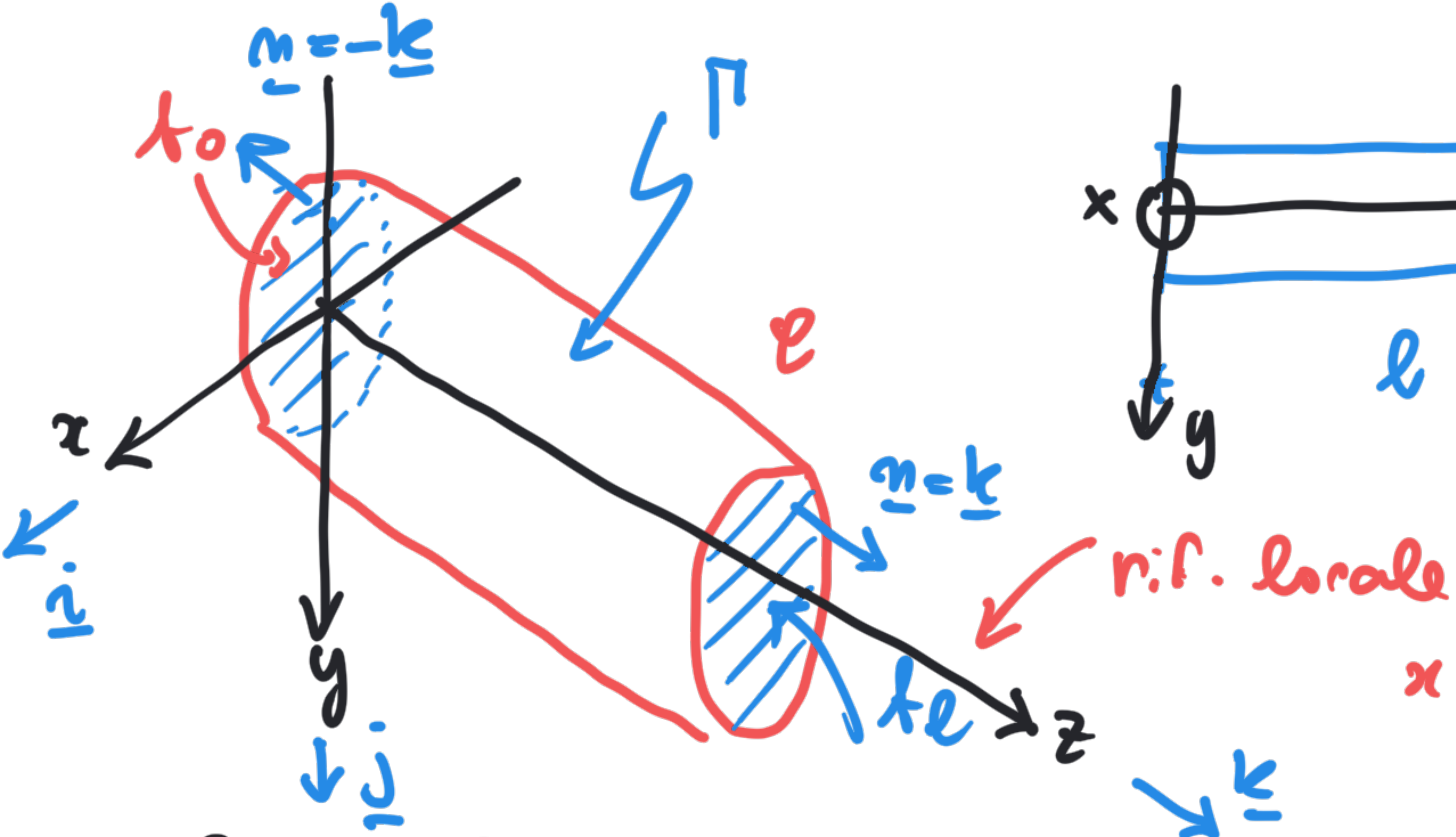
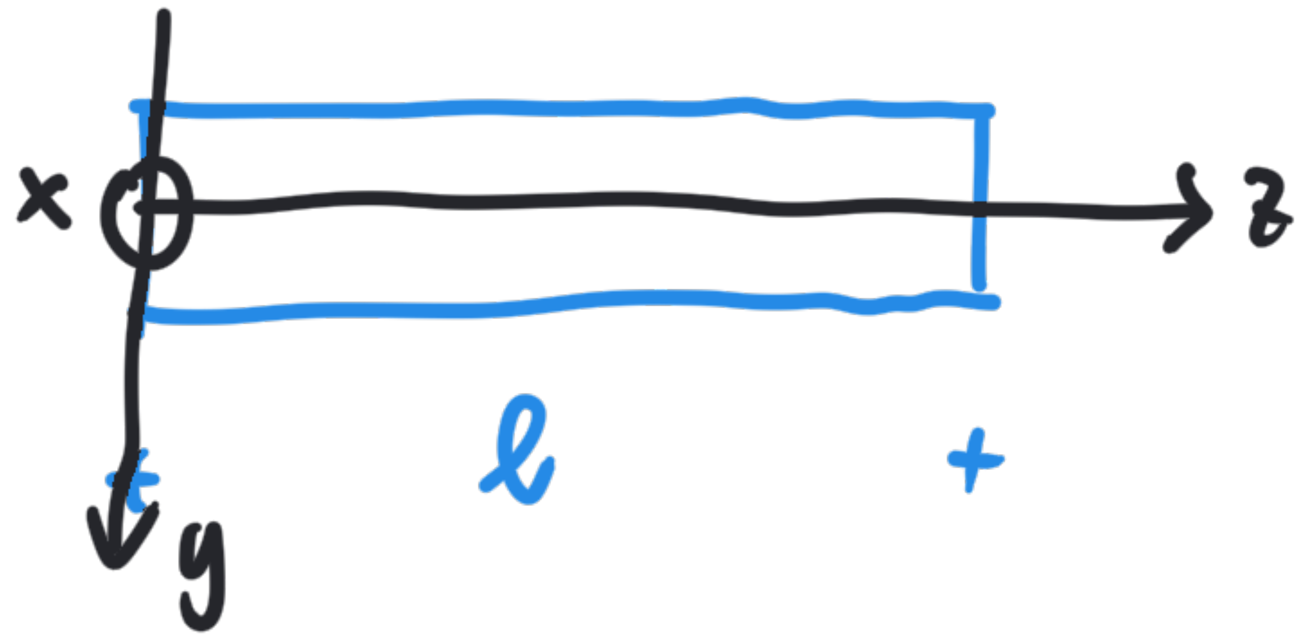
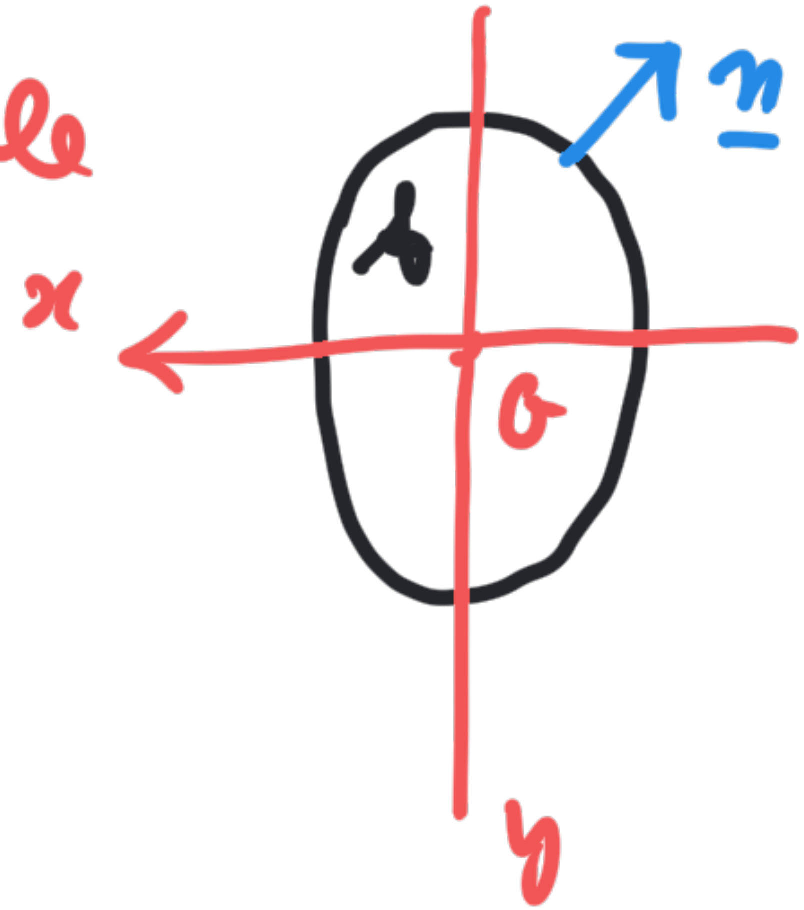
Prob. Saint Venant Cap. 12





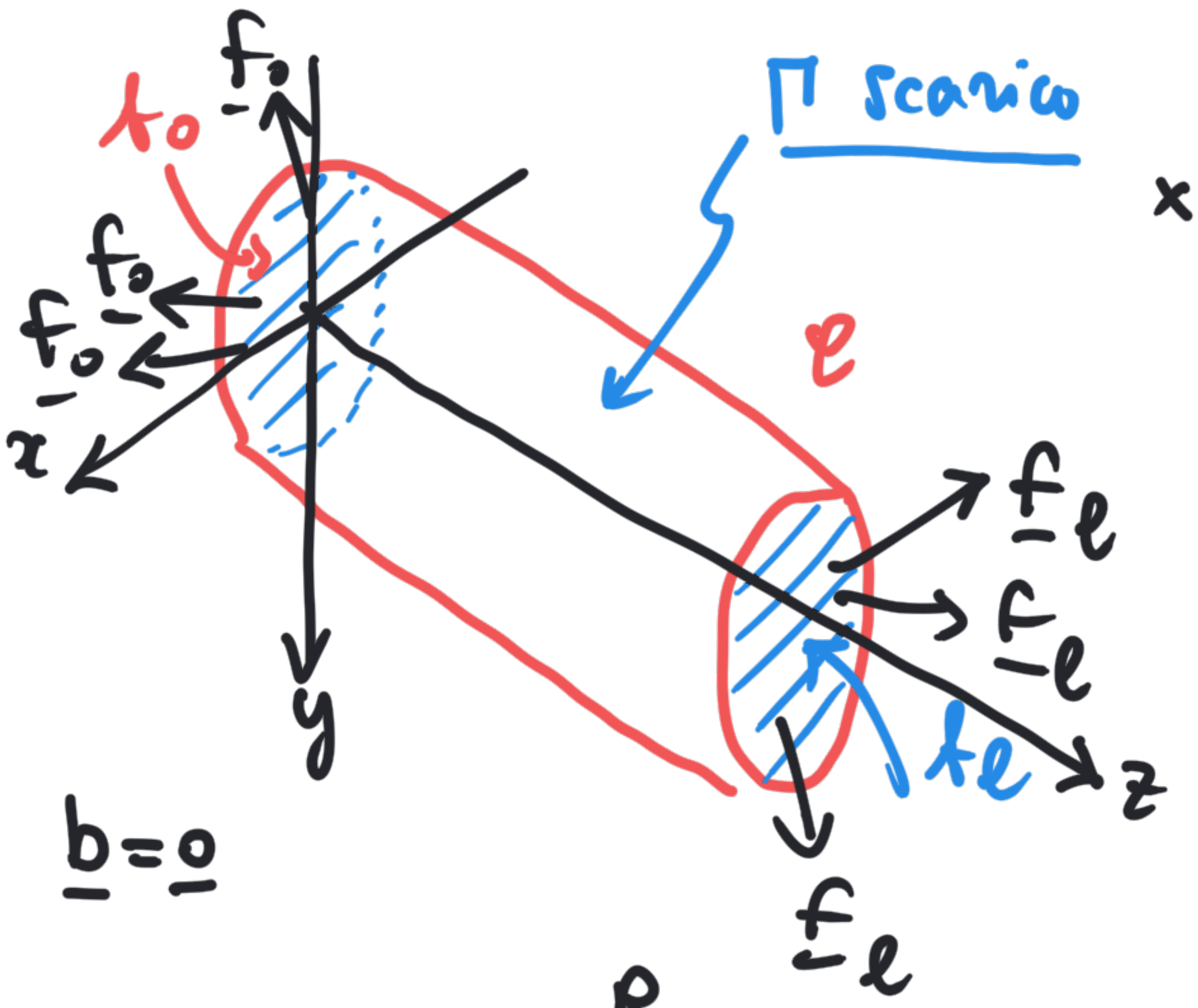


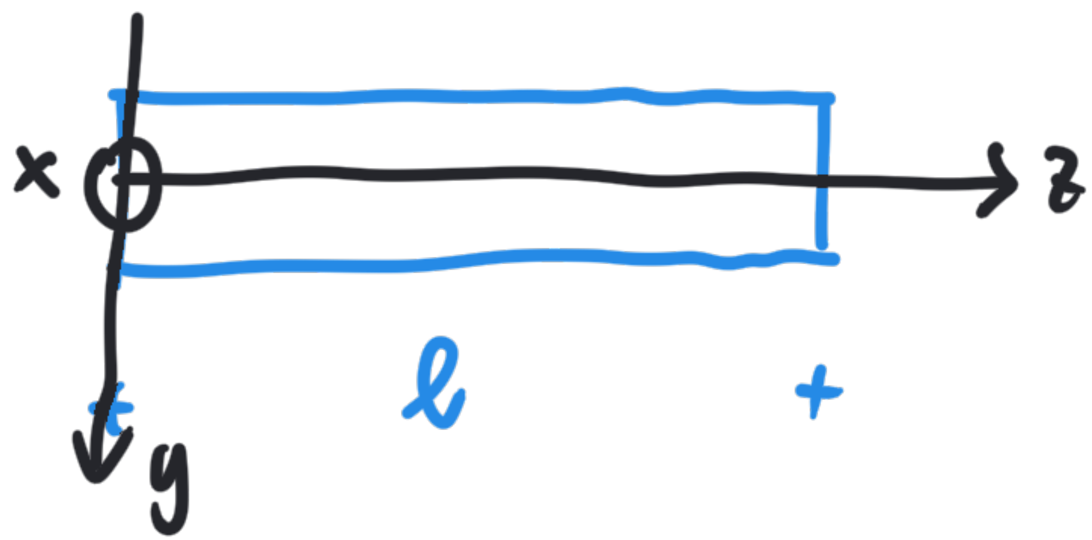
$S_x = 0 \quad S_y = 0$

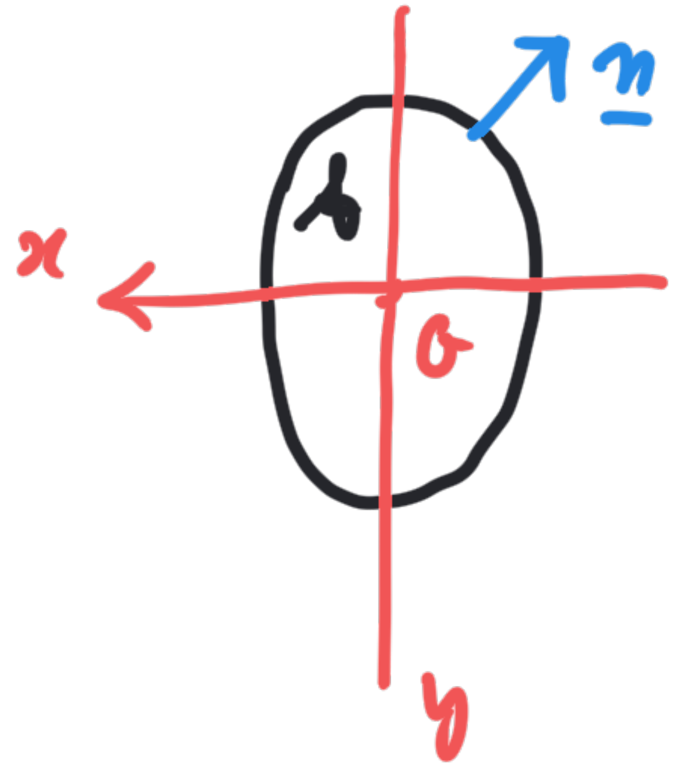
$I_{xy} = 0$

Prob. Saint Venant Cap. 12

Geometria







$\underline{b} = \underline{0}$

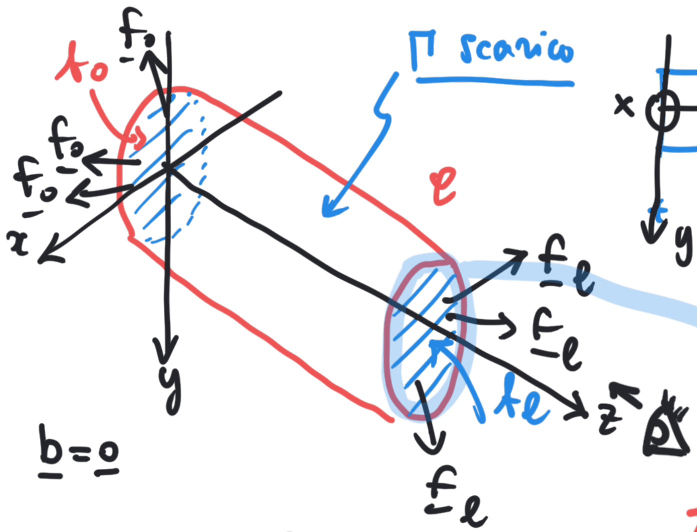
$\underline{R}_0 \longleftrightarrow \underline{R}_\ell$

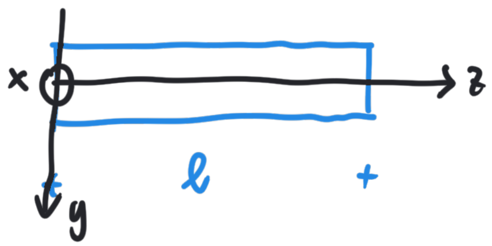
$$\underline{R}_\ell = \int_{A_\ell} \underline{f}_\ell \, dA$$

$$\underline{R}_0 = \int_{A_0} \underline{f}_0 \, dA$$

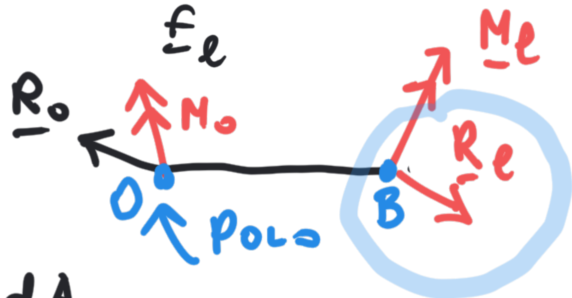
$$\underline{R}_\ell + \underline{R}_0 = \underline{0}$$

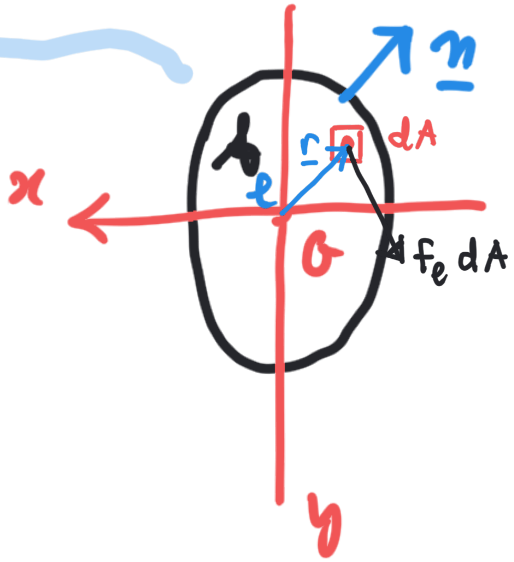
Prob. Saint Venant   Cap. 17   Geometria





$\underline{b} = \underline{0}$





$$d\underline{M}_\ell = \underline{r} \times \underline{f}_\ell \, dA$$

$$\underline{R}_\ell = \int_{A_\ell} \underline{f}_\ell \, dA$$

$$\underline{R}_o = \int_{A_o} \underline{f}_o \, dA \qquad (17.1)$$

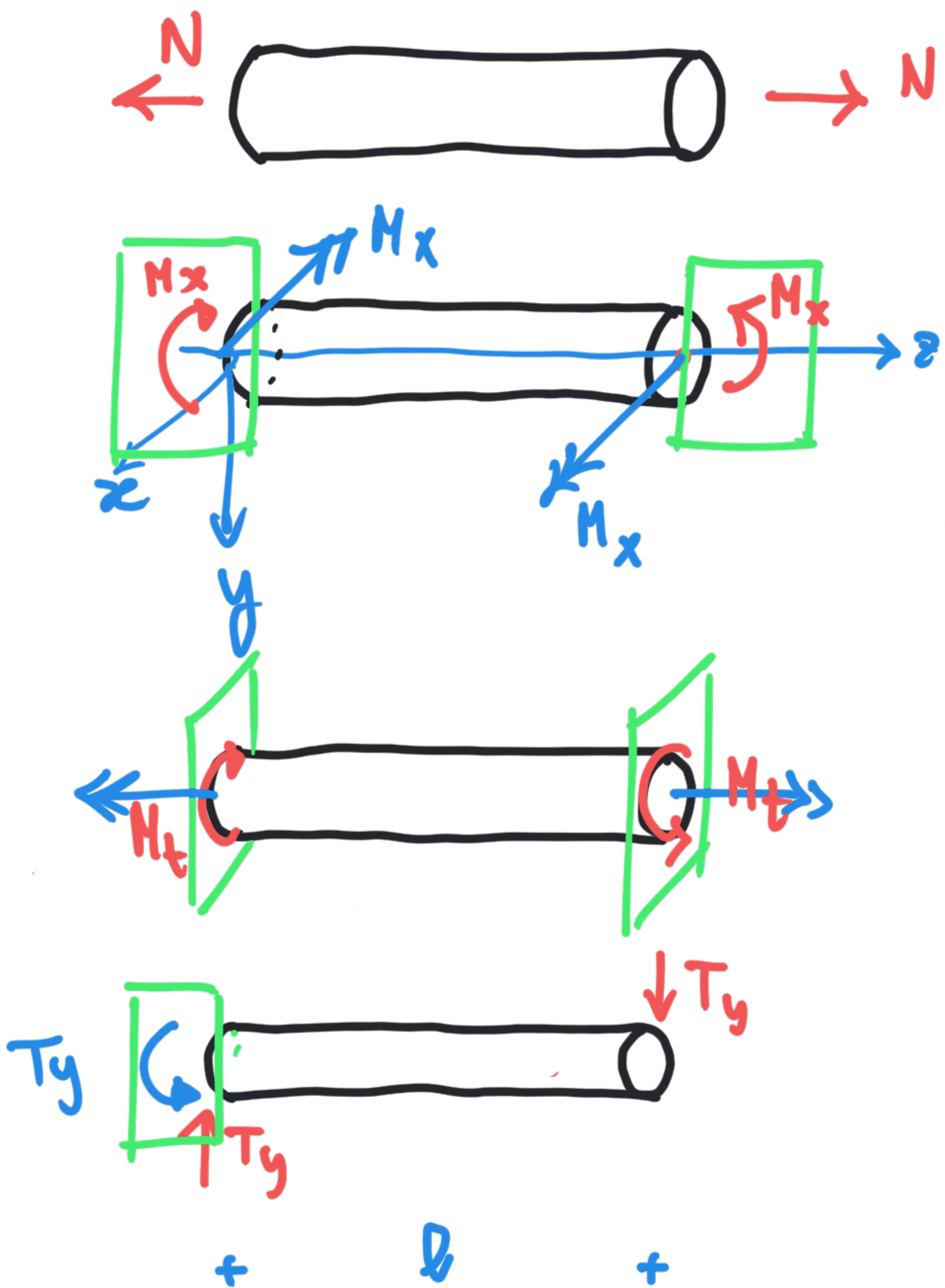
$$\underline{R}_\ell + \underline{R}_o = \underline{0}$$

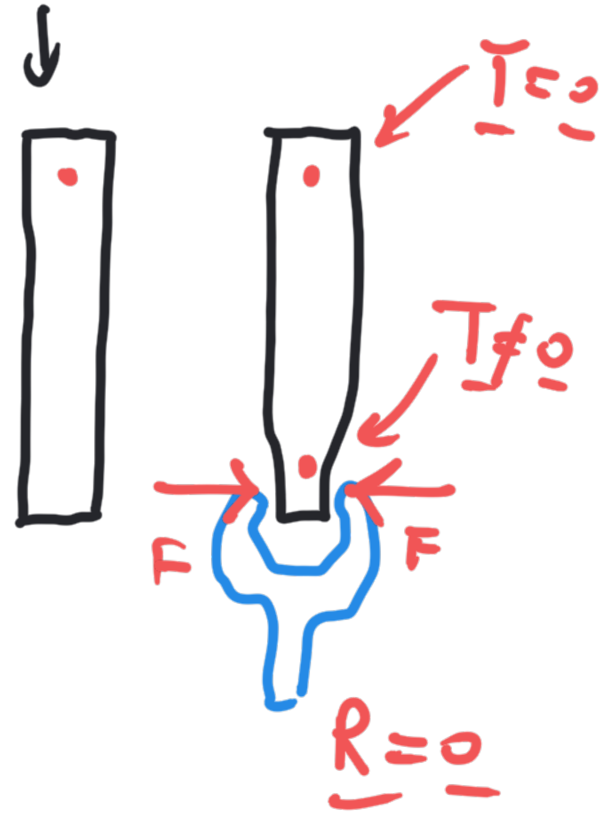
$$\underline{M}_o + \underline{M}_\ell + \underline{OB} \times \underline{R}_\ell = \underline{0}$$

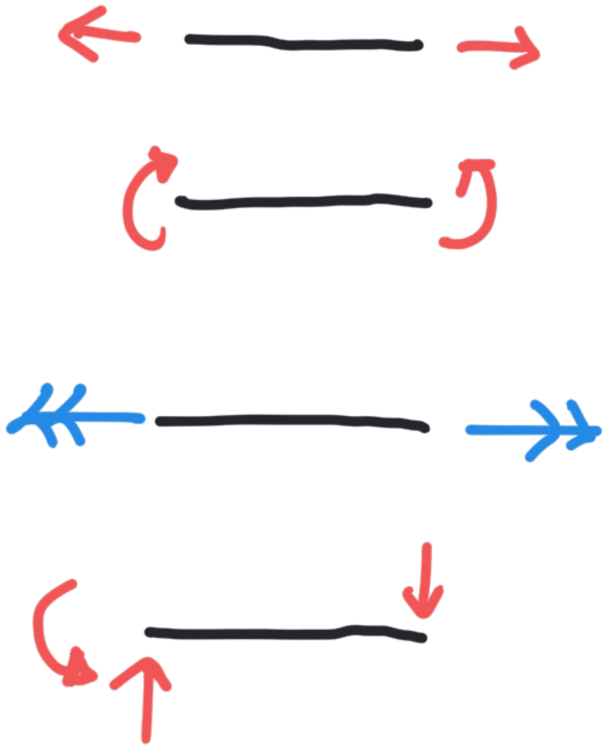
$$\underline{M}_\ell = \int_{A_\ell} \underline{r} \times \underline{f}_\ell \, dA \qquad (17.2)$$

$$\underline{M}_o = \int_{A_o} \underline{r} \times \underline{f}_o \, dA$$

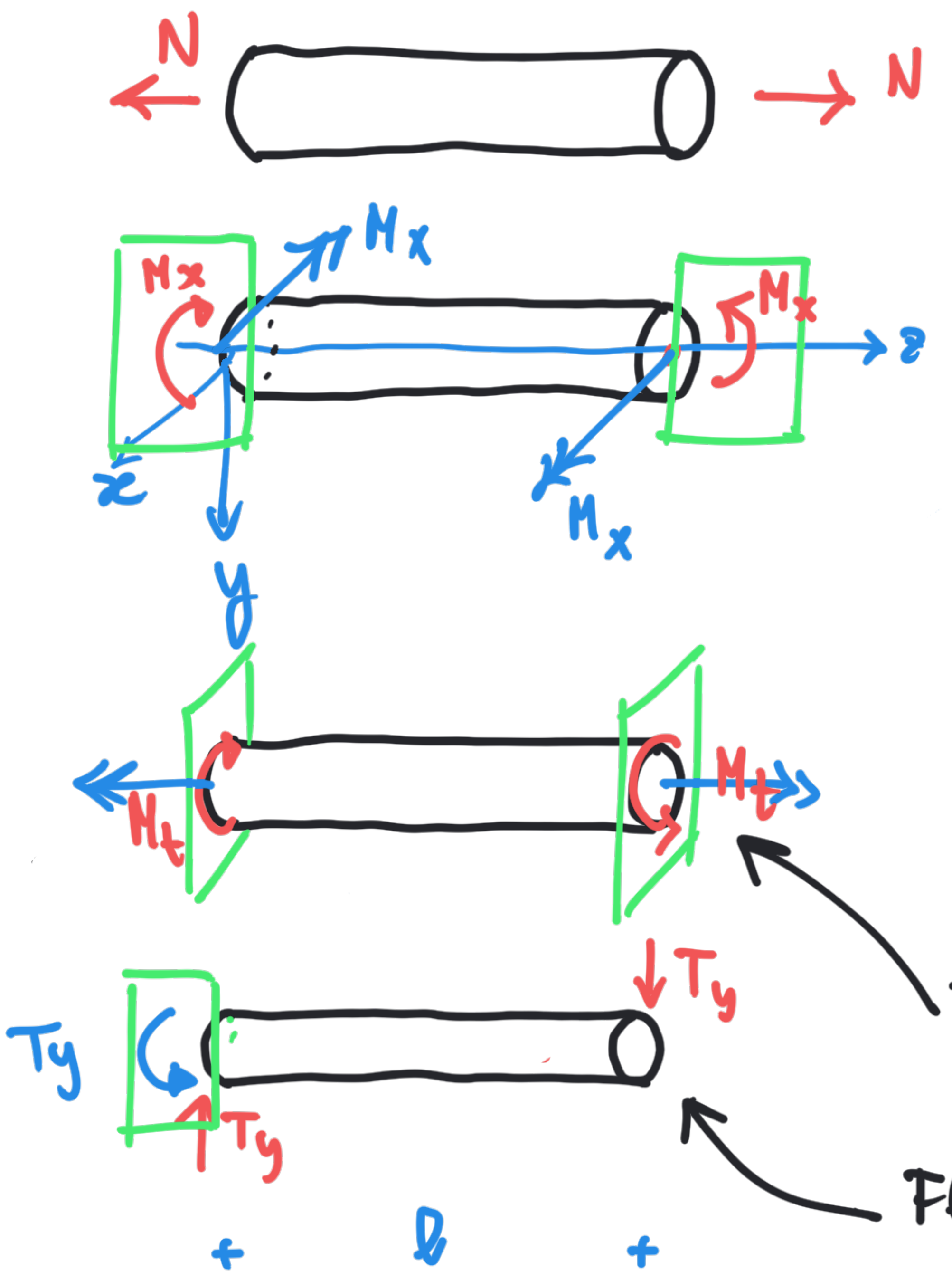
SOLLECITAZIONI SEMPLICI



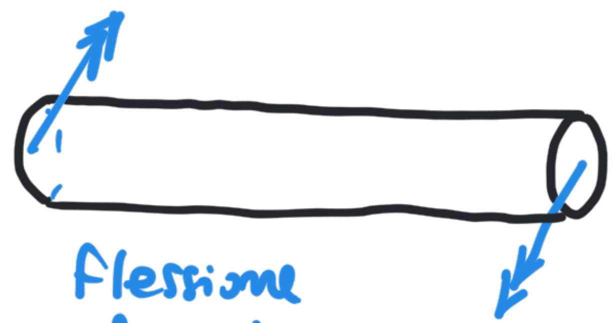




SOLLECITAZIONI SEMPLICI



forza normale
Cap. 18
flessione retta



Flessione deviata
cap. 19

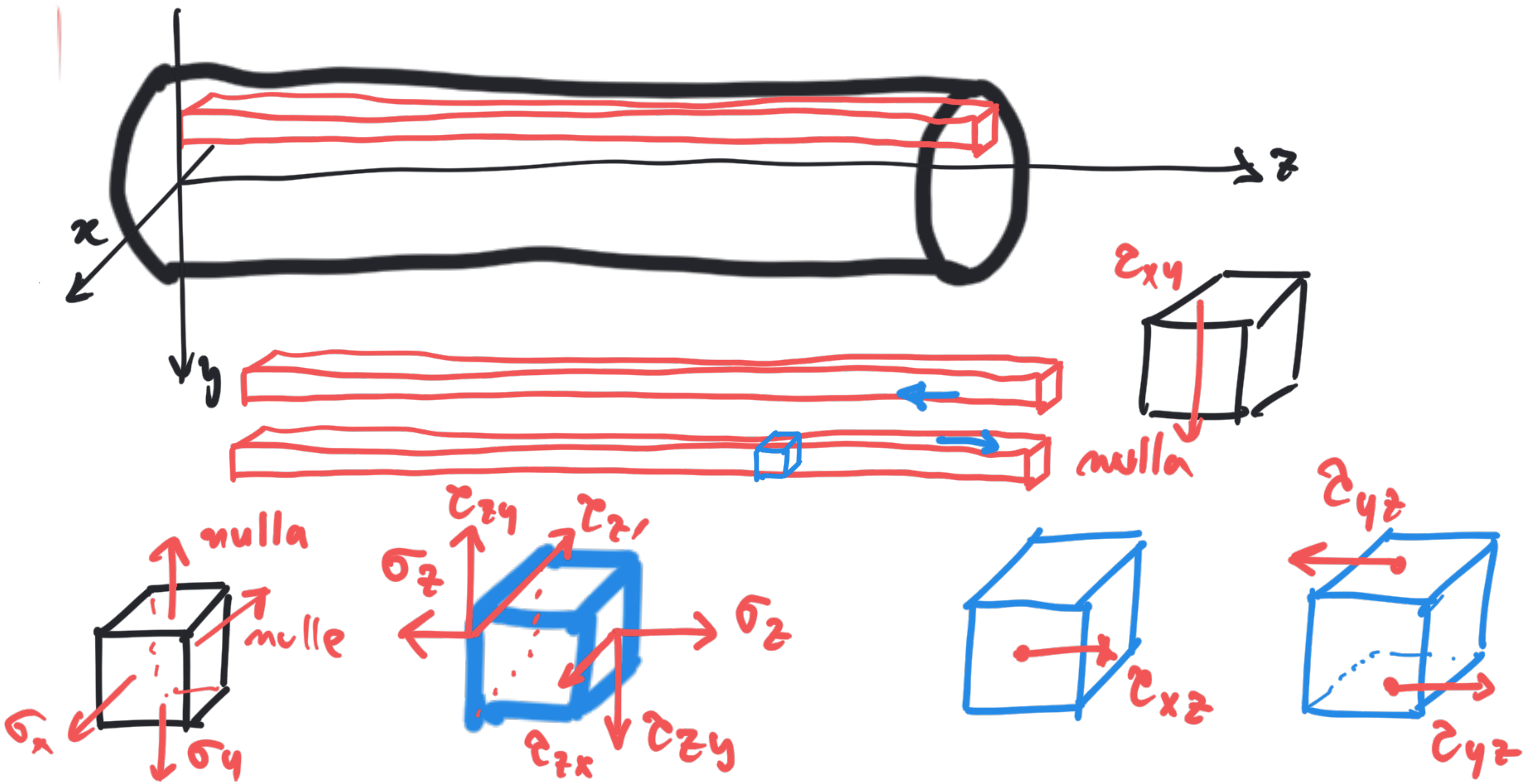
mom. flettente arbitrario

Torsione uniforme Cap. 20

Flessione e taglio Cap. 21

Metodo semi-inverso

$$\underline{T} = \begin{bmatrix} 0 & 0 & \tau_{zx} \\ 0 & 0 & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$



Metodo semi-inverso

$$\underline{T} = \begin{bmatrix} 0 & 0 & \tau_{zx} \\ 0 & 0 & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

eq. indef. di equil.

$$\cancel{\frac{\partial \sigma_x}{\partial x}} + \cancel{\frac{\partial \tau_{yx}}{\partial y}} + \frac{\partial \tau_{zx}}{\partial z} + \cancel{b_x} = 0 \qquad (14.16)_1$$

$$\frac{\partial \tau_{zx}}{\partial z} = 0 \qquad \tau_{zx}(x, y)$$

$$\frac{\partial \tau_{zy}}{\partial z} = 0 \qquad \tau_{zy}(x, y)$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0$$

$$\underline{\tau} = \tau_{zx}\,\underline{i} + \tau_{zy}\,\underline{j}$$
$$= \tau_{xz}\,\underline{i} + \tau_{xz}\,\underline{j}$$

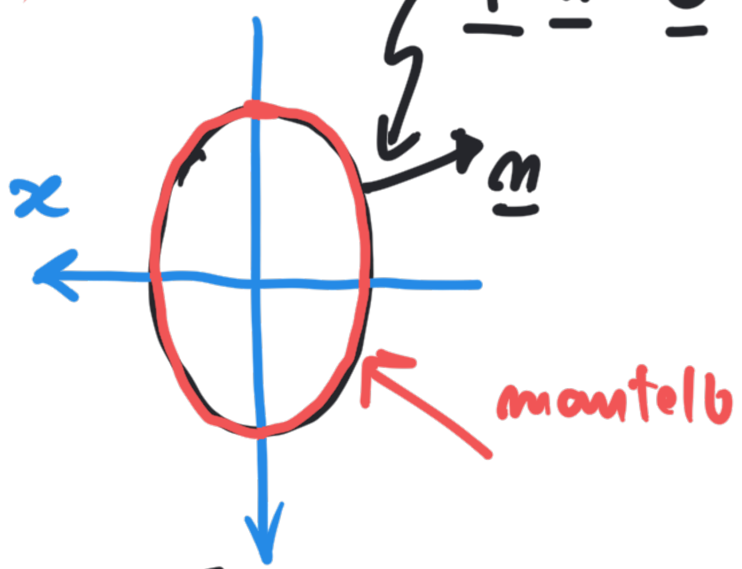
$$\boxed{\mathrm{div}\,\underline{\tau} = -\frac{\partial \sigma_z}{\partial z}}$$

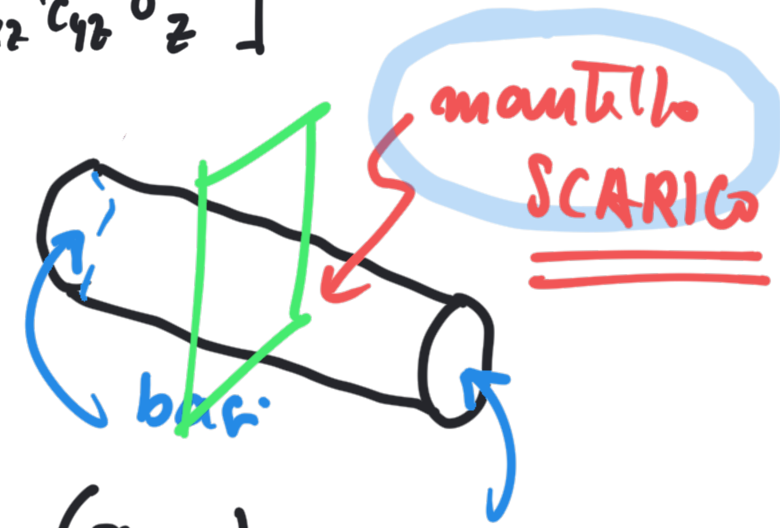
Metodo semi-inverso

$$\underline{\underline{T}} = \begin{bmatrix} 0 & 0 & \tau_{zx} \\ 0 & 0 & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

Condizioni al contorno

$\underline{\underline{T}}\,\underline{n} = \underline{0}$





$\underline{n} = \begin{bmatrix} \\ \\ 0 \end{bmatrix}$

$\tau_{zx}(x,y)$

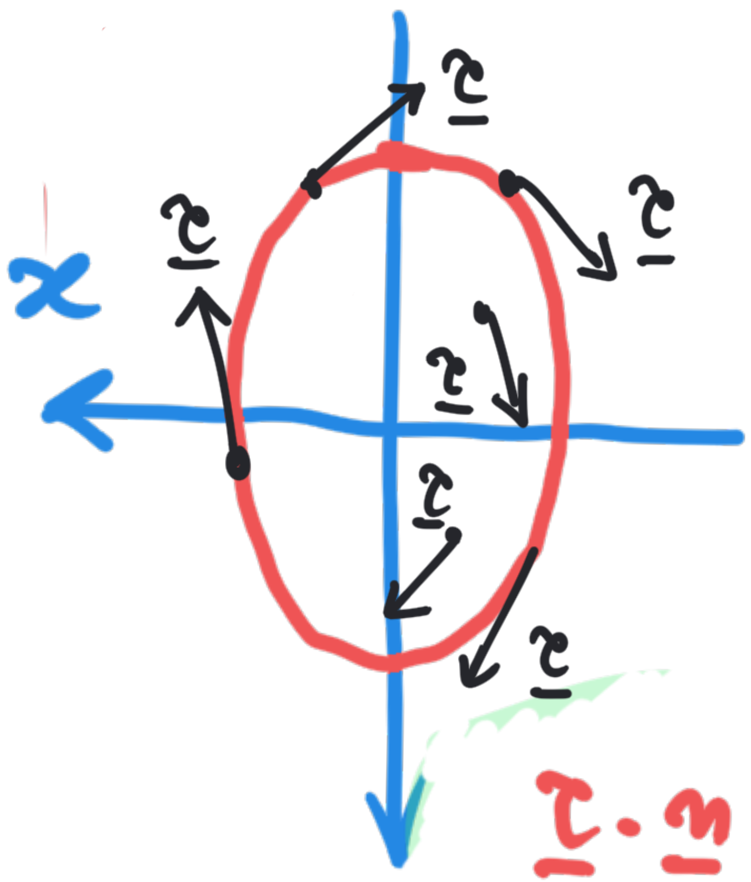
$\tau_{zy}(x,y)$

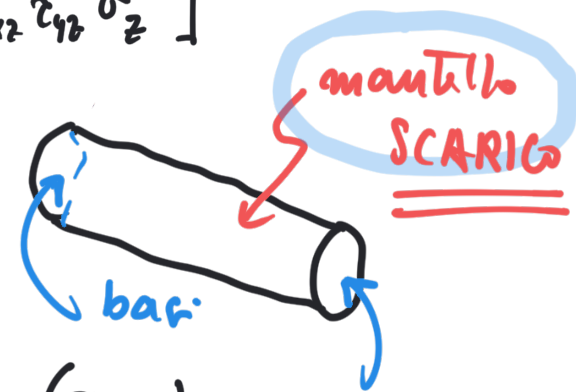
$$\underline{\tau} = \tau_{zx}\,\underline{i} + \tau_{zy}\,\underline{j}$$
$$= \tau_{xz}\,\underline{i} + \tau_{xz}\,\underline{j}$$

Metodo semi-inverso

Condizioni al contorno

$$\underline{T} = \begin{bmatrix} 0 & 0 & \tau_{zx} \\ 0 & 0 & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$





$\tau_{zx}(x,y)$

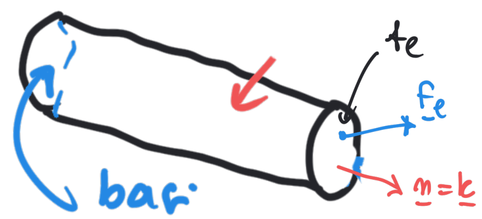
$\tau_{zy}(x,y)$

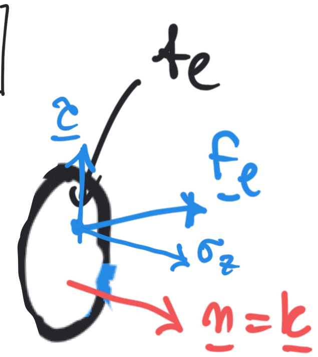
$$\underline{\tau} = \tau_{zx}\,\underline{i} + \tau_{zy}\,\underline{j}$$
$$= \tau_{xz}\,\underline{i} + \tau_{xz}\,\underline{j}$$

$$\underline{\tau} \cdot \underline{n} = 0$$

Le condizioni al contorno sul mantello si traducono nella condizione che $\underline{\tau}$ sia tangente al bordo della sezione condizioni al contorno

$$\underline{T} = \begin{bmatrix} 0 & 0 & \tau_{zx} \\ 0 & 0 & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$





su te $\quad \underline{T}\,\underline{k} = \underline{f_e}$

$$\underline{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \boxed{\underline{f_e} = \begin{bmatrix} 0 & 0 & \tau_{zx} \\ 0 & 0 & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \tau_{zx} \\ \tau_{zy} \\ \sigma_z \end{bmatrix} = \begin{bmatrix} \tau_{xz} \\ \tau_{xy} \\ \sigma_z \end{bmatrix}}$$

$\underline{\tau}$

$\sigma_z = f_{e_z} = \underline{f_e} \cdot \underline{k}$

$\underline{\tau}$ = componente di $\underline{f_e}$ tangente a te

Legame costitutivo

$$\underline{T} = \begin{bmatrix} 0 & 0 & \tau_{zx} \\ 0 & 0 & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

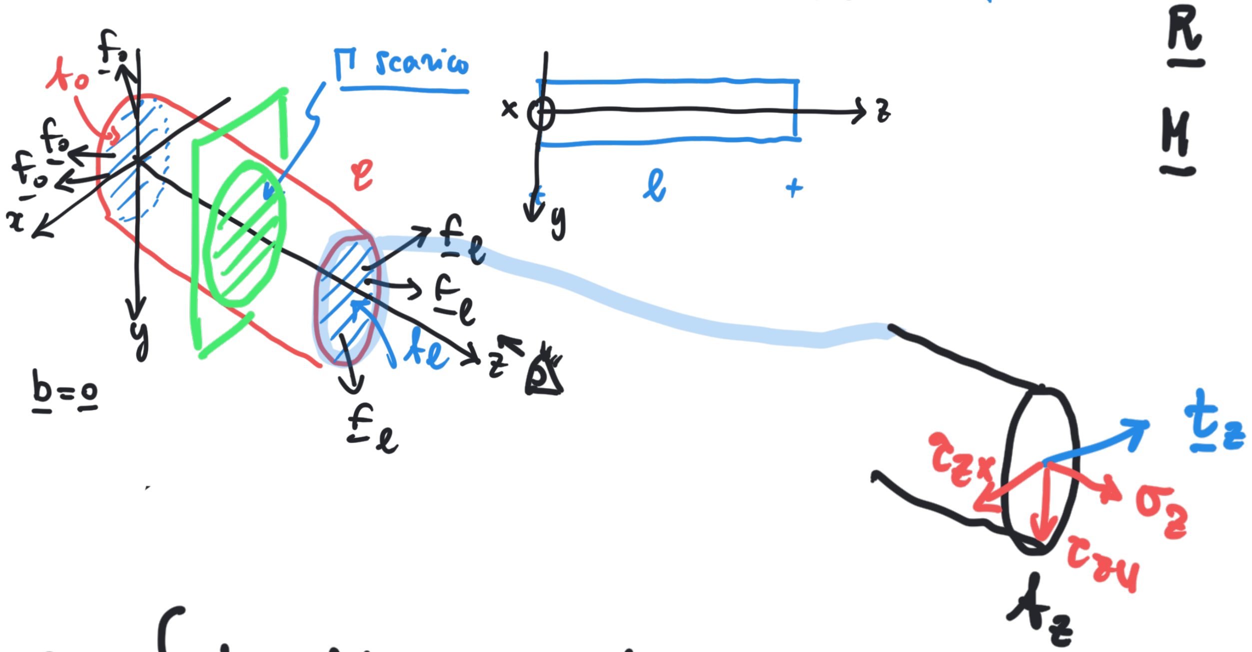
$$\varepsilon_z = \frac{\sigma_z}{E} \qquad \varepsilon_x = \varepsilon_y = -\nu \frac{\sigma_z}{E}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G}$$

Prob. Saint Venant Cap. 12 Geometria

$\underline{R}$

$\underline{M}$



$\underline{b} = \underline{0}$

$$\underline{R} = \int_{A_z} \underline{t}_z \, dA \Leftarrow \underline{t}_z = \tau_{zx}\underline{i} + \tau_{zy}\underline{j} + \sigma_z \underline{k}$$

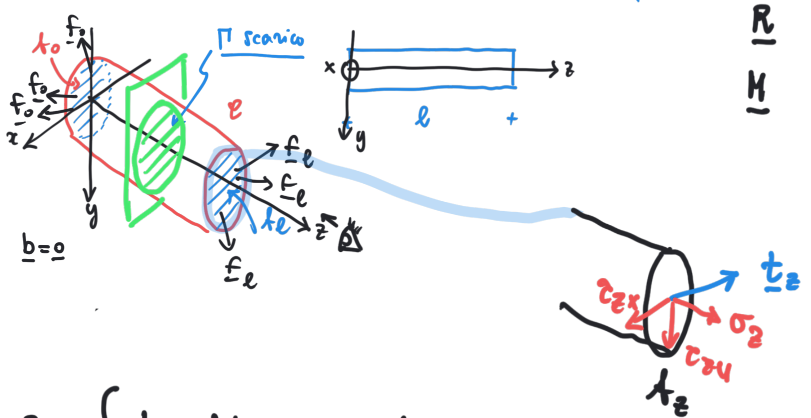
$$\underline{M} = \int_{A_z} \underline{r} \times \underline{t}_z \, dA$$

$$\underline{R} = \left(\int_{A_z} \tau_{zx} dA\right)\underline{i} + \left(\int_{A_z} \tau_{zy} dA\right)\underline{j} + \left(\int_{A_z} \sigma_z dA\right)\underline{k}$$

$$\underline{R} = T_x \underline{i} + T_y \underline{j} + N \underline{k}$$

$$T_x = \int_{A_z} \tau_{zx} dA \quad T_y = \int_{A_y} \tau_{zy} dA \quad N = \int_{A_z} \sigma_z dA$$

Prob. Saint Venant   Cap. 12   Geometria



$b = 0$

$\underline{R}$

$\underline{M}$

$\underline{i} \times \underline{j} = \underline{k}$ $\underline{j} \times \underline{i} = -\underline{k}$

$\underline{j} \times \underline{k} = \underline{i}$

$\underline{k} \times \underline{i} = \underline{j}$ $\underline{i} \times \underline{k} = -\underline{j}$

$$\underline{R} = \int_{A_z} \underline{t}_z \, dA \qquad \underline{t}_z = \tau_{zx}\underline{i} + \tau_{zy}\underline{j} + \sigma_z \underline{k}$$

$$\underline{M} = \int_{A_z} \underline{r} \times \underline{t}_z \, dA$$

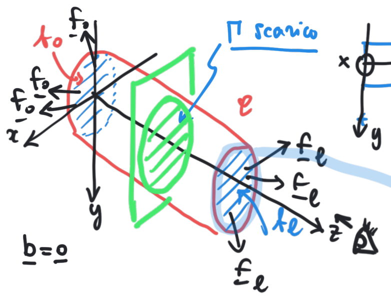
$$\underline{M} = \int_{A_z} (x\underline{i} + y\underline{j}) \times (\tau_{zx}\underline{i} + \tau_{zy}\underline{j} + \sigma_z \underline{k})$$

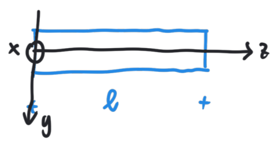
$$= \int_{A_z} x\tau_{zx} \underbrace{\underline{i}\times\underline{i}}_{0} + x\tau_{zy} \underbrace{\underline{i}\times\underline{j}}_{\underline{k}} + x\sigma_z \underbrace{\underline{i}\times\underline{k}}_{-\underline{j}} + y\tau_{zx} \underbrace{\underline{j}\times\underline{i}}_{-\underline{k}} + y\tau_{zy} \underbrace{\underline{j}\times\underline{j}}_{0} + y\sigma_z \underbrace{\underline{j}\times\underline{k}}_{\underline{i}}$$

$$= \left[\int_A (x\tau_{zy} - y\tau_{zx})\,dA\right]\underline{k} + \left(\int_A y\sigma_z\,dA\right)\underline{i} + \int_A x\sigma_z(-\underline{j})$$

$$T_x = \int_{A_z} \tau_{zx}\,dA \qquad T_y = \int_{A_y} \tau_{zy}\,dA \qquad N = \int_{A_z} \sigma_z\,dA$$

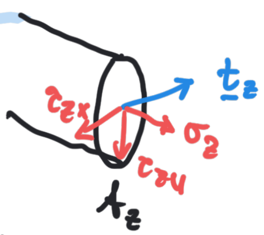
Prob. Saint Venant   Cap. 12   Geometria



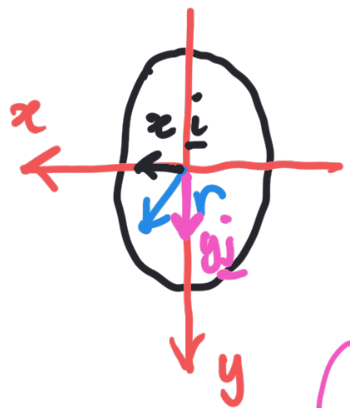


$\underline{R}$

$\underline{M}$



$b = 0$



$\underline{i} \times \underline{j} = \underline{k} \qquad \underline{j} \times \underline{i} = -\underline{k}$

$\underline{j} \times \underline{k} = \underline{i}$

$\underline{k} \times \underline{i} = \underline{j} \qquad \underline{i} \times \underline{k} = -\underline{j}$

$$\underline{R} = \int_{A_z} \underline{t}_z \, dA \qquad \underline{t}_z = \tau_{zx}\underline{i} + \tau_{zy}\underline{j} + \sigma_z \underline{k}$$

$$\underline{M} = \int_{A_z} \underline{r} \times \underline{t}_z \, dA$$

$$T_x = \int_{A_z} \tau_{zx} dA \qquad T_y = \int_{A_y} \tau_{zy} dA \qquad N = \int_{A_z} \sigma_z dA$$

(17.12)

$$\underline{M} = \underbrace{\left[\int_A (x\tau_{zy} - y\tau_{zx}) dA\right]}_{= M_z} \underline{k} + \underbrace{\left(\int_A y\sigma_z \, dA\right)}_{= M_y} \underline{i} + \int_A x\sigma_z (-\underline{j})$$

$$M_z = -\int_A x\sigma_z$$