{2}x_2^2 + C_3 x_2 + C_4 \qquad \textbf{(5)}$$

Boundary Conditions:

$\dfrac{dv_1}{dx_1} = 0$ at $x_1 = 0$. From Eq. (2), $C_1 = 0$

$v_1 = 0$ at $x_1 = 0$. From Eq. (3), $C_2 = 0$

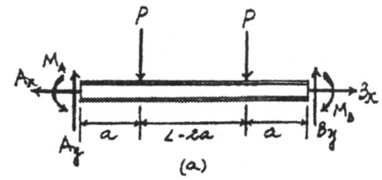
Due to symmetry, $\dfrac{dv_2}{dx_2} = 0$ at $x_2 = \dfrac{L}{2}$. From Eq. (4),

$$0 = \frac{A_y}{2}\left(\frac{L}{2}\right)^2 - \frac{P}{2}\left(\frac{L}{2}\right)^2 + Pa\left(\frac{L}{2}\right) - M_A\left(\frac{L}{2}\right) + C_3$$

$$C_3 = -\frac{A_y L^2}{8} + \frac{PL^2}{8} - \frac{PaL}{2} + \frac{M_A L}{2}$$

***12–104. Continued**

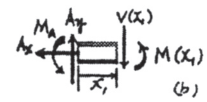
Due to symmetry, $\dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$ at $x_1 = a$ and $x_2 = L - a$. From Eqs. (2) and (4),



$$\frac{A_y a^2}{2} - M_A a = -\frac{A_y}{2}(L-a)^2 + \frac{P}{2}(L-a)^2 - Pa(L-a)$$

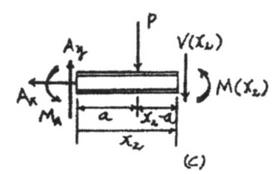
$$+ M_A(L-a) + \frac{A_y L^2}{8} - \frac{PL^2}{8} + \frac{PaL}{2} - \frac{M_A L}{2}$$

$$-A_y a^2 - \frac{3A_y L^2}{8} + A_y aL + \frac{3PL^2}{8} - \frac{3PaL}{2} + \frac{3Pa^2}{2} + \frac{M_A L}{2} = 0 \qquad \textbf{(6)}$$



Continuity Conditions:

At $x_1 = x_2 = a$, $\dfrac{dv_1}{dx_1} = \dfrac{dv_2}{dx_2}$. From Eqs. (2) and (4),



$$\frac{A_y a^2}{2} - M_A a$$

$$= \frac{A_y a^2}{2} - \frac{Pa^2}{2} + Pa^2 - M_A a - \frac{A_y L^2}{8} + \frac{PL^2}{8} - \frac{PaL}{2} + \frac{M_A L}{2}$$

$$\frac{Pa^2}{2} - \frac{A_y L^2}{8} + \frac{PL^2}{8} - \frac{PaL}{2} + \frac{M_A L}{2} = 0 \qquad \textbf{(7)}$$

Solving Eqs. (6) and (7) yields,

$$M_A = \frac{Pa}{L}(L-a) \qquad \textbf{Ans.}$$

$$A_y = P$$

Substitute the value of M_A and A_y obtained into Eqs. (1),

$$M_B = \frac{Pa}{L}(L-a) \qquad \textbf{Ans.}$$

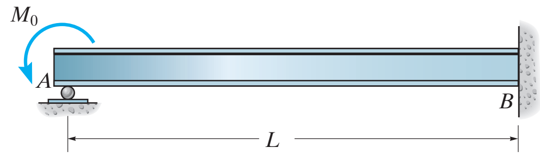
Ans:

$M_A = \dfrac{Pa}{L}(L-a)$,

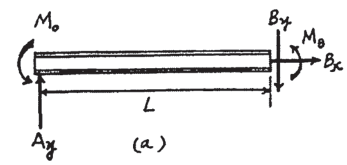
$M_B = \dfrac{Pa}{L}(L-a)$

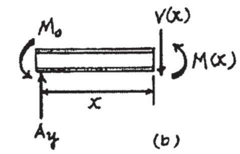
12–105.

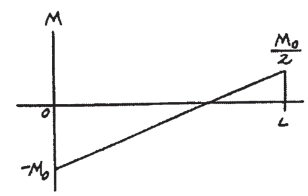
Determine the reactions at the supports A and B, then draw the moment diagram. EI is constant.



SOLUTION







Support Reactions: FBD(a).

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad A_y - B_y = 0$ **(1)**

$\circlearrowleft + \Sigma M_B = 0; \quad M_0 - A_y L + M_B = 0$ **(2)**

Moment Function: FBD(b)

$\circlearrowleft + \Sigma M_{NA} = 0; \quad M(x) + M_0 - A_y x = 0$

$$M(x) = A_y x - M_0$$

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = A_y x - M_0$$

$$EI\frac{dv}{dx} = \frac{A_y}{2}x^2 - M_0 x + C_1 \quad \textbf{(3)}$$

$$EIv = \frac{A_y}{6}x^3 - \frac{M_0}{2}x^2 + C_1 x + C_2 \quad \textbf{(4)}$$

Boundary Conditions:

At $x = 0, v = 0$. From Eq. (4), $C_2 = 0$

At $x = L, \dfrac{dv}{dx} = 0$. From Eq. (3),

$$0 = \frac{A_y L^2}{2} - M_0 L + C_1 \quad \textbf{(5)}$$

At $x = L, v = 0$. From Eq. (4),

$$0 = \frac{A_y L^3}{6} - \frac{M_0 L^2}{2} + C_1 L \quad \textbf{(6)}$$

Solving Eqs. (5) and (6) yields,

$$A_y = \frac{3M_0}{2L} \quad \textbf{Ans.}$$

$$C_1 = \frac{M_0 L}{4}$$

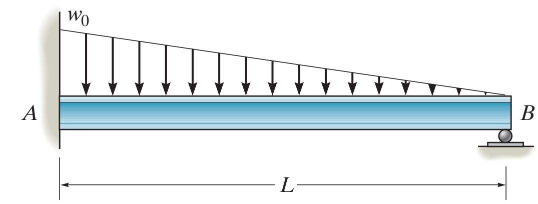
Substituting A_y, into Eqs. (1) and (2) yields:

$$B_y = \frac{3M_0}{2L} \qquad M_B = \frac{M_0}{2} \quad \textbf{Ans.}$$

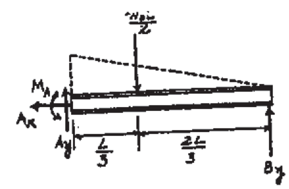
Ans:
$A_x = 0$,
$A_y = \dfrac{3M_0}{2L}$,
$B_y = \dfrac{3M_0}{2L}$,
$M_B = \dfrac{M_0}{2}$

12–106.

Determine the reactions at the support A and B. EI is constant.



SOLUTION

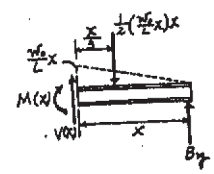


Support Reactions: FBD(a).

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y - \dfrac{w_0 L}{2} = 0$ **(1)**

$\circlearrowleft + \Sigma M_A = 0; \qquad B_y L + M_A - \dfrac{w_0 L}{2}\left(\dfrac{L}{3}\right) = 0$ **(2)**



Moment Function: FBD(b).

$\circlearrowleft + \Sigma M_{NA} = 0; \qquad -M(x) - \dfrac{1}{2}\left(\dfrac{w_0}{L}x\right)x\left(\dfrac{x}{3}\right) + B_y x = 0$

$$M(x) = B_y x - \frac{w_0}{6L}x^3$$

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = B_y x - \frac{w_0}{6L}x^3$$

$$EI\frac{dv}{dx} = \frac{B_y}{2}x^2 - \frac{w_0}{24L}x^4 + C_1 \qquad \textbf{(3)}$$

$$EIv = \frac{B_y}{6}x^3 - \frac{w_0}{120L}x^5 + C_1 x + C_2 \qquad \textbf{(4)}$$

Boundary Conditions:

At $x = 0, \quad v = 0.$ From Eq. (4), $\quad C_2 = 0$

At $x = L, \dfrac{dv}{dx} = 0.$ From Eq. (3),

$$0 = \frac{B_y L^2}{2} - \frac{w_0 L^3}{24} + C_1$$

$$C_1 = \frac{B_y L^2}{2} + \frac{w_0 L^3}{24}$$

At $x = L, \quad v = 0.$ From Eq. (4),

$$0 = \frac{B_y L^3}{6} - \frac{w_0 L^4}{120} + \left(-\frac{B_y L^2}{2} + \frac{w_0 L^3}{24}\right)L$$

$$B_y = \frac{w_0 L}{10} \qquad \textbf{Ans.}$$

Substituting B_y into Eqs. (1) and (2) yields,

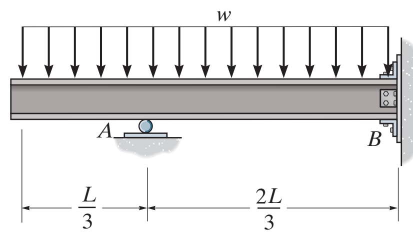
$$A_y = \frac{2w_0 L}{5} \qquad M_A = \frac{w_0 L^2}{15} \qquad \textbf{Ans.}$$

Ans:

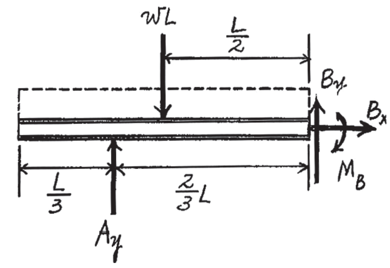
$A_x = 0, B_y = \dfrac{w_0 L}{10}, A_y = \dfrac{2w_0 L}{5}, M_A = \dfrac{w_0 L^2}{15}$

12–107.

Determine the reactions at roller support A and fixed support B.



SOLUTION



Equations of Equilibrium: Referring to the free-body diagram of the entire beam, Fig. *a*,

$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y - wL = 0$ **(1)**

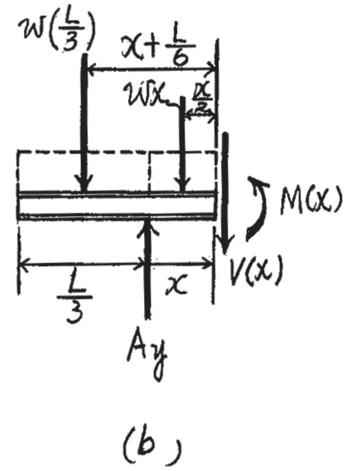
$$\circlearrowleft + \Sigma M_B = 0; \qquad wL\left(\frac{L}{2}\right) - A_y\left(\frac{2}{3}L\right) - M_B = 0$$

$$M_B = \frac{wL^2}{2} - \frac{2}{3}A_y L \qquad (2)$$

Moment Functions: Referring to the free-body diagram of the beam's segment, Fig. *b*,

$$\circlearrowleft + \Sigma M_O = 0; \qquad M(x) + wx\left(\frac{x}{2}\right) + w\left(\frac{L}{3}\right)\left(x + \frac{L}{6}\right) - A_y x = 0$$

$$M(x) = A_y x - \frac{w}{2}x^2 - \frac{wL}{3}x - \frac{wL^2}{18}$$



Equations of Slope and Elastic Curves:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = A_y x - \frac{w}{2}x^2 - \frac{wL}{3}x - \frac{wL^2}{18}$$

$$EI\frac{dv}{dx} = \frac{A_y}{2}x^2 - \frac{w}{6}x^3 - \frac{wL}{6}x^2 - \frac{wL^2}{18}x + C_1 \qquad (3)$$

$$EIv = \frac{A_y}{6}x^3 - \frac{w}{24}x^4 - \frac{wL}{18}x^3 - \frac{wL^2}{36}x^2 + C_1 x + C_2 \qquad (4)$$

Boundary Conditions: At $x = 0, v = 0$. Then Eq. (4) gives

$0 = 0 - 0 - 0 - 0 + 0 + C_2 \quad C_2 = 0$ At $x = \frac{2}{3}L, \frac{dv}{dx} = 0$. Then Eq. (3) gives

$$0 = \frac{A_y}{2}\left(\frac{2}{3}L\right)^2 - \frac{w}{6}\left(\frac{2}{3}L\right)^3 - \frac{wL}{6}\left(\frac{2}{3}L\right)^2 - \frac{wL^2}{18}\left(\frac{2}{3}L\right) + C_1$$

$$C_1 = \frac{13wL^3}{81} - \frac{2A_yL^2}{9} \qquad (5)$$

At $x = \frac{2}{3}L, v = 0$. Then Eq. (4) gives

$$0 = \frac{A_y}{6}\left(\frac{2}{3}L\right)^3 - \frac{w}{24}\left(\frac{2}{3}L\right)^4 - \frac{wL}{18}\left(\frac{2}{3}L\right)^3 - \frac{wL^2}{36}\left(\frac{2}{3}L\right)^2 + C_1\left(\frac{2}{3}L\right)$$

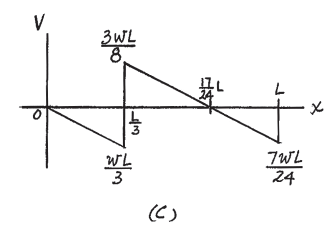
$$C_1 = \frac{wL^3}{18} - \frac{2A_yL^2}{27} \qquad (6)$$

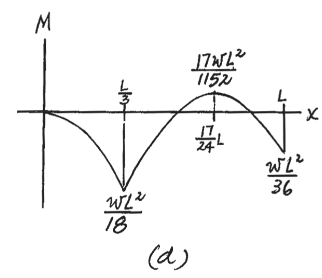
Solving Eqs. (5) and (6),

$A_y = \dfrac{17wL}{24}$ **Ans.** $C_1 = \dfrac{wL}{324}$ Substituting the result of A_y into Eqs. (1) and (2),

$B_y = \dfrac{7wL}{24} \qquad M_B = \dfrac{wL^2}{36}$ **Ans.**

The shear and moment diagrams are shown in Figs. *c* and *d*, respectively.

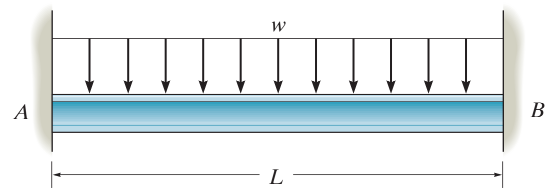




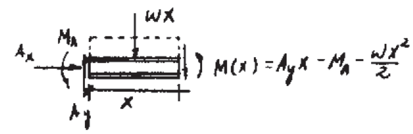
Ans:
$B_x = 0$,
$A_y = \dfrac{17wL}{24}$,
$B_y = \dfrac{7wL}{24}$,
$M_B = \dfrac{wL^2}{36}$

***12–108.**

Determine the moment reactions at the supports A and B, then draw the shear and moment diagrams. Solve by expressing the internal moment in the beam in terms of A_y and M_A. EI is constant.



SOLUTION



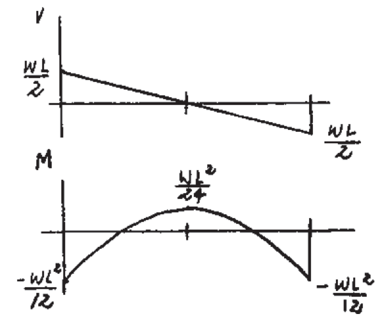
$$M(x) = A_y x - M_A - \frac{wx^2}{2}$$

Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x) = A_y x - M_A - \frac{wx^2}{2}$$

$$EI\frac{dv}{dx} = \frac{A_y x^2}{2} - M_A x - \frac{wx^3}{6} + C_1 \qquad \textbf{(1)}$$

$$EIv = \frac{A_y x^3}{6} - \frac{M_A x^2}{2} - \frac{wx^4}{24} + C_1 x + C_2 \qquad \textbf{(2)}$$



Boundary Conditions:

$$\frac{dv}{dx} = 0 \quad \text{at} \quad x = 0$$

From Eq. (1)

$$C_1 = 0$$

$$v = 0 \quad \text{at} \quad x = 0$$

From Eq. (2)

$$C_2 = 0$$

$$\frac{dv}{dx} = 0 \quad \text{at} \quad x = L$$

From Eq. (1)

$$0 = \frac{A_y L^2}{2} - M_A L - \frac{wL^3}{6} \qquad \textbf{(3)}$$

$$v = 0 \quad \text{at} \quad x = L$$

From Eq. (2)

$$0 = \frac{A_y L^3}{6} - \frac{M_A L^2}{2} - \frac{wL^4}{24} \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields:

$$A_y = \frac{wL}{2}$$

$$M_A = \frac{wL^2}{12} \qquad \textbf{Ans.}$$

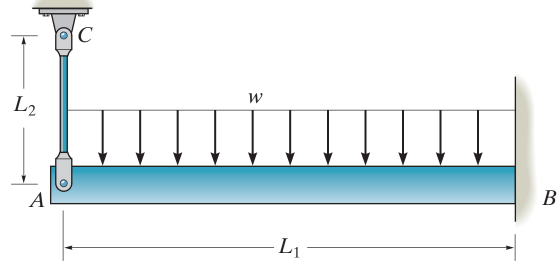
Due to symmetry:

$$M_B = \frac{wL^2}{12} \qquad \textbf{Ans.}$$

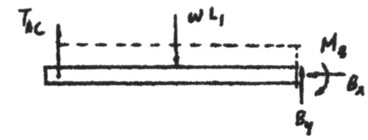
Ans:
$M_A = \frac{wL^2}{12}$,
$M_B = \frac{wL^2}{12}$

12–109.

The beam has a constant E_1I_1 and is supported by the fixed wall at B and the rod AC. If the rod has a cross-sectional area A_2 and the material has a modulus of elasticity E_2, determine the force in the rod.



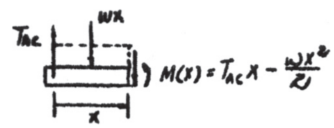
SOLUTION



$$+\uparrow \Sigma F_y = 0 \qquad T_{AC} + B_y - wL_1 = 0$$

$$\circlearrowleft + \ \Sigma M_B = 0 \qquad T_{AC}(L_1) + M_B - \frac{wL_1{}^2}{2} = 0 \tag{1}$$

$$M_B = \frac{wL_1{}^2}{2} - T_{AC}L_1 \tag{2}$$



Bending Moment $M(x)$:

$$M(x) = T_{AC}x - \frac{wx^2}{2}$$

Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x) = T_{AC}x - \frac{wx^2}{2}$$

$$EI\frac{dv}{dx} = \frac{T_{AC}x^2}{2} - \frac{wx^3}{6} + C_1 \tag{3}$$

$$EIv = \frac{T_{AC}x^3}{6} - \frac{wx^4}{24} + C_1x + C_2 \tag{4}$$

Boundary Conditions:

$$v = \frac{T_{AC}L_2}{A_2E_2} \qquad x = 0$$

From Eq. (4)

$$-E_1I_1\left(\frac{T_{AC}L_2}{A_2E_2}\right) = 0 - 0 + 0 + C_2$$

$$C_2 = \left(\frac{-E_1I_1L_2}{A_2E_2}\right)T_{AC}$$

$$v = 0 \qquad \text{at} \qquad x = L_1$$

From Eq. (4)

$$0 = \frac{T_{AC}L_1{}^3}{6} - \frac{wL_1{}^4}{24} + C_1L_1 - \frac{E_1I_1L_2}{A_2E_2}T_{AC} \tag{5}$$

$$\frac{dv}{dx} = 0 \qquad \text{at} \qquad x = L_1$$

From Eq. (3)

$$0 = \frac{T_{AC}L_1{}^2}{2} - \frac{wL_1{}^3}{6} + C_1 \tag{6}$$

Solving Eqs. (5) and (6) yields:

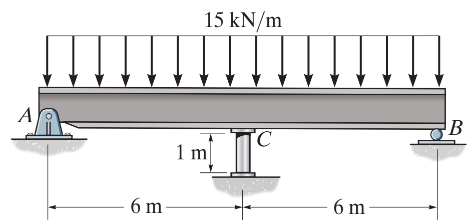
$$T_{AC} = \frac{3A_2E_2wL_1{}^4}{8\left(A_2E_2L_1{}^3 + 3E_1I_1L_2\right)} \qquad \textbf{Ans.}$$

Ans:

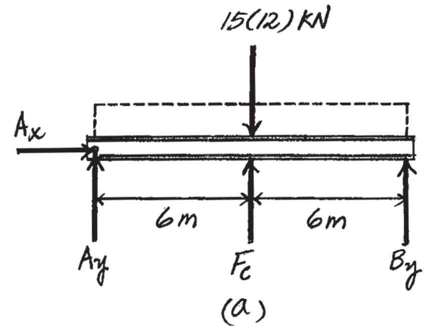
$$T_{AC} = \frac{3A_2E_2wL_1^4}{8(A_2E_2L_1^3 + 3E_1I_1L_2)}$$

12–110.

The beam is supported by a pin at A, a roller at B, and a post having a diameter of 50 mm at C. Determine the support reactions at A, B, and C. The post and the beam are made of the same material having a modulus of elasticity $E = 200$ GPa, and the beam has a constant moment of inertia $I = 255(10^6)\ \text{mm}^4$.



SOLUTION



Equations of Equilibrium: Referring to the free-body diagram of the entire beam, Fig. *a*,

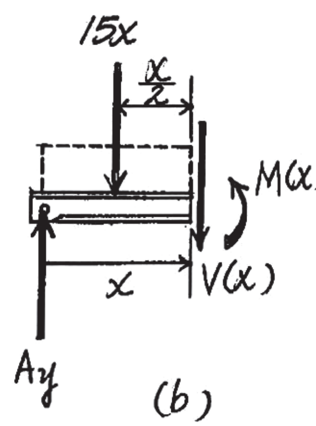
$\xrightarrow{+} \Sigma F_x = 0;\qquad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad A_y + B_y + F_C - 15(12) = 0$ **(1)**

$\circlearrowleft + \Sigma M_B = 0;\qquad 15(12)(6) - F_C(6) - A_y(12) = 0$

$2A_y + F_C = 180$ **(2)**

Moment Functions: Referring to the free-body diagram of the beam's segment, Fig. *b*,



$\circlearrowleft + \Sigma M_O = 0;\qquad M(x) + 15x\left(\frac{x}{2}\right) - A_y x = 0$

$$M(x) = A_y x - 7.5x^2$$

Equations of Slope and Elastic Curves:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = A_y x - 7.5x^2$$

$$EI\frac{dv}{dx} = \frac{A_y}{2}x^2 - 2.5x^3 + C_1 \quad \textbf{(3)}$$

$$EIv = \frac{A_y}{6}x^3 - 0.625x^4 + C_1 x + C_2 \quad \textbf{(4)}$$

Boundary Conditions: At $x = 0$, $v = 0$. Then Eq. (4) gives

$$0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$$

At $x = 6$ m, $v = -\Delta_C = -\dfrac{F_C L_C}{A_C E} = -\dfrac{F_C(1)}{\frac{\pi}{4}\left(0.05^2\right)E} = -\dfrac{1600F_C}{\pi E}$. Then Eq. (4) gives

$$E\left[255\left(10^{-6}\right)\right]\left(-\frac{1600F_C}{\pi E}\right) = \frac{A_y}{6}\left(6^3\right) - 0.625\left(6^4\right) + C_1(6)$$

$$C_1 = 135 - 6A_y - 0.02165F_C$$

Due to symmetry, $\dfrac{dv}{dx} = 0$ at $x = 6$ m. Then Eq. (3) gives

$$0 = \frac{A_y}{2}\left(6^2\right) - 2.5\left(6^3\right) + 135 - 6A_y - 0.02165F_C$$

$$12A_y - 0.02165F_C = 405 \quad \textbf{(5)}$$

Solving Eqs. (2) and (5),

$F_C = 112.096\ \text{kN} = 112\ \text{kN}\qquad A_y = 33.95\ \text{kN} = 34.0\ \text{kN}$ **Ans.**

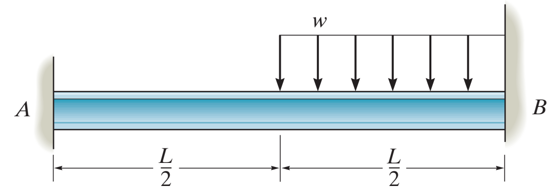
Substituting these results into Eq. (1),

$B_y = 33.95\ \text{kN} = 34.0\ \text{kN}$ **Ans.**

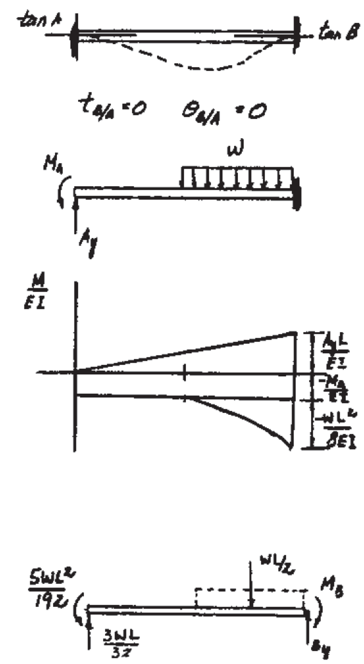
Ans:
$A_x = 0$, $F_C = 112$ kN, $A_y = 34.0$ kN, $B_y = 34.0$ kN

12–111.

Determine the moment reactions at the supports A and B. EI is constant.



SOLUTION



$$\theta_{B/A} = 0 = \frac{1}{2}\left(\frac{A_y L}{EI}\right)(L) + \left(\frac{-M_A}{EI}\right)(L) + \frac{1}{3}\left(\frac{-wL^2}{8EI}\right)\left(\frac{L}{2}\right)$$

$$0 = \frac{A_y L}{2} - M_A - \frac{wL^2}{48} \qquad (2)$$

$$t_{B/A} = 0 = \frac{1}{2}\left(\frac{A_y L}{EI}\right)(L)\left(\frac{L}{3}\right) + \left(\frac{-M_A}{EI}\right)(L)\left(\frac{L}{2}\right) + \frac{1}{3}\left(\frac{-wL^2}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{8}\right)$$

$$0 = \frac{A_y L}{6} - \frac{M_A}{2} - \frac{wL^2}{384} \qquad (3)$$

Solving Eqs. (2) and (3) yields:

$$A_y = \frac{3wL}{32}$$

$$M_A = \frac{5wL^2}{192} \qquad \textbf{Ans.}$$

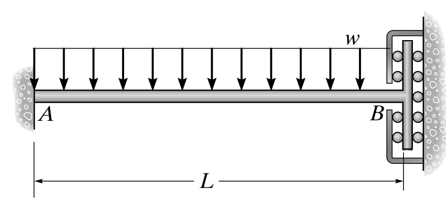
$$\circlearrowleft + \Sigma M_B = 0; \quad M_B + \frac{3wL}{32}(L) - \frac{5wL^2}{192} - \frac{wL}{2}\left(\frac{L}{4}\right) = 0$$

$$M_B = \frac{11wL^2}{192} \qquad \textbf{Ans.}$$

Ans:

$M_A = \dfrac{5wL^2}{192}$, $M_B = \dfrac{11wL^2}{192}$

***12–112.** The rod is fixed at A, and the connection at B consists of a roller constraint which allows vertical displacement but resists axial load and moment. Determine the moment reactions at these supports. EI is constant.



SOLUTION

Support Reaction: FBD(a).

$$\circlearrowleft + \Sigma M_A = 0; \qquad M_B + M_A - wL\left(\frac{L}{2}\right) = 0 \qquad [1]$$

Elastic Curve: As shown.

M/EI Diagrams: *M/EI* diagrams for M_B and the uniform distributed load acting on a cantilever beam are shown.

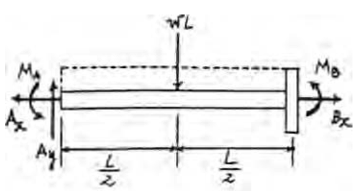
Moment-Area Theorems: Since both tangents at A and B are horizontal (parallel), $\theta_{B/A} = 0$.

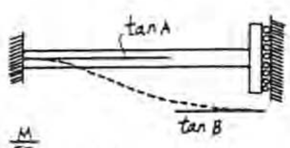
$$\theta_{B/A} = 0 = \left(\frac{M_B}{EI}\right)(L) + \frac{1}{3}\left(-\frac{wL^2}{2EI}\right)(L)$$

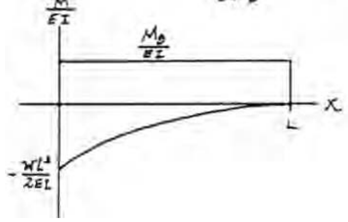
$$M_B = \frac{wL^2}{6} \qquad \textbf{Ans.}$$

Substituting M_B into Eq.[1],

$$M_A = \frac{wL^2}{3} \qquad \textbf{Ans.}$$



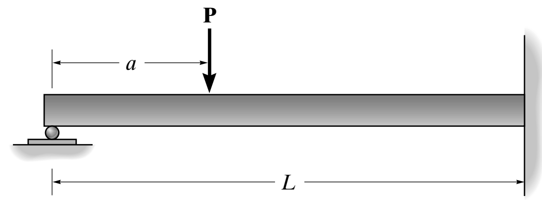




Ans:

$M_A = \dfrac{wL^2}{3}$; $M_B = \dfrac{wL^2}{6}$

12–113. Determine the value of a for which the maximum positive moment has the same magnitude as the maximum negative moment. EI is constant.



SOLUTION

$$(t_{A/B})_1 = \frac{1}{2}\left(\frac{-P(L-a)}{EI}\right)(L-a)\left(a + \frac{2(L-a)}{3}\right) = \frac{-P(L-a)^2(2L+a)}{6EI}$$

$$(t_{A/B})_2 = \frac{1}{2}\left(\frac{A_y L}{EI}\right)(L)\left(\frac{2L}{3}\right) = \frac{A_y L^3}{3EI}$$

$$t_{A/B} = 0 = (t_{A/B})_1 + (t_{A/B})_2$$

$$0 = \frac{-P(L-a)^2(2L+a)}{6EI} + \frac{A_y L^3}{3EI}$$

$$A_y = \frac{P(L-a)^2(2L+a)}{2L^3}$$

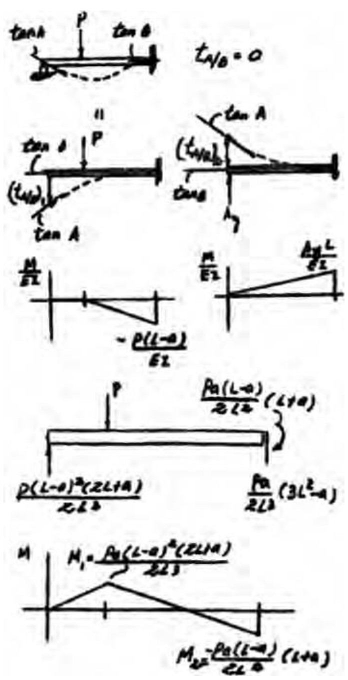
Require:

$$|M_1| = |M_2|$$

$$\frac{Pa(L-a)^2(2L+a)}{2L^3} = \frac{Pa(L-a)(L+a)}{2L^2}$$

$$a^2 + 2La - L^2 = 0$$

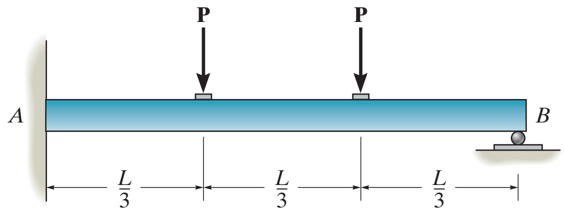
$$a = 0.414L$$ **Ans.**



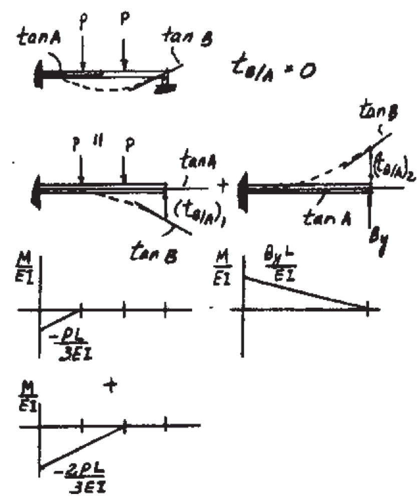
Ans:
$a = 0.414L$

12–114.

Determine the reactions at the supports A and B, then draw the shear and moment diagrams. EI is constant.



SOLUTION



$$(t_{B/A})_1 = \frac{1}{2}\left(\frac{-PL}{3EI}\right)\left(\frac{L}{3}\right)\left(\frac{2L}{3}+\frac{2L}{9}\right)+\frac{1}{2}\left(\frac{-2PL}{3EI}\right)\left(\frac{2L}{3}\right)\left(\frac{L}{3}+\frac{4L}{9}\right) = -\frac{2PL^3}{9EI}$$

$$(t_{B/A})_2 = \frac{1}{2}\left(\frac{B_yL}{EI}\right)(L)\left(\frac{2L}{3}\right) = \frac{B_yL^3}{3EI}$$

$$t_{B/A} = 0 = (t_{B/A})_1 + (t_{B/A})_2$$

$$0 = -\frac{2PL^3}{9EI} + \frac{B_yL^3}{3EI}$$

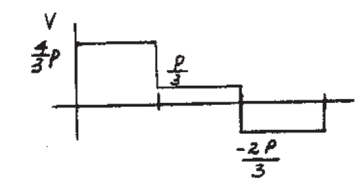
$$B_y = \frac{2}{3}P \qquad \textbf{Ans.}$$

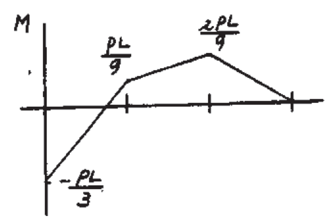
From the free-body diagram,

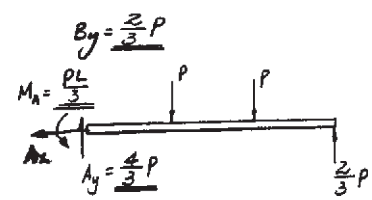
$$M_A = \frac{PL}{3} \qquad \textbf{Ans.}$$

$$A_y = \frac{4}{3}P \qquad \textbf{Ans.}$$

$$A_x = 0 \qquad \textbf{Ans.}$$





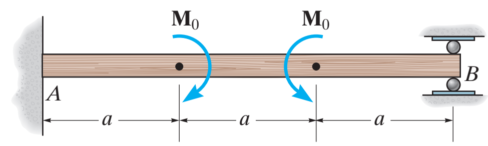


Ans:

$B_y = \frac{2}{3}P,\ M_A = \frac{PL}{3},\ A_y = \frac{4}{3}P,\ A_x = 0$

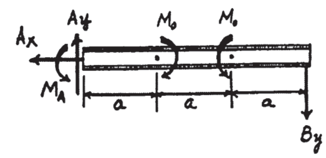
12–115.

Determine the reactions at the supports. EI is constant.



SOLUTION

Support Reaction: FBD(a).



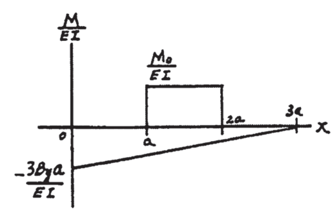
$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad -B_y + A_y = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad -B_y(3a) + M_A = 0 \qquad \textbf{(2)}$$

Elastic Curve: As shown.

***M*/*EI* Diagrams:** M/EI diagrams for B_y and M_0 acting on a cantilever beam are drawn.



Moment - Area Theorems: From the elastic curve, $t_{B/A} = 0$.

$$t_{B/A} = 0 = \frac{1}{2}\left(-\frac{3B_y a}{EI}\right)(3a)\left(\frac{2}{3}\right)(3a) + \left(\frac{M_0}{EI}\right)(a)\left(a + \frac{a}{2}\right)$$

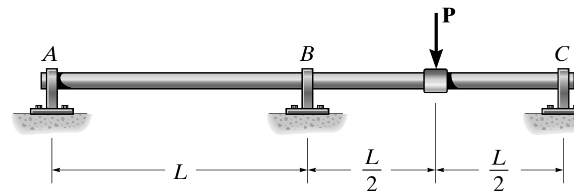
$$B_y = \frac{M_0}{6a} \qquad \textbf{Ans.}$$

Substituting B_y into Eqs. (1) and (2) yields,

$$A_y = \frac{M_0}{6a} \qquad M_A = \frac{M_0}{2} \qquad \textbf{Ans.}$$

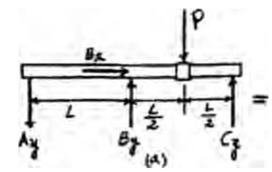
Ans:
$A_x = 0$,
$B_y = \frac{M_0}{6a}$,
$A_y = \frac{M_0}{6a}$,
$M_A = \frac{M_0}{2}$

***12–116.** Determine the reactions at the supports, then draw the shear and moment diagrams. *EI* is constant. Support *B* is a thrust bearing.



SOLUTION

Support Reactions: FBD(a).



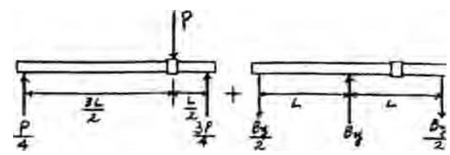
$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad -A_y + B_y + C_y - P = 0$ [1]

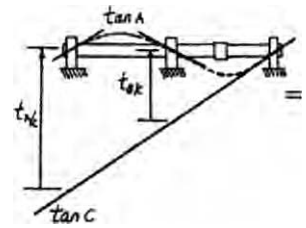
$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(L) + C_y(2L) - P\left(\frac{3L}{2}\right) = 0$ [2]

Elastic Curve: As shown.

M/EI Diagrams: *M/EI* diagrams for *P* and B_y acting on a simply supported beam are drawn separately.



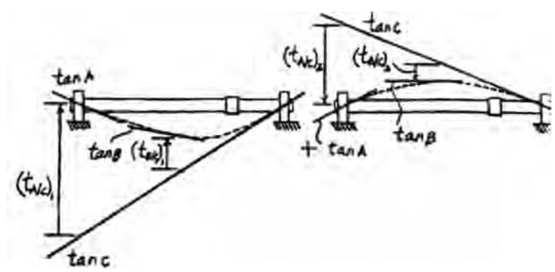
Moment-Area Theorems:



$$(t_{A/C})_1 = \frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{3L}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3L}{2}\right) + \frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{3L}{2} + \frac{L}{6}\right)$$

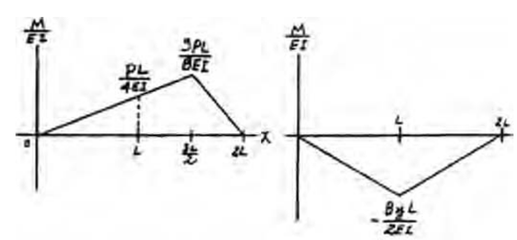
$$= \frac{7PL^3}{16EI}$$

$$(t_{A/C})_2 = \frac{1}{2}\left(-\frac{B_y L}{2EI}\right)(2L)(L) = -\frac{B_y L^3}{2EI}$$



$$(t_{B/C})_1 = \frac{1}{2}\left(\frac{PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{2}{3}\right)\left(\frac{L}{2}\right) + \left(\frac{PL}{4EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) + \frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2} + \frac{L}{6}\right)$$

$$= \frac{5PL^3}{48EI}$$

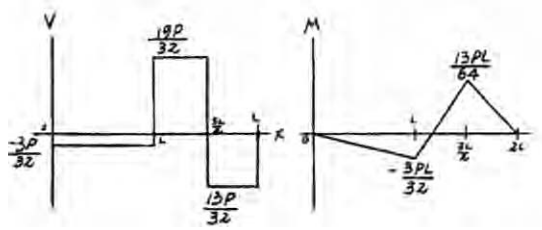


$$(t_{B/C})_2 = \frac{1}{2}\left(-\frac{B_y L}{2EI}\right)(L)\left(\frac{L}{3}\right) = -\frac{B_y L^3}{12EI}$$

$$t_{A/C} = (t_{A/C})_1 + (t_{A/C})_2 = \frac{7PL^3}{16EI} - \frac{B_y L^3}{2EI}$$

$$t_{B/C} = (t_{B/C})_1 + (t_{B/C})_2 = \frac{5PL^3}{48EI} - \frac{B_y L^3}{12EI}$$

From the elastic curve,



$$t_{A/C} = 2t_{B/C}$$

$$\frac{7PL^3}{16EI} - \frac{B_y L^3}{2EI} = 2\left(\frac{5PL^3}{48EI} - \frac{B_y L^3}{12EI}\right)$$

$B_y = \dfrac{11P}{16}$ **Ans.**

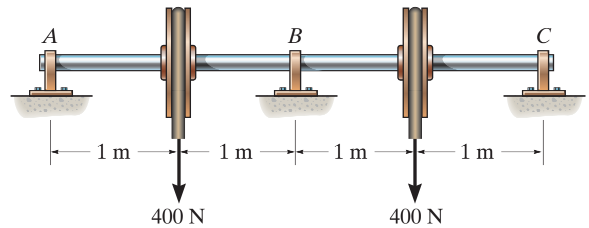
Substituting B_y into Eqs. [1] and [2] yields,

$C_y = \dfrac{13P}{32} \qquad A_y = \dfrac{3P}{32}$ **Ans.**

Ans:
$A_y = \dfrac{3P}{32}$; $B_y = \dfrac{11P}{16}$; $C_y = \dfrac{13P}{32}$

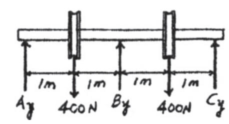
12–117.

Determine the reactions at the journal bearing supports A, B, and C of the shaft, then draw the shear and moment diagrams. EI is constant.



SOLUTION

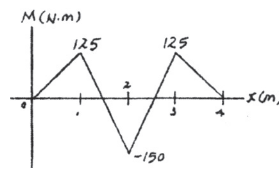
Support Reactions: FBD(a).



$+\uparrow\Sigma F_y = 0; \qquad A_y + B_y + C_y - 800 = 0$ **(1)**

$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(2) + C_y(4) - 400(1) - 400(3) = 0$ **(2)**

Method of Superposition: Using the table in Appendix C, the required displacements are

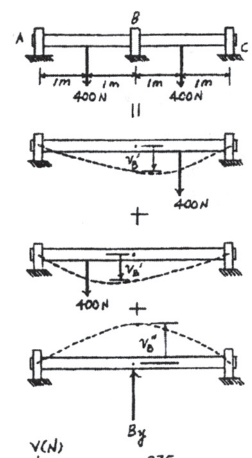


$$v_B{}' = \frac{Pbx}{6EIL}\left(L^2 - b^2 - x^2\right)$$

$$= \frac{400(1)(2)}{6EI(4)}\left(4^2 - 1^2 - 2^2\right)$$

$$= \frac{366.67 \text{ N}\cdot\text{m}^3}{EI} \downarrow$$

$$v_B{}'' = \frac{PL^3}{48EI} = \frac{B_y\left(4^3\right)}{48EI} = \frac{1.3333B_y \text{ m}^3}{EI} \uparrow$$



The compatibility condition requires

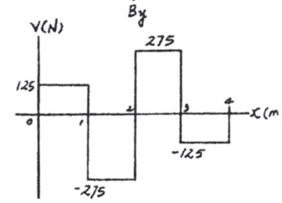
$(+\downarrow) \qquad 0 = 2v_B{}' + v_B{}''$

$$0 = 2\left(\frac{366.67}{EI}\right) + \left(-\frac{1.3333B_y}{EI}\right)$$

$B_y = 550 \text{ N}$ **Ans.**

Substituting B_y into Eqs. (1) and (2) yields,

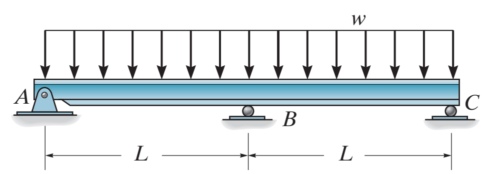
$A_y = 125 \text{ N} \qquad C_y = 125 \text{ N}$ **Ans.**



Ans:
$B_y = 550 \text{ N}, A_y = 125 \text{ N}, C_y = 125 \text{ N}$

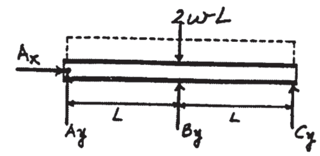
12–118.

Determine the reactions at the supports, then draw the shear and moment diagrams. EI is constant.



SOLUTION

Support Reactions: FBD(a).

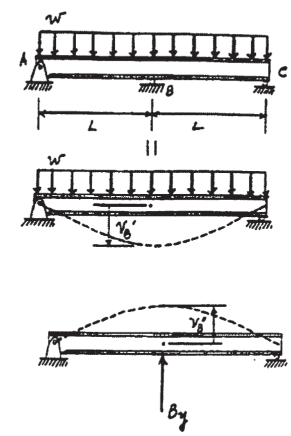


$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y + C_y - 2wL = 0 \qquad \textbf{(1)}$$

$$+\Sigma M_A = 0; \qquad B_y(L) + C_y(2L) - (2wL)(L) = 0 \qquad \textbf{(2)}$$

Method of Superposition: Using the table in Appendix C, the required displacements are



$$v_B{}' = \frac{5wL_{AC}^4}{384EI} = \frac{5w(2L)^4}{384EI} = \frac{5wL^4}{24EI} \downarrow$$

$$v_B{}'' = \frac{PL_{AC}^3}{48EI} = \frac{B_y(2L)^3}{48EI} = \frac{B_yL^3}{6EI} \uparrow$$

The compatibility condition requires

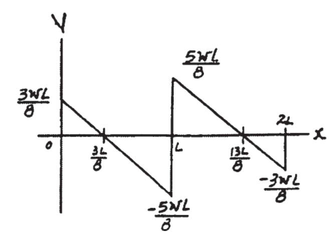
$$(+\downarrow) \qquad 0 = v_B{}' + v_B{}''$$

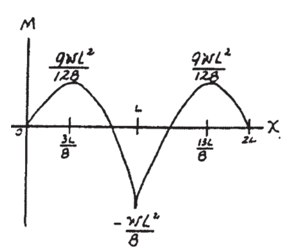
$$0 = \frac{5wL^4}{24EI} + \left(-\frac{B_yL^3}{6EI}\right)$$

$$B_y = \frac{5wL}{4} \qquad \textbf{Ans.}$$

Substituting the value of B_y into Eqs. (1) and (2) yields,

$$C_y = A_y = \frac{3wL}{8} \qquad \textbf{Ans.}$$

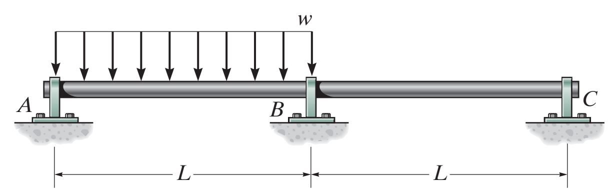




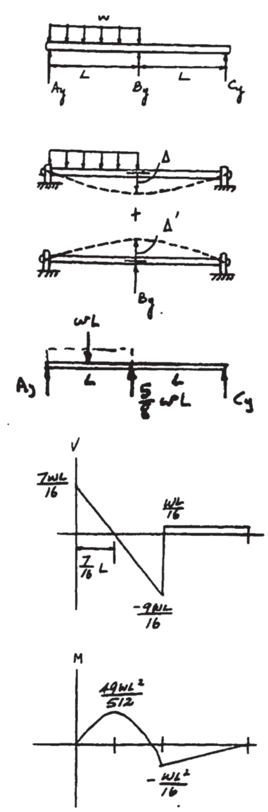
Ans:
$A_x = 0$,
$B_y = \frac{5wL}{4}$,
$C_y = \frac{3wL}{8}$

12–119.

Determine the reactions at the supports, then draw the shear and moment diagrams. EI is constant.



SOLUTION



$$\Delta = \frac{5w(2L)^4}{768EI} = \frac{5wL^4}{48EI}\downarrow$$

$$\Delta' = \frac{B_y(2L)^4}{48EI} = \frac{B_yL^3}{6EI}\uparrow$$

Require:

$$(+\downarrow) \qquad 0 = \Delta - \Delta'$$

$$0 = \frac{5wL^4}{48EI} - \frac{B_yL^3}{6EI}$$

$$B_y = \frac{5}{8}wL\uparrow$$ **Ans.**

$$\circlearrowleft + \Sigma M_A = 0; \qquad wL\left(\frac{L}{2}\right) - \frac{5}{8}wL(L) - C_y(2L) = 0$$

$$C_y = -\frac{wL}{16} = \frac{wL}{16}\downarrow$$ **Ans.**

$$+\uparrow\Sigma F_y = 0; \qquad -wL - \frac{wL}{16} + \frac{5}{8}wL + A_y = 0$$

$$A_y = \frac{7}{16}wL\uparrow$$ **Ans.**

Ans:

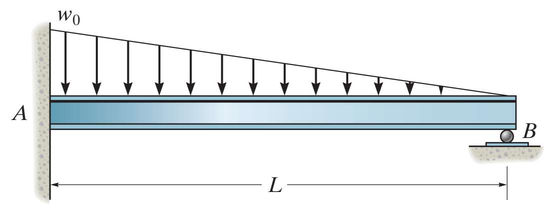
$B_y = \frac{5}{8}wL\uparrow$,

$C_y = \frac{wL}{16}\downarrow$,

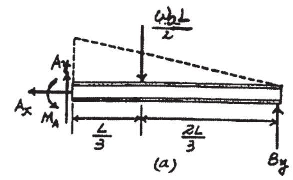
$A_y = \frac{7}{16}wL\uparrow$

***12–120.**

Determine the reactions at the supports A and B. EI is constant.



SOLUTION

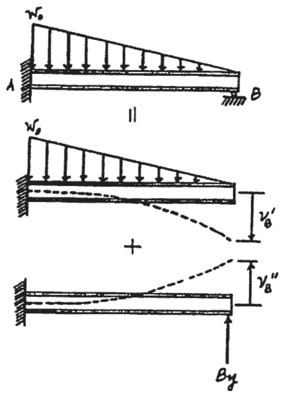


Support Reactions: FBD(a).

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y - \frac{w_0 L}{2} = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y L + M_A - \frac{w_0 L}{2}\left(\frac{L}{3}\right) = 0 \qquad \textbf{(2)}$$



Method of Superposition: Using the table in Appendix C, the required displacements are

$$v_B{}' = \frac{w_0 L^4}{30EI}\downarrow \qquad v_B{}'' = \frac{B_y L^3}{3EI}\uparrow$$

The compatibility condition requires

$$(+\downarrow) \qquad 0 = v_B{}' + v_B{}''$$

$$0 = \frac{w_0 L^4}{30EI} + \left(-\frac{B_y L^3}{3EI}\right)$$

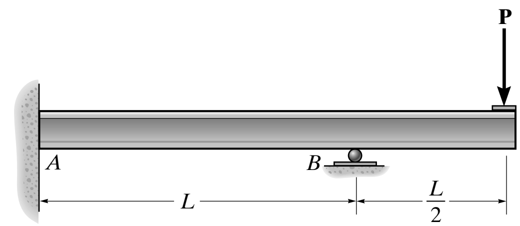
$$B_y = \frac{w_0 L}{10} \qquad \textbf{Ans.}$$

Substituting B_y into Eqs. (1) and (2) yields,

$$A_y = \frac{2w_0 L}{5} \qquad M_A = \frac{w_0 L^2}{15} \qquad \textbf{Ans.}$$

Ans:
$A_x = 0$,
$B_y = \frac{w_0 L}{10}$,
$A_y = \frac{2w_0 L}{5}$,
$M_A = \frac{w_0 L^2}{15}$

12–121. Determine the reactions at the supports A and B. EI is constant.



SOLUTION

Referring to the FBD of the beam, Fig. a

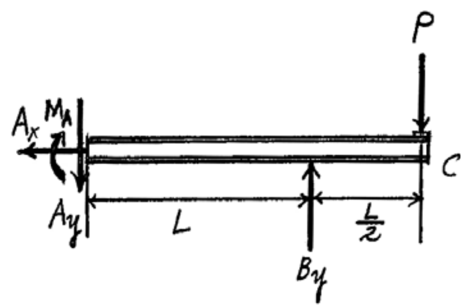
$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad B_y - P - A_y = 0$

$$A_y = B_y - P \tag{1}$$

$$\curvearrowleft + \Sigma M_A = 0; \quad -M_A + B_y L - P\left(\frac{3}{2}L\right) = 0$$

$$M_A = B_y L - \frac{3}{2}PL \tag{2}$$



(a)

Referring to Fig. b and the table in appendix, the necessary deflections are computed as follow:

$$v_P = \frac{Px^2}{6EI}(3L_{AC} - x)$$

$$= \frac{P(L^2)}{6EI}\left[3\left(\frac{3}{2}L\right) - L\right]$$

$$= \frac{7PL^3}{12EI} \quad \downarrow$$

$$v_{B_y} = \frac{PL_{AB}^3}{3EI} = \frac{B_y L^3}{3EI} \quad \uparrow$$

The compatibility condition at support B requires that

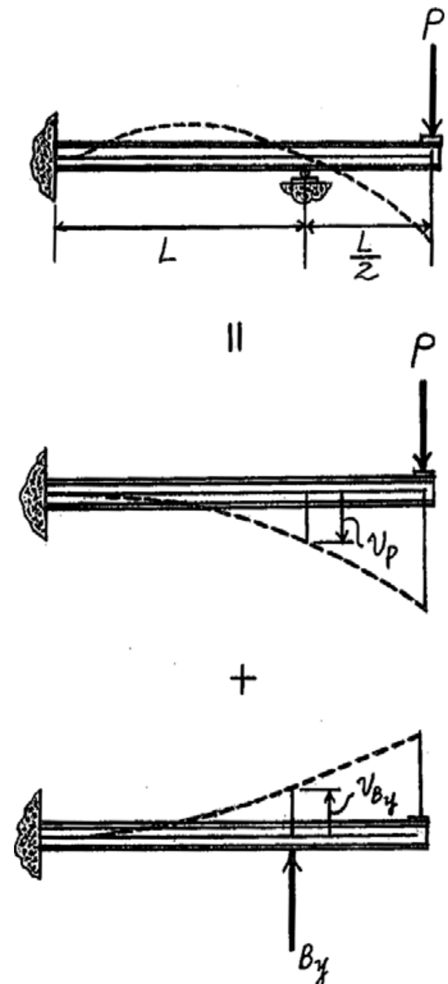
$$(+\downarrow) \qquad 0 = v_P + v_{B_y}$$

$$0 = \frac{7PL^3}{12EI} + \left(\frac{-B_y L^3}{3EI}\right)$$

$$B_y = \frac{7P}{4}$$ **Ans.**

Substitute this result into Eq (1) and (2)

$$A_y = \frac{3P}{4} \qquad M_A = \frac{PL}{4}$$ **Ans.**



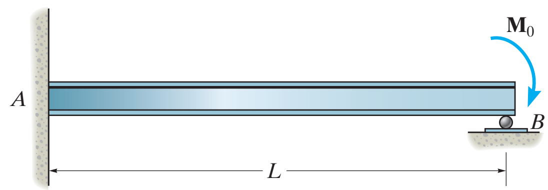
(b)

Ans:

$B_y = \frac{7P}{4}, A_y = \frac{3P}{4}, M_A = \frac{PL}{4}$

12–122.

Determine the reactions at the supports A and B. EI is constant.



SOLUTION

Referring to the FBD of the beam, Fig. *a*

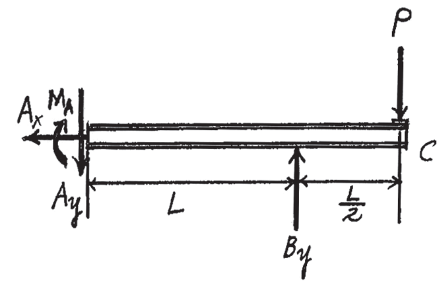
$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad B_y - P - A_y = 0$

$$A_y = B_y - P \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_A = 0; \quad -M_A + B_y L - P\left(\frac{3}{2}L\right) = 0$$

$$M_A = B_y L - \frac{3}{2}PL \qquad \textbf{(2)}$$



(a)

Referring to Fig. *b* and the table in appendix, the necessary deflections are computed as follow:

$$v_P = \frac{Px^2}{6EI}(3L_{AC} - x)$$

$$= \frac{P(L^2)}{6EI}\left[3\left(\frac{3}{2}L\right) - L\right]$$

$$= \frac{7PL^3}{12EI}\downarrow$$

$$v_{B_y} = \frac{PL_{AB}^3}{3EI} = \frac{B_y L^3}{3EI}\uparrow$$

The compatibility condition at support B requires that

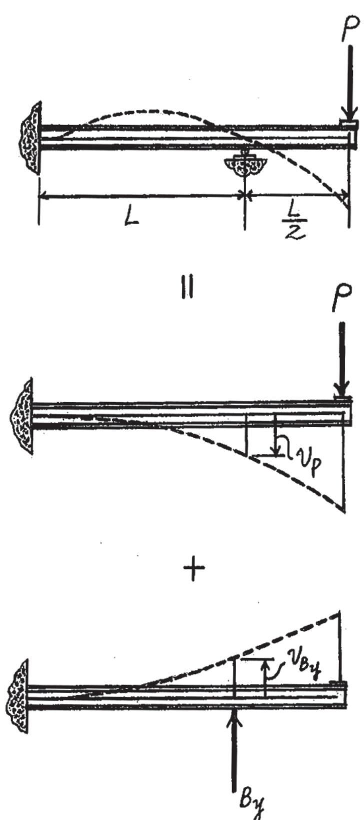
$$(+\downarrow) \qquad 0 = v_P + v_{B_y}$$

$$0 = \frac{7PL^3}{12EI} + \left(\frac{-B_y L^3}{3EI}\right)$$

$$B_y = \frac{7P}{4}$$ **Ans.**

Substitute this result into Eqs. (1) and (2)

$$A_y = \frac{3P}{4} \qquad M_A = \frac{PL}{4}$$ **Ans.**

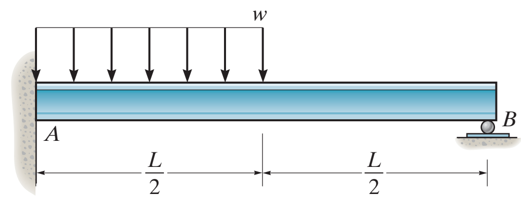


(b)

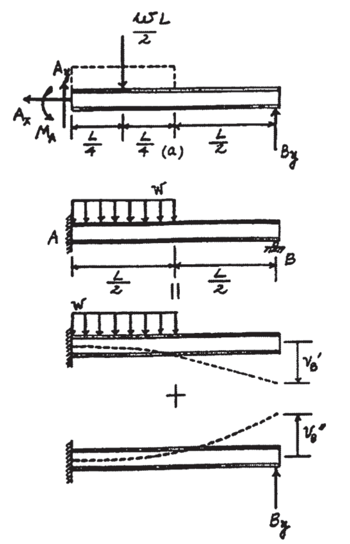
Ans:
$A_x = 0,$
$B_y = \frac{7P}{4},$
$A_y = \frac{3P}{4},$
$M_A = \frac{PL}{4}$

12–123.

Determine the reactions at the supports A and B. EI is constant.



SOLUTION



Support Reactions: FBD(a).

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad A_y + B_y - \dfrac{wL}{2} = 0$ **(1)**

$\circlearrowleft + \Sigma M_A = 0;$ $\quad B_y(L) + M_A - \left(\dfrac{wL}{2}\right)\left(\dfrac{L}{4}\right) = 0$ **(2)**

Method of Superposition: Using the table in Appendix C, the required displacements are

$$v_B{}' = \frac{7wL^4}{384EI}\downarrow \qquad v_B{}'' = \frac{PL^3}{3EI} = \frac{B_yL^3}{3EI}\uparrow$$

The compatibility condition requires

$$(+\downarrow) \qquad 0 = v_B{}' + v_B{}''$$

$$0 = \frac{7wL^4}{384EI} + \left(-\frac{B_yL^3}{3EI}\right)$$

$$B_y = \frac{7wL}{128}$$ **Ans.**

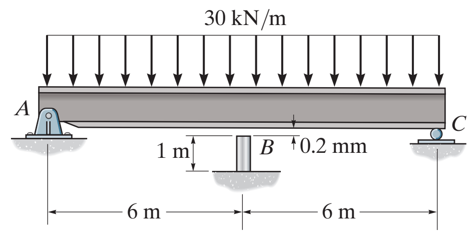
Substituting B_y into Eqs. (1) and (2) yields

$$A_y = \frac{57wL}{128} \qquad M_A = \frac{9wL^2}{128}$$ **Ans.**

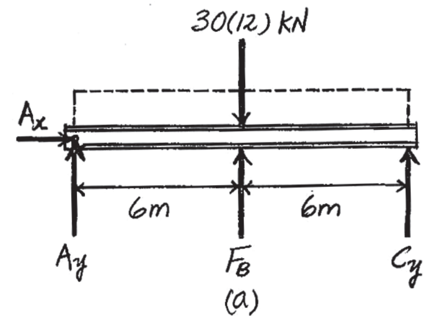
Ans:
$A_x = 0,$
$B_y = \dfrac{7wL}{128},$
$A_y = \dfrac{57wL}{128},$
$M_A = \dfrac{9wL^2}{128}$

***12–124.**

Before the uniform distributed load is applied to the beam, there is a small gap of 0.2 mm between the beam and the post at B. Determine the support reactions at A, B, and C. The post at B has a diameter of 40 mm, and the moment of inertia of the beam is $I = 875(10^6)\ \text{mm}^4$. The post and the beam are made of material having a modulus of elasticity of $E = 200$ GPa.



SOLUTION



Equations of Equilibrium: Referring to the free-body diagram of the beam, Fig. a,

$\xrightarrow{+} \Sigma F_x = 0;\qquad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad A_y + F_B + C_y - 30(12) = 0$ **(1)**

$\circlearrowleft + \Sigma M_A = 0;\qquad F_B(6) + C_y(12) - 30(12)(6) = 0$ **(2)**

Method of Superposition: Referring to Fig. b and the table in the appendix, the necessary deflections are

$$(v_B)_1 = \frac{5wL^4}{384EI} = \frac{5(30)\left(12^4\right)}{384EI} = \frac{8100\ \text{kN}\cdot\text{m}^3}{EI}\ \downarrow$$

$$(v_B)_2 = \frac{PL^3}{48EI} = \frac{F_B\left(12^3\right)}{48EI} = \frac{36F_B}{EI}\ \uparrow$$

The deflection of point B is

$$v_B = 0.2\left(10^{-3}\right) + \frac{F_B L_B}{AE} = 0.2\left(10^{-3}\right) + \frac{F_B(1)}{AE}\ \downarrow$$

The compatibility condition at support B requires

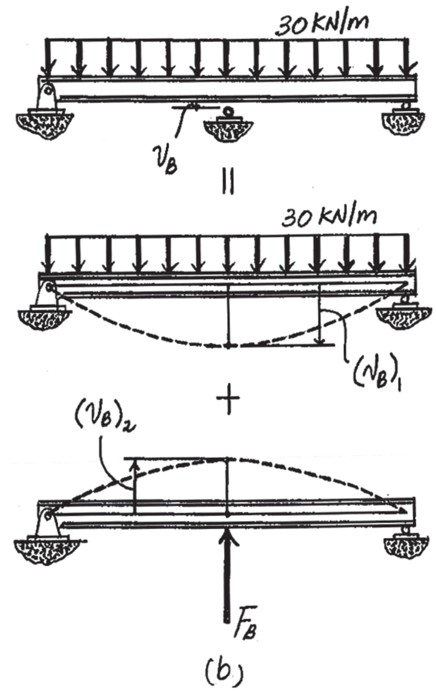
$$(+\downarrow)\qquad v_B = (v_B)_1 + (v_B)_2$$

$$0.2\left(10^{-3}\right) + \frac{F_B\,(1)}{AE} = \frac{8100}{EI} + \left(-\frac{36F_B}{EI}\right)$$

$$0.2\left(10^{-3}\right)E + \frac{F_B}{A} = \frac{8100}{I} - \frac{36F_B}{I}$$

$$\frac{F_B}{\frac{\pi}{4}\left(0.04^2\right)} + \frac{36F_B}{875\left(10^{-6}\right)} = \frac{8100}{875\left(10^{-6}\right)} - \frac{0.2\left(10^{-3}\right)\left[200\left(10^9\right)\right]}{1000}$$

$F_B = 219.78\ \text{kN} = 220\ \text{kN}$ **Ans.**



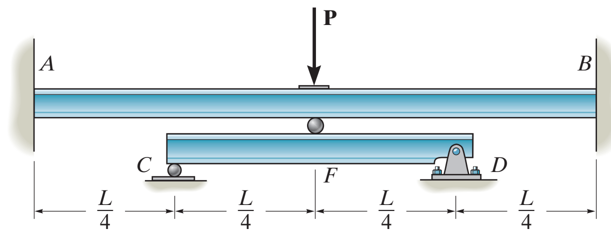
Substituting the result of F_B into Eqs. (1) and (2),

$A_y = C_y = 70.11\ \text{kN} = 70.1\ \text{kN}$ **Ans.**

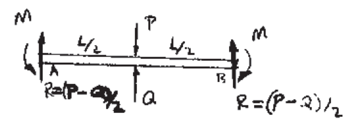
Ans:
$A_x = 0$,
$F_B = 220\ \text{kN}$,
$A_y = C_y = 70.1\ \text{kN}$

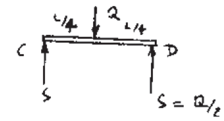
12–125.

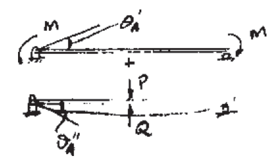
The fixed supported beam AB is strengthened using the simply supported beam CD and the roller at F which is set in place just before application of the load **P**. Determine the reactions at the supports if EI is constant.



SOLUTION







$\delta_F =$ Deflection of top beam at F

$\delta'_F =$ Deflection of bottom beam at F

$$\delta_F = \delta'_F$$

$$(+\downarrow)\quad \frac{(P-Q)(L^3)}{48EI} - \frac{2M\left(\frac{L}{2}\right)}{6\,EIL}\left[L^2 - \left(\frac{L}{2}\right)^2\right] = \frac{Q\left(\frac{L}{2}\right)^3}{48EI}$$

$$\frac{(P-Q)L}{48} - \frac{1}{6}M\frac{3}{4} = \frac{QL}{48(8)}$$

$$8PL - 48M = 9QL \qquad \textbf{(1)}$$

$$\theta_A = \theta'_A + \theta''_A = 0$$

$$\circlearrowleft+ \quad -\frac{ML}{6EI} - \frac{ML}{3EI} + \frac{(P-Q)L^2}{16EI} = 0$$

$$8M = (P-Q)L \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2):

$M = QL/16$

$Q = 2P/3$

$S = P/3$

$R = P/6$

$M = PL/24$

Thus,

$$M_A = M_B = \frac{1}{24}PL \qquad \textbf{Ans.}$$

$$A_y = B_y = \frac{1}{6}P \qquad \textbf{Ans.}$$

$$C_y = D_y = \frac{1}{3}P \qquad \textbf{Ans.}$$

$$D_x = 0 \qquad \textbf{Ans.}$$

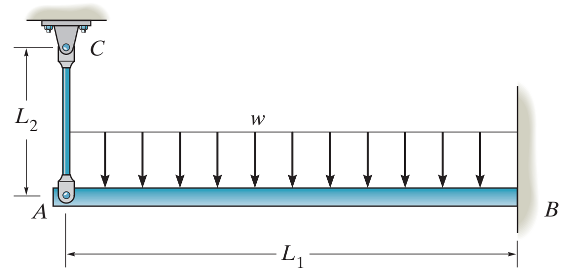
Ans:

$M_A = M_B = \dfrac{1}{24}PL,\ A_y = B_y = \dfrac{1}{6}P,$

$C_y = D_y = \dfrac{1}{3}P,\ D_x = 0$

12–126.

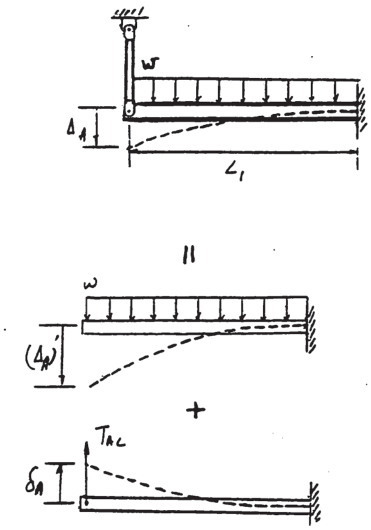
The beam has a constant E_1I_1 and is supported by the fixed wall at B and the rod AC. If the rod has a cross-sectional area A_2 and the material has a modulus of elasticity E_2, determine the force in the rod.



SOLUTION

$$(\Delta_A)' = \frac{wL_1^4}{8E_1I_1}; \qquad \Delta_A = \frac{T_{AC}L_2}{A_2E_2}$$

$$\delta_A = \frac{T_{AC}L_1^3}{3E_1I_1}$$



By Superposition:

$$(+\downarrow) \qquad \Delta_A = (\Delta_A)' - \delta_A$$

$$\frac{T_{AC}L_2}{A_2E_2} = \frac{wL_1^4}{8E_1I_1} - \frac{T_{AC}L_1^3}{3E_1I_1}$$

$$T_{AC}\left(\frac{L_2}{A_2E_2} + \frac{L_1^3}{3E_1I_1}\right) = \frac{wL_1^4}{8E_1I_1}$$

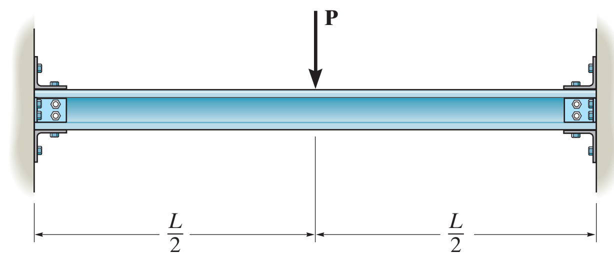
$$T_{AC} = \frac{3wA_2E_2L_1^4}{8(3E_1I_1L_2 + A_2E_2L_1^3)}$$ **Ans.**

Ans:

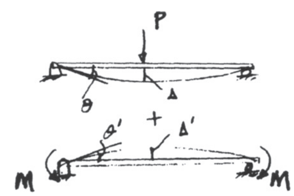
$$T_{AC} = \frac{3wA_2E_2L_1^4}{8(3E_1I_1L_2 + A_2E_2L_1^3)}$$

12–127.

The beam is supported by the bolted supports at its ends. When loaded these supports initially do not provide an actual fixed connection, but instead allow a slight rotation α before becoming fixed after the load is fully applied. Determine the moment at the supports and the maximum deflection of the beam.



SOLUTION



$$\theta - \theta' = \alpha$$

$$\frac{PL^2}{16EI} - \frac{ML}{3EI} - \frac{ML}{6EI} = \alpha$$

$$ML = \left(\frac{PL^2}{16EI} - \alpha\right)(2EI)$$

$$M = \frac{PL}{8} - \frac{2EI}{L}\alpha \qquad \textbf{Ans.}$$

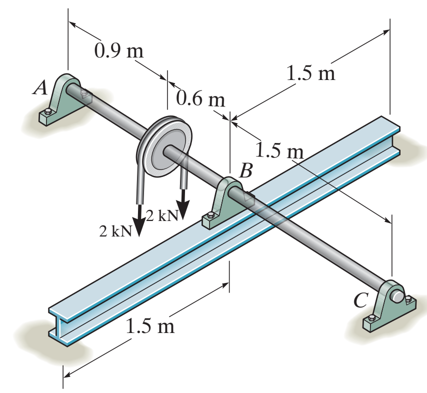
$$\Delta_{\max} = \Delta - \Delta' = \frac{PL^3}{48EI} - 2\left[\frac{M(\frac{L}{2})}{6EIL}\left[L^2 - (L/2)^2\right]\right]$$

$$\Delta_{\max} = \frac{PL^3}{48EI} - \frac{L^2}{8EI}\left(\frac{PL}{8} - \frac{2EI\alpha}{L}\right)$$

$$\Delta_{\max} = \frac{PL^3}{192EI} + \frac{\alpha L}{4} \qquad \textbf{Ans.}$$

Ans:
$M = \dfrac{PL}{8} - \dfrac{2EI}{L}\alpha, \Delta_{\max} = \dfrac{PL^3}{192EI} + \dfrac{\alpha L}{4}$

***12–128.** The 25-mm-diameter A-36 steel shaft is supported by unyielding bearings at A and C. The bearing at B rests on a simply supported steel wide-flange beam having a moment of inertia of $I = 195(10^6)$ mm^4. If the belt loads on the pulley are 2 kN each, determine the vertical reactions at A, B, and C.



For the shaft:

$$(\Delta_b)_1 = \left[\frac{4(10^3)(0.9)(1.5)}{6EI_s(3)}\right](3^2 - 0.9^2 - 1.5^2) = \frac{1782}{EI_s} \downarrow$$

$$(\Delta_b)_2 = \frac{B_y(3^3)}{48EI_s} = \frac{0.5625B_y}{EI_s} \uparrow$$

For the beam:

$$\Delta_b = \frac{B_y(3^3)}{48EI_b} = \frac{0.5625B_y}{EI_b}$$

Compatibility condition:

$$+\downarrow \Delta_b = (\Delta_b)_1 - (\Delta_b)_2$$

$$\frac{0.5625B_y}{EI_b} = \frac{1782}{EI_s} - \frac{0.5625B_y}{EI_s}$$

$$I_s = \frac{\pi}{4}(0.0125^4) = 19.175(10^{-9}) \text{ m}^4$$

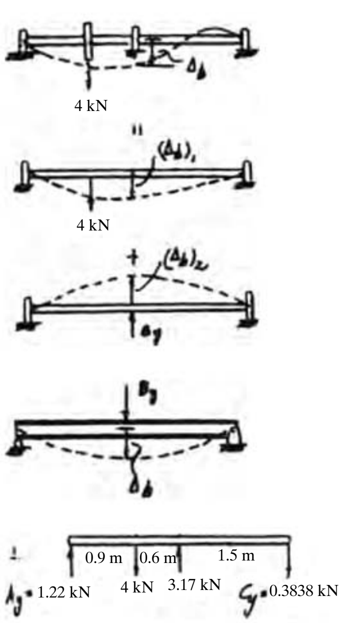
$$\frac{0.5625B_y}{E[195(10^{-6})]} = \frac{1782}{E[19.175(10^{-9})]} - \frac{0.5625B_y}{E[19.175(10^{-9})]}$$

$B_y = 3167.69 \text{ N} = 3.17 \text{ kN}$ **Ans.**

Form the free-body digram,

$A_y = 1.22$ kN **Ans.**

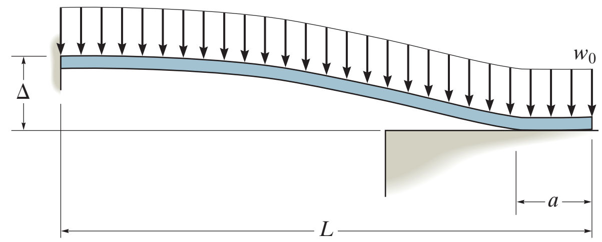
$C_y = 0.384$ kN **Ans.**



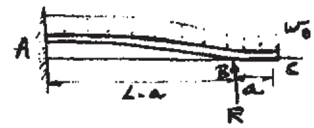
Ans:
$B_y = 3.17$ kN; $A_y = 1.22$ kN; $C_y = 0.384$ kN

12–129.

The beam is made from a soft linear elastic material having a constant EI. If it is originally a distance Δ from the surface of its end support, determine the length a that rests on this support when it is subjected to the uniform load w_0, which is great enough to cause this to happen.



SOLUTION



The curvature of the beam in region BC is zero, therefore there is no bending moment in the region BC. The reaction R is at B where it touches the support. The slope is zero at this point and the deflection is Δ where

$$\Delta = \frac{w_0(L-a)^4}{8EI} - \frac{R(L-a)^3}{3EI}$$

$$\theta_x = 0 = \frac{w_0(L-a)^3}{6EI} - \frac{R(L-a)^2}{2EI}$$

Thus,

$$R = \frac{w_0(L-a)}{3}$$

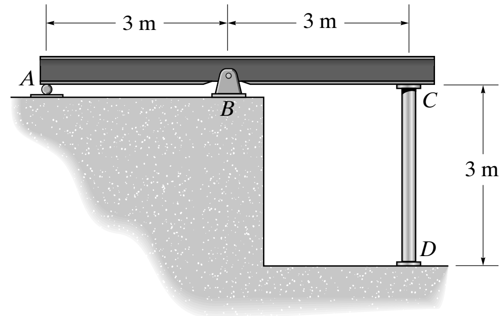
$$\Delta = \frac{w_0(L-a)^4}{(72EI)}$$

$$L - a = \left(\frac{72\Delta EI}{w_0}\right)^{\frac{1}{4}}$$

$$a = L - \left(\frac{72\Delta EI}{w_0}\right)^{\frac{1}{4}}$$ **Ans.**

Ans:
$a = L - \left(\frac{72\Delta EI}{w_0}\right)^{1/4}$

12–130. If the temperature of the 75-mm-diameter post CD is increased by 60°C, determine the force developed in the post. The post and the beam are made of A-36 steel, and the moment of inertia of the beam is $I = 255(10^6)\ \text{mm}^4$.



SOLUTION

Method of Superposition. Referring to Fig. a and the table in the Appendix, the necessary deflections are

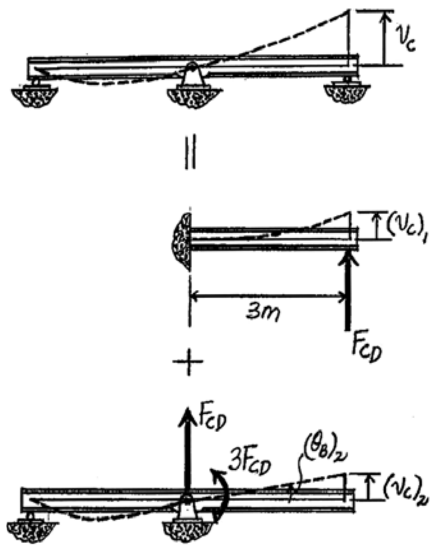
$$(v_C)_1 = \frac{PL_{BC}{}^3}{3EI} = \frac{F_{CD}\left(3^3\right)}{3EI} = \frac{9F_{CD}}{EI}\ \uparrow$$

$$(v_C)_2 = (\theta_B)_2 L_{BC} = \frac{M_O L_{AB}}{3EI}(L_{BC}) = \frac{3F_{CD}\,(3)}{3EI}(3) = \frac{9F_{CD}}{EI}\ \uparrow$$

The compatibility condition at end C requires

$$(+\uparrow) \qquad v_C = (v_C)_1 + (v_C)_2$$

$$= \frac{9F_{CD}}{EI} + \frac{9F_{CD}}{EI} = \frac{18F_{CD}}{EI}\ \uparrow$$



(a)

Referring to Fig. b, the compatibility condition of post CD requires that

$$\delta_{F_{CD}} + v_C = \delta_T \qquad (1)$$

$$\delta F_{CD} = \frac{F_{CD}\,L_{CD}}{AE} = \frac{F_{CD}\,(3)}{AE}$$

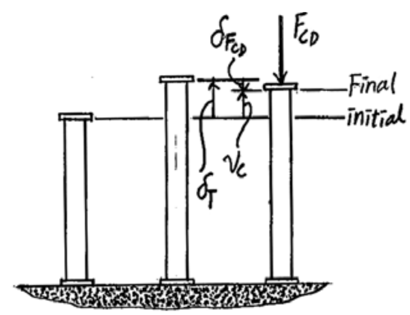
$$\delta_T = \alpha \Delta T L = 12\left(10^{-6}\right)(60)(3) = 2.16\left(10^{-3}\right)\text{m}$$

Thus, Eq. (1) becomes

$$\frac{3F_{CD}}{AE} + \frac{18F_{CD}}{EI} = 2.16\left(10^{-3}\right)$$

$$\frac{3F_{CD}}{\frac{\pi}{4}\left(0.075^2\right)} + \frac{18F_{CD}}{255\left(10^{-6}\right)} = 2.16\left(10^{-3}\right)\left[200\left(10^9\right)\right]$$

$$F_{CD} = 6061.69\text{N} = 6.06\ \text{kN}$$ **Ans.**

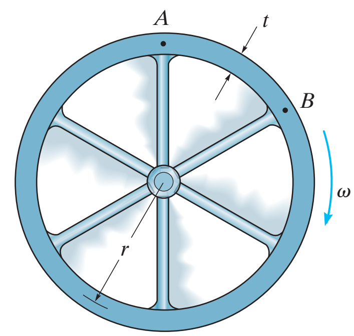


(b)

Ans:
$F_{CD} = 6.06\ \text{kN}$

12–131.

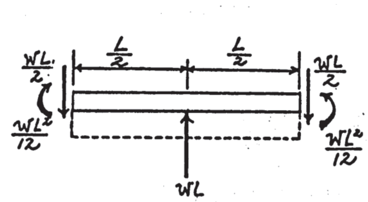
The rim on the flywheel has a thickness t, width b, and specific weight γ. If the flywheel is rotating at a constant rate of ω, determine the maximum moment developed in the rim. Assume that the spokes do not deform. *Hint*: Due to symmetry of the loading, the slope of the rim at each spoke is zero. Consider the radius to be sufficiently large so that the segment AB can be considered as a straight beam fixed at both ends and loaded with a uniform centrifugal force per unit length. Show that this force is $w = bt\gamma\omega^2 r/g$.



SOLUTION

Centrifugal Force: The centrifugal force acting on a unit length of the rim rotating at a constant rate of ω is

$$w = m\omega^2 r = bt\left(\frac{\gamma}{g}\right)\omega^2 r = \frac{bt\gamma\omega^2 r}{g} \qquad \textbf{(Q.E.D)}$$

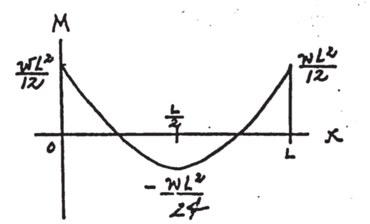


Elastic Curve: Member AB of the rim is modeled as a straight beam with both of its ends fixed and subjected to a uniform centrifugal force w.

Method of Superposition: Using the table in Appendix C, the required displacements are

$$\theta_B' = \frac{wL^3}{6EI} \qquad \theta_B'' = \frac{M_B L}{EI} \qquad \theta_B''' = \frac{B_y L^2}{2EI}$$

$$v_B' = \frac{wL^4}{8EI}\uparrow \qquad v_B'' = \frac{M_B L^2}{2EI}\uparrow \qquad v_B''' = \frac{B_y L^3}{3EI}\downarrow$$



Compatibility requires

$$0 = \theta_B' + \theta_B'' + \theta_B'''$$

$$0 = \frac{wL^3}{6EI} + \frac{M_B L}{EI} + \left(-\frac{B_y L^2}{2EI}\right)$$

$$0 = wL^2 + 6M_B - 3B_y L \qquad \textbf{(1)}$$

$$(+\uparrow) \qquad 0 = v_B' + v_B'' + v_B'''$$

$$0 = \frac{wL^4}{8EI} + \frac{M_B L^2}{2EI} + \left(-\frac{B_y L^3}{3EI}\right)$$

$$0 = 3wL^2 + 12M_B - 8B_y L \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$B_y = \frac{wL}{2} \qquad M_B = \frac{wL^2}{12}$$

Due to symmetry, $\quad A_y = \dfrac{wL}{2} \qquad M_A = \dfrac{wL^2}{12}$

Maximum Moment: From the moment diagram, the maximum moment occurs at the two fixed end supports. With $w = \dfrac{bt\gamma\omega^2 r}{g}$ and $L = r\theta = \dfrac{\pi r}{3}$,

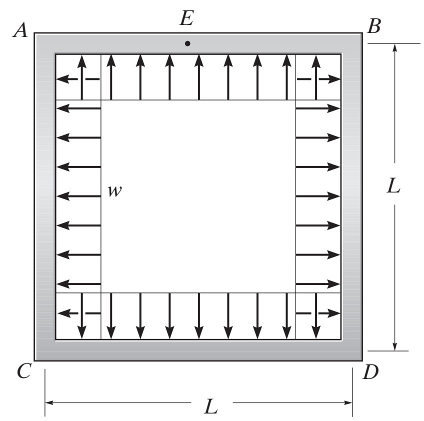
$$M_{max} = \frac{wL^2}{12} = \frac{\frac{bt\gamma\omega^2 r}{g}\left(\frac{\pi r}{3}\right)^2}{12} = \frac{\pi^2 bt\gamma\omega^2 r^3}{108g} \qquad \textbf{Ans.}$$

Ans:

$$M_{max} = \frac{\pi^2 bt\gamma\omega^2 r^3}{108g}$$

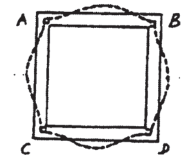
***12–132.**

The box frame is subjected to a uniform distributed loading w along each of its sides. Determine the moment developed in each corner. Neglect the deflection due to axial load. EI is constant.



SOLUTION

Elastic Curve: In order to maintain the right angle and zero slope (due to symmetrical loading) at the four corner joints, the box frame deforms into the shape shown when it is subjected to the internal uniform distributed load. Therefore, member AB of the frame can be modeled as a beam with both ends fixed.



Method of Superposition: Using the table in Appendix C, the required displacements are

$$\theta_B{}' = \frac{wL^3}{6EI} \qquad \theta_B{}'' = \frac{M_B L}{EI} \qquad \theta_B{}''' = \frac{B_y L^2}{2EI}$$

$$v_B{}' = \frac{wL^4}{8EI}\uparrow \qquad v_B{}'' = \frac{M_B L^2}{2EI}\uparrow \qquad v_B{}''' = \frac{B_y L^3}{3EI}\downarrow$$

Compatibility conditions require

$$0 = \theta_B{}' + \theta_B{}'' + \theta_B{}'''$$

$$0 = \frac{wL^3}{6EI} + \frac{M_B L}{EI} + \left(-\frac{B_y L^2}{2EI}\right)$$

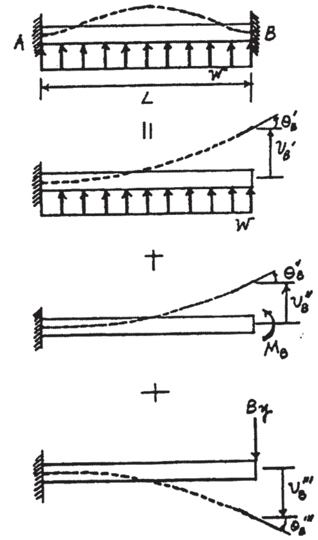
$$0 = wL^2 + 6M_B - 3B_y L \qquad \textbf{(1)}$$

$(+\uparrow)$

$$0 = v_B{}' + v_B{}'' + v_B{}'''$$

$$0 = \frac{wL^4}{8EI} + \frac{M_B L^2}{2EI} + \left(-\frac{B_y L^3}{3EI}\right)$$

$$0 = 3wL^2 + 12M_B - 8B_y L \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

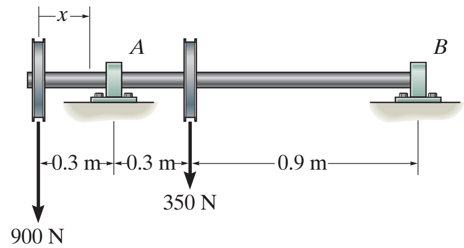
$$B_y = \frac{wL}{2}$$

$$M_B = \frac{wL^2}{12} \qquad \textbf{Ans.}$$

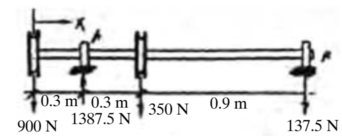
Ans:

$$M_B = \frac{wL^2}{12}$$

R12–1. The shaft supports the two pulley loads shown. Using discontinuity functions, determine the equation of the elastic curve. The bearings at A and B exert only vertical reactions on the shaft. EI is constant.



SOLUTION



$M = -900 < x - 0 > - (-1387.5) < x - 0.3 > - 350 < x - 0.6 >$

$M = -900x + 1387.5 < x - 0.3 > - 350 < x - 0.6 >$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M = -900x + 1387.5 < x - 0.3 > - 350 < x - 0.6 >$$

$$EI\frac{dv}{dx} = -450x^2 + 693.75 < x - 0.3 >^2 - 175 < x - 0.6 >^2 + C_1$$

$$EIv = -150x^3 + 231.25 < x - 0.3 >^3 - 58.333 < x - 0.6 >^3 + C_1x + C_2 \qquad (1)$$

Boundary conditions:

$v = 0 \quad \text{at} \quad x = 0.3\text{ m}$

From Eq. (1)

$$0 = -4.05 + 0.3C_1 + C_2$$

$$0.3C_1 + C_2 = 4.05 \qquad (2)$$

$v = 0 \quad \text{at} \quad x = 1.5\text{ m}$

From Eq.(1)

$$0 = -506.25 + 399.6 - 42.525 + 1.5C_1 + C_2$$

$$1.5C_1 + C_2 = 149.175 \qquad (3)$$

Solving Eqs. (2) and (3) yields:

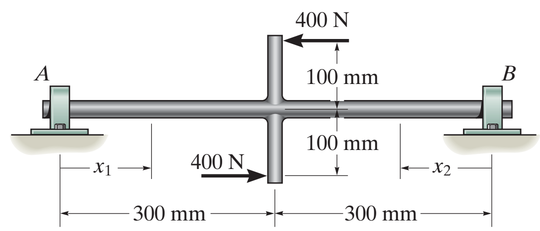
$C_1 = 120.94 \qquad C_2 = -32.23$

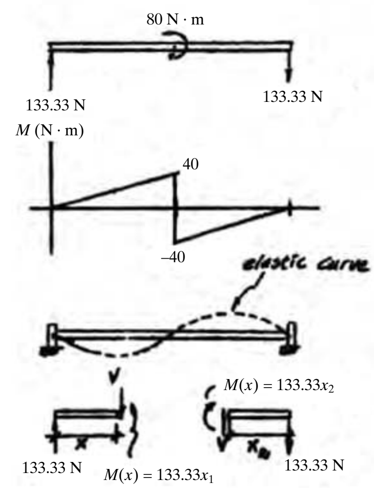
$$v = \frac{1}{EI}[-150x^3 + 231\langle x - 0.3\rangle^3 - 58.3\langle x - 0.6\rangle^3 + 121x - 32.2]\ \text{N}\cdot\text{m}^3$$ **Ans.**

Ans:

$$v = \frac{1}{EI}[-150x^3 + 231\langle x - 0.3\rangle^3 - 58.3\langle x - 0.6\rangle^3 + 121x - 32.2]\ \text{N}\cdot\text{m}^3$$

R12–2. The shaft is supported by a journal bearing at A, which exerts only vertical reactions on the shaft, and by a thrust bearing at B, which exerts both horizontal and vertical reactions on the shaft. Draw the bending-moment diagram for the shaft and then, from this diagram, sketch the deflection or elastic curve for the shaft's centerline. Determine the equations of the elastic curve using the coordinates x_1 and x_2. EI is constant.





For $M_1(x) = 133.33x_1$

$$EI\frac{d^2v_1}{dx_1^2} = 133.33x_1$$

$$EI\frac{dv_1}{dx_1} = 66.67x_1^2 + C_1 \qquad (1)$$

$$EIv_1 = 22.22x_1^3 + C_1x_1 + C_2 \qquad (2)$$

For $M_2(x) = -133.33x_2$

$$EI\frac{d^2v_2}{dx_2^2} = -133.33x_2$$

$$EI\frac{dv_2}{dx_2} = -66.67x_2^2 + C_3 \qquad (3)$$

$$EIv_2 = -22.22x_2^3 + C_3x_2 + C_4 \qquad (4)$$

Boundary conditions:

$v_1 = 0$ at $x_1 = 0$

From Eq.(2)

$C_2 = 0$

$v_2 = 0$ at $x_2 = 0$

$C_4 = 0$

Continuity conditions:

$$\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2} \quad \text{at} \quad x_1 = x_2 = 0.300\text{ m}$$

From Eqs. (1) and (3)

$$6 + C_1 = -(-6 + C_3)$$

$$C_1 = -C_3 \qquad (5)$$

$v_1 = v_2$ at $x_1 = x_2 = 0.300$ m

$$0.6 + 0.3C_1 = -0.6 + 0.3C_3$$

$$C_3 - C_1 = 4 \qquad (6)$$

Solving Eqs. (5) and (6) yields:

$C_3 = 2\text{ N}\cdot\text{m}^2 \qquad C_1 = -2\text{ N}\cdot\text{m}^2$

$$v_1 = \frac{1}{EI}(22.2x_1^3 - 2x_1)\text{N}\cdot\text{m}^3 \qquad \textbf{Ans.}$$

$$v_2 = \frac{1}{EI}(-22.2x_2^3 + 2x_2)\text{ N}\cdot\text{m}^3 \qquad \textbf{Ans.}$$

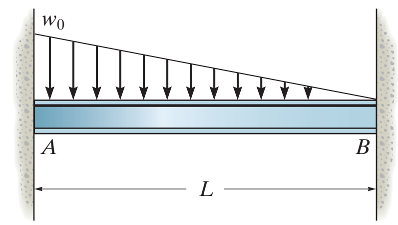
Ans:

$v_1 = \dfrac{1}{EI}(22.2x_1^3 - 2x_1)\text{N}\cdot\text{m}^3$,

$v_2 = \dfrac{1}{EI}(-22.2x_2^3 + 2x_2)\text{ N}\cdot\text{m}^3$

R12–3.

Determine the moment reactions at the supports A and B. Use the method of integration. EI is constant.

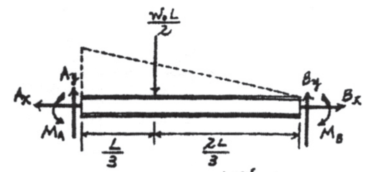


SOLUTION

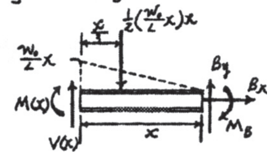
Support Reactions: FBD(a).

$$+\uparrow\Sigma F_y = 0; \qquad A_y + B_y - \frac{w_0 L}{2} = 0 \tag{1}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y L + M_A - M_B - \frac{w_0 L}{2}\left(\frac{L}{3}\right) = 0 \tag{2}$$



(a)



(b)

Moment Function: FBD(b).

$$\circlearrowleft + \Sigma M_{\text{NA}} = 0; \qquad -M(x) - \frac{1}{2}\left(\frac{w_0}{L}x\right)x\left(\frac{x}{3}\right) - M_B + B_y x = 0$$

$$M(x) = B_y x - \frac{w_0}{6L}x^3 - M_B$$

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = B_y x - \frac{w_0}{6L}x^3 - M_B$$

$$EI\frac{dv}{dx} = \frac{B_y}{2}x^2 - \frac{w_0}{24L}x^4 - M_B x + C_1 \tag{3}$$

$$EI\,v = \frac{B_y}{6}x^3 - \frac{w_0}{120L}x^5 - \frac{M_B}{2}x^2 + C_1 x + C_2 \tag{4}$$

Boundary Conditions:

At $x = 0, \dfrac{dv}{dx} = 0$ From Eq. (3), $C_1 = 0$

At $x = 0, v = 0.$ From Eq. (4), $C_2 = 0$

At $x = L, \dfrac{dv}{dx} = 0.$ From Eq. (3),

$$0 = \frac{B_y L^2}{2} - \frac{w_0 L^3}{24} - M_B L$$

$$0 = 12B_y L - w_0 L^2 - 24M_B \tag{5}$$

At $x = L, v = 0.$ From Eq. (4),

$$0 = \frac{B_y L^3}{6} - \frac{w_0 L^4}{120} - \frac{M_B L^2}{2}$$

$$0 = 20B_y L - w_0 L^2 - 60M_B \tag{6}$$

R12–3. Continued

Solving Eqs. (5) and (6) yields,

$$M_B = \frac{w_0 L^2}{30}$$ **Ans.**

$$B_y = \frac{3w_0 L}{20}$$

Substituting B_y and M_B into Eqs. (1) and (2) yields,

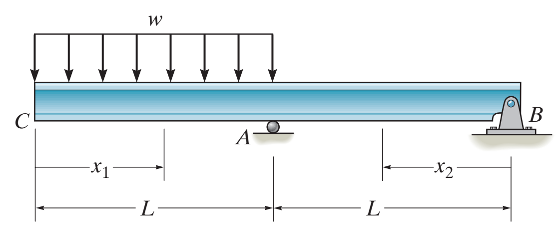
$$M_A = \frac{w_0 L^2}{20}$$ **Ans.**

$$A_y = \frac{7w_0 L}{20}$$

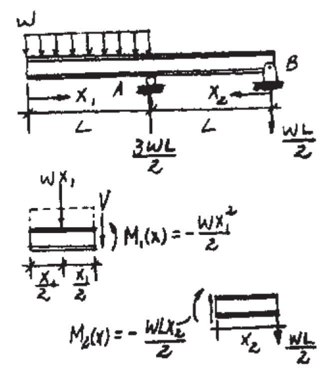
Ans:
$M_B = \dfrac{w_0 L^2}{30}$, $M_A = \dfrac{w_0 L^2}{20}$

***R12–4.**

Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. Specify the slope at A and the maximum deflection. Use the method of integration. EI is constant.



SOLUTION



Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = \dfrac{-wx_1^1}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = \frac{-wx_1^2}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{-wx_1^3}{6} + C_1 \qquad \textbf{(1)}$$

$$EIv_1 = \frac{-wx_1^4}{24} + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M_2(x) = \dfrac{-wLx_2}{2}$

$$EI\frac{d^2v_2}{dx_2^2} = \frac{-wLx_2}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{-wLx_3^2}{4} + C_3 \qquad \textbf{(3)}$$

$$EIv_2 = \frac{-wLx_2^3}{12} + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

$v_2 = 0 \quad$ at $\quad x_2 = 0$

From Eq. (4):

$C_4 = 0$

$v_2 = 0 \quad$ at $\quad x_2 = L$

From Eq. (4):

$$0 = \frac{-wL^4}{12} + C_3L$$

$$C_3 = \frac{wL^3}{12}$$

$v_1 = 0 \quad$ at $\quad x_1 = L$

From Eq. (2):

$$0 = -\frac{wL^4}{24} + C_1L + C_2 \qquad \textbf{(5)}$$

***R12–4. Continued**

Continuity Conditions:

$$\frac{dv_1}{dx_1} = \frac{dv_2}{-dx_2} \quad \text{at} \quad x_1 = x_2 = L$$

From Eqs. (1) and (3)

$$-\frac{wL^3}{6} + C_1 = -\left(-\frac{wL^3}{4} + \frac{wL^3}{12}\right)$$

$$C_1 = \frac{wL^3}{3}$$

Substitute C_1 into Eq. (5)

$$C_2 = -\frac{7wL^4}{24}$$

$$\frac{dv_1}{dx_1} = \frac{w}{6EI}(2L^3 - x_1^3)$$

$$\frac{dv_2}{dx_2} = \frac{w}{12EI}(L^3 - 3Lx_2^2) \qquad \textbf{(6)}$$

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1 = L} = -\left.\frac{dv_2}{dx_2}\right|_{x_2 = L} = \frac{wL^3}{6EI} \qquad \textbf{Ans.}$$

$$v_1 = \frac{w}{24EI}(-x_1^4 + BL^3x_1 - 7L^4) \qquad \textbf{Ans.}$$

$$(v_1)_{\max} = \frac{-7wL^4}{24EI} \quad (x_1 = 0)$$

The negative sign indicates downward displacement.

$$v_2 = \frac{wL}{12EI}(L^2x_2 - x_2^3) \qquad \textbf{(7)} \qquad \textbf{Ans.}$$

$$(v_2)_{\max} \text{ occurs when } \frac{dv_2}{dx_2} = 0$$

From Eq. (6)

$$L^3 - 3Lx_2^2 = 0$$

$$x_2 = \frac{L}{\sqrt{3}}$$

Substitute x_2 into Eq. (7),

$$(v_2)_{\max} = \frac{wL^4}{18\sqrt{3}EI} \qquad \textbf{(8)}$$

$$v_{\max} = (v_1)_{\max} = -\frac{7wL^4}{24EI} \qquad \textbf{(9)} \qquad \textbf{Ans.}$$

Ans:

$\theta_A = \dfrac{wL^3}{6EI}$,

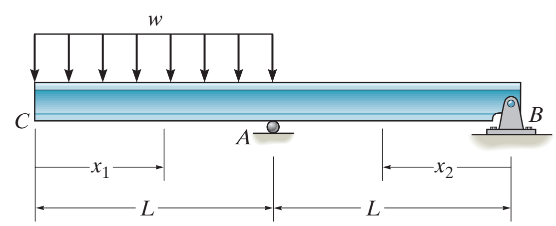
$v_1 = \dfrac{w}{24EI}(-x_1^4 + BL^3x_1 - 7L^4)$,

$v_2 = \dfrac{wL}{12EI}(L^2x_2 - x_2^3)$,

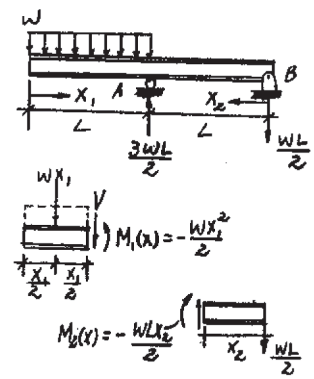
$v_{\max} = -\dfrac{7wL^4}{24EI}$

R12–5.

Determine the maximum deflection between the supports A and B. Use the method of integration. EI is constant.



SOLUTION



Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = \dfrac{-wx_1^2}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = \frac{-wx_1^2}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{-wx_1^3}{6} + C_1 \qquad \textbf{(1)}$$

$$EIv_1 = \frac{-wx_1^4}{24} + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M_2(x) = \dfrac{-wLx_2}{2}$

$$EI\frac{d^2v_2}{dx_2^2} = \frac{-wLx_2}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{-wLx_2^2}{4} + C_3 \qquad \textbf{(3)}$$

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Boundary Conditions:

$v_2 = 0 \quad$ at $\quad x_2 = 0$

From Eq. (4):

$C_4 = 0$

$v_2 = 0 \quad$ at $\quad x_2 = L$

From Eq. (4):

$$0 = \frac{-wL^4}{12} + C_3L$$

$$C_3 = \frac{wL^3}{12}$$

$v_1 = 0 \quad$ at $\quad x_1 = L$

From Eq. (2):

$$0 = -\frac{wL^4}{24} + C_1L + C_2 \qquad \textbf{(5)}$$

R12–5. Continued

Continuity Conditions:

$$\frac{dv_1}{dx_1} = \frac{dv_2}{-dx_2} \qquad \text{at} \qquad x_1 = x_2 = L$$

From Eqs. (1) and (3)

$$-\frac{wL^3}{6} + C_1 = -\left(-\frac{wL^3}{4} + \frac{wL^3}{12}\right)$$

$$C_1 = \frac{wL^3}{3}$$

Substitute C_1 into Eq. (5)

$$C_2 = -\frac{7wL^4}{24}$$

$$\frac{dv_1}{dx_1} = \frac{w}{6EI}(2L^3 - x_1^3)$$

$$\frac{dv_2}{dx_2} = \frac{w}{12EI}(L^3 - 3Lx_2^2) \qquad \textbf{(6)}$$

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1 = L} = -\left.\frac{dv_2}{dx_2}\right|_{x_2 = L} = \frac{wL^3}{6EI}$$

$$v_1 = \frac{w}{24EI}(-x_1^4 + 8L^3x_1 - 7L^4)$$

$$(v_1)_{\max} = \frac{-7wL^4}{24EI} \qquad (x_1 = 0)$$

The negative sign indicates downward displacement.

$$v_2 = \frac{wL}{12EI}(L^2x_2 - x_2^3) \qquad \textbf{(7)}$$

$(v_2)_{\max}$ occurs when $\frac{dv_2}{dx_2} = 0$

From Eq. (6)

$$L^3 - 3Lx_2^2 = 0$$

$$x_2 = \frac{L}{\sqrt{3}}$$

Substitute x_2 into Eq. (7),

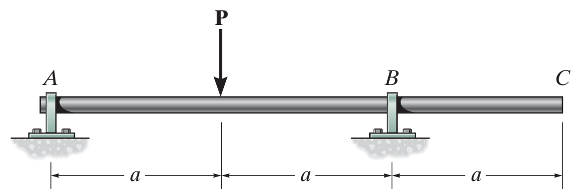
$$(v_2)_{\max} = \frac{wL^4}{18\sqrt{3}EI} \qquad \textbf{Ans.}$$

Ans:

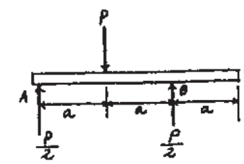
$$(v_2)_{\max} = \frac{wL^4}{18\sqrt{3}EI}$$

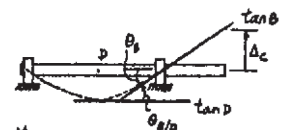
R12–6.

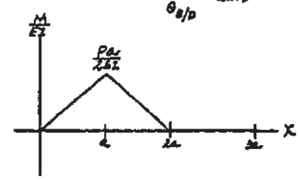
Determine the slope at B and the deflection at C. Use the moment-area theorems. EI is constant.



SOLUTION







Support Reaction and Elastic Curve: As shown.

***M*/*EI* Diagram:** As shown.

Moment-Area Theorems:

$$\theta_{B/D} = \frac{1}{2}\left(\frac{Pa}{2EI}\right)(a) = \frac{Pa^2}{4EI}$$

Due to symmetry, the slope at point D is zero. Hence, the slope at B is

$$\theta_B = \left|\theta_{B/D}\right| = \frac{Pa^2}{4EI} \qquad \textbf{Ans.}$$

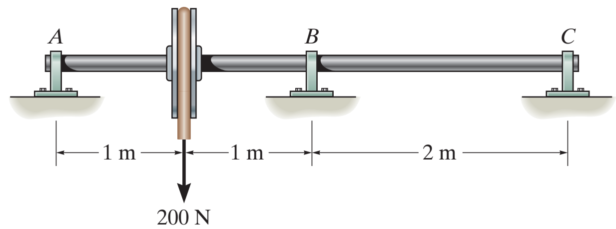
The displacement at C is

$$\Delta_C = \theta_B L_{BC} = \frac{Pa^2}{4EI}(a) = \frac{Pa^3}{4EI}\uparrow \qquad \textbf{Ans.}$$

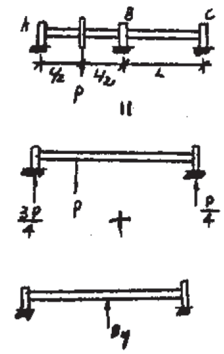
Ans:
$\theta_B = \dfrac{Pa^2}{4EI}$, $\Delta_C = \dfrac{Pa^3}{4EI}\uparrow$

R12–7.

Determine the reactions, then draw the shear and moment diagrams. Use the moment-area theorems. *EI* is constant.



SOLUTION

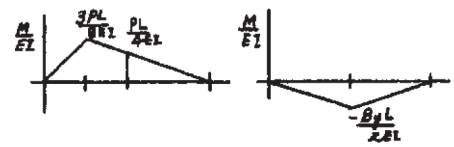


$$(t_{B/A})_1 = \frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2}+\frac{L}{6}\right) + \frac{1}{2}\left(\frac{PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) + \frac{PL}{4EI}\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) = \frac{5PL^3}{48EI}$$

$$(t_{C/A})_1 = \frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{3L}{2}+\frac{L}{6}\right) + \frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{3L}{2}\right)(L) = \frac{7PL^3}{16EI}$$

$$(t_{B/A})_2 = \frac{1}{2}\left(\frac{-B_yL}{2EI}\right)(L)\left(\frac{L}{3}\right) = \frac{-B_yL^3}{12EI}$$

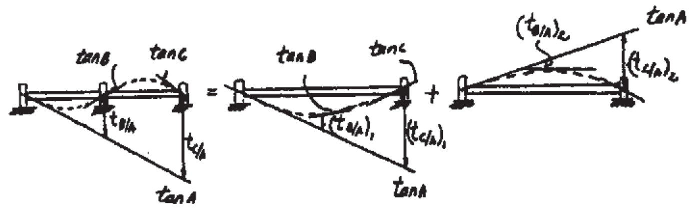
$$(t_{C/A})_2 = \frac{1}{2}\left(\frac{-B_yL}{2EI}\right)(2L)(L) = \frac{-B_yL^3}{2EI}$$



$$2t_{B/A} = t_{C/A}$$

$$2[(t_{B/A})_1 + (t_{B/A})_2] = (t_{C/A})_1 + (t_{C/A})_2$$

$$2\left[\frac{5PL^3}{48EI} + \left(\frac{-B_yL^3}{12EI}\right)\right] = \frac{7PL^3}{16EI} + \left(\frac{-B_yL^3}{2EI}\right)$$



$$B_y = \frac{11}{16}P$$

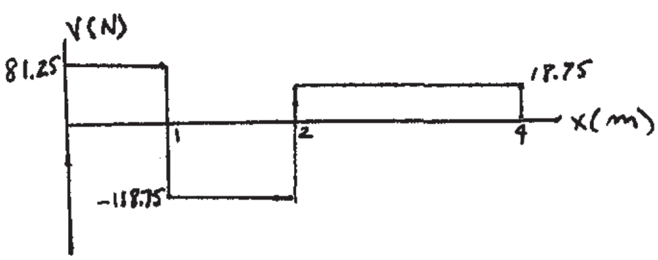
Thus,

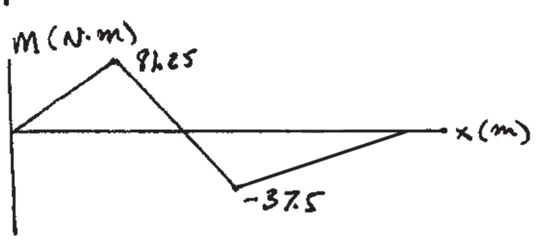
$B_y = \frac{11}{16}(200) = 138 \text{ N}\uparrow$ **Ans.**

As shown on the free-body diagram

$A_y = 81.3 \text{ N}\uparrow$ **Ans.**

$C_y = 18.8 \text{ N}\downarrow$ **Ans.**

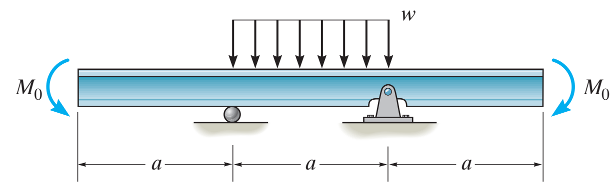




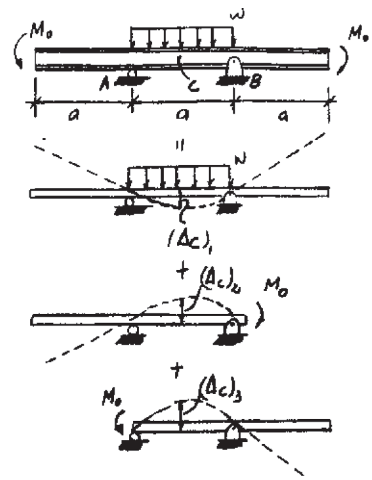
Ans:
$B_y = 138 \text{ N}\uparrow$, $A_y = 81.3 \text{ N}\uparrow$, $C_y = 18.8 \text{ N}\downarrow$

***R12–8.**

Using the method of superposition, determine the magnitude of $\mathbf{M}_0$ in terms of the distributed load w and dimension a so that the deflection at the center of the beam is zero. EI is constant.



SOLUTION



$$(\Delta_C)_1 = \frac{5wa^4}{384EI}\downarrow$$

$$(\Delta_C)_2 = (\Delta_C)_3 = \frac{M_0 a^2}{16EI}\uparrow$$

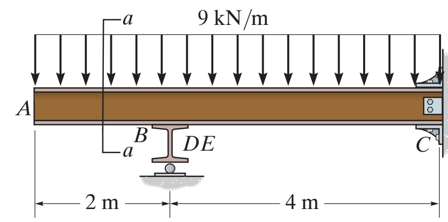
$$\Delta_C = 0 = (\Delta_C)_1 + (\Delta_C)_2 + (\Delta_C)_3$$

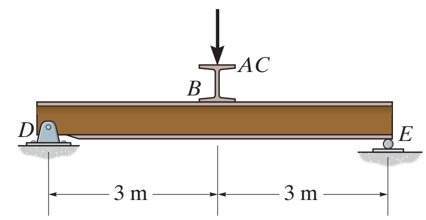
$$+\uparrow \qquad 0 = \frac{-5wa^4}{384EI} + \frac{M_0 a^2}{8EI}$$

$$M_0 = \frac{5wa^2}{48}$$ **Ans.**

Ans:
$M_0 = \dfrac{5wa^2}{48}$

***12–9.** Beam ABC is supported by beam DBE and fixed at C. Determine the reactions at B and C. The beams are made of the same material having a modulus of elasticity $E = 200$ GPa, and the moment of inertia of both beams is $I = 25.0(10^6)$ mm^4.





Section a – a

SOLUTION

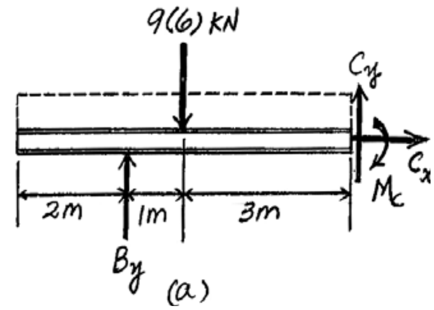
Equation of Equilibrium. Referring to the free-body diagram of the beam, Fig. a,

$\xrightarrow{+} \Sigma F_x = 0;\qquad C_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad B_y + C_y - 9(6) = 0 \qquad (1)$

$\curvearrowleft + \Sigma M_C = 0;\qquad 9(6)(3) - B_y(4) - M_C = 0$

$$M_C = 162 - 4B_y \qquad (2)$$



Method of superposition: Referring to Fig. b and the table in the appendix, the deflections are

$$v_B = \frac{PL_{DE}{}^3}{48EI} = \frac{B_y\left(6^3\right)}{48EI} = \frac{4.5B_y}{EI} \downarrow$$

$$(v_B)_1 = \frac{wx^2}{24EI}\left(x^2 - 4Lx + 6L^2\right) = \frac{9\left(4^2\right)}{24EI}\left[4^2 - 4(6)(4) + 6\left(6^2\right)\right]$$

$$= \frac{816 \text{ kN}\cdot\text{m}^3}{EI} \downarrow$$

$$(v_B)_2 = \frac{PL_{BC}{}^3}{3EI} = \frac{B_y\left(4^3\right)}{3EI} = \frac{21.3333B_y}{EI} \uparrow$$

The compatibility condition at support B requires that

$$(+\downarrow)\qquad v_B = (v_B)_1 + (v_B)_2$$

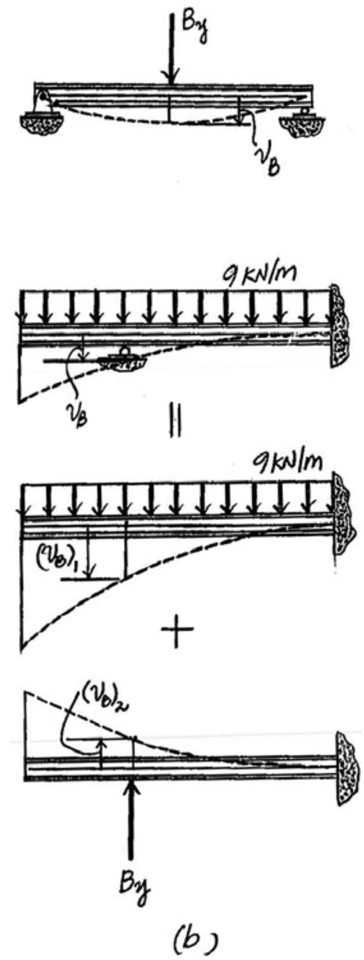
$$\frac{4.5B_y}{EI} = \frac{816}{EI} + \left(-\frac{21.3333B_y}{EI}\right)$$

$B_y = 31.59 \text{ kN} = 31.6 \text{ kN}$ **Ans.**

Substituting the result of B_y into Eqs. (1) and (2),

$M_C = 35.65 \text{ kN}\cdot\text{m} = 35.7 \text{ kN}\cdot\text{m}$ **Ans.**

$C_y = 22.41 \text{ kN} = 22.4 \text{ kN}$ **Ans.**



Ans:

$C_x = 0$, $B_y = 31.6$ kN, $M_C = 35.7$ kN·m,
$C_y = 22.4$ kN