

## Formulazione del problema dell'equilibrio per le trave

A B

Incoquite

spostamenti	deformazioni	forze
$v: (0, L) \rightarrow \mathbb{R}$	$\varepsilon: (0, L) \rightarrow \mathbb{R}$	$N: (0, L) \rightarrow \mathbb{R}$
$w: (0, L) \rightarrow \mathbb{R}$	$\kappa: (0, L) \rightarrow \mathbb{R}$	$M: (0, L) \rightarrow \mathbb{R}$
$\vartheta: (0, L) \rightarrow \mathbb{R}$		$V: (0, L) \rightarrow \mathbb{R}$

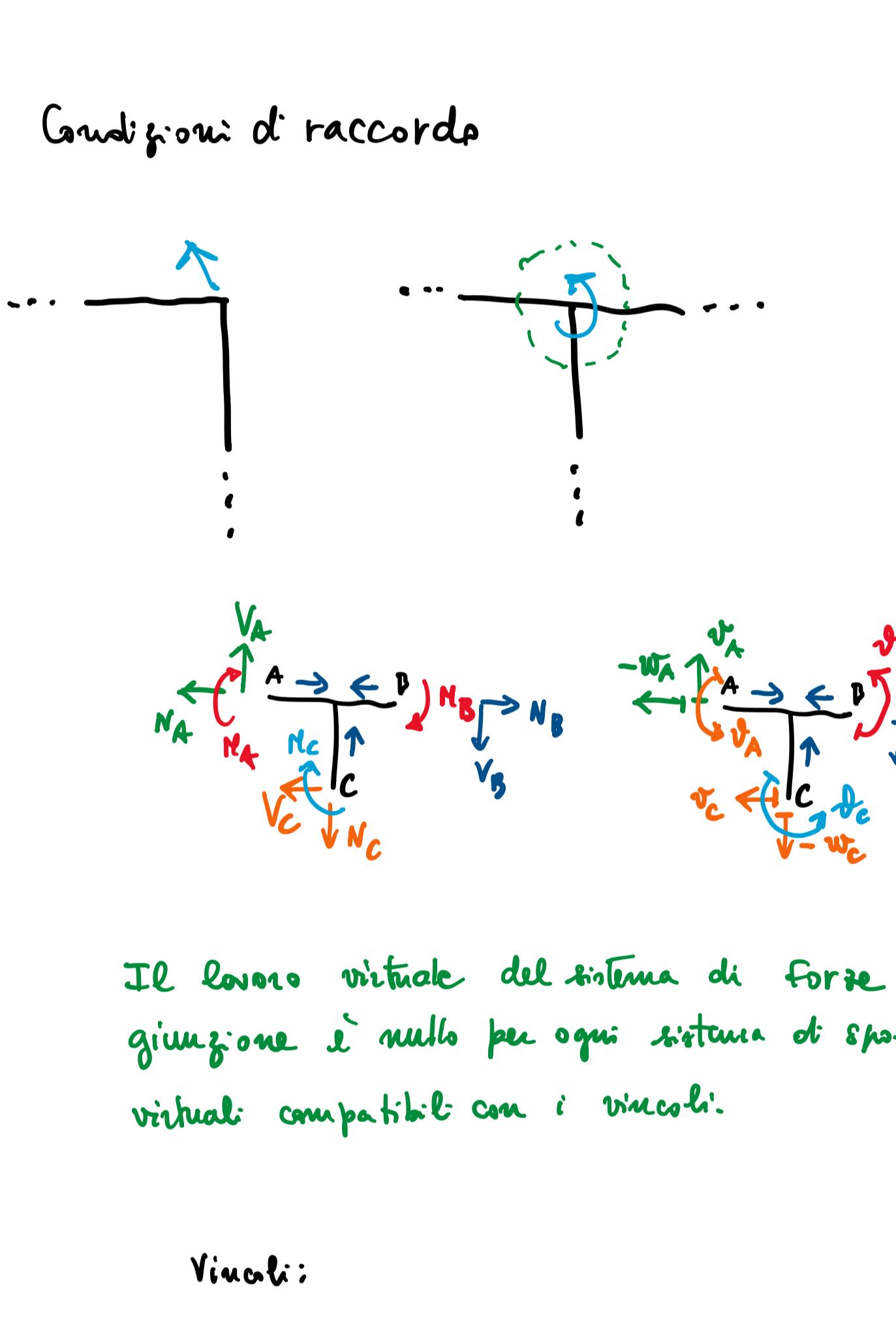
$$\text{opp: se } M \text{ è nato, allora } V = \frac{dM}{dx}$$

Equazioni in AB

$\frac{dN}{dx} + q = 0$	$N = EA\varepsilon$	$\varepsilon = \frac{dw}{dx}$
$\frac{d^2M}{dx^2} + p = 0$	$M = EI\kappa$	$\kappa = \frac{d^2\vartheta}{dx^2}$

$$V = \frac{dM}{dx} \quad \vartheta = \frac{d\vartheta}{dx}$$

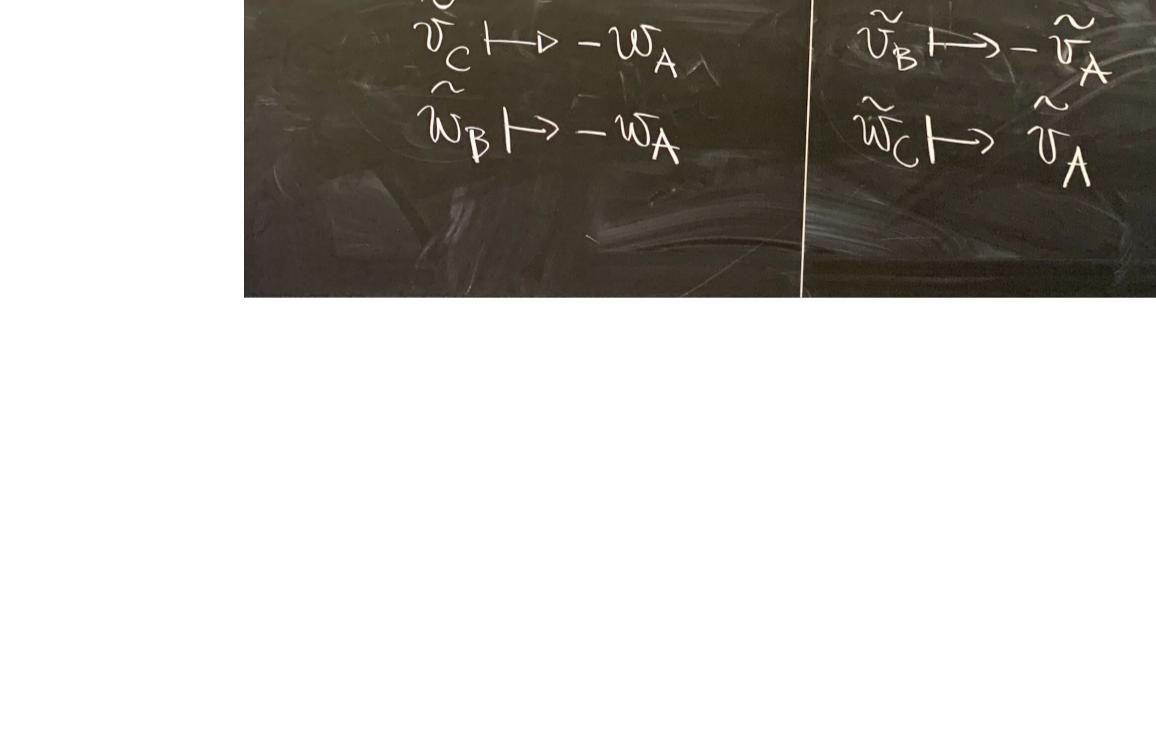
Condizioni agli estremi.

Sistemi di travi

Casini - Vasta

- 8 problema  
9 metodo spost.  
10 principio lavoro virtuale  
11 metodo delle forze  
12 travat. reticolari/travi continue

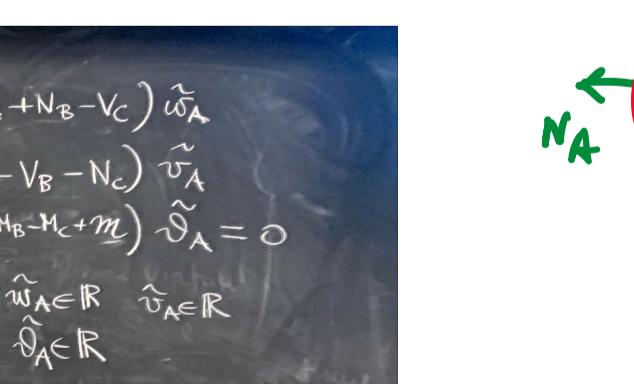
Sistemi di raccordo



Il lavoro virtuale del sistema di forze agenti sulla giungure è nullo per ogni sistema di spostamenti virtuali compatibile con i vincoli.

Vincoli:  
Le componenti orizzontali degli spostamenti in A, B e C sono uguali:

$$w_A = -w_B \\ w_A = -w_C$$



$$v_A = v_B \\ v_A = v_C$$



$$-N_A + N_B - N_C = 0$$

$$V_A - V_B - V_C = 0$$

$$-M_A - M_B - M_C = 0$$

$$0 = -N_A \tilde{w}_A + V_A \tilde{v}_A - M_A \tilde{\vartheta}_A \\ + N_B \tilde{w}_A - V_B \tilde{v}_A - M_B \tilde{\vartheta}_A \\ - N_C \tilde{w}_A + V_C \tilde{v}_A - M_C \tilde{\vartheta}_A = 0$$

$$(-N_A + N_B - N_C) \tilde{w}_A \\ + (V_A - V_B - V_C) \tilde{v}_A \\ + (-M_A - M_B - M_C) \tilde{\vartheta}_A = 0$$

$$\tilde{w}_A \in \mathbb{R} \quad \tilde{v}_A \in \mathbb{R} \quad \tilde{\vartheta}_A \in \mathbb{R}$$

le rotazioni delle regioni A, B e C sono uguali:

$$\vartheta_A = \vartheta_B \\ \vartheta_A = \vartheta_C$$

$$\tilde{v}_A = \tilde{v}_B \\ \tilde{v}_A = \tilde{v}_C$$

$$\tilde{\vartheta}_A = \tilde{\vartheta}_B \\ \tilde{\vartheta}_A = \tilde{\vartheta}_C$$

$$-N_A + N_B - N_C = 0$$

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