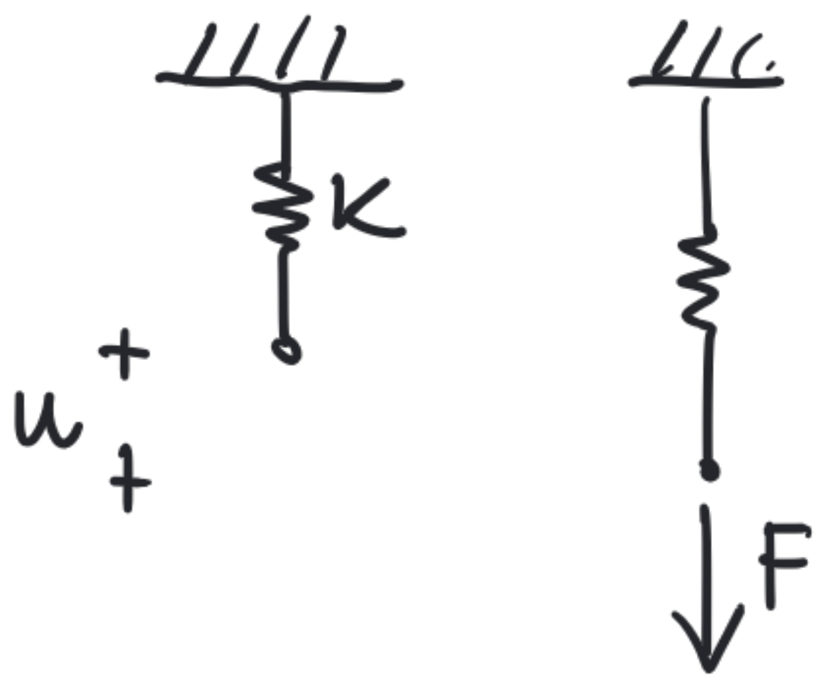
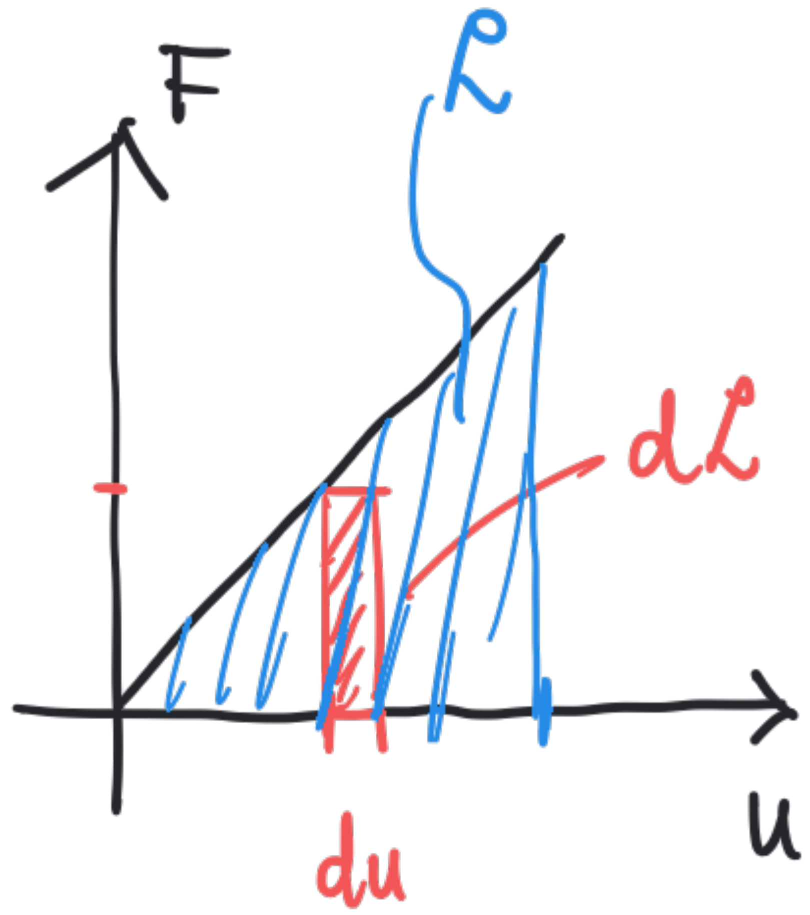
ENERGIA DI DEFORMAZIONE

Hibbeler 14.1

Lavoro di deformazione di una molla





$$F = k u$$

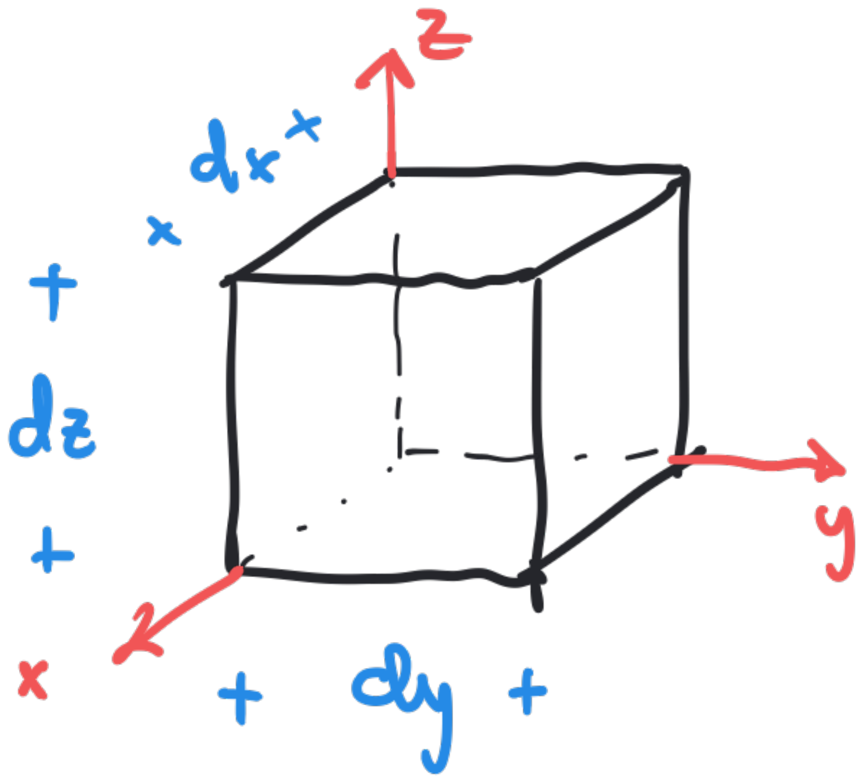
$$d\mathcal{L} = F\,du = k u\,du$$

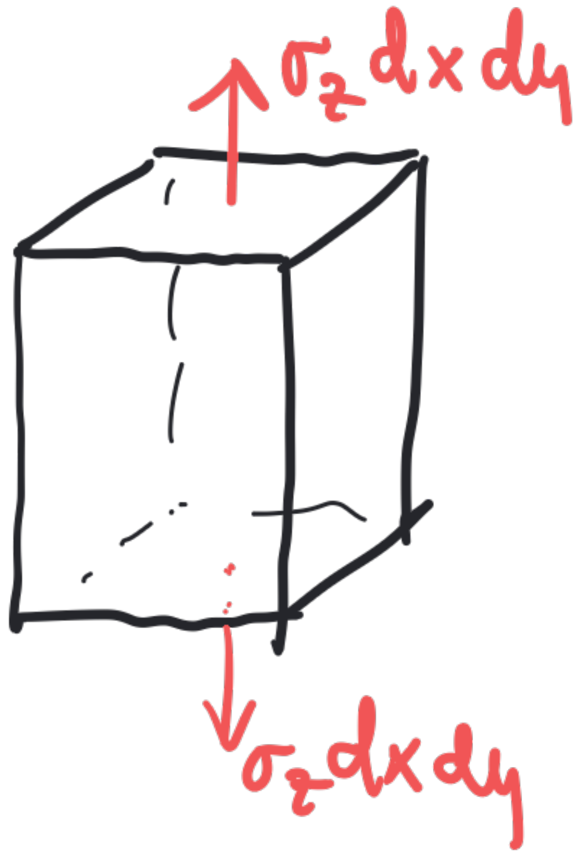
$$\mathcal{L} = \int_0^u d\mathcal{L} = \int_0^u k \bar{u}\,d\bar{u} = \frac{1}{2} k u^2 = \frac{1}{2} F u = \frac{1}{2}\frac{F^2}{k}$$

IL FATTORE ½ È DOVUTO AL FATTO CHE LA FORZA APPLICATA NON È COSTANTE DURANTE IL PROCESSO DI CARICO

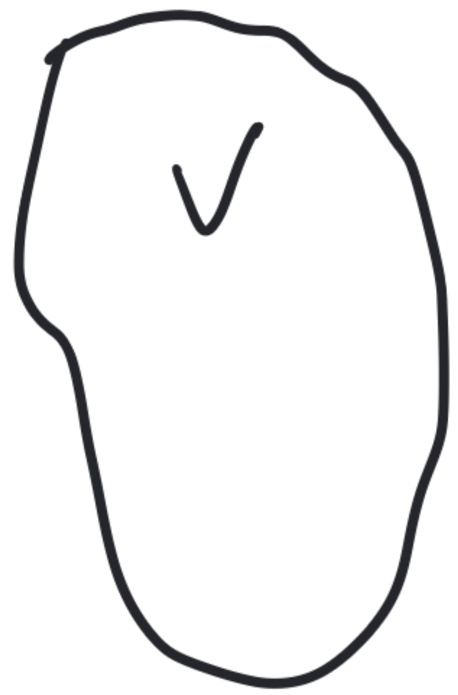
Lavoro di deformazione di un corpo elastico

Sforzo normale





$$\varepsilon_z = \frac{\sigma_z}{E}$$



$$U = \int_V \frac{\sigma^2}{2E} \, dV$$

$$dU_i = \int_0^{\varepsilon_z} \bar{\sigma}_z \, dx \, dy \;\; d\bar{\varepsilon}_z \, dz$$

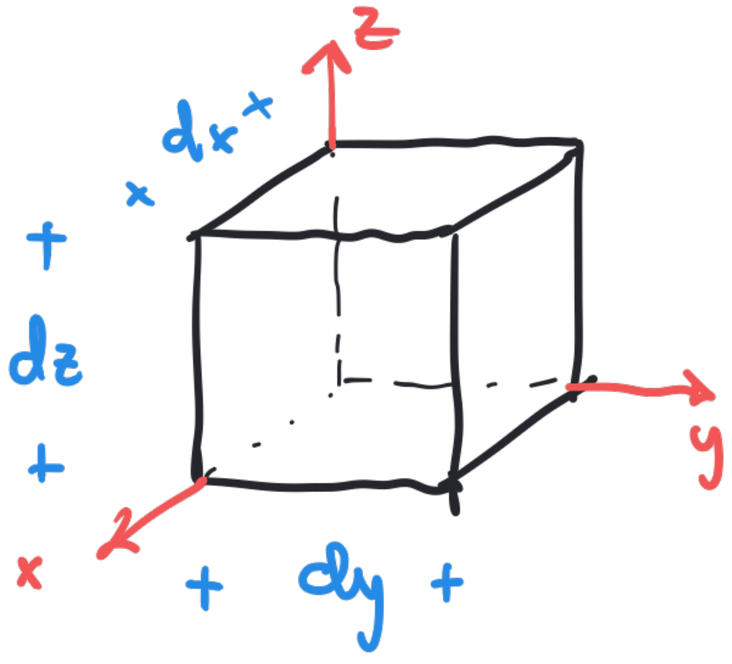
$$= \left(\int_0^{\varepsilon_z} \bar{\sigma}_z \, d\bar{\varepsilon}_z \right) dV$$

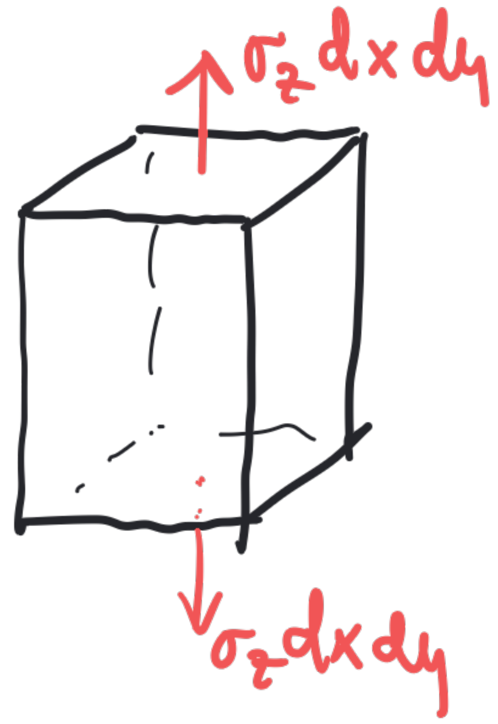
$$= \left(\int_0^{\varepsilon_z} E \bar{\varepsilon}_z \, d\bar{\varepsilon}_z \right) dV$$

$$= \frac{1}{2} E \varepsilon_z^2 \, dV = \frac{1}{2} \frac{\sigma_z^2}{E} dV = \frac{1}{2} \sigma_z \varepsilon_z$$

Lavoro di deformazione di un corpo elastico

Sforzo normale





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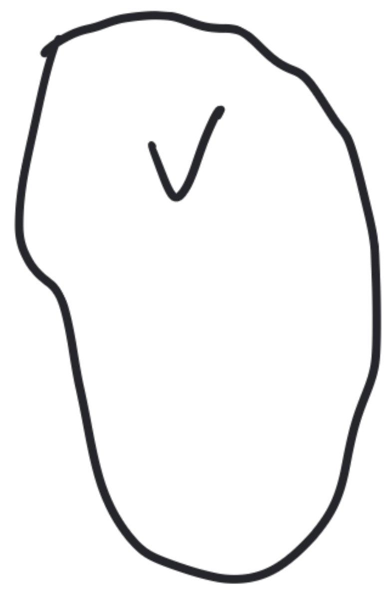
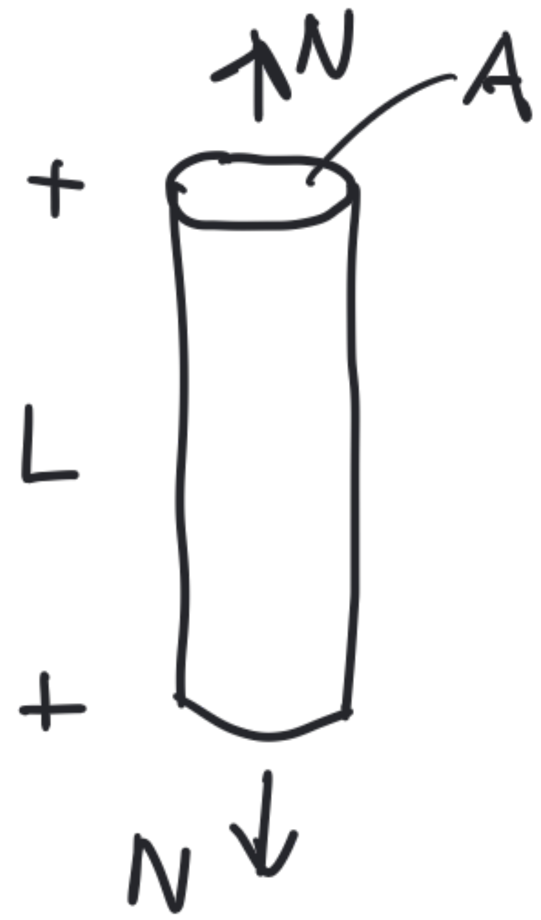
$$= \frac{1}{2} E \varepsilon_z^2 \, dV = \frac{1}{2} \frac{\sigma_z^2}{E} dV$$



$$U = \int_V \frac{\sigma^2}{2E} dV$$



$$U = \frac{\sigma^2}{2E} A \cdot L = \left(\frac{N}{A}\right)^2 \frac{AL}{2E}$$

$$= \frac{N^2}{2EA} L$$

Se N non è costante

$$U = \int_0^L \frac{N^2}{2EA} dx$$

Nel caso della flessione, la tensione normale è:

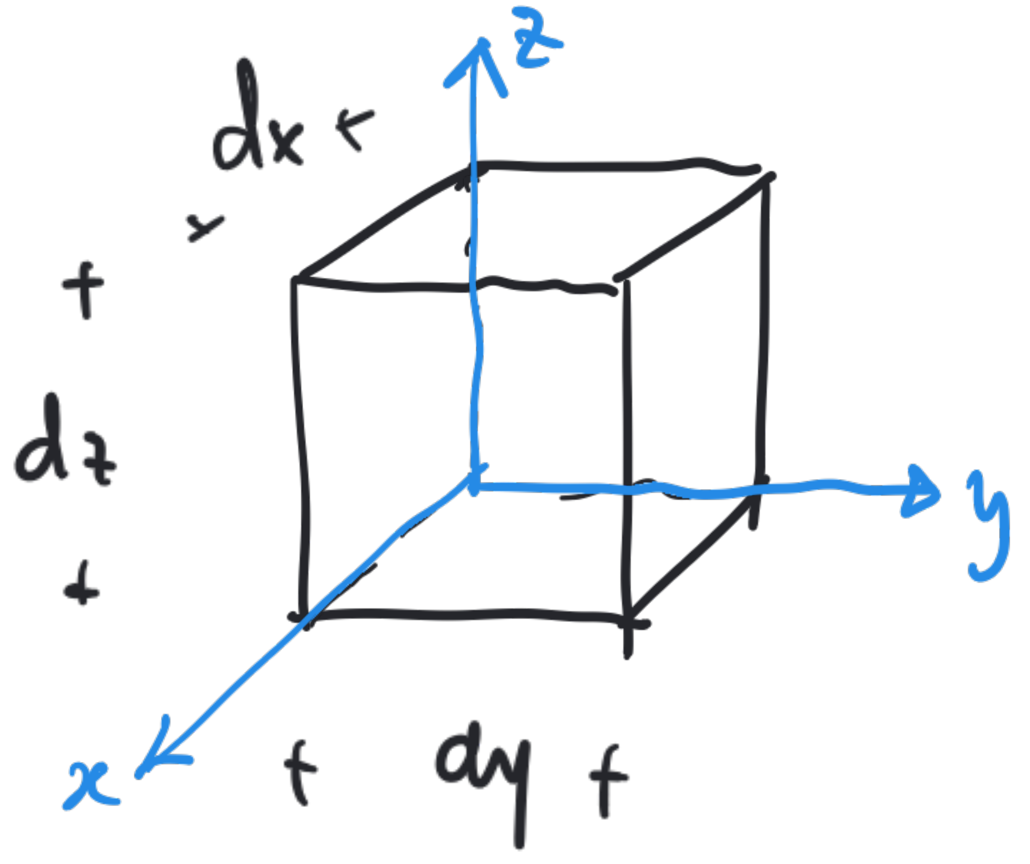
$$\sigma = \frac{M}{I} y$$

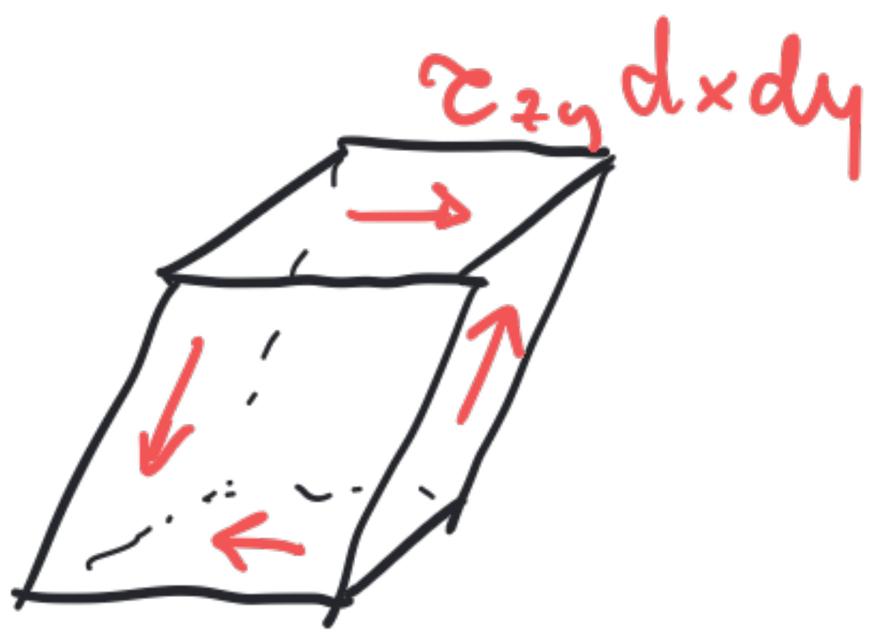
l'energia di deformazione per unità di lunghezza

è data da:

$$\int_A \frac{\sigma^2}{2E} = \int_A \frac{M^2}{I^2} y^2 \frac{1}{2E} = \frac{M^2}{2EI}$$

Sforzo di taglio





$$\gamma_{zy} = \frac{\tau_{zy}}{G}$$

$$dU_i = \int_0^{\gamma_{zy}} \bar{\tau}_{zy}\,dx\,dy\;d\bar{\gamma}_{zy}\,dz$$

$$= \left(\int_0^{\gamma_{zy}} G\,\bar{\gamma}_{zy}\,d\bar{\gamma}_{zy}\right) dx\,dy\,dz$$

$$= \frac{1}{2} G \gamma_{zy}^2\, dV = \frac{1}{2}\frac{\tau_{zy}^2}{G}\, dV = \frac{1}{2}\tau_{zy}\gamma_{zy}\, dV$$

$$U_i = \int_V \frac{\tau^2}{2G}\, dV$$

FATTORE DI TAGLIO

$$\int_A \frac{\tau^2}{2G} dA = \int_A \frac{1}{2G}\left(\frac{VQ}{It}\right)^2 dA = \frac{1}{2G}\frac{V^2}{I^2}\int_A \left(\frac{Q}{t}\right)^2 dA = \frac{V^2}{2GA}\underbrace{\frac{A}{I^2}\int_A\left(\frac{Q}{t}\right)^2 dA}_{f_s}$$

f_s fattore di taglio

$$U = \int_0^L \int_A \frac{\tau^2}{2G}\, dA\, dx = \int_0^L f_s \frac{V^2}{2GA} dx$$

<u>Es</u>: sezione rettangolare $f_s = 6/5$

TORSIONE

$$U_i = \int_V \frac{\tau^2}{2G}\,dV = \int_0^L \int_A \frac{1}{2G}\left(\frac{T\rho}{J}\right)^2 dA\,dx = \int_0^L \frac{T^2}{2GJ^{\cancel{2}}} \underbrace{\left(\int_A \rho^2\,dA\right)}_{\cancel{J}} dx$$

$$= \int_0^L \frac{T^2}{2GJ}\,dx$$

energia di deformazione
per unità di lunghezza

UTILIZZO DEI METODI ENERGETICI PER IL CALCOLO DI SPOSTAMENTI E ROTAZIONI

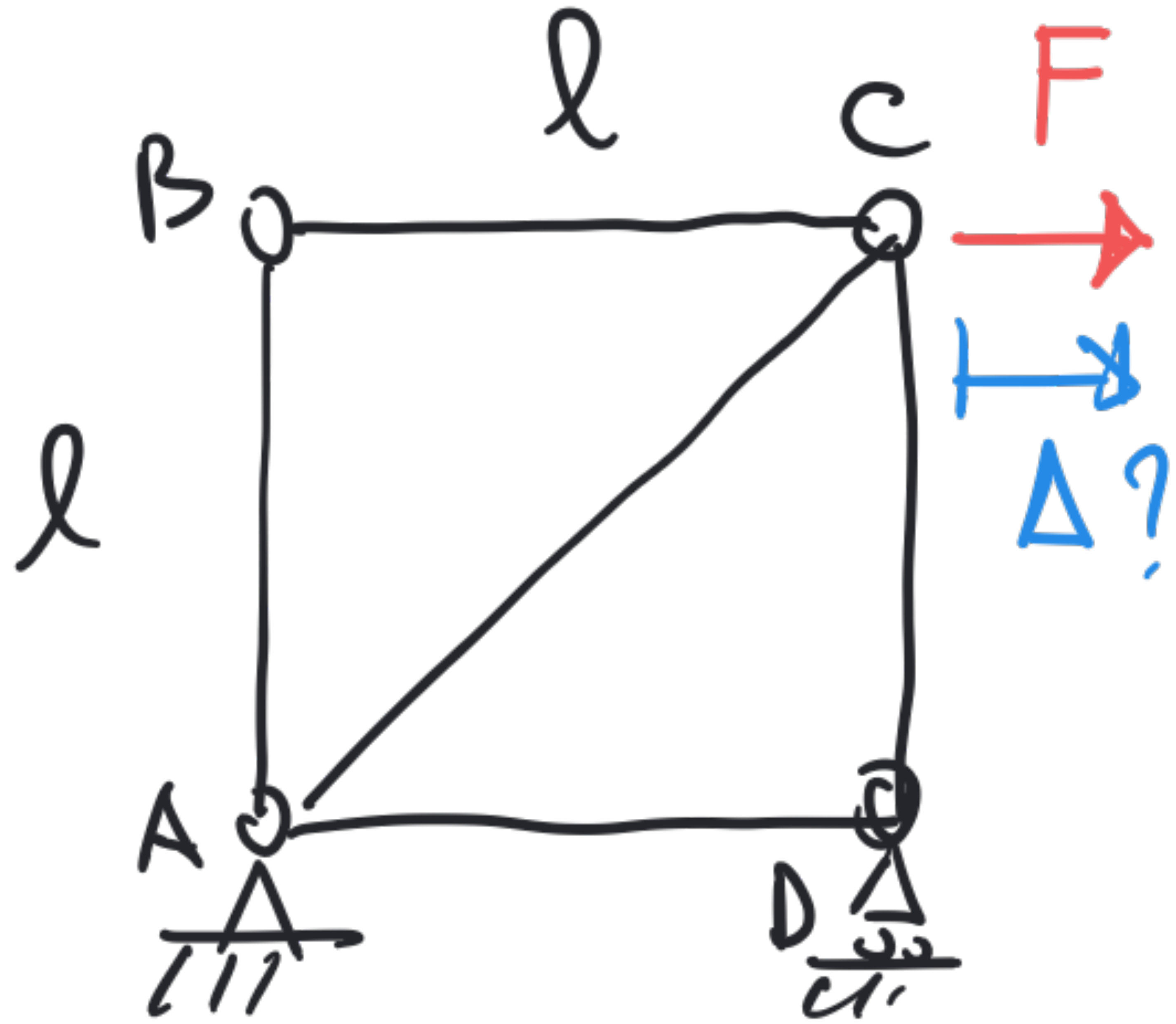
? Rotazione del punto di applicazione della coppia?

$$U_e = \frac{1}{2} C \vartheta$$

$$\parallel$$

$$U_i = l \frac{M^2}{2EI} = l \frac{C^2}{2EI} \qquad \vartheta = \frac{Cl}{EI}$$

$$\left.\begin{array}{l} EI v'' = M = C \\ v'(0) = 0 \end{array}\right\} \Rightarrow \quad v'(L) = \frac{Cl}{EI} = \vartheta$$



$$N_{AB} = N_{BC} = N_{AD} = 0$$

$$N_{AC} = \sqrt{2}\,F$$

$$N_{CD} = -F$$

$$U_e = \tfrac{1}{2} F \Delta$$

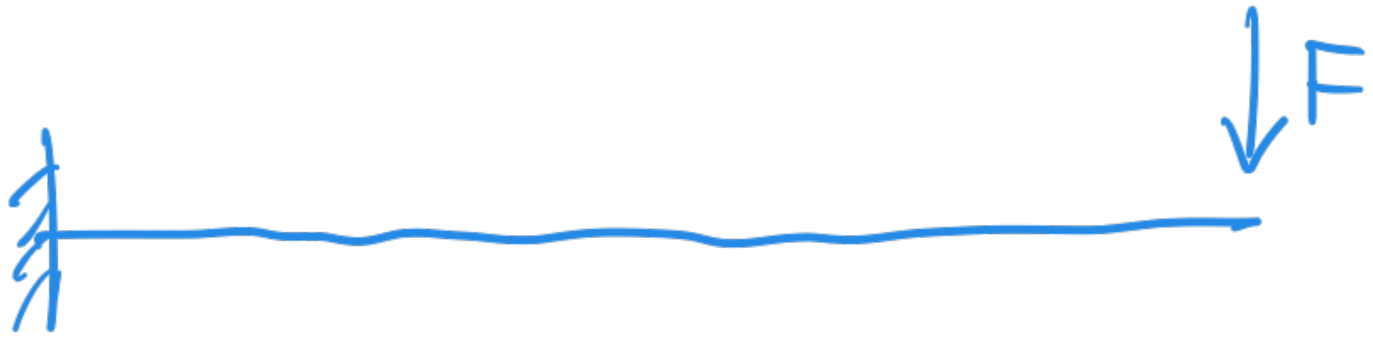
$$\parallel$$

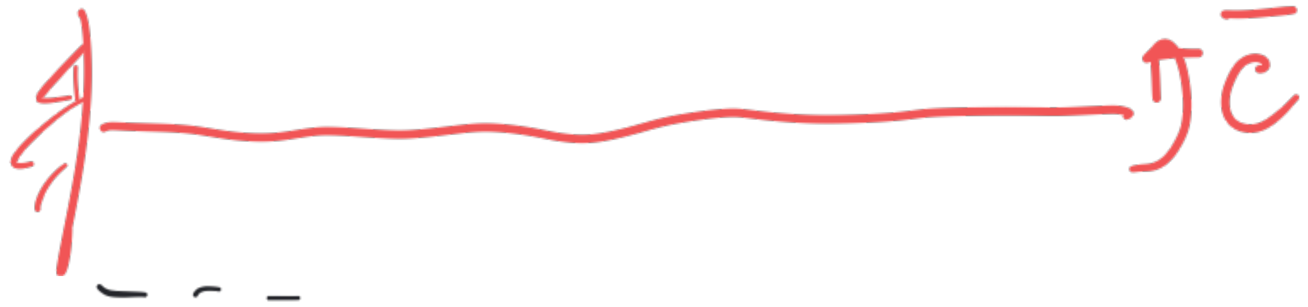
$$U_i = \sqrt{2}\,\ell \, \frac{N_{AC}^2}{2EA} + \ell \, \frac{N_{CD}^2}{2EA} = \sqrt{2}\,\ell \, \frac{2F^2}{2EA} + \ell \, \frac{F^2}{2EA}$$

$$\Delta = \left(2\sqrt{2}+1\right) \frac{F\ell}{EA}$$

ESEMPIO SUL TESTO

$$\Delta = \frac{PL^3}{3EI}$$



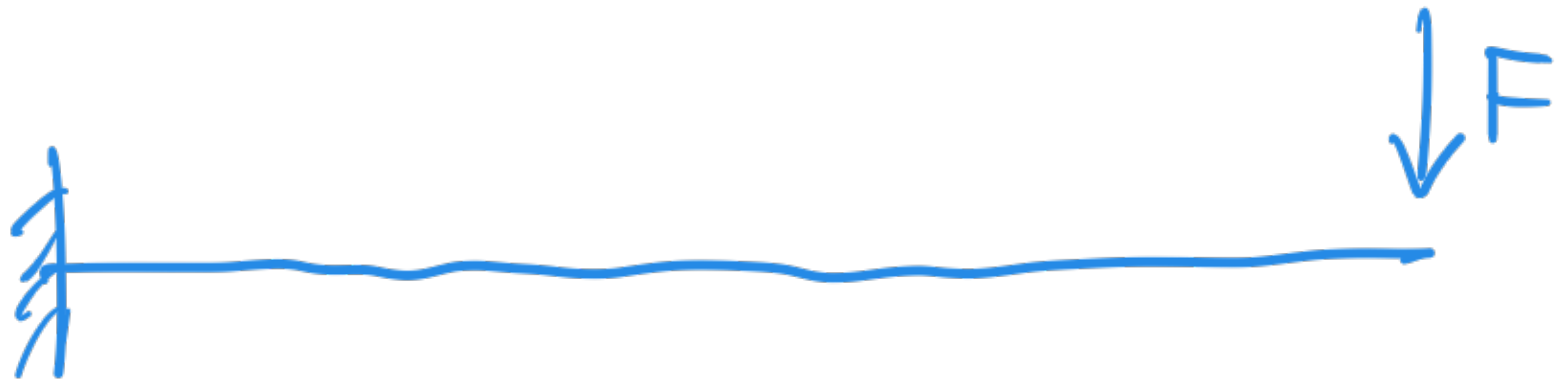


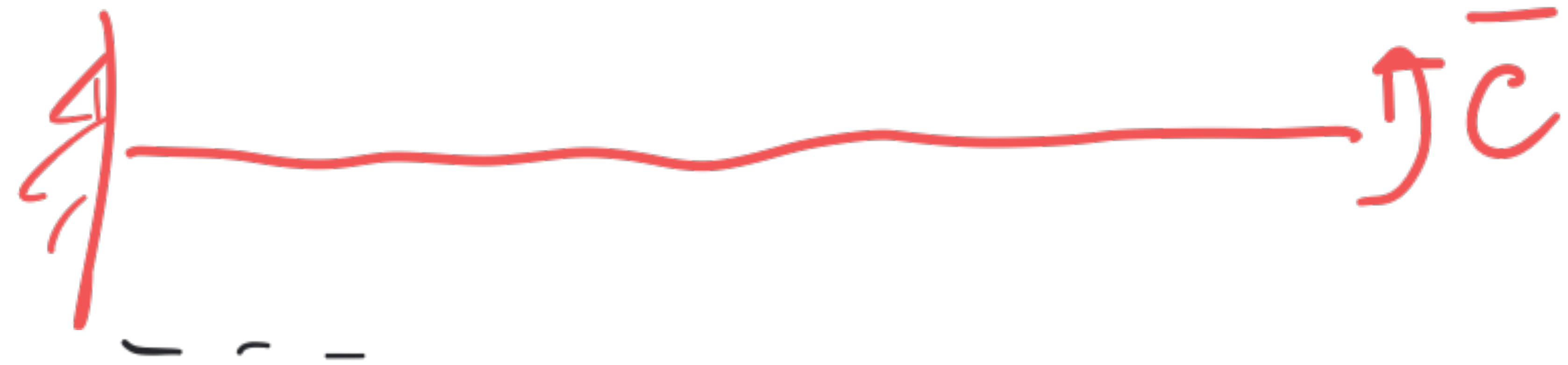
$$\vartheta(L) = ?$$

$$\mathcal{L}_{v\,est} = \bar{C}\,\vartheta(L)$$

$$\mathcal{L}_{v\,int} = \int_0^L \bar{M}\,k = \int_0^L \bar{C}\,\frac{M}{EI} = \int_0^L \bar{C}\,\frac{-F(L-x)}{EI}\,dx$$

$$= -\frac{\bar{C}F}{EI}\int_0^L (L-x)\,dx = -\bar{C}\,\frac{1}{2}\,\frac{FL^2}{EI}$$





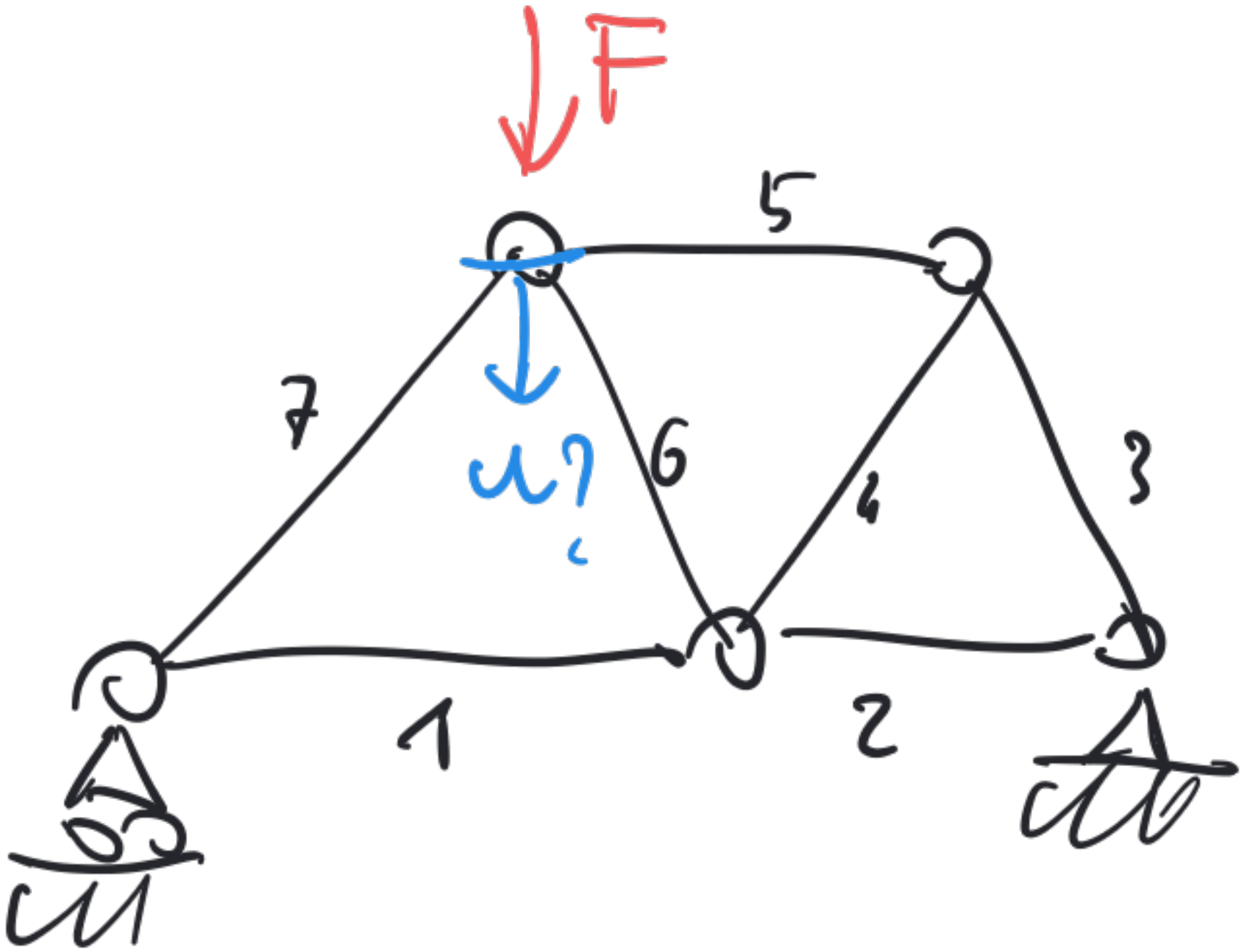
$$\vartheta(L) = \vartheta(0) + \int_0^L \frac{d\vartheta}{dx}\,dx = \int_0^L \frac{M}{EI}\,dx$$

$$= \frac{1}{EI}\int_0^L \left(-F(L-x)\right)dx = -\tfrac{1}{2}\,FL^2/EI$$

$$\mathcal{L}_{virt} = \int_0^L \bar{M}\,k = \int_0^L \bar{C}\,\frac{M}{EI} = \int_0^L \bar{C}\;\frac{-F(L-x)}{EI}\,dx$$

$$= -\frac{\bar{C}\,F}{EI}\int_0^L (L-x)\,dx = -\bar{C}\,\frac{1}{2}\,\frac{FL^2}{EI}$$

TRAVATURE RETICOLARI



$$\frac{1}{2} F u = \sum_{i=1}^{7} \int_0^{L_i} \frac{1}{2} \varepsilon_i N_i \, d\cdot$$