8–1.

A spherical gas tank has an inner radius of $r = 1.5$ m. If it is subjected to an internal pressure of $p = 300$ kPa, determine its required thickness if the maximum normal stress is not to exceed 12 MPa.

SOLUTION

$$\sigma_{\text{allow}} = \frac{p\,r}{2\,t}; \qquad 12(10^6) = \frac{300(10^3)(1.5)}{2\,t}$$

$t = 0.0188 \text{ m} = 18.8 \text{ mm}$ **Ans.**

Ans:
$t = 18.8$ mm

8–2. A pressurized spherical tank is to be made of 12-mm-thick steel. If it is subjected to an internal pressure of $p = 1.4$ MPa, determine its outer radius if the maximum normal stress is not to exceed 105 MPa.

SOLUTION

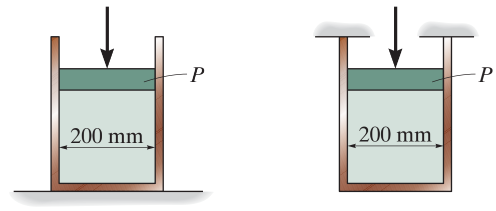
$$\sigma_{\text{allow}} = \frac{p\,r}{2\,t}; \qquad 105(10^6) = \frac{[1.4(10^6)]r_i}{2(0.012)}$$

$$r_i = 1.80 \text{ m}$$

$$r_o = 1.80 + 0.012 = 1.812 \text{ m}$$ **Ans.**

Ans.
$r_o = 1.812$ m

8–3. The thin-walled cylinder can be supported in one of two ways as shown. Determine the state of stress in the wall of the cylinder for both cases if the piston P causes the internal pressure to be 0.5 MPa. The wall has a thickness of 6 mm and the inner diameter of the cylinder is 200 mm.



SOLUTION

Case (a):

$$\sigma_1 = \frac{pr}{t}; \qquad \sigma_1 = \frac{0.5(100)}{6} = 8.33 \text{ MPa}$$ **Ans.**

$$\sigma_2 = 0$$ **Ans.**

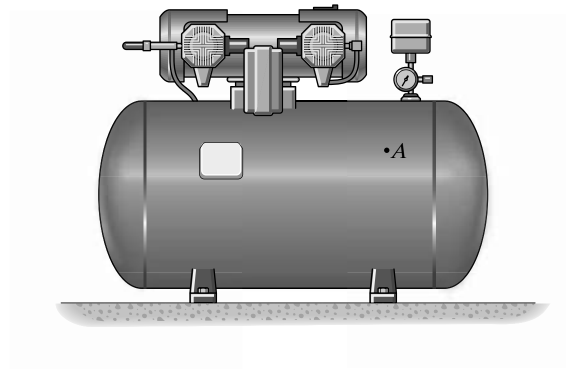
Case (b):

$$\sigma_1 = \frac{pr}{t}; \qquad \sigma_1 = \frac{0.5(100)}{6} = 8.33 \text{ MPa}$$ **Ans.**

$$\sigma_2 = \frac{pr}{2t}; \qquad \sigma_2 = \frac{0.5(100)}{2(6)} = 4.17 \text{ MPa}$$ **Ans.**

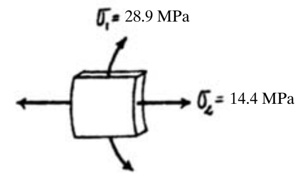
Ans.
Case (a): $\sigma_1 = 8.33$ MPa; $\sigma_2 = 0$
Case (b): $\sigma_1 = 8.33$ MPa; $\sigma_2 = 4.17$ MPa

***8–4.** The tank of the air compressor is subjected to an internal pressure of 0.63 MPa. If the internal diameter of the tank is 550 mm, and the wall thickness is 6 mm, determine the stress components acting at point *A*. Draw a volume element of the material at this point, and show the results on the element.



SOLUTION

Hoop Stress for Cylindrical Vessels: Since $\frac{r}{t} = \frac{275}{6} = 45.8 > 10$, then *thin wall* analysis can be used. Applying Eq. 8–1



$$\sigma_1 = \frac{pr}{t} = \frac{0.63(275)}{6} = 28.875 \text{ MPa} = 28.9 \text{ MPa} \qquad \textbf{Ans.}$$

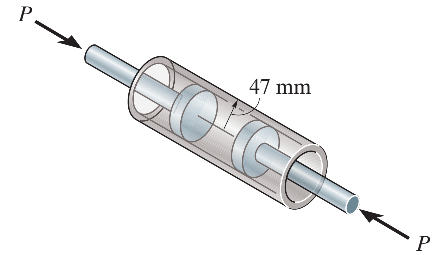
Longitudinal Stress for Cylindrical Vessels: Applying Eq. 8–2

$$\sigma_2 = \frac{pr}{2t} = \frac{0.63(275)}{2(6\ )} = 14.4375 \text{ MPa} = 14.4 \text{ MPa} \qquad \textbf{Ans.}$$

Ans.
$\sigma_1 = 28.9$ MPa, $\sigma_2 = 14.4$ MPa

8–5.

Air pressure in the cylinder is increased by exerting forces $P = 2$ kN on the two pistons, each having a radius of 45 mm. If the cylinder has a wall thickness of 2 mm, determine the state of stress in the wall of the cylinder.



SOLUTION

$$p = \frac{P}{A} = \frac{2(10^3)}{\pi(0.045^2)} = 314\,380.13 \text{ Pa}$$

$$\sigma_1 = \frac{p\,r}{t} = \frac{314\,380.13(0.045)}{0.002} = 7.07 \text{ MPa}$$ **Ans.**

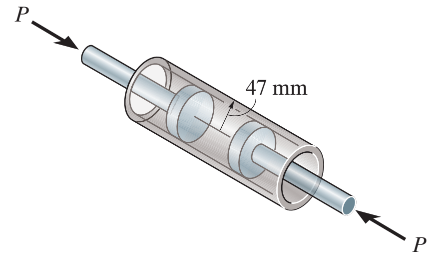
$$\sigma_2 = 0$$ **Ans.**

The pressure P is supported by the surface of the pistons in the longitudinal direction.

Ans:
$\sigma_1 = 7.07$ MPa, $\sigma_2 = 0$

8–6.

Determine the maximum force P that can be exerted on each of the two pistons so that the circumferential stress in the cylinder does not exceed 3 MPa. Each piston has a radius of 45 mm and the cylinder has a wall thickness of 2 mm.



SOLUTION

$$\sigma = \frac{p\,r}{t}; \qquad 3(10^6) = \frac{p(0.045)}{0.002}$$

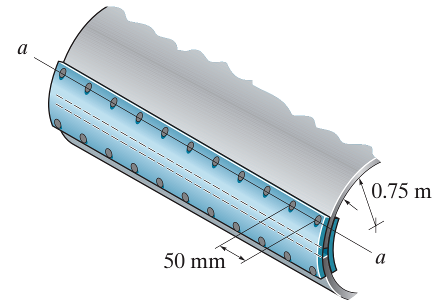
$$p = 133.3 \text{ kPa}$$

$$P = pA = 133.3\left(10^3\right)(\pi)(0.045)^2 = 848 \text{ N}$$ **Ans.**

Ans:
$P = 848$ N

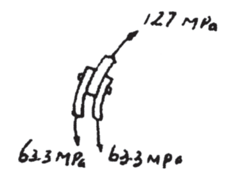
8–7.

A boiler is constructed of 8-mm-thick steel plates that are fastened together at their ends using a butt joint consisting of two 8-mm cover plates and rivets having a diameter of 10 mm and spaced 50 mm apart as shown. If the steam pressure in the boiler is 1.35 MPa, determine (a) the circumferential stress in the boiler's plate away from the seam, (b) the circumferential stress in the outer cover plate along the rivet line *a–a,* and (c) the shear stress in the rivets.



SOLUTION

a) $$\sigma_1 = \frac{pr}{t} = \frac{1.35(10^6)(0.75)}{0.008} = 126.56(10^6) = 127\text{ MPa}$$ **Ans.**



b) $$126.56\,(10^6)(0.05)(0.008) = \sigma_1{}'(2)(0.04)(0.008)$$

$$\sigma_1{}' = 79.1\,\text{MPa}$$ **Ans.**

c) From FBD(a)

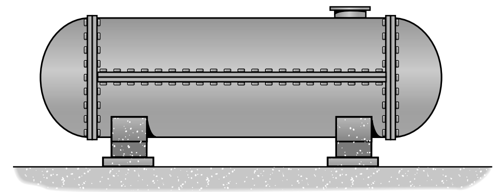
$+\uparrow\Sigma F_y = 0;\qquad F_b - 79.1(10^6)[(0.008)(0.04)] = 0$

$$F_b = 25.3\text{ kN}$$

$$(\tau_{\text{avg}})_b = \frac{F_b}{A} - \frac{25312.5}{\frac{\pi}{4}(0.01)^2} = 322\text{ MPa}$$ **Ans.**

Ans:
(a) $\sigma_1 = 127$ MPa,
(b) $\sigma_1{}' = 79.1\,$MPa,
(c) $(\tau_{avg})_b = 322$ MPa

***8–8.** The gas storage tank is fabricated by bolting together two half cylindrical thin shells and two hemispherical shells as shown. If the tank is designed to withstand a pressure of 3 MPa, determine the required minimum thickness of the cylindrical and hemispherical shells and the minimum required number of longitudinal bolts per meter length at each side of the cylindrical shell. The tank and the 25 mm diameter bolts are made from material having an allowable normal stress of 150 MPa and 250 MPa, respectively. The tank has an inner diameter of 4 m.



Normal Stress: For the cylindrical portion of the tank, the hoop stress is twice as large as the longitudinal stress.

$$\sigma_{\text{allow}} = \frac{pr}{t}; \qquad 150\left(10^6\right) = \frac{3\left(10^6\right)(2)}{t_c}$$

$$t_c = 0.04\,\text{m} = 40\,\text{mm} \qquad \textbf{Ans.}$$

For the hemispherical cap,

$$\sigma_{\text{allow}} = \frac{pr}{t}; \qquad 150\left(10^6\right) = \frac{3\left(10^6\right)(2)}{2t_s}$$

$$t_s = 0.02\,\text{m} = 20\text{ mm} \qquad \textbf{Ans.}$$

Since $\frac{r}{t} < 10$, thin-wall analysis is valid.

Referring to the free-body diagram of the per meter length of the cylindrical portion, Fig. *a*, where $P = pA = 3\left(10^6\right)[4(1)] = 12\left(10^6\right)\text{N}$, we have

$$+\uparrow \Sigma F_y = 0; \qquad 12\left(10^6\right) - n_c(P_b)_{\text{allow}} - n_c(P_b)_{\text{allow}} = 0$$

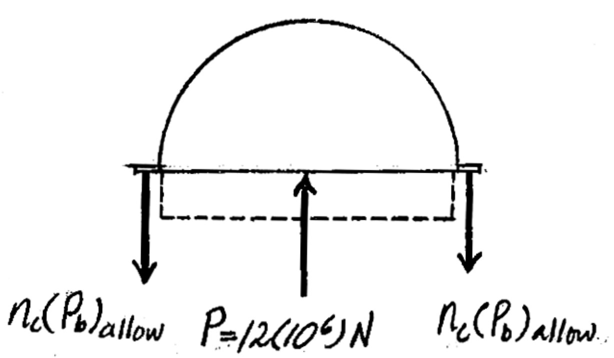
$$n_c = \frac{6\left(10^6\right)}{(P_b)_{\text{allow}}} \qquad \textbf{(1)}$$

The allowable tensile force for each bolt is

$$(P_b)_{\text{allow}} = \sigma_{\text{allow}} A_b = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.025^2\right)\right] = 122.72\left(10^3\right)\text{N}$$

Substituting this result into Eq. (1),

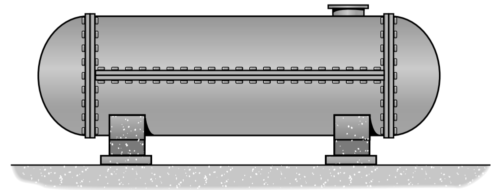
$$n_c = 48.89 = 49 \text{ bolts/meter} \qquad \textbf{Ans.}$$



(a)

Ans.
$t_c = 40\,\text{mm}$, $t_s = 20$ mm,
$n_c = 49$ bolts/meter

8–9. The gas storage tank is fabricated by bolting together two half cylindrical thin shells and two hemispherical shells as shown. If the tank is designed to withstand a pressure of 3 MPa, determine the required minimum thickness of the cylindrical and hemispherical shells and the minimum required number of bolts for each hemispherical cap. The tank and the 25 mm diameter bolts are made from material having an allowable normal stress of 150 MPa and 250 MPa, respectively. The tank has an inner diameter of 4 m.



Normal Stress: For the cylindrical portion of the tank, the hoop stress is twice as large as the longitudinal stress.

$$\sigma_{\text{allow}} = \frac{pr}{t}; \qquad 150\left(10^6\right) = \frac{3\left(10^6\right)(2)}{t_c}$$

$$t_c = 0.04 \text{ m} = 40 \text{ mm} \qquad \textbf{Ans.}$$

For the hemispherical cap,

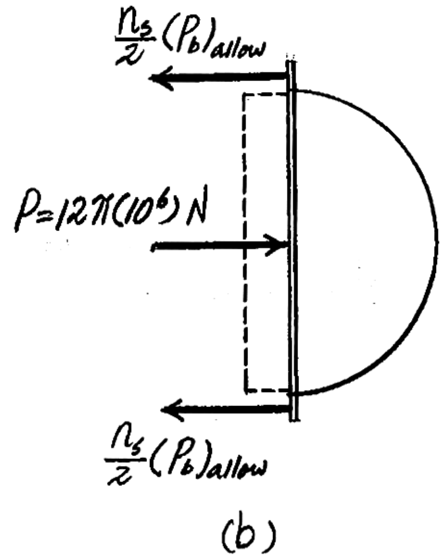
$$\sigma_{\text{allow}} = \frac{pr}{t}; \qquad 150\left(10^6\right) = \frac{3\left(10^6\right)(2)}{2t_s}$$

$$t_s = 0.02 \text{ m} = 20 \text{ mm} \qquad \textbf{Ans.}$$

Since $\frac{r}{t} < 10$, thin-wall analysis is valid.

The allowable tensile force for each bolt is

$$(P_b)_{\text{allow}} = \sigma_{\text{allow}} A_b = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.025^2\right)\right] = 122.72\left(10^3\right)\text{N}$$



Referring to the free-body diagram of the hemispherical cap, Fig. *b*, where $P = pA = 3\left(10^6\right)\left[\frac{\pi}{4}\left(4^2\right)\right] = 12\pi\left(10^6\right)\text{N}$,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 12\pi\left(10^6\right) - \frac{n_s}{2}(P_b)_{\text{allow}} - \frac{n_s}{2}(P_b)_{\text{allow}} = 0$$

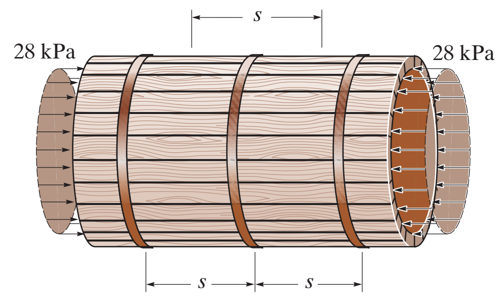
$$n_s = \frac{12\pi\left(10^6\right)}{(P_b)_{\text{allow}}} \qquad \textbf{(1)}$$

Substituting this result into Eq. (1),

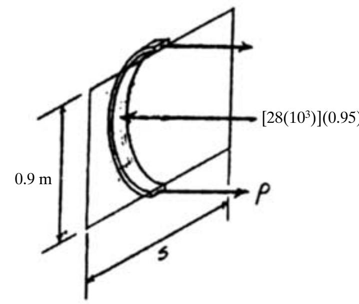
$$n_s = 307.2 = 308 \text{ bolts} \qquad \textbf{Ans.}$$

Ans.
$t_c = 40$ mm, $t_s = 20$ mm,
$(P_b)_{\text{allow}} = 122.72(10^3)$ N, $n_s = 308$ bolts

8–10. A wood pipe having an inner diameter of 0.9 m is bound together using steel hoops each having a cross-sectional area of 125 mm^2. If the allowable stress for the hoops is $\sigma_{allow} = 84$ MPa, determine their maximum spacing s along the section of pipe so that the pipe can resist an internal gauge pressure of 28 kPa. Assume each hoop supports the pressure loading acting along the length s of the pipe.



SOLUTION



Equilibrium for the steel Hoop: From the FBD

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 2P - [28(10^3)](0.93) = 0 \qquad P = 12.6(10^3)s$$

Hoop Stress for the Steel Hoop:

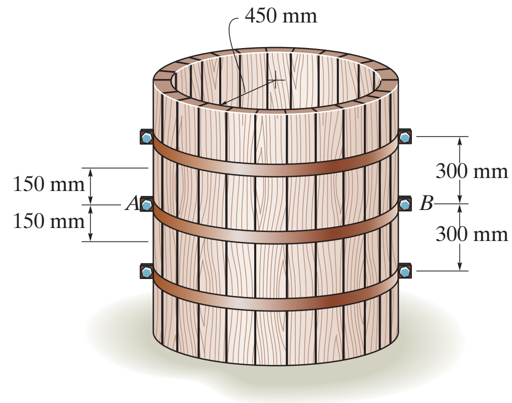
$$\sigma_1 = \sigma_{allow} = \frac{P}{A}$$

$$84(10^6) = \frac{12.6(10^3)s}{125(10^{-6})}$$

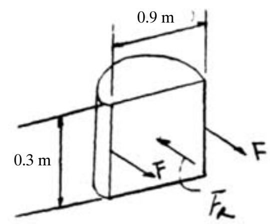
$$s = 833.33 \text{ mm} = 0.833 \text{ m}$$ **Ans.**

Ans.
$s = 0.833$ m

8–11. The staves or vertical members of the wooden tank are held together using semicircular hoops having a thickness of 12 mm and a width of 50 mm. Determine the normal stress in hoop AB if the tank is subjected to an internal gauge pressure of 14 kPa and this loading is transmitted directly to the hoops. Also, if 6-mm-diameter bolts are used to connect each hoop together, determine the tensile stress in each bolt at A and B. Assume hoop AB supports the pressure loading within a 300-mm length of the tank as shown.



SOLUTION



$$F_R = [14(10^3)][(0.9)(0.3^3)] = 3.78(10^3)\text{ N}$$

$$\Sigma F = 0; \quad 3.78(10^3) - 2F = 0; \quad F = 1.89(10^3)\text{ N}$$

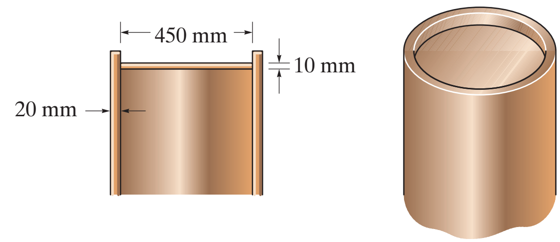
$$\sigma_h = \frac{F}{A_h} = \frac{1.89(10^3)}{0.012(0.05)} = 3.15(10^6)\text{ N/m}^3 = 3.15\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_b = \frac{F}{A_b} = \frac{1.89(10^3)}{\frac{\pi}{4}(0.006^2)} = 66.84(10^6)\text{ N/m}^3 = 66.8\text{ MPa} \qquad \textbf{Ans.}$$

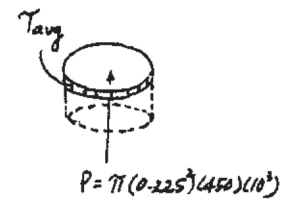
Ans.
$\sigma_h = 3.15$ MPa, $\sigma_b = 66.8$ MPa

***8–12.**

A pressure-vessel head is fabricated by welding the circular plate to the end of the vessel as shown. If the vessel sustains an internal pressure of 450 kPa, determine the average shear stress in the weld and the state of stress in the wall of the vessel.



SOLUTION



$+\uparrow \Sigma F_y = 0; \quad \pi(0.225)^2 450(10^3) - \tau_{avg}(2\pi)(0.225)(0.01) = 0;$

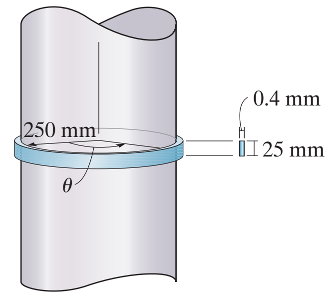
$\tau_{avg} = 5.06 \text{ MPa}$ **Ans.**

$$\sigma_1 = \frac{p\,r}{t} = \frac{450(10^3)(0.225)}{0.02} = 5.06 \text{ MPa}$$ **Ans.**

$$\sigma_2 = \frac{p\,r}{2\,t} = \frac{450(10^3)(0.225)}{2(0.02)} = 2.53 \text{ MPa}$$ **Ans.**

Ans:
$\tau_{avg} = 5.06$ MPa,
$\sigma_1 = 5.06$ MPa,
$\sigma_2 = 2.53$ MPa

8–13. The 304 stainless steel band initially fits snugly around the smooth rigid cylinder. If the band is then subjected to a nonlinear temperature drop of $\Delta T = 12 \sin^2 \theta$ °C, where θ is in radians, determine the circumferential stress in the band.



SOLUTION

Compatibility: Since the band is fixed to a rigid cylinder (it does not deform under load), then

$$\delta_F - \delta_T = 0$$

$$\frac{P(2\pi r)}{AE} - \int_0^{2\pi} \alpha \Delta T r d\theta = 0$$

$$\frac{2\pi r}{E}\left(\frac{P}{A}\right) = 12\alpha r \int_0^{2\pi} \sin^2 \theta d\theta \qquad \text{however, } \frac{P}{A} = \sigma_c$$

$$\frac{2\pi}{E}\sigma_c = 6\alpha \int_0^{2\pi} (1 - \cos 2\theta) d\theta$$

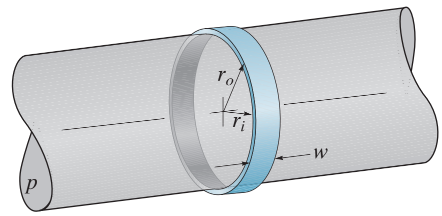
$$\sigma_c = 6\alpha E$$

$$= 6[17(10^{-6})][193(10^9)] = 19.686(10^6) \text{ N/m}^2 = 19.7 \text{ MPa} \qquad \textbf{Ans.}$$

Ans.
$\delta_F - \delta_T = 0, \sigma_c = 19.7$ MPa

8–14.

The ring, having the dimensions shown, is placed over a flexible membrane which is pumped up with a pressure *p*. Determine the change in the inner radius of the ring after this pressure is applied. The modulus of elasticity for the ring is *E*.



SOLUTION

Equilibrium for the Ring: From the FBD

$$\overset{+}{\rightarrow} \Sigma F_x = 0; 2P - 2pr_i w = 0 \qquad P = pr_i w$$

Hoop Stress and Strain for the Ring:

$$\sigma_1 = \frac{P}{A} = \frac{pr_i w}{(r_o - r_i)w} = \frac{pr_i}{r_o - r_i}$$

Using Hooke's Law

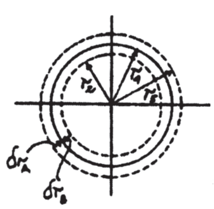
$$\epsilon_1 = \frac{\sigma_1}{E} = \frac{pr_i}{E(r_o - r_i)} \qquad \textbf{(1)}$$

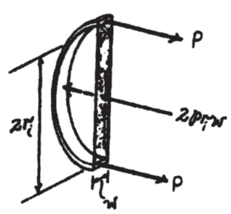
However, $\epsilon_1 = \dfrac{2\pi(r_i)_1 - 2\pi r_i}{2\pi r_i} = \dfrac{(r_i)_1 - r_i}{r_i} = \dfrac{\delta r_i}{r_i}$.

Then, from Eq. (1)

$$\frac{\delta r_i}{r_i} = \frac{pr_i}{E(r_o - r_i)}$$

$$\delta r_i = \frac{pr_i^2}{E(r_o - r_i)} \qquad \textbf{Ans.}$$



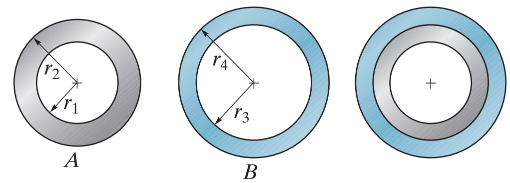


Ans:

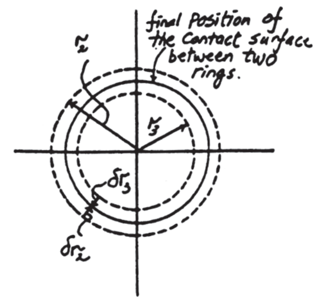
$\delta r_i = \dfrac{pr_i^2}{E(r_o - r_i)}$

8–15.

The inner ring A has an inner radius r_1 and outer radius r_2. The outer ring B has an inner radius r_3 and an outer radius r_4, and $r_2 > r_3$. If the outer ring is heated and then fitted over the inner ring, determine the pressure between the two rings when ring B reaches the temperature of the inner ring. The material has a modulus of elasticity of E and a coefficient of thermal expansion of α.



SOLUTION



Equilibrium for the Ring: From the FBD

$$\overset{+}{\rightarrow}\Sigma F_x = 0; 2P - 2pr_i w = 0 \qquad P = pr_i w$$

Hoop Stress and Strain for the Ring:

$$\sigma_1 = \frac{P}{A} = \frac{pr_i w}{(r_o - r_i)w} = \frac{pr_i}{r_o - r_i}$$

Using Hooke's Law

$$\epsilon_1 = \frac{\sigma_1}{E} = \frac{pr_i}{E(r_o - r_i)} \qquad \textbf{(1)}$$

However, $\epsilon_1 = \dfrac{2\pi(r_i)_1 - 2\pi r_i}{2\pi r_i} = \dfrac{(r_i)_1 - r_i}{r_i} = \dfrac{\delta r_i}{r_i}$.

Then, from Eq. (1)

$$\frac{\delta r_i}{r_i} = \frac{pr_i}{E(r_o - r_i)}$$

$$\delta r_i = \frac{pr_i^2}{E(r_o - r_i)}$$

Compatibility: The pressure between the rings requires

$$\delta r_2 + \delta r_3 = r_2 - r_3 \qquad \textbf{(2)}$$

From the result obtained above

$$\delta r_2 = \frac{pr_2^2}{E(r_2 - r_1)} \qquad \delta r_3 = \frac{pr_3^2}{E(r_4 - r_3)}$$

Substitute into Eq. (2)

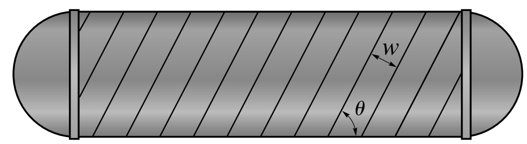
$$\frac{pr_2^2}{E(r_2 - r_1)} + \frac{pr_3^2}{E(r_4 - r_3)} = r_2 - r_3$$

$$p = \frac{E(r_2 - r_3)}{\dfrac{r_2^2}{r_2 - r_1} + \dfrac{r_3^2}{r_4 - r_3}} \qquad \textbf{Ans.}$$

Ans:

$$p = \frac{E(r_2 - r_3)}{\dfrac{r_2^2}{r_2 - r_1} + \dfrac{r_3^2}{r_4 - r_3}}$$

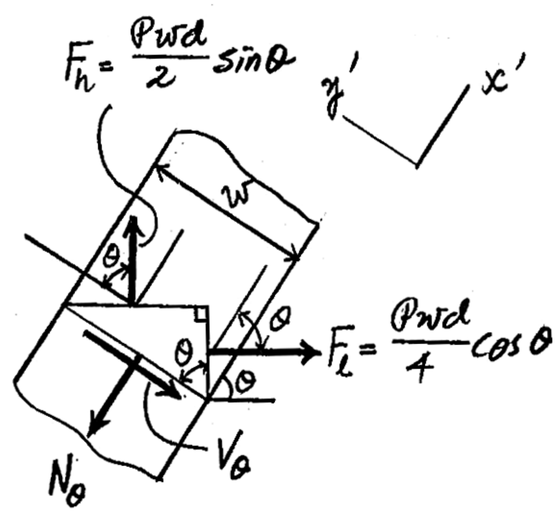
***8–16.** The cylindrical tank is fabricated by welding a strip of thin plate helically, making an angle θ with the longitudinal axis of the tank. If the strip has a width w and thickness t, and the gas within the tank of diameter d is pressured to p, show that the normal stress developed along the strip is given by $\sigma_\theta = (pd/8t)(3 - \cos 2\theta)$.



Normal Stress:

$$\sigma_h = \sigma_1 = \frac{pr}{t} = \frac{p(d/2)}{t} = \frac{pd}{2t}$$

$$\sigma_l = \sigma_2 = \frac{pr}{2t} = \frac{p(d/2)}{2t} = \frac{pd}{4t}$$



(a)

Equilibrium: We will consider the triangular element cut from the strip shown in Fig. *a*. Here,
$A_h = (w \sin\theta)t$ and $A_l = (w \cos\theta)t$. Thus,
$F_h = \sigma_h A_h = \frac{pd}{2t}(w \sin\theta)t = \frac{pwd}{2} \sin\theta$ and

$F_l = \sigma_l A_l = \frac{pd}{4t}(w \cos\theta)t = \frac{pwd}{4} \cos\theta.$

Writing the force equation of equilibrium along the x' axis,

$$\Sigma F_{x'} = 0; \quad \left[\frac{pwd}{2} \sin\theta\right] \sin\theta + \left[\frac{pwd}{4} \cos\theta\right] \cos\theta - N_\theta = \theta$$

$$N_\theta = \frac{pwd}{4}\left(2\sin^2\theta + \cos^2\theta\right)$$

However, $\sin^2\theta + \cos^2\theta = 1$. This equation becomes

$$N_\theta = \frac{pwd}{4}\left(\sin^2\theta + 1\right)$$

Also, $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$, so that

$$N_\theta = \frac{pwd}{8}(3 - \cos 2\theta)$$

Since $A_\theta = wt$, then

$$\sigma_\theta = \frac{N_\theta}{A_\theta} = \frac{\frac{pwd}{8}(3 - \cos 2\theta)}{wt}$$

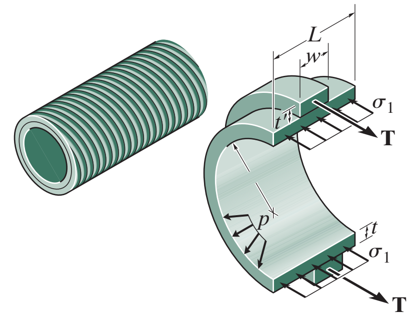
$$\sigma_\theta = \frac{pd}{8t}(3 - \cos 2\theta) \qquad \textbf{(Q.E.D.)}$$

Ans.

$\sigma_\theta = \frac{pd}{8t}(3 - \cos 2\theta)$

8–17.

In order to increase the strength of the pressure vessel, filament winding of the same material is wrapped around the circumference of the vessel as shown. If the pretension in the filament is T and the vessel is subjected to an internal pressure p, determine the hoop stresses in the filament and in the wall of the vessel. Use the free-body diagram shown, and assume the filament winding has a thickness t' and width w for a corresponding length L of the vessel.



SOLUTION

Normal Stress in the Wall and Filament Before the Internal Pressure is Applied: The entire length L of wall is subjected to pretension filament force T. Hence, from equilibrium, the normal stress in the wall at this state is

$$2T - (\sigma')_w\,(2Lt) = 0 \qquad (\sigma')_w = \frac{T}{Lt}$$

and for the filament the normal stress is

$$(\sigma')_{fil} = \frac{T}{wt'}$$

Normal Stress in the Wall and Filament After the Internal Pressure is Applied: In order to use $\sigma_1 = pr/t$, developed for a vessel of uniform thickness, we redistribute the filament's cross-section as if it were thinner and wider, to cover the vessel with no gaps. The modified filament has width L and thickness $t'w/L$, still with cross-sectional area wt' subjected to tension T. Then the stress in the filament becomes

$$\sigma_{fil} = \sigma + (\sigma')_{fil} = \frac{pr}{(t + t'w/L)} + \frac{T}{wt'} \qquad \textbf{Ans.}$$

And for the wall,

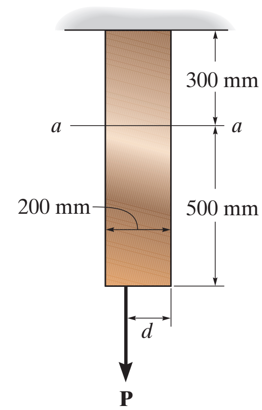
$$\sigma_w = \sigma - (\sigma')_w = \frac{pr}{(t + t'w/L)} - \frac{T}{Lt} \qquad \textbf{Ans.}$$

Check: $2wt'\sigma_{fil} + 2Lt\sigma_w = 2rLp$ OK

Ans:
$\sigma_{fil} = \dfrac{pr}{t + t'w/L} + \dfrac{T}{wt'}$,
$\sigma_w = \dfrac{pr}{t + t'w/L} - \dfrac{T}{Lt}$

8–18.

Determine the shortest distance d to the edge of the plate at which the force **P** can be applied so that it produces no compressive stresses in the plate at section a–a. The plate has a thickness of 10 mm and **P** acts along the centerline of this thickness.



SOLUTION

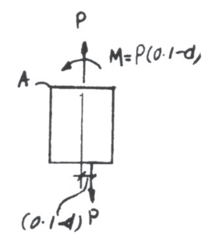
$$\sigma_A = 0 = \sigma_a - \sigma_b$$

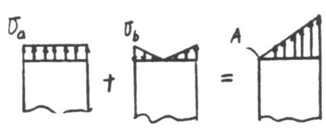
$$0 = \frac{P}{A} - \frac{M\,c}{I}$$

$$0 = \frac{P}{(0.2)(0.01)} - \frac{P(0.1 - d)(0.1)}{\frac{1}{12}(0.01)(0.2^3)}$$

$$P(-1000 + 15000\,d) = 0$$

$$d = 0.0667 \text{ m} = 66.7 \text{ mm}$$ **Ans.**

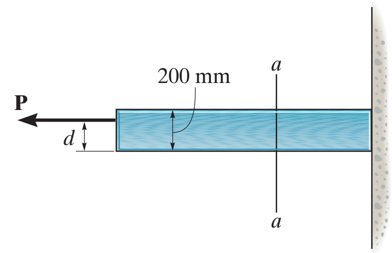




Ans:
$d = 66.7$ mm

8–19.

Determine the maximum distance d to the edge of the plate at which the force **P** can be applied so that it produces no compressive stresses on the plate at section a–a. The plate has a thickness of 20 mm and **P** acts along the centerline of this thickness.

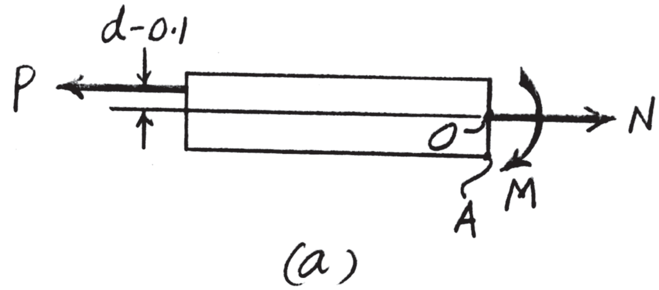


SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the plate sectioned through section a–a, Fig. a,

$\xrightarrow{+} \Sigma F_x = 0; \quad N - P = 0 \quad N = P$

$\circlearrowleft + \Sigma M_o = 0; \quad P(d - 0.1) - M = 0 \quad M = P(d - 0.1)$



Section Properties: For the rectangular cross section,

$$A = 0.2(0.02) = 0.004 \text{ m}^2$$

$$I = \frac{1}{12}(0.02)\left(0.2^3\right) = 13.3333\left(10^{-6}\right) \text{ m}^4$$

Normal Stress: It is required that $\sigma_A = 0$. For the combined loadings,

$$\sigma_A = \frac{N}{A} - \frac{Mc}{I}$$

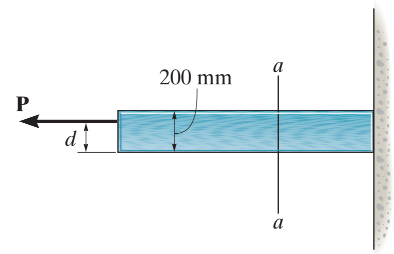
$$0 = \frac{P}{0.004} - \frac{P(d - 0.1)(0.1)}{13.3333\left(10^{-6}\right)}$$

$$d = 0.1333 \text{ m} = 133 \text{ mm}$$ **Ans.**

Ans:
$d = 133$ mm

***8–20.**

The plate has a thickness of 20 mm and the force $P = 3$ kN acts along the centerline of this thickness such that $d = 150$ mm. Plot the distribution of normal stress acting along section a–a.



SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the plate sectioned through section a–a, Fig. a,

$\xrightarrow{+} \Sigma F_x = 0;\quad N - 3 = 0 \quad\quad N = 3.00 \text{ kN}$

$\circlearrowleft + \Sigma M_o = 0;\quad 3(0.05) - M = 0 \quad\quad M = 0.150 \text{ kN} \cdot \text{m}$

Section Properties: For the rectangular cross section,

$$A = 0.2(0.02) = 0.004 \text{ m}^2$$

$$I = \frac{1}{12}(0.02)\left(0.2^3\right) = 13.3333\left(10^{-6}\right) \text{ m}^4$$

Normal Stress: For the combined loadings, the normal stress at points A and B can be determined from

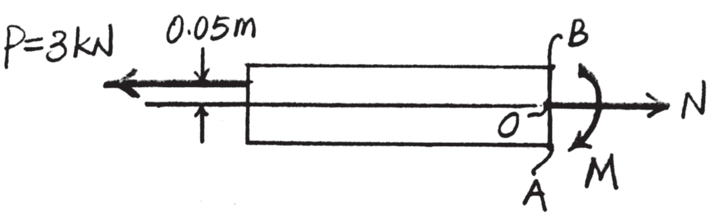
$$\sigma = \frac{N}{A} \pm \frac{Mc}{I} = \frac{3.00\left(10^3\right)}{0.004} \pm \frac{0.150\left(10^3\right)(0.1)}{13.3333\left(10^{-6}\right)}$$

$\sigma_A = 750\left(10^3\right) - 1.125\left(10^6\right) = -0.375\left(10^6\right) \text{ Pa} = 0.375 \text{ MPa (C)}$ **Ans.**

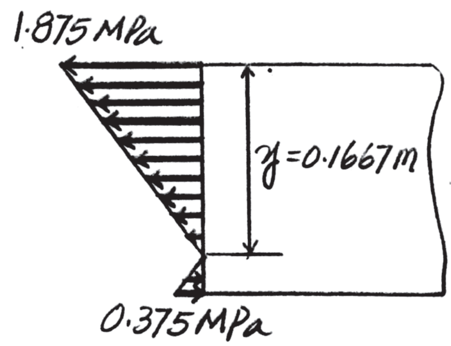
$\sigma_B = 750\left(10^3\right) + 1.125\left(10^6\right) = 1.875\left(10^6\right) \text{ Pa} = 1.875 \text{ MPa (T)}$ **Ans.**

Using similar triangles, the location of the neutral axis can be determined

$$\frac{y}{1.875} = \frac{0.2 - y}{0.375}; \quad y = 0.1667 \text{ m}$$



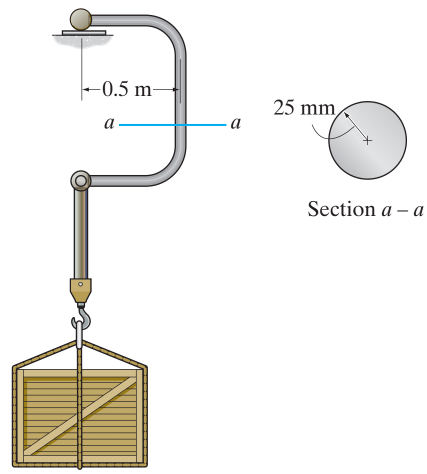
(a)



(b)

Ans:
$\sigma_A = 0.375$ MPa (C),
$\sigma_B = 1.875$ MPa (T)

8–21. If the load has a weight of 2700 N, determine the maximum normal stress developed on the cross section of the supporting member at section *a–a*. Also, plot the normal stress distribution over the cross-section.



SOLUTION

Internal Loadings: Consider the equilibrium of the free-body diagram of the bottom cut segment shown in Fig. a.

$+\uparrow\Sigma F_y = 0;\qquad N - 2700 = 0\qquad N = 2700\text{ N}$

$\circlearrowleft + \Sigma M_C = 0;\qquad 2700(0.5) - M = 0\qquad M = 1350\text{ N}\cdot\text{m}$

Section Properties: The cross-sectional area and the moment of inertia about the centroidal axis of the member are

$$A = \pi(0.025^2) = 0.625(10^{-3})\pi\text{ m}^2\qquad I = \frac{\pi}{4}(0.025^4) = 0.30680(10^{-6})\text{ m}^4$$

Normal Stress: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

By observation, the maximum normal stress occurs at point *B*, Fig. *b*. Thus,

$$\sigma_{\max} = \sigma_B = \frac{2700}{0.625(10^{-3})\pi} + \frac{1350(0.025)}{0.30680(10^{-6})}$$

$$= 111.38(10^6)\text{ N/m}^2 = 111\text{ MPa (T)}\qquad\textbf{Ans.}$$

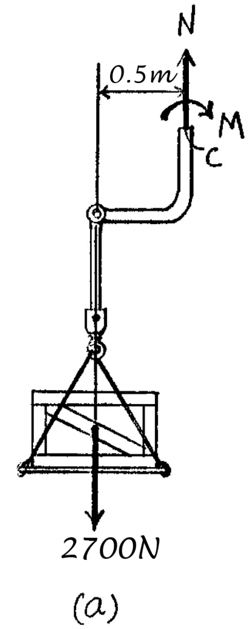
For Point *A*,

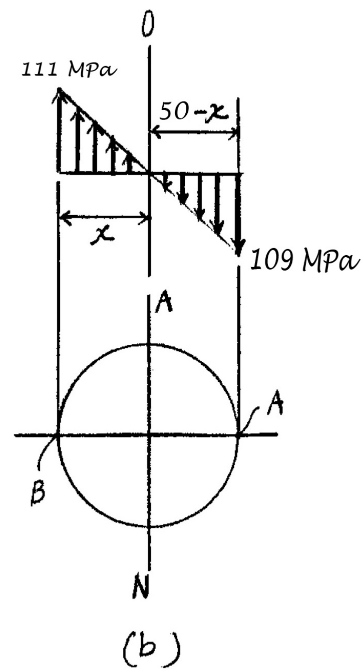
$$\sigma_A = \frac{2700}{0.625(10^{-3})\pi} - \frac{1350(0.025)}{0.30680(10^{-6})}$$

$$= -108.63(10^6)\text{ N/m}^2 = 109\text{ MPa (C)}\qquad\textbf{Ans.}$$

Using these results, the normal stress distribution over the cross section is shown in Fig. *b*. The location of the neutral axis can be determined from

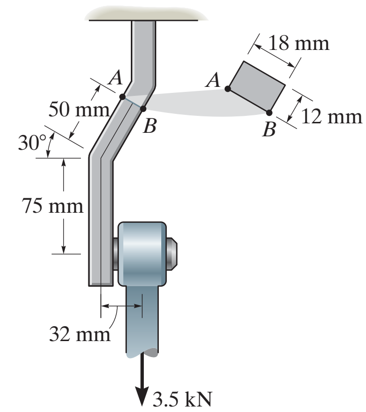
$$\frac{50 - x}{108.63} = \frac{x}{111.38};\qquad x = 25.3\text{ mm}$$





Ans:
$\sigma_{\max} = 111\text{ MPa (T)}$

8–22. The bearing pin supports the load of 3.5 kN. Determine the stress components in the support member at point A. The support is 12 mm thick.

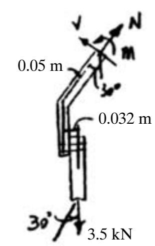


SOLUTION

$\Sigma F_x = 0; \quad N - 3.5 \cos 30° = 0; \quad N = 3.0311 \text{ kN}$

$\Sigma F_y = 0; \quad V - 3.5 \sin 30° = 0; \quad V = 1.75 \text{ kN}$

$\circlearrowleft + \Sigma M = 0; \quad M - 3.5(0.032 - 0.05 \sin 30°) = 0; \quad M = 0.0245 \text{ KN} \cdot \text{m} = 24.5 \text{ N} \cdot \text{m}$



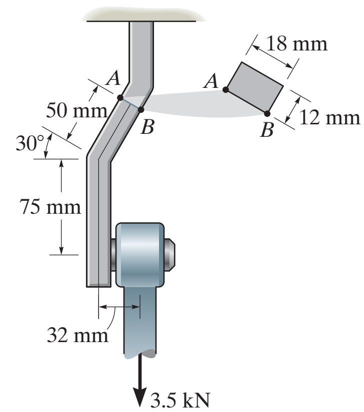
$$\sigma_A = \frac{N}{A} - \frac{Mc}{I} = \frac{3.0311(10^3)}{0.012(0.018)} - \frac{24.5(0.009)}{\frac{1}{12}(0.012)(0.018^3)}$$

$\sigma_A = -23.78(10^6) \text{ N/m}^2 = 23.8 \text{ MPa (C)}$ **Ans.**

$\tau_A = 0 \quad (\text{since } Q_A = 0)$ **Ans.**

Ans.
$\sigma_A = 23.8 \text{ MPa(C)}, T_A = 0$

8–23. The bearing pin supports the load of 3.5 kN. Determine the stress components in the support member at point B. The support is 12 mm thick.

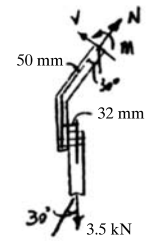


SOLUTION

$\Sigma F_x = 0; \quad N - 3.5\cos 30° = 0; \quad N = 3.0311 \text{ kN}$

$\Sigma F_y = 0; \quad V - 3.5\sin 30° = 0; \quad V = 1.75 \text{ kN}$

$\circlearrowleft + \Sigma M = 0; \quad M - 3.5(0.032 - 0.05\sin 30°) = 0; \quad M = 0.0245 \text{ KN}\cdot\text{m} = 24.5 \text{ N}\cdot\text{m}$



$$\sigma_B = \frac{N}{A} + \frac{Mc}{I} = \frac{3.0311(10^3)}{0.012(0.018)} + \frac{24.5(0.009)}{\frac{1}{12}(0.012)(0.018^3)}$$

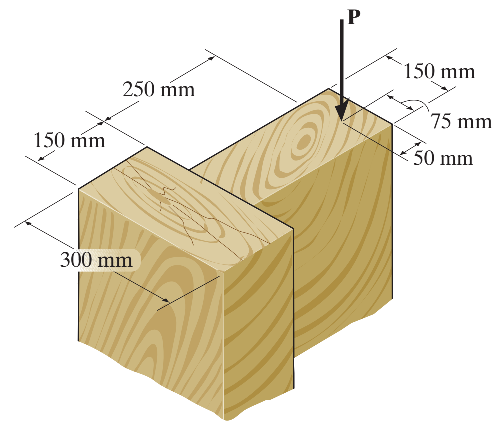
$\sigma_B = 51.84(10^6) \text{ N/m}^2 = 51.8 \text{ MPa (T)}$ **Ans.**

$\tau_B = 0 \quad (\text{since } Q_B = 0)$ **Ans.**

Ans.
$N = 3.031$ kN, $V = 1.75$ kN,
$M = 24.5 \text{ N}\cdot\text{m}$,
$\sigma_B = 51.8$ MPa (T), $\tau_B = 0$

***8–24.**

The column is built up by gluing the two boards together. Determine the maximum normal stress on the cross section when the eccentric force of $P = 50$ kN is applied.



SOLUTION

Section Properties: The location of the centroid of the cross section, Fig. *a*, is

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.075(0.15)(0.3) + 0.3(0.3)(0.15)}{0.15(0.3) + 0.3(0.15)} = 0.1875 \text{ m}$$

The cross-sectional area and the moment of inertia about the z axis of the cross section are

$$A = 0.15(0.3) + 0.3(0.15) = 0.09 \text{ m}^2$$

$$I_z = \frac{1}{12}(0.3)(0.15^3) + 0.3(0.15)(0.1875 - 0.075)^2 + \frac{1}{12}(0.15)(0.3^3) + 0.15(0.3)(0.3 - 0.1875)^2$$

$$= 1.5609(10^{-3}) \text{ m}^4$$

Equivalent Force System: Referring to Fig. *b*,

$+\uparrow \Sigma F_x = (F_R)_x;$ $\qquad -50 = -F \qquad F = 50 \text{ kN}$

$\Sigma M_z = (M_R)_z;$ $\qquad -50(0.2125) = -M \qquad M = 10.625 \text{ kN}\cdot\text{m}$

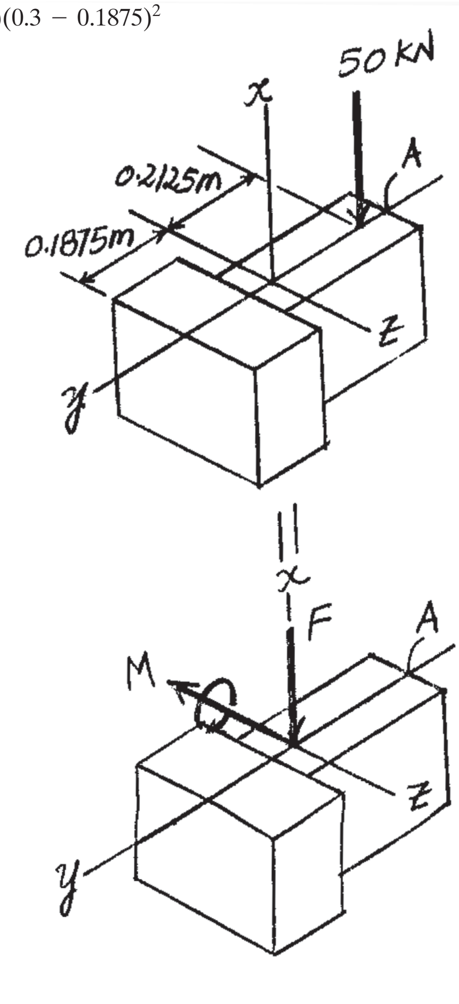
Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{My}{I}$$

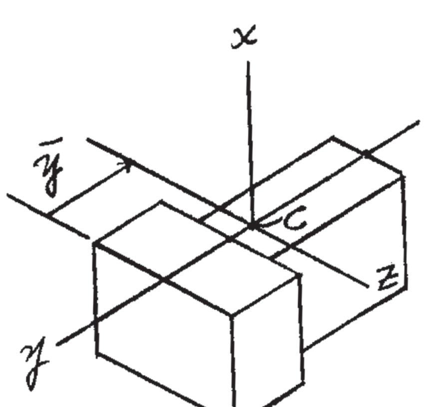
By inspection, the maximum normal stress occurs at points along the edge where $y = 0.45 - 0.1875 = 0.2625$ m such as point A. Thus,

$$\sigma_{\max} = \frac{-50(10^3)}{0.09} - \frac{10.625(10^3)(0.2625)}{1.5609(10^{-3})}$$

$$= -2.342 \text{ MPa} = 2.34 \text{ MPa (C)} \qquad \textbf{Ans.}$$



(b)

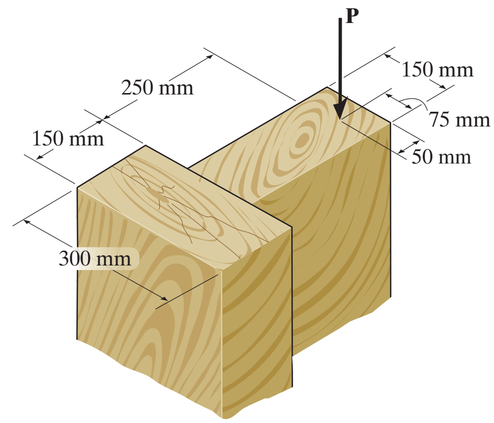


(a)

Ans:
$\sigma_{\max} = 2.34$ MPa (C)

8–25.

The column is built up by gluing the two boards together. If the wood has an allowable normal stress of $\sigma_{\text{allow}} = 6$ MPa, determine the maximum allowable eccentric force **P** that can be applied to the column.



SOLUTION

Section Properties: The location of the centroid c of the cross section, Fig. a, is

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{0.075(0.15)(0.3) + 0.3(0.3)(0.15)}{0.15(0.3) + 0.3(0.15)} = 0.1875 \text{ m}$$

The cross-sectional area and the moment of inertia about the z axis of the cross section are

$$A = 0.15(0.3) + 0.3(0.15) = 0.09 \text{ m}^2$$

$$I_z = \frac{1}{12}(0.3)(0.15^3) + 0.3(0.15)(0.1875 - 0.075)^2 + \frac{1}{12}(0.15)(0.3^3) + 0.15(0.3)(0.3 - 0.1875)^2$$

$$= 1.5609(10^{-3}) \text{ m}^4$$

Equivalent Force System: Referring to Fig. b,

$+\uparrow \Sigma F_x = (F_R)_x;\qquad -P = -F \qquad F = P$

$\Sigma M_z = (M_R)_z;\qquad -P(0.2125) = -M \qquad M = 0.2125P$

Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

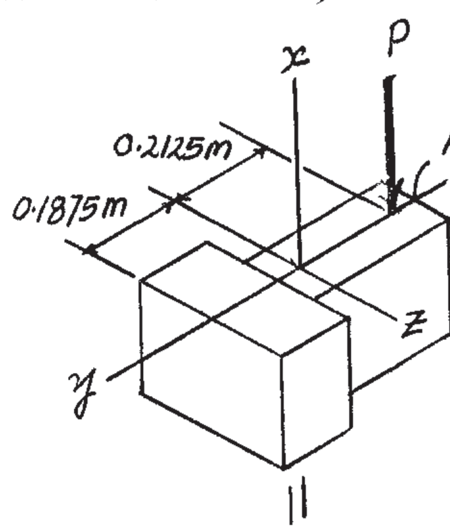
$$F = \frac{N}{A} + \frac{My}{I}$$

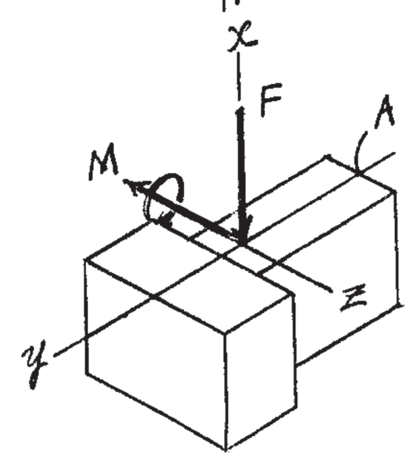
By inspection, the maximum normal stress, which is compression, occurs at points along the edge where $y = 0.45 - 0.1875 = 0.2625$ m such as point A. Thus,

$$-6(10^6) = \frac{-P}{0.09} - \frac{0.2125P(0.2625)}{1.5609(10^{-3})}$$

$P = 128\,076.92 \text{ N} = 128 \text{ kN}$ **Ans.**

(a)

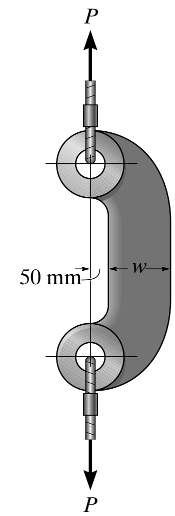




(b)

Ans:
$P_{\max} = 128$ kN

8–26. The offset link supports the loading of $P = 30$ kN. Determine its required width w if the allowable normal stress is $\sigma_{allow} = 73$ MPa. The link has a thickness of 40 mm.



SOLUTION

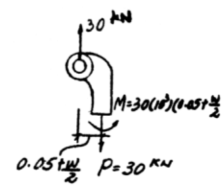
σ due to axial force:

$$\sigma_a = \frac{P}{A} = \frac{30(10^3)}{(w)(0.04)} = \frac{750(10^3)}{w}$$

σ due to bending:

$$\sigma_b = \frac{Mc}{I} = \frac{30(10^3)(0.05 + \frac{w}{2})(\frac{w}{2})}{\frac{1}{12}(0.04)(w)^3}$$

$$= \frac{4500\,(10^3)(0.05 + \frac{w}{2})}{w^2}$$



$$\sigma_{max} = \sigma_{allow} = \sigma_a + \sigma_b$$

$$73(10^6) = \frac{750(10^3)}{w} + \frac{4500(10^3)(0.05 + \frac{w}{2})}{w^2}$$

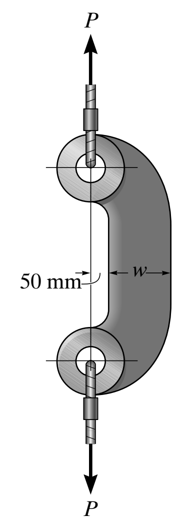
$$73\,w^2 = 0.75\,w + 0.225 + 2.25\,w$$

$$73\,w^2 - 3\,w - 0.225 = 0$$

$$w = 0.0797 \text{ m} = 79.7 \text{ mm}$$ **Ans.**

Ans.
$w = 79.7$ mm

8–27. The offset link has a width of $w = 200$ mm and a thickness of 40 mm. If the allowable normal stress is $\sigma_{\text{allow}} = 75$ MPa, determine the maximum load P that can be applied to the cables.



SOLUTION

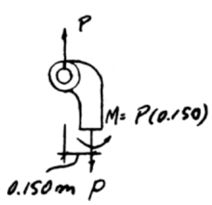
$$A = 0.2(0.04) = 0.008\ \text{m}^2$$

$$I = \frac{1}{12}(0.04)(0.2)^3 = 26.6667(10^{-6})\ \text{m}^4$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I}$$

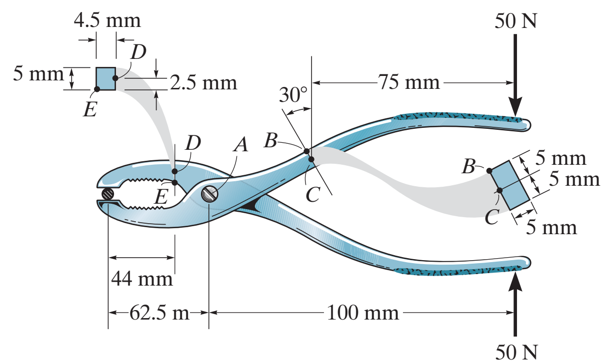
$$75(10^6) = \frac{P}{0.008} + \frac{0.150\,P(0.1)}{26.6667(10^{-6})}$$

$P = 109$ kN **Ans.**



Ans.
$P = 109$ kN

***8–28.** The pliers are made from two steel parts pinned together at A. If a smooth bolt is held in the jaws and a gripping force of 50 N is applied at the handles, determine the state of stress developed in the pliers at points B and C. Here the cross section is rectangular, having the dimensions shown in the figure.



SOLUTION

$$\nearrow^{+}\ \Sigma F_x = 0; \qquad N - 50\sin 30^\circ = 0; \quad N = 25\ \text{N}$$

$$\nwarrow^{+}\ \Sigma F_y = 0; \qquad V - 50\cos 30^\circ = 0; \quad V = 43.30\ \text{N}$$

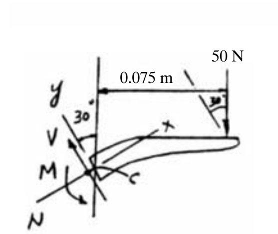
$$\circlearrowleft + \Sigma M_C = 0; \qquad M - 50(0.075) = 0; \quad M = 3.75\ \text{N}\cdot\text{m}$$

$$A = (0.005)(0.01) = 50(10^{-6})\ \text{m}^2$$

$$I = \frac{1}{12}(0.005)(0.01^3) = 0.41667(10^{-9})\ \text{m}^4$$

$$Q_B = 0$$

$$Q_C = \bar{y}'A' = (0.0025)(0.005)(0.005) = 62.5(10^{-9})\ \text{m}^3$$

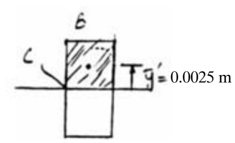


Point B:

$$\sigma_B = \frac{N}{A} + \frac{My}{I} = \frac{-25}{50(10^{-6})} + \frac{3.75(0.005)}{0.41667(10^{-9})}$$

$$= 44.5(10^6)\ \text{N/m}^2 = 44.5\ \text{MPa (T)} \qquad \textbf{Ans.}$$

$$\tau_B = \frac{VQ}{It} = 0 \qquad \textbf{Ans.}$$



Point C:

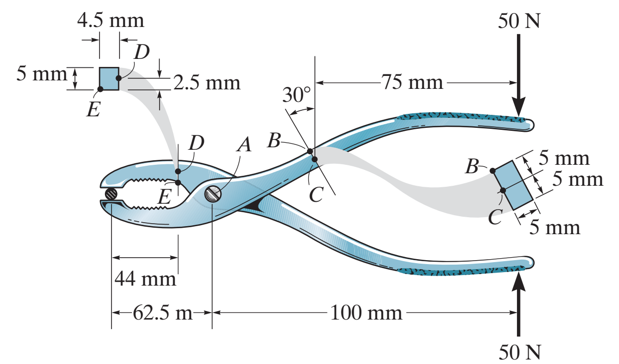
$$\sigma_C = \frac{N}{A} + \frac{My}{I} = \frac{-25}{50(10^{-6})} + 0 = -0.500(10^6)\ \text{N/m}^2 = 0.500\ \text{MPa (C)} \qquad \textbf{Ans.}$$

Shear Stress :

$$\tau_C = \frac{VQ}{It} = \frac{43.30[62.5(10^{-9})]}{[0.41667(10^{-9})](0.005)} = 1.299(10^6)\ \text{N/m}^2 = 1.30\ \text{MPa} \qquad \textbf{Ans.}$$

Ans.
$\sigma_B = 44.5$ MPa, $\tau_B = 0$,
$\sigma_C = 0.50$ MPa, $\tau_C = 1.30$ MPa

8–29. Solve Prob. 8–33 for points D and E.



SOLUTION

$\zeta + \Sigma M_A = 0; \qquad -F(0.0625) + 50(0.1) = 0; \qquad F = 80 \text{ N}$

Point D:

$$\sigma_D = 0 \qquad \textbf{Ans.}$$

$$\tau_D = \frac{VQ}{It} = \frac{80[(0.00125)(0.0045)(0.0025)]}{\left[\frac{1}{12}(0.0045)(0.005^3)\right](0.0045)}$$

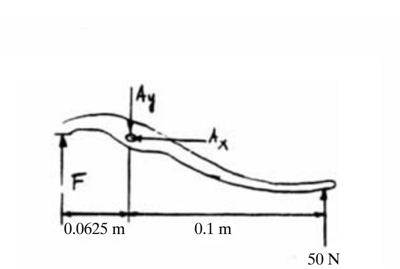
$$= 5.333(10^6) \text{ N/m}^2 = 5.33 \text{ MPa} \qquad \textbf{Ans.}$$

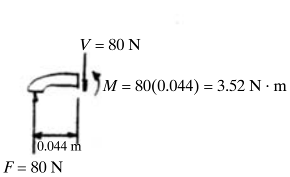
Point E:

$$\sigma_E = \frac{My}{I} = \frac{3.52(0.0025)}{\frac{1}{12}(0.0045)(0.005^3)}$$

$$= 187.73(10^6) \text{ N/m}^2 = 188 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_E = 0 \qquad \textbf{Ans.}$$

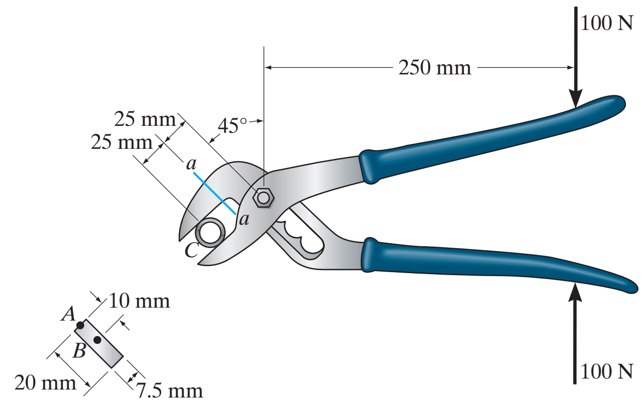




Ans.
$\sigma_D = 0, \tau_D = 5.33 \text{ MPa}, \sigma_E = 188 \text{ MPa}, \tau_E = 0$

8–30.

The rib-joint pliers are used to grip the smooth pipe C. If the force of 100 N is applied to the handles, determine the state of stress at points A and B on the cross section of the jaw at section a–a. Indicate the results on an element at each point.



Section a – a

SOLUTION

Support Reactions: Referring to the free-body diagram of the handle shown in Fig. *a*,

$\circlearrowleft + \Sigma M_D = 0;$ $\quad 100(0.25) - F_C(0.05) = 0 \qquad F_C = 500\text{ N}$

Internal Loadings: Consider the equilibrium of the free-body diagram of the segment shown in Fig. *b*.

$\Sigma F_{y'} = 0;$ $\quad 500 - V = 0 \qquad V = 500\text{ N}$

$\circlearrowleft + \Sigma M_C = 0;$ $\quad M - 500(0.025) = 0 \qquad M = 12.5\text{ N}\cdot\text{m}$

Section Properties: The moment of inertia of the cross section about the centroidal axis is

$$I = \frac{1}{12}(0.0075)(0.02^3) = 5(10^{-9})\text{ m}^4$$

Referring to Fig. *c*, Q_A and Q_B are

$$Q_A = 0$$

$$Q_B = \bar{y}'A' = 0.005(0.01)(0.0075) = 0.375(10^{-6})\text{ m}^3$$

Normal Stress: The normal stress is contributed by bending stress only. Thus

$$\sigma = \frac{My}{I}$$

For point A, $y = 0.01$ m. Then

$$\sigma_A = -\frac{12.5(0.01)}{5(10^{-9})} = -25\text{ MPa} = 25\text{ MPa (C)} \qquad \textbf{Ans.}$$

For point B, $y = 0$. Then

$$\sigma_B = 0 \qquad \textbf{Ans.}$$

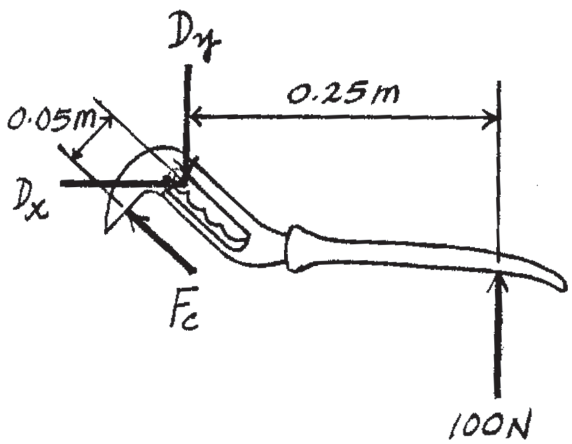
8–30. Continued

Shear Stress: The shear stress is contributed by transverse shear stress only. Thus,

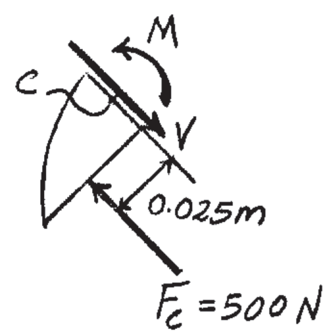
$$\tau_A = \frac{VQ_A}{It} = 0 \qquad \textbf{Ans.}$$

$$\tau_B = \frac{VQ_B}{It} = \frac{500[0.375(10^{-6})]}{5(10^{-9})(0.0075)} = 5 \text{ MPa} \qquad \textbf{Ans.}$$

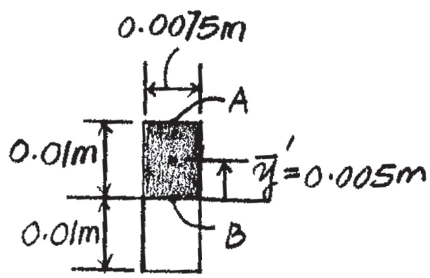
The state of stress of points A and B are represented by the elements shown in Figs. d and e respectively.



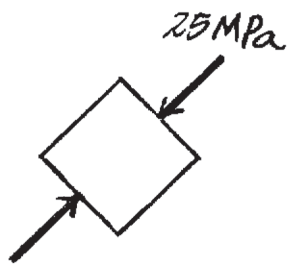
(a)



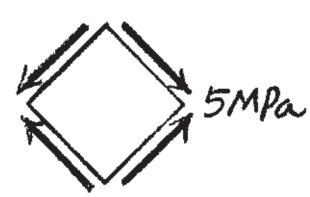
(b)



(c)



(d)

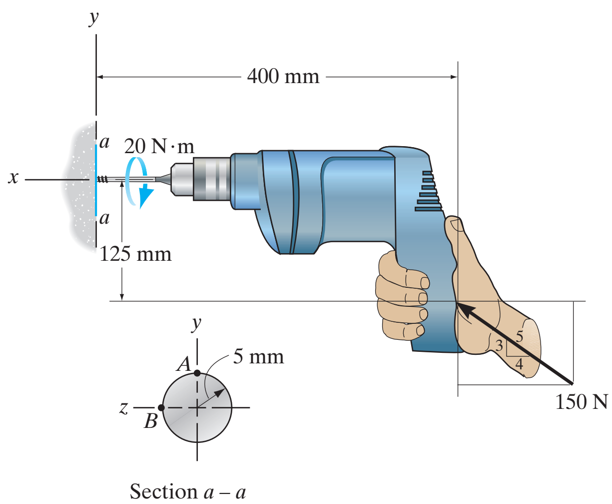


(e)

Ans:
$\sigma_A = 25$ MPa (C), $\sigma_B = 0$,
$\tau_A = 0$, $\tau_B = 5$ MPa

8–31.

The drill is jammed in the wall and is subjected to the torque and force shown. Determine the state of stress at point A on the cross section of the drill bit at section a–a.



SOLUTION

Internal Loadings: Consider the equilibrium of the free-body diagram of the drill's right cut segment, Fig. *a*.

$$\Sigma F_x = 0; \quad N - 150\left(\frac{4}{5}\right) = 0 \qquad N = 120\text{ N}$$

$$\Sigma F_y = 0; \quad 150\left(\frac{3}{5}\right) - V_y = 0 \qquad V_y = 90\text{ N}$$

$$\Sigma M_x = 0; \quad 20 - T = 0 \qquad T = 20\text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0; \quad -150\left(\frac{3}{5}\right)(0.4) + 150\left(\frac{4}{5}\right)(0.125) + M_z = 0$$

$$M_z = 21\text{ N}\cdot\text{m}$$

Section Properties: The cross-sectional area, the moment of inertia about the z axis, and the polar moment of inertia of the drill's cross section are

$$A = \pi\left(0.005^2\right) = 25\pi\left(10^{-6}\right)\text{ m}^2$$

$$I_z = \frac{\pi}{4}\left(0.005^4\right) = 0.15625\pi\left(10^{-9}\right)\text{ m}^4$$

$$J = \frac{\pi}{2}\left(0.005^4\right) = 0.3125\pi\left(10^{-9}\right)\text{ m}^4$$

Referring to Fig. *b*, Q_A is

$$Q_A = 0$$

Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z}$$

For point A, $y = 0.005$ m. Then

$$\sigma_A = \frac{-120}{25\pi\left(10^{-6}\right)} - \frac{21(0.005)}{0.15625\pi\left(10^{-9}\right)} = -215.43\text{ MPa} = 215\text{ MPa (C)} \qquad \textbf{Ans.}$$

8–31. Continued

Shear Stress: The transverse shear stress developed at point A is

$$\left[\left(\tau_{xy}\right)_V\right]_A = \frac{V_y Q_A}{I_z t} = 0$$

The torsional shear stress developed at point A is

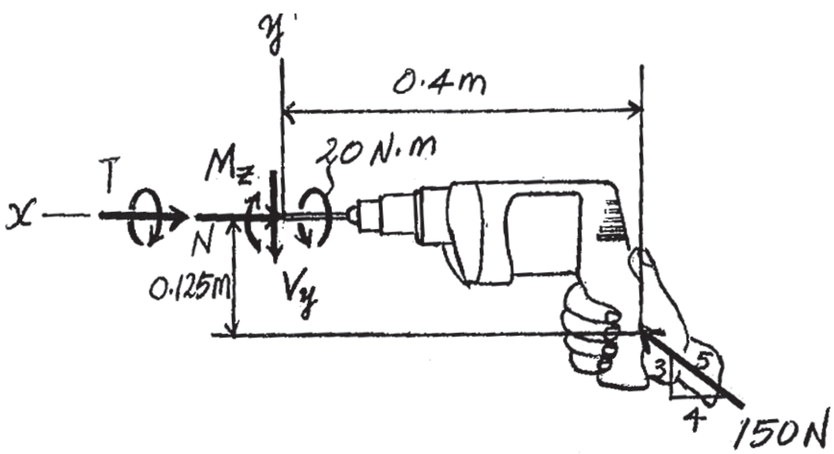
$$\left[(\tau_{xz})_T\right]_A = \frac{Tc}{J} = \frac{20(0.005)}{0.3125\pi\left(10^{-9}\right)} = 101.86 \text{ MPa}$$

Thus,

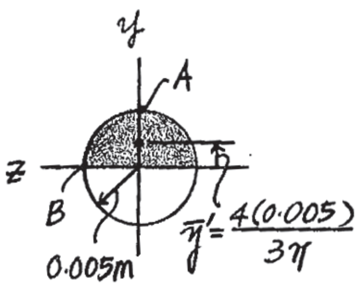
$$\left(\tau_{xy}\right)_A = 0 \qquad \textbf{Ans.}$$

$$\left(\tau_{xz}\right)_A = \left[\left(\tau_{xz}\right)_T\right]_A = 102 \text{ MPa} \qquad \textbf{Ans.}$$

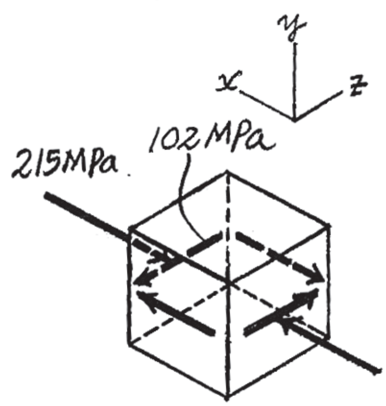
The state of stress at point A is represented on the element shown in Fig. *c*.



(a)



(b)

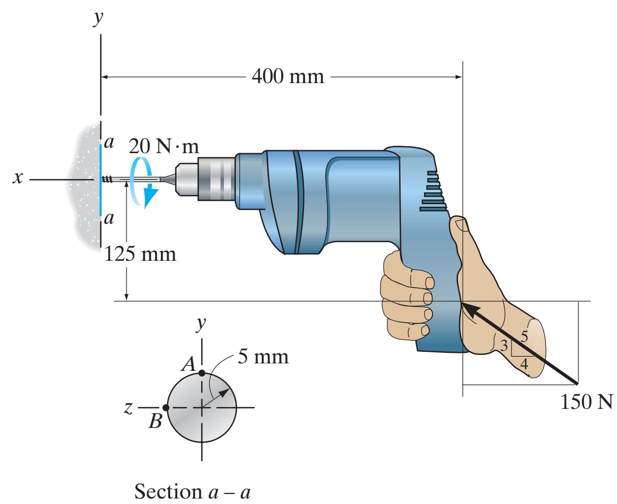


(c)

Ans:
$\sigma_A = 215$ MPa (C),
$\left(\tau_{xy}\right)_A = 0$,
$\left(\tau_{xz}\right)_A = 102$ MPa

***8–32.**

The drill is jammed in the wall and is subjected to the torque and force shown. Determine the state of stress at point B on the cross section of the drill bit at section a–a.



SOLUTION

Internal Loadings: Consider the equilibrium of the free-body diagram of the drill's right cut segment, Fig. a.

$$\Sigma F_x = 0; \quad N - 150\left(\frac{4}{5}\right) = 0 \qquad N = 120 \text{ N}$$

$$\Sigma F_y = 0; \quad 150\left(\frac{3}{5}\right) - V_y = 0 \qquad V_y = 90 \text{ N}$$

$$\Sigma M_x = 0; \quad 20 - T = 0 \qquad T = 20 \text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0; \; -150\left(\frac{3}{5}\right)(0.4) + 150\left(\frac{4}{5}\right)(0.125) + M_z = 0$$

$$M_z = 21 \text{ N}\cdot\text{m}$$

Section Properties: The cross-sectional area, the moment of inertia about the z axis, and the polar moment of inertia of the drill's cross section are

$$A = \pi\left(0.005^2\right) = 25\pi\left(10^{-6}\right) \text{ m}^2$$

$$I_z = \frac{\pi}{2}\left(0.005^4\right) = 0.15625\pi\left(10^{-9}\right) \text{ m}^4$$

$$J = \frac{\pi}{2}\left(0.005^4\right) = 0.3125\pi\left(10^{-9}\right) \text{ m}^4$$

Referring to Fig. b, Q_B is

$$Q_B = \bar{y}'A' = \frac{4(0.005)}{3\pi}\left[\frac{\pi}{2}\left(0.005^2\right)\right] = 83.333\left(10^{-9}\right) \text{ m}^3$$

Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z}$$

For point B, $y = 0$. Then

$$\sigma_B = \frac{-120}{25\pi\left(10^{-6}\right)} - 0 = -1.528 \text{ MPa} = 1.53 \text{ MPa (C)} \qquad \textbf{Ans.}$$

***8–32. Continued**

Shear Stress: The transverse shear stress developed at point B is

$$\left[\left(\tau_{xy}\right)_V\right]_B = \frac{V_y Q_B}{I_z t} = \frac{90\left[83.333\left(10^{-9}\right)\right]}{0.15625\pi\left(10^{-9}\right)(0.01)} = 1.528 \text{ MPa}$$

The torsional shear stress developed at point B is

$$\left[\left(\tau_{xy}\right)_T\right]_B = \frac{Tc}{J} = \frac{20(0.005)}{0.3125\pi\left(10^{-9}\right)} = 101.86 \text{ MPa}$$

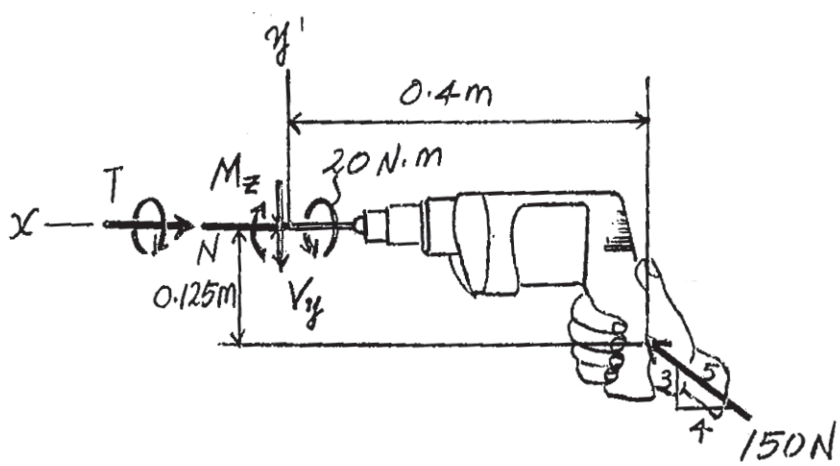
Thus,

$$\left(\tau_{xz}\right)_B = 0 \qquad \textbf{Ans.}$$

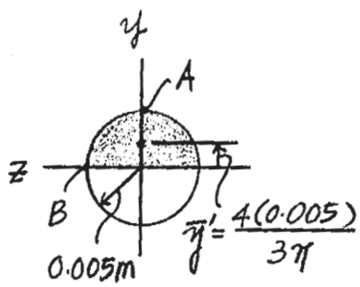
$$\left(\tau_{xy}\right)_B = \left[\left(\tau_{xy}\right)_T\right]_B - \left[\left(\tau_{xy}\right)_V\right]_B$$

$$= 101.86 - 1.528 = 100.33 \text{ MPa} = 100 \text{ MPa} \qquad \textbf{Ans.}$$

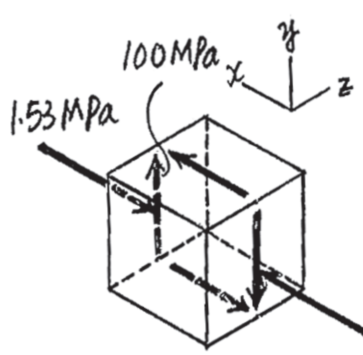
The state of stress at point B is represented on the element shown in Fig. d.



(a)



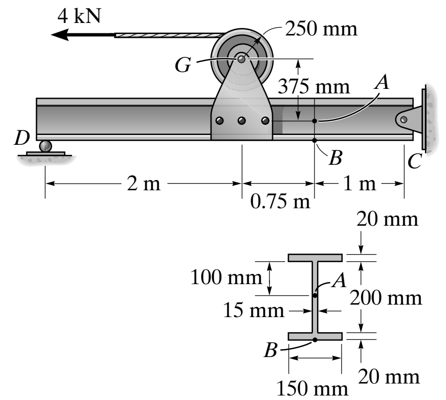
(b)



(d)

Ans:
$\sigma_B = 1.53$ MPa (C), $\left(\tau_{xz}\right)_B = 100$ MPa

8–33. Determine the state of stress at point A when the beam is subjected to the cable force of 4 kN. Indicate the result as a differential volume element.

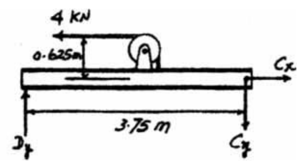


SOLUTION

Support Reactions:

$\circlearrowleft + \Sigma M_D = 0; \qquad 4(0.625) - C_y(3.75) = 0$

$C_y = 0.6667 \text{ kN}$



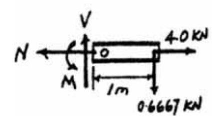
$\xrightarrow{+} \Sigma F_x = 0; \qquad C_x - 4 = 0 \qquad C_x = 4.00 \text{ kN}$

Internal Forces and Moment:

$\xrightarrow{+} \Sigma F_x = 0; \qquad 4.00 - N = 0 \qquad N = 4.00 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \qquad V - 0.6667 = 0 \qquad V = 0.6667 \text{ kN}$

$\circlearrowleft + \Sigma M_o = 0; \qquad M - 0.6667(1) = 0 \qquad M = 0.6667 \text{ kN} \cdot \text{m}$



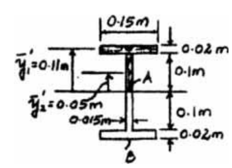
Section Properties:

$$A = 0.24(0.15) - 0.2(0.135) = 9.00\left(10^{-3}\right) \text{m}^2$$

$$I = \frac{1}{12}(0.15)\left(0.24^3\right) - \frac{1}{12}(0.135)\left(0.2^3\right) = 82.8\left(10^{-6}\right) \text{m}^4$$

$$Q_A = \Sigma \bar{y}' A' = 0.11(0.15)(0.02) + 0.05(0.1)(0.015)$$

$$= 0.405\left(10^{-3}\right) \text{m}^3$$

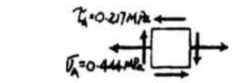


Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_A = \frac{4.00(10^3)}{9.00(10^{-3})} + \frac{0.6667(10^3)(0)}{82.8(10^{-6})}$$

$$= 0.444 \text{ MPa } (\mathbf{T}) \qquad \textbf{Ans.}$$



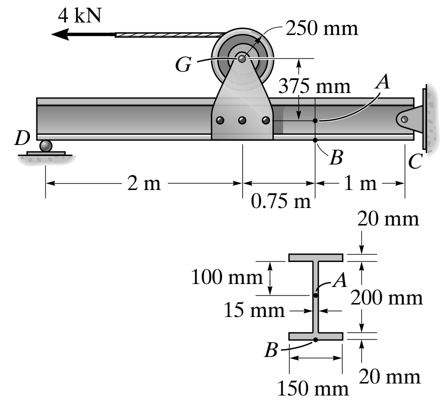
Shear Stress: Applying shear formula.

$$\tau_A = \frac{VQ_A}{It}$$

$$= \frac{0.6667(10^3)\left[0.405(10^{-3})\right]}{82.8(10^{-6})(0.015)} = 0.217 \text{ MPa} \qquad \textbf{Ans.}$$

Ans.
$\sigma_A = 0.444 \text{ MPa (T)}, \tau_A = 0.217 \text{ MPa}$

8–34. Determine the state of stress at point B when the beam is subjected to the cable force of 4 kN. Indicate the result as a differential volume element.

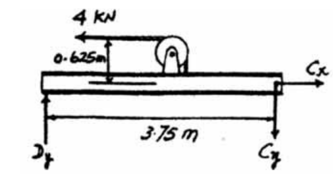


SOLUTION

Support Reactions:

$\circlearrowleft + \Sigma M_D = 0; \qquad 4(0.625) - C_y(3.75) = 0$

$$C_y = 0.6667 \text{ kN}$$



$\xrightarrow{+} \Sigma F_x = 0; \qquad C_x - 4 = 0 \qquad C_x = 4.00 \text{ kN}$

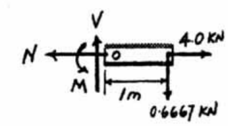
Internal Forces and Moment:

$\xrightarrow{+} \Sigma F_x = 0; \qquad 4.00 - N = 0 \qquad N = 4.00 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \qquad V - 0.6667 = 0 \qquad V = 0.6667 \text{ kN}$

$\circlearrowleft + \Sigma M_o = 0; \qquad M - 0.6667(1) = 0 \qquad M = 0.6667 \text{ kN} \cdot \text{m}$

Section Properties:

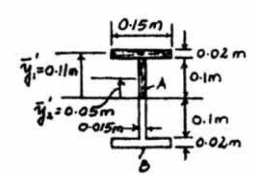


$$A = 0.24(0.15) - 0.2(0.135) = 9.00\left(10^{-3}\right) \text{ m}^2$$

$$I = \frac{1}{12}(0.15)\left(0.24^3\right) - \frac{1}{12}(0.135)\left(0.2^3\right) = 82.8\left(10^{-6}\right) \text{m}$$

$$Q_B = 0$$

Normal Stress:



$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_B = \frac{4.00(10^3)}{9.00(10^{-3})} - \frac{0.6667(10^3)(0.12)}{82.8(10^{-6})}$$

$$= -0.522 \text{ MPa} = 0.522 \text{ MPa (C)} \qquad \textbf{Ans.}$$

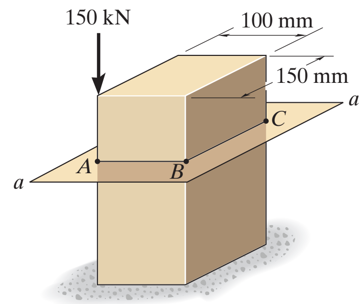
Shear Stress: Since $Q_B = 0$, then

$$\tau_B = 0 \qquad \textbf{Ans.}$$

Ans.
$A = 9.00\,(10^{-3})\text{ m}^2, I = 82.8\,(10^{-6})\text{ m},$
$Q_B = 0, \sigma_B = 0.522 \text{ MPa (C)}, \tau_B = 0$

8–35.

The block is subjected to the eccentric load shown. Determine the normal stress developed at points A and B. Neglect the weight of the block.



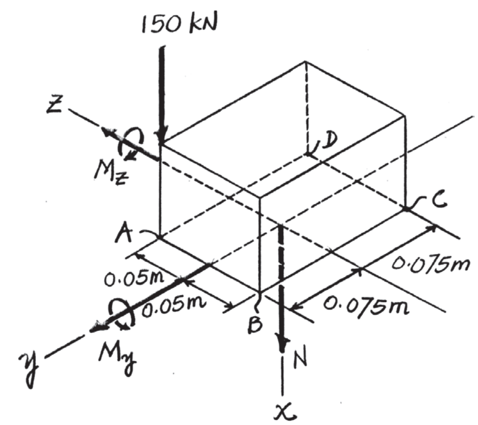
SOLUTION

Internal Loadings: Consider the equilibrium of the upper segment of the block sectioned through a–a, Fig. a.

$\Sigma F_x = 0;\quad N + 150 = 0\quad N = -150\text{ kN}$

$\Sigma M_y = 0;\quad M_y + 150(0.05) = 0\quad M_y = -7.50\text{ kN}\cdot\text{m}$

$\Sigma M_z = 0;\quad M_z - 150(0.075) = 0\quad M_z = 11.25\text{ kN}\cdot\text{m}$



(a)

Section Properties: For the rectangular cross section,

$$A = 0.1(0.15) = 0.015\text{ m}^2$$

$$I_y = \frac{1}{12}(0.15)\left(0.1^3\right) = 12.5\left(10^{-6}\right)\text{ m}^4$$

$$I_z = \frac{1}{12}(0.1)\left(0.15^3\right) = 28.125\left(10^{-6}\right)\text{ m}^4$$

Normal Stresses: For the combined loadings, the normal stress can be determined from

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

For point A, $y_A = 0.075$ m and $z_A = 0.05$ m

$$\sigma_A = \frac{-150\left(10^3\right)}{0.015} - \frac{11.25\left(10^3\right)(0.075)}{28.125\left(10^{-6}\right)} + \frac{\left[-7.50\left(10^3\right)\right](0.05)}{12.5\left(10^{-6}\right)}$$

$$= -70.0\left(10^6\right)\text{ Pa} = 70.0\text{ MPa (C)}$$ **Ans.**

For point B, $y_B = 0.075$ m and $z_B = -0.05$ m

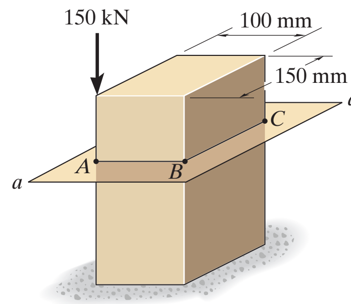
$$\sigma_B = \frac{-150\left(10^3\right)}{0.015} - \frac{11.25\left(10^3\right)(0.075)}{28.125\left(10^{-6}\right)} + \frac{\left[-7.50\left(10^3\right)\right](-0.05)}{12.5\left(10^{-6}\right)}$$

$$= -10.0\left(10^6\right)\text{ Pa} = 10.0\text{ MPa (C)}$$ **Ans.**

Ans:
$\sigma_A = 70.0$ MPa (C),
$\sigma_B = 10.0$ MPa (C)

***8–36.**

The block is subjected to the eccentric load shown. Sketch the normal-stress distribution acting over the cross section at section *a–a*. Neglect the weight of the block.



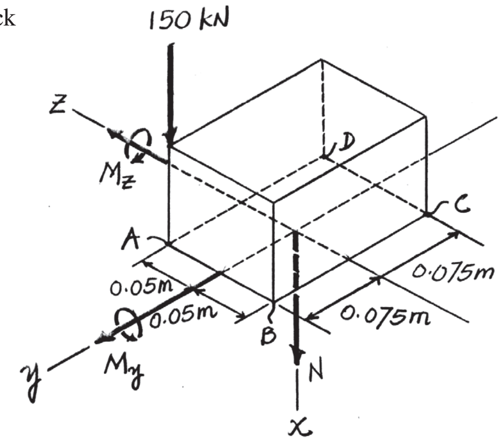
SOLUTION

Internal Loadings: Consider the equilibrium of the upper segment of the block sectioned through *a–a*, Fig. *a*.

$$\Sigma F_x = 0; \qquad N + 150 = 0 \qquad N = -150 \text{ kN}$$

$$\Sigma M_y = 0; \qquad M_y + 150(0.05) = 0 \qquad M_y = -7.50 \text{ kN}\cdot\text{m}$$

$$\Sigma M_z = 0; \qquad M_z - 150(0.075) = 0 \qquad M_z = 11.25 \text{ kN}\cdot\text{m}$$



(a)

Section Properties: For the rectangular cross section,

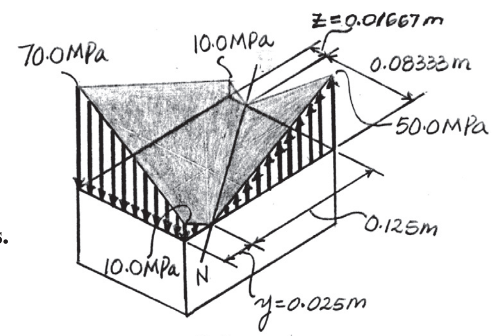
$$A = 0.1(0.15) = 0.015 \text{ m}^2$$

$$I_y = \frac{1}{12}(0.15)\left(0.1^3\right) = 12.5\left(10^{-6}\right) \text{ m}^4$$

$$I_z = \frac{1}{12}(0.1)\left(0.15^3\right) = 28.125\left(10^{-6}\right) \text{ m}^4$$

Normal Stress: For the combined loadings, the normal stress can be determined from

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$



(b)

For point A, $y_A = 0.075$ m and $z_A = 0.05$ m

$$\sigma_A = \frac{-150\left(10^3\right)}{0.015} - \frac{11.25\left(10^3\right)(0.075)}{28.125\left(10^{-6}\right)} + \frac{(-7.50)\left(10^3\right)(0.05)}{12.5\left(10^{-6}\right)}$$

$$= -70.0\left(10^6\right) \text{ Pa} = 70.0 \text{ MPa (C)} \qquad \textbf{Ans.}$$

For point B, $y_B = 0.075$ m and $z_B = -0.05$ m

$$\sigma_B = \frac{-150\left(10^3\right)}{0.015} - \frac{11.25\left(10^3\right)(0.075)}{28.125\left(10^{-6}\right)} + \frac{(-7.50)\left(10^3\right)(-0.05)}{12.5\left(10^{-6}\right)}$$

$$= -10.0\left(10^6\right) \text{ Pa} = 10.0 \text{ MPa (C)} \qquad \textbf{Ans.}$$

For point C, $y_C = -0.075$ m and $z_C = -0.05$ m

$$\sigma_C = \frac{-150\left(10^3\right)}{0.015} - \frac{11.25\left(10^3\right)(-0.075)}{28.125\left(10^{-6}\right)} + \frac{(-7.50)\left(10^3\right)(-0.05)}{12.5\left(10^{-6}\right)}$$

$$= 50.0\left(10^6\right) \text{ Pa} = 50.0 \text{ MPa (T)} \qquad \textbf{Ans.}$$

For point D, $y_D = -0.075$ m and $z_D = 0.05$ m

$$\sigma_D = \frac{-150\left(10^3\right)}{0.015} - \frac{11.25\left(10^3\right)(-0.075)}{28.125\left(10^{-6}\right)} + \frac{(-7.50)\left(10^3\right)(0.05)}{12.5\left(10^{-6}\right)}$$

$$= -10.0\left(10^6\right) \text{ Pa} = 10.0 \text{ MPa (C)} \qquad \textbf{Ans.}$$

***8–36. Continued**

The location of neutral axis can be found using similar triangles.

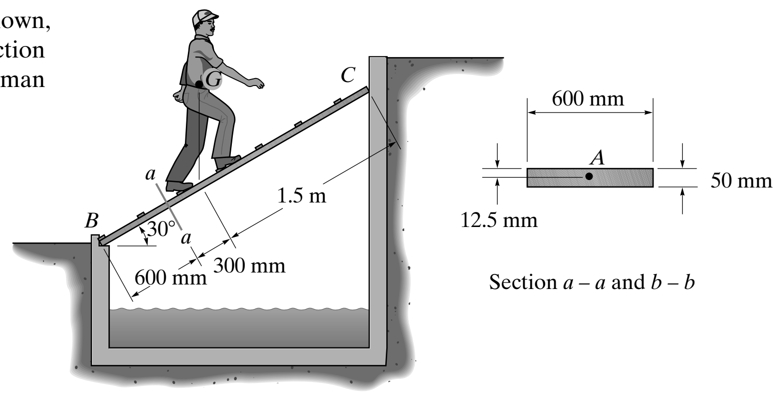
$$\frac{y}{10.0} = \frac{0.15 - y}{50.0}; \qquad y = 0.025 \text{ m}$$

$$\frac{z}{10.0} = \frac{0.1 - z}{50.0}; \qquad z = 0.01667 \text{ m}$$

Using these result, the normal stress distribution over the cross section shown in Fig. *b* can be sketched.

Ans:
$\sigma_A = 70.0$ MPa (C),
$\sigma_B = 10.0$ MPa (C),
$\sigma_C = 50.0$ MPa (T),
$\sigma_D = 10.0$ MPa (C)

8–37. If the 75-kg man stands in the position shown, determine the state of stress at point A on the cross section of the plank at section a–a. The center of gravity of the man is at G. Assume that the contact point at C is smooth.



Section a – a and b – b

SOLUTION

Support Reactions: Referring to the free-body diagram of the entire plank, Fig. a,

$$\circlearrowleft + \Sigma M_B = 0; \qquad F_C \sin 30°(2.4) - 75(9.81)\cos 30°(0.9) = 0$$

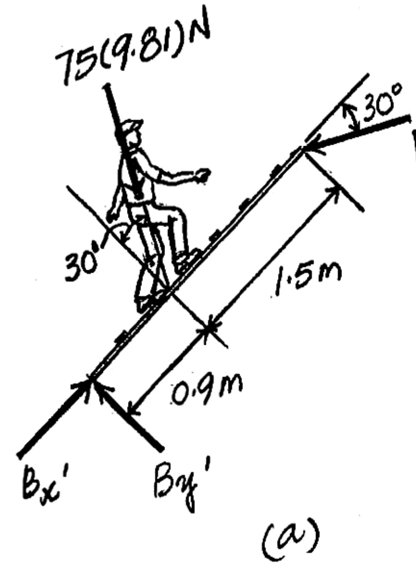
$$F_C = 477.88 \text{ N}$$

$$\Sigma F_{x'} = 0; \quad B_{x'} - 75(9.81)\sin 30° - 477.88 \cos 30° = 0$$

$$B_{x'} = 781.73 \text{ N}$$

$$\Sigma F_{y'} = 0; \quad B_{y'} + 477.88 \sin 30° - 75(9.81)\cos 30° = 0$$

$$B_{y'} = 398.24 \text{ N}$$



Internal Loadings: Consider the equilibrium of the free-body diagram of the plank's lower segment, Fig. b,

$$\Sigma F_{x'} = 0; \quad 781.73 - N = 0 \qquad N = 781.73 \text{ N}$$

$$\Sigma F_{y'} = 0; \quad 398.24 - V = 0 \qquad V = 398.24 \text{ N}$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad M - 398.24(0.6) = 0 \qquad M = 238.94 \text{ N}\cdot\text{m}$$

Section Properties: The cross-sectional area and the moment of inertia about the centroidal axis of the plank's cross section are

$$A = 0.6(0.05) = 0.03 \text{ m}^2$$

$$I = \frac{1}{12}(0.6)\left(0.05^3\right) = 6.25\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. c, Q_A is

$$Q_A = \bar{y}'A' = 0.01875(0.0125)(0.6) = 0.140625\left(10^{-3}\right) \text{m}^3$$

Normal Stress: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

For point A, $y = 0.0125$ m. Then

$$\sigma_A = \frac{-781.73}{0.03} - \frac{238.94(0.0125)}{6.25\left(10^{-6}\right)}$$

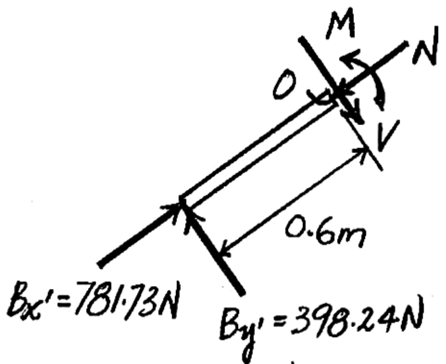
$$= -503.94 \text{ kPa} = 504 \text{ kPa (C)} \qquad \textbf{Ans.}$$

8–37. Continued

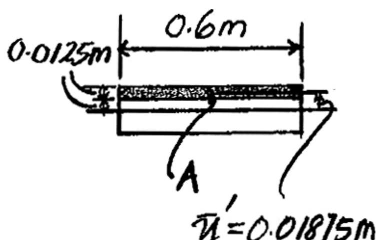
Shear Stress: The shear stress is contributed by transverse shear stress. Thus,

$$\tau_A = \frac{VQ_A}{It} = \frac{398.24\left[0.140625\left(10^{-3}\right)\right]}{6.25\left(10^{-6}\right)(0.6)} = 14.9 \text{ kPa} \qquad \textbf{Ans.}$$

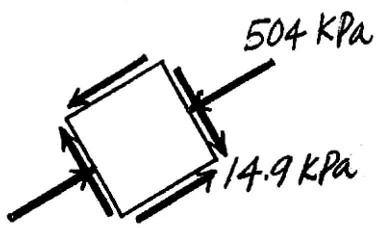
The state of stress at point A is represented on the element shown in Fig. *d*.



(b)



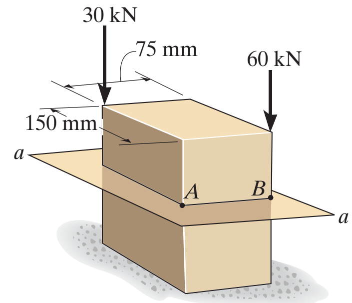
(c)



(d)

Ans.
$\sigma_A = 504$ kPa (C), $\tau_A = 14.9$ kPa

8–38. Determine the normal stress developed at points A and B. Neglect the weight of the block.



Referring to Fig. a,

$\Sigma F_x = (F_R)_x;$ $\quad -30 - 60 = F \quad F = -90\text{ kN}$

$\Sigma M_y = (M_R)_y;$ $\quad 30(0.0375) - 60(0.0375) = My \quad My = -1.125\text{ kN}\cdot\text{m}$

$\Sigma M_z = (M_R)_z;$ $\quad 60(0.075) - 30(0.075) = Mz \quad Mz = 2.25\text{ kN}\cdot\text{m}$

The cross-sectional area and moment of inertia about the y and z axes of the cross-section are

$$A = 0.075(0.15) = 0.01125\text{ m}^2$$

$$I_y = \frac{1}{12}(0.15)(0.075^3) = 5.27344(10^{-6})\text{ m}^4$$

$$I_z = \frac{1}{12}(0.75)(0.15^3) = 21.09375(10^{-6})\text{ m}^4$$

The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{F}{A} - \frac{M_z\,y}{I_z} + \frac{M_y\,z}{I_y}$$

For point A, $y = 0.075$ m and $z = -0.0375$ m

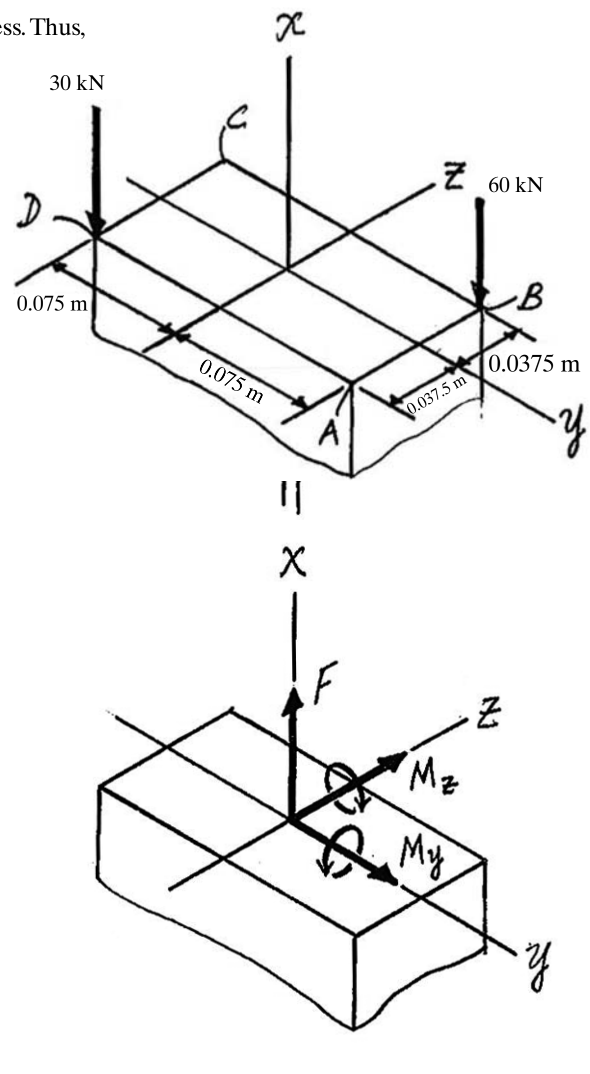
$$\sigma_A = \frac{-90(10^3)}{0.01125} - \frac{[2.25(10^3)](0.075)}{21.09375(10^{-6})} + \frac{[-1.125(10^3)](-0.0375)}{5.27344(10^{-6})}$$

$$= -8.00(10^6)\text{ N/m}^2 = 8.00\text{ MPa (C)}$$ **Ans.**

For point B, $y = 0.075$ m and $z = 0.0375$ m

$$\sigma_B = \frac{-90(10^3)}{0.01125} - \frac{[2.25(10^3)](0.075)}{21.09375(10^{-6})} + \frac{[-1.125(10^3)](0.0375)}{5.27344(10^{-6})}$$

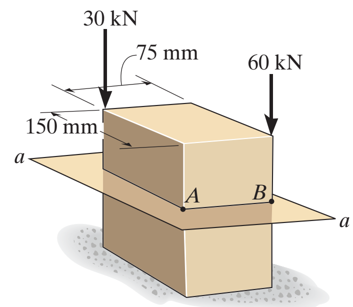
$$= -24.00(10^6)\text{ N/m}^2 = 24.00\text{ MPa (C)}$$ **Ans.**



(a)

Ans.
$\sigma_A = 8.00\text{ MPa (C)}, \sigma_B = 240\text{ MPa (C)}$

8–39. Sketch the normal stress distribution acting over the cross section at section *a–a*. Neglect the weight of the block.



SOLUTION

Referring to Fig. *a*,

$\Sigma F_x = (F_R)_x;\quad -30 - 60 = F \qquad F = -90 \text{ kN}$

$\Sigma M_y = (M_R)_y;\quad 30(0.0375) - 60(0.0375) = My \qquad My = -1.125 \text{ kN}\cdot\text{m}$

$\Sigma M_z = (M_R)_z;\quad 60(0.075) - 30(0.075) = Mz \qquad Mz = 2.25 \text{ kN}\cdot\text{m}$

The cross-sectional area and the moment of inertia about the y and z axes of the cross-section are

$$A = 0.075(0.15) = 0.01125 \text{ m}^2$$

$$I_y = \frac{1}{12}(0.15)(0.075^3) = 5.27344(10^{-6}) \text{ m}^4$$

$$I_z = \frac{1}{12}(0.75)(0.15^3) = 21.09375(10^{-6}) \text{ m}^4$$

The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{F}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

For point A, $y = 0.075$ m and $z = -0.0375$ m

$$\sigma_A = \frac{-90(10^3)}{0.01125} - \frac{[2.25(10^3)](0.075)}{21.09375(10^{-6})} + \frac{[-1.125(10^3)](-0.0375)}{5.27344(10^{-6})}$$

$$= -8.00(10^6) \text{ N/m}^2 = 8.00 \text{ MPa (C)}$$

For point B, $y = 0.075$ m and $z = 0.0375$ m

$$\sigma_B = \frac{-90(10^3)}{0.01125} - \frac{[2.25(10^3)](0.075)}{21.09375(10^{-6})} + \frac{[-1.125(10^3)](0.0375)}{5.27344(10^{-6})}$$

$$= -24.00(10^6) \text{ N/m}^2 = 24.00 \text{ MPa (C)}$$

8–39. Continued

For point C, $y = -0.075$ m and $z = 0.0375$ m

$$\sigma_C = \frac{-90(10^3)}{0.01125} - \frac{[2.25(10^3)](-0.075)}{21.09375(10^{-6})} + \frac{[-1.125(10^3)](0.0375)}{5.27344(10^{-6})}$$

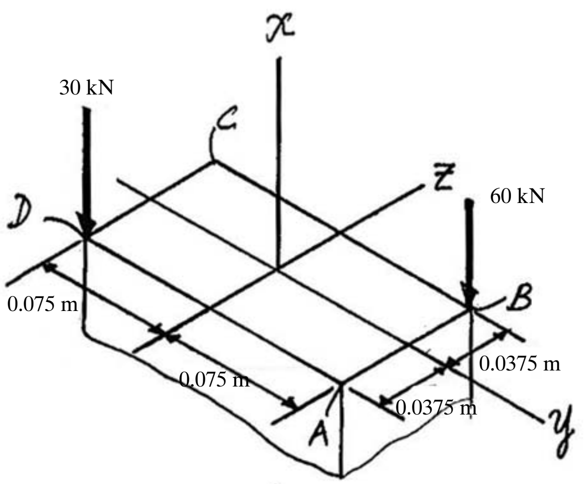
$$= -8.00(10^6)\ \text{N/m}^2 = 8.00\ \text{MPa (C)}$$

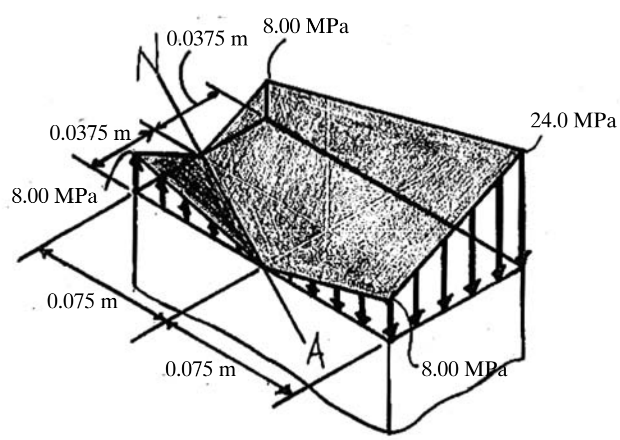
For point D, $y = -0.075$ m and $z = -0.0375$ m

$$\sigma_D = \frac{-90(10^3)}{0.01125} - \frac{[2.25(10^3)](-0.075)}{21.09375(10^{-6})} + \frac{[-1.125(10^3)](-0.0375)}{5.27344(10^{-6})}$$

$$= 8.00(10^6)\ \text{N/m}^2 = 8.00\ \text{MPa (T)}$$

The normal stress distribution over the cross-section is shown in Fig. *b*



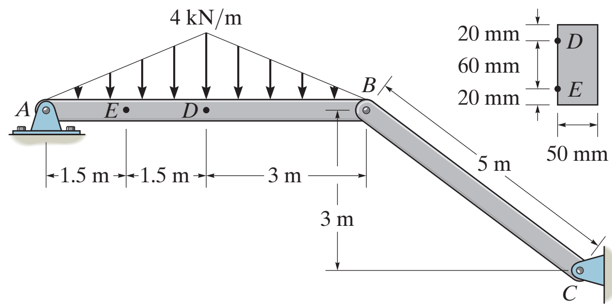


(b)

Ans.
$A = 0.01125$ m, $I_y = 5.27344(10^{-6})$ m^4,
$I_z = 21.09375\ (10^{-6})$ m^4, $\sigma_C = 8.00$ MPa (C)

***8–40.**

The frame supports the distributed load shown. Determine the state of stress acting at point D. Show the results on a differential element at this point.

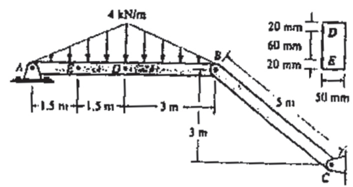


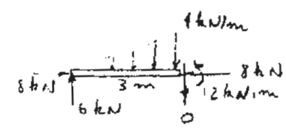
SOLUTION

$$\sigma_D = -\frac{P}{A} - \frac{My}{I} = -\frac{8(10^3)}{(0.1)(0.05)} - \frac{12(10^3)(0.03)}{\frac{1}{12}(0.05)(0.1)^3}$$

$\sigma_D = -88.0 \text{ MPa}$ **Ans.**

$\tau_D = 0$ **Ans.**

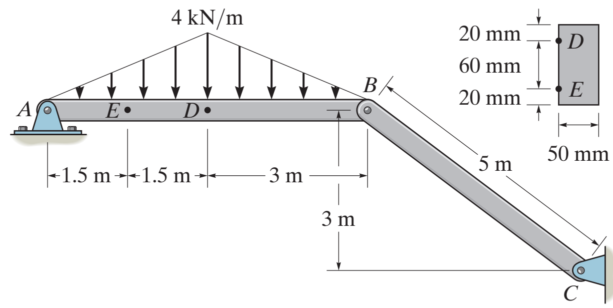




Ans:
$\sigma_D = -88.0 \text{ MPa}, \tau_D = 0$

8–41.

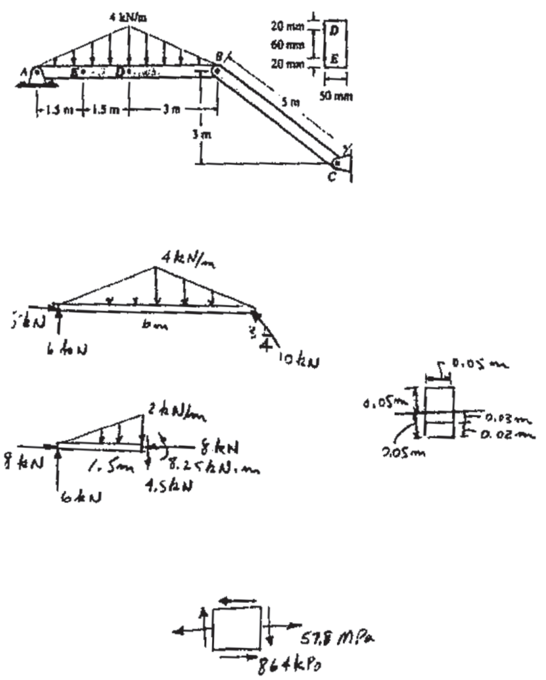
The frame supports the distributed load shown. Determine the state of stress acting at point E. Show the results on a differential element at this point.



SOLUTION

$$\sigma_E = -\frac{P}{A} - \frac{My}{I} = \frac{8(10^3)}{(0.1)(0.05)} + \frac{8.25(10^3)(0.03)}{\frac{1}{12}(0.05)(0.1)^3} = 57.8 \text{ MPa}$$ **Ans.**

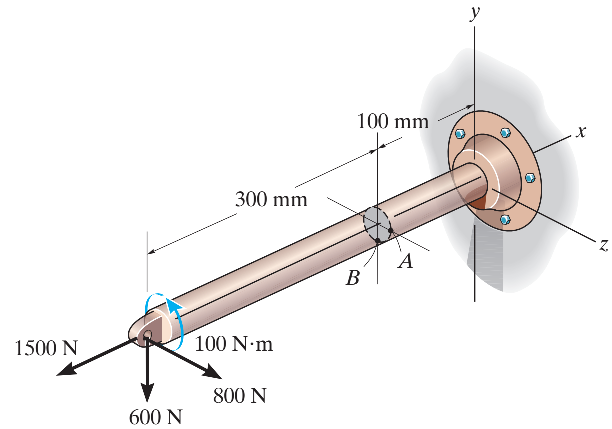
$$\tau_E = \frac{VQ}{It} = \frac{4.5(10^3)(0.04)(0.02)(0.05)}{\frac{1}{12}(0.05)(0.1)^3(0.05)} = 864 \text{ kPa}$$ **Ans.**



Ans:
$\sigma_E = 57.8$ MPa, $\tau_E = 864$ kPa

8–42.

The rod has a diameter of 40 mm. If it is subjected to the force system shown, determine the stress components that act at point A, and show the results on a volume element located at this point.



SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the rod being sectioned, Fig. *a*.

$$\Sigma F_x = 0; \quad N_x - 1500 = 0 \quad N_x = 1500\text{ N}$$

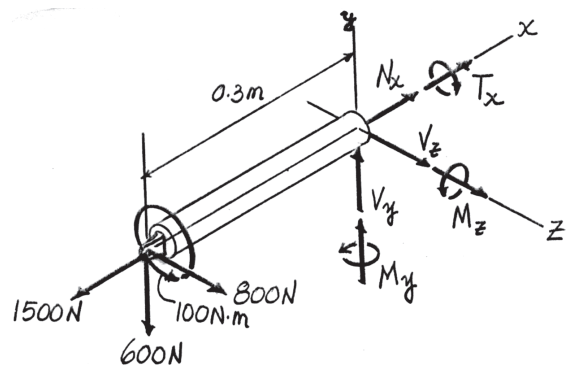
$$\Sigma F_y = 0; \quad V_y - 600 = 0 \quad V_y = 600\text{ N}$$

$$\Sigma F_z = 0; \quad V_z + 800 = 0 \quad V_z = -800\text{ N}$$

$$\Sigma M_x = 0; \quad T_x - 100 = 0 \quad T_x = 100\text{ N}\cdot\text{m}$$

$$\Sigma M_y = 0; \quad M_y + 800(0.3) = 0 \quad M_y = -240\text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0; \quad M_z + 600(0.3) = 0 \quad M_z = -180\text{ N}\cdot\text{m}$$



(a)

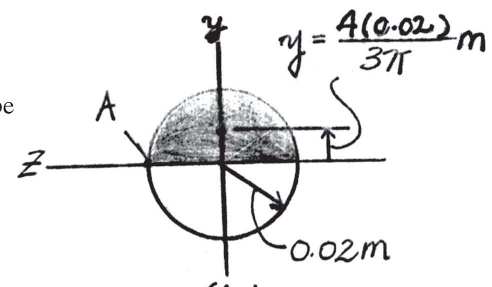
Section Properties: For the circular cross section, Fig. *b*,

$$A = \pi c^2 = \pi\left(0.02^2\right) = 0.400\left(10^{-3}\right)\pi\text{ m}^2$$

$$I_y = I_z = \frac{\pi}{4}c^4 = \frac{\pi}{4}\left(0.02^4\right) = 40.0\left(10^{-9}\right)\pi\text{ m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(0.02^4\right) = 80.0\left(10^{-9}\right)\pi\text{ m}^4$$

$$(Q_A)_z = 0 \qquad (Q_A)_y = \bar{y}'A' = \frac{4(0.02)}{3\pi}\left[\frac{\pi}{2}\left(0.02^2\right)\right] = 5.3333\left(10^{-6}\right)\text{ m}^3$$



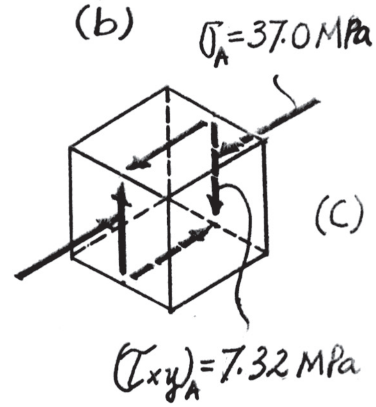
(b)

Normal Stress: For the combined loadings, the normal stress at point A can be determined from

$$\sigma_x = \sigma_A = \frac{N_x}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

$$= \frac{1500}{0.400\left(10^{-3}\right)\pi} - \frac{(-180)(0)}{40.0\left(10^{-9}\right)\pi} + \frac{(-240)(0.02)}{40.0\left(10^{-9}\right)\pi}$$

$$= -37.00\left(10^6\right)\text{ Pa} = 37.0\text{ MPa (C)}$$ **Ans.**



(c)

Shear Stress: The transverse shear stress in z and y directions and the torsional shear stress can be obtained using the shear formula $\tau_V = \frac{VQ}{It}$ and the torsion formula $\tau_T = \frac{T\rho}{J}$ respectively.

$$(\tau_{xy})_A = (\tau_V)_y - \tau_T$$

$$= \frac{600\left[5.3333\left(10^{-6}\right)\right]}{40.0\left(10^{-9}\right)\pi(0.04)} - \frac{100(0.02)}{80.0\left(10^{-9}\right)\pi}$$

$$= -7.321\left(10^6\right)\text{ Pa} = -7.32\text{ MPa}$$ **Ans.**

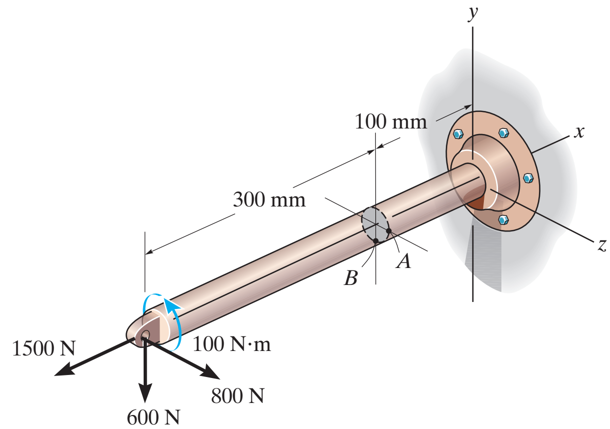
$$(\tau_{xz})_A = (\tau_V)_z = 0$$ **Ans.**

Using these results, the state of stress at point A can be represented by the differential volume element show in Fig. *c*.

Ans:
$\sigma_A = 37.0$ MPa (C),
$(\tau_{xy})_A = -7.32$ MPa,
$(\tau_{xz})_A = 0$

8–43.

Solve Prob. 8–40 for point B.



SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the rod being sectioned, Fig. a.

$$\Sigma F_x = 0; \quad N_x - 1500 = 0 \quad N_x = 1500\text{ N}$$

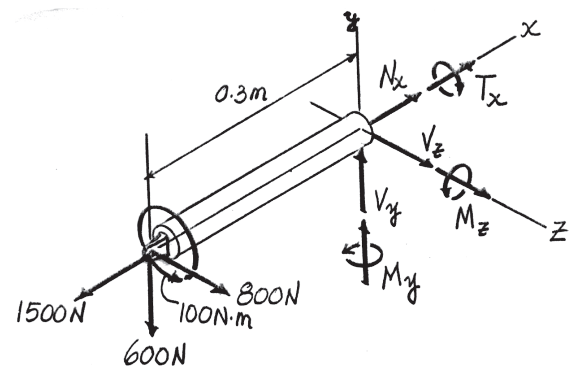
$$\Sigma F_y = 0; \quad V_y - 600 = 0 \quad V_y = 600\text{ N}$$

$$\Sigma F_z = 0; \quad V_z + 800 = 0 \quad V_z = -800\text{ N}$$

$$\Sigma M_x = 0; \quad T_x - 100 = 0 \quad T_x = 100\text{ N}\cdot\text{m}$$

$$\Sigma M_y = 0; \quad M_y + 800(0.3) = 0 \quad M_y = -240\text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0; \quad M_z + 600(0.3) = 0 \quad M_z = -180\text{ N}\cdot\text{m}$$



(a)

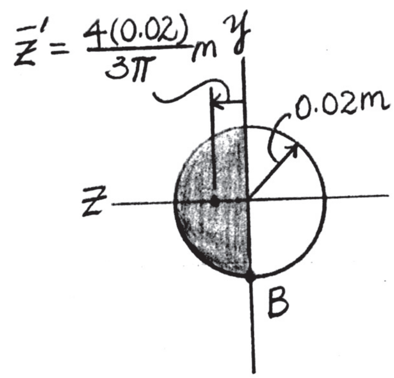
Section Properties: For the circular cross section,

$$A = \pi c^2 = \pi\left(0.02^2\right) = 0.400\left(10^{-3}\right)\pi \text{ m}^2$$

$$I_y = I_z = \frac{\pi}{4}c^4 = \frac{\pi}{4}\left(0.02^4\right) = 40.0\left(10^{-9}\right)\pi \text{ m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(0.02^4\right) = 80.0\left(10^{-9}\right)\pi \text{ m}^4$$

$$(Q_B)_z = \bar{z}'A' = \frac{4(0.02)}{3\pi}\left[\frac{\pi}{2}\left(0.02^2\right)\right] = 5.3333\left(10^{-6}\right)\text{ m}^3 \quad (Q_B)_y = 0$$



(b)

Normal Stress: For the combined loadings, the normal stress at point B can be determined from

$$\sigma_x = \sigma_B = \frac{N_x}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y}$$

$$= \frac{1500}{0.400\left(10^{-3}\right)\pi} - \frac{(-180)(-0.02)}{40.0\left(10^{-9}\right)\pi} + \frac{(-240)(0)}{40.0\left(10^{-9}\right)\pi}$$

$$= -27.45\left(10^6\right) = 27.5\text{ MPa (C)}$$ **Ans.**

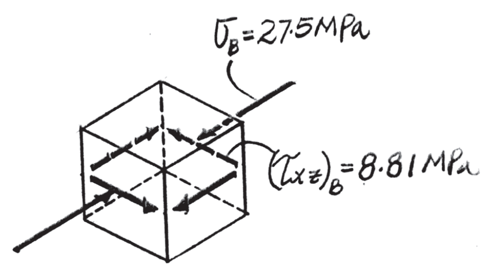
Shear Stress: The transverse shear stress in z and y directions and the torsional shear stress can be obtained using the shear formula $\tau_V = \dfrac{VQ}{It}$ and the torsion formula $\tau_T = \dfrac{T\rho}{J}$, respectively.

$$(\tau_{xz})_B = (\tau_V)_z - \tau_T$$

$$= \frac{-800\left[5.3333\left(10^{-6}\right)\right]}{40.0\left(10^{-9}\right)\pi(0.04)} - \frac{100(0.02)}{80.0\left(10^{-9}\right)\pi}$$

$$= -8.807\left(10^6\right)\text{ Pa} = -8.81\text{ MPa}$$ **Ans.**

$$(\tau_{xy})_B = (\tau_V)_y = 0$$ **Ans.**

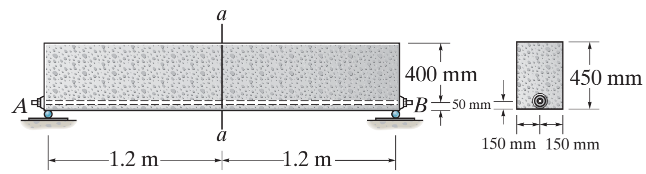


(c)

Using these results, the state of stress at point B, can be represented by the differential volume element shown in Fig. c.

Ans:
$\sigma_B = 27.5$ MPa (C),
$(\tau_{xz})_B = -8.81$ MPa,
$(\tau_{xy})_B = 0$

***8–44.** Since concrete can support little or no tension, this problem can be avoided by using wires or rods to *prestress* the concrete once it is formed. Consider the simply supported beam shown, which has a rectangular cross section of 450 mm by 300 mm. If concrete has a specific weight of 24 kN/m^3, determine the required tension in rod AB, which runs through the beam so that no tensile stress is developed in the concrete at its center section a–a. Neglect the size of the rod and any deflection of the beam.



SOLUTION

Support Reactions: As shown on FBD.

Internal Force and Moment:

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad T - N = 0 \qquad N = T$$

$$\circlearrowleft + \Sigma M_o = 0; \qquad M + T(0.175) - 3.888(10^3)(0.6) = 0$$

$$M = \{2.3328(10^3) - 0.175T\}\ \text{N}\cdot\text{m}$$

Section Properties:

$$A = 0.3(0.45) = 0.135\ \text{m}^2$$

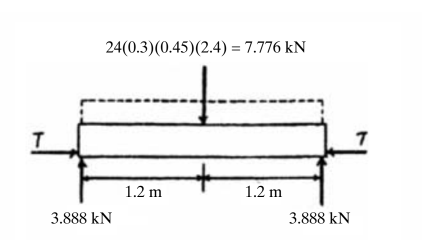
$$I = \frac{1}{12}0.3(0.45^3) = 2.278125(10^{-3})\ \text{m}^4$$

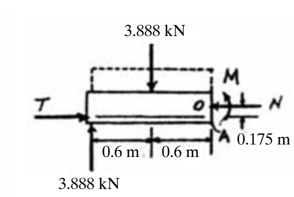
Normal Stress: Requires $\sigma_A = 0$

$$\sigma_A = 0 = \frac{N}{A} + \frac{Mc}{I}$$

$$0 = \frac{-T}{0.135} + \frac{[2.3328(10^3) - 0.175T](0.225)}{2.278125(10^{-3})}$$

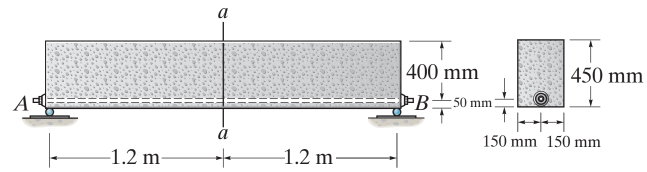
$$T = 9.3312(10^3)\ \text{N} = 9.33\ \text{kN}$$ **Ans.**





Ans.
$T = 9.33$ kN

8–45. Solve Prob. 8–38 if the rod has a diameter of 12 mm. Use the transformed area method discussed in Sec 6.6. $E_{st} = 200$ GPa, $E_c = 25$ GPa.



SOLUTION

Support Reactions: As shown on FBD.

Section Properties:

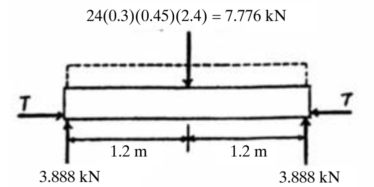
$$n = \frac{E_{st}}{E_{con}} = \frac{200}{25} = 8.00$$

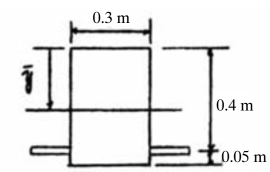
$$A_{con} = (n-a)A_{st} = (8.00-1)\left[\left(\frac{\pi}{4}\right)(0.012^2)\right] = 0.79168(10^{-3})\text{ m}^2$$

$$A = 0.3(0.45) + 0.79168(10^{-3}) = 0.135792\text{ m}^2$$

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{0.225(0.3)(0.45) + 0.4[0.79168(10^{-3})]}{0.135792} = 0.22602\text{ m}$$

$$I = \frac{1}{12}0.3(0.45^3) + (0.3)(0.45)(0.22602 - 0.225)^2$$
$$+\, 0.79168(10^{-3})(0.4 - 0.22602)^2 = 2.30223(10^{-3})\text{ m}^4$$



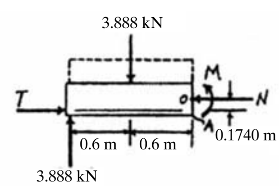


Internal Force and Moment:

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T - N = 0 \qquad N = T$$

$$\circlearrowleft + \Sigma M_o = 0; \qquad M + T(0.1740) - [3.888(10^6)](0.6) = 0$$

$$M = \{2.3328(10^3) - 0.1740T\}\text{ N}\cdot\text{m}$$



Normal Stress: Requires $\sigma_A = 0$

$$\sigma_A = 0 = \frac{N}{A} + \frac{Mc}{I}$$

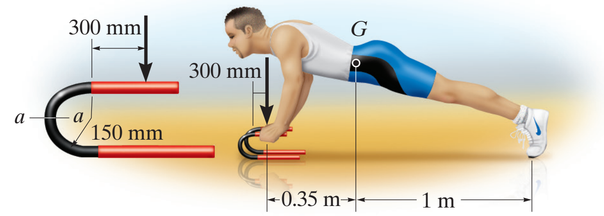
$$0 = \frac{-T}{0.135792} + \frac{[2.3328(10^3) - 0.1740T](0.45 - 0.22602)}{2.30223(10^{-3})}$$

$$T = 9.343(10^3)\text{ N} = 9.34\text{ kN}$$ **Ans.**

Ans.
$T = 9.34$ kN

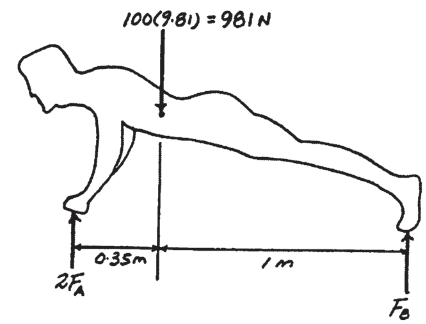
8–46.

The man has a mass of 100 kg and center of mass at G. If he holds himself in the position shown, determine the maximum tensile and compressive stress developed in the curved bar at section a–a. He is supported uniformly by two bars, each having a diameter of 25 mm. Assume the floor is smooth. Use the curved-beam formula to calculate the bending stress.



SOLUTION

Equilibrium: For the man



$$\circlearrowleft + \Sigma M_B = 0; \quad 981(1) - 2F_A(1.35) = 0 \qquad F_A = 363.33 \text{ N}$$

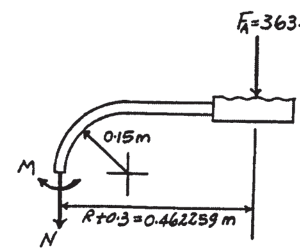
Section Properties:

$$\bar{r} = 0.15 + \frac{0.025}{2} = 0.1625 \text{ m}$$

$$\int_A \frac{dA}{r} = 2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right)$$

$$= 2\pi\left(0.1625 - \sqrt{0.1625^2 - 0.0125^2}\right)$$

$$= 3.02524\left(10^{-3}\right) \text{ m}$$



$$A = \pi\left(0.0125^2\right) = 0.490874\left(10^{-3}\right) \text{ m}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.490874\left(10^{-3}\right)}{3.02524\left(10^{-3}\right)} = 0.162259 \text{ m}$$

$$\bar{r} - R = 0.1625 - 0.162259 = 0.240741\left(10^{-3}\right) \text{ m}$$

Internal Force and Moment: The internal moment must be computed about the neutral axis.

$$+\uparrow \Sigma F_y = 0; \quad -363.33 - N = 0 \qquad N = -363.33 \text{ N}$$

$$\circlearrowleft + \Sigma M_o = 0; \quad -M - 363.33(0.462259) = 0 \qquad M = -167.95 \text{ N}\cdot\text{m}$$

Normal Stress: Apply the curved-beam formula.
For tensile stress

$$(\sigma_t)_{\max} = \frac{N}{A} + \frac{M(R - r_2)}{Ar_2(\bar{r} - R)}$$

$$= \frac{-363.33}{0.490874\left(10^{-3}\right)} + \frac{-167.95(0.162259 - 0.175)}{0.490874\left(10^{-3}\right)(0.175)0.240741\left(10^{-3}\right)}$$

$$= 103 \text{ MPa (T)} \qquad \textbf{Ans.}$$

For compressive stress,

$$(\sigma_c)_{\max} = \frac{N}{A} + \frac{M(R - r_1)}{Ar_1(\bar{r} - R)}$$

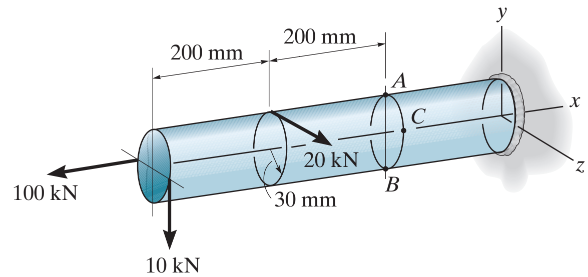
$$= \frac{-363.33}{0.490874\left(10^{-3}\right)} + \frac{-167.95(0.162259 - 0.15)}{0.490874\left(10^{-3}\right)(0.15)0.240741\left(10^{-3}\right)}$$

$$= -117 \text{ MPa} = 117 \text{ MPa (C)} \qquad \textbf{Ans.}$$

Ans:
$(\sigma_t)_{\max} = 103$ MPa (T),
$(\sigma_c)_{\max} = 117$ MPa (C)

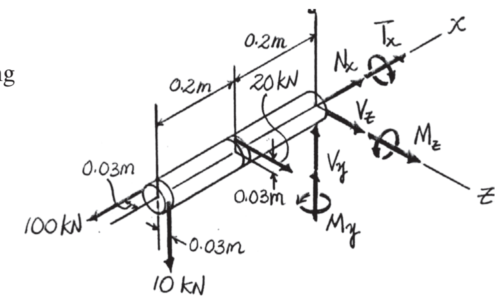
8–47.

The solid rod is subjected to the loading shown. Determine the state of stress at point A, and show the results on a differential volume element located at this point.



SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the rod being sectioned, Fig. a.



(a)

$$\Sigma F_x = 0; \qquad N_x - 100 = 0 \qquad N_x = 100\text{ kN}$$

$$\Sigma F_y = 0; \qquad V_y - 10 = 0 \qquad V_y = 10\text{ kN}$$

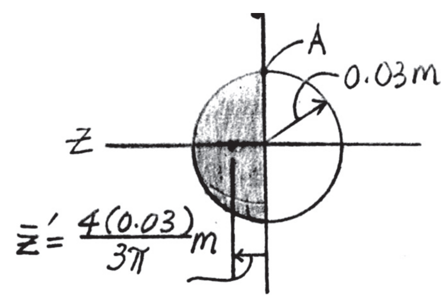
$$\Sigma F_z = 0; \qquad V_z + 20 = 0 \qquad V_z = -20\text{ kN}$$

$$\Sigma M_x = 0; \qquad T_x + 20(0.03) + 10(0.03) = 0 \qquad T_x = -0.9\text{ kN}\cdot\text{m}$$

$$\Sigma M_y = 0; \qquad M_y + 20(0.2) + 100(0.03) = 0 \qquad M_y = -7.00\text{ kN}\cdot\text{m}$$

$$\Sigma M_z = 0; \qquad M_z + 10(0.4) = 0 \qquad M_z = -4.00\text{ kN}\cdot\text{m}$$

Section Properties: For the circular cross section, Fig. b,



(b)

$$A = \pi c^2 = \pi\left(0.03^2\right) = 0.9\left(10^{-3}\right)\pi\text{ m}^2$$

$$I_y = I_z = \frac{\pi}{4}c^4 = \frac{\pi}{4}\left(0.03^4\right) = 0.2025\left(10^{-6}\right)\pi\text{ m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(0.03^4\right) = 0.405\left(10^{-6}\right)\pi\text{ m}^4$$

$$(Q_A)_z = \bar{z}'A' = \frac{4(0.03)}{3\pi}\left[\frac{\pi}{2}\left(0.03^2\right)\right] = 18.0\left(10^{-6}\right)\text{ m}^3$$

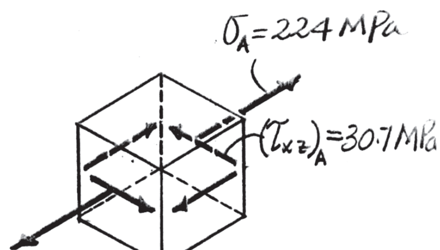
$$(Q_A)_y = 0$$

Normal Stress: For the combined loadings, the normal stress at point A can be determined from

$$\sigma_x = \sigma_A = \frac{N_x}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

$$= \frac{100\left(10^3\right)}{0.9\left(10^{-3}\right)\pi} - \frac{\left[-4.00\left(10^3\right)\right](0.03)}{0.2025\left(10^{-6}\right)\pi} + \frac{\left[-7.00\left(10^3\right)\right](0)}{0.2025\left(10^{-6}\right)\pi}$$

$$= 224.00\left(10^6\right)\text{ Pa} = 224\text{ MPa (T)} \qquad \textbf{Ans.}$$



(c)

Shear Stress: The transverse shear stress in z and y directions and the torsional shear stress can be obtained using the shear formula $\tau_V = \frac{VQ}{It}$ and the torsion formula $\tau_T = \frac{T\rho}{J}$, respectively.

$$(\tau_{xz})_A = (\tau_V)_z + \tau_T$$

$$= \frac{-20\left(10^3\right)\left[18.0\left(10^{-6}\right)\right]}{0.2025\left(10^{-6}\right)\pi(0.06)} + \frac{-0.9\left(10^3\right)(0.03)}{0.405\left(10^{-6}\right)\pi}$$

$$= -30.65\left(10^6\right)\text{ Pa} = -30.7\text{ MPa} \qquad \textbf{Ans.}$$

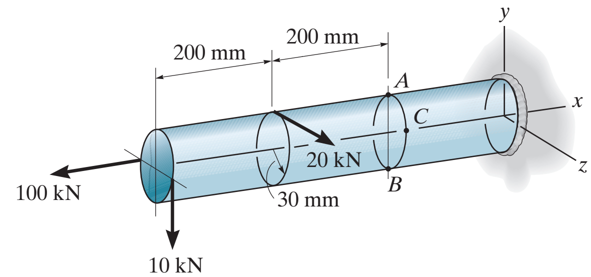
$$(\tau_{xy})_A = (\tau_V)_y = 0 \qquad \textbf{Ans.}$$

Using these results, the state of stress at point A can be represented by the differential volume element shown in Fig. c.

Ans:
$\sigma_A = 224$ MPa (T),
$(\tau_{xz})_A = -30.7$ MPa,
$(\tau_{xy})_A = 0$

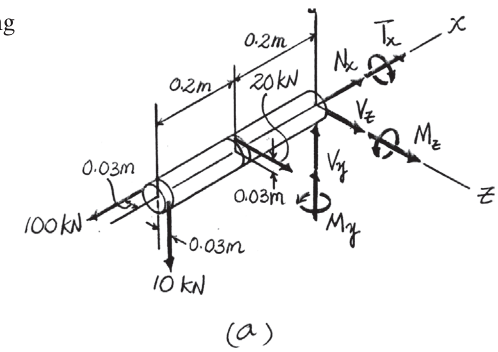
***8–48.**

The solid rod is subjected to the loading shown. Determine the state of stress at point B, and show the results on a differential volume element at this point.



SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the rod being sectioned, Fig. *a*.



$$\Sigma F_x = 0; \quad N_x - 100 = 0 \quad N_x = 100\text{ kN}$$

$$\Sigma F_y = 0; \quad V_y - 10 = 0 \quad V_y = 10\text{ kN}$$

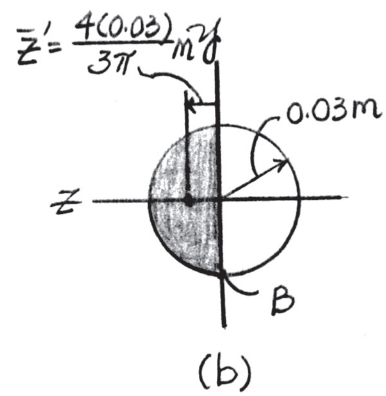
$$\Sigma F_z = 0; \quad V_z + 20 = 0 \quad V_z = -20\text{ kN}$$

$$\Sigma M_x = 0; \quad T_x + 20(0.03) + 10(0.03) = 0 \quad T_x = -0.9\text{ kN}\cdot\text{m}$$

$$\Sigma M_y = 0; \quad M_y + 20(0.2) + 100(0.03) = 0 \quad M_y = -7.00\text{ kN}\cdot\text{m}$$

$$\Sigma M_z = 0; \quad M_z + 10(0.4) = 0 \quad M_z = -4.00\text{ kN}\cdot\text{m}$$

Section Properties: For the circular cross section, Fig. *b*,



$$A = \pi c^2 = \pi(0.03^2) = 0.9(10^{-3})\pi\text{ m}^2$$

$$I_y = I_z = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.03^4) = 0.2025(10^{-6})\pi\text{ m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\pi\text{ m}^4$$

$$(Q_B)_z = \bar{z}'A' = \frac{4(0.03)}{3\pi}\left[\frac{\pi}{2}(0.03^2)\right] = 18.0(10^{-6})\text{ m}^3$$

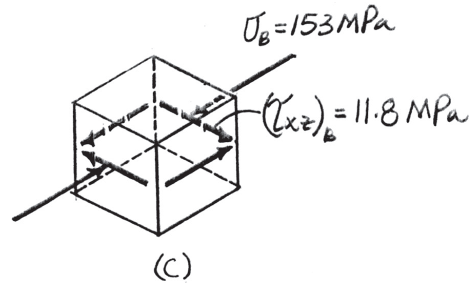
$$(Q_B)_y = 0$$

Normal Stress: For the combined loadings, the normal stress at point B can be determined from

$$\sigma_x = \sigma_B = \frac{N_x}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y}$$

$$= \frac{100(10^3)}{0.9(10^{-3})\pi} - \frac{\left[-4.00(10^3)\right](-0.03)}{0.2025(10^{-6})\pi} + \frac{\left[-7.00(10^3)\right](0)}{0.2025(10^{-6})\pi}$$

$$= -153.26(10^6)\text{ Pa} = 153\text{ MPa (C)} \qquad \textbf{Ans.}$$



Shear Stress: The transverse shear stress in z and y directions and the torsional shear stress can be determined using the shear formula $\tau_V = \dfrac{VQ}{It}$ and the torsion formula $\tau_T = \dfrac{T\rho}{J}$, respectively.

$$(\tau_{xz})_B = -\tau_T + (\tau_V)_z$$

$$= \frac{0.9(10^3)(0.03)}{0.405(10^{-6})\pi} + \frac{-20(10^3)\left[18.0(10^{-6})\right]}{0.2025(10^{-6})\pi(0.06)}$$

$$= 11.79(10^6)\text{ Pa} = 11.8\text{ MPa} \qquad \textbf{Ans.}$$

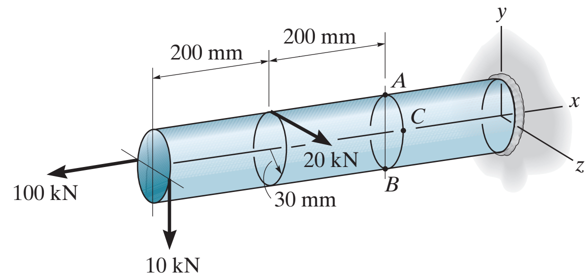
$$(\tau_{xy})_B = (\tau_V)_y = 0 \qquad \textbf{Ans.}$$

Using these results, the state of stress at point B can be represented by the differential volume element shown in Fig. *c*.

Ans:
$\sigma_B = 153$ MPa (C),
$(\tau_{xz})_B = 11.8$ MPa,
$(\tau_{xy})_B = 0$

8–49.

The solid rod is subjected to the loading shown. Determine the state of stress at point C, and show the results on a differential volume element at this point.



SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the rod being sectioned, Fig. *a*.

$$\Sigma F_x = 0; \quad N_x - 100 = 0 \quad N_x = 100 \text{ kN}$$

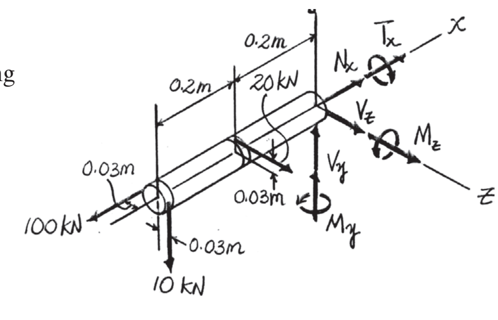
$$\Sigma F_y = 0; \quad V_y - 10 = 0 \quad V_y = 10 \text{ kN}$$

$$\Sigma F_z = 0; \quad V_z + 20 = 0 \quad V_z = -20 \text{ kN}$$

$$\Sigma M_x = 0; \quad T_x + 20(0.03) + 10(0.03) = 0 \quad T_x = -0.9 \text{ kN}\cdot\text{m}$$

$$\Sigma M_y = 0; \quad M_y + 20(0.2) + 100(0.03) = 0 \quad M_y = -7.00 \text{ kN}\cdot\text{m}$$

$$\Sigma M_z = 0; \quad M_z + 10(0.4) = 0 \quad M_z = -4.00 \text{ kN}\cdot\text{m}$$



(a)

Section Properties: For the circular cross section, Fig. *b*,

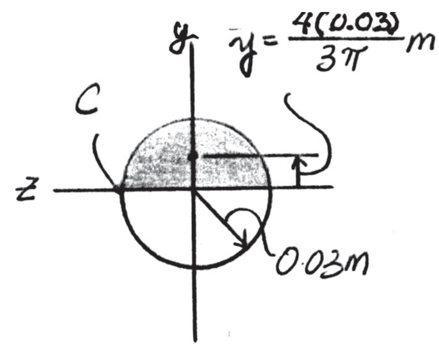
$$A = \pi c^2 = \pi(0.03^2) = 0.9(10^{-3})\pi \text{ m}^2$$

$$I_y = I_z = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.03^4) = 0.2025(10^{-6})\pi \text{ m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\pi \text{ m}^4$$

$$(Q_C)_y = \bar{y}'A' = \left[\frac{4(0.03)}{3\pi}\right]\left[\frac{\pi}{2}(0.03^2)\right] = 18.0(10^{-6}) \text{ m}^3$$

$$(Q_C)_z = 0$$



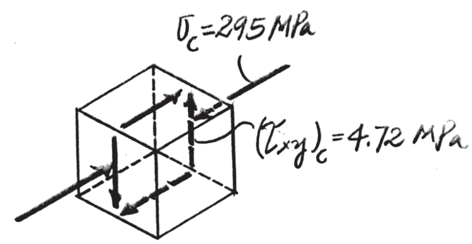
(b)

Normal Stress: For the combine loadings, the normal stress at point C can be determined from

$$\sigma_x = \sigma_C = \frac{N_x}{A} - \frac{M_z y_C}{I_z} + \frac{M_y z_C}{I_y}$$

$$= \frac{100(10^3)}{0.9(10^{-3})\pi} - \frac{[-4.00(10^3)](0)}{0.2025(10^{-6})\pi} + \frac{[-7.00(10^3)](0.03)}{0.2025(10^{-6})\pi}$$

$$= -294.73(10^6) \text{ Pa} = 295 \text{ MPa (C)} \qquad \textbf{Ans.}$$



(C)

Shear Stress: The transverse shear stress in z and y directions and the torsional shear stress can be determined using the shear formula $\tau_V = \dfrac{VQ}{It}$ and the torsion formula $\tau_T = \dfrac{T\rho}{J}$, respectively.

$$(\tau_{xy})_C = (\tau_V)_y - \tau_T$$

$$= \frac{10(10^3)[18.0(10^{-6})]}{0.2025(10^{-6})\pi(0.06)} - \frac{-0.9(10^3)(0.03)}{0.405(10^{-6})\pi}$$

$$= 25.94(10^6) \text{ Pa} = 25.9 \text{ MPa} \qquad \textbf{Ans.}$$

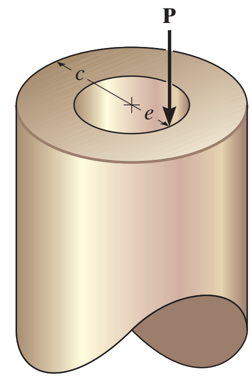
$$(\tau_{xz})_C = (\tau_V)_z = 0 \qquad \textbf{Ans.}$$

Using these results, the state of stress at point C can be represented by the differential volume element shown in Fig. *c*.

Ans:
$\sigma_C = 295$ MPa (C),
$(\tau_{xy})_C = 25.9$ MPa,
$(\tau_{xz})_C = 0$

8–50.

The post has a circular cross section of radius c. Determine the maximum radius e at which the load **P** can be applied so that no part of the post experiences a tensile stress. Neglect the weight of the post.

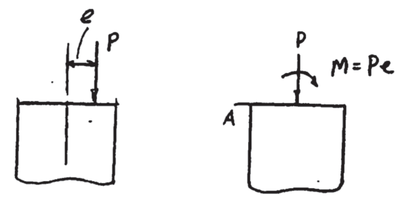


SOLUTION

Require $\sigma_A = 0$

$$\sigma_A = 0 = \frac{P}{A} + \frac{Mc}{I}; \qquad 0 = \frac{-P}{\pi c^2} + \frac{(Pe)c}{\frac{\pi}{4}c^4}$$

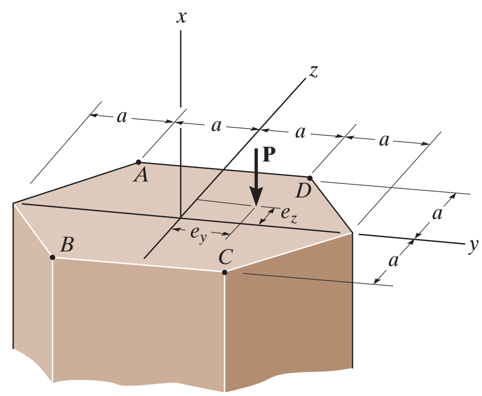
$$e = \frac{c}{4} \qquad \textbf{Ans.}$$



Ans:
$e = \frac{c}{4}$

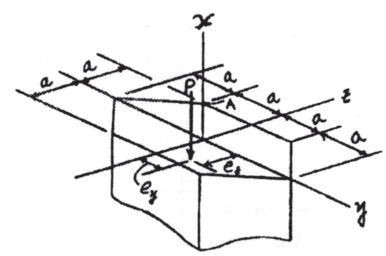
8–51.

The post having the dimensions shown is subjected to the load **P**. Specify the region to which this load can be applied without causing tensile stress at points A, B, C, and D.



SOLUTION

Equivalent Force System: As shown on FBD.



Section Properties:

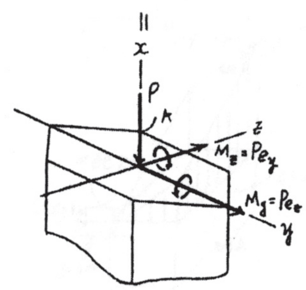
$$A = 2a(2a) + 2\left[\frac{1}{2}(2a)a\right] = 6a^2$$

$$I_z = \frac{1}{12}(2a)(2a)^3 + 2\left[\frac{1}{36}(2a)\,a^3 + \frac{1}{2}(2a)\,a\left(a + \frac{a}{3}\right)^2\right]$$

$$= 5a^4$$

$$I_y = \frac{1}{12}(2a)(2a)^3 + 2\left[\frac{1}{36}(2a)\,a^3 + \frac{1}{2}(2a)\,a\left(\frac{a}{3}\right)^2\right]$$

$$= \frac{5}{3}a^4$$

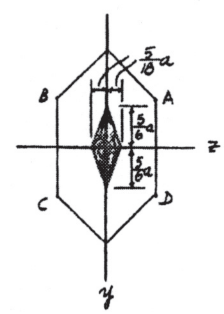


Normal Stress:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$= \frac{-P}{6a^2} - \frac{Pe_y y}{5a^4} - \frac{Pe_z z}{\frac{5}{3}a^4}$$

$$= \frac{P}{30a^4}\left(-5a^2 - 6e_y y - 18e_z z\right)$$



At point B where $y = -a$ and $z = -a$, we require $\sigma_B < 0$.

$$0 > \frac{P}{30a^4}\left[-5a^2 - 6(-a)\,e_y - 18(-a)\,e_z\right]$$

$$0 > -5a + 6e_y + 18e_z$$

$$6e_y + 18e_z < 5a$$ **Ans.**

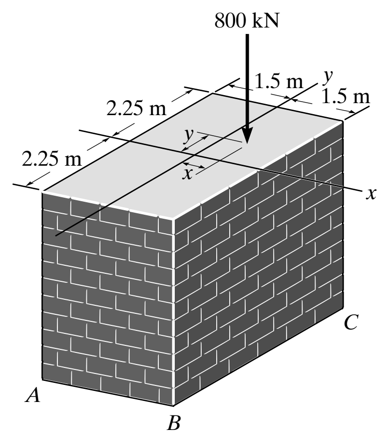
When $e_z = 0$, $\quad e_y < \frac{5}{6}a$

When $e_y = 0$, $\quad e_z < \frac{5}{18}a$

Repeat the same procedures for point A, C and D. The region where P can be applied without creating tensile stress at points A, B, C, and D is shown shaded in the diagram.

Ans:
$6e_y + 18e_z < 5a$

***8–52.** The masonry pier is subjected to the 800-kN load. Determine the equation of the line $y = f(x)$ along which the load can be placed without causing a tensile stress in the pier. Neglect the weight of the pier.



SOLUTION

$A = 3(4.5) = 13.5 \text{ m}^2$

$I_x = \dfrac{1}{12}(3)(4.5^3) = 22.78125 \text{ m}^4$

$I_y = \dfrac{1}{12}(4.5)(3^3) = 10.125 \text{ m}^4$

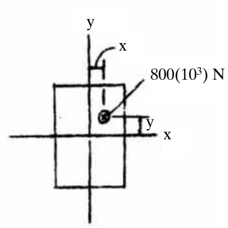
Normal Stress: Require $\sigma_A = 0$

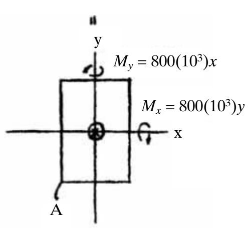
$$\sigma_A = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y x}{I_y}$$

$$0 = \frac{-800(10^3)}{13.5} + \frac{[800(10^3)y](2.25)}{22.78125} + \frac{[800(10^3)x](1.5)}{10.125}$$

$$0 = 79.01y + 118.52x - 59.26$$

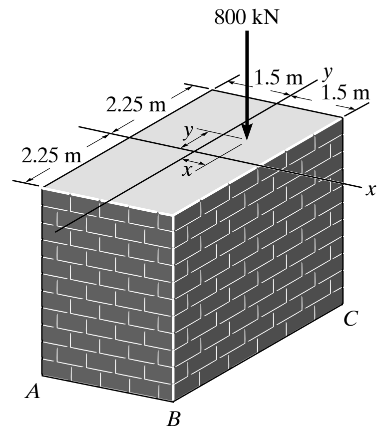
$y = 0.75 - 1.5\,x$ **Ans.**





Ans.
$y = 0.75 - 1.5\,x$

8–53. The masonry pier is subjected to the 800-kN load. If $x = 0.25$ m and $y = 0.5$ m, determine the normal stress at each corner A, B, C, D (not shown) and plot the stress distribution over the cross section. Neglect the weight of the pier.



$$A = 3(4.5) = 13.5 \text{ m}^2$$

$$I_x = \frac{1}{12}(3)(4.5^3) = 22.78125 \text{ m}^4$$

$$I_y = \frac{1}{12}(4.5)(3^3) = 10.125 \text{ m}^4$$

$$\sigma = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y x}{I_y}$$

$$\sigma_A = \frac{-800(10^3)}{13.5} + \frac{400(10^3)(2.25)}{22.78125} + \frac{200(10^3)(1.5)}{10.125}$$

$= 9.88$ kPa (T) **Ans.**

$$\sigma_B = \frac{-800(10^3)}{13.5} + \frac{400(10^3)(2.25)}{22.78125} - \frac{200(10^3)(1.5)}{10.125}$$

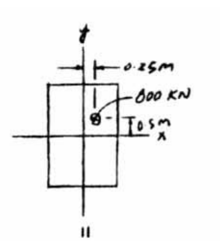
$= -49.4$ kPa $= 49.4$ kPa (C) **Ans.**

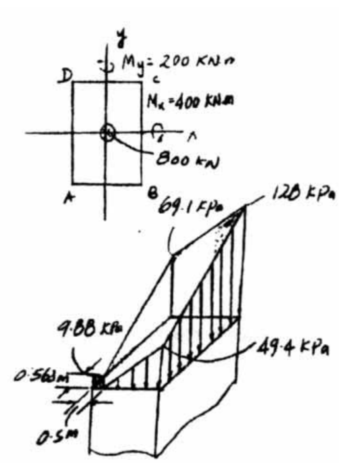
$$\sigma_C = \frac{-800(10^3)}{13.5} - \frac{400(10^3)(2.25)}{22.78125} + \frac{200(10^3)(1.5)}{10.125}$$

$= -128$ kPa $= 128$ kPa (C) **Ans.**

$$\sigma_D = \frac{-800(10^3)}{13.5} - \frac{400(10^3)(2.25)}{22.78125} + \frac{200(10^3)(1.5)}{10.125}$$

$= -69.1$ kPa $= 69.1$ kPa (C) **Ans.**

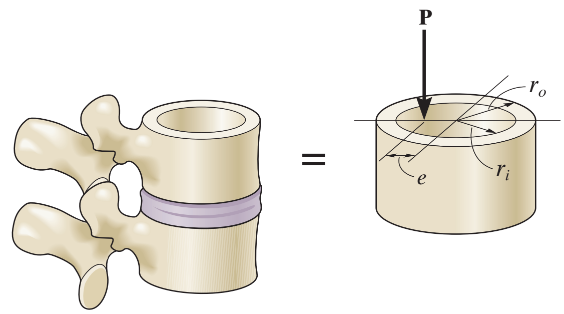




Ans.
$\sigma_A = 9.88$ kPa (T), $\sigma_B = 49.4$ kPa (C), $\sigma_C = 128$ kPa (C), $\sigma_D = 69.1$ kPa (C)

8–54.

The vertebra of the spinal column can support a maximum compressive stress of σ_{max}, before undergoing a compression fracture. Determine the smallest force P that can be applied to a vertebra, if we assume this load is applied at an eccentric distance e from the centerline of the bone, and the bone remains elastic. Model the vertebra as a hollow cylinder with an inner radius r_i and outer radius r_o.



SOLUTION

$$\sigma_{max} = \frac{P}{A} + \frac{Per_o}{\frac{\pi}{4}(r_0^4 - r_i^4)}$$

$$\sigma_{max} = P\left[\frac{1}{\pi(r_0^2 - r_i^2)} + \frac{4er_0}{\pi(r_0^4 - r_i^4)}\right]$$

$$\sigma_{max} = \frac{P}{\pi(r_0^2 - r_i^2)}\left[1 + \frac{4er_0}{(r_0^2 + r_i^2)}\right]$$

$$\sigma_{max} = \frac{P(r_0^2 + r_i^2 + 4er_0)}{\pi(r_0^2 - r_i^2)(r_0^2 + r_i^2)}$$

$$\sigma_{max} = \frac{P(r_0^2 + r_i^2 + 4er_0)}{\pi(r_0^4 - r_i^4)}$$

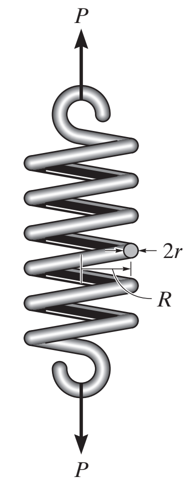
$$P = \frac{\delta_{max}\pi(r_0^4 - r_i^4)}{r_0^2 + r_i^2 + 4er_0}$$ **Ans.**

Ans:

$$P = \frac{\delta_{max}\pi(r_0^4 - r_i^4)}{r_0^2 + r_i^2 + 4er_0}$$

8–55.

The coiled spring is subjected to a force P. If we assume the shear stress caused by the shear force at any vertical section of the coil wire to be uniform, show that the maximum shear stress in the coil is $\tau_{max} = P/A + PRr/J$, where J is the polar moment of inertia of the coil wire and A is its cross-sectional area.



SOLUTION

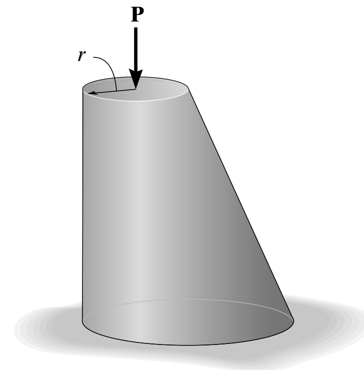
$$\frac{Tc}{J} = \text{max on perimeter} = \frac{PRr}{J}$$

$$\tau_{max} = \frac{V}{A} + \frac{Tc}{J} = \frac{P}{A} + \frac{PRr}{J}$$

QED

Ans:
N/A

***8–56.** The support is subjected to the compressive load **P**. Determine the maximum and minimum normal stress acting in the material. All horizontal cross sections are circular.



Section Properties:

$$d' = 2r + x$$

$$A = \pi(r + 0.5x)^2$$

$$I = \frac{\pi}{4}(r + 0.5x)^4$$

Internal Force and Moment: As shown on FBD.

Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

$$= \frac{-P}{\pi(r + 0.5x)^2} \pm \frac{0.5Px(r + 0.5x)}{\frac{\pi}{4}(r + 0.5)^4}$$

$$= \frac{P}{\pi}\left[\frac{-1}{(r + 0.5x)^2} \pm \frac{2x}{(r + 0.5x)^3}\right]$$

$$\sigma_A = -\frac{P}{\pi}\left[\frac{1}{(r + 0.5x)^2} + \frac{2x}{(r + 0.5x)^3}\right]$$

$$= -\frac{P}{\pi}\left[\frac{r + 2.5x}{(r + 0.5x)^3}\right] \qquad \textbf{[1]}$$

$$\sigma_B = \frac{P}{\pi}\left[\frac{-1}{(r + 0.5x)^2} + \frac{2x}{(r + 0.5x)^3}\right]$$

$$= \frac{P}{\pi}\left[\frac{1.5x - r}{(r + 0.5x)^3}\right] \qquad \textbf{[2]}$$

In order to have maximum normal stress, $\frac{d\sigma_A}{dx} = 0.$

$$\frac{d\sigma_A}{dx} = -\frac{P}{\pi}\left[\frac{(r + 0.5x)^3(2.5) - (r + 2.5x)(3)(r + 0.5x)^2(0.5)}{(r + 0.5x)^6}\right] = 0$$

$$-\frac{P}{\pi(r + 0.5x)^4}(r - 2.5x) = 0$$

Since $\frac{P}{\pi(r + 0.5x)^4} \neq 0$, then

$$r - 2.5x = 0 \qquad x = 0.400r$$

Substituting the result into Eq. [1] yields

$$\sigma_{\max} = -\frac{\mathrm{P}}{\pi}\left[\frac{\mathrm{r} + 2.5(0.400\mathrm{r})}{[\mathrm{r} + 0.5(0.400\mathrm{r})]^3}\right]$$

$$= -\frac{0.368P}{r^2} = \frac{0.368P}{r^2}\ \text{(C)} \qquad \textbf{Ans.}$$

***8–56. Continued**

In order to have minimum normal stress, $\dfrac{d\sigma_B}{dx} = 0$.

$$\frac{d\sigma_B}{dx} = \frac{P}{\pi}\left[\frac{(r + 0.5x)^3(1.5) - (1.5x - r)(3)(r + 0.5x)^2(0.5)}{(r + 0.5x)^6}\right] = 0$$

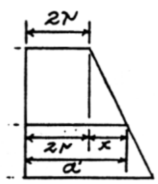
$$\frac{P}{\pi(r + 0.5x)^4}(3r - 1.5x) = 0$$

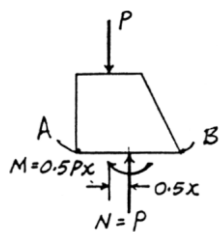
Since $\dfrac{P}{\pi(r + 0.5x)^4} \neq 0$, then

$$3r - 1.5x = 0 \qquad x = 2.00r$$

Substituting the result into Eq. [2] yields

$$\sigma_{\text{min}} = \frac{P}{\pi}\left[\frac{1.5(2.00r) - r}{[r + 0.5(2.00r)]^3}\right] = \frac{0.0796P}{r^2}\ \textbf{(T)} \qquad \textbf{Ans.}$$

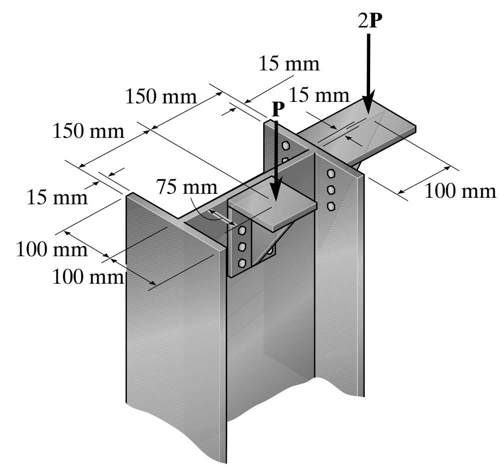




Ans.

$\sigma_{\text{max}} = \dfrac{0.368P}{r^2}$ (C), $\sigma_{\text{min}} = \dfrac{0.0796P}{r^2}$ (T)

8–57. If $P = 60$ kN, determine the maximum normal stress developed on the cross section of the column.



SOLUTION

Equivalent Force System: Referring to Fig. *a*,

$$+\uparrow \Sigma F_x = \left(F_R\right)_x; \qquad -60 - 120 = -F \qquad F = 180 \text{ kN}$$

$$\Sigma M_y = (M_R)_y; \qquad -60(0.075) = -M_y \qquad M_y = 4.5 \text{kN} \cdot \text{m}$$

$$\Sigma M_z = (M_R)_z; \qquad -120(0.25) = -M_z \qquad M_z = 30 \text{kN} \cdot \text{m}$$

Section Properties: The cross-sectional area and the moment of inertia about the y and z axes of the cross section are

$$A = 0.2(0.3) - 0.185(0.27) = 0.01005 \text{ m}^2$$

$$I_z = \frac{1}{12}(0.2)\left(0.3^3\right) - \frac{1}{12}(0.185)\left(0.27^3\right) = 0.14655\left(10^{-3}\right) \text{ m}^4$$

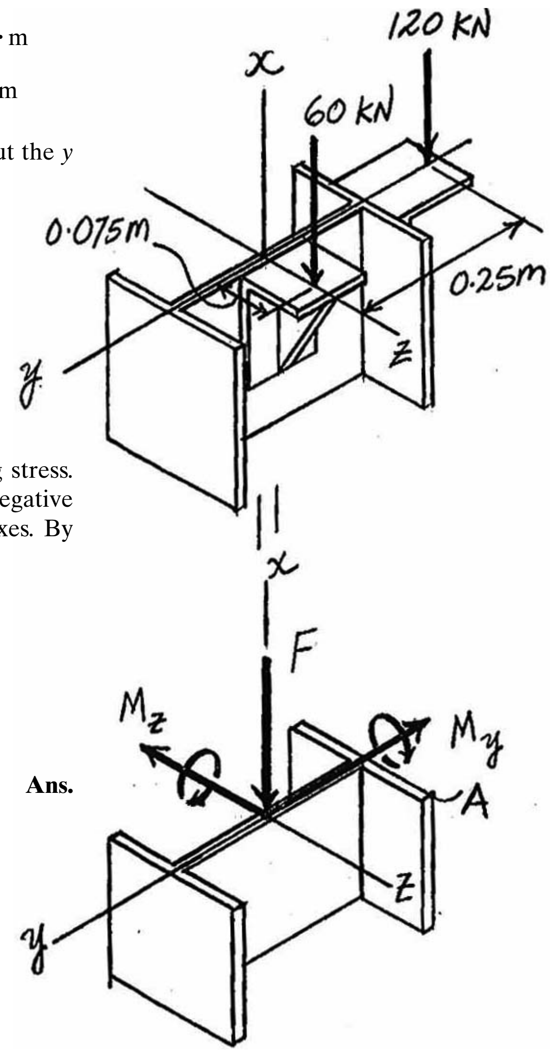
$$I_y = 2\left[\frac{1}{12}(0.015)\left(0.2^3\right)\right] + \frac{1}{12}(0.27)\left(0.015^3\right) = 20.0759\left(10^{-6}\right)\text{m}^4$$

Normal Stress: The normal stress is the combination of axial and bending stress. Here, **F** is negative since it is a compressive force. Also, $\mathbf{M}_y$ and $\mathbf{M}_z$ are negative since they are directed towards the negative sense of their respective axes. By inspection, point A is subjected to a maximum normal stress. Thus,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

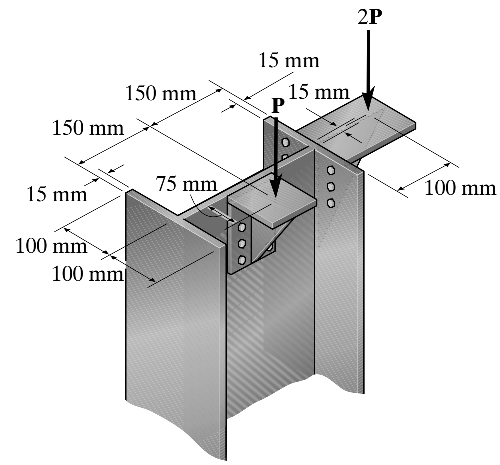
$$\sigma_{\max} = \sigma_A = \frac{-180\left(10^3\right)}{0.01005} - \frac{\left[-30\left(10^3\right)\right](-0.15)}{0.14655\left(10^{-3}\right)} + \frac{\left[-4.5\left(10^3\right)\right](0.1)}{20.0759\left(10^{-6}\right)}$$

$$= -71.0 \text{ MPa} = 71.0 \text{ MPa(C)}$$ **Ans.**



Ans.
$\sigma_{\max} = 71.0$ MPa (C)

8–58. Determine the maximum allowable force **P**, if the column is made from material having an allowable normal stress of $\sigma_{\text{allow}} = 100$ MPa.



SOLUTION

Equivalent Force System: Referring to Fig. *a*,

$+\uparrow \Sigma F_x = (F_R)_x; \qquad -P - 2P = -F$

$$F = 3P$$

$\Sigma M_y = (M_R)_y; \qquad -P(0.075) = -M_y$

$$M_y = 0.075\,P$$

$\Sigma M_z = (M_R)_z; \qquad -2P(0.25) = -M_z$

$$M_z = 0.5P$$

Section Properties: The cross-sectional area and the moment of inertia about the *y* and *z* axes of the cross section are

$$A = 0.2(0.3) - 0.185(0.27) = 0.01005 \text{ m}^2$$

$$I_z = \frac{1}{12}(0.2)\left(0.3^3\right) - \frac{1}{12}(0.185)\left(0.27^3\right) = 0.14655\left(10^{-3}\right)\text{ m}^4$$

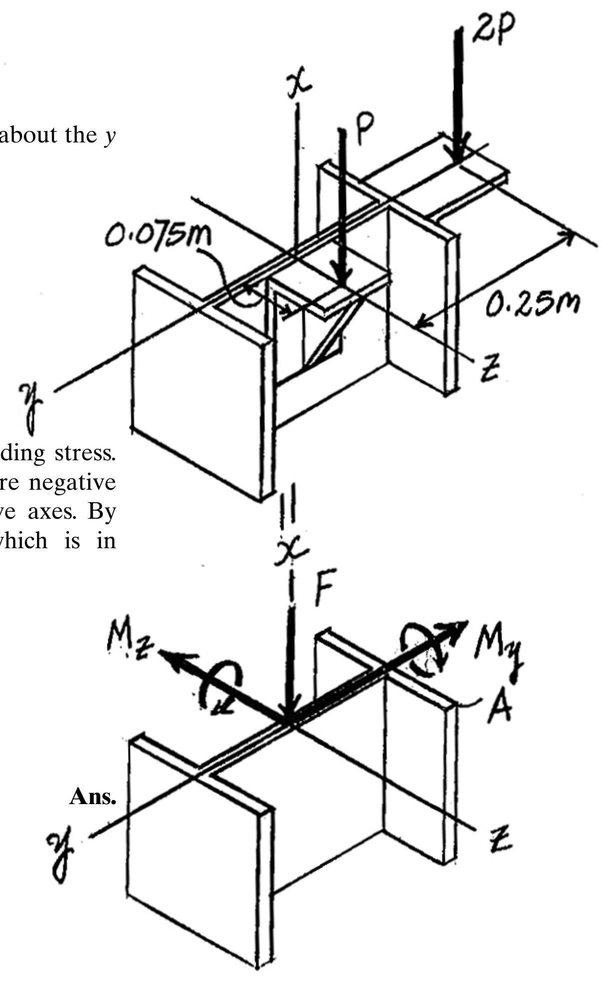
$$I_y = 2\left[\frac{1}{12}(0.15)\left(0.2^3\right)\right] + \frac{1}{12}(0.27)\left(0.015^3\right) = 20.0759\left(10^{-6}\right)\text{ m}^4$$

Normal Stress: The normal stress is the combination of axial and bending stress. Here, **F** is negative since it is a compressive force. Also, $\mathbf{M}_y$ and $\mathbf{M}_z$ are negative since they are directed towards the negative sense of their respective axes. By inspection, point *A* is subjected to a maximum normal stress, which is in compression. Thus,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$-100\left(10^6\right) = -\frac{3P}{0.01005} - \frac{(-0.5P)(-0.15)}{0.14655\left(10^{-3}\right)} + \frac{-0.075P(0.1)}{20.0759\left(10^{-6}\right)}$$

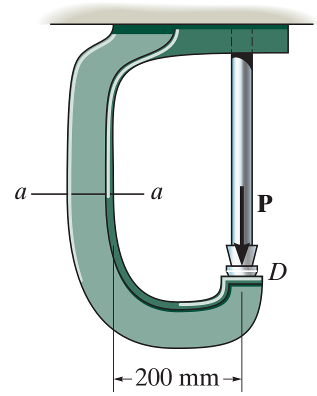
$$P = 84470.40 \text{ N} = 84.5 \text{ kN}$$ **Ans.**

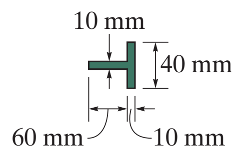


Ans.
$P = 84.5$ kN

8–59.

The C-frame is used in a riveting machine. If the force at the ram on the clamp at D is $P = 8$ kN, sketch the stress distribution acting over the section a–a.



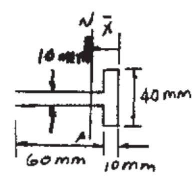


SOLUTION

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{(0.005)(0.04)(0.01) + 0.04(0.06)(0.01)}{0.04(0.01) + 0.06(0.01)} = 0.026 \text{ m}$$

$$A = 0.04(0.01) + 0.06(0.01) = 0.001 \text{ m}^2$$

$$I = \frac{1}{12}(0.04)(0.01^3) + (0.04)(0.01)(0.026 - 0.005)^2 + \frac{1}{12}(0.01)(0.06^3) + 0.01(0.06)(0.040 - 0.026)^2 = 0.4773(10^{-6}) \text{ m}^4$$

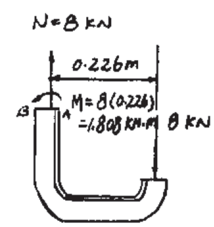


$$(\sigma_{\max})_t = \frac{P}{A} + \frac{Mx}{I} = \frac{8(10^3)}{0.001} + \frac{1.808(10^3)0.026}{0.4773(10^{-6})}$$

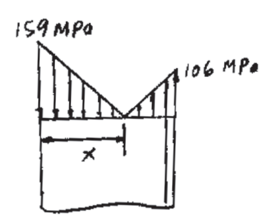
$$= 106.48 \text{ MPa} = 106 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\sigma_{\max})_c = \frac{P}{A} - \frac{Mc}{I} = \frac{8(10^3)}{0.001} - \frac{1.808(10^3)(0.070 - 0.026)}{0.4773(10^{-6})}$$

$$= -158.66 \text{ MPa} = -159 \text{ MPa} \qquad \textbf{Ans.}$$



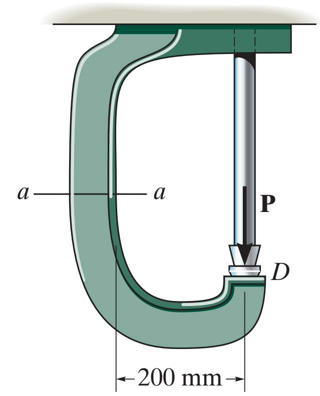
$$\frac{x}{158.66} = \frac{70 - x}{106.48}; \qquad x = 41.9 \text{ mm}$$

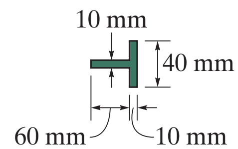


Ans:
$(\sigma_{\max})_t = 106$ MPa, $(\sigma_{\max})_c = -159$ MPa

***8–60.**

Determine the maximum ram force P that can be applied to the clamp at D if the allowable normal stress for the material is $\sigma_{allow} = 180$ MPa.





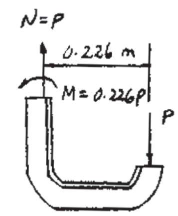
SOLUTION

$$\bar{x} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{(0.005)(0.04)(0.01) + 0.04(0.06)(0.01)}{0.04(0.01) + 0.06(0.01)} = 0.026 \text{ m}$$

$$A = 0.04(0.01) + 0.06(0.01) = 0.001 \text{ m}^2$$

$$I = \frac{1}{12}(0.04)(0.01^3) + (0.04)(0.01)(0.026 - 0.005)^2 + \frac{1}{12}(0.01)(0.06^3) + 0.01(0.06)(0.040 - 0.026)^2 = 0.4773(10^{-6}) \text{ m}^4$$

$$\sigma = \frac{P}{A} \pm \frac{Mx}{I}$$



Assume tension failure,

$$180(10^6) = \frac{P}{0.001} + \frac{0.226P(0.026)}{0.4773(10^{-6})}$$

$P = 13524 \text{ N} = 13.5 \text{ kN}$

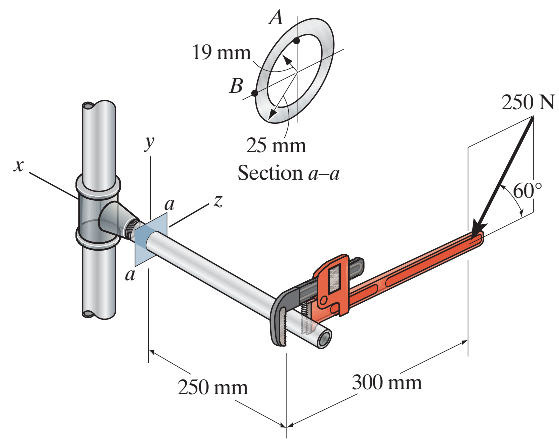
Assume compression failure,

$$-180(10^6) = \frac{P}{0.001} - \frac{0.226P(0.070 - 0.026)}{0.4773(10^{-6})}$$

$P = 9076 \text{ N} = 9.08 \text{ kN}$ (controls) **Ans.**

Ans:
$P_{max} = 9.08$ kN

8–61. Determine the state of stress at point A on the cross section of the pipe at section a–a.



SOLUTION

Internal Loadings: Referring to the free - body diagram of the pipe's right segment, Fig. *a*,

$\Sigma F_y = 0;\quad V_y - 250\sin 60^\circ = 0 \qquad V_y = 216.51\text{ N}$

$\Sigma F_z = 0;\quad V_z - 250\cos 60^\circ = 0 \qquad V_z = 125\text{ N}$

$\Sigma M_x = 0;\quad T + (250\sin 60^\circ)(0.300) = 0 \qquad T = -64.95\text{ N}\cdot\text{m}$

$\Sigma M_y = 0;\quad M_y - (250\cos 60^\circ)(0.250) = 0 \qquad M_y = 31.25\text{ N}\cdot\text{m}$

$\Sigma M_z = 0;\quad M_z + (250\sin 60^\circ)(0.250) = 0 \qquad M_z = -54.13\text{ N}\cdot\text{m}$

Section Properties: The moment of inertia about the y and z axes and the polar moment of inertia of the pipe are

$$I_y = I_z = \frac{\pi}{4}(0.025^4 - 0.019^4) = 65.076(10^{-9})\pi\text{ m}^4$$

$$J = \frac{\pi}{2}(0.025^4 - 0.019^4) = 0.130152(10^{-6})\pi\text{ m}^4$$

Referring to Fig. *b*,

$$(Q_y)_A = 0$$

$$(Q_z)_A = \bar{y}_1'A_1' - \bar{y}_2'A_2' = \left[\frac{4(0.025)}{3\pi}\right]\left[\frac{\pi}{2}(0.025^2)\right] - \left[\frac{4(0.019)}{3\pi}\right]\left[\frac{\pi}{2}(0.019^2)\right] = 5.844(10^{-6})\text{ m}^3$$

Normal Stress: The normal stress is contributed by bending stress only. Thus,

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

For point A, $y = 0.019$ m and $z = 0$. Then

$$\sigma_A = \frac{(-54.13)(0.019)}{65.076(10^{-9})\pi} + 0 = 5.030(10^6)\text{ N/m}^2 = 5.03\text{ MPa (T)} \qquad \textbf{Ans.}$$

Shear Stress: The torsional shear stress developed at point A is

$$\left[(\tau_{xz})_T\right]_A = \frac{T\rho_A}{J} = \frac{64.95(0.019)}{0.130152(10^{-6})\pi} = 3.0182(10^6)\text{ N/m}^2 = 3.0182\text{ MPa}$$

8–61. Continued

The transverse shear stress developed at point A is

$$\left[\left(\tau_{xy}\right)_V\right]_A = 0$$

$$\left[\left(\tau_{xz}\right)_V\right]_A = \frac{V_z\left(Q_z\right)_A}{I_y t} = \frac{125[5.844(10^{-6})]}{[65.076(10^{-9})\pi][2(0.006)]}$$

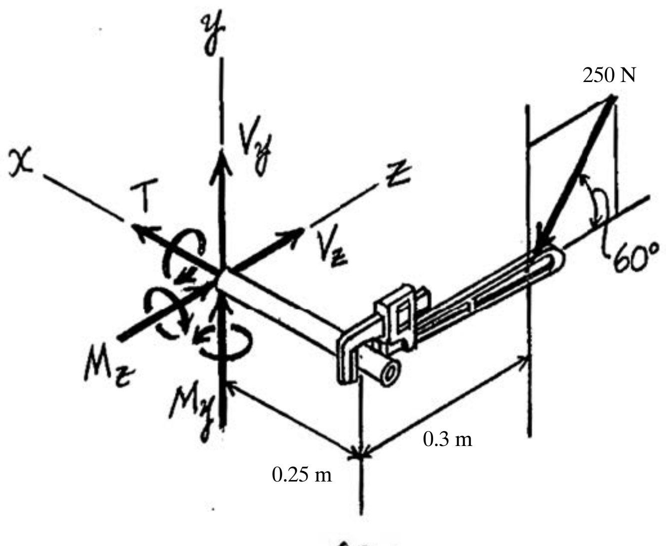
$$= 0.2978(10^6)\ \mathrm{N/m^2} = 0.2978\ \mathrm{MPa}$$

Combining these two shear stress components,

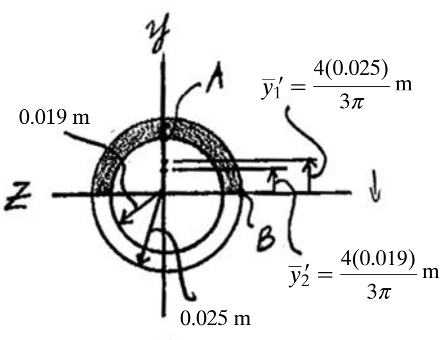
$$\left(\tau_{xy}\right)_A = 0 \qquad \textbf{Ans.}$$

$$\left(\tau_{xz}\right)_A = \left[\left(\tau_{xz}\right)_T\right]_A - \left[\left(\tau_{xz}\right)_V\right]_A$$

$$= 3.0182 - 0.2978 = 2.72\ \mathrm{MPa} \qquad \textbf{Ans.}$$



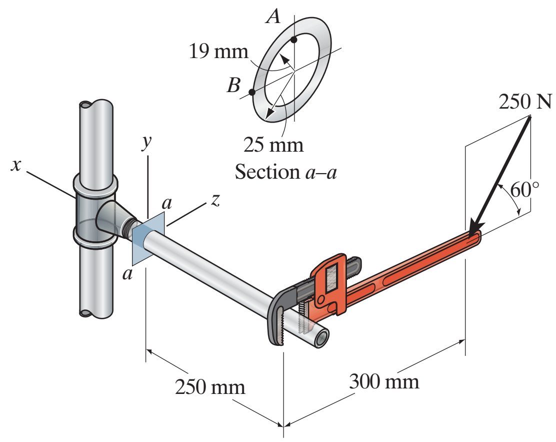
(a)



(b)

Ans.
$T = -64.95\ \mathrm{N\cdot m}$, $M_y = 31.25\ \mathrm{N\cdot m}$,
$M_z = -54.13\ \mathrm{N\cdot m}$, $I_y = I_z = 65.076\ (10^{-9})\ \pi\mathrm{m}^4$
$J = 0.130152\ (10^{-6})\ \pi\mathrm{m}^4$
$(Q_z)_A = 5.844\ (10^{-6})\ \mathrm{m}^3$
$\sigma_A = 5.03\ \mathrm{MPa\ (T)}$, $\tau_A = 2.72\ \mathrm{MPa}$

8–62. Determine the state of stress at point B on the cross section of the pipe at section a–a.



SOLUTION

Internal Loadings: Referring to the free - body diagram of the pipe's right segment, Fig. *a*,

$\Sigma F_y = 0;\quad V_y - 250 \sin 60° = 0 \qquad V_y = 216.5 \text{ kN}$

$\Sigma F_z = 0;\quad V_z - 250 \cos 60° = 0 \qquad V_z = 125 \text{ kN}$

$\Sigma M_x = 0;\quad T + 250 \sin 60°(0.300) = 0 \qquad T = -64.95 \text{ N} \cdot \text{m}$ b · in

$\Sigma M_y = 0;\quad M_y - 250 \cos 60°(0.250) = 0 \qquad M_y = 31.25 \text{ N} \cdot \text{m}$

$\Sigma M_z = 0;\quad M_z + 250 \sin 60°(0.250) = 0 \qquad M_z = -54.13 \text{ N} \cdot \text{m}$ b · in

Section Properties: The moment of inertia about the y and z axes and the polar moment of inertia of the pipe are

$$I_y = I_z = \frac{\pi}{4}(0.025^4 - 0.019^4) = 65.076(10^{-9})\pi \text{ m}^4$$

$$J = \frac{\pi}{2}(0.025^4 - 0.019^4) = 0.130152(10^{-6})\pi \text{ m}^4$$

Referring to Fig. *b*,

$$(Q_z)_B = 0$$

$$(Q_y)_B = \bar{y}_1'A_1' - \bar{y}_2'A_2' = \left[\frac{4(0.025)}{3\pi}\right]\left[\frac{\pi}{2}(0.025^2)\right] - \left[\frac{4(0.019)}{3\pi}\right]\left[\frac{\pi}{2}(0.019^2)\right] = 5.844(10^{-6}) \text{ m}^3$$

Normal Stress: The normal stress is contributed by bending stress only. Thus,

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

For point B, $y = 0$ and $z = -0.025$ m. Then

$$\sigma_B = -0 + \frac{31.25(-0.025)}{65.076(10^{-9})\pi} = -3.8214(10^6) \text{ N/m}^2 = 3.82 \text{ MPa (C)} \qquad \textbf{Ans.}$$

Shear Stress: The torsional shear stress developed at point B is

$$\left[(\tau_{xy})_T\right]_B = \frac{T\rho_C}{J} = \frac{64.95(0.025)}{0.130152(10^{-6})\pi} = 3.9713(10^6) \text{ N/m}^2 = 3.97 \text{ MPa}$$

8–62. Continued

The transverse shear stress developed at point B is

$$\left[(\tau_{xz})_V\right]_B = 0$$

$$\left[(\tau_{xy})_V\right]_B = \frac{V_y(Q_y)_B}{I_z t} = \frac{216.51[5.844(10^{-6})]}{[65.076(10^{-9})\pi][2(0.006)]}$$

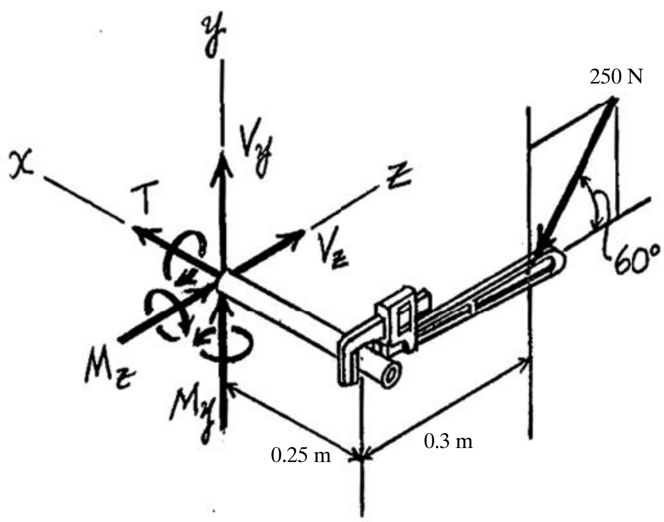
$$= 0.5157(10^6)\ \text{N/m}^2 = 0.5157\ \text{MPa}$$

Combining these two shear stress components,

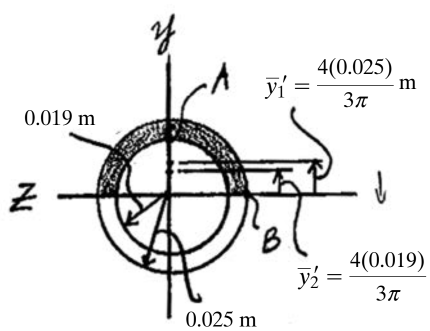
$$(\tau_{xy})_B = \left[(\tau_{xy})_T\right]_B - \left[(\tau_{xy})_V\right]_B$$

$$= 3.9713 - 0.5157 = 3.4555\ \text{MPa} = 3.46\ \text{MPa}$$ **Ans.**

$$(\tau_{xz})_B = 0$$ **Ans.**



(a)

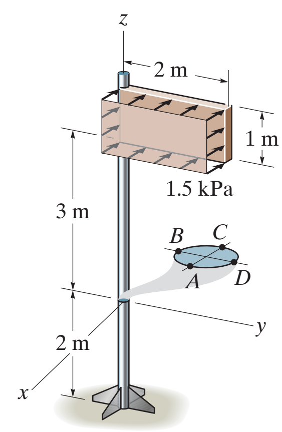


(b)

Ans.
$\sigma_B = 3.82$ MPa (C), $\tau_B = 3.46$ MPa

8–63.

The sign is subjected to the uniform wind loading. Determine the stress components at points A and B on the 100-mm-diameter supporting post. Show the results on a volume element located at each of these points.

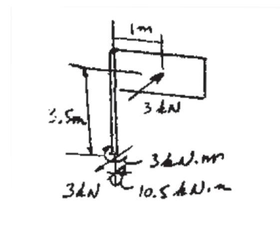


SOLUTION

Point A:

$$\sigma_A = \frac{Mc}{I} = \frac{10.5(10^3)(0.05)}{\frac{\pi}{4}(0.05)^4} = 107 \text{ MPa (T)}$$ **Ans.**

$$\tau_A = \frac{Tc}{J} = \frac{3(10^3)(0.05)}{\frac{\pi}{4}(0.05)^4} = 15.279(10^6) = 15.3 \text{ MPa}$$ **Ans.**

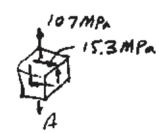


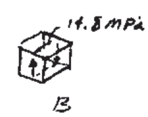
Point B:

$$\sigma_B = 0$$ **Ans.**

$$\tau_B = \frac{Tc}{J} - \frac{VQ}{It} = 15.279(10^6) - \frac{3000(4(0.05)/3\pi))(\frac{1}{2})(\pi)(0.05)^2}{\frac{\pi}{4}(0.05)^4(0.1)}$$

$$\tau_B = 14.8 \text{ MPa}$$ **Ans.**

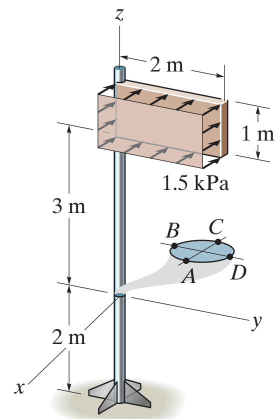




Ans:
$\sigma_A = 107$ MPa (T), $\tau_A = 15.3$ MPa,
$\sigma_B = 0$, $\tau_B = 14.8$ MPa

***8–64.**

The sign is subjected to the uniform wind loading. Determine the stress components at points C and D on the 100-mm-diameter supporting post. Show the results on a volume element located at each of these points.



SOLUTION

Point C:

$$\sigma_C = \frac{Mc}{I} = \frac{10.5(10^3)(0.05)}{\frac{\pi}{4}(0.05)^4} = 107 \text{ MPa (C)}$$ **Ans.**

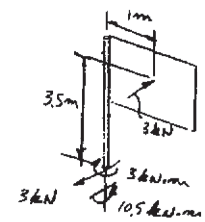
$$\tau_C = \frac{Tc}{J} = \frac{3(10^3)(0.05)}{\frac{\pi}{2}(0.05)^4} = 15.279(10^6) = 15.3 \text{ MPa}$$ **Ans.**

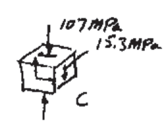
Point D:

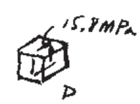
$$\sigma_D = 0$$ **Ans.**

$$\tau_D = \frac{Tc}{J} + \frac{VQ}{It} = 15.279(10^6) + \frac{3(10^3)(4(0.05)/3\pi)(\frac{1}{2})(\pi)(0.05)^2}{\frac{\pi}{4}(0.05)^4(0.1)} = 15.8 \text{ MPa}$$

Ans.



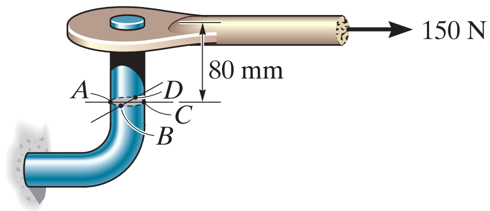




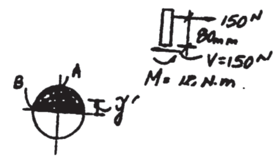
Ans:
$\sigma_C = 107$ MPa (C), $\tau_C = 15.3$ MPa,
$\sigma_D = 0$, $\tau_D = 15.8$ MPa

8–65.

The pin support is made from a steel rod and has a diameter of 20 mm. Determine the stress components at points A and B and represent the results on a volume element located at each of these points.



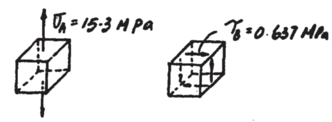
SOLUTION



$$I = \frac{1}{4}(\pi)(0.01^4) = 7.85398(10^{-9})\ \text{m}^4$$

$$Q_B = \bar{y}A' = \frac{4(0.01)}{3\pi}\left(\frac{1}{2}\right)(\pi)(0.01^2) = 0.66667(10^{-6})\ \text{m}^3$$

$$Q_A = 0$$



$$\sigma_A = \frac{Mc}{I} = \frac{12(0.01)}{7.85398(10^{-9})} = 15.3\ \text{MPa}$$ **Ans.**

$$\tau_A = 0$$ **Ans.**

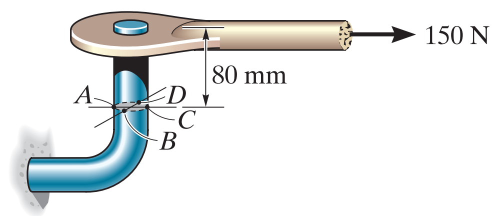
$$\sigma_B = 0$$ **Ans.**

$$\tau_B = \frac{VQ_B}{It} = \frac{150(0.6667)(10^{-6})}{7.85398(10^{-9})(0.02)} = 0.637\ \text{MPa}$$ **Ans.**

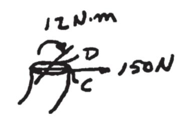
Ans:
$\sigma_A = 15.3$ MPa, $\tau_A = 0$,
$\sigma_B = 0$, $\tau_B = 0.637$ MPa

8–66.

Solve Prob. 8–65 for points C and D.

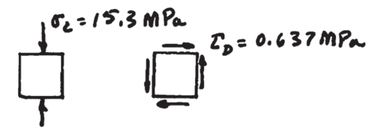


SOLUTION



$$I = \frac{1}{4}(\pi)(0.01^4) = 7.85398(10^{-9})\ \text{m}^4$$

$$Q_D = \bar{y}'A' = \frac{4(0.01)}{3\pi}\left(\frac{1}{2}\right)(\pi)(0.01^2) = 0.66667(10^{-6})\ \text{m}^3$$



$$Q_C = 0$$

$$\sigma_C = \frac{Mc}{I} = \frac{12(0.01)}{7.85398(10^{-9})} = 15.3\ \text{MPa}$$ **Ans.**

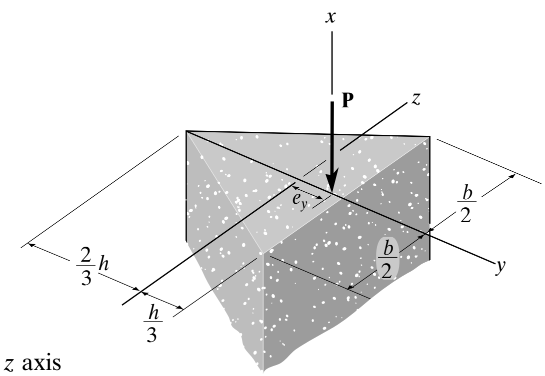
$$\tau_C = 0$$ **Ans.**

$$\sigma_D = 0$$ **Ans.**

$$\tau_D = \frac{VQ_D}{It} = \frac{150(0.6667)(10^{-6})}{7.8539(10^{-9})(0.02)} = 0.637\ \text{MPa}$$ **Ans.**

Ans:
$\sigma_C = 15.3$ MPa, $\tau_C = 0$,
$\sigma_D = 0$, $\tau_D = 0.637$ MPa

8–67. The eccentric force **P** is applied at a distance e_y from the centroid on the concrete support shown. Determine the range along the y axis where **P** can be applied on the cross section so that no tensile stress is developed in the material.



SOLUTION

Internal Loadings: As shown on the free - body diagram, Fig. *a.*

Section Properties: The cross-sectional area and moment of inertia about the z axis of the triangular concrete support are

$$A = \frac{1}{2}bh \qquad I_z = \frac{1}{36}bh^3$$

Normal Stress: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z}$$

$$\sigma = \frac{-P}{\frac{1}{2}bh} - \frac{\left(Pe_y\right)y}{\frac{1}{36}bh^3}$$

$$\sigma = -\frac{2P}{bh^3}\left(h^2 + 18e_y y\right) \tag{1}$$

Here, it is required that $\sigma_A \leq 0$ and $\sigma_B \leq 0$. For point A, $y = \frac{h}{3}$, Then. Eq. (1) gives

$$0 \geq -\frac{2P}{bh^3}\left[h^2 + 18e_y\left(\frac{h}{3}\right)\right]$$

$$0 \leq h^2 + 6he_y$$

$$e_y \geq -\frac{h}{6}$$

For Point B, $y = -\frac{2}{3}h$. Then. Eq. (1) gives

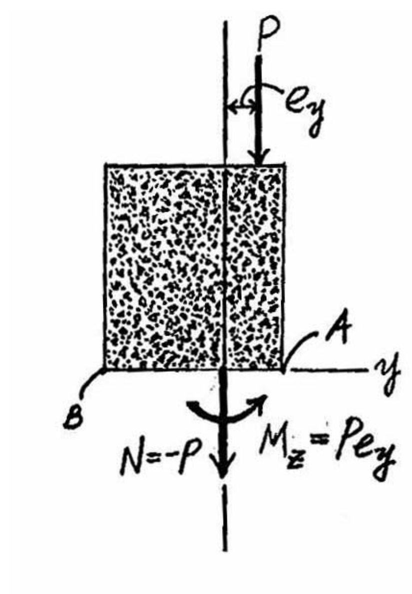
$$0 \geq -\frac{2P}{bh^3}\left[h^2 + 18e_y\left(-\frac{2}{3}h\right)\right]$$

$$0 \leq h^2 - 12he_y$$

$$e_y \leq \frac{h}{12}$$

Thus, in order that no tensile stress be developed in the concrete support, e_y must be in the range of

$$-\frac{h}{6} \leq e_y \leq \frac{h}{12} \qquad \textbf{Ans.}$$



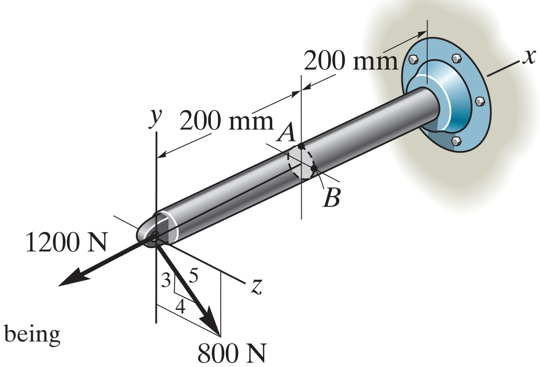
(a)

Ans.

$$\sigma = -\frac{2P}{bh^3}(h^2 + 18e_y y),\ -\frac{h}{6} \leq e_y \leq \frac{h}{12}$$

***8–68.**

The bar has a diameter of 40 mm. Determine the state of stress at point A and show the results on a differential volume element located at this point.



SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the rod being sectioned, Fig. *a*.

$$\Sigma F_x = 0; \qquad N_x - 1200 = 0 \qquad N_x = 1200\text{ N}$$

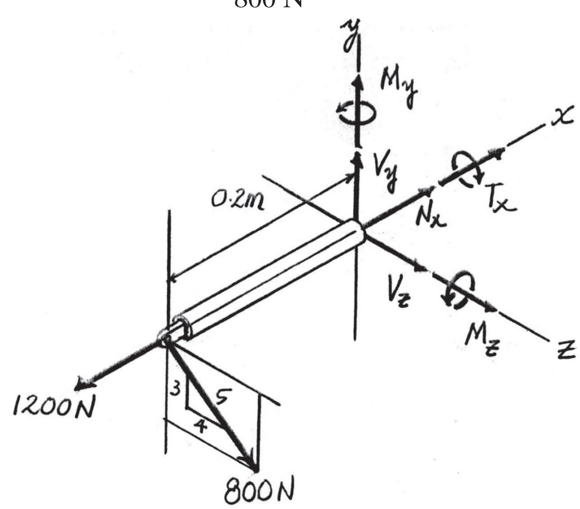
$$\Sigma F_y = 0; \qquad V_y - 800\left(\frac{3}{5}\right) = 0 \quad V_y = 480\text{ N}$$

$$\Sigma F_z = 0; \qquad V_z + 800\left(\frac{4}{5}\right) = 0 \quad V_z = -640\text{ N}$$

$$\Sigma M_x = 0; \qquad T_x = 0$$

$$\Sigma M_y = 0; \qquad M_y + 800\left(\frac{4}{5}\right)(0.2) = 0 \quad M_y = -128\text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0; \qquad M_z + 800\left(\frac{3}{5}\right)(0.2) = 0 \quad M_z = -96\text{ N}\cdot\text{m}$$



(a)

Section Properties: For the circular cross section, Fig. *b*,

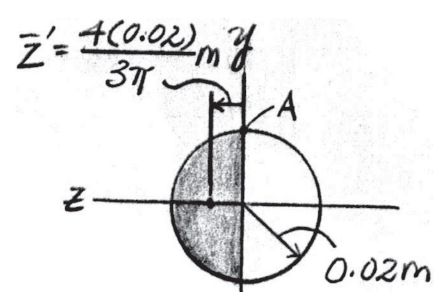
$$A = \pi c^2 = \pi(0.02^2) = 0.4\left(10^{-3}\right)\pi\text{ m}^2$$

$$I_y = I_z = \frac{\pi}{4}c^4 = \frac{\pi}{4}\left(0.02^4\right) = 40.0\left(10^{-9}\right)\pi\text{ m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(0.02^4\right) = 80.0\left(10^{-9}\right)\pi\text{ m}^4$$

$$(Q_A)_z = \bar{z}'A' = \frac{4(0.02)}{3\pi}\left[\frac{\pi}{2}\left(0.02^2\right)\right] = 5.3333\left(10^{-6}\right)\text{ m}^3$$

$$(Q_A)_y = 0$$



(b)

Normal Stress: For the combined loading, the normal stress at point A can be determined from

$$\sigma_x = \sigma_A = \frac{N_x}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

$$= \frac{1200}{0.4\left(10^{-3}\right)\pi} - \frac{(-96)(0.02)}{40.0\left(10^{-9}\right)\pi} + \frac{(-128)(0)}{40.0\left(10^{-9}\right)\pi}$$

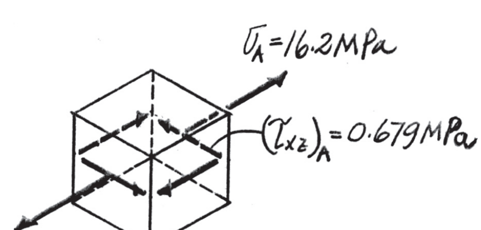
$$= 16.23\left(10^6\right)\text{ Pa} = 16.2\text{ MPa (T)} \qquad \textbf{Ans.}$$

Shear Stress: Since $T_x = 0$, the shear stress in the z and y directions is contributed by transverse shear stress only which can be obtained using the shear formula, $\tau_V = \dfrac{VQ}{It}$

$$(\tau_{xz})_A = (\tau_V)_z = \frac{-640\left[5.3333\left(10^{-6}\right)\right]}{40.0\left(10^{-9}\right)\pi(0.04)} = -0.6791\left(10^6\right)\text{ Pa} = -0.679\text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{xy})_A = (\tau_V)_y = 0 \qquad \textbf{Ans.}$$

Using these results, the state of stress at point A can be represented by the volume element shown in Fig. *c*.

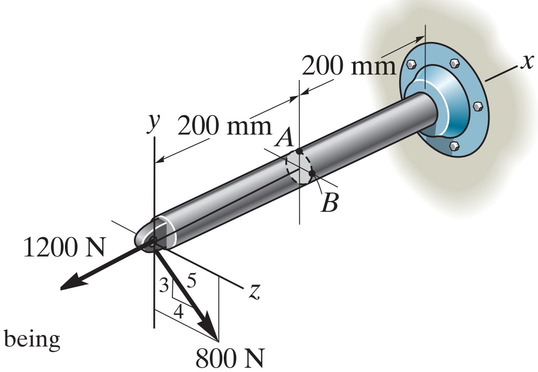


(c)

Ans:
$\sigma_A = 16.2$ MPa (T),
$(\tau_{xz})_A = -0.679$ MPa,
$(\tau_{xy})_A = 0$

8–69.

Solve Prob. 8–68 for point *B*.



SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the rod being sectioned, Fig. *a*.

$$\Sigma F_x = 0; \quad N_x - 1200 = 0 \quad N_x = 1200\text{ N}$$

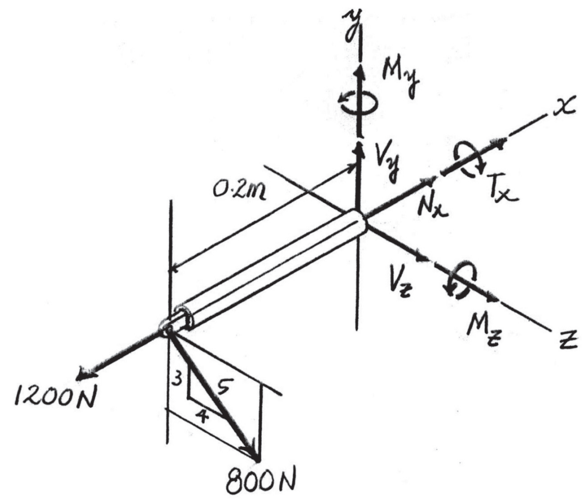
$$\Sigma F_y = 0; \quad V_y - 800\left(\frac{3}{5}\right) = 0 \quad V_y = 480\text{ N}$$

$$\Sigma F_z = 0; \quad V_z + 800\left(\frac{4}{5}\right) = 0 \quad V_z = -640\text{ N}$$

$$\Sigma M_x = 0; \quad T_x = 0$$

$$\Sigma M_y = 0; \quad M_y + 800\left(\frac{4}{5}\right)(0.2) = 0 \quad M_y = -128\text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0; \quad M_z + 800\left(\frac{3}{5}\right)(0.2) = 0 \quad M_z = -96\text{ N}\cdot\text{m}$$



(a)

Section Properties: For the circular cross section, Fig. *b*,

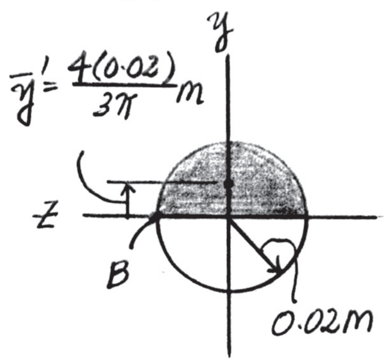
$$A = \pi c^2 = \pi(0.02^2) = 0.4\left(10^{-3}\right)\pi\text{ m}^2$$

$$I_y = I_z = \frac{\pi}{4}c^4 = \frac{\pi}{4}\left(0.02^4\right) = 40.0\left(10^{-9}\right)\pi\text{ m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(0.02^4\right) = 80.0\left(10^{-9}\right)\pi\text{ m}^4$$

$$(Q_B)_y = \bar{y}'A' = \frac{4(0.02)}{3\pi}\left[\frac{\pi}{2}\left(0.02^2\right)\right] = 5.3333\left(10^{-6}\right)\text{ m}^3$$

$$(Q_B)_z = 0$$



(b)

Normal Stress: For the combined loading, the normal stress at point *B* can be determined from

$$\sigma_x = \sigma_B = \frac{N_x}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y}$$

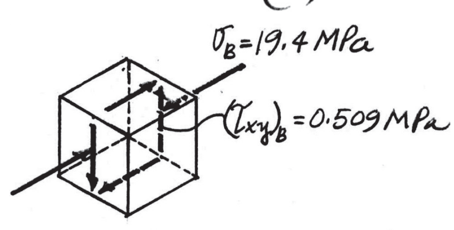
$$= \frac{1200}{0.4\left(10^{-3}\right)\pi} - \frac{(-96)(0)}{40.0\left(10^{-9}\right)\pi} + \frac{(-128)(0.02)}{40.0\left(10^{-9}\right)\pi}$$

$$= -19.42\left(10^6\right)\text{ Pa} = 19.4\text{ MPa (C)}$$ **Ans.**

Shear Stress: Since $T_x = 0$, the shear stress in z and y directions is contributed by transverse shear stress only, which can be obtained using the shear formula, $\tau_V = \frac{VQ}{It}$.

$$(\tau_{xy})_B = (\tau_V)_y = \frac{480\left[5.3333\left(10^{-6}\right)\right]}{40.0\left(10^{-9}\right)\pi(0.04)} = 0.5093\left(10^6\right)\text{ Pa} = 0.509\text{ MPa}$$ **Ans.**

$$(\tau_{xz})_B = (\tau_V)_z = 0$$ **Ans.**

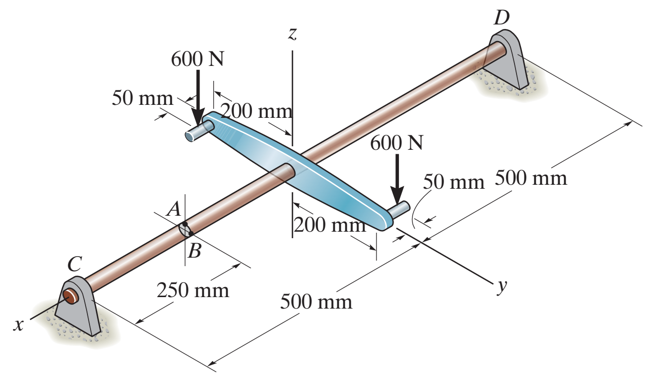


(c)

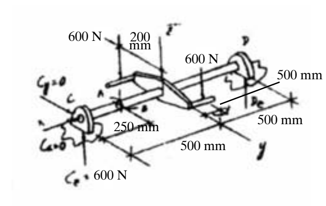
Using these results, the state of stress at point *B* can be represented by the volume element shown in Fig. *c*.

Ans:
$\sigma_B = 19.4$ MPa (C),
$(\tau_{xy})_B = 0.509$ MPa,
$(\tau_{xz})_B = 0$

8-70. The 18 mm-diameter shaft is subjected to the loading shown. Determine the stress components at point A. Sketch the results on a volume element located at this point. The journal bearing at C can exert only force components $\mathbf{C}_y$ and $\mathbf{C}_z$ on the shaft, and the thrust bearing at D can exert force components $\mathbf{D}_x$, $\mathbf{D}_y$, and $\mathbf{D}_z$ on the shaft.



SOLUTION

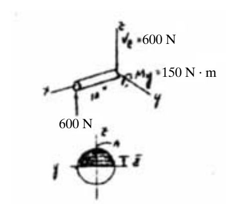


$$A = \frac{\pi}{4}(0.018^2) = 81.0(10^{-6})\pi \text{ m}^2$$

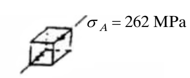
$$I = \frac{\pi}{4}(0.009^4) = 1.64025(10^{-9})\pi \text{ m}^4$$

$$Q_A = 0$$

$$\tau_A = 0 \qquad \textbf{Ans.}$$



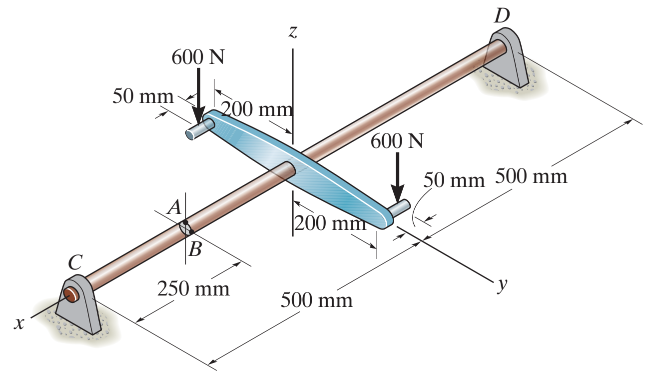
$$\sigma_A = -\frac{M_y c}{I} = -\frac{150(0.009)}{1.64025(10^{-9})\pi} = -261.98(10^6) \text{ N/m}^2 = 262 \text{ MPa (C)} \qquad \textbf{Ans.}$$



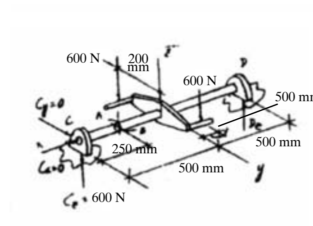
Ans.

$\tau_A = 0, \sigma_A = 262.0$ MPa (C)

8–71. Solve Prob. 8–70 for the stress components at point *B*.



SOLUTION



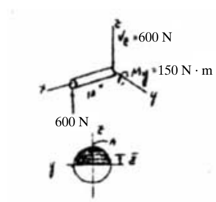
$$A = \frac{\pi}{4}(0.018^2) = 81.0(10^{-6})\pi \text{ m}^2$$

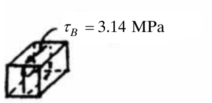
$$I = \frac{\pi}{4}(0.009^4) = 1.64025(10^{-9})\pi \text{ m}^4$$

$$Q_B = y'A' = \left[\frac{4(0.009)}{3\pi}\right]\left[\frac{\pi}{2}(0.009^2)\right] = 0.486(10^{-6}) \text{ m}^3$$

$$\sigma_B = 0$$ **Ans.**

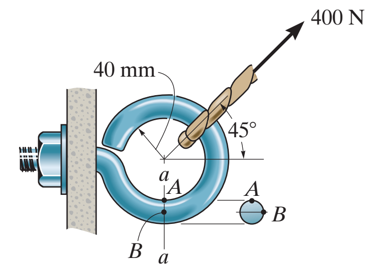
$$\tau_B = \frac{V_z Q_B}{I\,t} = \frac{600[0.486(10^{-6})]}{\left[1.64025(10^{-9})\pi\right](0.018)} = 3.144(10^{-6}) \text{ N/m}^2 = 3.14 \text{ MPa}$$ **Ans.**





Ans.
$\sigma_B = 0, \tau_B = 3.14$ MPa

***8–72.** The hook is subjected to the force of 400 N. Determine the state of stress at point A at section a–a. The cross section is circular and has a diameter of 12 mm. Use the curved-beam formula to compute the bending stress.



The location of the neutral surface from the center of curvature of the hook, Fig. a, can be determined from

$$R = \frac{A}{\Sigma \displaystyle\int_A \frac{dA}{r}}$$

where $A = \pi(0.006^2) = 36.0(10^{-6})\pi \text{ m}^2$

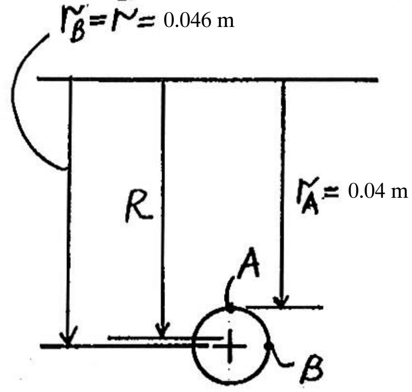
$$\Sigma \int_A \frac{dA}{r} = 2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right) = 2\pi\left(0.046 - \sqrt{0.046^2 - 0.006^2}\right) = 2.4692(10^{-3}) \text{ m}$$

Thus,

$$R = \frac{36.0(10^{-6})\pi}{2.4692(10^{-3})} = 0.0458035 \text{ m}$$

Then

$$e = \bar{r} - R = 0.046 - 0.0458035 = 0.0001965 \text{ m}$$

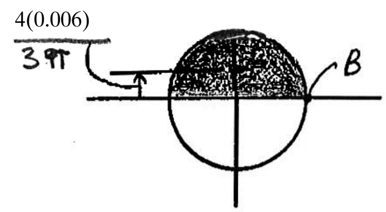


(a)

Referring to Fig. b, I and Q_A are

$$I = \frac{\pi}{4}(0.006^4) = 0.324(10^{-9})\pi \text{ m}^4$$

$$Q_A = 0$$



(b)

Consider the equilibrium of the FBD of the hook's cut segment, Fig. c,

$\overset{+}{\leftarrow} \Sigma F_x = 0;$	$N - 400 \cos 45° = 0$	$N = 282.84$ N
$+\uparrow \Sigma F_y = 0;$	$400 \sin 45° - V = 0$	$V = 282.84$ N
$\circlearrowleft + \Sigma M_o = 0;$	$M - (400 \cos 45°)(0.0458035) = 0$	$M = 12.9552 \text{ N} \cdot \text{m}$

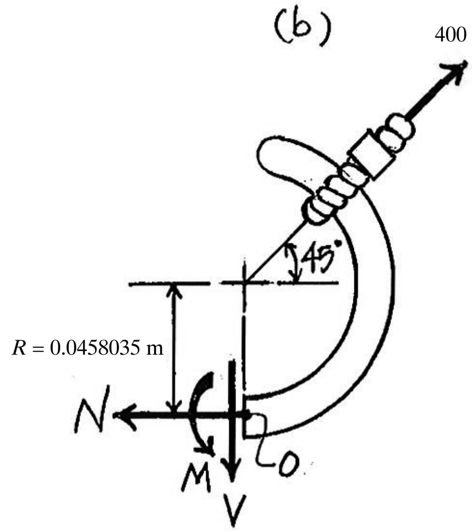
The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{M(R - r)}{Ae\,r}$$

Here, $M = 12.9552 \text{ N} \cdot \text{m}$ since it tends to reduce the curvature of the hook. For point A, $r = 0.04$ m. Then

$$\sigma = \frac{282.84}{36.0(10^{-6})\pi} + \frac{12.9552(0.0458035 - 0.04)}{[36.0(10^{-6})\pi](0.0001965)(0.04)}$$

$$= 87.08(10^{-6}) \text{ N/m}^2 = 87.1 \text{ MPa (T)} \qquad \textbf{Ans.}$$

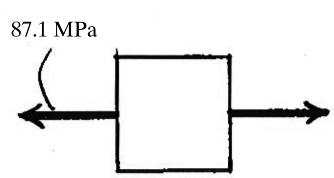


(c)

The shear stress in contributed by the transverse shear stress only. Thus

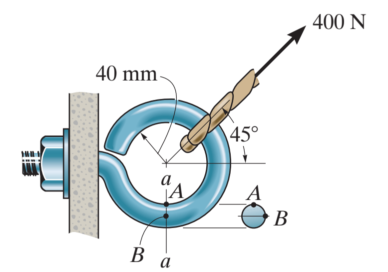
$$\tau = \frac{VQ_A}{It} = 0 \qquad \textbf{Ans.}$$

The state of strees of point A can be represented by the element shown in Fig. d.



(d)

8–73. The hook is subjected to the force of 400 N. Determine the state of stress at point B at section a–a. The cross section has a diameter of 12 mm. Use the curved-beam formula to compute the bending stress.



The location of the neutral surface from the center of curvature of the hook, Fig. a, can be determined from

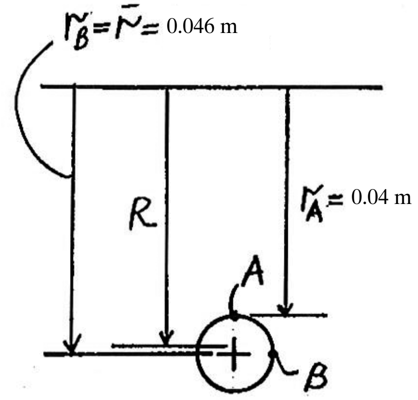
$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}}$$

where $A = \pi(0.006^2) = 36.0(10^{-6})\pi \text{ m}^2$

$$\Sigma \int_A \frac{dA}{r} = 2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right) = 2\pi\left(0.046 - \sqrt{0.046^2 - 0.006^2}\right) = 2.4692(10^{-3}) \text{ m}$$

Thus,

$$R = \frac{36.0(10^{-6})\pi}{2.4692(10^{-3})} = 0.0458035 \text{ m}$$



(a)

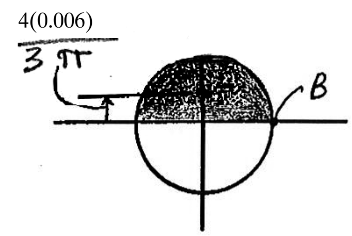
Then

$$e = \bar{r} - R = 0.046 - 0.0458035 = 0.0001965 \text{ m}$$

Referring to Fig. b, I and Q_A are

$$I = \frac{\pi}{4}(0.006^4) = 0.324(10^{-9})\pi \text{ m}^4$$

$$Q_B = \bar{y}'A' = \left[\frac{4(0.006)}{3\pi}\right]\left[\frac{\pi}{2}(0.006^2)\right] = 0.144(10^{-6}) \text{ m}^3$$



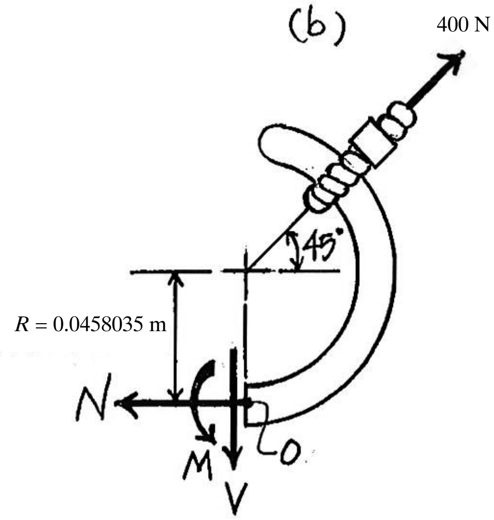
(b)

Consider the equilibrium of the FBD of the hook's cut segment, Fig. c,

$\xleftarrow{+} \Sigma F_x = 0;$ $\quad N - 400 \cos 45° = 0 \quad\quad N = 282.84 \text{ N}$

$+\uparrow \Sigma F_y = 0;$ $\quad 400 \sin 45° - V = 0 \quad\quad V = 282.84 \text{ N}$

$\circlearrowleft + \Sigma M_o = 0;$ $\quad M - (400 \cos 45°)(0.0458035) = 0 \quad\quad M = 12.9552 \text{ N}\cdot\text{m}$



(c)

The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{M(R - r)}{Aer}$$

Here, $M = 12.9552 \text{ N}\cdot\text{m}$ since it tends to reduce the curvature of the hook. For point B, $r = 0.04$. m. Then

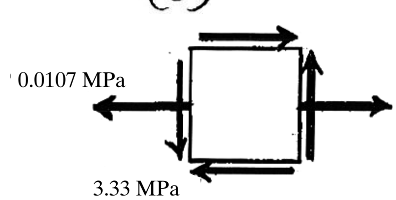
$$\sigma = \frac{282.84}{36.0(10^{-6})\pi} + \frac{12.9552(0.0458035 - 0.046)}{[36.0(10^{-6})\pi](0.0001965)(0.046)}$$

$$= 0.01068(10^6) \text{ N/m}^2 = 0.0107 \text{ MPa} \quad\quad \textbf{Ans.}$$

The shear stress is contributed by the transverse shear stress only. Thus,

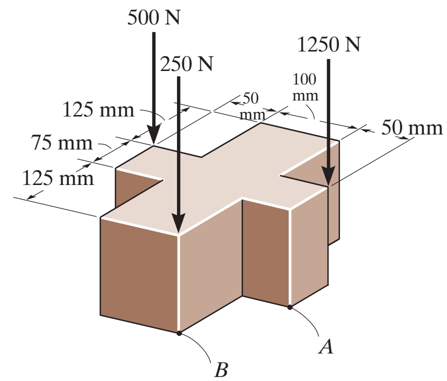
$$\tau = \frac{VQ_B}{It} = \frac{282.84[0.144(10^{-6})]}{[0.324(10^{-9})\pi](0.012)} = 3.3345(10^6) \text{ N/m}^2 = 3.33 \text{ MPa} \quad\quad \textbf{Ans.}$$

The state of stress of point B can be represented by the element shown in Fig. d.



(d)

R8–1. The block is subjected to the three axial loads shown. Determine the normal stress developed at points *A* and *B*. Neglect the weight of the block.



SOLUTION

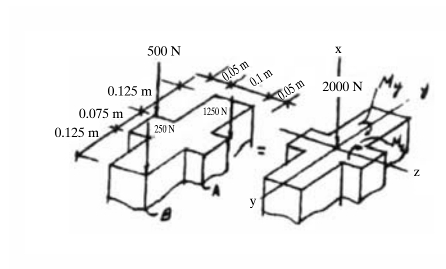
$$M_z = -1250(0.0375) - 500(0.0375) + 250(0.1625) = -25.0 \text{ N}\cdot\text{m}$$

$$M_y = -1250(0.1) - 250(0.05) + 500(0.1) = -87.5 \text{ N}\cdot\text{m}$$

$$I_z = \frac{1}{12}(0.1)(0.325^3) + \frac{1}{12}(0.1)(0.075^3) = 0.28958(10^{-3}) \text{ m}^4$$

$$I_y = \frac{1}{12}(0.75)(0.2^3) + \frac{1}{12}(0.25)(0.1^3) = 70.8333(10^{-6}) \text{ m}^4$$

$$A = 0.1(0.325) + 0.1(0.075) = 0.04 \text{ m}^2$$



$$\sigma = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = \frac{-2000}{0.04} - \frac{(-25.0)(0.0375)}{0.28958(10^{-3})} + \frac{(-87.5)(0.1)}{70.8333(10^{-6})}$$

$$= -170.29(10^3) \text{ N/m}^2 = 170 \text{ kPa (C)}$$ **Ans.**

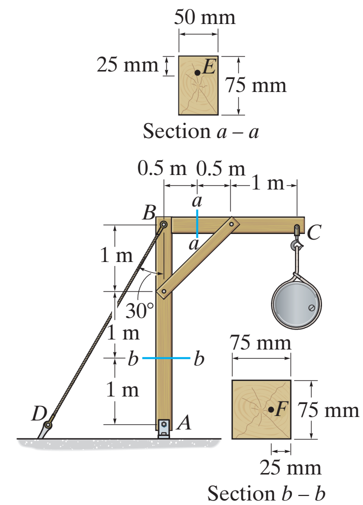
$$\sigma_B = \frac{-2000}{0.04} - \frac{(-25.0)(0.1625)}{0.28958(10^{-3})} + \frac{(-87.5)(0.1)}{70.8333(10^{-6})}$$

$$= 97.74(10^3) \text{ N/m}^2 = 97.7 \text{ kPa (C)}$$ **Ans.**

Ans.
$\sigma_A = 170 \text{ kPa (C)}, \sigma_B = 97.7 \text{ kPa (C)}$

R8–2.

The 20-kg drum is suspended from the hook mounted on the wooden frame. Determine the state of stress at point E on the cross section of the frame at section a–a. Indicate the results on an element.



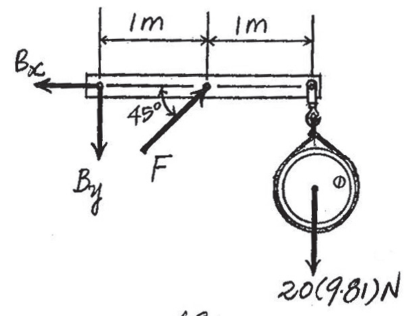
SOLUTION

Support Reactions: Referring to the free-body diagram of member BC shown in Fig. a,

$$\circlearrowleft + \Sigma M_B = 0; \quad F \sin 45°(1) - 20(9.81)(2) = 0 \quad F = 554.94 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 554.94 \cos 45° - B_x = 0 \quad B_x = 392.4 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad 554.94 \sin 45° - 20(9.81) - B_y = 0 \quad B_y = 196.2 \text{ N}$$



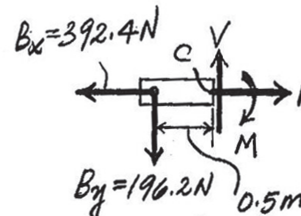
(a)

Internal Loadings: Consider the equilibrium of the free-body diagram of the right segment shown in Fig. b.

$$\xrightarrow{+} \Sigma F_x = 0; \quad N - 392.4 = 0 \quad N = 392.4 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad V - 196.2 = 0 \quad V = 196.2 \text{ N}$$

$$\circlearrowleft + \Sigma M_C = 0; \quad 196.2(0.5) - M = 0 \quad M = 98.1 \text{ N} \cdot \text{m}$$



(b)

Section Properties: The cross-sectional area and the moment of inertia of the cross section are

$$A = 0.05(0.075) = 3.75\left(10^{-3}\right) \text{ m}^2$$

$$I = \frac{1}{12}(0.05)\left(0.075^3\right) = 1.7578\left(10^{-6}\right) \text{ m}^4$$

Referring to Fig. c, Q_E is

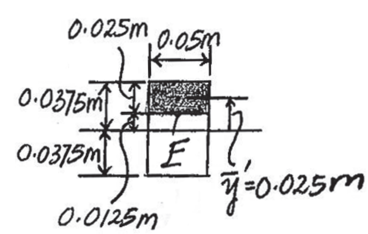
$$Q_E = \bar{y}'A' = 0.025(0.025)(0.05) = 31.25\left(10^{-6}\right) \text{ m}^3$$

Normal Stress: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

For point E, $y = 0.0375 - 0.025 = 0.0125$ m. Then

$$\sigma_E = \frac{392.4}{3.75\left(10^{-3}\right)} + \frac{98.1(0.0125)}{1.7578\left(10^{-6}\right)} = 802 \text{ kPa} \qquad \textbf{Ans.}$$

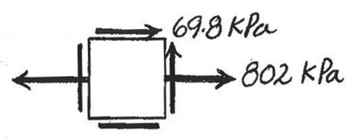


(c)

Shear Stress: The shear stress is contributed by transverse shear stress only. Thus,

$$\tau_E = \frac{VQ_A}{It} = \frac{196.2\left[31.25\left(10^{-6}\right)\right]}{1.7578\left(10^{-6}\right)(0.05)} = 69.8 \text{ kPa} \qquad \textbf{Ans.}$$

The state of stress at point E is represented on the element shown in Fig. d.

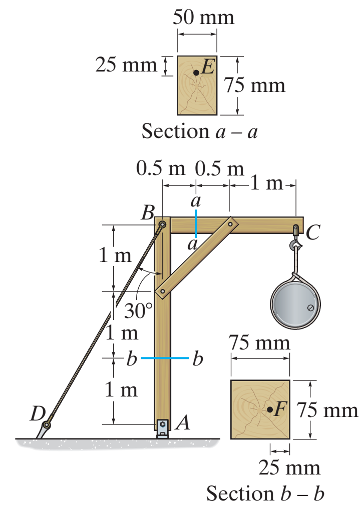


(d)

Ans:
$\sigma_E = 802$ kPa, $\tau_E = 69.8$ kPa

R8–3.

The 20-kg drum is suspended from the hook mounted on the wooden frame. Determine the state of stress at point F on the cross section of the frame at section b–b. Indicate the results on an element.



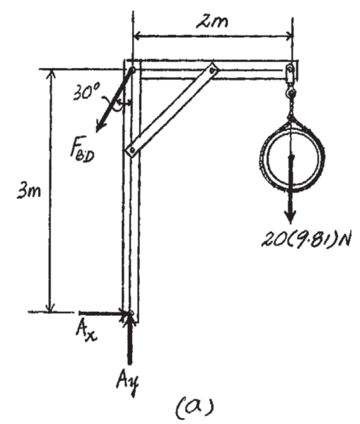
SOLUTION

Support Reactions: Referring to the free-body diagram of the entire frame shown in Fig. a,

$\circlearrowleft + \Sigma M_A = 0;\quad F_{BD}\sin 30°(3) - 20(9.81)(2) = 0 \qquad F_{BD} = 261.6\text{ N}$

$+\uparrow \Sigma F_y = 0;\quad A_y - 261.6\cos 30° - 20(9.81) = 0 \qquad A_y = 422.75\text{ N}$

$\xrightarrow{+} \Sigma F_x = 0;\quad A_x - 261.6\sin 30° = 0 \qquad A_x = 130.8\text{ N}$



Internal Loadings: Consider the equilibrium of the free-body diagram of the lower cut segment, Fig. b.

$\xrightarrow{+} \Sigma F_x = 0;\quad 130.8 - V = 0 \qquad V = 130.8\text{ N}$

$+\uparrow \Sigma F_y = 0;\quad 422.75 - N = 0 \qquad N = 422.75\text{ N}$

$\circlearrowleft + \Sigma M_C = 0;\quad 130.8(1) - M = 0 \qquad M = 130.8\text{ N}\cdot\text{m}$

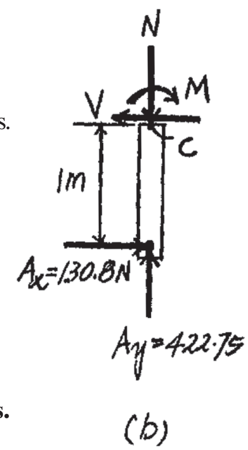
Section Properties: The cross-sectional area and the moment of inertia about the centroidal axis of the cross section are

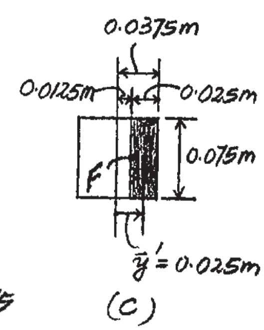
$$A = 0.075(0.075) = 5.625\left(10^{-3}\right)\text{ m}^2$$

$$I = \frac{1}{12}(0.075)\left(0.075^3\right) = 2.6367\left(10^{-6}\right)\text{ m}^4$$

Referring to Fig. c, Q_E is

$$Q_F = \bar{y}'A' = 0.025(0.025)(0.075) = 46.875\left(10^{-6}\right)\text{ m}^3$$





Normal Stress: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

For point F, $y = 0.0375 - 0.025 = 0.0125$ m. Then

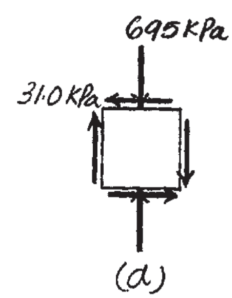
$$\sigma_F = \frac{-422.75}{5.625\left(10^{-3}\right)} - \frac{130.8(0.0125)}{2.6367\left(10^{-6}\right)}$$

$$= -695.24\text{ kPa} = 695\text{ kPa (C)} \qquad \textbf{Ans.}$$

Shear Stress: The shear stress is contributed by transverse shear stress only. Thus,

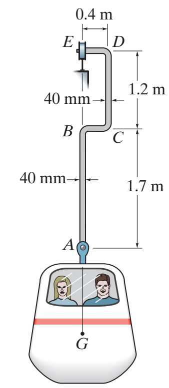
$$\tau_A = \frac{VQ_A}{It} = \frac{130.8\left[46.875\left(10^{-6}\right)\right]}{2.6367\left(10^{-6}\right)(0.075)} = 31.0\text{ kPa} \qquad \textbf{Ans.}$$

The state of stress at point A is represented on the element shown in Fig. d.



Ans:
$\sigma_F = 695\text{ kPa (C)}$,
$\tau_A = 31.0\text{ kPa}$

***R8–4.** The gondola and passengers have a weight of 7.5 kN and center of gravity at G. The suspender arm AE has a square cross-sectional area of 40 mm by 40 mm, and is pin connected at its ends A and E. Determine the largest tensile stress developed in regions AB and DC of the arm.

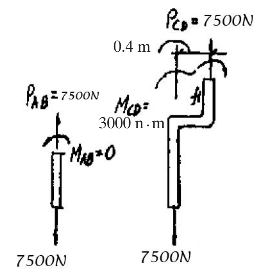


SOLUTION

Segment AB:

$$(\sigma_{\max})_{AB} = \frac{P_{AB}}{A} = \frac{7500}{0.04(0.04)} = 4.6875(10^6)\text{ N/m}^2 = 4.69\text{ MPa}$$ **Ans.**

Segment CD:



$$\sigma_a = \frac{P_{CD}}{A} = \frac{7500}{0.04(0.04)} = 4.6875(10^6)\text{ N/m}^2 = 4.6875\text{ MPa}$$

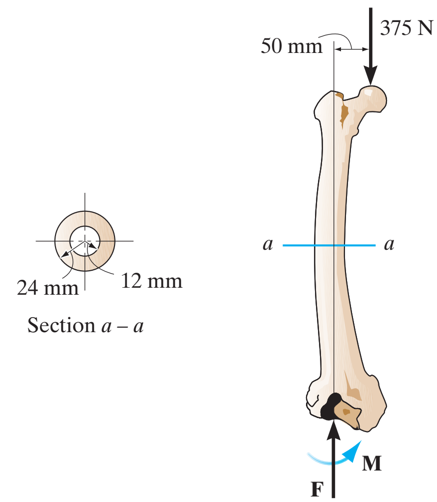
$$\sigma_b = \frac{Mc}{I} = \frac{3000(0.02)}{\frac{1}{12}(0.04)(0.04^3)} = 281.25(10^6)\text{ N/m}^2 = 281.25\text{ MPa}$$

$$(\sigma_{\max})_{CD} = \sigma_a + \sigma_b = 4.6875 + 281.25$$

$$= 285.9375\text{ MPa} = 286\text{ MPa}$$ **Ans.**

Ans:
$(\sigma_{\max})_{AB} = 4.69$ MPa, $(\sigma_{\max})_{CD} = 286$ MPa

R8–5. If the cross section of the femur at section *a–a* can be approximated as a circular tube as shown, determine the maximum normal stress developed on the cross section at section *a–a* due to the load of 375 N.



SOLUTION

Internal Loadings: Considering the equilibrium for the free-body diagram of the femur's upper segment, Fig. *a*,

$+\uparrow \Sigma F_y = 0; \qquad N - 375 = 0 \qquad N = 375 \text{ N}$

$\circlearrowleft + \Sigma M_O = 0; \qquad M - 375(0.05) = 0 \qquad M = 18.75 \text{ N} \cdot \text{m}$

Section Properties: The cross-sectional area, the moment of inertia about the centroidal axis of the femur's cross section are

$$A = \pi(0.024^2 - 0.012^2) = 0.432(10^{-3})\pi \text{ m}^2$$

$$I = \frac{\pi}{4}(0.024^2 - 0.012^2) = 77.76(10^{-9})\pi \text{ m}^4$$

Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{My}{I}$$

By inspection, the maximum normal stress is in compression.

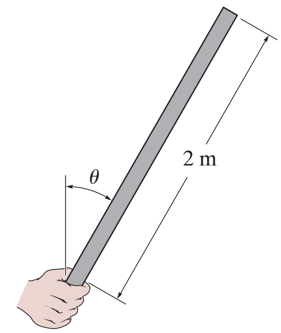
$$\sigma_{\max} = \frac{-375}{0.432(10^{-3})\pi} - \frac{18.75(0.024)}{77.76(10^{-9})\pi}$$

$$= -2.118(10^6) \text{ N/m}^2 = 2.12 \text{ MPa (C)} \qquad \textbf{Ans.}$$

Ans.
$\sigma_C = 93.7$ MPa (T), $\tau_C = 0$,
$\sigma_D = 187$ MPa (C), $\tau_D = 0$

R8–6.

A bar having a square cross section of 30 mm by 30 mm is 2 m long and is held upward. If it has a mass of 5 kg/m, determine the largest angle θ, measured from the vertical, at which it can be supported before it is subjected to a tensile stress along its axis near the grip.



SOLUTION

$$A = 0.03(0.03) = 0.9(10^{-3})\ \text{m}^2$$

$$I = \frac{1}{12}(0.03)(0.03^3) = 67.5(10^{-9})\ \text{m}^4$$

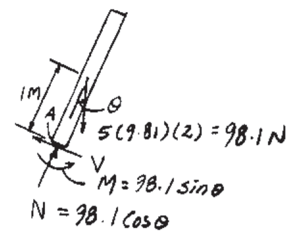
Require $\sigma_A = 0$

$$\sigma_A = 0 = \frac{P}{A} + \frac{Mc}{I}$$

$$0 = \frac{-98.1\cos\theta}{0.9(10^{-3})} + \frac{98.1\sin\theta(0.015)}{67.5(10^{-9})}$$

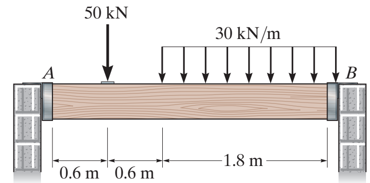
$$0 = -1111.11\cos\theta + 222222.22\sin\theta$$

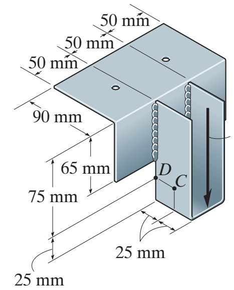
$\tan\theta = 0.005;\qquad \theta = 0.286°$ **Ans.**



Ans:
$\theta = 0.286°$

R8–7. The wall hanger has a thickness of 6 mm and is used to support the vertical reactions of the beam that is loaded as shown. If the load is transferred uniformly to each strap of the hanger, determine the state of stress at points C and D on the strap at A. Assume the vertical reaction **F** at this end acts in the center and on the edge of the bracket as shown.

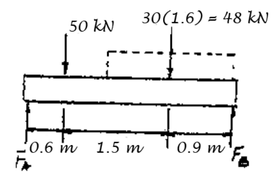




SOLUTION

$\zeta + \Sigma M_B = 0;$ $\qquad 54(0.9) + 50(2.4) - F_A(3) = 0$

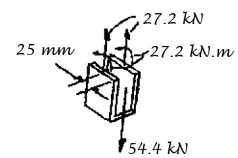
$$F_A = 56.2 \text{ kN}$$



$$I = 2\left[\frac{1}{12}(0.006)(0.05^3)\right] = 0.125(10^{-6}) \text{ m}^4$$

$$A = 2(0.006)(0.05) = 0.6(10^{-3}) \text{ m}^2$$

At point C,



$$\sigma_C = \frac{P}{A} = \frac{2[28.1(10^3)]}{0.6(10^{-3})} = 93.67(10^6) \text{ N/m}^2 = 93.7 \text{ MPa (T)}$$ **Ans.**

$$\tau_C = 0$$ **Ans.**

At point D,

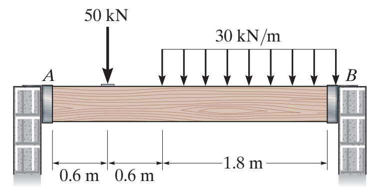
$$\sigma_D = \frac{P}{A} - \frac{Mc}{I} = \frac{2[28.1(10^3)]}{0.6(10^{-3})} - \frac{2[0.7025(10^3)](0.025)}{0.125(10^{-6})}$$

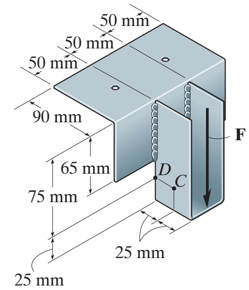
$$= -187.33(10^6) \text{ N/m}^2 = 187 \text{ MPa (C)}$$ **Ans.**

$$\tau_D = 0$$ **Ans.**

Ans:
$\sigma_C = 93.7$ MPa (T), $\tau_C = 0$,
$\sigma_D = -187$ MPa (C), $\tau_D = 0$

***R8–8.** The wall hanger has a thickness of 6 mm and is used to support the vertical reactions of the beam that is loaded as shown. If the load is transferred uniformly to each strap of the hanger, determine the state of stress at points C and D on the strap at B. Assume the vertical reaction $\mathbf{F}$ at this end acts in the center and on the edge of the bracket as shown.

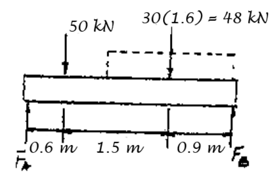




SOLUTION

$\zeta + \Sigma M_A = 0;\qquad -54(2.1) - 50(0.6) + F_B(3) = 0$

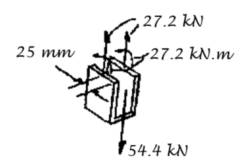
$$F_B = 47.8 \text{ kN}$$



$$A = 2(0.006)(0.05) = 0.6(10^{-3}) \text{ m}^2$$

$$I = 2\left[\frac{1}{12}(0.006)(0.05^3)\right] = 0.125(10^{-6}) \text{ m}^4$$

At point C,



$$\sigma_C = \frac{P}{A} = \frac{2[23.9(10^3)]}{0.6(10^{-3})} = 79.67(10^6) \text{ N/m}^2 = 79.7 \text{ MPa (T)}$$ **Ans.**

$$\tau_C = 0$$ **Ans.**

At point D,

$$\sigma_D = \frac{P}{A} - \frac{Mc}{I} = \frac{2[23.9(10^3)]}{0.6(10^{-3})} - \frac{2[0.5975(10^3)](0.025)}{0.125(10^{-6})}$$

$$= -159.33(10^6) \text{ N/m}^2 = 159 \text{ MPa (C)}$$ **Ans.**

$$\tau_D = 0$$ **Ans.**

Ans:
$\sigma_C = 79.7$ MPa (T), $\tau_C = 0$,
$\sigma_D = 159$ MPa (C), $\tau_D = 0$