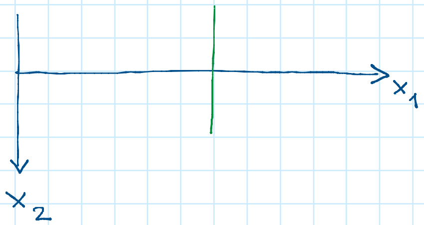
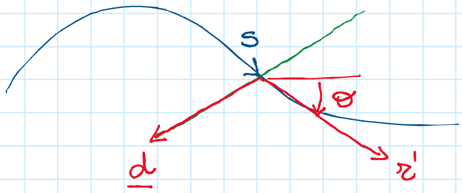
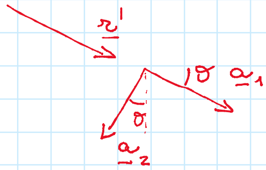




$$\underline{y}(x_1,x_2) = \underline{r}(x_1) + x_2\, \underline{d}(x_1) \qquad (1)$$

$$\underline{a}_1 := \frac{\underline{r}'}{|\underline{r}'|} =: \cos\vartheta\, \underline{e}_1 + \mathrm{sen}\,\vartheta\, \underline{e}_2$$

$$\underline{a}_2 := -\mathrm{sen}\,\vartheta\, \underline{e}_1 + \cos\vartheta\, \underline{e}_2$$

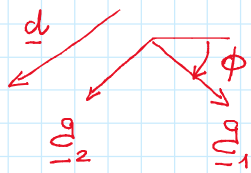


$$\underline{r}' =: (1+\varepsilon)\, \underline{a}_1 \qquad (2)$$

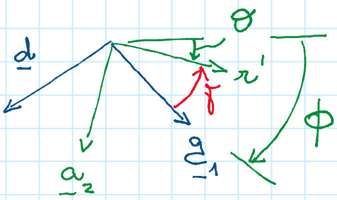
$$s = \int_0^{x_1} |\underline{r}'|\, dx_1 \longrightarrow \frac{ds}{dx_1} = |\underline{r}'| \longrightarrow \frac{ds}{dx_1} = 1+\varepsilon \qquad (3)$$

$$\underline{g}_1 := \cos\phi\, \underline{e}_1 + \mathrm{sen}\,\phi\, \underline{e}_2$$

$$\underline{g}_2 := \frac{\underline{d}}{|\underline{d}|} = -\mathrm{sen}\,\phi\, \underline{e}_1 + \cos\phi\, \underline{e}_2$$



$$\underline{d} =: (1+p)\, \underline{g}_2 \qquad (4)$$



$$\gamma := \phi - \vartheta \qquad (5)$$

$$\underline{F} = \nabla \underline{y} = (\underline{r}' + x_2\, \underline{d}') \otimes \underline{e}_1 + \underline{d} \otimes \underline{e}_2$$

$$\det F = (\underline{r}' + x_2\, \underline{d}') \wedge \underline{d} \cdot \underline{e}_3$$

Richiedo che $\displaystyle\int_{-\ell/2}^{\ell/2} \det F\, dx_2 = \ell$

$$\Rightarrow \ell = \int_{-\ell/2}^{\ell/2} (\underline{r}' + \cancel{x_2\, \underline{d}'}) \wedge \underline{d} \cdot \underline{e}_3\, dx_2 = \ell\, \underline{r}' \wedge \underline{d} \cdot \underline{e}_3$$

$$\Rightarrow \ell = \int_{-\ell/2} \left(\underline{r}' + \cancel{x_2 \underline{d}'}\right) \wedge \underline{d} \cdot \underline{e}_3 \, dx_2 = \ell \, \underline{r}' \wedge \underline{d} \cdot \underline{e}_3$$

$$= \ell \, (1+\varepsilon)\, \underline{a}_1 \wedge (1+p)\, \underline{g}_2 \cdot \underline{e}_3$$

$$= \ell \, (1+\varepsilon)(1+p) \begin{vmatrix} \cos\vartheta & \operatorname{sen}\vartheta & 0 \\ -\operatorname{sen}\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \ell \, (1+\varepsilon)(1+p) \left(\cos\vartheta \cos\phi + \operatorname{sen}\vartheta \operatorname{sen}\phi\right)$$

$$= \ell \, (1+\varepsilon)(1+p) \cos(\phi - \vartheta)$$

$$= \ell \, (1+\varepsilon)(1+p) \cos\gamma$$

$$\Rightarrow \quad (1+\varepsilon)(1+p)\cos\gamma = 1 \qquad (6)$$

$$\underline{y}^a(x_1) := \underline{r}(x_1) - \frac{\ell}{2}\underline{d}(x_1) \qquad \underline{y}^b(x_1) := \underline{r}(x_1) + \frac{\ell}{2}\underline{d}(x_1)$$

$$\underline{d} = (1+p)\,\underline{g}_2 = (1+p)\left(-\operatorname{sen}\phi\, \underline{e}_1 + \cos\phi\, \underline{e}_2\right)$$

$$\underline{d}' = p'\underline{g}_2 + (1+p)\left(-\cos\phi\, \underline{e}_1 - \operatorname{sen}\phi\, \underline{e}_2\right)\phi'$$

$$\underline{d}' = -(1+p)\phi'\,\underline{g}_1 + p'\,\underline{g}_2 \qquad (7)$$

$$(\underline{y}^a)' = \underline{r}' - \frac{\ell}{2}\underline{d}' = (1+\varepsilon)\,\underline{a}_1 + \frac{\ell}{2}(1+p)\phi'\,\underline{g}_1 - \frac{\ell}{2}p'\,\underline{g}_2$$

$$\left|(\underline{y}^a)'\right|^2 = (1+\varepsilon)^2 + \left(\frac{\ell}{2}(1+p)\phi'\right)^2 + \left(\frac{\ell}{2}p'\right)^2 + 2(1+\varepsilon)\frac{\ell}{2}(1+p)\phi'\, \underline{a}_1 \cdot \underline{g}_1 +$$
$$- 2(1+\varepsilon)\frac{\ell}{2}p'\, \underline{a}_1 \cdot \underline{g}_2$$

$$\underline{a}_1 \cdot \underline{g}_1 = \left(\cos\vartheta\, \underline{e}_1 + \operatorname{sen}\vartheta\, \underline{e}_2\right) \cdot \left(\cos\phi\, \underline{e}_1 + \operatorname{sen}\phi\, \underline{e}_2\right)$$
$$= \cos\vartheta \cos\phi + \operatorname{sen}\vartheta \operatorname{sen}\phi$$

$$\underline{a}_1 \cdot \underline{g}_1 = \ldots$$

$$= \cos\vartheta \cos\phi + \operatorname{sen}\vartheta \operatorname{sen}\phi$$

$$= \cos(\phi - \vartheta) = \cos\gamma$$

$$\underline{a}_1 \cdot \underline{g}_2 = (\cos\vartheta\, \underline{e}_1 + \operatorname{sen}\vartheta\, \underline{e}_2) \cdot (-\operatorname{sen}\phi\, \underline{e}_1 + \cos\phi\, \underline{e}_2)$$

$$= \operatorname{sen}\vartheta \cos\phi - \cos\vartheta \operatorname{sen}\phi$$

$$= \operatorname{sen}(\vartheta - \phi) = -\operatorname{sen}(\phi - \vartheta) = -\operatorname{sen}\gamma$$

$$|(\underline{y}^a)'|^2 = (1+\varepsilon)^2 + \left(\frac{\ell}{2}(1+p)\phi'\right)^2 + \left(\frac{\ell}{2}p'\right)^2 + \ell\, \underbrace{(1+\varepsilon)(1+p)\cos\gamma}_{= 1 \text{ dalla } (6)}\, \phi' + $$

$$+ \ell(1+\varepsilon)p' \operatorname{sen}\gamma$$

$$= (1+\varepsilon)^2 + \left(\frac{\ell}{2}(1+p)\phi'\right)^2 + \left(\frac{\ell}{2}p'\right)^2 + \ell\phi' + \ell(1+\varepsilon)p'\operatorname{sen}\gamma$$

$$\frac{ds^b}{dx_1} := |(\underline{y}^b)'| \qquad s^b = \text{lunghezza d'arco curva basale}$$

$$\underline{t}^b := \frac{(\underline{y}^b)'}{|(\underline{y}^b)'|} \qquad \kappa^b := \frac{d\underline{t}^b}{ds^b} \cdot (\underline{e}_3 \wedge \underline{t}^b)$$

$$|(\underline{y}^b)'|^2 = (1+\varepsilon)^2 + \left(\frac{\ell}{2}(1+p)\phi'\right)^2 + \left(\frac{\ell}{2}p'\right)^2 - \ell\phi' - \ell(1+\varepsilon)p'\operatorname{sen}\gamma$$

$$\kappa^b = e_3 \cdot t^b \wedge \frac{dt^b}{ds^b} = \underline{e}_3 \cdot \underline{t}^b \wedge \frac{d\,t^b}{dx_1}\frac{dx_1}{ds_b}$$

$$= \frac{dx_1}{ds_b}\, \underline{e}_3 \cdot \frac{(y^b)'}{|(y^b)'|} \wedge \frac{(y^b)''|y_b'| - \cancel{y_b' |y_b'|'}}{|y^{b\prime}|^2}$$

$$= \frac{1}{|y_b'|^3}\, \underline{e}_3 \cdot y_b' \wedge y_b''$$

$$\underline{y}_b' = (1+\varepsilon)\,\underline{a}_1 - \ell/2\,(1+\rho)\,\phi'\,\underline{g}_1 + \ell/2\,\rho'\,\underline{g}_2$$

$$\underline{y}_b'' = \varepsilon'\,\underline{a}_1 + (1+\varepsilon)\,\underline{a}_1' - \ell/2\,\rho'\,\phi'\,\underline{g}_1 - \ell/2\,(1+\rho)\,\phi''\,\underline{g}_1 +$$

$$- \ell/2\,(1+\rho)\,\phi'\,\underline{g}_1' + \ell/2\,\rho''\,\underline{g}_2 + \ell/2\,\rho'\,\underline{g}_2'$$

$$\underline{a}_1' = \vartheta'\left(-\operatorname{sen}\vartheta\,\underline{e}_1 + \cos\vartheta\,\underline{e}_2\right) = \vartheta'\,\underline{a}_2$$

$$\underline{g}_1' = \phi'\,\underline{g}_2$$

$$\underline{g}_2' = \phi'\left(-\cos\phi\,\underline{e}_1 - \operatorname{sen}\phi\,\underline{e}_2\right) = -\phi'\,\underline{g}_1$$

$$\underline{y}_b'' = \varepsilon'\,\underline{a}_1 + (1+\varepsilon)\,\vartheta'\,\underline{a}_2 - \ell/2\,\rho'\,\phi'\,\underline{g}_1 - \ell/2\,(1+\rho)\,\phi''\,\underline{g}_1 +$$

$$- \ell/2\,(1+\rho)(\phi')^2\,\underline{g}_2 + \ell/2\,\rho''\,\underline{g}_2 - \ell/2\,\rho'\,\phi'\,\underline{g}_1$$

$$= \varepsilon'\,\underline{a}_1 + (1+\varepsilon)\,\vartheta'\,\underline{a}_2 - \ell/2\left(2\rho'\phi' + (1+\rho)\phi''\right)\underline{g}_1 +$$

$$+ \ell/2\left(-(1+\rho)(\phi')^2 + \rho''\right)\underline{g}_2$$